PURE MATHEMATICS 1 **MOCK 1v1** **SPRING 2021**

TIME ALLOWED: 1 hour

ANSWER ALL THE QUESTIONS

TOTAL MARKS = 50

INSTRUCTIONS: You can use a calculator.

You must show your method.

Answer all the questions.

You must make sure the work you present is yours and is not copied from somewhere else.

Formulae

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$a^{-n} = \frac{1}{a^n}$$

Questions

1. Give two examples each of
   a) An irrational number

   b) A rational number to 2 significant figures

   c) A perfect square

   [6 marks]

   Ans
   a) Pi, √3
   b) 0.123, 23.9
   c) 16, 25
   .

2. A visitor to this country buys a bag costing 55.00 including VAT of 15%. How much VAT can be reclaimed? [4 marks]

   Ans
   Item + VAT = 55.00 [original value (100%) + VAT (15%=115%]
   Original value = 100/115 x 55
   VAT value = Item+VAT- original value
   VAT = 7.174

3. Find the value of [2 marks]

$$4 \times 9^2 - 27 \div 3$$

Ans

4 x 81 – 9 = 324 – 9 = 315

4. Rationalise [2 marks]

$$\frac{2}{\sqrt{5}+\sqrt{3}}$$

Ans

$$\frac{2}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$\frac{2\sqrt{5}-2\sqrt{3}}{2}$$

$$\sqrt{5}-\sqrt{3}$$

5. Write the equation using only positive indices. [2 marks]

$$9x^{-4}y^{-1}$$

Ans

$$\frac{9}{x^4 y}$$

6. Expand [2 marks]

$$\frac{(6x-3)}{3}(x-3)$$

Ans

$$(2x-1)(x-3)$$
$$2x^2 - 6x - 1x + 3$$
$$2x^2 - 7x + 3$$

7. Use Pascal's triangle to show that [5 marks]

$$(-2a+b)^4 = (b-2a)^4$$

Ans

$$1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4$$
$$a = -2a, b$$
$$b = b, -2a$$
$$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$a = -2a, b = b$$

$$(-2a)^4 + 4(-2a)^3(b) + 6(-2a)^2(b)^2 + 4(-2a)(b)^3 + (b)^4$$
$$16a^4 - 32a^3b + 24a^2b^2 - 8ab^3 + b^4 \text{ - - - } \textit{(Equation 1)}$$

$$(b)^4 + 4(b)^3(-2a) + 6(b)^2(-2a)^2 + 4(b)(-2a)^3 + (-2a)^4$$
$$b^4 - 8b^3a + 24b^2a^2 - 32ba^3 + 16a^4$$

By rearranging we get equation (1) above. Hence $(-2a + b)^4 = (b - 2a)^4$

8. Rearrange the equation to make x the subject. [3 marks]

$$y = \frac{x + 3}{3x + 5}$$

Ans

$$y(3x + 5) = x + 3$$
$$y3x + 5y = x + 3$$
$$y3x - x = 3 - 5y$$
$$x(3y - 1) = 3 - 5y$$
$$x = \frac{3 - 5y}{3y - 1}$$

9. Solve the simultaneous equation [4 marks]

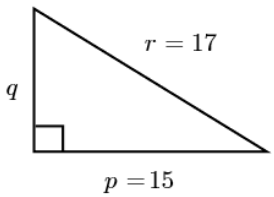
$$x + 4y = 10$$
$$2x + 3y = 5$$

Ans

$$x + 4y = 10 - - - (1) \times 2$$
$$2x + 3y = 5 - - - (2) \times 1$$

$$2x + 8y = 20 - - - (3)$$
$$2x + 3y = 5 - - - (4)$$
$$\textit{(3) minus (4)}$$
$$5y = 15 => y = 3 \quad (\textit{substitute into } (1))$$
$$x + 4y = 10 - - - (1)$$
$$x + 4 \cdot 3 = 10$$
$$x = 10 - 12 = -2$$

10. Find the value of q in the figure below [2 marks]



Ans

$$r^2 = p^2 + q^2$$
$$q^2 = r^2 - p^2$$
$$q^2 = 17^2 - 15^2$$
$$q = \sqrt{17^2 - 15^2} = 8$$

11. Write 10.09476 in
    a. 3 s.f

    b. 3 d.p [4 marks]

Ans

$$a. 10.1 \qquad b. 10.095$$

12. Convert [2 marks]
    a. 45 degrees to radian

    b. 1.6 radian to degrees

Ans

$$a)\ \frac{\pi}{180} \times 45 = \frac{\pi}{4}\ radians$$
$$b)\ \frac{180}{\pi} \times \frac{8}{5} = \frac{36 \times 8}{\pi} = 91.673$$

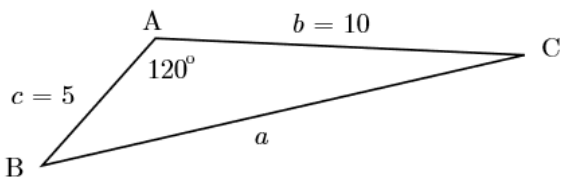
13. Determine all the angles between 0° and 360° whose cosecant is 1.6586. [4 marks]

Ans

Sine is positive in the first and second quadrant. But

$$\sin\theta = \frac{1}{cosec\ \theta} = \frac{1}{1.6586} = 0.6029545 \therefore$$
$$First\ quadrant, \theta = 37.08179$$
$$Second\ quadrant, 180 - \theta = 142.92$$

14. Find the length of side BC in the figure below. [2 marks]



Ans

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 10^2 + 5^2 - 2 \cdot 10 \cdot 5\cos 120$$
$$a^2 = 100 + 25 - 100 \cdot (-0.5)$$
$$a^2 = 125 + 50$$
$$a = \sqrt{175} = 5\sqrt{7} = 13.229$$

15. The angle of depression of a ship viewed from the top of a vertical cliff 75m high is 30 degrees.
    a. Find the distance of the ship from the base of the cliff at this time.
    b. If the ship is sailing away from the cliff at a constant speed and a minute and a half later, its angle of depression from the top of the cliff is 15 degrees. Find the speed of the ship in meters per second.

[6 marks]

Ans

$$\tan 30 = \frac{75}{y} => y = \frac{75}{\tan 30} = 75\sqrt{3} = 129.9038$$

$$\tan 15 = \frac{75}{z} => z = \frac{75}{\tan 15} = 279.9038$$

$$speed = \frac{distance}{time}, but\ time = 1.5 \times 60 = 90\ seconds$$

$$and\ distance = \frac{279.7038 - 75\sqrt{3}}{90} = \frac{150}{90} = \frac{5}{3} = 1.67 m/s$$