CSCI 677: Homework 1

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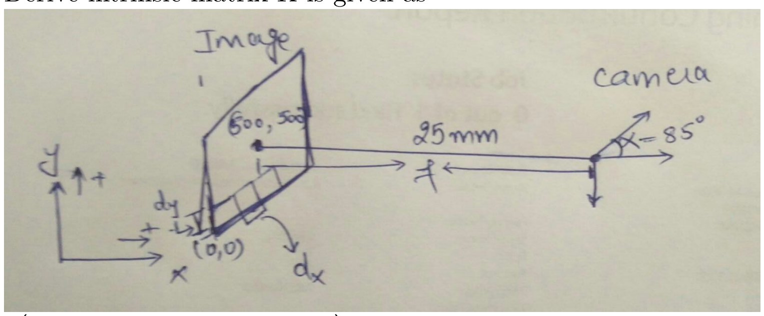
1. Derive intrinsic matrix K is given as



$$\begin{pmatrix} f/d_x & (-f/d_x)*cot\theta & x_0 \\ 0 & f/d_y * sin\theta & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

In this case
$f = 25mm$
$\theta = 85°$
$d_x = 0.04mm$
$d_y = 0.05mm$
$x0 = 500 * d_x y0 = 500 * d_y$

Putting these values back in the matrix K, we get

$$\begin{pmatrix} 25/0.04 & (-25/0.04)*0.08 & 500*0.04 \\ 0 & 25/0.05*0.99 & 500*0.05 \\ 0 & 0 & 1 \end{pmatrix}$$

Hence, K matrix is given as $\begin{pmatrix} 625 & -50 & 20 \\ 0 & 500 & 25 \\ 0 & 0 & 1 \end{pmatrix}$

2. Part (A) To prove that vanishing point depends on just the orientation and not position.
Consider a line in 3D given in camera coordinate system:
$X_c = a + \lambda b$
where a represents position and b represents orientation

But x, y in image plane is given as fX/Z, fY/Z.

Hence we can write

$$x = \frac{f*(a_x + \lambda b_x)}{a_z + \lambda b_z} \tag{1}$$

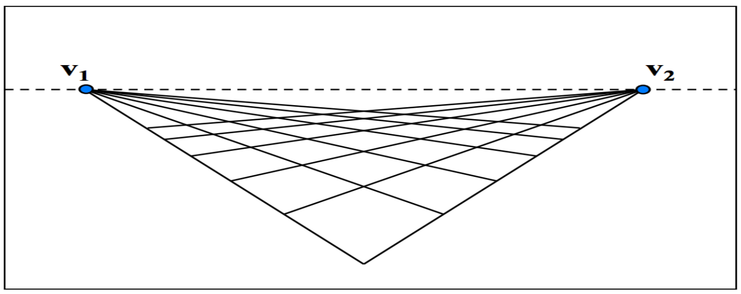
As λ approaches ∞, a becomes irrelevant and hence Vanishing point (x,

y) can be given as

$$f(\frac{b_x}{b_z}), f(\frac{b_y}{b_z}) \tag{2}$$

Hence we see that coordinates of vanishing point just depends on orientation and not position.

3. (Part B)



The dotted line shown in the above image is called Vanishing Line. To derive the equation of this line in 3D, consider a 3D plane with the equation:

$$X_c.n = d \tag{3}$$

$n = (n_x, n_y, n_z)$ i.e it is normal to plane.
Now, using perspective projection,

$$\begin{bmatrix} x \\ y \\ f \end{bmatrix} = \frac{f * X_c}{Z_c}$$

Now multiple both sides with direction vector n, to get

$$\begin{bmatrix} x \\ y \\ f \end{bmatrix} . \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \frac{f * X_c.n}{Z_c}$$

As already discussed in equation (3)

$$\begin{bmatrix} x \\ y \\ f \end{bmatrix} . \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \frac{f * d}{Z_c}$$

As, Z_c tends to ∞.The RHS of the above equation tends to 0.

Hence,

$$\begin{bmatrix} x \\ y \\ f \end{bmatrix} . \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = 0$$

Thus, equation of line is written as,

$$x * n_x + y * n_y + f * n_z = 0 \tag{4}$$

Hence, the vanishing line only depends on orientation of the plane and not its position.