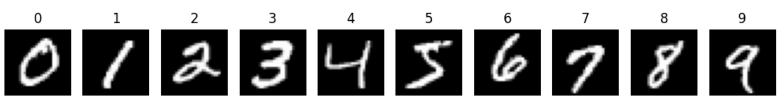
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_openml
from sklearn.decomposition import PCA, KernelPCA, IncrementalPCA
from sklearn.datasets import make_swiss_roll
from sklearn.linear_model import LogisticRegression
from sklearn.pipeline import Pipeline
from sklearn.model_selection import GridSearchCV
```

```
# Retrieve and load the MNIST dataset
mnist = fetch_openml('mnist_784')
```

```
/usr/local/lib/python3.10/dist-packages/sklearn/datasets/_openml.py:968: FutureWarning: The default value of `parser` will change from `
  warn(
```

```
# Split the data into a training set (first 60,000 instances)
x_train, y_train = mnist.data[:60000].to_numpy(), mnist.target[:60000].to_numpy()
```

```
# Display each digit using subplots
fig, axes = plt.subplots(1, 10, figsize=(10, 2))
for i in range(10):
    idx = np.where(y_train == str(i))[0][0]
    axes[i].imshow(x_train[idx].reshape(28, 28), cmap='gray')
    axes[i].axis('off')
    axes[i].set_title(str(i))
plt.tight_layout()
plt.show()
```



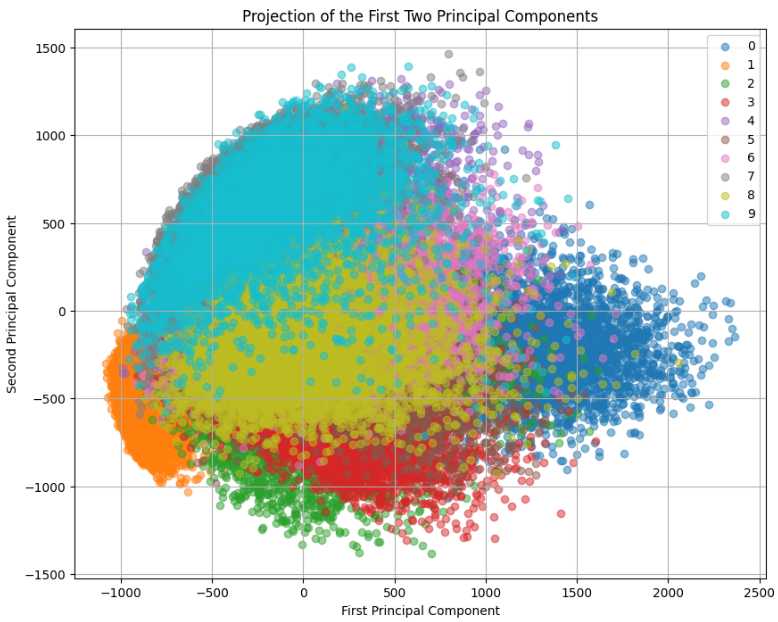

```
# Split the data into a test set (last 10,000 instances)
x_test, y_test = mnist.data[60000:], mnist.target[60000:]
```

```
# Use PCA to extract the first two principal components
pca = PCA(n_components=2)
x_pca = pca.fit_transform(x_train)
```

```
# Print the explained variance ratio of the two principal components
explained_variance_ratio = pca.explained_variance_ratio_
print(explained_variance_ratio)
```

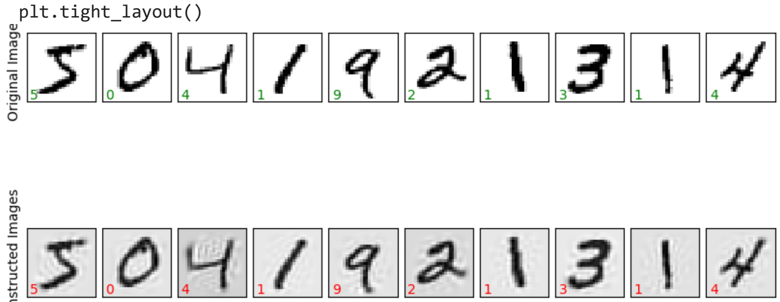
```
[0.09704664 0.07095924]
```

```
# Plot the projections of the first two principal components on a 2D plane
plt.figure(figsize=(10, 8))
for i in range(10):
    plt.scatter(x_pca[y_train == str(i), 0], x_pca[y_train == str(i), 1], alpha=0.5, label=str(i))
plt.xlabel('First Principal Component')
plt.ylabel('Second Principal Component')
plt.title('Projection of the First Two Principal Components')
plt.legend()
plt.grid(True)
plt.show()
```

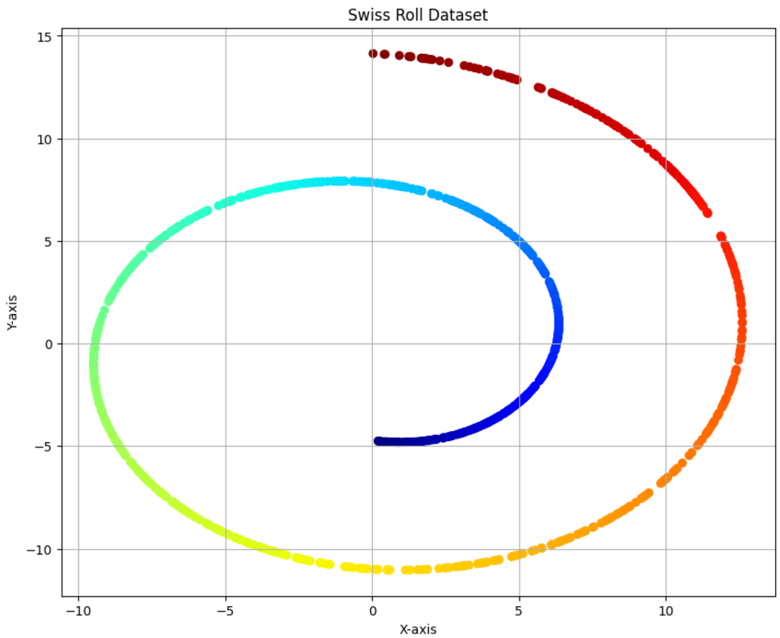



```
# Reducing the dimensionality of the MNIST dataset to 154 dimensions using Incremental PCA
n_batches = 100
ipca = IncrementalPCA(n_components=154)
for x_batch in np.array_split(x_train, n_batches):
    ipca.partial_fit(x_batch)
x_ipca = ipca.transform(x_train)


# Display the original and the compressed digits
x_reconstructed = ipca.inverse_transform(x_ipca)
fig, axes = plt.subplots(2, 10, figsize=(10, 5), subplot_kw={'xticks': [], 'yticks': []}, gridspec_kw=dict(hspace=0.1, wspace=0.1))
for i in range(10):
    axes[0, i].imshow(x_train[i].reshape(28, 28), cmap='binary', interpolation='nearest')
    axes[0, i].text(0.05, 0.05, str(y_train[i]), transform=axes[0, i].transAxes, color='green')
    axes[1, i].imshow(x_reconstructed[i].reshape(28, 28), cmap='binary', interpolation='nearest')
    axes[1, i].text(0.05, 0.05, str(y_train[i]), transform=axes[1, i].transAxes, color='red')
axes[0, 0].set_ylabel('Original Images')
axes[1, 0].set_ylabel('Reconstructed Images')
plt.tight_layout()
plt.show()
```

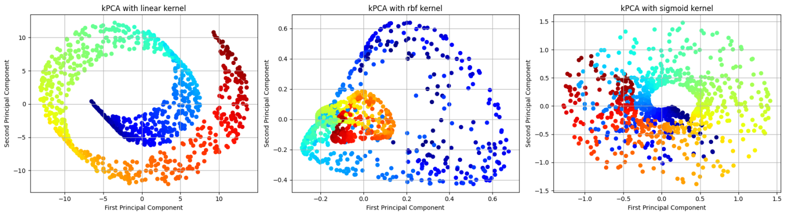
```
<ipython-input-10-0f1dcf7e8e55>:11: UserWarning: This figure includes Axes th
  plt.tight_layout()
```



```
# --- Swiss Roll Dataset Tasks ---

# Generate the Swiss roll dataset
X_swiss, y_swiss = make_swiss_roll(n_samples=1000, noise=0.0, random_state=42)



# Plot the generated Swiss roll dataset
plt.figure(figsize=(10, 8))
plt.scatter(X_swiss[:, 0], X_swiss[:, 2], c=y_swiss, cmap=plt.cm.jet)
plt.title("Swiss Roll Dataset")
plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.grid(True)
plt.show()
```




```
# Apply Kernel PCA (kPCA) with different kernels: linear, RBF, and sigmoid
kernels = ['linear', 'rbf', 'sigmoid']
kpca_results = {}
for kernel in kernels:
    kpca = KernelPCA(n_components=2, kernel=kernel, gamma=0.04)
    X_kpca = kpca.fit_transform(X_swiss)
    kpca_results[kernel] = X_kpca


  # Plot the results of applying kPCA with different kernels
  fig, axes = plt.subplots(1, 3, figsize=(18, 5))
  for i, kernel in enumerate(kernels):
      axes[i].scatter(kpca_results[kernel][:, 0], kpca_results[kernel][:, 1], c=y_swiss, cmap=plt.cm.jet)
      axes[i].set_title(f"kPCA with {kernel} kernel")
      axes[i].set_xlabel("First Principal Component")
      axes[i].set_ylabel("Second Principal Component")
      axes[i].grid(True)
  plt.tight_layout()
  plt.show()
```



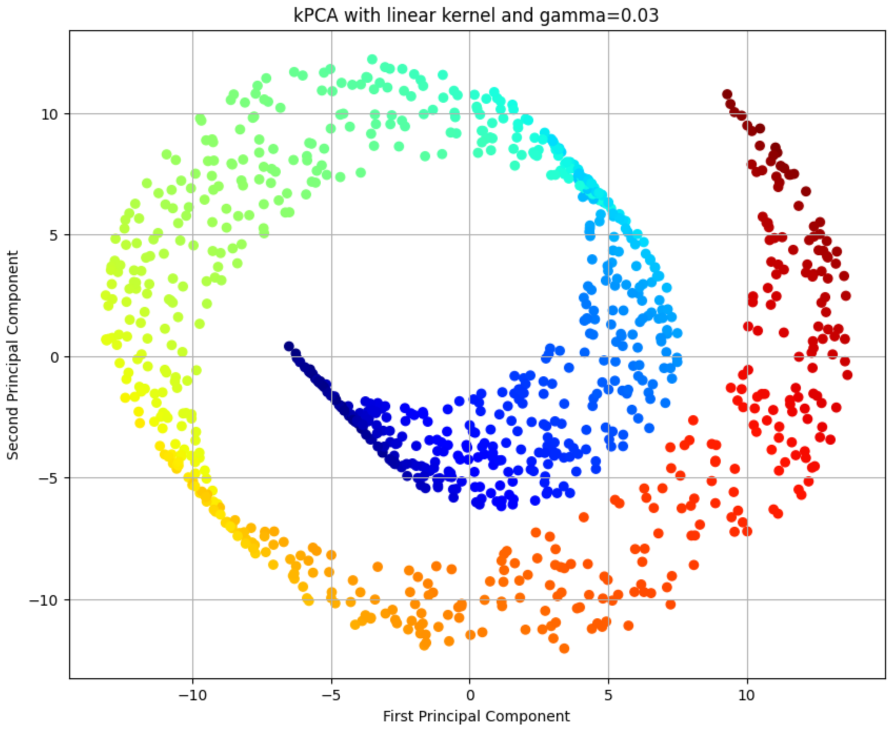

```
import warnings

# Ignore any warning messages
warnings.filterwarnings("ignore")
# Use kPCA with Logistic Regression for classification and apply GridSearchCV
y_binned = np.digitize(y_swiss, np.linspace(y_swiss.min(), y_swiss.max(), 50))
pipeline = Pipeline([
    ("kpca", KernelPCA(n_components=2)),
    ("log_reg", LogisticRegression())
])
param_grid = {
    "kpca__gamma": np.linspace(0.03, 0.05, 10),
    "kpca__kernel": ["linear", "rbf", "sigmoid"]
}
grid_search = GridSearchCV(pipeline, param_grid, cv=3)
grid_search.fit(X_swiss, y_binned)
```

```
GridSearchCV

GridSearchCV(cv=3,
             estimator=Pipeline(steps=[('kpca', KernelPCA(n_components=2)),
                                       ('log_reg', LogisticRegression())]),
             param_grid={'kpca__gamma': array([0.03      , 0.03222222, 0.034
       0.04111111, 0.04333333, 0.04555556, 0.04777778, 0.05      ]),
                         'kpca__kernel': ['linear', 'rbf', 'sigmoid']})

estimator: Pipeline

Pipeline(steps=[('kpca', KernelPCA(n_components=2
                ('log_reg', LogisticRegression()

KernelPCA
KernelPCA(n_components=2)

LogisticRegression
LogisticRegression()
```

```
# Print best parameters found by GridSearchCV
best_params = grid_search.best_params_
print(best_params)
```

```
{'kpca__gamma': 0.03, 'kpca__kernel': 'linear'}
```

```
# Plot the results using the best parameters from GridSearchCV
kpca_best = KernelPCA(n_components=2, kernel=best_params['kpca__kernel'], gamma=best_params['kpca__gamma'])
X_kpca_best = kpca_best.fit_transform(X_swiss)
plt.figure(figsize=(10, 8))
plt.scatter(X_kpca_best[:, 0], X_kpca_best[:, 1], c=y_swiss, cmap=plt.cm.jet)
plt.title(f"kPCA with {best_params['kpca__kernel']} kernel and gamma={best_params['kpca__gamma']}")
plt.xlabel("First Principal Component")
plt.ylabel("Second Principal Component")
plt.grid(True)
plt.show()
```


kPCA with linear kernel and gamma=0.03
Second Principal Component
First Principal Component
10
5
0
−5
−10
−10
−5
0
5
10