Assignment 1 Problem

Each question carries 2 marks. Discussion among students allowed, but copying from each other strictly prohibited.
Note: Meaning of a variable is same as we use in the class.

Due Date : 29 March, 2023 Marks: 10

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1. We derived a linear relationship between pressure (P) and depth (z) for water at rest : $P(z) = P_0 - \rho g z$. Now, if the water is accelerating along x-axis and gravity, as usual, along z-axis, how will the pressure $P(x, z)$ expression be modified?

2. In the Mid-sem exam following problem was given: If an object heavier than water is fully immersed in water, the net force exerted on the object by surrounding water is $-Mg$, where M is mass of water displaced by the object. How does this net force change when water is accelerating? Results from prob. 1 may help.

3. Starting from the following energy conservation equation

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2 + \rho\epsilon\right) + \frac{\partial}{\partial x_i}\left[\left(\frac{1}{2}\rho u^2 + \rho\epsilon + P\right)u_i\right] = \rho u_i F_i,$$

show that one can arrive at

$$\frac{\partial \epsilon}{\partial t} + u_i\frac{\partial \epsilon}{\partial x_i} + \frac{P}{\rho}\frac{\partial u_i}{\partial x_i} = 0.$$

4. In many simulations, we use scaled units instead of real units and the underlying equations gets modified as per our re-definition of units.

a) Consider the ideal, non-viscous hydrodynamics equations in conservative form (e.g., mass, momentum and energy density equations). Write these equations in spherical polar coordinates (r, θ, ϕ) and consider 1D flow along radial direction only (i.e., throw away $\frac{\partial}{\partial\theta}$ and $\frac{\partial}{\partial\phi}$ terms).

b) Now, we wish to study the accretion problem onto a compact star of mass M by solving these equations. We measure lengths in units of GM/c^2, velocity in units of c. Also, we measure density in units of a reference density $\rho_{\rm ref}$ such that at certain radius $R_{\rm max}$, the radial mass flux is equal to the the accretion rate $\dot{m}$ g/sec. Write down the above 1D hydro-equations in this unit system. (Here, G is gravitational constant, c is speed of light, we used ideal gas and adiabatic equation of state $P = K\rho^\gamma$. Also, explicitly mention any assumption that you make while solving this problem.)

5. Find out the solution of spherical accretion/wind using adiabatic condition. These are famous Bondi accretion and Parker wind solutions.