- Attempt any 5 problems and submit before the discussion session.
- You are free to discuss the problems with others. However, plagiarism will result in serious penalties, such as an F grade.

1. When is the outer product $\mathbf{x}\mathbf{y}^\mathsf{T}$ symmetric? Derive the conditions on $\mathbf{x}$ and $\mathbf{y}$ such that $\mathbf{x}\mathbf{y}^\mathsf{T} = \mathbf{y}\mathbf{x}^\mathsf{T}$. You may assume that all the entries of $\mathbf{x}$ and $\mathbf{y}$ are non-zero.

2. Let $\mathbf{A}$ be the matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

   (a) For a general $\mathbf{x} \in \mathbb{R}^4$, how is $\mathbf{A}\mathbf{x}$ related to $\mathbf{x}$?

   (b) For a general $\mathbf{x} \in \mathbb{R}^4$, obtain the general formula for $\mathbf{A}^n\mathbf{x}$ for $n \geq 1$.

3. Suppose that $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. Prove the parallelogram law:

$$\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2 = 2\left(\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2\right)$$

   This inequality states that in every parallelogram, the sum of squares of the lengths of the diagonals equals the sum of squares of the four sides.

4. Suppose that $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. Prove that $\|\alpha\mathbf{u} + \beta\mathbf{v}\| = \|\alpha\mathbf{v} + \beta\mathbf{u}\|$ for all $\alpha, \beta \in \mathbb{R}$ if and only if $\|\mathbf{u}\| = \|\mathbf{v}\|$. One approach to proving this statement is by evaluating the difference $\|\alpha\mathbf{u} + \beta\mathbf{v}\|^2 - \|\alpha\mathbf{v} + \beta\mathbf{u}\|^2$ and identifying the conditions when it is zero.

5. Find vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^2$ such that $\mathbf{u}$ is a scalar multiple of $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\mathbf{v}$ is orthogonal to $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

6. Show the reverse triangle inequality for two vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$:

$$\|\mathbf{u} - \mathbf{v}\| \geq \|\mathbf{u}\| - \|\mathbf{v}\|$$