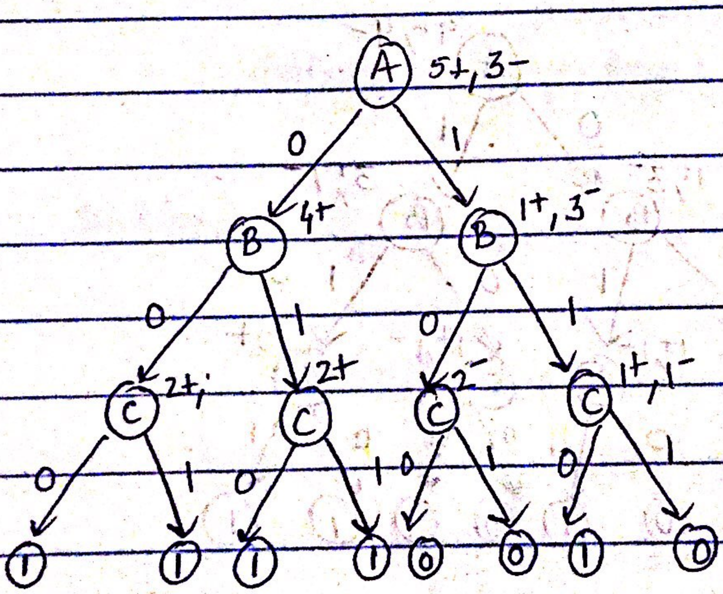
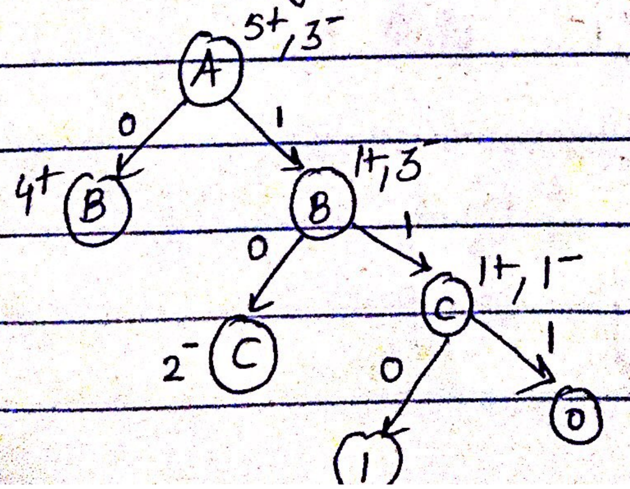
1. a) $Y = (\neg A \vee B) \wedge \neg(C \wedge A)$

A	B	C	$\neg A$	$(\neg A \vee B)$	$(C \wedge A)$	$\neg(C \wedge A)$	$(\neg A \vee B) \wedge \neg(C \wedge A)$
0	0	0	1	1	0	1	1
0	0	1	1	1	0	1	1
0	1	0	1	1	0	1	1
0	1	1	1	1	0	1	1
1	0	0	0	0	0	1	0
1	0	1	0	0	1	0	0
1	1	0	0	1	0	1	1
1	1	1	0	1	1	0	0

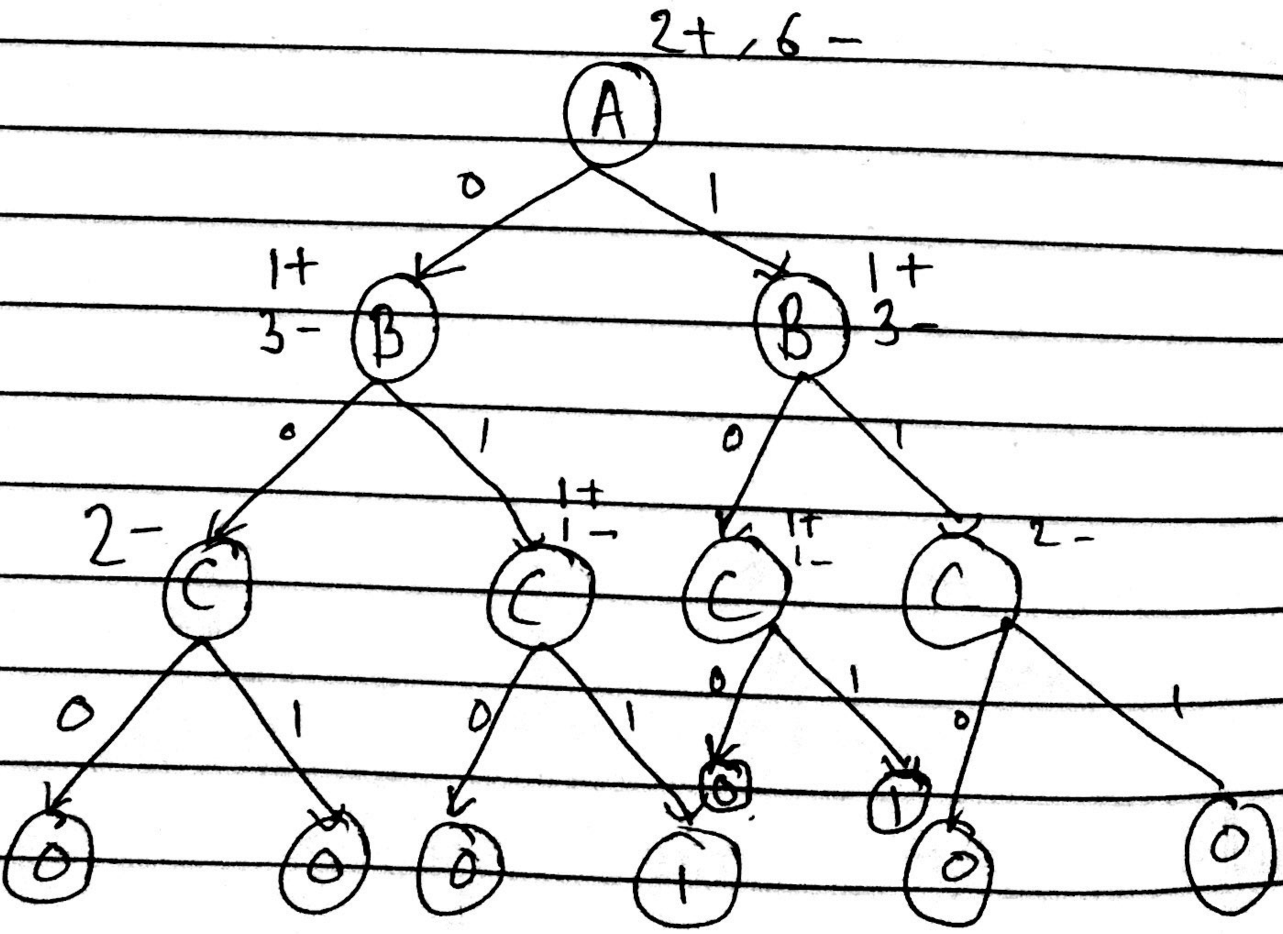


Minimization, (to get pure nodes)

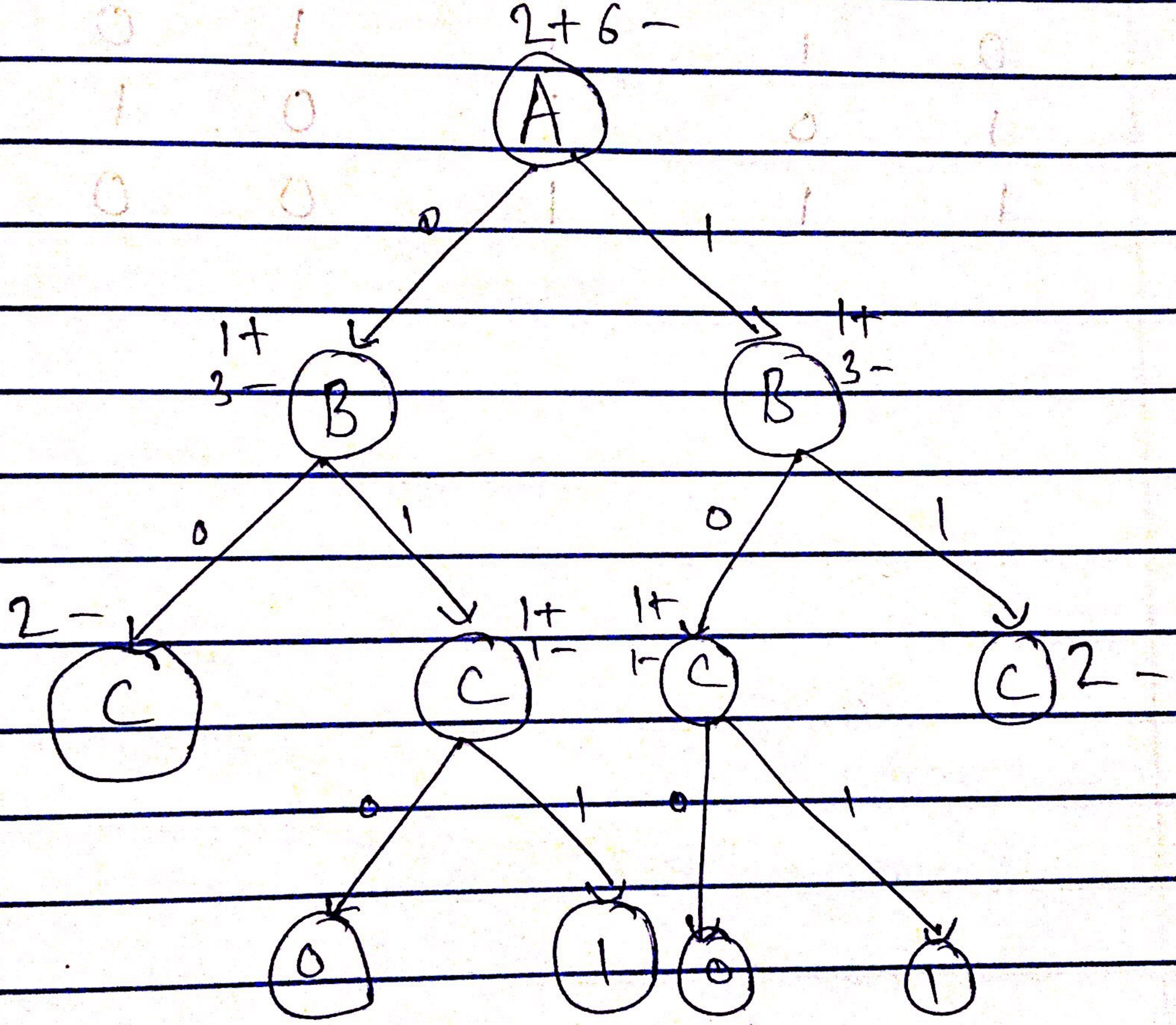


b) $Y = (A \oplus B) \wedge C$

A	B	C	$A \oplus B$	$A \oplus B \wedge C$
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0

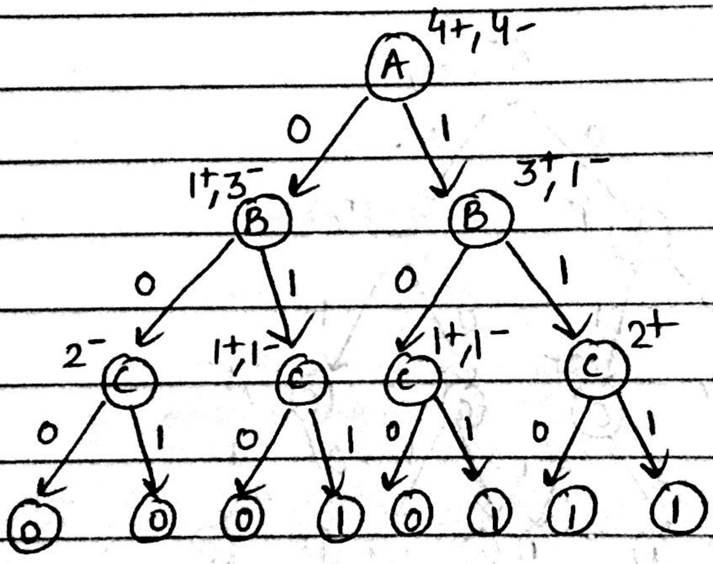


Hence, now we can minimize the nodes in the decision tree. Hence, we can stop splitting when the nodes become 'pure'.

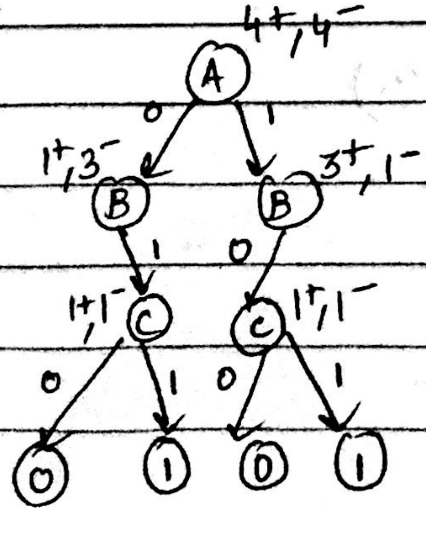


1\. c) $Y = (A \vee B) \wedge (B \vee C) \wedge (A \vee C)$

A	B	C	$(A \vee B)$	$(B \vee C)$	$(A \vee C)$	$(A \vee B) \wedge (B \vee C) \wedge (A \vee C)$
0	0	0	0	0	0	0
0	0	1	0	1	1	0
0	1	0	1	1	0	0
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1



Minimization



$Y = (A \vee B) \wedge \neg A \wedge \neg B$

A	B	$A \vee B$	$\neg A$	$\neg B$	$(A \vee B) \wedge \neg A \wedge \neg B$
0	0	0	1	1	0
0	1	1	1	0	0
1	0	1	0	1	0
1	1	1	0	0	0

Since the output class has same label
thus no decision tree required.

2.

Instance	x1	x2	x3	class
1	1	0	0	1
2	0	1	0	1
3	0	0	0	0
4	1	0	1	0
5	0	0	0	0
6	1	1	0	1
7	0	1	1	0
8	1	0	0	1
9	0	0	0	0
10	1	0	0	1

$5^+, 5^-$

ID3 Algorithm.

$$\text{Entropy} = -\sum_{i=1}^{10} p_i \log p_i$$

$$H(\text{class}) = -\frac{5}{10}\log\frac{5}{10} - \frac{5}{10}\log\frac{5}{10}$$

$$= -2 \times \frac{1}{2}\log\frac{1}{2}$$

STAGE 1

$$= +1$$

$5 \rightarrow 1^+$, $5 \rightarrow 4^-$

$$H(\text{class}/x_1 = 0) = -\frac{1}{5}\log\frac{1}{5} - \frac{4}{5}\log\frac{4}{5}$$

$$= 0.72$$

$5 \rightarrow 4^+$, $5 \rightarrow 1^-$

$$H(\text{class}/x_1 = 1) = -\frac{4}{5}\log\frac{4}{5} - \frac{1}{5}\log\frac{1}{5}$$

$$= 0.72$$

$$x_1 \begin{array}{l} \nearrow 5^+ \\ \searrow 5^- \end{array}$$

$$H(class/x_1) = \frac{5}{10}(0.72) + \frac{5}{10}(0.72)$$
$$= 0.72$$

$\Rightarrow$ Information Gain, $IG = 1 - 0.72$

$$\underline{IG_{x_1} = 0.28}$$

$$7 \begin{array}{l} \nearrow 3^+ \\ \searrow 4^- \end{array}$$

$$H(class/x_2=0) = -\frac{3}{7}\log\frac{3}{7} - \frac{4}{7}\log\frac{4}{7}$$

$$= 0.98$$

$$3 \begin{array}{l} \nearrow 2^+ \\ \searrow 1^- \end{array}$$

$$H(class/x_2=1) = -\frac{2}{3}\log\frac{2}{3} - \frac{1}{3}\log\frac{1}{3}$$

$$= 0.92$$

$$x_2 \begin{array}{l} \nearrow 3^+ \\ \searrow 7^- \end{array}$$

$$H(class/x_2) = \frac{3}{10}\times 0.92 + \frac{7}{10}\times 0.98$$
$$= 0.96$$

$\Rightarrow$ Information Gain, $IG_{x_2} = 1 - 0.96$

$$\underline{IG_{x_2} = 0.04}$$

$$8 \begin{array}{l} \nearrow \cancel{-3}\,3^+ \\ \searrow 5^+ \end{array}$$

$$H(class/x_3=0) = -\frac{5}{8}\log\frac{5}{8} - \frac{3}{8}\log\frac{3}{8}$$

$$= 0.95$$

$$2 \begin{array}{l} \nearrow 2^- \\ \searrow 0^+ \end{array}$$

$$H(class/x_3=1) = 0 - \frac{2}{2}\log\frac{2}{1}$$

$$= 0$$

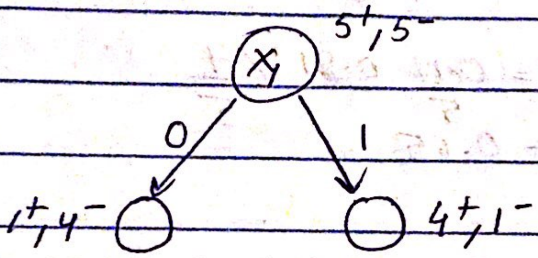
$x_3 \nearrow 8^-$, $\searrow 2^+$

$$H(class|x_3) = \frac{2}{10} \times 0 + \frac{8}{10} \times 0.95 = 0.76$$

$\Rightarrow$ Information Gain, $IG_{x_3} = \cancel{IG}\ 1 - 0.76$

$$\underline{IG_{x_3} = 0.24}$$

IG_{x_1} is the highest $\therefore$ x_1 is the first node



<u>STAGE 2</u> (Left child $(x_1 = 0)$)

$$H(\text{Left Node } (x_1 = 0)) = -\frac{1}{5} \log \frac{1}{5} - \frac{4}{5} \log \frac{4}{5} = 0.72$$

$3 \nearrow 0^+$, $\searrow 3^-$

$$\cancel{H(cla}\ H(x_1 = 0 | x_2 = 0) = 0 - \frac{3}{3} \log \frac{3}{3} = 0$$

$2 \nearrow 1^+$, $\searrow 1^-$

$$H(x_1 = 0 | x_2 = 1) = -\frac{1}{2} \log \frac{1}{2} - \frac{1}{2} \log \frac{1}{2} = 1$$

$x_2 \nearrow 2^+$, $\searrow 3^-$

$$H(x_1 = 0 | x_2) = \cancel{-\frac{2}{5} \log \frac{2}{5}}\ 0 \times \frac{3}{5} + 1 \times \frac{2}{5} = 0.4$$

→ Information Gain, $IG_{x_2} = 0.72 - 0.4$

$$IG_{x_2} = \cancel{0.68}\ 0.32$$

$4 \nearrow 1^+$, $\searrow 3^-$

$$H(x_1 = 0 \mid x_3 = 0) = -\frac{1}{4}\log\frac{1}{4} - \frac{3}{4}\log\frac{3}{4}$$
$$= 0.81$$

$1 \nearrow 0^+$, $\searrow 1^-$

$$H(x_1 = 0 \mid x_3 = 1) = 0 - \frac{1}{1}\log\frac{1}{1}$$
$$= 0$$

$x_3 \nearrow 0^+$, $\searrow 4^-$

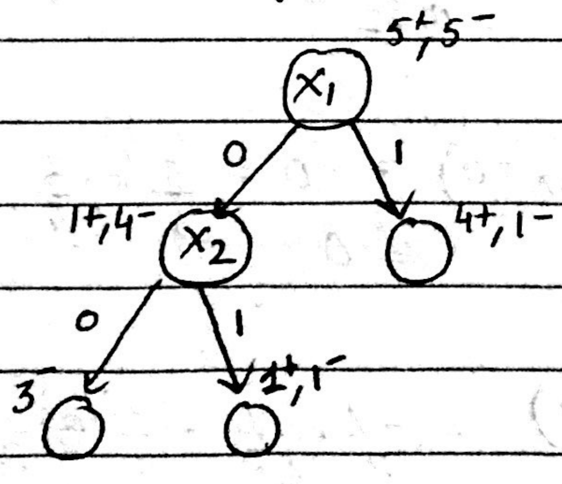
$$H(x_1 = 0 \mid x_3) = 0 \times \frac{1}{5} + 0.81 \times \frac{4}{5}$$
$$= 0.65$$

→ Information Gain, $IG_{x_3} = 0.72 - 0.65$

$$IG_{x_3} = 0.07$$

$IG_{x_2} > IG_{x_3}$

Therefore considering x_2 as the next node to split



Right child, $(x_1 = 1)$

$$H(x_1=1) = -\frac{4}{5}\log\frac{4}{5} - \frac{1}{5}\log\frac{1}{5}$$
$$= 0.72$$

$4 \begin{cases} \to 3^+ \\ \to 1^- \end{cases}$

$$H(x_1=1/x_2=0) = -\frac{3}{4}\log\frac{3}{4} - \frac{1}{4}\log\frac{1}{4}$$
$$= 0.81$$

$1 \begin{cases} 1^+ \\ 0^- \end{cases}$

$$H(x_1=1/x_2=1) = -\frac{1}{1}\log\frac{1}{1} - 0$$
$$= 0$$

$x_2 \begin{cases} 1^+ \\ 4^- \end{cases}$

$$H(x_1=1/x_2) = \frac{1}{5}\times 0 + \frac{4}{5}\times 0.81$$
$$= 0.65$$

∴ Information Gain, $IG_{x_2} = 0.72 - 0.65$

$$\underline{IG_{x_2} = 0.07}$$

$4 \begin{cases} \to 4^+ \\ \to 0^- \end{cases}$

$$H(x_1=1/x_3=0) = -\frac{4}{4}\log\frac{4}{4} - 0$$
$$= 0$$

$1 \begin{cases} \to 0^+ \\ \to 1^- \end{cases}$

$$H(x_1=1/x_3=1) = 0 - \frac{1}{1}\log\frac{1}{1}$$
$$= 0$$

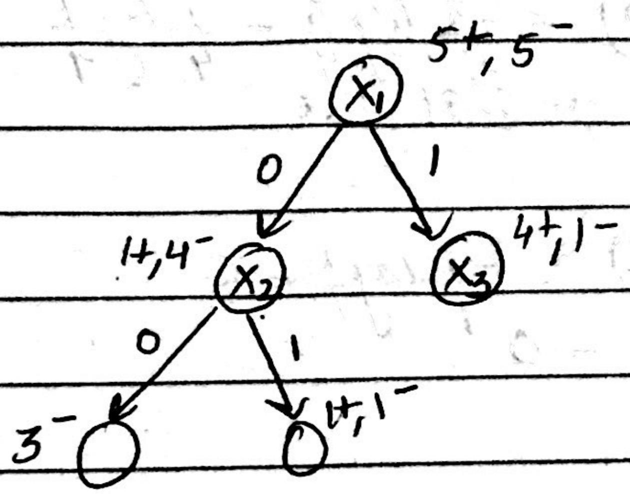
$$H(x_1=1/x_3) = \frac{1}{5}\times 0 + \frac{4}{5}\times 0$$
$$= 0$$

⇒ Information Gain, $IG_{x_3} = 0.72 - 0$

$\underline{IG_{X_3} = 0.72}$

$IG_{X_3} > IG_{X_2}$

⇒ X_3 is the next node to split.



Now we are left with one attribute X_3.

Final Decision Tree