Solutions to Worktext Exercises

Chapter 5

Visualizing Continuous Distributions

Basic Learning Exercises

1. It slides to the right or left and the shape does not change.

2. As the standard deviation increases, the distribution becomes more spread out and shorter. As it decreases, the distribution becomes more compact and taller. The curve returns to the bell-shape because the horizontal and vertical axes are redrawn to a scale that allows you to see a proper perspective of the distribution.

3. All three measures of central tendency equal the first parameter (mean = mode = median). Changing the standard deviation has no effect on central tendency. No, skewness is 0 and kurtosis is 3 (mesokurtic) for *any* normal distribution.

4. Between 98% and 98.5% of the time. Between 1% and 1.5% of the time.

5.

	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$
Area = 0.01	Z Value = 2.326	Value = 4.653	Value = 6.979
Area = 0.025	Z Value = 1.960	Value = 3.920	Value = 5.880

6. For a right-tail area of 0.025, the X value is 1.960 σ. For a right-tail area of 0.01, the X value is 2.326 σ.

7. Normal distribution value = 4 + 1.645 (3) = 8.935. In general, $X = \mu + 1.645\sigma$.

8. Z = 0 is a carbon monoxide reading of X = 50 ppm, and Z = 1 is a reading of X = 60.

9. a) About 95% of the area is within $\mu \pm 2\sigma$. b) About 99% of the area is within $\mu \pm 3\sigma$. c) Observations beyond $\mu \pm 3\sigma$ would be sufficiently unlikely that they might appear to be from a different population than the one under consideration.

10. Since a normal variate would almost never exceed $\mu \pm 6\sigma$, a "6-sigma" defect rate is extremely low. Such a low probability of non-conformance to specifications would be desirable. However, not every product can achieve a "6-sigma" defect rate because of limitations in manufacturing technology.

11. a) We can see that $P(Z < 0) = 0.50$, as would be expected for a symmetric distribution (half the area is below the mean). b) From the graph, we estimate that $P(Z < 1) \approx 0.84$, $P(Z < 2) \approx 0.98$, and $P(Z < 3) \approx 1.00$. c) If you scored two standard deviations above the mean, then about 98 percent of the class scored below you. d) If you scored three standard deviations above the mean, then you are probably the top student in the class.

Intermediate Learning Exercises

12. DF = 1 6.314   DF = 6 1.943   DF = 11 1.796   DF = 16 1.746   DF = 21 1.721
The tails become thinner and the middle of the distribution becomes thicker as degrees of freedom increase.

13. Mean = mode = median = 0, skewness = 0, standard deviation > 1, and kurtosis > 3. As DF increases, the SD and kurtosis both decrease, approaching 1 and 3, respectively.

14. The t is more peaked, has thicker tails, and is not as wide in the body of the distribution. The difference between the distributions has decreased because, as the DF increases, the t distribution becomes a standard normal.

15. With 10 DF between -1.5 and –0.5 the cumulative t distribution is below the cumulative standard normal. Between 0.5 and 1.5 the opposite is true. The difference has decreased, since Student's t is more nearly mesokurtic for 20 DF.

16. The difference is more noticeable in the probability distribution that shows the density of x. The cumulative distribution integrates from $-\infty$ to x, which tends to hide the differences. The t probability distribution has longer tails and a narrower body but is taller at 0 than the normal probability distribution. Since both have a mean and median of 0, their cumulatives equal 0.5 at 0. Therefore, while the t cumulative starts out above the normal cumulative, they are equal at 0.

17. If DF increases, the distribution slides to the right and becomes more symmetric. If DF decreases, the distribution slides to the left and becomes more asymmetric.

18. Mean = DF, mode = DF – 2, median = DF – 0.667, and standard deviation = $\sqrt{2 \text{ x DF}}$.
The distribution is always skewed right (skewness > 0) and leptokurtic (kurtosis > 3).

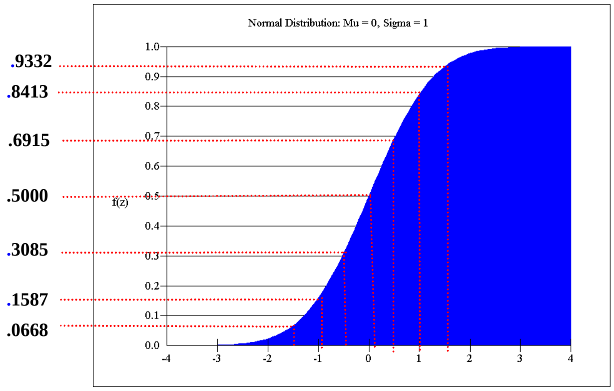
19. The mean is 5 and the standard deviation is 3.16. The cumulative integrates the probability distribution from left to right, so at any value the taller function adds more to its cumulative. The normal distribution is above the chi-square up to 0.5; below it between 0.5 and about 5; and again above it for values greater than 5.

20. The distribution changes very little between 10 and 20 degrees of freedom, and hardly changes at all between 20 and 200 degrees of freedom. The axis scale did not change.

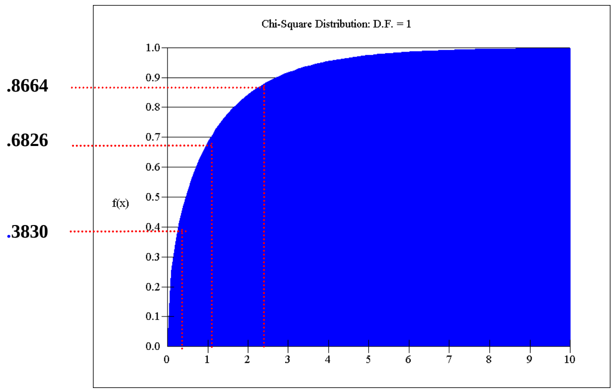
21. (a) The distribution becomes more symmetric, though is it never quite normal. (b) Its right tail shortens dramatically between 3 and 10 degrees of freedom, is reduced slightly more as you move from 10 to 20 degrees of freedom, and changes very little between 20 and 200 degrees of freedom. (c) The mean and mode approach 1 (the mean from above and the mode from below). The skewness becomes smaller, approaching 0 (symmetry). The kurtosis becomes smaller, approaching 3 (mesokurtic).

Advanced Learning Exercises

22.



23.



24. Normal Prob = Cumulative Upper Value – Cumulative Lower Value = Chi-Square Prob
0.6915 – 0.3085 = 0.3830, 0.8413 – 0.1587 = 0.6826, 0.9332 – 0.0668 = 0.8664

25. Squaring a normal random variable between –0.5 and 0.5 translates to a chi-square random variable between 0 and 0.25. Therefore the probability of a normal variable being between –0.5 and 0.5 is exactly the same as a chi-square random variable being between 0 and 0.25. Similarly, a normal random variable between –1 and 1 translates to a chi-square variable between 0 and 1. Therefore the probabilities in exercise 24 are consistent with the fact that a standard normal squared is a chi-square random variable.

26. First, draw the cumulative of the t distribution with k DF (k is your choice) and show the probability for t equal to –1.5, –1, –.5, .5, 1, and 1.5. Second, draw the cumulative of the F distribution with 1 and k DF and show the probability that F is less than .25, 1, and 2.25. Finally, show that Pr($-1.5 \le t \le 1.5$) = Pr ($F \le 2.25$), Pr($-1 \le t \le 1$) = Pr ($F \le 1$), and Pr($-0.5 \le t \le 0.5$) = Pr ($F \le 0.25$).

27. Values along horizontal axis that correspond to an area of 0.05 in the right tail:

	$F_{1,d}$	$F_{10,d}$	$F_{200,d}$
d=1	4.24	2.24	1.75
d=50	4.03	2.03	1.48
d=200	3.89	1.88	1.26
	$\chi^2_1/1=$ 3.841	$\chi^2_{10}/10 =$ 1.831	$\chi^2_{200}/200 =$ 1.170

As the denominator DF increases the F value gets closer to the χ^2 value. The difference in each column between the F value (DF = 200) and the χ^2 value is 0.049, 0.049 and 0.09. The F distribution is the ratio of two independent chi-squares divided by their degrees of freedom. Hence, chi-square must be divided by its DF to make a valid comparison.