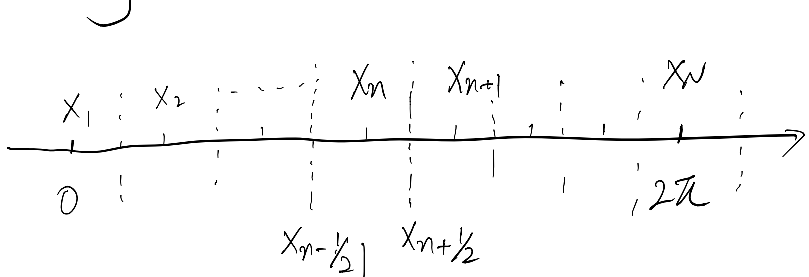
$$\frac{\partial v}{\partial t} = -\frac{\partial v}{\partial x} + \eta \frac{\partial^2 v}{\partial x^2} + f(x,t) \qquad (1)$$

Writing in conservation form:

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial x} J(x) + f(x,t) \qquad (2) \quad J = \frac{v^2}{2} + \eta \frac{\partial v}{\partial x}$$

Considering disretization:



Periodic boundary

Integrate Eq. 2 on $[x_{n-1/2}, x_{n+1/2}]$: length $= \Delta x$

$$\frac{\partial}{\partial t} \int_{x_{n-1/2}}^{x_{n+1/2}} v\,dx = \int_{x_{n-1/2}}^{x_{n+1/2}} \frac{\partial}{\partial x} J(x)\,dx + \int_{x_{n-1/2}}^{x_{n+1/2}} f(x,t)\,dx \qquad (3)$$

$$\frac{\partial}{\partial t} \bar{v} = \frac{J(x_{n+1/2}) - J(x_{n-1/2})}{\Delta x} + \bar{f}(x,t) \qquad (4)$$

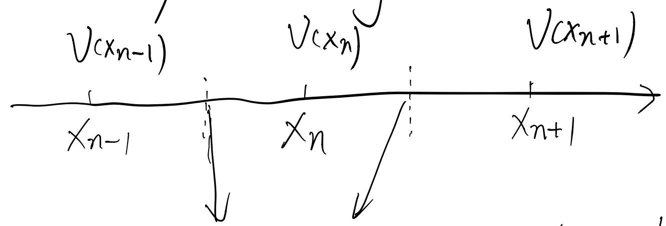
$$\bar{v} = \frac{1}{\Delta x} \int_{x_{n-1/2}}^{x_{n+1/2}} v\,dx$$

Assuming $V(x_n) = \bar{V}$

To update numerically, we will calculate $\frac{\partial}{\partial t} U$.

The problem is how to get $J(x_{n+1/2})$ and $J(x_{n-1/2})$ from $V(x_1)$ ...... $V(x_N)$.

Consider only one segment:



The value here is what we want.

Actually, to get the values on boundary is to get the distribution of V or $\frac{\partial V}{\partial x}$ on the segment.

* This step is called as "reconstruction" in Finite Volume Method (FVM)
* This step is the key point in FVM.
* To design a high order reconstruction scheme is difficult in 3D problem.

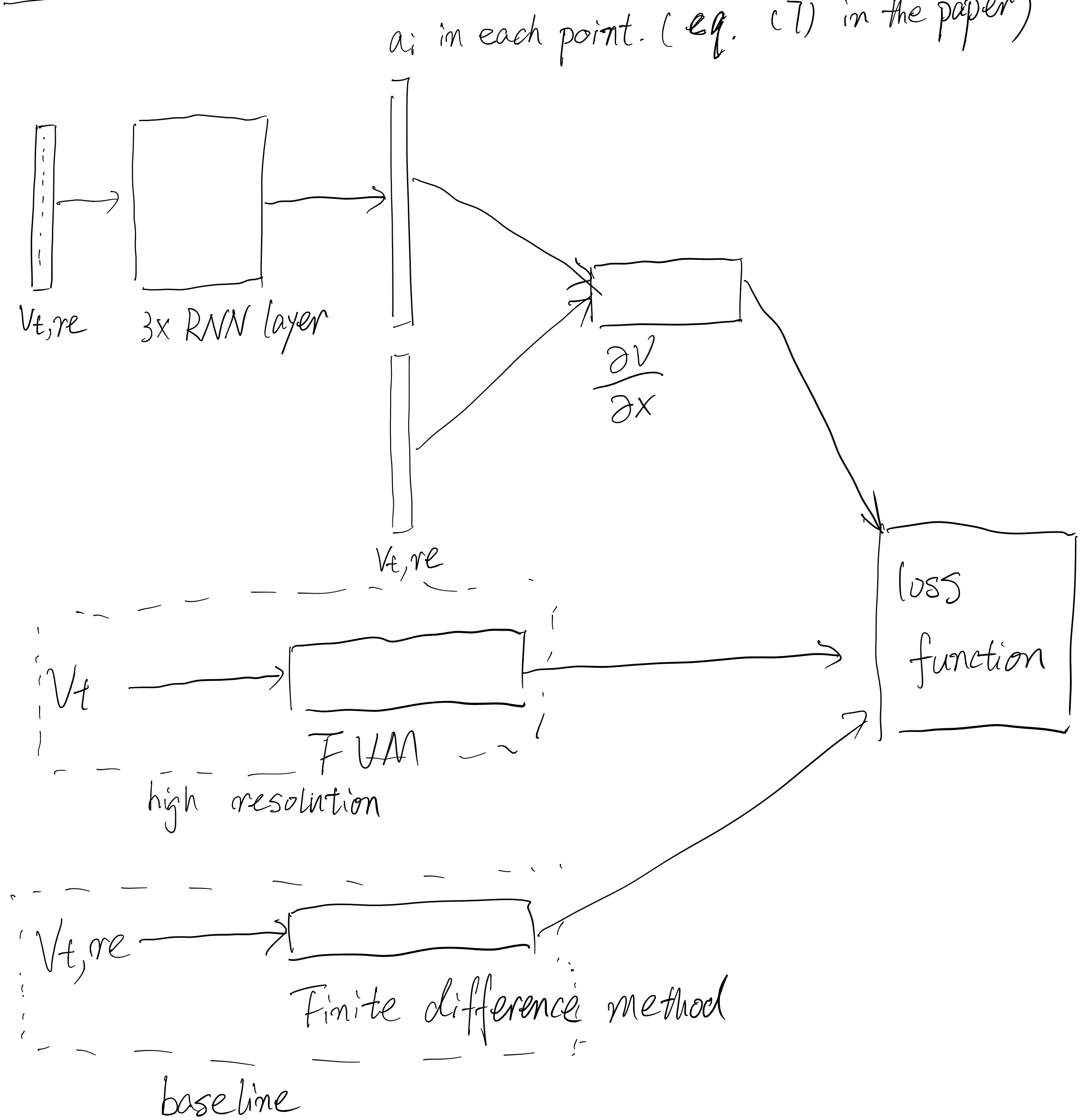
In this paper, the authors try to reconstruct $\frac{\partial V}{\partial x}$ with deep learning method.

Assuming $\frac{\partial v}{\partial x} = \sum_{i=1}^{N} a_i v(x_i)$ Eq.(7) in the paper.

The structure of deep learning in this paper (?)

$V_{t,re}$ is the V_t after resampling (which has large Δx).

V_t is the original high resolution result.



The choice of loss function:

$$\alpha (y-\hat{y})^2 + \beta \frac{(y-\hat{y})^2}{(y-\tilde{y})^2+\gamma} \quad \text{(Eq. (10) in the paper)}$$

y: high-resolution

$\hat{y}$: deep learning method

$\tilde{y}$: base line

$(y-\tilde{y})^2+\gamma$ is kind of difficulty coefficient to evaluate the difficulty level of numerical simulation for a region or time step.

For Burgers' equation, the shock wave region has higher difficulty level than others.