

$A \rightarrow B$   $A \rightarrow B$   $B \rightarrow C$   $C \rightarrow D$  and  $D \rightarrow A$

$$\begin{aligned} \{A\}^+ &= \{A, B, C, D\} \\ \{B\}^+ &= \{A, B, C, D\} \\ \{C\}^+ &= \{A, B, C, D\} \\ \{D\}^+ &= \{A, B, C, D\} \end{aligned}$$

$R_1(A, B)$

$$L(R_1) = \{A \rightarrow B\}$$

All non trivial FDs from subset of  $L(R_1)$  under  $S$   
(since these include all attributes in taking  $AB$ )  
 $\{A \rightarrow B\}$  of  $R_1$  there is no point in taking  $AB$  FDs of  $R_1(A, B)$

This is also the minimal basis of set  $\mathcal{F}_1$  of  $R_1(A, B)$

$R_2(B, C)$

$$L(R_2) = \{B \rightarrow C\}$$

All non trivial FDs from subset of  $L(R_2)$  under  $S$   
 $B \rightarrow C$   
 $C \rightarrow B$

This is also the minimal basis of set  $\mathcal{F}_2$  of FDs of  $R_2(B, C)$

$R_3(C, D)$

$$L(R_3) = \{C \rightarrow D\}$$

All non trivial FDs from subset of  $L(R_3)$  under  $S$   
 $C \rightarrow D$   
 $D \rightarrow C$

This is also the minimal basis of set  $\mathcal{F}_3$  of FDs of  $R_3(C, D)$

2) We take the union of FDs of  $S_1, S_2, S_3$

$$S_1 \cup S_2 \cup S_3 = \{A \rightarrow B, B \rightarrow A, B \rightarrow C, C \rightarrow B, C \rightarrow D, D \rightarrow C\} \quad (A, E)$$

We need to check if  $S_1 \cup S_2 \cup S_3$  imply  $S$ .

§ We can see from  $S_1 \cup S_2 \cup S_3$

$$\cancel{\{A\}^+ = \{A, B, C\}}; \{A\}^+ = \{B\}^+ = \{C\}^+ = \{D\}^+ = \{A, B, C, D\}$$

Thus all  $\{A, B, C, D\}$  are contained in the closure.

Therefore  $S_1 \cup S_2 \cup S_3$  implies  $S$ .

Therefore the decomposition is dependency preserving

$3 = C_1 =$

$= d$

A2+ R(A|B|C|D|E)

S = set of FDs

A → C    B → C    C → D    DE → C    CE → A

R<sub>1</sub>(A|D)    R<sub>2</sub>(A|B)    R<sub>3</sub>(B|E)    R<sub>4</sub>(C|D|E)    R<sub>5</sub>(A|E)

We use chain test

let t = (a|b|c|d|e) be a tuple in the schema  
R(A|B|C|D|E)

Tableau Obtained →

	A	B	C	D	E
t <sub>1</sub> →	a	b <sub>1</sub>	c <sub>1</sub>	d	e <sub>1</sub>
t <sub>2</sub> →	a	b	c <sub>2</sub>	d <sub>1</sub>	e <sub>2</sub>
t <sub>3</sub> →	a <sub>1</sub>	b	c <sub>3</sub>	d <sub>2</sub>	e
t <sub>4</sub> →	a <sub>2</sub>	b <sub>2</sub>	c	d	e
t <sub>5</sub> →	a	b <sub>3</sub>	c <sub>4</sub>	d <sub>3</sub>	e

Using A → C    c<sub>1</sub> = c<sub>2</sub> = c<sub>4</sub>    B → C    c<sub>2</sub> = c<sub>3</sub> = c<sub>1</sub> = c<sub>4</sub>

	A	B	C	D	E
t <sub>1</sub>	a	b <sub>1</sub>	c <sub>1</sub>	d	e <sub>1</sub>
t <sub>2</sub>	a	b	c <sub>1</sub>	d <sub>1</sub>	e <sub>2</sub>
t <sub>3</sub>	a <sub>1</sub>	b	c <sub>1</sub>	d <sub>2</sub>	e
t <sub>4</sub>	a <sub>2</sub>	b <sub>2</sub>	c	d	e
t <sub>5</sub>	a <sub>2</sub>	b <sub>3</sub>	c <sub>1</sub>	d <sub>3</sub>	e

Using ~~DE → C~~    ~~CE → A~~    C → D    d<sub>1</sub> = d<sub>2</sub> = d<sub>3</sub> = d

	A	B	C	D	E
$t_1$	$a$	$b_1$	$c_1$	$d$	$e_1$
$t_2$	$a$	$b$	$c_1$	$d$	$e_2$
$t_3$	$a_1$	$b$	$c_1$	$d$	$e$
$t_4$	$a_2$	$b_2$	$c$	$d$	$e$
$t_5$	$a$	$b_3$	$c$	$d$	$e$

Using  $DE \rightarrow C$   $\boxed{c_1 = c}$  and  $CE \rightarrow A$   $a_1 = a_2$

	A	B	C	D	E
$t_1$	$a$	$b_1$	$c$	$d$	$e_1$
$t_2$	$a$	$b$	$c$	$d$	$e_2$
$t_3$	$a$	$b$	$c$	$d$	$e$
$t_4$	$a_2$	$b_2$	$c$	$d$	$e$
$t_5$	$a$	$b_3$	$c$	$d$	$e$

The tuple  $t_3 = (a|b|c|d|e)$  is the 3<sup>rd</sup> row of the tableau.  
This proves lossless decomposition.

A3  $\rightarrow R(A|B|C|D)$

Set  $S$  of FDs

$A \rightarrow C$        $BC \rightarrow A$        $CD \rightarrow B$

1)  $\rightarrow$  Phase I Closure  $\{A\}^+$  under  $S' = S - \{A \rightarrow C\}$

$$\{A\}^+ = \{A\}$$

So  $A \rightarrow C$  is not implied, hence not redundant.

$\Rightarrow$  check FD  $BC \rightarrow A$  using only  $A \rightarrow C, CD \rightarrow B$

$$\{B, C\}^+ = \{B, C\}$$

$BC \rightarrow A$  is not implied, hence not redundant

$\Rightarrow$  check FD  $CD \rightarrow B$ , using only  $A \rightarrow C, BC \rightarrow A$

$$\{C, D\}^+ = \{C, D\}$$

$CD \rightarrow B$  is not implied, not redundant

Phase II check

Given FD set  $S$ , is the LHS of every FD minimal.

To check for FDs  $BC \rightarrow A$  and  $CD \rightarrow B$

check for  $BC \rightarrow A$

if  $B$  can be removed from  $BC \rightarrow A$

$\{C\}^+ = \{C\}$  under  $S \Rightarrow$  does not imply  $C \rightarrow A$

if  $C$  can be removed from  $BC \rightarrow A$

$\{B\}^+ = \{B\}$  under  $S \Rightarrow$  does not imply  $B \rightarrow A$

check for  $CD \rightarrow B$

if  $C$  can be removed from  $CD \rightarrow B$

$\{D\}^+ = \{D\}$  under  $S \Rightarrow$  does not imply  $C \rightarrow B$

7) if  $D$  can be removed from  $CD \rightarrow B$

$\{C\}^+ = \{C\}$  under  $S \Rightarrow$  does not imply  $D \rightarrow B$

Hence  $S$  is a minimal basis.

$$\boxed{A \rightarrow C, BC \rightarrow A, CD \rightarrow B}$$

2)  $R(A, B, C, D)$

$$\{A\}^+ = \{A\}$$

$$\{B\}^+ = \{B\}$$

$$\{C\}^+ = \{C\}$$

$$\{D\}^+ = \{D\}$$

$$\{A, B\}^+ = \{A, B, C\}$$

$$\{B, C\}^+ = \{A, B, C\}$$

$$\{C, D\}^+ = \{B, C, D\}$$

$$\{D, A\}^+ = \{A, B, C, D\}$$

$$\{A, C\}^+ = \{A, C\}$$

$$\{B, D\}^+ = \{B, D\}$$

$$\{A, B, C\}^+ = \{A, B, C\}$$

$$\{B, C, D\}^+ = \{A, B, C, D\}$$

Therefore we observe the keys for  $R(A, B, C, D)$  under the set of FDs  $S$  are  $\{A, D\}$ ,  $\{B, C, D\}$ .

3) 3NF decomposition

Since  $S$  is a minimal basis, decomposing

$S$  into

$$R_1(A, C) \quad R_2(A, B, C) \quad R_3(B, C, D)$$

Also  $R_3(B, C, D)$  contains  $\{B, C, D\}$  whose schema is a key for  $R$ .

$\Rightarrow$  Final 3NF decomposition.

$$R_1(A, C) \quad R_2(A, B, C) \quad R_3(B, C, D)$$

A3)

4)

$R(A, B, C, D)$

$BC \rightarrow A$

we ~~can~~ break  $R(A, B, C, D)$

into  $R_1(B, C, D)$

and  $R_2(A, B, D)$

(using BCNF decomposition)

~~R<sub>2</sub>~~

Consider  $R_x(A, C, D)$

$A \rightarrow C$

we decompose  $R_x(A, C, D)$  (using BCNF decomposition)

into  $R_1(A, C)$   $R_2(C, D)$

Therefore obtained relations using BCNF are

$R_1(A, C)$   $R_2(A, B, C)$   $R_3(B, C, D)$

This is the same as that obtained using BCNF decomposition.

Therefore (B) is BCNF decomposition.



A4 →

CIATRS G

$CI \rightarrow A$

$AC \rightarrow I$

$ATI \rightarrow R$

$ATS \rightarrow R$

$ATR \rightarrow C$

$ATC \rightarrow R$

$CS \rightarrow G$

1)  $\{C, I, T\}^+ = \{C, I, T, A, R\}$   
 $\{A, T, I\}^+ = \{A, T, I, R, C\}$   
 $\{A, T, S\}^+ = \{A, T, S, R, C, G, I\}$

2) we can see from 1) that  $\{A, T, S\}^+ = \{C, I, A, T, R, S, G\}$   
Therefore it is a key for the schema

Also  ~~$\{C, I, A, T, R, S, G\}^+ = \{C, I, A, T, R, S, G\}$~~   
 ~~$\{I, C, S, T\}^+ = \{C, I, A, T, R, S, G\}$~~   
 ~~$\{C, I, A, T, R, S, G\}^+ = \{C, I, A, T, R, S, G\}$~~

Therefore 2 keys exist for the given schema