1. A jar contains 5 red, 9 blue and 14 white marbles. If a marble is drawn from the jar at random, then what is the probability that the marble is neither red nor white?

1. 5 red, 9 blue, 14 white

Neither red nor white $\Rightarrow$ Blue marble

$\therefore$ P (Blue) = $\dfrac{9}{5+9+14}$

2. I toss a coin three times. What is the probability of getting a head at most two times?

Sample Space

$\{$ HHH, HHT, HTH, THH, HTT, TTH, THT, TTT $\}$

Atmost 2 heads implies that we have either 1 head

or 2 heads.

$\therefore$ Favorable cases are

$\{$ HTT, TTH, THT, HHT, HTH, THH $\}$

$\therefore$ P(Atmost 2 heads) = $\dfrac{6}{8}$ = $\dfrac{3}{4}$

3. If I roll two dice simultaneously, then what is the probability of getting the numbers such that their sum is odd?

Sample Space for two dice:

(1,1)   (1, 2)   (1, 3)   (1,4)   (1,5)   (1,6)

(2, 1)   (2, 2)   -   -   -   .   (2,6)

(6,1)   (6,2)   -   -   -   .   .   (6,6)

Now, for the sum to be odd one of the dice has an even number and the other an odd number

Case-1 : Die 1 : odd   Die 2 : even

we have $3 \times 3 \to 9$ cases

Case-2 : Die 2 : odd   Die : even

$3 \times 3 \to 9$ cases

$\therefore$ $\dfrac{18}{36}$ is the required probability

4. Two cards are drawn randomly from a pack of 52 cards. Find the probability that both cards are aces?
   a. For the case where cards are drawn with replacement.
   b. For the case where cards are drawn without replacement.

(a) we have 4 aces.

With replacement probability is

$$\frac{4C_2}{52C_2}$$

(b) Without replacement probability is

$$\frac{4C_1}{52C_1} \times \frac{3C_1}{51C_1} \qquad = \frac{4}{52} \times \frac{3}{51}$$

5. There are two events A and B. Assume P(A) = 0.25, P(B) = 0.45, and P(A ∩B) =0.1.
   a. What is P(A U B)?
   b. What is P(A' ∩ B)?

$$P(A) = 0.25 \qquad P(B) = 0.45 \qquad P(A \cap B) = 0.1$$

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.25 + 0.45 - 0.1$$

$$= 0.6$$

b. $P(A' \cap B) = P(B) - P(A \cap B)$

$$= 0.45 - 0.1$$

$$= 0.35$$

6. Consider two sets X and Y.
X = {A, B, C, D, E}
Y = {P, Q, R, S, T, U}
If you select one letter each from X and Y, then find the probability of
a. Getting an A and T.
b. Getting an A or T.

a. $\frac{1}{5} \times \frac{1}{6}$ $\left[P(A) = \frac{1}{5} \quad P(T) = \frac{1}{6} \right]$

$\underbrace{\qquad\qquad}$
Nothing but $P(A \cap T)$

b. $P(A \cup T) = P(A) + P(T) - P(A \cap T)$

$= \frac{1}{5} + \frac{1}{6} - \frac{1}{30}$

$= \frac{1}{3}$

7. In ABC, there are 300 IT employees and 100 Management employees. 20% of IT employees have a college education and 40% of Management employees have college education. If one employee is randomly chosen for promotion, then find the probability that the employee is Management employee and has college education?

ABC

300 IT   100 Management
                40%
20%   College education
College education

Total employees
= 300 + 100
= 400
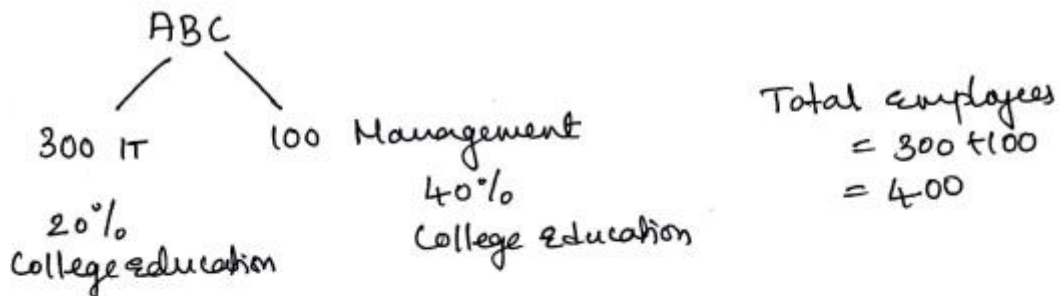
Prob { of randomly choosing an employee } = $\frac{1}{400}$

P { Management employee } = $\frac{100}{400}$ = $\frac{1}{4}$

P { College education | Management } = 0.4

∴ our probability of interest is

$\frac{1}{400} \times \frac{1}{4} \times 0.4$