# Contents



# 1 differential equations

## 1.1 heat

we evolve the total energy balance equation since the flow of energy between the participating particles is dependent on heating/cooling and exchange of particles themselves.

Assumptions:

- electron beam current / density unchanged during time evolution

The set of equations for the energy is given by:

$$\frac{d}{dt}\left[\frac{3}{2}N_iT_i\right] = \frac{d}{dt}\left[\frac{3}{2}N_iT_i\right]^{Spitzer} + \frac{d}{dt}\left[\frac{3}{2}N_iT_i\right]^{Exchange} + \frac{d}{dt}\left[\frac{3}{2}N_iT_i\right]^{Escape} \tag{1}$$

### 1.1.1 Spitzer heating:

Spitzer heating is given by:

$$\frac{d}{dt}E^{Spitzer} = f_{e,i}N_iJ_e\frac{eq_i^2\ln\Lambda_{e,i}}{4\pi\epsilon_0^2m_iv_e^2} = f_{e,i}N_i\ln\Lambda_{e,i}\cdot 1.569\cdot 10^{15}\frac{J_e\hat{q}_i^2}{\mu_iv_e^2}\quad\left[\frac{eV}{s}\right] \tag{2}$$

with the electron-ion Coulomb logrithm assumed to be $ln\Lambda_{e,i} = 10$ and the current density $J_e$ given in A/cm^2 and the electron speed v_e given in cm/s:

### 1.1.2 Escape cooling

$$\left[\frac{dE}{dt}\right]^{escape} = -\left(\frac{2}{3}E_i + N_i q_i V_t\right) R_i^{Esc} \tag{3}$$

### 1.1.3 Heat exchange

$$\left[\frac{dE}{dt}\right]^{\text{exchange}} = N_i \sum_{j,\alpha} f_{i,j} \tau_{i,j}^{-1} k(T_j - T_i) = \frac{2}{3} \sum_{j,\alpha} \tau_{i,j}^{-1} f_{i,j} \left(E_j - E_i\right) \tag{4}$$

### 1.1.4 overlap factor

the overlap factor is defined as:

$$f_{e,i} = \left(\frac{r_e}{r_i}\right)^2 \tag{5}$$

and the characteristic radius $r_i$ given as:

$$r_i = \begin{cases} r_e \sqrt{\frac{k_b T_i}{q_i e V_e}}, & \text{if } q_i e V_e > k_b T_i \\ r_e \exp\left[\frac{k_b T_i}{2 q_i e V_e} - \frac{1}{2}\right], & \text{if } q_i e V_e \leq k_b T_i \end{cases} \tag{6}$$

ignoring the difference and assuming the 1/x dpendence, we define the overlap as:

$$f_{e,i} = \frac{q_i V_e}{k_b T_i} = \frac{3}{2} q_i V_e \frac{N_i}{E_i} \tag{7}$$

using the same simplification we arrive for the ion-ion overlap, at:

$$f_{i,j} = \frac{N_j q_j E_i}{N_i q_i E_j} \tag{8}$$

### 1.1.5 density

$$n_i\left(E_i, N_i\right) = \frac{q_i V_e}{L \pi r_e^2} \cdot \frac{N_i^2}{E_i} \tag{9}$$

#### 1.1.6 ion relaxation time:

$$\tau_{ij} = \frac{3(2\pi)^{3/2}\epsilon_0 m_i m_j}{2q_i^2 q_j^2 n_j \ln\Lambda_{ij}} \left(\frac{kT_i}{m_i} + \frac{kT_j}{m_j}\right)^{3/2} \tag{10}$$

$$\tau_{ij} = 7.37{\cdot}10^{12} \frac{A_i A_j}{\hat{q}_i^2 \hat{q}_j^2 n_j \ln\Lambda_{i,j}} \times \left(\frac{kT_i}{A_i} + \frac{kT_j}{A_j}\right)^{3/2} [s] = 4.01{\cdot}10^{12} \frac{A_i A_j}{\hat{q}_i^2 \hat{q}_j^2 n_j \ln\Lambda_{i,j}} \times \left(\frac{E_i}{N_i A_i} + \frac{E_j}{N_j A_j}\right)^{3/2} [s]\,. \tag{11}$$

#### 1.1.7 Rate of escape

$$R_i^{Esc} = \frac{3}{\sqrt{3}} \nu_i \frac{e^{-\omega_i}}{\omega_i}, \tag{12}$$

where $\nu_i = \sum_{j,\alpha} f_{i,j} \tau_{i,j}^{-1}$ is the Coulomb collision frequency for ions of charge state $q_i$ with *all* other ion species and $\omega_i$ is given by:

$$\omega_i = \frac{q_i V_t}{kT_i}, \tag{13}$$

### 1.2 todos

#### 1.2.1 TODO include charge exchange