

# Multi-channel Images derived Priors for Image Restoration and Reconstruction

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# Outline

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**Target:** How to utilize higher-dimensional prior in reconstruction ?

**Motivation:**

1. Network for color-to-grayscale (TIP, INFFUS)
2. Multi-filters guided tensor for image Rec (TMM)

**Works:**

1. Utilizing prior in color space to grayscale IR (TIP)
2. Utilizing self-copy prior in MR Rec (MRM)
3. Utilizing color/multi-coil MR prior to CT Rec (TMI)

**Conclusions**

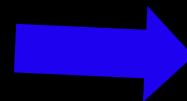
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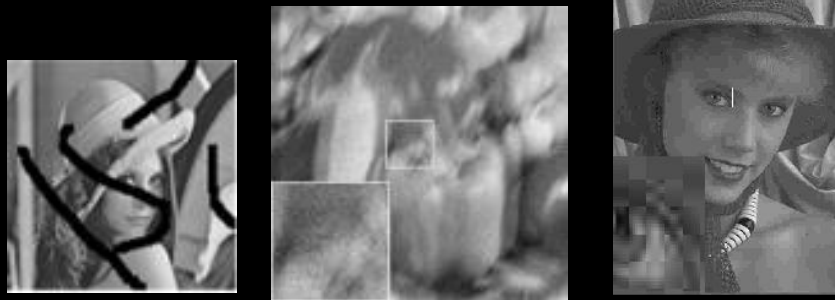
# Target: How to utilize higher-dimensional prior in reconstruction ?



Color images

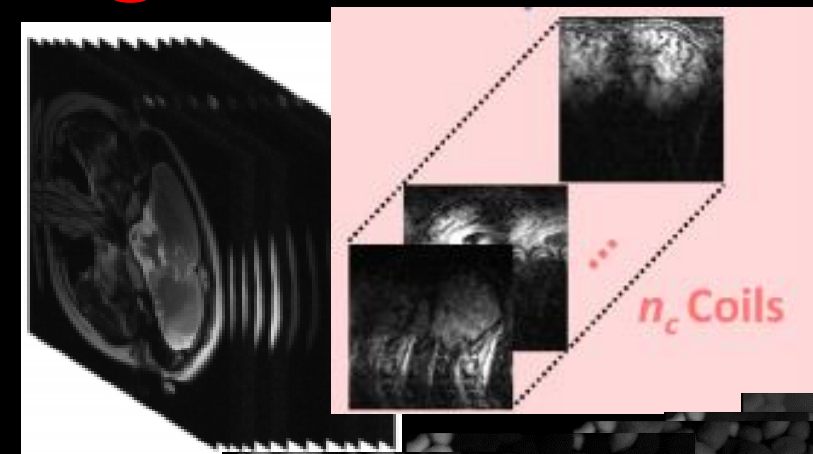


Color prior  
knowledge



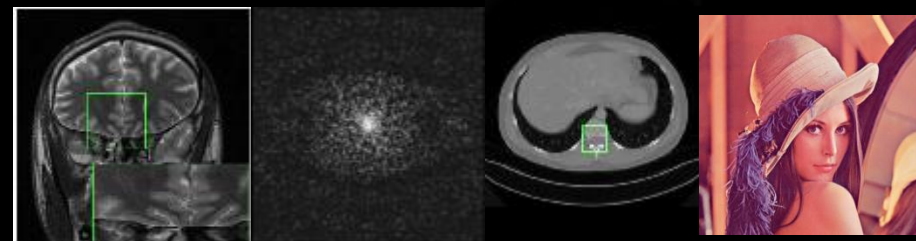
Grayscale images

**Target:** How to utilize higher-dimensional prior in reconstruction ?



Multi-coil MR images, Hyperspectral images, ....

Multi-channel prior knowledge



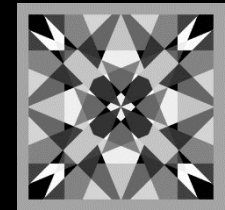
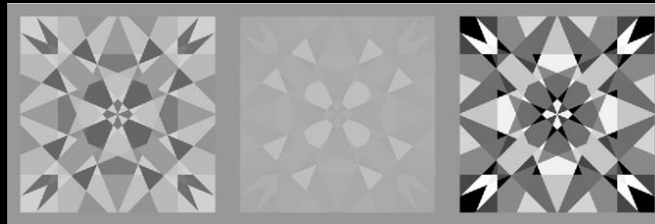
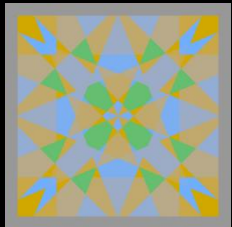
Single-coil MR/CT images, Color images

# **Part I-1 – Motivation** **from** **color-to-grayscale problem**

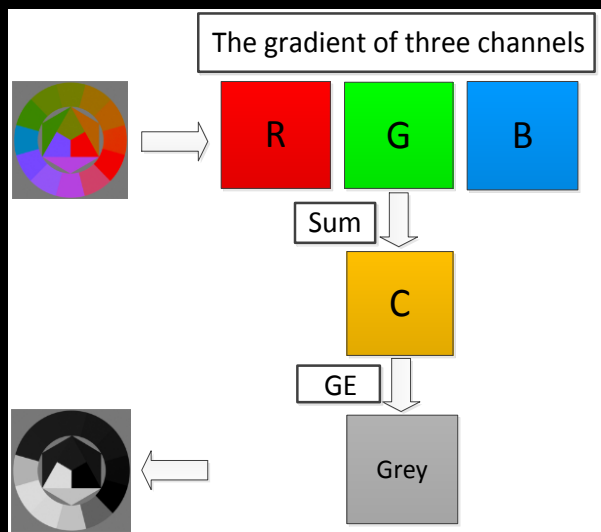


# Decolorization (color-to-grayscale)

color image  $I$     three-channels  $I_r$      $I_g$      $I_b$     single-channel grayscale image  $g$



## Conventional strategies



$$\min_g \sum_{(x,y) \in P} (g_x - g_y - \delta_{x,y})^2, \quad |\delta_{x,y}| = \sqrt{\sum_{c=\{r,g,b\}} (I_{c,x} - I_{c,y})^2}$$

$$\min_g - \sum_{(x,y) \in P} \ln \{ \alpha_{x,y} N_{\sigma}(g_x - g_y + \delta_{x,y}) + (1 - \alpha_{x,y}) N_{\sigma}(g_x - g_y - \delta_{x,y}) \}$$

.....

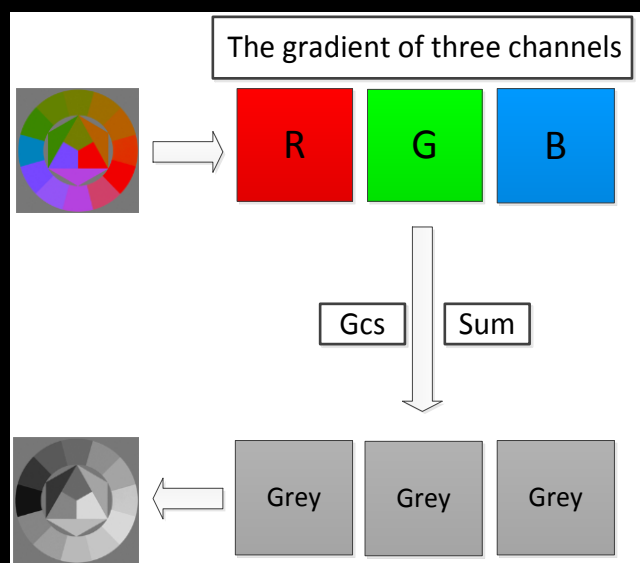
Many of them are based on the gradient error (GE) measure, calculated in the single-channel data space.



Refs: Gooch et al. 2005; Kim et al., 2009; Kuk et al., 2010; Lu et al., 2012; Song et al, 2010; Lu et al, 2012.

# GcsDecolor (Gradient Correlation Similarity)

## Our strategy



$$\min_{w_c} - \sum_{(x,y) \in P} \sum_{c=\{r,g,b\}} \frac{2|I_{c,x} - I_{c,y}| |\nabla g_{x,y}|}{|I_{c,x} - I_{c,y}|^2 + |\nabla g_{x,y}|^2}$$

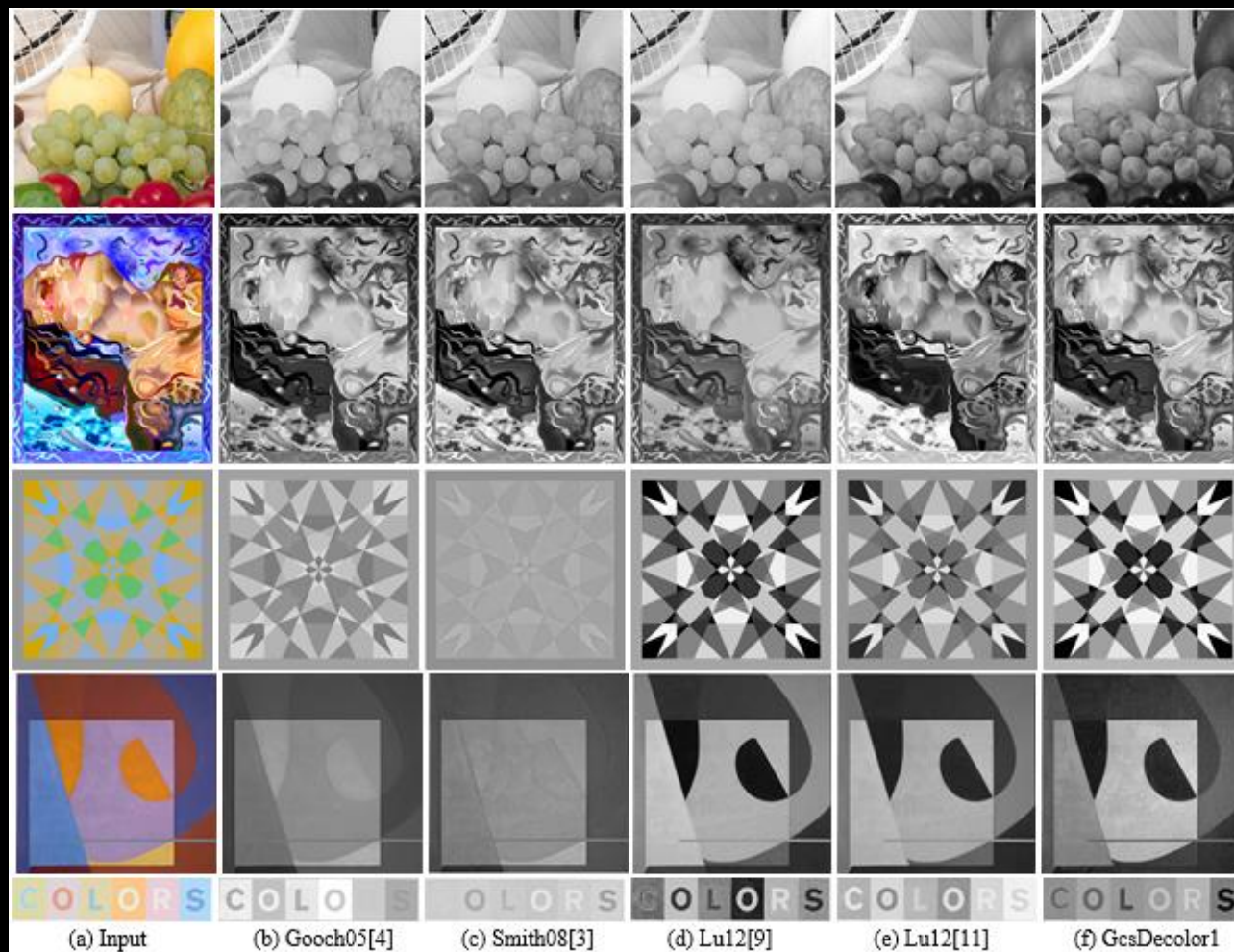
The proposed measure adaptively calculates the gradient correlation for each channel rather than the whole channels at one time. It is calculated in the three-channel data space.

Gcs measure computes the overall pixel-wise similarity between the gradient magnitudes in each channel of the original color image and the resulting grayscale image





# GcsDecolor (Gradient Correlation Similarity)



Some results





# DecolorNet (Network for Decolorization)

Similar to Gcs, we introduce the L1-norm with variable augmentation technique:

Cost function of the network:  $\min_w |F_w(\nabla I) - \nabla G|_1$

where  $I = \{I_R, I_G, I_B\}$  and  $G = \{g, g, g\}$  is an auxiliary variable.

$F_w(\bullet)$  is the network that we need to design and train.

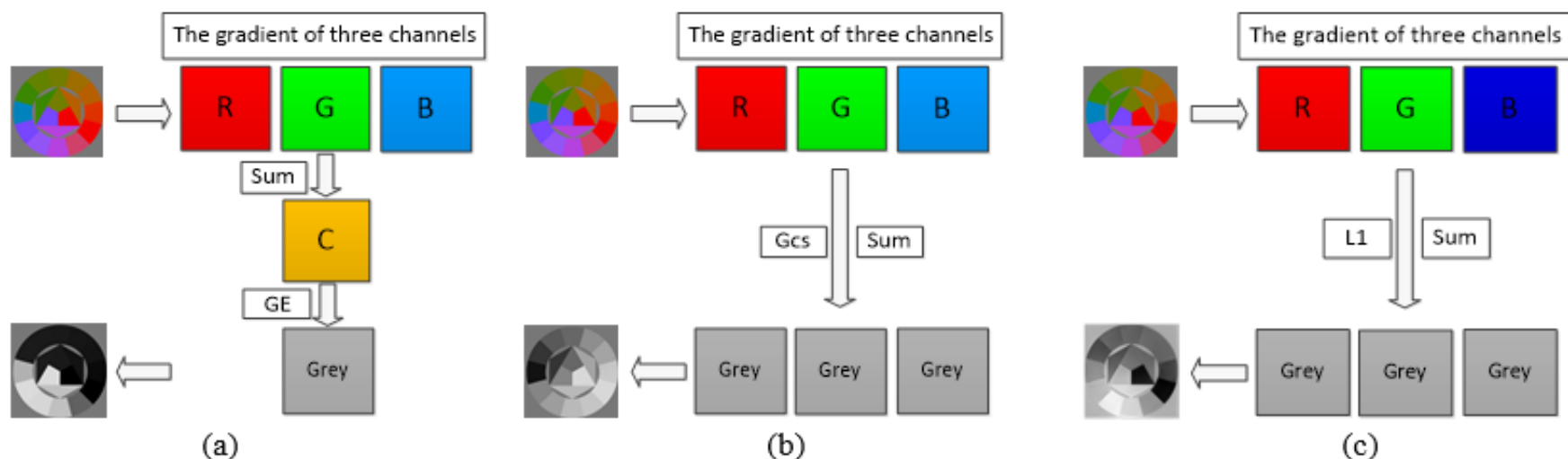
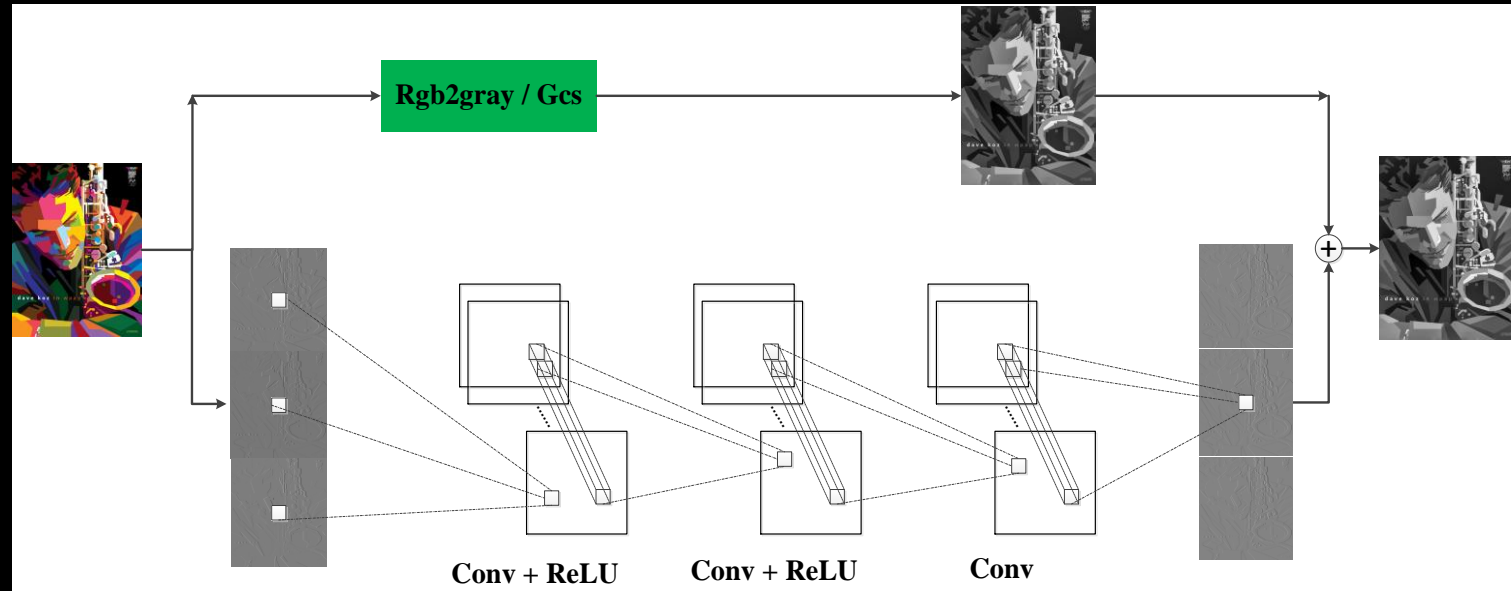


Fig. 10. Example confirming the auxiliary variables technique. (a) GE measure. (b) Gcs measure. (c) Proposed L1-norm measure.



# DecolorNet (Network for Decolorization)

Network design



Demonstration the flowchart of the proposed DecolorNet architecture.

The proposed network is formulated as a shallow and easily-trainable CNN:

$$F_0(\partial I) = \partial I, n = 0$$

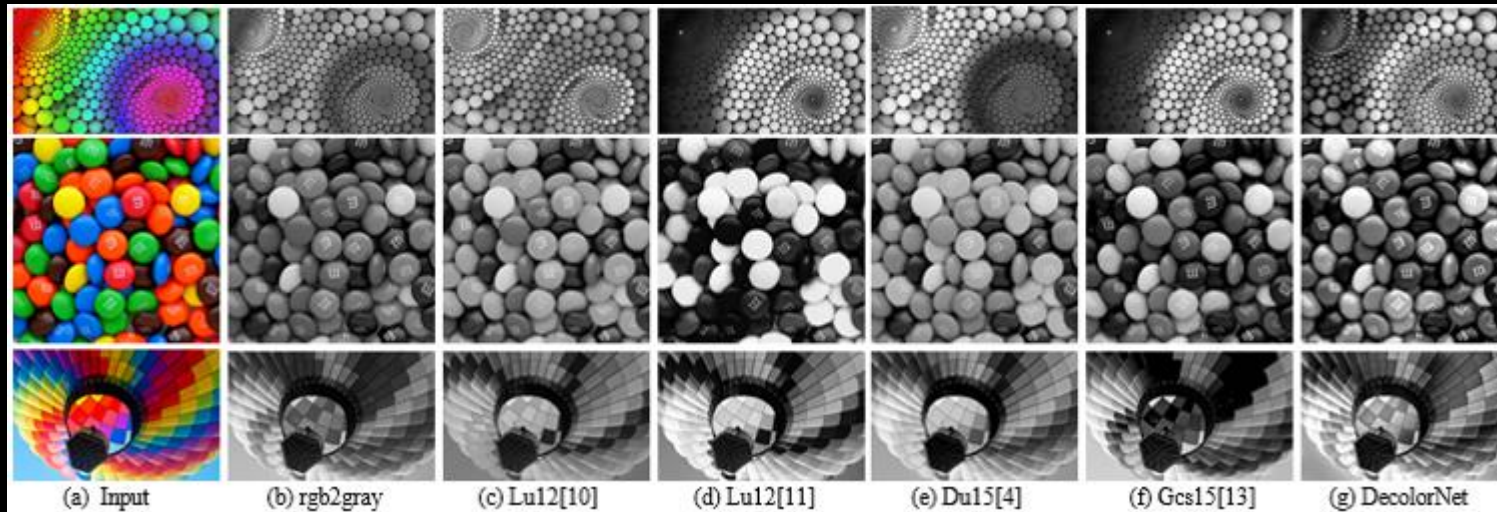
$$F_w^n(\partial I) = \sigma(W^n * F_w^{n-1}(\partial I) + b^n), n = 1, 2$$

$$F_w^n(\partial I) = W^n * F_w^{n-1}(\partial I) + b^n, n = 3$$



**Q. Liu, H. Leung, “Variable augmented neural network for decolorization and multi-exposure fusion,”**  
*Information Fusion*, vol. 46, pp.114-127, 2019.

# DecolorNet (Network for Decolorization)



Decolorization

Multi-exposure  
image fusion:  
FusionNet

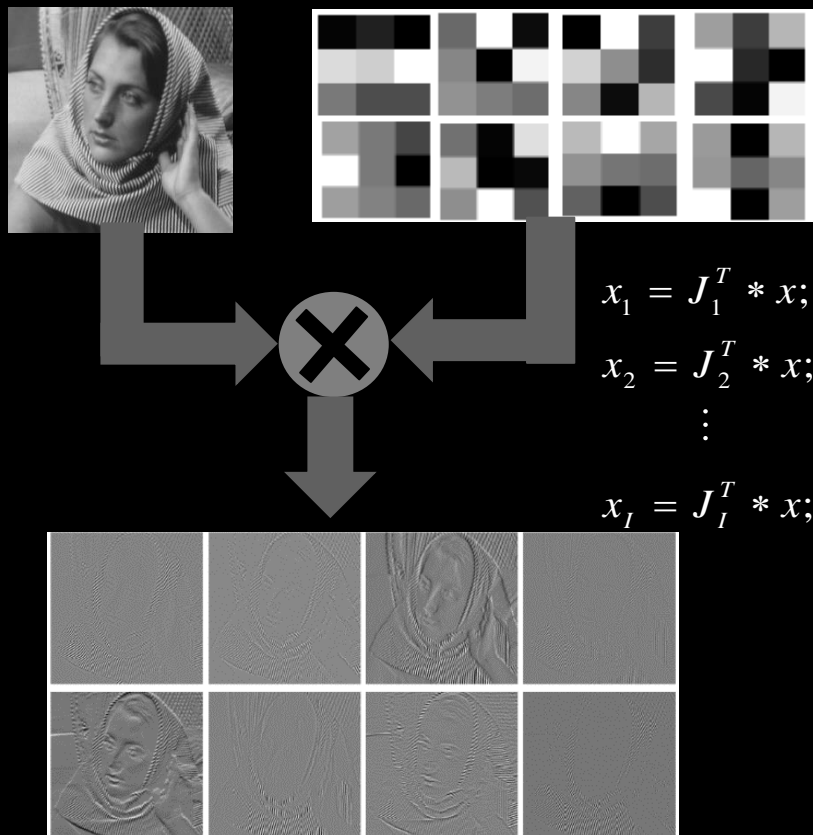


Q. Liu, H. Leung, "Variable augmented neural network for decolorization and multi-exposure fusion," *Information Fusion*, vol. 46, pp.114-127, 2019.

**Part I-2 – Motivation**  
from  
Multi-filters guided tensor  
optimization for image restoration



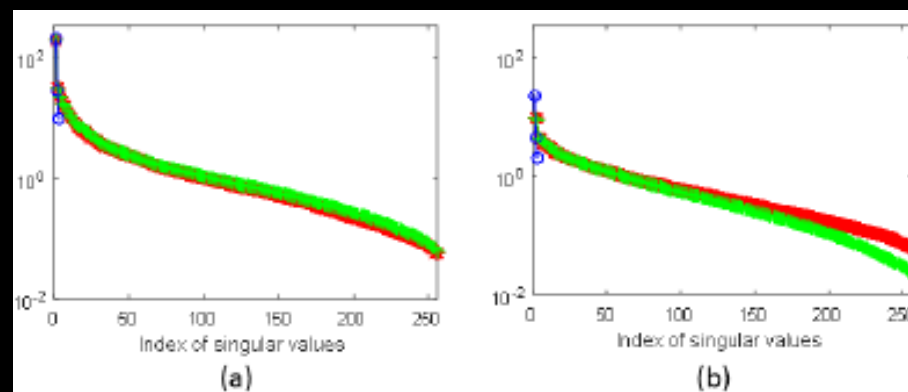
# Field-of-Experts Filters Guided Tensor Completion



**Proposition 1**  $\| [J_1^T, \dots, J_l^T] * \mathcal{X} \|_* \leq \sum_{i=1}^l \| J_i^T * \mathcal{X} \|_*$

**Proposition 2.**  $\| J_i^T * \mathcal{X} \|_* \leq \| \mathcal{X} \|_*, \quad \forall i$

Considering a color image “Barbara”  $\mathcal{X}$  and its one-FoE filter resulting tensor  $J_1^T * \mathcal{X}$



Singular values of three unfolding matrices of (a) the original tensor "Barbara" with size 256×256×3, and (b) the associated feature tensor.

Multi-view features seamlessly contain rich information such as high-frequency information, edges and textures at various orientations and scales.



**Ref. [1]** S. Roth, and M. J. Black, “Fields of Experts,” *IJCV*, 2009.

**[2]** U. Schmidt and S. Roth, “Shrinkage fields for effective image restoration,” in *CVPR*, 2014.

# Field-of-Experts Filters Guided Tensor Completion

## FoE-STDC:

Incorporate the FoE filters into the STDC model (Chen *et al.*, “Simultaneous tensor decomposition and completion using factor priors,” *IEEE TPAMI*, 2014), it yields:

### Algorithm 2 FoE-STDC

```

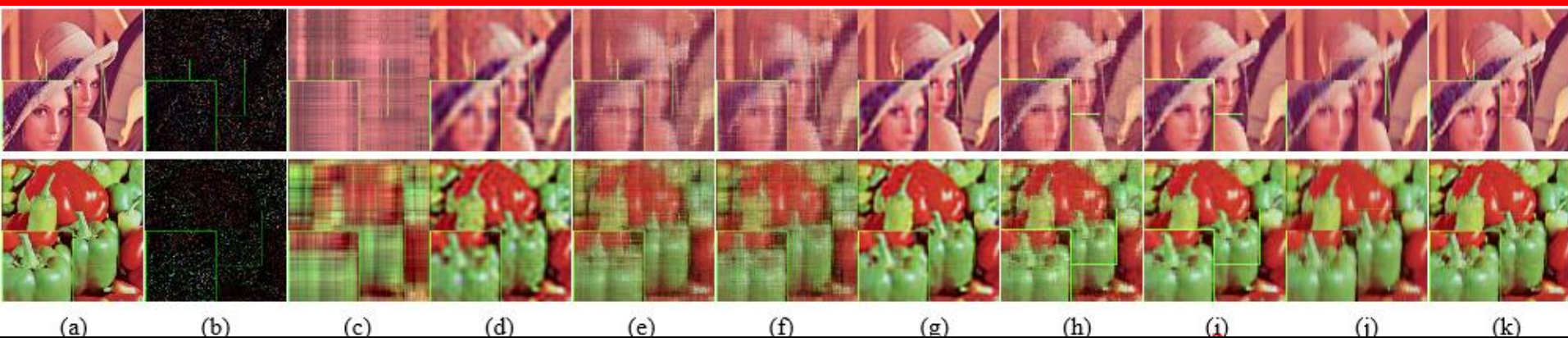
1: Input : an incomplete tensor  $\mathcal{X}_0 \in \mathbb{R}^{I_1 \times I_2 \times \dots \times I_n}$ , matrix  $\mathbf{L}$ 
2: Initialize  $\mathcal{X}, \mathbf{V}_1^1, \dots, \mathbf{V}_n^1, \mathbf{V}_1^2, \dots, \mathbf{V}_n^2, \mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Y}_1, \mathcal{Y}_2, \mathcal{B}, \mathcal{W}$  by
    $\mathbf{V}_k$ : identity matrix ( $1 \leq k \leq n$ ),  $\mathcal{Z} = \mathcal{X} = \mathcal{X}_0$  and  $\mathcal{Y} = 0$ 
3: For  $t = 1, 2, \dots$ , repeat until a stop-criterion is satisfied
4:   update  $\mathbf{V}_k^1$  and  $\mathbf{V}_k^2$  via Eq. (19)
5:   update  $\mathcal{Z}_1$  and  $\mathcal{Z}_2$  via Eq. (21)
6:   update  $\mathcal{B}$  via Eq. (23)
7:   update  $\mathcal{X}$  via Eq. (25)
8:   update  $\mathcal{Y}_1$  and  $\mathcal{Y}_2$  via
        $\mathcal{Y}_1^{t+1} = \mathcal{Y}_1^t + \mu_1^t (\mathcal{X} - \mathcal{Z}_1 \times_1 \mathbf{V}_1^{1T} \dots \times_n \mathbf{V}_n^{1T})$ 
        $\mathcal{Y}_2^{t+1} = \mathcal{Y}_2^t + \mu_2^t (\mathcal{B} - \mathcal{Z}_2 \times_1 \mathbf{V}_1^{2T} \dots \times_n \mathbf{V}_n^{2T})$ 
9:   update  $\mathcal{W}$  via
        $\mathcal{W}^{t+1} = \mathcal{W}^t + \lambda^t (\mathcal{B} - [\mathbf{J}_1^T, \dots, \mathbf{J}_I^T] * \mathcal{X})$ 
10:   $\mu_1^{t+1} = \rho \mu_1^t, \mu_2^{t+1} = \rho \mu_2^t, \rho \in [1.1, 1.2]$ 
11: End
12: Output:  $\mathcal{X}, \mathbf{V}_1^1, \dots, \mathbf{V}_n^1, \mathbf{V}_1^2, \dots, \mathbf{V}_n^2, \mathcal{Z}_1, \mathcal{Z}_2$  and  $\mathcal{B}$ 
    
```

$$\begin{aligned}
 & \min_{\mathcal{X}, \mathcal{Z}_1, \mathcal{Z}_2, \mathbf{V}_1^1, \dots, \mathbf{V}_n^1, \mathbf{V}_1^2, \dots, \mathbf{V}_n^2} \sum_{k=1}^n \alpha_k^1 \|\mathbf{V}_k^1\|_* + \beta_1 \text{tr}((\mathbf{V}_1^1 \otimes \dots \otimes \mathbf{V}_n^1) \mathbf{L} (\mathbf{V}_1^1 \otimes \dots \otimes \mathbf{V}_n^1)^T) + \gamma_1 \|\mathcal{Z}_1\|_F^2 \\
 & + \eta \left( \sum_{k=1}^n \alpha_k^2 \|\mathbf{V}_k^2\|_* + \gamma_2 \|\mathcal{Z}_2\|_F^2 + \beta_2 \text{tr}((\mathbf{V}_1^2 \otimes \dots \otimes \mathbf{V}_n^2) \mathbf{L} (\mathbf{V}_1^2 \otimes \dots \otimes \mathbf{V}_n^2)^T) \right) \\
 & \text{s.t. } \mathcal{X} = \mathcal{Z}_1 \times_1 \mathbf{V}_1^{1T} \dots \times_n \mathbf{V}_n^{1T}, [\mathbf{J}_1^T, \dots, \mathbf{J}_I^T] * \mathcal{X} = \mathcal{Z}_2 \times_1 \mathbf{V}_1^{2T} \dots \times_n \mathbf{V}_n^{2T} \text{ and } \Omega(\mathcal{X}) = \Omega(\mathcal{X}_0)
 \end{aligned}$$

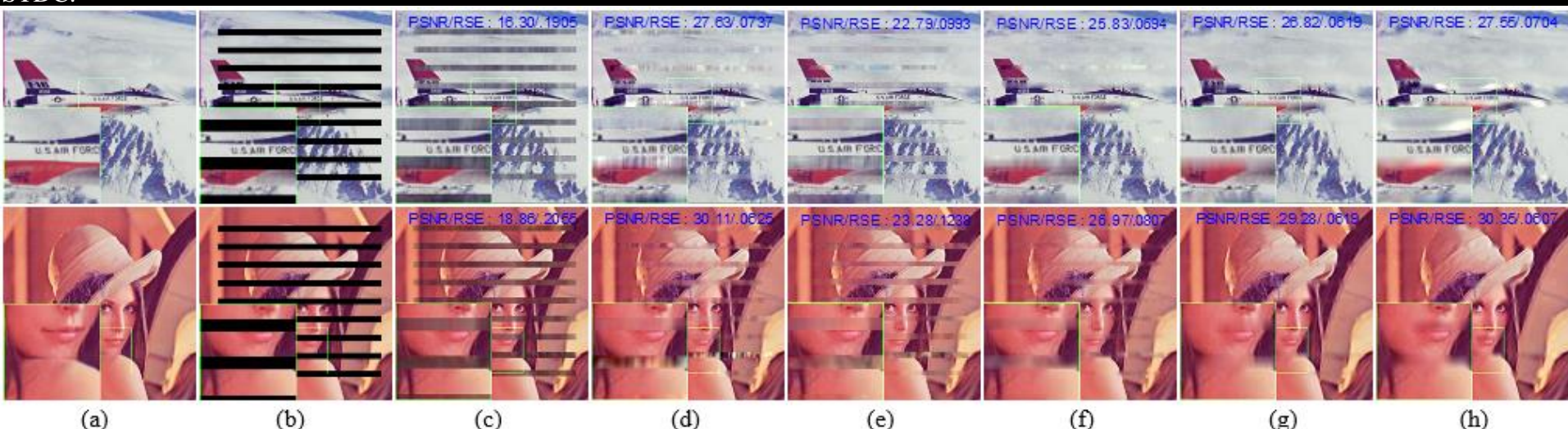




# Field-of-Experts Filters Guided Tensor Completion



(a) Original image. (b) Observed data. (c)  $M^2SA$ . (d)  $M^2SA-G$ . (e) LRTC. (f) HaLRTC. (g) STDC. (h) SPCTV. (i) SPCQV. (j) FoE-LRTC. (k) FoE-STDC.



(a) Original image. (b) Observed data. (c) LRTC. (d) STDC. (e) SPCTV. (f) SPCQV. (g) FoE-LRTC. (h) FoE-STDC.



**Biao Xiong<sup>#</sup>, Qiegen Liu<sup>#</sup>, J. Xiong, S. Li, S. Wang, Dong Liang<sup>\*</sup>**, Field-of-Experts Filters Guided Tensor Completion, *IEEE Transactions on Multimedia*, 20(9): 2316-2329, 2018.

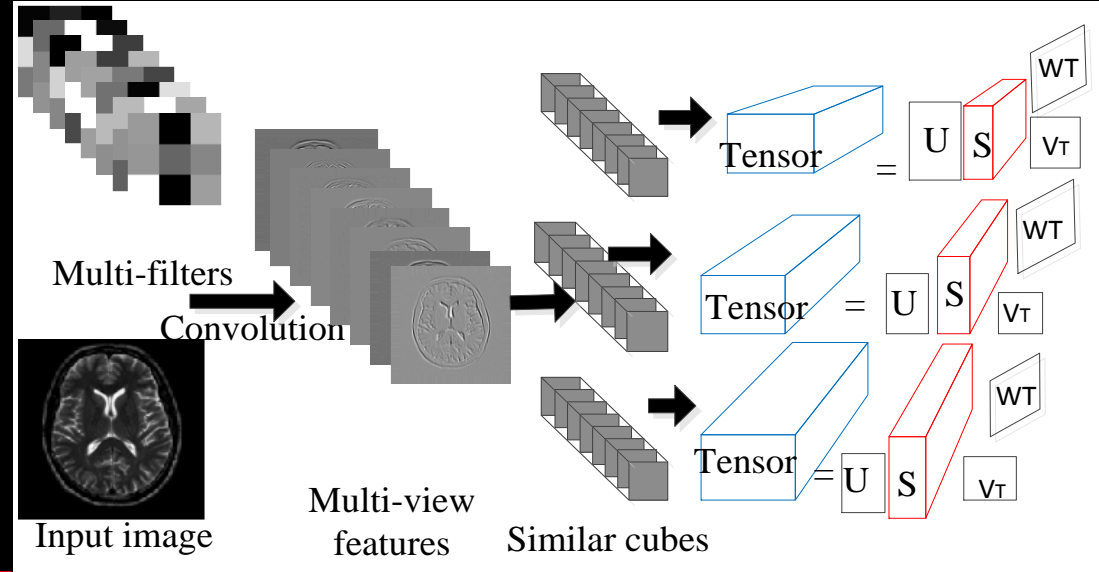
# MF-LRTC (Multi-filters guided low-rank tensor coding)

## Motivation:

- ❑ Reduce the redundancy between feature vectors at neighboring locations.
- ❑ Improve the efficiency of the overall sparse representation.

## Model:

- ❑ Observed data:  $y = Au + n$   
 $A$  is measurement operator and  $n$  is noise.
- ❑ The recovery of  $u$  from  $y$  can be achieved by:



Exploration of multi-filters guided low-rank tensor coding for image restoration

$$\{u, \mathcal{Y}\} =$$

$$\arg \min_{u, \mathcal{Y}} \sum_{i=1}^N \left\{ \frac{1}{2} \left\| \tilde{\mathbf{R}}_i [J_1^T u, \dots, J_K^T u] - \mathcal{Y}_i \right\|_2^2 + \tau \text{rank}(\mathcal{Y}_i) \right\} + \frac{\nu_2}{2} \|y - Au\|_2^2$$



# MF-LRTC (Multi-filters guided low-rank tensor coding)

## Image Deblurring

PSNR results of five methods at Gaussian blur kernel with different noise levels (top line: std=sqrt(1); bottom line: std=2)

Test image	TV	L0-Abs	Bayesian-TV	ASDS-TD2	MF-LRTC
Cameraman	23.08	23.51	22.59	23.90	<b>25.83</b>
	22.93	23.25	22.36	23.76	<b>24.82</b>
Peppers	25.96	26.61	24.94	26.79	<b>27.69</b>
	25.72	26.24	24.38	26.06	<b>26.97</b>

## CS Recovery

Reconstruction PSNR values of four methods at under-sampling percentages with 63%, 73%, and 80%

	Test image	under-sampling ratio	DLMRI	Grad-DL	NLR-CS-base	MF-LRTC
2D Random	T2axialbrain	63%	38.52	43.75	48.72	<b>48.81</b>
		73%	36.77	40.84	45.22	<b>45.43</b>
		80%	35.22	38.42	42.26	<b>42.95</b>
	Herniateddiscspine	63%	39.45	42.21	46.53	<b>47.60</b>
		73%	37.40	39.67	43.28	<b>44.60</b>
		80%	35.64	37.33	39.31	<b>41.47</b>
Pseudo Radial	T2axialbrain	63%	36.68	39.54	42.96	<b>44.01</b>
		73%	34.87	36.65	39.24	<b>39.55</b>
		80%	32.75	33.88	36.31	<b>36.73</b>
	Herniateddiscspine	63%	36.77	37.08	42.83	<b>43.55</b>
		73%	34.92	34.85	38.90	<b>39.12</b>
		80%	33.03	32.30	35.44	<b>35.99</b>



# Part II-1 –Multi-channel and Multi-model based Autoencoding Prior for Grayscale Image Restoration



S. Li, B. Qin, Q. Liu\*, Y. Wang, D. Liang\*, Multi-channel and multi-model based autoencoding prior for grayscale image restoration, *IEEE Trans. Image Process.*, vol. 29, 142-156, 2020.

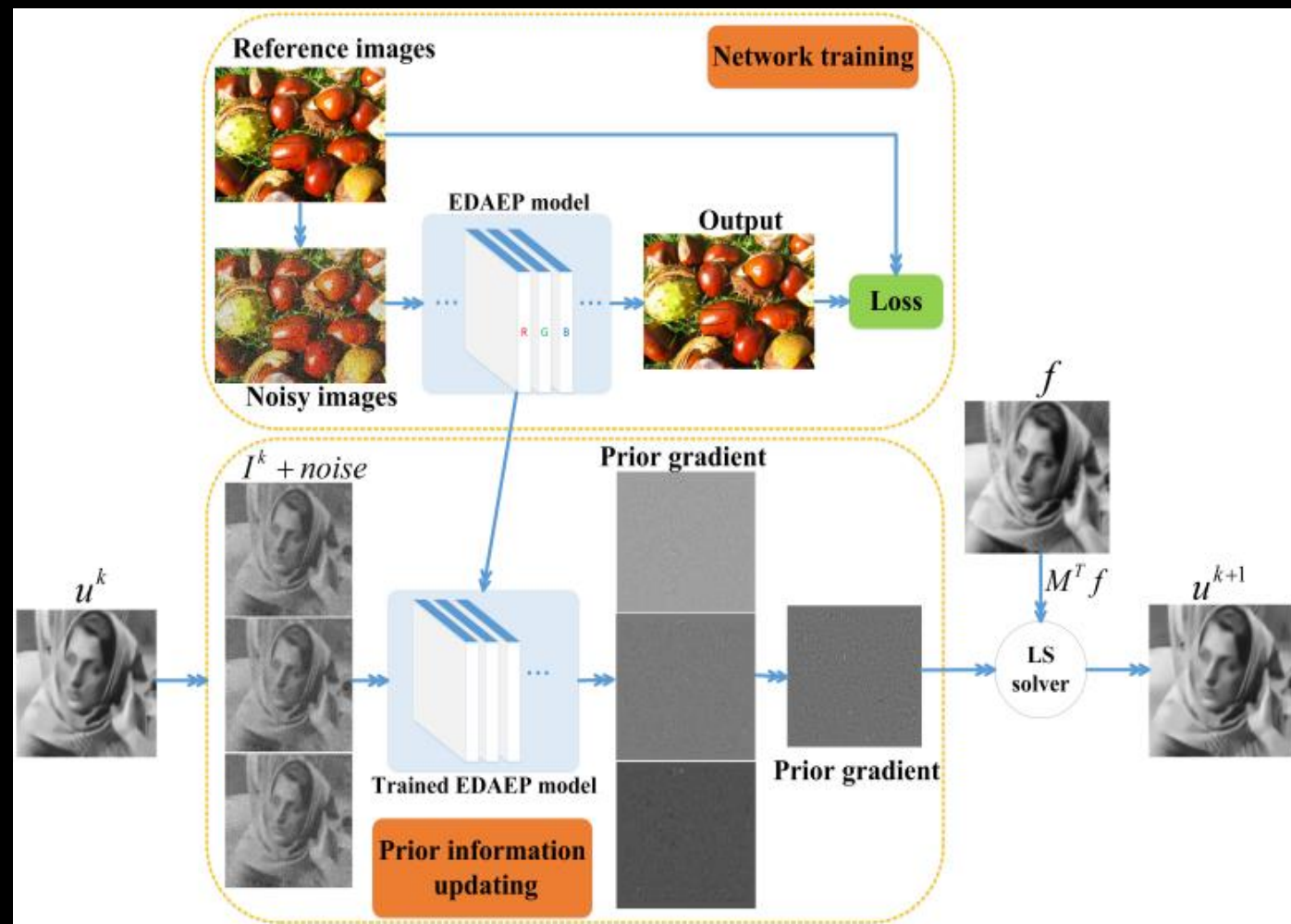


# Algorithm overview

□ Multi-channel learning scheme  $\{I(u) = [u, u, u]\} \subset \{I \mid I = [I_r, I_g, I_b]\}$

**Training images:**

$$\{I \mid I = [I_r, I_g, I_b]\}$$



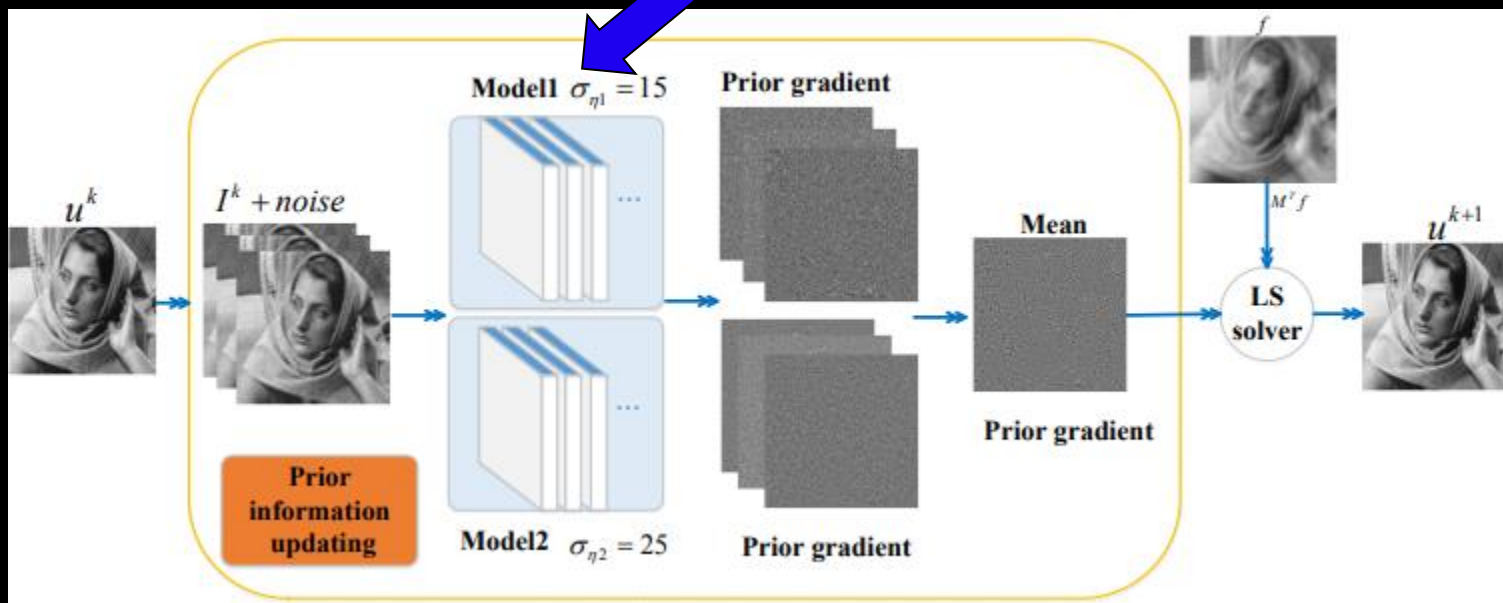
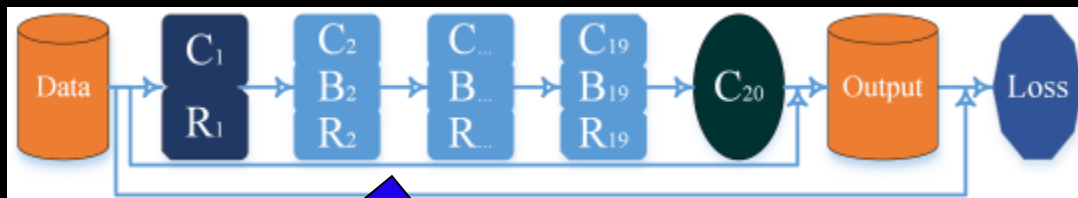
**Testing stage:**

$$I^k = [u^k, u^k, u^k]$$



# Algorithm overview

## Multi-model strategy



Mathematical model for IR is: 
$$\min_u \|Mu - f\|^2 + \frac{\lambda}{N} \sum_{i=1}^N \|I(u) - A_{\sigma_{\eta_i}}(I(u))\|^2, \quad N = 2$$





# Theoretical analysis

- The autoencoder error  $A_{\sigma_\eta}(u) - u$  is proportional to the gradient of the log likelihood of the smoothed:

$$A_{\sigma_\eta}(u) - u = \sigma_\eta^2 \nabla \log[g_{\sigma_\eta} * q](u)$$

where the data distribution is  $Probability(u) = \int q(u + \eta) d\eta$

- The autoencoder error  $A_{\sigma_\eta}(I) - I$  is proportional to the gradient of the log likelihood of the smoothed:

$$A_{\sigma_\eta}(I) - I = \sigma_\eta^2 \nabla \log[g_{\sigma_\eta} * q](I)$$

where the data distribution is  $Probability(I) = \int q(I + \eta) d\eta$

**The effectiveness of prior (i.e., the autoencoder error) depends on the distribution of training data!**



# Experimental results

## Image Deblurring



Fig. 9. Deblurring comparison on image "Barbara". (a) Original image; (b) Noisy and blurred image (Gaussian kernel:  $19 \times 19$ ,  $\delta_e = 2.25$ ); (c) LevinSps (PSNR=30.14dB; SSIM=0.885), (d) EPLL (PSNR=28.21dB; SSIM=0.904), (e) DMSP (PSNR=28.78dB; SSIM=0.897), (f) DPE (PSNR=30.94dB; SSIM=0.889), (g) DAEP (PSNR=28.50dB; SSIM=0.798), (h) EDAEP (PSNR=31.83dB; SSIM=0.910) and (i) MEDAEP (PSNR=31.88dB; SSIM=0.912).

## Image Deblocking

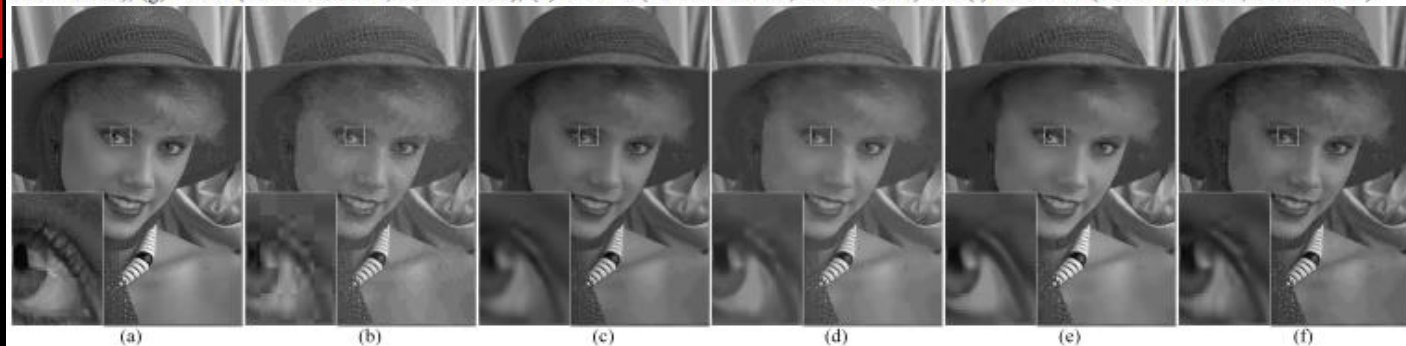


Fig. 13. Visual comparison of image deblocking for "Womanhat" in the case of QF = 10. (a) Original image, (b) JPEG compressed image (30.49, 28.18, 0.772), (c) CONCOLOR (31.89, 0.815, 31.89), (d) ARCNN (31.71, 0.810, 31.59), (e) DnCNN-3 (31.49, 0.803, 31.49), (f) MEDAEP (31.72, 0.817, 31.67).



S. Li, B. Qin, Q. Liu\*, Y. Wang, D. Liang\*, Multi-channel and multi-model based autoencoding prior for grayscale image restoration, *IEEE Trans. Image Process.*, vol. 29, 142–156, 2020.

# Part II-2 –Undersampled MRI Reconstruction using Autoencoding Priors



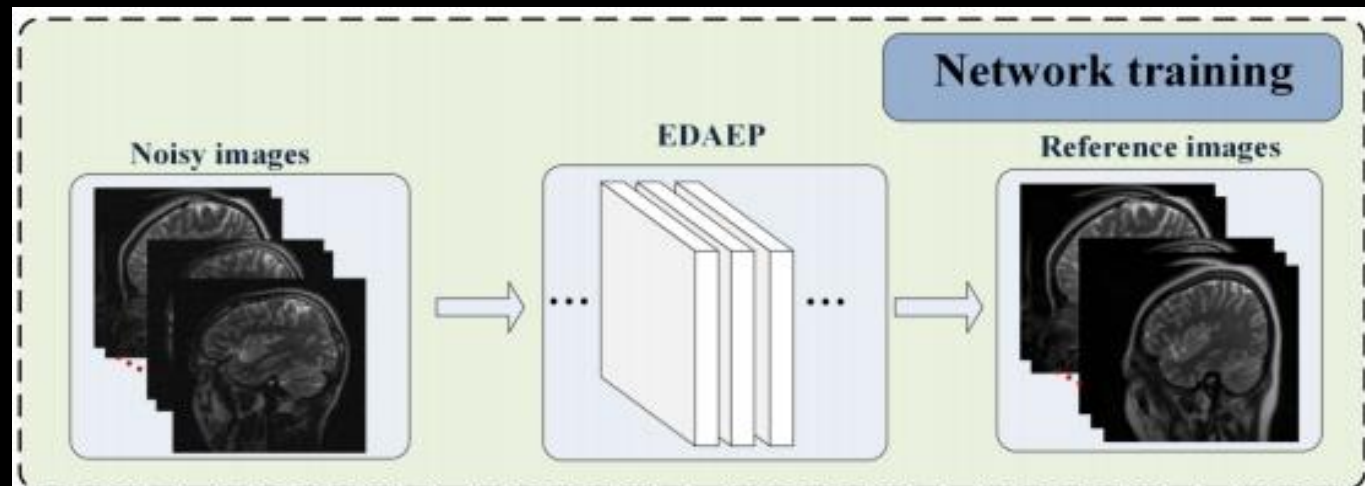
Q. Liu, Q. Yang, H. Cheng, S. Wang, M. Zhang, D. Liang\*, Highly undersampled magnetic resonance imaging reconstruction using autoencoding priors, *Magn. Reson. Med.*, vol. 83, no. 1, pp. 322–336, 2020.

# Algorithm overview

## □ Multi-channel learning scheme (self-copy)

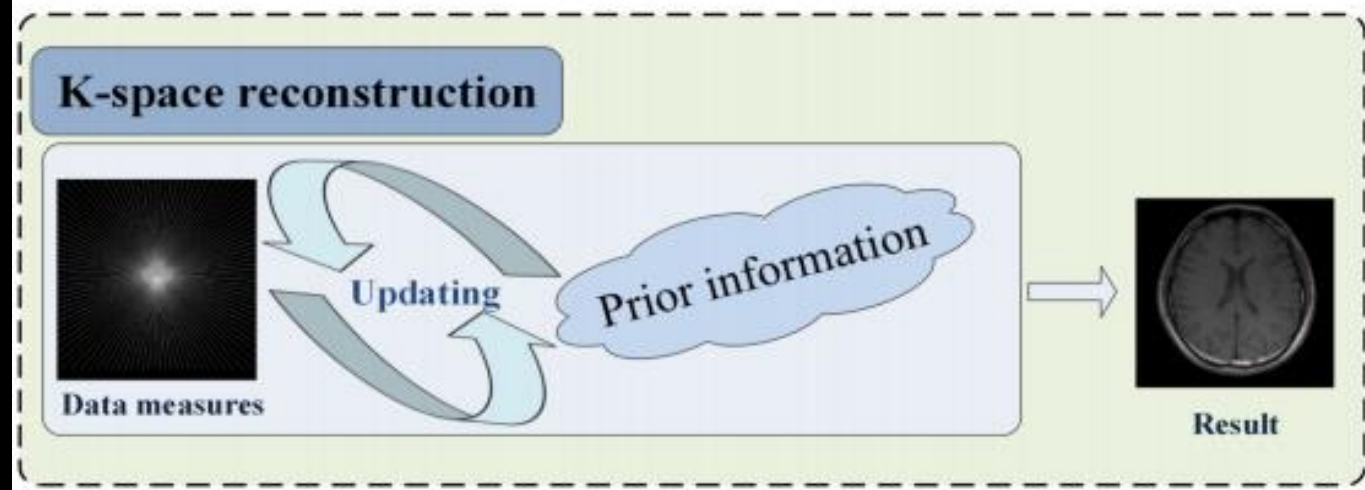
**Training stage:**

$$\{U = [u, u, u]\}$$



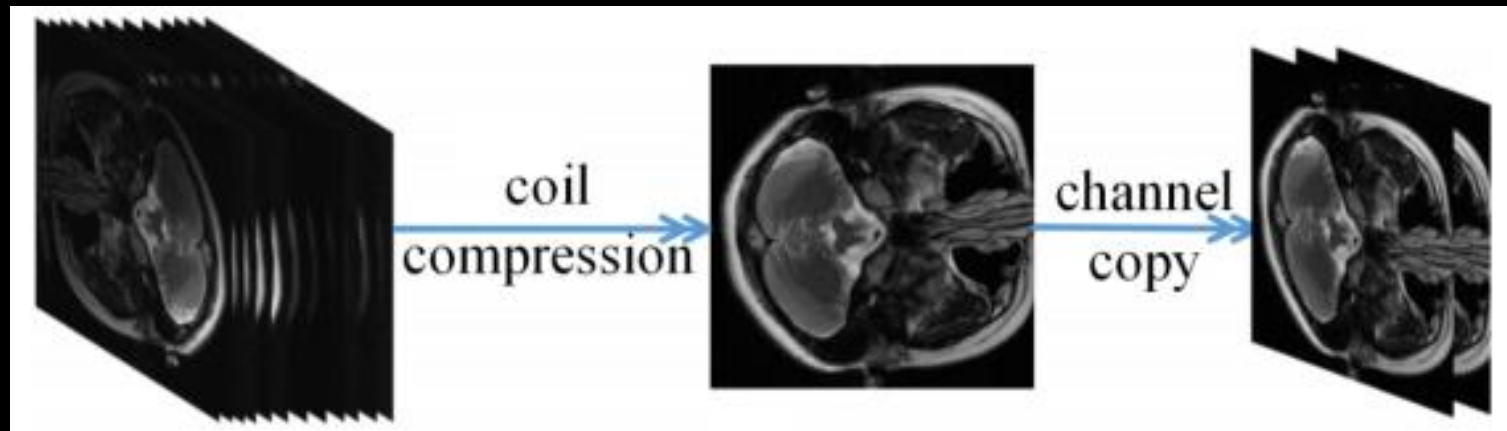
**Testing stage:**

$$U^k = [u^k, u^k, u^k]$$



# Algorithm overview

## □ Multi-channel learning scheme (self-copy)



Schematic illustration of generating the three-channel training data.

Mathematical model for MRI Recon is:

$$\text{Min}_u \frac{1}{2} \|U - D_{\sigma_{\eta_1}}(U)\|^2 + \frac{1}{2} \|U - D_{\sigma_{\eta_2}}(U)\|^2 + \nu \|F_p u - f\|^2$$

We convert the input of EDAEP from  $C^{m \times n \times 3}$  space into  $R^{m \times n \times 6}$  space.



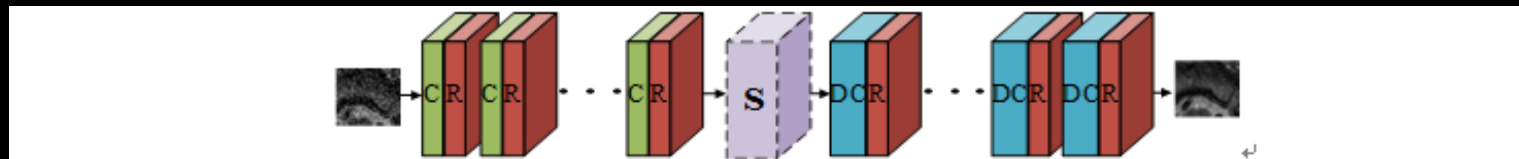
# Theoretical analysis

## Step1: Rate-optimal Bound on Deep-prior based Denoisers

Heckel *et al.* analytically quantified the recovery performance of deep-prior based denoisers. Given a  $d$ -layer generative neural network  $G : R^S \rightarrow R^N$  with  $S < N$  and random weights, the authors presented a gradient method with a tweak that aims to minimize the last-square loss  $\|G(x) - y\|^2$  between the corrupted image  $y$  and the network output  $G(x)$ . They proved that the proposed algorithm yields an estimate  $\hat{x}$  obeying:

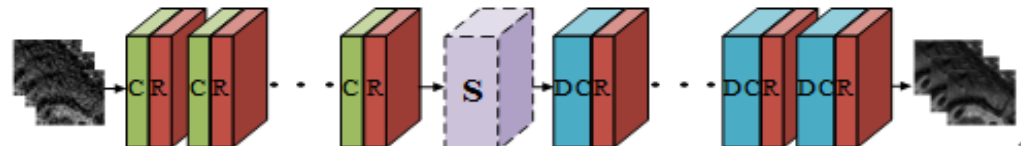
$$\|G(\hat{x}) - y_*\|^2 \lesssim \sigma^2 \frac{S}{N}$$

## Step2: Experimental Comparisons of Learned Prior on 1-channel and 3-channel Data



**Figure S1\_2.** The architecture of DWTA-1 network. Each convolutional/deconvolutional layer is followed by its rectified linear units (ReLU). The channel number is 1 at input and output layer, and 64 for the rest of the layers. C, DC, R and S represent convolutional, deconvolutional, ReLU layers and Sparse, respectively.

Various  $S$ -value



**Figure S1\_3.** The architecture of DWTA-3 network. Each convolutional/deconvolutional layer is followed by its rectified linear units (ReLU). The channel number is 3 at input and output layer, and 64 for the rest of the layers. C, DC, R and S represent convolutional, deconvolutional, ReLU layers and Sparse, respectively.

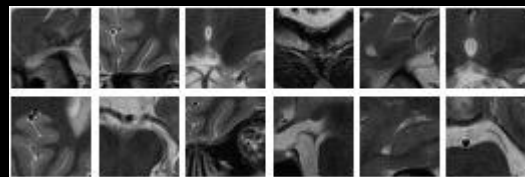
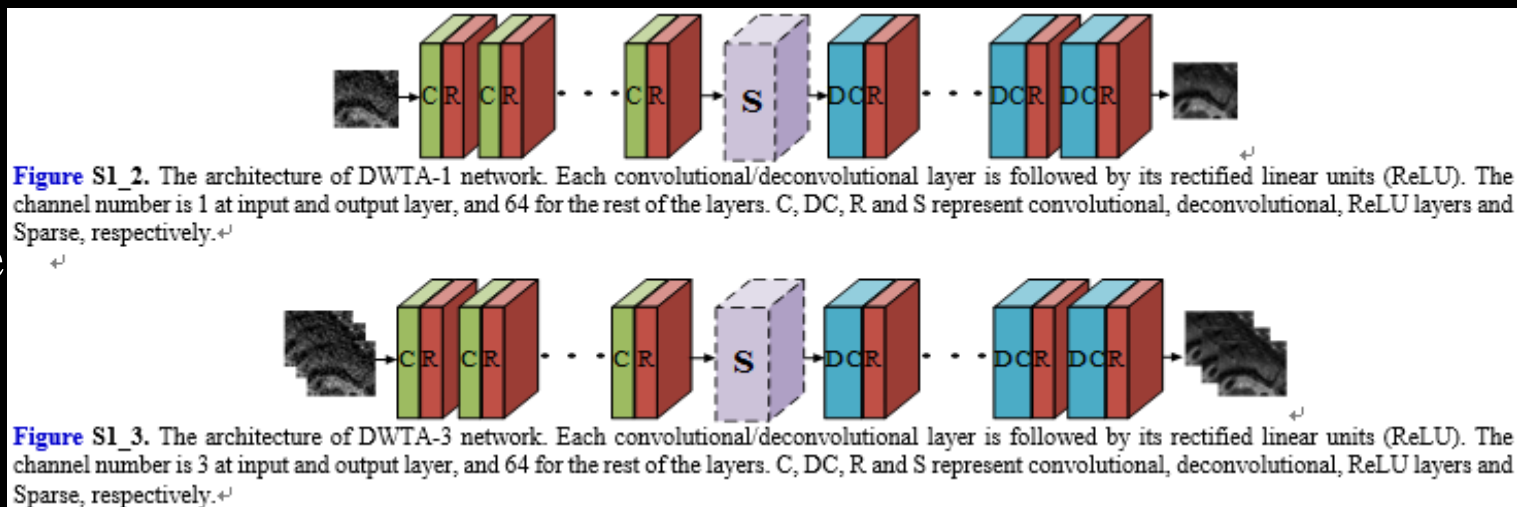




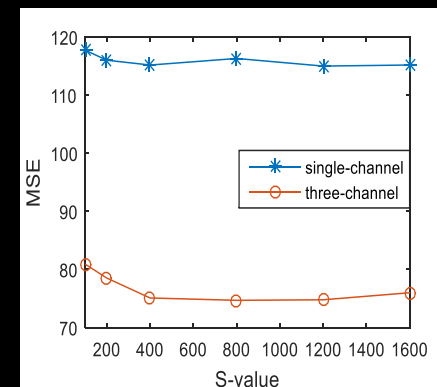
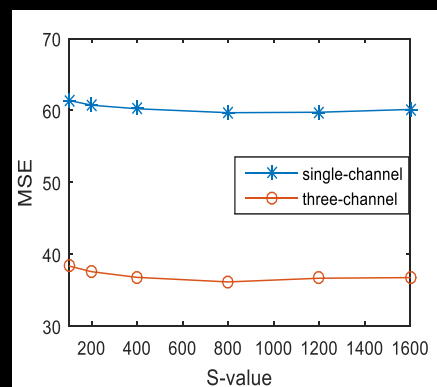
# Theoretical analysis

## Step2: Experimental Comparisons of Learned Prior on 1-channel and 3-channel Data

### Various $S$ -value



Set12 test images used for image recovery from noise corrupted images.



Average MSE values vs.  $S$ -sparse values in DWTA-1 and DWTA-3 network at noise level of (a) 15 and (b) 25 conducted on Set 12.



Q. Liu, Q. Yang, H. Cheng, S. Wang, M. Zhang, D. Liang\*, Highly undersampled magnetic resonance imaging reconstruction using autoencoding priors, *Magn. Reson. Med.*, vol. 83, no. 1, pp. 322-336, 2020.

# Experimental results

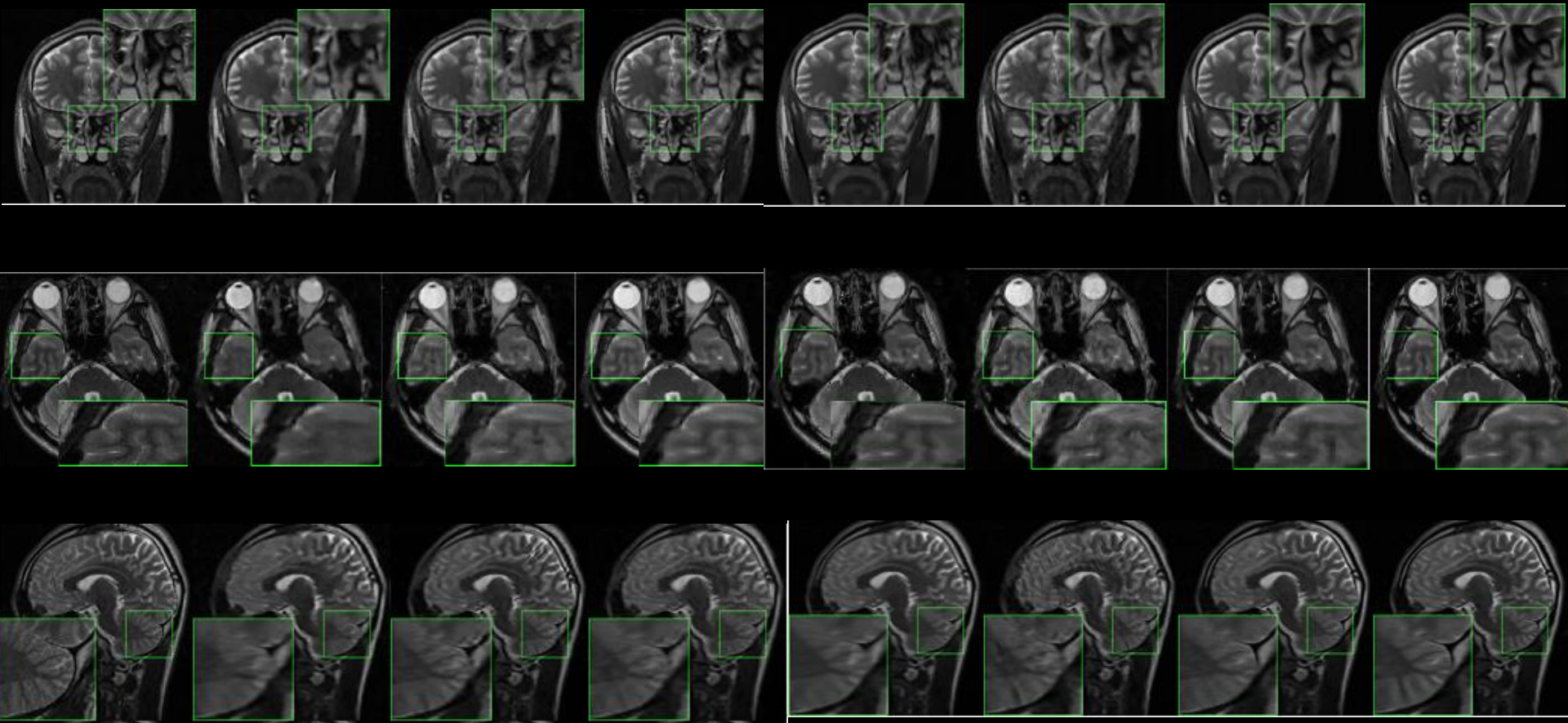
Average PSNR, SSIM and HFEN values (mean  $\pm$  std) of reconstructing 31 test images.

(a)	DLMRI	PANO	FDLCP	NLR-CS	DC-CNN	DAEPRrec	EDAEPRec
$R=3.3$	33.43( $\pm 0.87$ )	34.64( $\pm 1.26$ )	34.89( $\pm 1.20$ )	35.31( $\pm 1.48$ )	<b>35.71(<math>\pm 1.38</math>)</b>	34.04( $\pm 1.00$ )	35.62( $\pm 1.16$ )
	0.9054( $\pm 0.0101$ )	0.9152( $\pm 0.0161$ )	0.9135( $\pm 0.0170$ )	0.9099( $\pm 0.2333$ )	0.9234( $\pm 0.0176$ )	0.9111( $\pm 0.0122$ )	<b>0.9279(<math>\pm 0.0130</math>)</b>
	0.63( $\pm 0.0544$ )	0.56( $\pm 0.0799$ )	0.50( $\pm 0.0670$ )	0.47( $\pm 0.0778$ )	0.44( $\pm 0.0659$ )	0.62( $\pm 0.0589$ )	<b>0.42(<math>\pm 0.0676</math>)</b>
$R=4$	32.41( $\pm 0.87$ )	33.65( $\pm 1.23$ )	34.04( $\pm 1.18$ )	34.35( $\pm 1.43$ )	34.07( $\pm 1.26$ )	33.21( $\pm 0.98$ )	<b>34.49(<math>\pm 1.15</math>)</b>
	0.8866( $\pm 0.0113$ )	0.8995( $\pm 0.0180$ )	0.8980( $\pm 0.0196$ )	0.8938( $\pm 0.0256$ )	0.8992( $\pm 0.0256$ )	0.8973( $\pm 0.0135$ )	<b>0.9151(<math>\pm 0.0146</math>)</b>
	0.84( $\pm 0.0852$ )	0.73( $\pm 0.0999$ )	0.62( $\pm 0.0821$ )	0.61( $\pm 0.0950$ )	0.69( $\pm 0.0950$ )	0.76( $\pm 0.0695$ )	<b>0.64(<math>\pm 0.0822</math>)</b>
$R=5$	31.21( $\pm 0.83$ )	32.44( $\pm 1.16$ )	32.97( $\pm 1.16$ )	33.32( $\pm 1.24$ )	32.68( $\pm 1.10$ )	32.29( $\pm 0.96$ )	<b>33.49(<math>\pm 1.14</math>)</b>
	0.8602( $\pm 0.0135$ )	0.8777( $\pm 0.0194$ )	0.8770( $\pm 0.224$ )	0.8812( $\pm 0.0243$ )	0.8791( $\pm 0.0243$ )	0.8797( $\pm 0.0150$ )	<b>0.8990(<math>\pm 0.0167</math>)</b>
	1.10( $\pm 0.0933$ )	0.96( $\pm 0.1217$ )	0.80( $\pm 0.1037$ )	0.79( $\pm 0.1119$ )	0.95( $\pm 0.1119$ )	0.94( $\pm 0.0840$ )	<b>0.79(<math>\pm 0.1015</math>)</b>
$R=10$	27.39( $\pm 0.98$ )	28.58( $\pm 1.00$ )	29.34( $\pm 1.13$ )	29.51( $\pm 1.16$ )	28.39( $\pm 0.93$ )	29.65( $\pm 0.94$ )	<b>30.30(<math>\pm 1.17</math>)</b>
	0.7444( $\pm 0.0288$ )	0.7805( $\pm 0.0254$ )	0.7856( $\pm 0.0333$ )	0.7845( $\pm 0.0243$ )	0.7710( $\pm 0.0243$ )	0.8160( $\pm 0.0213$ )	<b>0.8319(<math>\pm 0.0254</math>)</b>
	2.18( $\pm 0.2198$ )	1.90( $\pm 0.2014$ )	1.60( $\pm 0.2008$ )	1.65( $\pm 0.2140$ )	1.93( $\pm 0.2140$ )	1.56( $\pm 0.1469$ )	<b>1.40(<math>\pm 0.1842</math>)</b>
(b)	DLMRI	PANO	FDLCP	NLR-CS	DC-CNN	DAEPRrec	EDAEPRec
$R=6.7$ , 2D Random	27.63( $\pm 0.98$ )	29.12( $\pm 1.06$ )	30.14( $\pm 1.19$ )	30.34( $\pm 1.21$ )	28.78( $\pm 1.02$ )	29.90( $\pm 0.91$ )	<b>30.68(<math>\pm 1.20</math>)</b>
	0.7518( $\pm 0.0257$ )	0.7964( $\pm 0.0243$ )	0.8004( $\pm 0.0328$ )	0.8087( $\pm 0.0327$ )	0.7873( $\pm 0.0327$ )	0.8232( $\pm 0.0201$ )	<b>0.8433(<math>\pm 0.0258</math>)</b>
	2.02( $\pm 0.1849$ )	1.77( $\pm 0.1976$ )	1.44( $\pm 0.1890$ )	1.46( $\pm 0.1992$ )	1.83( $\pm 0.1992$ )	1.49( $\pm 0.1312$ )	<b>1.31(<math>\pm 0.1921</math>)</b>
$R=6.7$ , Pseudo Radial	29.36( $\pm 0.99$ )	30.60( $\pm 1.12$ )	31.31( $\pm 1.15$ )	31.35( $\pm 1.05$ )	30.57( $\pm 1.04$ )	30.99( $\pm 0.96$ )	<b>32.00(<math>\pm 1.19</math>)</b>
	0.8103( $\pm 0.0243$ )	0.8372( $\pm 0.0231$ )	0.8391( $\pm 0.0275$ )	0.8494( $\pm 0.0232$ )	0.8348( $\pm 0.0232$ )	0.8512( $\pm 0.0177$ )	<b>0.8716(<math>\pm 0.0213</math>)</b>
	1.58( $\pm 0.1991$ )	1.37( $\pm 0.1694$ )	1.13( $\pm 0.1463$ )	1.17( $\pm 0.1341$ )	1.38( $\pm 0.1341$ )	1.22( $\pm 0.1102$ )	<b>1.05(<math>\pm 0.1470</math>)</b>
$R=6.7$ , 1D Cartesian	26.50( $\pm 1.08$ )	27.51( $\pm 0.98$ )	27.91( $\pm 1.07$ )	28.23( $\pm 1.10$ )	27.05( $\pm 0.89$ )	28.67( $\pm 1.20$ )	<b>28.85(<math>\pm 1.32</math>)</b>
	0.7390( $\pm 0.0416$ )	0.7683( $\pm 0.0320$ )	0.7776( $\pm 0.0335$ )	0.7798( $\pm 0.0361$ )	0.7506( $\pm 0.0361$ )	0.8012( $\pm 0.0304$ )	<b>0.8041(<math>\pm 0.0348</math>)</b>
	2.51( $\pm 0.2223$ )	2.28( $\pm 0.2214$ )	2.15( $\pm 0.2448$ )	2.03( $\pm 0.2391$ )	2.44( $\pm 0.2391$ )	1.86( $\pm 0.2526$ )	<b>1.81(<math>\pm 0.2807</math>)</b>



Q. Liu, Q. Yang, H. Cheng, S. Wang, M. Zhang, D. Liang\*, Highly undersampled magnetic resonance imaging reconstruction using autoencoding priors, *Magn. Reson. Med.*, vol. 83, no. 1, pp. 322–336, 2020.

# Experimental results



Q. Liu, Q. Yang, H. Cheng, S. Wang, M. Zhang, D. Liang\*, Highly undersampled magnetic resonance imaging reconstruction using autoencoding priors, *Magn. Reson. Med.*, vol. 83, no. 1, pp. 322–336, 2020.

# **Part II-3 – Sparse-view CT**

## **Reconstruction via Robust and**

## **Multi-channels Autoencoding**

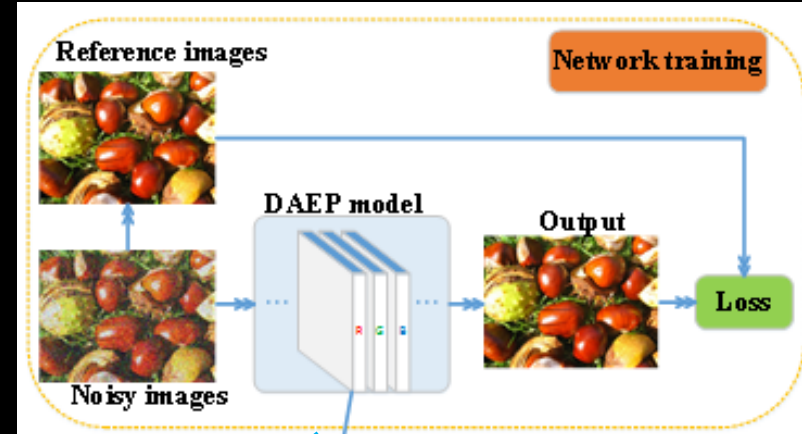
## **Priors**



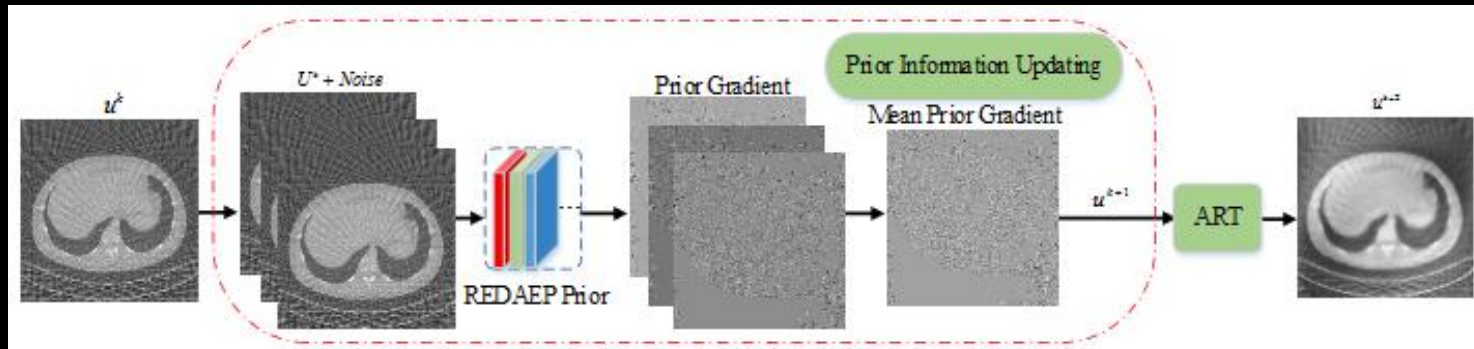


# Priors learning procedure

- At the prior learning stage, we train a three-channels network with input-output pairs of the ground-truth color natural image and its noisy version, the EDAEP prior can be denoted as:  $L_{EDAE} = E_{\eta, U} [\|U - A_{\sigma_{\eta}}(U)\|^2]$  where the training dataset is  $\{U \mid U = [U_r, U_g, U_b]\}$



- The joint learning of the (R, G, B)-channel noisy images exhibits structural information, due to the inherent channel priors.



# CT reconstruction procedure

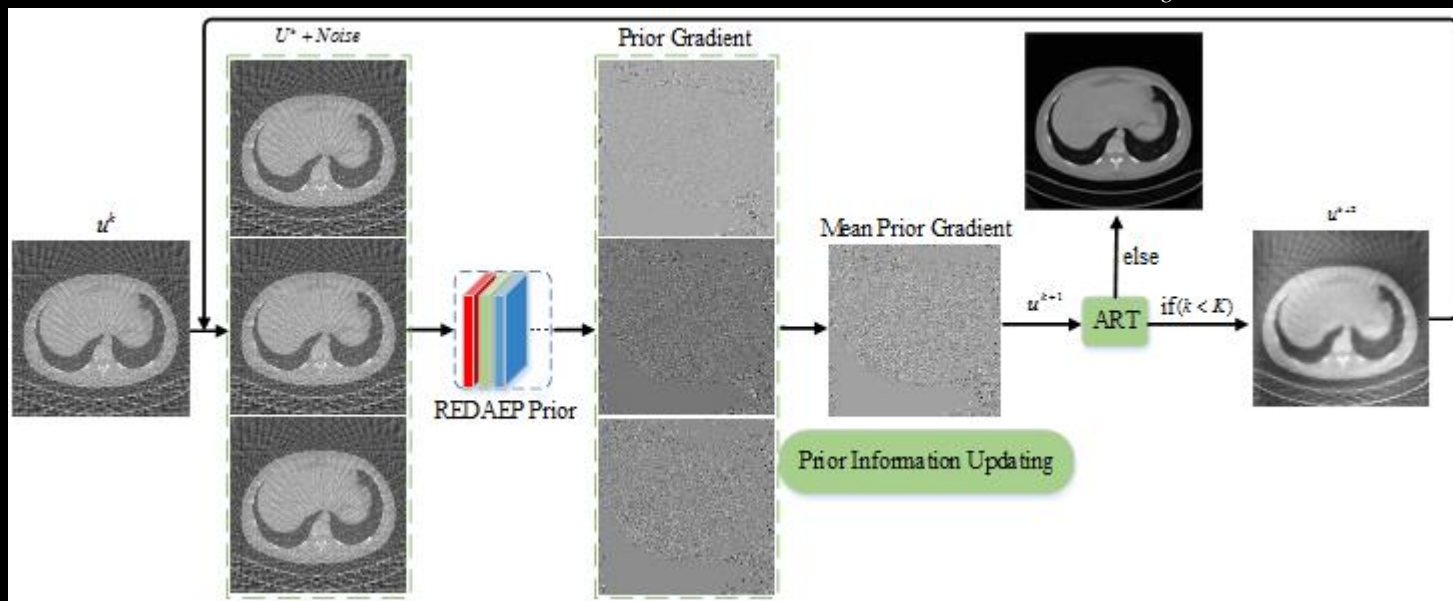
- After the prior is learned, the mathematical reconstruction model can be achieved by solving the following minimization:

$$\min_u \|Mu - f\|^2 + \lambda \|U - A_{\sigma_n}(U)\|^p$$

here the auxiliary variable  $U = [u, u, u]$  is caused from the desired solution variable  $u$ .

- The testing samples space falls into the learning samples space, i.e.,

$$\{U = [u, u, u]\} \subset \{U \mid U = [U_r, U_g, U_b]\}$$



Schematic flowchart of REDAEP algorithm for CT reconstruction.





# Algorithm

## Algorithm 1 REDAEP

**Initialization:**  $u^0 = M^T f$

**Loop** #iterations  $k = 1, 2, \dots, K$

1: Add the auxiliary variable  $U^k = [u^k, u^k, u^k]$ ;

2: Compute the gradient of REDAEP term:

$$\nabla_U G^k = \frac{1}{\|U^k - A_{T_q}(U^k)\|} \odot [\nabla_U A_{T_q}^T(U^k)[A_{T_q}(U^k) - U^k] + U^k - A_{T_q}(U^k)];$$

3:  $U^{k+1} = U^k - \gamma \nabla_U G^k$ ;  $u^{k+1} = \text{Mean}(U^{k+1})$ ;

4: Compute the gradient of data-fidelity term (ART iteration):

$$5: u^{k+2} = u^{k+1} + \beta M_i \frac{f_i - M_i u^{k+1}}{\|M_i\|^2}, \quad i = 1, \dots, I;$$

**End loop**

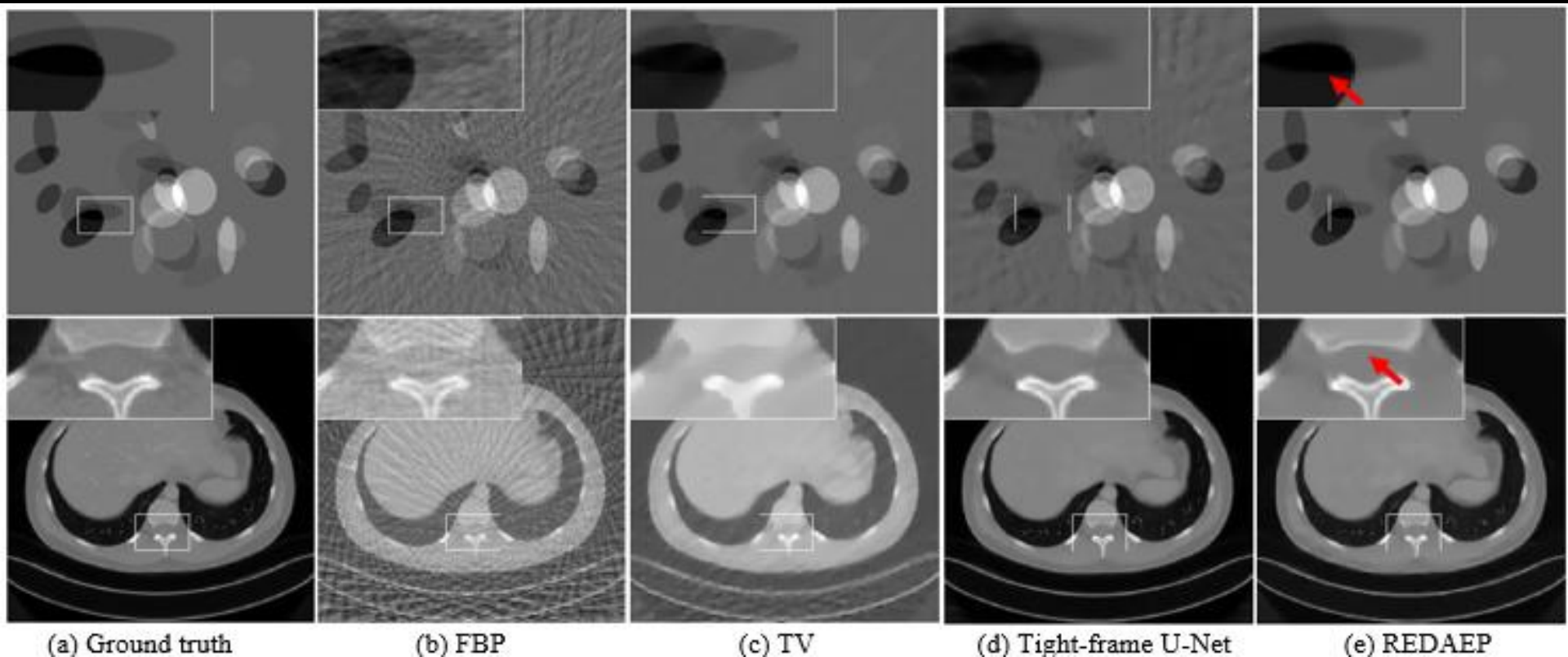
Regularization  
term

Data-fidelity  
term

The overall testing phase of REDAEP algorithm ( $p = 1$ ).



# Experiments



**Reconstructed images of simulated ellipsoidal and real CT1 data from 60 views using FBP, TV, Tight-frame U-Net and the proposed REDAEP.**



M. Zhang, F. Zhang, Q. Liu and D. Liang, Sparse-View CT reconstruction via Robust and Multi-channels Autoencoding Priors, ISICDM 2018, Chengdu, China.

# Experiments and Summary

Reconstruction PSNRs/SSIMs of numerical ellipsoidal data and real CT data.

Views	Images	TV	Tight-frame U-Net	REDAEP
60	Ellipsoid	40.61/0.9786	33.25/0.9563	<b>45.00/0.9853</b>
	CT1	30.33/0.8470	<b>38.92</b> /0.9354	38.27/ <b>0.9470</b>
120	Ellipsoid	44.56/0.9877	38.85/0.9779	<b>48.09/0.9889</b>
	CT2	36.93/0.9056	41.17/0.9453	<b>41.23/0.9556</b>

## Summary :

- A multi-channels network for single-channel CT reconstruction is proposed. The principle is to exploit high-dimensional structural prior via enhancing the DAE priors.
- We modify the objective function in the basic DAEP model from L2-norm to L1-norm. Subsequently, it provides the reconstructions with more texture details.



M. Zhang, F. Zhang, Q. Liu and D. Liang, Sparse-View CT reconstruction via Robust and Multi-channels Autoencoding Priors, ISICDM 2018, Chengdu, China..

# Part III – Summary and Conclusions



# Today We Have Seen that ...

**Two successful applications** in decolorization and IR problem

Motivation

**Metrics or/and learning in higher-dimensional** data space is better for data representation

**Grayscale IR**  
**MRI reconstruction**  
**Sparse-View CT**  
**Reconstruction**

Applications of deep learning

**Color prior**  
**Self-copy prior**  
Robust p-norm prior

Apply to Rec via enhanced DAEP?

More details (including the papers and relevant codes) can be found in  
<http://www.escience.cn/people/liuqiegen/index.html>; <https://github.com/yqx7150>



 **Code :**

<http://www.escience.cn/people/liuqiegen>

<https://github.com/yqx7150>



信息工程学院

School of Information Engineering

*Thanks all!*



**Qiegen Liu (刘且根)**

Nanchang University , Associate professor

研究兴趣: Dictionary learning , compressed sensing , Image Processing , Deep Learning

Love to imaging science!

#### Code

Code of TDAEP (Transformed Denoising Autoencoding Priors for Imaging Inverse Problems)

Code of MWDMSP (Multi-Wavelet Guided Deep Mean-shift Prior for Image Restoration)

Code of M2DAEP (High-dimensional embedding denoising autoencoding prior for color Image restoration)

Code of MEDAEP (Multi-channels and Multi-models based Autoencoding Priors for Grayscale Image Restoration)

Code of VST-Net (VST-Net: Variance-stabilizing Transformation Inspired Network for Poisson Denoising)

Code of EDAEPRec (Highly Undersampled Magnetic Resonance Imaging Reconstruction using Autoencoding Priors)

Code of RicianNet (Progressively distribution-based Rician noise removal for magnetic resonance imaging)

Code of MDAEP-SR (Learning Multi-Denoising Autoencoding Priors for Image Super-Resolution)

Code of Iterative-scheme Inspired Network (Iterative-scheme Inspired Network for Impulse Noise Removal)

Matlab code of MFLRTC\_inpainting (Multi-filters guided low-rank tensor coding for image inpainting)

Matlab code of Dictionary Learning (predual dictionary learning (PDL) / augmented Lagrangian multi-scale dictionary learning(ALM))

And also thanks MICS-2019 organizers,  
Zhipei Liang, Dong Liang, Henry Leung, Shanshan Wang, Minghui Zhang,  
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Fengqin Zhang, Qinxin Yang, Wenzhao Zhao

