Monte Carlo Simulation for European Option Pricing

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Project: Monte Carlo Option Pricing with Greeks, Confidence Intervals, and Put-Call Parity

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1. Introduction

This report implements a Monte Carlo simulation to price European call and put options.
The stock selected is NVDA, with data spanning 2020-01-01 to 2021-01-01.

The study includes:

- Monte Carlo option pricing
- Control variate technique
- Comparison with Black-Scholes analytical prices
- Calculation of option Greeks
- Put-call parity verification
- Geometric Brownian Motion (GBM) simulations of stock price paths

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2. Data Collection and Preprocessing

Stock data is downloaded from Yahoo Finance using Python's `yfinance` package.
Adjusted closing prices are used to account for splits and dividends.

Let P_t denote the stock price at time t, with $t = 0, 1, \dots, N$.

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3. Returns and Volatility

Log returns are calculated as:

$$r_t = \ln \frac{P_t}{P_{t-1}}$$

The **annualized return** and **annualized volatility** are computed as:

$$\mu_{\text{annual}} = 252 \cdot \bar{r}, \quad \sigma_{\text{annual}} = \sqrt{252} \cdot \text{std}(r_t)$$

where 252 is the typical number of trading days in a year.

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4. Option Parameters and Hoeffding Bound

Define the option parameters:

- Strike price: $K = 1.2 \cdot P_0$ (slightly out-of-the-money)
- Time to maturity: $T = 1$ year
- Risk-free rate: $r_f = 3\%$

To ensure a desired accuracy in Monte Carlo, the **Hoeffding inequality** provides a lower bound on the number of simulations:

$$N \geq \frac{\ln \frac{2}{\delta} \cdot (b-a)^2}{2\varepsilon^2}$$

where ε is the tolerance, $[a, b]$ is the payoff range, and $1-\delta$ is the confidence level.

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5. Monte Carlo Simulation of Final Prices

The stock price is modeled using **Geometric Brownian Motion (GBM)**:

$$S_{t+\Delta t} = S_t \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t} Z_t\right]$$

where $Z_t \sim \mathcal{N}(0,1)$ are standard normal random variables.
Antithetic variates are used to reduce simulation variance.

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6. Monte Carlo Option Pricing with Control Variate

For a European call option, the payoff is:

$$C_T = \max(S_T - K, 0)$$

and for a put option:

$$P_T = \max(K - S_T, 0)$$

Discounted payoffs are averaged to obtain the Monte Carlo estimate:

$$\hat{C}_0 = e^{-rT}\frac{1}{N}\sum_{i=1}^{N} C_T^{(i)}, \quad \hat{P}_0 = e^{-rT}\frac{1}{N}\sum_{i=1}^{N} P_T^{(i)}$$

The **control variate technique** adjusts the estimate using the analytical Black-Scholes price:

$$\hat{C}_0^{\text{adj}} = \hat{C}_0 + \left(C_{\text{BS}} - \hat{C}_0\right)$$

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7. Black-Scholes Analytical Prices

The Black-Scholes formulas for European options:

- Call:

$$C_0 = S_0 N(d_1) - K e^{-rT} N(d_2)$$

- Put:

$$P_0 = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$

where

$$d_1 = \frac{\ln(S_0/K) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

and $N(\cdot)$ is the standard normal CDF.

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8. Option Greeks

The primary Greeks are computed from Black-Scholes:

- Delta:

$$\Delta_{\text{call}} = N(d_1)$$

- Vega:

$$\nu = S_0\, \phi(d_1)\sqrt{T}$$

- Gamma:

$$\Gamma = \frac{\phi(d_1)}{S_0 \sigma \sqrt{T}}$$

where $\phi(\cdot)$ is the standard normal PDF.

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9. Simulation Plots

- Histogram of final simulated prices with strike marked.
- Sample GBM paths illustrating stochastic evolution.

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10. Put-Call Parity

Verified using:

$$C_0 - P_0 \approx S_0 - Ke^{-rT}$$

A small difference confirms numerical consistency.

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11. Summary and Insights

- Monte Carlo with control variate closely matches Black-Scholes prices.
- Greeks provide measures for risk management.
- GBM simulations illustrate stock price uncertainty.
- Put-call parity holds within tolerance, validating computations.

References

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