Solutions to Worktext Exercises
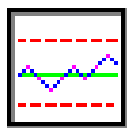
Chapter 21

Visualizing Statistical Process Control

Basic Learning Exercises

1. Desired mean _____2,000_____   Units of measurement ___milliliters___
    Sampling frequency __hourly__   Sample size _____5_____

2. $\overline{X}$-chart   R-chart
    Upper Control Limit (UCL) __2006.7__   Upper Control Limit (UCL) __25.8__
    Centerline (Actual Mean) __2000.0__   Centerline (Actual Mean) __11.6__
    Lower Control Limit (UCL) __1993.3__   Lower Control Limit (UCL) __0.0__
    Since the process is in control and the UCL and LCL are set from the true parameters, the
    sample statistics should lie between the UCL and LCL over 99 percent of the time since these
    are 3 sigma limits.

3. UCL and LCL are sometimes called 3 sigma limits because they are derived from control
    chart factors based on the normal distribution. "Sigma" is a shorthand reference to the
    standard deviation of the sample mean (also called the standard error of the mean). If the
    process is in control, the sample means will lie within $\pm$ 3 sigmas of the mean about 99.73%
    of the time , within $\pm$ 2 sigmas of the mean about 95.44% of the time, and within $\pm$ 1 sigmas
    of the mean about 68.26% of the time. The normal distribution is symmetric, so half the
    means on the X-bar chart should be below the centerline, and half above.

4. $\overline{X}$-chart   R-chart
    Frequency above centerline __25 (17 to 33)__   Frequency above centerline __25 (15 to 35)__
    Frequency below centerline __25 (17 to 33)__   Frequency below centerline __25 (15 to 35)__
    Frequency outside limits __0__   Frequency outside limits __0__
    Sample results will vary. The frequency above and below the centerline should be roughly the
    same. Since the process is in control, no observations outside the UCL and LCL are expected
    (although such a result is possible). Yes, 50 samples is a reasonable basis for generalization,
    although more samples would be even better.

5. The histogram should resemble the normal curve. However, there can be variation from
    normality, even when the process is in control.

6.
	Frequency	Percent	Percent if Normal
Above centerline	50 (45 to 55)	50	50.00
Below centerline	50 (45 to 55)	50	50.00
Within $\pm$ 1 sigma	68 (65 to 80)	68	68.26
Within $\pm$ 2 sigma	95 (93 to 97)	95	95.44
Within $\pm$ 3 sigma	100	99	99.73
Outside $\pm$ 3 sigma	0	0	0.27

Results will vary. The frequencies should be roughly symmetric and normally distributed.
Although the process is in control, an observation outside the UCL and LCL would not be
terribly unusual. With 100 replications, generalizations can be made, although 500
replications would be needed to ensure reasonable stability. Note that the "sigma limits" are

based on $\mu \pm k\,\sigma/\sqrt{n}$, where μ and σ are the true process parameters. The Help file contains formulas, examples, and a table of normal percentages.

Intermediate Learning Exercises

7. a) Instability is a larger-than-normal amount of variation (the actual process standard deviation has increased). b) Unless something is done, quality will suffer because output is more variable than it is supposed to be. c) In manufacturing, typical causes are untrained machine operators, overadjustment of equipment, equipment in need of repair, tool wear, or defective material. In the service sector, instability may indicate employee distractions, poor job design, or untrained employees. Instability is a common problem with many causes. d) It shows up most clearly on the R-chart (points outside the control limits) but can also be seen on the $\overline{X}$-chart (too many means beyond the ± 1 sigma limits). There will also be higher-than-expected frequencies in the tails of the histogram of means (and fewer than 68.26% within the ± 1 sigma).

8. When the Instability scroll bar is in the middle, 12 to 25 points outside the control limits are likely (above UCL or below LCL). The evidence of instability is very strong.

9.

	Frequency	Percent	Percent if Normal
Above centerline	50 (45 to 55)	50.0	50.00
Below centerline	50 (45 to 55)	50.0	50.00
Within ± 1 sigma	48 (43 to 50)	48.5	68.26
Within ± 2 sigma	83 (78 to 90)	83.5	95.44
Within ± 3 sigma	96 (93 to 97)	96.0	99.73
Outside UCL and LCL	4 (3 to 7)	4.0	0.27

The histogram should be roughly symmetric. But visually, there is too much in the tails and not enough in the middle, compared with the normal curve.

10. a) Trend is a gradual drift of measurements either up or down from the centerline. b) It is a problem because control limit violations eventually will result. We want to detect trend early, before severe quality loss occurs. c) In manufacturing, trend may be due to tool wear, inadequate maintenance, worker fatigue, gradual clogging of machines (dirt, shavings, etc.), or drying out of lubricant. In the service sector, it may signify gradual relaxing of employee attention, increasing task flow, or developing bottlenecks. d) Trend is detected visually on the $\overline{X}$-chart if enough measurements are taken. Process variance is unchanged, but the cumulative histogram gradually is skewed in one tail.

11. a) The R-chart will not indicate a problem because, within each sample, variation is not changing much. However, within the first 50 samples, the process mean begins to drift above the $\overline{X}$-chart centerline. b) A slow upward trend may be masked by random variation, but you probably will get at least 1 control limit violations within the first 50 samples. By 100 samples, the upward trend will be apparent. If sampling continues beyond 100 samples, some means will drift entirely off the chart. In real life, the process would have been stopped long before then! A slow trend is like an ongoing level shift, so a small group of samples may not reveal the trend. c) The X-bar chart shows a trend.

12. Violations of Rule 1 (single mean outside 3 sigma) and Rule 2 (two out of three means outside 2 sigma) are fairly easy to detect. Violations of Rule 4 (eight means on same side of centerline) could be missed if the observer is only checking if the means are close to the centerline. Violations of Rule 3 (four out of five means outside 1 sigma) are hardest to detect because the violation is not extreme and points could lie on opposite sides of the centerline. Yes, there may be more than one violation. The chart is created normally, and then a violation is introduced. The laws of chance may lead to multiple violations.

Advanced Learning Exercises

13. a) A complex process may be composed of a number of subprocesses. b) Even if engineers have designed each individual process, the quality control analyst usually would not know the overall process mean and standard deviation. c) It is the nature of random sampling that sample estimates vary, even after taking a large number of samples. There is no guarantee that the empirical control limits will be perfect.

14. Smallest column mean   8.6744   Largest column mean   8.6836
     Smallest column range   0.002   Largest column range   0.027
     Smallest column std. dev.   0.0019   Largest column std. dev.   0.0085

     Cumulative average mean   8.679 (8.678 to 8.680)
     Cumulative average range   0.0142 (0.0130 to 0.0155)
     Cumulative average std. dev.   0.0050 (0.0046 to 0.0054)
     Column results will vary but will almost always fall within the limits shown. The cumulative averages will vary only slightly from the averages shown.

15. Cumulative average mean   8.6790
     Cumulative average range   0.0142
     Cumulative average std. dev.   0.0050
     Number of samples taken   About 100
     Most learners will take at least 100 samples, but there is no "right" answer.

16. a. Your estimated upper control limit for sample mean:

$$UCLX = \overline{\overline{X}} + 3\frac{\overline{R}}{d_2\sqrt{n}} = 8.679 + 3\frac{0.0142}{2.847\sqrt{8}} = 8.684$$

     b. Your estimated lower control limit for sample mean:

$$LCLX = \overline{\overline{X}} - 3\frac{\overline{R}}{d_2\sqrt{n}} = 8.679 - 3\frac{0.0142}{2.847\sqrt{8}} = 8.674$$

     c. Your estimated upper control limit for sample range:

$$UCLR = D_4\overline{R} = (1.864)(0.0142) = 0.027$$

     d. Your estimated lower control limit for sample range:

$$LCLR = D_3\overline{R} = (0.136)(0.0142) = 0.002$$

17. $\overline{X}$-chart true control limits:

True UCL   8.6836
True LCL   8.6744

R-chart true control limits:

True UCL   0.027
True LCL   0.002

Assuming the number of samples was 100 or more, the limits calculated from empirical samples should be very close to the theoretical limits based on the true process parameters. However, it is the nature of random sampling that differences may exist, even after taking a large number of samples. Therefore, there is no guarantee that the empirical control limits will be perfect.

18. True process mean   8.6790   Your cumulative mean   8.6790
True process std. dev.   0.0050   Your cumulative std. dev.   0.0050

Minor differences are inevitable, due to sampling error. If the cumulative mean overstates the true process mean, the $\overline{X}$-chart will be incorrectly centered and the histogram of means will be shifted down (and vice-versa). If the cumulative standard deviation (and hence the range) overstates the true process standard deviation, both the R-chart and the $\overline{X}$-chart control limits will be too wide (and vice-versa). If both process parameters were estimated incorrectly, the distortion of the control charts will combine both types of error. In order to make exercises 13–17 as realistic as possible, the question wasn't asked earlier (real world process parameters are often unknown).

19.
True process parameters	Mean 500.0	Standard Deviation	50.00
10 samples accumulated	Mean 502.1	Standard Deviation	62.95
20 samples accumulated	Mean 502.0	Standard Deviation	53.00
30 samples accumulated	Mean 498.6	Standard Deviation	52.36
40 samples accumulated	Mean 497.0	Standard Deviation	50.95
50 samples accumulated	Mean 497.0	Standard Deviation	51.32
60 samples accumulated	Mean 497.4	Standard Deviation	50.50
70 samples accumulated	Mean 497.1	Standard Deviation	51.45
80 samples accumulated	Mean 496.9	Standard Deviation	51.65
90 samples accumulated	Mean 497.8	Standard Deviation	50.26
100 samples accumulated	Mean 498.3	Standard Deviation	50.47

Results will vary. The cumulative estimates do not always move closer to the true process parameters. After 100 samples, gross errors are unlikely, but noticeable differences may persist (in this example, the standard deviation is still off by almost 1%). Firms may be unable to take as many samples as they would like when sampling is destructive and/or expensive (in dollars or in time).

20. a) Level shift is a change in the process mean. The process standard deviation is unchanged. b) In manufacturing, typical causes would include a new work shift, changes in equipment, new inspectors, changed instructions to employees, a changed machine setting, or a new lot of material. In the service sector, level shifts could reflect a changed work environment, new supervision, or new rules. c) It does not show up on the range chart, but can be seen on the $\overline{X}$-chart (shift in means above or below the centerline) and the histogram will be shifted and somewhat skewed. Mild level shift is difficult to detect until many samples are taken. d) Depending on severity, it might be detected by Rule 4 (8 means on same side of centerline) but other rules might reveal it.

21. The R-chart does not reveal level shift, because only the mean has changed (not the variance within each sample). The level shift begins within the first 20 samples, and it will be evident on the $\overline{X}$-chart. In the next 50 samples, the histogram is clearly shifted and is skewed (too many means above the centerline). There will probably be violations of the UCL (and perhaps violations of other rules of thumb).

22. a) A cycle is a short, repeated series of above-centerline measurements followed by a series of below-centerline measurements. b) It is detected visually on the X-bar chart, or by higher-than-expected frequencies in the tails of the histogram. c) In manufacturing, likely causes may include humidity or temperature fluctuations, voltage fluctuations, or operator over-adjustment. In the service sector, causes may include duty rotations, uneven work scheduling, or periodic distractions.

23. a) The R-chart does not a reveal cycle, because only the mean is changing (not the variance within each sample). b) Cycles are seen on the $\overline{X}$-chart. c) A strong cycle may violate Rules 2, 3, and possibly 4. A fairly powerful visual test is to count the number of times the means graph crosses the centerline (it should be about half the number of samples). A cycle would create too few crossings.