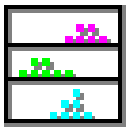
Solutions to Worktext Exercises



Chapter 12

Visualizing Analysis of Variance

Basic Learning Exercises

1. H_0: $\mu_1 = \mu_2 = \mu_3$ (The means are the same)
   H_a: Not all the means are the same.
   Based on the scroll bar, the means differ. Therefore, the null hypothesis is false.

2. Assumption 1 – The variances from each group are equal to one another.
   Assumption 2 – The populations underlying each sample are normally distributed.
   Both assumptions are met in this scenario because the population distribution is normal and the difference in variances is set to zero.

3. Depending upon the sample drawn, the sample items could appear similar or different. Generally, dispersion will look similar, but one group is likely to appear shifted slightly right or left. In this scenario it is unusual for there to be outliers since an outlier must be 3 standard deviations from the mean. But with small sample sizes, observations away from the rest may *appear* to be outliers.

4. Depending on the sample, in this scenario there is a good chance that at least two (and often all three) confidence intervals will overlap. This suggests that H_0 is unlikely to be rejected.

5. Given the very small sample sizes, the data generally looks more uniformly distributed than normally distributed. This is often the case when a small sample is displayed in a dot plot because there are many more "bins" than data points.

6. The differences in standard deviations and means are likely to appear small.

7.

DF Between:	$c-1 = 3 - 1 = 2$
DF Within:	$n-c = 28 - 3 = 25$
Total DF:	$n-1 = 28 - 1 = 27$
Mean Square Between:	MSB = SSB / (c–1) = Vary from 1,000 to 12,000
Mean Square Within:	MSW = SSW / (n–c) = Vary from 250 to 950
F ratio:	F = MSB / MSW = Vary from near 2 to 18

8. MSW measures the average variation *within* each dot plot. MSB measures the variation *between* the dot plots. If H_0 is true, the variation *within* the dot plots should be about the same as the variation *between* the dot plots; that is, MSB and MSW are about equal.

9. Samples may vary, but rejection of H_0 is very unlikely. Since the Difference in Means scroll bar is set to None, we would not expect to reject the hypothesis of equal means.

10. When the means are not equal the variation *between* the dot plots is generally greater than the variation *within* the dot plots. Therefore, MSB will be greater than MSW.

11. Rejection of H_0 is extremely likely. Since the Difference in Means scroll bar is set to Lots, we expect to reject the hypothesis of equal means

Intermediate Learning Exercises

12. Results will vary, but you most likely will get either 0 or 1 rejection out of 10. The empirical Type I error is thus 0% or 10%. No, 10 samples would not be enough.

13. Results will vary, but in 1,000 samples you should get about 50 rejections at $\alpha = 0.05$, and about 10 rejections at $\alpha = 0.01$. Thus, empirical Type I error would be about 5% at $\alpha = 0.05$, and about 1% at $\alpha = 0.01$. Yes, the estimate should be reasonable.

14. Empirical Type I error will generally not equal the α-level because of sample variation.

15. The histogram is skewed right, and matches the shape of the theoretical distribution. The theoretical F distribution assumes a true null hypothesis. Since H_0 is true (equal means) the F statistics should follow the theoretical F distribution.

16. As predicted by the Central Limit Theorem, all three histograms are bell-shaped. They appear to have equal means and variances. This is expected, since you used the scroll bars to create a true null hypothesis ($\mu_1 = \mu_2 = \mu_3$). The same is true for variances.

17. The experiment averages about 3 rejections in 10 trials (corresponding to an empirical power of 0.30 or 30%). However, it may vary from about 1 to 4 rejections.

18. One of the histograms is building to the left of the overall mean line, one to its right, and one centered on top of it. Further trials confirm the initial observations. The final histograms are bell-shaped, with similar variances but dissimilar means.

19. There are too few F statistics in the left tail, and too many in the right tail. This preponderance of large F statistics suggests a false null hypothesis.

20. Results will vary, but you should get about 250 rejections in 1,000 replications, so empirical power is about 0.25. Empirical Type II error is about 0.75 (recall that power is $1 - \beta$). The small difference in means is hard to detect, so the test has low power.

21. Empirical power has increased to about 0.55 and empirical Type II error has decreased to about 0.45. When the null hypothesis is false, as sample size increases power increases and empirical Type II error decreases.

Advanced Learning Exercises

22. MSW <u>about 250,000</u> Hartley's Statistic <u>1–5</u> p-value of F <u>0–1</u>
Since the means are equal, it is very unlikely that the p-value for the F statistic is below 0.05. Hartley's statistic is the ratio of the largest to the smallest variance. A large value of Hartley's statistic would indicate rejection of the null hypothesis of equal variances. For this scenario, its critical value is 4.85.

23. The dot plots may give the appearance of unequal variances about 30% of the time. The average of the MSWs is about 250,000 (the square of each standard deviation). It is unlikely to have more than one rejection with Hartley's statistic since the variances are equal. It is unlikely to get a p-value below 0.05 in the F-test since the means are equal.

24. About 5% and 1% rejections are likely from the experiment. The expected results are 5% and 1%. Yes, most experiment results will be plausible.

25. About 40% of the dot plots may suggest unequal variances; about the same as when they were equal. You will probably get 1, 2, or 3 rejections using Hartley's statistic. This means that the power of Hartley's statistic to detect slightly unequal variances is around 20%. Even with the equal variance assumption violated, you are unlikely to get more than one p-value below 0.05 in the F-test.

26. The expected results are *slightly above* 5% and 1%. This means that one-way ANOVA is robust against slightly unequal variances. You can identify the group with a standard deviation of 400, but not with the other two.

27. About half the time the variances appear unequal, slightly more than when the variances were equal. You are likely to get 4, 5, or 6 rejections using Hartley's statistic, resulting in a power of about 0.50 to detect moderately unequal variances. But you still are unlikely to see more than one p-value below 0.05 in the F-test. The dot plot is used if Hartley's statistic is unavailable. It also provides a visual check of the data.

28. About 6% and 1.2% rejections are likely in this experiment. Yes, one-way ANOVA appears to be robust against moderately unequal variances. Yes, you can tell from the histogram which group has the smallest and largest variance.

29. The dot plots will suggest unequal variances about 90% of the time. You would expect 9 or 10 rejections of Hartley's Statistic. When the variances are very unequal, the power of Hartley's statistic to detect very unequal variances is high (about 95%). Even so, p-values below 0.05 are rare in the F-test. Type I error is affected very little.

30. About 6.5% and 2% rejections are likely in this experiment. Since Type I error is still close to 5% and 1% respectively, this result suggests that one-way ANOVA is fairly robust against very unequal variances.

31. The dot plots reveal unequal variances all of the time. About 10% and 4.5% rejections are likely in this experiment. The result means that one-way ANOVA is *not* robust to extremely unequal variances.

32. When the variances are extremely unequal Hartley's statistic can be over 1,000. One-way ANOVA is robust up to very unequal variances. Beyond that point, the empirical Type I error no longer is equal the α-level at which the test was conducted.

33. Power is about 0.40. Power is about 0.30. Power is reduced because the variances are unequal. Yes, violating the equal variances assumption reduces the power of ANOVA.

34. The rejection rates will be about 5% and 1% (quite close to the desired α values). This suggests that one-way ANOVA is robust against sampling from a uniform distribution.

35. The rejection rates will be about 5% and 1% (quite close to the desired α values). One-way ANOVA appears to be robust to samples drawn from *some* skewed distributions. The histograms are slightly skewed. In general, one-way ANOVA appears robust to *some* violations of the normality assumption.

36. Power $\approx$ 0.40. In *some* cases, violating the normality assumption does not reduce power.