Quadratic Proof from Square Completion

09 February 2021 19:12

$Given\ that\ (x+d)^2 = x^2 + 2dx + d^2$ - - - Equation (1)

And

$ax^2 + bx + c = 0$ - - - Equation (2)

$$\begin{aligned} Where\ a &= coefficient\ of\ x^2 \\ b &= coefficient\ of\ x^1 \\ c &= coefficeint\ of\ x^0 \end{aligned}$$

$We\ would\ like\ to\ rewrite\ LHS\ of\ equation\ (2)\ to\ look\ like\ the$
$LHS\ of\ equation\ (1)\ by\ comparing\ coefficients\ of\ LHS\ of\ equation\ 2\ to$
$RHS\ of\ equation\ 1.\ \ So\ we\ first\ of\ all\ divide\ through\ by\ a\ so\ that\ the\ coefficient\ of$
$x^2 is\ 1\ just\ as\ in\ equation\ 1\ (RHS)$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ - - - Equation (3)

$Next\ we\ would\ lilke\ to\ make\ the\ first\ 2\ terms\ of\ Equation\ (3)\ LHS\ to\ be\ equivalent\ to$
$Equation\ 1\ (LHS)\ by\ comparing\ Equation\ 1\ (RHS)\ to\ first\ 2\ terms\ of\ equation\ 3\ (LHS).$
$First\ let\ us\ get\ rid\ of\ the\ last\ term \frac{c}{a} in\ equation\ (3)\ by\ taking \frac{c}{a} away\ from\ both\ sides$

$$x^2 + \frac{b}{a}x + \frac{c}{a} - \frac{c}{a} = -\frac{c}{a}$$

$x^2 + \frac{b}{a}x = -\frac{c}{a}$ - - - Equation (4)

$Now\ the\ coefficient\ of\ x^2 is\ 1$

$$\begin{aligned} &Coefficient\ of\ x^1 = \frac{b}{a},\ and \\ &Cofeficient\ of\ x^0 = 0\ \ i.e. \end{aligned}$$

$$x^2 + \frac{b}{a}x = 1x^2 + \frac{b}{a}x^1 + 0x^0$$

Let us now equate LHS of Equation (4) to Equation (1)

$x^2 + \frac{b}{a}x = (x + d)^2 = x^2 + 2dx + d^2$ - - - - Equation (5)

$We\ want\ an\ expression\ for\ d\ such\ that\ this\ equation\ will\ be\ balanced.\ \ We\ get\ this\ by\ comparing$
$coefficients\ of\ the\ Left\ and\ Right\ hand\ sides\ of\ equation\ (5).\ \ \ already\ x^2 = x^2,\ hence\ no\ expression$

$for\ d\ in\ the\ first\ term,\ which\ is\ the\ coefficientof\ x^2.\ \ In\ the\ second\ term\ (coefficieint\ of\ x)\ \frac{b}{a} = 2d$

$hence\ d\ = \dfrac{b}{2a} and\ in\ the\ 3rd\ term\ the\ coefficient\ of\ 0 + x^0 is\ d^2,\ hence\ d^2 will\ be\ made\ to\ be\ zero.$
$We\ make\ d^2 equal\ zero\ by\ subtracting\ d^2 from\ both\ sides\ i.e.,\ from\ the\ MHS\ and\ \ RHS$

$$x^2 + \frac{b}{a}x\ =\ (x\ +\ d)^2 - d^2\ =\ x^2 +\ 2dx\ +\ d^2 - d^2$$
$$x^2 + \frac{b}{a}x\ =\ (x\ +\ d)^2 - d^2\ =\ x^2 +\ 2dx$$

$$but\ d\ = \frac{b}{2a} and\ d^2\ = \left(\frac{b}{2a}\right)^2 = \frac{(b)^2}{(2a)^2} = \frac{b^2}{(2)^2(a)^2} = \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x\ = \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}$$ ---- Equation (6)

$We\ can\ now\ substitute\ x^2 + \dfrac{b}{a}x\ from\ Equation\ \ (6) back\ into\ equation\ \ (4)$

$$\left(x\ + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\ =\ -\ \frac{c}{a}$$ --- Equation (7)

$We\ can\ now\ add\ \left(\dfrac{b^2}{4ac^2}\right)\ \ to\ both\ sides$ to get

$$\left(x + \frac{b}{2a}\right)^2\ = \frac{b^2}{4a^2} - \frac{c}{a}$$ ---- Equation (8)

By resolving the fractional component of the RHS we get that for RHS

$$\left(x + \frac{b}{2a}\right)^2\ = \frac{b^2}{4a^2} - \frac{c}{a}\ \ = \frac{\{\ b^2 - 4ac\}}{4a^2}$$

By taking the root of both sides we have

$$x + \frac{b}{2a} = \pm\frac{\sqrt{\{b^2 - 4ac\}}}{2a}$$

$Next\ subtract \dfrac{b}{2a} from\ both\ sides$

$$x\ \ =\ \ \pm\frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$

Since 2a is a common denominator, $Therefore$

$$x\ \ = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$