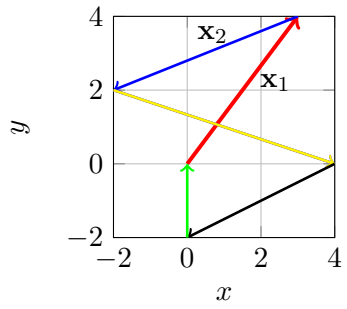
- Attempt all problems and submit before the discussion session on Sunday.
- You are free to discuss the problems with others. However, plagiarism will result in serious penalties, such as an F grade.

1. Which of the following operations are valid, given that $\mathbf{a} \in \mathbb{R}^{10}$, $\mathbf{b} \in \mathbb{R}^{10}$, and $\mathbf{c} \in \mathbb{R}^{20}$:
   (a) $\mathbf{a} + \mathbf{b} - \mathbf{c}_{5:14}$
   (b) $\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{bmatrix}$
   (c) $2\mathbf{a} + \mathbf{c}$
   (d) $\begin{bmatrix} \mathbf{a} \\ 0 \end{bmatrix} + \begin{bmatrix} c_1 \\ \mathbf{b} \end{bmatrix}$
   (e) $[\mathbf{a}\ \mathbf{b}] + \mathbf{c}$
   (f) $\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} + \mathbf{c}$

2. Given $\mathbf{a} \in \mathbb{R}^{10}$ and $\mathbf{b} \in \mathbb{R}^{15}$, is it possible to determine the size of the all-zero or all-one vectors in each of the following mathematical expressions:
   (a) $\mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{a} \end{bmatrix}$
   (b) $\mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{a} \\ \mathbf{0} \end{bmatrix}$
   (c) $\begin{bmatrix} \mathbf{0}\ \mathbf{a} \end{bmatrix}$
   (d) $\mathbf{a} + \mathbf{1}$

3. Suppose that we have 100 vectors, each of length $10^8$. Answer the following:
   (a) What is the total storage space required if each vector is dense and contains numbers in 64 bit precision format.
   (b) How many flops would it take to add these 100 vectors together?
   (c) How much time would the process take on a processor which is capable of carrying out 1 Gflops/sec ($10^9$ flops per second).

4. Consider a vacuum cleaning robot in a closed space, initially located at the origin. It starts to move in a straight line and continues to move till it encounters an obstacle. Let us denote the displacement vector (from the starting point to the obstacle) by $\mathbf{x}_1$. After encountering an obstacle, the robot chooses another direction and starts moving in a straight line along that direction, till it encounters another obstacle. We can denote the second displacement vector by $\mathbf{x}_2$. The process continues for an hour, at which point the robot ends up at the origin. What is the sum of all the displacement vectors $\mathbf{x}_1 + \mathbf{x}_2 + \ldots$? An example path is shown in the figure below.




5. Determine which of the following subsets of $\mathbb{R}^3$ are subspaces of $\mathbb{R}^3$
   (a) $\{\mathbf{x} \in \mathbb{R}^3 \mid x_1 + 2x_2 + 3x_3 = 0\}$
   (b) $\{\mathbf{x} \in \mathbb{R}^3 \mid x_1 = 2x_2 = 3x_3\}$

   Is $\mathbb{R}^2$ a subspace of $\mathbb{C}^2$ over the field $\mathbb{F} = \mathbb{C}$?.

6. Consider the block matrix:

$$\mathbf{A} = \begin{bmatrix} \mathbf{I} & \mathbf{B}^\mathsf{T} \\ \mathbf{B} & \mathbf{0} \end{bmatrix}.$$

   Given no other information and without making any additional assumptions, which of the following must be true in general:
   (a) The matrix $\mathbf{A}$ is square.
   (b) The matrix $\mathbf{A}$ is symmetric.
   (c) The identity and zero submatrices in $\mathbf{A}$ are of the same sizes.
   (d) The zero submatrix is square.