# Chapter 2: Vector Operations

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# Outline

## Vector Addition

- Properties
- Displacements and Positions

## Scaling

- Scaling Displacements

## Linear Combinations

## Complexity of Vector Operations

# Vector Addition

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 2 + 1 \\ 3 + (-2) \\ (-1) + 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

- Vector subtraction is similar

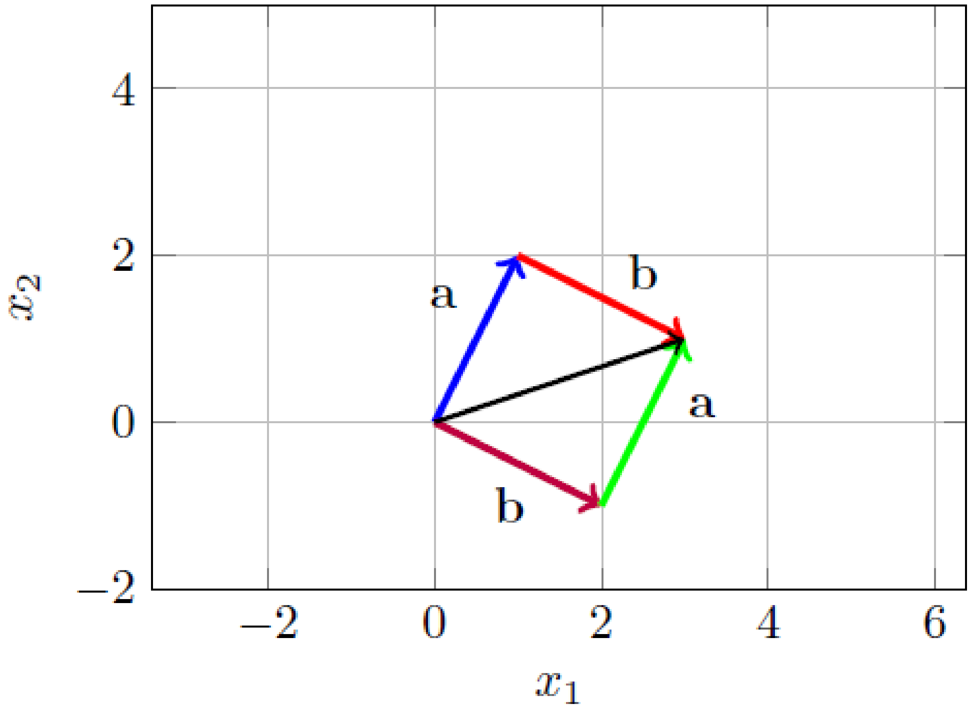
$$\mathbf{x} = \mathbf{u} - \mathbf{v} = \begin{bmatrix} 2 - 1 \\ 3 - (-2) \\ (-1) - 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -5 \end{bmatrix}$$

# Vector Addition: Properties

- Commutative $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
- Associative $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
- Additive Identity $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$
- Additive Inverse $\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = \mathbf{0}$

# Displacements

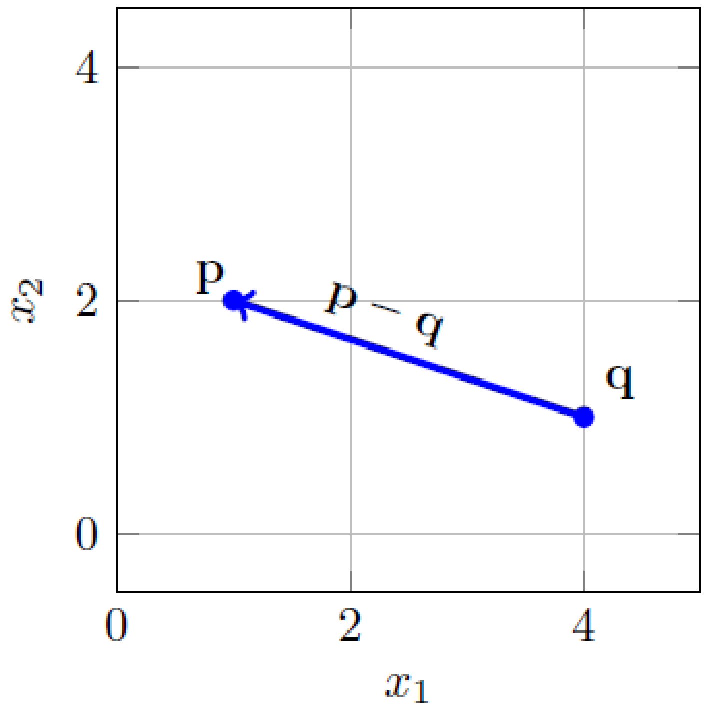
- If $\mathbf{a}$ and $\mathbf{b}$ represent displacements, then
- $\mathbf{a} + \mathbf{b}$ represents the effect of displacing first by $\mathbf{a}$ and then by $\mathbf{b}$
- By commutativity, one can also first displace by $\mathbf{b}$ and then by $\mathbf{a}$



# Displacement and position

- If $\mathbf{p}$ is the position of a point and $\mathbf{a}$ is the displacement, then
- $\mathbf{p} + \mathbf{a}$ is the final position

- If $\mathbf{p}$ and $\mathbf{q}$ are positions, then
- $\mathbf{p} - \mathbf{q}$ is the displacement from $\mathbf{q}$ to $\mathbf{p}$



# Scaling

- Scaling = multiplication of vector by a scalar

$$\mathbf{v} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \text{ and } \alpha = 3 \qquad\qquad \alpha\mathbf{v} = \alpha \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix}$$

- Commutative $\alpha\mathbf{v} = \mathbf{v}\alpha$
- Zero element $0\mathbf{u} = \mathbf{0}$
- Associative $(\alpha\beta)\mathbf{v} = \alpha(\beta\mathbf{v}) = (\alpha\mathbf{v})\beta$
- Distributive $(\alpha + \beta)\mathbf{v} = \alpha\mathbf{v} + \beta\mathbf{v}$

$$\alpha(\mathbf{v} + \mathbf{w}) = \alpha\mathbf{v} + \alpha\mathbf{w}$$

# Scaling Displacements

- If $\mathbf{v}$ represents displacement, then
  - $\alpha\mathbf{v}$ for $\alpha > 0$: displacement in same direction, length scaled by $\alpha$
  - $\beta\mathbf{v}$ for $\beta < 0$: displacement in opposite direction, length scaled by $|\beta|$

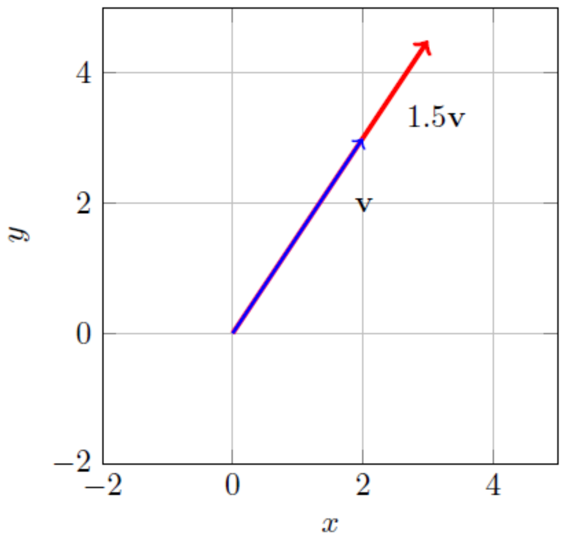



Figure 2.3: Scaling a vector $\mathbf{v}$ by a factor of 1.5.

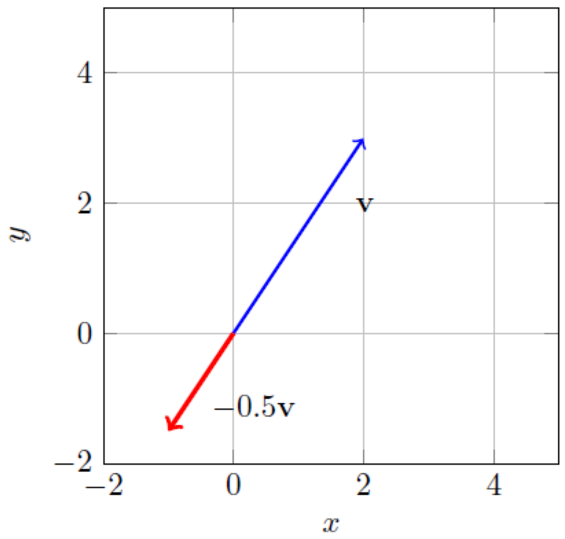



Figure 2.4: Scaling a vector $\mathbf{v}$ by a factor of -0.5.

# Linear Combinations

- Sum of scalar multiples of vectors

$$\underbrace{\alpha_1}_{\text{Coefficients/weights}}\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \cdots + \alpha_m\underbrace{\mathbf{v}_m}_{\text{Component vectors}}$$

- Any vector = linear combination of standard unit vectors

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = v_1\mathbf{e}_1 + v_2\mathbf{e}_2 + \cdots + v_n\mathbf{e}_n \qquad \mathbf{v} = 3\mathbf{e}_1 + (-2)\mathbf{e}_2 + 4\mathbf{e}_3 = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$$

# Complexity

- Real numbers stored as data types, e.g. 64-bit floating point numbers
- For each operation, we want to know memory & computational cost
- Memory complexity: memory required
  - E.g., load two vectors in memory to add them
  - Memory complexity = $2n \times 64$ bits
- Computational complexity: number of floating point operations (flops)
- Includes addition, subtraction, multiplication, division

| **Operation** | **Flop count** |
|---|---|
| Scaling | $n$ |
| Vector Addition | $n$ |

# Thank You

Next: Vector Spaces