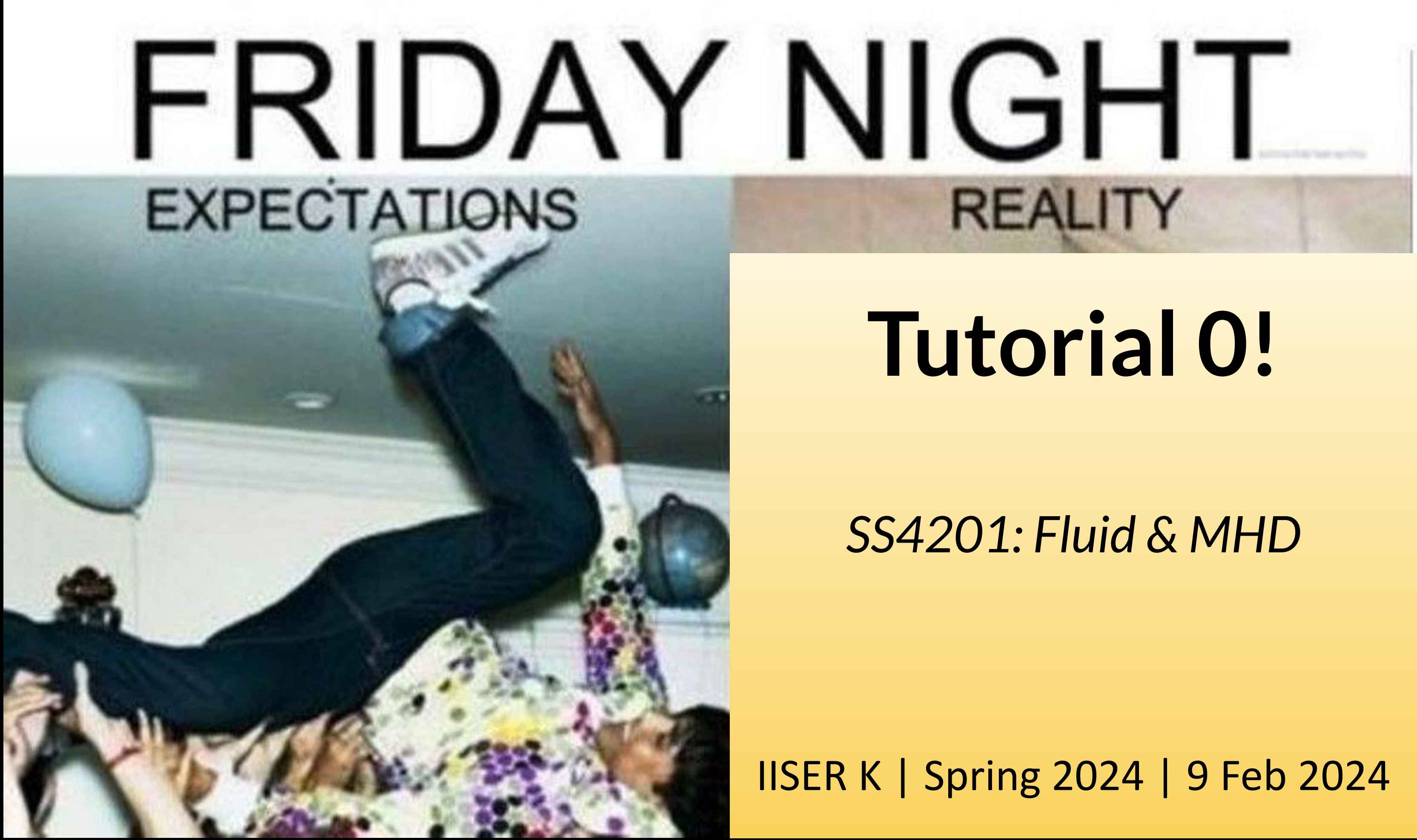

Tutorial 0!

SS4201: Fluid & MHD

IISER K | Spring 2024 | 9 Feb 2024

MidSem Exam - Spring 2024
SS4201: Fluid and Magneto-hydrodynamics

Answer Question 1 and any 4 out of the 5 remaining questions.

Time : 90 min Marks: 25

Q1. a) 1 M

b) 1 M

c) 1 M

d) 1 M

e) 1 M

1 mark x 5 questions = 5

Q2. 5 M

Q3. 5 M

Q4. 5 M

Q5. 5 M

Q6. 5 M

5 marks x 4 questions = 20

Addressing the **queries/doubts** received so far …

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I had a doubt 2:14 pm

The-physics-of-fluids-and-plasmas-an-introduction-for-astrophysicists.pdf

probability of deflection in a certain direction. We are interested here in a statistical treatment so that it is enough for us to know the probability of deflection in different directions. This can be handled by introducing the concept of a *differential scattering cross-section* discussed below. We shall not concern ourselves here with the question of calculating this cross-section from the interaction potential. We only discuss how the dynamics of our system can be studied when the scattering cross-section is given.

For the definition of the differential scattering cross-section in the context of classical mechanics, let us consider a beam of particles of number density n_1 and velocity $\mathbf{u}_1$ colliding with another beam of particles of number density n and velocity $\mathbf{u}$. A particle in the second beam experiences a flux $I = |\mathbf{u} - \mathbf{u}_1| n_1$ of particles from the first beam. We consider the number of collisions δn_c per unit volume per unit time which deflect particles from the second beam into a solid angle $d\Omega$. This number must be proportional to the number density n of particles in the second beam, proportional to the flux I these particles are exposed to and proportional to the solid angle $d\Omega$. Hence we write

$$\delta n_c = \sigma(\mathbf{u}, \mathbf{u}_1 | \mathbf{u}', \mathbf{u}_1') \cdot n \cdot |\mathbf{u} - \mathbf{u}_1| n_1 \cdot d\Omega, \qquad (2.6)$$

How is this true? 2:14 pm

19 January 2024

$$ff_1 = f'f'_1. \tag{2.16}$$

Since the distribution function for a uniform gas in equilibrium depends on the particle velocity alone, we need not explicitly indicate the dependences on $\mathbf{x}$ and t so that the logarithm of (2.16) would give

$$\log f(\mathbf{u}) + \log f(\mathbf{u}_1) = \log f(\mathbf{u}') + \log f(\mathbf{u}'_1), \tag{2.17}$$

where $\mathbf{u}$ and $\mathbf{u}_1$ are the velocities of two particles before a binary collision, and $\mathbf{u}'$ and $\mathbf{u}'_1$ are the velocities after the collision. If $\chi(\mathbf{u})$ is a quantity which is conserved during a binary collision, then we must also have

$$\chi(\mathbf{u}) + \chi(\mathbf{u}_1) = \chi(\mathbf{u}') + \chi(\mathbf{u}'_1). \tag{2.18}$$

Comparing (2.17) and (2.18), we conclude that the most general expression for the distribution function must be of the form

$$\log f(\mathbf{u}) = C_0 + \sum_r C_r \chi_r(\mathbf{u}), \tag{2.19}$$

where $\chi_r(\mathbf{u})$-s should include all the independently conserved quantities and C_r-s are constants. If conservations of energy and the three components of momenta are all the independent conservation laws

HD

Dada bol6i eai line ta ARC er boi thky bujte parchilam

12:24 am

help me out here 10:28 pm

Let Q be any quantity associated with each particle. The average value of Q is defined by

$$\langle Q \rangle = \frac{1}{n} \int d^3u\, Q f, \tag{2.36}$$

where

$$n = \int d^3u\, f$$

why are we only integrating over velocity space here? 10:33 pm

Evolution of volume density of any conserved quantity in a system of particles with binary collisions

$$\frac{\partial}{\partial t}(n\langle\chi\rangle) + \frac{\partial}{\partial x_i}(n\langle u_i\chi\rangle) - n\left\langle u_i \frac{\partial \chi}{\partial x_i}\right\rangle - \frac{n}{m}\left\langle F_i \frac{\partial \chi}{\partial u_i}\right\rangle - \frac{n}{m}\left\langle \underline{\frac{\partial F_i}{\partial u_i}}\chi\right\rangle = 0. \quad (2.37)$$

For $\chi = mu_j$

$$\frac{\partial}{\partial t}(nm\langle u_j\rangle) + \frac{\partial}{\partial x_i}(nm\langle u_i u_j\rangle) - nF_j = 0. \quad (3.5)$$

Let us define the pressure tensor,

$$P_{ij} = nm\langle(u_i - v_i)(u_j - v_j)\rangle. \quad (3.6)$$

$$\Big\downarrow$$

$$P_{ij} = mn\left(\frac{m}{2\pi\kappa_B T}\right)^{3/2}\int d^3U\, U_i U_j \exp\left(-\frac{mU^2}{2\kappa_B T}\right), \quad (3.18)$$

Momentum-average

$$\langle Q\rangle = \frac{1}{n}\int d^3u\, Qf, \quad (2.36)$$

Zero-order distribution

$$f^{(0)}(\mathbf{x},\mathbf{u},t) = n(\mathbf{x},t)\left[\frac{m}{2\pi\kappa_B T(\mathbf{x},t)}\right]^{3/2}\exp\left[-\frac{m\{\mathbf{u}-\mathbf{v}(\mathbf{x},t)\}^2}{2\kappa_B T(\mathbf{x},t)}\right], \quad (3.17)$$

Can we derive ideal gas equation from here?

Homework!

$$\epsilon = \tfrac{1}{2}\langle|\mathbf{u}-\mathbf{v}|^2\rangle \tag{3.10}$$

$$\downarrow$$

$$\epsilon = \frac{3}{2}\frac{\kappa_B T}{m}. \tag{3.22}$$

Momentum-average

$$\langle Q\rangle = \frac{1}{n}\int d^3u\, Qf, \tag{2.36}$$

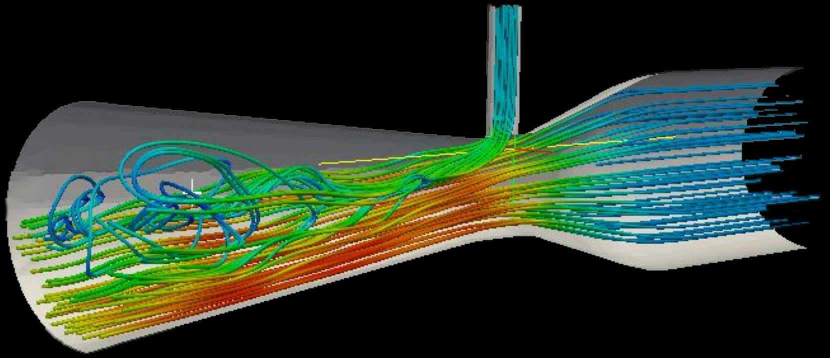
Zero-order distribution

$$f^{(0)}(\mathbf{x},\mathbf{u},t) = n(\mathbf{x},t)\left[\frac{m}{2\pi\kappa_B T(\mathbf{x},t)}\right]^{3/2}\exp\left[-\frac{m\{\mathbf{u}-\mathbf{v}(\mathbf{x},t)\}^2}{2\kappa_B T(\mathbf{x},t)}\right], \tag{3.17}$$

*Special tool: Harnessing momentum sphere may make your job easier!

Start with the Euler equation:

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{2}\nabla(\mathbf{v}\cdot\mathbf{v}) - \mathbf{v}\times(\nabla\times\mathbf{v}) = -\frac{1}{\rho}\nabla p + \mathbf{F}.$$



To derive the vorticity equation for an incompressible fluid:

$$\frac{\partial \omega}{\partial t} = \nabla\times(\mathbf{v}\times\omega). \qquad (4.20)$$

Some MAGIC* numbers!

Problems 4.1, 4.4, 4.5,....

*Including but not limited to.

Memorize

Euler equations in cartesian/vector form

Perform a scaling analysis of the Navier-Stokes equation to derive the definition of Reynolds Number

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{F} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}, \tag{5.10}$$

Flow through a circular pipe: Poiseuille's Equation

The basic accretion disk dynamics

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https://github.com/deephysics1729