# Probability:

1) (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

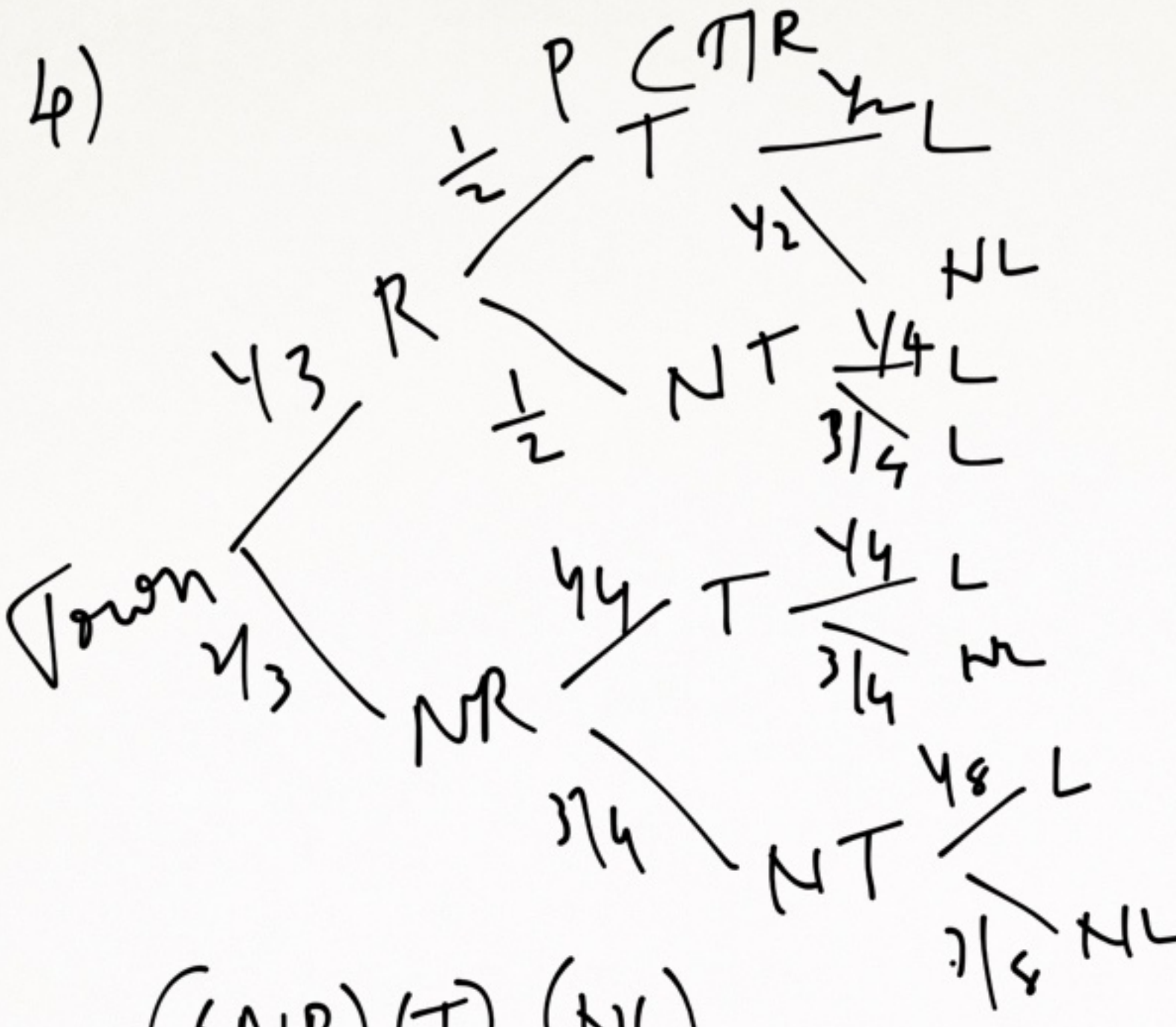
$P(\text{sum} = 7 \cap 6) = 5/30$

2) $P(\text{sum} < 7) = 15/36$

3) $P(h \geq 2 \mid h \geq 1) = \frac{4}{7}$

HHH TTT
HTH
HHT
THH
THT
TTH
HTT

4)




a) $P((NR)(T)(NL)) = \frac{2}{8} \times \frac{1}{4} \times \frac{3}{4} = \frac{1}{8}$

b) $p(L) = p(RTL) + p(R(NT)(L)) + p(NRTL) + p(NRNTL)$

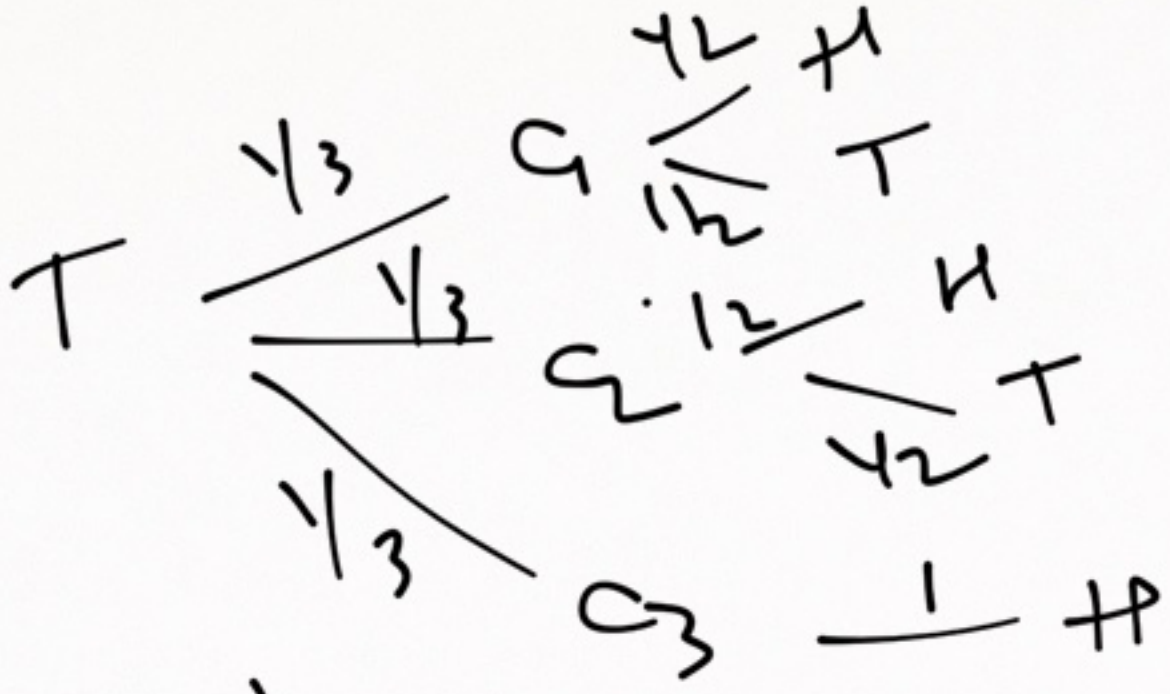
$= \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{4} + \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{1}{8}$

$= \frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16} = \frac{4+2+2+3}{48} = 11/48$

c) $p(R|L) = \frac{p(L \cap R)}{p(L)} = \frac{\frac{1}{24} + \frac{1}{12}}{\frac{4}{48}} = \frac{\frac{1}{8}}{\frac{11}{48}}$

$= \frac{6}{11}$

i)




ii) $p(H) = \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{3} = 2/3$

b) $p(C_3|H) = \frac{p(C_3 \cap H)}{p(H)} = \frac{\frac{1}{3} \cdot 1}{\frac{2}{3}} = \frac{1}{2}$

6) (S — 70% Coffee; 40% Coffee; 20% Coffee ∧ Coffee)

$P(Cf) = 0.7$

$P(Ck) = 0.4$

$P(Cf \mid Ck) = 0.2$

$$P(Cf \mid Ck) = \frac{P(Cf \wedge Ck)}{P(Ck)}$$

$$= \frac{0.2}{0.4} = 1$$

7) $\mu_p = 50 \quad \sigma_p = 6 \qquad \mu_{\bar{x}} = \mu_s$

a) $n = 16 \quad \sigma_s = \sigma_p / \sqrt{n} \quad \mu_s = 50$

$= 6 / \sqrt{16} = 3/2 = 1.5$

b) $n = 20$

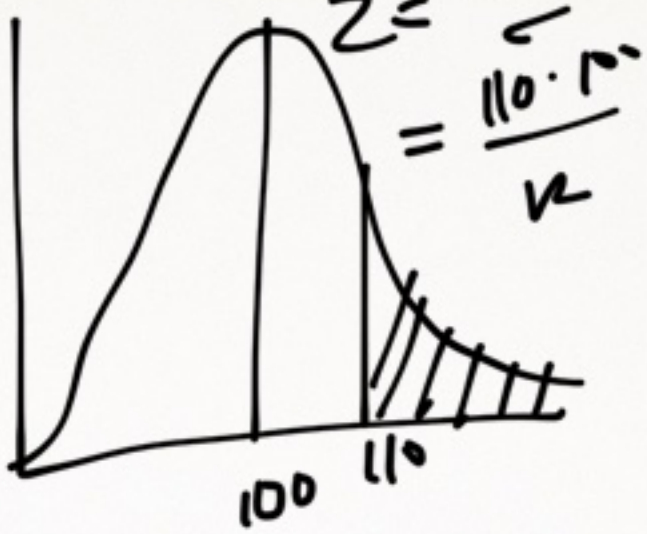
$\sigma_s = 6 / \sqrt{20}$

8) $\mu = 100 \quad \sigma = 12$

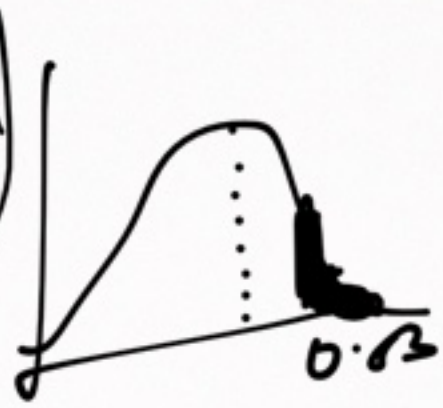
$$P(X > 110) = P(X - \mu > 110 - 100)$$

$$= P\left(\frac{X-\mu}{\sigma} > \frac{110-100}{12}\right)$$

$$P(X > 110) = P(Z > 0.83)$$




$$P(Z \leq 0.83) = 0.7967 \quad \text{(normal table)}$$




$$P(Z > 0.83) = 1 - 0.7967$$

$$= 0.2033.$$

$$P(X > 110) = 0.2033$$

b) $P(\bar{X}_{25} > 105) = P\left(Z > \frac{105-100}{2.4}\right)$ $\quad \sigma_{\bar{x}} = \frac{12}{\sqrt{25}}$

$$= P(Z > 2.083) \quad = \frac{12}{5} = 2.4$$

$$P(Z < 2.083) = 0.9812$$

$$P(\bar{X}_{25} > 105) = 1 - 0.9812$$

$$= 0.0188$$

c) $p(X_{64} > 105) = p\left(z > \frac{5}{1.5}\right)$

$= p(z > 3.33)$

$p(z < 3.33) = 0.9996$

$p(z > 3.33) = 1 - 0.9996$

$= 0.0004$

$\sigma_{64} = \frac{12}{\sqrt{64}} = \frac{12}{8} \cdot 1.5$

$= 1.5$

d) $p(95 < X_{16} < 105) =$

$z_{95} = \frac{95 - 100}{3} = \frac{-5}{3} = -1.67$ $\sigma_{16} = \frac{12}{\sqrt{16}} = \frac{12}{4}$

$z_{105} = \frac{5}{3} = 1.67 = 0.9525$

$-1.67 = 0.0475$

$p(95 < X_{16} < 105) = 0.9525 - 0.0475$

$= 0.905$

10) $\sigma = 2.8$

a) 95 %. $\rightarrow \mu \pm 2\sigma$

$$\mu = \frac{8+9+10+13+14+16+17+20+21}{9}$$

$$= 14.22$$

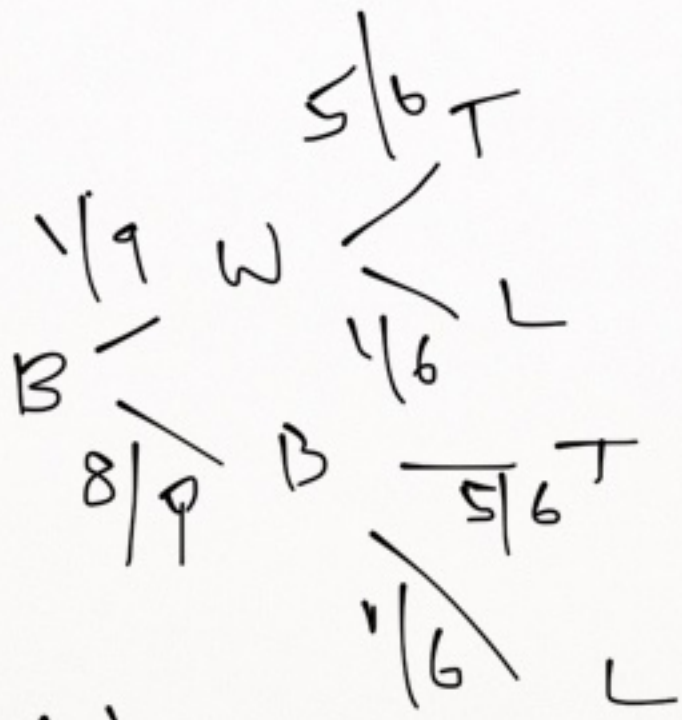
95 %. $\rightarrow 14.22 \pm 2(2.8)$

$\Rightarrow \underline{8.62 - 19.82}$

b) 99 %. $\rightarrow \mu \pm 3\sigma = 14.22 \pm 3(2.8)$

$\rightarrow \underline{5.82 - 22.62}$

11)

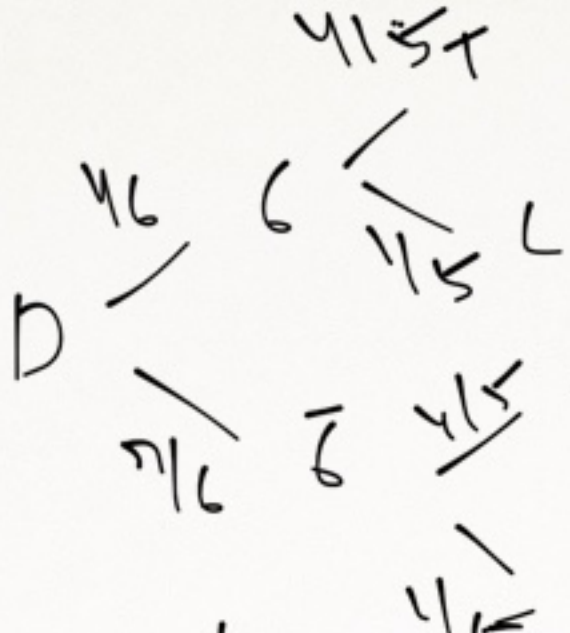



$$P(W) = \frac{1}{9} \times \frac{5}{6} + \frac{8}{9} \times \frac{1}{6}$$

$$= \frac{13}{54}$$

$$P(W|T) = \frac{\frac{1}{9} \times \frac{5}{6}}{\frac{13}{54}} = 5/13$$

12)




$$p(6) = \frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \times \frac{1}{5}$$

$$= \frac{9}{30} \quad \frac{3}{10}$$

$$p(T|6) = \frac{p(T \cap 6)}{p(6)} = \frac{\frac{1}{6} \times \frac{4}{5} \times 1}{\frac{3}{10}} = \frac{4}{9}$$