SMAI Assignment 2 Report

1. **Convergence Proof of Single-Sample Correction:**

Using the fixed increment rule , that is ette is constant.If samples are linearly separable, then the previous algorithm will succeed and return a solution vector.

We've already seen

$a(k+1) = a(k) + y^k$ .

$a_2$ is any solution vector such that $a_2^t y_i$ is positive for all i. We can add a scale factor α and rewrite this equation.

$a(k+1) - \alpha a_2 = (a(k) - \alpha a_2 ) + y^k$

$||a(k+1) - \alpha a^2||^2 = ||a(k) - \alpha a_2||^2 + 2(a(k) - \alpha a_2)^t y^k + ||y^k||^2$

Because $y^k$ was misclassified, $a_t(k)y^k <= 0$,

giving:

$||a(k+1) - \alpha a_2||^2 <= ||a(k) - \alpha a^2||^2 + 2\alpha a_2^t y^k + ||y^k||^2$

Because $a_2^t y^k$ is strictly positive, the second term will overpower the third term if α is large enough.

For example, let β be the maximum pattern vector length ( $\beta = \max_i ||y_i||^2$ ), and γ be the smallest inner product of the solution vector with any pattern vector ( $\gamma = \min_i [a_2^t y_i] > 0$ ).

We now have the inequality:

$||a(k+1) - \alpha a_2||^2 <= || a(k) - \alpha a_2||^2 - 2\alpha\gamma + \beta^2$

Choosing $\alpha = \beta^2 / \gamma$ gives:

$||a(k+1) - \alpha a_2 ||^2 <= || a(k) - \alpha a_2 ||^2 - \beta^2$

So, the squared distance between a(k) and $\alpha a_2$ is reduced by at least $\beta^2$ after each correction. After k corrections, we have:

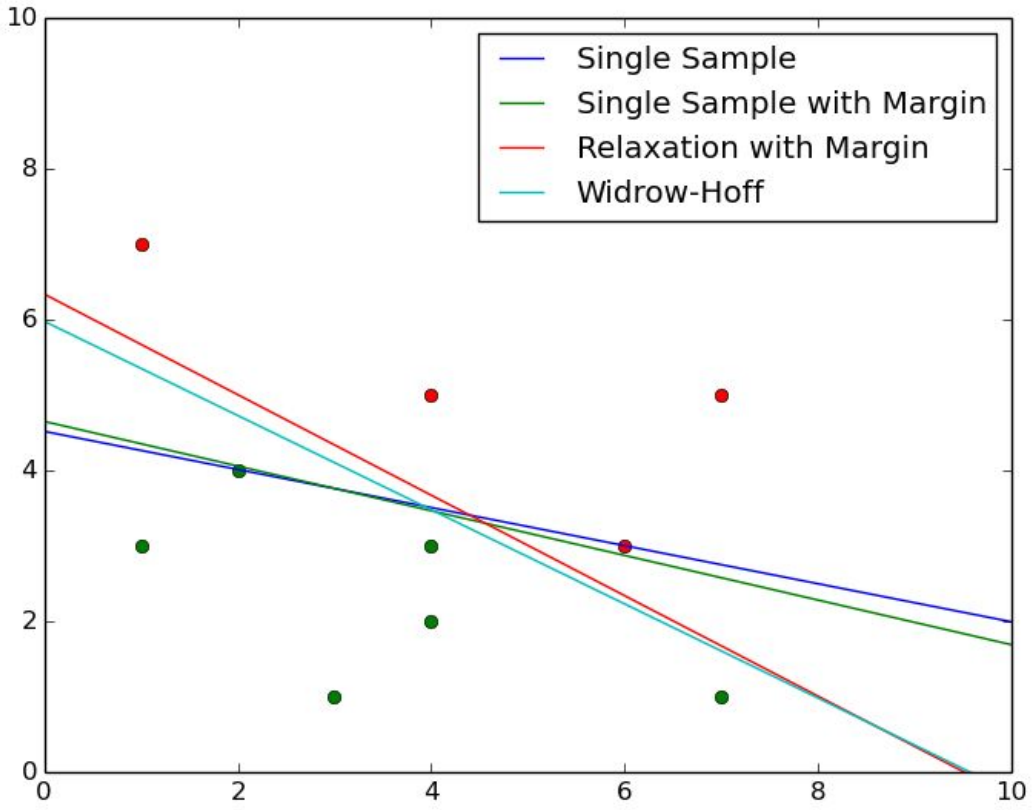
$||a(k+1) - \alpha a_2 ||^2 <= ||a(1) - \alpha a_2 ||^2 - k\beta^2$

The squared distance can not be negative, so there must be at most k 0 corrections, where $k_0 = ||a(1) - \alpha a_2 ||^2 /\beta^2$..

The weight vector that results after the corrections are done must classify all samples correctly, because there are a finite number of corrections that only occur at each misclassification.

In summary, $k_0$ is a bound on the number of corrections,and there will always be a finite number of corrections using the fixed - increment rule if the samples are linearly separable.

2.




Algorithm for Single-Sample:

begin initialize a, k = 0
do k ← (k + 1)modn
if $y_k$ is misclassified by a then a ← a − $y_k$
until all patterns properly classified
return a
end

Algorithm for Single-Sample with Margin:

begin initialize a, criterion θ, margin b, η(·), k = 0
do k ← k + 1
if $a^t y_k + b < 0$ then a ← a − η(k)$y_k$
until $a^t y_k + b \leq 0$ for all k
return a
end

Algorithm for Relaxation with Margin:

begin initialize a, $\eta(\cdot)$, k = 0

do k ← k + 1

if $y_k$ is misclassified then $a \leftarrow a + \eta(k)\, y_k * (b - a^t y)/||y_k||^2$

until all patterns properly classified

return a

end

Algorithm for Widrow-Hoff:

begin initialize a, b, criterion θ, $\eta(\cdot)$, k = 0

do k ← k + 1

$a \leftarrow a + \eta(k)(b_k - a^t y_k)y_k$

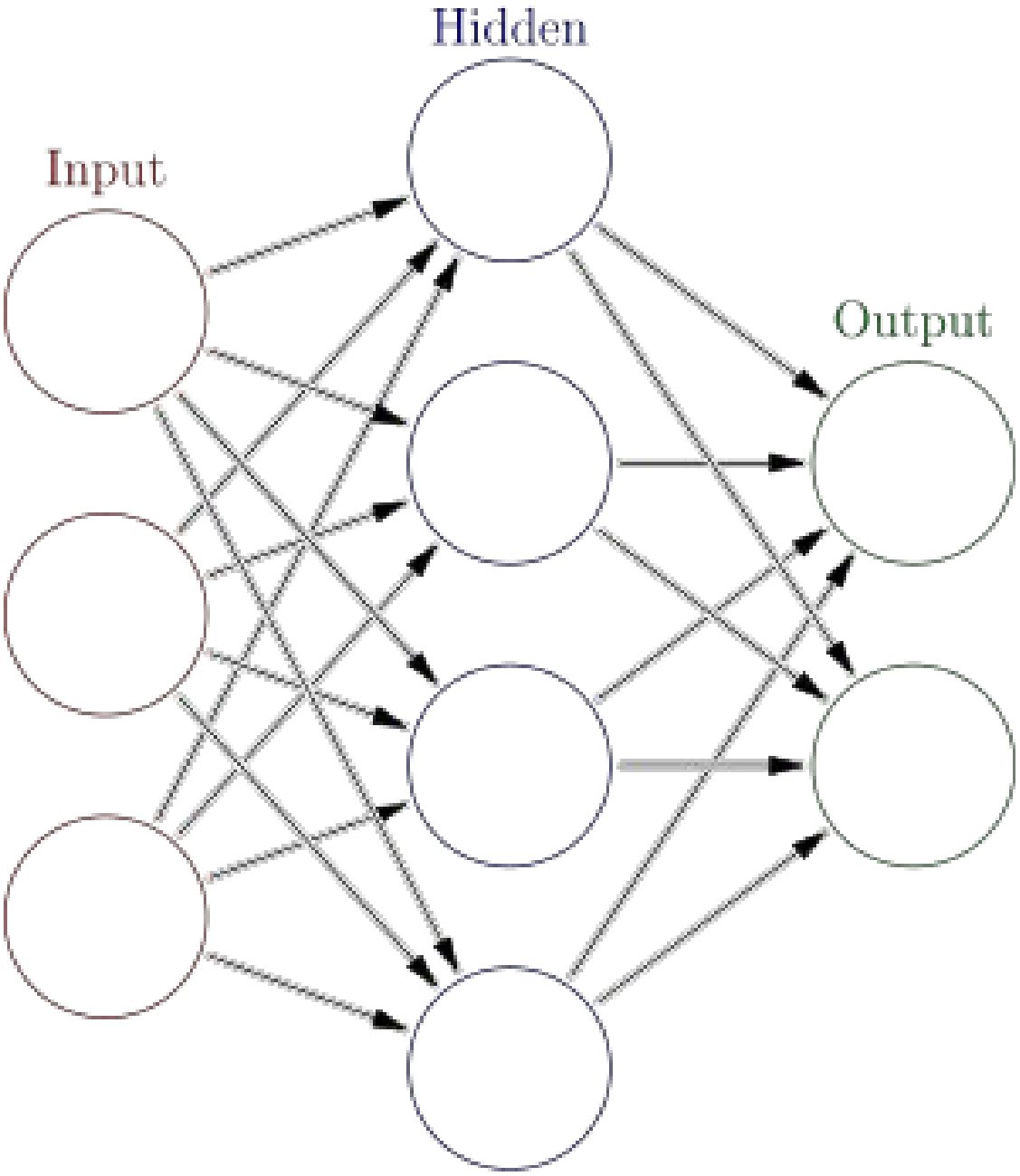
until $\eta(k)(b_k - a^t y_k)y_k < \theta$

return a

end

**Increasing margin increases the converge time and better solution and eventually leading to no solution. There is not much dependence of converge time on initialization. It solely depends on the case. In general, if weight is close to the final result, the faster it will be. Since, we don't know the final result, it is best that we keep it random.**

3.




Sigmoid functions are used in hidden layers and random weights are assigned initially. Then errors are calculated in output layer and then back propagation is done to calculate errors in hidden layer. The weights are then updated.

Using 3 hidden units and 2 output units, the weights are as follows:

Input weights:
[0.055370231697730199, -0.13431635980350981, 0.080190898627293339]

[-4.7684127827650151, -4.4556085960818379, 5.7928439203910598]
[-55.589743279468223, -51.097507276391859, 50.296762240126931]
[-57.608709327932353, -52.724833312317855, 2.836451648003198]
[-102.63497716471595, -94.227638154916406, 80.102068066525248]
[-128.92551693393057, -118.3821431678732, 152.90185713745728]
[-145.68530333164554, -133.47034761170931, 219.66971736900197]
[-20.826035797591167, -19.271427056845155, 30.98534043270881]
[0.074653379112581741, 0.10747822200513829, 0.094741744728646493]
[-24.136289773950605, -21.716470641845753, 34.712800033537036]
[-31.16155182179336, -27.834632273105527, -19.044282950830997]
[-11.283935179230488, -10.320665862250829, -62.162795622757002]
[-6.2254286998146897, -5.7514620797721143, -71.731388832862748]
[-26.180053335511378, -24.760388177872585, -46.83980687732204]
[-113.55205842124172, -104.3430634871261, 156.34840029814995]
[-30.335993428903027, -27.422200030181518, 48.408549834325989]
[0.13699296009421641, -0.075489619057704349, -0.12079547112750709]
[20.746100629610918, 19.701302971199066, -46.409515194735654]
[131.67782186865963, 122.04706379574712, -265.70370824332582]
[100.15356499275266, 91.821650380737083, -187.6932286844241]
[-58.213066701243022, -53.690640811989304, 60.571941489704344]
[17.516833111722796, 15.435367969892052, -121.47285811648706]
[-20.319108247845225, -18.549648751216733, 5.7943361817157566]
[-9.3629991832727963, -8.7405719726800832, 16.105403554739997]
[0.046542931115079733, 0.062391324605450271, -0.044269752386357497]
[34.188024315751612, 31.926023617224697, -72.392568850947754]
[127.34780959470204, 116.39432657678709, -264.52281445255983]
[-39.055082616376779, -36.812555169811489, 28.005822379125906]
[-200.52035471612641, -184.15366412429518, 280.00735603796056]
[-68.804619914055635, -63.704558221609531, 35.864009392706187]
[25.921475729988455, 23.57160159725904, -80.523794293201959]
[-0.11568933724183605, -0.074044555800451453, 0.27235450306770842]
[0.063531305485140688, -0.034474272833024011, -0.1776580951746809]
[8.8996703751515405, 8.9447028883750495, -35.107036270936618]
[33.058232564498049, 29.835502665313491, -127.26141602017465]
[-164.41847411299042, -151.14955695641925, 209.20544663200425]
[-252.17398041678453, -231.13520731462631, 355.9063936284133]
[-118.44232494748573, -108.64699914685947, 122.37517818084022]
[-3.4755987179933276, -2.5611123853662727, -35.428782051213744]
[0.0040440596560590569, 0.10663308913884306, 0.061583279841601168]
[0.059351138773232726, -0.061492244269718194, 0.14739078340274347]
[17.959558121692798, 16.409742790736821, -41.854228274793677]
[104.52030624647811, 96.801777154027107, -230.54684351915407]
[-115.51585430232558, -106.61084503618636, 132.0780977514915]

[-149.6552716099155, -137.00846964649236, 200.56632624498744]
[112.41133565247902, 103.14623171511769, -237.02862235991768]
[45.861644449486874, 43.008718899792093, -97.253021063269571]
[-0.08524205654417899, 0.048790520970315887, -0.013504737987770715]
[-0.044951870225124091, 0.11083564576894983, -0.077452802390702902]
[11.157443074729517, 10.067422162484641, -21.580837336355174]
[54.410712916054386, 50.032180506278024, -145.27613322963711]
[-35.731254605257135, -33.19522544140829, -30.600747457947186]
[62.48927130815558, 57.869536643512674, -155.43537571602423]
[201.09471276539108, 184.30504925702547, -377.73386571123137]
[27.636335573140482, 25.867288177775887, -54.489933054233362]
[0.095048067245869117, 0.11213264194468381, -0.15917583097874075]
[-0.11713078911051196, -0.11382252483654264, -0.019191389205576892]
[-3.8979452164074786, -3.4606385512247848, 4.6788903633650341]
[-66.427722099028301, -61.255980419548806, 62.545582614978002]
[-24.930172707456901, -23.167545868888453, -43.214116048908366]
[113.67773664452926, 104.83362124707047, -233.23725721295716]
[52.246245582303736, 48.41676170040666, -103.40142899756354]
[0.97252113284663644, 0.95668071056503623, -2.3384957505392703]
[0.067848104813491017, -0.13015887663712045, -0.088539373912555952]
[-3.8720200051105271, -3.6122469404540065, -0.09573747643532049]

Output weights:
[5.8082546552969285, -4.7256630324008411]
[4.2479216401293618, -5.5731172939169076]
[-9.5852306308836699, 9.3446541910766587]

Using 4 hidden units and 2 output units, weights are:

Input weights:
[0.088193504000056377, 0.036784118607101096, 0.13979853173655532, 0.18004099403123502]
[1111.922748441119, 2443.9247009969863, 2844.8190107732889, 2569.2133319921563]
[569.90289063509658, 1254.1043813711622, 1459.3260940674186, 1318.201922820685]
[-15662.063411427527, -34415.135826439, -40065.106418472802, -36184.671646964634]
[-4735.1808322436164, -10399.992820038176, -12106.940873881074, -10934.326575368088]
[18824.263980531668, 41377.112298793145, 48170.417608261683, 43504.802170315968]

[11212.928536079708, 24639.493875960714, 28684.610373206651, 25906.2303128323]
[1468.4780012942829, 3228.4250713187498, 3758.4445713406781, 3394.3830386064324]
[0.032668523723479753, 0.015537358471337914, -0.15422161634805376, 0.19514217769127157]
[-1283.3718566026207, -2820.4800279868368, -3283.3435248824157, -2965.5400913369804]
[-22405.557938106067, -49241.754346251582, -57326.308864725419, -51773.639550246771]
[-19881.183633081862, -43690.617896008422, -50863.069587363367, -45937.027719704718]
[-14695.717899625246, -32295.463039746573, -37597.506234184424, -33956.051416223556]
[-5829.3473781261755, -12805.584872481057, -14907.460742200263, -13463.890815064979]
[12058.193341788885, 26501.969791737316, 30853.086540777851, 27864.857965204614]
[1440.1213845453408, 3165.7479822058526, 3685.6432763722651, 3328.6764309118748]
[0.055507737191363038, -0.17731510100438266, 0.11833325034359382, 0.11359427425636631]
[-12658.848990961704, -27821.908978759151, -32389.466996890715, -29252.339047401336]
[-46842.723863085557, -102948.11423343183, -119848.72273272506, -108241.15036713875]
[-14147.891909769056, -31092.379694379375, -36196.826089278984, -32691.126077162913]
[7983.1530211329291, 17547.810108461184, 20427.998862805576, 18449.524330716889]
[-16762.758755380139, -36834.031597336878, -42879.94199298889, -38727.443693292298]
[-2401.2976737397175, -5277.4204925211443, -6143.8926369247101, -5548.7830098711374]
[309.19335358318318, 679.87808725182379, 791.33030202816906, 714.70464364932661]
[0.17065404049959437, -0.19537727322417023, 0.017667481603874657, 0.079531204506400555]
[-17367.2414726264, -38168.732233019262, -44434.60650483043, -40131.097870904334]
[-36816.596454609695, -80914.693611941577, -94197.636195821106, -85074.807605516369]

[10836.740317850587, 23811.270994529328, 27719.984446806902, 25035.383519600298]
[29949.192836965634, 65820.847962346932, 76625.664601317025, 69204.910879274845]
[-4467.4330361821012, -9815.7139542231653, -11426.762979596111, -10320.179817354319]
[-13924.655735099961, -30606.136740696253, -35630.535980279223, -32179.708328557314]
[9.732356978719201, 21.288320291695097, 24.884185543553322, 22.216865653508492]
[-0.14309145275230084, -0.10286017521425445, -0.063462851839329565, 0.11965303313792625]
[-15013.496153424267, -32996.09461329646, -38413.14337352632, -34692.596696283508]
[-20334.504604078305, -44682.194925382988, -52016.750902973268, -46979.265982770812]
[30822.246128051869, 67742.774893973474, 78864.109168127761, 71225.609984720242]
[36738.099047880518, 80745.139186410306, 94000.639102509391, 84896.472933884535]
[176.73979987672348, 392.56379715273221, 457.09417455593967, 412.97423741900189]
[-14207.111972134588, -31221.282349686695, -36346.651916852381, -32826.546656133483]
[0.18923084212039326, 0.078699172839396758, 0.1436728790870469, 0.11442405617063595]
[0.16004287233912518, 0.1325988442676429, -0.196282000607648, -0.086924581773682175]
[-9480.0825425423209, -20834.668695021996, -24254.624183855809, -21905.504185824877]
[-36471.416888818458, -80150.94152041206, -93308.606183980723, -84271.689577921657]
[23690.058167409559, 52072.967749598567, 60621.582986401168, 54749.827506377187]
[22002.270482544802, 48354.947649353548, 56293.0333682376, 50841.142532173282]
[-28860.508767559368, -63427.506163784485, -73839.821799358659, -66688.188297891204]
[-20956.381573205821, -46056.657332810864, -53617.993719911057, -48424.58270312268]
[0.041532760618736564, -0.11789509833150796, 0.18286145925843067, -0.18535497140836366]

[0.029471567073552085, -0.011739894948086915, 0.088247455264881536, 0.10304679005940143]
[-2242.4070859962217, -4927.8063034678817, -5737.0198760492467, -5181.2455912854502]
[-35459.950432130521, -77932.184628788105, -90726.037629016268, -81938.897111399434]
[-6685.9985472758644, -14689.60103067003, -17100.818906730128, -15444.614498792384]
[-24464.647283092956, -53771.451249407313, -62598.635867322453, -56536.099440234662]
[-48092.005484764166, -105691.42835309132, -123041.90249811365, -111125.3544918213]
[-9779.4462326087832, -21492.985026474511, -25021.866717785906, -22598.375301336517]
[0.10252955358384563, 0.079806203445235646, -0.015119297390389697, 0.088156942874142941]
[0.19302701707090464, 0.17766187870053707, 0.14613003143272457, -0.15828680365288036]
[1167.865022588082, 2566.8631764014949, 2988.0055001412684, 2698.758050567254]
[2912.2752497003339, 6403.5245304560285, 7454.4693483745241, 6732.4786020669899]
[-21616.787943645922, -47506.439962834251, -55305.616075888698, -49948.837459717564]
[-43228.118483294107, -95001.856652791786, -110597.73612258378, -99886.469050198837]
[-19783.403171856051, -43477.190553350658, -50614.771916066493, -45712.717807966088]
[-734.17901368222954, -1613.8609782931471, -1879.0685716980629, -1696.9633677778083]
[0.017340214830465189, 0.13187145868246969, 0.049866027228582188, 0.15550355720339137]
[-1173.3233182371735, -2578.8366456052809, -3001.925753217075, -2711.421482437519]

Output weights:
[-1.5063390333818543, 1.1484502670759178]
[-3.2576382914624284, 2.0699883753073989]
[-2.6916599769595408, 3.4762051037817168]
[-2.5707611789559977, 3.0331465765194268]

Using 5 hidden units and 2 output units, weights are:

Input weights:
[-0.19894540458760024, 0.17280041387733019, 0.053703945527201313, 0.01008844171823714, -0.17397399772484132]
[3.5362353537484608, -4.3492915441282927, -6.3804910372519554, 2.776569121135271, 4.4695009226745999]
[6.8840842802506295, -18.088375030770447, -25.399537874539316, 5.3072964699100584, 8.1368784511582675]
[-4.2387127236144133, -21.368926153411991, -28.180088141508868, -3.064005865219404, -4.9077613675976934]
[17.120330581881163, -42.571989923436831, -58.921395521131572, 13.542536539297346, 19.710553211728715]
[34.907670127903963, -49.18853418508197, -69.796931648147236, 26.411291421516456, 41.30814132567734]
[48.810554457656117, -60.563335328228376, -83.553007807581167, 38.729337788309465, 57.067321406852898]
[6.7733866732758967, -8.253902741528, -11.456792507846377, 5.154523948954802, 7.6619982113488376]
[-0.18099954965325826, -0.073846749988450894, 0.093541247686415585, -0.013340614842297566, -0.046837610913850469]
[8.0620366180114651, -11.641754106962381, -16.026867231737146, 6.3645414351204943, 9.2602418848384964]
[-7.8350427322380405, -14.86133559928675, -19.705245739996251, -5.6167230520110039, -9.1304302155581123]
[-12.479330870918085, -9.1685169074284438, -13.472553024651784, -9.4747903244251397, -14.621646134656004]
[-14.932984940580885, -4.4742416291914706, -6.8819369509073676, -12.237383061222435, -17.652382250878979]
[-21.963297563056233, 4.8847035369554259, 6.2321796480161451, -18.293601886899069, -25.388802575608139]
[39.216054286425141, -49.743665996715812, -69.886857331638282, 30.267339916109634, 45.911164910156785]
[11.971385442067279, -14.627008497178430, -19.714229950643567, 9.6347983627637674, 13.960525651223064]
[0.090587701461602943, 0.065226062742731572, 0.048926720819120473, 0.10627874379402957, 0.072382702440205504]
[-17.33778430339175, 12.894846843203627, 19.596021292039303, -13.16174608122315, -20.248088502522322]
[-63.49673218001837, 50.123434332587635, 72.379685653716422, -49.111324303557886, -74.877034410134698]
[-32.75950081571483, 31.910203253874148, 42.391343793654414, -25.764618263389337, -39.01759218487679]

[20.243324016171105, -29.317427488083855, -42.944863939420664,
15.373060189665525, 23.551365651171281]
[-42.551141741660352, 25.908990281587375, 36.102749508567079,
-34.564949055741494, -49.726880028026841]
[6.7397028905295091, -14.783650150835077, -20.866501642659649,
5.3933427057937902, 7.8664041585855626]
[4.1155141732995286, -5.2915525453853247, -7.0586475472414651,
3.5045218781794212, 4.9604132014917894]
[0.136191964526517, -0.0035374778907367455, 0.022207542062341046,
-0.015226086896859892, -0.10057739560013951]
[-21.081204638358383, 15.588717057698336, 23.317992644777412,
-16.305300392188691, -24.404787121886908]
[-68.232922424004286, 58.433592047387457, 82.818931890319178,
-53.423300695472882, -80.409465210730488]
[17.631153011827337, -25.260751988902182, -37.198363054936436,
13.971749479500229, 20.519749385285685]
[68.437114659502299, -84.551013652225294, -119.99875404213284,
53.341777486277195, 80.966578786014935]
[0.67515193536413221, -18.687451116357046, -27.125010243688603,
0.11724797573952979, 0.61924573009915718]
[-19.105071485967716, 9.2430342263760608, 14.026916010089586,
-14.675380647413943, -22.259503218498029]
[-0.15950986303077333, -0.1577654907480916, -0.096513574991173579,
0.096854489870672522, -0.0025262341016923616]
[-0.17949211744714369, 0.14275348375507324, -0.16795917066236687,
-0.18190590124934253, 0.055779324699998356]
[-10.615656114411282, 1.9386192266241287, 4.7262140374478028,
-7.7348478806549519, -12.263889631425075]
[-37.47060815536306, 23.037530137487334, 32.221397874357862,
-29.678498674002956, -44.199836040797017]
[53.123364542863449, -66.434817193631972, -96.220935987315784,
40.519364830691181, 62.378582765421505]
[84.751435337839297, -103.73265933320172, -147.41714860160445,
65.976016172920993, 99.98428254624946]
[30.904872516028533, -53.322116696201149, -75.524220945061884,
24.298581886145229, 36.137012697516376]
[-8.1695396244281664, -4.1993194174660351, -4.5301446869041708,
-6.0383761270645619, -9.6202214762412268]
[-0.12988879319477303, 0.0061559798789483366, -0.042046331674006343,
-0.15119729197746415, -0.11615619580190902]
[0.196047865295573, 0.014764700570534472, 0.022467064950751181,
-0.13276351906871581, 0.15662923741806384]

[-14.565830991089834, 11.229889406335726, 16.772394629853334,
-11.399041745238828, -17.46516372135493]
[-53.66746232107905, 39.601113825588222, 57.091337885763863,
-41.40567732358344, -63.288528149599742]
[38.812223021899371, -50.644003883394326, -74.000226003409779,
29.289543686852252, 45.897947466028462]
[55.04133840333111, -70.060258959474183, -99.845226310420472,
43.098470479591676, 64.851896299003499]
[-52.705727131895799, 43.031120645625755, 59.283253489148102,
-41.429993149680762, -62.334045660794281]
[-26.805072122463837, 19.917052168497523, 29.719042158803656,
-20.612314113904954, -31.493425945851605]
[0.063620738247727571, -0.17022685231441309, -0.17103894208182499,
-0.043654683764463298, 0.093259415643776433]
[-0.18146776991222097, 0.022998833138776209, -0.058732291908556528,
-0.15454648630020318, -0.016020256826200396]
[-3.8461243954479891, 3.3163631252081123, 4.5772787729973219,
-3.0517860632583607, -4.6502264669589914]
[-42.178648229012857, 25.57935720622676, 38.032180052504593,
-32.138944632576774, -49.751398234093067]
[-6.9762990757864465, -13.277565862353086, -19.028895246049025,
-5.6914498568399692, -7.9456027846592869]
[-28.848805886252563, 14.885507535527152, 19.99268279236848,
-21.911549840362539, -34.314045859085752]
[-100.76301936144549, 95.883734559980496, 136.32368247190274,
-79.147953863760009, -118.35365032018875]
[-16.308991006102815, 14.535000909427758, 21.080796980950375,
-12.575378743896898, -19.409005603265847]
[-0.060884935216585501, -0.12822255149102926, 0.045755391915432042,
-0.0017866545635887854, 0.13783105665853318]
[-0.0027434745916455339, 0.059101464068033738, 0.020050748336625318,
0.15691458642344974, 0.14992741484196398]
[4.6710805866057594, -5.3717880254541104, -7.9051905449155679,
3.619508322673441, 5.1841744768080966]
[5.8680189693791185, -17.103145145564941, -24.406228357593008,
4.5017716553572198, 7.1900684292785071]
[-18.626159172613384, -4.5765545850712437, -4.8703991199695835,
-13.872587480317957, -21.823409389911248]
[-58.765974599973006, 46.063423678324455, 67.357556375902291,
-45.123335724948596, -69.148657469582332]
[-34.768039429935335, 31.313732988684588, 45.369375435564343,
-27.251314625637825, -40.759780531189548]

[-1.1547361512428906, 0.96063388469299182, 1.3468731372203102,
-0.83214540215399724, -1.1426078170803009]
[0.081165164604610207, 0.047402253551416695, -0.06810898597337034,
-0.083974520746239192, -0.021184702756545409]
[-0.27718124467079108, -1.4987568396097193, -1.984657648467846,
-0.4846222756208558, -0.59523827736896717]

Output weights:
[-3.709018494697871, 3.4180233699038887]
[3.6356183804801319, -5.308689297899142]
[6.3754586884082318, -4.9574615260434767]
[-3.2255848294286302, 3.0185923468128095]
[-3.864112650412487, 4.1087228583970576]