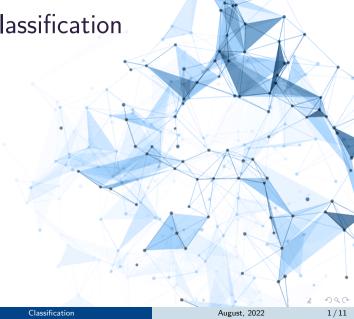
From Regression to Classification



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Classification

Classification

Classification is one type of supervised learning where the output is categorical.

Categorical Variable

A categorical variable means that the variable has only finite many possible values of the output, denoted by \mathcal{Y} . W.l.o.g $\mathcal{Y}=\{1,\cdots,n_o\}$, where n_o denotes the number of possible categories.

Regression v.s. Classification

The main difference between classification and regression is the type of output variable.

- Regression: Continuous output $(\mathcal{Y} = \mathbb{R}^d)$.
- Classification: Categorical output (\mathcal{Y} is a finite set, i.e. $\mathcal{Y} = \{1, \cdots, n_o\}$, where n_o denotes the number of possible categories.)
 - **1** binary classification ($n_o = 2$).
 - 2 multi-class classification $(n_o > 2)$.

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Categorical Variables and Representation

Two representations of the categorical variable

- Integer encoding (the i^{th} class is represented using an integer i);
- One-hot vector encoding (the i^{th} class is represented using a binary vector of length n_o , which has the unique non-zero element at the i^{th} position).

Example

The representation of the price movement direction are given below:

Price Movement Direction	up	down	no change
Integer encoding	1	2	3
One-hot encoding	100	010	001

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Models

Remark

The expectation of a random categorical variable does not make sense!! Hence the main objective of the classification is to estimate the probability of the output being y on conditional on an input x.

Models

In classification framework, a model $f_{\theta}: \mathcal{X} \to \mathbb{R}^{n_o}$ is to approximate the conditional probability of the output being y given input x, in formula,

$$\langle f_{\theta}(x), \bar{y} \rangle \approx \mathbb{P}[y|x],$$

where \bar{y} is one-hot encoding of the class y and $\langle .,. \rangle$ is the inner product of two vectors of length n_o .

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Loss Function

Cross Entropy

For discrete probability distributions p and q with the same support \mathcal{Y} , the cross entropy is defined to be

$$H(p,q) := -\sum_{i \in \mathcal{V}} p(j) \log(q(j)).$$

Definition (Cross Entropy Loss Function)

The cross entropy loss function $Q_{\theta}: \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$ is defined to be

$$Q_{\theta}(x, y) = -\langle \bar{y}, \log f_{\theta}(x) \rangle,$$

where $x \in \mathcal{X}$, \bar{y} is one-hot encoding in \mathcal{Y} , θ are model parameters of f_{θ} , and $\langle .,. \rangle$ is the inner product.

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Empirical Cross Entropy Loss

The empirical cross entropy loss function is given as the average of the above cross entropy loss function evaluated at all samples:

$$L(\theta|\mathcal{D}) = -\frac{1}{N} \sum_{i=1}^{N} \langle y_i, \log f_{\theta}(x_i) \rangle.$$

Maximum Likelihood Estimation (MLE) Interpretation of Cross Entropy

$$\prod_{i=1}^N \langle f_{\theta}(x_i), \bar{y}_i \rangle o \mathsf{max},$$

which is equivalent to minimize the negative log-likelihood ratio, i.e.

$$-\sum_{i=1}^N (\langle \log f_{ heta}(x_i), ar{y_i}
angle)
ightarrow \mathsf{min} \ .$$

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Prediction

Prediction

Let θ_* denote the optimal model parameter. Then for any new given input x_* , the estimated output class is given by the class i, which achieves the highest estimated probability, i.e.

$$\hat{y}_* = \arg\max_{i \in \mathcal{Y}} f_{\theta_*}^{(i)}(x_*),$$

where $f_{\theta_*}^{(i)}(x_*)$ is the i^{th} coordinate of $f_{\theta_*}(x_*)$.

Example (Prediction of Multi-class Classification)

class index	1	2	3	4		
predicted prob.	0.1	0.2	0.6	0.1		
						
		predicted label $= 3$				

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Imbalanced classification

Imbalanced classification refers to the classification tasks, where the distribution of output class is far from the even distribution. [1]

Remark

The prediction of the imbalance classification might be different from the method mentioned in the above.

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Conclusion

Summary

In this video, we cover

- what is the classification?
- How to extend the regression framework to classification, including the model, loss function and prediction.

We will follow by discussing the test metric of the classification in the next step.

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Thanks for your attention!

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References I



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