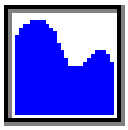
Solutions to Worktext Exercises



Chapter 3

Visualizing Shapes of Distributions

Basic Learning Exercises

1. The population consists of the six discrete values 1, 2, 3, 4, 5 and 6. The probability distribution is six bars or sticks of height 1/6.

2. a) It shows the probability of the random variable equaling a specific value. b) In this scenario, it is the probability of rolling a specific number on a die. c) $P(X = 3) = 1/6$.

3. a) It shows the probability of X being less than or equal to a specific value X_0; written $P(X \le X_0)$. b) It gives the probability of rolling a number less than or equal to X_0. c) The cumulative at X_0 is the sum of the probabilities of the random variable that are less than or equal to X_0. d) $P(X \le 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = 5 \times 1/6 = 5/6$ since $P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = P(X = 5) = 1/6$.

4. The population is all real values between 0 and 2. It is a continuous population because there are an infinite number of values between 0 and 2.

5. It is the area under the probability distribution from its minimum value to X_0. It is also the integral of the probability distribution from $-\infty$ to X_0 in calculus. At X = 2 the cumulative equals 1 since this is the area under the entire probability distribution.

6. a) The mode is at 1.0. b) The cumulative distribution equals 0.5 at the mode because the probability distribution is symmetric and the mode is at the center of the distribution. c) It means there is a 0.5 probability of the sum being less than or equal to 1.0.

7. No, it did not change the shape of either distribution. The centrality parameter only changes the location of the distribution. If a distribution has a centrality or location parameter, changing it will shift the distribution. This is equivalent to adding or subtracting a value to the random variable.

8. Midrange 1 Mean about 0.85 Median about 0.8 Mode about 0.5
   All three statistics are the same because this is a symmetric distribution. In a positively skewed distribution Midrange > Mean > Median > Mode, while in a negatively skewed distribution Midrange < Mean < Median < Mode

9. a) The height is 1.0 and the range is 2.0. b) The new height is 0.667 and the range is 3.0. c) Increasing dispersion (or multiplying the random variable by a value greater than one), increases the population's range and decreases the height of its distribution (i.e., pushes the distribution down). However, its basic shape is unchanged. This is evident when the scale of both axes is changed. The opposite occurs if dispersion is decreased.

10. Standard Deviation 0.41 Interquartile Range 0.59 Range 2
    Range ÷ Standard Deviation 4.878 Range ÷ Interquartile Range 3.390

Intermediate Learning Exercises

11. The range is all real numbers between 0 and 2. It is a continuous distribution. Its shape is a rectangle with a length of 2. It has a height of ½ since its area is one.

12. The cumulative is a straight line with a slope of ½. Since the probability distribution is a rectangle, the cumulative probability at any point is the product of the probability distribution's height (½) and the length of the rectangle at the point being evaluated. Therefore, the $P(X \le 0.5) = 0.5 \times ½ = 0.25$ and $P(X \le 1.2) = 1.2 \times ½ = 0.6$.

13. a) $P(1 \le X \le 2) = 0.5$ b) $P(1 \le X \le 1.1) = 0.05$ c) $P(1 \le X \le 1.01) = .005$
d) $½ \times 0 = 0$ e) It equals zero because there is no area between X_0 and X_0.

14. A positively skewed probability distribution has the mode left of the midrange; hence, it has a short left tail and a long right tail. The corresponding cumulative is shifted left and is bowed upwards. A negatively skewed probability distribution has a short right tail and a long left tail. Its cumulative is shifted right and is bowed downwards.

15. Standard Deviation 0.25 Interquartile Range 0.34 Range 1.64
Range ÷ Standard Deviation 6.56 Range ÷ Interquartile Range 4.82

16. Standard Deviation 0.2 Interquartile Range 0.08 Range 2
Range ÷ Standard Deviation 10 Range ÷ Interquartile Range 25
A peaked probability distribution has longer tails and its cumulative is steeper.

17. Standard Deviation 0.56 Interquartile Range 0.94 Range 2
Range ÷ Standard Deviation 3.57 Range ÷ Interquartile Range 2.13
A platykurtic probability distribution has short tails and its cumulative is a straight line.

18. As peakedness increases, the standard deviation (SD) and interquartile range (IQR) decrease because the data is concentrated in the middle of the distribution. Therefore, since the range did not change, more SDs or IQRs are needed to cover the range.

19. Mean 1 Standard Deviation 6.0 Skewness 0 Kurtosis 3.0
Only the mean changes. Only the standard deviation changes.

20. Changing symmetry shifts the mode of the distribution changing its overall shape and hence all of the shape statistics. Changing peakedness changes the concentration of the data within the distribution. When peakedness is increased, the data is more concentrated; hence, the standard deviation is reduced. Conversely when peakedness is decreased.

Advanced Learning Exercises

21. The distribution is a rectangle. The population is continuous because there are an infinite number of values between 0 and 2. A distribution with no modes has no tails; hence, its symmetry and peakedness cannot be changed.

22. It has two modes because there is a morning and afternoon rush hour. The random variable is the time of day since we are examining traffic distribution by time of day. It is a continuous population since there are an infinite number of times within a day. The cumulative distribution's tails are short, almost flat, and connected with a slightly wavy line. You can tell there are two modes because the cumulative distribution has three inflection points: one at each mode and one at the minimum value between the modes.

23. As asymmetry increases, the local mode becomes smaller relative to the global mode. In addition, both modes move toward the left (positively skewed) or right (negatively skewed)

side of the probability distribution. The cumulative distribution, with three inflection points, is bowed outward as the distribution becomes more negatively skewed and bowed downwards as the distribution becomes more positively skewed.

24. As the probability distribution becomes more peaked, the two modes are pushed together. This causes the cumulative distribution to become steeper and straighter in the middle and its tails to become longer. The two modes are still inflection points (although they are difficult to see) on the cumulative distribution.

25. As the probability distribution becomes less peaked, the two modes separate. When the two modes are at the minimum and maximum population values, the distribution has a "U" shape. As the modes separate, (1) the cumulative distribution's flat tails become shorter, becoming almost vertical when the distribution is ∪-shaped; (2) the inflection points move to the ends of the tails and disappear when the distribution is ∪-shaped; and (3) the middle of the cumulative distribution becomes flatter.

26. The distribution could be trimodal as more people come in for breakfast, lunch, and dinner. The random variable is the time of day since we are examining the distribution of customers by time of day. The population consists of all real numbers between 0 and 24. It is a continuous population because there are an infinite number of times in a day.

27. The answer should identify where the two or three modes are and why they would occur. Either the explanation or the sketch should clearly show if one of the modes is global.

28. It is a very skewed, very peaked distribution.

29. It is a symmetric distribution with no tails; almost a rectangular distribution.