Ant Colony Optimization for Design of Water Distribution Systems

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Abstract: During the last decade, evolutionary methods such as genetic algorithms have been used extensively for the optimal design and operation of water distribution systems. More recently, ant colony optimization algorithms (ACOAs), which are evolutionary methods based on the foraging behavior of ants, have been successfully applied to a number of benchmark combinatorial optimization problems. In this paper, a formulation is developed which enables ACOAs to be used for the optimal design of water distribution systems. This formulation is applied to two benchmark water distribution system optimization problems and the results are compared with those obtained using genetic algorithms (GAs). The findings of this study indicate that ACOAs are an attractive alternative to GAs for the optimal design of water distribution systems, as they outperformed GAs for the two case studies considered both in terms of computational efficiency and their ability to find near global optimal solutions.

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Introduction

Genetic algorithms (GAs) are an evolutionary optimization method based on the concept of survival of the fittest that have been used extensively for the optimal design and operation of water distribution systems (WDS) (e.g., Goldberg and Kuo 1987; Simpson et al. 1994; Halhal et al. 1997). More recently, Dorigo et al. (1996) developed an evolutionary optimization algorithm based on the foraging behavior exhibited by ant colonies in their search for food. Ant colony optimization algorithms (ACOAs) have been successfully applied to a number of benchmark combinatorial optimization problems, such as the traveling salesman and quadratic assignment problems (Dorigo et al. 2000), and have been shown to outperform other evolutionary optimization algorithms, including GAs (e.g., Dorigo and Gambardella 1997). In the late 1990s, Dorigo and Di Caro (1999) introduced a general framework for developing ACOAs; the ant colony meta-heuristic. This enables ACOAs to be applied to a range of combinatorial optimization problems, provided problem specific formulations can be developed (Stützle and Hoos 2000). Thus far, the use of ACOAs in water resources has been limited (Abbaspour et al. 2001), and in this research, the utility of ACOAs for the optimal design of WDS is explored.

The objectives of this paper are (1) to introduce ACOAs to the hydraulics and water resources community; (2) to compare and contrast the formulation methodology of ACOAs with that of GAs; (3) to develop a formulation for using ACOAs for the optimal design of WDS; and (4) to compare the performance of ACOAs and GAs for two benchmark WDS optimization case studies.

Ant Colony Optimization

Ant colony optimization algorithms are inspired by the fact that ants are able to find the shortest route between their nest and a food source, even though they are almost blind. This is accomplished by using pheromone (chemical) trails as a form of indirect communication. Ants deposit pheromone trails whenever they travel. The path taken by individual ants from the nest in search for a food source is essentially random (Dorigo et al. 1996). However, when many ants are searching for a food source simultaneously, the paths taken are affected by the pheromone trails laid by other ants. When ants encounter pheromone trails, there is a higher probability that trails with higher pheromone intensities will be chosen. As more ants travel on paths with higher pheromone intensities, the pheromone on these paths builds up further,

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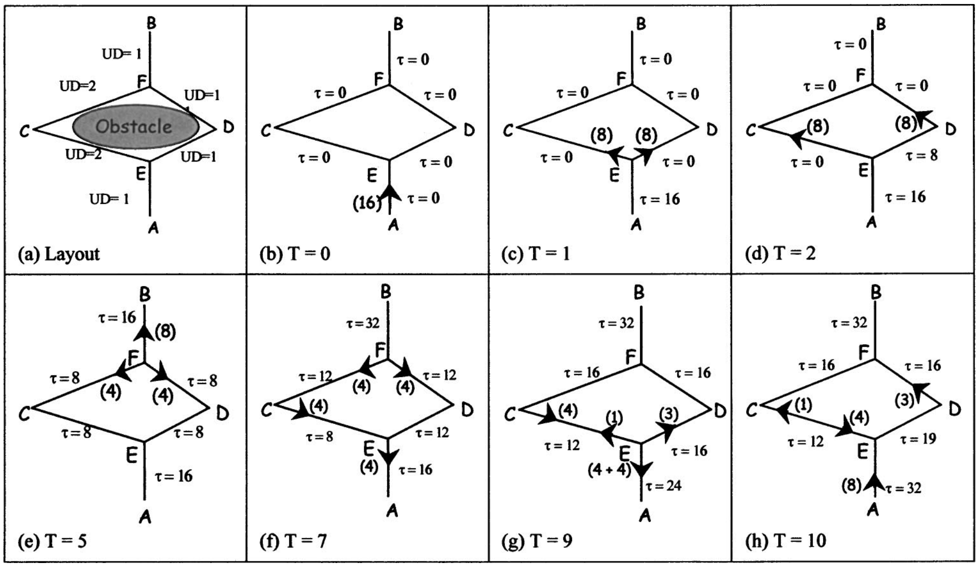

Fig. 1. Example of reinforcement of shorter routes as ants travel from their nest to food source and back

making it more likely to be chosen by other ants. The way this form of positive reinforcement can be used to find the shortest path between the nest and a food source can best be illustrated with an example.

Consider the scenario shown in Fig. 1(a), where an obstacle has been placed between the ants' nest (A) and the food source (B) so that one route from the nest to the food source (*AEDFB*) is shorter than the other (*AECFB*). In the example considered, the length of the shorter route is 4 unit distances (UDs), whereas the length of the longer route is 6 UDs [Fig. 1(a)]. Let us assume that 16 ants leave the nest at time $T=0$, that the initial pheromone concentration (τ) on each path segment is zero, that each ant moves one unit distance per unit time (T), and that each ant deposits one unit of pheromone on the path after reaching the next node. The numbers of ants (shown in brackets) and the pheromone concentrations on each path segment at times $T=0$, 1, 2, 5, 7, 9, and 10 are shown in Figs. 1(b–h).

At $T=1$ [Fig. 1(c)], 16 ants arrive at E and have deposited 16 units of pheromone on *AE*. As there is no pheromone on *EC* and *ED*, there is an equal probability that the ants will choose either path. Consequently, it is assumed that eight ants choose *EC* and eight ants choose *ED*. At $T=2$ [Fig. 1(d)], the eight ants following path *ED* have reached D and have deposited eight units of pheromone on *ED*. Since path *EC* is twice as long as path *ED*, the ants following path *EC* have not yet reached C, and thus pheromone has not yet been deposited on *EC* (assuming that the pheromone is only deposited on the path once *EC* has been traversed completely). At $T=5$ [Fig. 1(e)], the eight ants traveling on the longer route are at F on their way to the food source (B) and have deposited 8 units of pheromone on *EC* and *CF*. At the same time, the eight ants traveling on the shorter route are also at F on their way back to the nest (A), having already reached the food source (B) at $T=4$. Consequently, they have deposited 8 units of pheromone on *DF* and 16 units of pheromone on *FB* (8 units going from F to B and 8 units going from B to F). At this stage, the pheromone intensities on *FD* and *FC* are 8 units each, and thus, by equal probability, it is assumed that four returning ants choose path *FC* and the other four choose path *FD*.

At $T=7$ [Fig. 1(f)], the four returning ants that have chosen the shorter route to the nest (*FDEA*) have reached E and have deposited an additional 4 units of pheromone on *FD* and *DE*. In contrast, the four returning ants that have chosen the longer route to the nest (*FCEA*) have reached C and have deposited an additional 4 units of pheromone on *FC*. At the same time, the eight ants that chose the longer route from the nest to the food source initially have returned to F, after reaching the food source (B) at $T=6$. Consequently, they have deposited an additional 16 units of pheromone on *BF* (8 units going from F to B and 8 units going from B to F). At F, there is an equal probability that the eight ants will choose paths *FC* and *FD*, as each has a pheromone intensity of 12 units. Consequently, it is assumed that four ants travel on each of these paths on their way back to the nest (A).

At $T=9$ [Fig. 1(g)], eight ants are at E on their way from the food source (B) back to the nest (A); four ants that took the long route to and the short route from the food source, and four ants that took the short route to and the long route from the food source. The former have deposited an additional 4 units of pheromone on *FD* and *DE*, while the latter have deposited an additional 4 units of pheromone on *CE*. At the same time, the four ants that took the long route to and from the nest have reached C on their way back to the nest (A), and have deposited an additional 4 units of pheromone on *FC*. In addition, the first four returning ants (i.e., the ants that chose the shorter route to and from the food source) are at E, having reached the nest (A) at $T=8$, and have deposited an additional 8 units of pheromone on *EA* (4 units going from E to A and 4 units going from A to E). At E, there is now a greater probability that the ants will choose the shorter path (*ED*), as the pheromone concentration on *ED* is 16 units, compared with 12 units on *EC*. Consequently, it is assumed that three ants choose *ED*, while only one ant chooses *EC*. This further reinforces the shorter route, as shown in Fig. 1(h). At $T=10$, the gap between the pheromone intensity on *EC* and *ED* has widened, increasing the probability that *ED* will be chosen by the eight ants leaving the nest (A) at that time. In this way, the probability that the shorter route is chosen increases with time.

Ant Colony Optimization Algorithms

The basic principle of positive reinforcement via the use of pheromone trails discussed above also underlies ACOAs. In addition,

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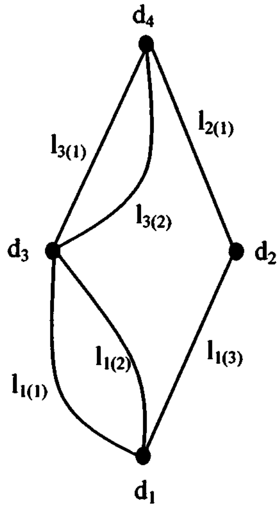

option	cost
$l_{1(1)}$	$c_{1(1)}$
$l_{1(2)}$	$c_{1(2)}$
$l_{1(3)}$	$c_{1(3)}$
$l_{2(1)}$	$c_{2(1)}$
$l_{3(1)}$	$c_{3(1)}$
$l_{3(2)}$	$c_{3(2)}$

Fig. 2. Typical representation of optimization problems in terms of graph

ACOAs make use of a colony of cooperating individuals and adopt a stochastic decision-making policy using local information. However, as the main purpose of artificial ant systems is to find solutions to combinatorial optimization problems, they also incorporate features that are not found in their natural counterparts. For example, artificial ants are generally given some memory and sight, and live in an environment where time is discrete (Dorigo et al. 1996). In addition, pheromone updates may only occur once one or more ants have completed their tour.

In order to implement ACOAs, the combinatorial optimization problem under consideration has to be able to be mapped onto a graph $G=(\mathbf{D},\mathbf{L},\mathbf{C})$, where $\mathbf{D}=\{d_1,d_2,\ldots,d_n\}$ is a set of points at which decisions have to be made, $\mathbf{L}=\{l_{i(j)}\}$ is the set of options (j) available at each decision point (i), and $\mathbf{C}=\{c_{i(j)}\}$ is the set of costs associated with options $\mathbf{L}=\{l_{i(j)}\}$. A set of finite constraints $\Omega(\mathbf{D},\mathbf{L})$ may be assigned over the elements of $\mathbf{D}$ and $\mathbf{L}$. A feasible path over G is called a solution (φ) and a minimum cost path is an optimal solution (φ^*). The cost of a solution is denoted by $f(\varphi)$ and the cost of the optimal solution by $f(\varphi^*)$ (Dorigo and Di Caro 1999). In the example in Fig. 2, there are four decision points, d_1-d_4. At d_1, three options are available, denoted by $l_{1(1)}$, $l_{1(2)}$ and $l_{1(3)}$. If it is assumed that the graph is traversed from d_1 to d_4, only one option ($l_{2(1)}$) is available at d_2, and two options ($l_{3(1)}$ and $l_{3(2)}$) are available at d_3. It should be noted that the reference numbers for the various options (i.e., the numbers in brackets) are assigned arbitrarily. A cost (e.g., $c_{1(1)}$) is associated with each of the available options, as shown in Fig. 2. For example, if the various options represent distances between cities, the costs associated with the various options would be the lengths of the respective paths. Solutions would consist of the different paths connecting d_1 and d_4 (e.g., $l_{1(1)}$ and $l_{3(2)}$ or $l_{1(3)}$ and $l_{2(1)}$) and the costs of different solutions would be the total lengths of the paths.

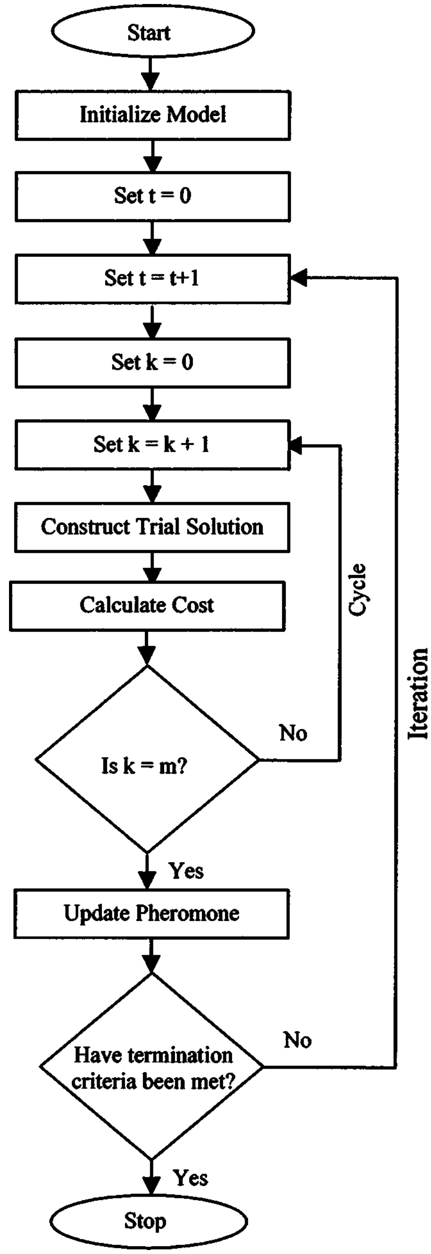

Fig. 3. Steps in ant colony optimization algorithm

Once the problem has been defined in the terms set out above, the ACOA can be applied. The discussion in this paper is based on both the original Ant System algorithm developed by Dorigo et al. (1996), as well as some modifications to the algorithm suggested by Stüzle and Hoos (2000). The main steps in the algorithm are shown in Fig. 3 and include:

1. Trial solutions are constructed incrementally as artificial ants move from one decision point to the next until all decision points have been covered.
2. The cost of the trial solution generated is calculated. The generation of a complete trial solution and calculation of the corresponding cost is referred to as a cycle (k).
3. Pheromone is updated after the completion of one iteration (t), which consists of m cycles (i.e., the construction of m trial solutions), where m is the number of ants used.

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At each decision point, the component to be added to the trial solution is chosen stochastically in accordance with the following equation (Dorigo et al. 1996):

$$p_{i(j)}(k,t)=\frac{[\tau_{i(j)}(t)]^{\alpha}\cdot[\eta_{i(j)}]^{\beta}}{\sum_{l_{i(j)}}[\tau_{i(j)}(t)]^{\alpha}\cdot[\eta_{i(j)}]^{\beta}} \tag{1}$$

where $p_{i(j)}(k,t)$ = probability that option $l_{i(j)}$ is chosen at cycle k and iteration t; $\tau_{i(j)}(t)$ = concentration of pheromone associated with option $l_{i(j)}$ at iteration t; $\eta_{i(j)}=l/c_{i(j)}$ = heuristic factor favoring options that have smaller "local" costs; and α and β = exponent parameters that control the relative importance of pheromone and the local heuristic factor, respectively. It should be noted that the addition of the local heuristic factor ($\eta_{i(j)}$) is analogous to providing real ants with sight, and is sometimes called "visibility" (Dorigo et al. 1996). Artificial ants can also be provided with memory to ensure that each decision point is only visited once.

Once an iteration has been completed, and m trial solutions have been constructed, the pheromone trails are updated in a way that reinforces good solutions. The general form of the pheromone update equation is as follows (Dorigo et al. 1996):

$$\tau_{i(j)}(t+1)=\rho\tau_{i(j)}(t)+\Delta\tau_{i(j)} \tag{2}$$

where $\tau_{i(j)}(t+1)$ = concentration of pheromone associated with option $l_{i(j)}$ at iteration $t+1$; $\tau_{i(j)}(t)$ = concentration of pheromone associated with option $l_{i(j)}$ at iteration t; ρ = coefficient representing pheromone persistence; and $\Delta\tau_{i(j)}$ = change in pheromone concentration associated with option $l_{i(j)}$ as a function of the trial solutions found at iteration t. The pheromone persistence coefficient (ρ) has to be less than one and simulates pheromone evaporation. This enables greater exploration of the search space and avoids premature convergence to suboptimal solutions as it reduces the difference in pheromone concentration between options at each decision point. In addition, evaporation reduces the likelihood that high cost solutions will be selected in future cycles.

In this research, two alternative methods for calculating the change in pheromone concentration $\Delta\tau_{i(j)}$ are considered. In the first method, $\Delta\tau_{i(j)}$ is given by (Dorigo et al. 1996)

$$\Delta\tau_{i(j)}=\sum_{k=1}^{m}\Delta\tau_{i(j)}^{k} \tag{3}$$

where $\Delta\tau_{i(j)}^{k}$ = change in the concentration of pheromone associated with option $l_{i(j)}$ at cycle k during iteration t. In the second method, $\Delta\tau_{i(j)}$ is given by (Stützle and Hoos 2000)

$$\Delta\tau_{i(j)}=\Delta\tau_{i(j)}^{k^*} \tag{4}$$

where k^* = cycle number that results in the best solution during iteration t.

The value of $\Delta\tau_{i(j)}^{k}$ is given by (Dorigo et al. 1996)

$$\Delta\tau_{i(j)}^{k}=\begin{cases}\dfrac{R}{f(\varphi)^{k}} & \text{if option } l_{i(j)} \text{ is chosen at cycle } k\\ 0 & \text{otherwise}\end{cases} \tag{5}$$

where R = pheromone reward factor; and $f(\varphi)^{k}$ = cost of the trial solution generated at cycle k. It should be noted that the amount of pheromone added to each of the options chosen during a cycle is a function of the cost of the trial solution obtained; the better the trial solution, and hence the lower the cost, the larger the amount of pheromone added. Consequently, solution components (options) that are used by many ants and form part of lower cost solutions receive more pheromone and are more likely to be chosen in future cycles (Stützle and Hoos 2000). The steps of generating trial solutions, calculating the costs of the chosen solutions and updating the pheromone concentrations are repeated until certain stopping criteria are met (Fig. 3). The stopping criterion generally used is the completion of a certain number of cycles.

Comparison with Genetic Algorithms

Both GAs and ACOAs are global optimization methods that belong to the class of evolutionary algorithms in the sense that they generate a population of trial solutions. In GAs, the number of trial solutions generated is a function of population size, whereas the number of trial solutions generated by ACOAs is a function of the number of ants. Consequently, the population size in GAs is equivalent to the number of ants in ACOAs. In addition, one generation in GAs is equivalent to an iteration in ACOAs and one cycle in ACOAs is equivalent to the evaluation of an individual member of a population in GAs.

In both GAs and ACOAs, trial solutions are generated using biologically inspired methods. Genetic algorithms utilize the principle of survival of the fittest, whereas ACOAs are based on the foraging behavior of ant colonies. Both algorithms construct trial solutions in a probabilistic manner. In GAs, this process is governed by the probabilities of crossover and mutation and in ACOAs, by pheromone intensities and local heuristic information. Both GAs and ACOAs have a mechanism for encouraging wider exploration of the search space. In GAs, this is facilitated by the mutation operator, whereas ACOAs use pheromone evaporation to achieve the same goal. However, it should be noted that pheromone evaporation in ACOAs is deterministic, whereas mutation in GAs is stochastic.

The main difference between GAs and ACOAs is in the way the trial solutions are generated. In GAs, trial solutions are represented as strings of genetic material, and new solutions are obtained by modifying previous solutions. Consequently, the memory of the system is embedded in the actual trial solutions. In ACOAs, system memory is contained in the environment, rather than the trial solutions. As ants step through this environment, trial solutions are constructed incrementally based on the information contained in the environment. Improved trial solutions are obtained by modifying the environment via a form of indirect communication called stigmergy (Dorigo et al. 2000). As a result of this difference, ACOAs may have advantages over GAs in certain types of applications. For example, ACOAs may be more useful in an operational setting, where the system is dynamically changing (e.g., pipe breakages, valve blockages, pump failures, etc.). By maintaining pheromone trails and continuously exploring different options, ants serendipitously set up backup plans and are therefore prepared to respond to changes in their environment (Bonabeau and Theraulaz 2000). Once a disruption to the system occurs, weak links can be reinforced quickly and used to replace missing or damaged links (Bonabeau et al. 2000). These concepts have already been successfully used in telecommunications routing problems (Bonabeau and Theraulaz 2000). Ant colony optimization algorithms may also have an advantage in situations where sequential decisions have to be made in order to construct a trial solution, and the selection of some component solutions restricts subsequent choices. In such instances, the graph $G=(\mathbf{D},\mathbf{L},\mathbf{C})$ may take the form of a decision tree, and IF ... THEN operators may be incorporated into the algorithm to restrict the available choices at each decision point.

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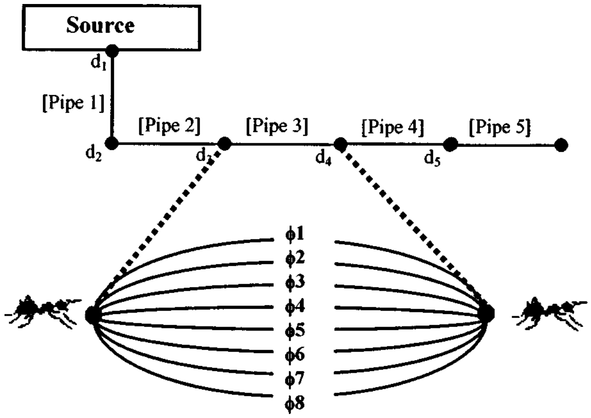

Fig. 4. Representation of water distribution system optimization problems in terms of graph

Application of Ant Colony Optimization to Water Distribution Systems

As pointed out by Dorigo and Gambardella (1997), the key to the application of ACOAs to new problems is to identify an appropriate representation of the problem in terms of a graph $G=(\mathbf{D},\mathbf{L},\mathbf{C})$. In order to apply ACOAs to WDS optimization problems, the graph $G=(\mathbf{D},\mathbf{L},\mathbf{C})$ needs to take the form shown in Fig. 4. One decision point is associated with each pipe. In the example in Fig. 4, there are five pipes and hence five decisions points $(d_1,d_2,\ldots,d_5)$. At each decision point, there are a number of options, corresponding to the available pipe diameters (ϕ_j). In the example shown in Fig. 4, there are eight possible pipe diameters $(\phi_1,\phi_2,\ldots,\phi_8)$, corresponding to eight choices at each decision point $(l_{i(1)},l_{i(2)},\ldots,l_{i(8)},i=1,2,...,5)$. The costs corresponding to each of these choices $(c_{i(1)},c_{i(2)},\ldots,c_{i(8)},i=1,2,...,5)$ are the product of the unit cost per meter length of each of the pipe diameters (UC_{ϕ_j}) and the length of the pipe segment under consideration (LE_i). The cost associated with a particular trial solution $(f(\varphi))$ is therefore given by

$$f(\varphi)=\sum_{i=1}^{n}\mathrm{UC}_{\phi j}\times\mathrm{LE}_i \tag{6}$$

The problem formulation required for the optimization of WDS is different from that used for other combinational optimization problems, such as the traveling salesman problem, in the way the problem is constrained. For example, in the traveling salesman problem, the only constraints are that each city must be visited once only and that the finishing point must be the same as the starting point. In this situation, tabu lists are used to ensure that only feasible solutions are generated (see Dorigo et al. 1996). However, the constraints that need to be satisfied in the optimal design of WDS are of a different nature. The feasibility of a particular trial solution (e.g., whether minimum pressure constraints have been satisfied) can only be assessed after it has been constructed in its entirety, and consequently, the constraints cannot be taken into account explicitly during the construction of trial solutions. The approach taken in this research to deal with this problem is to modify Eq. (5) so as to give negative reinforcement to pipe diameter options that result in solutions that violate the pressure constraints

$$\Delta\tau_{i(j)}^{k}=\begin{cases}\dfrac{R}{f(\varphi)^k}-P_{\mathrm{pher}}\times\Delta H_{\max} & \text{if option } I_{i(j)} \text{ is chosen at cycle } k,\\ 0 & \text{otherwise}\end{cases} \tag{7}$$

where P_{pher} = pheromone penalty factor and $\Delta H_{\max}$ = maximum pressure deficit in the WDS, which is obtained using a hydraulic solver for each trial solution (i.e., combination of pipes) generated. This approach has not been used previously in ACOA applications.

In order to calculate the change in pheromone concentration $\Delta\tau_{i(j)}$ using Eq. (4), Eq. (6) was modified as follows in order to determine which cycle results in the best solution

$$f(\varphi)=\begin{cases}\displaystyle\sum_{i=1}^{n}\mathrm{UC}_{\phi j}\times\mathrm{LE}_i+\mathrm{PC}\times\Delta H_{\max} & \text{if any of the pressure constraints are violated}\\ \displaystyle\sum_{i=1}^{n}\mathrm{UC}_{\phi j}\times\mathrm{LE}_i & \text{otherwise}\end{cases} \tag{8}$$

where PC = penalty cost multiplier (\$/m head violated).

Case Studies

In order to test the utility of ACOAs for the optimization of WDS, they were applied to two benchmark case studies to which GAs have been applied previously, thus providing a direct basis of comparison between the two approaches. The first case study is the 14-pipe network expansion problem studied by Simpson et al. (1994). This case study was chosen as it is relatively simple and because the global optimum solution to the problem is known. The second case study is the New York City Water Supply Tunnels (NYCWST) problem. This case study was chosen as it is more complex than the 14-pipe network expansion problem and because it has been used as a benchmark for a number of optimization algorithms in previous studies (e.g., Schaake and Lai 1969; Morgan and Goulter 1985; Murphy et al. 1993; Dandy et al. 1996; Savic and Walters 1997; Lippai et al. 1999; Wu et al. 2001; Wu and Simpson 2001).

The software code required to implement the ACOA was developed in *Fortran 77*. This code was linked with the code for the hydraulic solver *WADISO* in order to calculate the maximum pressure deficit for each trial solution generated. All final solutions obtained were also checked using *EPANET* version 2.0.

14-Pipe Problem

This case study network is shown in Fig. 5. The system has a total of 14 pipes supplied by two water sources—a tank and a reservoir. Both water supplies are assumed to be at a constant elevation. Eight pipes are existing while there are five new pipes to be sized (with at least the minimum size pipe). Three of the existing pipes may be duplicated with a new pipe in parallel (not necessarily of the same diameter as the existing pipe). Three water demand loading cases need to be considered—a peak hour case and two fire loading cases as tabulated for each of the nodes in Fig. 5. The associated minimum allowable heads for each of the water demand loading cases are shown in Fig. 5. The possible pipe choices are shown in Table 1.

It should be noted that the size of the search space used in this research is different from that used by Simpson et al. (1994). The

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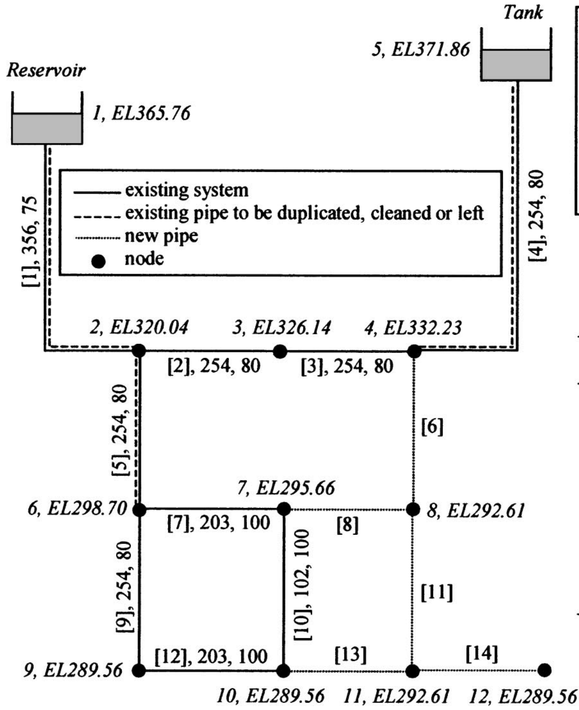

[1], 356, 75 [pipe number], diameter (mm), present day Hazen-Williams roughness coefficient (C)

2, EL320.04 node number, node elevation (m)

Note: 1. All pipe lengths are 1609m, except pipe [1] and pipe [4] which are 4828m and 6437m, respectively.
2. C is 120 for new and cleaned pipes.

Node	Demand Pattern 1		Demand Pattern 2		Demand Pattern 3	
	Q (L/s)	H_{min} (m)	Q (L/s)	H_{min} (m)	Q (L/s)	H_{min} (m)
2	12.62	28.18	12.62	14.09	12.62	14.09
3	12.62	17.61	12.62	14.09	12.62	14.09
4	0	17.61	0	14.09	0	14.09
6	18.93	35.22	18.93	14.09	18.93	14.09
7	18.93	35.22	82.03	10.57	18.93	14.09
8	18.93	35.22	18.93	14.09	18.93	14.09
9	12.62	35.22	12.62	14.09	12.62	14.09
10	18.93	35.22	18.93	14.09	18.93	14.09
11	18.93	35.22	18.93	14.09	18.93	14.09
12	12.62	35.22	12.62	14.09	50.48	10.57

Fig. 5. 14-pipe network expansion problem (after Simpson et al. 1994)

reason for this is that Simpson et al. (1994) did not consider the two largest available pipe diameter options for the rehabilitation of the existing pipes, in order that the available rehabilitation options could be represented by a three-bit binary substring. As a result, the search space in the problem studied by Simpson et al. (1994) consisted of 16,777,216 possible pipe rehabilitation options, whereas the search space considered in this study is 32,768,000. The fact that the search space is not confined to a predetermined set of sizes is one advantage ACOAs have over GAs in which the decision variables are represented by binary strings. However, this potential limitation of GAs can be overcome by using real number coding, for example.

The impact of using the trial solutions found by all ants [Eq. (3)], as well as only using the trial solution found by the "iteration best" ant [Eq. (4)] in the pheromone updating process was investigated. Some preliminary trials on the sensitivity of the results obtained to the parameters that control the ACOA were conducted for both cases. As a result of these trials, the following model parameters were adopted for the case where the trial solutions found by all ants were used in the pheromone updating process: $\alpha=5$, $\beta=3.5$, $\rho=0.8$, $m=100$, $R=200{,}000$, $P_{\text{pher}}=0.005$, and PC$=70{,}000$. Additional information on these sensitivity analyses is given in Simpson et al. (2001). The model

Table 1. Pipe Options for 14-Pipe Case Study Design

Diameter (mm)	Cost of new pipe ($/m)
152	49.54
203	63.32
254	94.82
305	132.87
356	170.93
407	194.88
458	232.94
509	264.10

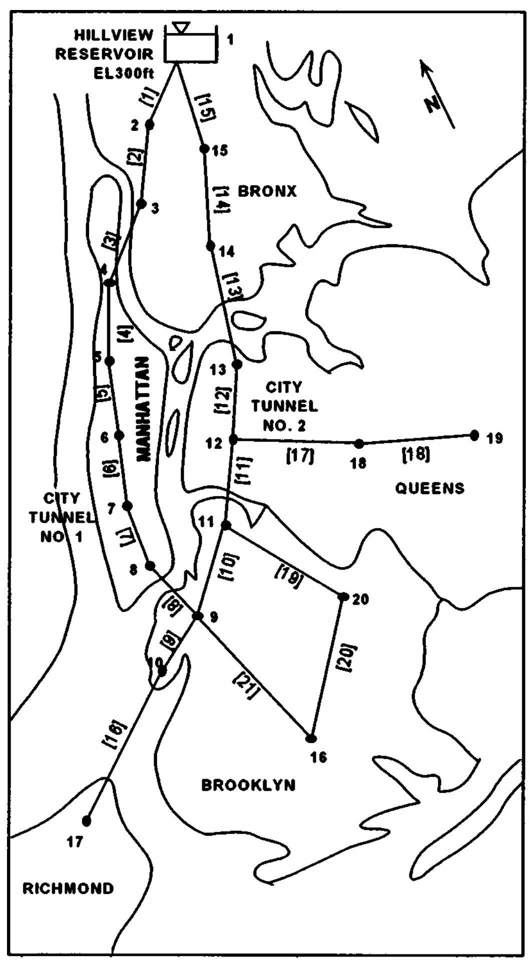

Fig. 6. New York City water supply tunnels problem

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Table 2. Duplicate Tunnel Options for New York City Water Supply Tunnels Problem

Option no.	Diameter (in.)	Cost ($/ft)
1	0	0
2	36	93.5
3	48	134
4	60	176
5	72	221
6	84	267
7	96	316
8	108	365
9	120	417
10	132	469
11	144	522
12	156	577
13	168	632
14	180	689
15	192	746
16	204	804

Note: Hazen–Williams coefficient is 100 for all new tunnels (except the zero diameter option).

parameters that were found to be optimal for the case when only the trial solution produced by the iteration best ant was used in the pheromone updating process were identical to those given above with the following exceptions: $\beta=1.5$, and $P_{\text{pher}}=0.01$. In addition, a ''virtual'' unit cost of $20/m was assigned to zero pipe options for both cases for the purpose of calculating $\eta_{i(j)}$.

To be consistent with Simpson et al. (1994), the number of trial solutions generated was 50,000. Consequently, the number of iterations used was 500, as $m=100$. The analysis was repeated for ten different random number seeds, which is in agreement with Simpson et al. (1994).

New York City Water Supply Tunnels Problem

The New York Tunnels network, as first considered by Schaake and Lai (1969), consists of 20 nodes connected via 21 tunnels (Fig. 6). The system is formed into two primary tunnels, City Tunnel No. 1 and City Tunnel No. 2. Details of the network including tunnel lengths, diameters, and nodal elevations are given in Dandy et al. (1996). Only a single demand loading case supplied by Hillview Reservoir is considered.

As of 1969, increased demands were required to cater for an increase in population growth. Details of the demand pattern and minimum head constraints are given in Dandy et al. (1996). The network in its original state has pressure violations at nodes 16, 17, 18, 19, and 20. In order to meet the minimum allowable total head requirements, the network is to be modified using duplicate tunnels in parallel with the existing tunnels. The objective of the optimization is to decide on which of the 21 tunnels need to be duplicated, and if so, what diameter of tunnels should be constructed. For each duplicate tunnel there are 16 allowable options; 15 different diameter sizes and the option of no tunnel, as shown in Table 2. The search space for this problem is 16^{21} (1.934×10^{25}) possible combinations.

Only the ''iteration best'' ant pheromone updating scheme [Eq. (4)] was used for this case study as a result of its superior performance on the 14-pipe problem (see ''Results and Discussion''). As a result of preliminary sensitivity analyses, the following model parameters were adopted: $\alpha=3.5$, $\beta=0.5$, $\rho=0.95$, $m=100$, $R=15{,}000{,}000$, $P_{\text{pher}}=0.1$, and PC$=500{,}000{,}000$. A ''virtual'' unit cost of $70/m was assigned to zero pipe options for the purpose of calculating $\eta_{i(j)}$. As for the 14-pipe problem, 500 iterations were used (i.e., a total of 50,000 evaluations). The analyses were repeated with different random number seeds.

Results and Discussion

14-Pipe Problem

The results obtained using the two different forms of the ACOA are given in Table 3. It can be seen that the ''iteration best'' algorithm performed significantly better than the algorithm that used all trial solutions found during one iteration in the pheromone updating process. When the former algorithm was used, the global optimum solution of $1.750 million, which was obtained by Simpson et al. (1994) by complete enumeration, was found for all of the ten random number seeds used, whereas when the latter

Table 3. Results Obtained Using GAs and Ant Colony Optimization Algorithms for 14-Pipe Problem

	MINIMUM COST NETWORK ($ MILLION)			
	GA		ACOA	
Seed	Proportionate selection (Simpson et al. 1994)	Tournament selection (Simpson and Goldberg 1994)	All ants [Eq. (3)]	Iteration best ant [Eq. (4)]
1	1.773[a]	1.750	1.750	1.750
2	1.750	1.750	1.750	1.750
3	1.750	1.750	1.813[a]	1.750
4	1.812[a]	1.750	1.750	1.750
5	1.750	1.750	1.750	1.750
6	1.750	1.750	1.813[a]	1.750
7	1.750	1.750	1.750	1.750
8	1.750	1.750	1.750	1.750
9	1.750	1.750	1.750	1.750
10	1.750	1.750	1.812[a]	1.750
Average cost	1.759	1.750	1.769	1.750
Average No. of evaluations to reach the minimum	20,790	8,700	12,455	8,509

[a]Nonoptimal solutions.

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Table 4. New York City Water Supply Tunnels Tunnel Duplicate Combinations

Pipe no.	Pipe Diameters (in.) Dandy et al. GA1 (1996)	Lippai et al. NYD1 (1999)	Wu et al. fmGA2 (2001)	ACOA
[1]	0	0	0	0
[2]	0	0	0	0
[3]	0	0	0	0
[4]	0	0	0	0
[5]	0	0	0	0
[6]	0	0	0	0
[7]	0	132	108	144
[8]	0	0	0	0
[9]	0	0	0	0
[10]	0	0	0	0
[11]	0	0	0	0
[12]	0	0	0	0
[13]	0	0	0	0
[14]	0	0	0	0
[15]	120	0	0	0
[16]	84	96	96	96
[17]	96	96	96	96
[18]	84	84	84	84
[19]	72	72	72	72
[20]	0	0	0	0
[21]	72	72	72	72
Total cost ($M)	38.80	38.13	37.13	38.64
Evaluation No. (other authors)	96,750	46,016	37,186	—
Evaluation No. (ACOA)	13,273[a]	—	—	13,928[b]
Feasibility	feasible	not feasible	not feasible	feasible

[a]Average of (12,306, 13,701, 13,813).

[b]Average of (7,014, 11,725, 23,045).

algorithm was used, the global optimum was only found in seven out of ten trials. In addition, the ''iteration best'' algorithm was computationally more efficient, using 8,509 evaluations on average to find the global optimum compared with 12,455 evaluations when Eq. (3) was used in the pheromone updating process.

Table 3 also indicates that the performance of the ACOA in which all trial solutions found during one iteration were used in the pheromone updating process performed similar to the GA in which proportionate selection was used. The GA was able to find the global optimum solution for eight of the ten random number seeds tried, compared with seven out of ten when the ACOA was used. On average, the least cost solution found by the GA was $1.756 million, which is slightly less than the optimal solution of $1.769 million obtained using the ACOA. However, the ACOA was able to reach the global optimum more quickly than the GA. On average, the ACOA found the optimum solution within 12,455 function evaluations, whereas the GA required 20,790 function calls to the hydraulic solver. This is despite the larger space searched by the ACOA.

The results in Table 3 also indicate that the performance of the ACOA using the ''iteration best'' algorithm was very similar to that of the GA using tournament selection. Both algorithms were able to find the global optimum solution irrespective of the random number seed used with an average number of evaluations of just under 9,000. This indicates that ACOAs appear to be a suitable method for the optimal design of WDS.

New York City Water Supply Tunnels Problem

The results obtained using the ACOA and those previously reported in the literature are shown in Table 4. It should be noted that the combinations of pipes shown in Table 4 are attributed to the authors who have reported the lowest number of evaluations to find the optimal solutions. For example, the $38.80 million solution found by Dandy et al. (1996) was also originally reported by Murphy et al. (1993) at approximately 200,000 evaluations, while the $37.13 million solution found by Wu et al. (2001) was originally reported by Savic and Walters (1997) at approximately 1,000,000 evaluations. It should also be noted that another optimal design that has been reported in the literature but is not included in Table 4 is the $37.83 million solution first reported by Lippai et al. (1999) and then by Wu et al. (2001). The reason for its exclusion is that it is not a discrete solution to the NYCWST problem (as defined by Table 2), as it includes a tunnel at pipe no. [7] with a diameter of 124 in.

As can be seen in Table 4, the least cost feasible solution found by the ACOA was $38.64 million, compared with optimal costs of $38.80, $38.13, and $37.13 million found by Dandy et al. (1996), Lippai et al. (1999), and Wu et al. (2001), respectively. However, a rigorous hydraulic analysis of all networks in Table 4 using *EPANET* version 2.0 revealed that the two solutions that were lower cost than that obtained using the ACOA were infeasible, as shown in Table 5. It can be seen that whereas the solutions obtained by Dandy et al. (1996) and the ACOA have minimum pressure excesses of 0.11 and 0.05 ft, respectively, the solutions obtained by Lippai et al. (1999) and Wu et al. (2001) have maximum pressure deficits of 0.02 and 0.22 ft, respectively. The infeasible solutions of Lippai et al. (1999) and Wu et al. (2001) are based on inaccurate coefficients relative to the original formulation of the Hazen–Williams head loss equation as shown in Eq. (9) (Brater and King 1976)

Table 5. Total Head at Critical Nodes for New York City Water Supply Tunnels Network Combinations

Critical node	Minimum allowable total head (ft)	Actual Total Heads, *Head Difference from Minimum* (ft) Dandy et al. GA1 (1996)		Lippai et al. NYD1 (1999)		Wu et al. fmGA2 (2001)		ACOA	
16	260.0	260.59	*0.59*	259.998	*−0.002*	259.79	*−0.21*	260.08	*0.08*
17	272.8	272.91	*0.11*	272.79	*−0.01*	272.58	*−0.22*	272.87	*0.07*
19	255.0	255.78	*0.78*	254.98	*−0.02*	254.80	*−0.20*	255.05	*0.05*
Total cost ($M)		38.80		38.13		37.13		38.64	
Feasibility		feasible		infeasible		infeasible		feasible	

Note: Hydraulic analysis was performed in *EPANET* version 2.0 with an accuracy of 0.000001 and a maximum of 500 iterations.

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$$V = 1.318 C_{\mathrm{HW}} R_H^{0.63} S_f^{0.54} \tag{9}$$

where V = velocity (fps); C_{HW} = Hazen–Williams coefficient; R_H = hydraulics radius (ft); S_f = friction slope [h_f/LE, where h_f = head loss (ft); and LE = length of pipe (ft)]. Solving for the head loss in Eq. (9) in terms of the discharge gives

$$h_f = 4.7279 \frac{\mathrm{LE} Q^{1.852}}{C_{\mathrm{HW}}^{1.852} D_P^{4.871}} \tag{10}$$

where Q = discharge (cfs) and D_p = pipe diameter (ft). This equation is implemented in *EPANET 2.0*. If the coefficients in the Hazen–Williams equation are not carried to sufficient significant figures or are incorrect, then losses through the network may be calculated to be less than what is calculated when coefficients associated with the original Hazen–Williams formula are used. Clearly, if comparable versions of the Hazen–William equation are not used, then it is difficult to fairly compare the results of different optimization studies.

The results in Table 4 show that the solution found by the ACOA belongs to the family of solutions that duplicates tunnels [7], [16], [17], [18], [19], and [21] (see Lippai et al. 1999; Wu et al. 2001), as opposed to the family that duplicates tunnels [15], [16], [17], [18], [19], and [21] (see Dandy et al. 1996). Table 4 also shows that the results obtained in this study compare favorably with those obtained in previous studies in terms of computational efficiency. The optimal solution obtained using the ACOA was reached with an average number of evaluations of 13,938 from three runs. This compares with evaluation numbers of 96,750, 46,016, and 37,186 reported by Dandy et al. (1996); Lippai et al. (1999); and Wu et al. (2001), respectively. The ACOA was also able to find the $38.80 million solution found by Dandy et al. (1996), which is the lowest cost feasible solution reported in the literature thus far, with an average number of evaluations of 13,273, which is less than 14% of the number of evaluations taken by the GA used by Dandy et al. (1996).

Conclusions and Recommendations

A formulation for applying ACOAs to the optimal design of WDS has been presented in this paper. In order to apply ACOAs to the optimal design of WDS, a decision point was placed on each potential pipe in the system. At each decision point, the available choices corresponded to the available pipe diameters (or pipe rehabilitation options). Pheromone intensities and heuristic values were associated with each of these choices. The heuristic value was taken as the inverse of the cost of each choice. Pheromone intensities were modified in a way that favors choices that result in smaller total network costs. In addition, the pheromone levels associated with choices that result in systems that violate the required pressure constraints were decreased.

Based on the results obtained in this research, in which ACOAs were applied to two benchmark WDS optimization problems, ACOAs are an attractive alternative to GAs for the optimal design of WDS. For a simple 14-pipe network expansion problem, the performance of ACOAs and GAs was very similar, both in terms of their ability to find the global optimum solution of $1.750 million from different starting positions in decision variable space and in terms of computational efficiency (~9,000 evaluations). However, for the NYCWST problem, the ACOA found a feasible solution of $38.64 million, which is lower cost than the lowest cost feasible solution of $38.80 million reported in the literature obtained using a GA. In addition, the ACOA was found to be significantly more computationally efficient, taking 13,928 evaluations on average to find its best solution compared with 96,750 evaluations for the GA. It should be noted that lower cost solutions to this problem have been reported in the literature. However, these solutions are infeasible as they violate the minimum pressure constraints when analyzed using *EPANET* version 2.0.

Future research efforts in this field should focus on (1) the application of ACOAs to other WDS optimization problems; (2) investigations into of the sensitivity of ACOAs to the parameters that control their operation (e.g., α, β), with the aim of developing guidelines for users; (3) investigations into the effect of alternative penalty cost formulations; (4) the development of formulations that enable ACOAs to be applied to other water resources optimization problems; (5) the determination of which problems can be efficiently solved by ACOAs (Stützle and Hoos 2000) and under what circumstances ACOAs should be used in preference to GAs and vice versa; and (6) the evaluation of the effectiveness of algorithmic improvements to ACOAs (e.g., Stützle and Hoos 2000) for water resources problems.

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Notation

The following symbols are used in this paper:

- $\mathbf{C}$ = set of costs associated with each decision made;
- C_{HW} = Hazen–Williams coefficient;
- $c_{i(j)}$ = cost associated with option $l_{i(j)}$;
- $\mathbf{D}$ = set of points at which decisions have to be made;
- D_p = pipe diameter;
- d_i = decision point i
- $f(\varphi)$ = cost of trial solution;
- $f(\varphi)^k$ = cost of trial solution generated at cycle k;
- $f(\varphi^*)$ = cost of optimal solution;
- G = graph;
- h_f = head loss;
- k = cycle number;
- k^* = cycle that results in best solution during iteration t;
- $\mathbf{L}$ = set of options available at each decision point;
- LE = length of pipe;
- LE_i = length of pipe segment i;
- $l_{i(j)}$ = option j at decision point i;
- n = number of decision points, number of pipes;
- P_{pher} = pheromone penalty factor;
- PC = penalty cost multiplier;
- $p_{i(j)}(k,t)$ = probability that option $l_{i(j)}$ is chosen at cycle k and iteration t;
- R = pheromone reward factor;
- R_H = hydraulic radius;
- S_f = friction slope;
- T = time;

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t = iteration number;
UD = unit distance;
UC_{ϕ_j} = unit cost per meter length for pipe with diameter ϕ_j;
V = velocity;
α = parameter controlling relative importance of pheromone;
β = parameter controlling relative importance of local heuristic factor;
$\Delta H_{\max}$ = maximum pressure deficit;
$\Delta\tau_{i(j)}$ = change in pheromone concentration associated with option $l_{i(j)}$;
$\Delta\tau^k_{i(j)}$ = change in pheromone concentration associated with option $l_{i(j)}$ at cycle k;
$\eta_{i(j)}$ = heuristic factor favoring options that have smaller ''local'' costs;
ρ = pheromone persistence coefficient;
τ = pheromone concentration;
$\tau_{i(j)}(t)$ = pheromone concentration associated with option $l_{i(j)}$ at iteration t;
$\tau_{i(j)}(k+1)$ = pheromone concentration associated with option $l_{i(j)}$ at iteration $t+1$;
ϕ_j = available pipe diameters;
φ = trial solution, feasible path over G;
φ^* = optimal solution, optimal path over G; and
$\Omega(\mathbf{D},\mathbf{L})$ = set of finite constraints assigned over elements of **D** and **L**.

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