CS 631: DATA MANAGEMENT SYSTEMS DESIGN
ASSIGNMENT 4

EXERCISE

Consider a relation R(ABCDEFGHIJ) with the following set of functional dependencies
F = { F → AB, CD → E, C → FG, H → IJ, D → H }
1. Is CDE a superkey of R?

2. Is CDE a key of R?

3. Apply the appropriate algorithm to determine a key for R.

4. Apply the appropriate algorithm to determine all the keys for R.

5. Determine the prime attributes of R.

6. Is R in BCNF?

7. Is R in 3NF?

8. Determine whether the decomposition **D** = { CDE, CFG, DH, HIJ, FAB } has (i) the dependency preservation property and (ii) the lossless join property, with respect to **F**. Also determine which normal form each relation in the decomposition is in.

Please justify (briefly) your answers. Answers without justification will not get any points.

1. Is CDE a superkey of R?

Yes. Since CDE^+ = ABCDEFGHIJ = R
2. Is CDE a key of R?

CD is a proper subset of CDE (CD ⊂ CDE).
CD is a superkey of R: CD^+ = R.
Since a proper superset of CDE is a superkey of R, CDE is not a key of R.
3. Apply the appropriate algorithm to determine a key for R.

C and D do not appear in the RHS of any FD in **F**. Therefore, every key of R must contain C and D. We will not try to remove them from K.
Initially K = R.
If $(K – X)^+$ = R for some attribute X in K – {CD} then we remove X from K; otherwise, we leave X in K and continue with another attribute in K – {CD}, until all attributes in K – {CD} are considered:
We try to remove E from R: (CDFGHIJAB)+ = R. ∴ K := CDFGHIJAB
We try to remove F from R: (CDGHIJAB)+ = R. ∴ K := CDGHIJAB

We try to remove G from R: (CDHIJAB)+ = R. ∴ K := CDHIJAB
We try to remove H from R: (CDIJAB)+ = R. ∴ K := CDIJAB
We try to remove I from R: (CDJAB)+ = R. ∴ K := CDJAB
We try to remove J from R: (CDAB)+ = R. ∴ K = CDAB
We try to remove A from R: (CDB)+ = R. ∴ K := CDB
We try to remove B from R: (CD)+ = R. ∴ K := CD
Thus, CD is a key of R.
4. Apply the appropriate algorithm to determine all the keys for R.

C and D do not appear in the RHS of any FD in **F**. Therefore, every key of R must contain C and D. From question 2, CD is a superkey of R. Thus, CD is the unique key of R.
5. Determine the prime attributes of R.

The prime attribute of R are C and D.
6. Is R in BCNF?

Consider H → IJ in **F**.
H → IJ violates BCNF in R: H is not a superkey of R ($\mathbf{F}^+$ = IJ ≠ R).
R is not in BCNF.
7. Is R in 3NF?

Consider H → IJ in **F**.
H → IJ violates 3NF in R: H is not a superkey of R (H^+ = IJ ≠ R) and neither I nor J is a prime attribute of R.
8. Determine whether the decomposition **D** = { CDE, CFG, DH, HIJ, FAB } has (i) the dependency preservation property and (ii) the lossless join property, with respect to **F**. Also determine which normal form each relation in the decomposition is in.

(i) Yes, since every FD in **F** is applicable to at least one schema in **D**. For instance, CD → E ∈ Π_{CDE}(**F**).
(ii)
We use the property of binary LJ decomposition and the property of repeated LJ decompositions.
Let R1 = CDE, R2 = CFG and R3 = R1 ∪ R2 = CDEFG
Since **F** ⊨ C → FG, **F** ⊨ R1 ∩ R2 → R2 – R1. Thus, the decomposition {R1, R2} of R3 is LJ w.r.t. Π_{R3}(**F**).
Let R4 = DH and R5 = R3 ∪ R4 = CDEFGH
Since **F** ⊨ D → H, **F** ⊨ R3 ∩ R4 → R4 – R3. Thus, the decomposition {R3, R4} of R5 is LJ w.r.t. Π_{R5}(**G**). Therefore, the decomposition {R1, R2, R4} of R5 is LJ w.r.t. Π_{R5}(**G**).
Let R6 = HIJ and R7 = R5 ∪ R6 = CDEFGHIJ
Since **F** ⊨ H → IJ, **F** ⊨ R5 ∩ R6 → R6 – R5. Thus, the decomposition {R5, R6} of R7 is LJ w.r.t. Π_{R7}(**F**). Therefore, the decomposition {R1, R2, R4, R6} of R7 is LJ w.r.t. Π_{R7}(**F**).
Let R8 = FAB. Notice that R7 ∪ R8 = R.

Since $\mathbf{F} \models F \rightarrow AB$, $\mathbf{F} \models R7 \cap R8 \rightarrow R8 - R7$. Thus, the decomposition {R7, R8} of R is LJ w.r.t. **F**. Therefore, the decomposition {R1, R2, R4, R6, R8} of R is LJ w.r.t. **F**.

(iii)

CDE: $\Pi_{CDE}(\mathbf{F}) = \{ CD \rightarrow E \}$. Since CD is a superkey of CDE, CDE is in BCNF.

Similarly we show that the rest of the schemas in **D** are in BCNF.