Problems

Expansion of a Boolean expression to standard SOP form (CANONICAL FORM)

1. Write down all the terms
2. If one or more variables are missing in any term, expand that term by multiplying it with the sum of each one of the missing variable and its complement
3. Drop out the redundant terms

Another method

1. write down all the terms
2. Put Xs in terms where variables must be inserted to form a min term
3. Replace complemented variables by 0s and noncomplemented variables by 1s. and use all combinations of Xs in terms of 0s and 1s to generate min terms.
4. Drop out all the redundant terms.

Qn. Expand $\bar{A}+\bar{B}$ to min terms and max terms ?

missing variable to 1st term

$$\bar{A}+\bar{B} = \bar{A}(B+\bar{B}) + \bar{B}(A+\bar{A})$$

$$= \bar{A}B+\bar{A}\bar{B}+A\bar{B}+\bar{A}\bar{B}$$

$$= 01 + 00 + 10 + 00$$

$$= m_1 + m_0 + m_2 + m_0$$

$$= \Sigma m(0, 1, 2)$$

<u>another method</u>

$\bar{A}+\bar{B} \Rightarrow$

1st term $\rightarrow$ $\bar{A}$

$\rightarrow 0X \rightarrow$ $00 \rightarrow m_0$, $01 \rightarrow m_1$

2nd term

$\bar{B} \rightarrow X0 \rightarrow$ $00 \rightarrow m_0$, $10 \rightarrow m_2$

$$= \Sigma m(m_0, m_1, m_2) = \Pi M(3)$$

In the above eq: $\boxed{\Sigma m(0,1,2) = \Pi M(3)}$

Qn: Expand $Y = A(\bar{A}+B)(\bar{A}+B+\bar{C})$ to max terms and min terms?

$A \rightarrow 0XX = (000)(001)(010)(011) = M_0 \cdot M_1 \cdot M_2 \cdot M_3$.

$\bar{A}+B \rightarrow 10X = (100)(101) = M_4 \cdot M_5$

$\bar{A}+B+\bar{C} = 101 - M_5$ There fore, $\Pi M(0, 1, 2, 3, 4, 5)$

Minterm

For n binary variables, one can obtain 2^n distinct minterms. The minterms whose sum defines the Boolean function are those that give the 1's of the function in the truth table.

eg: Express the Boolean function $F = A + \bar{B}C$ in a sum of minterms.

Ans) The function has 3 variables, A B & C. The first term A is missing two other variables. therefore

$$A = A(B+\bar{B}) = AB + A\bar{B}$$

$AB + A\bar{B}$ → This term missing one variable C. So

$$(AB + A\bar{B})(C+\bar{C}) = ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$$

The 2^{nd} term $\bar{B}C$ is missing one variable ie A. So

$$\bar{B}C = \bar{B}C(A+\bar{A})$$

$$= \bar{B}CA + \bar{B}C\bar{A}.$$

$$F = ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$$

$$F = ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C$$

$$F(A,B,C) = \sum_m (1, 4, 5, 6, 7) = \prod_M (0, 2, 3)$$

This is known as <u>CANONICAL FORM</u> of representation.

eg: Reduce the boolean expression $(\overline{A+\overline{B}C})(A\overline{B}+ABC)$

$= (\bar{A}\cdot\overline{\overline{B}C})(A\overline{B}+ABC)$

$= (\bar{A}\cdot BC)(A\overline{B}+ABC)$

$= \cancel{\bar{A}A\overline{B}+}\ \bar{A}ABC\overline{B} + \bar{A}BCABC$

$= \underline{\underline{0}}$

eg: Show that ~~$A\overline{B}C + B + B\overline{D} + AB\overline{D} + \bar{A}C =$~~ .

eg: ~~Demorganize $\overline{AB+A\overline{C}}$~~ .

$\cancel{= A\overline{B}C + \bar{A}C + B(1+\overline{D}+A\overline{D})}$

$= A\overline{B}C + \bar{A}C + B$

$= C(A\overline{B}+\bar{A}) + B$

$= C(\bar{A}+\overline{B}) + B.$

$= \bar{A}C + \overline{B}C + B.$

~~$= A$~~

Qn: Simplify the boolean expression

$Y = (A+AB)(B+BC)(C+AB)$

$= A(1+B)\,B(1+C)(C+AB)$

$= AB(C+AB)$

$= ABC + AB\cdot AB$

$= ABC + AB$

$= AB(\underbrace{1+C}_{1})$

$= \underline{\underline{AB}}$

Qn: Expand $F = A + B\overline{C} + AB\overline{D} + ABCD$ to minterm and maxterms.

first variable A doesnot contain other 3 variables. so multiply with

$A = A(B+\overline{B})(C+\overline{C})(D+\overline{D})$

$= (AB + A\overline{B})(C+\overline{C})(D+\overline{D})$

$= ABCD + ABC\overline{D} + A\overline{B}CD + A\overline{B}C\overline{D} + AB\overline{C}D + AB\overline{C}\overline{D} + A\overline{B}\overline{C}D + A\overline{B}\overline{C}\overline{D}$

or

A B C D

A → 1 X X X

1 0 0 0 → 8

1 0 0 1 → 9

1 0 1 0 → 10

1 0 1 1 → 11

1 1 0 0 → 12

1 1 0 1 → 13

1 1 1 0 → 14

1 1 1 1 → 15

2nd term $B\bar{C} \Rightarrow B\bar{C}(A+\bar{A})(D+\bar{D})$ (or) ↗

$B\bar{C} \rightarrow$

A	B	C	D	
X	1	0	X	
0	1	0	0	→ 4
0	1	0	1	→ 5
1	1	0	0	→ 12
1	1	0	1	→ 13

$= B\bar{C}AD + B\bar{C}A\bar{D} + B\bar{C}\bar{A}D + B\bar{C}\bar{A}\bar{D}$

$= AB\bar{C}D + AB\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D}$

3rd term $AB\bar{D} = AB\bar{D}(C+\bar{C})$ or $AB\bar{D} \rightarrow 11X0$

$= ABC\bar{D} + AB\bar{C}\bar{D}$

$1100 \rightarrow 12$

$1110 \rightarrow 14$

The function $F = ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} +$

$\underbrace{A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D}}_{1^{st}\text{ term}} + \underbrace{AB\bar{C}D + AB\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D}}_{2nd\text{ term}} +$

$\underbrace{ABC\bar{D} + AB\bar{C}\bar{D}}_{3^{rd}\text{ term}} + \underbrace{ABCD}_{4^{th}\text{ term}}$

$F = ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} +$

$\bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D}$

$= m_{15} + m_{14} + m_{13} + m_{12} + m_{11} + m_{10} + m_9 + m_8 + m_5 + m_4$

SOP form $= \Sigma m(4, 5, 8, 9, 10, 11, 12, 13, 14, 15) \rightarrow$ Canonical SOP form.

Therefore the Pos form is $\Pi M(0, 1, 2, 3, 6, 7) \rightarrow$ Canonical POS form

eg: Expand $\bar{A} + \bar{B}$ to minterms and max terms.

Ans) $\bar{A} = \bar{A}(B+\bar{B}) = \bar{A}B + \bar{A}\bar{B}$

$\bar{B} = \bar{B}(A+\bar{A}) = A\bar{B} + \bar{A}\bar{B}$.

$y = \bar{A}B + \bar{A}\bar{B} + A\bar{B} + \bar{A}\bar{B}$

$= \bar{A}B + A\bar{B} + \bar{A}\bar{B}$

01, 10, 00

$y = \Sigma m(0, 1, 2) \rightarrow$ SOP expression (Minterms)

$y = \Pi M(3)$