MA 691: Statistical Simulation and Data Analysis
Term Project - 1
Results

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Data Analysis using the EM Algo as given in the paper: "Statistical Inference for a New Class of Multivariate Pareto Distributions" by Alexandru et al.

The results of all the trials are tabulated below:

0.1 For parameter: N = 5000, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1798.7

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.96972	0.43393	0.56117	0.00012	0.00004	1.40406	0.50694
MSE	0.00778	0.00717	0.01232	0.00000	0.00000	0.00956	0.00136

0.2 For parameter: N = 5000, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 532.16

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.98085	1.01270	1.42828	0.00015	0.00004	1.39380	0.50361
MSE	0.00204	0.00483	0.00891	0.00000	0.00000	0.00742	0.00090

0.3 For parameter: N = 5000, $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1151.36

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.94363	1.30207	1.46015	1.00003	2.00004	0.40698	0.50042
MSE	0.01409	0.03793	0.02442	0.00000	0.00000	0.00105	0.00169

0.4 For parameter: N = 5000, $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 926.36

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.97949	0.32483	1.40673	0.00014	0.00004	1.00485	0.49595
MSE	0.00178	0.00199	0.00871	0.00000	0.00000	0.00237	0.00126

0.5 For parameter: N = 4000, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1794.94

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.98192	0.43185	0.54351	0.00015	0.00005	1.40896	0.50551
MSE	0.01061	0.00494	0.00949	0.00000	0.00000	0.01052	0.00189

0.6 For parameter: N = 4000, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 531.48

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.97356	1.02311	1.44156	0.00019	0.00006	1.40048	0.50225
MSE	0.00309	0.00665	0.01620	0.00000	0.00000	0.00853	0.00136

0.7 For parameter: N = 4000, $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1171.62

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.94370	1.30046	1.43634	1.00004	2.00004	0.40916	0.49598
MSE	0.01836	0.04605	0.01892	0.00000	0.00000	0.00180	0.00156

0.8 For parameter: N = 4000, $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1894.8

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.97296	0.33504	1.41345	0.00021	0.00005	1.00584	0.49545
MSE	0.00301	0.00369	0.00990	0.00000	0.00000	0.00335	0.00154

0.9 For parameter: N = 3000, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1857.18

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.95833	0.44064	0.52896	0.00022	0.00007	1.39929	0.49592
MSE	0.01153	0.00973	0.01093	0.00000	0.00000	0.01139	0.00162

0.10 For parameter: N = 3000, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 535.8

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.98368	1.02332	1.43653	0.00024	0.00008	1.41120	0.50830
MSE	0.00235	0.00525	0.01175	0.00000	0.00000	0.00774	0.00113

0.11 For parameter: N = 3000, $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 11148.1

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.91564	1.25688	1.51406	1.00004	2.00005	0.39550	0.50464
MSE	0.02022	0.02505	0.07241	0.00000	0.00000	0.00131	0.00257

0.12 For parameter: N = 3000, $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 887.16

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.97343	0.31736	1.42278	0.00031	0.00008	0.98522	0.49807
MSE	0.00284	0.00224	0.01552	0.00000	0.00000	0.00374	0.00177

0.13 For parameter: N = 2000, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1810.04

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.96631	0.47190	0.52527	0.00034	0.00010	1.42693	0.49713
MSE	0.02645	0.01669	0.00934	0.00000	0.00000	0.02423	0.00323

0.14 For parameter: N = 2000, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 522.24

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.95594	1.07265	1.40799	0.00031	0.00009	1.41867	0.49012
MSE	0.00574	0.02263	0.02098	0.00000	0.00000	0.01956	0.00242

0.15 For parameter: N = 2000, $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1227.9

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.96767	1.30879	1.53919	1.00005	2.00007	0.41059	0.51735
MSE	0.01554	0.06234	0.07882	0.00000	0.00000	0.00155	0.00276

0.16 For parameter: N = 2000, $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 914.32

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.96846	0.31997	1.43379	0.00034	0.00009	0.98432	0.49973
MSE	0.00357	0.00215	0.02253	0.00000	0.00000	0.00640	0.00236

0.17 For parameter: N = 1000, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1821.4

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.90956	0.43422	0.61967	0.00051	0.00020	1.35270	0.50661
MSE	0.04836	0.01562	0.05716	0.00000	0.00000	0.04344	0.00683

0.18 For parameter: N = 1000, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 549.84

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.97878	1.07073	1.49395	0.00071	0.00018	1.46347	0.52483
MSE	0.00816	0.03104	0.08129	0.00000	0.00000	0.05189	0.00859

0.19 For parameter: N = 1000, $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1379.36

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.91956	1.27014	1.69893	1.00010	2.00011	0.39339	0.53478
MSE	0.07128	0.05627	0.38646	0.00000	0.00000	0.00365	0.01516

0.20 For parameter: N = 1000, $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 841.34

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.95270	0.36797	1.47673	0.00060	0.00029	1.02770	0.50857
MSE	0.00961	0.01397	0.06767	0.00000	0.00000	0.01362	0.00680

0.21 For parameter: N = 500, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1859.26

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.95618	0.46742	0.73620	0.00117	0.00039	1.42407	0.55308
MSE	0.07022	0.03127	0.14380	0.00000	0.00000	0.06179	0.01992

0.22 For parameter: N = 500, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 533.7

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.90050	1.14344	1.50343	0.00109	0.00037	1.45169	0.49413
MSE	0.02772	0.07866	0.12047	0.00000	0.00000	0.08810	0.01221

0.23 For parameter: N = 500, $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1604.9

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.86473	1.55816	1.71571	1.00023	2.00025	0.43426	0.53351
MSE	0.13336	0.45512	1.03595	0.00000	0.00000	0.01558	0.04851

0.24 For parameter: N = 500, $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 819.12

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.93768	0.35206	1.50287	0.00146	0.00043	0.97830	0.51588
MSE	0.01456	0.01088	0.11049	0.00000	0.00000	0.02300	0.01056

0.25 For parameter: N = 450, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1860.64

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.91070	0.52748	0.57160	0.00158	0.00047	1.45425	0.49844
MSE	0.05030	0.05707	0.03983	0.00000	0.00000	0.07447	0.00563

0.26 For parameter: N = 450, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 607.06

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.90767	1.11320	1.68839	0.00151	0.00052	1.41224	0.54954
MSE	0.02390	0.09853	0.29713	0.00000	0.00000	0.07482	0.02072

0.27 For parameter: N = 450, $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1597.92

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.82057	1.56514	1.82241	1.00028	2.00033	0.42783	0.54503
MSE	0.15430	0.72119	0.91778	0.00000	0.00000	0.01829	0.04252

0.28 For parameter: N = 450, $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 822.22

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.93220	0.38246	1.58845	0.00146	0.00039	1.02558	0.53575
MSE	0.01619	0.02312	0.27038	0.00000	0.00000	0.02446	0.02371

0.29 For parameter: N = 350, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1985.84

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.95681	0.63506	0.66958	0.00128	0.00045	1.54458	0.52922
MSE	0.11245	0.19393	0.17753	0.00000	0.00000	0.15812	0.02200

0.30 For parameter: N = 350, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 659.52

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.95044	1.11060	1.65182	0.00284	0.00066	1.46801	0.54903
MSE	0.03938	0.14395	0.55777	0.00002	0.00000	0.16724	0.04801

0.31 For parameter: N = 350, $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 2238.7

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.85857	1.71729	1.93863	1.00037	2.00038	0.45526	0.56246
MSE	0.19776	1.40713	2.60180	0.00000	0.00000	0.04631	0.09872

0.32 For parameter: N = 350, $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 886.06

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.91284	0.35202	1.47468	0.00229	0.00046	0.94811	0.48738
MSE	0.02569	0.01957	0.13425	0.00002	0.00000	0.03913	0.01322

0.33 For parameter: N = 250, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1818.86

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.89482	0.68652	0.77714	0.00157	0.00079	1.55876	0.54650
MSE	0.17735	0.29100	0.36991	0.00000	0.00000	0.35973	0.04611

0.34 For parameter: N = 250, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 612.3

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.89139	1.11346	1.53932	0.00216	0.00060	1.38735	0.49828
MSE	0.06835	0.15333	0.32457	0.00001	0.00000	0.17258	0.02681

0.35 For parameter: N = 250, $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 3001.34

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.82182	1.58229	2.31611	1.00042	2.00069	0.42448	0.63001
MSE	0.24033	1.24484	5.78014	0.00000	0.00000	0.04144	0.21566

0.36 For parameter: N = 250, $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 978.62

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.95464	0.38226	1.51248	0.00296	0.00087	1.02160	0.51108
MSE	0.01838	0.04626	0.25408	0.00002	0.00000	0.04718	0.02585

0.37 For parameter: N = 150, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 2084.24

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.92345	0.66563	0.64336	0.00328	0.00108	1.51346	0.51486
MSE	0.24887	0.42604	0.17580	0.00002	0.00000	0.31287	0.02869

0.38 For parameter: N = 150, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 991.78

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.98316	1.36645	1.81702	0.00512	0.00118	1.81903	0.61865
MSE	0.10438	1.16796	0.87107	0.00006	0.00000	1.56462	0.11024

0.39 For parameter: N = 150, $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 7382.14

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	1.81386	2.09745	2.93554	1.00071	2.00096	0.50445	0.73492
MSE	0.46035	6.82995	18.62076	0.00000	0.00000	0.12581	0.56824

0.40 For parameter: N = 150, $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

Average number of iterations (AI) = 1354.5

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.96768	0.44585	1.82548	0.00551	0.00129	1.10858	0.59932
MSE	0.04027	0.12275	0.91930	0.00006	0.00000	0.15461	0.07822