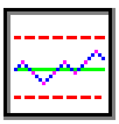
CHAPTER 21



Visualizing Statistical Process Control

CONCEPTS

- Statistical Process Control, Control Chart, Control Limits, Control Chart Factors, Mean, Range, X-Bar Chart, R-Chart, Centerline, In-Control Process, Out-of-Control Process, Rules of Thumb

OBJECTIVES

- Understand how control limits are constructed
- Be able to interpret simple control charts
- Recognize out-of-control processes (level shift, trend, cycle, oscillation, instability, mixtures) and their typical causes
- Understand pattern recognition rules

Overview of Concepts

Products or services can be defined by measurable characteristics (such as diameter, weight, or waiting time) or by customer perceptions (such as color, courtesy, or defects). **Statistical process control** (SPC) allows monitoring of process parameters to ensure that quality standards are met. The broader goal of continuous quality improvement (CQI) is to increase customer satisfaction and reduce cost through better conformance to customer requirements. To this end, organizations must analyze customer needs, design their products and services to meet these needs, monitor their processes of production and distribution, and implement appropriate management systems. Variation is a natural phenomenon that cannot be eliminated, but it can be reduced toward the *asymptotic* goal of zero variation. The attainable degree of variation depends on equipment, technology, processes, management, and worker training.

An **in-control process** performs as expected, exhibiting only *common cause* variation (normal or random variation). An **out-of-control process** exhibits *special cause* anomalies (not due to random variation) such as instability, trend, level shift, cycle, mixture, or oscillation. Samples are used to monitor a process when measurements are costly, time-consuming, or destructive (100 percent inspection is not discussed here). Sample size and frequency of sampling will depend on the situation, but small samples (under 10) are typical. This module focuses on two well-known statistics: the **mean** (a measure of centrality) and the **range** (a measure of variation). These sample statistics are plotted over time on **control charts**, which guide decisions to continue, adjust, or halt the process.

The control chart for a mean is an **X-bar chart** (or $\bar{X}$-chart) while the control chart for the range is an **R-chart**. The **centerline** of a control chart is the expected value of the statistic. Statistical theory is used to establish an upper control limit (UCL) and lower control limit (LCL) around the desired centerline. **Control limits** in this module are based on **control chart factors** (found in tables) so that, when a process is in control, the sample statistic will lie within the control limits about 99 percent of the time. The X-bar chart's control limits are symmetric (based on a normal distribution), while the R-chart's control limits are asymmetric.

The position of sample statistics in relation to these control limits will tell us whether or not the process is in control. We are willing to accept a certain amount of variation, which is normal and expected. Excessive variation would indicate that a process may be out of control. Statisticians use **rules of thumb** to detect abnormal patterns so that management can take corrective action. The initial hypothesis is that the process is in control. Rejecting this hypothesis when it is true is *Type I error*, while accepting this hypothesis when it is false is *Type II error*. Failure to make timely corrections (Type II error) may lead to poor quality, excess scrap, re-work, and customer dissatisfaction. But unnecessary adjustments (Type I error) may cause loss of production and unnecessary expense.

Upper and lower control limits are established empirically by taking repeated samples from a new process once it is stable (even if engineering specifications are available). The overall mean becomes the assumed centerline of the X-bar chart, while the average range becomes the centerline for the R-chart. Walter A. Shewhart pioneered techniques for control charts, but SPC was further developed by other twentieth century experts, including W. Edwards Deming, Joseph M. Juran, Armand V. Feigenbaum, and Genichi Taguchi.

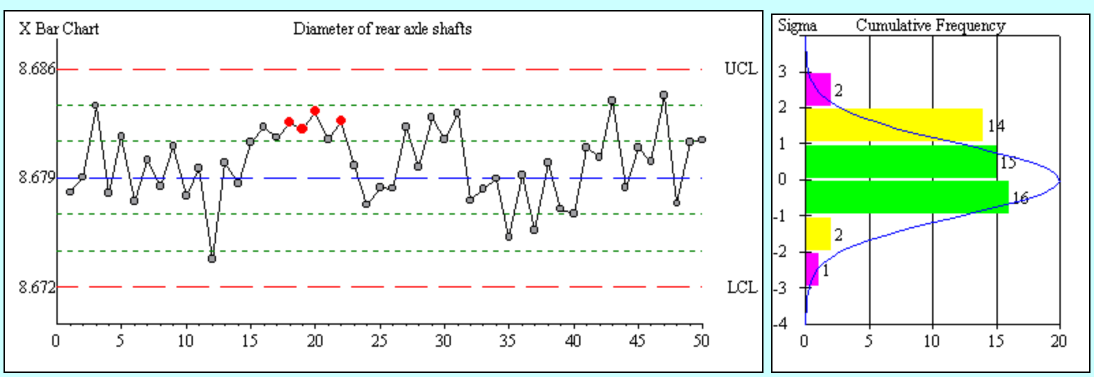
Illustration of Concepts

Rear axle shafts for a certain truck are supposed to have a diameter of 8.679 centimeters. Experience has shown the process standard deviation to be 0.005 centimeters. **Statistical process control** recognizes that variation exists in actual diameters because the hard-turn machining cell that produces the axle shafts is subject to vibration, wear, and other random perturbations. Samples of five axles are taken every hour and are measured to determine the actual diameter to the nearest 0.001 centimeters.

Control charts are constructed for the **mean** and **range**, using **control chart factors** for n = 5 to set the **control limits**. The 99 percent upper and lower control limits (UCL and LCL) are shown as dashed blue lines. On the X-bar chart, dashed green lines are also used to display ± 1 sigma limits and ± 2 sigma limits on either side of the **centerline** ("sigma" refers to the standard deviation of the sample mean). To check for patterns that might indicate an **out-of-control process**, the quality monitoring staff applies these **rules of thumb** to the **X-bar chart**:

Rule 1: Single mean outside the 3 sigma limits (UCL and LCL)
Rule 2: 2 of 3 means outside the 2 sigma limits (same side of centerline)
Rule 3: 4 of 5 means outside the 1 sigma limits (same side of centerline)
Rule 4: 8 means on same side of centerline

One Thursday, the X-bar chart below showed four of five consecutive observations on the same side of its centerline (a violation of Rule 3). While this could be due to chance, it might also signal increased variation or a developing trend. Should the process be stopped?



As a further test, the histogram of the sample means was compared with the normal bell-shaped curve. This histogram is based on 50 sample means, so inferences about the distribution are guarded. Its general appearance resembles the normal curve that is superimposed, yet upon closer inspection it has too many means above zero and too few means below zero. The **R-chart** (not shown) indicated no violations.

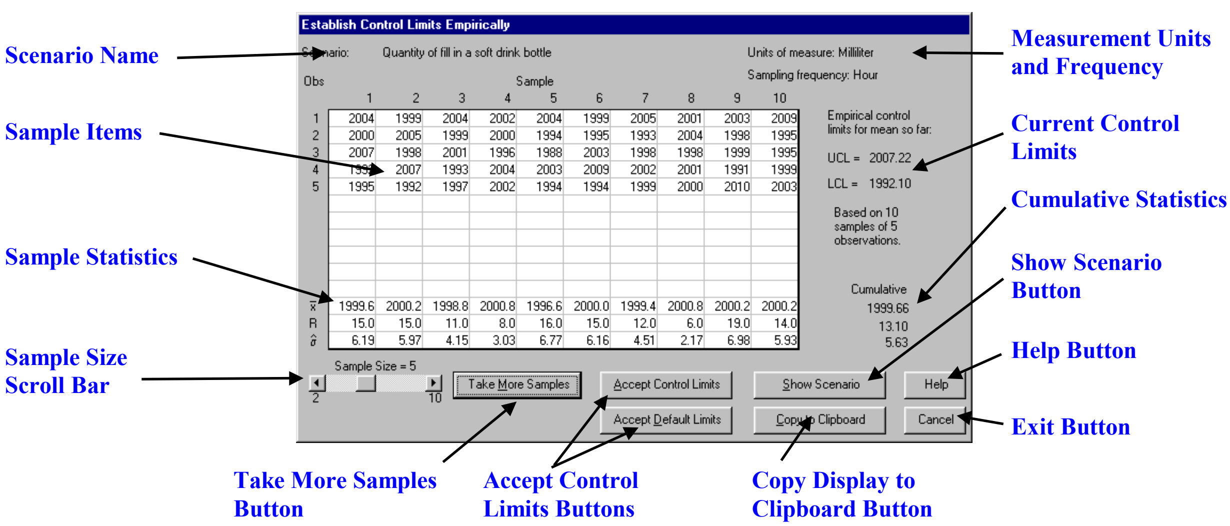
A decision was made to halt production and to inspect the machine tools. It was discovered that a small adjustment screw had loosened, allowing axle diameter to increase slightly. Only 10 minutes of equipment downtime were lost. If an adjustment had not been made (i.e., if an **in-control process** had been assumed) the axle shafts would have been slightly oversized, on average, leading to excessive friction, overheating, and shorter axle life.

Orientation to Basic Features

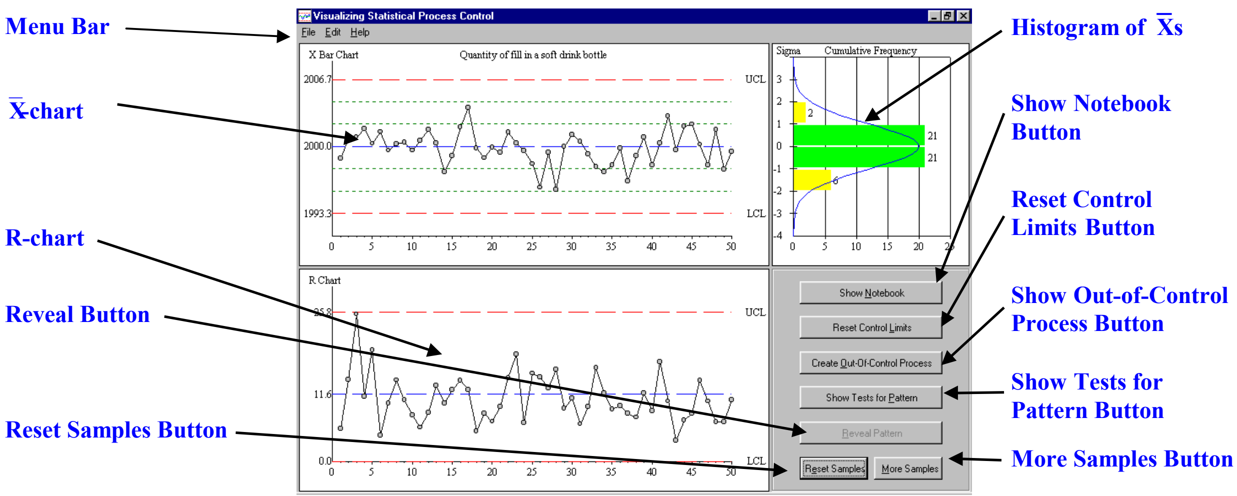
This module helps you learn how to create and read control charts. It illustrates both in-control and out-of-control processes using a variety of scenarios that represent manufacturing and service sector applications. You can use samples from processes to establish control limits and display samples on the resulting control charts.

1. **Select a Scenario**
   Start the module by clicking on the module's icon, title, or chapter number in the *Visual Statistics* menu and pressing the Run Module button. When the module is loaded, you will be on the introduction page of the Notebook. Read the questions and then click the Concepts tab to see the concepts that you will learn. Click the Scenarios tab. Select the Quantity of fill in a soft drink bottle scenario, read it, and press OK.

2. **Establish Control Limits**
   The Establish Control Limits Empirically screen contains a blank data matrix. Use the Sample Size scroll bar to alter the sample size (from 2 to 10). Changing the sample size changes the number of observations used to calculate each $\overline{X}$. Click the flashing Take More Samples button. At the bottom of each column you will see each sample's mean, range, and standard deviation. At the right are the control limits based on the samples taken so far. Below them are shown the cumulative mean, average range, and average standard deviation. Click the Take More Samples button to increase the number of samples used to calculate the control limits (the limit is 1,000). Click Accept Control Limits when you feel you have taken enough samples (or click Accept Default Limits to get control limits based on the true process parameters). Click the Help button to get more information.



3. **Main Screen**
The main screen shows the $\overline{X}$ chart (upper left), R-chart (lower left), and cumulative histogram of sample means (upper right) with a superimposed normal curve. On the $\overline{X}$ chart and R-chart, the centerline is a blue dashed line, and the upper control limit (UCL) and lower control limit (LCL) are shown as dashed red lines. The dotted green lines on the X-bar chart represent the 1 sigma and 2 sigma limits (here, "sigma" refers to the standard error of the mean $\sigma/\sqrt{n}$ where σ is estimated when you establish the control limits empirically or is given by the scenario if you choose Accept Default Limits). Sampling experiments are controlled by the buttons in the Control Panel (lower right), as well as by the menu bar at the top of the screen.



4. **Taking Samples**
Click the More Samples button to generate 50 more samples of means (you are limited to 1,000 samples). Examine the $\overline{X}$-chart and R-chart to determine if the process is in control. The color-coded histogram shows the number of sample means in each zone: green (within ±1 sigma), yellow (within ±2 sigmas), magenta (within 3 sigmas), and red (outside 3 sigmas). These results can be compared with what is predicted by the normal distribution. Click the Reset Samples button to start over using the same control limits.

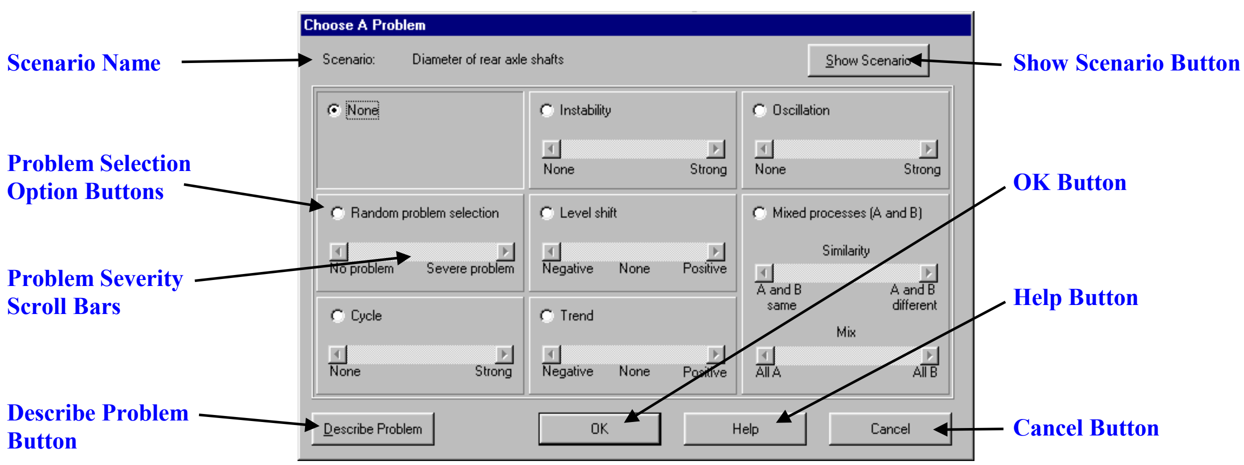
5. **Copying a Display**
Click on the display you wish to copy. Its window title will be highlighted. Select Copy from the Edit menu (on the menu bar at the top of the screen) or Ctrl-C to copy the display. It can then be pasted into other applications, such as Word or WordPerfect, so it can be printed.

6. **Help**
Click on Help on the menu bar at the top of the screen. Search for Help lets you search an index for this module, Contents shows a table of contents for this module, Using Help gives instructions on how to use Help, and About gives licensing and copyright information about *Visual Statistics*.

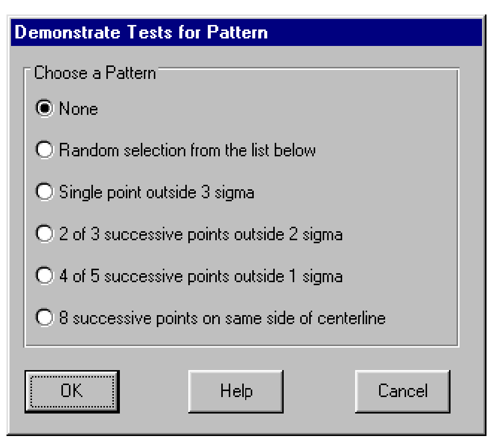
7. **Exit**
Close the module by selecting Exit in the File menu (or click ☒ in the upper right-hand corner of the window). You will be returned to the *Visual Statistics* main menu.

Orientation to Additional Features

1. **Create an Out-of-Control Process**
   Click the Create Out-of-Control Process button to obtain the choices for out-of-control processes shown on the next page. The default is None. Click the Describe Problem button to get more information about the problem you have selected. Drag or click the scroll bar(s) for the problem selected to set its level of severity (often, a severe problem is easiest to see). Click on Help if you want more information about the causes and consequences of out-of-control processes. Choose OK or Cancel to return to the main screen. Select Random Problem to have a random problem chosen with the severity you indicated. On the main screen, click on the Reveal Problem button to see if you correctly identified the problem.



2. **Show Tests for Pattern**
   Click the Show Test for Pattern button to illustrate various tests for special cause. Choose a test and click OK. The Random selection tells the computer to choose a pattern (so you can test your ability to recognize them). Click the Reveal Pattern or Reveal Random Pattern button to highlight the points that demonstrate the pattern (there may be other patterns, but the points that were created to demonstrate the pattern are the ones highlighted).



Basic Learning Exercises Name ________________

Press the Show Notebook button, select the Scenarios tab, click on Manufacturing I, and select Quantity of Fill in a Soft Drink Bottle. Read the scenario and click OK.

In-Control Processes

1. What is the desired mean? What are the units of measurement? What is the frequency of sampling? What is the sample size?

   Desired mean ________________ Units of measurement _____________
   Sampling frequency ___________ Sample size __________

2. From the Establish Control Limits Empirically screen click Accept Default Limits. This will cause the control limits to be set using the true process parameters (usually unknown to the statistician but used initially in this exercise to be sure the control limits are exact). The main screen will appear. Record the centerline (dashed blue line) and control limits (dashed red lines) from the $\overline{X}$-chart and R-charts. How often should the sample statistics lie between the upper control limit (UCL) and lower control limit (LCL)?

	$\overline{X}$-chart		R-chart
Upper Control Limit (UCL)	__________	Upper Control Limit (UCL)	_______
Centerline (Actual Mean)	__________	Centerline (Actual Mean)	_______
Lower Control Limit (UCL)	__________	Lower Control Limit (UCL)	_______

3. Click Help on the menu bar at the top of the screen. Choose Search for Help and enter the key word "sigma limits." The titles of relevant screens appear in the window below. In the lower window, double-click on the title of a screen that sounds relevant (Histogram of Sample Means, Histogram Bar Colors). Explain why the UCL and LCL on the $\overline{X}$-chart are sometimes called "3 sigma limits," and explain how they are related to the normal distribution. What percent of the means are within ± 1 sigma, ± 2 sigma, and ± 3 sigma? Close Help by selecting Exit from the File menu on the Help screen.

4. Press the More Samples button once to take 50 samples. From the $\overline{X}$-chart, count the number of sample means that lie above the centerline of the chart. From the R-chart, count the number of sample ranges that lie above the centerline of the chart. Are any sample statistics outside the control limits (dashed red lines)? Do these sample frequencies differ significantly from what you would expect? Should we generalize from 50 samples?

	$\overline{X}$-chart		R-chart
Frequency above centerline	____________	Frequency above centerline	____________
Frequency below centerline	____________	Frequency below centerline	____________
Frequency outside limits	____________	Frequency outside limits	____________

5. Press the More Samples button again, to accumulate 100 samples. Look at the cumulative histogram of sample means. How closely does it resemble the bell-shaped curve?

6. Based on the cumulative histogram, record the number of sample means that lie above and below the centerline of the $\overline{X}$-chart. Referring to the “sigma limits” (dotted green lines), record the number of sample means within ± 1 sigma (sum of the green bars), ± 2 sigma (sum of the green and yellow bars) and ± 3 sigma (sum of the green, yellow, and magenta bars). Are any sample means outside the control limits (dashed red lines)? Do these sample frequencies differ significantly from a normal distribution? Should we generalize from 100 samples? **Hint:** Use Help on the menu bar to find out more about the normal distribution.

	Frequency	Percent	Percent if Normal
Above centerline	____________	___________	50.00
Below centerline	____________	___________	50.00
Within ± 1 sigma	____________	___________	68.26
Within ± 2 sigma	____________	___________	95.44
Within ± 3 sigma	____________	___________	99.73
Outside ± 3 sigma	____________	___________	0.27

Intermediate Learning Exercises Name ________________

Press the Show Notebook button, select the Scenarios tab, click on Manufacturing I, and select Quantity of Fill in a Soft Drink Bottle. Read the scenario and click OK.

Out-Of-Control Processes

7. Press the Reset Samples button. Click Create Out-of-Control Process and then click Instability. Click Describe Problem. a) What is instability? b) Why is it a problem? c) What might cause it (in the service sector as well as in manufacturing)? d) How might we detect it?

8. Make sure the Instability scroll bar is in the exact middle between None and Strong (the scroll bar should initially appear in this position, but make sure). This will generate a problem of moderate instability. Click OK. How many of the 50 samples violate the R-chart control limits? Using only the R-chart, do you see clear indications of instability? Explain.

9. Press More Samples again (now you have 100 samples). How does the cumulative histogram of sample means compare with the normal curve? From the histogram, record the number of sample means that lie above and below the centerline of the $\overline{X}$-chart, within ± 1 sigma (sum of the green bars), ± 2 sigma (sum of the green and yellow bars), within ± 3 sigma (sum of the green, yellow, and magenta bars), and outside UCL and LCL. Do these sample frequencies differ greatly from a normal distribution? What does this reveal about instability's effects?

	Frequency	Percent	Percent if Normal
Above centerline	____________	__________	50.00
Below centerline	____________	__________	50.00
Within ± 1 sigma	____________	__________	68.26
Within ± 2 sigma	____________	__________	95.44
Within ± 3 sigma	____________	__________	99.73
Outside ± 3 sigma	____________	__________	0.27

10. Click Create Out-of-Control Process. Click on Trend, then press Describe Problem. a) What is trend? b) Why is it a problem? c) What might cause it (in the service sector as well as in manufacturing)? d) How might you detect it?

11. Set the Trend scroll bar all the way to the end labeled Positive (the scroll bar should initially be set just to the right of None). This will generate strong upward trend. Click OK. Examine the R-chart and then the $\overline{X}$-chart. a) For these 50 samples, do you see any indication of trend on either chart? Explain. b) Click More Samples again until the trend is obvious. How many samples did you have to take to see the trend? c) Which chart reveals the problem?

Show Tests for Pattern

12. Click Show Tests for Pattern (this will automatically reset the process to being in control). Each rule offers a method for detecting problems on the $\overline{X}$-chart (which is supposed to be free of any patterns). Click Single Point Outside 3 Sigma and look at the $\overline{X}$-chart to see if you can spot the violation. Click Reveal Pattern to verify that you have found the problem. Repeat this exercise for each of the other rules of thumb. Evaluate the degree of difficulty in detecting each pattern. Why might an observer overlook some of the patterns? Do you see more than one violation (that is, a violation other than the one the computer revealed)?

Advanced Learning Exercises Name ________________

Setting Control Limits

Press the Show Notebook Button, select the Scenarios tab, click on Manufacturing II, and select Diameter of Rear Axle Shafts. Read the scenario and click OK.

13. a) Why might a firm need to set its control limits empirically (that is, by taking a number of preliminary samples to estimate the overall process mean and its average range)? b) Haven't engineers or systems analysts already designed the process and its parameters? c) Even after extensive sampling, might the resulting control limits be imperfect?

14. On the Establish Control Limits Empirically screen, use the Sample Size scroll bar to set the sample size to n = 8. Press the Take More Samples button once. This will display 10 samples. Examine the statistics (mean, range, standard deviation) at the bottom of each column and the cumulative statistics over all 10 samples (at the right side of the screen) and record the information requested below. Choose any sample and verify its column statistics to be sure you understand how they were calculated.

Smallest column mean	___________	Largest column mean	___________
Smallest column range	___________	Largest column range	___________
Smallest column std. dev.	___________	Largest column std. dev.	___________

Cumulative average mean ________________________
Cumulative average range ________________________
Cumulative average std. dev. ________________________

15. Continue to press the Take More Samples button until you are satisfied that you have obtained a good estimate of the true mean, range, and standard deviation. (**Hint:** Watch the cumulative statistics.) Record the cumulative statistics here.

Cumulative average mean ___________
Cumulative average range ___________
Cumulative average std. dev. ___________
Number of samples taken ___________

16. Use the formulas shown below to calculate the estimated control limits, plugging in your cumulative mean ($\overline{\overline{X}}$) and range ($\overline{R}$) from exercise 15. Show each step of your calculations clearly in the space provided. **Hint:** Find control chart factors for d_2, D_3, and D_4 in the Glossary (or click Help).

    a. Your estimated upper control limit for sample mean:

    $$UCLX = \overline{\overline{X}} + 3 \frac{\overline{R}}{d_2 \sqrt{n}} =$$

    b. Your estimated lower control limit for sample mean:

    $$LCLX = \overline{\overline{X}} - 3 \frac{\overline{R}}{d_2 \sqrt{n}} =$$

    c. Your estimated upper control limit for sample range:

    $$UCLR = D_4 \overline{R} =$$

    d. Your estimated lower control limit for sample range:

    $$LCLR = D_3 \overline{R} =$$

17. To assess the accuracy of your estimates (exercise 16), from the Establish Control Limits Empirically screen click Accept Default Limits to find the control limits using the true process parameters (rather than using your sample estimates). These true control limits will appear on the main screen. How close were your calculated control limits to the true UCL and LCL that are displayed on the screen? If there are differences, explain why they might exist.

    $\overline{X}$-chart true control limits:

    True UCL ____________
    True LCL ____________

    R-chart true control limits:

    True UCL ____________
    True LCL ____________

18. Press the Show Notebook button, select the Scenarios tab, click on Manufacturing II, and select Diameter of Rear Axle Shafts. Read the scenario and record the true process parameters. How do they compare with your cumulative mean and standard deviation from exercise 15? If there is a difference, how might the difference have arisen? What difference would it make? Why wasn't this question asked earlier?

True process mean	__________	Your cumulative mean	__________
True process std. dev.	__________	Your cumulative std. dev.	__________

19. Press the Show Notebook button, select the Do-It-Yourself tab, and click OK. Enter 500 for the mean and 50 for the standard deviation, and then click OK. On the Establish Control Limits Empirically screen, choose a sample size of 5 and press Take More Samples once (to generate 10 samples). Record the cumulative mean and standard deviation (shown on the right-hand side of the screen) in the spaces below. Repeat this process until you have accumulated 100 samples. After each round of sampling, do the cumulative estimates always move closer to the true process parameters? Is the 100th estimate better than the 10th? Are 100 samples enough that you feel comfortable accepting the control limits? Why might a firm be unable to take as many samples as it would like?

True process parameters	Mean	__________	Standard Deviation	__________
10 samples accumulated	Mean	__________	Standard Deviation	__________
20 samples accumulated	Mean	__________	Standard Deviation	__________
30 samples accumulated	Mean	__________	Standard Deviation	__________
40 samples accumulated	Mean	__________	Standard Deviation	__________
50 samples accumulated	Mean	__________	Standard Deviation	__________
60 samples accumulated	Mean	__________	Standard Deviation	__________
70 samples accumulated	Mean	__________	Standard Deviation	__________
80 samples accumulated	Mean	__________	Standard Deviation	__________
90 samples accumulated	Mean	__________	Standard Deviation	__________
100 samples accumulated	Mean	__________	Standard Deviation	__________

Out-Of-Control Processes

20. Click Create Out-of-Control Process and then click Level shift. Click Describe Problem. a) What is level shift? b) What might cause it (in the service sector as well as in manufacturing)? c) Which chart would show it? d) Which rule might detect it?

21. Set the Level shift severity scroll bar about halfway between None and Positive to create a moderate level shift. Click OK. From these 50 samples, does the R-chart reveal this level shift? Does the $\overline{X}$-chart reveal it? Click More Samples again to accumulate 100 samples. Does the histogram of means reveal this level shift? Discuss.

22. Click Create Out-of-Control Process and then click Cycle. Click Describe Problem. a) What is a cycle? b) How is it detected? c) What might cause it?

23. Set the Cycle severity scroll bar to Strong (all the way to the right) to create a strong cycle. Click OK. a) In these 50 samples, does the R-chart reveal the cycle? b) Does the $\overline{X}$-chart reveal it? If not, click More Samples again. c) Which rule is most likely to reveal a cycle?

Individual Learning Projects

Write a report on one of the three topics listed below. Use the cut-and-paste facilities of the module to place the appropriate graphs in your report. Include in your report a copy of at least one X-bar chart, R-chart, and histogram for each different experimental setup.

1. Investigate out-of-control processes by choosing any scenario that interests you, accepting the default control limits, and then using the Create Out-of-Control Process button to select one of the following problems: oscillation, level shift, or cycle. Include three different scroll bar settings for problem severity (low, moderate, strong) and take 50 samples for each severity level. Can you see the problem? If not, take more samples until you are sure. Explain how you detected the problem (or if it was undetectable, why not). Which control chart shows the problem most clearly? Are both charts needed? Repeat using a different problem from this list. Based on your experience, which problem was hardest to detect?
2. Investigate out-of-control processes by choosing any scenario that interests you, accepting default control limits, and then using the Create Out-of-Control Process button and selecting Random Problem Selection. Start with the problem severity scroll bar in the Strong position (to create a severe problem) and take 50 samples. Can you see the problem? If not, take more samples until you are sure. Explain how you detected the problem (or if it was undetectable, why). Which control chart shows the problem most clearly? Are both charts needed? Do this again, until you have seen at least three *different* random problems. Click the Create Out-of-Control Process button, change the problem severity to moderate (scroll bar halfway between None and Strong) and repeat these questions. Compare the ease of evaluating the two sets of three experiments.
3. Investigate rules of thumb. Select a scenario that interests you and accept the default control limits. Press Show Tests for Pattern and Random Selection. Each time you press More Samples you will get a new illustration of the same pattern, which you must try to identify. Look at all three charts. Tell which ones reveal the pattern, how clearly it is revealed, and whether more than one rule of thumb is violated. In the context of the scenario, what could cause this type of pattern? When you have identified the pattern, press the Reset Samples button to get a new random pattern. Do this until you have seen each rule of thumb illustrated twice. Which pattern violations are easiest to detect, and why? Why do you suppose this project asked you to use default control limits instead of setting the control limits empirically?

Team Learning Projects

Select one of the three projects listed below. In each case, produce a team project that is suitable for an oral presentation. Use presentation software or large poster boards to display your results. Graphs should be large enough for your audience to see. Each team member should be responsible for producing some of the graphs. Include in your report a copy of *relevant* X-bar charts, R-charts, and histograms you evaluated. Ask your instructor if a written report is also expected.

1. This is a project for a team of three or more. Investigate mixtures by choosing any scenario that interests you, accepting default control limits, and clicking Create Out-of-Control Process. Select Mixture. Each team member should choose a different setting on the Similarity scroll bar (ranging from A and B Same to A and B Different) and should investigate five settings for the Mix scroll bar (including All A, 50-50, and All B). What are the symptoms of a mixture? When is it difficult to detect? Which charts are needed?
2. This is a project for a team of two. Investigate two new rules of thumb that may help you detect cycles and oscillation:

   Rule 5: Six points in a row steadily increasing or decreasing
   Rule 6: Fourteen points in a row alternating up and down

   Choose a scenario, accept default control limits, and click Create Out-of-Control Process. One team member should select Cycle and the other should select Oscillation. Try five settings for the problem severity scroll bar (starting just above None and ending at Strong). Take 50 samples for each severity level. Can you see the problem? If not, take 50 more samples. Continue until you are sure. Explain how you detected the problem (or if it was undetectable, why). Did Rule 5 and Rule 6 help you detect the out-of-control processes, or were Rules 1 through 4 sufficient?
3. This is a project for a team of two. Press the Show Notebook button, select the Do-It-Yourself tab, and click OK. Enter any mean and standard deviation, make up a scenario that fits these parameters, and click OK. On the Establish Control Limits Empirically screen, choose any sample size and press Take More Samples once (to generate 10 samples). Record the cumulative mean and standard deviation (shown on the right-hand side of the screen). Repeat this process until you have accumulated at least 500 samples. Plot your cumulative sample statistics results in a graph and discuss the pattern. Do the cumulative estimates move closer to the true process parameters? How many samples are enough that you would feel comfortable accepting the control limits?

Self-Evaluation Quiz

1. Process control charts are primarily associated with which statistical expert?
   a. Joseph M. Juran.
   b. W. Edwards Deming.
   c. Armand V. Feigenbaum.
   d. Genichi Taguchi.
   e. Walter A. Shewhart.

2. Control limits are *least likely* to be based on
   a. detailed knowledge of engineering technology.
   b. repeated sample measurements of the mean.
   c. knowledge of the normal distribution.
   d. tables of control chart factors.
   e. repeated sample measurements of the range.

3. Quality improvement would probably *not* entail
   a. reducing variance.
   b. identifying sources of variance.
   c. blaming employees who do poor work.
   d. improving technology.
   e. monitoring processes continually.

4. Quality in a product is best assessed by
   a. trained statisticians.
   b. skilled mechanical engineers.
   c. attorneys who handle product liability.
   d. government quality inspectors.
   e. customers.

5. The R-chart is most likely to reveal which problem?
   a. Instability.
   b. Cycle.
   c. Oscillation.
   d. Trend.
   e. Level shift.

6. Which is *not* a characteristic of instability?
   a. A larger than normal amount of variation.
   b. Higher-than-expected frequencies in tails of the histogram of means.
   c. Could be caused by untrained operators.
   d. Could indicate that equipment tolerances are too tight.
   e. Could indicate that two processes are mixed.

7. A slow drift of measurements either up or down from the process centerline suggests
   a. mixed processes.
   b. instability.
   c. oscillation.
   d. cycle.
   e. trend.

8. Which is *not* a characteristic of a trend?
   a. Variance is essentially unchanged in each sample.
   b. The cumulative histogram grows skewed in one tail.
   c. It is easily detected visually if enough measurements are taken.
   d. Rules of thumb can be established to detect it.
   e. It is often due to mixing two batches of materials.

9. Which is *not* a rule of thumb to indicate an out-of-control process on the X-bar chart?
   a. three out of four points beyond ±2 sigma.
   b. two out of three points beyond ±2 sigma.
   c. four out of five points beyond ±1 sigma.
   d. one point outside ±3 sigma.
   e. eight consecutive points on same side of the centerline.

10. Likely reasons for inaccurate control limits would include which of the following?
   a. The engineering parameter for variance is unknown.
   b. The engineers were underpaid for their work.
   c. There was insufficient preliminary sampling.
   d. Process variation was not zero, as expected.
   e. None of the above.

11. A cycle
   a. is a series of high measurements followed by a series of low measurements.
   b. is usually detectable visually.
   c. is somewhat sinusoidal in shape.
   d. is all of the above.
   e. is none of the above.

12. If we see measurements near the upper or lower edges of the chart with fewer than expected near the centerline, we would be *least likely* to suspect which problem?
   a. Cycle.
   b. Oscillation.
   c. Instability.
   d. Stratification.
   e. Mixed processes.

Glossary of Terms

Centerline Expected value of a process parameter (for example, mean or range). On a control chart, it is shown as a horizontal line.

Common cause Random variation in sample measurements that is inherent in the underlying process. It does not require action, because it is normal and expected. It cannot be reduced unless the process itself changes.

Control chart Plot of sample statistics (such as the mean or range) over time on a graph that shows the process centerline and its control limits.

Control limits For each process parameter, an upper control limit (UCL) and lower control limit (LCL) are established above and below the centerline. If the sample statistics lie within these control limits, we conclude that the process is in control. See **Control chart factors**.

Control chart factors D_3 and D_4 are based on percentiles of the distribution of the range, while d_2 relates the average range to the process standard deviation ($\sigma = \overline{R} / d_2$). They are derived from the normal and chi-square distributions.

n	d_2	D_3	D_4
2	1.128	0	3.267
3	1.693	0	2.575
4	2.059	0	2.282
5	2.326	0	2.215
6	2.534	0	2.004
7	2.704	0.076	1.924
8	2.847	0.136	1.864
9	2.970	0.184	1.816
10	3.078	0.223	1.777

Control limits for X-bar chart

$$UCLX = \overline{\overline{X}} + 3 \frac{\overline{R}}{d_2 \sqrt{n}} = \text{upper control limit of sample mean}$$

$$LCLX = \overline{\overline{X}} - 3 \frac{\overline{R}}{d_2 \sqrt{n}} = \text{lower control limit of sample mean}$$

where

$\overline{\overline{X}}$ is the empirical sample mean (centerline of $\overline{X}$ chart)

$\overline{R}$ is the empirical average range (centerline of R chart)

n is the size of each sample taken

d_2 is the control chart factor for samples of size n (from a table)

Control limits for R chart

$$UCLR = D_4 \overline{R} = \text{upper control limit of sample range}$$

$$UCLR = D_3 \overline{R} = \text{lower control limit of sample range}$$

where

$\overline{R}$ is the empirical average range based on repeated sampling

D_3 is the control chart factor for samples of size n (from a table)

D_4 is the control chart factor for samples of size n (from a table)

Cycle An out-of-control process in which means tend to follow a cyclic pattern (runs of high and then low values in relation to the centerline). It may be difficult to identify on the X-bar chart unless it is extreme. The R-chart typically is unaffected. See **Oscillation**.

Histogram of sample means If the process is in control, the histogram of sample means should form a normal (bell-shaped) distribution centered on the centerline of the control chart.

In-control process A certain amount of variation is normal and acceptable. As long as variation is within the control limits, the process is in control. See **Out-of-control process**.

Instability An out-of-control process in which sample means vary more than expected (in effect, the control limits are too narrow relative to the actual process). It may be detected directly from inspection of the R-chart, or by applying various rules of thumb to the X-bar chart.

Level shift An out-of-control process in which the sample means shift abruptly to a new level either above or below centerline, and stay there. It differs from a trend, which is a slow, gradual movement. The R-chart typically is unaffected. See **Trend**.

Lower control limit See **Control limits**.

Mean Average of n sample observations ($\overline{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$), often referred to as X-bar. It is a measure of central tendency. Its theoretical distribution is normal if the population is normal or if n is large enough. Its expected value is the process mean μ.

Mixture An out-of-control process in which sample items come from two different populations that may differ. Very difficult to detect, and may be confused with instability. The R-chart may be affected, depending on how the samples are drawn.

Normal distribution If the sampling distribution is normal, the histogram of sample means should be symmetric about the centerline, with 68.26% within $\mu \pm 1\,\sigma/\sqrt{n}$, 95.44% within $\mu \pm 2\,\sigma/\sqrt{n}$, and 99.73% within $\mu \pm 3\,\sigma/\sqrt{n}$ where σ is the process standard deviation. The quantity $\sigma/\sqrt{n}$ is called the standard error of the mean. See **Sigma limits**.

Oscillation An out-of-control process in which the sample means tend to alternate (high-low-high-low) in "sawtooth" fashion. It may be difficult to identify on the X-bar chart unless it is extreme. It is an uncommon problem. The R-chart typically is unaffected. See **Cycle**.

Out-of-control process Process characterized by special-cause variation, detected from control charts (often by applying rules of thumb).

Parameter In the context of quality control, a numerical descriptor of a process, such as the its mean (μ, a measure of central tendency) or its standard deviation (σ, a measure of variation).

Process Method of producing a good or service, characterized by one or more measurable parameters. See **Parameter**.

Quality Quality of products or services can be defined by measurable characteristics (such as diameter, weight, or waiting time) or by customer perceptions (such as color variation, employee courtesy, or defects). Slight variation may not affect customer satisfaction, but variation that affects real or perceived performance of the product or service requires remedy.

Range Difference between the largest and smallest sample data values ($R = X_{max} - X_{min}$). It is a measure of dispersion.

R-chart Control chart for the sample range. The centerline is the average range (from past experience). The upper and lower control limits (UCLR and LCLR) are not symmetric, and are calculated using a table. See **Control chart factors**.

Rules of thumb Rules of pattern recognition to decide when a process is out of control:
Rule 1: Single mean outside 3 sigma limits
Rule 2: Two of three means outside 2 sigma limits (on the same side of the centerline)
Rule 3: Four of five means outside 1 sigma limits (on the same side of the centerline)
Rule 4: Eight consecutive means on same side of centerline.

Sample size In some situations (e.g., destructive testing) the sample size for a measurable characteristic is usually small (typically about five items). Sample size can be increased if more accuracy is needed or decreased in order to reduce sampling cost. In many modern factories every item may be inspected automatically ($n = 1$).

Sigma limits Informal way of referring to control limits. For example, the "3 sigma" control limits are $\mu \pm 3\ \sigma/\sqrt{n}$, where σ is the process standard deviation (often estimated from a sample) and n is the sample size. Similarly, $\mu \pm 2\ \sigma/\sqrt{n}$ would be called the "2-sigma" limits.

Special cause Variation in sample measurements attributed to nonrandom factors that are *not* inherent in the underlying process. It requires that action be taken, because it signifies that there is something wrong with the system.

Statistical process control The entire collection of statistical techniques that are applicable to quality management. More narrowly, it often refers to control charts.

Trend An out-of-control process whose means drift slowly either upward or downward, forming a trend. Trends may usually be detected visually on a control chart.

Type I error Making unnecessary adjustments to a process that is in control. It causes downtime, loss of production, added expense, diminished profit, and stockholder disappointment.

Type II error Failing to correct a process that is out of control. It leads to poor quality, excess scrap, unnecessary re-work, customer dissatisfaction, or litigation due to defective products.

Upper control limit See **Control limits**.

Variation Normal, expected deviation from an intended process parameter, due to a multiplicity of random causes, such as differences in raw material, equipment, and/or worker skill. The role of statistics is to measure variation, define attainable quality standards, and identify problems.

Variance reduction Multi-step process to improve quality: (1) analyze the process until it is well-understood and its parameters are known; (2) ensure that the process is stable and in control; (3) decide whether the process is capable of meeting the desired specifications; (4) over time, work to reduce variation so that measured variation approaches the unattainable ideal of zero.

X-bar chart A control chart for a sample mean. The centerline is the mean (from product specifications or past experience). The upper and lower control limits (UCLX and LCLX) are symmetric and are calculated using the normal distribution. When a process is in control, the sample mean will lie within the control limits almost all the time, and will be free of patterns. See **Sigma limits**.

Solutions to Self-Evaluation Quiz

1. e Read the Overview of Concepts.
2. a Do Exercises 2 and 13. Read the Overview of Concepts.
3. c Consult the Glossary (see Variance reduction).
4. e Read the Overview of Concepts.
5. a Do Exercises 7 and 8. Consult the Glossary.
6. d Do Exercises 7–9. Consult the Glossary.
7. e Do Exercises 10 and 11. Consult the Glossary.
8. e Do Exercises 10 and 11. Consult the Glossary.
9. a Do Exercise 12. Consult the Glossary.
10. c Do Exercises 13–19. Read the Overview of Concepts.
11. d Do Exercises 22 and 23. Consult the Glossary. Read the Overview of Concepts. Do Team Learning Project 2.
12. d Consult the Glossary.