Height of Trangle from sides using Pythagoras theorem

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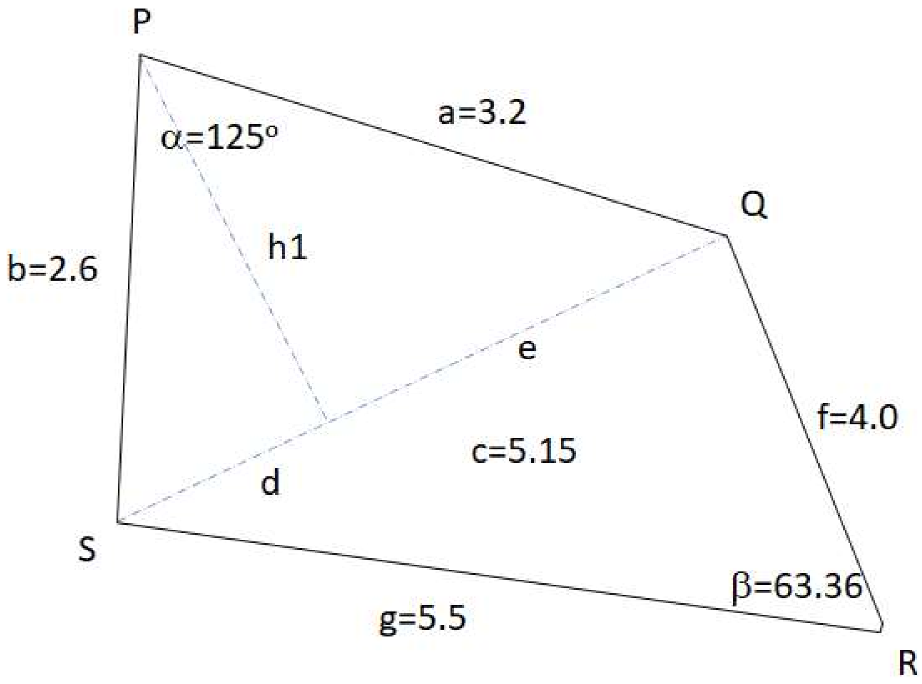
Given the irregular quadrilateral below



We have the following information

1. PQ = a = 3.2
2. PS = b = 2.6
3. QS = c = 5.15 (value obtained from previous calculation)
4. QR = f = 4.0
5. SR = g = 5.5
6. <QRS = β =63.36(value obtained from previous calculation)
7. <QPS = α = 125

We also have the following unknowns

- h1 = height of triangle 1 (QPS)
- h2 = height of triangle 2 (QRS, not plotted)
- h1 bisects QS such that c = d + e ---- equation (1)

Our aim is to obtain an expression for h1 in terms of a, b, and c, which we can also extend to h2 in terms of c, f, and g such that we can calculate the area of the parallelogram from the following formula

$$Area = \frac{1}{2}c(h1 + h2) \qquad - - - - \quad equation\ (2)$$

By Pythagoras theorem

$$b^2 = h1^2 + d^2 \qquad - - - \quad equation\ (3)$$
$$a^2 = h1^2 + e^2 \qquad - - - \quad equation\ (4)$$

Make $h1^2$ the subject in equation (3) and (4) thus

$$h1^2 = b^2 - d^2 = a^2 - e^2 \quad --- \quad \text{equation (5)}$$

Collect the known and unknown terms together. Since a nd b are known hence:

$$d^2 - e^2 = b^2 - a^2 \quad --- \quad \text{equation (6)}$$

Equation (1) and equation (6) are now 2 simultaeous equation having two off our unkown variables d, and e. So let us use the substitution method

So, from equation (1) d = c – e, therefore, substituting d into equation (6)

$$(c - e)^2 - e^2 = b^2 - a^2 \quad --- \quad \text{equation (7)}$$

Using the binomial expanstion $(x - a)^2 = x^2 - 2ax + a^2$, we can expand $(c - e)^2$ to become

$$\left[c^2 - 2ce + e^2\right] - e^2 = b^2 - a^2 \quad --- \quad \text{equation (8)}$$

Once we open the bracket, $+e^2 - e^2 = 0$ therefore,

$c^2 - 2ce = b^2 - a^2$ add 2ce and subtract $\left(b^2 - a^2\right)$ on both giving

$2ce = c^2 + a^2 - b^2$ dividing both cides by 2c, therefore,

$$e = \frac{\left(c^2 + a^2 - b^2\right)}{2c} \quad --- \quad \text{equation (9)}$$

Substituting e into $h1^2 = a^2 - e^2$ in equation (5) we have that

$$h1 = \sqrt{a^2 - \left[\frac{c^2 + a^2 - b^2}{2c}\right]^2} \quad - \ - \ - \quad \text{equation (10)}$$

We can therefore extend the same definition to h2 in terms of c, f, and g. being careful of the correct position of c, f and g, such that the line being cut by the perpendicular bisector remains the denominator in the equation and the first squared term is same as the one being added to the square of the line being cut by the bisector.

$$h2 = \sqrt{f^2 - \left[\frac{c^2 + f^2 - g^2}{2c}\right]^2} \quad - \ - \ - \quad \text{equation (11)}$$

We now have all the unkowns in terms of the knowns required to calculate the area of the irregular quadilateral. Let's plug h1 and h2 into the area calculation

$$Area = \frac{1}{2}c\left[\sqrt{a^2 - \left[\frac{c^2 + a^2 - b^2}{2c}\right]^2} + \sqrt{f^2 - \left[\frac{c^2 + f^2 - g^2}{2c}\right]^2}\right]$$

--- equation (12)