Vibration Mechanics: Report 2
Rotor on flexible supports

Federico Pavone S305830

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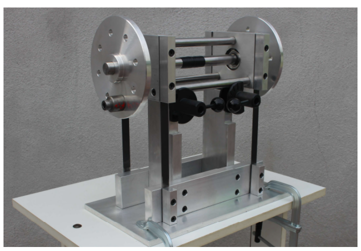
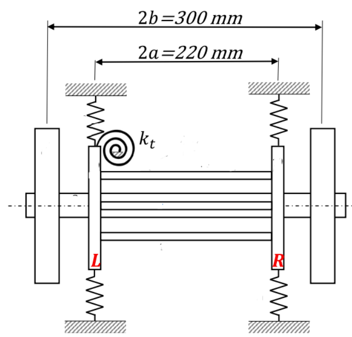
Abstract

The purpose of this report is to briefly analyse a dynamic system composed of a rotor supported by flexible supports.

All the code, images and data are available on my Github repository



(a) Real system



(b) Scheme of the system

Figure 1

1 Free body diagram

The system can be modelled as a conservative system with two degrees of freedom, respectively the horizontal displacement x and the rotation around the baricenter of shaft θ in the plane xy, and under the assumption of small displacement ($\sin\theta = \theta$)

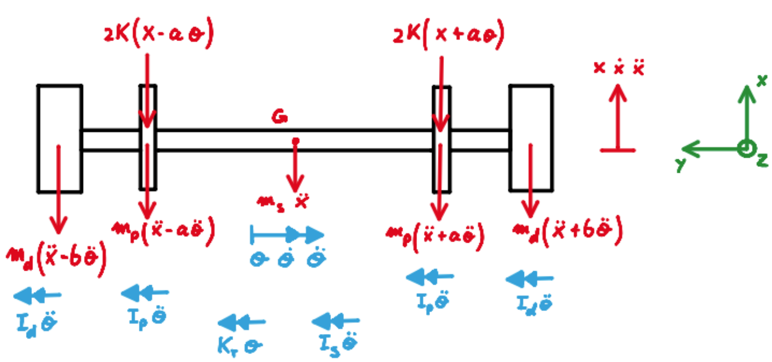


Figure 2: Free body diagram of the system

2 Equations of motion

From the free body diagram, the equations of motion can be obtained:

$$\begin{cases} (\mathrm{m}_{shaft} + 2m_{plate} + 2m_{disk})\ddot{x} + 4Kx = 0 \\ (\mathrm{I}_{shaft} + 2I_{plate} + 2I_{disk} + 2m_{plate}a^2 + 2m_{disk}b^2)\ddot{\theta} + (K_t + 4Ka^2)\theta = 0 \end{cases}$$

where the symmetric contribution of the inertial terms of the masses have been simplified, due to the system being symmetric, K is the stiffness of the thin beam that support the shaft and K_t is the torsional stiffness of the system, given as data and calculated by FEM analysis.

3 Physical parameters

From the drawing provided and having made some assumptions, such as neglecting the influence of the holes and the eccentric masses on the rotors, the physical parameters needed by the model have been calculated.

Moments of inertia can be calculated from the technical drawings as:

$I_{shaft} = \frac{mL^2}{12}$

$I_{plate} = \frac{m(a^2+h^2)}{12}$

$I_{disk} = \frac{md^2}{16}$

Table 1: Physical parameters

	Mass (kg)	Moment of inertia (kg m^2)
Shaft	0.86	$8.8 \cdot 10^{-3}$
Plate	0.45	$4.6 \cdot 10^{-4}$
Disk	0.48	$6.7 \cdot 10^{-4}$
Rod	0.28	$9.2 \cdot 10^{-4}$

4 Estimate of stiffness K

The estimation of the stiffness of the thin beam for the motion in the horizontal direction (previously denoted as x) has been made using the formula:

$K = \frac{12 \cdot E \cdot I_{thinbeam}}{L^3}$

where $I_{thinbeam} = \frac{bh^3}{12}$ is the moment of area of the section.

As a result, the stiffness of one beam is $K = 1.74 \cdot 10^3 \frac{N}{m}$

5 Natural frequencies

Since the equation of motion of the rotation and the translation are decoupled, the natural frequencies have been evaluated by solving the two equations previously derived by the free body diagram:

$\omega_t = \sqrt{\frac{4K}{m_{shaft}+2m_{plate}+2m_{disk}}}$

$\omega_r = \sqrt{\frac{K_t+4Ka^2}{I_{shaft}+2I_{plate}+2I_{disk}+2m_{plate}a^2+2m_{disk}b^2}}$

and knowing that $f = \frac{\omega}{2\pi}$

$f_t = 8.07Hz \quad f_r = 39.12Hz$

6 Comments

Table 2: Experimental and theoretical frequencies

	Experimental (Hz)	Theoretical (Hz)
f_t	8.07	8.06
f_r	39.12	31.9

Compared to the experimental frequencies, the vibration of the horizontal translation of the system is well predicted by the model, while the rotational frequency shows a 18% error.

The relative large error can be explained by the approximation made on the behaviour of the constrains (no damping), the simplification made on the geometry of the various components of the systems and the eccentric masses on the disks neglected.