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A method for modelling operational risk with fuzzy cognitive maps and Bayesian belief networks

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ABSTRACT

A main concern of risk management in financial institutions is measurement of operational risk and its value at risk as a requirement of Basel II accord. Besides risk quantification, identifying causal mechanism leading to operational loss is necessary to plan risk mitigation activities. Bayesian belief networks (BBN) is a causal modelling method able to achieve both goals simultaneously. Eliciting BBN causal model and its parameters from expert knowledge is an alternative to data driven models in case of data scarcity. However, there is still a problem with parameter extraction for complex models with a number of multi parent and multi state nodes. In this paper, we proposed a method combining fuzzy cognitive maps (FCM) and BBN in order to improve BBN capability in modelling operational risks. In the first phase, a causal model is constructed by applying FCM and then a new migration method is proposed to translate FCM parameters to BBN ones. A case study of an Iranian private bank is then given to examine and validate the proposed method.

1. Introduction

Operational risk has been a part of any financial service business since its evolution. Unlike market and credit risk, operational risk is so wide in source (Akkizidis & Bouchereau, 2005), frequency and severity which makes it more and more sophisticated and less measurable. Any person, system or process even external events such as natural crises could be a source of operational risk. Moreover, severity and frequency of operational risk takes place in a very wide range. A few sever or a great number of neglected low severity operational risks could result in the collapse of a big bank such as Barings. In spite of its damaging power and inclusion in all aspects of business, operational risk was not counted in regulatory frameworks and risk management accords until 2006. Inadequacy of traditional qualitative risk management methods (Akkizidis & Bouchereau, 2005; Xie, Wu, & Hu, 2011) leads the Basel committee to propose three approaches for measurement of operational risk and calculation of its regulatory capital in Basel II Capital Accord (BCBS, 2004). BCBS introduced three approaches for calculation of operational risk's regulatory capital: the Basic Indicator approach, the Standardized Approach, and the Advanced Measurement Approaches (AMA). AMA is the most complicated one while it has the ability to reduce the regulatory capital up to 20–40% (Neil, Andersen, & Hager, 2009). Various methods have been proposed under AMA such as time series, Loss distribution approach, Extreme value theory and causal modelling methods. Causal models generally serve risk management in two ways. First, they provide loss distribution estimation and then provide a deep understanding of causal mechanism creating risk events which could help assessing risk mitigation activities and improving risk management decisions. BBNs as a causal modelling approach is used in a number of operational risk measurement studies (Adusei-Poku, Van den Brink, & Zucchini, 2007; Aquaro et al., 2010; Cowell et al., 2007; Mittnik & Starobinskayare, 2010; Neil et al., 2009; Neil, Fenton, & Tailor, 2005; Sanford & moosa, 2012, 2015) in recent years. A BBN consists of two parts: a casual model demonstrating variables and their connecting edges, and a set of joint probability distributions. The BBN causal model could be learned from data or constructed manually from expert knowledge. Scarcity of data is the main challenge in measurement of operational risk. Almost all banks have no operational risk data base, however, a few number of banks has only a weak database, which is not enough to construct the causal model of BBN automatically from data. Extracting the causal model from expert knowledge is an alternative to automatic extraction which is encountered with the challenge of eliciting the BBNs parameters (conditional probability tables). Extracting CPTs from experts is going to be a more and more difficult and time consuming task due to the growing complexity of the causal model. Although the merit of BBN ability for causal modelling and measurement of operational risk has been insisted in previous researches, improving the

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capability of BBN at parameter extraction step has been neglected. In this paper, a new method is proposed which integrates fuzzy cognitive maps (FCM) and BBN in order to improve BBN capability in real complex domains with poor data such as operational risk. Fuzzy cognitive maps and Bayesian belief networks are both causal modelling tools which could be merged to improve the efficiency and capability in comparison with each individually. In the proposed method, FCM is used as a problem structuring method to construct the casual model and then a new procedure is proposed to migrate from FCM to BBN. The migration method describes a procedure to construct CPTs based on causal strength of relationships among each child variable and its parents which could accelerate and facilitate the parameter setting of a BBN.

The paper is structured as follows: Section 2 contains an overview of operational risk measuring approaches with emphasis on BBNs approach and discusses the drawbacks associated with BBN in modelling operational risk. In Section 3, a new method is proposed to construct a Bayesian model from a fuzzy cognitive map. Section 4 explains the results obtained from application of the proposed method to a real case of operational risk modelling. The conclusions derived from this study are presented in Section 5.

2. Literature study

This section consists of three parts. The concept of operational risk and its measurement approaches reported in the literature is discussed in the first part. Then, BBN approach, its applications to operational risk and some drawbacks is mentioned. Finally, a literature review for combining BBN and FCM is provided.

2.1. Operational risk

Calculation of capital charge for operational risk was mentioned in the second capital accord (Basel II) proposed by Basel Committee on Banking Supervision for the first time. Basel Committee (2004) defined operational risk as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. The Basel II accord insists on what must be measured, operational risk capital, although does not make restriction on how it should be done. In other words, Basel II proposed three approaches for calculation of operational risk capital. Although the first two approaches i.e. the basic indicator approach and the standardized approach calculate the operational risk capital by means of a simple function, financial institutes are allowed to develop their own operational risk measurement model under the advanced measurement approach. In recent years, a variety of approaches have been introduced to measure operational risk under AMA (Chernobai, Rachev, & Fabozzi, 2007). To mention a few: (Brechmann, Czado, & Paterlini, 2014; Chapelle, Crama, Hübner, & Peters, 2008; Chavez-Demoulin, Embrechts, & Hofert, 2016; Dutta & Perry, 2006; Fontnouvelle & Rosengren, 2004; Li, 2009; Mignola & Ugoccioni, 2005; Shevchenko, 2010) have paid a good deal of attention to actuarial models specially the loss distribution approach (LDA). The LDA uses two statistical distributions -frequency and severity- to estimate final loss distribution for each business line independently or the whole of the bank. Both frequency and severity distributions are constructed based on internal or external historical data which leads to a data driven model. Since the scarcity of historical data is a main challenge of modelling operational risk (Brechmann et al., 2014), data driven models would be ill-equipped to integrate into day-to-day risk management process which is a main requirement of Basel II for an AMA model (Hager & Andersen, 2010). However, a statistical model approximates the loss distribution in the best way in the case of sufficient data. Neglecting causal mechanism leading to operational loss events is another drawback of statistical models (Neil et al., 2005; Neil et al., 2009). Causal models that take in to account the root causal factors as risk drivers and their causal relationships with loss events are good alternatives to statistical models. In some cases, they are preferred to actuarial data driven models from the viewpoint of risk management since they could account for idiosyncratic factors affecting operational risk frequency and or severity (Hager & Andersen, 2010) and provide a deeper understanding of causation that helps with planning and managing risk control activities (Sanford & Moosa, 2012). Various linear and nonlinear techniques have been applied to causal modelling in risk management domain. Among all, more flexible nonlinear and nonparametric ones such as Bayesian belief networks are more popular (Cowell, Verral, & Yoon, 2007).

2.2. Bayesian belief network

A BBN model consists of two parts: a graphical construction which depicts a set of variables and their dependence relations. The second part is a set of conditional probability tables (CPTs) as BBN parameters which assigns a node's probability distribution conditional on a specific set of its parents states (Jensen & Nielsen, 2007). Output of a BBN model is a set of posterior probabilities which is calculated based on Bayes theorem and through different inference methods. BBN models have been applied to diverse domains. However, there is still a small literature report on its application to operational risk (Sanford & Moosa, 2015). Neil et al. (2005) are among the first ones who paid attention to BBN applicability to modelling operational risk. Although their suggested model is still based on Loss Distribution Approach (LDA), but introduces process effectiveness with three parents as the main cause of loss frequency and severity. Identification of these risk factors means changing the loss modelling approach from capital allocation to cause management of risk events (Sanford & Moosa, 2012). Cowell et al. (2007) demonstrated how BBN can be used to model operational risk through a combination of past data and expert input. They emphasis the importance of causal models like BBN to understand the arising mechanism of operational losses. However the efficiency of BBN decreases by increasing model complexity since complex models need to specify a significant volume of conditional probabilities which requires a large amount of data or a cost and time consuming process to elicit them from experts (Cowell et al., 2007). Mittnik and Starobinskaya (2010) defined three variables (loss frequency, loss severity and operational loss) per each business line to develop a BBN model determining operational losses following Loss Distribution Approach. Neil et al. (2009) proposed a new method for applying Hybrid Dynamic Bayesian Networks (HDBNs) to model the operational risk. They tried to improve BBN applicability taking in to account time dynamics and applied the dynamic discretization algorithm to approximate continues loss distribution. Aquaro et al. (2010) gave attention to correlated operational losses. In their proposed BBN model, each node corresponds to the operational loss of a specific process and the links represent the causal relationships between the processes. Hager and Andersen (2010) introduced BBN as a powerful framework for identification and analysis of causal mechanisms leading to operational loss. The main objective of their proposed approach is to reduce data dependency in modelling operational risk and improve scenario based methods. Sanford and Moosa (2012) concentrated on the first step of BBN method and developed the causal model of operational risk for a functioning finance unit within an Australian bank. Later Sanford and Moosa (2015) developed an operational risk forecasting tool based on the previous network.

Previous research denoted BBN as a causal model and a suitable tool in modelling operational risk and assessing the impact of the risk mitigation activities. But there is still some problems with structuring causal models through BBN approach in actual situations. In a real case each node may have more than two or three parents which makes CPTs extraction a more complex and time consuming task in the case of data scarcity. Providing CPTs through a systematic procedure could improve BBN applicability to operational risk which is investigated in this paper.

2.3. Combining BBN and FCM

Both Fuzzy Cognitive Map (FCM) and BBN are root cause analysis approaches which have been applied to various fields separately. FCM introduced by Kosko (1988) for the first time, is a problem structuring method (Rosenhead & Mingers, 2001) which provides a systemic view of the subject problem (Ackerman & Eden, 2011). Problem structuring methods could be efficient tools to manage complexity (Ackermann, Howick, Quigley, Walls, & Houghton, 2014) such as the one arised from the above mentioned problems in measurement of operational risk. Application of FCM in modelling and managing risk have been investigated previously in various domains such as project management (Ackermann et al., 2014), ERP systems (Salmeron & Lopez, 2012), health care (Bevilacqua, Ciarapica, Mazzuto, & Paciarotti, 2013; Büyükavcu, Albayrak, & Göker, 2016), systems analysis (Jamshidi, Rahimi, Ruiz, Ait-kadi, & Rebaiaia, 2016; Rezaee, Yousefi, & Babaei, 2017), supply chain (Xiao, Chen, & Li, 2012) and natural environment(Kontogiann, Papageorgiou, Salomatina, Skourtos, & Zanou, 2012). In spite of the applicability of both methods in homogeneous issues very few authors focused on combining these two approaches (Wee, Cheah, Tan, & Wee, 2015) to improve their capability. The idea of combining cognitive maps (CM) and BBN was proposed by Nadkarni and Shenoy (2001). They introduced a procedure for making inference in cognitive maps by BBN approach in order to improve CM capability. Later, Nadkarni and Shenoy (2004) proposed a procedure to convert a cognitive map in to the causal model of BBN by making some structural changes to the cognitive map. In spite of their contribution in structural modification of CM to become a BBN compatible model, they didn't consider transformation of parameters. Then extraction of CPTs from expert's knowledge was still a challenging task. Cheah, Kim, Yang, Choi, and Lee (2007) combined FCM and BBN for the first time. They introduced a procedure to migrate from FCM to BBN by converting the strength of causal relations in FCM to the CPTs of BBN. They considered algebraic sum of the individual effects on each variable as prior probability and believed that each element of a CPT should fall exactly in between the related prior probability and 1. Then the counterpart probability can be simply calculated as they defined only two states for each variable. Their proposed procedure is simple to use but it is not applicable to variables with more than two states or continues ones. However, there are still no enough proofs to consider the sum of causal effects as prior conditional probabilities or to calculate the mid-point of the mentioned interval as probability of a variable being at a specific state conditional on a combination of its parents' states. Sedki and Beaufort (2012) tried to transform FCM parameters into BBN ones by defining merely two increasing and decreasing states for each variable and two positive and negative weights for each causal relation followed by calculating CPTs based on weights of the causal relations. This migration method is also limited to variables with two states and can only determine the probability of increase or decrease of a variable and not a probability distribution which is required to be estimated as BBN parameters.

Discrepancy between uncertainty theories underlying parameters of the causal model is a fundamental difference between FCM and BBN. The parameters of FCM and BBN are assigned based on possibility and probability theories respectively (Wee et al., 2015). None of the proposed migration methods take into account this main difference except the one proposed by Wee et al. (2015) which defines a migration procedure from BBN to FCM. In their proposed method, the BBN's structure and parameters is learned from data. Then CPTs are resolved into individual causal effects and Pearson's correlation coefficient is used to calculate causal influences between variables. Finally, Pearson's coefficient value of causal strength is converted to degrees of possibility which is compatible with uncertainty theory underlying FCM. Wee and his colleagues used learning power of BBN to produce CPTs and subsequently, the parameters of FCM. The learning power of Bayesian framework has been used formerly by Tipping in 2001 to estimate hyperparameters of his proposed probabilistic sparse model, named as Relevance vector machine (RVM) that was functionally identical to Support vector machine. RVM appears successful in producing accurate prediction models which employee fewer basis functions than a comparable SVM. In spite of the efficiency of BBN in learning causal model of a data-rich issue as used by Tipping (2001) and Wee et al. (2015), it is very difficult, if possible, to learn the model in the case of complex issues with scarce data such as operational risk. In such cases, extracting CPTs from expert knowledge becomes more and more complex and time-consuming by growing the number of parents and their states. Using FCM to construct the causal model of BBN is a suitable idea for modelling such issues. However, there are still some questions about how to migrate from FCM to BBN while taking into account the possibility-probability consistency, and how to develop a migration method applicable to variables with more than two states?

In this paper, we proposed a migration method from FCM to BBN which could be used to reduce complexity of CPT eliciting task and to make it less time-consuming which would help to improve the BBN capability in actual sophisticated problems with poor data such as operational risk modelling. A comparison of the proposed method and previous researches used to combine FCM and BBN is presented in Table 1.

3. The proposed method

A Bayesian causal map is constructed through two phases. First, the causal model should be structured and then the quantitative parameters should be determined. In this paper we used the guidelines proposed by Nadkarni and Shenoy (2001) to convert the causal map to the qualitative causal model of BBN after eliciting FCM from expert knowledge. Then, a new migration method was proposed to extract CPTs from strength of causal relations.

3.1. Constructing causal model

The causal model, in other words the risk cognitive map, could be extracted from expert knowledge by means of individual interviewees or group workshops. Requirements and implementation procedures is explained in detail by Bryson, Ackermann, Eden, and Finn (2004). In this research, we provided a risk modelling workshop to identify the risk events, their causal factors and relations throughout group mapping. The criterion for selecting participants is that they have been dealing with the under-studied business process. To this end, experts from facility department of bank branches and central office, head of branch and operational risk department were invited to attend a workshop lasting for 3–4 hours. Once the primary causal model was constructed, the first round of modelling was completed. Then, the model was shared with participants individually through 30–45 interview sections to modify if necessary and to ensure its validity. Once the map was finalized structurally, it was time to determine causal weights by means of linguistic variables. Generally in the FCM contex, cauasl weight is interpreted as the influence of cause variable on the effect one wich is measured by linguistic variable taking values

Table 1
Comparison of methods that combined FCM and BBN.

Research	Migration approach	Pros	Cons	Our proposed method
Nadkarni and Shenoy (2001)	From CM to BBN	introduced a procedure for making inference in cognitive maps by BBN approach	Time-consuming and difficulty of extracting CPTs from experts	CPTs are extracted from FCM's parameters which is less time-consuming and requires less knowledge elicitation activities.
Nadkarni and Shenoy (2004)	From CM to BBN	Proposed a procedure to convert the graphical structure of a cognitive map to a Bayesian network.	Time consuming and difficulty of extracting CPTs from experts	
cheah et al. (2007)	From FCM to BBN	Combined FCM and BBN through a simple procedure for the first time	There is not enough proofs to consider the sum of causal effects as prior conditional probabilities or to calculat the mid-point in between the related prior probability and 1 as a CPT's element.	A procedure is defined to produce the conditional possibility functions which is then transformed to CPTs.
			Not considering discrepancy of uncertainty theories underlying parameters of FCM and BBN	Considering discrepancy of uncertainty theories underlying parameters of FCM and BBN
Sedki and Beaufort (2012)	From FCM to BBN	calculating the probability of increase and decrease invariables based on FCM's parameters	This migration method could just determine probability of increase or decrease of a variable and not a probability distribution.	There is no limitation about number of the states of the variables and full probability distribution is estimated.
		Not considering discrepancy of uncertainty theories underlying parameters of FCM and BBN	Not considering discrepancy of uncertainty theories	
Wee et al. (2015)	From BBN to FCM	Proposed an efficient migration method for data-rich domains Considered discrepancy between uncertainty theories underlying parameters of FCM and BBN	Hardly applicable to complex issues with scarce data	Improve BBN capability in complex or data-poor issues.

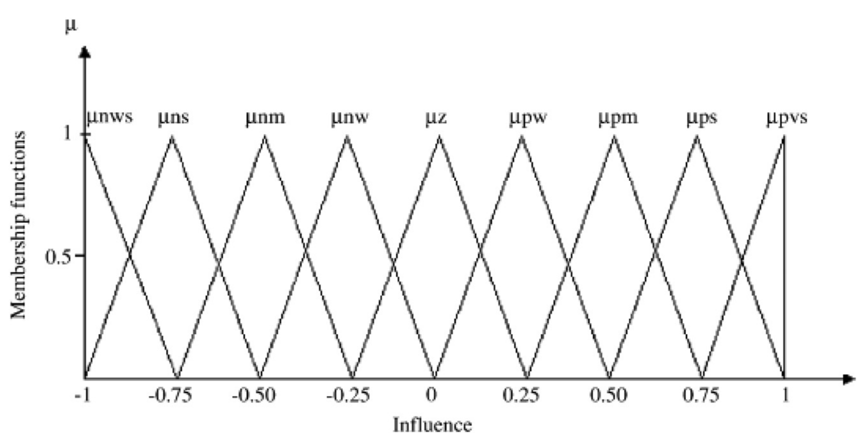

Fig. 1. Membership functions of the linguistic variable Influence(Papageorgiou et al., 2006).

in the interval [−1, 1] which is suggested to include nine variables as: T(influence) = {negatively very strong, negatively strong, negatively medium, negatively weak, zero, positively weak, positively medium, positively strong and positively very strong}. The corresponding memberships functions for these terms are labelled as μ_{nvs}, μ_{ns}, μ_{nm}, μ_{nw}, μ_{z}, μ_{pw}, μ_{pm}, μ_{ps} and μ_{pvs}, respectively, (Papageorgiou, Stylios, & Groumpos, 2006) as shown in Fig. 1.

3.2. Structural modification of fuzzy cognitive map

In spite of their similarities, there are some structural differences between FCM and BBN in terms of circular relations and variables dependency or conditional independency which means FCM and BBN are not always necessarily isomorphic graphs. Once the fuzzy cognitive map is constructed, it is necessary to imply some structural modifications as proposed by Nadkarni and Shenoy (2001) to convert the cognitive map to the causal model of BBN which includes:

- *Conditional independence*: A FCM depicts dependency between variables although the absence of relation between two variables doesn't imply independency, whereas the causal model of a BBN is an independent map which ensures conditional independence between unconnected variables. In order to transform the Fuzzy cognitive map to the causal model of BBN, a D-map should be modified to an I-map. This modification could be performed by expert participation and by ensuring that all dependency relations are depicted.
- *Circular relations*: Unlike BBN, FCM allows causal loops among variables which destroys acyclic structure required in a Bayesian network. Regardless of whether they are caused by mistake or representing some dynamic relations between vari-

ables, these causal loops should be investigated and modified to transform a cognitive map to a Bayesian network. In case of dynamic loops, de-aggregating the variables into two time frames can often solve the problem. Modification of causal loops and ensuring the acyclic structure of the model in this stage would ensure avoiding circular inference later at quantification steps.

Once these changes are applied, a BBN compatible causal model would be achieved having a topology which may differ from primary model of FCM. In other words, although the number of vertices of the modified graph would be just as the primary causal model, modification of some edges and subsequently, the adjacency matrix would result in non-isomorphic graphs.

3.3. Extraction of conditional possibilities

Once the fuzzy cognitive map is modified structurally, it's time to imply quantitative modifications which relates to model parameters. The causal strength of relations in a Fuzzy cognitive map could be determined in the form of fuzzy numbers which demonstrates the possibility degree of the effect variable being affected by the cause variable However the parameters of a Bayesian network (CPTs) demonstrate the probability of different states of a specific variable conditional on various combinations of its parents states. Thus, production of conditional possibility functions should be considered as the first step of quantitative modification process. We used the following procedure to produce the possibility function of each node conditional on each one of its parent nodes individually.

The possibility distribution of variable X taking values in a set named U is a function from U to [0, 1] which is denoted by $\pi_x(u)$. This function links a possibility degree to each possible amount of X. If we know X = u, then the conditional possibility distribution of Y given X = u would be denoted as $\pi_{y|x}(v|u)$ and demonstrates the possibility that Y = v while we know X = u (Zadeh, 1978). Since the quantity of an effect variable is a function of the amount of its cause variable and strength of the causal relation between the effect node and its cause, we defined the width of the support of $\pi(y|x)$ as a function of the strength of causal relation and the amount of the cause variable i.e. product of the causal strength and the value of the cause variable

. The conditional possibility distribution would map a possibility degree to each state of the effect variable conditional on a specific state of its cause. Herein, we used triangular fuzzy numbers to determine the strength of causal relations and the different states of each variable. If $\tilde{M} = (m, a, b)$ represents strength of the causal relation between X and Y, given $X = \tilde{s}_i$ where $\tilde{s}_i = (n, a', b')$ then $\pi(y|x = \tilde{s}_i)$ would be calculated as:

$$\pi(y|x = \tilde{s}_i) \begin{cases} 1 - \frac{m.n-y}{n.a+m.a'} & m.n - \left(n.a + m.a'\right) \le y \le m.n \\ 1 - \frac{y-m.n}{n.b+m.b'} & m.n \le y \le m.n + \left(n.b + m.b'\right) \\ 0 & otherwise \end{cases} \tag{1}$$

Almost always variables have at least two states which means if variable Y have K possible states (s_1,s_2,..., s_k) then the above function should be repeated for K times, each time demonstrating the possibility function of Y conditional on a specific state of X($\pi(y|x = \tilde{s}_i)$; $i = 1, 2, ..., k$).

3.4. From possibility to probability

Once the conditional possibility distributions has been extracted for each effect node conditional on all its causes individually, it's time to transform the conditional possibility distributions to CPTs which are probability based parameters. In the conversion process of possibility to probability some information would be added to the possibilistic incomplete knowledge as going from interval-valued to point-valued probabilities (Dubois, Prade, & Sandri, 1993). Possibility degree was first introduced by Zadeh (1978). He suggested a possibility- probability consistency principle which has been the starting point and basis of many possibility-probability conversion methods. According to his consistency principle, the possibility of an event is always higher than its probability. Dubois and Prade (1988) added two more postulates to this principle. They claimed that the order of events based on possibility would be the same as the order defined based on probability. In other words, the transformation rule should satisfy the constraint: $\pi(x) \succ \pi(x') \Leftrightarrow p(x) \succ p(x')$. Selection of the most specific possibility distribution is the second postulate. Dubois et al. (1993) proposed a transformation rule based on the mentioned principles. In the continues case, let $X = [a, b] \subseteq R$ and π represent the membership function of an upper semi-continuous, unimodal, support-bounded fuzzy number, then $p(x)$ would be computed as follows:

$$\forall x \in [a, b],\ p(x) = \int_0^{\pi(x)} d\alpha / |A_\alpha| \tag{2}$$

Where $|A_\alpha|$ is the width of α-cut of π and for $A_\alpha = [m_\alpha, M_\alpha]$ the width of α-cut would be calculated as: $|A_\alpha| = M_\alpha - m_\alpha$

Given the conditional possibility function defined in the previous section, $|A_\alpha|$ would be calculated as follows:

$$\begin{aligned} m_\alpha &= (m.n - m.a' - n.a) + (m.a' + n.a)\alpha \\ M_\alpha &= (m.n + m.b' + n.b) - (m.b' + n.b)\alpha \\ |A_\alpha| &= (m.a' + n.a + m.b' + n.b)(1 - \alpha) \end{aligned} \tag{3}$$

Then, $\pi(y|x = \tilde{s})$ would be transformed to $p(y|x = \tilde{s})$ as:

$$p(y|x = \tilde{s}_i) = \int_0^{\pi(y|x)} \frac{d\alpha}{(m.a' + n.a + m.b' + n.b)(1 - \alpha)}; \quad i = 1, 2, \dots, k \tag{4}$$

As $\pi(y|x = \tilde{s}_i)$ is constructed of two; left and right hand side functions, a two side probability function would form by inserting each conditional possibility function in the above equation:

$$\mathrm{p(y|x{=}\tilde{s}_i)} = \begin{cases} -\frac{1}{(m.a' + n.a + m.b' + n.b)} \log\left|\frac{m.n - y}{n.a + m.a'}\right| \\ \mathrm{m.n - (n.a + m.a') \le y \le m.n} \\ -\frac{1}{(m.a' + n.a + m.b' + n.b)} \log\left|\frac{y - m.n}{n.b + m.b'}\right| \\ \mathrm{m.n \le y \le m.n + (n.a + m.a')} \end{cases} \tag{5}$$

A new column of the conditional probability table of $p(y|x)$ would be determined each time calculating $p(y|x = \tilde{s}_i)$. In other words, the above probability function assigns a probability degree to each state of the child variable "y" conditional on a specific state of one of its parents named "x". Estimation of $p(y = \tilde{s}_j|x = \tilde{s}_i)$ depends on the width of the fuzzy number used to describe the state of $\tilde{s}_j$. Given $\tilde{s}_j = (r, c, d)$, a triangular fuzzy number, then the possibility of "y" being at state $\tilde{s}_j$ conditional on "x" being at state $\tilde{s}_i$ would be estimated as follows where Choosing one of the following equations or using both of them depends on the left and right intervals of $\tilde{s}_j$.

$$\mathrm{p}\left(y = \tilde{s}_j|x = \tilde{s}_i\right) = \int_{r-c}^{r+d} -\frac{1}{(m.a' + n.a + m.b' + n.b)} \log\left|\frac{m.n - y}{n.a + m.a'}\right| \tag{6}$$

And or

$$\mathrm{p}\left(y = \tilde{s}_j|x = \tilde{s}_i\right) = \int_{r-c}^{r+d} -\frac{1}{(m.a' + n.a + m.b' + n.b)} \log\left|\frac{y - m.n}{n.b + m.b'}\right| \tag{7}$$

At the end of this step, all the elements of the table $p(y|x)$ would be estimated.

3.5. Construction of CPTs

Since each BBN node has usually more than one parent, the separate tables calculated in the previous step should be merged to approximate probability of each child node conditional on all of its parents simultaneously. Kim and Pearl (1983) proposed that when a node A has two parents B and C, its probability conditional on B and C could be approximated by $p(A|B,C) = \alpha p(A|B)p(A|C)$, Where α is a normalization factor to ensure $\sum_{a\in A} p(a|B,C) = 1$. Herein, we used the generalized form of the above result as Chin and his colleagues used in 2009.

For a given variable "y" with m states and n parents, the probability of $y = \tilde{s}_j$ conditional on $x_1 = \tilde{s}_{1i}, x_2 = \tilde{s}_{2i}, ..., x_n = \tilde{s}_{ni}$ would be approximated as:

$$p(y = \tilde{s}_j|x_1 = \tilde{s}_{1i}, x_2 = \tilde{s}_{2i}, ..., x_n = \tilde{s}_{ni}) = \alpha\, p(y = \tilde{s}_j|x_1 = \tilde{s}_{1i}). \times p(y = \tilde{s}_j|x_2 = \tilde{s}_{2i})...p(y = \tilde{s}_j|x_n = \tilde{s}_{ni}) \tag{8}$$

As mentioned by Chin and his colleagues in 2009, the normalization factor is calculated as $\alpha = 1/k$

Where

$$k = \sum_{j=1}^{m}\left[p\big(y = \tilde{s}_j|x_1 = \tilde{s}_{1i}\big).p\big(y = \tilde{s}_j|x_2 = \tilde{s}_{2i}\big)...p\big(y = \tilde{s}_j|x_n = \tilde{s}_{ni}\big)\right] \tag{9}$$

Then, given Eq. (8) we have:

$$p(y = \tilde{s}_1|x_1 = \tilde{s}_{1i}, x_2 = \tilde{s}_{2i}, ..., x_n = \tilde{s}_{ni}) = \frac{1}{k}.p(y = \tilde{s}_1|x_1 = \tilde{s}_{1i}). \times p(y = \tilde{s}_1|x_2 = \tilde{s}_{2i})...p(y = \tilde{s}_1|x_n = \tilde{s}_{ni})$$

$$p(y = \tilde{s}_2|x_1 = \tilde{s}_{1i}, x_2 = \tilde{s}_{2i}, ..., x_n = \tilde{s}_{ni}) = \frac{1}{k}.p(y = \tilde{s}_2|x_1 = \tilde{s}_{1i}). \times p(y = \tilde{s}_2|x_2 = \tilde{s}_{2i})...p(y = \tilde{s}_2|x_n = \tilde{s}_{ni})$$

$$p(y = \tilde{s}_m|x_1 = \tilde{s}_{1i}, x_2 = \tilde{s}_{2i}, ..., x_n = \tilde{s}_{ni}) = \frac{1}{k}.p(y = \tilde{s}_m|x_1 = \tilde{s}_{1i}). \times p(y = \tilde{s}_m|x_2 = \tilde{s}_{2i})...p(y = \tilde{s}_m|x_n = \tilde{s}_{ni})$$

And then, sum of the probability of different states of a variable conditional on a specific combination of all parents' states would be calculated as:

$$\begin{aligned}\sum_{j=1}^{m} & p(y = \tilde{s}_j|x_1 = \tilde{s}_{1i}, x_2 = \tilde{s}_{2i}, ..., x_n = \tilde{s}_{ni}) \\ &= \frac{1}{k}.\sum_{j=1}^{m}\left[p(y = \tilde{s}_j|x_1 = \tilde{s}_{1i}).p(y = \tilde{s}_j|x_2 = \tilde{s}_{2i})...p(y = \tilde{s}_j|x_n = \tilde{s}_{ni})\right] \\ &= \frac{1}{k}.k = 1\end{aligned}$$

Therefore we can conclude that the above Eqs. (8) and (9) would produce the CPT elements one by one while ensuring sum of the probability of different states of a variable conditional on a specific combination of all parents' states is equal to 1.

Traditional extraction of CPTs directly from expert knowledge suffers from a problem, called "curse of dimension" (Chin, Tang, Yang, & Wong, 2009) which means the task will get difficult and time consuming by increasing the number of the states of a node or by growing the model complexity in which a node has many parents. In the proposed method, extraction of parameters from experts is facilitated since, CPTs are calculated based on causal strengths. Traditionally for a given node with k parents the number of elements that must be specified by experts would be equal to: $e = m.\prod_{p=1}^{k} n_p$, where m and n_p denote the number of states of the given child node and its pth parent, respectively. However constructing CPT of the same node under the proposed method requires only k elements to be specified as strengths of the causal relations.

4. Case study

In order to examine and validate the proposed method, a private Iranian bank was chosen as a case study. In this part, operational risks of the process of providing bank guarantee services which is one of a main process of retail banking line of the studied bank is modelled. First, the causal model depicting operational loss determinants, their causal factors, causal links and causal strength of links is extracted through fuzzy cognitive maps approach. Fig. 2 shows a part of the causal model. Erroneous credit assessment of the customer and preparation of a letter of guarantee with undermined support are the two main determinants of operational loss in the studied process. Manipulation of accounts by customer is the first causal factor leading to errors in the credit assessment result. Another cause is misevaluation of financial statements originating from two root causes: providing the forged statements by customer which could lead to the positive deviation of its assessment score, and human mistakes that is sometimes the main cause of misevaluation. Undermining the support of the guarantee letter is recognized as the second loss determinant. Indeed, accepting a weak collateral and/or a non-accredited sponsor are the primary reasons of the weakness of the support. Although there are lots of regulations and instructions supposed to ensure adequate support of the guarantee letter provided by the bank, there are still several factors such as accepting non-accredited sponsor or misevaluating and overestimating the collaterals which could undermine the support of the guarantee letter. Inadequate monitoring, customer collusion with staff and human mistakes are recognized as root causes of the above-mentioned risk factors.

4.1. Extraction of conditional possibilities

The strength of causal relation is extracted from the expert knowledge by means of linguistic variables usually taking values in the universe U = [−1, 1] as shown in Fig. 1.

States of variables are also shown by triangular fuzzy numbers. For example, (0,0,0.6) and (1,0.6,0) are two numbers referring respectively to "no" and "yes" states of "Erroneous credit assessment" (EA), " statements misevaluation" (SM) and "Manipulated accounts" (MA) nodes. As shown by Fig. 2, the causal strength of the relation between "SM" and "EA" is estimated equal to (0.95, 0.25, 0.05). Thus, when "SM" is at "yes" or "no" states then the possibility function of "EA" conditional on "SM" would form, respectively, as follows:

$$\pi(EA|SM = yes) = \begin{cases} 1 - \frac{0.95-EA}{0.82} & 0.13 \le EA \le 0.95 \\ 1 - \frac{EA-0.95}{0.05} & 0.95 \le EA \le 1 \\ 0 & Otherwise \end{cases} \tag{10}$$

$$\pi(EA|SM = no) = \begin{cases} 1 - \frac{0-EA}{0} & 0 \le EA \le 0 \\ 1 - \frac{EA-0}{0.84} & 0 \le EA \le 0.6 \\ 0 & Otherwise \end{cases} \tag{11}$$

4.2. From possibility to probability

Using the equation proposed by Dubois et al. (1993) the above possibility function would be transformed to a probability function as follows:

$$\begin{aligned} p(EA|SM = yes) &= \int_0^{\pi(EA|SM=no)} \frac{d\alpha}{0.87(1-\alpha)} \\ &= -\frac{100}{87}Log(|\alpha - 1|)\Big|_0^{\pi(EA|SM=no)} \\ &= \begin{cases} -\frac{100}{87}Log\left(\left|\frac{0.95-EA}{0.82}\right|\right) & 0.13 \le EA \le 0.95 \\ -\frac{100}{87}Log\left(\left|\frac{EA-0.95}{0.05}\right|\right) & 0.95 \le EA \le 1 \end{cases} \end{aligned} \tag{12}$$

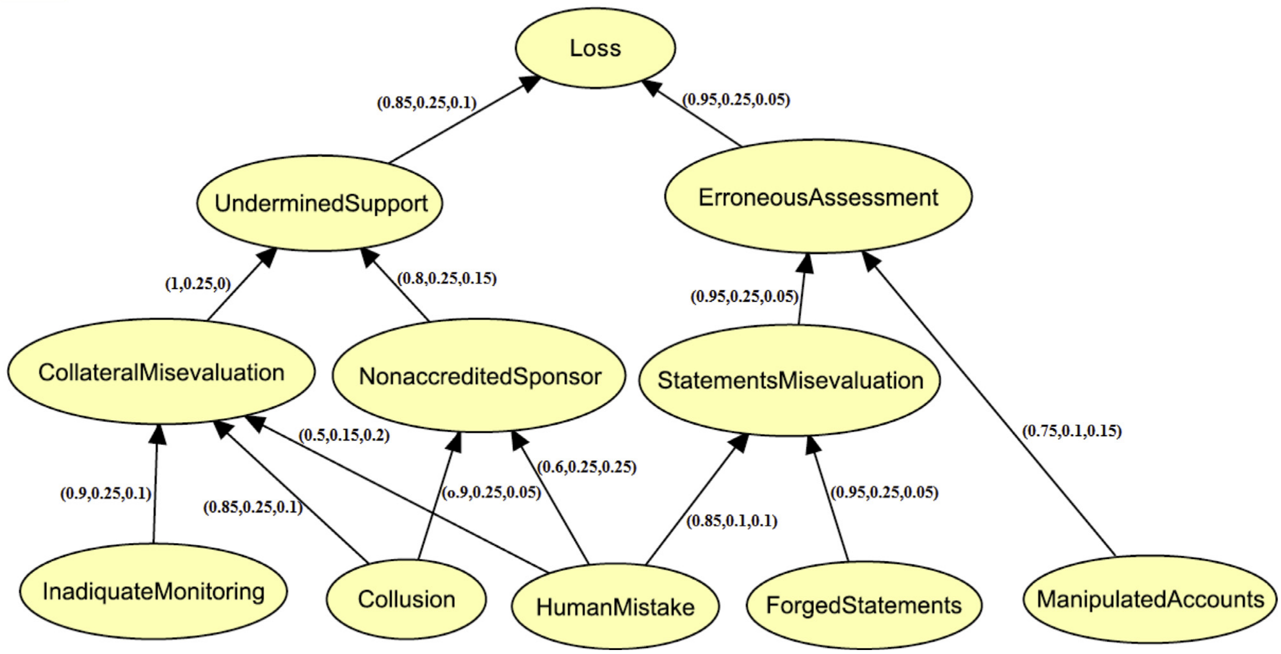

Fig. 2. The operational risk fuzzy causal model.

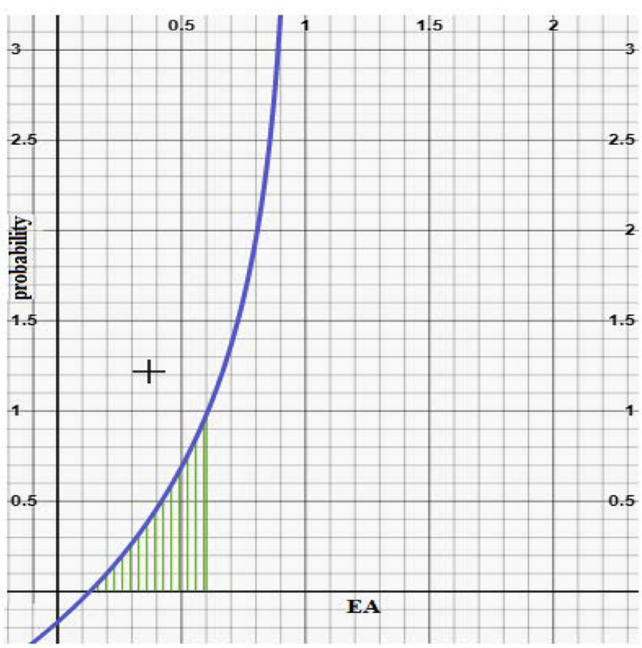

Fig. 3. The probability function of EA conditional on SM = yes.

$$p(EA|SM=no) = \int_0^{\pi(EA|SM=no)} \frac{d\alpha}{.57(1-\alpha)}$$
$$= -\frac{100}{57}Log(|\alpha - 1|)\Big|_0^{\pi(EA|SM=no)}$$
$$= -\frac{100}{57}Log\left(\left|\frac{100EA}{84}\right|\right) \quad (13)$$

The above function depicts probability of all states of "EA" conditional on "yes" state of "SM" (Fig. 3). To estimate probability of "EA" being at "no" state conditional on "yes" state of "SM" we should calculate the space under the conditional probability chart at the interval of [0, 0.6]. The determined space is hachured in Fig. 4 and the related probability would be calculated as follows:

$$p(EA=no|SM=yes) = \int_0^{0.6} \left(-\frac{100}{87}Log\left(\left|\frac{0.95-EA}{0.82}\right|\right)\right)dEA = 0.1977 \quad (14)$$

$$p(EA=yes|SM=yes)$$
$$= \int_{0.4}^{0.95} \left(-\frac{100}{87}Log\left(\left|\frac{EA-0.95}{0.05}\right|\right)\right)dEA$$
$$+ \int_{0.95}^{1} \left(-\frac{100}{87}Log\left(\left|\frac{0.95-EA}{0.82}\right|\right)\right)dEA = 0.9421 \quad (15)$$

Similarly $p(EA=no|SM=no)$ and $p(EA=yes|SM=no)$would be calculated as:

$$p(EA=no|SM=no) = \int_0^{0.6} \left(-\frac{100}{57}Log\left(\left|\frac{100EA}{57}\right|\right)\right)dEA$$
$$= \frac{100EA\left(Log\left(\frac{100EA}{57}\right)-1\right)}{57}\Big|_0^{0.6} = 1 \quad (16)$$

$$p(EA=yes|SM=no) = \int_{0.4}^{0.6} \left(-\frac{100}{57}Log\left(\left|\frac{100EA}{57}\right|\right)\right)dEA = 0.0497 \quad (17)$$

Thus, the primary probability table of EA conditional on different states of SM is formed as shown in Table 2 which needs to be normalized as Table 3.

As shown in causal model (Fig. 2), the node "EA" has another parent named manipulated accounts (MA). The probability table of "EA" conditional on different states of "MA" is obtained as listed in Table 4.

Table 2
The primary probability table of EA conditional on different states of SM.

	Statements misevaluation	No	Yes
Erroneous assessment	No	1	0.1977
	Yes	0.0497	0.9421

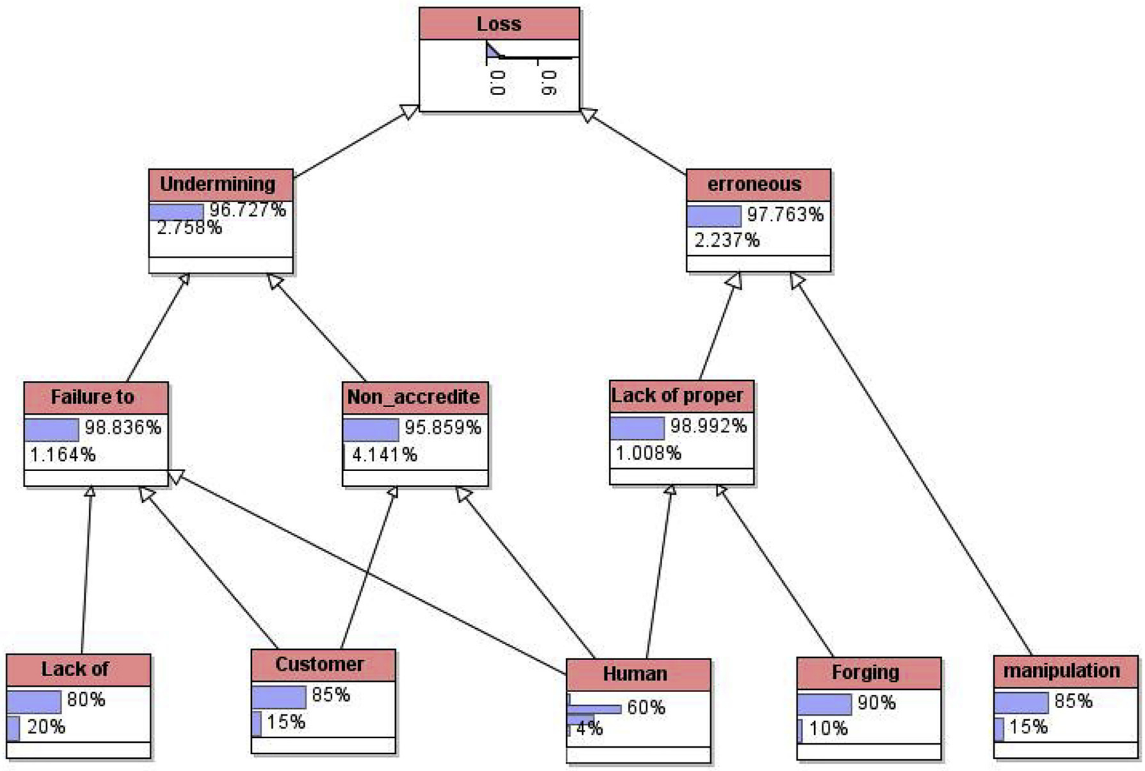

Fig. 4. The operationa risk FCBN of studied process.

Table 3
The normalized probability table of EA conditional on different states of SM.

	Statements misevaluation	No	Yes
Erroneous assessment	No	0.9527	0.1735
	Yes	0.0473	0.8265

Table 4
The normalized probability table of EA conditional on different states of MA.

	Manipulated accounts	No	Yes
Erroneous assessment	No	0.9936	0.2376
	Yes	0.0064	0.7624

4.3. Constructing CPTs

The normalized probability tables of each child conditional on different parents should be combined to form its CPT as a BBN parameter. For instance, when both parents of "EA" are at "No" state we will have:

$$p(EA = No|SM = No, MA = No) = \alpha . p(EA = No|SM = No). \times p(EA = No|MA = No) \quad (18)$$

As mentioned by Chin et al. (2009) $\alpha = 1/k$

Here we have:

$$\begin{aligned} k &= p(EA = No|SM = No).p(EA = No|MA = No) \\ &\quad + p(EA = Yes|SM = No).p(EA = Yes|MA = No) \\ &= (0.9527 * 0.9936) + (0.0473 * 0.0064) = 0.9469 \\ \alpha &= 1.05607 \end{aligned} \quad (19)$$

Then, we get the first column of the CPT of "EA" as follows, and the whole CPT as listed in Table 5:

$$p(EA = No|SM = No, MA = No) = 0.9997 \quad (20)$$

Table 5
The CPT of "EA".

	Statements misevaluation	No		Yes	
	Manipulated accounts	No	Yes	No	Yes
Erroneous assessment	No	0.9997	0.9704	0.8624	0.0614
	Yes	0.0003	0.0296	0.1376	0.9386

$$p(EA = Yes|SM = No, MA = No) = 0.0003 \quad (21)$$

Once all CPTs are constructed the parameters of BBN are determined, and Bayes inference mechanisms could be used to estimate the loss distribution as the ultimate output. Fig. 4 depicts the final fuzzy cognitive Bayesian network of operational risks in the studied business process.

4.4. Model validation

In order to verify the reliability of a probability produced through a Bayesian network, sensitivity analysis could be performed (Coupé & van der Gaag, 2002) which is a prevalent quantitative validation test of BBN assessing how predictive is the behaviour of a BBN model (Pitchforth & Mengersen, 2013). Sensitivity analysis depicts the model behaviour and sensitivity of output nodes to variation in model inputs (Bednarski, Cholewa, & Frid, 2004; Coupé & van der Gaag, 2002). Since the output of extreme conditions is more predictable, defining scenarios based on extreme conditions and assessing changes occurred in the output nodes could be regarded as another predictive validity test (Pitchforth & Mengersen, 2013). In this paper, we used both predictive validity tests to evaluate the efficiency of the model yielded from our proposed method.

In this step, a one-way sensitivity analysis is performed and then two scenarios are defined based on extreme conditions in or-

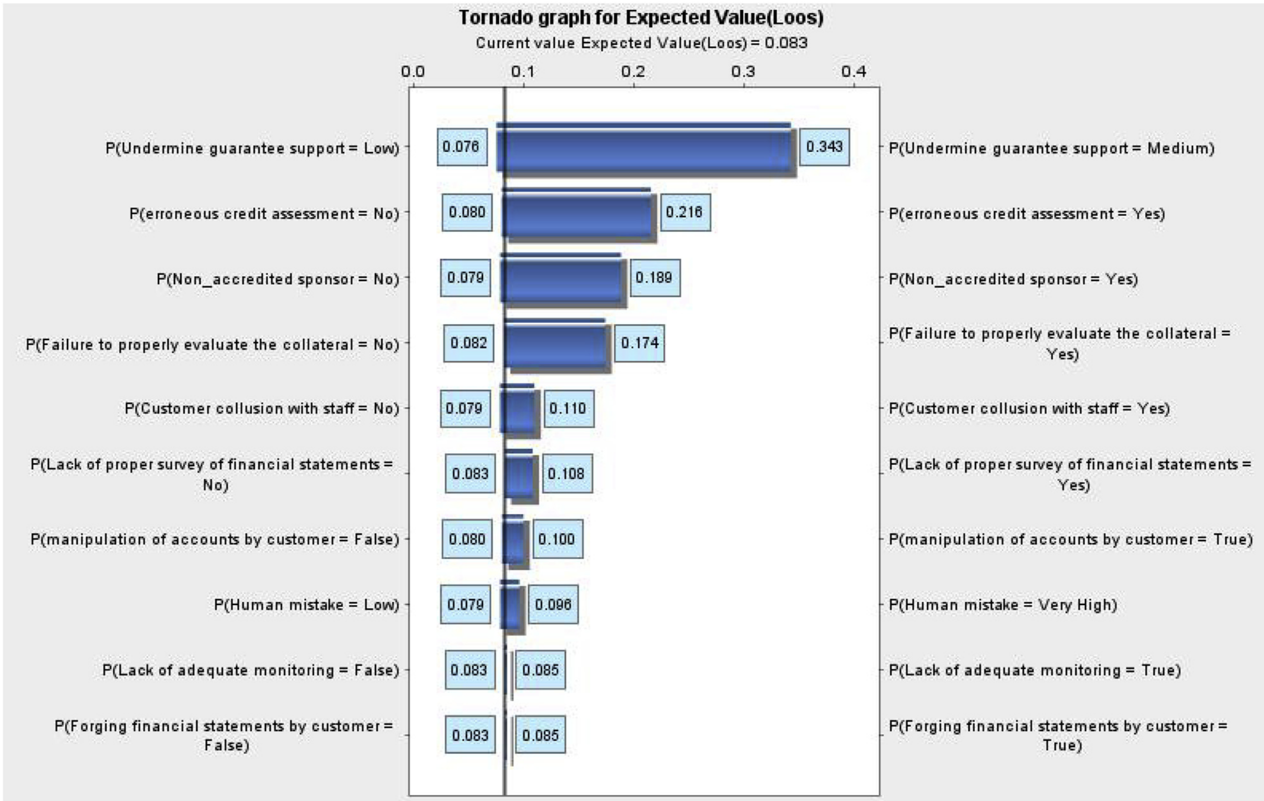

Fig. 5. Tornado graph for expected value of Loss.

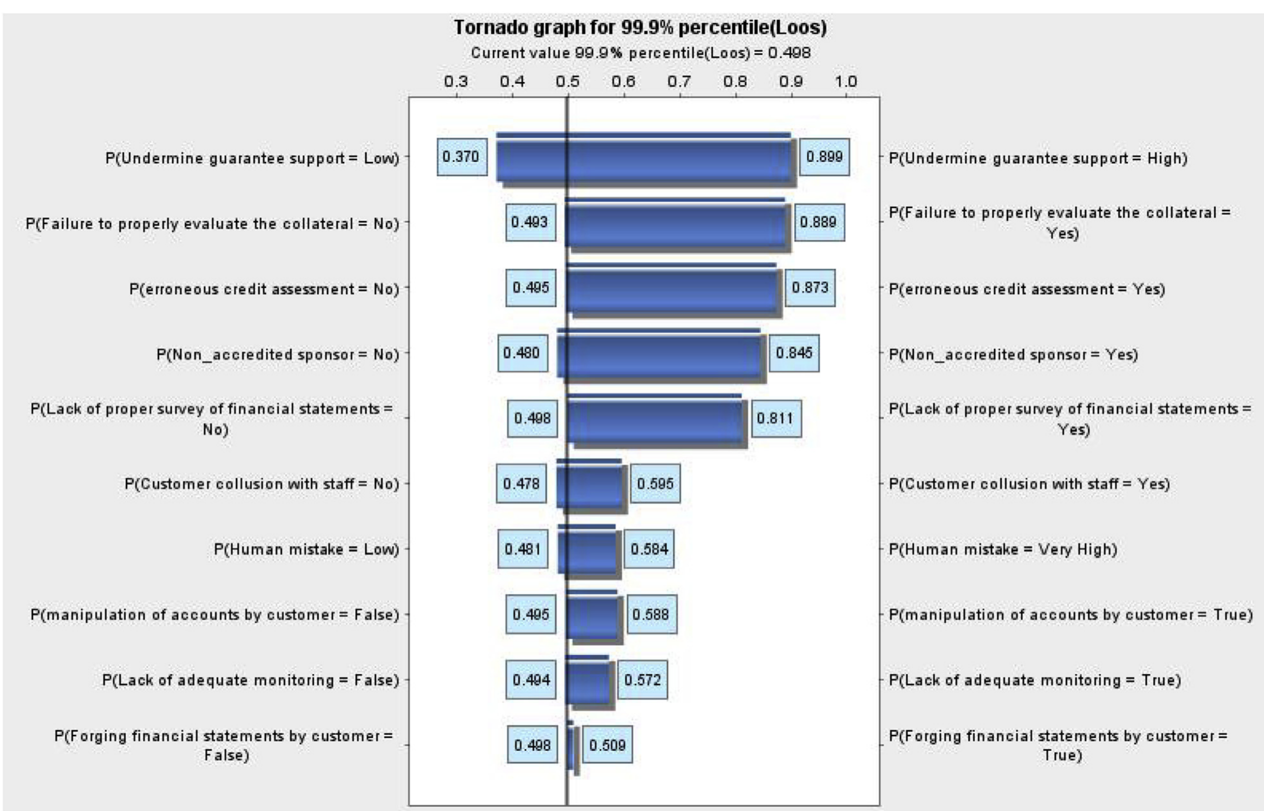

Fig. 6. Tornado graph for value at risk of Loss distribution.

Table 6
Comparing outputs of the extreme conditions scenarios.

Scenario	expected value of loss distribution	value at risk
The optimistic scenario	0.0783	0.0950
The pessimistic scenario	0.1937	0.3518

der to assess predictive validity of the model. Since the loss node is the main output of an operational risk BBN, it is chosen as the target node of the predictive validity tests.

Fig. 5 depicts the tornado graph demonstrating the most effective nodes on the expected amount of loss.

In the context of operational risk modelling, the amount of 99.9% of loss distribution is a consequential quantity as it denotes the operational value at risk. Loss distribution variation according to its value at risk is depicted in Fig. 6.

The results of sensitivity analysis shows four variables (undermining Guarantee support, failure to properly evaluate collateral, erroneous credit assessment and non-accredited sponsor) as the most influential ones on the operational losses of the studied process which is admitted from experts point of view.

Results obtained from the second predictive test are listed in Table 6 providing a comparison between optimistic and the pessimistic scenarios in terms of expected value of loss distribution and the value at risk. A comparison between outputs indicates the sensitivity of loss node as the main output of operational risk model to extreme conditions.

5. Conclusions

In this paper a method has been proposed to combine FCM and BBN for modelling operational risk in the case of data scarcity. Determination of CPT parameters is the main challenge of applying BBN to a data-poor domain such as operational risk which needs expert elicitation. We used FCM as a problem structuring method to improve BBN capability in the case of data scarcity. Once the FCM is constructed, the causal strength of relations in FCM were transformed to the CPTs of a Bayesian network through the proposed migration method. Then, the method was examined through modelling operational risk of a retail banking process of an Iranian private bank.

The main contribution of this paper is providing a procedure for making the BBN approach applicable to complex problems having poor data. The proposed method enables Bayesian inference engine to work using verbally expressed knowledge of experts which is significantly beneficial to causal modeling in various domains such as risk analysis, quality management and change analysis. In other words when constructing a Bayesian network of a complex problem, the proposed method in this paper can be used to produce CPTs just from verbally expressed parameters of FCM. The process of extracting FCM parameters from experts needs less knowledge input and also, it is more cost and time-effective since merely a linguistic variable is required to determine strength of causal relations instead of specifying all the CPT's elements directly by experts. Decreasing the amount of assessments required to be made by experts for CPT elicitation could improve precision and prevent the task to be more time consuming even if the dimensions of the CPT are growing in terms of the number of parents or node's states. In other words, the proposed method doesn't suffer from "curse of dimension" because although the activity of CPT construction is based on experts knowledge and experiences, experts are required to specify only one more element whenever a new parent is added to the parent set of a given node. Estimation of conditional probability distributions instead of just approximating the probability of the two decrease and increase states of a given node, is another contribution of the proposed method that is an advantage to the previous migration methods from FCM to BBN.

The proposed method is a good alternative to many existing operational risk modeling methods since it provides causal mechanism of risk evolution which is necessary to plan risk mitigation activities besides estimation of loss distribution.

Contributions of the paper

The main contribution of this paper is providing a procedure for making the BBN approach applicable to complex problems having poor data. The proposed method enables Bayesian inference engine to work using verbally expressed knowledge of experts which is significantly beneficial to causal modeling in various domains such as risk analysis, quality management and change analysis. In other words when constructing a Bayesian network of a complex problem, the proposed method in this paper can be used to produce CPTs just from verbally expressed parameters of FCM. The process of extracting FCM parameters from experts needs less knowledge input and also, it is more cost and time-effective since merely a linguistic variable is required to determine strength of causal relations instead of specifying all the CPT's elements directly by experts. Decreasing the amount of assessments required to be made by experts for CPT elicitation could improve precision and prevent the task to be more time consuming even if the dimensions of the CPT are growing in terms of the number of parents or node's states. In other words, the proposed method doesn't suffer from "curse of dimension" because although the activity of CPT construction is based on experts knowledge and experiences, experts are required to specify only one more element whenever a new parent is added to the parent set of a given node. Estimation of conditional probability distributions instead of just approximating the probability of the two decrease and increase states of a given node, is another contribution of the proposed method that is an advantage to the previous migration methods from FCM to BBN.

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