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Ant colony optimization for disaster relief operations

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Abstract

This paper presents a meta-heuristic of ant colony optimization (ACO) for solving the logistics problem arising in disaster relief activities. The logistics planning involves dispatching commodities to distribution centers in the affected areas and evacuating the wounded people to medical centers. The proposed method decomposes the original emergency logistics problem into two phases of decision making, i.e., the vehicle route construction, and the multi-commodity dispatch. The sub-problems are solved in an iterative manner. The first phase builds stochastic vehicle paths under the guidance of pheromone trails while a network flow based solver is developed in the second phase for the assignment between different types of vehicle flows and commodities. The performance of the algorithm is tested on a number of randomly generated networks and the results indicate that this algorithm performs well in terms of solution quality and run time.

Keywords: Emergency logistics; Pickup and delivery; Split delivery; Ant colony optimization; Multi-commodity network flow problem

1. Introduction

Logistics support and evacuation are the two major activities in disaster response. Evacuation activities take place during the initial response phase, whereas logistics support operations tend to continue for a longer time for sustaining the basic needs of survivors who remain in the affected area. The timely availability of commodities such as food, shelter and medicine and the effective transportation of the wounded have an effect on the survival rate in the affected areas.

The problem described here deals with the coordination of commodity transportation from major supply centers to distribution centers in the affected areas and the transport of wounded people from those areas to the emergency medical centers. The goal is to minimize delay in the provision of prioritized commodities to survivors and health care services to injured people, where different types of vehicles are utilized to serve the transportation needs.

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Emergency logistics support and vehicle dispatch have features different from the established dispatch settings. Supply availability is limited in the initial phases of disaster response: the exact impact of the phenomenon is not known and it takes time to explore the affected regions, communicate the impacts of the disaster and coordinate national and international help. Furthermore, there always occurs the transportation delay from major support centers due to infrastructure failures. Different from commercial situations where several depots serve several customers, the number of supply nodes in this problem is large. Supply nodes may represent warehouses, but most often, if they are located in the affected areas, they are mere tent shelters where food and other materials are distributed to survivors at those locations. If located in unaffected zones, then they may represent district collection centers, shelters and hospitals.

The following vehicle routing and availability conditions prevail. (1) During the first response and throughout the ongoing relief operations, a vehicle is not required to return immediately to a supply node (depot) once its current assignment is completed. It can wait for the next instruction at its last destination or may move towards a depot at the end of the shift if drivers are required to change shifts. The depot where the vehicle ends needs not be the one from which its itinerary starts. Due to the latter reason and the fact that the supply nodes are numerous and dispersed among demand nodes, the tour definition may be given by an itinerary that starts at the beginning of the shift and continues without specific restriction on the nodes to be visited until the end of planning horizon. (2) Since it is not logical to assume that vehicle capacity is sufficient to carry a "customer's" demand, the same nodes have to be visited multiple times. This implies split delivery. (3) An affected node may have both commodity demand and wounded people waiting to be evacuated at the same time; while commodity supply nodes may become medical facilities. In demand nodes, commodities are delivered, and the wounded people are picked up and transported to hospitals (most hospitals lie at supply nodes). In supply nodes, supplies are picked up and wounded people are delivered if hospitals exist at the supply nodes. Hence, simultaneous split pickup–delivery with mixed order service defines the service strategy. Furthermore, it is not necessary that the vehicle picking up some goods from a supply node should be the one to deliver them to their last destination. These may be dropped at an intermediary location and then picked up by another vehicle to the last destination. Hence, the cooperation among vehicles is possible.

Time plays a crucial role in managing the response to a particular emergency. The logistic plan that deals with time-variant demand and supply involves a time horizon consisting of a given number of time periods. It is updated at regular time intervals incorporating new information on demand, supplies and vehicle availability, and accounting for the status of the logistics system resulting from the plan implemented previously. Thus, the system is designed to have a time-dependent structure.

From the above analysis, there is a general dynamic routing problem that can handle various practical complexities. It integrates features of many conventional discrete optimization problems, such as vehicle routing problems and integer multi-commodity flow problems. The existence of heterogeneous vehicles adds more complexity to the problem. Moreover, in the real emergency situation, re-planning needs to be conducted in a timely manner to account for the frequently updated information. Therefore, the study on effective solution method is of crucial importance in emergency logistics management. This paper presents an ACO meta-heuristic with several trail updating strategies proposed for this problem. It is organized as follows: In Section 2, the related work from literature is reviewed. The mathematical formulation of the model is provided in Section 3. Section 4 depicts the meta-heuristic and the computational results are presented in Section 5. The paper is concluded in Section 6.

2. Literature review

The most relevant routing problem would be the simultaneous pickup–delivery problem (Min, 1989; Gendreau et al., 1999a; Nagy and Salhi, 2005) where some goods are delivered to customers from a single depot whereas others (re-cycled packages) are picked up from them and transported back to the depot. The standard definition of the latter problem necessitates that the customer is visited only once. Min (1989) solves the problem with clustering followed by TSP solutions after which infeasibilities are penalized and TSPs are resolved. Gendreau et al. (1999a) solve the TSP first and then order the pickups and deliveries in the TSP. Nagy and Salhi (2005) establish a weakly feasible solution first (one that checks only the total load delivered or picked up, but does not check vehicle capacity in-between nodes on the tour), and then remove infeasibilities by a combination of moves and an iterative procedure that reduces those infeasibilities in a controlled manner. Multi- depot extension is also introduced to this problem by the authors.

A close variation of simultaneous pickup–delivery problem is the mixed pickup–delivery problem (Golden et al., 1985; Kontoravdis and Bard, 1995; Salhi and Nagy, 1999). Similar to simultaneous pickup–delivery problem, maintaining the feasibility of vehicle capacity is difficult in this problem since the available capacity fluctuates during the tour. The solution approach developed in Nagy and Salhi (2005) for simultaneous pickup–delivery problem is applied to this problem as well. Ropke and Pisinger (2006) transform all backhaul problems into a given generic form and propose a unified heuristic based on Large Neighborhood Search method incorporating heuristics with different properties and probabilistic move acceptance scheme.

A special case of the simultaneous pickup–delivery problem is the problem where customers are either delivery (linehaul) or collection (backhaul) nodes where linehaul customers have to be first in a tour (Deif and Bodin, 1984; Yano et al., 1987; Goetschalckx and Jacobs-Blecha, 1989; Toth and Vigo, 1997; Osman and Wassan, 2002). Proposed solution approaches include saving methods, set covering, VRP plus insertion, clustering and routing, and tabu search. A survey of the various models and techniques utilized on this problem can be found in Savelsbergh and Sol (1995). More recently, Lu and Dessouky (2004) propose a branch and cut based algorithm for the multiple vehicle version of this problem. Bent and Henteryck (2006) propose a simulated annealing approach for assigning customers to vehicles first with minimized number of routes and then use Large Neighbourhood Search method to minimize the total travel cost.

Finally, dial-a-ride problems involve taking up on-line requests for picking up and delivering customers at their desired locations by maintaining the capacity feasibility of a vehicle that is en route on cyclic trips. A survey on such dynamic routing problems are found in Gendreau and Potvin (1998) and there are a large variety of solution techniques proposed, including insertion heuristics (Madsen et al., 1995; Diana and Dessouky, 2004), local search (Healy and Moll, 1995), clustering (Ioachim et al., 1995), simulated annealing (Hart, 1996), branch and price (Savelsbergh and Sol, 1998), and tabu search (Gendreau et al., 1999b; Cordeau and Laporte, 2003).

Similar to the standard assumption of unlimited supply quantities, the usual assumptions made in these problems discussed above are that vehicle capacity is sufficient to meet individual customer demand/supply quantities, and that vehicle availability is abundant. These assumptions are difficult to satisfy in emergencies where immediate response is required from as many vehicles as possible. Hence, split delivery and the limitation on the number of vehicles and supplied quantities are valid in the emergency logistics problem. VRP with split delivery is first introduced by Dror and Trudeau (1989), in which they show split deliveries could result in significant savings in terms of total distance and the number of vehicles. Frizzell and Giffin (1995) extend the split delivery routing problem to the situation with time windows, and three heuristics are implemented considering multiple time windows and grid network distances. Mullaseril et al. (1997) study several heuristics for the split delivery capacitated rural postman problem with time windows. Belenguer et al. (2000) study the polyhedron and develop a lower bound for the problem from a new class of valid inequalities. Recently, Ho and Haugland (2004) propose a tabu search based heuristic for the problem with time windows in which the split delivery options are not imposed but decided by a pool of solutions maintained in the solution process.

Although some specific features of the emergency logistics problem were studied in the routing literature by previous researchers, the general logistics problem integrating these characteristics received far less attention. The mathematical formulations for commodities transportation in emergency can be found in Haghani and Oh (1996) and Özdamar et al. (2004), and the latter authors also proposed a Lagrangean relaxation based iterative algorithm for small test instances. Yi and Özdamar (2007) extend the commodity logistics model to integrate the wounded evacuation and emergency medical center location problems and the logistics operations are illustrated by a concrete application on earthquake scenario. The formulation below is extracted from the formulation presented in Yi and Özdamar (2007), however, the facility location problem is not considered here.

3. Formulation of the problem

The mathematical formulation of the problem and the notation are given below. It is a network flow based model with real valued and integer commodities (injured people), i.e., a mixed integer multi-commodity network flow model in which the vehicles themselves are treated as integral commodities that accompany other commodities. The evacuation problem is integrated by modeling the service rates in hospitals as the demands for wounded people, while the wounded in affected nodes are considered as the supplies. For the sake of simplicity, a single transportation mode and a corresponding heterogeneous fleet valid for that mode is assumed.

Sets and parameters

T	length of the planning horizon; t (or s) denotes a specific time period in T
A	set of commodities types; a denotes a specific commodity type
H	set of wounded people (heavy, light); h denotes a specific people type
V	set of different vehicle types; v denotes a specific vehicle type
C	set of nodes. o (or p, i, j) denotes a specific node
CD	set of commodities demand nodes, $\mathrm{CD} \subset C$
CS	set of commodities supply nodes, $\mathrm{CS} \subset C \backslash \mathrm{CD}$
CH	set of available hospitals whose emergency units can receive wounded people, $\mathrm{CH} \subset C \backslash \mathrm{CD}$
t_{op}	time required to traverse arc (o,p)
d_{aot}	amount demanded of commodity type a at node $o \in \mathrm{CD}$ at time t; $d_{aot} = 0$ for $o \in C \backslash \mathrm{CD}$
dw_{hot}	number of wounded people in type h waiting at node $o \in \mathrm{CD}$ at time t; $dw_{\mathrm{hot}} = 0$ for $o \in C \backslash \mathrm{CD}$
sup_{aot}	amount supplied of commodity type a at node $o \in \mathrm{CS}$ at time t; $\mathrm{sup}_{aot} = 0$ for $o \in C \backslash \mathrm{CS}$
av_{ovt}	number of type v vehicles added to the fleet at node $o \in \mathrm{CS}$ at time t; $av_{ovt} = 0$ for $o \in C \backslash \mathrm{CS}$
srv_{ho}	per period service rate for wounded people in type h at hospital at node $o \in \mathrm{CH}$; $\mathrm{srv}_{ho} = 0$ for $o \in C \backslash \mathrm{CH}$
cap_v	load capacity of vehicle type v
w_a	unit weight of commodity type a
w_h	average unit weight of wounded person in type h
M	a big number
P_a	priority of satisfying demand of commodity type a
P_h	priority of serving wounded people type h
K_{ospt}	binary parameter matrix indicating if node p is reachable at time t from node o at time s: if $t - s < t_{op}$, then $K_{ospt} = 0$, else $K_{ospt} = 1$.

Decision variables

Z_{aopvt}	amount of commodity type a traversing arc (o,p) at time t using vehicle type v
X_{hopvt}	integer number of wounded people in type h traversing arc (o,p) at time t using vehicle type v
dev_{aot}	amount of unsatisfied demand of commodity type a at node o at time t
dew_{ht}	integer number of unserved wounded people in type h at time t
Y_{opvt}	integer number of vehicles of type v traversing the arc (o,p) at time t
sp_{hot}	integer number of wounded people in type h who are served at node o at time t

The objective aims at minimizing the weighted sum of unsatisfied demand over all commodities and that of unserved wounded people waiting at demand nodes and emergency units. Commodities are represented in their people equivalents. This objective is compatible with the goal of minimizing service delay.

Model P:

$$\text{Minimize} \sum_{a \in A} \sum_{o \in \mathrm{CD}} \sum_{t} P_a \mathrm{dev}_{aot} + \sum_{h \in H} \sum_{t} P_h \mathrm{dew}_{ht}. \tag{0}$$

Subject to:

Constraint set (1) balances the material flow on demand nodes and explicitly reports unsatisfied demand, dev_{aot}, in each time period. Constraint set (2) enforces material flow balance on all other nodes. Knowledge on future demand is predicted based on current demand and information reaching the coordination center. Confirmed commodity arrivals represent supplies in future periods, thereby enabling continuity of routing plans throughout multiple planning horizons.

$$\sum_{s=0}^{t} d_{aos} - \sum_{v\in V}\sum_{s=0}^{t}\sum_{p\in C}[K_{psot}Z_{apovs} - Z_{aopvs}] = \mathrm{dev}_{aot} \quad (\forall a \in A, o \in \mathrm{CD}, t \in T) \tag{1}$$

$$\sum_{v\in V}\sum_{s=0}^{t}\sum_{p\in C}[Z_{aopvs} - K_{psot}Z_{apovs}] \leqslant \sum_{s=0}^{t}\mathrm{sup}_{aos} \quad (\forall a \in A, o \in C \setminus \mathrm{CD}, t \in T) \tag{2}$$

Constraints (3) restrict the itinerary of each vehicle type with respect to the traveling time between the pair of nodes. Constraints (4) restrict the load transported by the capacity of vehicles traversing the arc.

$$Y_{opvt} \leqslant M^{*}\sum_{s=t}^{|T|}K_{otps} \quad (\forall o \in C, p \in C, v \in V, t \in T), \tag{3}$$

$$Y^{*}_{opvt}\mathrm{cap}_v \geqslant \sum_{a\in A} w^{*}_{a}Z_{aopvt} + \sum_{h\in H} w^{*}_{h}X_{hopvt} \quad (\forall o \in C, p \in C, t \in T). \tag{4}$$

Constraints (5) balance the flow of vehicles over nodes and restrict the number of vehicles moving through the network by their cumulative availability over time. Thus, it is also possible to plan ahead with a dynamic number of available vehicles varying over time.

$$\sum_{s=0}^{t}\sum_{p\in C}[Y_{opvs} - K_{psot}Y_{povs}] \leqslant \sum_{s=0}^{t} av_{ovs} \quad (\forall o \in C, v \in V, t \in T). \tag{5}$$

Constraints (6) and (7) balance the flow of wounded people at all nodes whereas constraints (8) define the number of wounded people that are not served till time period t (waiting in the affected area or hospital queue; or on the way to hospital). Constraints (7) state that the people served at hospitals cannot exceed the number arrived. Here, queue size at hospitals is reduced by those who have already been served and sent out of the emergency system.

$$\sum_{v\in V}\sum_{s=0}^{t}\sum_{p\in C}[X_{hopvs} - K_{psot}X_{hpovs}] \leqslant \sum_{s=0}^{t} dw_{hos} \quad (\forall h \in H, o \in CD, t \in T), \tag{6}$$

$$\sum_{v\in V}\sum_{s=0}^{t}\sum_{p\in C}[K_{psot}X_{hpovs} - X_{hopvs}] \geqslant \sum_{s=0}^{t} sp_{hos} \quad (\forall h \in H, o \in C \setminus \mathrm{CD}, t \in T), \tag{7}$$

$$\sum_{s=0}^{t}\sum_{o\in C}[dw_{hos} - sp_{hos}] = \mathrm{dew}_{ht} \quad (\forall h \in H, t \in T). \tag{8}$$

Constraints (9) define variable domains, and especially restrict the number of wounded served in each period by the service rate of the medical center.

$$sp_{hot}, Y_{opvt}, X_{hopvt}, Z_{aopvt}, \mathrm{dev}_{aot}, \mathrm{dew}_{hot} \geqslant 0; \; sp_{hot} \leqslant \mathrm{srv}_{ho}; \quad (\forall h \in H, a \in A, v \in V, o \in C, t \in T) \tag{9}$$

4. The ACO meta-heuristic for disaster relief operations

The embodied routing sub-problem in emergency logistics planning has higher degrees of freedom as compared to conventional routing models. Capacity feasibility becomes an essential issue because not only does the vehicle capacity fluctuate throughout the tour, but also the assignment of the multiple types of load to available vehicles with different capacity and type. Hence, the feasible space becomes much more relaxed with an increase in the number of alternatives in a local neighborhood search, making the problem more difficult to solve than the general integer multi-commodity flow problem.

Given the hybrid characteristics of the problem, it is a natural way to decompose the model into two components: the vehicle route construction and the multi-commodity dispatch. They are solved sequentially where the first phase constructs the vehicles' route, and then the multi-commodity problem is solved based on the resulting vehicle flows. Thus a solution to the original problem is given. Generally, a one-pass process may not produce a good solution; an iteration framework must be developed, which possesses both the diversification on vehicle paths building and the efficiency on multiple commodities dispatch, as well as smooth communication between the two phases for the continuous improvement of solution quality. An ACO meta-heuristic is proposed for the problem in this paper. As an extension of constructive heuristic, it builds stochastic vehicle paths under the guidance of pheromone trails while a successive maximum flow (SMF) algorithm is developed in the second phase for the commodities dispatch to different types of vehicle flows. Pheromone trails are updated according to the dispatch result by SMF. Thus, the two sub-problems are coordinated through trails and thereby integrated into the overall solution framework. The notations used in the algorithm are defined as follows:

l	a specific vehicle label
(t,o,i)	an arc with tail node o at time t and head node i; the arrival time at head node i is implicitly decided as $t+t_{oi}$
L_{tov}	set of the neighborhood nodes of current node o at time t by vehicle in type v
at_{ol}	arrival time at node o by vehicle l
τ_{toiv}	amount of accumulated pheromone trails of vehicles in type v on the arc (t,o,i); τ^{μ}_{toiv} denotes the pheromone trail in solution μ
p_{tojv}	probability for choosing the next node j by vehicle in type v
DEM	set of demand types defined jointly by the original commodity (or wounded) demand type and the time period it emerges, dem_{at} or dem_{ht}denotes a specific type in the set
U^{μ}_{toil}	utility achieved by vehicle l on arc (t,o,i) in solution μ
σ	elite set of best solutions

4.1. Route construction

Ant colony optimization is a meta-heuristic approach inspired by the behavior of ants in nature that communicate with pheromones trails. It is proposed for solving hard combinatorial optimization problems and was first used on the traveling salesman problem, and has been successfully applied to other problems such as vehicle routing problem (Bullnheimer et al., 1999; Bell and McMullen, 2004), quadratic assignment problem (Maniezzo, 1999), scheduling problem (Merkle et al., 2000), and so on. Detailed description of ACO theoretical results and applications review can be found in the recent papers by Dorigo and Blum (2005) and Dorigo and Stützle (2002).

Artificial ants used in the ACO meta-heuristic conduct stochastic vehicle paths construction procedures that iteratively add arcs to partial itinerary using the probability generated from pheromone trails, which change dynamically to reflect the ants' past search experience. To facilitate the information sharing and thereby the cooperative behavior among vehicles in the same type, multiple types of ants are employed and the pheromone trails are aggregated based on the type.

Initially, the algorithm assigns a ready time at_{ol} to all ants (vehicles) available at node o based on the parameter set av_{ovt} $(l \in v)$. Each ant is tracked individually, and it selects the next customer node j to visit from the list of feasible locations L_{tov} and the path decision time is advanced to $at_{jl} = at_{ol} + t_{oj}$ till it reaches the end of planning horizon T. The neighborhood size of L_{tov} is set to all adjacent nodes in node set C, and it is tractable because not all pairs of nodes are adjacent to each other in a practical network. Moreover, the sparse network is represented in adjacency-list form for space efficiency. To select the next customer j for vehicle $l \in v$, the ant uses the following probabilistic formula – also called the transition probabilities (Dorigo and Blum, 2005):

$$p_{tojv} = \frac{\tau_{tojv}}{\sum_{i \in L_{tov}} \tau_{toiv}} \quad \text{if } j \in L_{tov}. \tag{10}$$

The algorithm constructs a complete tour for the current ant and prior to the next ant starting its tour. This continues until each ant constructs a feasible route and reaches the end of planning horizon. The route selection rule in Eq. (10) is different from the usual ACO implementation. The greedy selection of the most favorable path is observed being dominated in the computation test and therefore forbidden in this problem. In fact, due to the very large dimensionality of the problem and the converging search space in ACO, diversification (exploration) plays a key role in solution improvement process. It is observed that a real time (or frequent) probability updating scheme (strong exploitation) usually worsens the solution quality although convergence is accelerated. The results are generally better when probability updating (Eq. (10)) is conducted regularly by a limited elite set (σ) of best solutions. It is performed only if the following two conditions are satisfied: (1) new elite solution μ enters set σ; and (2) a predetermined number ($0.5 * |\sigma|$ in our implementation) of iterations are performed. Condition (1) is the common idea in ACO literature, while condition (2) ensures a wide search in the current solution stage so as to reduce the chance of missing good solution area.

In addition, one may note that the desirability is not employed in this problem. Different from the conventional routing problem where length (cost) reveals the attractiveness of an arc, the desirability in this problem should be defined as the expectation on utility to be achieved on that arc. However, as to be discussed in the next subsection, these values could be drastically uneven among arcs and result in myopic vehicle route choice. Hence, instead of using very small influencing factor to counteract this effect, the desirability is not explicitly defined but included in pheromone trails and a backward arc utility updating procedure (next section) is applied to reveal the relative fair measure of arc attractiveness.

In this problem, pheromone trail τ^{μ}_{toiv} from a provisional solution μ is calculated as the utilities over all vehicles in type v on the arc (t,o,i):

$$\tau^{\mu}_{toiv} = \sum_{l \in v} U^{\mu}_{toil}, \tag{11}$$

where the vehicle utility U^{μ}_{toil} is evaluated by the contribution to the objective value and calculated by Eq. (12) in the next commodities dispatch phase.

4.2. Commodities dispatch and trail updating strategies

After the vehicle routing is settled, the dispatch of commodities to be addressed in this phase is an integer multi-commodity flow problem. It contributes to the final solution quality by directly affecting the provisional objective value and the pheromone trail updating. Compared to the vehicle routes construction in phase one, this phase poses more computational burden due to the complexity in the integer multi-commodity flow problem. Moreover, the ACO meta-heuristic may run over a large number of iterations. As discussed before, the solution speed is a major concern in real emergency situations for the accommodation of frequent re-planning incurred by information updates. Hence, a tradeoff between dispatch quality and speed must be made.

Given these considerations, a successive maximum flow heuristic algorithm (SMF) is developed to solve the commodity dispatch problem. Initially a set DEM of demand types is constructed, which gives out at most total $(|H| + |A|) * |T|$ kinds of demand since the service rates at medical facilities are formulated in Model P as demands for wounded on a time period basis. Then SMF decomposes the multi-commodity flow problem into maximal flow components regarding each demand type, which are sorted and solved sequentially with the following heuristic procedure:

(i) For each demand type (dem_{at} or dem_{ht}) in DEM, calculate the unit weight utility ($\text{uw}_{at} = P_a*(|T| - t)/w_a$; $\text{uw}_{ht} = P_h*(|T| - t)/w_h$);
(ii) Sort DEM in descending order according to uw_{at} or uw_{ht};
(iii) For each demand dem_{at} (or $\text{dem}_{ht} > 0$) in DEM:
  (a) Allocate the existing flow for the last $\text{dem}_{a't'}$(or $\text{dem}_{h't'}$) to vehicle flows, update vehicle capacity and utility U^{μ}_{toil}: for each vehicle l and arc (t,o,i) in its path $(t + t_{oi} = t')$, suppose x units of flow a' (or h') is assigned, then:

$$U^{\mu}_{toil} = U^{\mu}_{toil} + P_{a'} * (|T| - t') * x \ (\text{or } U^{\mu}_{toil} = U^{\mu}_{toil} + P_{h'} * (|T| - t') * x). \tag{12}$$

  (b) Build a provisional network: identify vehicles compatible with dem_{at} (or dem_{ht}), extract the arcs in their paths, as well as the pass-forward arcs (the arc connecting a node from period t' to $t'+1$, $\forall\, t' \in T$), from the beginning of planning horizon to period t. Calculate the capacity of each arc with respect to the demand type a (or h); set capacity to ∞ for pass-forward arcs.

(c) Apply maximum flow algorithm to the provisional network; update the supplies in type $a(h)$ and demand accordingly.
(d) If $\text{dem}_{ht} > 0$ (surplus service rate), then the unsatisfied demand vanishes by setting $\text{dem}_{ht} = 0$; else if $\text{dem}_{at} > 0$, then the unsatisfied demand transfers to the next period by setting $\text{dem}_{a(t+1)} = \text{dem}_{a(t+1)} + \text{dem}_{at}$, and $\text{dem}_{at} = 0$.

The push-relabel method is employed in step (iii).(c) in SMF, which is known as the most efficient algorithm so far for the maximum flow problem (Cherkassky and Goldberg, 1997). In addition, the provisional networks constructed for each demand are quite compact due to the restriction of vehicle flows. Hence, the SMF algorithm will not pose any problem on the computation cost.

In order to improve future solutions, the pheromone trails must be updated to reflect the ant's performance and the quality of the solutions found. Promising solution space should be marked and favored in the future search. The pheromone trail is updated whenever a solution μ enters elite set. For each vehicle type v on arc (t, o, i), the following rule is applied:

$$\tau_{toiv} = (1 - \rho)\tau_{toiv} + \rho\tau^{\mu}_{toiv}. \tag{13}$$

A convex combination of the existing and added trails is employed here instead of using the evaporation process. By setting a small parameter ρ and a large initial trail value, Eq. (13) suggests a diversified search at the initial stages of solution process (the accumulated trails on visited arcs decrease) whereas intensification is emphasized in the later as the accumulated trails on visited arcs begin to increase themselves. The balance scheme between intensification and diversification can be adjusted at ease by parameter ρ. In addition, the initial trail τ^{0}_{toiv} is set to the maximal possible utility of vehicle type v on the arc:

$$\tau^{0}_{toiv} = (|T| - t - t_{oi}) * \max_{h \in H, a \in A} \{p_h * (\text{cap}_v / w_h), p_a * (\text{cap}_v / w_a)\}. \tag{14}$$

In emergency logistics operations, a vehicle may be fully utilized only in some stages of its route spanning the entire planning horizon, for example, an ambulance is loaded only in backhaul. Furthermore, cooperation among vehicles could result in a higher overall performance, so it is not unusual to find in the optimal solution of the model that some vehicles are dedicated to the transfer work between other vehicles during some periods. In these situations, an arc choice based merely on the utilities achieved on the candidate arcs may lead to inferior results according to our observation, even though it is a probabilistic process. Therefore, a backward path utility updating procedure is called to address this issue before trail updating Eq. (13), by which pheromone trails on the posterior arcs are revealed to the front so that the ant could avoid the myopic path choice: for each vehicle, scan the path from the end to the beginning, record the maximal utility found so far, and use it to replace the utility on the current arc.

In addition to the ACO algorithm, a post-optimization procedure is also developed which improves the existing solution by exploiting the most promising part of each elite and re-combining them into new solution. Since each solution may be biased on some vehicles, the combination of all those favored vehicles may produce a good solution or even the best solution to replace the elite item and therefore enhance the convergence speed. The procedure is conducted whenever the probability updating is eligible to perform and at least one vehicle makes improvement on its best path. It can be described as follows:

(i) Construct a new set of vehicle paths by combining the best path of each vehicle;
(ii) Run SMF and evaluate the solution quality: if a replacement happens, then update the elite set and trails.

5. Numerical results

The performance of the ACO meta-heuristic is tested on 28 randomly generated test problems constructed on grid networks with integer arc travel times. All nodes in the network are connected first by constructing a minimum spanning tree, and then, node degrees are increased randomly by pairwise node connection. The arc number is limited to simulate the sparse road network in practice. The instances are generated as follows. The number of nodes ranges between [20, 80]. About 30% of the nodes are allocated to supply and hospital nodes (all hospitals are assumed to be overlapped with supply nodes). The networks are generated on a 12×12 grid, where each cell is probabilistically allocated to a node. The total number of vehicles ranges between [20, 65], and there are three types of vehicles, the first and last types having the ability to carry commodities and type 2 can carry wounded persons. The vehicle type (type 1) that can carry both wounded and commodities, provides a joint total capacity. We categorize two levels of injury and two kinds of commodities in all instances. Demand and supply are given in the number of persons. Demand/supply quantities as well as the numbers of service rate and wounded people are assigned to each corresponding node randomly according to a uniform distribution while the total supply sufficiency is ensured. For each test problem, the number of vehicles lies within a given interval. Then, the overall vehicle tightness with regard to commodity and people is calculated by the total transportation quantity over the total compatible capacity. A value greater than 1.0 implies tight transportation capacity and vice versa. We take the maximum of commodity and people tightness and then depending on whether the value is larger than 1.0, the problem is classified as capacity tight. Half of the test problems are designed as capacity tight (average tightness for capacity tight problems is 2.06) and the remaining half as capacity loose (average tightness for loose problems is 0.82).

Details of the problem characteristics are provided in Table 1. The second and third columns present the node number and actual arc number which consists of all the node pairs reachable throughout the whole planning horizon ($K_{\mathrm{ospt}} = 1$, $\forall o \in C$, $p \in C$, $s \in T$, $t \in T$). The actual arc number is obtained by running a search algorithm that fans out from vehicle depots and identifies all the reachable arcs. To get more extensive results, two problems in the same group (for example, problems 1 and 1′) are generated for each network structure while having different vehicle distribution and arc connection. Consequently, the capacity tightness and actual arc numbers in the resulting expanded network are different. Two groups of instances are generated for each problem size (node number); hence, there are a total of 4 instances for a certain node number. The number of

Table 1
Characteristics of test problems

Problem	No. of nodes	No. of arcs	No. of vehicles	Tightness
1	20	351	20	2.53
1′	20	364	25	1.54
2	20	414	21	0.79
2′	20	373	16	1.23
3	30	606	20	2.30
3′	30	555	12	4.13
4	30	630	25	0.86
4′	30	536	18	1.17
5	40	871	26	1.61
5′	40	895	18	2.73
6	40	837	40	0.89
6′	40	853	27	1.39
7	50	975	32	2.46
7′	50	968	25	3.30
8	50	987	36	0.80
8′	50	1038	43	0.62
9	60	1134	41	1.97
9′	60	1354	49	1.51
10	60	1014	38	0.85
10′	60	1166	46	0.73
11	70	1580	46	1.79
11′	70	1633	40	2.18
12	70	1598	49	0.79
12′	70	1828	40	1.00
13	80	1657	55	1.68
13′	80	1719	65	1.47
14	80	1686	65	0.77
14′	80	1600	55	0.77

vehicles is given in aggregate for the three vehicle types. The planning horizon is set to 10 periods. Here, it has been noted that the problem size is limited by the memory requirements of model P solution in CPLEX. In all ACO solutions, search parameters are set to the following values: $\rho = 0.05$, $|\sigma| = 20$. The algorithm terminates when there is no solution entering the elite set in 30 iterations.

The ACO algorithm is implemented in C++ and all runs are taken on a PC (3.2 GHz CPU and 512 MB RAM). To evaluate the solution efficiency of the algorithm, direct solutions of model P are provided by executing the MIP solver ILOG CPLEX 7.5 on the same computer. CPLEX has been shown as a highly efficient solver for many multi-commodity flow problems (Castro, 2003) and therefore it serves as the reference solver in this study. All results are given in detail in Table 2. We note the optimal solution value as well as CPU time in seconds. The solution quality of the ACO meta-heuristic is presented by the gap between algorithm solution value and the optimal one. Model P optima are obtained by applying the following relative termination criterion: 0% for problems with nodes $\leqslant$ 40, and 0.5% for larger problems, except that 1% for problems 5′, 9′, 12 and 12′. These very small tolerances do not weaken the solution quality much as they save substantial CPU time on branch and bound process to prove that a solution found is the best. The respective CPU times for finding optimum and heuristic solution are shown in columns 3 and 5. However, the direct comparison between these two columns makes no sense because the solutions from both the solvers are not of identical quality. For the equitable measure of solution speed, the objective value gaps given by ACO (column 6 in Table 2) are fed back to CPLEX as tolerances, in which all the problems are re-solved. The results are presented in the last two columns in Table 2.

Tables 3–5 present the summary of results. Table 3 measures performance robustness by the average value and standard deviation (SD) of CPU time on the problems grouped in same size. Columns 5, 8 and 11 show the relative standard deviation (RSD). The RSD values are high for both the solvers because the size of

Table 2
Numerical results

Problem	Optimal model solution		ACO meta-heuristic			Model solution with tolerance	
	Obj. value	Runtime	Obj. value	Runtime	Optimality gap (%)	Runtime	Runtime gap (%)
1	51689	1.56	51858	0.97	0.33	1.38	42.19
1′	49477	1.52	49683	0.88	0.42	1.23	40.71
2	19748	2.25	20272	2.38	2.65	1.19	−50.15
2′	20362	36.16	20877	2.17	2.53	2.73	25.93
3	52144	447.25	53307	6.79	2.23	12.75	87.78
3′	54000	105.31	54568	2.67	1.05	31.97	1095.96
4	17103	4.20	17157	6.86	0.32	4.03	−41.24
4′	19475	5.00	19509	4.11	0.17	5.28	28.62
5	41041	589.77	42396	11.41	3.30	17.63	54.43
5′	42886	1282.44	45451	4.50	5.98	34.41	664.75
6	38610	89.52	39556	28.38	2.45	9.17	−67.68
6′	40091	36.44	41001	8.69	2.27	9.36	7.71
7	99554	70.25	102465	10.88	2.92	53.19	388.86
7′	102745	733.02	105568	13.27	2.75	44.19	232.88
8	35222	221.50	35596	6.33	1.06	135.47	2040.43
8′	32943	55.41	33363	11.17	1.28	24.05	115.27
9	90082	246.89	93501	28.18	3.79	96.02	240.74
9′	82851	1181.94	87490	30.93	5.60	99.41	221.43
10	28643	149.47	29604	10.46	3.35	74.11	608.57
10′	25496	156.95	26759	11.14	4.95	85.44	666.94
11	96443	152.67	102107	24.62	5.87	110.83	350.15
11′	90317	703.00	95699	21.75	5.96	117.28	439.35
12	45655	1951.23	48408	38.00	6.03	126.98	234.21
12′	43604	4033.97	46308	28.26	6.20	111.69	295.23
13	106694	367.31	111859	24.50	4.84	163.92	569.15
13′	102024	593.86	105817	31.84	3.72	168.42	428.89
14	56769	289.05	57394	30.82	1.10	200.48	550.56
14′	57820	363.64	59282	33.82	2.53	208.734	517.21

Table 3
Summary of results – average and standard deviation of runtimes

Group	Problem	Optimal model solution			ACO meta-heuristic			Model solution with tolerance		
		Runtime average	Runtime SD	RSD (%)	Runtime average	Runtime SD	RSD (%)	Runtime average	Runtime SD	RSD (%)
1	1–2′	10.37	17.19	165.79	1.60	0.79	49.24	1.63	0.74	45.25
2	3–4′	140.44	209.98	149.51	5.11	2.07	40.49	13.51	12.90	95.47
3	5–6′	499.54	578.40	115.79	13.25	10.48	79.14	17.64	11.85	67.19
4	7–8′	270.04	317.64	117.63	10.41	2.92	28.08	64.22	49.03	76.35
5	9–10′	433.81	500.71	115.42	20.18	10.89	53.97	88.74	11.43	12.88
6	11–12′	1710.22	1722.24	100.70	28.15	7.08	25.15	116.70	7.43	6.37
7	13–14′	403.46	131.95	32.70	30.24	4.03	13.32	170.65	38.30	22.44
All problems		495.41	833.17	168.18	15.56	11.94	76.74	69.69	65.59	94.12

Table 4
Summary of results – solution quality

	Optimality gap of ACO		
	All problems (%)	Tight (%)	Not tight (%)
All problems	3.06	3.48	2.64
Nodes $\leqslant 40$	1.97	2.22	1.73
Nodes > 40	3.87	4.43	3.31

Table 5
Summary of results – runtime

	Runtime-ACO meta-heuristic			Runtime-model solution with tolerance			Gap of runtime		
	All problems	Tight	Not tight	All problems	Tight	Not tight	All problems (%)	Tight (%)	Not tight (%)
All problems	15.56	15.23	15.90	69.69	68.04	71.34	349.60	346.95	352.26
Nodes $\leqslant 40$	6.65	4.54	8.76	10.93	16.56	5.29	157.42	330.97	−16.13
Nodes > 40	22.25	23.25	21.25	113.76	106.66	120.87	493.74	358.93	628.55

problem affects CPU time significantly; besides, the data sets are quite different for the problems in a specific group. Nevertheless, the ACO meta-heuristic shows a relatively more robust performance with a lower RSD value of 76.74% than 94.12% in model solution with tolerance. This stable performance may result from the more compact representation of the network (adjacency list) in ACO implementation compared to the incidence matrix in model P. The RSD value in model optimum is the highest (168.18%) because proving a solution found is the best possible can use enormous amounts of resources.

The problems are classified according to the tightness and problem size in Table 4 with the corresponding solution optimality gaps. It is observed that the ACO meta-heuristic produces solutions with an average gap of around 3%. The ACO solution quality is affected by the transportation capacity tightness, where the average gaps are 3.48% and 2.64% for tight and non-tight problems, respectively. Problem size is the other impacting factor as expected, which results in a double average gap on larger problems. Furthermore, the influence of tightness increases with the problem size. For instance, considering smaller sized problems with less than 40 nodes, there is only a small difference (0.49%) between tight and non-tight problems in the average gap from the optimum. This difference grows to 1.12% in larger scale problems over 40 nodes.

The solution speeds of both solvers are shown in Table 5. The small instances with less tightness can be solved 16.13% faster with CPLEX than ACO meta-heuristic given the same tolerance. Although the problem size contributes to the increment in runtime for both the methods, one can see that model P solution time (column 5) increases faster than ACO (column 2). Hence, the heuristic speed outperforms the direct model solution on larger instances. Furthermore, capacity tightness has virtually no influence on ACO algorithm speed. For the small scale instances, model solution with tolerance responds to capacity tightness while this effect diminishes on larger problems, which indicates that the dominant factor affecting CPU time lies on the network configuration rather than tightness so that CPLEX is capable of finding solutions within the stable CPU time for both tight (68.04 s) and non-tight (71.34 s) problems.

To summarize, due to the effective exploration scheme in ACO and exploitation of the network structure, the meta-heuristic substantially saves the computational time on larger scale instances and it makes a good compromise between solution quality and speed. Hence, this approach could be an alternative to the exact model solution. The result presented also suggests that this approach is promising to solve the larger emergency logistics problems.

6. Conclusion

In this paper, a quick solution approach is proposed to a complex logistics support and evacuation coordination problem that arises in emergencies. The ACO meta-heuristic decomposes the original emergency logistics problem into two sequential phases and iterates between them. Several trails updating strategies are proposed for this special problem setting. The solution to this problem is compared with the model solution produced by CPLEX. Analyzing the overall solution quality and times, one can say that ACO solution quality achieved within a minute of runtime is acceptable for the planner in real emergency situation where there is continuous uncertainty and information dynamism.

The proposed meta-heuristic exploits the ACO's effective exploration scheme of large search space. The local search based meta-heuristics, such as tabu search, need further study to prove their efficiency on this problem. It is noted that an introduction of local search into the post-optimization procedure does not enhance the overall solution efficiency although some provisional solutions quality are improved in the process. The computational results suggest that this decomposition approach may be efficient for other complex combinatorial problems with interdependent decision variables. Hence, it can be the core of a suite of heuristics developed for the extended models such as location-routing problem in emergency logistics, which are left for future work.

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