

MA 691: Statistical Simulation and Data
Analysis
Results

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Data Analysis: “Statistical Inference for a New Class of Multivariate Pareto Distributions” by Alexandru et al.

In this analysis, the expected log-likelihood is taken to be the stopping criteria for all the iterations of the Expectation-Maximation (EM) Algorithm :

$$\begin{aligned} Q(\alpha, \alpha_0; x_1, \dots, x_m, \alpha^{(k)}, \alpha_0^{(k)}) \\ \propto m * \ln(\alpha_0 \alpha_1 \alpha_2) - \alpha_0 \left(\sum_{i=1}^m \ln(1 + z_{(2)i}^{(k)}) + \frac{\alpha_2^{(k)} w_2^{(k)}}{\alpha_0^{(k)} \alpha_{02}^{(k)}} + \frac{\alpha_1^{(k)} w_1^{(k)}}{\alpha_0^{(k)} \alpha_{01}^{(k)}} \right) \\ - \sum_{j=1}^2 \alpha_j \left(\sum_{i=1}^m \ln \left(1 + z_{j,i}^{(k)} + \frac{\alpha_0^{(k)} w_j^{(k)}}{\alpha_j^{(k)} \alpha_{0j}^{(k)}} + \frac{w_0^{(k)}}{\alpha_j^{(k)}} \right) \right) \end{aligned}$$

The results of all the trials are tabulated below:

For parameter : N = 500 , $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

No. of iterations = 866.0

For parameter : N = 500 , $\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$

No. of iterations (AI) = 917.3

**For parameter : $N = 500$, $\alpha_0 = 1.0$, $\alpha_1 = 1.0$,
 $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$**

No. of iterations (AI) = 416.02

**For parameter : $N = 500$, $\alpha_0 = 2.0$, $\alpha_1 = 0.4$,
 $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$**

No. of iterations (AI) = 833.16

1 $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

1.1 N = 450

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.93572	0.39066	1.46446	0.00128	0.00044	1.02905	0.50616
MSE	0.01988	0.02811	0.12185	0.00000	0.00000	0.02734	0.01516

1.2 N = 350

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.93300	0.42142	1.65687	0.00242	0.00061	1.05975	0.55259
MSE	0.02123	0.03943	0.30231	0.00002	0.00000	0.03981	0.02894

1.3 N = 250

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.94219	0.38053	1.61029	0.00278	0.00076	1.03565	0.54951
MSE	0.02234	0.03282	0.55713	0.00002	0.00000	0.06323	0.04281

1.4 N = 150

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.93173	0.41856	1.79789	0.00541	0.00126	1.04286	0.57701
MSE	0.05238	0.05854	1.10001	0.00007	0.00000	0.10261	0.09340