MA 691: Statistical Simulation and Data Analysis
Results

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Data Analysis : "Statistical Inference for a New Class of Multivariate Pareto Distributions" by Alexandru et al.

1 Using the Euclidean norm as the stopping criteria (on $\alpha_0, \alpha_1, \alpha_2$) for EM Algorithm

S.No	Parameters	No. of iterations
N = 450	$\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$	859.56
	$\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$	1669.04
	$\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	558.12
	$\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	1824.2
N = 350	$\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$	932.04
	$\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$	1644.08
	$\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	634.34
	$\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	1843.26
N = 250	$\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$	949.46
	$\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$	2216.88

	$\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	861.76
	$\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	1897.72
N = 150	$\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$	1061.46
	$\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$	41350.88
	$\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	718.58
	$\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	1966.34

NOTE: For N=50 the EM Algorithm was not converging

1.1 Parameters : $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

1.1.1 N = 450

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.91200	0.38099	1.43253	0.00122	0.00047	0.98716	0.47251
MSE	0.01923	0.02455	0.09275	0.00000	0.00000	0.02906	0.01018

1.1.2 N = 350

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.94737	0.35374	1.60712	0.00245	0.00063	1.00813	0.54788
MSE	0.01358	0.02397	0.29346	0.00001	0.00000	0.03705	0.02848

1.1.3 N = 250

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.96341	0.37196	1.52900	0.00364	0.00098	1.02310	0.52744
MSE	0.02642	0.02182	0.20270	0.00003	0.00000	0.05023	0.02040

1.1.4 N = 150

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.97515	0.41675	2.07989	0.00521	0.00183	1.07867	0.68065
MSE	0.06218	0.09441	1.89394	0.00004	0.00001	0.12573	0.19376

1.1.5 N = 50

The EM algorithm is not converging for N=50

2 Using stopping criteria $(Q(X;\theta^{(k+1)})-Q(X;\theta^{(k)}))/Q(X;\theta^{(k)})$

In this analysis, the expected log-likelihood is taken to be the stopping criteria for all the iterations of the Expectation-Maximation (EM) Algorithm :

$$Q(\alpha,\alpha_0;x_1,...,x_m,\alpha^{(k)},\alpha_0^{(k)})$$

$$\propto m*ln\Big(\alpha_0\alpha_1\alpha_2\Big)-\alpha_0\Big(\sum_{i=1}^{m}ln(1+z_{(2)i}^{(k)})+\frac{\alpha_2^{(k)}w_2^{(k)}}{\alpha_0^{(k)}\alpha_{02}^{(k)}}+\frac{\alpha_1^{(k)}w_1^{(k)}}{\alpha_0^{(k)}\alpha_{01}^{(k)}}\Big)$$

$$-\sum_{j=1}^{2}\alpha_j(\sum_{i=1}^{m}ln\Big(1+z_{j,i}^{(k)}+\frac{\alpha_0^{(k)}w_j^{(k)}}{\alpha_j^{(k)}\alpha_{0j}^{(k)}}+\frac{w_0^{(k)}}{\alpha_j^{(k)}}\Big)$$

The results of all the trials are tabulated below:

S.No	Parameters	No. of iterations
N = 450	$\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$	877.3
	$\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$	979.48
	$\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	410.54
	$\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	843.78
N = 350	$\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$	935.14
	$\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$	1024.3
	$\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	430.9
	$\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	771.6
N = 250	$\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$	916.56
	$\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$	1598.34
	$\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	419.84
	$\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	920.64
N = 150	$\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$	1668.66
	$\alpha_0 = 2.0$, $\alpha_1 = 1.2$, $\alpha_2 = 1.4$, $\mu_1 = 1.0$, $\mu_2 = 2.0$, $\sigma_1 = 0.4$, $\sigma_2 = 0.5$	4036.78

	$\alpha_0 = 1.0$, $\alpha_1 = 1.0$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	460.06
	$\alpha_0 = 2.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.5$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.4$, $\sigma_2 = 0.5$	1754.94

NOTE : For N=50 the EM algorithm was not converging.

2.1 $\alpha_0 = 1.0$, $\alpha_1 = 0.3$, $\alpha_2 = 1.4$, $\mu_1 = 0.0$, $\mu_2 = 0.0$, $\sigma_1 = 1.0$, $\sigma_2 = 0.5$

2.1.1 N = 450

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.93572	0.39066	1.46446	0.00128	0.00044	1.02905	0.50616
MSE	0.01988	0.02811	0.12185	0.00000	0.00000	0.02734	0.01516

2.1.2 N = 350

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.93300	0.42142	1.65687	0.00242	0.00061	1.05975	0.55259
MSE	0.02123	0.03943	0.30231	0.00002	0.00000	0.03981	0.02894

2.1.3 N = 250

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.94219	0.38053	1.61029	0.00278	0.00076	1.03565	0.54951
MSE	0.02234	0.03282	0.55713	0.00002	0.00000	0.06323	0.04281

2.1.4 N = 150

Value	α_0	α_1	α_2	μ_1	μ_2	σ_1	σ_2
AE	0.93173	0.41856	1.79789	0.00541	0.00126	1.04286	0.57701
MSE	0.05238	0.05854	1.10001	0.00007	0.00000	0.10261	0.09340

2.1.5 N = 50

The EM algorithm is not converging for N=50