# Plasma Parameters

## Plasma frequency, Debye length, etc.

Teaching Assistant: Chitradeep Saha

---

Plasmas are **macroscopically neutral** substances containing many **interacting free electrons and ionized atoms or molecules**, which exhibit **collective behaviour** due to long-range Coulomb forces.

### Debye length

The Debye length is an important physical parameter for the description of a plasma. It provides a measure of the distance over which the influence of the electric field of an individual charged particle (or of a surface at some nonzero potential) is felt by the other charged particles inside the plasma. The charged particles arrange themselves in such a way as to effectively shield any electrostatic fields within a distance of the order of the Debye length. This shielding of electrostatic fields is a consequence of the collective effects of the plasma particles.

Debye length ($\lambda_D$) is directly proportional to the square root of the temperature ($T$) and inversely proportional to the square root of the electron number density ($n_e$) according to,

$$\lambda_D = \sqrt{\frac{\epsilon_0 \kappa_B T}{\mathbf{2} n_e e^2}}$$

In CGS system of units,

$$\lambda_D = \sqrt{\frac{\kappa_B T}{\mathbf{8}\pi n_e e^2}}$$

In the neighborhood of any surface inside the plasma there is a layer of width of the order of $\lambda_D$, known as the plasma sheath, inside which the condition of macroscopic electrical neutrality may not be satisfied. Beyond the plasma sheath region there is the plasma region, where macroscopic neutrality is maintained.

A necessary and obvious requirement for the existence of a plasma is that the physical dimensions of the system be large compared to $\lambda_D$. Otherwise there is just not sufficient space for the collective shielding effect to take place, and the collection of charged particles will not exhibit plasma behavior. If $L$ is a characteristic dimension of the plasma, a ***first criterion*** for the definition of a plasma is therefore,

$$L >> \lambda_D$$

### Debye sphere

It is convenient to define a Debye sphere as a sphere inside the plasma of radius equal to $\lambda_D$. Any electrostatic fields originated outside a Debye sphere are effectively screened by the charged particles and do not contribute significantly to the electric field existing at its center. Consequently, each charge in the plasma interacts collectively only with the charges that lie inside its Debye sphere, its effect on the other charges being effectively negligible. The number of electrons $N_D$, inside a Debye sphere, is given by

$$N_D = \frac{4}{3}\pi\lambda_D^3 n_e$$

Since the shielding effect is the result of the collective particle behavior inside a Debye sphere, it is also necessary that the number of electrons inside a Debye sphere be very large. Or in other words, the average distance between electrons, which is roughly given by $n_e^{-1/3}$, must be very small compared to $\lambda_D$. A ***second criterion*** for the definition of a plasma is therefore,

$$n_e\lambda_D^3 >> 1$$

## Plasma parameter, $g$

The quantity defined by

$$g = \frac{1}{n_e\lambda_D^3}$$

is known as the plasma parameter and the condition $g << 1$ is called the ***plasma approximation***.

**Another interpretation of the plasma parameter is that it is a measure of the potential energy of interactions compared to the kinetic energy.** When $g$ is small (as for a low-density plasma), the interaction amongst particles is weak, but a large number of particles interact simultaneously. On the other hand, a larger $g$ implies few particles interacting collectively, but interacting strongly. The limit of small $g$ is referred to as the **plasma limit**.

## Plasma frequency

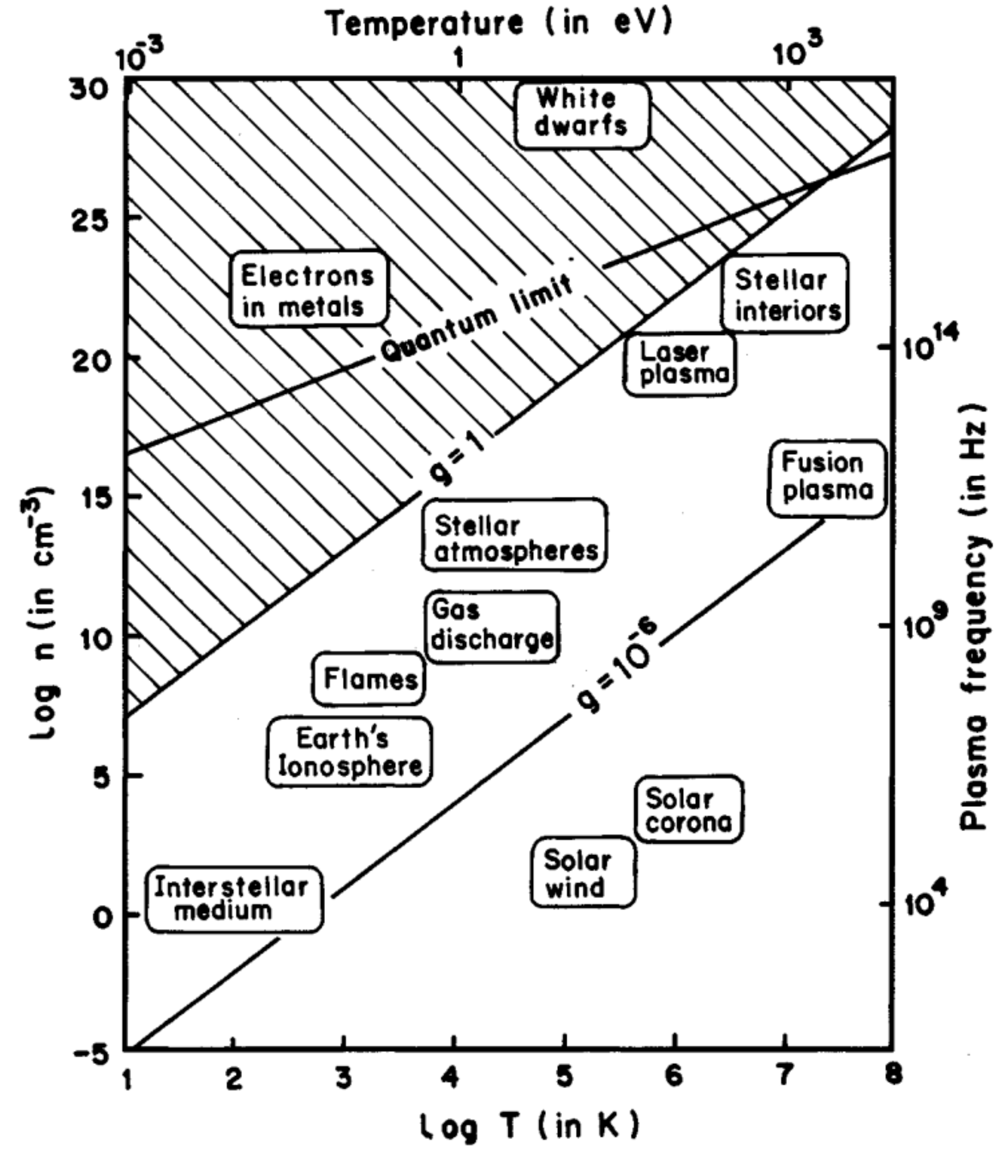
An important plasma property is the stability of its macroscopic space charge neutrality. When a plasma is instantaneously disturbed from the equilibrium condition, the resulting internal space charge fields give rise to collective particle motions that tend to restore the original charge neu- trality. These collective motions are characterized by a natural frequency of oscillation known as the plasma frequency,

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

## Practice problem:

Find out the Debye length, plasma parameter $g$, plasma frequency for the following physical systems and comment if the *plasma approximation* holds in each cases,

- Solar corona ($n_e = 10^8\text{cm}^{-3}; T_e = 10^6 K$)
- Solar wind ($n_e = 10\,\text{cm}^{-3}; T_e = 10^5 K$)
- Earth's Ionosphere ($n_e = 10^{12}\text{m}^{-3}; T_e = 10^3 K$)
- Keplerian accretion disk around a stellar mass black hole ($n_e = 10^{17}\text{cm}^{-3}; T_e = 10^7 K$)


Temperature (in eV)
$10^{-3}$
1
$10^{3}$
30
25
20
15
10
5
0
-5
Log n (in $cm^{-3}$)
1
2
3
4
5
6
7
8
Log T (in K)
$10^{14}$
$10^{9}$
$10^{4}$
Plasma frequency (in Hz)
White dwarfs
Electrons in metals
Quantum limit
Stellar interiors
Laser plasma
g = 1
Fusion plasma
Stellar atmospheres
Gas discharge
$g = 10^{-6}$
Flames
Earth's Ionosphere
Solar corona
Solar wind
Interstellar medium