Solutions to Worktext Exercises

Chapter 4
Visualizing Discrete Distributions

Basic Learning Exercises

1. Its parameters are n = 23 (number of trials) and p = 0.60 (probability of success on each trial). The distribution is slightly negatively skewed and nearly bell-shaped.

2. Mean 11.5 Standard Deviation 2.398 Skewness 0 Kurtosis 2.913
   The distribution is now symmetric and close to bell-shaped (kurtosis near 3.0). The light does not favor either direction.

3. Mean 9.2 Standard Deviation 2.349 Skewness 0.085 Kurtosis 2.920
   The distribution is slightly positively skewed. The light now favors the other street.

4. Mean 28 Standard Deviation 4.099 Skewness 0.049 Kurtosis 2.974
   Mean 49 Standard Deviation 3.834 Skewness - 0.104 Kurtosis 2.982
   Mean 21 Standard Deviation 3.834 Skewness 0.104 Kurtosis 2.982
   It became more symmetric and more nearly bell-shaped.

5. Mean 2.1 Standard Deviation 1.449 Skewness 0.690 Kurtosis 3.476
   The parameter Lambda (λ) defines the Poisson distribution.

6. a) The distribution is positively skewed and leptokurtic (more peaked than a normal).
   b) Lambda (λ) tells the expected number of billboards per mile. c) The mean and variance of the Poisson distribution both equal λ.

7. λ = 21
   Mean 21 Standard Deviation 4.583 Skewness 0.218 Kurtosis 3.048
   Lambda (λ) tells the expected number of billboards in 10 miles.

8. Mean 4.5 Standard Deviation 2.872 Skewness 0.0 Kurtosis 1.80
   Its has two parameters: maximum (upper limit) equals 9 and minimum (lower limit) equals 0. The uniform distribution is symmetric with no modes.

9. Mean 1.667 Standard Deviation 0.841 Skewness 0.079 Kurtosis 2.734
   b) The parameters are population size (N), sample size (n), and successes in population (A).
   c) The distribution is positively skewed. d) At the beginning Pr(Success) = 4/12.

10. Mean 2.0 Standard Deviation 0.853 Skewness 0 Kurtosis 2.750
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Intermediate Learning Exercises

11. a) The mean is $n \times p = 2.5$ and the variance is $n \times p \times q = 1.25$. The normal curve *almost* goes through the center of each bar. c) The approximation indicator for n = 5 is "good." d) The mean is 10 and the variance is 5. The normal curve now goes through the center of each bar. For n = 20, it is "excellent."

12.

X	Binomial	Normal	Absolute Difference
0	0.1216	0.1006	0.0210
1	0.2702	0.2229	0.0473
2	0.2852	0.2906	0.0054
3	0.1901	0.2229	0.0328
4	0.0898	0.1006	0.0108
5	0.0319	0.0267	0.0052
6	0.0089	0.0041	0.0048
7	0.0020	0.0004	0.0016
8	0.0004	0.0000	0.0004
9	0.0001	0.0000	0.0001

a) No, the normal misses the centers of most of the bars. b) X = 1 is the worst. c) The table shows that X=1 is worst, but X = 3 is nearly as bad. It's hard to be precise using the visual display. d) The approximation indicator says the quality is poor.

13. It changes to "adequate" at n = 47, "good" at n = 87 and to "excellent" at n = 175. Even at n = 175 the normal does not cut each bar in the middle, but its overall shape is normal.

14.

n	p	np	Quality Indicator
10	0.50	5	Excellent
15	0.35	5.25	Good
50	0.10	5	Adequate
100	0.05	5	Adequate
125	0.04	5	Adequate
167	0.03	5.01	Poor

The farther p is below 0.50, the more right-skewed the binomial distribution becomes. This causes the probability bars in the left tail to be taller that the normal, while those in the right tail are below the normal. The graph shows this, and also shows the failure of the normal to intersect each bar at its midpoint.

15. a) $\lambda = n \times p = 0.5$. b) The Number of Trials, n, equals 19. c) n = 81. d) It never becomes "adequate" or "poor." e) Use the Poisson approximation when p is small.

16. a) The mean is 5.1 and the standard deviation is 2.258. b) At 5.1, it is "poor." c) It is "adequate" at 5.2. d) Greater than 5 is close to 5.2. e) It is "good" when $\lambda = 9.9$.

Advanced Learning Exercises

17. a) It became its mirror image. b) If successes in the population is unchanged and the sample size (n) is changed to N – n, the mirror image of the distribution is obtained. c) It became its mirror image. d) If sample size is unchanged and the successes in the population (A) is changed to N – A, the mirror image of the distribution is obtained.

18. a) It is symmetric because A = N/2. b) The shape of the distribution would stay the same, but the scale and mean would change. c) The distribution remains symmetric as long as A = N/2. The mean remains the same as long as sample size is constant.

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20. The number of intervals decreases to 2. This happens because there is only one success in the population; hence, the only possible outcomes are zero or one success.

21. a) The number of intervals increases from 2 to 10 and then decreases back to 2. In addition, the distribution changes from positively skewed to symmetric (when number of successes is 25) and then becomes more negatively skewed. b) This happens because the probability of one success is changing from 0.02 (1/50) to 0.50 (25/50) to 0.98 (49/50) as the number of successes changes from 1 to 25 to 49.

22. In the hypergeometric distribution, we are sampling without replacement. All of the other distributions illustrated in this module sample with replacement.

23. p = A/N and n = sample size. The quality of the approximation increases as the ratio n/N decreases, i.e., as the sample size is a smaller portion of the population size. In general, the approximation is best if the proportion of successes in the population is 0.5 (A/N).

24. a) The mean is $A/N \times n = 25$, and the variance equals $A/N \times n \times (N-A)/N = 12.5$. b) The approximation deteriorates for small or large sample sizes, the same as the normal approximation to the binomial does. c) The approximation deteriorates because the proportion of successes in the population (A/N) is getting small. It is the same as the normal approximation to the binomial deteriorating as p gets smaller.

25. The uniform distribution is always rectangular, so no valid approximation exists.