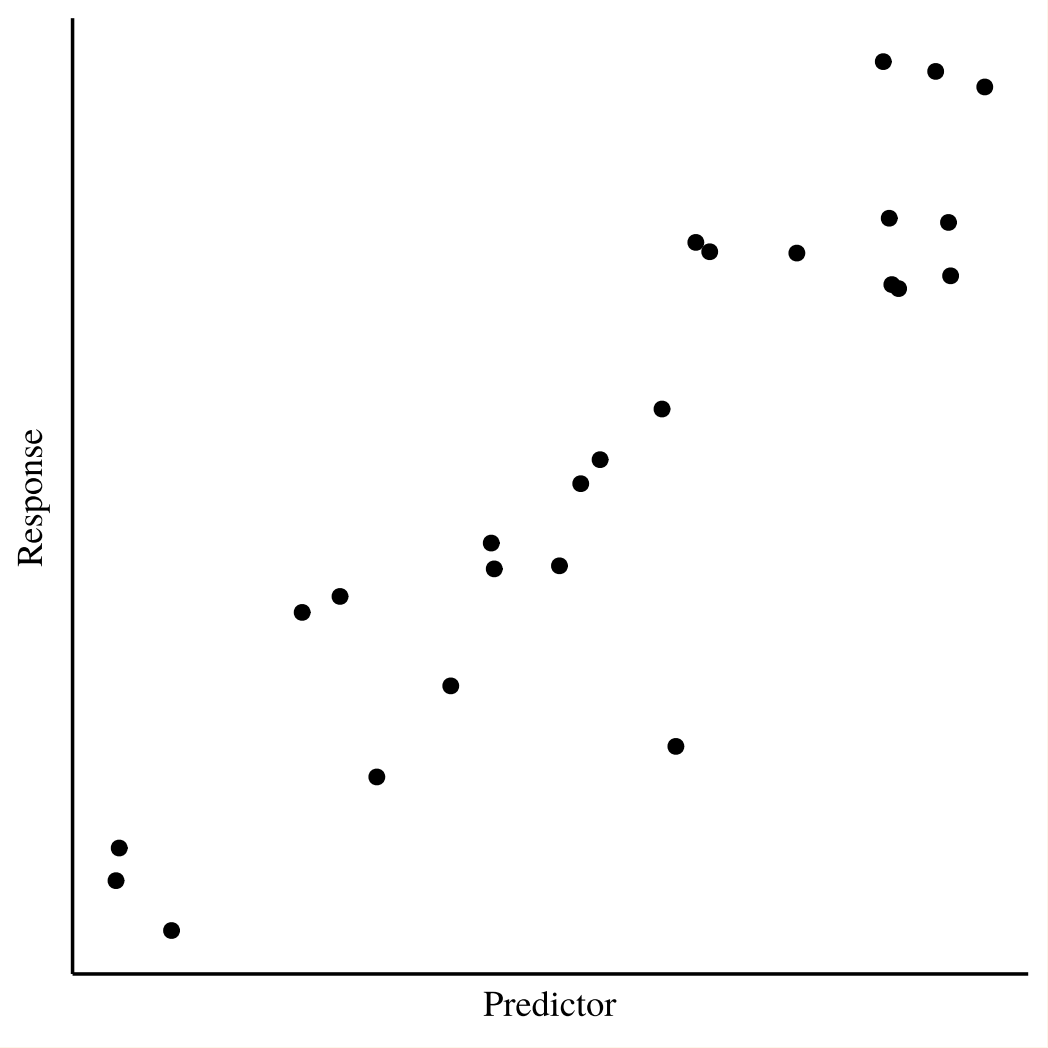
Simple Linear Regression (SLR) Model Inference

Stat 230

April 06 2022

Overview



Today:

Inference for the SLR model

- Residual plots
- Normal QQ plots
- Checks for Independence

The Simple Linear Regression (SLR) model

$$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i \qquad \epsilon_i \sim N(0, \sigma)$$

- **L**inear mean function: varies linearly with x
- **I**ndependence of errors
- **N**ormality of errors
- **E**quality of variance of the errors

"All models are wrong, but some are useful."

Professor George Box (1919-2013)

- Statistical models are used to model or describe potentially complex physical phenomenon

Need to strike a balance between

- model simplicity: easy to derive estimates/inference methods and interpret
- model complexity: more challenging to derive estimates/inference methods and interpret

For any model: we need check whether our data "fits" the model assumptions

Model error terms

Reframe the theoretical model:

$$\epsilon_i = Y_i - (\beta_0 + \beta_1 x_i) \quad \epsilon_i \sim N(0, \sigma)$$

ϵ_i are the key!

- mean and SD don't depend on x
- normally distributed
- and independent!

Problem: we only know ϵ_i if we know β_0 and β_1 !

Residuals

Residuals are what we get when we estimate β_0 and β_1 :

$$r_i = y_i - \left(\hat{\beta}_0 + \hat{\beta}_1 x_i\right) = y_i - \hat{y}_i$$

- $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$ are called **fitted** values

Residuals can be thought of as the error or the “lack-of-fit” between the observed value y_i and the fitted value $\hat{y}_i$ on the regression line

R package moderndive

Can get nice tables with built-in functions in moderndive. Previously used broom and tidy to get a nice coefficients table.

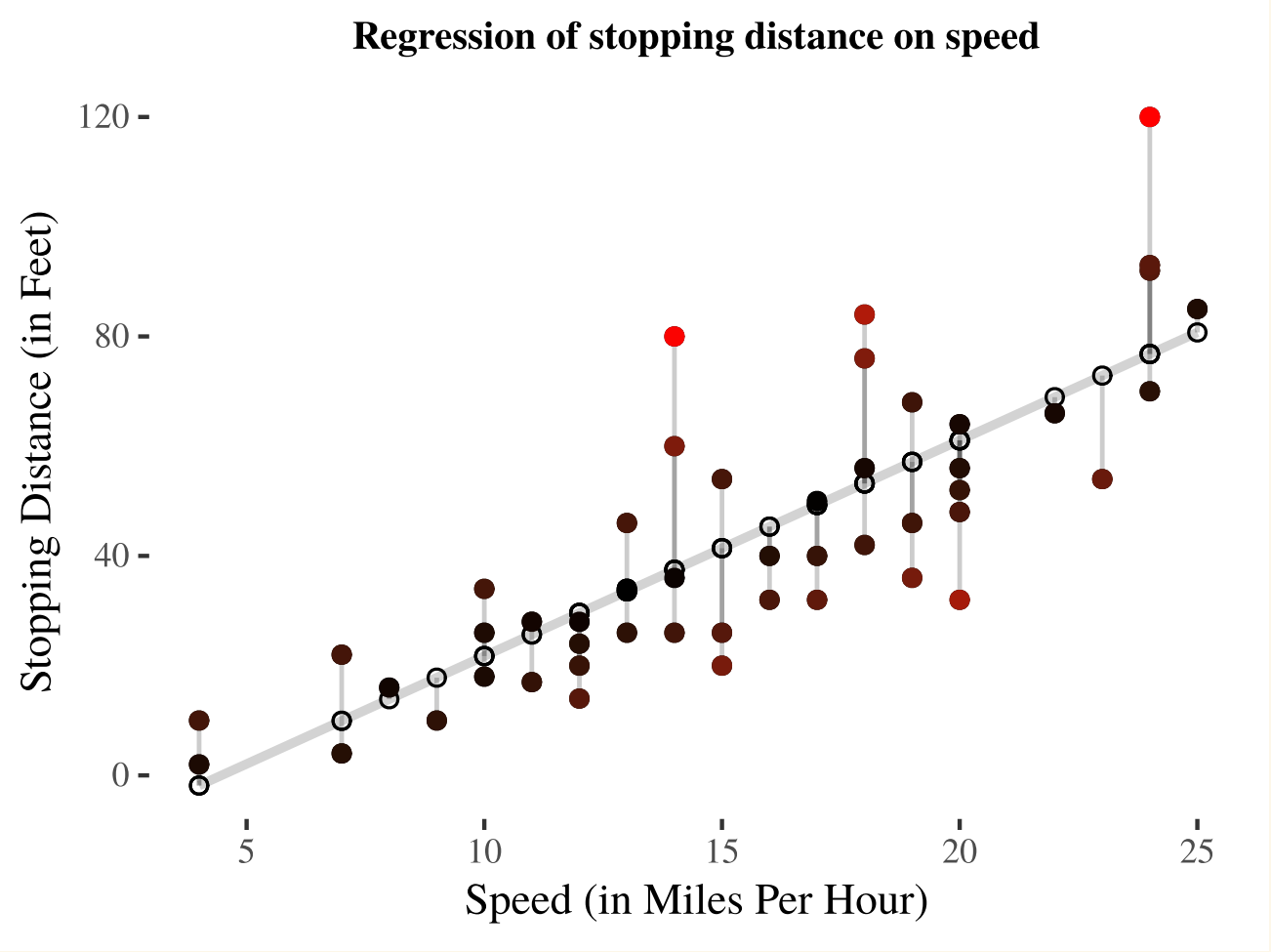
```
# call the library
# install.packages("moderndive")
library(moderndive)
```

```
# Get regression table:
regression_table <- get_regression_table(cars_lm)
knitr::kable(regression_table, digits = 4, format = "html") # does not work in .pdf
```

term	estimate	std_error	statistic	p_value	lower_ci	upper_ci
intercept	-17.579	6.758	-2.601	0.012	-31.168	-3.990
speed	3.932	0.416	9.464	0.000	3.097	4.768

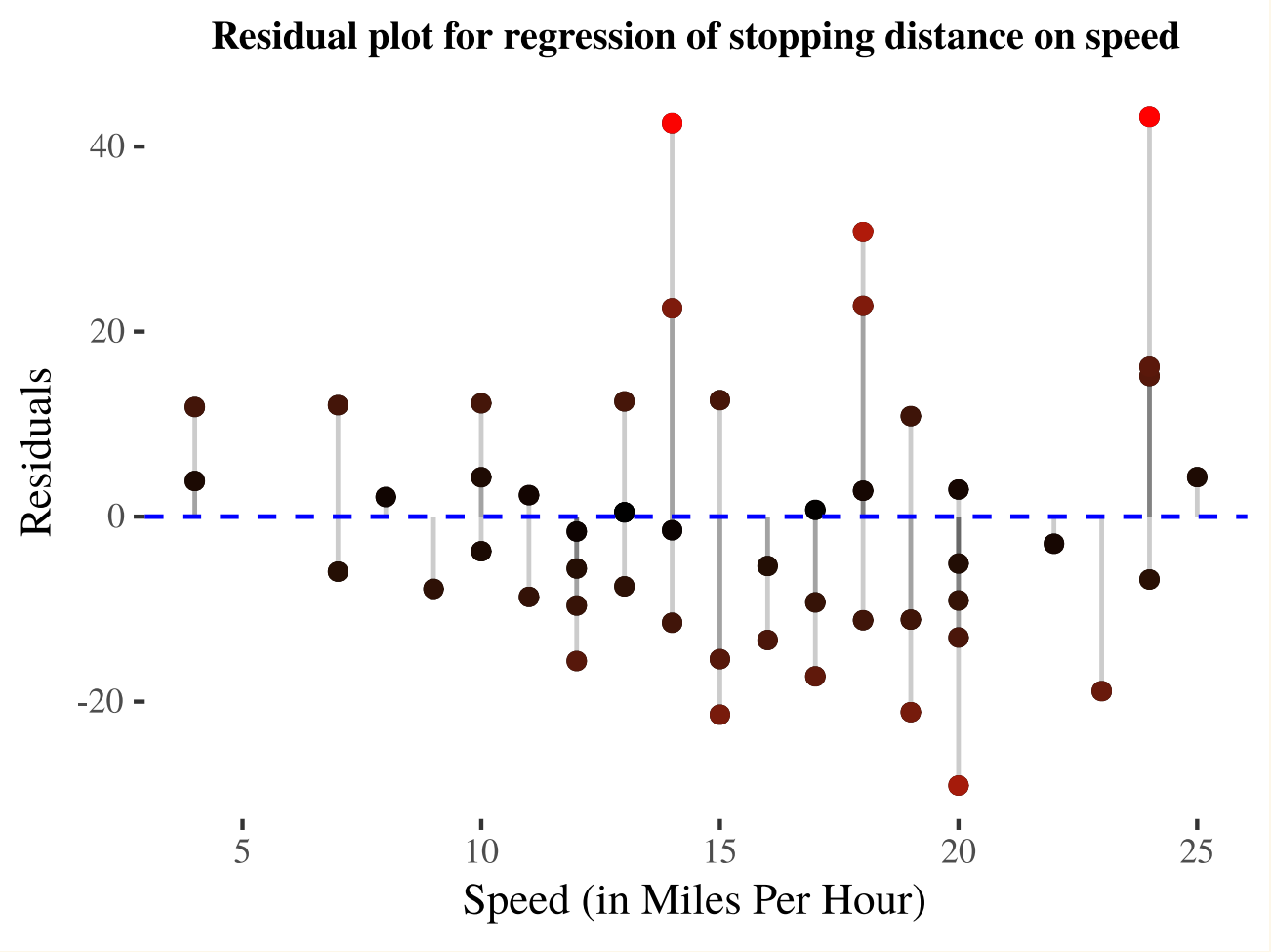
Residuals

```
ggplot(regression_points, aes(x = speed, y = dist))
  geom_point() +
  theme(legend.position = "none") +
  geom_smooth(method = "lm", se = FALSE, color = "l
  geom_point(aes(y = dist_hat), shape = 1) +
  geom_segment(aes(xend = speed, yend = dist_hat),
  scale_color_continuous(low = "black", high = "red
  geom_point(aes(color = abs(residual))) +
  labs(x='Speed (in Miles Per Hour)',
       y='Stopping Distance (in Feet)',
       title='Regression of stopping distance on sp
  theme(plot.title = element_text(hjust=0.5, size=9
```

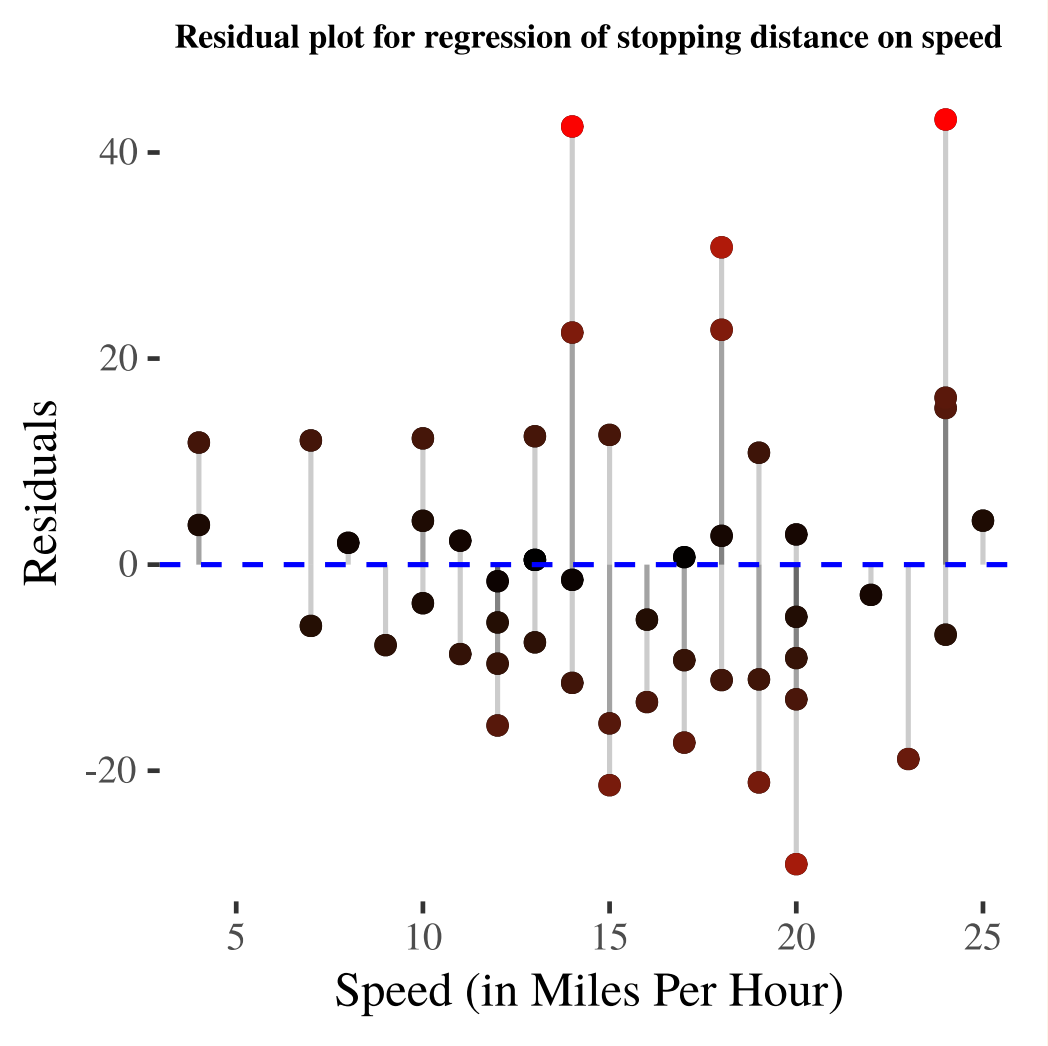


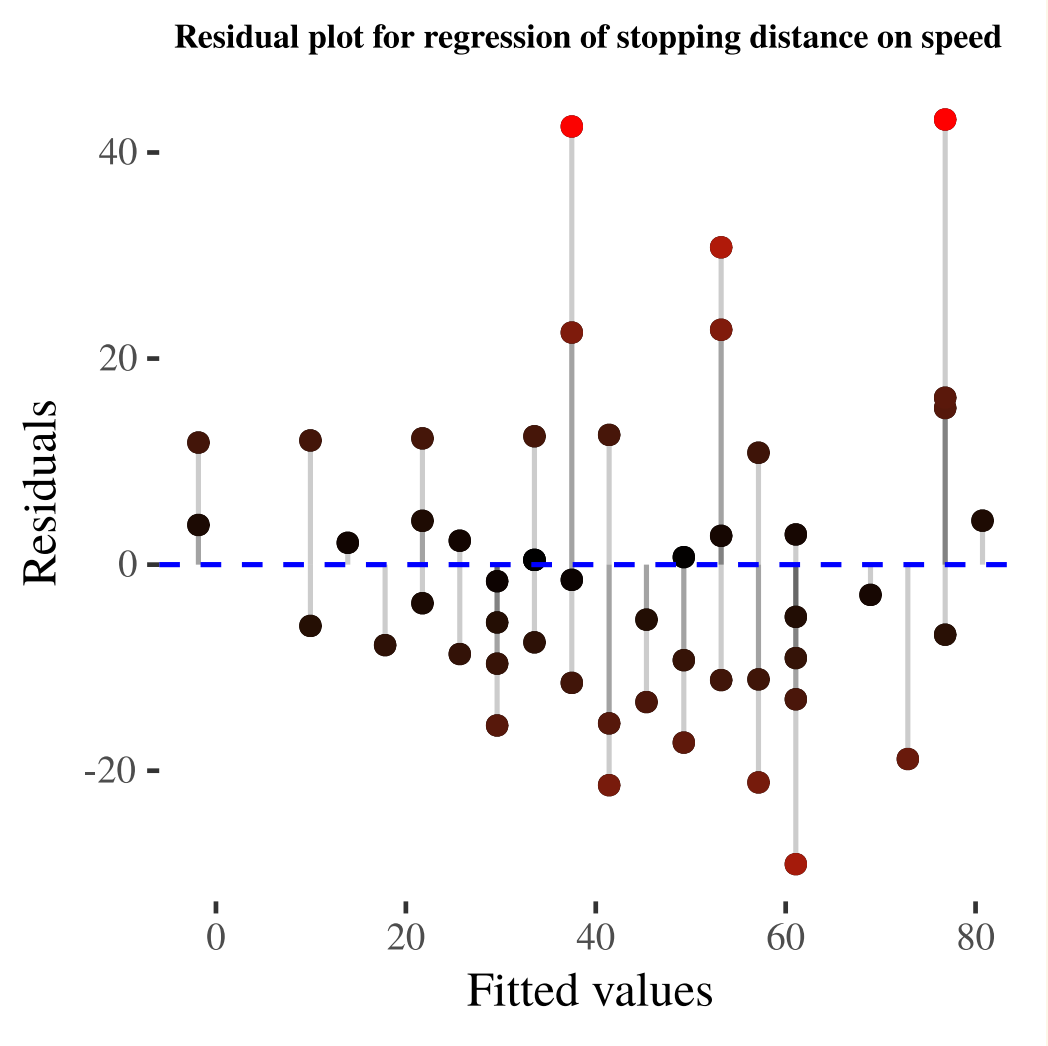
Residual plot (explanatory variable)

```
ggplot(regression_points, aes(x = speed, y = residu
  geom_point() +
  geom_segment(aes(xend = speed, yend = 0), alpha =
  theme(legend.position = "none") +
  scale_color_continuous(low = "black", high = "red
  geom_point(aes(color = abs(residual))) +
  geom_hline(yintercept = 0, col = "blue", size = 0
  labs(x='Speed (in Miles Per Hour)',
       y='Residuals',
       title='Residual plot for regression of stopp
  theme(plot.title = element_text(hjust=0.5, size=9
```



Scatterplot of r_i against $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$





Residual plot: interpretation

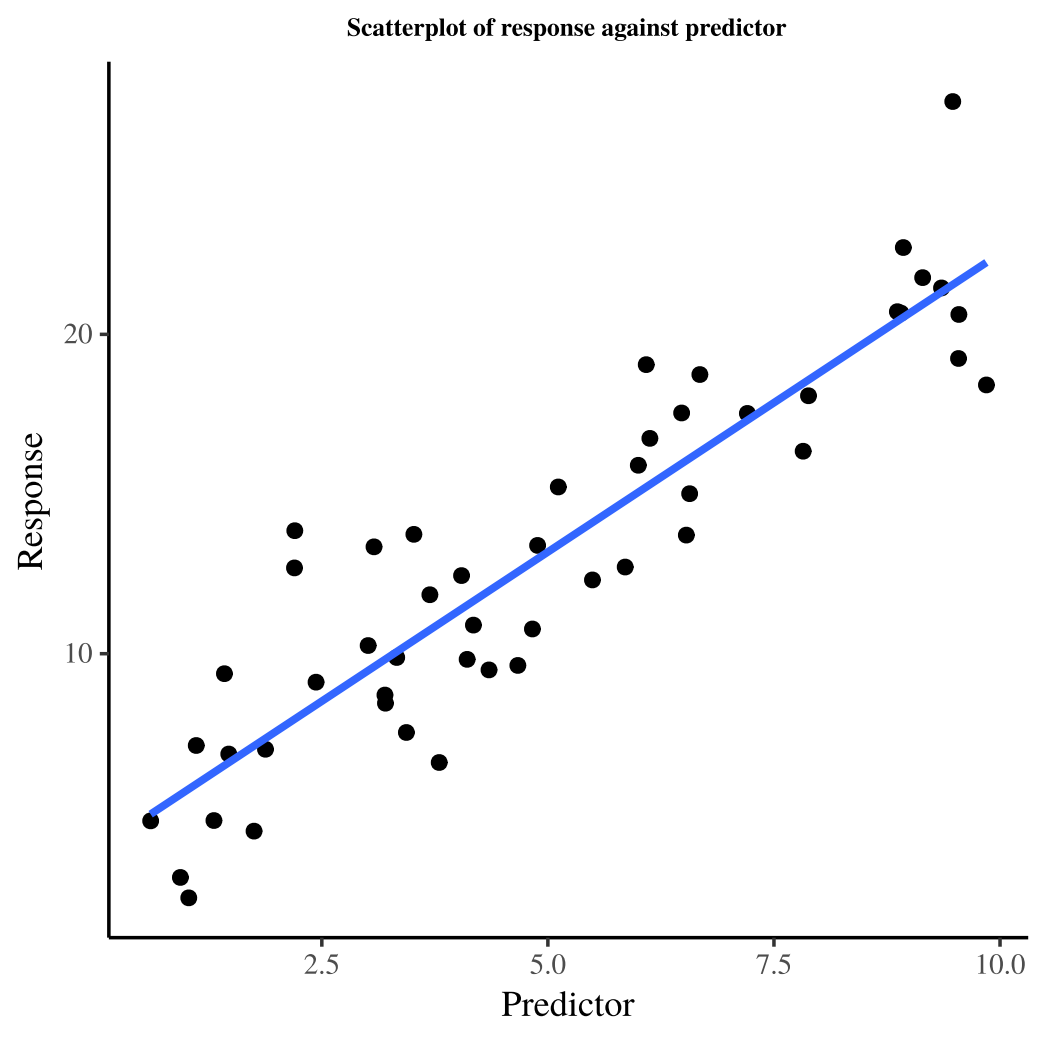
SLR assumptions met if we see a "null" plot:

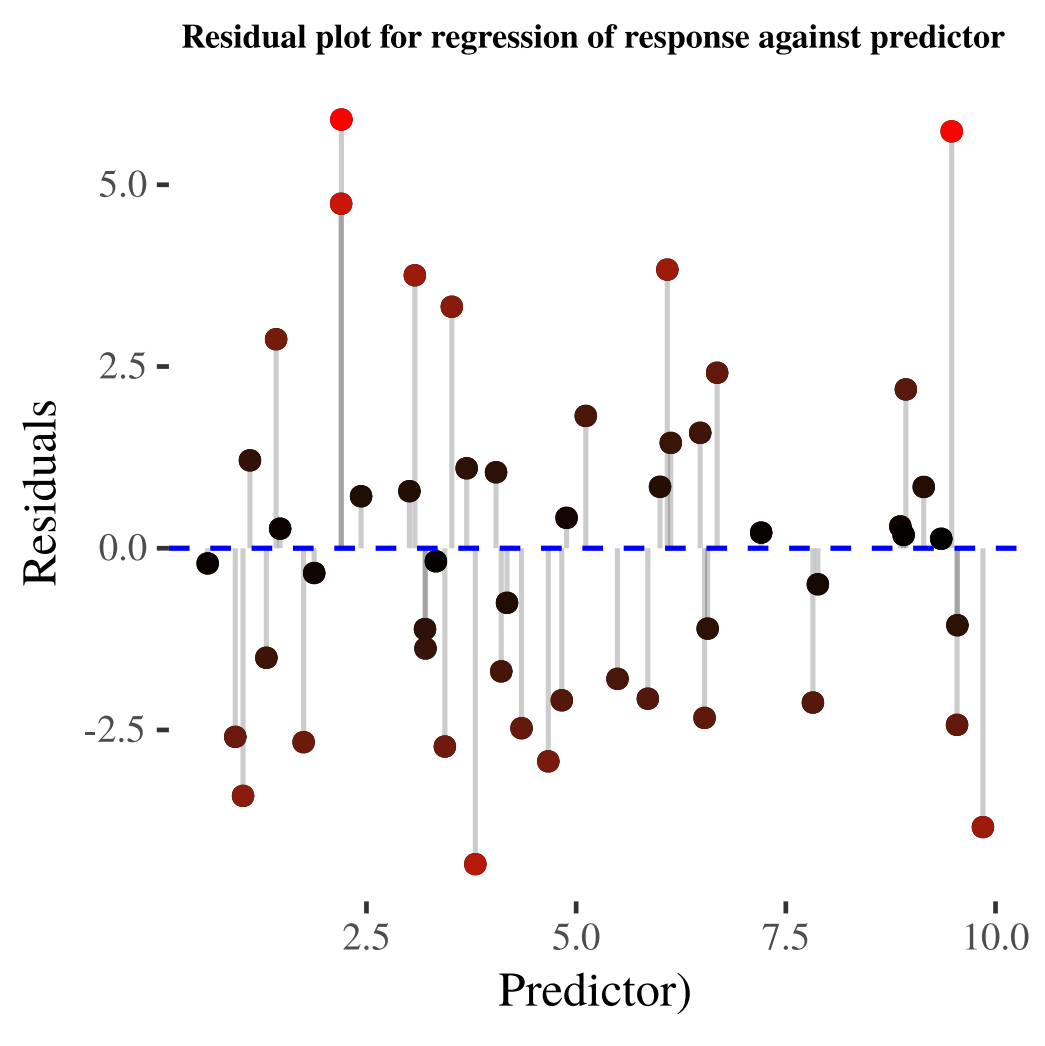
1. **Linearity met:** similar scatter above and below the 0-line, for all values of x
2. **Equal/Constant variance met:** magnitude of residual values around the 0-line doesn't depend on x

Watch out for:

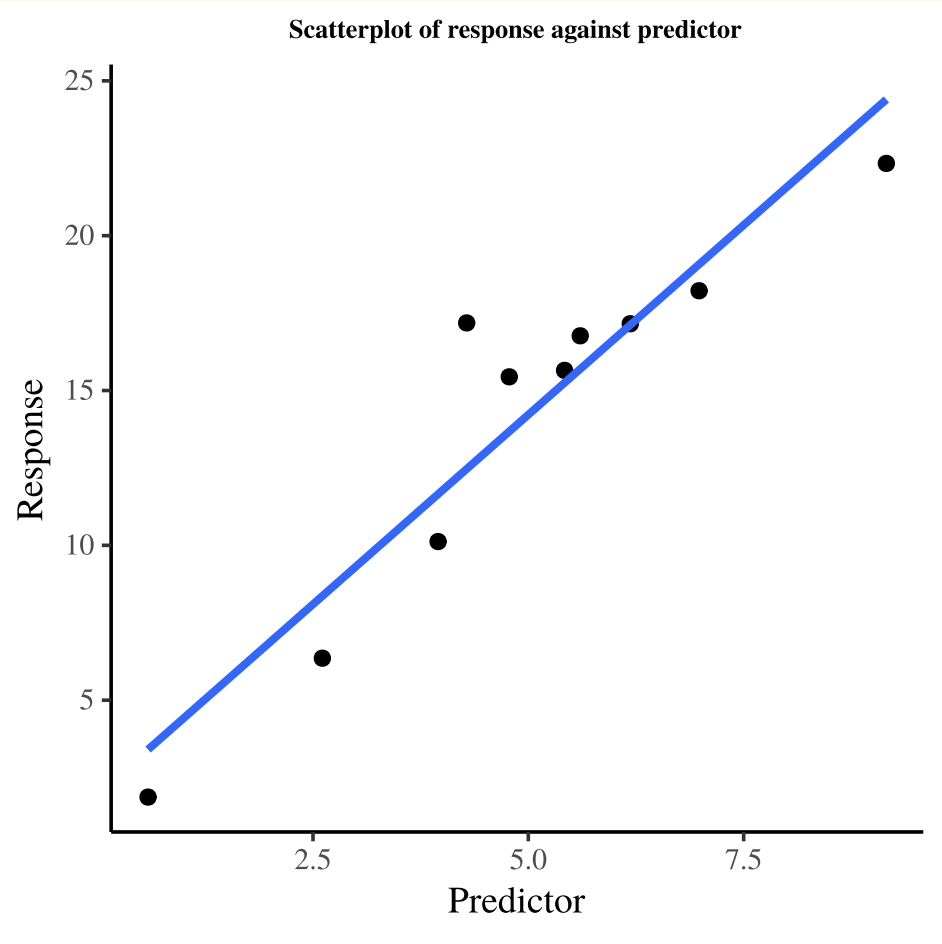
- **Curvature** - indicates mean function may not be linear
- **Fan shapes** - indicates nonconstant variance
- **Outliers** (may have large effect on estimates)

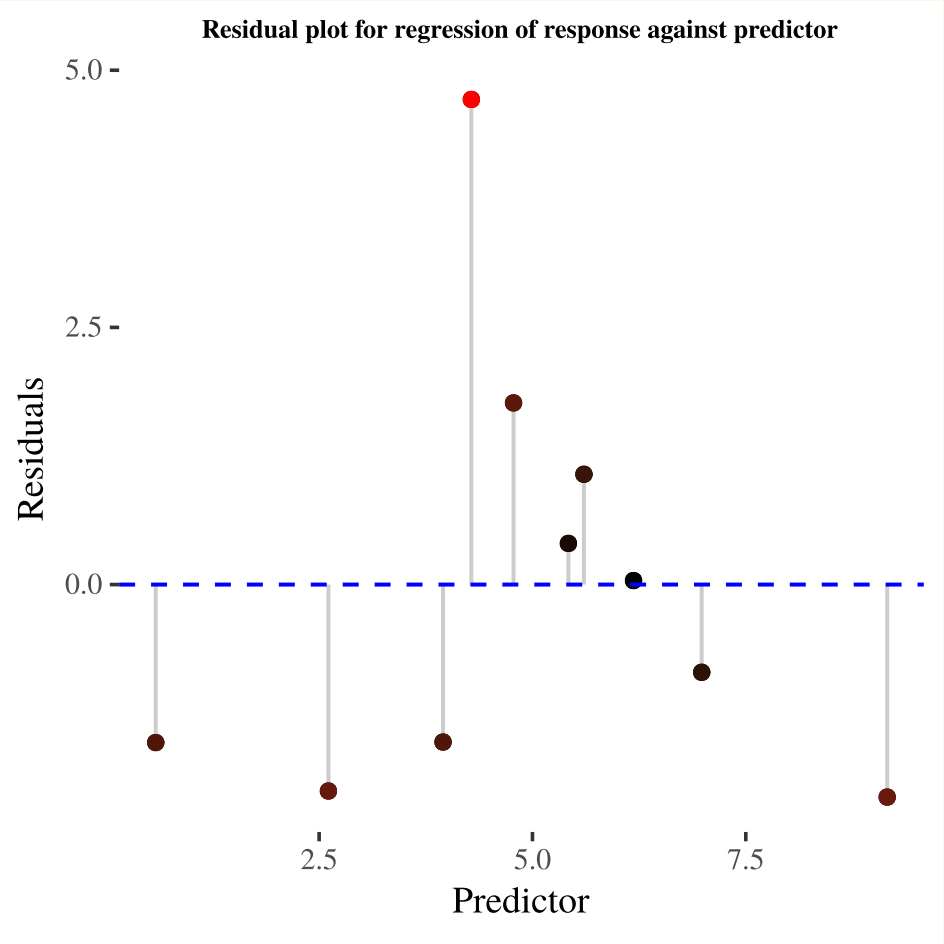
Residual plot: null plot





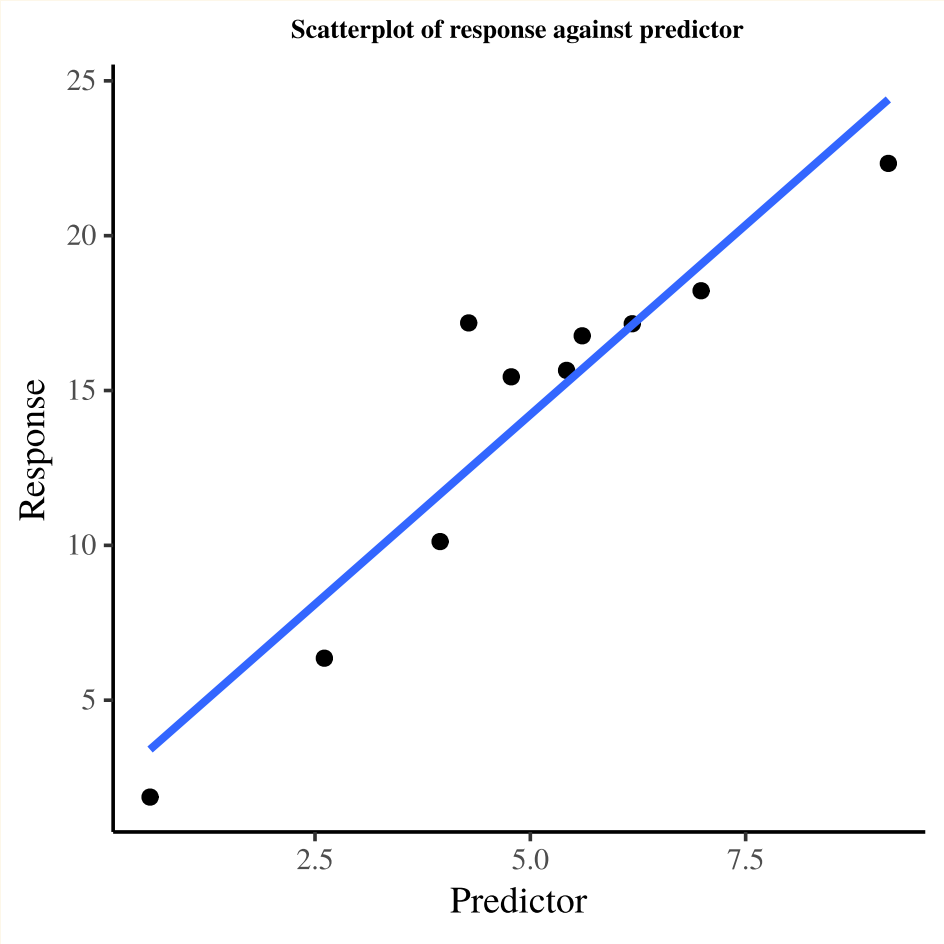
Residual plot: null plot (n=10)

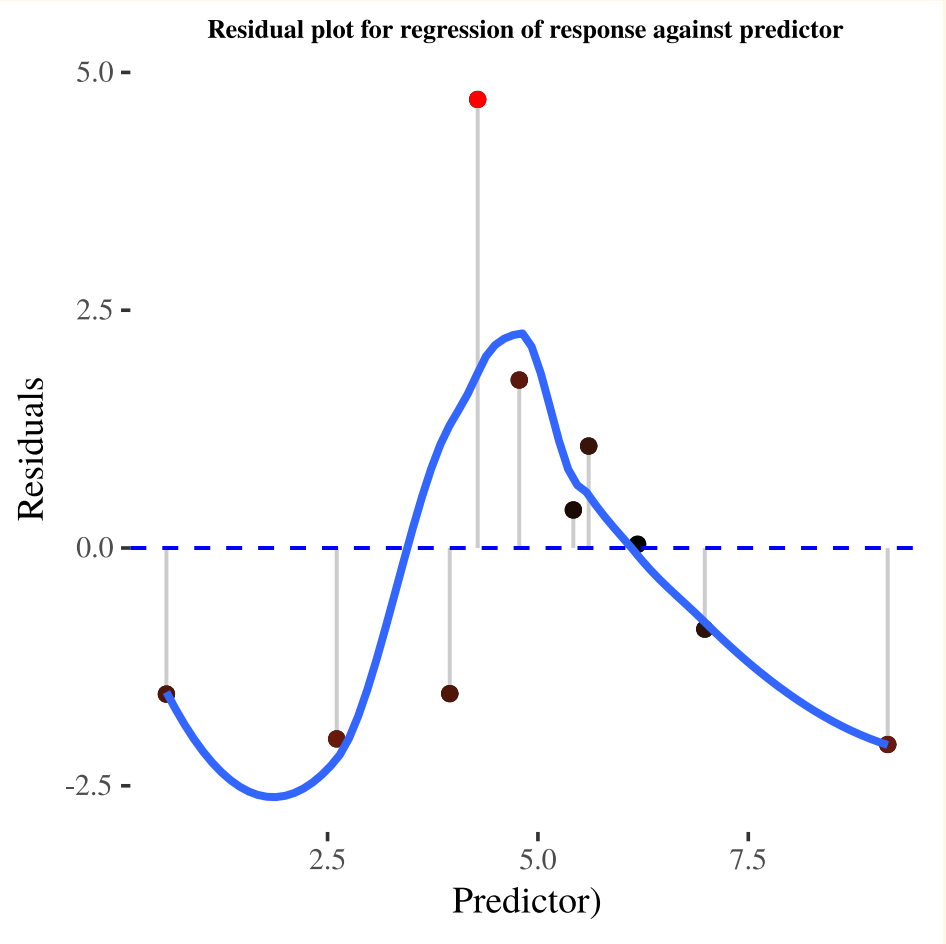




Warning: residual plots can be hard to interpret if you don't have a lot of data!!

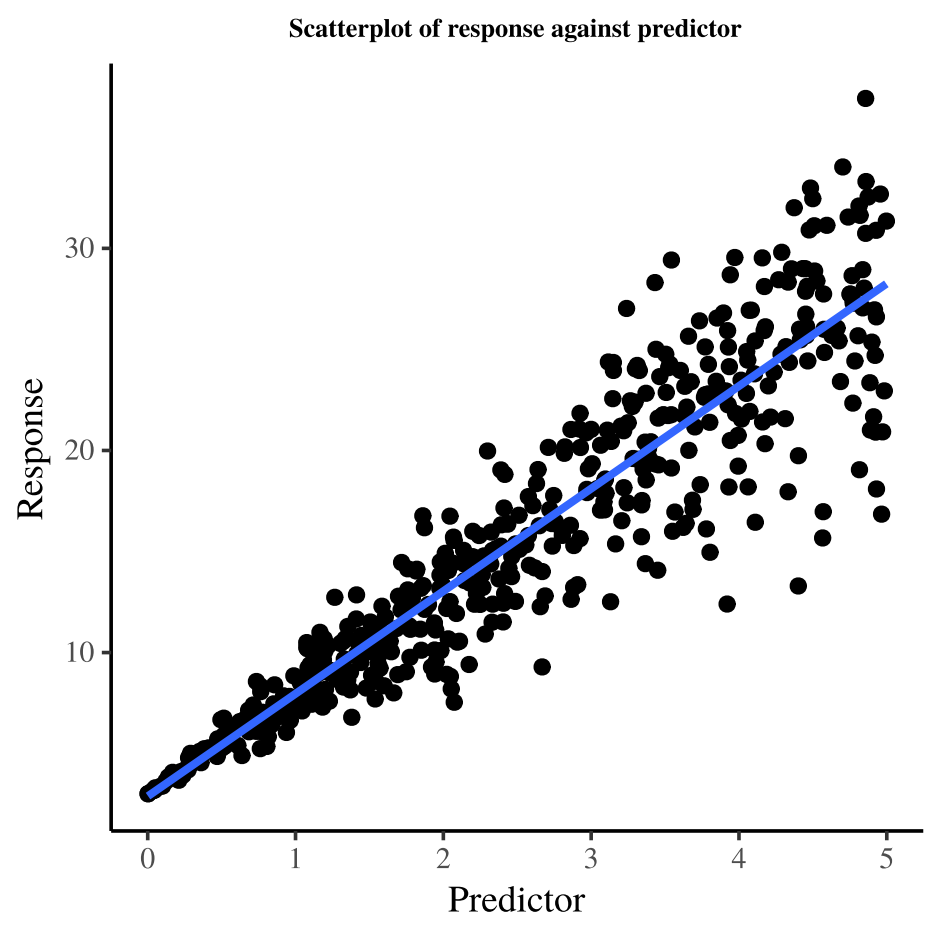
Residual plot: null plot (n=10)

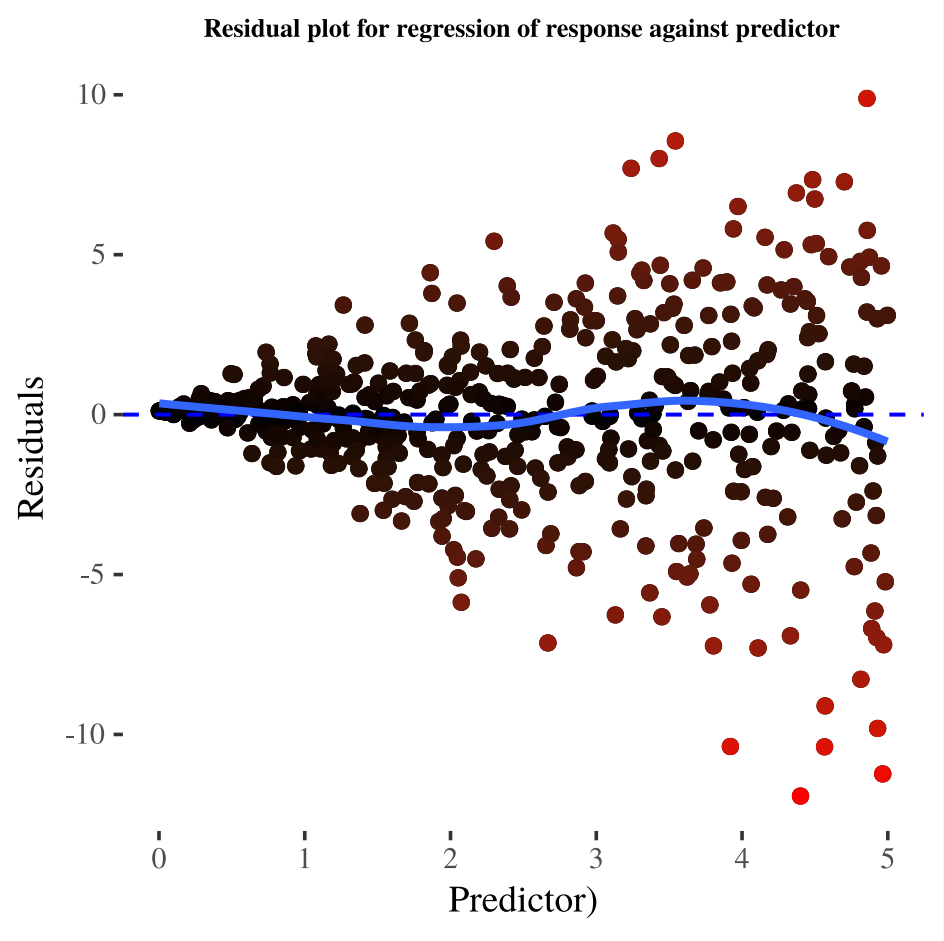




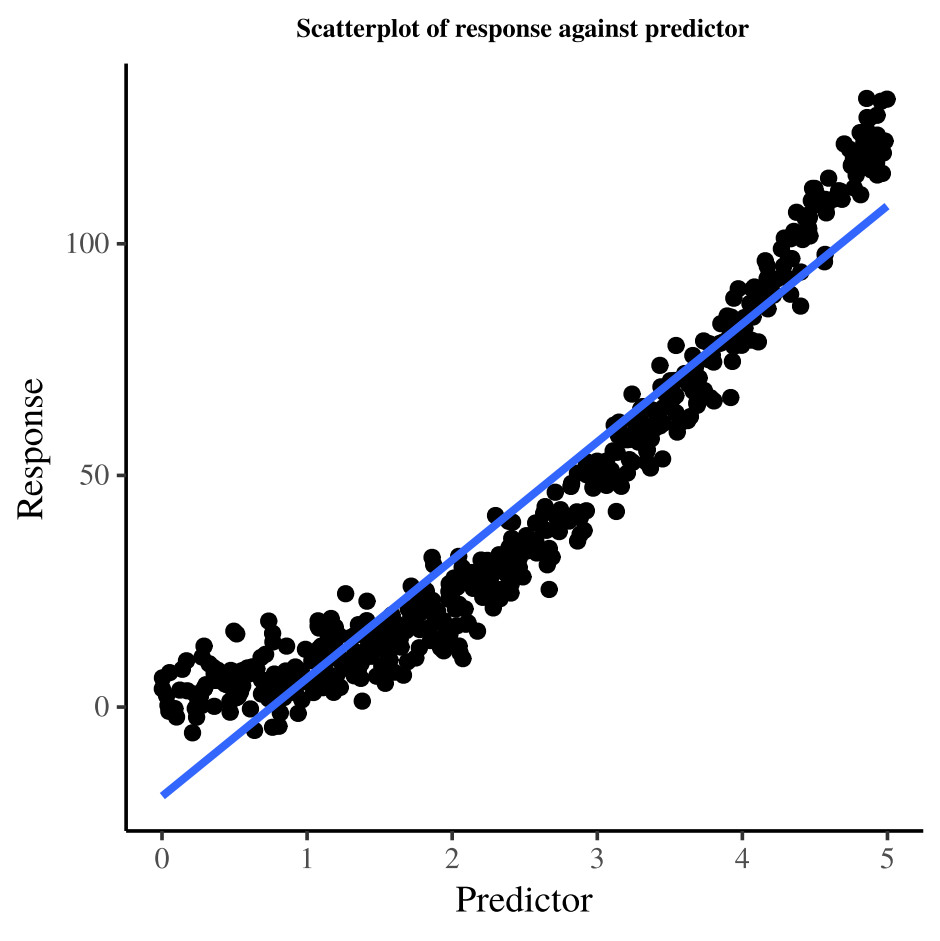
Smoothers are very sensitive to individual points when data is sparse

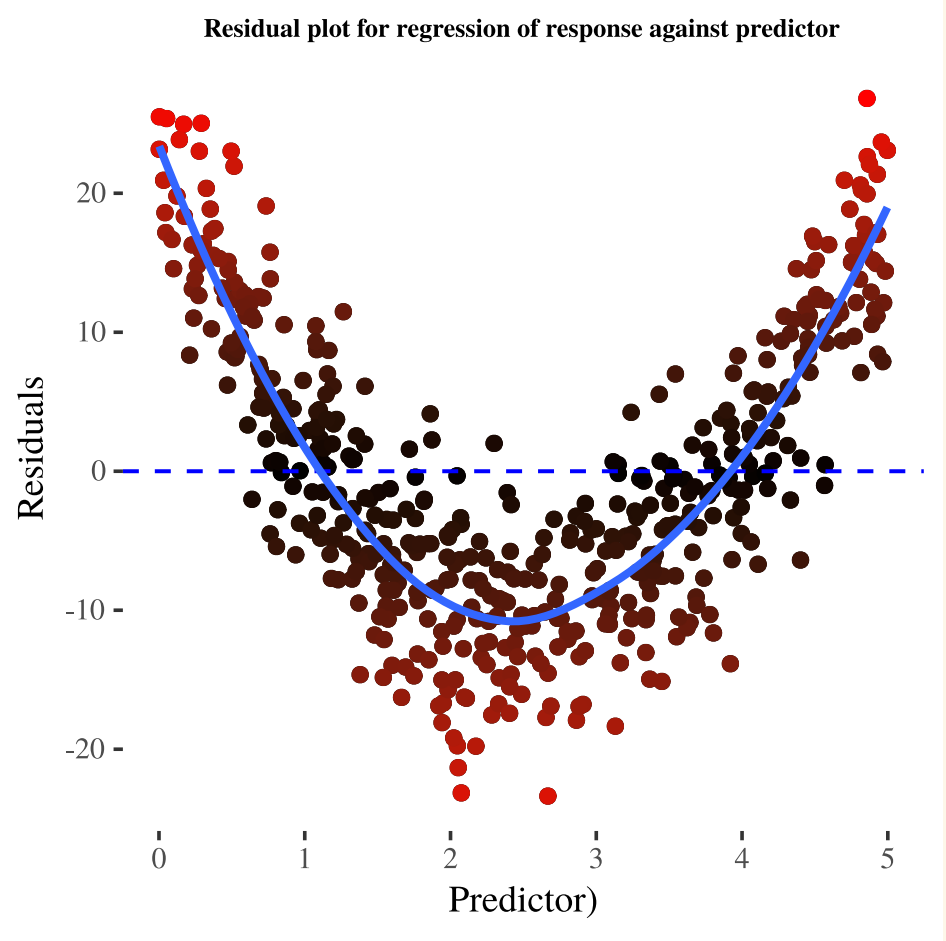
Violation 1: Guess the assumption violation ...



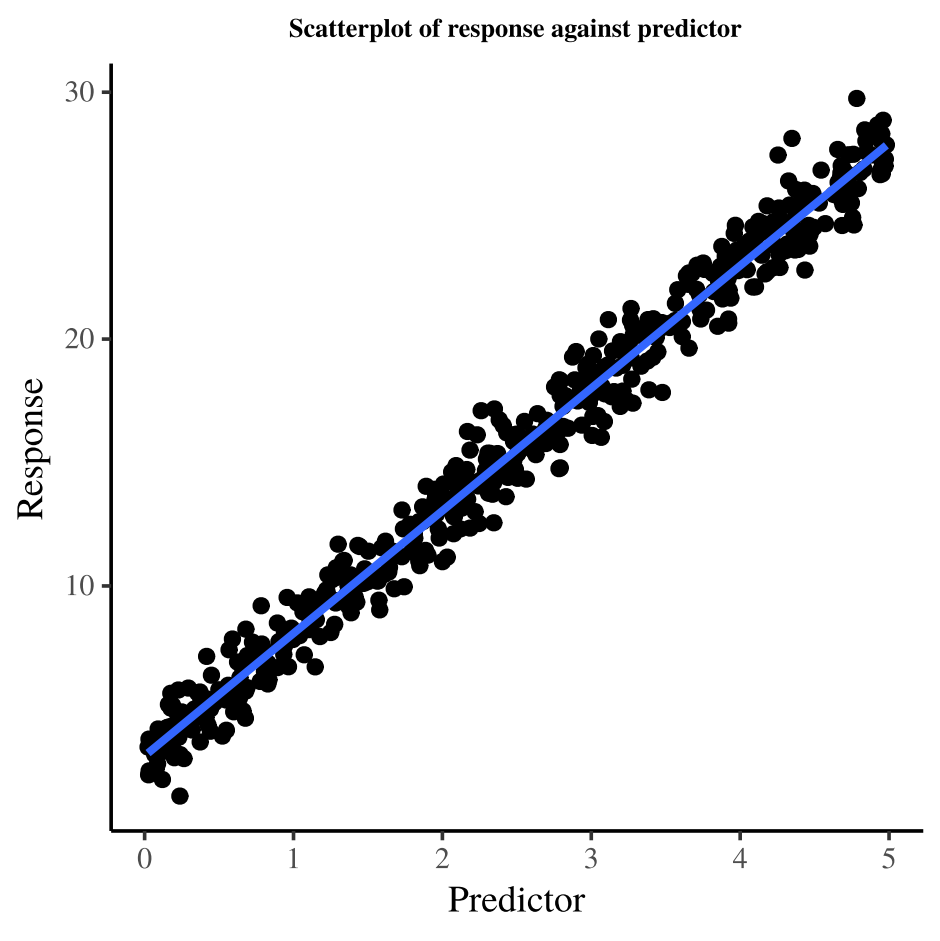


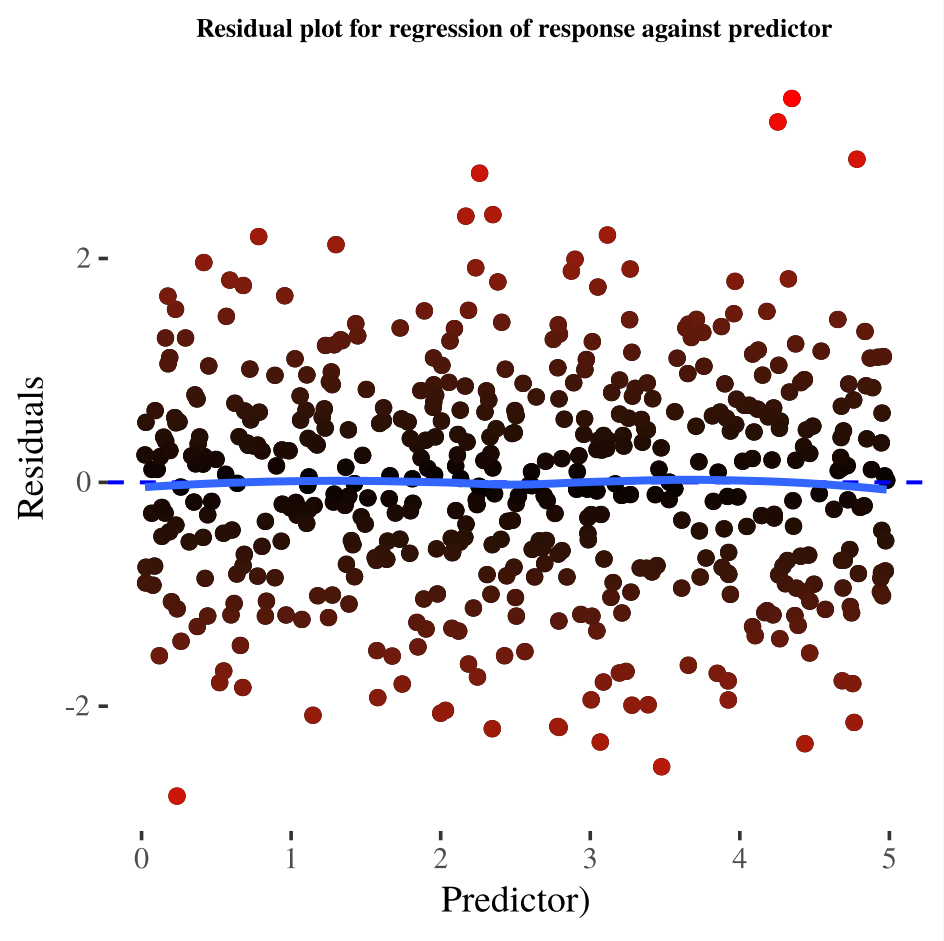
Violation 2: Guess the assumption violation ...





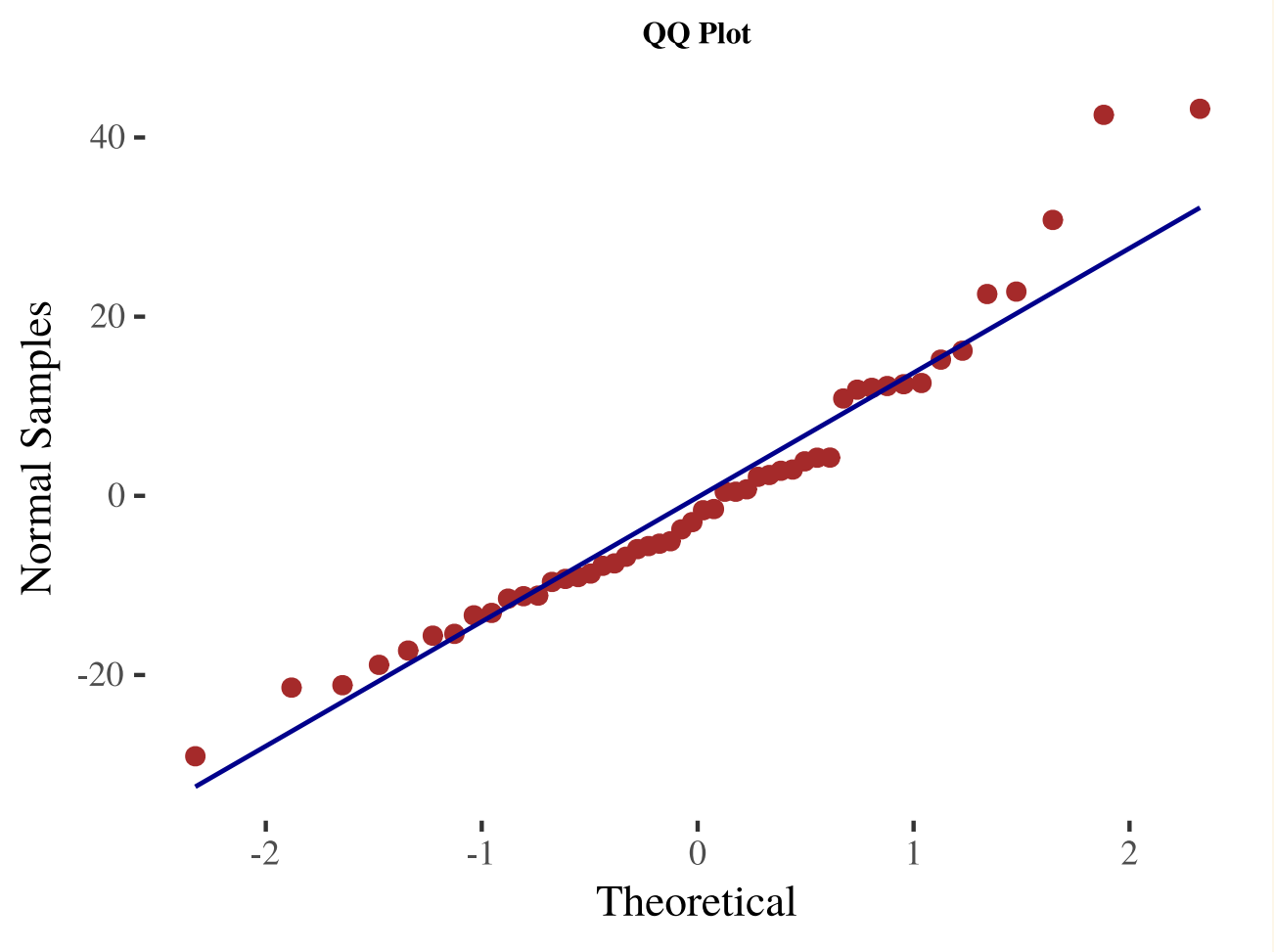
Good model


Scatterplot of response against predictor
Response
Predictor
30
20
10
0
1
2
3
4
5

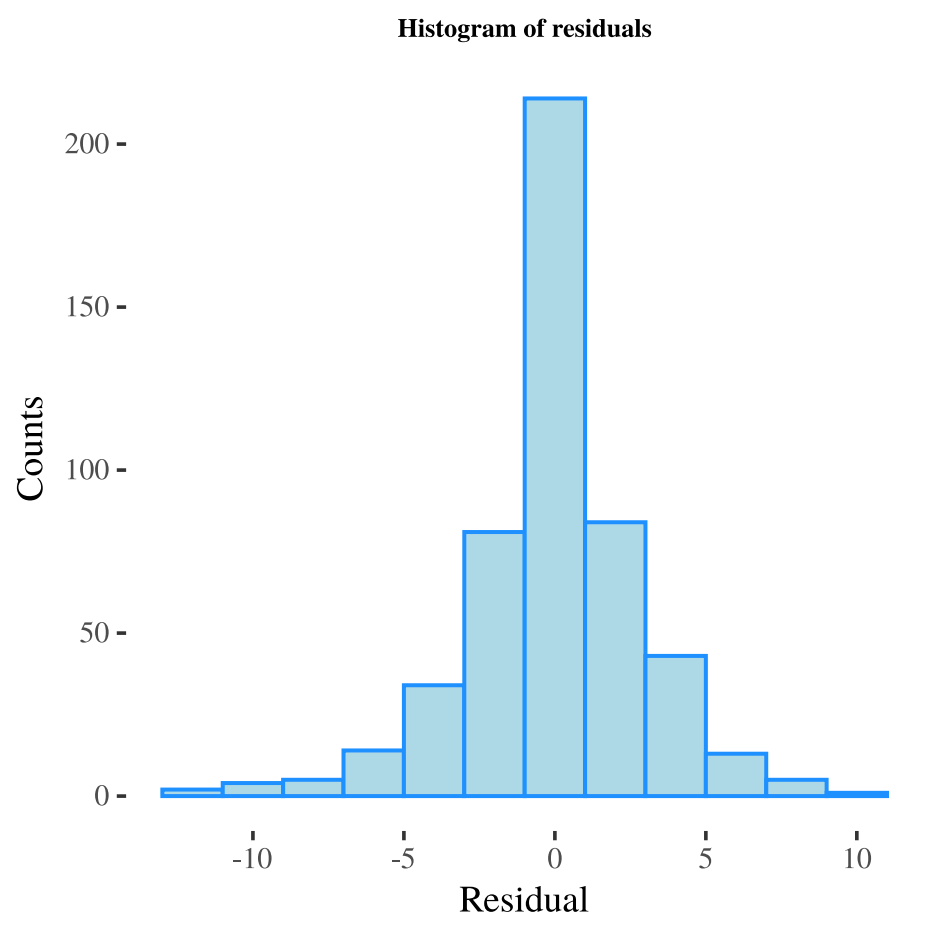

Residual plot for regression of response against predictor
Residuals
Predictor)
2
0
-2
0
1
2
3
4
5

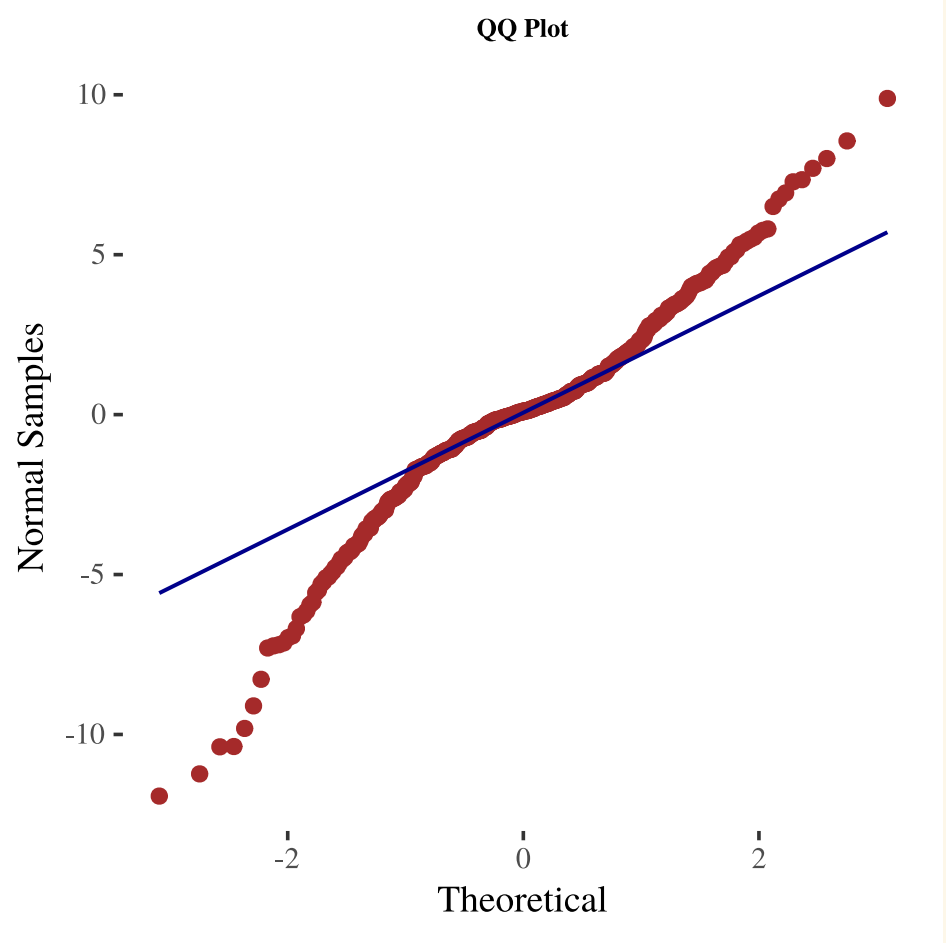
Q-Q Plot: Cars dataset

```
ggplot(regression_points, aes(sample = residual)) +
 stat_qq(col = "brown") +
  stat_qq_line(col = "darkblue")+
  labs(
    title = "QQ Plot",
    x = "Theoretical",
    y = "Normal Samples") +
  theme(plot.title = element_text(hjust=0.5, size=7
```

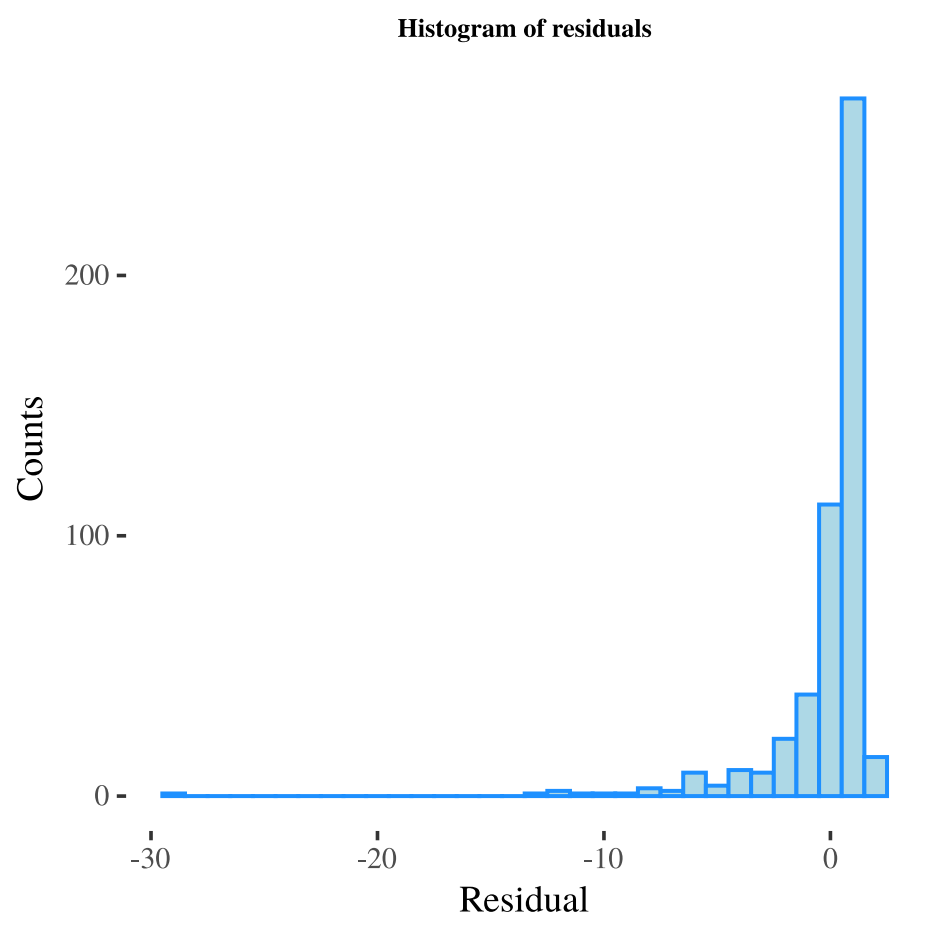


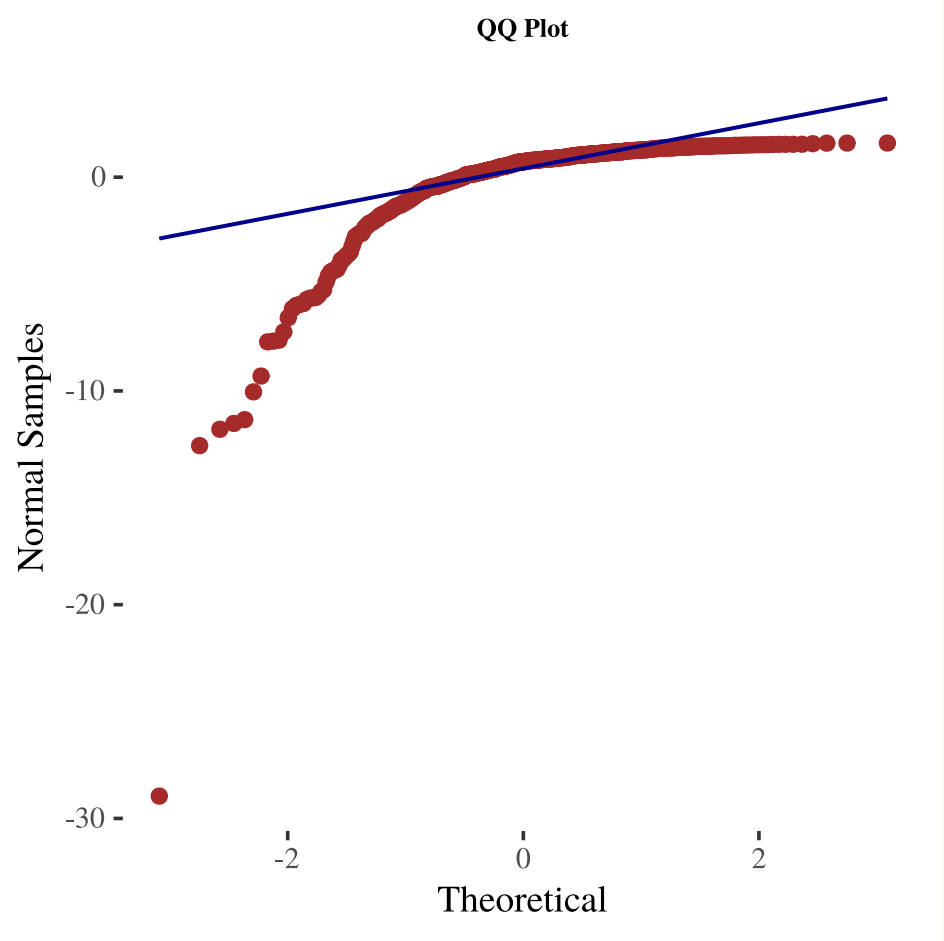
Normal Q-Q plot: Potential violations





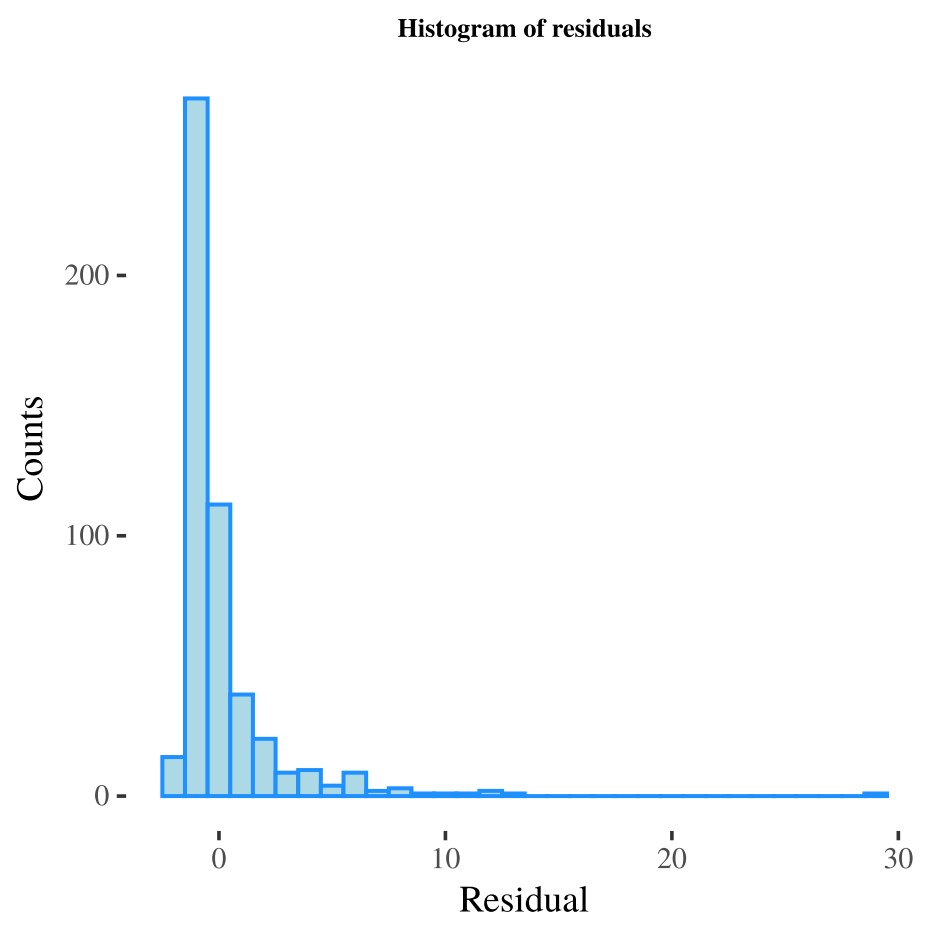
Normal Q-Q plot: Potential violations

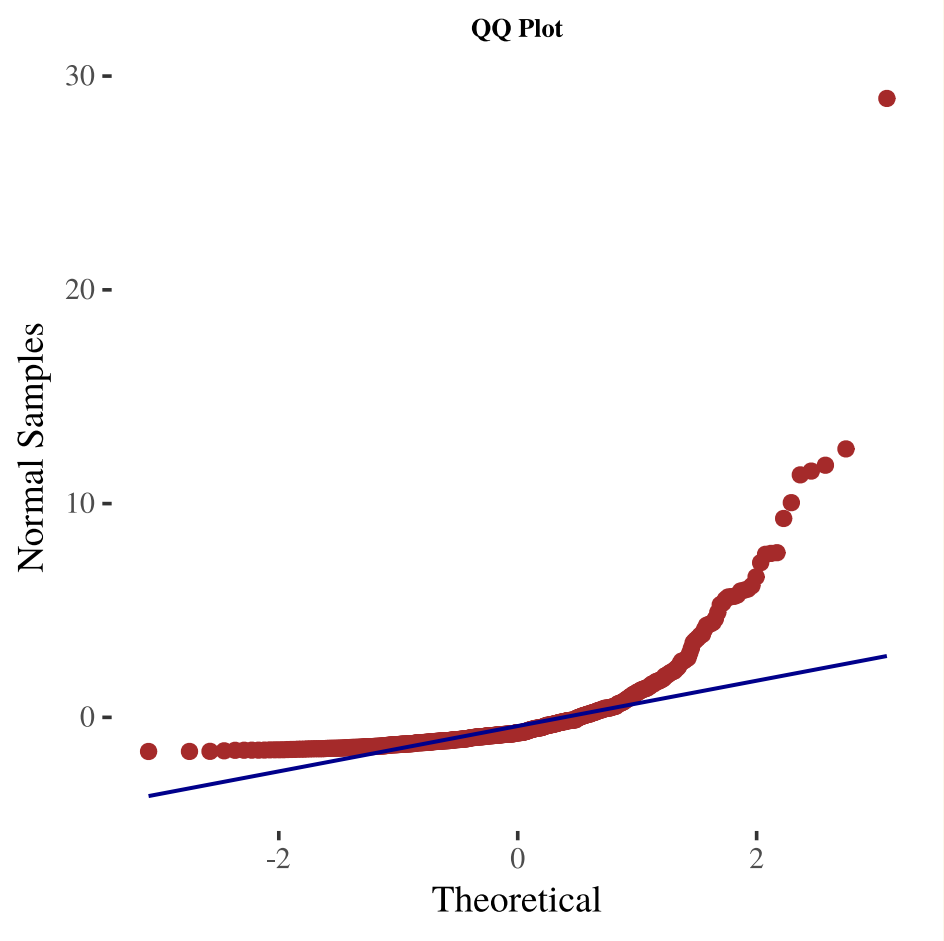




Concave down means left skewed!

Normal Q-Q plot: Potential violations

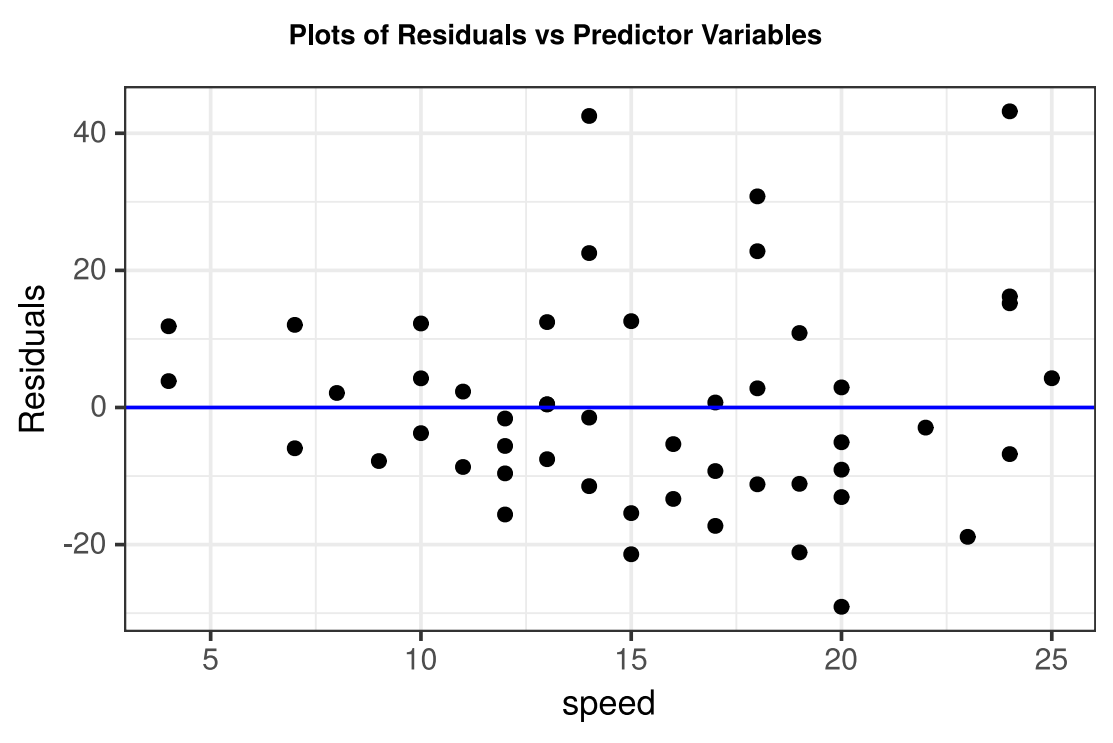




Concave up means right skewed!

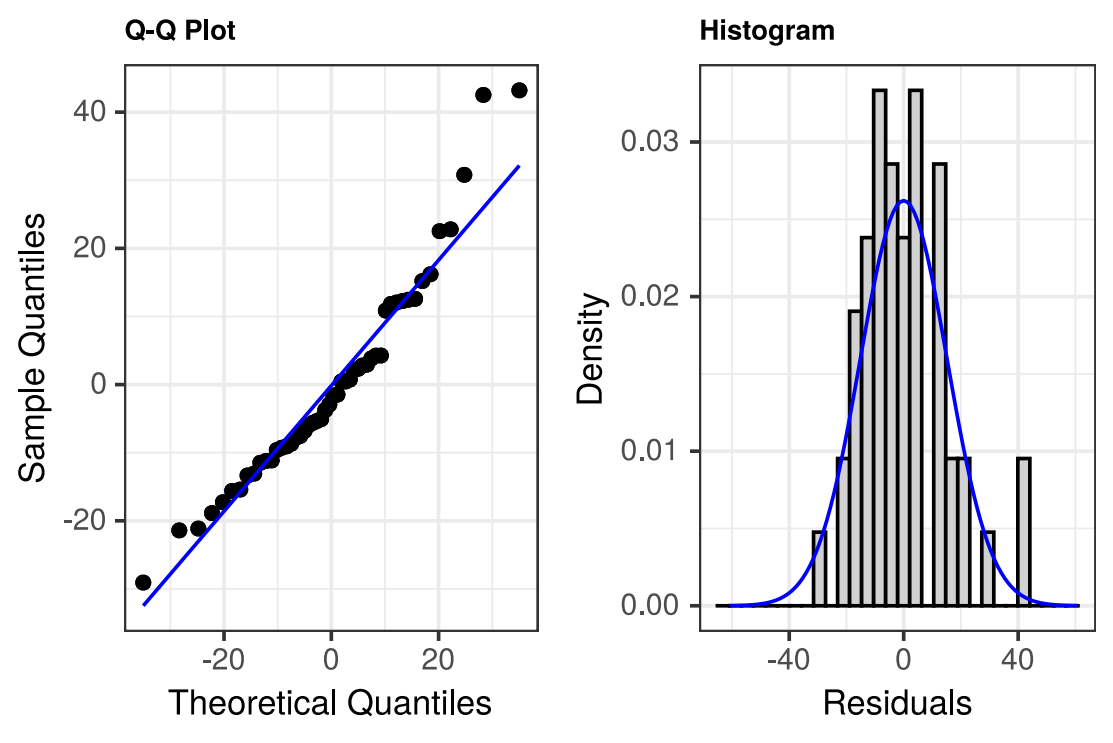
Diagnostics using ggResidpanel package

```
library(ggResidpanel) # need to install if you aren't using maize
resid_xpanel(cars_lm, axis.text.size = 10, title.text.size = 8, scale = 1) # residual against
```



Diagnostics using ggResidpanel package

```
# QQ plot and histogram of residuals
resid_panel(cars_lm, plots = c("qq", "hist"), axis.text.size = 10, title.text.size = 8, scale
```



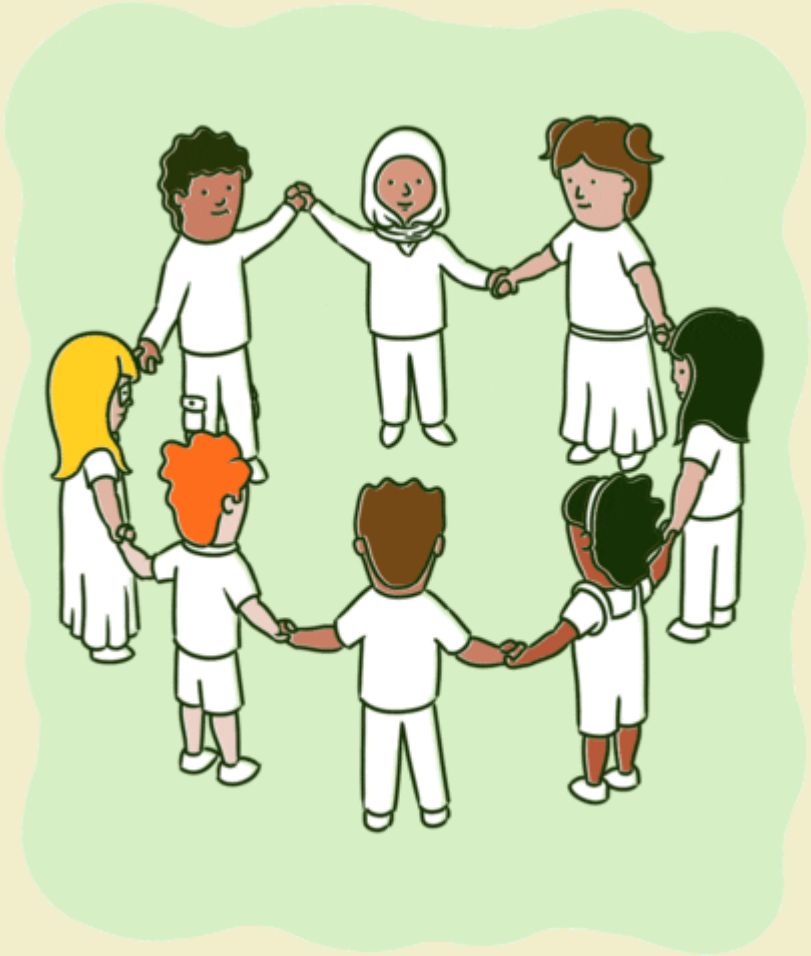
Assessing independence

Can only verified through an understanding of how the data was collected

- do measurements vary over time?
- do measurements vary over a space (geographic region)?
- are measurements naturally clustered together?
- If the answer is "yes", then try plotting residuals against any variables that measure these attributes (time/space/clustering)

Your Turn 1

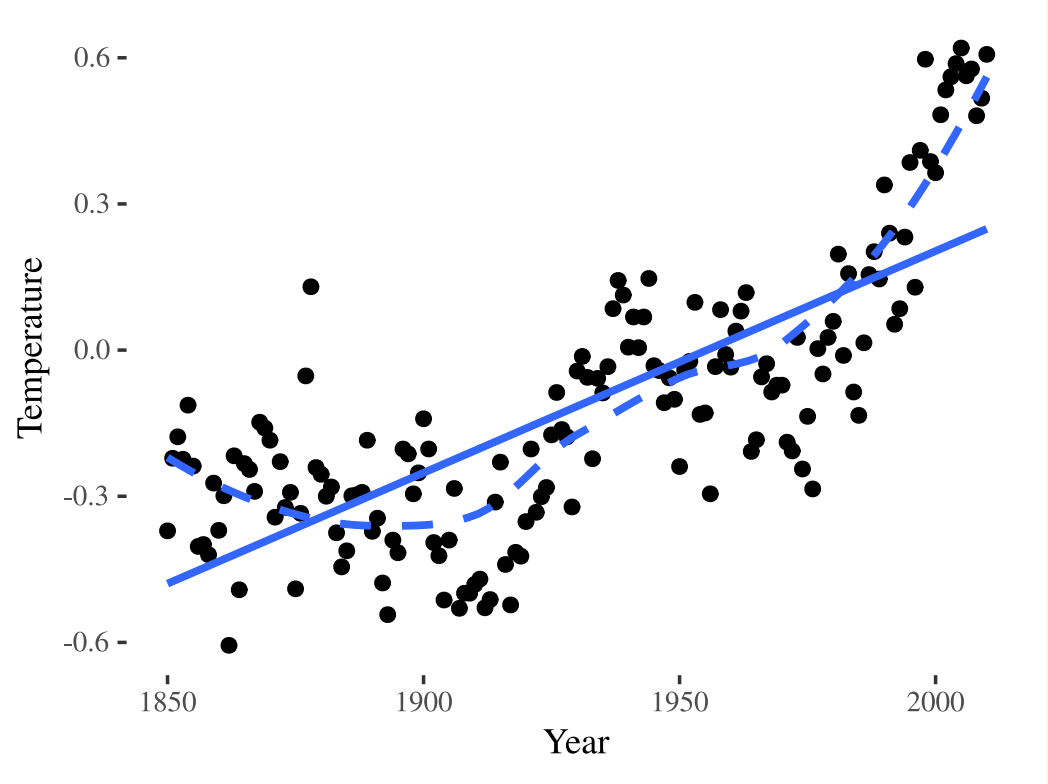
05:00



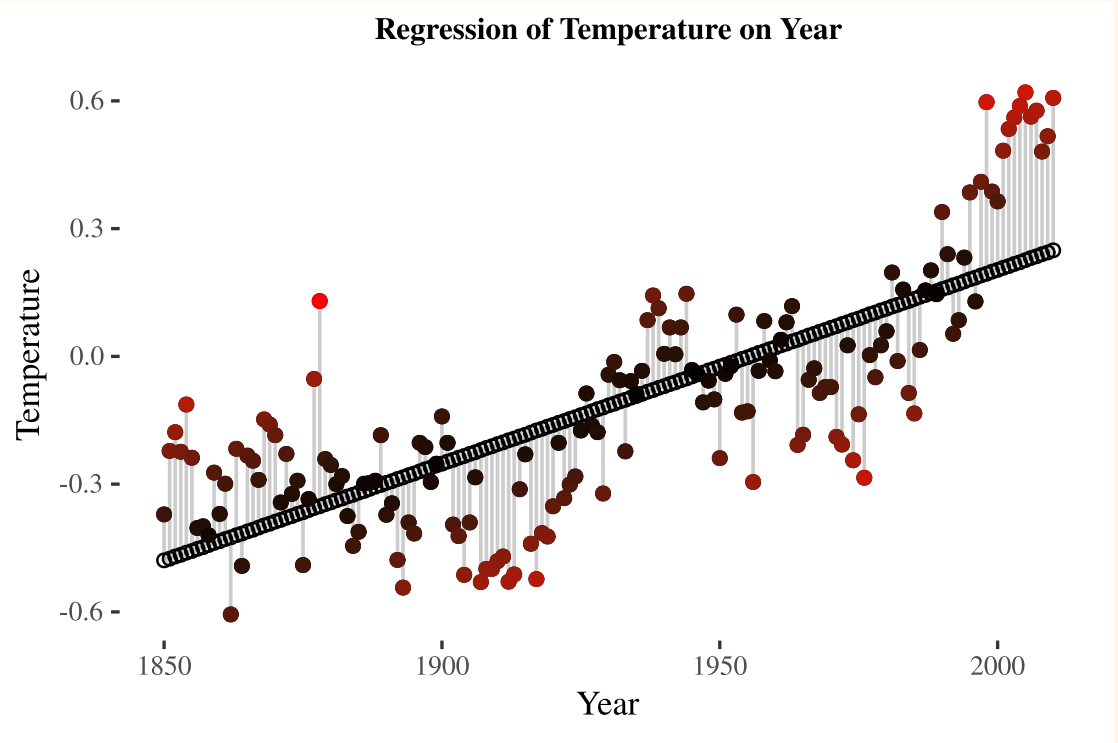
- Get the in class activity file from moodle
- We will do a case study of global warming
- Please skim through the activity .Rmd file in your group

Case Study 15.2 Global Warming

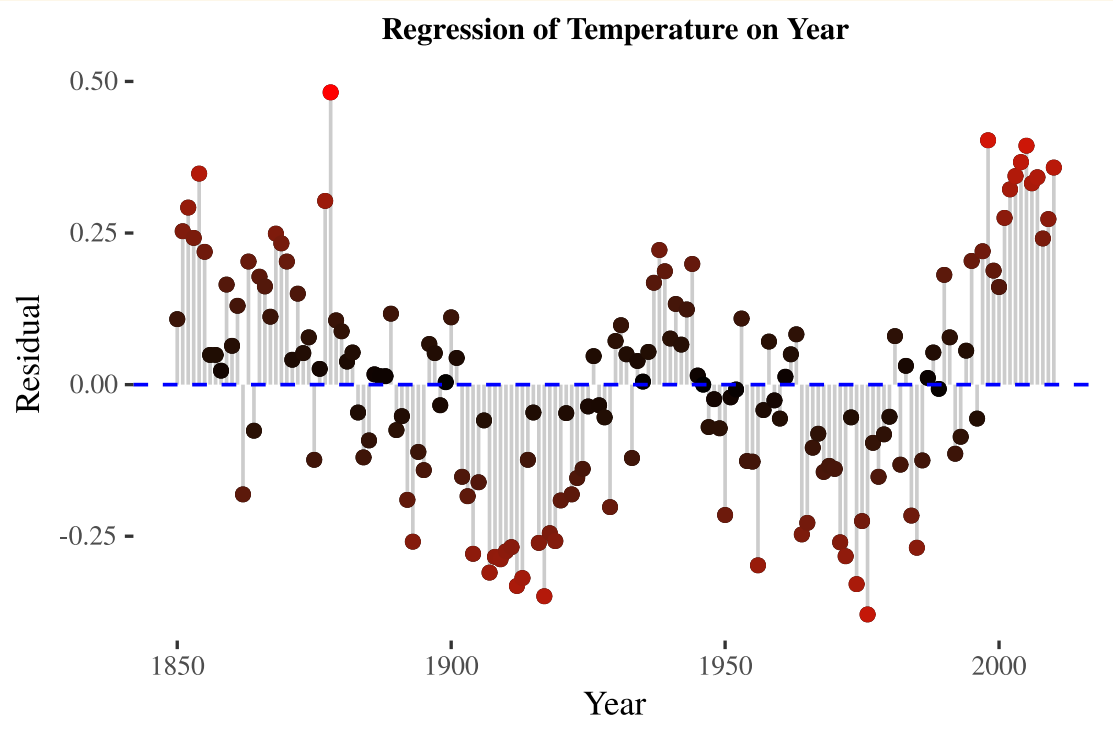
- $y =$ annual mean global temp
- $x =$ year
- $n = 161$ years from 1850 to 2010
- Model: SLR of global temp on year



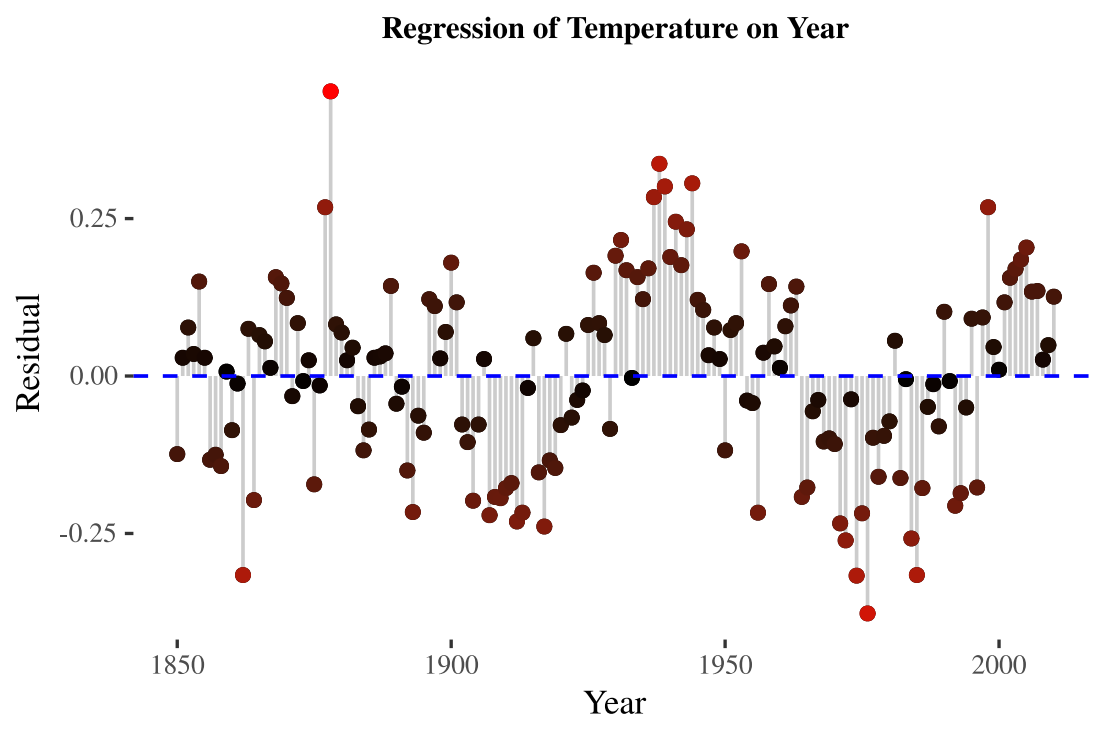
Case Study 15.2 Global Warming



Residual plot for SLR of global temp on year



Residual plot for regression of global temp on $year$ and $year^2$



- Residuals that are 1 year apart have a(n) (auto)correlation coefficient of 0.59!
- Even after accounting for year, temps are still correlated. A more complex model is needed!

How “robust” is regression against violation of the assumptions?

Robust = can violate an assumption and still get valid inference results

SLR models are robust against violations of

- **Normality:** the t-tests and CI for model parameters and the mean response are saved by the Central Limit Theorem when n is large, even if your subpopulation of responses are not normally distributed.

How “robust” is regression against violation of the assumptions?

SLR models are not robust against violations of

- **Linearity:** if the mean function is wrong then your estimated effects, mean response, or predicted response will be biased!
- **Equal/Constant variance and independence:** if you are not correctly modeling your response variability, then your SEs will not be an accurate reflection of your actual uncertainty (meaning CIs/tests might be misleading)
- **Normality when computing prediction intervals:** these intervals need the normal subpopulation assumption to hold

What if model assumptions are violated?

Start by finding the correct mean function form!

- **Linearity, Variance, Normality:** Transform one or both variables
- **Linearity:** change mean function, use non-linear regression
- **Variance:** weighted regression, “robust SEs"
- **Independence:** use a mixed-effects model (Stat 330), times series (Stat 320) or spatial regression models