# Amicable numbers

- **Amicable numbers** are like "best friends" among numbers.
- A pair $(a, b)$ is **amicable** if:
- Sum of proper divisors of $a = b$
- Sum of proper divisors of $b = a$
- The classic pair:
- 220 and 284
    - Proper divisors of 220: 1,2,4,5,10,11,20,22,44,55,110 $\rightarrow$ sum = **284**
    - Proper divisors of 284: 1,2,4,71,142 $\rightarrow$ sum = **220**

# Armstrong numbers

- A number that equals the sum of its digits each raised to the power of the number of digits.
- Example: 153
    - 3 digits $\rightarrow 1^3 + 5^3 + 3^3 = 1 + 125 + 27 = 153.$
- Other examples: 370, 371, 407.

# Perfect Numbers

A positive integer $n$ is called a *perfect number* if it is equal to the sum of its positive divisors other than itself.

(a) Show that 6 and 28 are perfect numbers.

(b) Let $p$ be a prime number such that $2^p - 1$ is also prime. Prove that
$$N = 2^{p-1}(2^p - 1)$$
is a perfect number.

# Happy Numbers

positive integer is called happy if the repeated process "replace the number by the sum of the squares of its digits" eventually reaches 1. Otherwise, it is called unhappy.

$n_{k+1}$=(sum of squares of the decimal digits of $n_k$)

Start with 19 => $1^2+9^2$=1+81=82 => $8^2+2^2$=64+4=68

    => $6^2+8^2$=36+64=100

    => $1^2+0^2+0^2$=1+0+0 => 1

# Automorphic Numbers

- A positive integer $n$ is called **automorphic** in base 10 if $n^2$ ends with the same digits as $n$.
  For example, $5^2 = 25$ and $6^2 = 36$, so, 5 and 6 are automorphic.
- Verify that 5, 6, 25, and 76 are automorphic numbers by explicitly computing their squares.

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# Harshad number

➤A positive integer $n$ is called a **Harshad number** (or **Niven number**) in base 10 if it is divisible by the sum of its decimal digits.

➤18 and 21 are Harshad numbers
  - 1+8= 9 properly divides 18
  - 2 + 1 = 3 properly divides 21

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