MidSem Exam – Spring 2024
SS4201: Fluid and Magneto-hydrodynamics
Model Answer

Answer Question 1 and any 4 out of the 5 remaining questions.

Note: Meaning of variables and symbols are same as we used in the class, unless stated otherwise. Clearly mention any assumption that you make while solving any problem.

Time : 90 min　　　　Marks: 25

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1. a) Derive continuity equation starting from the below equation:

$$\frac{\partial}{\partial t}(n\langle\chi\rangle) + \frac{\partial}{\partial x_i}\left(n\left\langle u_i\chi\right\rangle\right) - \frac{n}{m}\left\langle F_i\frac{\partial\chi}{\partial u_i}\right\rangle = 0$$

where, χ is a conserved quantity in a binary collision.

Solution 1a.

For $\chi = m$, where m is the fluid mass, we have the following relations –

$$n\langle\chi\rangle = nm = \rho; \quad \frac{\partial m}{\partial x_i} = 0 \;\text{ and, }\; \langle u_i\chi\rangle = m\left\langle v_i + w_i\right\rangle = m\left(\langle v_i\rangle + \langle w_i\rangle\right) = mv_i$$

Utilizing these relations in the provided equation,

$$\tfrac{\partial}{\partial t}(nm) + \tfrac{\partial}{\partial x_i}\left(nm\nu_i\right) = 0 \qquad \therefore \tfrac{\partial\rho}{\partial t} + \tfrac{\partial}{\partial x_i}\left(\rho\nu_i\right) = 0 \quad \text{(Mass continuity equation for fluids)}$$

b) Lorentz force $\vec{F} = q(\vec{u}\times\vec{B})$ is velocity dependent. Show that even for such situation, $\left\langle\frac{\partial F_i}{\partial u_i}\chi\right\rangle = 0$

Solution 1b.

In vector notation we can write the i-th component of Lorentz force as,

$$F_i = q\left(u_jB_k - v_kB_j\right) \qquad \therefore \quad \frac{\partial F_i}{\partial u_i} = 0 \qquad \therefore \quad \left\langle\frac{\partial F_i}{\partial u_i}\chi\right\rangle = 0$$