1. We let X be the number of times player 1 is a winner. Let's start with the case that player 1 never wins. That is, player 1 loses to player 2. If we construct the sample space, we would see that P(X = 0) = 1 2 because exactly one-half of the 5! permutations have the first number (player 1) greater than the second number (player 2).

$$P(X=0)=\frac{\frac{1}{2}5!}{5!}=\frac{1}{2}$$

Player 1 wins exactly 1 game if player 3 has a larger number than player 1, but player 1 has a larger number than player 2. The number of ways this can happen is the same as the number of ways that player 2 loses to player 1 and player 3 (hypothetically), etc. Therefore,

$$P(X=1)=P(Y_2<Y_1<Y_3)$$

Where Y_i denotes the number given to player i. When $i \neq j \neq k$ there are 3! Ways to arrange the inequality $Y_i<Y_j<Y_k$. Exactly one of them gives us the inequality $Y_2<Y_1<Y_3$. Therefore,

$$P(X=1)=\frac{\frac{1}{3!}5!}{5!}=\frac{1}{6}$$

Similarly,

$$P(X=2)=\frac{10}{5!}=\frac{1}{12}$$
$$P(X=3)=\frac{3!}{5!}=\frac{1}{20}$$
$$P(X=4)=\frac{4!}{5!}=\frac{1}{5}$$

2. To maximize the commodity, we should buy 0 ≤ k ≤ 500 ounce of the commodity at the beginning of the week and buy it with the remaining money at the end. Then the random variable Y of the amount of commodity at the end of the week satisfies P(Y = k + (1000 − 2k)/1) = P(Y = k + (1000 − 2k)/4) = 1/2. So the expectation of it is E[Y] = 1/2 (k + (1000 − 2k)/1 + k + (1000 − 2k)/4) = 625 − k/4. To maximize it, we should let k = 0 and use all the money to buy commodity at the end of the week.

3. We have E(X)=1 and Var(X)=5.
Therefore, $Var(X)=E(X^2)-E(X)^2=E(X^2)-1$
Which means that $E(X^2)=5+1=6$
Now,
a) $E((2+X)^2)=E(4+X^2+4X)=4+E(X)^2+4E(X)=4+6+4=14$
b) $Var(4+3X)=9*Var(X)=45$

4. We have n = 10 trials and each flip of a coin is independent. But what is the probability of success? Take a closer look at the problem. What is the probability that he would have done at least this well if he had no ESP? We solve this problem on the premise that he does not have ESP. Therefore, p = ½ , the probability of randomly guessing the outcome. We define a success as the event that the man guesses correctly. Since we have a finite number of independent Bernoulli trials with constant probability of success, we use the binomial. Let X be the number of times the man guesses the outcome correctly (the number of successes). We want to know the probability that he does at least as well as he did in the observed study where he guessed correctly 7 times out of 10. Then,

$P(X \geq 7) = P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$

$= \binom{10}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + \binom{10}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + \binom{10}{9}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1 + \binom{10}{107}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0$

$=176/1024$

5.

a) $\binom{50}{1}\left(\frac{1}{100}\right)^1\left(\frac{99}{100}\right)^{49}$

b) $1 - \binom{50}{0}\left(\frac{1}{100}\right)^0\left(\frac{99}{100}\right)^{50}$

c) $1 - \binom{50}{0}\left(\frac{1}{100}\right)^0\left(\frac{99}{100}\right)^{50} - \binom{50}{1}\left(\frac{1}{100}\right)^1\left(\frac{99}{100}\right)^{49}$

6. a) This person arrives at the bus stop at 10am. We want to find the probability that the person has to wait longer than 10 minutes. That is, the bus arrives after 10:10 but before or at 10:30. Let X be the waiting time for the bus and let $X \sim U(0,30)$. Then to wait 10 minutes means that the bus arrives in the last 20 minutes of this 30 minute domain. For the Uniform distribution, we know that $P(X \leq x) = \frac{x}{b-a}$. Therefore,

$$P(X \geq 10) = 1 - P(X \leq 10) = 1 - \frac{10}{30 - 0} = \frac{2}{3}$$

b) Let X again be the number of minutes the person waits for the bus. Since it is 10:15, we know the person has been waiting at least 15 minutes. That is, X ≥ 15. We want to find the probability that the person waits at least 10 more minutes until the bus arrives. In total, the person will have waited at least 25 minutes in total. That is, we want to find P(X ≥ 25|X ≥ 15). Using Bayes' rule,

$P(X \geq 25 | X \geq 15) = \frac{P(X \geq 25 \cap X \geq 15)}{P(X \geq 15)} = \frac{P(X \geq 25)}{P(X \geq 15)}$

The bus arrives uniformly within a 30 minute period so for a person to wait at least 25 minutes means that the bus still has 5 more minutes to arrive. If a person waits at least 15 minutes, then the bus still has 15 minutes to arrive. That is,

$P(X \geq 25 | X \geq 15) = \frac{P(X \geq 25)}{P(X \geq 15)} = \frac{\frac{5}{30}}{\frac{15}{30}} = \frac{5}{15} = \frac{1}{3}$

7. Let X denote the annual rainfall, and E denote the event that it will take over 10 years starting from this year before a year occurs having a rain fall of over 50 inches. Then
$P(X > 50) = 1 - P(X \leq 50) = 1 - \Phi(2.5) = 1 - 0.9938$
Therefore, $P(E) = \{1 - (1 - 0.9938)\}^{10} = 0.9397$.
We are assuming that the annual rainfall is independent from year to year