Practice Questions 3: Solutions to unsolved preoblems

1. We have the following information:

$$P(Math\ \cap Science) = 0.3\ and\ P(Math) = 0.7$$

We want $P(Science/Math) = \frac{P(Math\ \cap Science)}{P(Math\ )}$=0.3/0.7

2. Let E_i be the event that the first i cards have no pairs among them. We want the value of $P(E_4)$. We can see clearly that $E_4 \subset E_3 \subset E_2 \subset E_1$, which implies that $E_4 \cap E_3 \cap E_2 \cap E_1 = E_4$. Therefore, $P(E_4) = P(E_4 \cap E_3 \cap E_2 \cap E_1)$.

$$P(E_4 \cap E_3 \cap E_2 \cap E_1) = P(E_1) * P(E_2|E_1) * P(E_3|E_2 \cap E_1) * P(E_4|E_3 \cap E_2 \cap E_1)$$
$$= \frac{52}{52} * \frac{48}{51} * \frac{44}{50} * \frac{40}{49}$$

3. Let U_i: $be\ the\ event\ Up - to - date\ in\ week\ i\ and\ B_i$: $be\ the\ event\ behind\ in\ week\ i$
We know $P(U_1) = 0.8$ which means that $P(B_1) = 1 - 0.8 = 0.2$.
Now, $P(U_2) = P(U_1) * P(U_2|U_1) + P(B_1) * P(U_2|B_1) = 0.8 * 0.8 + 0.2 * 0.4 = 0.72$
$P(B_2) = P(B_1) * P(B_2|U_1) + P(B_1) * P(B_2|B_1) = 0.8 * 0.2 + 0.2 * 0.6 = 0.28$
We are interested in $P(U_3)$. Same as above, we have:
$$P(U_3) = P(U_2) * P(U_3|U_2) + P(B_2) * P(U_3|B_2) = 0.72 * 0.8 + 0.28 * 0.4 = 0.688$$

4. There are 52 * 51 * 50 possibilities of 3 cards. Of these there are 13 * 50 * 12 ways to choose the cards so that the first and third are spades and 13 * 12 * 11 where all three are spades. Our probability of interest is :
$$\frac{(13 * 12 * 11)/(52 * 51 * 50)}{(13 * 50 * 12)/(52 * 51 * 50)} = \frac{11}{50}$$

5. Let the probability that A wins is X.
The probability that A does not win at 1st draw is 2/3. If this happens, then B is exactly in the same situation as A was at the beginning, so now his chance to win is X. So probability that B wins is (2/3)*X.
Similarly, probability that C wins is $(2/3)^2$*X.
Sum of all probabilities is one.
X + (2/3)X + $(2/3)^2$X = 1
X = 9/16

Therefore the probabilities of interest are:
The probability that A wins is 9/19 = 0.4737
The probability that B wins is 6/19 = 0.3158
The probability that C wins is 4/19 = 0.2105

6. There are 4 possible outcomes here, corresponding to the 4 combinations of success and failure of the two chefs.
SS: Both succeed
FF: Both fail

SF: A succeeds B fails

FS: A fails B succeeds

We know the following:

P(SS)+P(SF) = 2/3

P(SS)+P(FS) = ½

P(SS)+P(FS)+P(SF) = ¾

We know that P(SS)+P(SF)+P(FS)+P(FF)=1. Using the 4 equations above we get the separate probabilities as: P(SS)=5/12, P(SF)=1/4, P(FS)=1/12, P(FF)=1/4.

The conditional probability of interest is:

P({SF}|{SF,FS})= $\frac{1/4}{\frac{1}{4}+\frac{1}{12}}$