24156244S - EE951 - Assignment 1

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(1) Addition of vectors is valid if all vectors have same dimensions

(a) $a+b-c_{5:14}$

→ Here, $a \in \mathbb{R}^{10}$, $b \in \mathbb{R}^{10}$, $c_{5:14} \in \mathbb{R}^{10}$. Hence valid

(b) $X = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \Rightarrow X \in \mathbb{R}^{10+10+20} \Rightarrow X \in \mathbb{R}^{40}$. → Valid

(c) $2a+c \Rightarrow$ Here $a \in \mathbb{R}^{10}$, $c \in \mathbb{R}^{20}$. → Not valid

d) $\begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} c_1 \\ b \end{bmatrix}$

Here $\begin{bmatrix} a \\ 0 \end{bmatrix} \in \mathbb{R}^{11}$

$\begin{bmatrix} c_1 \\ b \end{bmatrix} \in \mathbb{R}^{1+10} \Rightarrow \mathbb{R}^{11}$ Hence valid

(e) $[a\ b] + c \Rightarrow \mathbb{R}^{10\times 2} + \mathbb{R}^{20}$ → Not valid, as the first term is a matrix of (10, 2) dimension

(f) $\begin{bmatrix} a \\ b \end{bmatrix} + c \Rightarrow \mathbb{R}^{20} + \mathbb{R}^{20}$ → Hence valid.

∴ a, b, d & f are valid

(2) (a) $b = \begin{bmatrix} \mathbb{0} \\ a \end{bmatrix}$ since $b \in \mathbb{R}^{15}$, $a \in \mathbb{R}^{10}$

$\therefore \begin{bmatrix} \mathbb{0} \\ a \end{bmatrix} \in \mathbb{R}^{15}$

$\Rightarrow \mathbb{0} \in \mathbb{R}^{5}$

(b) $b = \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix} \in \mathbb{R}^{15}$.

$\because a \in \mathbb{R}^{10}$, the two all-zero vectors combined have a dimension of $\mathbb{R}^5$. However, it cannot be determined what the dimension of each of the all-zero vector is. Possible combinations are

(1, 4), (2, 3), (3, 2) & (4, 1)

(c) $[\mathbb{0} \ a] \Rightarrow$ Since 0 has to be the same dimension for this operation to be valid, $\mathbb{0} \in \mathbb{R}^{10}$

(d) $a + \mathbb{1} \Rightarrow$ for the operation to be valid, all-one vector has to be the same dimension as 'a', ie

$\mathbb{1} \in \mathbb{R}^{10}$

$\therefore$ a, c & d we can determine the dimension of the all-zero or all-one vectors.

(3)

(a) Since each number takes 64 bit storage, a vector of length 10^8 will take 64×10^8 bit storage. Hence, 100 such vectors will take

$100 \times 64 \times 10^8$

$= 64 \times 10^{10}$ bit storage

=

(b) Adding 2 vectors of size 'n' takes 'n' flops to add.
Adding 3 vectors takes $n+n = 2n$ flops.

⋮

Adding 'a' vectors takes ~~a×n~~ ~~(a-1) flops~~ $(a-1)n$ flops

$\therefore$ Adding 100 vectors of size 10^8

$= (100-1) \times 10^8$ flops

$= 99 \times 10^8$ flops

$= 9.9 \times 10^9$ flops

(c) A processor with 1 Gflops/sec (10^9 flops/sec) will take $9.9\times10^9 / 10^9$ sec $= 9.9$ sec to compute.

(4) ~~Sum of all the di~~

(4) Lets take the positions of all points as $a_0, a_1, a_2, \ldots$

where $a_0 = (0,0) =$ origin

$\therefore x_1 = a_1 - a_0 = a_1$

$x_2 = a_2 - a_1$

$x_3 = a_3 - a_2$

⋮

$x_n = a_n - a_{n-1}$

$\therefore x_1 + x_2 + \ldots + x_n = a_1 + (a_2 - a_1) + (a_3 - a_2) + \ldots + (a_n - a_{n-1})$

$= a_n$

$\because a_n =$ origin $= (0,0)$

$\therefore x_1 + x_2 + \ldots + x_n = 0$

(5) ~~(a) {a~~ (a) $\{X \in \mathbb{R}^3 \mid a_1 + 2a_2 + 3a_3 = 0\}$

① $0 \Rightarrow 0 + 2\cdot 0 + 3\cdot 0 = 0$. $\Rightarrow$ Origin is in the space

② ~~$A + B \Rightarrow$~~ Let $C = A + B$, where A & B satisfy the above condition.

$\therefore a_1 + 2a_2 + 3a_3 = 0$ & $b_1 + 2b_2 + 3b_3 = 0$

$\therefore C \Rightarrow c_1 + 2c_2 + 3c_3$

$= (a_1 + b_1) + 2(a_2 + b_2) + 3(a_3 + b_3)$

$= (a_1 + 2a_2 + 3a_3) + (b_1 + 2b_2 + 3b_3)$

$= 0 + 0 = 0$

$\therefore$ for every A, B, $A + B$ also ~~sa~~ is in the space

3) $\alpha A \Rightarrow \alpha a_1 + \alpha 2a_2 + \alpha 3a_3$

$= \alpha(a_1 + 2a_2 + 3a_3) = \alpha \cdot 0 = 0$

Hence, $\{X \in \mathbb{R}^3 \mid a_1 + 2a_2 + 3a_3 = 0\}$ is subspace of $\mathbb{R}^3$

(b) $\{X \in \mathbb{R}^3 \mid x_1 = 2x_2 = 3x_3\}$

① 0 is a part of the space as $0 = 2\cdot 0 = 3\cdot 0$

② If $A, B \in X$, $C = A + B$

$\therefore c_1 = a_1 + b_1 = 2a_2 + 2b_2 = 2(a_2 + b_2) = 2c_2$

& $c_1 = a_1 + b_1 = 3a_3 + 3b_3 = 3(a_3 + b_3) = 3c_3$

$\therefore C \in X$

③ If $A \in X$ then $\alpha A \Rightarrow$

$\alpha a_1 = 2\alpha a_2 = 3\alpha a_3$

$\therefore \alpha A \in X$

$\therefore \{X \in \mathbb{R}^3 \mid x_1 = 2x_2 = 3x_3\}$ is a subspace of $\mathbb{R}^3$

(c) Since all real numbers x can be represented as complex numbers $x + i0$ Hence $\mathbb{R}^2$

any $x \in \mathbb{R}^2$ also $\in \mathbb{C}^2$

Thus $\mathbb{R}^2$ is a subspace of $\mathbb{C}^2$

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(6) ~~Let~~ $\cancel{I \in \mathbb{R}^m}$, $\cancel{B \in \mathbb{R}^{p\times q}}$, $\cancel{O \in \mathbb{R}^{x \times y}}$

$\therefore$ ~~No. of rows in A~~ $\cancel{\Rightarrow m+p = q+x}$

~~No. of columns in A~~ $\cancel{\Rightarrow m+p = q+y}$

$\cancel{A = \begin{bmatrix} I & B^T \\ B & O \end{bmatrix} = [A_1 \; A_2]}$,

~~where~~ $\cancel{A_1 = \begin{bmatrix} I \\ B \end{bmatrix}}$, $\cancel{A_2 = \begin{bmatrix} B^T \\ O \end{bmatrix}}$

$\cancel{\therefore A_1 \in \mathbb{R}^{m+p,\, m+q}}$

$\cancel{A_2 \in \mathbb{R}^{q+x,\, p+y}}$

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(6) Let $I \in \mathbb{R}^m$, $B \in \mathbb{R}^{p \times q}$, $O \in \mathbb{R}^{x \times y}$

$\therefore A = \begin{bmatrix} I & B^T \\ B & O \end{bmatrix}$

~~A has~~ No. of rows in A can be written in 2 ways $m+p$ & $q+x$ $\Rightarrow m+p = q+x$

lly, No. of columns in $A \Rightarrow m+p = q+y$

$\therefore$ A is square with $(m+p)$ rows & columns.

Also $m+p = q+x = q+y \Rightarrow x = y$

$\therefore$ O is square $\in \mathbb{R}^x$

Hence, $A \in \mathbb{R}^{m+p}$ or $\mathbb{R}^{q+x}$ $\forall (m, x) \in \mathbb{N}$

$\Rightarrow m+p = q+x \quad \forall (m, x) \in \mathbb{N}$

$\Rightarrow p = x$ & $q = m$

$\therefore \mathbb{B} \in \mathbb{R}^{a \times m}$

$\therefore A = \begin{bmatrix} \mathbb{I} & \mathbb{B}^T \\ \mathbb{B} & \mathbb{O} \end{bmatrix}$ can be written as $[A_1 \ A_2]$

where $A_1 = \begin{bmatrix} \mathbb{I} \\ \mathbb{B} \end{bmatrix} \in \mathbb{R}^{m+a, m}$

$A_2 = \begin{bmatrix} \mathbb{B}^T \\ \mathbb{O} \end{bmatrix} \in \mathbb{R}^{m+a, a}$

$\therefore A^T = \begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} = \begin{bmatrix} \mathbb{I}^T & \mathbb{B}^T \\ (\mathbb{B}^T)^T & \mathbb{O}^T \end{bmatrix}$

$= \begin{bmatrix} \mathbb{I} & \mathbb{B}^T \\ \mathbb{B} & \mathbb{O} \end{bmatrix} = A$

$\therefore A$ is symmetric

Hence a, b & d cannot be true