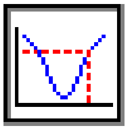
Solutions to Worktext Exercises



Chapter 11

Visualizing Power and Type I / Type II Error

Basic Learning Exercises

1. $H_0 : \mu = 7.0$ versus $H_1 : \mu < 7.0$

2. The critical value is 6.638, which equals $\mu_0 - Z_{0.05} \times \boldsymbol{\sigma}_{\bar{x}} = \mu_0 - Z_{0.05} \times \sigma/\sqrt{n} =$ $7 - 1.645 \times 1.1/\sqrt{25}$.

3. Left side

4. $H_0 : \mu = 7.0$ versus $H_1 : \mu > 7.0$ Right side

5. The rejection region agrees with the direction of the alternative hypothesis.

6. There are two critical values because we will reject the null hypothesis if there is evidence that the test statistic is too large or too small. The rejection region is left of the lower critical value and right of the upper critical value.

7. The power is 0.23. If the true quiz mean is 6.8, there is, on average, a 23% chance of correctly rejecting the null hypothesis that μ is 7.0. Power is the area under the distribution of the true mean to the left of the critical value (in the rejection region).

8. The range is 6.0 to 7.2. If the true mean μ is 6.6, the power is 0.57. If $\mu = 6.4$, then power is 0.86. If $\mu = 6.2$, then power is 0.98. If $\mu = 7.0$, then power is 0.05.

9. In this scenario power decreases as the true value increases from 6.0 to 7.0 because, if the true mean is closer to the hypothesized mean, it is less likely that there will be enough evidence to reject the null hypothesis. If the true mean is 7.0 the null hypothesis is true, so the probability of rejecting the null hypothesis equals the alpha level of the test. Type II error is 0.95 at X = 7.0 since Type II error + Power = 1.

10. Power would equal 0.5 since half of the distribution would be in the rejection region.

11. Power is 0.44. It is different because the critical value has changed; it was 6.638 and it is now 6.569 and 7.431. Since we are now doing a two-tailed test, the alpha level of 0.05 is divided between two tails of the distribution. Therefore, the *left* critical value has shifted to the left.

Intermediate Learning Exercises

12. The alpha level is 0.05. The power is reduced to 0.22.

13. The power is 0.57. It increased because the area in each tail of the hypothesized distribution increased to 0.05. This meant that the two critical values moved closer together and that the area to the left of the *left* critical value under the true distribution has increased.

14. $\alpha = 0.01$ 0.38 $\alpha = 0.05$ 0.62 $\alpha = 0.10$ 0.73

15. A decrease in alpha leads to a decrease in power.

16. As alpha is increased, Type II error is decreased.

17. There is an inverse relationship. When Type I error is increased, Type II error is decreased, other things being held constant.

18. The power is 0.66. If the battery voltage has a true mean of 1.45 volts there is, on average, a 66% chance of correctly rejecting the null hypothesis. The 1.45 represents the true mean of the distribution of battery voltage. Power is the area to the left of the left critical value (in the rejection region) under the distribution of the true mean.

19. The power curve has shifted up. At 1.45, the power has increased to 0.99.

20. $n/2 = 50$ 0.12 $n = 100$ 0.28 $2n = 200$ 0.60

21. There is a direct relationship. As sample size increases, power increases.

22. Type II error decreases.

23. There is an inverse relationship. Type II error decreases as sample size increases.

24. Total error decreases as sample size increases.

25. The power is 0.76. If the true mean of the diameter of the beer mugs is 3.60 inches, there is, on average, a 76% chance of correctly rejecting the null hypothesis. The 3.60 represents the true mean of the distribution of the diameter of the beer mugs. Power is the area to the right of the critical value under the distribution of the true mean.

26. The power curve has shifted down. At 3.60 the power has decreased to 0.39.

27. $\sigma/2 = 0.2$ 0.96 $\sigma = 0.4$ 0.59 $2\sigma = 0.8$ 0.30

28. As the standard deviation increases, power decreases.

29. Type II error increases.

30. Type II error increases as standard deviation increases. It is true for two-tailed tests.

31. Example: An assembly line will be shut down if a hole diameter exceeds 2 cm.
Standard Error = $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2/\sqrt{100} = 0.2$ Critical Value = 2.329 True Mean = 2.2

32. $Z > (2.329 - 2.2) \div 0.2 = 0.129 \div 0.2 = 0.645$. Power is 0.26.

33. Hypothesized Mean = 1.8 CV = 2.129 Z value = –0.355 Power = 0.66
Hypothesized Mean = 1.6 CV = 1.929 Z value = –1.355 Power = 0.92
Hypothesized Mean = 1.4 CV = 1.729 Z value = –2.355 Power = 0.99
Hypothesized Mean = 2.2 CV = 2.529 Z value = 1.645 Power = 0.05

34. a) Power increases as the difference between the true and hypothesized mean increases. b) This relation holds true regardless even if a left-tailed or two-tailed test is used.

35. Because the null hypothesis is true if the hypothesized mean equals 2.2, the area is the probability of incorrectly rejecting the null hypothesis. The true mean $\mu = 2.2$ is the same as the hypothesized mean $\mu_0 = 2.2$, so there can be no Type II error (accepting a false null hypothesis) because the null hypothesis is true. Therefore, instead of power, the risk should be called β or Type I error.

Advanced Learning Exercises

36. a) $\pi = 0.70$ versus $\pi \neq 0.79$. b) The power is 0.67. The range of true proportion values where the power curve is less than 1.0 is 0.6 to 0.8. If alpha is halved, the power curve shifts down and the power is reduced to 0.55. c) If $\alpha = 0.05$ and n = 200, power is 0.32.

37. The power is 0.08. The true distribution is slightly taller, and, since its area is one, it must have slightly less dispersion (not noticeable).

38. The power is 0.96. The true distribution is getting much taller and noticeably narrower than the hypothesized distribution.

39.

True Distribution	$\pi = 0.12$	0.0325	$\pi = 0.04$	0.0196
Hypothesized Distribution	$\pi = 0.13$	0.0336	$\pi = 0.13$	0.0336

40. Since the null is assumed true, the conditions $n\,\pi_0 \geq 5$ and $n\,(1 - \pi_0) \geq 5$ must be true in order to assume that estimated p is normally distributed. In this case $n\,\pi_0 = 4$, and hence estimated p may not be normally distributed.

41. The hypothesized distribution stops at zero because a negative value of π makes no sense. It does not meet the horizontal axis because a normal distribution with a mean of 0.04 and a standard error of 0.0196 does not have a height of zero at zero.

42. Yes, more of the distribution is truncated than before. The critical value is negative and it should be positive. Since the test statistic is always positive, a negative critical value means that the *actual* Type I error and power is 0.

43. Yes, because a right-tailed test for a proportion is equivalent to a left-tailed test if you subtract the value of the null hypothesis from one.

44. If these conditions are not met, then the distribution of the test statistic is not normal. Therefore, the hypothesis test is biased (Type I error is incorrect) and hence has less power and more Type II error than one expects.