Inference for multiple proportions

Stat 120

February 20 2023

Tests for Categorical Variable(s)

Goodness-of-fit test

- ✨Test a claim about the distribution of one categorical variable
- ✨E.g. are 6 M&M colors equally likely?
- ✨E.g. is Biden's approval rating 50%?

Chi-square test for association

- ✨Determine if a relationship between two categorical variables is statistically significant
- ✨E.g. Does M&M color distribution depend on type (chocolate vs. peanut)?

Test for one categorical variable

Seen single proportion tests before

✨*Example : Test if the proportion of Reese's Pieces that are orange is different from 1/3.*

$$H_0: p = 1/3$$
$$H_a: p \neq 1/3$$

What if we want to test proportions for several categories at once?

✨Example: Are the three colors (orange, yellow, brown) of Reese's Pieces equally likely?

H_0 specifies a proportion, p_i , for each category.

Test for one categorical variable

The proportions do not have to be the same. e.g: Grade distribution

$$
\begin{aligned}
&H_0 : p_A = 0.2, \quad p_B = 0.3, \quad p_C = 0.3, \quad p_D = 0.1, \quad p_F = 0.1 \\
&H_a : \text{ at least one proportion different}
\end{aligned}
$$

Rock-Paper-Scissors

ROCK	PAPER	SCISSORS	TOTAL
36	12	37	85

How would we test whether all of these categories are equally likely?

Conduct a hypothesis test

- ✨*State Hypothesis*
- ✨*Calculate a test statistic, based on your sample data*
- ✨*Create a distribution of this statistic, as it would be observed if the null hypothesis were true*
- ✨*Measure how extreme your test statistic is, as compared to the distribution generated under null*

Test Statistic

Why can't we use the familiar formula to get the test statistic?

$$\frac{\text{sample statistic - null value}}{\text{SE}}$$

✨More than one sample statistic

✨More than one null value

We need something a bit more complicated ...

Observed Counts

The observed counts are the actual counts observed in the study

ROCK	PAPER	SCISSORS	TOTAL
36	12	37	85

✨The expected counts are the expected counts if the null hypothesis were true
✨For each cell, the expected count is the sample size n times the null proportion, p_o

	ROCK	PAPER	SCISSORS	TOTAL
Observed	36	12	37	85
Expected	28.33	28.33	28.33	85

Chi-Square Statistic

✨A test statistic is one number, computed from the data, which we can use to assess the null hypothesis

✨The chi-square statistic is a test statistic for categorical variables:

$$\chi^2 = \sum \frac{(observed - expected)^2}{expected}$$

Rock-Paper-Scissors

	ROCK	PAPER	SCISSORS	TOTAL
Observed	36	12	37	85
Expected	28.33	28.33	28.33	85

$$\begin{aligned}\chi^2 &= \frac{(36-28.33)^2}{28.33} + \cdots + \cdots \\ &\approx 2.076 + \cdots + \cdots\end{aligned}$$

What next?

We have a test statistic. What else do we need to perform the hypothesis test?

✨*A distribution of the test statistic assuming H_0 is true*

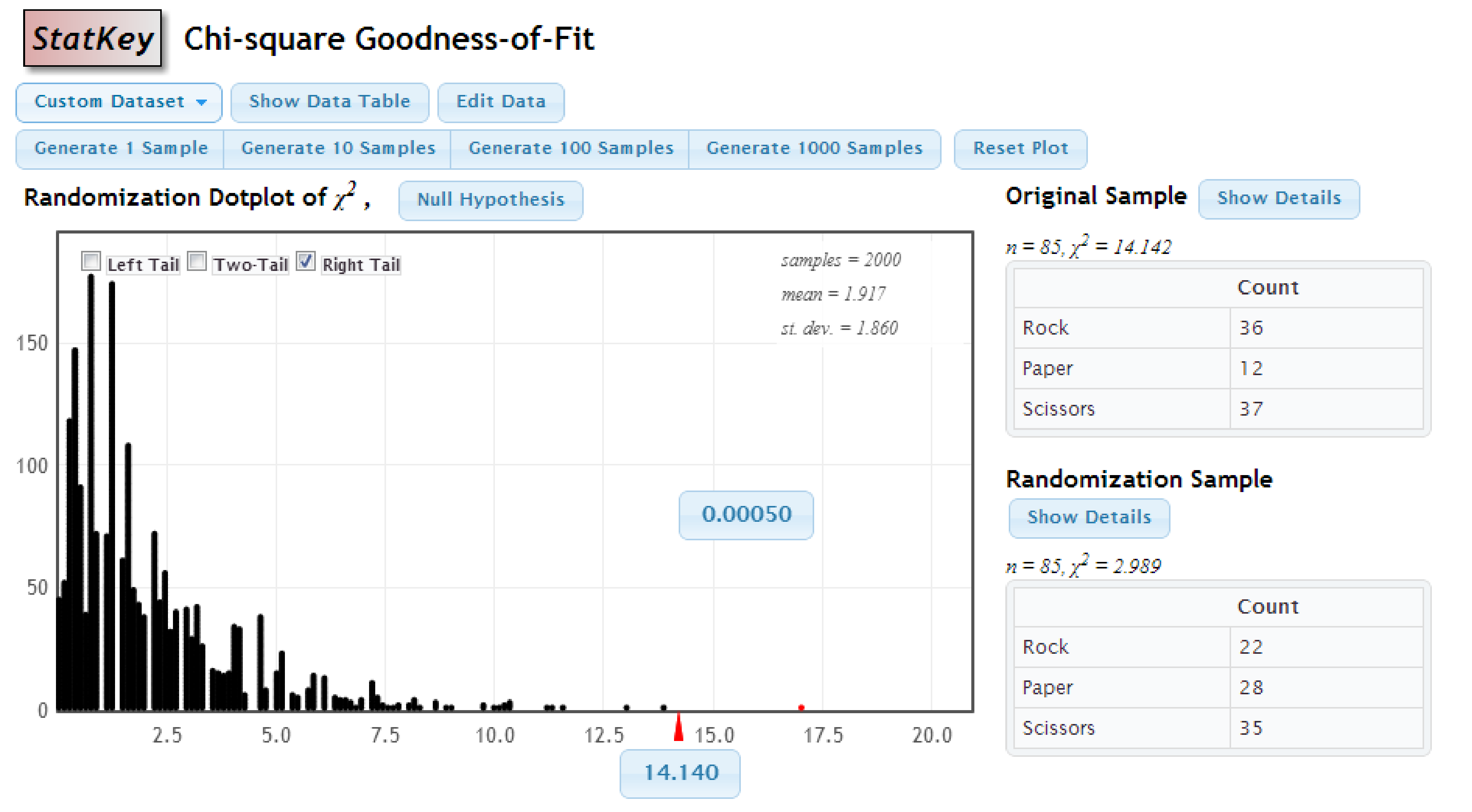
How do we get this? Two options:

1. *Simulation*
2. *Theoretical Distribution*

Simulation

1. Take 3 scraps of paper and label them Rock, Paper, Scissors. Fold or crumple them so they are indistinguishable. Choose one at random and record the result.
2. Repeat a number of times to match the original sample size and get a table of observed counts.
3. Calculate the χ^2-statistic.
4. Repeat this many times to get a randomization distribution of many χ^2-statistics.
5. How extreme is the actual test statistic in this randomization distribution?

Statkey: Chi-Square Distribution



Chi-Square Distribution

If each of the expected counts are at least 5, AND if the null hypothesis is true, then the χ^2 statistic follows a χ^2 distribution, with degrees of freedom equal to

$$df = \text{number of categories} - 1$$

Rock-Paper-Scissors:

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df = 3 - 1 = 2 # degrees of freedom
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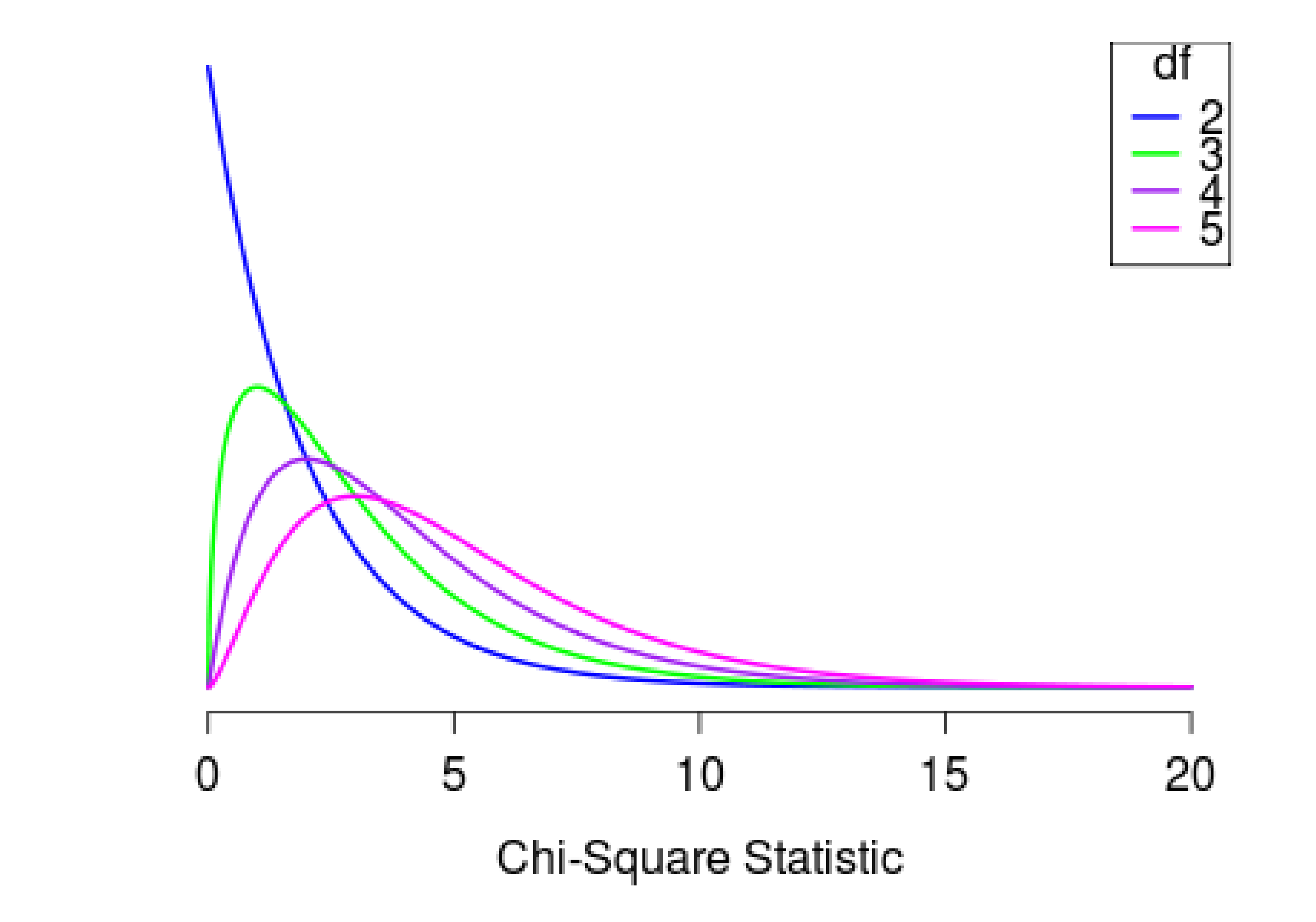
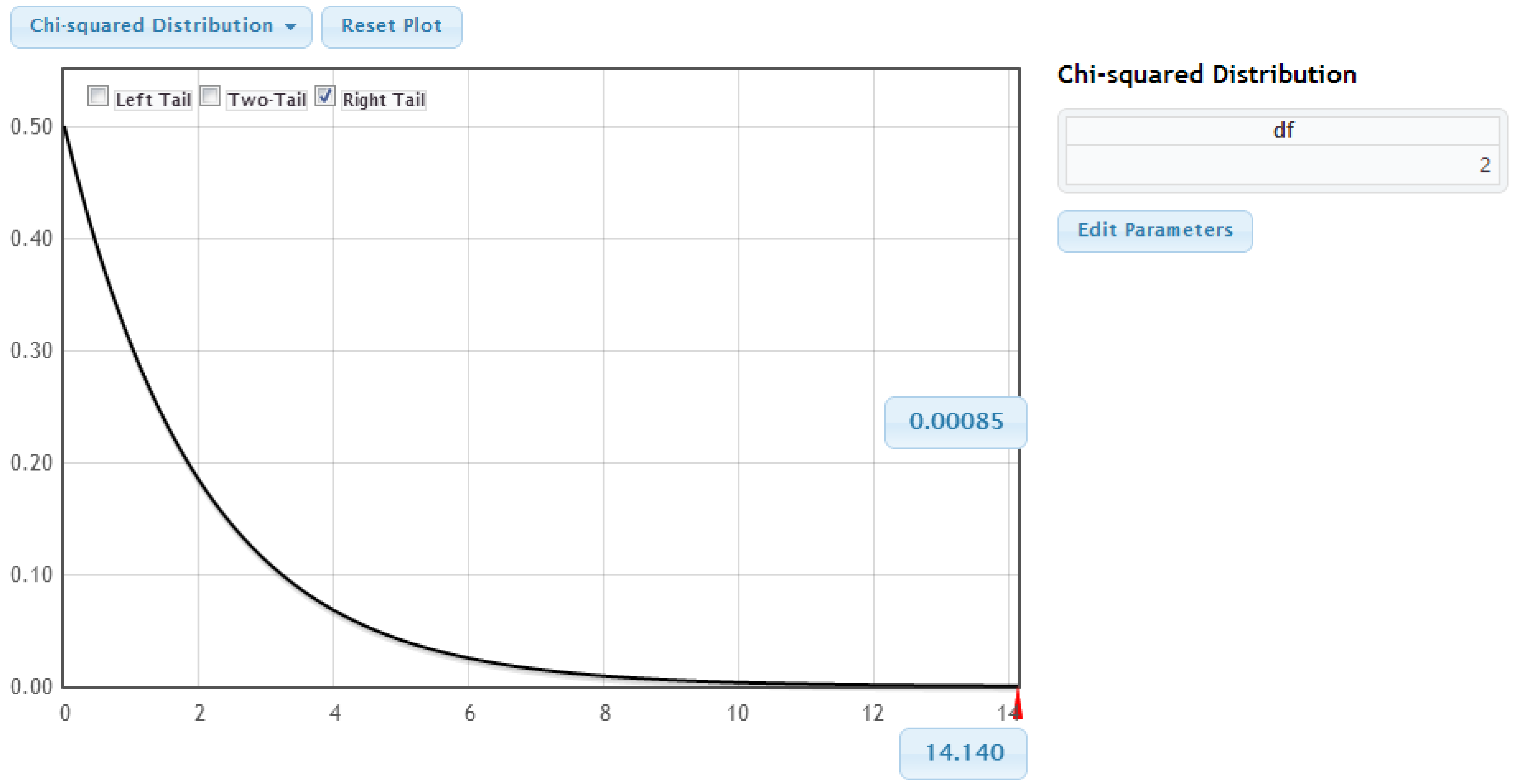
Chi-Square Distribution



Statkey: p-value using Chi-square distribution



Goodness of Fit

- ✨A χ^2 test for goodness of fit test determines whether the distribution of a categorical variable is the same as some null hypothesized distribution
- ✨The null hypothesized proportions for each category do not have to be the same

Chi-Square Test for Goodness of Fit

- ✨State null hypothesized proportions for each category, pi. Alternative is that at least one of the proportions is different than specified in the null.
- ✨Calculate the expected counts for each cell as $n \cdot p_i$. Make sure they are all greater than 5 to proceed.
- ✨Calculate the χ^2 statistic: $\chi^2 = \sum \frac{(observed - expected)^2}{expected}$
- ✨Compute the p-value as the area in the tail above the χ^2 statistic, for a χ^2 distribution with $df = (\text{number of categories - 1})$.
- ✨Interpret the p-value in context.

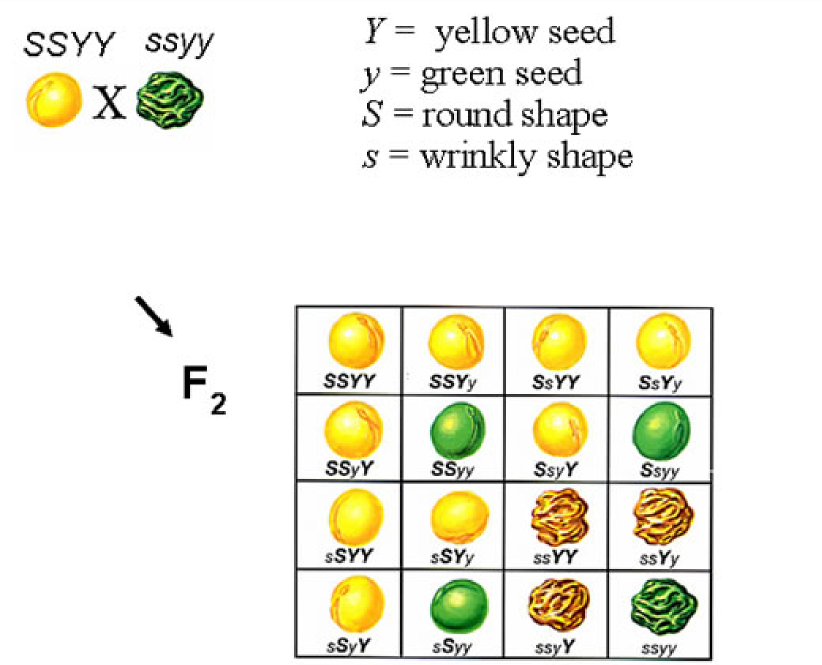
Mendel’s Pea Experiment

Between 1856 and 1863 Gregor Mendel cultivated and tested roughly 29,000 pea plants. This study showed that one in four pea plants was pure-bred recessive, two out of four were hybrid and one out of four were pure-bred dominant.

Mendel's work was rejected at first and was not widely accepted until after he died. He is now known as the “father of modern genetics"

Source: Mendel, Gregor. (1866). Versuche ber Pflanzen-Hybriden. Verh. Naturforsch. Ver. Brünn 4: 3–47 (in English in 1901, Experiments in Plant Hybridization, J. R. Hortic. Soc. 26: 1–32)

Mendel's Pea Experiment



S, Y: **Dominant** and s, y: **Recessive**

Mate SSYY with ssyy: 1st Generation: all Ss Yy

Mate 1st Generation: → 2nd Generation

Phenotype	Theoretical Proportion
Round, Yellow	9/16
Round, Green	3/16
Wrinkled, Yellow	3/16
Wrinkled, Green	1/16

Mendel's Pea Experiment

Phenotype	Theoretical Proportion	Observed Counts
Round, Yellow	9/16	315
Round, Green	3/16	101
Wrinkled, Yellow	3/16	108
Wrinkled, Green	1/16	32

Let's test this data against the null hypothesis of each p_i equal to the theoretical value, based on genetics

$$H_0 : p_1 = 9/16, p_2 = 3/16, p_3 = 3/16, p_4 = 1/16$$

$$H_a : \text{ At least one } p_i \text{ is not as specified in } H_0$$

Mendel’s Pea Experiment

What is the expected count for round green peas?

A. 0.182

B. 108

C. 104.25

D. 139

Phenotype	Theoretical Proportion	Observed Counts
Round, Yellow	9/16	315
Round, Green	3/16	101
Wrinkled, Yellow	3/16	108
Wrinkled, Green	1/16	32

▶ Click for answer

Mendel's Pea Experiment

Phenotype	Theoretical Proportion	Observed Counts
Round, Yellow	9/16	315
Round, Green	3/16	101
Wrinkled, Yellow	3/16	108
Wrinkled, Green	1/16	32

The χ^2 statistic is the sum of 4 components (because there are 4 categories). What is the contribution to the χ^2 statistic from the "Round, Yellow" category?

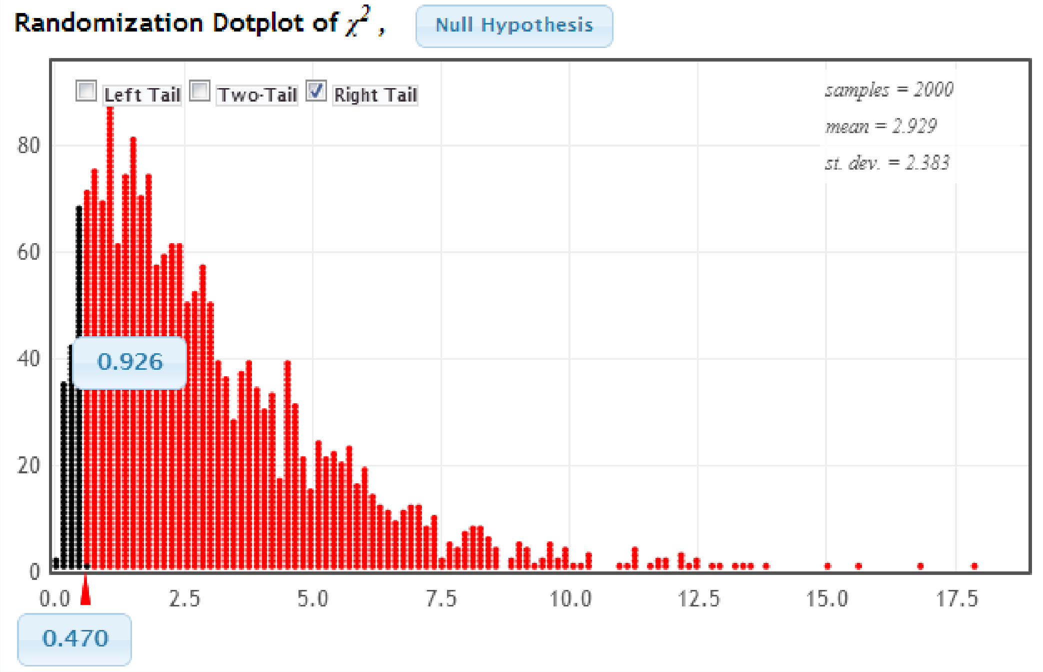
1. 0.016
2. 1.05
3. 5.21
4. 107.2

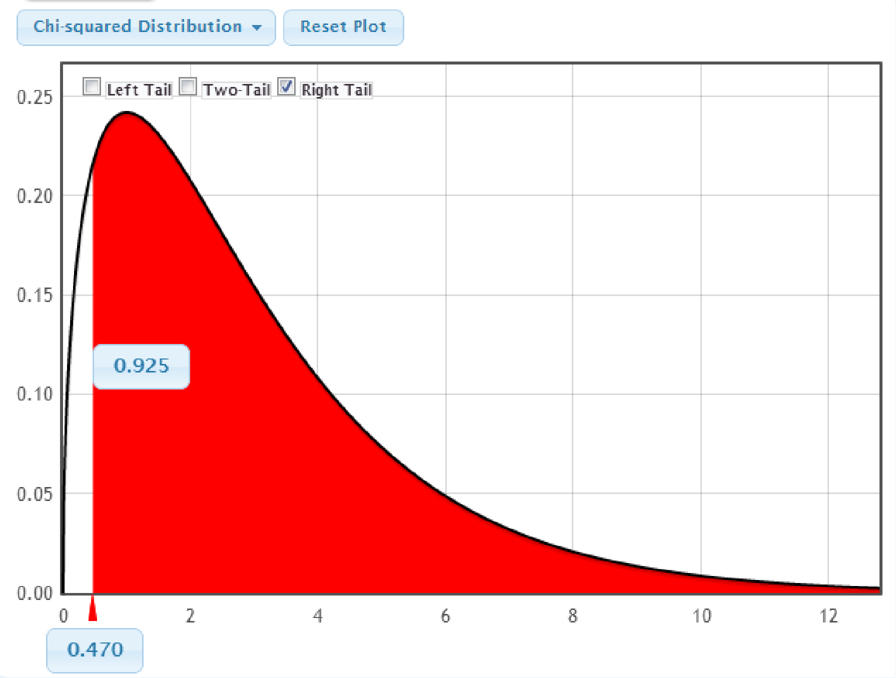
▶ Click for answer

Mendel's Pea Experiment: $\chi^2 = 0.47$

Two options:

- ✨*Simulate a randomization distribution*
- ✨*Compare to a χ^2 distribution with $df = 4 - 1$*





Mendel’s Pea Experiment

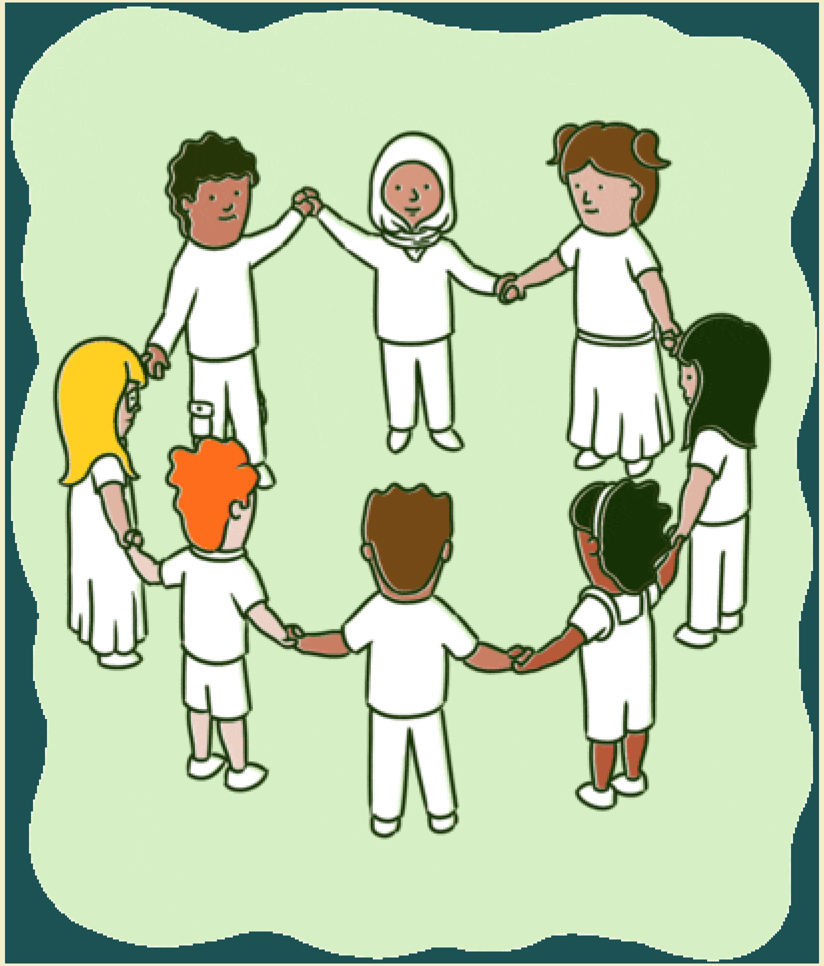
Does this prove Mendel’s theory of genetics? Or at least prove that his theoretical proportions for pea phenotypes were correct?

1. Yes
2. No

▶ Click for answer

Your Turn 1

05:00



✨Go over to the in class activity file

✨Complete the activity as much as possible