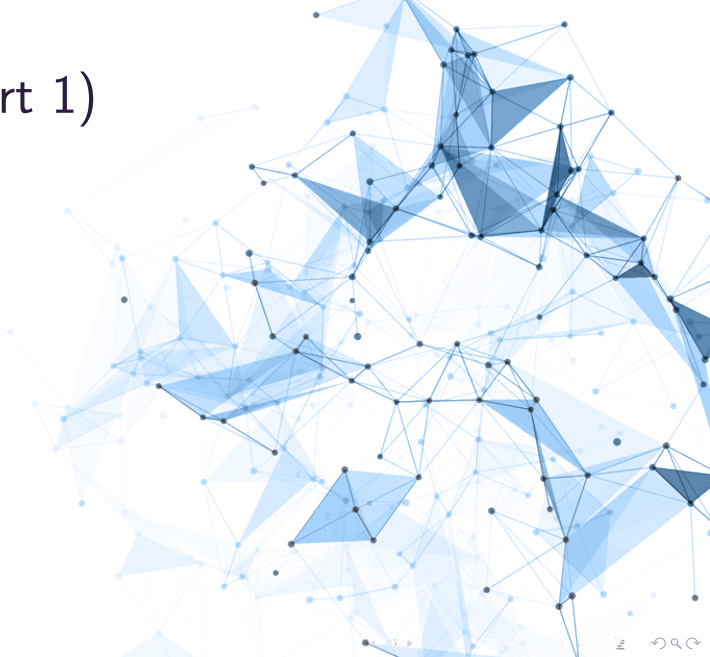


# Linear Regression (Part 1)



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- 1 A motivating example
- 2 Linear Regression

# Housing Data Example

Task: How to predict the median house value based on the factors, e.g. longitude, latitude, median age of the housing, etc.

Input X:

	longitude	latitude	housing_median_age	total_rooms	total_bedrooms	population	households	median_income
0	-122.23	37.88	41.0	880.0	129.0	322.0	126.0	8.3252
1	-122.22	37.86	21.0	7099.0	1106.0	2401.0	1138.0	8.3014
2	-122.24	37.85	52.0	1467.0	190.0	496.0	177.0	7.2574
3	-122.25	37.85	52.0	1274.0	235.0	558.0	219.0	5.6431
4	-122.25	37.85	52.0	1627.0	280.0	565.0	259.0	3.8462

Output Y:

median_house_value
452600.0
358500.0
352100.0
341300.0
342200.0

Figure: Housing data downloaded via the link have 20640 samples [1].

# Setup of Linear regression

## Linear Regression Setting

**Dataset:** Let  $\mathcal{D} = \{(x_i, y_i) \in \mathbb{R}^d \times \mathbb{R}\}_{i=1}^N$  such that

$$y_i = f_{\theta}(x_i) + \varepsilon_i,$$

where  $\theta := (\theta_i^{(1)}, \dots, \theta_i^{(d)})^T \in \mathbb{R}^d$  and  $x_i := (x_i^{(1)}, \dots, x_i^{(d)}) \in \mathbb{R}^d$  and  $f_{\theta} : \mathbb{R}^d \rightarrow \mathbb{R}$  is a linear function such that

$$f_{\theta}(x) = \sum_{j=1}^d \theta^{(j)} x^{(j)} \leftarrow \text{(Linear Model)}.$$

**Further Assumptions:** Assume that the noise term  $\varepsilon_i$  has Gaussian distribution and iid with  $\mathbb{E}[\varepsilon_i | x_i] = 0$ .

We also adopt the matrix form for  $\mathcal{D} = (X, Y)$ , where

$$X = \begin{pmatrix} x_1^{(1)} & x_1^{(2)} & \cdots & x_1^{(d)} \\ \vdots & \vdots & & \vdots \\ x_N^{(1)} & x_N^{(2)} & \cdots & x_N^{(d)} \end{pmatrix} \text{ and } Y = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}.$$

**Question:** What is the loss function of linear regression to choose the **best** estimator for  $\theta$  ?

**Answer:** Mean squared error of the model estimated output, i.e.

$$L(\hat{Y}, Y) = \frac{1}{N} \sum_{i=1}^N |\hat{y}_i - y_i|^2,$$

where  $\hat{Y} = (\hat{y}_1, \dots, \hat{y}_N)^T$  is the model estimated output.

**Remark:** Note that under the above model assumption, minimizing the loss function of OLS is equivalent to maximizing the log-likelihood ratio of the observation of the output  $Y$  conditional on  $X$ .

# Linear Regression (Ordinary Least Squares)

## Ordinary Least Squares (OLS)

Data  $\mathcal{D} = \{(x_i, y_i) \in \mathbb{R}^d \times \mathbb{R}\}_{i=1}^N$ .

Model:  $y = f_{\theta}(x) + \varepsilon = x\theta + \varepsilon$ .

Loss Function:  $L(\theta|X, Y) = (Y - X\theta)^T(Y - X\theta) \rightarrow \min$ .

Optimization:  $\hat{\theta} = (X^T X)^{-1} X^T y$ .

Prediction:  $\hat{y}_* = x_* \hat{\theta}$ .

Validation: Compute RMSE,  $R^2$ , the adjusted  $R^2$ ,  $p$ -value.

# Conclusion

- This video introduces Linear regression under the framework of supervised learning covered in Step 2.2.
- This video is based on Chapter 3 of my book [2] (Chinese version) and [3] (English version).
- One can refer to the code examples of linear regression to the Chapter3\_LinearRegression folder at the link <https://github.com/deepintomlf/mlfbook>.
- **Next steps:** the derivation of the optimal linear coefficient estimator (Step 2.6) and numerical experiments (Step 2.7).



Thanks for your attention!



# References I



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