CORE MATHEMATICS

Learning Outline

I. The coordinate system
II. Plotting linear graphs
III. Plotting quadratic graphs
IV. Gradient of a curve
V. Real life applications of graphs

1.0 THE COORDINATE SYSTEM

Coordinates are numbers that are used to tell us how to get to a certain point on a grid- e.g. a graph or map. The grid will have an x-axis, which runs horizontally and a y-axis that runs vertically.

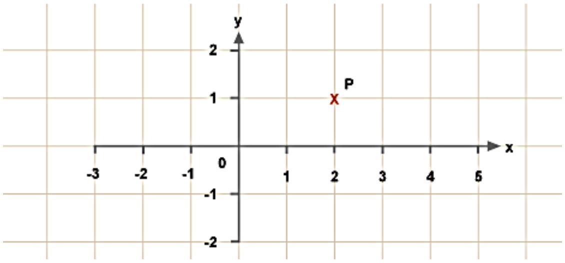


Figure 1: Coordinates of a point P on a graph

The coordinates of a point are written in pairs separated by a comma. For example, the coordinates of the point P on the graph shown in figure 1 is written as (2, 1). The first number in the pair is known as the **x** coordinate (**2**, 1) while the second number in the pair is known as the **y** coordinate (2, **1**).

The x coordinate tells us how many units to go from the origin to the left if it is a negative number, or to the right if it is a positive number. The y coordinate tells us how many units to go from the origin upward if the number is positive, or downward if the number is negative.

Worked Example 1

Using the graph shown in figure 2, work out which letter represents the following coordinates;

(i) (5, 2)
(ii) (-3,4)
(iii) (-6,-1)

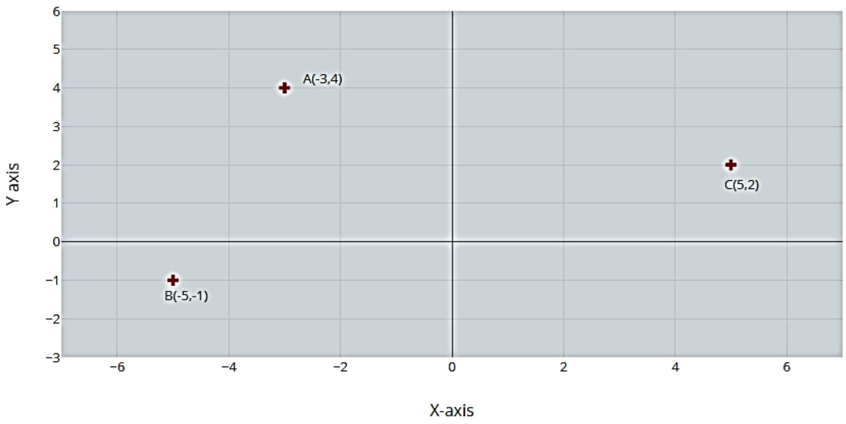
Solutions

(5,2) means starting from the origin, go 5 points across to the right and then 2 points up... C.

(-3,4) means starting from the origin go 3 points across to the left and 4 points up... A.

(-5,-1) means starting from the origin go 5 points across to the left and 1 points down... B.

Figure 2



Worked Example 2

Show on the graph the points represented by the following coordinates;

(i) (6,-1)

(ii) (2, 4)

(iii) (-3, -3)

Show the coordinate points A(6,-1), B(2,4) C(-3, -3) on the graph

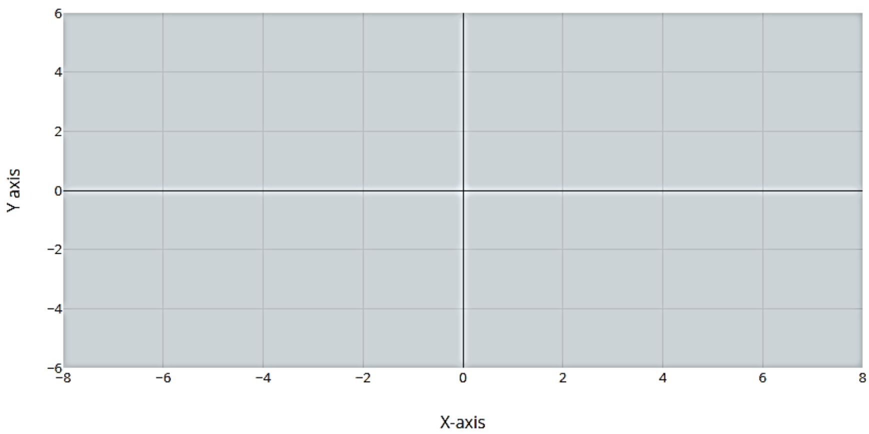


Figure 3: graph sheet for example 2

2.0 PLOTTING GRAPHS

A graph is a simple diagram that shows the relationship between variables, typically two variables. Graphs are a great way to represent data in a straightforward and simple way. Whether you're showing off the number of visitors to your Facebook page, or an accountant portraying the latest stock forecasts, graphs are essential.

2.1 HOW TO DRAW A GRAPH FROM AN EQUATION

Some key points to note when drawing a graph from a given equation are;

1. Make a table of the values of the coordinate points for the x and y variables using the equation of the line provided.

2. Draw the **x** and **y** axes and select appropriate scales for both axes. Always check to see that the scale selected will fit all the points in your dataset and you can easily work out the interval between each small box in your graph paper.

3. Plot the coordinates of the points obtained in (1) on the graph paper.

4. If the points are in a straight line, you must draw a straight line **through** them to fill the whole of your graph. Don't just start at the first point plotted and end at the last.

5. If the points are not in a straight line, draw a **smooth curve** through them. If you are not very good at this, you must practise to get your technique sorted out!

6. Always **label** your graph fully - the axes, and give your graph a title.

Note:

(1) If there is a point that doesn't look like it follows the line or curve then you've probably worked it out wrongly and need to recheck its calculation.

(2) Always draw your graphs using pencils- sharp for accuracy and pencil because if you make an error you could easily erase it off and replot your graph.

3.0 STRAIGHT LINE GRAPHS

One of the simplest graphs is that of a straight line. These are written in the form $y = mx + c$ where y is the dependent variable, x is the independent variable and m and c are constants. In this form, **m** is the gradient (slope) of the line, (the steepness), and **c** is the y intercept, (the point where the line cuts the y axis).

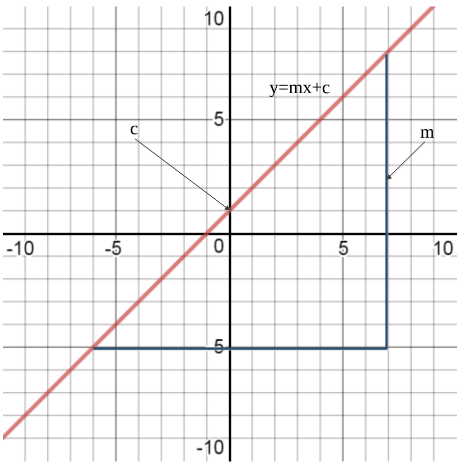


Figure 4: Plot of a straight line graph

Worked Example 3*:*

Plot the graph of $y = 2x + 5$ for values of x in the range $-3 \le x \le 3$

Step 1: Make a table for x and y for the range of x $-3 \le x \le 3$

when $x = -3$, $y = 2(-3) + 5$ $y = -6 + 5$ $y = -1$

when $x = -2$, $y = 2(-2) + 5$ $y = -4 + 5$ $y = 1$

when $x = -1$, $y = 2(-1) + 5$ $y = -2 + 5$ $y = 3$

when $x = 0$, $y = 2(0) + 5$ $y = 0 + 5$ $y = 5$

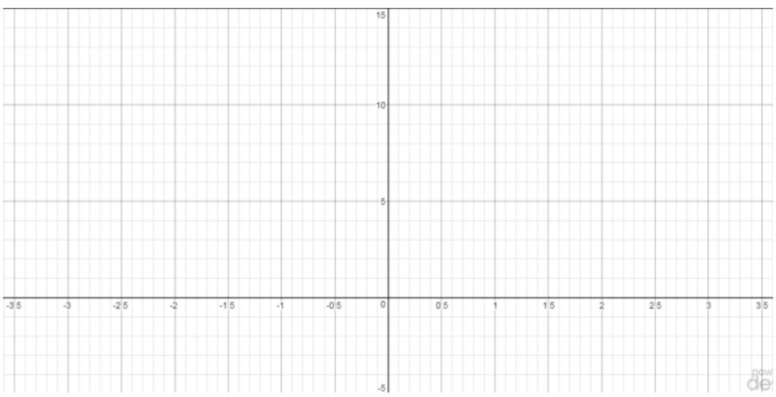
when $x = 1$, $y = 2(1) + 5$ $y = 2 + 5$ $y = 7$

when $x = 2$, $y = 2(2) + 5$ $y = 4 + 5$ $y = 9$

when $x = 3$, $y = 2(3) + 5$ $y = 6 + 5$ $y = 11$

x	-3	-2	-1	0	1	2	3
y=2x+5	-1	1	3	5	7	9	11

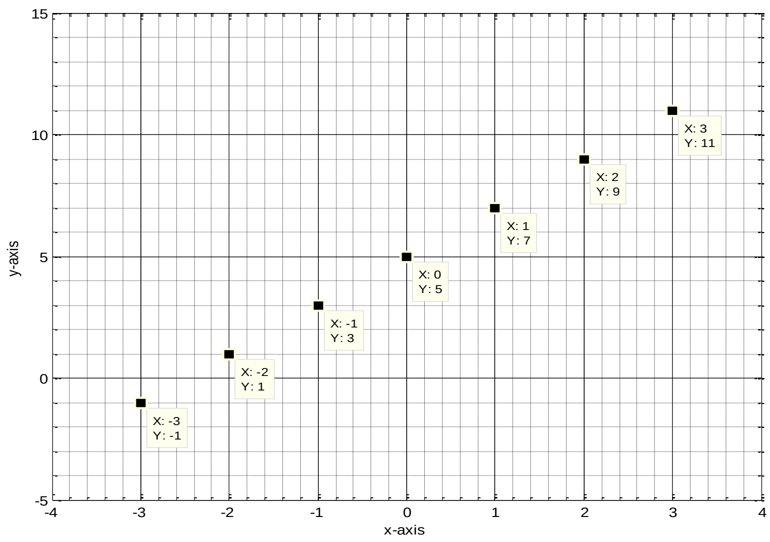
Step 2: Draw the **x** and **y** axes and select appropriate scales for both axes



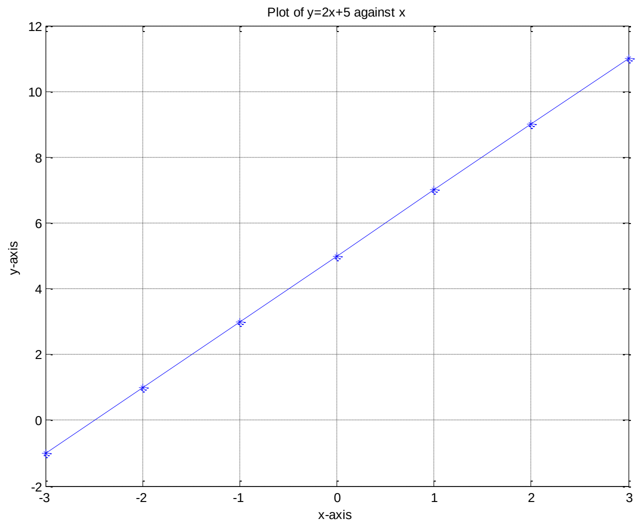
The scale for the y-axis is chosen such that each small rectangular box represents one unit while for the x-axis each small rectangular box represents 0.1unit.

The y-axis ranges from -5 to 15 which cover the span of the values of y in the table and the x-axis ranges from -4 to 4.

Step 3: Plot the coordinate points on the graph paper



Step 4: Join the coordinate points



<u>3.0 FINDING THE GRADIENT OF STRAIGHT LINES</u>

Gradient is another word for "slope". It shows the rate at which the dependent variable (y) is changing with respect to the independent variable (x). The higher the gradient of a graph, the steeper the graph line. A negative gradient means that the line slopes downwards. Sometimes you may be required to find the gradient of a straight line graph, this can be done using the following steps

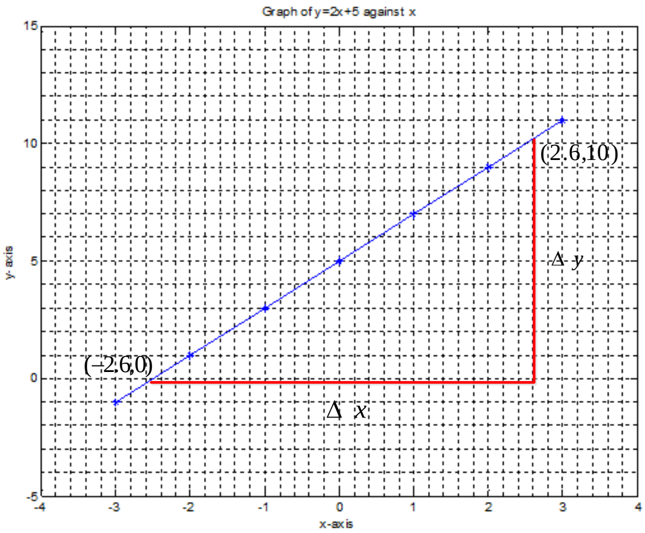
<u>To find the gradient of a straight line</u>:

i. choose any two coordinate points on the line that are (make sure the points are well apart)
ii. draw a right-angled triangle with the line as hypotenuse

iii. use the scale on each axis to find the triangle's vertical length and horizontal length

iv. work out the vertical length ÷ horizontal length

v. the result is the gradient of the line



$$m = \frac{change\ in\ y - values(\ \Delta y)}{change\ in\ x - values(\ \Delta x)}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{10 - 0}{2.6 - (-2.6)} = \frac{10}{5.2} = 1.92$$

Exercise 1

a) Plot the graph $2y = 4x - 5$ for values of x in the range $-3 \le x \le 3$ and calculate the gradient of the graph.

b) Plot the graph $y + 3x - 11 = 0$ for values of x in the range $-2 \le x \le 2$ and calculate the gradient of the graph.

c) Plot the graph $y = 5x$ for values of x in the range $-3 \le x \le 3$ and calculate the gradient of the graph.

4.0 CURVED GRAPHS (QUADRATIC AND CUBIC GRAPHS)

Drawing a curved graph is similar to drawing a straight-line graph and you also have to substitute numbers into the equation. With a curved graph the formula will include x^2 and x^3 for quadratic and cubic graphs respectively.

A quadratic function can be written in the form: $y = ax^2 + bx + c$ where a, b and c are constants and 'a' does not equal zero. Quadratic graphs are always parabolas ('U' shapes).

4.1 PLOTTING GRAPH OF A QUADRATIC EQUATION USING A GIVEN INTERVAL

Worked Example 4

Complete the table for y= $4x^2$ -3x+3 below and plot the graph of y= $4x^2$ -3x+3 against x

x	-3	-2	-1	0	1	2	3	4
y= $4x^2$ -3x+3								

Following the same steps given in example 3, complete the above table using the equation provided.

when $x = -3$ $\quad y = 4(-3)^2 - 3(-3) + 3 = 48$

when $x = -2$ $\quad y = 4(-2)^2 - 3(-2) + 3 = 25$

when $x = -1$ $\quad y = 4(-1)^2 - 3(-1) + 3 = 10$

when $x = 0$ $\quad y = 4(0)^2 - 3(0) + 3 = 3$

when $x = 1$ $\quad y = 4(1)^2 - 3(1) + 3 = 4$

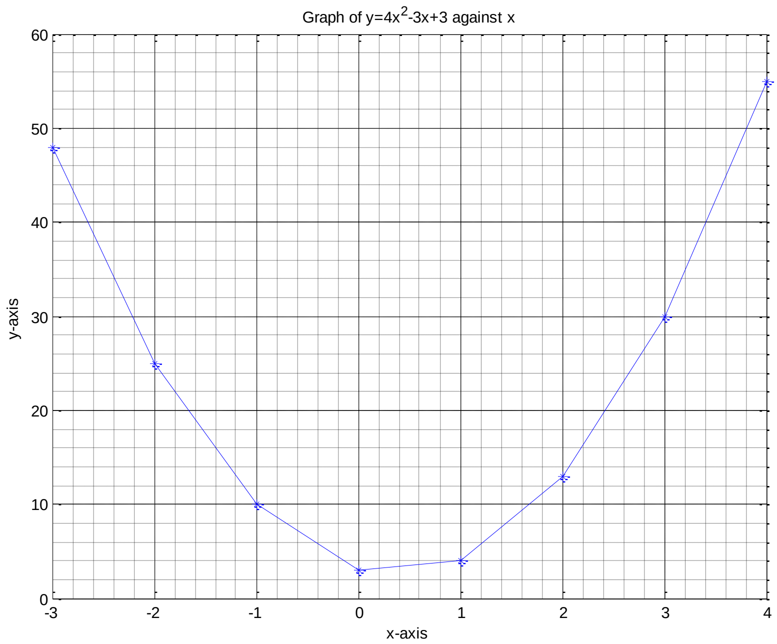
when $x = 2$ $\quad y = 4(2)^2 - 3(2) + 3 = 13$

when $x = 3$ $\quad y = 4(3)^2 - 3(3) + 3 = 30$

when $x = 4$ $\quad y = 4(4)^2 - 3(4) + 3 = 55$

x	-3	-2	-1	0	1	2	3	4
y= x^2 -4x-5	48	25	10	3	4	13	30	55

Select appropriate scales for the x-axis and y-axis and plot the coordinate points on the graph. Join the points together to form the quadratic curve.



The really important bits of a quadratic graph are:

Where it turns (the bottom of the 'U')- this is called its stationary point and it is maximum or minimum point.

Where it crosses the x-axis (if it does!)- these points are referred to as the solutions of the quadratic equation. Looking at our co-ordinates it appears that the curve does not touch cross the x axis hence it has no solution.

4.2 FINDING THE GRADIENT OF A CURVE

It is important to note firstly that unlike a straight line graph where the gradient of the line is constant, the gradient of a curve varies along the x-axis. To calculate the gradient of a curve at a given point on the x-axis, a tangent line is carefully drawn to touch the curve at that point. Once the tangent is drawn, you can then find the slope of the tangent line by selecting any two points on it and drawing a right angled triangle as shown in example 3.

***NB: A tangent is a straight line that touches a curve at one point only*.**

Worked Example 5

Find the gradient of the curve $y= 4x^2 -3x+3$ at the point where x=2

Solution

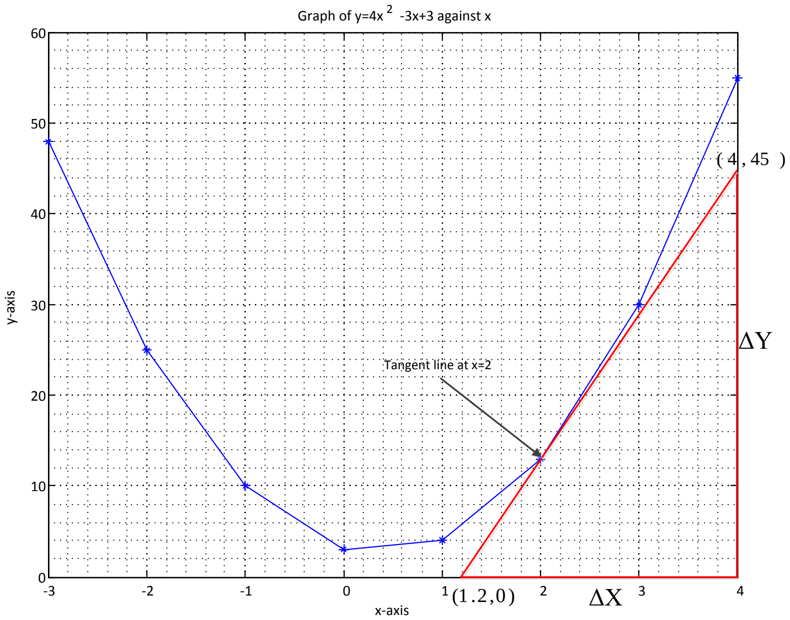
i. First draw the graph of $y= 4x^2 -3x+3$ following the steps in example 4,

ii. Draw a tangent to touch the curve at the point where x=2

iii. Draw a right angled triangle where the tangent forms the hypotenuse

iv. Select any two points and calculate the vertical length Δy and horizontal length Δx

v. Calculate the gradient at point where x=2 as $m_{x=2} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$



$$m_{x=2} = \frac{change\ in\ y - values(\ \Delta y)}{change\ in\ x - values(\ \Delta x)}$$

$$m_{x=2} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{x=2} = \frac{45-0}{4-(1.2)} = \frac{45}{2.8} = 16$$

Note that the accuracy of your answer is dependent on how correctly you have drawn your tangent to the curve at the point where x=2

Exercise 2

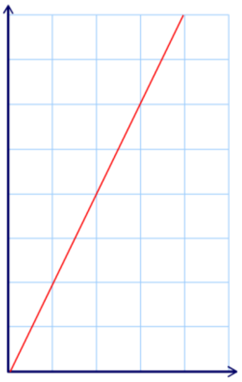
a) Plot the graph $y=3x^2-x-2$ for values of x in the range $-3 \leq x \leq 3$ and calculate the gradient of the graph at $x=1$.

b) Plot the graph $y=8x^2+6x-5$ for values of x in the range $-3 \leq x \leq 3$ and calculate the gradient of the graph at $x=1.5$.

c) Plot the graph $y=5x$ for values of x in the range $-3 \leq x \leq 3$ and calculate the gradient of the graph.

5.0 REAL LIFE APPLICATION OF GRAPHS

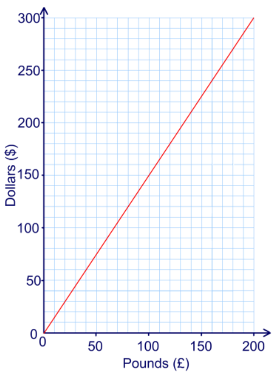
We often use graphs to illustrate real-life situations. Instead of plotting y-values against x-values, we plot one physical quantity against another physical quantity. The resulting graph shows the **rate** that one quantity changes with another.



When illustrating situations using graphs, we usually have control over one of the quantities. This is the independent variable. The other quantity is called the dependent variable. This is because it is determined by the outcome of the experiment or some mathematical relationship. The quantity on the y-axis is usually the dependent variable while the quantity on the x-axis is usually the independent variable.

Worked Example 6

The graph in the figure below shows the exchange rate from British pounds to American dollars. Use this graph to answer the following questions.



(1) Write the equation of the graph

(2) What is the gradient of the graph and what does it represents?

(3) How many dollars would you get if you had £150 to exchange?

Solution

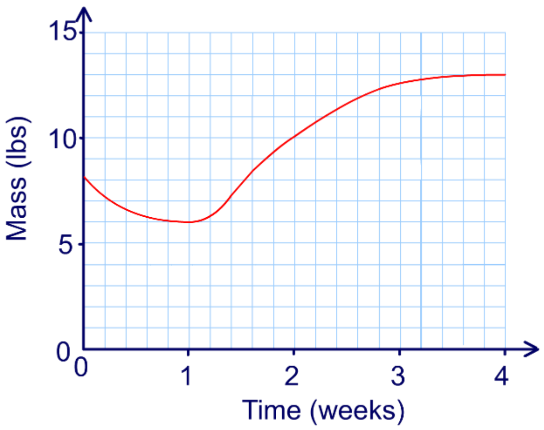
(1) It is a straight line graph that passes through the origin. The equation of the line would be of the form: $y = mx$.

(2) In this graph, m is the slope and it represents the rate of change of the dollar to the pound and its 1.5. This tells us that £1 is equal to $1.50.

(3) By reading from the graph, we can tell that if you had £150 to exchange, you would get $225. You could also check this by multiplying £150 by the gradient (1.5).

Worked Example 7

This graph shows the mass of a newborn baby over the first month from birth.



(1) What was the mass of the baby when it was first born**?** *The mass of the baby when it was first month was 8 lbs*

(2) What is the baby's mass at the end of the first month? *The mass of the baby at the end of the first month is 13 lbs*

(3) Use the information given to describe the graph in detail. *The baby's mass decreases slightly during the first week and by the end of week 1 the baby has dropped from its initial mass of 8 lbs to a mass of 6 lbs. From this point on its mass increases in decreasing amounts over the rest of the month until it reaches a mass of 13 lbs by the end of week 4. Between the end of week 1 and the end of week 2, its mass increases by 4 lbs. From the end of week 2 to the end of week 3, its mass increases by 2.5 lbs. Between the end of week 3 and the end of week 4, the baby's mass has increased by 0.5 lbs.*

Exercise 3

(1) Frank has been asked to draw a graph that illustrates the temperature relationship between °F and °C. Frank records the following information from his research.

Temp (°C)	0	20	40	60	80	100
Temp (°F)	32	68	104	140	176	212

i. Plot the graph of °F against °C

ii. What is the temperature in °F when it is 70°C?

iii. What is the gradient of the graph when °F is plotted on the vertical axis?

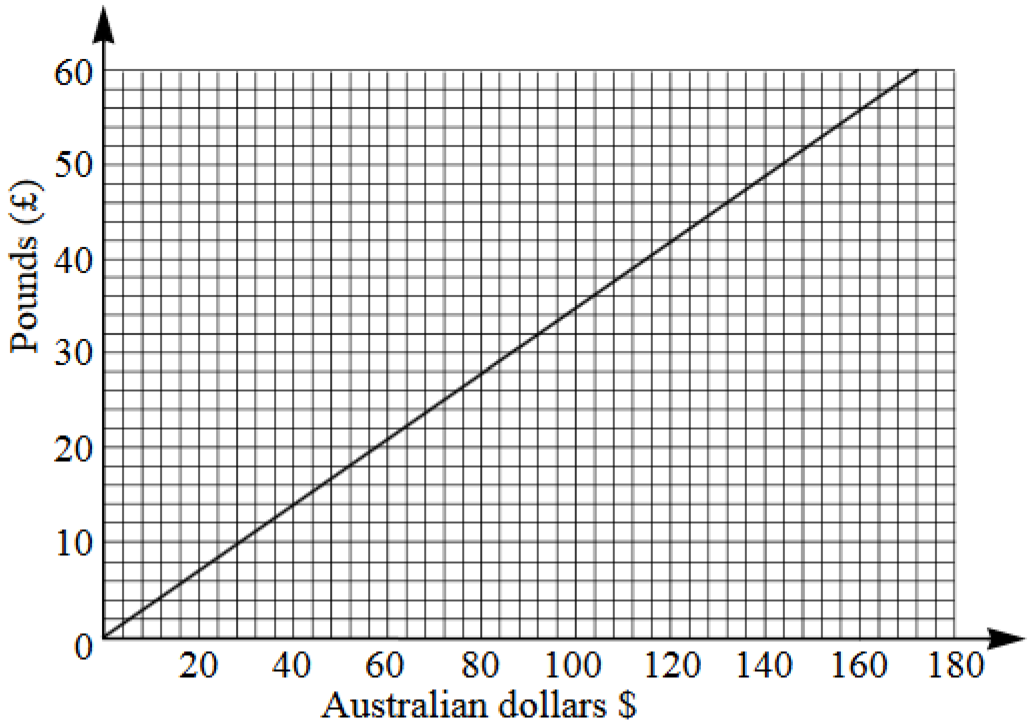
(2) Theo is looking for a new mobile phone and has seen the model he wants advertised on two different tariffs. (a) 12 month contract: £9.99 a month, FREE handset and texts! Calls only 10 p per minute! (b) PAYG £40 for handset FREE texts Calls only 5 p per minute! Using the graph below answer the following questions;

i. Which tariff is better value if Theo makes 200 minutes of calls in the first month?

ii. At what stage in the first month does the monthly contract cost more than the PAYG phone?

iii. How much will it cost to talk for 200 minutes on a pay as you go mobile?

(3) This graph shows the relationship between the Australian dollars and the pound sterling. From the graph determine:



a) The rate of exchange (i.e. the number of dollars to the pound)

b) The number of dollars that can be obtained for £22.

c) The amount of £ sterling that can be exchanged for $140.

(4) Christmas trees are priced according to their height. The table below shows some of the prices charged last Christmas.

Height (metres)	1.0	1.5	2.0	2.5	3.0
Price (£)	8.50	11.75	15.00	18.25	21.50

Draw a conversion graph using 8cm to represent 1 metre on the vertical axis and 1cm to represent £2 on the horizontal axis.

a) From your graph, calculate the cost of a tree measuring 2m 35cm.

b) What size tree can be purchased for £12.50?

(5) Bill is a craftsman who makes wooden bowls on his lathe. He advertises that he can make any size bowl between 20cm and 60cm diameter. In his shop he gives the price of five different bowls as an example.

Diameter of bowl (cm)	20	30	40	50	60
Price	£8.80	£20.00	£42.40	£79.00	£133.60

a) Use these figures to draw a conversion graph. Use a scale of 2cm to represent a diameter of 10cm on the horizontal axis and 2cm to represent £10 on the vertical axis.

b) Jane has £50 to spend. From your diagram, estimate the size of bowl she can buy.

c) What is the cost of a bowl of 34cm diameter?