Revision Question Day 4

1. If a matrix A is has dimension nxn. What are the dimensions of inv(A)?
2. If a matrix A has dimension nxn. And if its inverse exists what can you say about the rank of A?
3. Are ranks of A and inv(A) always the same?
4. Calculate the inverse of the following matrices. Do first by hand and second in R.
   a. $\begin{matrix} 3 & 7 \\ 2 & 1 \end{matrix}$

   b. $\begin{matrix} 1 & 0.5 & 0.25 \\ 5 & 2 & 7 \\ \sqrt{3} & \pi & 8 \end{matrix}$

5. For a diagonal matrix A what is its inverse?
   a. Invert this $\begin{matrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{matrix}$
6. For an upper triangular matrix, what can you comment on its inverse?
   a. Invert this $\begin{matrix} 1 & 3 & 7 \\ 0 & 2 & 4 \\ 0 & 0 & 8 \end{matrix}$
7. Prove the following
   a. $(A^T)^{-1} = (A^{-1})^T$
   b. $(AB)^{-1} = B^{-1}A^{-1}$
8. For a 3x3 matrix what are the maximum possible eigen values? What are the minimum possible eigen values?
9. For 3x3 diagonal matrix, with diagonal elements as d1,d2 and d3, what are the eigen values.
10. Find the eigen values and eigen vectors of the following
    a. $\begin{matrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{matrix}$
11. Is the following statement true?
    a. Det(A)=product of all the eigen values.
12. Can a singular matrix have eigen value = 0, ever?