1. $P(\omega_n) = 0.85$

$P(g_a) = 0.95$

$P(g_a / \omega_n) = 0.99$

a) $P(\omega_n | g_a) = ?$

$$P(\omega_n | g_a) = \frac{P(g_a | \omega_n) \times P(\omega_n)}{P(g_a)}$$

$$= \frac{0.99 \times 0.85}{0.95}$$

$$= 0.885$$

b) $P(g_a / \bar{\omega}_n) = ?$

$$P(g_a / \bar{\omega}_n) = \frac{P(g_a)\ P(\bar{\omega}_n / g_a)}{P(\bar{\omega}_n)}$$

$$= \frac{0.95 \times (1 - 0.885)}{(1 - 0.85)}$$

$$= 0.724$$

2. Given, 75% reliable

Let: AB – taxi appears blue

TB – taxi is blue

AG – taxi appears green

TG – taxi is green

$\Rightarrow$ $P(AB|TB) = 0.75$, $P(AG|TG) = 0.25$

Also, Given:

$P(TG) = 0.9$

$P(TB) = 0.9$

$$\Rightarrow P(TB|AB) = \frac{P(AB|TB)\,P(TB)}{P(TB)P(AB|TB) + P(AB|TG)\,P(TG)}$$

$$= \frac{0.75 \times 0.1}{0.75 \times 0.1 + 0.25 \times 0.9}$$

$$= 0.25$$

$\Rightarrow$ $P(TG|AB) = 1 - 0.25$

$\therefore$ The car is most likely green.

3. a)

Weather (W)	Happy (Y)	Happy (N)
Bad	2/3	2/5
Good	1/3	3/5

Study (S)	Happy (Y)	Happy (N)
fail	0	4/5
Pass	3/3 = 1	1/5

Neighbor (N)	Happy (Y)	Happy (N)
Home	2/3	2/5
out	1/3	3/5

$P(Y \mid Good, Pass, out)$

$= P(Good \mid Y)\ P(Pass \mid Y)\ P(out \mid Y)\ P(Y)$

$= 1/3 \times 1 \times \frac{1}{3} \times 3/8$

$= 0.0416$

$P(N \mid Good, Pass, out)$

$= P(Good \mid N)\ P(Pass \mid N)\ P(out \mid N)\ P(N)$

$= 3/5 \times 1/5 \times 3/5 \times 5/8$

$= 9/200 = 0.045$

∴ Jim is not happy based on above results.

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b) $P(Y \mid Good, Pass, out)$

$$= \frac{P(Good, Pass, out \mid Y)\, P(Y)}{P(Good, Pass, out)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{8}}{\frac{1}{8}}$$

$= 1$

$P(N \mid Good, \cancel{Go}\ Pass, out) = 1 - P(Y \mid Good, Pass, out)$

$= 1 - 1$

$= 0$

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4. $P(C{+}{+}) = 0.5$

$P(Java) = 0.4$

$P(Macrosoft) = 0.01$

$P(C{+}{+} / Macrosoft) = 0.99$

$P(Java / Macrosoft) = 0.98$

$$P(Macrosoft / C{+}{+}, Java) = P(C{+}{+} / Macrosoft)\, P(Java / Macrosoft) \times P(Macrosoft)$$

$$= 0.99 \times 0.98 \times 0.01$$

$$= 0.0097$$

~~P(C++~~ $P(C{+}{+}) = P(C{+}{+} / Macrosoft) + P(C{+}{+} / Macrosoft) \cdot P(Macrosoft)$

$$P(C{+}{+} / Macrosoft) = \frac{0.5 - 0.99\,(0.01)}{0.99}$$

$$= 0.495$$

$$P(Java) = P(Java / Macrosoft)\, P(Macrosoft) + P(Java / Macrosoft)\, P(Macrosoft)$$

$$= 0.495 \times 0.394 \times 0.99$$

$$= 0.195$$

$\Rightarrow$ Programmer who knows C++ & Java is not a Macrosoft employee.

5. $P(X_i | Y=1) = \theta_{i1}^{x_i} (1-\theta_{i1})^{1-x_i}$

$$P(Y=1/X) = \frac{P(X|Y=1)\, P(Y=1)}{P(X)}$$

$$P(X) = P(X|Y=1)\, P(Y=1) + P(X|Y=0)\, P(Y=0)$$

$$P(Y=1|X) = \frac{P(X|Y=1)\, P(Y=1)}{P(X|Y=1)\, P(Y=1) + P(X|Y=0)\, P(Y=0)}$$

$$= \frac{1}{1 + \frac{P(X|Y=0)\, P(Y=0)}{P(X|Y=1)\, P(Y=1)}}$$

$$= \frac{1}{1 + \exp\left(\ln\left(\frac{P(Y=0)}{P(Y=1)}\right) + \Sigma \ln\left(P\left(\frac{X_i | Y=0}{X_i | Y=1}\right)\right)\right)}$$

$$\Sigma \ln \frac{P(X_i | Y=0)}{P(X_i | Y=1)} = \Sigma \ln \frac{\theta_{i0}^{x_i} (1-\theta_{i0})^{1-x_i}}{\theta_{i1}^{x_i} (1-\theta_{i1})^{1-x_i}}$$

$$= \Sigma \ln \left(\theta_{i0}^{x_i} (1-\theta_{i0})^{(1-x_i)}\right) - \ln \left(\theta_{i1}^{x_i} (1-\theta_{i1})^{(1-x_i)}\right)$$

$$= \Sigma x_i \ln \theta_{i0} + (1-x_i) \ln (1-\theta_{i0}) - x_i \ln \theta_{i1} - (1-x_i) \ln (1-\theta_{i1})$$

$$= \sum x_i (\ln \theta_{i0} - \ln \theta_{i1}) + (1-x_i)(\ln(1-\theta_{i0}) - \ln(1-\theta_{i1}))$$

$$\ln\left(\frac{P(Y=0)}{P(Y=1)}\right) = \ln \frac{1-\pi}{\pi} \quad \text{consider } (Y=1) = \pi$$

$$P(Y=1/x) = \frac{1}{1+\exp\left[\ln\left(\frac{1-\pi}{\pi}\right) + \sum_i x_i \left(\ln \frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})}\right) + \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right]}$$

$$\omega_0 = \ln\left(\frac{1-\pi}{\pi}\right) + \sum_i \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)$$

$$\omega_i = \ln \frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})}$$

$$P(Y=1/x) = \frac{1}{1+\exp(\omega_0 + \sum_i x_i \omega_i)}$$

$$P(Y=0/x) = 1 - P(Y=1/x)$$

$$= 1 - \frac{1}{1+\exp(\omega_0 + \sum_i x_i \omega_i)}$$

$$P(Y=0/x) = \frac{\exp(\omega_0 + \sum_i x_i \omega_i)}{1+\exp(\omega_0 + \sum_i \omega_i x_i)}$$

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6. According to Baye's

$$P(Y=1/x) = \frac{P(x/Y=1)\quad P(Y=1)}{P(x/Y=1)\ P(Y=1) + P(X/Y=0)\,P(Y=0)}$$

$$= \frac{1}{1+ \frac{P(X/Y=0)\,P(Y=0)}{P(X/Y=1) \times P(Y=1)}}$$

$$= \frac{1}{1+\exp\left(\ln \frac{P(X/Y=0)P(Y=0)}{P(X/Y=1)\ P(Y=1)}\right)}$$

$$= \frac{1}{1+\exp\left(\ln \frac{1-\pi}{\pi} + \sum_{i=1} \ln \frac{P(X_i/Y=0)}{P(X_i/Y=1)}\right)}$$

$$\sum_i \ln \frac{P(x_i/Y=0)}{P(x_i/Y=1)} = \sum_i \ln \frac{h(x_i;\theta_{i0})\exp(-\theta_{i0}^T x_i + c)}{h(x_i,\theta_{i1})\ \exp(-\theta_{i1}^T x_i + c)}$$

$$= \sum_i \ln \frac{h(x_i,\theta_{i0})}{h(x_i,\theta_{i1})} + \ln\left(\exp(-\theta_{i0}^T x_i\, c - (-\theta_{i1}^T x_i + c)\right)\bigg)$$

$$= \sum_i \ln \frac{h(x_i\theta_{i0})}{h(x_i\,\theta_{i1})} + (\theta_{i1}^T - \theta_{i0}^T)\,x_i$$

$$P(Y=1|X) = \frac{1}{1+\exp\left(\ln\frac{1-\pi}{\pi} + \sum_i\left((\theta_{i1}^T - \theta_{i0}^T)x_i\right) + \ln\left(\frac{n(x_i\ \theta_{i0})}{n(x_i\theta_{i1})}\right)\right)}$$

$$w_0 = \ln\frac{1-\pi}{\pi} + \ln\left(\frac{n(x_i . \theta_{i0})}{n(x_i\theta_{i1})}\right)$$

$$w_1 = \theta_{i1}^T - \theta_{i0}^T$$