Linear Programming: Applications

The custom molder's problem

- 100 cases of six-ounce juice glasses require six production hours.
- 100 cases of ten-ounce fancy cocktail glasses require five production hours
- 60 hours of production capacity available per week.
- Effective storage capacity of 15,000 cubic feet is available per week.
- A case of six-ounce juice glasses requires 10 cubic feet of storage space,
- A case of ten-ounce cocktail glasses requires 20 cubic feet.
- The contribution of the six-ounce glasses is $5.00 per case.
- The contribution of ten-ounce cocktail glasses is $4.50 per case.
- Customer available will not accept more than 800 cases per week of six-ounce glasses.
- There is no limit on the amount that can be sold of ten-ounce glasses.

How many cases of each type of glass should be produced per week to maximize the total contribution?

Frosty distributors: Data

- 15 vegetables in cartons of size 1.25 cubic feet are procured every week.
- Rarely any inventory is left over at the end of the week.
- Warehouse space available is 18000 cubic feet.
- Credit available per week is $30000.
- Forecast is expressed in terms of a minimum and maximum anticipated sales quantity.
- The minimum quantity is based on a contractual agreement.
- The maximum quantity represents a conservative estimate of the sales potential per week.

  Write down a model that maximizes Frosty's weekly profit.

Vegetable	***Min***	***Max***	***Cost***	***Price***
Whipped potatoes	300	1500	2.15	2.27
Creamed corn	400	2000	2.20	2.48
Black-eyed peas	250	900	2.40	2.70
Artichokes	0	150	4.80	5.20
Carrots	300	1200	2.60	2.92
Succotash	200	800	2.30	2.48
Okra	150	600	2.35	2.20
Cauliflower	100	300	2.85	3.13
Green peas	750	3500	2.25	2.48
Spinach	400	2000	2.10	2.27
Lima beans	500	3300	2.80	3.13
Brussels sprouts	100	500	3.00	3.18
Green beans	500	3200	2.60	2.92
Squash	200	500	2.50	2.70
Broccoli	400	2500	2.90	3.13

TelecomOptics' transportation problem data

	Atlanta (A)	Boston (B)	Chicago (C)	Denver (D)	Omaha (O)	Portland (P)	*Supply (000's)*	
Baltimore (L)	1,675	400	685	1,630	1,160	3,800	18	
Cheyenne (H)	1,460	1,940	970	100	495	1,200	30	
Salt Lake (S)	1,925	2,400	100	500	950	800	22	
Memphis (M)	380	1,355	543	1,045	665	2,321	24	
Wichita (W)	922	1,646	700	508	311	1,797	31	Total demand
Demand (000's)	20	16	28	12	14	22		112
						Total supply	125	

The management is debating how to serve all the six markets at the least possible total production and transportation cost. Formulate a model and recommend a least total cost shipping solution.

Binary Problems

Knights, knaves, and werewolves

- The information we have:
- Knights always tell the truth and knaves always lie
- Of the three inhabitants you interview, exactly one of them is a werewolf.
- Each of three inhabitants interviewed is either a knight or a knave and could be a werewolf too.

The statements made:

A: I am a werewolf.
B: I am a werewolf.
C: At most one of us is a knight.

Construct a model to classify the A, B, C as knights and knaves.

The capacitated facility location problem

This problem is a frequently encountered problem in supply chain management while designing “facility networks”. Sometimes it is also referred to as the “fixed charge problem”. Instances of the fixed charge problem are seen in several applications including product line selection, portfolio design, and others. Let us consider the same set up we discussed earlier while studying the transportation model. However, now we have an additional challenge: It costs us to keep a supply (production) facility open. This cost is independent of the demand that will be served from the supply facility – hence it is a fixed cost. Of course, we can only serve any demand from a supply location if it is open, else we neither incur the fixed cost nor any operating costs.

	Atlanta (A)	Boston (B)	Chicago (C)	Denver (D)	Omaha (O)	Portland (P)	Supply (000's)	Fixed Cost	
Baltimore (L)	1,675	400	685	1,630	1,160	3,800	18	7650	
Cheyenne (H)	1,460	1,940	970	100	495	1,200	24	3500	
Salt Lake (S)	1,925	2,400	100	500	950	800	27	5000	
Memphis (M)	380	1,355	543	1,045	665	2,321	22	4100	
Wichita (W)	922	1,646	700	508	311	1,797	31	2200	Total demand
Demand (000's)	10	8	14	6	7	11			112
						Total supply	125		

Modelling piecewise linear functions

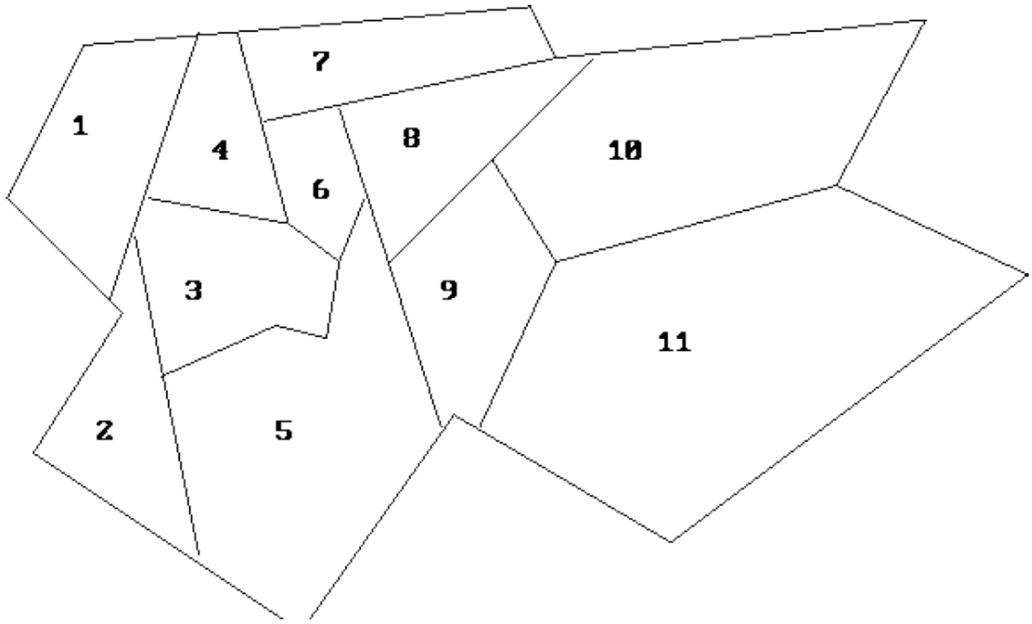
Our goal is to build a simple costing model, using a set of linear constraints, that computes the cost of a commodity exactly. Suppose we want to procure k units of the commodity and the marginal cost of procurement is as follows:

Marginal cost = \$2,000 per unit: for the first 10 units,
\$1,800 per unit: for the next 15 units,
\$1,700 per unit: for the remaining 25 units.

How can such a cost structure be modelled using binary variables.

Set covering formulation: Where to locate fire stations?

Consider the following location problem: A city is reviewing the location of its fire stations. The city made up of several neighbourhoods, as illustrated in the figure below.



A fire station can be placed in any neighbourhood. It can service both its neighbourhood and any adjacent neighbourhood (any neighbourhood with a non-zero border with its home neighbourhood). The objective is to minimize the number of fire stations used. How can this problem be solved as an integer problem?