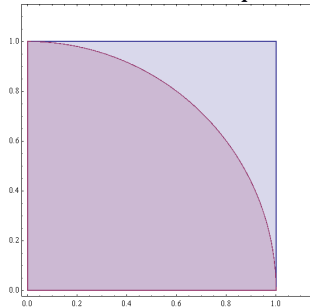


## EE 511 Simulation Methods for Stochastic Systems

### Project #4: Investigations on Monte Carlo Methods

#### [Pi-Estimation]

Generate  $n=100$  samples of i.i.d 2-dimensional uniform random variables in the unit-square. Count how many of these samples fall within the quarter unit-circle centered at the origin. This quarter circle inscribes the unit square as shown below:



- Use these random samples to estimate the area of the inscribed quarter circle. Use this area estimate to estimate the value of pi. Do  $k=50$  runs of these pi-estimations. Plot the histogram of the  $k=50$  pi-estimates (each estimate based on  $n=100$  2-D uniform samples).
- Repeat the experiment with different numbers of uniform samples,  $n$ . Plot the sample variance of the pi-estimates for these different values of  $n$ . Keep  $k=50$  for all these runs. Comment on the sample variance of your estimates.

#### [Monte Carlo Integration and Variance Reduction Strategies]

Use  $n=1000$  random samples to obtain Monte Carlo estimates for the definite integrals:

- $[1+\sinh(2x)\ln(x)]^{-1}$ ,  $x$  in  $[0.8, 3]$
- $\text{Exp}[-x^4 - y^4]$ ,  $(x, y)$  in  $[-\pi, \pi]$

Calculate the sample variance of the Monte Carlo estimates using a similar method as in problem 1.

Use the same number of random samples,  $n=1000$ , to obtain those Monte Carlo estimates. But this time incorporate stratification and importance sampling in the Monte Carlo estimation procedures. Compare the Monte Carlo estimates and their sample variances.

Discuss the quality of the Monte Carlo estimates from each method. Also discuss the strengths and weaknesses of stratification and importance sampling in Monte Carlo estimation.

Test your integral estimator on the following function with your own choice of  $n$  samples:

$$f(x, y) = 20 + x^2 + y^2 - 10(\cos[2\pi \times x] + \cos[2\pi \times y])$$

$(x, y)$  in  $[-5, 5]$  for  $f(x, y)$

Turn in:

- A summary of your experiments including histograms of your data, any relevant plots, and relevant numerical results
- brief discussions of the results
- a print out of your code.