EE961 — ASSIGNMENT-3 — 241562445 (Roll no)

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① $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $B = \begin{bmatrix} \cos w & -\sin w \\ \sin w & \cos w \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos w & -\sin w \\ \sin w & \cos w \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos w - \sin\theta\sin w & -\cos\theta\sin w - \sin\theta\cos w \\ \cos w\sin\theta + \sin w\cos\theta & -\sin\theta\sin w + \cos\theta\cos w \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+w) & -\sin(\theta+w) \\ \sin(\theta+w) & \cos(\theta+w) \end{bmatrix}$$

lly $BA = \begin{bmatrix} \cos w & -\sin w \\ \sin w & \cos w \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos w\cos\theta - \sin w\sin\theta & -\cos w\sin\theta - \sin w\cos\theta \\ \sin w\cos\theta + \cos w\sin\theta & -\sin w\sin\theta + \cos w\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+w) & -\sin(\theta+w) \\ \sin(\theta+w) & \cos(\theta+w) \end{bmatrix} = BA$$

Hence $AB = BA = \begin{bmatrix} \cos(\theta+w) & -\sin(\theta+w) \\ \sin(\theta+w) & \cos(\theta+w) \end{bmatrix}$ represents a rotation vector of any $(\theta+w)$

$$\|AB\|^2 = (AB)^T \cdot (AB) = \begin{bmatrix} \cos(\theta+w) & \sin(\theta+w) \\ -\sin(\theta+w) & \cos(\theta+w) \end{bmatrix} \begin{bmatrix} \cos(\theta+w) & -\sin(\theta+w) \\ \sin(\theta+w) & \cos(\theta+w) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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② $f(x) = \left(\|x-c\|^2 - \|c\|^2\right) - \left(\|x-d\|^2 - \|d\|^2\right)$

$= \left(\|x\|^2 - 2\langle c, x\rangle + \|c\|^2 - \|c\|^2\right) - \left(\|x\|^2 - 2\langle d, x\rangle + \|d\|^2 - \|d\|^2\right)$

$= 2\langle d, x\rangle - 2\langle c, x\rangle$

$= 2\langle d-c, x\rangle$

$= \langle 2(d-c), x\rangle = \langle a, x\rangle, \quad a = 2(d-c) \in \mathbb{R}^n$

~~(4)~~ (3) $\|p - \theta u - (1-\theta) v\|^2$

$$= \|(p-v) - \theta(u-v)\|^2$$

$$= \langle (p-v) - \theta(u-v), (p-v) - \theta(u-v) \rangle$$

$$= \|p-v\|^2 - 2\theta \langle p-v, u-v \rangle + \theta^2 \|u-v\|^2 = h(\theta)$$

At $\theta^* = \arg\min_{\theta} \|p - \theta u - (1-\theta) v\|^2$

$$\left.\frac{dh(\theta)}{d\theta}\right|_{\theta=\theta^*} = 0$$

$$\Rightarrow -2\langle p-v, u-v \rangle + 2\theta^* \|u-v\|^2 = 0$$

$$\Rightarrow \theta^* = \frac{\langle p-v, u-v \rangle}{\|u-v\|^2}$$

(4) $\langle a, b \rangle = a^T b = \|a\| . \|b\| \cos\theta$, θ is the angle between a & b

$$\therefore x^* = \frac{a^T b}{a^T a} = \frac{\|a\| . \|b\| \cos\theta}{\|a\|^2} = \frac{\|b\|}{\|a\|} \cos\theta \rightarrow [\text{scalar}]$$

$$\therefore \|ax^* - b\|^2 = \langle ax^* - b, ax^* - b \rangle$$

$$= \|ax^*\|^2 + \|b\|^2 - 2 \langle ax^*, b \rangle$$

$$= (x^*)^2 \|a\|^2 + \|b\|^2 - 2 \langle a, b \rangle x^*$$

$$= \frac{\|b\|^2}{\|a\|^2} . \cos^2\theta . \|a\|^2 + \|b\|^2 - 2 . \|a\| \|b\| \cos\theta . \frac{\|b\|}{\|a\|} \cos\theta$$

$$= \|b\|^2 \cos^2\theta + \|b\|^2 - 2\|b\|^2 \cos^2\theta$$

$$= \|b\|^2 (1 - \cos^2\theta)$$

$$\Rightarrow \|ax^* - b\|^2 = \|b\|^2 \sin^2\theta$$

(5) $A = \alpha \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$ $\quad \therefore A^T = A$

$\because A$ is orthogonal, $\quad A^T A = I$

$$\Rightarrow \alpha^2 \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix} = I$$

$$\Rightarrow \alpha^2 \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = I$$

$$\Rightarrow 4\alpha^2 \cdot I = I$$

$$\therefore \alpha^2 = 1/4$$

$$\Rightarrow \boxed{\alpha = \pm \frac{1}{2}}$$