Hypothesis Tests and Confidence Intervals using Normal Distribution!

Stat 120

May 02 2023

How do Malaria parasites impact mosquito behavior?

Experiment Design

- ***Mosquitoes are exposed to two groups of mice:***
    - ***Malaria-infected mice (experimental group)***
    - ***Healthy mice (control group)***
- ***Malaria-infected mice go through two stages of infection:***
    - ***Stage 1**: Not yet infectious (Days 1-8)*
    - ***Stage 2**: Infectious (Days 9-28)*
- ***Response Variable**: Whether the mosquito approaches a human.*

Research Questions: Does mosquito behavior differ when exposed to malaria-infected mice compared to healthy mice? Does mosquito behavior differ based on the infection stage of the malaria-infected mice?

Cator LJ, George J, Blanford S, Murdock CC, Baker TC, Read AF, Thomas MB. (2013). 'Manipulation' without the parasite: altered

Malaria Parasites and Mosquitoes

Malaria parasites would benefit if

- *Mosquitoes approached humans **less often** after being exposed, but before becoming infectious, because humans are risky*
- *Mosquitoes approached humans **more often** after becoming infectious, to pass on the infection*

Days 1-8

We'll first look at the mosquitoes before they become infectious (days 1-8).

p_C : ***proportion of controls to approach human***

p_E : ***proportion of exposed to approach human***

What are the relevant hypotheses?

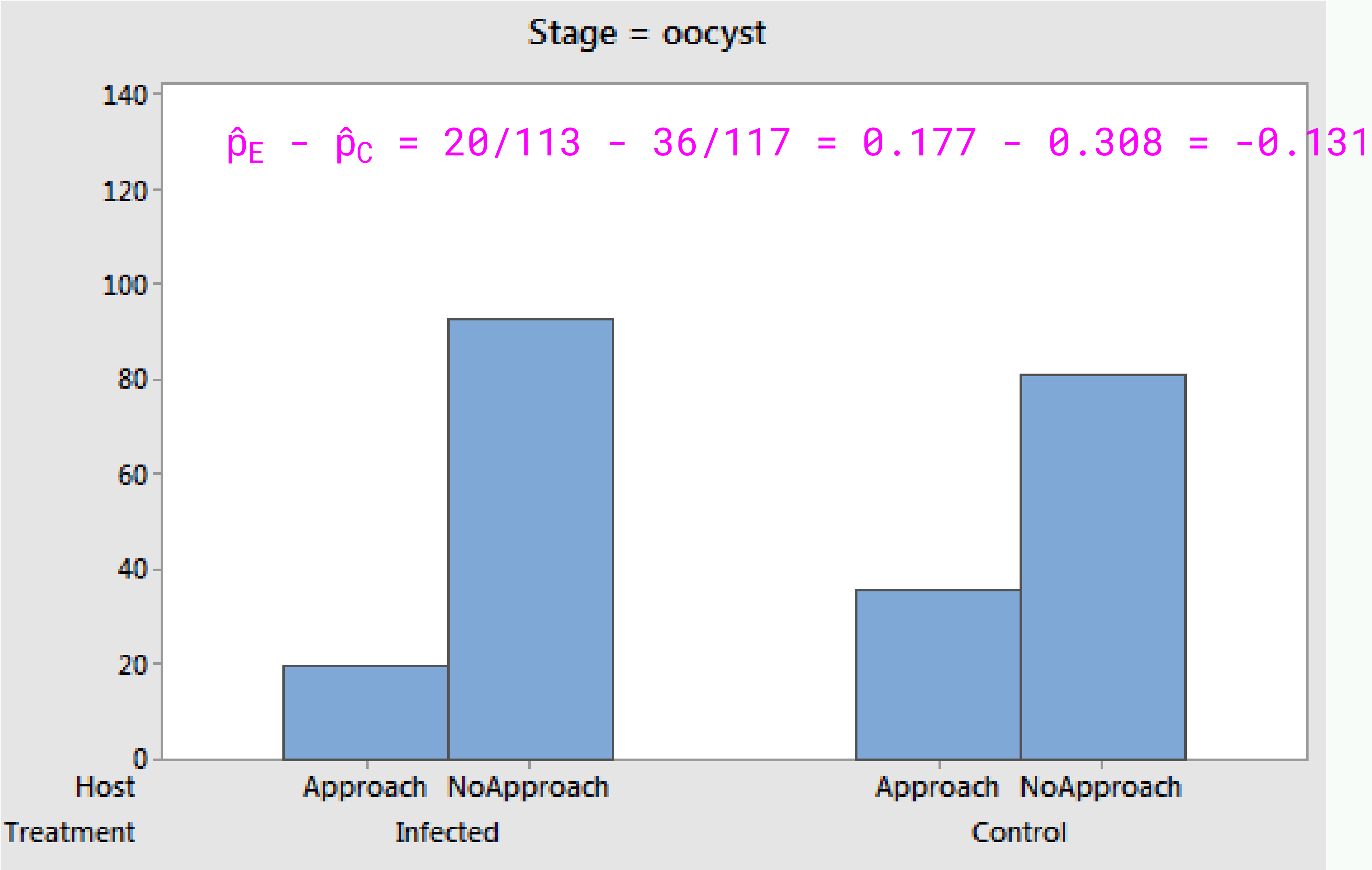
A. $\mathrm{H}_0 : p_\mathrm{E} = p_\mathrm{C}, \mathrm{H_a} : p_\mathrm{E} < p_\mathrm{C}$

B. $\mathrm{H}_0 : p_\mathrm{E} = p_\mathrm{C}, \mathrm{H_a} : p_\mathrm{E} > p_\mathrm{C}$

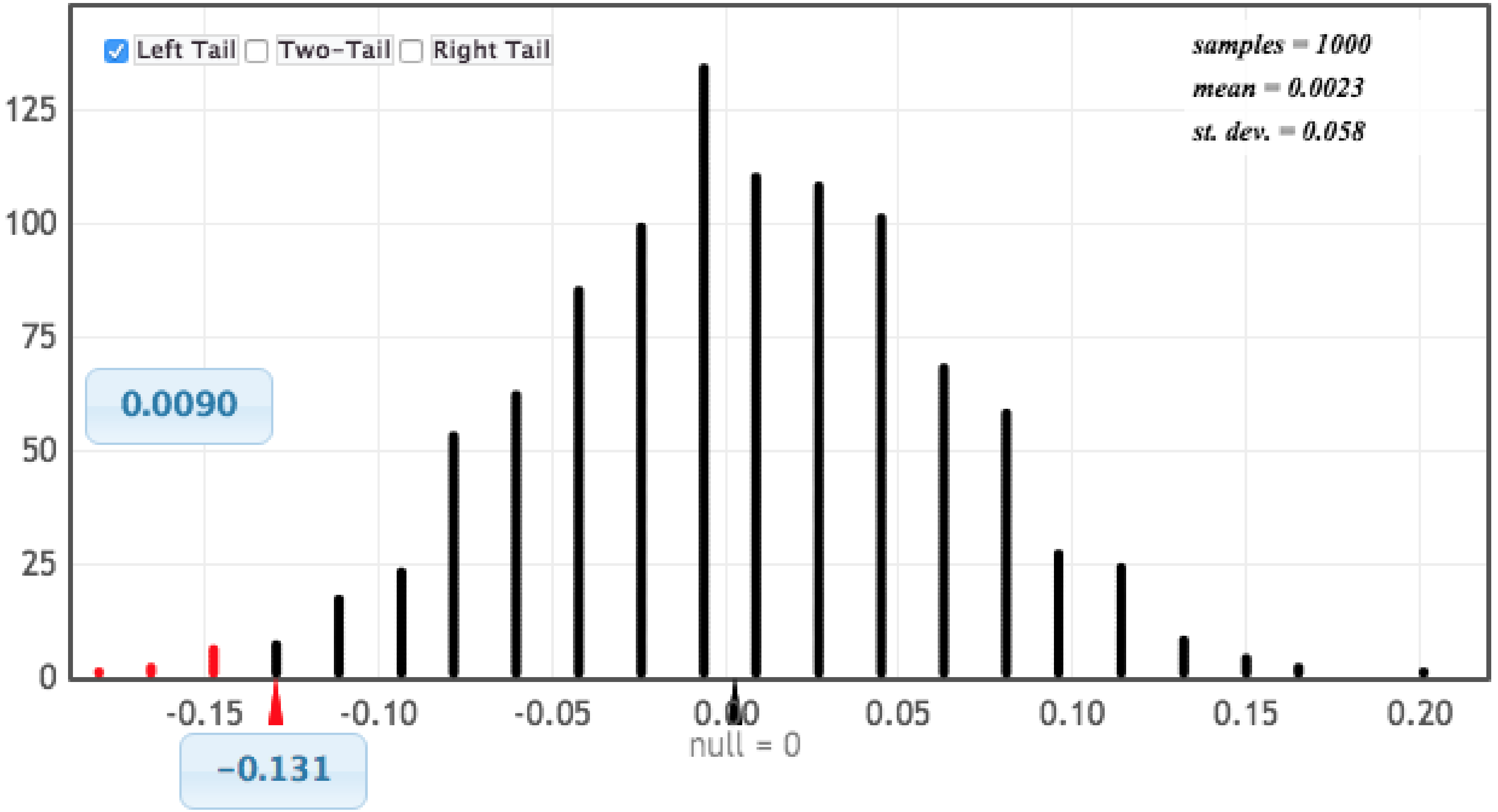
C. $\mathrm{H}_0 : p_\mathrm{E} < p_\mathrm{C}, \mathrm{H_a} : p_\mathrm{E} = p_\mathrm{C}$

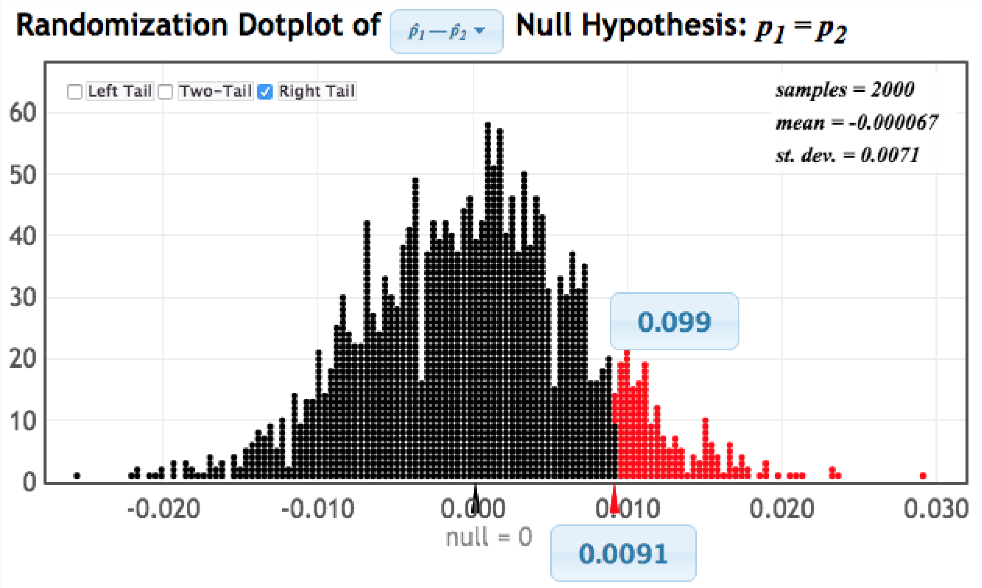
D. $H_0 : p_\mathrm{E} > p_\mathrm{C}, \mathrm{H_a} : p_\mathrm{E} = p_\mathrm{C}$

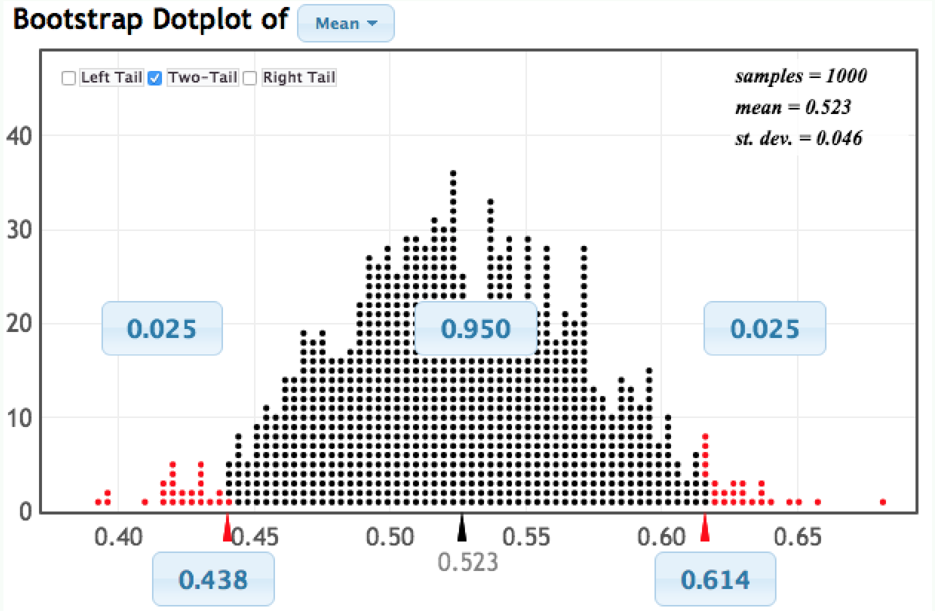
▶ Click for answer

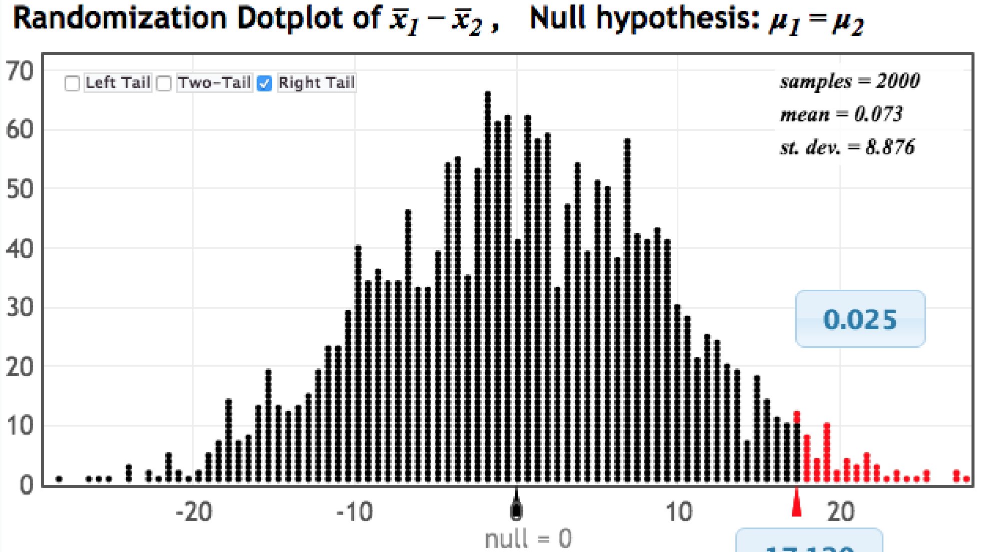

Stage = oocyst
p̂E - p̂C = 20/113 - 36/117 = 0.177 - 0.308 = -0.131
140
120
100
80
60
40
20
0
Host
Approach
NoApproach
Approach
NoApproach
Treatment
Infected
Control

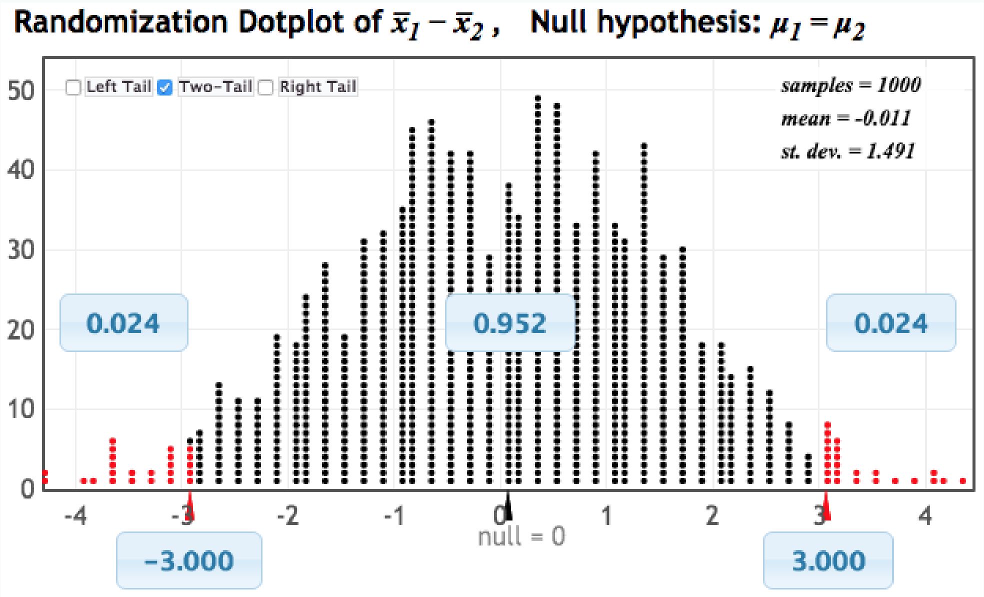
Randomization Dotplot of $\hat{p}_1 - \hat{p}_2$ Null Hypothesis: $p_1 = p_2$











What do you notice?

Central Limit Theorem (CLT)

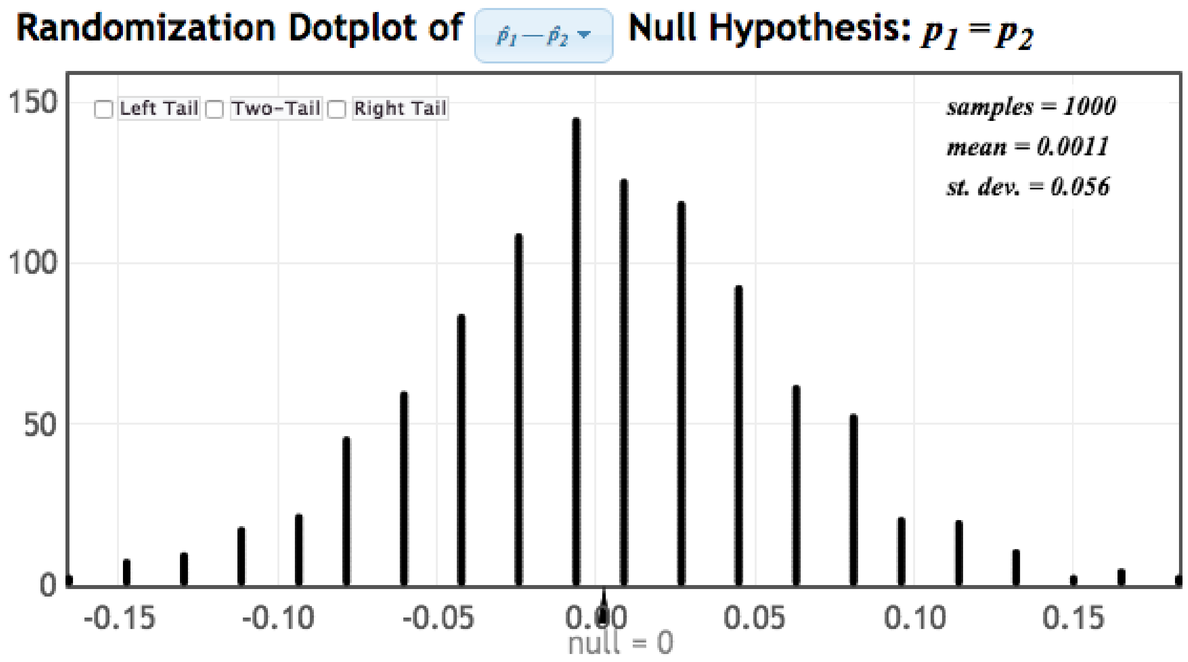
For random samples with a sufficiently large sample size, the distribution of sample statistics for a mean or a proportion is normally distributed

The catch: "sufficiently large sample size"

The more skewed the original distribution of data/population is, the larger n has to be for the CLT to work

- For quantitative variables that are not very skewed, $n \geq 30$ is usually sufficient
- For categorical variables, counts of at least 10 within each category is usually sufficient

Which normal distribution should we use to approximate this?



Original Sample

Group	Count	Sample Size	Proportion
Group 1	20	113	0.177
Group 2	36	117	0.308
Group 1–Group 2	-16	n/a	-0.131

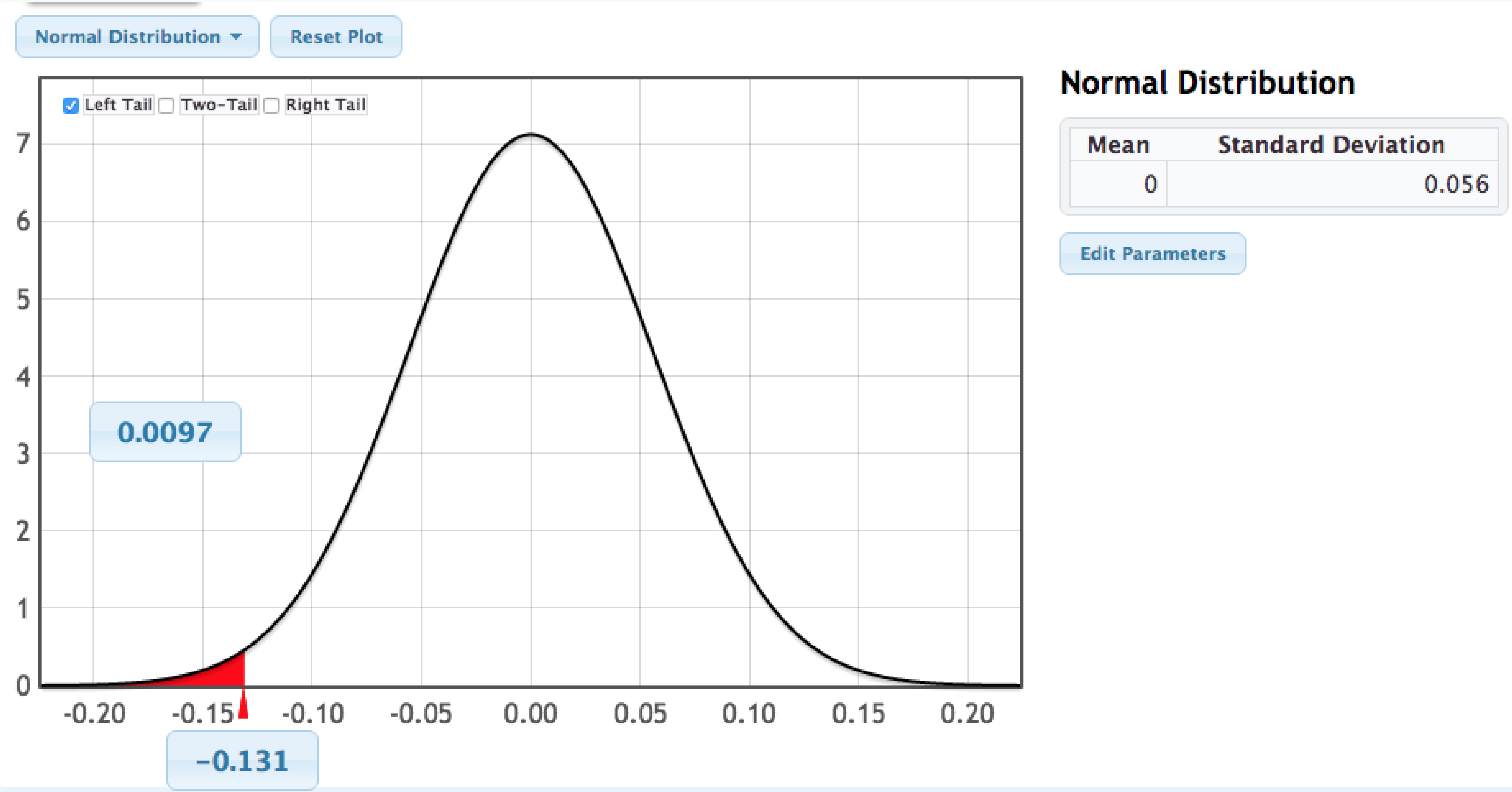
Randomization Sample

Group	Count	Sample Size	Proportion
Group 1	27	113	0.239
Group 2	29	117	0.248
Group 1–Group 2	-2	n/a	-0.0089

A. $\mathrm{N}(0, -0.131)$
B. $\mathrm{N}(0, 0.056)$
C. $\mathrm{N}(-0.131, 0.056)$
D. $\mathrm{N}(0.056, 0)$

▶ Click for answer

Statkey: p-value from N(null, SE)



Connecting Normal model to hypothesis tests

Suppose: randomization distribution is bell shaped.

- ***Center**: hypothesized null parameter value*
- ***Spread**: the standard error given in the randomization graph (or by formula)*
- ***P-value**: computed from the normal model the "usual" way - the chance of being as extreme, or more extreme, than the observed statistic.*

Standardized Statistic

The standardized test statistic (also known as a z-statistic) is

$$z = \frac{\text{statistic} - \text{null}}{SE}$$

Calculating the number of standard errors a statistic is from the null lets us assess extremity on a common scale.

Malaria and Mosquitos

Does infecting mosquitoes with Malaria actually impact the mosquitoes' behavior to favor the parasite?

- ***After the parasite becomes infectious, do infected mosquitoes approach humans more often, so as to pass on the infection?***

Days 9 – 28

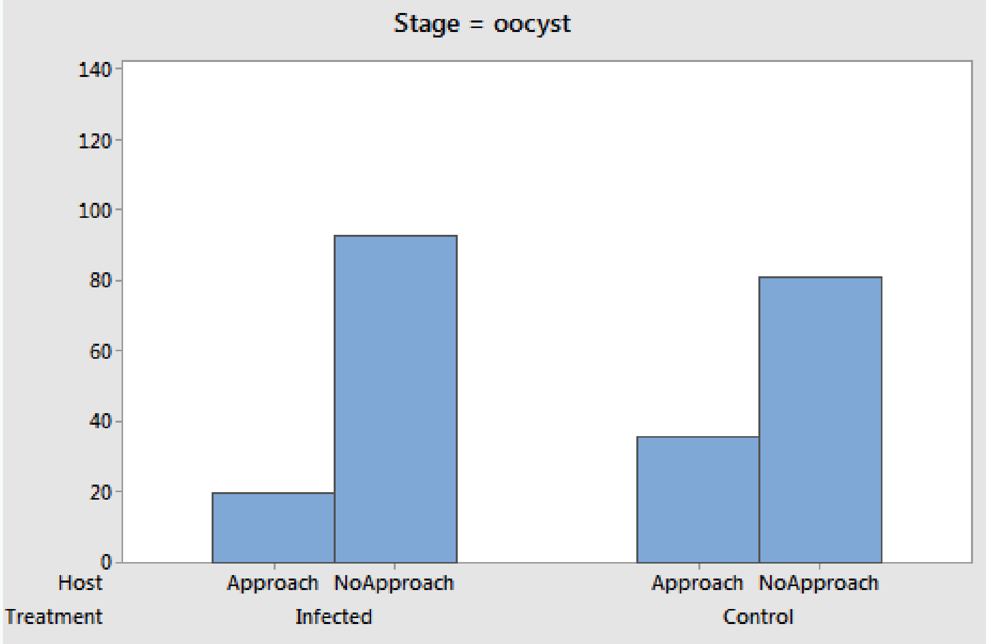
For the data after the mosquitoes become infectious (Days (9-28)), what are the relevant hypotheses?

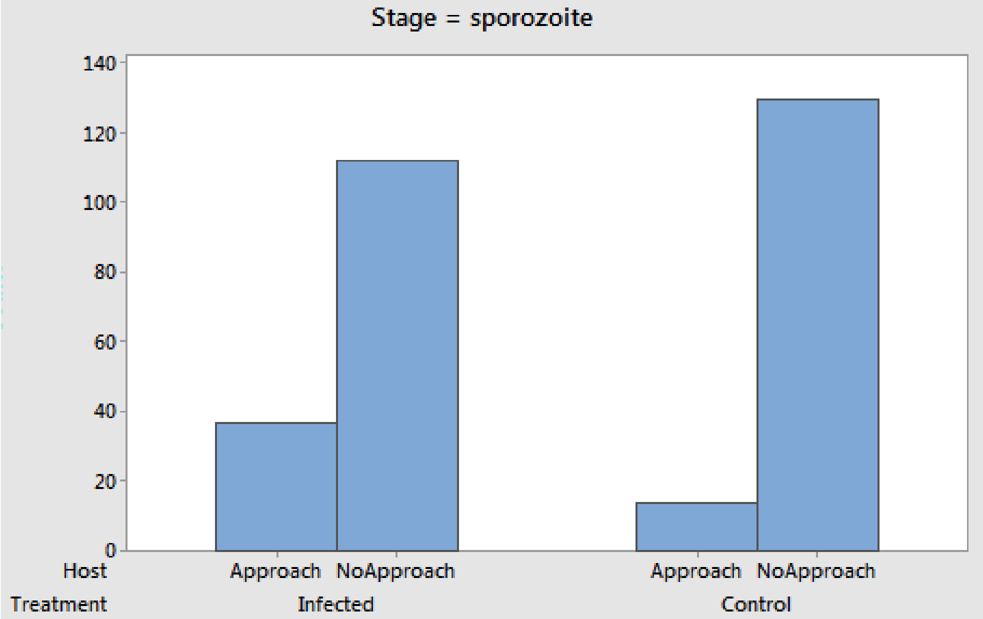
$$\boldsymbol{p_C} \text{ :proportion of controls to approach human}$$
$$\boldsymbol{p_E} \text{ :proportion of exposed to approach human}$$

$$
\begin{array}{ll}
A. & \mathrm{H_0} : p_\mathrm{E} = p_\mathrm{C}, \mathrm{H_a} : p_\mathrm{E} < p_\mathrm{C} \\
B. & \mathrm{H_0} : p_\mathrm{E} = p_\mathrm{C}, \mathrm{H_a} : p_\mathrm{E} > p_\mathrm{C} \\
C. & \mathrm{H_0} : p_\mathrm{E} < p_\mathrm{C}, \mathrm{H_a} : p_\mathrm{E} = p_\mathrm{C} \\
D. & \mathrm{H_0} : p_\mathrm{E} > p_\mathrm{C}, \mathrm{H_a} : p_\mathrm{E} = p_\mathrm{C}
\end{array}
$$

▶ Click for answer

Before and after



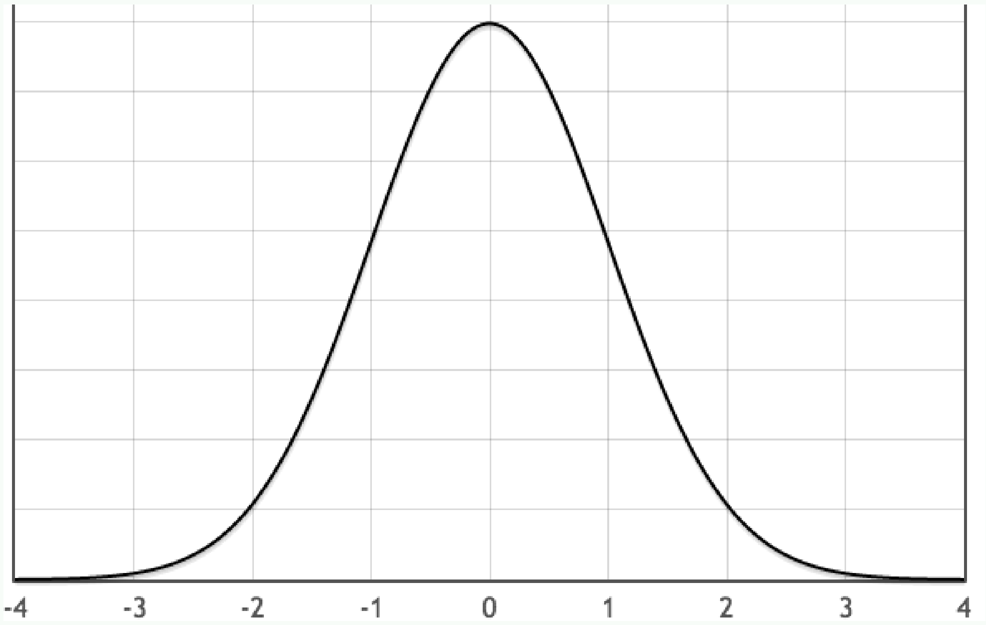


Is the difference significant?

The difference in proportions is 0.151 and the standard error is 0.05. Is this significant?

A. Yes

B. No



Malaria and Mosquitoes

It appears that mosquitoes infected by malaria parasites do, in fact, behave in ways advantageous to the parasites!

- *Exposed mosquitos are less likely to approach before becoming infectious (so more likely to stay alive)*
- *Exposed mosquitos are more likely to approach humans after becoming infectious (so more likely to pass on disease)*

Formula for p-values Using N(0,1)

From original data

From H_o

$$z = \frac{\text{sample statistic} - \text{null value}}{\text{SE}}$$

From randomization distribution

Connecting Normal model to Confidence Intervals

Suppose: bootstrap distribution is bell-shaped.

- *Center: sample statistic*
- *Spread: the standard error given in the bootstrap graph (or by formula)*

Connecting Normal model to Confidence Intervals

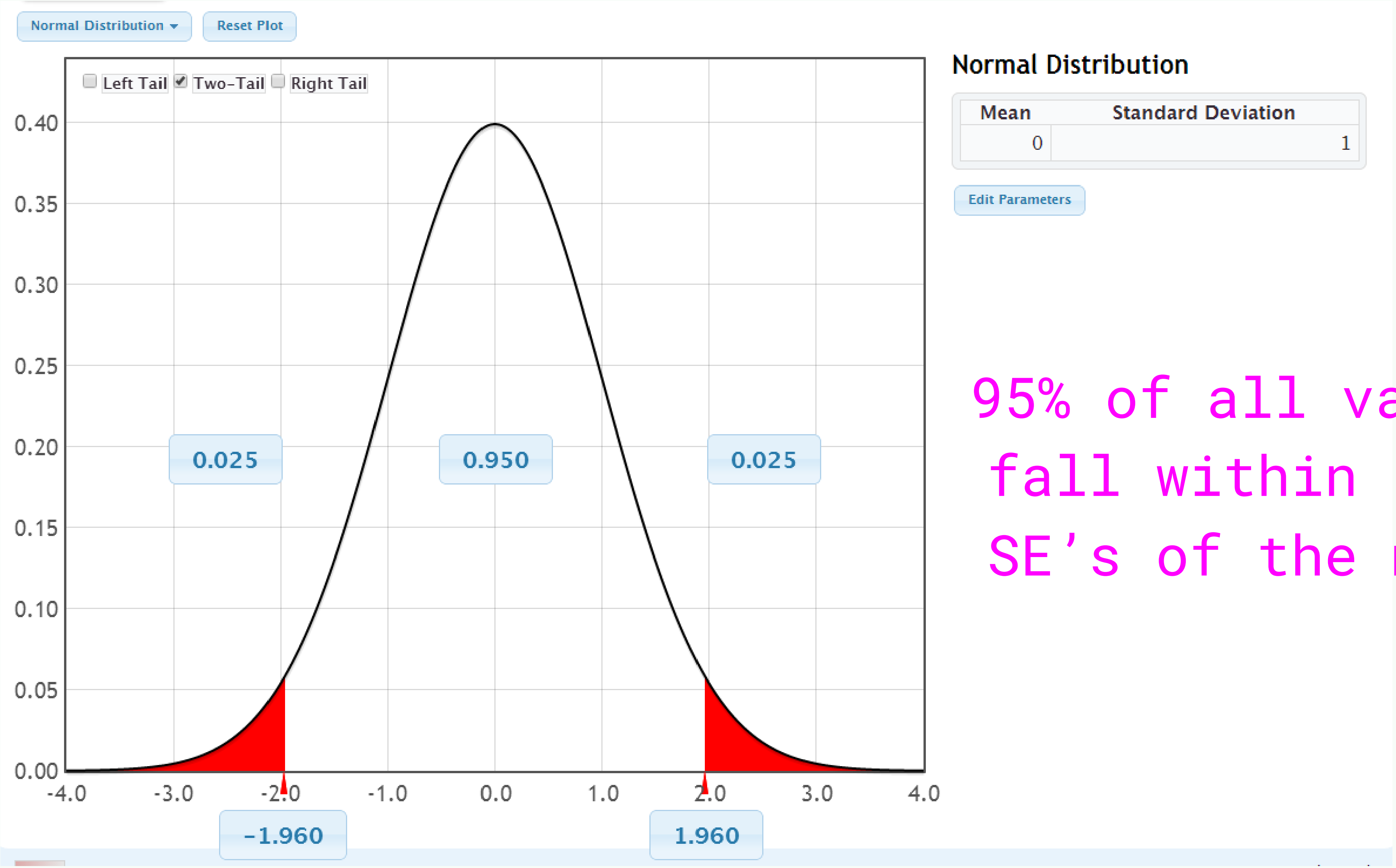
To get a 95% confidence interval we compute:

$$statistic \pm 2(SE)$$

Why 2 SE's?

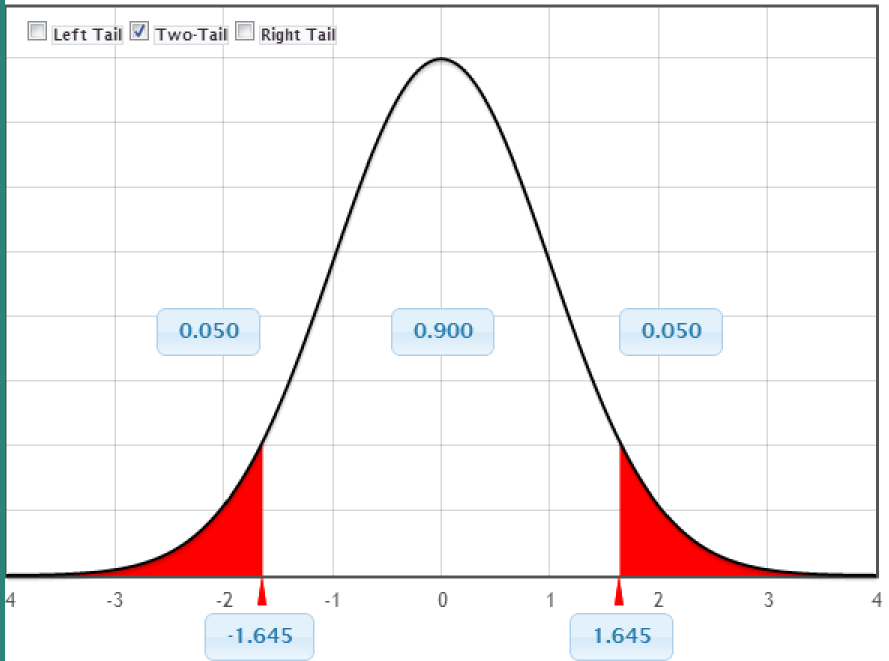
- 95% ***of all sample means fall within 2 SE's of the population mean***
- ***The value 2 is a z-score!***
- ***Well, actually the precise z-score under a normal model is*** $z = 1.96$ ***instead of 2 !***

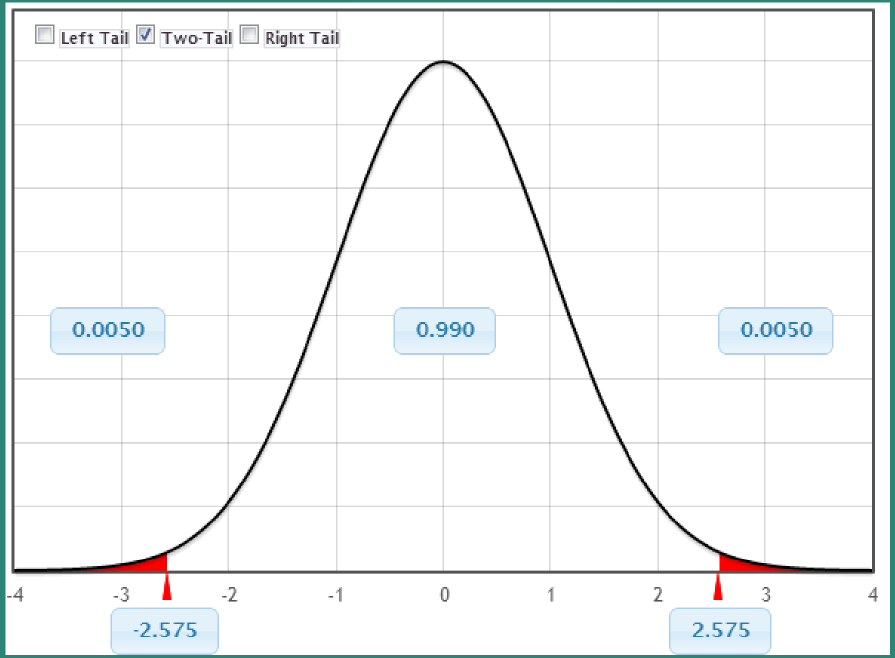
N(0,1) model



95% of all values fall within 1.96 SE's of the mean

What if we wanted a 90% CI? What z-score should we use to get the margin of error?






```
qnorm(0.95)
[1] 1.644854
```

```
qnorm(0.995)
[1] 2.575829
```

90% Confidence: $z^* = 1.645$

99% Confidence: $z^* = 2.576$

Confidence Interval using N(0,1)

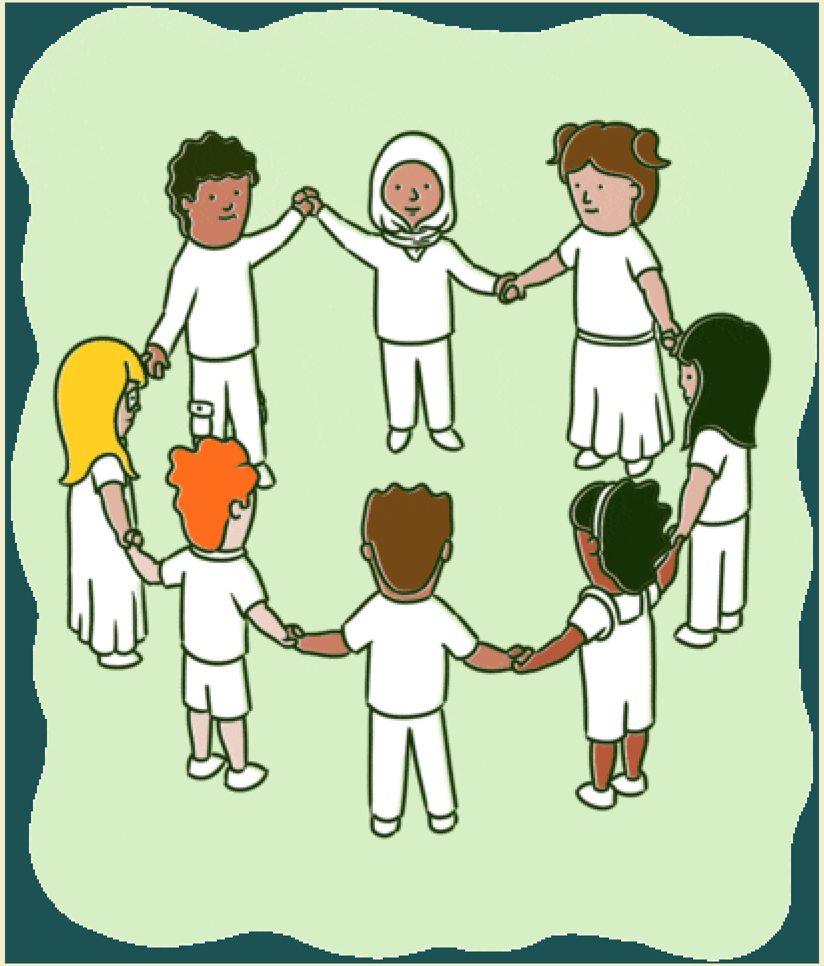
If a statistic is normally distributed, we find a confidence interval for the parameter using

$$statistic \pm z^* SE$$

where the area between $-z^*$ and $+z^*$ in the standard normal distribution is the desired level of confidence.

YOUR TURN 1

05:00



Let's go over to the class activity .Rmd file and complete the tasks for today.