Physics-informed deep learning: towards tire simulations

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CONTEXT AND BACKGROUND

- At Michelin, simulation tools are present at every stage of the design to improve the performance of tires and to optimize their manufacturing process. Both machine learning and numerical simulations are employed to model physical phenomena.
- Despite all that, some problems remain complicated and even impossible to solve accurately with conventional approaches. For example: ill-posed problems when lacking of boundary conditions or inverse problems to identify some parameters in a constitutive law from a few measurements.
- Hybrid modeling is considered promising technique as it combines the physics-based and data-driven modeling, and therefore able to fulfill the physical laws and also inherit the learning capabilities from machine learning algorithms.
- The objective of this work is to study and assess a hybrid approach, known as physics-informed machine learning (PIML) [4], and focus on its applicability in the context of tire simulations. Several questions are considered central: the performance in high-dimensional space, the impact of hyperparameters and noisy measurements, the applicability when dealing with geometrical variations, ill-posed problems and identification problems.
- We focus on the development of a class of deep learning methods, named as Physics-Informed Neural Networks (PINNs) [1], which have gained much attention and remarkable results in recent years.

INTRODUCTION

- We present the application of Physics-Informed Neural Networks (PINNs) to elasticity problem and thermal problem on a tire toy model.
- In this framework, deep neural networks (DNN) are employed to approximate the solution of a physical system by minimizing the loss function that embeds the physical information (PDEs, BCs,…)
- To validate the model, we compare the proposed framework to an analytical reference solution.
- Different strategies to increase the accuracy of PINNs are also investigated.

METHODOLOGY

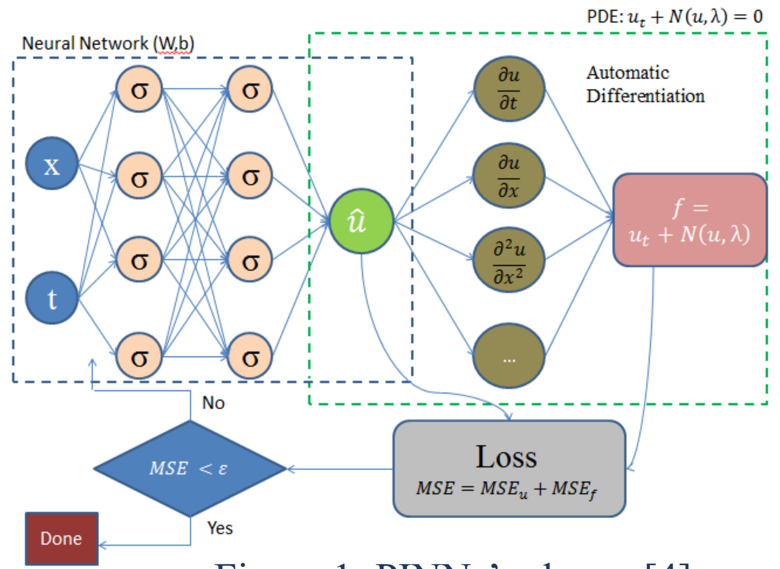


Figure 1: PINNs' scheme [4]

PINNs needs:
+ architecture NN
+ supervised data (N_u: boundary and initial data)
+ collocation points (N_f: unsupervised data in the entire domain)

- Supervised points: $\{x_i^u, t_i^u, u_i\}_{i=1}^{N_u}$; collocation points: $\left\{x_i^f, t_i^f, \right\}_{i=1}^{N_f}$
- Minimize the loss: $MSE = MSE_u + MSE_f$ where

$$MSE_u = \frac{1}{N_u}\sum_{i=1}^{N_u} |\hat{u}(x_i^u, t_i^u) - u_i|^2 \text{ and } MSE_f = \frac{1}{N_f}\sum_{i=1}^{N_f} |\hat{f}(x_i^f, t_i^f)|^2$$

APPLICATION TO ELASTICITY PROBLEM

We apply PINNs to infer the solution of a 2D linear elasticity problem:

$$-(\lambda+\mu)\nabla(\nabla.\vec{u}) - \mu\nabla^2\vec{u} = \vec{g}$$

where the body force $\vec{g}$ and the boundary conditions are defined as in [2]. The analytical solution:

$$u_x(x,y) = \cos(2\pi x)\sin(\pi y) \qquad u_y(x,y) = \sin(\pi x) y^4$$

The number of supervised data $N_u = 200$, collocation points $N_f = 10000$. All these points are randomly distributed.

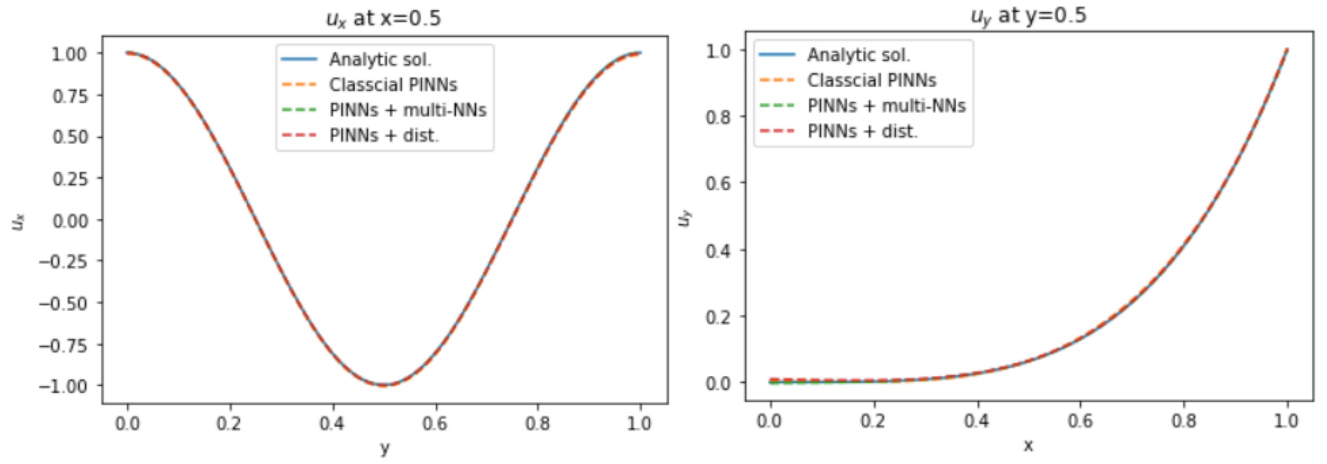


Figure 2: Analytical solution and prediction by different strategies for PINNs

We also study different strategies to improve the accuracy of PINNs, including using multi neural networks [2] or incorporating with a distance function [3].

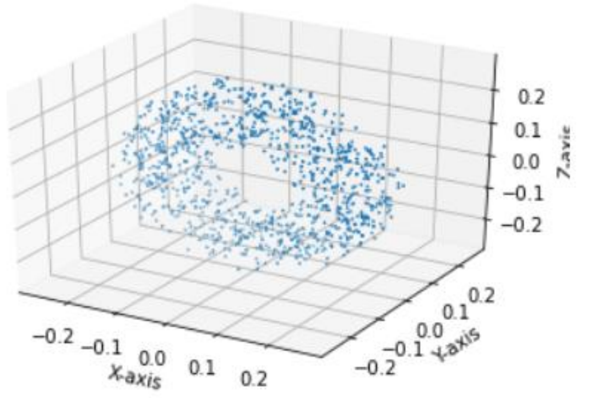
	Relative error	Training time (s)	Predicting time (s)
Classical PINNs	9.66e-03	3.38e+03	9.08e-03
PINNs + multi NNs	4.14e-03	4.61e+03	1.39e-02
PINNs + distance function	4.05e-03	8.73e+03	6.40e-02

APPLICATION ON A TIRE TOY MODEL

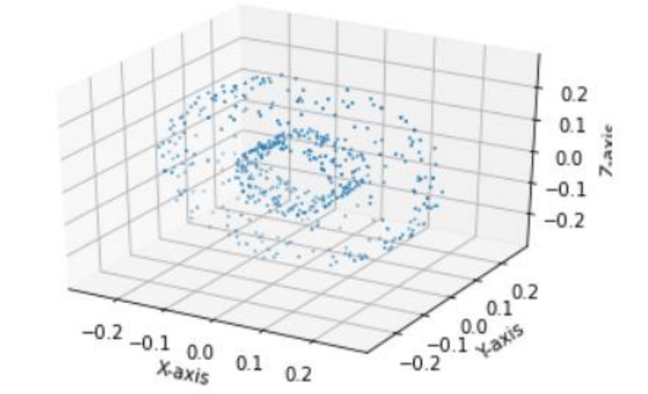
We consider the following heat equation:

$$\begin{cases} u_t - \mu(u_{xx} + u_{yy} + u_{zz}) = 0 \text{ for } x, y \in \Omega \text{ and } t \in [0, T] \\ u(x,y,z,0) = T_{init} \text{ for } x, y, z \in \Omega/\partial\Omega \\ u(x_{lb}, y_{lb}, z_{lb}, t) = T_{lb} \text{ for } x_{lb}, y_{lb}, z_{lb} \in \Omega_{lb} \text{ (Fig. 3.a)} \\ u(x_{ub}, y_{ub}, z_{ub}, t) = T_{ub} \text{ for } x_{ub}, y_{ub}, z_{ub} \in \Omega_{ub} \text{ (Fig. 3.a)} \end{cases}$$

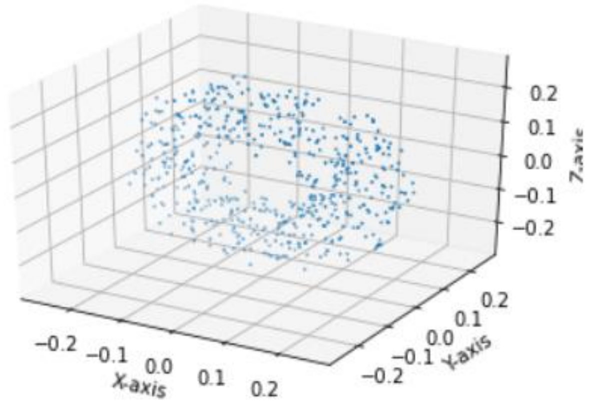
The number of supervised data $N_{init} = 10000$, $N_b = 50000$ and collocation points $N_f = 100000$. All these points are randomly distributed.



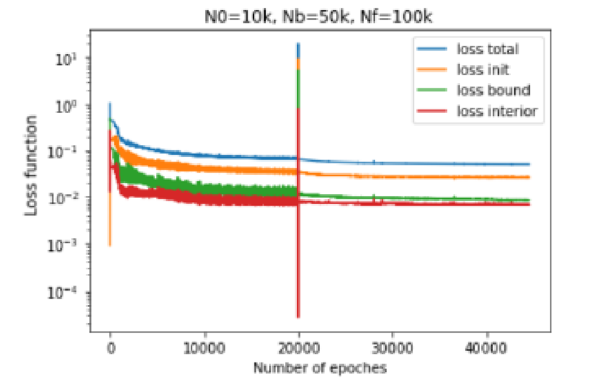
Supervised points on IC



Supervised points on BC at a given time (*)



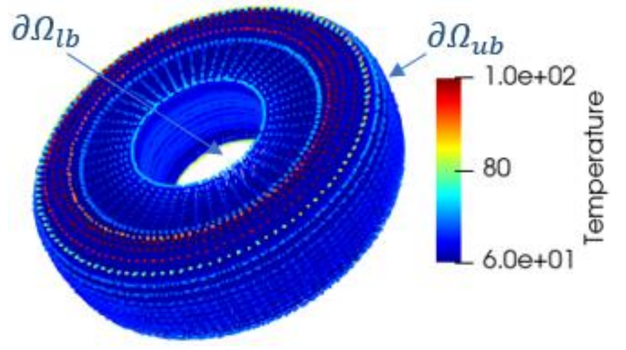
Collocation points at a given time(*)



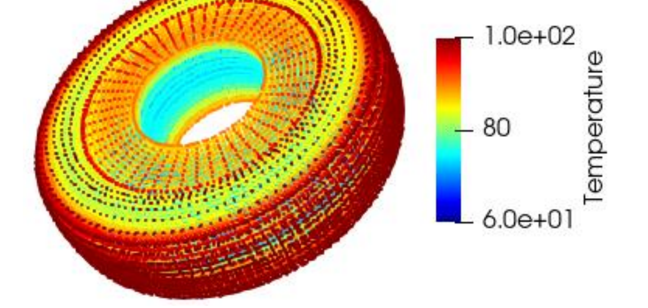
Loss function of the training network

(*) Note that these points are different at different time t.

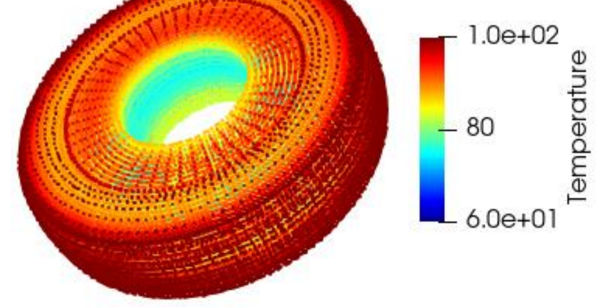
PINNs configuration: feedforward networks with 8 hidden layers and 30 neurons per layers, *Adam* (20000 epochs) and *L-BFGS-B* optimizer and *tanh* activation function.



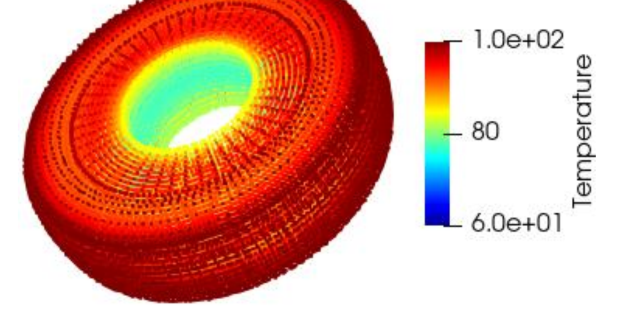
a) At $t = 0s$



b) At $t = 25s$



c) At $t = 50s$



d) At $t = 75s$

Figure 3: PINNs' prediction for the temperature on the toy model at different time

CONCLUSION AND PERSPECTIVES

- We have studied and applied PINNs approach with different strategies to enable reliable predictions. In our work, it is shown that:
  - For elasticity problem, PINNs gives accurate approximations for the solutions with a small relative error.
  - For thermal problem on tire toy model, PINNs gives predictions with expected behavior for a heat equation solution.
- Many questions are under investigation: impact of the initialisation the neural network, the architecture of NNs, number of learning points, weights for loss terms in the loss function, …
- Future work is also to focus on other physics-informed deep learning methods: *e.g.* PhyGeoNet [5]: a method using physics-informed convolutional NN to solve PDEs on irregular physical domain.
- In the industrial context, it is expected that PINNs will help to solve difficult problems for which the conventional methods are difficult (or impossible) to set up. For example:
  - Ill-posed problems where the boundary conditions or the constitutive laws are unknown.
  - Inverse problems where we want to find physical parameters from restricted measurements of a material.
  - Complex problems with multi-physics features and complex geometry.
  - Problems with geometrical variation.

ACKNOWLEDGEMENTS

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