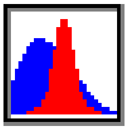
Solutions to Worktext Exercises



Chapter 7

Visualizing the Central Limit Theorem

Basic Learning Exercises

1. Population Being Sampled: Mean 25.4 Standard Deviation 0.60
   Sampling Distribution: Mean 25.4 Standard Error 0.200
   The standard deviation of the sampling distribution is smaller than the standard deviation of the population. From the Central Limit Theorem, the formula relating them is: Standard Error = Standard Deviation / $\sqrt{\text{Sample Size}}$

2. The upper diagram shows that some of the 9 individual observations fall below 25 mpg. But the lower diagram shows that the sample mean exceeds 25 mpg, so the company's expectations are met *on average*. Since the mileage is normally distributed with a mean of 25.4 inches and a standard deviation of 0.60, individual mileage will range between 23.6 mpg and 27.2 mpg (3 standard deviations from the mean). It is very likely that a sample of 9 will have at least one sample point less than 25.

3. Individual customers, who presumably do not know the Central Limit Theorem, may not understand that individual observations well below 25 mpg are likely. While legally correct, the claim might be misleading to some customers.

4. Standard Deviation 0.60 Standard Error 0.120
   The standard error is $\sigma/\sqrt{n}$, so when the sample size changed from 9 to 25 the standard error became smaller. The lower diagram shows that the sampling distribution for n = 25 is narrower and taller than for n = 9.

5. A standard error of 0.06 would require a sample of n = 100 since $\sigma/\sqrt{n}$ and $\sigma = 0.60$.

6. If one selects n independent observations from any population with finite mean μ and finite standard deviation σ, then the sampling distribution has a mean μ and standard error $\sigma/\sqrt{n}$. If the sample size is large enough, the sampling distribution is normally distributed. Your interpretation could be as simple as saying "If you take a sample from a population, the distribution of the sample means will be approximately normal with the same mean as the population sampled and a much smaller variance."

7. You have to know how the standard error related to the standard deviation.

8. Yes it is. Because the distribution being sampled is normally distributed, the sample mean is normally distributed regardless of sample size.

9. The populations overlap. Although machine two has a slightly greater mean, the most striking difference is that machine two has a much smaller variance than machine one.

10. You would select Machine 1 (the one with the smaller mean). You would need to sample about 100 groups of 100 pieces of fruit. Knowing the CLT leads you to increase the number of samples, thereby reducing the standard error.

Intermediate Learning Exercises

11. The distribution of delivery times would be positively skewed because sometimes traffic or weather conditions could add a considerable amount to the usual delivery time.

12. No, the experimental sampling distribution has a longer right tail then left tail, and the mode of the experimental sampling distribution is to the left of the mode of the normal sampling distribution. This is consistent with the Central Limit Theorem since the theorem says that, depending upon the population being sampled, the sample size may have to approach infinity before the sampling distribution is normally distributed. In this case the sample size is three. For the skewed population being sampled, this is too small a sample size to generate a normally distributed sampling distribution.

13. Yes. Since the sample size is much larger, the Central Limit Theorem would hold, even with this amount of skewness.

14. A sample size of 20 is large enough to generate a normal sampling distribution with this amount of skewness.

15. No. A very skewed distribution may require larger sample sizes than 20.

16. It is not clear which driver is preferred until you have collected a very large sample of data. In this case even 100 weeks of observed deliveries per driver might not be enough to show that the sampling distributions differ. This scenario illustrates the Central Limit Theorem because it shows two things. First, as the sample size gets larger, the sampling distribution becomes more normal. Second, as the sample size gets larger, the sampling distribution becomes narrower.

17. No student should take the wager since the chances of the sample mean being out of this range is very small and he is only offering a 10 to 1 payoff. Yes, he could have lost the wager. Since the standard error of the sampling distribution is $\sigma/\sqrt{n}$, the standard error is 0.0424. This means the range from 0.9 to 1.1 is the same as a range from $-2.358 \times \left(\frac{(0.9-1.0)}{.0424}\right)$ to $2.358 \times \left(\frac{(1.1-1.0)}{.0424}\right)$ if you use a standard normal distribution. Therefore, the sample mean will be outside this interval slightly less than 2% of the time. It shows how the standard error can be made very small by taking very large samples.

18. No it is not. Since the distribution being sampled has a uniform distribution, a sample size of 2 generates a triangular distribution. The sample size is not large enough to generate a normally distributed sampling distribution.

19. Yes. Since the distribution being sampled has a uniform distribution, a sample size of 6 is enough to generate a sampling distribution that is approximately normal.

20. Although the range and general frequency is clear, it is sometimes difficult to see the exact shape because the sampling distribution is so narrow.

Advanced Learning Exercises

21. It could be located almost anywhere within the sampling distribution.

22. It could have moved closer to the population mean or away from the population mean. Usually if it is very close to the population mean to begin with, it will move away, while if it is far away to begin with it will move toward the population mean.

23. $\overline{\overline{X}}$ would, in general, move closer to μ as more samples are taken. It is normal for there to be times when it moves away, but in general, it should be moving closer.

24. $\overline{\overline{X}}$ would, in general, move closer to μ as more samples are taken. After each group of 10 samples, there should be a noticeable movement of $\overline{\overline{X}}$ towards μ. The only exception will be if, at the beginning of exercise 19, the two were already very close.

25. The same general pattern still holds that $\overline{\overline{X}}$ moves closer to μ. However, the variation in $\overline{\overline{X}}$ is reduced considerably. It does not jump around as much, especially in the early part of the experiment.

26. Standard error if n = 4 ___10___ Standard error if n = 16 ___5___
Standard error if n = 64 ___2.5___ Standard error if n = 256 ___1.25___
Yes, the sample mean is a consistent estimator for μ.