Describing Quantitative Variables

Stat 120

April 06 2022

Describing Quantitative Variables

Numerically: measures of

- "center": what is a middle/common value?
  - common measures: mean, median
- "variability": how spread out are values?
  - common measures: standard deviation, IQR, range
- "shape": how are values distributed around the "middle"?
  - common measures: 5-number summary

Describing Quantitative Variables

Graphically: common visuals are

- dotplots
- histograms
- boxplots

Distribution

```
"The distribution of the variable Y"
```

- describes its center, variability and shape
- use both numbers and graphics

Center: Mean or Average

Mean: average value in a sample or population

- $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$ is an average of n values y_i in a sample
- μ is an average value of y in a population

Example: The data `StudentSurvey.csv` is a sample of student survey responses obtained by the textbook authors

```
survey <- read.csv("https://raw.githubusercontent.com/deepbas/statdatasets/main/StudentSurve
mean(survey$Pulse) # the command `mean` computes an average
[1] 69.57459
```

The mean pulse rate for this sample of students is $\bar{y} = 69.6$ beats per minute.

Center: Median

Median: the middle value when the data are ordered

- The median splits the data in half
- m is the median value in a sample
- M is the median value in a population

```
median(survey$Pulse) # the command `median` computes an median
[1] 70
```

The median pulse rate for this sample of students is $m = 70$ beats per minute.

- This means half the students have a rate below 70 and half have a rate about 70.

Variability: Standard Deviation

Standard Devation (SD): average value in a sample or population

- $s = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \overline{y})^2}{n-1}}$ is the SD of n values y_i in a sample
- σ is the SD of values of y in a population

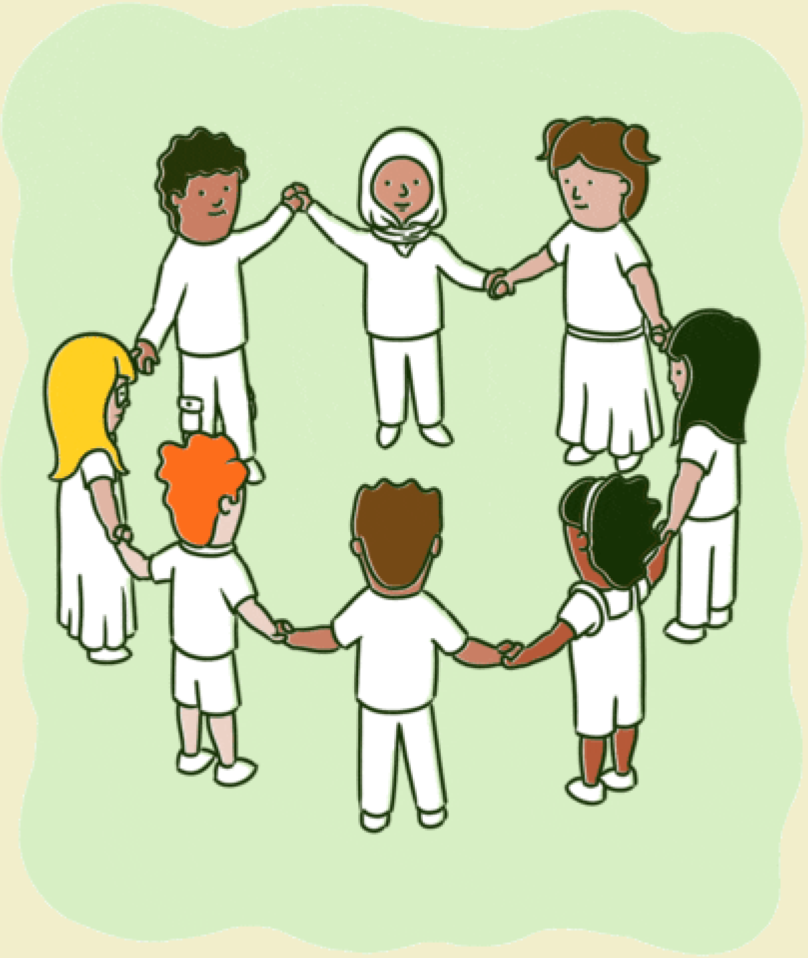
```
sd(survey$Pulse) # the command `sd` computes an average
[1] 12.20514
```

The SD of pulse rates for this sample of students is $s = 12.2$ beats per minute.

- The "average" deviation of individual pulse rates around the mean value is about 12.2 beats per minute.

Your Turn 1

05:00



Go to our class moodle and skim through the problems

Feel free to talk to your neighbor

Missing Data in R

- Missing data values in R are coded as `NA` values
- Many basic statistic functions in R return an `NA` value if variable has any missing values

```
movies <-read.csv("https://raw.githubusercontent.com/deepbas/statdatasets/main/HollywoodMovi
mean(movies$WorldGross)
[1] NA
sd(movies$WorldGross)
[1] NA
```

Why is this good?

- It lets the user (you) know that at least one value (maybe many, many values!) are missing

Missing Data in R

How many missing?

- Use the `summary` command:

```
summary(movies$WorldGross)
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.     NA's
   0.025   30.706   76.659  150.742  173.691 1328.111        2
```

There are 2 movies with missing world gross amounts.

Add the argument `na.rm = TRUE` to remove missing values and get your summary stats:

```
mean(movies$WorldGross, na.rm = TRUE)
[1] 150.7423
sd(movies$WorldGross, na.rm = TRUE)
[1] 215.0186
```

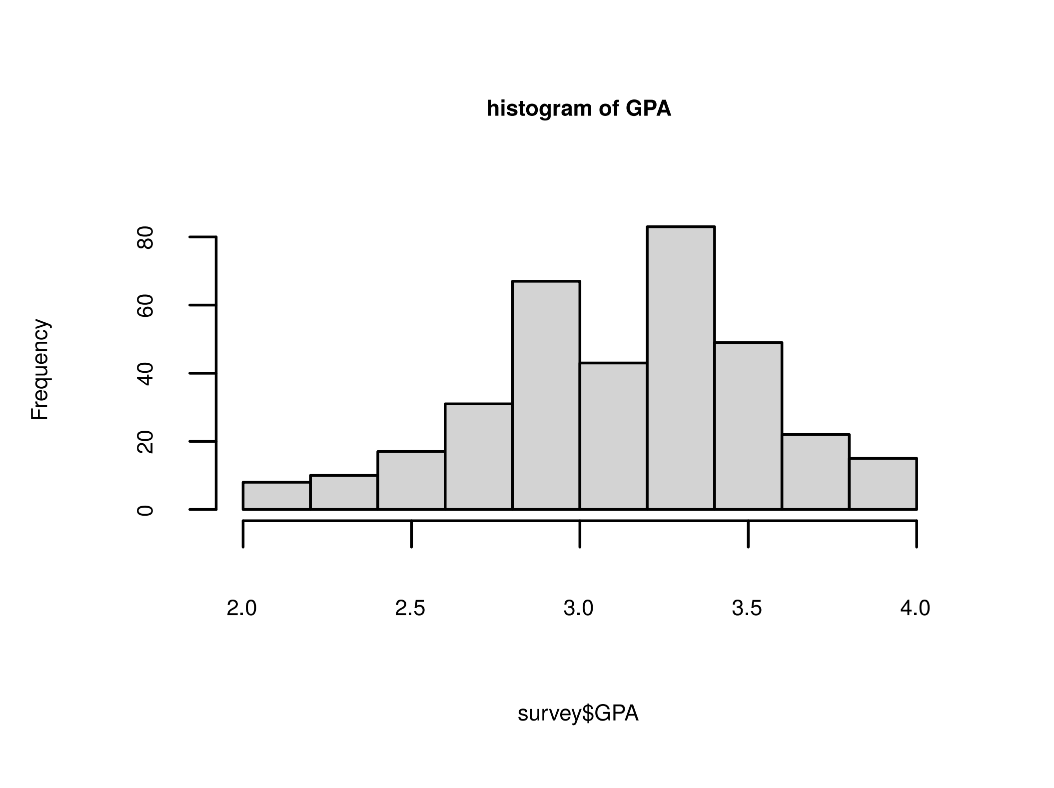
Shape: histogram

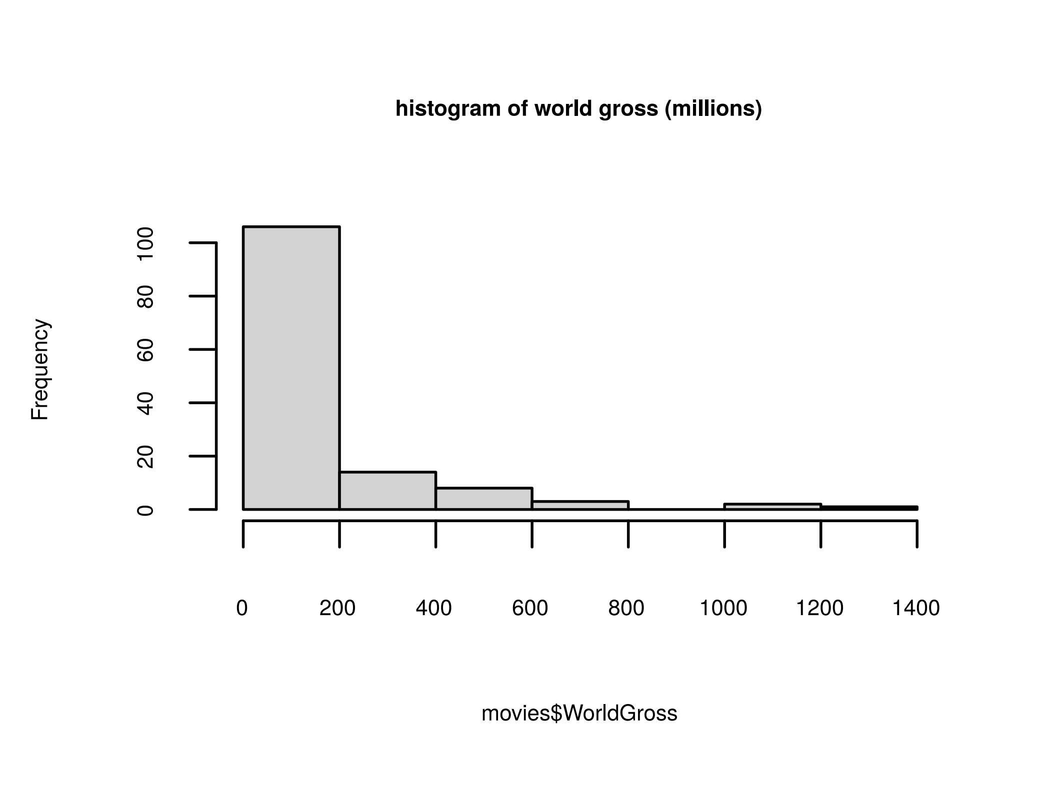
Histogram: aggregates values into bins and counts how many cases fall into each bin

```
hist(survey$Pulse,
     main = "Histogram of pulse rates",
     cex.lab=0.5, cex.main = 0.5, cex.axis=0.5)
```

- Pulse rates are **symmetrically** distributed around a rate of about 70 beats per minute.
- Symmetric distributions are "centered" around a mean and median that are roughly the same in value.

Shape: Left Skew & Right Skew






```
mean(survey$GPA, na.rm =T)
[1] 3.157942
median(survey$GPA, na.rm = T)
[1] 3.2
```

```
mean(movies$WorldGross, na.rm =T)
[1] 150.7423
median(movies$WorldGross, na.rm = T)
[1] 76.6585
```

Extreme values

outlier: an observed value that is notably distinct from most other values in the dataset

resistant: a statistic is resistant to outliers if it is relatively unaffected by outliers

- Median is resistant to outliers
- Mean and SD are not resistant

Movie world gross (millions of dollars) stats with and without Harry Potter movie:

	Mean	SD	Median
with HP	150.7	215.0	76.7
without HP	141.9	189.7	75.0

Identifying extreme values in R

`which` identifies the **row number** of cases that satisfy a given criteria

- Which movies had world gross bigger than 1200?

```
which(movies$WorldGross > 1200)  # gives row number
[1] 4
movies[4, c("Movie","WorldGross")]
                                        Movie WorldGross
4 Harry Potter and the Deathly Hallows Part 2   1328.111
```

Harry Potter (row number 4) had world gross of 1.328 billion dollars!

Identifying extreme values in R

What are stats without Harry Potter?

- omit **row 4** from the `WorldGross` variable with the "minus row 4" subset:

```
WorldGross.noHP <- movies$WorldGross[-4]
```

- Then compare stats

```
summary(movies$WorldGross) # with Harry Potter
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.     NA's
   0.025   30.706   76.659  150.742  173.691 1328.111        2
summary(WorldGross.noHP) # without Harry Potter
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.     NA's
   0.025   30.426   75.009  141.890  170.301 1123.195        2
sd(movies$WorldGross, na.rm = T) # with Harry Potter
[1] 215.0186
sd(WorldGross.noHP, na.rm = T) # without Harry Potter
[1] 189.7441
```

Adding a categorical variable: stats

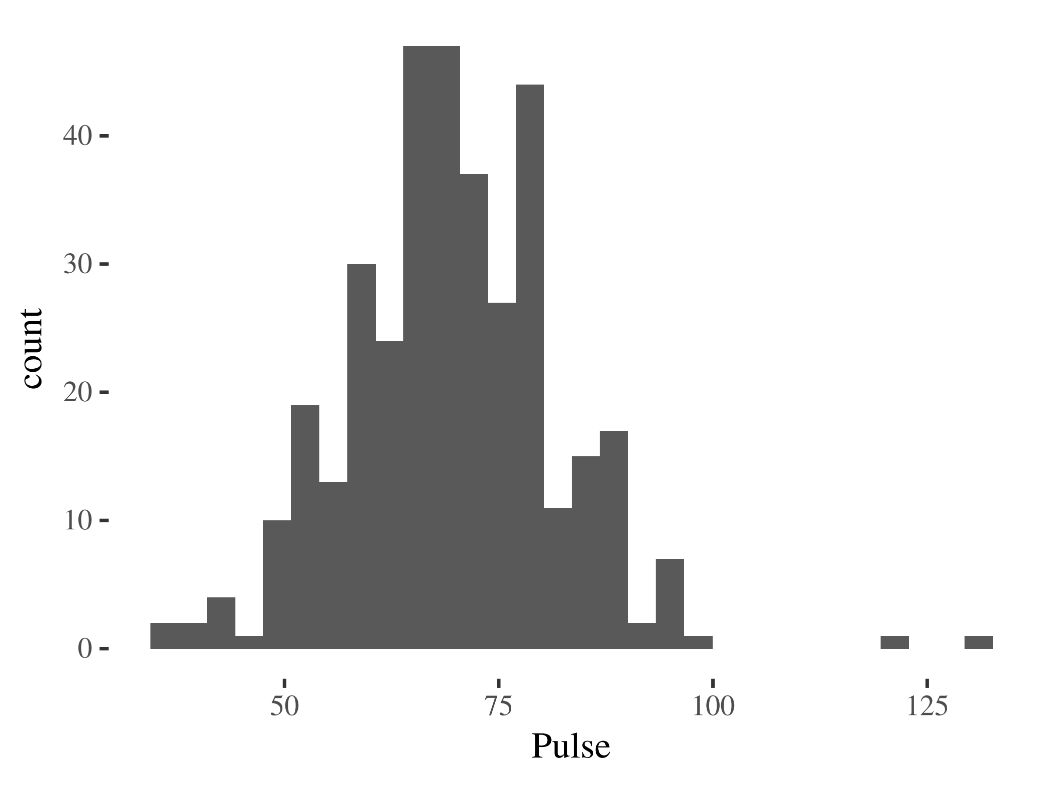
We can compare distributions across different levels of a categorical variable to explore whether the two variables are **associated**.

- Use `tapply(y,x,fun)` to apply the `fun` function to `y` for different levels of `x`
- Pulse rate stats by smoking status: Smoker have a slightly higher mean pulse rate than non-smokers (71.8 vs. 69.3).

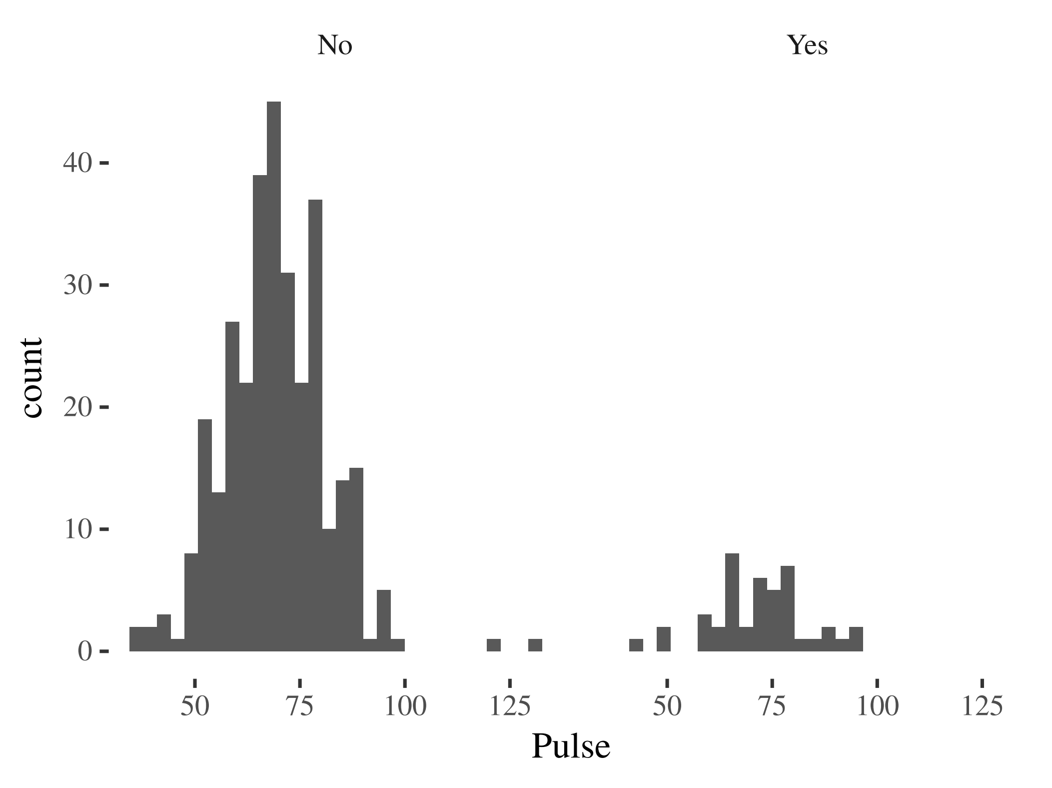
```
table(survey$Smoke)

 No Yes
319  43
tapply(survey$Pulse, survey$Smoke, summary)
$No
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  35.00   61.00   69.00   69.27   77.00  130.00

$Yes
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  42.00   65.00   72.00   71.81   79.00   96.00
```

Adding a categorical variable: graphics

```
library(ggplot2)
ggplot(survey, aes(x=Pulse)) + geom_histog
```

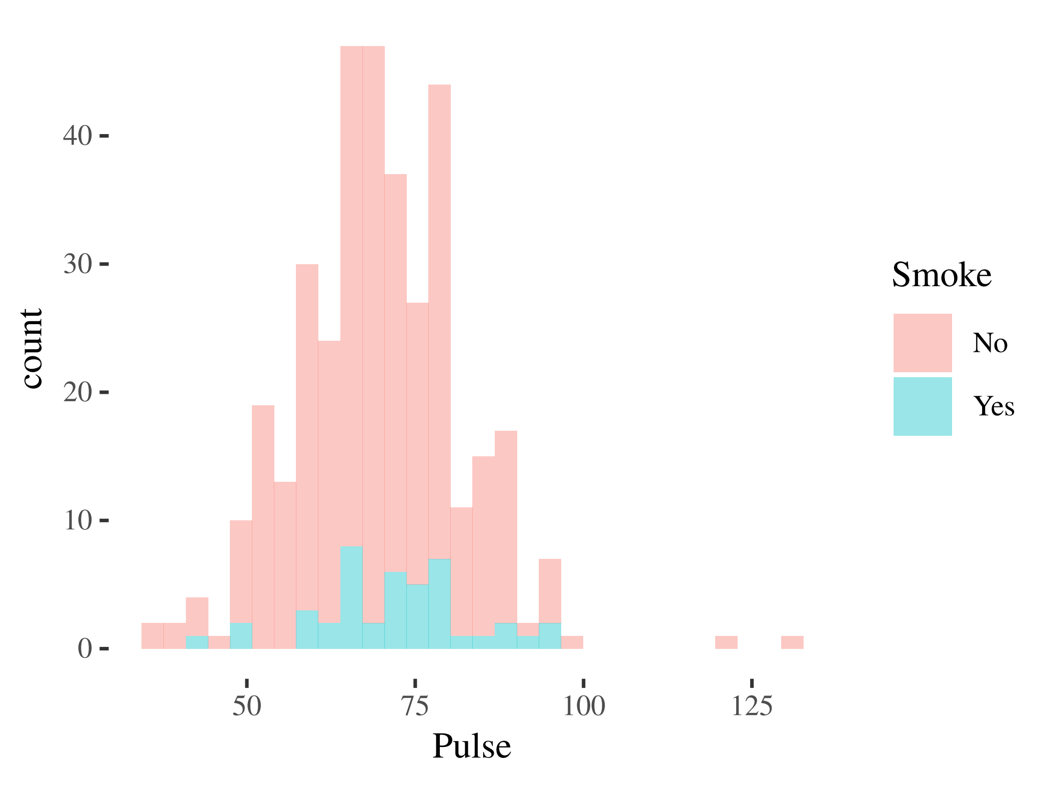



```
ggplot(survey, aes(x=Pulse)) + geom_histog
  facet_wrap(~Smoke)
```

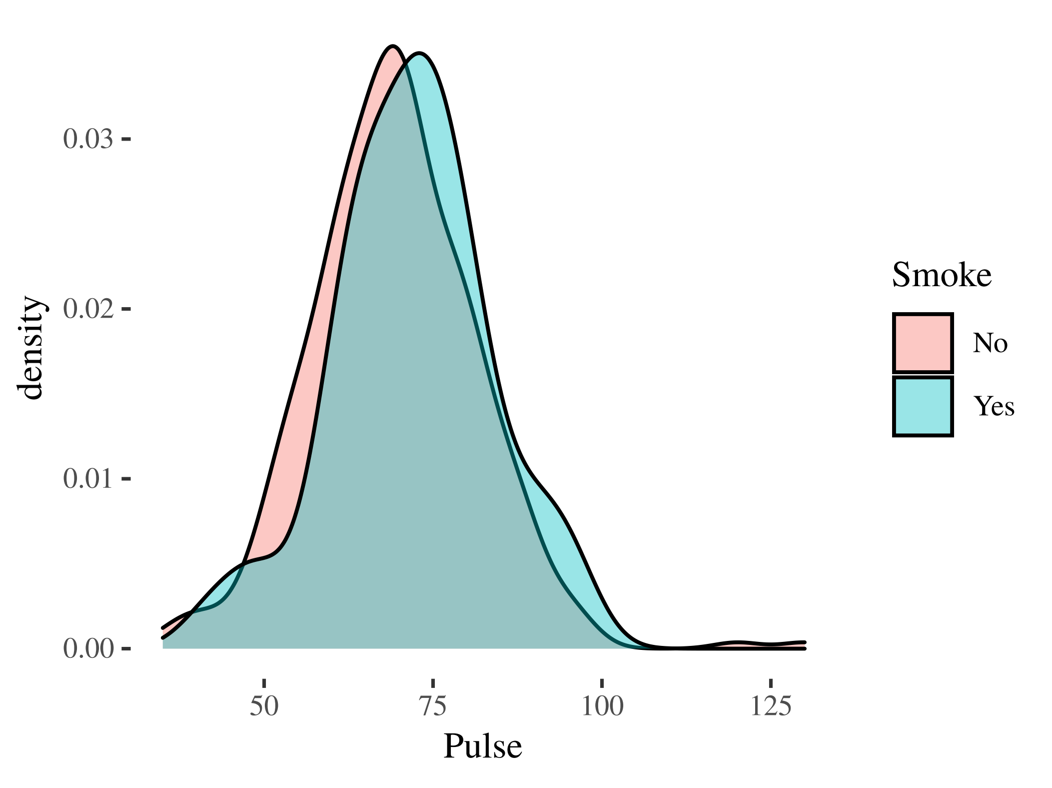


Adding a categorical variable: graphics

```
ggplot(survey, aes(x=Pulse, fill=Smoke)) +
  geom_histogram(alpha=.4)
```

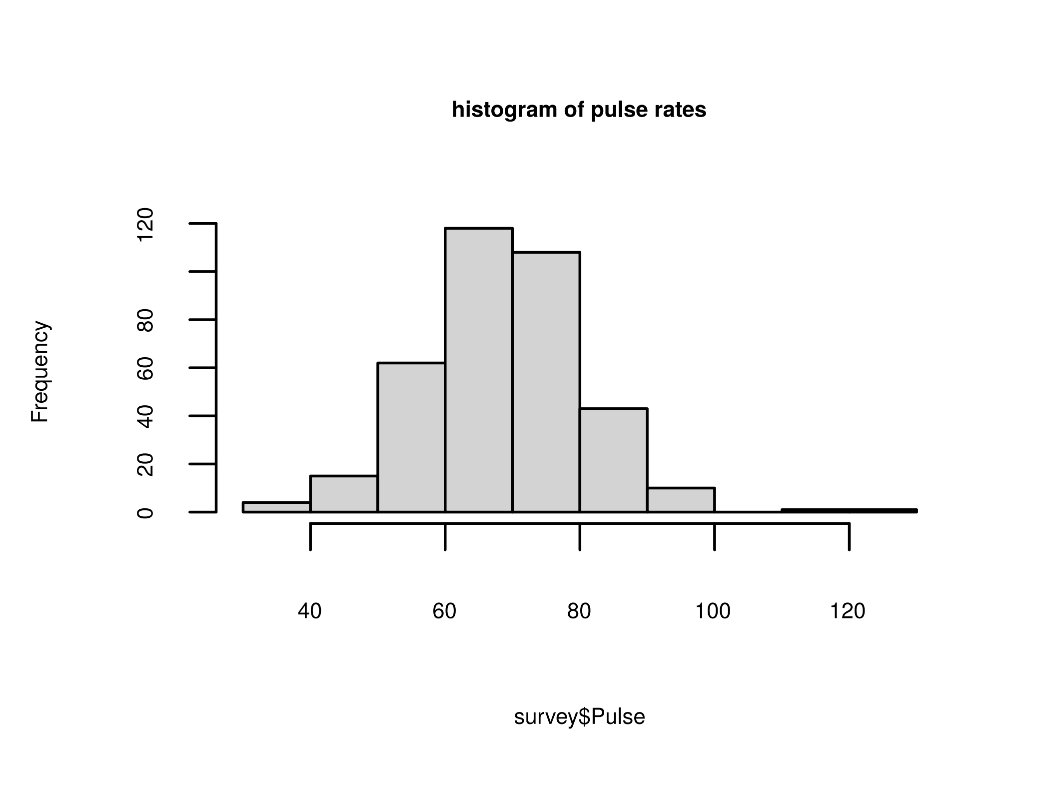



```
ggplot(survey, aes(x=Pulse, fill=Smoke)) +
  geom_density(alpha=.4)
```



Shape and Stats

Mean and standard deviation are good summary stats of a **symmetric** distribution.



Similar variation to the left and right of the mean so one measure of SD is fine.

```
# mean
mean(survey$Pulse)
[1] 69.57459
```

```
# standard deviation
sd(survey$Pulse)
[1] 12.20514
```

Shape

If a distribution of data is **approximately bell-shaped**, about 95% of the data should fall within two standard deviations of the sample mean.

- for a sample: 95% of values between $\bar{y} - 2s$ and $\bar{y} + 2s$
- for a population: 95% of values between $\mu - 2\sigma$ and $\mu + 2\sigma$

Shape

```
sleep <- read.csv("https://raw.githubusercontent.com/deepbas/statdatasets/main/SleepStudy.cs
```

Question The standard deviation for hours of sleep per night is closest to

(a) 0.5

(b) 1

(c) 2

(d) 4

Standardizing data: z-score

The z-score of a data value, x, tells us how many standard deviations the value is above or below the mean:

$$z = \frac{x - \text{mean}}{\text{SD}}$$

- E.g. if a value x has $z = -1.5$ then the value x is **1.5 standard deviations below** the mean.

Question: If we standardize all values in a bell-shaped distribution, 95% of all z-scores fall between what values?

Standardizing data: z-score

Z-scores put measurements on a common scale

- **Example 4:** Which is better, an ACT score of 28 or a combined SAT score of 2100?
  - ACT: $\mu = 21, \sigma = 5$
  - SAT: $\mu = 1500, \sigma = 325$

SAT score because it is **1.85 SDs above** the mean SAT score while the ACT score is only 1.4 SD above the mean ACT score.

$$z_{ACT} = \frac{28 - 21}{5} = 1.4 \qquad z_{SAT} = \frac{2100 - 1500}{325} = 1.85$$

Shape and Stats: Percentiles

Percentiles are good summary stats to describe a **skewed** distribution.

- The P^{th} percentile of a distribution is the value which is greater than $P\%$ of all other values.
- The median is the 50^{th} percentile

Example 4: used z-scores to determine whether a SAT score of 2100 or an ACT score of 28 is better

We could also have used percentiles:

- `ACT score of 28:` 91st percentile (you scored better than 91% of people)
- `SAT score of 2100:` 97th percentile (you scored better than 97% of people!)

Shape and Stats: Quartiles

Quartiles divide values in to quarters

- 1st Quartile: Q_1 is the 25th percentile
- 2nd Quartile: Q_2 is the 50th percentile (median)
- 3rd Quartile: Q_3 is the 75th percentile

5-number summary is quartiles along with min and max: $\min, Q_1, \mathrm{m}, Q_3, \max$

Interquartile Range (IQR) is the range of the middle 50% of values:

- $IQR = Q_3 - Q_1$
- the **range** is just $\max - \min$

Shape and Stats: Quartiles

```
summary(movies$WorldGross)
    Min.   1st Qu.    Median      Mean   3rd Qu.      Max.      NA's
   0.025    30.706    76.659   150.742   173.691  1328.111         2
```

The 5-number summary is

- $\min = 0.025, Q_1 = 30.7, \mathrm{m} = 76.7, Q_3 = 173.7, \max = 1328.1$
- right skewed: variation is upper 25 of movies is much larger than lower 25%
  - upper range: $\max - Q_3 = 1328.1 - 173.7 = 1154.4$
  - lower range: $Q_1 - \min = 30.7 - 0.025 = 30.675$

Shape and Stats: Boxplot

Boxplot: Visualization of 5-number summary

- Draw a numerical scale appropriate for the data
- Draw a box stretching from Q_1 to Q_3
- Divide the box with a line at the median
- Draw a line from each quartile to the most extreme data value that is not an outlier
- Identify each outlier individually by plotting with a symbol such as an asterisk or dot

Outlier rule of thumb: cases that are more extreme than

$$Q_1 - 1.5(IQR) \quad \text{or} \quad Q_3 + 1.5(IQR)$$

Shape and Stats: Boxplot

```
boxplot(survey$Pulse, ylab="pulse rate",
        cex.lab=0.5, cex.main = 0.5, cex.axis=0.5)
```

```
summary(survey$Pulse)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  35.00   62.00   70.00   69.57   77.75  130.00
```

- $IQR = 77.75 - 62 = 15.75$
- $1.5(15.75) = 23.625$
- lower "fence" = $62 - 23.625 = 38.375$
- upper "fence" = $77.75 + 23.625 = 101.375$

```
which(survey$Pulse < 38.375)
[1]  55 106 200
which(survey$Pulse > 101.375)
[1]   3 171
```

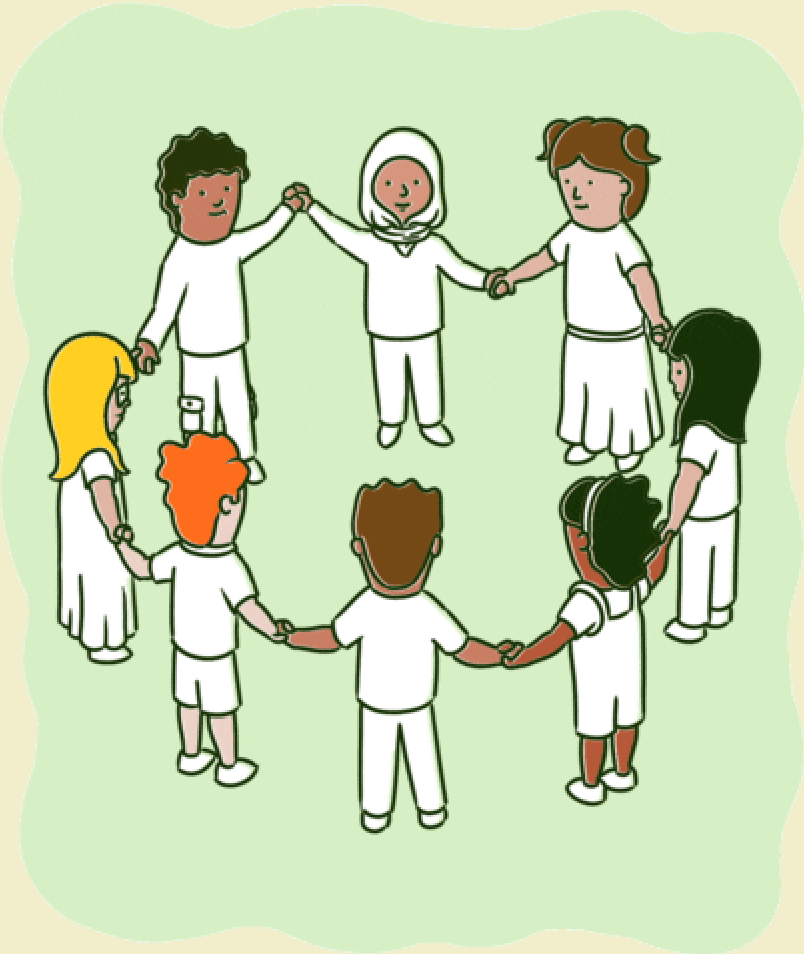
Shape and Stats: Side-by-side Boxplots

```
boxplot(Pulse ~ Smoke, data = survey,
        ylab="pulse rate",
        cex.lab=0.8, cex.axis=0.7)
```

- Median pulse rates are slightly higher for smokers than non-smokers (72 vs. 69 beats per minute) but variation is slightly lower (IQR 14 vs 16 beats per minute).
- Both distributions are roughly symmetric.
- Overall, just a slight association between smoking status and pulse rates.

Your Turn 2

05:00



Go over the remaining portion of in class activity and let me know if you have any questions!