THE EFFECT PROPAGATION PROCESS (EPP): AN AXIOMATIC FOUNDATION FOR DYNAMIC CAUSALITY

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ABSTRACT

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1 Introduction

The study of cause and effect has served humankind for millennia and provides a foundation for scientific inquiry and intervention. Contemporary frameworks for computational causality, particularly those based on Directed Acyclic Graphs, offer powerful tools for reasoning within this domain. These classical models, however, are predicated on a set of core assumptions, including a fixed background spacetime, linear temporal progression, and static causal structures. However, these assumptions become a limitation when addressing a class of complex, dynamic systems where the causal relationships themselves are subject to change.

@TODO: Write new introduction@

2 History of Causality

2.1 Plato

Plato is believed to be the first to have explored the cause in a systematic way, most notably in his dialogue Timaeus[1] (c. 360 BC). In this work, Plato explains the creation of the cosmos through the actions of a divine craftsman, the Demiurge. The Demiurge looks to the eternal Forms as a model to bring order to the pre-existing, chaotic matter. Plato's concept of causality is thus tied to his broader metaphysical theory of Forms, where true causes are the eternal patterns or blueprints of which the physical world is merely a copy. He identifies several contributing factors necessary for the creation of the world, including the Demiurge (the efficient cause), the Forms (the formal cause), and the Receptacle (the material space)[1].

2.2 Aristotle

A student of Plato, Aristotle (c. 350 BC) formalized the notion of causality in his Metaphysics[2] with the "Four Causes"[3]:

1. The material cause or that which is given in reply to the question, "What is it made out of?"
2. The formal cause or that which is given in reply to the question, "What is it?". What is singled out in the answer is the essence of the what-it-is-to-be something.
3. The efficient cause or that which is given in reply to the question, "Where does change (or motion) come from?". What is singled out in the answer is the whence of change (or motion).
4. The final cause, the end purpose, is given in reply to the question, "What is its good?". What is singled out in the answer is that for the sake of which something is done or takes place.

Aristotle's framework provided a comprehensive vocabulary for analyzing causality that became foundational to Western scientific and philosophical thought for nearly two millennia.

2.3 Seneca

Seneca (c. 56 AD) argues in Letter 65[4] that cause and effect operate within a stage (space) and follow an order (time). Remove the stage or the order, and the conventional understanding of 'making something' or 'causing something' breaks down. His argument highlights time and space as indispensable prerequisites for classical causality. His focus on space and time as necessary conditions served as a precursor for physical concepts that treat spacetime as a background for causal processes.

2.4 Gottfried Wilhelm Leibniz

The idea of space and time as a background for causality, however, did not remain unchallenged. Gottfried Wilhelm Leibniz (1646–1716) rejected the concept of absolute space and absolute time as independent, fundamental constructs. Instead, Leibniz proposed[5] a relational view in which space is the simultaneous relation of coexisting things and time is the relational order of successive things. Through rigorous first principles analysis, Leibniz argued that the concept of absolute space and time was logically untenable. His relational perspective offered a significant alternative to the preeminent Newtonian worldview of his time.

2.5 John Stuart Mill

John Stuart Mill (1806-1873) shifted the focus of causality from metaphysical inquiry to a more practical, methodological problem. In his influential work, *A System of Logic, Ratiocinative and Inductive*[6], Mill developed a set of principles, now known as "Mill's Methods," to serve as a guide for discovering causal connections through empirical observation. These methods are designed to systematically eliminate non-causal factors and isolate true causes. Mill's work provided a formal basis for the inductive reasoning that underpins modern experimental science.

2.6 Albert Einstein

Albert Einstein (1879–1955) departed from Newtonian physics with his theory of general relativity[7] (GR), in which he established that space and time are one manifold, spacetime, that is bent by the gravitational influence of large masses. General relativity preserves the prerequisite of a spatiotemporal context for causality, echoing Seneca's key insight. However, the notion of a dynamic spacetime requires a dynamic view of causality to fit into the dynamic spacetime manifold.

2.7 Bertrand Russell

Bertrand Russell (1872–1970) observed that successful physics has its roots in sophisticated, law-based descriptions of how a system evolves dynamically. In modern physics, the focus is on the state of a system (e.g., position, velocity, field strength across space) and how that entire state evolves continuously and dynamically. Therefore, for Russell, the idea of classical causality, a strict happen-before relation, no longer matches the reality of modern physics. Consequently, Russell wrote in his 1912 essay "On the Notion of Cause"[8]:

The law of causality, [...], is a relic of a bygone age." - Bertrand Russell

Many modern physics laws are time-symmetric, which means that if state S1 at time t1 is related to state S2 at time t2 by a law, it is equally true that state S2 at time t2 is related to state S1 at time t1. This relationship is not a simple, linear, one-way street from a necessary "cause" to a dependent "effect." Knowing the state at any time allows you to calculate the state at any other time, past or future. Therefore, which state is the "cause" and which one is the "effect" becomes arbitrary. Russell was not opposed to causality itself; instead, his primary argument was that the traditional philosophical interpretation of causality as a fundamental, temporally asymmetric, and directed link is not what he observed in physics. His critique resonates with challenges encountered in contemporary quantum research.

2.8 Quantum Physics

Quantum field theory[9] (QFT) stands out as one of the most rigorously tested theories, with unparalleled predictive accuracy in the history of science. QFT predictions in quantum electrodynamics have been experimentally verified up to an astonishing accuracy of one part in a billion or better. The standard model of physics, built on top of QFT, despite being one of the most successful theories of all time, accurately describes three of the four known fundamental forces: electromagnetism, the strong nuclear force, and the weak nuclear force. Notably, gravity remains absent due to complex discrepancies between Einstein's (pre-quantum) theory of relativity and quantum field theory.

The unification of general relativity with quantum field theory would complete the standard model of physics, but doing so faces a non-trivial impasse, as Lucian Hardy formulated: "Quantum theory is a probabilistic theory with a fixed causal structure. General relativity is a deterministic theory, but where the causal structure is dynamic"[10]. The fundamental dynamical laws of quantum physics (excluding the weak nuclear force) are accepted as T-symmetric. However, the transition of a quantum state from a probabilistic, T-symmetric state into a definite and time-asymmetric state, while observed, is not well understood and is the subject of ongoing research in the field of quantum measurement.

The quantum superposition of states, on the other hand, is well-observed, well-documented, and, as a result, is accepted as a fundamental building block of quantum physics. Quantum superposition inspires the exploration of concepts like indefinite causal structures in theories aiming to unify quantum mechanics and gravity. Therefore, the classical conceptualization of cause and effect embedded into a background spacetime might not exist at the fundamental level of quantum gravity. In quantum gravity, space and time are not external conditions but potential emergent properties of the internal quantum structure itself. The problem is no longer whether spacetime is static or dynamic, but that spacetime itself emerges from the quantum level and thus positions itself as a higher-order effect of a generative process.

Russell saw physics moving towards laws governing states, a view echoed in quantum gravity's search for fundamental rules governing the structure from which spacetime and causal order emerge. At this stage, the understanding of causality evolved from a structure that required the existence of space toward a dynamic generative process from which causality emerges. This emergent causality does not rely on a pre-existing spacetime but is grounded in a more fundamental level of reality—a set of underlying rules (i.e., conceptualized as a 'generating function') that determines the fundamental manifestation of spatiotemporal properties. The conceptualization of this fundamental level as a "generating function" captures the idea of a quantum process from which the necessary condition of classical causality's spatiotemporal structure arises. It is a shift from asking, "What causes X, given spacetime?" to "What process generates spacetime (and thus enables X to be caused)?".

3 Related Work

The study of causality and causal inference aims to distinguish genuine cause-and-effect relationships from mere associations. Traditionally, establishing causality often relied on carefully controlled randomized controlled trials. However, significant theoretical advancements have shown that causal knowledge can be inferred from observational data by examining patterns of conditional independence among variables, given explicit assumptions [11].

3.1 Foundational Theories of Causal Inference

The endeavor to formalize and compute causal relationships draws upon several influential theoretical frameworks. Understanding these foundations is crucial for situating contemporary advancements and appreciating the nuances of different approaches to causal reasoning.

3.1.1 Directed Acyclic Graphs (DAGs)

A foundational framework for representing causal structures is based on graphical causal models, most notably Directed Acyclic Graphs (DAGs) [12, 13, 14, 15]. In these models, variables are typically represented by nodes, and directed edges indicate direct causal influences [13]. The impact of interventions, conceptualized by operators like the `do`-operator which sets a variable's value independently of its usual causes, can be analyzed within this framework to predict outcomes under hypothetical scenarios [13, 16]. The theoretical underpinnings of Structural Causal Models (SCMs), which are closely related to graphical models, have been extensively studied [17, 18, 19, 20, 21]. Methods exist for handling complex scenarios, including incorporating latent variables [22, 23] and understanding the relationship between different causal models [24, 11]. Policy interventions in specific graphical structures, such as Lauritzen-Wermuth-Freydenburg (LWF) latent-variable chain graphs, have also been investigated [25]. This includes work providing a novel identification result for effects of policy interventions in these graphs [25].

3.1.2 Structural Causal Model (SCM)

The dominant paradigm in modern computational causality is arguably the Structural Causal Model (SCM) framework, extensively developed by Judea Pearl and his colleagues [16]. An SCM consists of a set of variables, some of which are designated as exogenous (external, uncaused within the model) and others as endogenous (their values are determined by other variables within the model). The relationships between these variables are represented by a set of structural equations, typically of the form $X_i = f_i(\mathbf{PA}_i, U_i)$, where $\mathbf{PA}_i$ are the direct causal parents of X_i in the associated causal graph, and U_i are exogenous error terms representing unmodeled influences or inherent stochasticity. These structural equations are considered to represent autonomous, invariant causal mechanisms. Graphically, SCMs are typically depicted using Directed Acyclic Graphs (DAGs), where nodes represent variables and directed edges $X_j \to X_i$ indicate that X_j is a direct cause of X_i (i.e., $X_j \in \mathbf{PA}_i$). This graphical representation provides an intuitive way to encode causal assumptions and to determine statistical independencies via the criterion of *d-separation*.

A cornerstone of Pearl's framework is the *do-calculus*, a set of three axiomatic rules that allows for the inference of the effects of interventions from a combination of observational data and the causal graph structure, even when direct experimentation is not possible [16]. An intervention, denoted $do(X_j = x'_j)$, represents an external manipulation that sets the variable X_j to a specific value x'_j, thereby severing the links from its original parents $\mathbf{PA}_j$ and altering the system's natural dynamics. The ability to calculate post-intervention distributions, $P(Y|do(X = x))$, is central to predicting the consequences of actions and policies. Bayesian Networks, which are DAGs coupled with conditional probability distributions $P(X_i|\mathbf{PA}_i)$, are closely related to SCMs and are often used to represent the observational probability distribution $P(\mathbf{X})$ entailed by an SCM under specific assumptions about the error terms U_i. They provide a powerful tool for probabilistic inference under passive observation, but require the *do-calculus* or similar interventional logic to reason about causal effects.

3.1.3 Counterfactuals: What if?

While discovering causal structures and predicting the effects of interventions ("What if we do $X = x$?") are fundamental tasks in causal inference, the ability to compute counterfactual queries represents a deeper and often more insightful level of causal reasoning [16]. Counterfactuals address questions about alternative realities or "what might have been" ("What if X had been x', given that we observed $X = x$ and $Y = y$?"). This form of reasoning is crucial for tasks such as understanding individual responsibility, learning from past mistakes, diagnosing failures, and fine-tuning policies. It requires moving beyond population-level effects of interventions to consider specific individuals or units in specific factual circumstances [16, 26].

Within Pearl's Structural Causal Model (SCM) framework, computing a counterfactual, denoted as $Y_x(u)$ (the value Y would have taken in unit u had X been x), involves a three-step algorithmic process [16]:

1. **Abduction:** Use the available factual evidence (e.g., observed values of some variables) to update the probability distribution over the exogenous variables U. This step accounts for the specific unit or situation under consideration by inferring the background conditions consistent with the observed facts.
2. **Action:** Modify the original SCM by replacing the structural equation for the counterfactual antecedent X with $X = x'$ (the hypothetical condition), effectively performing a "mini-surgery" on the model as in the *do*-calculus. The equations for other variables remain unchanged, reflecting the principle that interventions only alter the targeted mechanism directly.
3. **Prediction:** Compute the probability of the counterfactual consequent Y using the modified model and the updated distribution of U (from the abduction step). This yields the probability $P(Y_{x'} = y'|\text{evidence})$.

This process allows for a principled way to reason about hypothetical scenarios that differ from what was actually observed, effectively comparing parallel possible worlds [16, 27]. Foundational work also explored the bounding and identification of specific types of counterfactual queries related to probabilities of causation [28]. Formal systems like Pearl's *do*-calculus provide tools for determining if causal effects under intervention are identifiable from observational data [29], and algorithms exist to automate this process [30], which are often prerequisite steps before full counterfactual queries can be comprehensively addressed.

The extension of these concepts to practical applications and more complex settings remains an active area of research. For instance, model-agnostic approaches aim to enable counterfactual reasoning without full specification of the SCM, particularly in dynamic environments where systems evolve over time. Furthermore, the domain of causal bandits, which focuses on online decision-making and learning under uncertainty, increasingly incorporates causal background knowledge and aspects of counterfactual reasoning to optimize sequences of actions and learn policies more efficiently than purely correlational reinforcement learning approaches [31, 32, 33, 34]. The capacity for counterfactual reasoning thus forms a critical component of advanced intelligent systems that can not only predict and act, but also reflect, learn, and adapt based on a deep understanding of cause and effect in alternative scenarios.

More recent work explores model-agnostic approaches to counterfactual reasoning, particularly in dynamic environments [35], and investigates optimizing treatment effects in such settings [36]. Causal bandits also incorporate causal background knowledge into online decision-making problems [31, 32, 33, 34].

3.1.4 Potential Outcomes

Alongside SCMs, Potential Outcomes Framework, also known as the Rubin Causal Model (RCM) [37], offers another rigorous foundation for causal inference, with early conceptualizations by Neyman [38] and formally developed for observational studies by Rubin [39]. It has been particularly influential in statistics, econometrics, and the social sciences. This framework defines the causal effect of a treatment (or exposure) on an individual unit by considering the potential outcomes that unit would exhibit under different treatment assignments. For a binary treatment $T \in \{0, 1\}$, each unit i is conceptualized as having two potential outcomes: $Y_i(1)$, the outcome if unit i receives the treatment, and $Y_i(0)$, the outcome if unit i receives the control. The individual treatment effect (ITE) is then $Y_i(1) - Y_i(0)$. A core challenge, often termed the "fundamental problem of causal inference," is that only one of these potential outcomes can be observed for any given unit [37].

Inference in this framework hinges on crucial assumptions, such as the Stable Unit Treatment Value Assumption (SUTVA), which posits no interference between units and well-defined treatment versions. When all confounders are believed to be observed, the key assumption is **ignorability** (or unconfoundedness), which states that treatment assignment is independent of potential outcomes, conditional on the observed covariates [40]. Under this assumption, causal effects can be estimated using methods like matching, stratification, or inverse probability weighting based on propensity scores [40].

In many settings, the belief that all confounders have been measured is not plausible. To address this, applied researchers have developed a powerful toolkit of **quasi-experimental methods** that can enable causal identification even in the presence of unobserved confounding. These methods are cornerstones of modern computational causality and include:

- **Instrumental Variables (IV):** This technique is used to handle unobserved confounding by leveraging an "instrument": a variable that is correlated with the treatment but is not causally related to the outcome except through its effect on the treatment. The instrument provides a source of exogenous variation in the treatment, allowing for the estimation of a causal effect that is not biased by the unobserved confounders [41].
- **Difference-in-Differences (DiD):** Leveraging panel data (observations of the same units over time), DiD estimates the effect of a treatment by comparing the change in the outcome for a treated group before and after an intervention to the change in the outcome for an untreated group over the same time period. This method controls for unobserved confounders that are constant over time by differencing them out[42].
- **Regression Discontinuity Design (RDD):** RDD is applicable when the treatment is assigned based on a sharp cutoff in a continuous variable (the "running variable"). By comparing the outcomes of units just below the cutoff to those just above it, RDD can provide an unbiased estimate of the local causal effect at the threshold, mimicking a randomized experiment in a narrow window around the cutoff [43].

While SCMs provide an explicit language for encoding causal mechanisms, the Potential Outcomes framework, supported by these robust quasi-experimental methods, provides a powerful applied toolkit for estimating causal effects from complex observational data.

3.2 Invariant Prediction and Out-of-Distribution Generalization

A central challenge in modern machine learning remains robust out-of-distribution (OOD) generalization, as models trained under the standard I.I.D. assumption often fail when deployed in new or shifting environments. A powerful approach to this problem is rooted in the causal principle of invariance: the idea that while statistical correlations can be spurious and brittle, true causal mechanisms remain stable across different contexts [17].

This principle has been operationalized into a formal framework for both causal discovery and robust prediction. The seminal work in this area is Invariant Causal Prediction (ICP), developed by Peters, Bühlmann, and Meinshausen [44]. The ICP framework leverages data from multiple distinct "environments" or "settings." It posits that a set of variables constitutes the direct causes of a target if and only if the conditional distribution of the target given those variables remains invariant across all environments. By searching for a set of predictors that yields such a stable predictive model, ICP can identify causal relationships and produce a model that is robust to the types of distributional shifts observed during training.

This idea has been influential in the deep learning community, inspiring methods aimed at learning invariant representations. A prominent example is Invariant Risk Minimization (IRM), which seeks to learn a data representation such that the optimal classifier on top of that representation is the same for every training environment [45]. The goal is to isolate invariant causal features from spurious, environment-specific correlations. Other related approaches, such as Risk Extrapolation (REx) [46], also aim to improve OOD performance by enforcing penalties on models whose performance is unstable across environments. Collectively, this body of work formalizes the intuition that a model based on causal structure should generalize better than one that merely interpolates the training data, representing a major school of thought in building more reliable and robust machine learning systems.

3.3 Causal Discovery

A central task in the field is causal discovery, which focuses on learning the causal structure, represented by the graph, from observed data alone. A comprehensive survey categorizes existing methods for causal discovery on both independent and identically distributed (I.I.D.) data and time series data, including approaches for both types of data. According to this survey, categories include Constraint-based, Score-based, FCM-based, Hybrid-based, Continuous-Optimization-based, or Prior-Knowledge-based. Constraint-based methods infer relationships by testing for conditional independencies in the data [14, 47, 48]. Score-based methods search over potential graph structures and evaluate them based on how well they fit the data, often including a penalty for complexity [49]. The KGS method [50], for example, leverages prior causal information such as the presence or absence of a causal edge to guide a greedy score-based causal discovery process towards a more restricted and accurate search space. It demonstrates how incorporating different types of edge constraints can enhance both accuracy and runtime for graph discovery and candidate scoring, concluding that any type of edge information is useful. This method relates to the KCRL framework [50]. Continuous optimization techniques formulate causal discovery as an optimization problem, potentially involving differentiable approaches that can handle constraints like acyclicity. The NOTEARS framework is one such example [51], and studies have analyzed its performance and proposed post-processing algorithms to enhance its precision and efficiency. A study provides an in-depth analysis of the NOTEARS framework for causal structure learning, proposing a local search post-processing algorithm that significantly increased the precision of NOTEARS and other algorithms [52].

3.4 Causal Inference and Discovery for (Hyper)graphs

The representation of causal relationships via graphical models, predominantly Directed Acyclic Graphs (DAGs) as foundational to Structural Causal Models (SCMs) [16], is a cornerstone of computational causality. Much research has focused on discovering these graph structures from observational or interventional data (causal discovery) and subsequently estimating causal effects based on the identified graph (causal inference). While traditional methods often assume simpler pairwise relationships, the inherent complexity of many real-world systems necessitates considering more intricate relational structures.

Recent work has begun to explicitly tackle causal inference in settings involving multi-way interactions best represented by hypergraphs. Ma et al. [53] directly address the problem of estimating Individual Treatment Effects (ITE) on hypergraphs, specifically accounting for high-order interference where group interactions (modeled by hyperedges) influence individual outcomes. Their proposed HyperSCI framework leverages hypergraph neural networks to model these spillover effects and uses representation learning to control for confounders, demonstrating the utility of explicitly considering hypergraph topology for ITE estimation from observational data. This represents a significant step beyond assuming only pairwise interference, which is common in ordinary graph-based causal inference. While the work by Ma et al. focuses on statistical ITE estimation on a *given* hypergraph, it highlights the increasing recognition of hypergraph structures as vital for certain causal problems.

The broader field of graph mining and network science also provides a rich backdrop, with techniques for link prediction and understanding influence spread, though these often operate at a correlational level rather than a strictly causal one. The challenge remains to bridge network science concepts with formal causal reasoning in these complex relational systems.

3.5 Causal Inference for Time Series

Causal inference for time series data introduces a unique set of challenges and opportunities compared to static, cross-sectional settings. The inherent temporal ordering of observations provides strong, intuitive information about potential causal directionality—causes generally precede their effects—but also necessitates methods that can handle auto-correlation, non-stationarity, feedback loops, and varying time lags in causal influences.

A foundational concept in this domain is Granger Causality, originally developed by Clive Granger for economic time series [54, 55]. A time series X_t is said to Granger-cause another time series Y_t if past values of X_t contain information that helps predict future values of Y_t beyond the information already contained in past values of Y_t itself. This is typically tested using vector autoregression (VAR) models and statistical tests on the coefficients of lagged variables [56, 57]. While widely applied, standard Granger causality is primarily about predictive improvement and may not always align with true mechanistic causation, especially in the presence of unobserved confounders, instantaneous effects, or non-linear relationships. Extensions and refinements have been developed to address some of these limitations, including non-linear Granger causality tests and methods incorporating multivariate information criteria.

To explicitly model evolving causal relationships and dependencies over time, dynamic graphical models have been developed. The Dynamic Uncertain Causality Graph (DUCG) [58] is one such framework, specifically designed to represent and reason about causal relationships that themselves change as a system evolves. DUCGs find applications in complex dynamic systems, such as fault diagnosis in nuclear power plants where understanding the temporal progression of component failures is critical [59, 60]. These models often aim to unify diagnostic reasoning (what caused an observed state?) with treatment or control strategies (what intervention will lead to a desired future state?) [61].

More recently, deep learning techniques have been increasingly applied to causal discovery and inference in time series. For example, the Time-Series Causal Discovery Framework (TCDF) utilizes attention-based convolutional neural networks to learn causal relationships, explicitly trying to identify relevant time lags and dependencies [62]. Research in this direction often focuses on challenges such as optimizing hyperparameters for these complex models, ensuring robustness to varying noise levels and non-stationarities in the data, improving the interpretability of attention mechanisms to understand which past events are deemed causally salient, and developing robust causal validation methods beyond simple predictive accuracy. Other important research avenues in time series causality include:

- **Handling Unobserved Confounders:** Just as in static settings, unobserved common causes can induce spurious relationships between time series. Methods that attempt to detect or adjust for such confounding, perhaps using instrumental variable approaches adapted for time series or by searching for specific types of conditional independencies, are crucial.
- **State-Space Models and Causal Inference:** Integrating causal concepts with state-space models (e.g., Kalman filters and their non-linear extensions) allows for reasoning about causality between latent (unobserved) states as well as observed variables.
- **Interventional Time Series Analysis:** Developing methods to estimate the effect of specific interventions applied at certain points in time on the future trajectory of one or more time series. This is vital for policy evaluation and system control.
- **Causal Discovery from Irregularly Sampled or High-Dimensional Time Series:** Many real-world time series (e.g., medical patient data, sensor networks) are not regularly sampled or involve a very large number of variables, posing challenges for traditional methods.
- **Information-Theoretic Approaches:** Methods like Transfer Entropy [Schreiber, 2000, *Measuring information transfer*] provide a non-parametric way to quantify directed information flow between time series, offering an alternative perspective to Granger causality, especially for detecting non-linear interactions.

The temporal dimension thus adds significant complexity but also provides a powerful constraint (time ordering) that can be leveraged for causal reasoning, making this a vibrant and critical area of ongoing research.

3.6 The Role of Context in Causal Inference

While foundational causal frameworks like SCMs implicitly allow for conditioning variables, the explicit, structured, and dynamic modeling of *context* as a multi-faceted entity is a growing area of focus, crucial for applying causal inference to complex, real-world systems. The Jiao et al. survey [63] highlights numerous deep learning applications where contextual understanding is paramount, from visual commonsense reasoning to multimodal interactions, and notes the challenges posed by contextual shifts and confounders.

Berrevoets et al. [64, 65] propose a conceptual "map of causal deep learning" (CDL) that explicitly incorporates dimensions for structural knowledge, parametric assumptions, and significantly, a temporal dimension. They argue that time is not merely another variable but introduces unique considerations in causal settings, such as the fundamental principle that causes precede effects, and the potential for feedback loops or evolving relationships in dynamic systems. Their framework aims to help researchers and practitioners categorize CDL methods based on how they handle these dimensions, including whether they operate on static data or explicitly model temporal dynamics. For instance, they differentiate models based on whether they assume "no structure," "plausible causal structures" (often derived from statistical independencies), or a "full causal structure" as input, and similarly categorize parametric assumptions from non-parametric to fully known factors. The temporal axis distinguishes between static models and those designed for time-series data where variables are observed repeatedly. While this work by Berrevoets et al. primarily offers a *taxonomy and conceptual guide* for the emerging field of CDL rather than a specific implemented reasoning engine, it underscores the increasing recognition of structured context, and especially temporality, as a first-class concern in bridging deep learning with robust causal inference. This emphasis on temporal context aligns with established work in time series causality, such as Granger causality [54] and dynamic graphical models like DUCGs [58], which inherently focus on how relationships evolve over time. However, modern approaches, including those at the intersection of deep learning and causality, seek richer representations of temporal context beyond simple lagged variables. For example,

methods like TCDF [62] attempt to learn relevant temporal dependencies and attention patterns. The challenge remains to develop frameworks that can uniformly reason over diverse types of contextual information (static attributes, explicit temporal sequences, spatial relationships (both Euclidean and non-Euclidean), and even abstract conceptual states) and integrate this rich contextual understanding directly into the causal reasoning process. The ability to model multiple, potentially interacting contexts, and to allow these contexts to be dynamically updated, is key to building causal AI systems that can adapt to real-world complexities.

3.7 Hierarchical Causality

A significant frontier in causal inference is its extension to handle hierarchical or nested data structures, a common feature in fields from social science to biology. While hierarchical Bayesian modeling is a standard statistical tool for such data, causal modeling has traditionally forced a difficult choice: either aggregate the data to the unit level, losing valuable information, or ignore the group structure, risking incorrect inferences. Recent work by Weinstein and Blei[66] formalizes this problem by introducing Hierarchical Causal Models (HCMs). They extend Structural Causal Models (SCMs) by incorporating the concept of plates from graphical modeling to explicitly represent the nested structure. The central and powerful insight of their work is that disaggregating data and modeling the hierarchy can enable causal identification even in situations where it would be impossible with "flat" or aggregated data. For instance, with an unobserved unit-level confounder (e.g., a school's budget), the within-unit variation (e.g., student-level randomization) provides a "natural experiment" that can be leveraged to control for the confounder. To operationalize Hierarchical Causal Models, they develop a systematic, nonparametric identification procedure that extends the do-calculus. The HCM framework provides a formal causal justification for many existing methods, such as fixed-effects and difference-in-difference models, which can be seen as specific parametric instances of an HCM. Their work provides a broad and rigorous toolkit for analyzing cause and effect in multi-level systems, formally connecting the principles of hierarchical modeling with the inferential power of graphical causal models.

3.8 (Geometric) Deep Learning for Causal Inference and Representation

Deep learning has achieved remarkable success in various tasks,such as neural architecture search [67, 68] and techniques for handling complex relationships in data, such as those explored using hypergraphs [69, 70]. Hypergraphs, introduced by Berge in 1973 [70], can model multi-way relationships and have found applications in areas like visualization [71, 72], partitioning [73, 74, 75], and recommender systems [76, 77, 78, 79, 80, 81]. Link prediction, particularly in multiplex networks, is another active area where deep learning is applied [82, 83].

The intersection of deep learning with causal inference is a rapidly expanding research area, aiming to leverage the expressive power of neural networks to address challenges in causal representation learning, discovery, and effect estimation [84, 63]. Many approaches focus on adapting deep learning architectures to better estimate treatment effects from observational data, often by learning balanced representations of covariates to mitigate confounding bias or by modeling complex response surfaces. Ramachandra [85] proposes the use of deep autoencoders for generalized neighbor matching to estimate ITE, focusing on dimensionality reduction while preserving local neighborhood structure, and also suggests using Deep Neural Networks (DNNs) for improved propensity score estimation (PropensityNet). These methods exemplify the application of standard deep learning architectures to enhance specific statistical tasks within the potential outcomes framework, typically under assumptions such as the Stable Unit Treatment Value Assumption (SUTVA), which precludes interference.

A notable direction involves incorporating prior causal knowledge into deep generative models, enabling the generation of data that respects a given causal graph. While combining causal discovery with generative modeling is a goal, these methods are often constrained by the fundamental limitations of causal discovery [14]. Specific efforts include incorporating causal graphical prior knowledge into predictive modeling [86] and matching learned causal effects with domain priors in neural networks [87]. Applications in finance have also utilized informed machine learning frameworks based on a priori causal graphs for prediction tasks. The area of causal reinforcement learning and causal bandits also represents significant related work in combining causality with learning agents that interact with environments [31, 32, 33, 34, 88, 89].

More fundamentally, researchers are exploring how geometric deep learning principles can inform the design of causal models capable of handling complex data structures and respecting informational constraints. Acciaio et al. [90] introduce a "universal causal geometric DL framework," featuring the Geometric Hypertransformer (GHT). Their work is concerned with the universal approximation of causal maps between discrete-time path spaces, which may be non-Euclidean metric spaces such as Wasserstein spaces, while strictly respecting the forward flow of information inherent in causal processes. The GHT employs hypernetworks to adapt its parameters over time and aims to provide theoretical guarantees for approximating Hölder continuous functions between these complex spaces. Although highly

theoretical and with a focus on applications in stochastic analysis and mathematical finance, this line of work signifies a deep engagement with geometric structures, non-Euclidean spaces, and transformer-like attention mechanisms within a causal learning context. Their "geometric attention mechanism" operating on Quantizable and Approximately Simplicial (QAS) spaces represents a sophisticated approach to handling non-Euclidean output geometries. A survey by Jiao et al. [63] details various methods where deep learning is applied to causal discovery (e.g., leveraging neural networks for GraN-DAG or extensions of NOTEARS) or to augment specific causal inference tasks within existing deep learning modalities (e.g., developing causal attention mechanisms in computer vision, or applying causal methods to Graph Neural Networks). This body of work collectively seeks to imbue deep learning models with a degree of causal awareness or to use their representational power to overcome limitations in traditional causal inference techniques. The ongoing challenge is to move beyond enhancing specific sub-tasks towards building more integrated and principled frameworks for comprehensive causal reasoning using deep learning.

3.9 Causal Representation Learning

Causal Representation Learning (CRL) has emerged as a field dedicated to this problem, aiming to learn latent representations that are not just statistically useful but are also causally meaningful [91]. The central goal is to learn a mapping from high-dimensional observations to a latent space where the dimensions correspond to independent, underlying causal factors. Early efforts in deep learning focused on learning "disentangled" representations, with the hope that unsupervised methods could automatically separate data into its factors of variation. However, foundational work by Locatello et al. demonstrated that learning disentangled representations without inductive biases is theoretically impossible [92]. This finding has spurred the CRL community to propose that causal structure is the necessary inductive bias to achieve meaningful and identifiable disentanglement. The core assumption is that real-world changes often arise from interventions on a sparse set of high-level causal mechanisms. A model that captures these mechanisms in its latent space should therefore be more robust and generalize better out-of-distribution. A key technical challenge in this area is the identifiability of the latent causal variables—ensuring that the learned representation is unique and corresponds to the true underlying causal structure.

3.10 The Causaloid Framework

A significant effort towards establishing a framework for probabilistic theories with dynamic causal structure was presented by Hardy [93]. Hardy proposes a framework aimed at unifying quantum theory (QT) and general relativity (GR) as a step towards quantum gravity (QG). The core of this unification lies in a generalized theory of causality, capable of describing both the probabilistic nature and fixed causal structure of QT, as well as the deterministic nature and dynamic causal structure of GR, within a single formalism. The core of this new framework of unified causality is the "causaloid," a mathematical object designed to encapsulate all information about the causal relationships within a physical system. The framework begins from an operational standpoint, focusing on "recorded data" which consists of "actions" and "observations" associated with "elementary regions" of spacetime.

The causaloid itself is a theory-specific mathematical entity, primarily represented by a collection of "lambda matrices" (Λ). These matrices quantify how the complexity of describing a composite region (specifically, the number of fiducial measurements needed to determine its state) is reduced due to causal connections between its component elementary regions. Associated with any region R and an experimental procedure F_R resulting in outcome X_R are "r-vectors," denoted $r(X_R, F_R)(R)$, which are analogous to operators in QT [93]. A key innovation is the "causaloid product," which is governed by the causaloid (via the lambda matrices). This product combines r-vectors of sub-regions to form the r-vector for a composite region, e.g., $r(R_1 \cup R_2) = r(R_1)\hat{}r(R_2)$. This product aims to unify the different ways systems are composed in QT, such as tensor products for space-like separated systems and sequential (matrix) products for timelike evolutions. Probabilities for joint outcomes, conditioned on experimental settings, are then derived from these r-vectors. A crucial feature of the causaloid formalism is that it does not impose a fixed causal structure or a background time a priori. Instead, the causal relations are implicitly defined by the causaloid itself.

Hardy demonstrated how both classical probability theory and quantum theory can be cast within this framework, with the differences between theories being encoded entirely in the specification of their respective causaloids. The ultimate aim is to provide a structure wherein the dynamic causal aspects of GR can be consistently combined with the probabilistic nature of QT. Hardy also introduces "causaloid diagrams" as a visual tool to represent and compute the causaloid based on local lambda matrices for nodes (elementary regions) and links (pairwise connections), particularly under simplifying assumptions met by QT and classical probability [93].

3.11 Computational Causality Libraries

A vibrant ecosystem of Python libraries has emerged over time, providing tools for various aspects of causal inference, discovery, and analysis. These libraries typically build upon foundational causal theories and aim to make causal methods accessible to data scientists, researchers, and engineers. This report summarizes several key libraries shaping the Python landscape for computational causality.

3.11.1 DoWhy (Microsoft)

Developed by Microsoft Research, DoWhy[1] [94] is perhaps one of the most well-known libraries aiming to provide an end-to-end workflow for causal inference. Its philosophy centers on explicitly separating the causal modeling assumptions from the statistical estimation steps, adhering to a four-stage process: 1) Modeling the causal assumptions (often using graphical models), 2) Identifying the target causal estimand based on the model, 3) Estimating the causal effect using appropriate statistical methods (like propensity scores, regression, instrumental variables), and 4) Refuting the obtained estimate through robustness checks. DoWhy aims to unify concepts from both Pearl's Structural Causal Models (SCMs) and the Potential Outcomes framework. It integrates with other libraries like EconML and CausalML for specific estimation tasks and is designed to be a general-purpose tool for applied causal analysis.

3.11.2 EconML (Microsoft)

EconML[2] [95] focuses specifically on estimating heterogeneous treatment effects (HTE) – understanding how the effect of an intervention or treatment varies across different individuals or subgroups. It heavily leverages machine learning techniques to model complex conditional outcome expectations and propensity scores while incorporating causal identification strategies to ensure the validity of the effect estimates. Key methodologies implemented include Double Machine Learning (DML), Orthogonal Random Forests, Deep Instrumental Variables (DeepIV), and various "meta-learners" (S-learner, T-learner, X-learner) that adapt standard ML models for causal effect estimation. Meanwhile, DoWhy and EconML have been brought together in the PyWhy[3] project to foster industry wide collaboration.

3.11.3 CausalML (Uber)

Developed initially at Uber, CausalML[4] [96] is another library primarily focused on treatment effect estimation and, notably, uplift modeling. Uplift modeling specifically aims to estimate the incremental impact of an intervention on an individual's behavior – identifying who would be positively influenced by an action (e.g., receiving a promotion) compared to doing nothing. CausalML provides implementations of various uplift algorithms, including tree-based methods (causal trees/forests) and meta-learners similar to those in EconML. It's geared towards practical industry applications, especially in customer relationship management (CRM) and marketing, where optimizing interventions based on predicted individual uplift is a key objective.

3.11.4 CausalNex (McKinsey)

CausalNex[5] takes a different approach, focusing more strongly on causal discovery and the use of Bayesian Networks for causal reasoning. It provides tools to learn causal graph structures from data, potentially incorporating domain knowledge to constrain the search space. It implements structure learning algorithms (like NOTEARS) and allows users to fit Bayesian Networks to the data based on the learned (or provided) graph structure. Once the network is built, users can perform queries (e.g., conditional probability queries, interventions via the do-calculus if the graph assumptions hold) to understand relationships and simulate scenarios within the modeled system. CausalNex is particularly useful for exploring and visualizing complex systems where understanding the network of causal influences is a primary goal.

[1] https://github.com/py-why/dowhy
[2] https://github.com/py-why/EconML
[3] https://www.pywhy.org
[4] https://github.com/uber/causalml
[5] https://github.com/mckinsey/causalnex

3.11.5 Meridian (Google)

Meridian[6] is a marketing mix model (MMM) that uses Bayesian causal inference methods to offer better insights into online and offline marketing channels. Meridian provides methodologies to support calibration of MMM with experiments and other prior information, and to optimize target ad frequency by utilizing reach and frequency data.

3.12 Causal Inference with Large Language Models (LLMs)

The intersection of causal inference and Large Language Models (LLMs) has emerged as a vibrant and rapidly developing research frontier. Foundational work has mapped out the potential for using LLMs as interactive causal knowledge engines, capable of answering queries about causal relationships, interventions, and counterfactuals by drawing on the vast information embedded in their training data [97]. This opens up the possibility of automating parts of the causal modeling process that have traditionally been highly manual. However, a fundamental question shadows this potential: whether LLMs, trained on vast quantities of observational and correlational text, can distinguish true causation from spurious association. Research specifically investigating this issue has shown that LLMs often struggle to infer causation correctly when faced with scenarios where correlation and causation are deliberately misaligned, highlighting a significant risk of the models simply repeating the statistical patterns in their training data [98]. To address this challenge, the research community has focused on creating structured and comprehensive benchmarks to systematically assess the causal reasoning capabilities of LLMs. For instance, the CLADDER framework was developed to generate complex, language-based causal problems from underlying structural causal models, allowing for a controlled and fine-grained analysis of model performance and failure modes [99]. In a similar vein, CausalBench provides another comprehensive benchmark suite designed to evaluate a wide array of causal reasoning skills, from basic causal discovery to complex counterfactual inference [100]. Collectively, this body of work indicates that while LLMs show promise, they are not yet reliable causal reasoners.

3.13 In-context Causal Reasoning with Large Language Models

Recent advancements have seen the application of large-scale language models (LLMs) to tasks in causal inference, leveraging their ability for in-context learning. This emergent capability allows models to perform causal reasoning on the fly, based on the provided context, without requiring parameter updates. A notable direction of this research is in-context counterfactual reasoning. Miller, Schölkopf, and Guo[101] demonstrate that language models can perform counterfactual reasoning in a controlled synthetic environment. Their work suggests that for a wide variety of functions, counterfactual reasoning can be reduced to a transformation of in-context observations. The authors find that self-attention, model depth, and data diversity are key drivers of this capability in transformer architectures. This line of inquiry extends to sequential data, providing preliminary evidence for the potential of counterfactual story generation. Building on the concept of in-context learning, Schölkopf et al.[102] have proposed Do-PFN, a pre-trained foundation model designed to predict interventional outcomes from purely observational data. Their model is pre-trained on a wide variety of synthetic causal structures, which enables it to meta-learn how to perform causal inference. Do-PFN has shown strong performance in estimating causal effects, even without knowledge of the underlying causal graph, a significant departure from traditional methods that require such information.

3.14 Causal Inference at Industry Scale

An open challenge for the practical application of causal inference remains scalability. In enterprise environments such as Netflix, Google, and Meta, causal questions must be answered using datasets with millions or billions of observations. Causal inference at scale has exposed a significant gap between theoretical algorithms and practical feasibility due to two known limitations:
The first is causal discovery, where the goal is to learn the graph structure from data. Score-based search methods are generally NP-hard in the worst case [49], and even faster constraint-based methods like the PC algorithm face a combinatorial explosion of required conditional independence tests in dense or high-dimensional graphs [103].
The second is in causal effect estimation, especially in the presence of high-dimensional confounding. To address this, a significant body of work has emerged around Double/Debiased Machine Learning (DML) [104]. The DML framework provides a recipe for using powerful, arbitrary machine learning models to flexibly control for a large number of confounders without introducing bias into the final treatment effect estimate. This line of research is explicitly motivated by the need to apply causal inference in settings where the number of variables makes traditional statistical methods intractable. In response to these challenges, a major engineering focus has been the development of causal inference platforms. These internal systems are designed to automate and scale causal analyses, enabling data scientists to run thousands of experiments and quasi-experiments reliably and efficiently.

[6] https://github.com/google/meridian

4 Motivation

In his seminal 1985 essay, "Programming as Theory Building," [105], Turing Award winner Peter Naur argued that the act of creating a significant software system is not one of mere coding, but of building a deep, explanatory theory. The source code is just a formal notation; the real program is the mental model of how it works and, more importantly, why it was built that way. This monograph is a deliberate attempt to use programming and theory building as a joint force to apply the Naur principle synergetically where the implementation imposes new challenges on the theory and the theory guides a stronger, more principled implementation.

The pre-existing philosophy of causality served mankind for millennia, and one might be tempted to conclude that this is all there is to know about cause and effect. However, the origin of the Effect Propagation Process (EPP) did not start in philosophy, but in three distinct problems. The first problem is related to non-Euclidean data representation, the second problem is rooted in causal inference over non-linear time representations, and the third in handling dynamic causal structures.

4.1 Non-Euclidean representation

The first problem applies equally to computational causality and deep learning; therefore, it is best illustrated with familiar large language models. Language embeddings remain foundational to contemporary large language models (LLMs), but these require a reduction into a vector space because many prevalent LLM architectures operate efficiently in vector spaces, thus making the reduction from non-Euclidean realms (language) into a Euclidean representation (Vector space) mandatory. Instead of advancing LLM architectures to natively handle non-Euclidean representations, the industry has focused on leveraging Vector databases for storing and retrieving embeddings derived from LLMs. Graph neural networks operate on non-Euclidean spaces, but as these are not yet commonly adopted as core components in mainstream LLM architectures, the problem prevails.

4.2 Non-linear Time

However, when generalizing space beyond Euclidean, then the second problem emerges: how to represent time? More profoundly, can we separate time and space from data? Out of this insight, the idea emerged to separate space, time, and data into a separate context usable by multiple models. As space and time were elevated from an implicit background into an explicit first-class context representation, then moving beyond correlation towards causality became the next obvious step. At this moment, a profound problem emerged: When space is non-Euclidean, and time might not be a simple linear progression, then how do we establish a causal relationship? As it turned out, establishing a clear causal relationship became problematic within the classical definition of causality, which fundamentally relies on a linear time-asymmetric ordering (cause preceding effect) within its assumed background spacetime.

One might challenge the presumption of non-linear time progression, but in complex systems with dynamic feedback loops, it's perfectly possible to see context structures that entail non-linear time regions. Non-linear time regions can occur when the background time is represented as a temporal hypergraph that holds multiple time resolutions simultaneously. The simultaneous presence of time units at different scales breaks the simple time-linear assumption (all time has the same unit, and moves therefore at the same rate) that computational causality tools commonly make.

Furthermore, in a temporal hypergraph, the unit of time is scale dependent which means in order to compare temporal values one must consider the scale to make a valid comparison between equally scaled values (hour X compared to hour Y). Less obvious, a temporal hypergraph, by design, holds all past and present temporal values simultaneously within its structure. This co-existence of multiple temporal points simplifies non-trivial temporal arithmetic over hetero-scaled time units, yet it imposes a vexing problem: How do you know if a time value in a node of the graph is current or past?

The problem is non-trivial because, as time progresses, the engulfing context engine continually generates the non-Euclidean temporal hypergraph representation with the implication that, at one lookup, the value of a temporal node is current, but at the next one, it might be past; however, the exact temporal distance at which a "present" value becomes "past" depends on the node's time scale. A node holding a temporal value "hour" will be valid for 60 minutes; that means a lookup every ten minutes will yield the current hour 6 times, but handling the seventh lookup leads to a fundamental design decision that illustrates the implied complexity.

A "dynamic-position" design means, when a new hour starts, a new node will be added; therefore, the seventh lookup returns a past value. By implication, the index of the new node needs to be looked up to retrieve the value of the new current hour. Therefore, a "dynamic-position" design requires a dynamic index to locate current values.

A "static-position" design means, when a new hour starts, the previous current value will be overwritten with the understanding that the seventh lookup returns the new current value. By implication, the index of the current value always remains static. Therefore, a "static-position" design requires a fixed index, i.e., a lookup table to locate current values. Use cases exist for both scenarios and in practice, temporal hypergraphs use a mixture of static and dynamic positioning to handle different types of relative values, e.g., current day, last year, next hour, and similar..

Exacerbating the problem further, feedback loops across different time scales using different relative values may dynamically modify the temporal hypergraph itself to capture non-regular observations or to add estimations at future time values that have not yet occurred. At this point, it becomes abundantly clear that the assumption of a simple, unidirectional linear temporal order required for establishing cause-and-effect becomes untenable.

At the same time, causal relations remain valid in those non-linear regions. Additionally, the designation of a cause purely based on strict temporal order feels arbitrary in a temporal hypergraph in which past, present, and estimated future temporal values across multiple time scales all exist simultaneously. Regardless of static or dynamic location of relative values, the definition of causality had to evolve to match the reality of modeling causal structures across complex multimodal hypergraphs.

4.3 No a-priori causal structures

When modeling dynamic feedback loops across different time scales, the third problem emerges eventually: Not all causal structures are known upfront. There are cases where the causal structure emerges from the prevalent context predominantly in response to a change in externalities. For example, in the financial industry, a shifting volume imbalance indicates an emerging regime change. The exact cause for the shifting volume imbalance can be attributed to externalities such as breaking news. However, in absence of internal references, the subsequent causal structure emerges as part of the unfolding regime change. There is no possible way to know the new structure upfront therefore, it cannot reliably be modeled ahead of time. Likewise, in correlation-based methods, a similar phenomenon unfolds because, in absence of internal references, the deep learning model cannot predict correctly anymore because data during an emerging regime change fall outside its training data distribution. The mechanism is different, but the outcome is the same: that previously reliable models crater in novel situations.

The previously identified limitation of temporal order directly applies to dynamic regime changes because, if the new causal structure has not yet emerged, how can we know its temporal structure beforehand? In short, we cannot know reliably anything about emerging causality up to the moment it emerges.

These three problems, non-Euclidean representation, non-linear temporal structures, and emerging causality deeply interrelate with each other, thus defy overly simplistic solutions. For example, advanced graph neural networks work on non-Euclidean data representation, but fail on non-linear temporal structures. One might be tempted to build non-linear-time graph neural networks, but this does not address the problem of non-Euclidean data representation and emerging causality. There is research to combine methods from computational causality with deep learning, but these approaches are focused on non-Euclidean representation without integrating the challenges of non-linear temporality and emergent causal logic.

4.4 Boundaries of Classical Causal Models

The established methods of computational causality, particularly the frameworks developed by Pearl, Granger, and Rubin, represent monumental intellectual achievements that form the foundation of the field. The purpose of the following analysis is to carefully delineate the set of assumptions, such as a fixed spacetime and a static causal structure, upon which they were designed to operate. By clearly defining these boundaries, we can identify the emerging class of problems in modern dynamic systems that now fall outside this classical scope.

Granger Causality

Granger Causality[106] is used for time-series data where past values of one time-series are used to predict values of another time-series. In a strict sense, Granger Causality tests if one variable (say X) predicts another variable (Y) through a series of t-tests and F-tests on lagged values of variable X.

Assumptions:

Granger causality assumes a stable causal structure and a linear, uniform time representation with consistent intervals.

Implications:

- **Non-Linear Time:** Granger causality cannot handle non-linear time representation
- **Non-Euclidean Representation:** Granger causality operates on time-series values within a Euclidean representation. It cannot be applied to a non-Euclidean representation.
- **Emergent Causality:** Because of the assumption of a stable causal structure, Granger causality cannot operate on emergent causal structures.

Pearl's Causal DAGs and Structural Causal Models (SCMs)

Judea Pearl pioneered the formalization of causality that underpins the majority of contemporary methods of computational causality. The framework of Structural Causal Models[17] (SCMs) rests upon the assumptions of Directed Acyclic Graphs (DAGs).

Assumptions:

Pearl's causal framework is exceptionally powerful for reasoning about interventions given a known or discovered causal model. The framework assumes:

- Acyclicity (DAG): Causal relationships are acyclical in a directed acyclic graph.
- Static Causal Structure: The causal graph, once defined, is assumed to be static.
- Fixed background spacetime: Variables in the DAG are assumed to be embedded within a fixed background spacetime.

Implications:

- **Non-Linear Time:** The acyclical assumption explicitly prohibits feedback loops and the fixed background spacetime assumption prevents any form of non-linear time.
- **Non-Euclidean Representation:** The DAG structure could potentially allow for non-Euclidean representation, but existing tooling assumes Euclidean structures and thus does not allow non-Euclidean representation
- **Emergent Causality:** The assumption of static causal structure prevents any handling of causal emergence.

Rubin causal model (RCM)

The Rubin causal model[107] (RCM) is a statistical framework for estimating causal effects by comparing potential outcomes under different treatment assignments.

Assumptions:

RCM assumes stable units, no interference between units (SUTVA), and that treatment assignment is ignorable conditional on observed covariates. RCM also assumes a stable causal structure and a fixed background spacetime.

Implications:

- **Non-Linear Time:** RCM is not designed to handle non-linear or multi-scale time representation.
- **Non-Euclidean Representation:** RCM is designed for variables representing treatments, outcomes, and covariates, without any mechanism for complex relational types. Therefore, RCM cannot infer across non-Euclidean data representations.
- **Emergent Causality:** RCM estimates effects within a stable causal structure and therefore cannot handle emergent causality.

Dynamic Bayesian Networks

Dynamic Bayesian Networks[108] (DBN) extend Bayesian Networks to model temporal processes by defining transition probabilities between states.

Assumptions:

Dynamic Bayesian Networks, similar to Granger causality, require a linear and uniform time representation. DBNs model changes in the state of variables over time according to a fixed probabilistic causal structure assumed to be static.

Implications:

- **Non-Linear Time:** DBN cannot handle non-linear time.
- **Non-Euclidean Representation:** DBNs reason over probabilities, not non-Euclidean space itself.
- **Emergent Causality:** DBNs assume a static causal structure and thus cannot handle emergent causality

The analysis of classical models reveals a consistent set of boundaries, built upon assumptions of a fixed, linear spacetime and a static causal structure. While these assumptions enable powerful reasoning within their defined domains, they become untenable when confronted with a class of problems of increasing importance in safety-critical and complex systems.

4.5 Beyond Classical Causal Models

The need for the Effect Propagation Process arises directly from the imperative to model these systems, which are characterized by three deeply interrelated challenges: non-Euclidean representation, non-linear temporality, and dynamic causal emergence. The most critical of these is the challenge of dynamic causal emergence, where the causal laws of a system are themselves subject to change. This is not a theoretical abstraction but a practical reality in numerous domains. In financial markets, for instance, the causal relationships between assets are relatively stable during normal conditions but undergo a fundamental "regime shift" following a black-swan event or a major policy change. The old causal graph becomes obsolete, and a new one emerges in response to the new market dynamics. Similarly, in climate science, ecological systems can cross "tipping points" where gradual changes trigger new, powerful feedback loops, fundamentally altering the causal structure of the climate model. In such cases, a framework is required that does not assume a-priori knowledge of the causal structure, but is instead capable of instantiating new structures in response to emergent events.

This dynamic nature exacerbates the problem of non-linear time. In a system with emergent feedback loops, such as in climate modeling or high-frequency trading, the notion of a single, uniform temporal progression becomes untenable. Events on nanosecond, daily, and quarterly scales can all interact, forming a complex temporal hypergraph where causal influence is no longer a simple unidirectional arrow. A framework must therefore be capable of reasoning over these complex, multi-scale temporal structures without relying on the assumption of a linear time.

Complex causal relations are often expressed through non-Euclidean representations. For example, a medical record documenting a patient's health cannot be reduced to a simple vector of measurements. It is a complex graph of relationships between genomic data, electronic health records, comorbidities, and environmental factors. Likewise, a global supply chain is not a grid but a dynamic network of dependencies.

It is a complex graph of relationships between genomic data, electronic health records, comorbidities, and environmental factors. Likewise, a global supply chain is not a grid but a dynamic network of dependencies. Reducing these relational structures to a Euclidean vector space to fit classical models often destroys the very information needed for accurate causal inference.

These challenges, emergent causality, complex temporality, and non-Euclidean data, are not independent issues to be solved in isolation. They are deeply inter related and jointly point to a single, underlying problem: modeling reality in complex, adaptive, and dynamic systems. This requires a fundamental rethinking of causality itself, moving away from a static, linear interpretation and towards a dynamic, context-aware, and dynamic process.

5 Causality as Effect Propagation Process

The foundational premise of the EPP is the detachment of causality from a presupposed spacetime. This premise necessitates a re-evaluation of the causal relation itself, shifting its conceptualization to a more general process of effect propagation. From this re-evaluation, the core architectural components of the EPP, are derived. The theoretical foundations of the EPP results from a multi-disciplinary background.

5.1 Background

The Effect Propagation Process (EPP) builds upon a confluence of ideas from process philosophy, modern physics, and contemporary machine learning. Its conception follows a progression from a core philosophy to a specific architecture that addresses contemporary computational challenges.

The EPP's primary departure point is a fundamental rejection of the classical Newtonian conception of a static, absolute background spacetime. This move is deeply rooted in the tradition of process philosophy, which argues that reality is not composed of enduring, static substances but is a dynamic flow of interconnected events. This idea finds its clearest expression in the work of Alfred North Whitehead, who posited a universe of "actual occasions"[109], and Henri Bergson, who described reality as a continuous "creative evolution"[110]. Their shared insight of reality as a process inspires the EPP's foundational redefinition of causality itself, shifting from a static, happen-before relation to a dynamic process of effect propagation.

The EPP is inspired by Einstein's theory of General Relativity[7], which demonstrated that spacetime is a dynamic fabric, its geometry determined by the matter within it, which in turn dictates the motion of matter. The EPP's concept of a Contextual Relativity that is both influenced by and influences the entities within is a direct metaphysical analogue of this profound physical insight.

Luciano Floridi's view[111] that the design principles for dynamic systems require a relational paradigm was profoundly inspirational to the formalization of the Effect Propagation Process. The EPP leverages the hypergraph as its foundational structure to model rich and complex causality through relationships.

Bernhard Schölkopf[112] advocates that integrating causal methods into machine learning helps to navigate fairness, privacy, robustness, accuracy, and explainability. He argues that a causal approach is essential for balancing multiple competing objectives and, ideally, these objectives should ideally be satisfied simultaneously. The EPP embraces his line of reasoning and takes a principled integrated stance as a result.

Lucian Hardy introduced the "causaloid,"[10] a concept that encapsulates a spatial region and the causal connections within as foundation his work on finding a theory of Quantum Gravity. Critically, unlike all prior forms of causality, Hardy's causaloid is spacetime agonistic because it folds cause and effect into one entity and thus removes the need for temporal order. The EPP draws direct inspiration from Hardy's pioneering work by using the term Causaloid honoring Hardy's concept of a unified, self-contained unit of causality, though it has been adapted for a more general, computational context. However, to operationalize spacetime agnostic causality, a new definition of causality became necessary.

5.2 Definition

The notion of a generative process that underlies the fabric of spacetime leads to the implication that causality has to evolve beyond the strict "before-after" relation towards a spacetime-agnostic view. The critical question then becomes:

> "What is the minimum necessary and sufficient condition for a relationship to be considered causal, if one cannot use temporal precedence ('happen-before') in the definition?"

The answer to this question entails the very foundation upon which computational causality is build upon. The classical answer to this question for centuries has been: $TemporalPrecedence + CounterfactualDependence$. That means, the cause must come before the effect, AND the effect would not have happened without the cause.

The EPP, by its foundational act of detaching causality from spacetime, makes the "Temporal Precedence" condition untentable. It requires an answer that entails the essence of causality independent of spacetime. Finding that answer, however, starts with the classical definition of causality.

5.2.1 Classical Definition of Causality

The classical definition of causality, taken from Judea Pearl's foundational work[17]:

> IF (cause) A then (effect) B.
> AND
> IF NOT (cause) A, then NOT (effect) B.

When removing temporal order from causality, it is indeed no longer possible to discern cause from effect because, in the absence of time, there is no "happen-before" relation any longer, and therefore, the designation of cause or effect indeed becomes infeasible, just as Russell hinted at earlier on. When removing space from causality, the location of a cause or effect in space is not possible anymore because space itself is no longer available.

5.2.2 Deconstructing Classical Causality

The absence of spacetime raises the question: *What is the essence of causality?*

Logically, the answer comes in three parts:

1. Causality is a process.
2. Causality determines effects.
3. Causality describes how effects propagate.

The first one is self-explanatory because causality occurs in dynamic systems that change and therefore, causality must be a process. The idea goes back to Whitehead, has been further developed by Bergson, and finds precedent in Mill's method of concomitant variation[6].

The second one is less obvious, because one might think that causality is all about the "cause" that brings the effects into existence. However, let's think the other way around: We know that X is the cause of effect E, because E happens when X happens and because E does not happen when X does not happen either. Therefore, we can determine a cause in terms of its effects. An effect, in essence, is an observable change of state. Therefore, it is true that causality determines effects.

The third one, effect propagation, needs elaboration because it is commonly assumed that the cause is the dominant factor in causality. When we rewrite the previous definition of classical causality in terms of effect propagation, however, we see that there is no loss of information:

> If X happens, then its effect propagates to Effect E.
> AND
> If X does not happen, then its effect does not propagate to Effect E.

In this definition, X does not have a designated label and instead is described in terms of its emitting effect. Therefore, X can be seen as a preceding effect, which then propagates its effect further.

Thus, we can write without loss of information:

> If Effect E1 happens, then its effect propagates to Effect E2.
> AND
> If Effect E1 does not happen, then its effect does not propagate to Effect E2.

Therefore, causality becomes an effect propagation process. The effect propagation process definition is more general and treats the classical happen-before definition of causality as a specialized derived form. When the preceding effect is designated as a “cause”, then you can rewrite the general definition back into the classical definition of causality therefore the generalized and the specialized definition of causality remain congruent. The EPP formalizes the ’effect’ itself as a dedicated ’PropagatingEffect’ together with supporting formalism to operationalize the effect propagation process.

5.2.3 Generalized Definition of Causality

The generalized definition of causality used by the EPP is:

> If Effect E1 happens, then its effect propagates to Effect E2.
> AND
> If Effect E1 does not happen, then its effect does not propagate to Effect E2.

The re-definition of causality as "effect propagation" is a more general, process-oriented term for functional dependency. To illustrate why that is the case, lets deconstruct the new definition:

1. "If Effect E1 happens, then its effect propagates to Effect E2." Here, the state of E2 functionally depends on E1
2. "If Effect E1 does not happen, then its effect does not propagate to Effect E2." The counter-factual case for E2 is equally functional dependent on E1. That also means, a different input (NOT E1) necessarily leads to a different output (NOT E2), assuming the function is not a constant.

For the initial question, "What is the minimum necessary and sufficient condition for a relationship to be considered causal, if one cannot use temporal precedence (’happen-before’) in the definition?", the answer is that the relationship is causal because one effect state is a function of another’s. As a direct consequence of the generalized definition of causality, the single, minimum necessary and sufficient condition for a relationship to be causal is Functional Dependency.

5.2.4 Axiom of Causality

For the generalized definition of causality, the only applicable axiom is a spacetime independent functional dependency between Effect E1 and Effect E2. Therefore, we can write:

E2 = f(E1)

It is important to recognize that E1 and E2 are deliberately chosen to be unconstraint. That means, effects E1 and E2 might be deterministic, probabilistic, symbolic, or of arbitrary complex structure. Furthermore, f() does not assume and particular modality either and thus could be a simple logical expression, a SCM, a DBN, a differential equation, or even a neuronal net. The only practical constraint is that the function must compute within a time constraint deemed acceptable to meet its requirements.

5.2.5 Higher-Order Implications

The higher-Order Implications of a single axiomatic foundation of causality entail, among others:

- If causality is functional dependency, then the fundamental unit of causality must be a container for a function. More precisely, the resulting unit will be equivalent to a higher order function of $apply(f(E1))$
- If the function needs inputs to operate on, then the system must provide those inputs.
- If the output of one function is to be the input of another, then it follows that input and output are isomorphic. It also follows that effects propagating from one causal to another, hence this is the materialization of the effect propagation process.

- If the function interacts with the outside world, then the system must provide the context.
- If the outside world changes, then the system must provide a mechanism to reflect those changes in the context.
- If these function-containers are to relate to one another, there must be a structure to hold them and define their pathways. This logically necessitates a causal graph structure
- If the cauasal graph is to model complex, real-world systems, it must be scalable and composable.

The author needs to emphasize that the currently identified higher order effects most likely will remain incomplete for the foreseeable future despite the EPP's attempt to describe and formalize an initial set. A result of the an irreducible single axiomatic definition is a vast space of new possibilities and, while this monograph captures a selected set of relevant to computational causality, the full exploration of the new possibilities will take some meaningful time. However, the EPP accepts the premise of detaching causality from spacetime, it embraces the necessary and sufficient condition of causality as functional dependency as its core tenants and, as a result, many of the properties of the EPP are necessary consequences that arise from its single axiomatic foundation. This monograph presents and explores the EPP to further understand its higher order implication and how to utilize it for causal modelling.

5.3 Overview of the EPP

The starting point of the EPP is a generalized definition of causality as effect propagation, which detaches the concept from a pre-supposed spacetime. From the generalized definition of causality, a cascade of novelties followed to operationalize the EPP:

1. Explicite Assumptions
2. Externalized, Computable Context
3. Unified Causal Unit
4. Fractal, Self-Referential Causal Structure
5. Multi Modal Propagating Effect
6. Causal State Machine

Explicit Assumption:

The EPP defines a "Model" as a set that comprises of a the core logic encapsulated in one or more causaloids, one or more contexts used by the causaloids, and a set of explicit assumption that must hold true for the model to work. Conventionally, in classical causality, assumptions about the data are implicit and the decision whether a causal model can be transferred to a different data dataset requires additional methods such as Invariant Causal Prediction (ICP) to determine if a causal model applies to a dataset with a different distribution. In contrast, the EPP elevates assumptions as a first class entity that serves the purpose to decide upon model transferability by encoding assumptions into explicitly testable and computable units.

An Externalized, Computable Context:

The detachment from a fixed spacetime necessitates the externalization of the environment into a first-class Context. This Context is a rich and explicit representation of the world that the Causaloids read from and reason about. A context may be a static representation of facts, a complex multi-scale temporal graph, a complex coordinate system, or any combination thereof. A static context emits an invariant structure after it's defined whereas in a dynamic context, the context structure itself evolves e.g. new elements are added or remove. In both cases, elements of a context can be adjusted and updated to reflect either new values or correction of existing values. The adjustment mechanism allows for error correction or the adjustment for relativistic effects.

Unified Causal Unit:

The EPP resolves the untenability of the classical 'cause-effect' separation by introducing the Causaloid. The Causaloid is a single, computable entity that unifies the mechanism of a cause with its effect. The Causaloid comprises of a causal function and therefore establishes function theory and the related lambda calculus as the foundation for causality in the EPP.

Fractal, Self-referential Causal Structure:

The EPP inverts the classical relationship between a causal unit and its environment. In classical models, the causal structure is a pre-defined framework (such as a DAG) in which entities are placed. The EPP establishes a fundamentally entity-first, fractal and self-referential definition of causality by defining the structure as relationships between

causaloids and causaloids as the elements of the relationships. This mechanism is made operational through isomorphic recursive composition. The EPP establishes three distinct forms of causaloids that are all isomorphic, yet distinct: Singleton, Collection, Graph. The isomorphism allows a single Causaloid node to represent either a single causaloid, a collection of causaloids, or to encapsulate an entire, arbitrarily complex Causaloid Graph, enabling the concise expression of deeply layered systems. The collection exists as a specialized form of the graph to simplify common use cases without the complexity of a graph structure.

Multi Modal Propagating Effect:

To facilitate reasoning within the causal structure, the EPP unifies causal input (Evidence) and output (Effect) into a single, isomorphic type: the PropagatingEffect. The Causaloids unifies cause and effect into one logical unit and the EPP is achieving this by formulating the Causaloid as a higher order function, one that encapsulates a causal function and applies to data for reasoning. As a direct consequence, the distinction between causal input and output becomes quite arbitrary considering that the output of one causaloid is expected to be applied to another causaloid. Therefore, the deliberate decision was made to follow the same logic of the causaloid and fold causal input (Evidence) and output (Effect) into a single, isomorphic type that uniformly represent both and thus directly enable the application of causal output from one causaloid as causal input to another causaloid. This provides a single type that enables the principled unification of deterministic and probabilistic reasoning across arbitrary complex causal structures.

Causal State Machine:

The EPP bridges the gap between causal reasoning and intervention with the causal state machine (CSM). The CSM uses a causal state that connects to the reasoning outcome via the PropagatingEffect and ultimately converts all complex reasoning into a binary outcome to indicate whether to act according to the defined causal action. When the causal state of a CSM evaluates to true, it then fires its causal action. The action is a regular function that may interact with the context, the causal model, or any external system. The clear separation between context, causal logic, and intervention has been designed for regulatory reasons. In regulated industries, an auditable trail for each action is legally required and by making the causal reasoning fully explainable before deciding upon an intervention directly supports internal auditing and external regulatory reporting.

5.4 Explicit Assumptions

In the EPP, model assumptions are explicit and connected to the model. In practice, this allows to define a Model with a set of preconditions (the assumptions). Before using the model for reasoning or prediction, this allows to efficiently check if all its underlying assumptions hold true for the given data, ensuring the model is operating within its valid parameters. Furthermore, transferring a model to a new environment can be tested by sampling representative data and testing the samples against the model assumptions to gauge if a model transfer is feasible. In practice, even if an assumption test is positive, it is advised to test the model with an alternate context that closely resembles the new environment to gain more confidence in the degree of transferability.

5.5 Context

A key contribution of the EPP is the externalization of context as a first-class entity. The context of a causal model is a hypergraph, that encapsulates supporting data. Each node in this hypergraph is a Contextoid, a unit of information that can represent:

- Data
- Time, Space, and Spacetime
- Symbol

The causal logic is kept distinct from the contextual data it operates on. It also directly enables the agnosticism to the structure of space and time, accommodating Euclidean, non-Euclidean, and symbolic representations within the same architecture. Furthermore, causal logic may operate on one or more contexts and, equally important, a particular context might be shared between different causal logic thus enable efficient and salable context representation in complex dynamic systems.

5.6 Causaloid

In the Effect Propagation Process, due to the detachment from a fixed spacetime, the fundamental temporal order is absent. Consequently, the entire classical concept of causality, where a cause must happen before its effect, can no longer be fundamentally established. The distinction between a definitive 'Cause' and a definitive 'Effect' becomes untenable as Russell foresaw. When the separation between cause and effect becomes untenable, then the obvious question arises: why even preserve an untenable separation?

Therefore, the Effect Propagation Process framework adopts the causaloid, a uniform entity proposed by Hardy[10], that merges the 'cause' and 'effect' into a single entity. Instead of dealing with two nearly identical concepts discernible from each other by temporal order, the causaloid is a single entity that applies its "input" (cause), to a causal function that derives its "output" (effect), and whose relations to other causaloids define the causal structure and whose effect then propagates to the next causaloid. The nature of the causal function is not prescribed, allowing the Causaloid to encapsulate diverse logical forms, including but not imited to:

- A deterministic rule (IF temp > 100).
- A formal Structural Causal Model (SCM).
- A probabilistic estimate or Bayesian network.
- A specialized neural network.

For the deterministic case, the causal function takes some evidence as input, applies boolean operators (AND, OR) or comparators, and returns a boolean value as its PropagatingEffect. For a more complex causal scenario, the causal function encapsulates a set of structural causal equations, applies the corresponding calculus and returns a probability distribution value as its PropagatingEffect.

In case of a probabilistic estimate or a Bayesian network, the causal function implements a Conditional Probability Table (CPT) or a similar probabilistic model, applies a probabilistic calculus i.e. the chain rule of probability, and returns another probability as its PropagatingEffect. If a Causaloid receives multiple PropagatingEffects, each carrying a probability, the receiving Causaloid implements the aggregation of all probabilities. This is a deliberate architectural principle rooted in the EPP's primary role as a flexible, hybrid framework. A specialized neural network embedded into a causal function will most likely return a classification score as its PropagatingEffect. In case the output of the neural network results in a complex type, for example generative data, then it is sensible to write its output into the appropriate context as a contextoid and return the context and contextoid ID as the PropagatingEffect.

It is important to note that the EPP framework adopts the conceptual role of the Causaloid as a spacetime-agnostic unit of causal interaction, inspired by Hardy's work on Quantum Gravity, but it does not uses Hardy's formal definition that requires a complex process matrix. Instead, the EPP formulates the Causaloid as an abstract data structure that embeds a causal function, thereby decoupling it from any particular physical theory while preserving its core philosophical utility and making it practical implementable in software.

The term "propagation" refers to the fundamental process by which an effect is transferred within the structure from one Causaloid to another. This fundamental process is what gives rise to the appearance of propagation through spacetime in the classical view. Furthermore, while classical causality relies on a definite temporal order, the Effect Propagation Process treats temporal order as an emergent property, arising from the fundamental process itself.

While the Effect Propagation Process involves the transfer of effects within the fundamental structure, it is crucial to distinguish this from mere accidental correlation. The process reflects the fundamental way the underlying structure of reality establishes dependencies between its components and how it is gives rise to the non-accidental relationships we recognize as observed causal relations.This fundamental determination, rather than simple co-occurrence, is what the "Effect Propagation Process" captures at the deepest level.

The Effect Propagation Process fundamentally inverts the notion of causal structure. Before the EPP, the causal structure was pre-supposed (i.e. DAG, a set of equations) in which variables, events, objects are placed. The consequence of this order is the fixed and often flat causal structure observed in classical methods of computational causality. The EPP, however, inverts the order and puts the causal entities first as monoidic primitives, and then introduces structure as relationships between these. This results in the operationalization of the fractal, self-referential causal structure that directly follows from the adopted single axiomatic generalized definition of causality. Specifically, the self-referential structures materializes from the following:

- What is a causal structure? A set of relationships between Causaloids.
- What is a causaloid? An entity that defines its relationships to other Causaloids.

- What is a relationship? The pathway of propagating effects between Causaloids.

Therefore, a set of causaloids can represent arbitrary complex causal relationships because of this fractal, self-referential definition. It also follows from the definition that causality in the EPP has three primary modalities:

- When the causal relations are fixed, the structure is static.
- When the causal relations are changing, the structure is dynamic.
- When the causal relations are brought into being, the structure emerges.

The core principle, that the entities define the structure and the structure is defied by its relationships between entities, holds true for all three modalities and, with it, establishes a principled foundation for dynamic. Because of its flexibility, the EPP can express static causal relationships similar to Pearl's Causal DAG, it can handle probabilistic causal systems similar in spirit to Dynamic Bayesian Networks, but then goes further and adds causal emergence and unifies both paradigms into one that is static and dynamic, deterministic and probabilistic while remaining structurally agnostic and thus allows for flexible geometric representation.

5.7 Causaloid Collection

Many real-world causal scenarios are not defined by a single cause but by the interplay of multiple factors. In practice, multiple modalities of causal aggregation may occur. For example, from a set of known causes, all of them must be true for an effect to occur. Then, in some cases, only one of many potential causes may lead to an effect. In other cases, more than one potential causes might be needed to trigger an effect, but not all known causes are required. To model these common collective structures , the EPP provides the Causaloid Collection.

A Causaloid Collection is a first-class entity that encapsulates a set of Causaloids and an explicit, configurable Aggregate Logic. This logic dictates how the individual PropagatingEffects of the member Causaloids are combined into a single, definitive outcome for the collection as a whole. This provides an ergonomic and principled way to express common causal patterns:

- **Conjunction (All):** The collection is active only if all of its member Causaloids are active.
- **Disjunction (Any):** The collection is active if at least one of its member Causaloids is active.
- **Absence (None):** The collection is active only if none of its member Causaloids are active.
- **Threshold (Some(k)):** The collection is active only if at least a specific number, k, of its n member Causaloids are active.

In a conjunction, the causal collection is active only if all of its member Causaloids are active. For a disjunction, it is sufficient when just one cause of the causal collection is active. Negation inverts the conjunction in the sense that it's only true when all causes in a causal collection evaluate to false. For a threshold-based logic, the causal collection is active only when at least some K out of the collection of n causes are true.

The causal collection might seem to break from the EPP's geometric foundation, as the relationships between its members are not defined by explicit hyperedges. However, the relationship of a Causaloid within a collection is not to its peers, but to the collection's aggregate logic itself. For example, a collection with All logic is the formal equivalent of a single hyperedge connecting all member Causaloids as a source set to a single target representing a logical AND gate. Disjunctive and other logics follow a similar principle, allowing for a simpler and more intuitive alternative to modeling these common use cases with a full hypergraph. By providing the causal collection, the EPP allows for the concise and efficient modeling of the most frequent types of causal structures.

It is important to note that the configurable aggregate logic is only applicable to the causaloid collection. For a single causaloid, there is nothing to aggregate, and for the causal graph, the pathway through the graph provides the order of evaluation, therefore aggregation does not apply.

The Causaloid Collection proves particularly useful, for example, when modeling sensor fusion logic (k-of-n), multi-source object detection (All), or verifying safety interlocks (None). The verification of safety interlocks is a particular good example for causal collections because it allows for clear, verifiable, and auditable encoding of standard safety protocols commonly found in regulated industries such as robotics and avionics. For use cases that require nested or arbitrary complex causal relationships, the EPP provides the Causaloid Graph.

5.8 Causaloid Graph

Modeling real-world dynamic causal systems requires a mechanism capable of managing complexity. Classical computational causality relies on algebra, which is rich in formalization, but has its limits when complexity grows and thus limits scalability. The EPP adopts a geometric approach by expressing causal models as a hypergraph. The EPP exchanges the arithmetic complexity of solving large equation systems for the challenge of managing structural complexity that comes from the geomtrization of causality.

In the EPP, the fractal, self-referential definition of causality directly translates to isomorphic recursive composition that enable concise expression of complex causal structures to manage structural complexity. A causal hypergraph may contain any number of nodes with any number of relations to other nodes, with each node representing a causaloid. A causaloid uniformly represents three distinct levels of abstraction:

- Singleton Causaloid: The base case, representing a single, indivisible causal mechanism.
- A Collection of Causaloids: A set of Causaloids that can be evaluated with an aggregate logic.
- A Causaloid Graph: A node can encapsulate an entire graph

Recursive isomorphism allows to built causal models in a modular and hierarchical fashion. A complex sub-system can be modeled as a self-contained Causaloid Graph, then encapsulated into a single node to be used as a component in a larger, higher-level model. The causal graph enables the concise expression of deeply layered systems without sacrificing logical integrity.

The architecture of the Causaloid Graph is the direct physical manifestation of the EPP's core axioms. In accordance with the definition, the pathway of propagating effects, the relationship between Causaloids, is the hyperedge that connects the causaloid nodes. The Effect Propagation Process is the operational dynamic on this graph. When triggered, Causaloids are evaluated. Their outcomes, the PropagatingEffects, propagate along these hyperedges to other Causaloids, which in turn evaluate their own functions, thus continuing the process until the graph traversal completes and a final, reasoned inference is reached.

5.9 PropagatingEffect

The EPP emphasizes uniformity and, just like the causaloid folds cause and effect into one uniform unit, the PropagatingEffect folds causal input and output into one uniform unit. Its purpose is to distill the result of the node's reasoning into a clear, actionable directive that is passed to the next causaloid as its input. The PropagatingEffect represents a unified inference outcome across different reasoning modalities. By design, the EPP supports the following reasoning modalities:

- Deterministic
- Probabilistic
- Mixed, deterministic and probabilistic

The deterministic mode facilitate logical reasoning using boolean logic. A state is either true or false. The EPP recognizes that, while there is a clear use case for deterministic causal reasoning, there is also an equally important use case for probabilistic causal reasoning. Also, the EPP only provides the reasoning mechanism, but leaves the exact details of the reasoning mechanism as implementation details of the causaloid. The wisdom of this decision comes from the realization that the EPP may be used in different scenarios with different requirements and therefore it leaves the exact reasoning details to the practitioner. One important detail on the mixed reasoning. While it is designed internally to convert all boolean state to probabilities and as then reasons only over probabilities, its final outcome is actually a deterministic boolean relative to a probabilistic threshold. In the DeepCausality implementation, all reasoning modes are traits with a default implementation thus leaving the practitioner the option to overwrite the default mode with a custom implementation when the need arises. Because the mixed modalities require different input and output types, the PropagatingEffect is isomorphic recursive to represent a variety of different reasoning types such as:

- Primitive Types: Deterministic, Numerical, and Probability values
- Complex Structures: Map or Graph for passing complex, structured, or relational data between causaloids.
- Contextual Link: A reference to a specific fact in the context to find the PropagatingEffect .
- None: Explicitly represent no effect.

- Referral: A type that contains a PropagatingEffect and reference to another causaloid to process it.

The Contextual Link accommodates for advanced causal reasoning via non-numerical representations by writing a complex reasoning outcome directly into the context and then propagating the reference to the next reasoning stage which reads the complex reasoning outcome and processes it further. For example, the Symbolic context type combined with the Contextual Link establishes a foundation for a uniform integration of multi-modal causal reasoning with advanced neuro-symbolic reasoning in a future revision of the EPP.

5.10 Dynamic Reasoning Modalities

The EPP uses context, the causaloid, and the propagating effect to enables several distinct modes of causal reasoning:

- **Static Reasoning**
- **Dynamic Reasoning**
- **Adaptive Reasoning**
- **Emergent Reasoning**

5.10.1 Static Reasoning

The entire causal graph is traversed according to its pre-defined structure. The reasoning is static because the reasoning path is always fixed. There is a large category of static causal model in practice that needs this modality. Furthermore, just by making the static context dynamic, a fundamentally static causal model becomes more valuable without any modification as the core dynamics is shifted to the context.

5.10.2 Dynamic Reasoning

A sub-graph or a specific path (such as the shortest path) is evaluated, allowing for targeted, context-dependent reasoning within the larger graph. It is dynamic reasoning because the pathway through the graph is determines at runtime via one of the EPP methods for graph traversal. Furthermore, the context of a dynamic reasoning graph might be either static or dynamic. In practice, usually a combination of both is used. An immutable static context for factual data assumed to remain invariant. A classic example would be the ICD-10 catalogue. And then one or more dynamic contexts to model various data streams for real-time causal inference.

5.10.3 Adaptive Reasoning

The Causaloid itself determines the next step in the reasoning process conditional on its reasoning outcome. Based on its own internal logic, a Causaloid can dynamically dispatch the flow of causality to another Causaloid in the graph, enabling adaptive reasoning. To illustrate conditional reasoning, a clinical patent risk model may operate very differently for patients with normal blood pressure compared to high blood pressure patients. Therefore, two highly specialized models are defined and a dedicated dispatch causaloid queries its context for the latest blood pressure measurements and its recent history, uses its internal logic to decided the level of blood pressure and then dispatches all further reasoning to the specialized causal model for the detected blood pressure level. Adaptive Reasoning can be combined with either a static or dynamic causal graph. In a static graph, all potential reasoning pathways must be defined upfront. In a dynamic causal graph, new causaloids might be added dynamically i.e. in response to a changing context and then the dispatch causaloid gets replaced with a new one that can dispatch to the newly created reasoning causaloids to enable dynamic adaptive contextual reasoning.

5.10.4 Emergent Reasoning

In case a dynamic context is combined with a dynamic causal reasoning graph that may even use adaptive reasoning, the fundamental assumption of causality, determinism, may not hold true any longer. Intuitively, when the reasoning graph is dynamically constructed and possibly mixed with adaptive reasoning, one would guess that determinism might not be feasible any longer. However that is not necessarily the case. When the rules of dynamic graph modification are carefully crafted and verified, then even in this complex scenario determinism might hold true. However, the actual problem is the dynamic context in this case. When context is constructed dynamically in response to, say, a sensor data stream and relative to the detected shift in sensor data the reasoning graph is modified dynamically, then the entire premise of determinism hangs on the generative function of the context and that is exactly were formal verification falls apart. It is simply not possible to proof that a context, dynamically constructed from a sensor stream, can be

deterministic and therefore the inter-dependent dynamic causal graph, regardless of its reasoning modality, cannot be proven to be deterministic either.

It is important to understand that the "unprovability" roots in the system's coupling with an open, unpredictable world facilitated via the dynamic context. The precise problem isn't the existence of a dynamic adaptable context, but the door it opes to an open, unpredictable world that may produce states that fall outside the boundaries of the causal model that interacts with the open world through the dynamic context.

5.10.5 Trade-off: Adaptability vs. Verifiability

There is a fundamental trade-off between adaptive and emergent reasoning in terms of adaptability and verifiability.

- **Adaptive Reasoning (Deterministic Dynamics)**
- **Emergence (Non-deterministic Dynamics)**

Dynamic and adaptive reasoning in the EPP is fundamentally deterministic, but limited in adaptability. These systems may grow complex, but fundamentally remain deterministic and therefore can be formally verified. Here, the trade-off is to preserve determinism at the expense of dynamic adaptability to enable a large class of problems where the modalities are reasonable well known upfront. For example, a clinical model that can switch between a "Normal Response" subgraph and a "Drug Resistance" subgraph is adaptive relative to the known spectrum of modalities. A financial model that switches between "Bull Market" logic and "Bear Market" logic is adaptive relative to these two market modalities. Because the set of possible causal models is closed, pre-defined, and pre-validated, the system's dynamic behavior remains fully deterministic and verifiable. Here, the EPP provides a formal architecture for building systems that can adapt to changing contexts without sacrificing safety and predictability.

Emergence on the other hand inverts the trade-off and enables dynamic adaptability at the expense of determinism and formal verification. Emergence is for systems where the causal rules themselves must be generated in response to dynamically emerging situations not foreseen by the designers. Even though determinism is not tenable any longer, restoring verifiability of dynamic emergence requires an entirely new set of methods.

The EPP deliberately supports both modalities to enable practitioners to choose the mode of dynamic that best fits their requirements and constraints. In regulated industries, emergence might be a non-starter, but a verified static causal model with a mixture of static and dynamic context might be able to get certified within clearly defined boundaries. In advanced research of system dynamics, emergence might be a good choice to explore and simulated scenarios that are otherwise complex to model.

In practice, it is realistic to see a largely deterministic system that leverages the full spectrum of static and dynamic context, static, dynamic, and even adaptive causal reasoning and only in very specific corner cases, a very small portion of a system would be designed as emergent albeit with carefully crafted boundaries to the generating functions. The EPP already enables a large part of dynamic use cases with dynamic context, dynamic and adaptive reasoning and therefore it is most likely that dynamic emergence will be reserved for a small and highly specialized set of use cases. In that sense, the EPP scales with growing requirements, from simple static models, complex dynamic models interacting with multiple context, to most advanced use cases of dynamic emergence.

5.11 Causal State Machine

The causaloid and causal graph provides the mechanism for causal inference, but they lack the ability of intervention. The EPP addresses this through the Causal State Machine (CSM), which serves as the formal bridge between causal reasoning and deterministic intervention.

The CSM originates in Finite State Machine (FSM) in that it aims to formalize state transition. However, a defining property of the Finite State Machine is its explicit 'Finiteness': the entire set of possible system states must be known at design time. The FSM paradigm is highly effective for closed-world problems where all conditions are predictable and known. However, the finiteness of states becomes untenable when applied to dynamic causality. The Causal State Machine generalizes the FSM and adapts it to operationlize interventions for dynamic causality through two mechanism:

- A "Causal State" is an Inferred Predicate.
- The "Causal Action" is a Deterministic Intervention based on the Causal State.

In a classical FSM, a state is an identifier from a pre-defined list (e.g., "State A"). In the CSM, a Causal State is a inferential predicate defined as a specific Causaloid whose truthfulness is evaluated. The CSM does not need to know

all possible states in advance. It only requires the causal logic (the Causaloids) necessary to infer whether the encoded predicate in the "Causal State" is true.

Each Causal State is formally linked to a Causal Action. This is a deterministic, programmatic function that is executed if and only if its corresponding Causal State is inferred to be true. This action represents a real-world intervention.

The CSM is a inference-to-action state machine that is both deterministic in its execution and dynamic in its definition. The CSM is deterministic within its encoded caudal states and actions, but also dynamic in its definition as it can be extended at run-time by adding new Causal States and Actions, enabling the control logic of a system to evolve in tandem with the causal understanding of its environment.

5.12 Mapping Pearl's Ladder of Causation to the EPP

Judea Pearl's Ladder of Causation[17] defines three distinct levels of ability required for causal reasoning: Association, Intervention, and Counterfactuals. The EPP achieves these three rungs of the ladder by different means than the established methods of the SCM and Causal DAG.

Rung 1: Association

The first rung, Association, concerns reasoning from observational data. It answers the question, "What is the likelihood of Y, given that we have observed X?" In classical models, this is handled by conditional probabilities i.e $P(Y|X)$.

In the EPP, association is a structured, operational process:

1. The observation (X) is formalized as Evidence and presented as an input to a Causaloid.
2. The background condition $(|)$ is represented by the context, which provides the necessary supporting data for the reasoning process.
3. The causal inference of (Y) is performed by the causal function embedded within each Causaloid. This function takes the Evidence and any required information from the Context as its inputs and computes a result.
4. The inference result is emitted as a PropagatingEffect, which then travels along the graph's hyperedges, serving as Evidence for subsequent Causaloids.

Thus, "seeing" in the EPP is the operational dynamic where initial Evidence triggers a cascade of computations via the causal functions throughout the Causaloid Graph, leading to a final, reasoned inference.

Rung 2: Intervention

The second rung, Intervention, involves predicting the effects of deliberate actions. It answers the question, "What would Y be if we do X?" This is formalized in Pearl's framework by the do-operator, which simulates an intervention on the causal model itself.

The EPP provides a mechanism of intervention through the Causal State Machine (CSM). The CSM links causal inferences to deterministic actions:

- A Causal State is defined as an inferential predicate. It is a specific Causaloid evaluated against a specific Context.
- This Causal State is mapped to a Causal Action, a verifiable function that executes when its corresponding state is inferred to be true.

This Causal Action is the EPP's intervention. It is a programmatic function that changes state. This change is then reflected as an update to the context, creating a complete feedback loop of inference, action, and new observation.

Rung 3: Counterfactuals

The third rung, Counterfactuals, involves reasoning about alternative possibilities given a known outcome. It answers the retrospective question, "What would Y have been if X had been different, given that we actually observed Z?"

The EPP's architecture provides a Contextual Counterfactual mechanism, which leverages the EPP's externalization of context:

1. Abduction is Context Pinning

2. Action is Contextual Alternation
3. Prediction is Re-evaluation over an altered context.

The factual observation (Z) is already explicitly represented within the primary context. The abduction step is therefore equivalent to identifying and "pinning" this factual context.

The hypothetical premise ("if X had been different") is then established by creating a new, hypothetical context $(C_counterfactual)$ by cloning the primary context $C_factual$ and modifying the value of the relevant Contextoid.The system then executes the exact same, unmodified Causaloid Graph, but uses $C_counterfactual$ as its frame of reference. The resulting inference is the answer to the counterfactual query.

The EPP's mechanisms of causation differ from Structural Causal Models, but they fulfill the same fundamental goals of 'seeing,' 'doing,' and 'imagining' Judea Pearl established via the ladder of causation. Table 1 summarizes the comparison of the EPP to the existing methods of computational causality.

Table 1: Comparison of Causal Ladder Implementations

Ladder Rung	Pearl's Framework (SCM/DAG)	Effect Propagation Process (EPP)
1. Association	Statistical analysis; calculating conditional probability $P(Y\|X)$.	Execution of a `causal function` on `Evidence` within a factual `Context` (C_{factual}).
2. Intervention	The *do*-operator; surgical modification of the model's structural equations.	Execution of a `Causal Action` by the `Causal State Machine (CSM)` in response to an inferred state.
3. Counterfactuals	Three-step algorithm: Abduction (solving for latent variables), Action (model surgery), and Prediction.	Three-step process: Context Pinning, Contextual Alternation, and Re-evaluation of the *unmodified model* over an *altered context* ($C_{\text{counterfactual}}$).

The decision to separate causal logic (the Causaloid Graph) from its data (the Context) that underpins contextual alternation leads to some welcome properties. For example, through contextual alternation, counter-factual reasoning becomes an "embarrassingly parallel" problem because, if 100 alternate contexts are derived, all of them are independent from each other and thus can be evaluated in parallel.

5.13 Mapping Dynamic Bayesian Networks to the EPP

Dynamic Bayesian Network is the established framework for modeling dynamic causal systems. A DBN models a temporal process by "unrolling" a causal graph over discrete time slices, creating separate nodes for a variable at each point in time. The EPP represents this same process by evaluating a single, static causal model over a dynamic, temporal context hypergraph. The mapping of DBN to EPP components constructs as following:

Time Modeling:

In a DBN, time is an implicit index of the temporal variables. In the EPP, time is made explicit within a context by representing each time slice (e.g., X_{t-1}, X_t, X_{t+1}) as a Tempoid, a temporal type of Contextoid, with their sequential relationship defined by hyper-edges within the context hypergraph. In the EPP data might be attached to a Tempoid as a dedicted node of a different type i.e. Datoid.

State Variables:

state variable in a DBN (e.g., the concept of Weather across time) corresponds to a single Causaloid in the Causaloid Graph. The Causaloid represents the variable's underlying causal mechanism and the state of "Weather at time t" is the result of evaluating the "Weather" Causaloid contextual using the Tempoid that maps to the time t. A contextual temporal graph allows a Causaloid to access uniformly multiple slices of time at different scales i.e. weekly average rainfall and today's rainfall to inform its causal logic.

Dependencies:

The directed edges in a DBN can reference within the same time slice or across different time slice. For edges within the same time slice, (e.g., Weather$_t$ $\rightarrow$ Umbrella$_t$), the causal function simply references its causal logic to the one tempoid at time t in the graph. If "Weather" is a complex state object, then the causaloid may loads it from another context before evaluating the causal rule that would lead to Umbrella become true.

For edges between different time slices (e.g., Weather$_{t-1}$ $\rightarrow$ Weather$_t$), are represented by the hyperedges in the Causaloid Graph by referencing two different Tempoid nodes. Practically, one would implement a dynamic temporal graph index with accessors for frequently used "current" or "previous" values.

Conditional Probability Tables (CPT):

Conditional Probability Tables (CPTs): The CPT defining a variable's probability given its parents (e.g., $P(X_t \mid Z_t, X_{t-1})$) is implemented as the causal function within the Causaloid.

Execution Flow:

To compute the state of the system at time t, the causal function of a Causaloid queries the Context to determine the current time slice. It receives the PropagatingEffects from its parent Causaloids (representing their states at the appropriate time slices) and uses its internal CPT logic to compute a new probability distribution. This distribution is then emitted as a new probabilistic PropagatingEffect. This process maps the EPP to the "filtering" or "unrolling" inference of a DBN.

5.14 Mapping Granger Causality to the EPP

Granger causality is a foundational concept for causal inference in time-series data, particularly in econometrics. It is used to solve practical problems, for instance, determining if past changes in the price of oil can be said to "Granger-cause" future changes in shipping industry activity. A time series $X(oilprice)$ is said to "Granger-cause" another time $Y(shippingactivity)$ series if the past values of X contain information that helps predict the future values of Y better than using the past values of Y alone. It is fundamentally a test of predictive utility. The EPP's architecture, with its first-class treatment of time and context, provides a natural and powerful framework for modeling this scenario:

- **Time-Series as a Temporal Context:** The historical data for both oil prices and shipping activity are represented within the EPP context. This is modeled as a sequence of Tempoid nodes (representing months or quarters), each linked to Datoid nodes containing the respective values for $X(oilprice)$ and $Y(shippingactivity)$ at time t.
- **The Predictive Model as a Causaloid:** The predictive model for shipping activity is encapsulated within a dedicated 'Causaloid'. This Causaloid's purpose is to predict the next value of shipping activity, Y_{t+1}, by querying the 'Context' for past values.
- **The Granger Test as a Counterfactual Query:** The core question of "Do past oil prices improve the prediction of future shipping activity?" is a fundamentally counterfactual query. The EPP models this directly using its Contextual Alternation mechanism:
  1. A "Granger Test" Causaloid initiates two parallel, hypothetical evaluations.
  2. In the first evaluation, the Causaloid is allowed to query the complete, factual 'Context', which contains the history of both oil prices and past shipping activity.
  3. In the second evaluation, the Causaloid is evaluated against a hypothetical, alternate 'Context', C', where the history of oil prices is deliberately excluded or masked.
  4. The "Granger Test" Causaloid then compares the prediction error from both evaluations. If the error is significantly lower in the evaluation that included the history of oil prices, the test concludes that oil price Granger-causes shipping activity.

The EPP's temporal Context is not restricted to linear time steps; it can be a rich hypergraph representing multiple time scales (e.g., daily price volatility and quarterly shipping trends). Furthermore, the Causaloid is not restricted to the linear models typical in econometrics. This allows for the construction of more sophisticated, non-linear, and multi-scale versions of Granger-causal tests

5.15 Mapping Rubin Causal Model (RCM) to the EPP

The Rubin Causal Model (RCM), also known as the Potential Outcomes framework, is a cornerstone of modern causal inference, particularly in statistics, econometrics, and the social sciences. The RCM defines the causal effect on a specific unit, i, as the difference between two potential outcomes: $Y_i(1)$, the outcome if the unit receives the treatment, and $Y_i(0)$, the outcome if the unit does not eceives the treatment. The central challenge of the RCM, termed the "fundamental problem of causal inference," is that for any given unit, only one of these two potential outcomes can ever be factually observed.

The EPP models the scenario of potential outcomes as contextual alternations. From a base context, say C_i, two alternate contexts are derived, one C_i' with the treatment applied and another one, C_i'' with the control applied. From there, the two potential outcomes for unit i are therefore defined as:

- $Y_i(1)$ is the final 'PropagatingEffect' that results from evaluating the system's 'CausaloidGraph' against a context, C_i' where the treatment has been applied.
- $Y_i(0)$ is the final 'PropagatingEffect' that results from evaluating the exact same 'CausaloidGraph' against another hypothetical context, C_i'' where the control has been applied.
- Estimate the difference between the PropagatingEffect resulting from $Y_i(1)$ and the PropagatingEffect resulting $Y_i(0)$ as the causal effect on a unit i.

From the EPP's perspective, the "fundamental problem of causal inference" is physical constraint that only applies to physical measurements. The EPP, as a computational framework, treats counterfactuals via its contextual alternation mechanism. The EPP can instantiate and evaluate the causal outcomes for both the treatment and control contexts in parallel thus estimate the difference between two potential outcomes. This mapping demonstrates that the RCM's core concepts are a natural fit within the EPP's architecture and the EPP provides a mechanism to extend the RCM with a richer, dynamic, and non-Euclidean context.

5.16 Mapping Conditional Average Treatment Effects (CATE) Inference to the EPP

The estimation of Conditional Average Treatment Effects (CATE) is a critical goal of modern applied causal inference, particularly in domains like personalized medicine. CATE is defined as the expected causal effect of an intervention for a specific individual or sub-population, conditioned on their unique characteristics. The purpose of CATE is to move beyond population averages and determine for whom a treatment is beneficial, harmful, or ineffective.

It achieves this by leveraging the Rubin Causal Model (RCM). CATE is formally expressed as $\tau(x) = E[Y(1) - Y(0)|X = x]$, where $Y(1)$ and $Y(0)$ are the potential outcomes. While the definition is elegant, applying it to real-world observational data is a challenge, as only one potential outcome is ever observed. This challenge necessitates a two-stage process:

- Stage 1 (Discovery): Data science discovers and validates causal links.
- Stage 2 (Inference): The causal links are operationalized to answer specific CATE queries.

It is crucial to position the Effect Propagation Process correctly within the broader landscape of computational causality. The EPP is a foundational framework for causal reasoning, simulation, and operations. In its current form, it is not a tool for automated causal discovery from raw observational data. For contextual causal learning within the EPP, more work will be necessary. Therefore, all existing methods of causal discovery methods remain indispensable for inferring causal graphs from observational data. In an EPP-based workflow, these tools are invaluable for building and validating the initial causal graph. A function learned via a tool like DoWhy or EconML can be directly encapsulated within a Causaloid. The following mapping of CATE inference presumes a causal graph has been found and validated with existing methods and thus is scoped to operationalize the inference stage of CATE. The inference state of CATE maps to the EPP in two different modalities: Static and Dynamic.

Representing Static CATE in the EPP

The EPP models a static CATE query as a formal, computable, counterfactual simulation. This is achieved by mapping the core concepts of the query onto the EPP's architectural primitives:

- **The Condition $X = x$ as a Static Context:** The unit's complete set of pre-treatment covariates, $X = x$, is represented by a static `Context` (C_x). This context serves as the factual "ground truth" for the unit's state at the moment of the query.
- **The Learned Function as a 'Causaloid':** The causal relationships and the CATE function ('$\tau(x)$') that were discovered in Stage 1 (e.g., using DoWhy) are encapsulated within the causal logic of one or more `Causaloids`. For a simple case, a single 'CATEstimator' `Causaloid` might contain the entire learned function. In a more complex model, this might be a full `CausaloidGraph` representing the mechanistic pathways of the treatment.
- **Potential Outcomes via 'Contextual Alternation':** The EPP computes the potential outcomes, '$Y(1)$' and '$Y(0)$', not through statistical adjustment, but through direct, parallel simulation. This is achieved via the EPP's core counterfactual mechanism:

1. **Abduction:** The factual, pre-treatment state of the unit is established by "pinning" the static context, 'C_x'.
2. **Action:** The system then creates two hypothetical, parallel realities by cloning this base context:
   - A 'treatment' context ('C_1') is created where the treatment is applied (e.g., by introducing specific `Evidence` to a "treatment" `Causaloid`).
   - A 'control' context ('C_0') is created where the control is applied.
3. **Prediction:** The EPP evaluates the entire, unmodified `CausaloidGraph` twice—once against the treatment context 'C_1' to compute '$Y(1)$', and once against the control context 'C_0' to compute '$Y(0)$'.

- **The CATE as the Final 'PropagatingEffect':** The final CATE value, '$\tau(x)$', is the directly computed difference between the two 'PropagatingEffects' representing the potential outcomes. The entire workflow can be encapsulated in a single 'CATECausaloid' that orchestrates this simulation and returns the final estimate.

Representing Dynamic CATE in the EPP

In practice, complex systems evolve over time. Taking a patient's hospital stay as an example, diagnostic establishes a ground truth, treatment is administered, and depending on the effectiveness of the treatment, further treatment may follow until the patient's health has improved up to the stage at which a discharge is warranted. Medical professionals experience this reality every day but classical CATE struggles to capture the temporal progression because it can only provides a single estimate for a frozen moment in time, a property inherited from the underlying RCM methodology.

The EPP, by contrast, provides a complete architecture for Dynamic CATE that captures temporal causal progression by leveraging the existing EPP primitives:

- **Dynamic CATE ($\tau(x,t)$) via Evolving Contexts:** A patient's state is not static; it evolves. In the EPP, this is modeled by a dynamic `Context` that is updated with new clinical data over time. The EPP can therefore compute the CATE not just once, but continuously as the patient's `Context` changes. The result is a dynamic CATE estimate, $\tau(x,t)$, that provides a longitudinal, real-time view of treatment efficacy, allowing for intervention strategies that are continuously adapted to a patient's changing condition.
- **Context-Aware Model Selection:** The most profound challenge in long-term treatment is that the patient's underlying causal mechanisms can change. For example, a patient may develop a known pathway for drug resistance. The EPP can handles this not with unpredictable emergence, but with verifiable, dynamic model switching. A clinical EPP model can contain a library of multiple, pre-built, and independently validated causal sub-models (e.g., 'SubGraph_NormalResponse', 'SubGraph_ResistancePathwayA'). A high-level 'TriageCausaloid' continuously monitors the patient's evolving 'Context'. If it detects a specific biomarker that indicates the onset of drug resistance, it uses the EPP's Dynamic Dispatch ('RelayTo') mechanism to seamlessly and deterministically route all future reasoning through the appropriate 'SubGraph_ResistancePathwayA'.

  This provides the full benefit of an adaptive system by responding correctly to a fundamental change in the patient's state while remaining fully deterministic, auditable, and verifiable, as the set of all possible causal models is fixed and validated at design time. Also, in case the TriageCausaloid encounters an undecidable situation, it can trigger a predefined and validated safety protocol to inform medical experts of the detected anomaly and provide the full proceeding context.
- **Certified Assurance:** In a dynamic system, safety is paramount. The EPP integrates estimation with control at an architectural level. Any treatment recommendation, whether from a single model or as the result of a dynamic switch between models, is merely a proposal. The `Causal State Machine (CSM)` provides an architectural guarantee that this proposal will be deterministically verified against a `Certified Assurance Enclave` of immutable clinical safety rules (e.g., dose limits, contraindications, FDA rules) before any action is proposed to ensures the entire system remains verifiably safe at all times.

The EPP at this stage may have not causal structural learning, but it offers a different set of properties that makes it well suited to implement dynamical causality in domains that require dynamic adaptation, context aware model selection, and verifiable safety. By converting existing static CATE model to the EPP, first hand experience can be gained which then informs the exploration of gradual introduction of dynamic causality and thus offers a scalable pathway forward.

5.17 Discussion

The preceding sections have laid out the architectural components of the Effect Propagation Process. However, the capacity of the EPP to model dynamic systems where the causal structure itself can evolve introduces a class of

challenges that reach beyond the scope of formalism alone. There is an important distinction between the reasoning modalities the EPP introduces:

- Static
- Dynamic
- Adaptive
- Emergent

For static, dynamic and even adaptive reasoning determinism remains preserved. However, when a dynamic context is combined with a dynamic causal model, the resulting co-evolving of context and causal logic might not be deterministic any longer and thus amounts to dynamic causal emergence. In practice, dynamic and adaptive causal reasoning already cover the vast majority of practical problems the EPP is designed to address. However, in situations where the dynamic change cannot be known upfront, the emergent modality closes the gap albeit at the expense of determinism which imposes a profound challenge. To ensure that the EPP is built upon a sound, coherent, and trustworthy foundation, it is required to first understand the inherent higher-order consequences of its dynamic design from first principles.

Therefore, the subsequent philosophical chapters establish the first principles of the EPP. The Metaphysics in section 6 will establish the EPP's first principles of being and change. The Epistemology in section 7 will explore the far-reaching consequences for knowledge and truth in such a system. Finally, the Ontology in secction 8 will use these insights to structure a safe and sound set of primitives that are ready for rigorous formalization in section 9 and implementation in section 10.

6 The Metaphysics of the Effect Propagation Process

The metaphysics of the Effect Propagation Process establishes a set of principles underlying the ontology. We begin by defining the three core metaphysical concepts: Monoidic Primitives, Isomorphic Recursive Composition, and Contextual Relativity and then proceed to derive from these first principles the metaphysic of dynamics and becoming of the EPP. Conbined, these form the foundation of ontological design used throughout the EPP and its implementation DeepCausality. Ontological design is a constructive, engineering-oriented form of philosophy to specify the necessary and sufficient conditions of a new system to exist and operate coherently. The EPP's metaphysics is, therefore, the blueprint for a computable reality. It establishes the axiomatic foundation upon which the ontology is built, the epistemology is derived, and the implementation is realized.

6.1 Monoidic Primitives

A monoid is defined as an abstract algebraic structure that comprises of:

1. A set of elements with a certain type.
2. A binary operation that combines any two elements of the set results in a third element of the same set.
3. An identity element, which, when combined with any other element, leaves it unchanged.

Monoidic elements that can be combined with each other is fundamental to the composability of the EPP.

6.2 Isomorphic Recursive Composition

Isomorphism means "having the same shape" in the sense that isotropic elements all share the same form. Recursion refers to a structure containing itself. Isomorphism enables the combination of different types of monoidic primitives whereas recursion allows self-referential nesting. Combining these two concepts results in isomorphic recursive composition. This composition enables a monoidic primitive to derive its significance from its structural relation to other monoidic primitives. Critically, to ensure the non-reducible of form of monoidic primitives within the isomorphic recursive composition, it must have a singleton representation of itself.

6.3 The Metaphysics of Being

The metaphysics of being is structures in a classical Aristotelian notion (Book Zeta[113]):

- Monoidic Primitives (Matter): The EPP is composed of fundamental, identifiable elements.
- Isomorphic Recursive Composition (Form): Gives the primitive elements its form.

Combined, the monoidic primitives and the isomorphic recursive composition amount form an Aristotelian hylomorphic compound. The 'Monoidic Primitives' constitute the "matter" (*hyle*) of and the 'Isomorphic Recursive Composition' provides the "form" (*morphe*) that arranges its elements into a structured, meaningful whole.

This hylomorphic compound is the *archê kai aitia* of the system's existence. The specific form imposed upon the primitives is the foundational principle and explanation for its properties. Therefore, the hylomorphic compound is by definition static and describes what the system is at a snapshot in time, but it contains no inherent mechanism of change.

6.4 The Metaphysics of Dynamics

The classical hylomorphism describes a static substance as the combination of its matter and form. Therefore, to account for the dynamism inherent in the EPP, the *archê kai aitia* needs to capture the dynamics within an existing substance. However, because of the spacetime agnostic design of the EPP, dynamics can only be defines relative to its engulfing context. Therefore, the principle of contextual relativity operationalizes the expression of Substantial-Structural co-determination relative to its context. This is an inherent principle of how a substance with a fixed identity can exhibit variable states. By its definition, dynamics is predictable and bound to an existing substance and context.

6.5 The Metaphysics of Structural Change

The principle of Contextual Relativity (Dynamics) accounts for how a system can change its state predictably within a fixed structure. However, it cannot account for how the structure itself might evolve, as is required in advanced use cases like autonomous navigation. For structural change, the principle of emergence allows for two modalities.

- Fixed structural change
- Dynamic structural change

Fixed structural changes limit systems to pre-defined all possible structural changes that are triggered by known conditions. Consequently, the system remains fully deterministic and verifiable, but at the expense of . Conversely, dynamic structural changes enable adaptability by generating novel causal structures that were not explicitly encoded before, but at the loss of determinism and verifiability. This fundamental trade-off determinism at the expense of adaptability versus adaptability at the expense of determinism demands a fundamental decision. Also, fixed structural change can be expressed as a specialized form of dynamic structural change with a momentum of change hold constant. The reverse, however, is not possible.

The Effect Propagation Process as a theory of dynamic causality deliberately embraces dynamic structural changes. The limitations of fixed structural changes are too restrictive for the next generation of intelligent systems. The consequences of defining and implementing dynamic structural changes, while substantial, are accepted as the price of enabling truly dynamic causality in a dynamic environment. Therefore, the metaphysics of becoming defines the required principle of emergence, the subsequent ontology structures the mechanism of emergence, and the epistemology captures the implications. To capture both, fixed and static structural change, the principle of Higher-Order Emergence becomes necessary.

6.6 The Metaphysics of Becoming

The principle of Higher-Order Emergence captures the profound process of creation of new substance from within an existing substance. Emergence brings into being a new mater, a new form, both of it, or new dynamics. The principle of Higher-Order Emergence operationalizes two different modalities.

First-Order Emergence explains the creation of new substance. While contextual relativity applies to an existing substance, it cannot bring into being a new substance. The principle of ‘First-Order Emergence ̀posits a generative capacity within the EPP, a capability of imposing a novel *archê kai aitia* upon a set of primitives. This is the mechanism by which a new substance from within an existing context becomes into being.

In this modality, the distinction between the system and its context begins to dissolve. The result is a non-deterministic co-evolution where the spectrum of subsequent causal structures and contextual facts cannot be predicted any longer. The "reason for being" *aitia* of any given state is no longer a fixed principle but is itself an emergent property of the ongoing, self-modifying process. However, the process of becoming is invariant because of the first order designation.

Higher-Order Emergence refers to moves beyond the creation of a new substance from within an existing one. Instead, it describes a state where the generative capacity that enables ‘Emergence‘ acts upon itself recursively in what amounts to dynamic co-mergence of form, matter, and dynamics through recursive higher order emergence.

In this modality, the process of becoming itself becomes dynamic. This higher order emergence represents the EPP’s most advanced state of becoming, one that necessitates a new epistemology of emergence to explore its inherent emergent properties.

6.7 Discussion

The metaphysics of the Effect Propagation Process defines the substance of the EPP, its dynamics, and its emergence. Metaphysically, the principal of Higher-Order Emergence demands further elaboration. Fundamentally, it implies that first-order emergence, the capacity to create new substance, is simply a less recursive specialization of the governing higher order principle.The principal of Higher-Order Emergence leads to outcomes that are not necessarily guaranteed to be decidable let alone deterministic any longer and therefore it foreshadows three crisis:

1. The Crisis of Justification
2. The Crisis of Truth
3. The Crisis of Explainability

The crisis of justification immediately results from the fact that it must be decided how to chose one state of emergence over another one and how? The justification must be there otherwise the decision cannot be made, but this would render higher-order emergence fundamentally undecidable.

The crisis of truth results from the fact that, if emergence generates a new context, then how do we know that the facts in the newly generated context are true? If emergence generates new causal rules that uses new facts from a generated context, how do we know the outcome is true? If facts are fluid, verification is impossible. Therefore, truth must be re-established otherwise it undermines trust in operational safety of the EPP.

The crisis of explainability means that in co-emergence, it might not be possible any longer to explain the outcome because of the previous crisis of truth and the crisis of justification. Furthermore, if the process of emergence itself cannot be explained, how could possible derived artifacts be explained? How can a system be held accountable when its not explainable. It is not possible, and therefore the crisis of explainability roots in the very core of higher-order emergence.

The introduction of higher order emergence also raises the question of the genesis process, the origin of the emergence itself. Fundamentally, the genesis process imposes a decision: Do we allow any kind of machine intelligence to modify if its genesis process or not? The author argues for a unequivocal no. Considering the alternative, when a system that can evolve its own genesis process, it fundamentally becomes uncontrollable and unexplainable.

Therefore, the genesis process of emergence has to remain at the sole discretion of a human designer to ensure its explainability and a fundamental alignment with human values. The metaphysics itself cannot establish the core ethos or telos, only a human designer can do that. Under no circumstances should the genesis process in parts or in its entirety ever be created or modified by a machine intelligence because it cannot possible have the innate ethos of a human being and thus cannot possible align itself with humanity.

The existence of the genesis process also raises the thorny issue of whose ethics and values to codify and why? Which human designer? Who guards the guardians of the genesis? How to balance conflict demands from different stakeholders? These are immense normative and political challenge and, a metaphysics alone, cannot possibly answer a fundamentally societal set of questions.

As a consequence, the genesis process itself is a decision with far fetching higher order effects. The mitigation of unintended higher order consequences, will lead to the necessity of an immutable genesis telos, an underlying intent that serves as a criterion to discern whether emerging states are intended.

6.8 Summary

The complete metaphysics of the EPP is the synthesis of three core principles:

1. Substance (Being): The hylomorphic compound of Monoidic Primitives and Isomorphic Recursive Composition.
2. Dynamics (Changing): Governed by the principle of Contextual Relativity.
3. Emergence (Becoming): Governed by the principle of Higher-Order Emergence

The EPP's core metaphysical principles, Monoidic Primitives, Isomorphic Recursive Composition combined with the principles of and Contextual Relativity and Higher-Order Emergence form the foundation of the EPP. The The EPP provides a framework that distinguishes between two tiers of emergence. First-Order Emergence describes the system's capacity to generate new substance from a stable set of generative rules. Higher-Order Emergence, describes the system's ultimate capacity to evolve itself via a recursive and open-ended process of generative emergence. The

introduction of higher-order emergence implies that the ultimate outcome of the emergent process is not guaranteed to be decidable let alone deterministic any longer. This is a deliberate design decision to provide multiple modalities for different requirements. For dynamic systems, contextual relativistic dynamics should suffice. For handling regime change where the new structure can be decided a-priori, first-order emergence should suffice. However, when handling dynamic relativistic regime change in response to an evolving context, higher-order emergence becomes necessary to capture the dynamic co-emergence. The EPP, through the introduction of Higher-Order Emergence, establishes a foundation for exploring the interrelation between emergent systems and human value. However, it also presents a new set of unique challenges that demand a new epistemology and ontology of the EPP.

7 The Epistemology of the Effect Propagation Process

Epistemological approaches to acquiring knowledge in research fall into three categories: positivism, interpretivism, and pragmatism. Positivism concerns itself with observable facts based on the scientific method and thus seeks to achieve generalizability and objectivity. Interpretivism maintains that our knowledge depends greatly on our interpretation of observations of human actions, experiences, and environments thus making interpretive research more subjective. Pragmatism focuses on practical effects or solutions to address problems that are suitable for existing situations or conditions. The epistemology of pragmatism is that knowledge is a self-correcting process based on experience thus, it must be evaluated and revised in view of subsequent experience.

The presented EPP epistemology changes depending on whether the context is static or dynamic, and, equally profound, whether the EPP is static, dynamic, or emergent.

Ontology of Knowledge sources

In the EPP, the context is designed as the source of factual knowledge. For context, facts may remain invariant (e.g. the value of Pi) or receive continuous updates. The designation whether a context is static or dynamic refers to its structure, not to the factual data in it. Furthermore, a context might be shared between two or more defined EPP and an EPP may use one or more context(s) thus simplifying modeling complex domains. The EPP encodes each causal relationship in a designated Causaloid. The Causaloid encodes the causal rule, whereas the context encodes supporting data required to apply the rules. The Causaloid may use external data or data from the context to apply its rule. For example, a context may encode a continuous signal feed from a LIDAR sensor and the Causaloid encodes a rule to infer whether an obstacle has been detected. In this case, the context provides all data. In another scenario, a context may encode several known defect patterns, a Causaloid tests incoming image data for the defect data from the context, but uses incoming real-time image feeds from a manufacturing monitoring system to determine if any of the produced items contain known defects. In this case, the Causaloid relies on context and external data. Therefore, the Effect Propagation Process emits a flexible knowledge ontology by relying on one or more contexts and potentially multiple external data sources.

Knowledge Derivation

The EPP derivates knowledge by applying data to the Causaloid that models that causal relationship to determine whether the causal relation holds true within the applicable context. Consequently, multi-stage reasoning maps directly to the topology of the EPP itself because each effect from a Causaloid propagates further through the EPP topology, which is the structure of the EPP manifested as all connected Causaloids. From this perspective, a "line of reasoning" literally becomes a pathway through the EPP topology. Through the topological approach of knowledge derivation, the Effect Propagation Process provides a flexible way to model complex, contextual, multi-causal domains.

Justification of Derived Knowledge

Discerning the truthfulness of knowledge is one key element of epistemology. The EPP with its explicit context, explicit causal function (Causaloid), and explicit support for external data provides all pre-requirements to support the full explainability of each inference. Furthermore, in the case of multi-stage reasoning, the sequence of applied Causaloids establishes the order or explainability. Fundamentally, the EPP leads to explainable causal inference because of complete data, context, and inference function when assuming a static EPP. For a dynamic or emergent EPP, explainability might not be guaranteed for all potential state transitions. An implementation of the EPP has to specify the exact details to support explainable inference and where to establish sensible constraints on explainability.

The gravitas of the EPP epistemology is rooted in its flexible, contextualized ontology, a powerful knowledge derivation mechanism, and its intrinsic support of explainable causal inference. The epistemology varies depending on whether the EPP process is static, dynamic, or emergent.

7.1 Static EPP Epistemology

For a static Effect Propagation Process, the knowledge is explicitly modeled during the design stage and confined in the context. The quantitative nature of explicitly modeled context and EPP leads to the positivism of the resulting epistemology.

Static context

A static context emits an invariant structure after it's defined, therefore a static EPP combined with a static context allows for the strongest deterministic guarantees albeit at the expense of flexibility. Static contexts remain an invaluable tool to model contextual data that remain structurally invariant, which is a common situation when integrating

external data sources. The content, structure, richness, and accuracy of that static context profoundly determine the epistemology of what can be known through the EPP.

Dynamic context

In a dynamic context, the context structure itself evolves e.g. new elements (i.e. quarter of a year) are added as the data feed progresses. By definition, a dynamic context relies on a generating function to gauge the dynamic changes of the context. The impact on the epistemology of a static EPP remains minimal though. Fundamentally, dynamic contexts are used when structural elements occur at either regular intervals or otherwise determinable occurrences, and therefore, the EPP can model these elements regardless of whether they have been added to the context yet. For example, a Causaloid that determines whether the sum of the previous three monthly financial reports matches the quarterly financial reports for the current quarter might be a precondition if the "current" quarter in the context has been updated. Therefore, dynamic contexts simplify domain modeling while leaving the epistemology modeled in a static EPP intact.

7.2 Dynamic EPP Epistemology

For a dynamic Effect Propagation Process, the dynamics are captured in generative functions that evolve either the EPP, the context, or both. Conceptually, these generative functions could range from deterministic, rule-based algorithms that construct or modify Causaloids and Context structures based on predefined logic or specific triggers, to more adaptive mechanisms. For instance, a generative function for a dynamic Context might be a higher-order function that, given the current state and new inputs, returns an updated Context graph, a practice well established in functional programming to build dynamic systems.

The ontology of knowledge may evolve as a result of the evolving EPP and the impact of the epistemology remains deepening not only the EPP evolution itself but the interaction with its context as it can happen that both, the EPP and its context evolve dynamically.

Static context

For a dynamic EPP, a static context may serve as the foundational layer that captures core data that remain structurally invariant. As with a static EPP, the static context determines fundamentally what determines the epistemology of what can be known through the dynamic EPP.

Dynamic context

For a dynamic context, though, the impact on the epistemology captured in a dynamic EPP can be profound. For example, with the advent of model context protocol (MCP), which lets LLMs call into tools to retrieve or modify data, a causaloid in a dynamic EPP may trigger an MCP invocation, which then updates the context by expanding its structure, and then triggers a generative function that creates a new Causaloid based on the retrieved contextual data, which then analyzes either a newly created part of the context, or a new external data feed created by the MCP. As a consequence, the epistemology in this case depends on a dynamic EPP-Context co-evolution.

Dynamic Co-Evolution

When both, the EPP and the engulfing context evolve dynamically in what can be seen as a co-evolution, then no fixed epistemology can be established anymore because the inference based on generated Causaloid over newly added sub-structures of the context may or may not occur depending on the occurrence of the underlying trigger event(s). One could estimate a potential epistemology by using a Rubin causal model[107] (RCM) by comparing potential reasoning outcomes under different scenarios in which a Causaloid was generated versus when not.

More profoundly, it might not be possible any longer to use automated explainability to discern the appropriateness and relevance of the generated Causaloids and contextual shifts in response to external changes. This introduces an element of interpretivism to the resulting epistemology: the derived knowledge requires the observer to apply a conceptual framework for understanding the system's complex and dynamic evolving behavior.

7.3 Emergent EPP Epistemology

Unlike a dynamic EPP, an emergent EPP does not evolve anymore based on pre-determined triggers that initiate pre-defined generative functions. Instead, an EPP is considered emergent when the underlying generation process leads to novel causal configurations, reasoning pathways, or new generative principles for Causaloids context interactions that were not explicitly encoded beforehand. The generation process may incorporate principles from evolutionary computation, novelty search, or machine learning embedded in the EPP itself. While the full exploration of AI-

driven generative functions for EPP remains future work, the foundational idea of using programmable functions to dynamically define and evolve both the EPP and its context is a natural extension of EPP's core design philosophy.

Regardless of the mechanism, emergent EPP does not interact with its Context using pre-defined procedures but instead relies on procedures generated by the EPP itself. The key indicator of emergence is that its novel behavior was not foreseeable by its initial designers.

Static context

Like a static or dynamic EPP, when the static context has been defined upfront, it determines fundamentally what determines the epistemology of what can be known through the dynamic EPP within the contextual boundaries.

Unlike a static or dynamic EPP, an emergent EPP may or may not generate a new static context and that indeed alters the Epistemology emerging from an emerging EPP.

Dynamic context

Likewise, when an emerging EPP creates or modifies a dynamic context, the emerging Epistemology cannot be determined any longer because of the resulting co-emergence of the EPP and its context.

Dynamic Co-Emergence

An EPP co-emerges with its context when the underlying generation process leads to novel causal configurations that were not explicitly encoded beforehand. This can happen when the EPP contains methods of machine learning that evolve the EPP itself in response to a dynamically changing context. As a result of the dynamic, non-deterministic self-modification of the EPP itself, the spectrum of subsequent factual representation in the context and the emerging causal structures cannot be predicted any longer.

Therefore, determining the truthfulness of the emerging causality imposes a non-trivial challenge that adds an elevated level of pragmatism to the epistemology. The pragmatism becomes necessary because it is not guaranteed that the underlying dynamic context always leads to a truthful representation of the world it seeks to model, but the generated causal relationships may not always be correct either. Both can happen due to generative errors during the EPP. Generative errors may result from complex interactions that contain steps that, in isolation, are correct, but when combined in a certain order may lead to an incorrect outcome. This is a typical characteristic of increasingly complex dynamic systems that need to be considered by taking an operational stance on truth.

7.4 Operationalized Meaning of Truth

The meaning of truth evolves depending on the modality of the EPP because the underlying reference for a true statement varies depending on the chosen modality. Per the definition of knowledge, a true belief must entail a high degree of justification and come from a reliable source to count as knowledge. It is the underlying justification process that depends on the modality of the EPP that causes the shift in the meaning of truth to vary.

For a static Effect Propagation Process, the meaning of truth aligns with the classical correspondence theory. That means, that if the context encodes accurate facts and the causal relationships are true, all derived forms of knowledge must be true. Justification rests upon the verifiable mapping between the EPP's explicit model encoded in its context and the part of reality it purports to represent. The static EPP implicitly operates under the assumption that its model is a faithful mirror of objective facts. Here, the truth of an inference is determined by the adherence to the contextual facts and encoded causal relationships. As a result, a static EPP leads to deterministic verifiability within the confined boundaries of its context. As the EPP transitions into a dynamic modality, the meaning of truth begins to shift towards a coherent adaptability to dynamic interaction with a changing context. A dynamically modified causal relationship is deemed true if it maintains consistency with the facts in its evolving context. In a dynamic modality, the justification of knowledge becomes contextually and temporally aware. Therefore, truth is assessed by the EPP's capacity to maintain a relevant and internally consistent causal understanding amidst navigating a temporal dynamic context. This leans towards a coherence theory of truth, where coherence itself must be evaluated relative to the EPP's intricate temporal structures.

For a contextual co-emergent EPP, the meaning of truth shifts further toward pragmatic efficacy. This shift becomes necessary because of the emergence of relativistic causal relationships from the EPP that co-evolve with its context. Here, establishing an objective a priori truth becomes elusive since the fabric that would traditionally serve as a stable reference for truth is itself emerging dynamically alongside the causal inference made from it. Instead, the truth of an emergent causal inference is established by its utility in enabling the EPP to navigate its environment within its temporally complex context. This pragmatic efficacy means that truth, defined by its functional value, becomes inherently system-relative and context-dependent.

Indeed, a functional value could serve as a fitness function guiding the emergent process itself thus raising fundamental questions about alignment. Consequently, pragmatic efficacy can lead to multiple, functionally 'true' yet distinct causal understandings, each valid within its own emergent trajectory and its relativistic interrelation with its context.

This dynamic interplay, where the EPP generates both its context and the Causaloids that encode the causal relationships that operate within that context presents a research opportunity. It allows for the exploration of relativistic emergent causality and how coherent and pragmatically effective causal understandings can arise in systems that lack a fixed predefined spacetime. This might be of interest in theories of fundamental physics where spacetime itself may be an emergent phenomenon arising from more fundamental processes.

7.5 Causal Emergence

The problem of modeling no a-priori causal structures motivates a different view of causality that sets the stage for tackling causal emergence. The Effect Propagation Process framework's detachment from fixed spacetime and its focus on a generative function establish the foundation" for causal emergence. Static causal discovery, for example Pearl's DAGs framework, assumes a fixed causal structure and thus cannot handle causal emergence. Granger causality assumes that time-dependent variables change, but the causal structure remains fixed and therefore cannot handle causal emergence either. This argument holds true for any dynamic system with fixed causality because of the inability of the underlying methodology to handle spacetime-agnostic causal structure. The Effect Propagation Process instead proposes that the causal relationships themselves emerge, change, and may even disappear. The implications of this approach lead to a fundamental reassessment of how to operationalize causality:

Causal discovery

Instead of trying to find a fixed causal structure, EPP models how causal relationships emerge from underlying (dynamic) processes. The existing work on causal discovery remains valid; the EPP, however, takes the idea one step further by incorporating a dynamic generative process. Further research will verify the utility of this perspective, but at least it expands the notion of discovering a static structure to describing a dynamic process.

Causal transferability

Instead of trying to capture the exact conditions under which a causal relationship holds true, the EPP specifies all presumptions as a generative function, which makes it fundamentally testable and thus transferability can be decided.

Causal dynamics

The inspiration from causality in quantum gravity was carefully chosen because of its unique ability to reconcile dynamic and static structures and its handling of deterministic and probabilistic modality. The underlying idea in quantum gravity is that the spacetime fabric of reality itself emerges from an underlying process. While we do not have the scientific methods and technology to verify this idea on the quantum level, we can carefully, within boundaries, transfer the idea to the EPP notion that causality itself emerges from an underlying generative process and, therefore, model the dynamics of causal emergence. The properties of EPP become apparent when looking from the lens of modeling the dynamics of causal emergence.

- **Temporal order becomes irrelevant:** Because causal relations can emerge from an underlying process
- **Spacetime-agnostic becomes necessary:** Because the generative process is concerned with establishing relations of effect propagation, the exact fabric through which those effects propagate is conceptualized as an external context; therefore, EPP itself has to be spacetime-agnostic.
- **Hardy's Causaloids are necessary:** Because the EPP itself is spacetime-agnostic, a different representation of causal relationship that is also spacetime-agnostic becomes necessary and the causaloid proposed by Lucian Hardy has been deemed the best fit.
- **Centrality of the generative function:** Because the EPP can represent causal relations as either static, dynamic, or emergent, the generative function takes on a central role to express those causal relations. Furthermore, a generative function may generate the engulfing context as a specific fabric for the effect propagation process.

8 The Ontology of the Effect Propagation Process

8.1 The Ontology of Being

The static ontology defines the concrete primitives that are the fundamental building blocks, the "matter" (hyle) and "form" (morphe) of the EPP, as defined by the Metaphysics of Being. Each primitive is a direct instantiation of the Monoidic Primitives and Isomorphic Recursive Composition, designed to represent distinct categories of existence within the EPP's universe. This section details these foundational entities, which, when combined, form the foundation upon which all dynamic processes and emergent phenomena are built.

8.1.1 The Causaloid: A Unit of Causality

Causality is fractal. A high-level cause (e.g., "economic recession") can be decomposed into a network of smaller, interacting causes (e.g., "inflation," "supply chain disruption," "interest rate hikes"), each of which can be further decomposed into smaller causes. Classical causality has always struggled with this reality because structural composition is fundamentally at odds with both, the SCM representation and the causal DAG.

The EPP proposes to represent causality closer to its fractal nature: The Causaloid. In line with the EPP metaphysics, the Causaloid isa monoid.

The identity element of the monoid is the Singleton 'Causaloid'. This is the fundamental unit of causality It has an id as identity and a causal function that captures the causal relationship. The form of the Causaloid is Isomorphic recursive to enable uniform composition. The causaloid is isomorphic in the sense that a causaloid can have different types and yet share the shape of a causaloid. For example, a ccausaloid can be:

1. A singleton causaloid that is a single cause
2. A collection of causaloids
3. A graph in which each node is another causaloid

Applying the concept of isomorphic recursive composition means that any number of causaloids of different types uniformly compose. A complex causal graph can be encapsulate into a single Causaloid, which then becomes a node in another causal graph that itself is part of a causal collection. This creates a mechanism where:

- Complexity is manageable
- Scalability is inherent
- Abstraction is first-class

Complexity is manageable because each causaloid can be build and tested in isolation, or as part of a specific subgraph. The mechanism scales from a singular unit, to larger collections up to complex graphs. While the system complexity scales, the conceptual overhead remains constant because a single concept, the causaloid, scales from simple to complex, from small to large. Abstraction is first-class because causal reasoning happens uniformly regardless of the underlying complexity. The reasoning mechanism remains the same regardless of whether a causaloid is a singleton, a collection or a graph.

The monodic binary operation for composition results from the fact that causaloids are isomorphic which allows to combine a causal collection into a mew singleton causaloid. Likewise, an arbitrary complex causal graph composes uniformly any number of causaloids as its nodes and is in itself a causaloid that composes with other causaloids.

Isomorphic Recursive Composition for causality defines an ontology where the distinction between "part" and "whole" is fluid. Every "whole" (a composite Causaloid) can become a "part" in a larger whole without ever changing its fundamental nature as a Unit of Causality.

8.1.2 The Contextoid: A Unit of Context

The Contextoid represents the monoidic primitive of state, an atomic, non-recursive, and identifiable unit of factual information. The EPP ontology makes a strict distinction between these two primitives: Causaloids are recursive and represent reasoning; Contextoids are isomorphic and represent the ground truth upon which reasoning operates. The ontology of the Contextoid does not enforce a single, fixed representation for concepts like "space" or "time." Instead, it defines abstract categories:

- Datoid - A monoidic unit of data-like context

- Spaceoid - A monoidic unit of space-like context
- SpaceTempoid - A monoidic unit of spacetime-like context
- Symboid - A monoidic unit of symbol-like context
- Tempoid - A monoidic unit of time-like context

The categorical isomorphism leads to the most critical design principle of this primitive: Contextoids are structurally non-recursive. A Tempoid cannot contain another Tempoid; a Spaceoid cannot contain another Spaceoid. The structural prohibition of recursion guarantees the logical consistency of the Context. It makes it impossible to construct a paradoxical state, such as a time loop where a moment in time is defined as preceding itself. By ensuring the factual bedrock is non-paradoxical and acyclic, the EPP provides the stable, non-contradictory ground truth required for sound, higher-level reasoning to operate.

Conversely, the same categorical isomorphism enables heterogeneous composition within each category. For example, the Spaceoid category can represent a point in a flat, EuclideanSpace alongside another representing a point in a non-Euclidean GeoSpace. From the perspective of the ontology, both are valid "spatial fact i.e. instances of a Spaceoid and thus uniformly treated as space-like. This categorical isomorphism enables the EPP to model heterogeneous systems where different parts of reality demand different mathematical representations of space, time, spacetime, symbol, and data. However, the absence of isomorphic recursion deprives the contextuid of its structure.

8.1.3 The Context: A structure for Contextoids

The Context is the structure that gives meaning through relations to the Contextoids. In the EPP, the Context is a first-class ontological entity: a hypergraph structure that holds all Contextoid primitives and the explicit, typed relationships between them. It is partially monodic in the sense that it has an identity, but unlike other primitives, it has no monoidic composition hence its name “Context” reflects that it is not monoidic and thus does not compose. The deliberate choice roots in the fact that, while its elements, the contextoids, are structurally isomophic and compose, the engulfing structure, the context, does not compose to prevent the complexity that arises from merging graphs, but more importantly, to prevent incorrect states in this critical structure. Despite this decision, the context, however, is neither singular nor absolute.

Static vs Dynamic Context

A static context is established upfront and is structurally assumed to remain invariant during its lifetimes. The utility comes from encoding static knowledge, for example the ICD-10 medical ontology that is standardized for a given version. A static representation for each version of the ICD ensures there is no mixing of different standards.

A dynamic context is one whose structure is dynamically build and modified during its lifetime. The utility applies to situations where context is fluent but structurally known. For example, a monitoring system that receives data feeds from maintain drones has to add contextoids for drones coming online and streaming data and, likewise, remove contextoids for drones that get out of range and stop the data stream. The dynamic context can only add, modify, or remove contextoids of known types to ensure strict operational safety.

Single vs Multiple Contexts

The EPP ontology explicitly supports multiple frames of reference via multiple contexts. A causal model can be linked to a primary context representing the current, observable reality, while also having access to any number of additional contexts. These auxiliary contexts can represent simulated worlds, historical states, counterfactual scenarios, or backup data sources for real-time data feeds.

Contextual Counterfactuals

This capacity for multiple contexts establishes the foundation for relativistic contextual counterfactual analysis. It allows the system to ask "what if" questions by creating hypothetical realities (a new extra context), modifying specific Contextoids within them (e.g., "what if temperature were 5% higher?”), and then evaluating the same Causaloid logic against these altered frames of reference. This can be achieved through architectural patterns such as "hot/cold" context partitioning, where the subset of Contextoid relevant for the counter factual analysis are centralized into one dedicated contest, from which the alternate versions are derived. When combined with high-performance data structures, this enables the execution of thousands of counterfactual simulations in real-time, even on resource-constrained embedded devices.

8.1.4 The Evidence: A Unit of facts

Evidence represents a specific monoidic fact presented to a Causaloid for evaluation that may originate from outside the system i.e. from a sensor reading, were extracted from a contextoid, or is derived from a previous chain of reasoning from a causaloid. A causaloid uses Evidence as input and the context for supporting data used during the analysis. The output of a Causaloid can then be directly verified against the specific Evidence it received, which is fundamental for explainability in complex systems.

Evidence is a generalized isomorphic recursive container designed to support unified reasoning across multiple modalities:

- Deterministic: Boolean values ("true/false").
- Numerical values: Numbers (e.g., sensor readings).
- Probabilistic values (e.g., confidence scores, likelihoods).
- Contextual Links (a Contextoid within a Context), enabling the Causaloid to access complex, structured, and non-numerical facts.

Evidence is recursive and may contain:

- Maps of other Evidence primitives
- Graphs of Evidence primitives enables a Causaloid to access complex, relational data.

The multi-modal Evidence enables the EPP to reason over the full spectrum of information found in complex systems.

8.1.5 The Propagating Effect: A Unit of Influence

The output of a causal evaluation is the PropagatingEffect, a monoidic primitive of influence. The PropagatingEffect is the operational heart of the "Effect Propagation Process" itself, a unit of influence that travels through the causal graph, from one Causaloid to the next, driving the continuous effect propagation process. The PropagatingEffect is a unified inference outcome across different modalities. It is isomorphic in that it can represent:

- Deterministic effect: A definitive boolean outcome ("true/false").
- Probabilistic effect: A quantitative outcome, such as a probability score or an estimate.
- Contextual Link: A reference to a specific Contextoid within a Context.

For the Contextual Link, a causaloid writes its reasoning outcome into a contextoid and then propagates its "effect" as a Contextual link to direct the next Causaloid to use the structured information in the contextoid for further analysis. This Contextoid then becomes the Evidence for the subsequent step in the reasoning chain, enabling dynamic, data-driven causal pathways for non numerical representation as required, for example, for causal symbolic reasoning enabled by the symboid contextoid type.

8.2 The Ontology of Becoming: Dynamics

The ontology of dynamics in the EPP is governed by the metaphysical principle of contextual relativity.Contextual means that the significance of a monoidic primitive is derived from its relation to its context. Contextual relativity means that state is an emergent properties arising relative to its context with the implication that the same object may have different states when used in different context, but, more profoundly, the same object may alters its state when context itself is relativistic altered and populates its contextual adjustment to all connected monoidic primitives. Contextual relativity represents the duality a monoidic primitive being influenced relative to its context while simultaneously the context itself is being influenced by relativistic effects of the engulfing fabric itself.

Contextual Relativity is expressed in the EPP in two ways:

First, a Causaloid's reasoning is relative to the 'Context' it is evaluated against. The EPP ontology explicitly supports multiple frames of reference for each causaloid. A Causaloid asking "Is the pressure critical?" can return true when evaluated against a Context representing a high-altitude environment and false when evaluated against one representing sea level. The causal logic is the same, but the truth it produces is relative to the frame of reference. This is very much in line with the observer principle in the general theory of relativity.

Second, the state of a Contextoid itself is subject to relativistic forces imbued on its engulfing fabric. For example, a spacetime contextoid may need to adjust its temporal value for gravity induced time dilation. A quaternion contextoid may need to adjust its rotation value relative to incoming sensor data. Therefore, the EPP ontology defines two critical operations to handle contextual relativity via two defined operations:

1. Adjust
2. Update

Adjust means, an existing value is adjusted for relativistic effects or corrected for sensor drift. The core property of an adjustment operation is to take a correction value, say an offset, and applies it to the existing value.

Update means and existing value is replaced with a new value, fort example a new sensor reading. The core property of an update operation is that to take a as data value and replace the current one with the new one.

These two operations, adjust and update, determine the difference between a strictly static contextoid and a dynamic contextoid. A static one is assumed to be invariant throughout its lifetime, for example by holding a set of immutable facts i.e. thermal threshold. A dynamic contextoid is one that gets either adjusted or updated, for example when new sensor reading becomes available. A simple causaloid then reads the thermal threshold from one contextoid, and reads the current thermal sensor data from a dynamic contextoid, and determines if it's getting closer to the thermal limit. Critically, when transferring the model to a different opertatonal environment that requires a different thermal threshold, the replacement of one contextoid is sufficient to ensure the correct functionality of the modeled thermal system.

8.3 The Ontology of Emergence

The ontology of Emergence is captured in the mechanism of the Generative Process, a four-stage command-execution cycle that transforms external stimuli or internal states into structural modifications of the EPP itself.

1. The Generative Trigger
2. The Generative Command
3. The Generative Process
4. The Generative Outcome

This four stage process applies to both, first order emergence and higher-order emergence because of its recursive design. Ontologically, each step results in a specific and verifiable object, which lays the foundation of a principled implementation at a later stage.

8.3.1 The Generative Trigger

Emergence begins with a Generative Trigger. This primitive represents the initial impetus for change, signaling to the EPP that a modification of its internal structure becmomes necessary. Triggers can originate from external stimuli, the passage of time, or an explicit external commands. A generative trigger can arise from internal states, such as the detection of an anomaly, a deviation from a desired goal, or from a change in external state such as the detection of a

fundamental change in the environment i.e. transiting from day to night. The Generative Trigger acts as the catalyst to initiate the generative process.

8.3.2 The Generative Command

Upon receiving a Generative Trigger, a Generative Command is constructed. This primitive is a declarative, verifiable blueprint of a desired action or structural modification. It is the formal expression of the EPP's intent to alter its own substance. Generative Commands are explicit instructions for change, such as the creation, update, or deletion of a Causaloid or Contextoid, the modification of contextual relationships by adding or removing edges in the hypergraph.

The Generative Command is both isomorphic and recursive. It is isomorphic in that it unifies a diverse set of possible actions under a single primitive type. These actions include, but are not limited to:

- No operation. *NoOp*:
- Commands for managing causal logic: *CreateCausaloid*, *UpdateCausaloid*, *DeleteCausaloid*
- Commands for managing contexts : *CreateBaseContext*, *CreateExtraContext*, *UpdateContext*, *DeleteContext*:
- Commands for managing factt within a context. *AddContextoidToContext*, *UpdateContextoidInContext*, *DeleteContextoidFromContext*:
- A user-defined command for higher-order emergence: *Evolve*.

The Generative Command is recursive through its Composite variant. This variant allows a single Generative Command to contain an ordered sequence (a "stack") of other Generative Commands. This recursive capability enables the EPP to express complex, multi-step transformations as a single, atomic unit of intent. For example, a single Composite command could specify: "add root node, then add Causaloid A, then add an edge between root and A." This ensures that even intricate sequences of operations are treated as a coherent, atomic transaction, maintaining the integrity and verifiability of the system's emergence.

The Evolve command is one notable outlier in that is not an atomic command. Instead, Evolve is a meta-command, it's an instruction to replace the mechanism that generates commands itself. By definition, evolve is no longer guaranteed to be deterministic. If the choice of what to evolve into is not itself deterministically derived, then the system's future behavior becomes non-deterministic and potentially unpredictable. Even if the new the command is deterministic, the decision to evolve and how to evolve represents a point where the system's fundamental archê kai aitia of change is altered and it is conceptually unclear what this may entail in practice.

8.3.3 The Generative Process

The Generative Process is the operation responsible for actuating the Generative Command. It interprets xommand and translates it into concrete, structural modifications within the EPP's ontology.

This stage represents the materialization of the intended change, where new Causaloids are instantiated, Context graphs are reconfigured, or Contextoids are added, modified or removed. The Generative Process transforms the abstract intent into tangible alterations of the EPP's substance.

The Generative process is designed to be robust and auditable. It processes each Generative Command deterministically, ensuring that the system's state transitions are predictable given a specific command. Its primary function is to ensure the integrity of the EPP's internal structure during the process of self-modification.

8.3.4 The Generative Outcome

The generative outcome is a primitive that holds both, the modified or generated artifact and relevant metadata about the process itself. It attest, the 'what happened" during the process in form of a log detailing every state transition and asserts whether the outcome is in accordance with the stated intention of the generative command. This explicit record of the system's self-modification means that the EPP maintains a complete and verifiable history of its own evolution; a crucial step for auditability, debugging, and the continuous learning process, providing the ground truth for subsequent development of emergence in complex dynamic systems.

8.4 Discussion

The ontology of Effect Propagation Process addresses the limitations of classical causal models in dynamic, complex systems. The EPP's ontology deeply integrates a set of primitives: the Causaloid as a recursive unit of logic, the Contextoid as a non-recursive unit of fact, and the Context as a dynamic, multi-frame relational environment. These primitives, combined with the multi-modal Evidence and PropagatingEffect, form a robust foundation for expressing causality not as a static, linear chain, but as a continuous process of effect propagation.

The EPP's explicit ontological primitives and its four-stage Generative Process provide a unique computational platform for exploring the nature of emergence itself. While the principles of Dynamic State Mutation and Dynamic Structural Evolution offer clear pathways for predictable adaptation, the concept of Dynamic Co-Emergence, through the Evolve command, opens a frontier for investigating self-referential self-modification of dynamic systems.

The Evolve command, as defined within the EPP's ontology, represents the system's capacity to fundamentally alter its own generative principles. This capability, while theoretically profound, introduces complex questions regarding predictability, control, and safety, the EPP's transparent, auditable architecture where every Generative Command and Execution Result is explicit offers an unprecedented opportunity for empirical investigation into these phenomena.

By designing and observing systems that engage in self-referential self-modification, practitioners can gain firsthand insights into the dynamics of emergent behavior, the challenges of maintaining verifiability in adaptive systems, and the potential pathways towards engineering truly autonomous and self-adapting dynamic systems. This area represents a rich vein for future research, where the EPP's foundational principles can be tested and expanded through direct computational experimentation

9 The Formalization of the Effect Propagation Process

9.1 Axioms

The theoretical foundation of the Effect Propagation Process is derived from a single axiom, which is itself a necessary consequence of detaching causality from a presupposed spacetime.

> **Axiom of Causality:** Every causal phenomenon is an instance of a spacetime-agnostic functional dependency. Let $\mathcal{C}$ be the universal set of all causal phenomena. The axiom states:
>
> $$\forall c \in \mathcal{C}, \exists f \text{ such that } c \equiv (E_2 = f(E_1)) \tag{1}$$
>
> where E_1 and E_2 are effects and f is a function. The nature of E and f are deliberately unconstrained.

9.2 Formal Derivation of the EPP Primitives

The core architectural primitives of the Effect Propagation Process are not a collection of independent design choices, but are necessary logical consequences that follow from the single causal axiom. This section provides a formal summary of that derivation.

1. **The Contextual Fabric ($\mathcal{H}$): The Background Condition.** A function $f(E_1)$ may be conditioned on a set of background variables, K. The general form of the axiom is thus $E_2 = f(E_1, K)$. A mechanism to hold and provide this background information K is therefore necessary.

   $\implies$ Let the **Contextual Fabric** ($\mathcal{H}$) be this mechanism.

2. **The Causaloid (χ): The Functional Primitive.** Since every causal phenomenon c is defined by a function f, a computational primitive to encapsulate f is a logical necessity.

   $\implies$ Let the **Causaloid** (χ) be this primitive, whose primary component is the **causal function** (f_χ).

3. **The PropagatingEffect (Π_E): The Isomorphic Message.** A function requires inputs and produces an output. Given the compositional nature of functions (e.g., $g(f(E))$), where the output of one serves as the input to the next, a single, uniform message type is logically necessary.

   $\implies$ Let the **PropagatingEffect** (Π_E) be this isomorphic primitive serving as input to and output from a Causaloid.

4. **The Causaloid Graph ($\mathcal{L}$): The Structural Composition.** A system may contain multiple causal phenomena $(c_1, c_2, \ldots, c_n)$ that are themselves causally related. A structure is required to represent the set of Causaloids $\{\chi_1, \chi_2, \ldots, \chi_n\}$ and the dependency pathways of their 'PropagatingEffect's.

   $\implies$ Let the **Causaloid Graph** ($\mathcal{L}$) be this structure, defined as a hypergraph where nodes are Causaloids and hyperedges are

The subsequent sections of this chapter provide the detailed set-theoretic definitions of these logically necessary primitives, which together form the operational core of the Effect Propagation Process.

9.3 Contextual Fabric ($\mathcal{C}$)

The Effect Propagation Process (EPP), being spacetime-agnostic, requires a formally defined structure through which effects propagate and within which causal relationships are conditioned. This structure is termed the **Contextual Fabric**, denoted abstractly as $\mathcal{C}$. The Contextual Fabric is not a monolithic entity but can be composed of multiple, distinct contextual realms, each potentially representing different aspects of a system's environment or internal state (e.g., physical space, temporal scales, relational networks, data streams). This section provides a set-theoretic formalization of this fabric, starting from its most granular component, the Contextoid, and building up to collections of Context Hypergraphs.

9.3.1 The Contextoid (v)

The atomic unit of contextual information within the EPP framework is the **Contextoid**. A Contextoid encapsulates a single, identifiable piece of data, which can be temporal, spatial, spatiotemporal, or a general data value.

Contextoid Definition

A Contextoid v is an element of a set of all possible contextoids $V_{\mathcal{C}}$ within a given Context Hypergraph (defined in Section 9.3.2). It is formally defined as a tuple:

$$v = (id_v, \text{payload}_v, \text{adj}_v)$$

Contextoid Identifier

Let $\mathbb{I}$ be a universal set of unique identifiers. Then $id_v \in \mathbb{I}$ is a unique identifier for the contextoid v. This ensures each piece of contextual information can be uniquely referenced within its encompassing Context Hypergraph C.

Contextoid Payload

The payload_v represents the actual contextual information encapsulated by the Contextoid. It is defined as a tagged union, allowing for diverse types of context:

$$\text{payload}_v \in \{\text{Data}(d) \mid d \in \mathcal{D}_T\} \cup \{\text{Time}(t) \mid t \in \mathcal{T}\} \cup \{\text{Space}(s) \mid s \in \mathcal{S}\} \cup \{\text{SpaceTime}(st) \mid st \in \mathcal{ST}\}$$

where $\mathcal{D}_T, \mathcal{T}, \mathcal{S}$, and $\mathcal{ST}$ are predefined sets representing the domains of all possible data, temporal, spatial, and spatiotemporal values, respectively.

Data Payload ($Data(d), d \in \mathcal{D}_T$) The set $\mathcal{D}_T$ represents the domain of all possible data values that a Data Contextoid can hold. The specific nature of d is generic (of type T) and can encapsulate arbitrary information relevant to the causal model, such as sensor readings, calculated metrics, textual features, or symbolic states.

Time Payload ($Time(t), t \in \mathcal{T}$) The set $\mathcal{T}$ represents the domain of all possible temporal values. A temporal value $t \in \mathcal{T}$ is typically structured as an ordered pair:

$$t = (\text{scale}, \text{unit})$$

Formalizing Temporal Scale (T_{scale}) and Unit (T_{unit}) Let $\mathcal{T}_{\text{scale}}$ be an enumerated set of possible temporal granularities, e.g.,

$$\mathcal{T}_{\text{scale}} = \{\text{Year, Month, Day, Hour, Minute, Second, Nanosecond, EventStep}\}$$

Let $\mathcal{T}_{\text{unit}}$ be the domain of values for the temporal unit, typically non-negative integers ($\mathbb{N}_0$) or a suitable ordered set, representing the count at the given scale. Thus, scale $\in \mathcal{T}_{\text{scale}}$ and unit $\in \mathcal{T}_{\text{unit}}$.

Space Payload ($Space(s), s \in \mathcal{S}$) The set $\mathcal{S}$ represents the domain of all possible spatial values. A spatial value $s \in \mathcal{S}$ can be represented by coordinates, for example, as a tuple for up to three dimensions:

$$s = (x, y, z)$$

where x, y, z are optional components.

Formalizing Coordinates (T_{coord}) Let T_{coord} be the domain for coordinate values (e.g., $\mathbb{R}$, $\mathbb{Z}$, or a generic type V). Each coordinate component $x, y, z \in T_{coord} \cup \{\text{null}\}$, allowing for variable dimensionality.

Formalizing Spatial Interpretation (Euclidean/Non-Euclidean via `Spatial<V>`) The geometric interpretation of spatial coordinates (e.g., distance metrics, neighborhood relations) is not fixed but is governed by functions associated with the specific instantiation of the spatial context. In a computational framework like DeepCausality, this is typically managed through a trait or interface, analogous to the `Spatial<V>` trait discussed in the EPP philosophy [114]. This allows s to represent points in Euclidean spaces, nodes in a graph (non-Euclidean), or other abstract relational structures.

SpaceTime Payload ($SpaceTime(st), st \in \mathcal{ST}$) The set $\mathcal{ST}$ represents the domain of all possible spatiotemporal values. A value $st \in \mathcal{ST}$ combines spatial and temporal information, typically as a composite structure:

$$st = (\text{space_value}, \text{time_value})$$

where space_value $\in \mathcal{S}$ and time_value $\in \mathcal{T}$. This represents a specific spatial configuration at a particular temporal point or interval.

Contextoid Adjustability Protocol

The component adj_v represents optional functions implementing the EPP's Adjustable protocol, allowing for the dynamic modification of a Contextoid's payload payload_v. Let $\mathcal{V}_{\text{payload}}$ be the set of all possible payload values and $\mathcal{V}_{\text{adj_factor}}$ be the set of all possible adjustment factors. The functions are:

- $\text{update}_v : \mathcal{V}_{\text{payload}} \to \text{void}$: This function replaces the current payload_v of the Contextoid with a new payload value.
- $\text{adjust}_v : \mathcal{V}_{\text{adj_factor}} \to \text{void}$: This function modifies the current payload_v based on an adjustment factor.

The 'void' return type indicates that these functions perform a stateful update on the Contextoid v. Their absence implies the Contextoid's payload is immutable post-instantiation.

9.3.2 The Context Hypergraph (C)

Individual Contextoids are organized into a structured collection called a **Context Hypergraph**. This hypergraph structure allows for the representation of complex, N-ary relationships between different pieces of contextual information, which is crucial for modeling the intricate dependencies found in real-world systems.

Context Hypergraph Definition. An individual Context Hypergraph C is defined as a tuple:

$$C = (V_C, E_C, ID_C, \text{Name}_C)$$

where V_C is its finite set of **Contextoid** nodes (each conforming to Definition 3.1, thus $V_C = \{v_1, v_2, \ldots, v_n\}$), E_C is its finite set of **Hyperedges**, $ID_C \in \mathbb{N}$ is a unique identifier for C, and Name_C is a descriptive name.

Set of Context Hyperedges (E_C). A Hyperedge $e \in E_C$ represents a relationship among a subset of Contextoids in V_C. It is defined as a tuple:

$$e = (V_e, \text{kind}_e, \text{label}_e)$$

where:

- $V_e \subseteq V_C$ is a non-empty subset of Contextoids in C connected by this hyperedge, with $|V_e| \geq 1$.
- $\text{kind}_e \in \mathcal{K}_{\text{relation}}$ specifies the type or nature of the relationship, drawn from a predefined set of possible relation kinds $\mathcal{K}_{\text{relation}}$ (e.g., $\mathcal{K}_{\text{relation}} = \{\text{containment, proximity, temporal_sequence, logical_and, synonymy}\}$).
- label_e is an optional descriptive label for the hyperedge (e.g., a string).

The use of hyperedges allows a single relational link (e) to connect an arbitrary number of Contextoids (V_e), enabling the representation of multi-way contextual dependencies.

9.3.3 Context Collection (C_{sys})

In many complex scenarios, causal reasoning may need to draw upon multiple, potentially distinct but interacting, contextual realms. EPP accommodates this through the concept of a **Context Collection**.

Definition 3.4 (Context Collection): A System Context, or Context Collection, C_{sys}, is defined as a finite set of distinct Context Hypergraphs:

$$C_{sys} = \{C_1, C_2, \ldots, C_k\}$$

where each C_i is an individual Context Hypergraph as defined in Definition 3.2. Each $C_i \in C_{sys}$ possesses a unique identifier $ID_{C_i} \in \mathbb{N}$ (distinct from other Context Hypergraphs in the collection) and a descriptive Name_{C_i}.

9.3.4 Context Accessor (ContextAccessor($\mathcal{C}_{refs}$))

To enable Causaloids (defined in Section 9.4) to interact with the Contextual Fabric, a mechanism for querying and retrieving contextual information is required. This is conceptualized as a **Context Accessor**.

Definition 3.5 (Context Accessor): A Context Accessor, denoted ContextAccessor($\mathcal{C}_{refs}$), is a functional interface that provides read-access to a specified subset of Context Hypergraphs $\mathcal{C}_{refs} \subseteq \mathcal{C}_{sys}$. Its operations, denoted abstractly, would typically include functions f_{access} such that:

- getContextoid$(id_v, ID_C) \rightarrow v \cup \{\text{null}\}$: Retrieves a Contextoid v by its identifier id_v from a specific Context Hypergraph C (identified by ID_C) within $\mathcal{C}_{refs}$.
- getHyperedges$(v_i, \text{kind}_e) \rightarrow \{e_1, \ldots, e_m\}$: Retrieves all hyperedges of a specific kind kind_e that involve a given Contextoid v_i.

The precise set of functions within the Context Accessor depends on the requirements of the Causaloid functions that utilize it. This formalism defines the accessor as a means to obtain relevant contextual data for causal evaluation based on identifiers and relational queries.

9.4 Causal Units and Structures ($\mathcal{G}$)

Having formalized the Contextual Fabric ($\mathcal{C}$) within which effects propagate, we now turn to the formalization of the causal entities themselves. The Effect Propagation Process (EPP) posits that causal influence is mediated by discrete, operational units which can be composed into complex structures. This section defines these Causal Units and Structures, denoted abstractly as $\mathcal{G}$. The central entity is the Causaloid (χ), representing an individual causal mechanism, and these are organized into CausaloidGraphs (G) to model intricate webs of causal relationships.

In the EPP philosophy, Causaloids are grounded in a process of observation, assumption validation, and inference. While a full formalization of this empirical grounding process is extensive, we briefly define its key components as they inform the structure of a Causaloid, particularly its linkage to underlying evidence or hypotheses. Let $\mathbb{I}$ be a universal set of identifiers.

Observation Instance (o_data)

An **Observation Instance** o_{data} represents an empirical data point or a collection of related measurements relevant to a potential causal link. It typically includes an identifier, the observed value(s), and any associated outcome or effect. Formally, one might define $o_{data} = (id_{obs} \in \mathbb{I}, \text{val}_{obs}, \text{eff}_{obs})$. A dataset $\mathcal{D}_{obs}$ would be a set of such observations.

Assumption Instance (A_smp)

An **Assumption Instance** A_{smp} articulates a condition believed to hold, under which causal interpretations are made. It includes an identifier, a description, and critically, an evaluation function f_{asmp} that tests the assumption against observational data $\mathcal{D}_{obs}$ and/or contextual information $\mathcal{C}_{sys}$. Formally, $A_{smp} = (id_{asmp} \in \mathbb{I}, \text{desc}_{asmp}, f_{asmp}, \text{status}_{asmp})$, where $f_{asmp} : \mathcal{P}(\mathcal{D}_{obs}) \times \text{ContextAccessor}(\mathcal{C}_{relevant}) \rightarrow \{\text{true}, \text{false}\}$. ($\mathcal{P}(\mathcal{D}_{obs})$ denotes the power set or a relevant subset of observations).

Inference Instance (I_inf)

An **Inference Instance** I_{inf} represents a tested hypothesis about a potential causal link, derived from observations under validated assumptions. It typically includes an identifier, the inferential question, the strength of the observed relationship, and a threshold for asserting the inference. Formally, $I_{inf} = (id_{inf} \in \mathbb{I}, \text{question}_{inf}, \text{strength}_{obs}, \text{threshold}_{inf}, \text{status}_{inf})$.

9.4.1 The Causaloid (χ)

The fundamental unit of causal interaction within EPP is the **Causaloid**, inspired by Hardy's work [10] but adapted here as an abstract, operational entity for generalized causal modeling. It encapsulates a single, testable causal mechanism or hypothesis, whose activation depends on input observations and contextual information. Its definition reflects the versatile structural forms found in the DeepCausality implementation.

Causaloid Definition

Definition 4.1 (Causaloid): A Causaloid χ is defined as a tuple:

$$\chi = (id_\chi, \text{type}_\chi, f_\chi, \mathcal{C}_{refs}, \text{desc}_\chi, \mathcal{A}_{linked}, I_{linked})$$

where:

- $id_\chi \in \mathbb{I}$ is a unique identifier for the Causaloid.
- $\text{type}_\chi \in \{\text{Singleton, Collection, Graph}\}$ specifies the structural nature of the Causaloid, determining how its causal function f_χ is realized. This corresponds to the `CausalType` enum in the Rust implementation.
- f_χ is the **causal function or logic** associated with the Causaloid. Its precise signature and operation depend on type_χ (detailed in Section **??**). It fundamentally maps inputs and context to an activation status $\{\text{true (active)}, \text{false (inactive)}\}$.

- $\mathcal{C}_{refs} \subseteq \mathcal{C}_{sys}$ is a set of references to Context Hypergraphs that provide contextual information for the evaluation of f_χ.
- desc_χ is a human-readable description (e.g., a string) of the causal mechanism or hypothesis. This corresponds to the output of an `explain()` method in an implementation.
- $\mathcal{A}_{linked}$ is an optional set of identifiers $\{id_{asmp_1}, \dots\}$ referring to Assumption Instances (Section **??**) upon which this Causaloid's validity depends.
- I_{linked} is an optional identifier id_{inf} referring to an Inference Instance (Section **??**) that may have led to this Causaloid's formulation.

Causal Function Logic (f_chi)

The causal function f_χ embodies the specific operational logic of the Causaloid χ. Its behavior is contingent on type_χ:

- If type_χ = Singleton: f_χ is a direct evaluation function, $f_{\chi,S} : \mathcal{O}_{\text{type}} \times \text{ContextAccessor}(\mathcal{C}_{refs}) \to \{\text{true}, \text{false}\}$. This function directly tests a specific causal hypothesis against an input observation (of type $\mathcal{O}_{\text{type}}$, often a `NumericalValue` in implementations) and the accessed context.
- If type_χ = Collection: The Causaloid χ encapsulates an ordered or unordered set of other Causaloids, $\mathcal{X}_{coll} = \{\chi'_1, \chi'_2, \dots, \chi'_p\}$. In this case, f_χ represents an aggregate reasoning logic over the activation states of the Causaloids in $\mathcal{X}_{coll}$. For example, $f_{\chi,C}$ might evaluate to true if all $\chi'_j \in \mathcal{X}_{coll}$ are active, or if any one of them is active. The evaluation of each χ'_j would itself involve its own causal function $f_{\chi'_j}$. The input $\mathcal{O}_{\text{type}}$ might be a collection of observations, one for each member of $\mathcal{X}_{coll}$, or a shared observation.
- If type_χ = Graph: The Causaloid χ encapsulates an entire CausaloidGraph $G' = (V_{G'}, E_{G'}, \dots)$ (as defined in Section 9.4.2). Here, f_χ represents the outcome of the full Effect Propagation Process Π_{EPP} (defined in Section **??**) operating on the encapsulated graph G'. For instance, $f_{\chi,G}$ might evaluate to true if a specific target node in G' becomes active after propagation, or if the overall graph reaches a certain state. The input $\mathcal{O}_{\text{type}}$ would serve as the initial trigger(s) for propagation within G'.

Regardless of type_χ, the causal function f_χ ultimately determines the binary activation state of χ, providing the "testable effect transfer" mechanism central to EPP.

9.4.2 The CausaloidGraph (G)

Individual Causaloids are composed into **CausaloidGraphs** to represent complex webs of interconnected causal relationships. A CausaloidGraph is itself a hypergraph.

CausaloidGraph Definition

Definition 4.2 (CausaloidGraph): A CausaloidGraph G is defined as a tuple:

$$G = (V_G, E_G, ID_G, \text{Name}_G)$$

where:

- V_G is a finite set of causal nodes.
- E_G is a finite set of causal hyperedges, representing functional relationships or propagation pathways between nodes in V_G.
- $ID_G \in \mathbb{I}$ (or $\mathbb{N}$) is a unique identifier for the CausaloidGraph.
- Name_G is a descriptive name for the CausaloidGraph (e.g., a string).

Set of Causal Nodes (V_G)

Each causal node $v_g \in V_G$ is defined as a tuple $v_g = (id_g, \text{payload}_g)$, where $id_g \in \mathbb{I}$ is its unique identifier within G. The payload_g embodies the principle of **recursive isomorphism** central to EPP, allowing for hierarchical model construction. It can be one of:

- A single Causaloid χ (as per Definition 4.1).
- A collection of Causaloids $\{\chi_1, \chi_2, \dots, \chi_m\}$, where the collection itself might have aggregate evaluation logic (e.g., "active if any χ_i is active").
- Another entire CausaloidGraph G', enabling the nesting of causal sub-models.

Set of Causal Hyperedges

Each causal hyperedge $e_g \in E_G$ represents a directed functional relationship or pathway for effect propagation. It is defined as a tuple:

$$e_g = (V_{\text{source}}, V_{\text{target}}, \text{logic}_e)$$

where:

- $V_{\text{source}} \subseteq V_G$ is a non-empty set of source causal nodes.
- $V_{\text{target}} \subseteq V_G$ is a non-empty set of target causal nodes.
- logic_e defines the functional relationship or condition under which effects propagate from V_{source} to V_{target}. This logic might range from simple conjunction/disjunction of source node states to more complex functions that determine how the activation of source nodes influences target nodes.

This hypergraph structure allows for many-to-many causal relationships.

State of the CausaloidGraph (S_G)

The state of a CausaloidGraph G at any given point in its evaluation is defined by the activation states of its constituent causal nodes.

Definition 4.3 (State of CausaloidGraph): The state S_G is a function mapping each causal node in V_G to an activation status:

$$S_G : V_G \to \{\text{active}, \text{inactive}\}$$

This state evolves as effects propagate through the graph according to the Causaloid functions and the logic of the hyperedges, as will be detailed in Section **??**.

9.5 Dynamics and Emergence

The preceding sections formalized the static structural components of the Contextual Fabric ($\mathcal{C}$) and Causal Units/Structures ($\mathcal{G}$). This section now delves into the heart of the Effect Propagation Process (EPP): its core dynamics, the mechanisms for generating and evolving these structures, and how these formalisms embody the foundational philosophical principles of EPP. This operationalization is crucial for understanding how EPP moves beyond classical causality to address complex, dynamic, and emergent systems.

The Effect Propagation Process (Pi_EPP)

The core dynamic of EPP is the propagation of effects through a CausaloidGraph G, influenced by the Contextual Fabric $\mathcal{C}_{sys}$ and triggered by observations or internal states. This process, denoted Π_{EPP}, describes how the activation state S_G of the CausaloidGraph evolves from an initial state S_G to an updated state S'_G.

Defining an Effect (epsilon) within EPP

Within the EPP formalism, an **"Effect"** (ε) is primarily represented by:

1. The activation state (active or inactive) of a Causaloid (χ) or a causal node (v_g) within a CausaloidGraph, as determined by its causal function f_χ.
2. The transfer of this activation status, or information derived from it, to other connected Causaloids according to the graph structure and hyperedge logic.

An active state $S_G(v_g) = \text{active}$ signifies that the causal mechanism encapsulated by v_g has met its conditions for effect transfer. The *nature* of the effect is qualitatively described by desc_χ, and its *consequence* is how its activation (or inactivation) influences other parts of the CausaloidGraph.

Input Observations and Triggers (O_input)

The propagation process Π_{EPP} is typically initiated or influenced by external inputs or observations.

Definition 5.1 (Input Trigger Set): Let $\mathcal{O}_{input}$ be the set of all possible input observations relevant to the CausaloidGraph G. An input $o_{in} \in \mathcal{O}_{input}$ is an element compatible with the $\mathcal{O}_{\text{type}}$ expected by one or more Causaloid functions f_χ within G. At any given evaluation cycle, a set of specific input observations $O_{trig} \subseteq \mathcal{O}_{input}$ serves as triggers. These triggers might be:

- Directed at specific Causaloids in G (e.g., root nodes, or nodes representing sensors).

- Representing updates to the Context Collection $\mathcal{C}_{sys}$ which, in turn, affect the evaluation of context-aware Causaloids.

The Propagation Transition Function

The propagation of effects within a CausaloidGraph G is formalized as a state transition function Π_{EPP}. This function describes how the graph's state evolves.

Definition 5.2 (EPP Transition Function): The EPP Transition Function Π_{EPP} is a mapping:

$$\Pi_{EPP} : (G, S_G, \mathcal{C}_{sys}, O_{trig}) \to S'_G$$

where $G = (V_G, E_G, \dots)$ is a CausaloidGraph, $S_G : V_G \to \{\text{active, inactive}\}$ is its current state, $\mathcal{C}_{sys}$ is the Context Collection, O_{trig} is the set of current input triggers, and $S'_G : V_G \to \{\text{active, inactive}\}$ is the updated state of the CausaloidGraph.

The computation of S'_G from S_G is achieved through an iterative evaluation process driven by the structure of G and the logic of its components):

1. **Initialization:** Some nodes in V_G may have their states in S_G initially set or updated directly by O_{trig}.
2. **Iterative Evaluation & Propagation:** The process iteratively (or recursively, depending on the traversal strategy) considers nodes $v_g \in V_G$:
   (a) The activation of a node v_g (i.e., $S'_G(v_g)$) is determined by evaluating its causal function $f_{\chi_{v_g}}$. This evaluation takes into account:
      - Relevant observations from O_{trig} or the activation states of its source nodes (elements of V_{source} from incoming hyperedges e_g).
      - Contextual information obtained via ContextAccessor($\mathcal{C}_{refs_{\chi_{v_g}}}$).

   (b) The influence of source nodes V_{source} on target nodes V_{target} is mediated by the logic$_e$ of the connecting hyperedge $e_g = (V_{\text{source}}, V_{\text{target}}, \text{logic}_e)$. This logic$_e$ determines if and how the states of source nodes contribute to triggering the evaluation of target nodes.
3. **Convergence:** The process continues until no further changes in node states occur for the given O_{trig} and $\mathcal{C}_{sys}$, resulting in the stable updated state S'_G. For CausaloidGraphs with cycles, specific convergence criteria or iteration limits may be necessary.

This operational definition emphasizes that effect propagation is a structured traversal and evaluation, where the "rules" of propagation are embedded in both the individual Causaloid functions f_χ and the connective logic logic$_e$ of the CausaloidGraph's hyperedges.

9.5.1 The Operational Generative Function (Φ_{gen})

The EPP philosophy posits that the Contextual Fabric and even the Causal Structures themselves can be dynamic and emergent, shaped by an underlying "generative function." This formalism operationalizes this concept as Φ_{gen}, a function or set of functions that govern the evolution of $\mathcal{C}_{sys}$ and/or G.

Conceptual Role of Phi_gen in EPP

Φ_{gen} represents the meta-level rules or processes that can alter the EPP's structural components. It embodies the system's capacity for adaptation, learning, or emergence beyond simple state changes within a fixed structure. While the EPP framework itself does not mandate a specific physical interpretation for an ultimate generative function, it provides the means to model systems where such generative dynamics are at play operationally.

Generation and Dynamics of Contexts (Phi_gen_C)

Let Φ_{gen_C} be the component of Φ_{gen} responsible for the evolution of the Context Collection.

Definition 5.3 (Context Generative Function):

$$\Phi_{gen_C} : (\mathcal{C}_{sys}, O_{in}, E_{ext}) \to \mathcal{C}'_{sys}$$

where O_{in} represents relevant system inputs/observations and E_{ext} represents external events or triggers. $\mathcal{C}'_{sys}$ is the updated Context Collection. This function can:

- Modify Contextoids $v \in V_C$ within a $C_i \in \mathcal{C}_{sys}$ via their adj_v protocol.

- Modify the structure of a Context Hypergraph C_i, e.g., by adding/removing Contextoids from V_{C_i} or Hyperedges from E_{C_i}.
- Add new Context Hypergraphs to $\mathcal{C}_{sys}$ or remove existing ones.

The specific rules defining Φ_{gen_C} are domain-dependent but allow for contexts that structurally adapt to new information or changing environmental conditions (e.g., the dynamic temporal hypergraph described in the EPP philosophy [114]).

Generation and Dynamics of Causal Structures (Phi_gen_G)]

Let Φ_{gen_G} be the component of Φ_{gen} responsible for the evolution of CausaloidGraphs, enabling emergent causality.

Definition 5.4 (Causal Structure Generative Function):

$$\Phi_{gen_G} : (G, S_G, \mathcal{C}_{sys}, O_{in}, E_{ext}) \to (G', S'_G)$$

This function can modify the CausaloidGraph $G = (V_G, E_G, \dots)$ into a new graph $G' = (V'_G, E'_G, \dots)$ with a corresponding initial state S'_G. Modifications can include:

- Adding or removing causal nodes v_g from V_G.
- Modifying the payload of existing nodes (e.g., changing a Causaloid's function f_χ or its linked assumptions $\mathcal{A}_{linked}$).
- Adding or removing causal hyperedges e_g from E_G, or altering their logic$_e$.

This function formalizes the EPP's capacity for causal emergence, where the causal "rules of the game" themselves evolve in response to the system's experience and context.

Co-evolution of Context and Causal Structure (Phi_gen_Total)]

In the most general case, context and causal structure can co-evolve.

Definition 5.5 (Total Generative Function):

$$\Phi_{gen_Total} : (G, S_G, \mathcal{C}_{sys}, O_{in}, E_{ext}) \to (G', S'_G, \mathcal{C}'_{sys})$$

This represents the combined action of Φ_{gen_C} and Φ_{gen_G}, allowing for feedback between structural changes in context and structural changes in causal models.

9.5.2 Core EPP Principles

The formalism presented supports and makes explicit the core philosophical principles of EPP.

Spacetime Agnosticism

The EPP formalism is inherently spacetime-agnostic. Neither the Contextoid (Def 3.1), Context Hypergraph (Def 3.2), Causaloid (Def 4.1), nor CausaloidGraph (Def 4.2) definitions mandate a spatiotemporal nature for the underlying fabric or relations. Spatial or temporal properties are introduced only if specific payload$_v$ types (Time, Space, SpaceTime) are used within Contextoids. The core propagation function Π_{EPP} (Def 5.2) and generative function Φ_{gen} (Defs 5.3-5.5) operate on these abstract structures; they only interact with spatiotemporal concepts if explicitly encoded within particular Causaloid functions f_χ or the rules of Φ_{gen}. This formally detaches EPP causality from a presupposed spacetime, fulfilling a key requirement for modeling non-Euclidean systems or systems where spacetime is emergent.

Emergence of Temporal Order

While EPP does not assume a universal linear temporal order, an operational or effective temporal order can emerge within the system through several mechanisms:

- **Contextual Sequencing:** Sequences of Time(t) Contextoids, or relations (kind$_e$) like "temporally_precedes" defined in E_C, can establish a local or domain-specific temporal order within $\mathcal{C}_{sys}$.
- **Propagation Dynamics:** The iterative application of Π_{EPP} inherently defines evaluation steps. While these steps are not necessarily universal time, they form a sequence of state transitions $S_G \to S'_G \to S''_G \dots$ that constitutes an internal process time.
- **Generative Function Dynamics:** Changes orchestrated by Φ_{gen} also occur in sequence, creating a history of structural evolution.

Causaloid functions f_χ can then be designed to query and utilize this emergent temporal information from the context or process history when relevant.

Congruence with Classical Causality

Classical causality (A causes B if A precedes B and influences B) can be shown as a special case within the EPP formalism under specific conditions:

1. The Contextual Fabric $\mathcal{C}_{sys}$ is structured to represent a classical (possibly dynamic, as in GR) spacetime with a well-defined linear temporal order.
2. Causaloids χ_A and χ_B represent events A and B.
3. A causal hyperedge e_g connects χ_A to χ_B, with logic$_e$ such that χ_B is activated if χ_A is active AND the context (queried via f_{χ_B} or through logic$_e$) confirms that the event corresponding to χ_A occurred at an earlier time than the event corresponding to χ_B within the defined spacetime context.
4. The Causaloid χ_A can be designated "the cause" and χ_B "the effect" based on this emergent, contextually-defined temporal precedence.

Under these (and potentially other simplifying) assumptions, EPP's effect propagation aligns with classical cause-and-effect chains.

[Handling Indefinite Causal Order

While a full formal treatment of quantum indefinite causal order is beyond the scope of this foundational paper, the EPP framework's spacetime agnosticism and its use of Causaloids (which do not inherently encode a fixed temporal role as "cause" or "effect") provide conceptual compatibility. Future work could explore:

- Extending the CausaloidGraph G to allow for superpositions of different hyperedge structures (E_G) representing different causal orderings.
- Modifying the propagation process Π_{EPP} to evaluate such superposed pathways, perhaps drawing from formalisms like process matrices.

The EPP's detachment from a fixed causal structure is a prerequisite for such extensions.

9.6 The Causal State Machine (CSM)

The Effect Propagation Process (EPP) framework provides mechanisms for modeling how effects propagate and causal structures evolve. To translate the insights derived from such processes into concrete operations or interventions, the **Causal State Machine (CSM)** is introduced. The CSM serves as an orchestrator, linking specific causal conditions (represented as Causal States) to predefined deterministic Actions. This section formalizes the CSM, reflecting its implementation within systems like DeepCausality[7].

Let $\mathbb{I}_{CSM}$ be a set of unique identifiers for Causal States within a CSM.

Definition 6.1 (Causal State): A **Causal State** q represents a specific condition whose truthfulness is determined by the evaluation of an associated Causaloid. It is defined as a tuple:

$$q = (id_q, \text{data}_q, \chi_q, \text{version}_q)$$

where:

- $id_q \in \mathbb{I}_{CSM}$ is a unique identifier for this Causal State.
- data$_q$ $\in$ $\mathcal{O}_{\text{type}}$ is the specific input observation or data value (compatible with the Causaloid's expected observation type $\mathcal{O}_{\text{type}}$) used for evaluating this state. This data may be intrinsic to the state or provided externally during evaluation.
- χ_q is a reference to a Causaloid (as defined in Section 9.4.1), $\chi_q = (id_{\chi_q}, f_{\chi_q}, \mathcal{C}_{refs_q}, \dots)$. The function f_{χ_q} embodies the predicate for this state.
- version$_q$ $\in \mathbb{N}$ is an optional version number for the state definition.

The activation of Causal State q is determined by eval$(q) = f_{\chi_q}(\text{data}_q, \text{ContextAccessor}(\mathcal{C}_{refs_q}))$.

[7] `deepcausality.com`

Definition 6.2 (Causal Action): A **Causal Action** a represents a deterministic operation. It is defined as a tuple:

$$a = (\text{exec}_a, \text{descr}_a, \text{version}_a)$$

where:

- exec_a : void $\rightarrow$ Result(void, ActionError) is the executable function representing the action. Its invocation may modify a WorldState implicitly.
- descr_a is a descriptive label for the action.
- $\text{version}_a \in \mathbb{N}$ is an optional version number for the action definition.

The successful execution of exec_a yields Ok(void); failure yields an Err(ActionError).

Definition 6.3 (Causal State Machine): A Causal State Machine M is defined by its collection of Causal State-Action pairs. Let $\mathcal{Q}$ be the set of all possible Causal States and $\mathcal{A}$ be the set of all possible Causal Actions.

$$M = (\mathcal{SA}_M)$$

where:

- $\mathcal{SA}_M \subseteq \{(q, a) \mid q \in \mathcal{Q}, a \in \mathcal{A}\}$ is a finite set of ordered pairs, where each pair (q_j, a_j) associates a unique Causal State q_j (identified by id_{q_j}) with a Causal Action a_j. In implementation, this is often represented as a map from $id_q \rightarrow (q, a)$.

The set of CausaloidGraphs $\mathcal{G}_M$ supervised by M is implicitly defined as the set of all CausaloidGraphs containing any Causaloid χ_q referenced by any q in $\mathcal{SA}_M$. Similarly, the set of relevant Context Hypergraphs $\mathcal{C}_M$ is the union of all $\mathcal{C}_{refs_q}$ for all q in $\mathcal{SA}_M$.

9.6.1 CSM Operation: State Evaluation and Action Triggering

The operation of the CSM involves evaluating its Causal States and triggering associated Causal Actions.

Single State Evaluation and Action (eval_single_state): Given a CSM $M = (\mathcal{SA}_M)$, an identifier $id_q^* \in \mathbb{I}_{CSM}$, and potentially new input data $d_{in} \in \mathcal{O}_{\text{type}}$ for that state:

1. Retrieve the state-action pair (q^*, a^*) from $\mathcal{SA}_M$ such that $id_{q^*} = id_q^*$. If no such pair exists, an error is indicated.
2. The Causal State $q^* = (id_{q^*}, \text{data}_{q^*}, \chi_{q^*}, \dots)$ is evaluated. The evaluation function is $f_{\chi_{q^*}}$. The input data used is either d_{in} (if provided externally for this evaluation) or the state's intrinsic data_{q^*}. Let trigger $= f_{\chi_{q^*}}(\text{data}_{\text{eval}}, \text{ContextAccessor}(\mathcal{C}_{refs_{q^*}}))$.
3. If trigger = true, then the action a^* is executed by invoking its function $\text{exec}_{a^*}()$.

All States Evaluation and Action (eval_all_states): Given a CSM $M = (\mathcal{SA}_M)$:

1. For each state-action pair $(q_j, a_j) \in \mathcal{SA}_M$:
   (a) The Causal State $q_j = (id_{q_j}, \text{data}_{q_j}, \chi_{q_j}, \dots)$ is evaluated using its intrinsic data data_{q_j}. Let $\text{trigger}_j = f_{\chi_{q_j}}(\text{data}_{q_j}, \text{ContextAccessor}(\mathcal{C}_{refs_{q_j}}))$.
   (b) If trigger_j = true, then the action a_j is executed by invoking its function $\text{exec}_{a_j}()$.

The order and concurrency of action execution in step 1(b) depend on the CSM's specific execution semantics (e.g., sequential, parallel, prioritized), which are an implementation detail beyond this core formalism.

9.6.2 CSM Dynamics: Managing State-Action Pairs

The set of state-action pairs $\mathcal{SA}_M$ within a CSM M can be dynamic, allowing the CSM to adapt its stimulus-response behavior. Formal operations on $\mathcal{SA}_M$ include:

- **AddStateAction**(M, q_{new}, a_{new}): $\mathcal{SA}'_M = \mathcal{SA}_M \cup \{(q_{new}, a_{new})\}$, provided $id_{q_{new}}$ is not already a key in $\mathcal{SA}_M$.
- **RemoveStateAction**(M, id_q): $\mathcal{SA}'_M = \mathcal{SA}_M \setminus \{(q, a) \mid id_q \text{ is the identifier of } q\}$.
- **UpdateStateAction**$(M, id_q, q_{updated}, a_{updated})$: Replaces the pair (q, a) associated with id_q with $(q_{updated}, a_{updated})$.

9.7 Example: Smoking, Tar, and Cancer

To illustrate the core concepts of the Effect Propagation Process (EPP) formalism in a more concrete manner, we consider the well-known causal chain: Smoking $\rightarrow$ Tar in Lungs $\rightarrow$ Lung Cancer. This example demonstrates how Causaloids, Contexts (implicitly for this simplification), and a CausaloidGraph can represent this system, and how effect propagation leads to an inference. This example is simplified and inspired by the structure typically modeled as a Directed Acyclic Graph (DAG) in classical causal inference [17], here adapted to EPP principles.

9.7.1 Defining the Causal Problem

We aim to model a system to infer the likelihood of lung cancer based on smoking habits, mediated by tar accumulation. The core causal hypotheses are:

1. Smoking (represented by nicotine levels) leads to increased tar in the lungs.
2. Increased tar in the lungs leads to a higher risk of lung cancer.

We will represent these as individual Causaloids within a CausaloidGraph. For simplicity, any contextual dependencies (e.g., thresholds for "high" nicotine or tar) are assumed to be encapsulated within the Causaloid functions themselves or drawn from an implicitly defined context for this illustrative purpose. Observations will be represented as numerical values.

9.7.2 Formalizing the Components

The Causaloid (chi)

We define two primary Causaloids:

- $\chi_{S\rightarrow T}$ **(Smoking $\rightarrow$ Tar):**
  - $id_{\chi_{S\rightarrow T}} = 1$ (a unique identifier)
  - $\text{type}_{\chi_{S\rightarrow T}} = \text{Singleton}$
  - $f_{\chi_{S\rightarrow T}} : \mathcal{O}_{\text{nicotine}} \rightarrow \{\text{true, false}\}$, where $\mathcal{O}_{\text{nicotine}}$ is a numerical input representing nicotine level. The function $f_{\chi_{S\rightarrow T}}$ evaluates to true if the nicotine level exceeds a predefined threshold (e.g., 0.55), indicating a significant likelihood of tar presence due to smoking.
  - $\mathcal{C}_{refs} = \emptyset$ (assuming context-free logic for this example, or that thresholds are part of $f_{\chi_{S\rightarrow T}}$).
  - $\text{desc}_{\chi_{S\rightarrow T}}$ = "Causal relation between smoking and tar in the lung."
  - $\mathcal{A}_{linked}, I_{linked}$ = Optional, representing linkage to foundational assumptions or prior inferences regarding this relationship.

- $\chi_{T\rightarrow C}$ **(Tar $\rightarrow$ Cancer):**
  - $id_{\chi_{T\rightarrow C}} = 2$
  - $\text{type}_{\chi_{T\rightarrow C}} = \text{Singleton}$
  - $f_{\chi_{T\rightarrow C}} : \mathcal{O}_{\text{tar}} \rightarrow \{\text{true, false}\}$, where $\mathcal{O}_{\text{tar}}$ is a numerical input representing tar level. The function $f_{\chi_{T\rightarrow C}}$ evaluates to true if the tar level exceeds a predefined threshold (e.g., 0.55).
  - $\mathcal{C}_{refs} = \emptyset$.
  - $\text{desc}_{\chi_{T\rightarrow C}}$ = "Causal relation between tar in the lung and lung cancer."
  - $\mathcal{A}_{linked}, I_{linked}$ = Optional.

The shared underlying logic for f_χ in this example (input observation $\geq$ threshold) is abstracted into each Causaloid's specific function.

The CausaloidGraph (G)

We construct a CausaloidGraph $G_{STC} = (V_G, E_G, ID_G, \text{Name}_G)$ to represent the causal chain.

- $V_G = \{v_{g1}, v_{g2}\}$, where:
  - $v_{g1} = (id_{g1}, \text{payload}_{g1} = \chi_{S\rightarrow T})$
  - $v_{g2} = (id_{g2}, \text{payload}_{g2} = \chi_{T\rightarrow C})$
- $E_G = \{e_{g1}\}$, representing the link from smoking/tar to tar/cancer. For this simple chain:

  - $e_{g1} = (V_{\text{source}} = \{v_{g1}\}, V_{\text{target}} = \{v_{g2}\}, \text{logic}_{e1})$
  - logic_{e1}: Specifies that the evaluation of v_{g2} is conditioned by, or follows, the evaluation of v_{g1}. In a simple direct propagation, if v_{g1} becomes active, this contributes to the conditions for evaluating v_{g2}. (In a computational system with distinct inputs for each step, this logic might simply define the sequence or dependency).
- ID_G, Name_G: Appropriate identifiers and names (e.g., "Smoking-Tar-Cancer Model").

This formal G_{STC} represents the structure. In a practical implementation, a collection of Causaloids (like a vector) reasoned over sequentially can instantiate such a linear graph.

9.7.3 Formalizing Effect Propagation (Π_{EPP})

Consider input observations $O_{trig} = \{(id_{\chi_{S\to T}}, o_{\text{nicotine}}), (id_{\chi_{T\to C}}, o_{\text{tar}})\}$, where o_{nicotine} is the observed nicotine level and o_{tar} is the observed tar level. Let the initial state be $S_G(v_{g1}) = \text{inactive}$, $S_G(v_{g2}) = \text{inactive}$.

The propagation $\Pi_{EPP}((G_{STC}, S_G, \emptyset, O_{trig})) \to S'_G$ conceptually proceeds as follows, reflecting a sequential evaluation for this chain:

1. **Evaluate $\chi_{S\to T}$ (node v_{g1}):** The function $f_{\chi_{S\to T}}$ is evaluated with its corresponding input o_{nicotine} from O_{trig}. If $f_{\chi_{S\to T}}(o_{\text{nicotine}}) = \text{true}$ (e.g., nicotine ≥ 0.55), then $S'_G(v_{g1}) = \text{active}$. Else, $S'_G(v_{g1}) = \text{inactive}$.
2. **Evaluate $\chi_{T\to C}$ (node v_{g2}):** The function $f_{\chi_{T\to C}}$ is evaluated with its corresponding input o_{tar} from O_{trig}. The evaluation of v_{g2} in this chained model might also be implicitly conditioned on v_{g1} being active if the overall inference requires the full chain to hold. If $f_{\chi_{T\to C}}(o_{\text{tar}}) = \text{true}$ (e.g., tar ≥ 0.55), then $S'_G(v_{g2}) = \text{active}$. Else, $S'_G(v_{g2}) = \text{inactive}$.
3. **Final Inference/System State:** The overall state of the system might be defined by a conjunction: Cancer risk is inferred if $S'_G(v_{g1}) = \text{active} \wedge S'_G(v_{g2}) = \text{active}$. This reflects whether the complete causal pathway from smoking, via tar, to cancer is deemed active for the given observations.

This illustrative propagation shows how individual Causaloid evaluations, based on specific inputs, contribute to the overall state of the causal model. The logic_e of the hyperedge e_{g1} here implies a dependency or sequential consideration in this chain.

9.7.4 Formalizing Observations

This simplified example illustrates:

- **Causaloids** (χ) as operational units (Section 9.4.1), encapsulating specific causal functions (f_χ).
- A simple **CausaloidGraph** (G) structure (Section 9.4.2) representing the causal chain.
- The **Effect Propagation Process** (Π_{EPP}) (Section **??**) as the evaluation of Causaloids in a sequence determined by the graph structure and input data.
- The **State of the CausaloidGraph** (S_G) (Definition **??**) evolving based on these evaluations.

While this example omits explicit dynamic Contexts ($\mathcal{C}_{sys}$) or a dynamic Generative Function (Φ_{gen}) for brevity, it demonstrates how the core EPP structural entities map to a well-understood causal scenario.

9.8 Discussion

The preceding sections have laid out a formal, set-theoretic definition of the Effect Propagation Process (EPP). This formalism is a direct response to the need for new conceptual and computational tools to understand causality in systems where classical assumptions of a fixed spatiotemporal background and linear temporal order are demonstrably insufficient. The motivation stems from both the frontiers of fundamental physics, where spacetime itself is considered emergent, and the practical challenges of modeling complex adaptive systems across various scientific and engineering domains.

The primary significance of the EPP formalism lies in its principled detachment from spacetime. By making Context ($\mathcal{C}$) an explicit, definable, and potentially non-Euclidean, dynamic fabric, and by defining Causaloids (χ) as operational units of effect transfer independent of a priori temporal ordering, EPP achieves a level of generality that classical causal formalisms cannot. This enables the modeling of causality in non-physical or abstract domains, systems with complex multi-scale temporal dynamics and feedback loops, and, crucially, systems where causal relationships themselves can

emerge, transform, or dissolve in response to evolving contexts (dynamic regime shifts). The recursive isomorphism of CausaloidGraphs (G) further enhances expressive power, allowing for modular construction of complex causal models, while the clear separation of causal logic, contextual data, and propagation mechanisms facilitates transparency.

This foundational formalism, while potent, has current limitations. It primarily defines a deterministic framework, though probabilistic behavior can be encapsulated within Causaloid functions or edge logic. A more deeply integrated probabilistic EPP, or a full formal treatment of quantum indefinite causal order, remains an area for future development. Furthermore, EPP does not, in itself, provide algorithms for automated causal discovery of its structures from raw data; it provides the language to represent and reason with such structures once hypothesized. Compared to established frameworks like Pearl's SCMs, EPP offers greater flexibility for cyclic and emergent systems but currently lacks an equivalent to the extensive *do*-calculus, though the Causal State Machine (CSM) provides a mechanism for linking EPP inferences to actions. EPP generalizes classical notions where necessary but does not seek to replace them where they are already sufficient.

10 The Implementation of the Effect Propagation Process

10.1 Overview

DeepCausality is an open-source framework, hosted at the Linux Foundation and accessible at `https://deepcausality.com`, designed to enable the construction, execution, and rigorous management of explicit, context-aware, and explainable causal models. It implements the Effect Propagation Process and with hat provides a pathway to reason about cause and effect within intricate, multi-dimensional, and dynamically evolving environments. The framework's unique hypergeometric nature refers to its core reliance on hypergraph structures for representing both the rich tapestry of context and the complex web of causal relationships, offering a new level of expressiveness and analytical depth.

DeepCausality's core contributions are design to provide a robust foundation for causally-grounded intelligence:

- **A Novel Hypergraph-based Context Engine:** At its heart, DeepCausality features a sophisticated engine for managing context. This moves beyond simple conditioning variables to enable the creation of intricate context hypergraphs populated by *Contextoids* – specialized nodes representing rich, multi-dimensional information encompassing Data, Time, Space, and SpaceTime. This inherently supports dynamically adjustable contexts, the simultaneous integration of information from multiple distinct context hypergraphs (potentially with differing Euclidean or non-Euclidean geometries), and grounds causal reasoning in a highly nuanced and comprehensive understanding of the operational environment.
- **Structurally Composable Causal Modeling:** DeepCausality introduces *Causaloids* – encapsulated, testable causal functions – as the fundamental building blocks of causal models. These are organized within *CausaloidGraphs*, which are themselves hypergraphs explicitly representing intricate causal relationships. Crucially, this architecture employs recursive isomorphic causal data structures: nodes within a CausaloidGraph can themselves be entire sub-graphs or collections of other causes. This enables the intuitive, modular construction of deeply complex, layered causal systems where macro-level phenomena can be decomposed into interacting micro-level mechanisms, ensuring transparent composability.
- **The Causal State Machine (CSM) for Actionable Intelligence:** Bridging the gap between causal understanding and effective intervention, the CSM is architected to manage interactions between causal models and their contexts. Based on the collective causal inference derived—the identification of specific active causes or system states—the CSM deterministically initiates predefined actions, facilitating the creation of complex, dynamic control and supervision systems that respond with causally-reasoned precision.
- **Implementation in Rust for Performance and Reliability:** Recognizing the demanding requirements of a sophisticated causal reasoning engine operating on potentially vast and dynamic data, DeepCausality is implemented in Rust. This choice leverages Rust's high-performance characteristics, memory safety guarantees, and expressive type system to build an efficient, robust, and reliable foundational causal engine.

This section details the complete architecture of DeepCausality, its conceptual foundations, and its Rust implementation.

10.2 UltraGraph

The importance of hypergraphs in the EPP lead to the decision to implement a hypergrah in a dedicated Rust crate called UltraGraph. Initially, UltraGraph wrapped the MatrixGraph implementation of the Petgraph[8] crate. This was a deliberate decision to speed up the bootstrapping of the DeepCausality project while preserving the option to replace the implementation when the need arises. And indeed, as the requirements of the DeepCausality project keep advancing with the introduction of emergence, a new hypergraph implementation became necessary. The UltraGraph v0.8 release adds a ground-up rewrite of UltraGraph inspired the NWHypergraph (NWHy)[115] architecture that was further optimized for Rust's memory model.

The key elements of the new UltraGraph implementation are:

- The introduction of a dual-state graph lifecycle.
- The introduction of a SoA CsrAdjacency type
- The separation of forward and backward CsrAdjacency.

10.2.1 Dual-state graph life-cycle

UltraGraph v0.8 introduces a dual-state architecture. This recognize that graph-based systems that implement the EPP have two distinct phases: a dynamic "Evolve" phase, where the structure is built and modified, and a stable "Analyze" phase, where high-speed queries are essential.

The Dynamic Graph State:

This is the default state for every new graph. It's an adjacency-list-based structure optimized for flexibility. Adding nodes and edges is a cheap O(1) operation, perfect for systems where the graph structure emerges dynamically over time.

The Static Graph State:

When the graph is ready, a on time call to *.freeze()* transforms the graph into a hyper-optimized, immutable Compressed Sparse Row (CSR) format designed for peak performance. All algorithms check if a graph is in a frozen state therefore it is impossible by design to run any graph algorithm on a dynamic graph.

The graph life-cycle:

A Graph begins in a flexible DynamicGraph state, optimized for fast, O(1) mutations as the structure evolves. When the state has been finalized, and causal reasoning ca begin, a single *.freeze()* call transforms the graph into a hyper-optimized, immutable CsmGraph based on a cache-friendly Struct of Arrays (SoA) memory layout. This step is the key eliminating cache misses and unlocking near-linear scaling. If the graph needs to evolve further, simply cal *.unfreeze()*. There is a memory trade-off in the state transition because, in each state, the memory usage is roughly $(e + v)$ whereas during a state transition, the memory usage temporarily peaks at $(e + v)^2$ and that warrants carefully consideration when a graph grows large i.e. beyond 1 billion nodes.

10.2.2 SoA CsrAdjacency Type

In conventional CSR-based graph representations (like NWHypergraph), adjacency information is typically packed together row-wise in a "single structure" per edge or neighbor, which is technically an Array of Structs (AoS) layout, or in simple terms, "rows of neighbors." UltraGraph, however, takes a different approach by introducing a CsrAdjacency<W> type that implements a Struct of Arrays (SoA) pattern:

Listing 1: UltraGraph: CsrAdjacency

```
#[derive(Default)]
pub(crate) struct CsrAdjacency<W> {
    pub(crate) offsets: Vec<usize>,
    pub(crate) targets: Vec<usize>,
    pub(crate) weights: Vec<W>,
}
```

[8] https://docs.rs/petgraph/latest/petgraph

The CsrAdjacency layout divides each component of the adjacency data (offsets, targets, and weights) into separate, contiguous memory regions:

- offsets: Starting positions of each node's adjacency list.
- targets: The target node indices for each edge.
- weights: Edge weights (optional).

The struct of Arrays design of the CsrAdjacency has two advantages:

Better cache utilization: When performing traversal or shortest path algorithms, the CPU can stream just the fields it needs (often offsets and targets) without pulling in unnecessary weight data and thus avoiding cache pollution and improving CPU data prefecht.

SIMD-friendliness: SoA layouts enable vectorized processing (e.g., with AVX) far more easily than Array of Structs (AoS). Also, a SOA layout is easier for the compiler to optimize and thus unlock meaningful performance gains without the need of complex optimization.

10.2.3 Separation of Forward and Backward CsrAdjacency.

Moreover, UltraGraph uses two separate CsrAdjacency instances: one for successor or outbound edges and another one for backward or inbound edges. This dual-CSR setup is more explicit and efficient than mixing directions within a single row layout because it reduces CPU cache pollution and thereby directly supports fast and efficient algorithm implementations. UltraGraph deliberately traded a bit more memory for better algorithm performance, as shown in the benchmarks.

Listing 2: UltraGraph: Forward and Backward CsrAdjacency

```
#[derive(Clone)]
pub struct CsmGraph<N, W>
where
    N: Clone,
    W: Clone + Default,
{
    // Node payloads, indexed directly by `usize`.
    nodes: Vec<N>,
    // CsrAdjacency structure for forward traversal (successors).
    forward_edges: CsrAdjacency<W>,
    // CSR structure for backward traversal (predecessors).
    backward_edges: CsrAdjacency<W>,
    // Index of the designated root node.
    root_index: Option<usize>,
}
```

The backward node list is particularly useful in causality-based inference algorithms, where backtracking is often required, and is thus particularly well suited for DeepCausality. Memory usage remains low due to the combined effects of the Struct of Arrays layout and the clean separation between forward and backward adjacency.This design leads to a predictable, flat memory layout with minimal overhead:

- No per-node allocation overhead
- No padding, no vtables, no boxed pointers
- No HashMaps, linked lists, or other complex types
- No indexing needed due to simple offset
- When a node has no inbound or outbound nodes or weights, there is zero allocation, thus saving memory.

Memory fragmentation is largely prevented because of the freeze/unfreeze operation in the graph evolution lifecycle. Calling *.freeze()* compacts the structure, which removes any prior allocation gaps from the dynamic phase and thus results in a clean, continuous memory structure.

10.2.4 Discussion

This new implementation of UltraGraph based on its innovative two stage graph life-cycle and it CPU cache friendly design enables causal graph reasoning at scale. While it is unlikely in practice to see causal graphs exceeding a million nodes, a context may grow large. UltraGraph confidently handles graphs up to a 100 million nodes on consumer grade hardware and would only need a specialized high memory server when the graph size is expected to exceed a billion node. At this scale, the underlying CSR implementation will most likely benefit from contemporary reram-based[116] hardware accelerators specifically designed to accelerate hypergraphs with sparse representation. Furthermore, the two state graph life-cycle directly supports the implementation of emergence in DeepCausality because it cleanly separates the mutation from the analysis phase and thus enables the four stage process outlined in the EPP Ontology.

10.3 DeepCausality

10.4 Benchmarks

10.4.1 UltraGraph Benchmarks

All benchmarks were completed on a 2023 Macbook Pro with a M3 Max CPU.

Dynamic Graph Benchmarks:

The dynamic graph structure, when the graph is in an unfrozen state, is optimized for efficient mutation. The table below summarizes the performance characteristics of the key operations.

Table 2: Dynamic Graph Mutation Performance

Benchmark Name	Graph Size	Operation	Estimated Time (Median)	Outliers Detected
`small_add_node`	10	`add_node`	29.099 ns	14% (14 / 100)
`medium_add_node`	100	`add_node`	45.864 ns	12% (12 / 100)
`large_add_node`	1,000	`add_node`	39.293 ns	11% (11 / 100)
`small_get_node`	10	`get_node`	3.9417 ns	8% (8 / 100)
`medium_get_node`	100	`get_node`	3.9849 ns	2% (2 / 100)
`large_get_node`	1,000	`get_node`	3.9916 ns	7% (7 / 100)

Benchmark source code is available in the UltrGraph Github repository[9].

Static Graph Benchmarks:

The new architecture causes the largest and most significant performance gains for algorithms running over large graphs (100k or more nodes) because of its close alignment with contemporary hardware. By combining an instantaneous O(1) lookup with a perfectly linear scan over a node's neighbors, Ultragraph creates the ideal scenario for the CPU's prefetcher to easily anticipates a straight-line sprint through memory. The result becomes more notable the more data the prefetcher can load ahead of time, thus the disproportional performance gains on larger graphs.

Table 3: Memory Usage and Scaling Performance on a Linear Graph

Number of Nodes	Memory Usage	`eval_subgraph`	`eval_path`	`eval_cause`
100,000	55 MB	0.68 ms	0.57 ms	**5.4 ns**
1,000,000	350 MB	11.12 ms	6.95 ms	**5.5 ns**
10,000,000	3 GB	114 ms	85.80 ms	**5.6 ns**
100,000,000	32 GB	1.23 s	0.98 s	**5.5 ns**

Benchmark source code is available in the DeepCausality Github repository[10].

Observations:

Constant Time to get a single node: The benchmark evaluate_single_cause returns always takes about 5.5. ns regardless of whether the node lookup happens before or during the benchmark loop and regardless of whether blackbox is used or not. The time does not change with the size of the graph because the implementation of the underlying get_node is just two O(1) array lookup to find the index and than a straight redirect to a virtual memory address, which in this case, is close to the physical limit of the hardware memory architecture.

Near-Linear Scalability: Both the memory usage and the execution time for the subgraph and shortest_path tasks appear to scale in a roughly linear fashion with the number of nodes. A 10x increase in nodes results in a roughly 10x increase in time and memory.

All benchmarks are single-threaded. The performance shown in all benchmarks is from a single core. Initial experiments showed that for graphs up to 1 million nodes, the overhead of even highly-optimized parallel libraries like rayon resulted in a net performance loss of 30% or more compared to the single-threaded version. This is a testament to the extreme efficiency of the CSR layout when paired with modern CPU caches and prefetchers. The results suggest that meaningful gains from concurrency will only appear on very large graphs (likely 10M-50M nodes and above). However, this requires a concurrency model carefully designed to avoid the cache-invalidation issues common in work-stealing schedulers (used by rayon and Tokio).

[9] `https://github.com/deepcausality-rs/deep_causality/tree/main/ultragraph/benches`

[10] `https://github.com/deepcausality-rs/deep_causality/tree/main/deep_causality/benches`

11 Validity

The validity of the Effect Propagation Process must be assessed according to the nature of its contribution. The EPP is a philosophy-informed, formal computational framework designed to provide a new, more expressive language for modeling dynamic, contextual causal systems. Therefore, its validity rests on the criteria appropriate for such a framework, analogous to how one would assess a mathematical logic or a new programming language:

- **Internal Validity** (Soundness & Consistency): The framework's internal validity is determined by its logical and mathematical soundness and coherence. As detailed in the accompanying specification, the EPP is built on a foundation of first-principles reasoning to ensure its components are consistent and its operations are sound.
- **External Validity** (Expressive Power & Scoped Generalization): The framework's external validity is demonstrated by its robustness against alternative interpretations of its premises and how well it defines the boundaries of its generalization.

11.1 Internal validity

The **soundness** of EPP derives from the first principled logical progression of the presented argument. The stated problem of inadequacy of classical causality in light of the challenges imposed by Quantum Gravity (emergent spacetime, indefinite causal order) is well-recognized and documented[117].. From this recognized problem, the argument for EPP progresses as stated below:

1. Establishes classical causality and its historical critiques.
2. Introduces the challenges from modern physics (GR, QG).
3. Shows why these challenges render classical causality insufficient.
4. Proposes EPP as a coherent response to these challenges.
5. Contrasts EPP with classical causality.
6. Discusses its ontological, epistemological, and teleological implications.
7. Acknowledges and addresses threats to its validity.

The soundness is further strengthened by its inspiration from established scientific theories (QFT, GR) and foundational work in quantum gravity. While quantum gravity as a scientific theory remains a work in progress, the conceptual challenges that arise from it are valid regardless of how any particular theory may explain the underlying quantum mechanisms.

The **consistency** of the EPP framework arises from a handful of carefully stated conclusions.

11.1.1 Spacetime Agnosticism & Causaloids

Premise: On a quantum level, spacetime may not exist.

Conclusion: Therefore, remove spacetime from EPP.

Premise: EPP does not have a defined spacetime.

Conclusion: Cause and effect cannot be separated anymore because there is no a priori temporal order.

Premise: Cause and effect cannot be discerned because of missing temporal order.

Conclusion: Fold cause and effect into one entity, the causaloid, that is independent of temporal (and spatial) order.

Note, the last conclusion holds because of the temporal order required for classical causality. The only logical alternative conclusion from the premise of missing temporal order would be to abandon causality altogether. However, this conflicts with the reality in which causal relationships indeed exist; therefore, the alternative has been deemed unsound.

11.1.2 Effect Propagation as the Essence of Causality

Premise: The causaloid, as a building block, is independent of temporal (and spatial) order.

Conclusion: Define causality by what it does (propagate effects) instead of what it was thought to be (a temporal order dependent atomic relationship).

The careful reader may raise a concern over the choice of words (i.e., propagate effects), since “transferring information” or “transmitting influence” might be equally valid choices. True, that is indeed a fair point. However, the term information has specific meaning in information theory and computer science, and likewise, the term influence has specific meaning in social science; therefore the author settled cautiously on “effect” mainly to prevent conflating different meanings.

11.1.3 Compatibility with Classical Causality

The full logical argument of how classical causality is derived from EPP is in the section “Causality as Effect Propagation Process”. While classical causality can be derived from EPP, the reverse is not true because EPP cannot be derived from classical causality because classical causality requires a background spacetime. That deduction proves that EPP is more abstract in the sense of more general than classical causality. Therefore, it follows that EPP naturally applies to areas where classical causality cannot be used any longer. The internal validity of EPP roots in its internal soundness and consistency that stems from its first-principles reasoning. Therefore, ambiguity, contradictory claims, and unjustified leaps in logic are avoided to the extent it is possible. Minor mistakes might be possible and the author is open to suggestions to improve EPP further.

11.2 External validity

Establishing the boundaries of generalization as a proxy for external validity requires a delicate balance of realistically acknowledging what EPP can address versus avoiding overstating any particular capability. Related to external validity is always the possibility of an alternative interpretation of the underlying premises.

11.2.1 Falsifiability

A crucial distinction must be made regarding the role of falsifiability. The EPP, as a formal foundation, is not itself an empirical scientific theory and therefore is not directly falsifiable. One cannot falsify the established philosophical foundation of the EPP—its metaphysics, ontology, and epistemology—without challenging its underlying axiomatic assumptions. Instead, the validity of the EPP rests on its internal consistency and its expressive power.

Falsifiability, however, is a critical property of the **specific, testable causal models that are constructed *within* the EPP.** A model of an avionics system, for instance, represents a set of falsifiable hypotheses encoded as a ‘CausaloidGraph‘ and its associated ‘Context‘. This model makes concrete predictions about system behavior that can be tested against simulation or real-world data. If the predictions fail, the *model* is falsified, not the framework.

The EPP provides the formal language and the computational primitives necessary to empower engineers and domain experts to express and test causal hypotheses in complex, contextual realities that were previously beyond the reach of formal modeling.

11.2.2 Alternative interpretations

There are several potential alternative interpretations of the premises underlying the Effect Propagation Process framework.

Russell was more right than acknowledged

One can take the position that, if Russell deemed causality as a relict of a bygone era, then why not openly ask to abandon causality altogether and focus solely on descriptive and correlation-based data science?

DARPA disagrees[118]:

> “In the real world, observations are often correlated and a product of an underlying causal mechanism, which can be modeled and understood.”

The problem is not a simple choice between correlation and causation. The deeper issue, as contemporary philosophers like Luciano Floridi have argued, is that the predominant mental model underlying complex system design remains rooted in an outdated Aristotelian and Newtonian "Ur-philosophy" of fixed space and time. This leads to tools that are inadequate for the intricate complexity of reality. To address this core critique, the EPP lifts causality into a contextual generalization required to model dynamic systems that no longer adhere to a fixed background spacetime. This includes systems with non-Euclidean geometries, multiple interacting contexts, and emergent causal structures, as found in domains like avionics. The question of falsifiability, therefore, applies not to the EPP framework itself, but to the specific, testable models that are constructed within it. A model of an avionics system built using the EPP

is indeed falsifiable against simulation data; the framework itself simply provides the language for expressing that model.

The author argues, a similar shift towards a richer contextualization of advanced dynamic models needs to happen in the correlation-based methodologies of deep learning as well to build tools more suitable for today's reality. A transfer of core EPP concepts, i.e., the contextual hypergraph, uniform Euclidean and non-Euclidean geometries, into the foundations of deep learning is welcomed by the author.

Pluralism of causal concepts

Instead of a unified framework like EPP, one can argue in favor of the existing pluralistic reality where multiple disjoint concepts of (computational) causality exist for different levels of causal analysis.

As stated before, EPP does not seek to replace classical causality and all tools that are built atop the classical definition of causality. Instead, EPP seeks to advance the core concept of causality to meet increasingly challenging demands. Due to the novelty of this foundational work, a single coherent framework is preferred until it becomes clear which parts may branch out and become more specialized domains over time.

11.2.3 Boundaries of Generalization

The primary boundary of the Effect Propagation Process is one of scope: it is a foundational framework, not a final, comprehensive theory of all dynamic causal phenomena. Its purpose is to provide the formal language and computational primitives necessary to begin a systematic exploration of dynamic causality, not to claim to have concluded it. The EPP foundation defines two clear boundaries:

- **A Lower Boundary of Complexity:** The EPP is designed for a class of problems characterized by non-linear temporality, non-Euclidean structures, multiple contexts, and the potential for emergence. For systems that align well with the classical assumptions of a fixed spacetime and static causal rules, simpler, established methodologies such as Pearl's SCMs are often more appropriate and should be preferred. The EPP's power is best reserved for the complex realities that lie beyond the classical scope.
- **An Upper Boundary of Knowledge:** The EPP does not claim to have "solved" dynamic causality. On the contrary, by providing the first formal tools to model phenomena like causal/contextual co-emergence, it reveals the vastness of our current ignorance. The framework, in its early stages, makes it possible to ask new kinds of questions and to begin building and testing the first generation of truly dynamic causal theories for which the specific characteristics need to be discovered

While the EPP draws conceptual inspiration from physics to address these new frontiers, it is a foundational and formal computational framework. It does not propose that macroscopic systems are quantum mechanical, but rather draws inspiration from these powerful concepts to engineer a more robust and flexible toolkit for modeling the complex, adaptive systems we face today.

11.2.4 Method Selection Criteria: Classical Causality vs. EPP

In cases where methods of classical causality and conventional machine learning do not solve the problem at hand, methodologies rooted in EPP might be preferred. The following decision matrix supports the assessment of when to use which methods. This matrix provides guidance on selecting an appropriate causal modeling approach based on the temporal and spatial complexity of the system under investigation.

Feature Assessed	System Characteristic	Classical Methods	EPP Methods
Temporal Complexity			
	Single time scale + linear progression	Sufficient	
	Multiple time scales OR non-linear temporal relationships	May struggle	Consider EPP
	Multiple time scales AND non-linear temporal relationships	Insufficient	EPP Required
Spatial Structure			
	Euclidean space with fixed coordinates	Sufficient	
	Non-Euclidean OR dynamic spatial relationships	Limited/Difficult	EPP Advantageous
	Non-Euclidean AND dynamic spatial relationships	Very Limited	EPP Required
Combined Complexity			
Scenario 1	Linear Time & Euclidean Space	Preferred	
Scenario 2	Non-Linear Time OR Complex Spatial	Limited/Difficult	Consider EPP
Scenario 3	Non-Linear Time AND Complex Spatial	Insufficient	EPP Required

Table 4: Causal Method Selection Matrix

Emergent Causality: If the problem involves emergent causal structures (where the causal graph itself is not fixed and changes dynamically based on context, i.e., dynamic regime shifts), EPP-based methodologies become the only viable option.

12 Future Work

The preceding chapters have established the Effect Propagation Process as a foundational framework for modeling dynamic contextual causality. The framework's philosophical underpinnings have informed its implementation in DeepCausality. However, because of the single axiomatic definition of generalized causality the EPP is build upon, the resulting vast space of new possibilities will inform a substantial amount of future work that clearly falls outside of the scope the foundational work presented in this monograph.

12.1 Contextual Causal Discovery:

A primary focus for future work is the development of a robust contextual causal discovery algorithm to automatically construct Causaloid Graphs for a given domain. The main challenge roots in the interaction between context and the causal model because and that results in triple challenge.

First, for a given domain, one context graph structure may work better than others for a given model, thus finding an "optimal" context graph w.r.t. supporting the causal model is the first major challenge.

Second, when holding the context graph constant, the structure of the graph has significant impact on the overall efficacy of the model and thus finding an optimal causal graph for a given context is the second major challenge. Unfortunately, the context and causal graph cannot be separated from each other therefore it is most likely that these two challenges are deeply interrelated and require an integrated approach that searches for context representation and the causal model in tandem.

Third, the causal function embedded into each causaloid also implies a not-trivial search, construct, and validate challenge. This challenge deeply interrelates with contemporary work on decomposing causality during the discovery process. However, one complication is that causaloids are isomorphic recursive and may container another causaloid, a causal collection, or another causaloid graph to represent arbitrary complex causal structure. It is not known to the author if sufficiently advanced algorithms that can discover complex nested causal structure exist.

Addressing the challenges imposed by contextual causal learning would require a hybrid approach. For example, for finding causal relation in raw data, an adaptation of the SURD algorithm[119] may result in a workable foundation. For the challenge of the entangled context and causal graph structure, algorithms from the field of search based procedural content generation[120] may offer a path forward. Dealing with the expected computational cost of resulting from the high dimensional nature of the problem of finding two interdependent hypergraphs suggest the exploration of deep neuroevolution adapted to high dimensional data[121]. Furthermore, the broader exploration of novelty search[122] algorithms may yield deep insights about robust contextual causal discovery.

12.2 Formalization of the Effect Ethos:

The Teloid and Effect Ethos are introduced as ontological primitives. The next logical step is the formalization of a calculus to establish the foundation for verifiable teleological reasoning that enables the design and validation of systems whose adherence to specified intent or regulations is mathematically provable. This line of study might be particular valuable for regulated industries such as robotics, avionics, and defense.

12.3 Exploration of Conceptual Blueprints

The ultimate purpose of a foundational framework is its ability to provide a clear path to solving problems that were previously intractable. The following conceptual blueprints are presented as demonstrations of the EPP's expressive power. They represent a class of dynamic, and context-aware systems that were previously infeasible to solve with classical methods of computational causality.

The engineering of these systems would require a major, dedicated effort to build the necessary domain-specific tooling (e.g., relativistic physics simulators, validated models of cancer biology, or resilient decentralized communication protocols). However, these blueprints serve to illustrate how the EPP and its implementation in DeepCausality could provide the foundation upon which such tooling and dynamic systems could be built.

13 Epilogue

The Effect Propagation Process, as detailed in the preceding sections, provides a consistent, first-principles foundation for dynamic causality and a new philosophy grounded language to study dynamic, emergent, and contextual causality. It is important to acknowledge that the higher order effects resulting from the single axiomatic foundation most likely will remain incomplete for the foreseeable future despite the EPP's attempt to describe and formalize an initial set. The vast space of new possibilities opened up by an irreducible single axiomatic definition will require some meaningful time to explore and understand. The presented 3 modalities of dynamic causality already demonstrated very clearly that the higher order consequences need to be carefully considered to ensure safe and reliable dynamic systems build upon the EPP.

The implementation of the EPP in the DeepCausality[11] demonstrates the practicality of hypergeometric computational causality for fast context-aware causal reasoning across Euclidean and non-Euclidean spaces. Furthermore, DeepCausality, as a reference implementation, provides an excellent foundation to study dynamic causality further. And indeed, the new foundation of dynamic causality already suggests several domains of inquiry:

- **The Theory of Hybrid Causal Models:** The EPP already enables uniform causal reasoning over deterministic and probabilistic modes and the Causaloid is a polymorphic container that can host SCMs, Bayesian networks, deterministic rules, and neural networks. One area the EPP opens up is the exploration of a theory of hybrid causal models to explore hybrid causal composition and discovery.
- **Formal verification of the CSM** The Causal State Machine (CSM) bridges insights from causal reasoning to actions and that opens up the area of formal verification of the CSM especially for dynamic causal models with a mixture of static and dynamic contexts.
- **The Formalization of Causal Emergence:** While the EPP introduced causal emergence, its foundation is far from settled therefore the study of causal emergence promises further discoveries.
- **The Dynamics of Context:** The EPP established external context and its modalities, but especially the intersection between a dynamically changing environment and a dynamic context offers promising area for study. This is particular valuable for control systems in autonomous unmanned vehicles that have to adapt to new terrain.
- **The Teleology of Emergence:** Directly related to the calculus of purpose follows the study to teleological boundaries of causal emergence which is particular relevant to the development of safe emergent algorithms.

The presented EPP serves as an open invitation. It is an invitation to the architects and engineers of complex systems who require formal proof of safety and explainability. It is an invitation to the underwriters of risk, who must quantify, model, and price complex risks that is contextual and dynamic. And it is an invitation to the regulators and policymakers tasked with ensuring the safe and compliant integration of complex systems into the fabric of society. Trust is the single most valuable asset any society has and the EPP exists to facilitate trust building and certification of complex dynamic causal systems.

[11] `https://deepcausality.com`

Glossary

This glossary provides definitions for the key formal terms and symbols used throughout this paper to describe the Effect Propagation Process (EPP) and its components.

Effect Propagation Process (EPP)
The overarching philosophical framework and the formalized dynamic process (Π_{EPP}) of effect transfer within a CausaloidGraph, conditioned by the Contextual Fabric.

Contextual Fabric ($\mathcal{C}$)
The structured environment, composed of Context Hypergraphs, within which effects propagate and causal relationships are conditioned. It is formalized as a Context Collection $\mathcal{C}_{sys}$.

Contextoid (v)
The atomic unit of contextual information, defined as a tuple $v = (id_v, \text{payload}_v, \text{adj}_v)$. It encapsulates data, time, space, or spacetime values. (See Definition 3.1)

Contextoid Payload (payload$_v$)
The tagged union representing the actual data, temporal, spatial, or spatiotemporal value held by a Contextoid.

Context Hypergraph (C)
A structured collection of Contextoids (V_C) and the N-ary relationships (Hyperedges E_C) between them, defined as $C = (V_C, E_C, ID_C, \text{Name}_C)$. (See Definition 3.2)

Context Collection ($\mathcal{C}_{sys}$)
A finite set of distinct Context Hypergraphs, $\mathcal{C}_{sys} = \{C_1, C_2, \ldots, C_k\}$, representing the total available contextual information. (See Definition 3.4)

Context Accessor (ContextAccessor($\mathcal{C}_{refs}$))
A functional interface providing read-access for Causaloids to a specified subset of Context Hypergraphs ($\mathcal{C}_{refs} \subseteq \mathcal{C}_{sys}$). (See Definition 3.5)

Causaloid (χ)
The fundamental, operational unit of causal interaction, defined as $\chi = (id_\chi, \text{type}_\chi, f_\chi, \mathcal{C}_{refs}, \text{desc}_\chi, \mathcal{A}_{linked}, I_{linked})$. It encapsulates a testable mechanism for effect transfer. (See Definition 4.1)

Causaloid Type (type$_\chi$)
Specifies the structural nature of a Causaloid (Singleton, Collection, or Graph), determining how its causal function f_χ is realized.

Causal Function (f_χ)
The operational logic associated with a Causaloid χ, which maps input observations ($\mathcal{O}_{\text{type}}$) and accessed context to an activation status ({true, false}). (See Section 4.2.2)

CausaloidGraph (G)
A hypergraph, defined as $G = (V_G, E_G, ID_G, \text{Name}_G)$, whose nodes ($V_G$) typically encapsulate Causaloids and whose hyperedges (E_G) define pathways and logic for effect propagation. Supports recursive isomorphism. (See Definition 4.2)

Causal Node (v_g)
A node within a CausaloidGraph G, defined as $v_g = (id_g, \text{payload}_g)$, where payload$_g$ can be a Causaloid, a collection of Causaloids, or another CausaloidGraph.

Causal Hyperedge (e_g)
A hyperedge within a CausaloidGraph G, defined as $e_g = (V_{\text{source}}, V_{\text{target}}, \text{logic}_e)$, representing a directed functional relationship.

State of CausaloidGraph (S_G)
A function $S_G : V_G \to \{\text{active}, \text{inactive}\}$ mapping each causal node in a CausaloidGraph to its current activation status. (See Definition 4.3)

Effect (ε)
Primarily the activation state (active/inactive) of a Causaloid within EPP, and the transfer of this status or derived information. (See Section 5.1.1)

Input Observations/Triggers (O_{input}, O_{trig})
External data or events (O_{input} being the set of all possible, O_{trig} being the current set) that can initiate or influence the Effect Propagation Process. (See Definition 5.1)

EPP Transition Function (Π_{EPP})
The core dynamic function $\Pi_{EPP} : (G, S_G, \mathcal{C}_{sys}, O_{trig}) \rightarrow S'_G$, describing how the state of a Causaloid-Graph evolves. (See Definition 5.2)

Operational Generative Function (Φ_{gen})
A formal function or set of functions representing meta-level rules for the dynamic evolution of the Contextual Fabric (Φ_{gen_C}), CausaloidGraphs (Φ_{gen_G}), or both (Φ_{gen_Total}). (See Section 5.2)

Causal State Machine (CSM, M)
An operational component, defined by its set of state-action pairs $\mathcal{SA}_M$, that links recognized Causal States (q) to deterministic Causal Actions (a). (See Definition 6.3)

Causal State (q)
A specific condition within a CSM, $q = (id_q, \text{data}_q, \chi_q, \text{version}_q)$, whose activation is determined by its associated Causaloid χ_q. (See Definition 6.1)

Causal Action (a)
A deterministic operation within a CSM, $a = (\text{exec}_a, \text{descr}_a, \text{version}_a)$, triggered by an active Causal State. (See Definition 6.2)

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