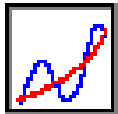


# CHAPTER 20

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## Visualizing Trends and Seasonality

### CONCEPTS

- Time series, Periodicity, Trend, Cycle, Seasonal, Error, Decomposition, Fitted Trend, Additive Model, Multiplicative Model

### OBJECTIVES

- Understand periodicity
- Recognize the difficulty in separating trend, seasonality and error, and why sample size is important
- Understand the difference between additive and multiplicative seasonality
- Recognize the difference between the true and fitted trend and the importance of selecting the correct functional form for the fitted trend
- Understand common fit statistics (MAPE,  $R^2$ , standard error) and what they tell us about the model

## Overview of Concepts

**Time series** data consists of observations on a variable over several periods of time. For example, a firm uses time series data to monitor its financial situation (e.g., unit sales, market share, return on assets, stock prices), the quality of its product (e.g., warranty claims, defect rates, customer satisfaction), or general economic conditions (e.g., inflation, prime rate, S&P 500 stock index). Time series **periodicity** is most commonly annual (1 observation per year), quarterly (4 observations per year), or monthly (12 observations per year). Time series **decomposition** seeks to separate a time series into four components: trend, cycle, seasonality, and error. These components are assumed to follow an additive or a multiplicative model:

### Additive Model

$$\text{Time Series} = \text{Trend} + \text{Cycle} + \text{Seasonality} + \text{Error}$$

### Multiplicative Model

$$\text{Time Series} = \text{Trend} \times \text{Cycle} \times \text{Seasonality} \times \text{Error}$$

**Trend** is the general pattern over many periods (change over a few periods is not a trend). The statistician estimates the trend by using a **fitted trend** model. We may imagine three general trends: growth, stability, or decline. Yet a trend may not be that simple, as illustrated in Figures 1, 2, and 3, which depict graphs whose Y-axes show the variable of interest and whose X-axes show time. Specific trend models (linear, quadratic, cubic, exponential) may be used to try to estimate such trends. Forecasters prefer the simplest trend model that adequately matches the trend (the principle of Occam's Razor) because simple models are easier to interpret and explain to others.

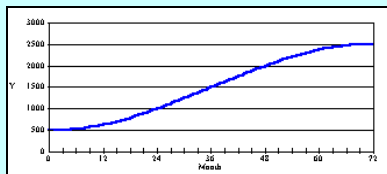


Figure 1: Growth Then Level Off

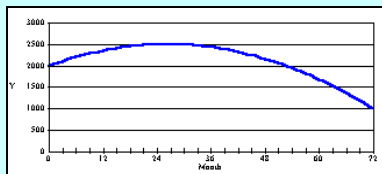


Figure 2: Peak Then Decline

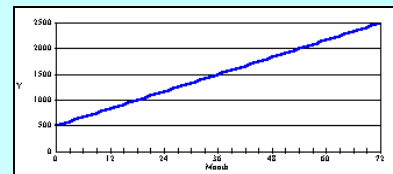


Figure 3: Constant Increase

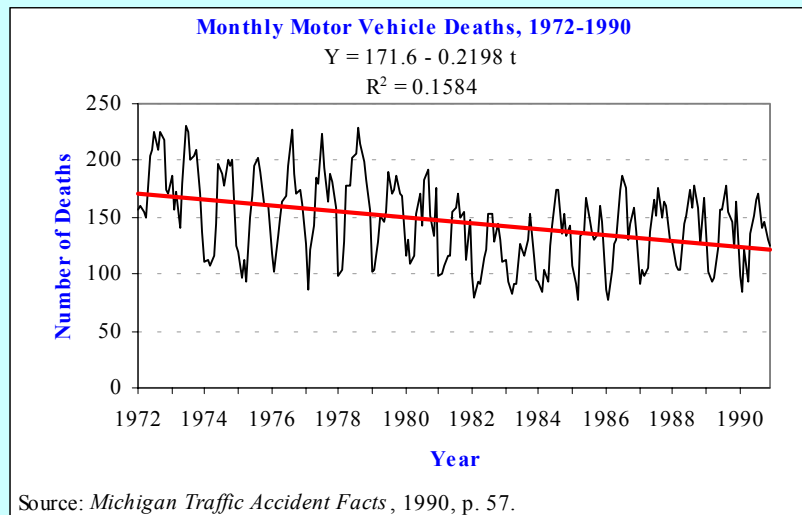
**Cycle** is a repetitive up-and-down movement about a trend over several years. For example, analysts have studied cycles for sales of new automobiles, new home construction, inventories, and investment. But this module omits cycles because their timing is erratic, their causes are complex, and over a small number of years they are undetectable or may resemble a trend.

**Seasonal** pattern is a repetitive cyclical movement within a year (months, quarters). For example, many retail businesses experience strong sales during the fourth quarter because of Christmas. Automobile sales rise when new models are released. Peak demand for airline flights to Europe occurs during summer vacation travel. Although often imagined as sine waves, seasonal patterns may not be smooth. Peaks and valleys can occur in any month or quarter, and each industry may face its own unique seasonal pattern.

**Error** is a random disturbance that follows no pattern. This “random noise” reflects all factors other than trend, cycle, and seasonality. Large error components are common. For example, daily prices of volatile stocks are important to financial analysts. When the error is large, it may be difficult to isolate other individual model components.

## Illustration of Concepts

Motor vehicle deaths are an example of **time series** data. Many states have enjoyed a downward trend in motor vehicle deaths over the past few decades. The graph below shows this **trend** using data with monthly **periodicity**. In this example, a linear **fitted trend** model is shown by the heavy red line. The slope of this line says that monthly traffic deaths have declined at an average rate of 0.2198 deaths per month from a starting point of 171.6 (forecasts could be rounded to integers). Other trend models (quadratic, cubic, or exponential) might give a better fit (note the low  $R^2$ ) but all would support the idea of a slow decline in deaths. The linear model has the advantage of simplicity.



In addition to trend, you can see a distinct **seasonal** pattern in this time series: Deaths are higher in the warm months and lower in the cool months. This is probably due to increased travel during the summer months. No **cycles** can be inferred from this graph (at least using “eyeball” methods). But you can see that trend and seasonality still do not account for all the variation in the data. Any remaining variation is nonsystematic **error** due to unpredictable events (such as mild or severe weather). For example, the summer “high” is not as pronounced in 1982 and 1983, and the winter “low” is less apparent in 1988 and 1989. Overall, the error component appears relatively small, and we might conclude that variation in monthly traffic deaths is mostly due to trend and seasonal patterns.

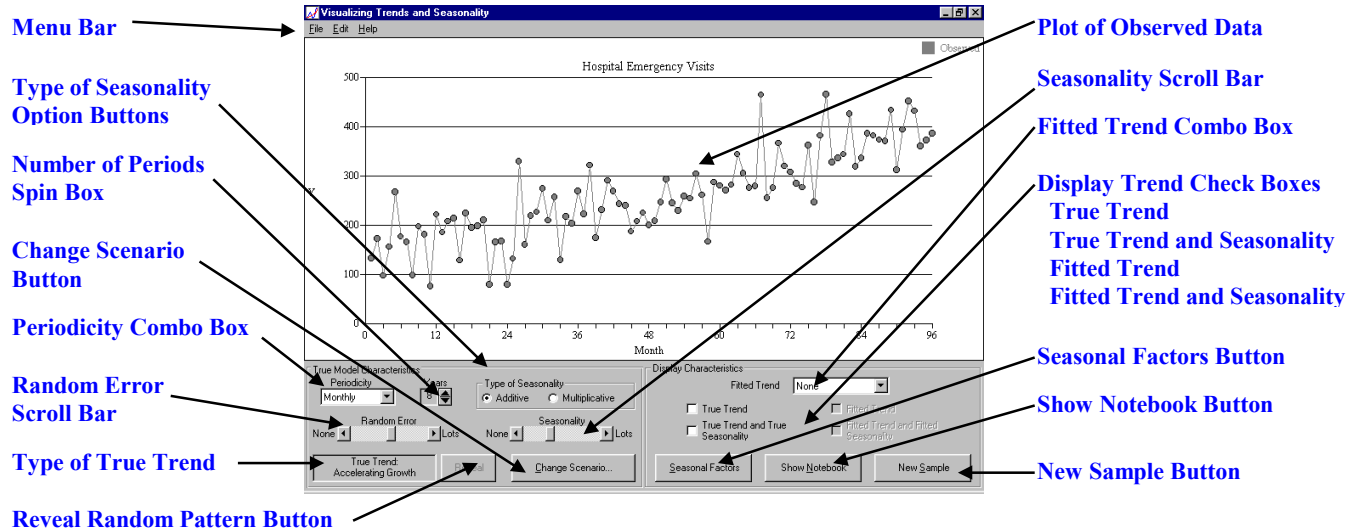
Time series **decomposition** separates these overlaid components and assesses their relative magnitude. Two general types of time series models are **additive models** and **multiplicative models**. The first step in estimating either of these models is to estimate the fitted trend models. The *linear* trend is the simplest (constant growth or decline with no turning points). Non-linear trends include the *quadratic* trend (growth or decline with one turning point), *cubic* trend (growth or decline with two turning points), or *exponential* trend (constant percent growth or decline with no turning points). Once we have a fitted trend, seasonal factors may be estimated in relation to the fitted trend. Estimates will be better if the researcher has more data to work with. Finally, the cyclical component could be estimated. The rest is error.

## Orientation to Basic Features

This module generates monthly, quarterly or yearly data that conforms to a model that you specify, consisting of trend, seasonality, and error. You can fit a linear, quadratic, cubic, or exponential model to the sample and estimate the seasonality. You can compare the estimated trend and seasonality to the true trend and seasonality.

### 1. Opening Screen

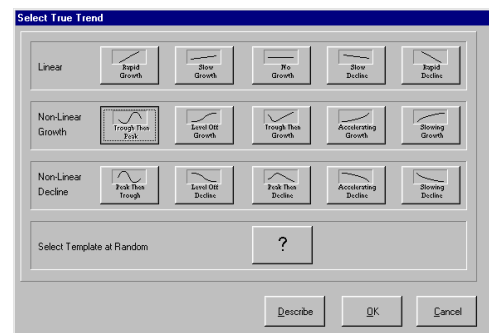
Start the module by clicking on the module's icon, title, or chapter number in the *Visual Statistics* menu and pressing the **Run Module** button. When the module is loaded, you will be on the introduction page of the Notebook. Read the questions and then click the **Concepts** tab to see the concepts that you will learn. Click on the **Scenarios** tab. Select **Monthly Data**, choose a scenario, and press **OK**. The top of the screen shows a graph of the sample data. The module's Control Panel appears on the bottom. A flashing **New Sample** button will indicate when you have changed one or more control settings.



### 2. Selecting a Trend

The true trend can be selected in two simple ways:

- You can use a scenario that you selected from the Notebook. To change this scenario click the **Change Scenario** button (lower left corner of display). Four situations (monthly, quarterly, and yearly data) are available.
- You can select from a menu of 15 different trends by clicking on the **Show Notebook** button, selecting the **Templates** tab, and pressing **OK**. A screen like the one to the right will appear. Any of these trends (or a random trend, indicated by the question mark button) can be selected. Click the **Describe** button to see an explanation of the selected trend. Click **OK** to use the trend selected, or **Cancel** to return to the Notebook. Once a template has been selected the **Change Scenario** button becomes a **Change Template** button, providing easy access to this template.



### 3. Seasonality and Random Error

When you return to the main screen, use the **Seasonality** and **Random Error** scroll bars to change the amount of seasonality and random error. Click **New Sample**. The graph shows a sample of data, based upon the true model, seasonality, and error you specified.

### 4. Fitted Trend

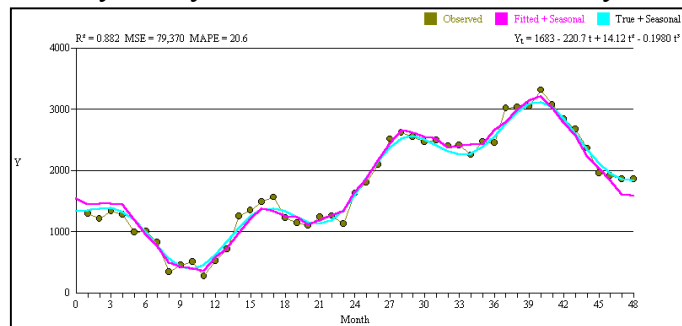
Click on the **Fitted Trend** combo box to select the type of trend you wish to fit (**None**, **Linear**, **Quadratic**, **Cubic**, **Exponential**). The equation of the fitted model is shown at the top of the graph, along with its fit statistics ( $R^2$ , MSE, and MAPE) and the **Fitted Trend** check box becomes active. Click it to display the fitted trend as a red line on the graph.

### 5. Fitted Trend and Seasonality

If you have chosen a trend from the **Fitted Trend** combo box, the **Fitted Trend and Seasonality** check box becomes active. Click it to see the fitted trend with seasonality as a magenta line on the graph. If seasonality is present, the magenta line should provide a better visual fit to your sample. There can be fitted seasonality even if you set the **Seasonality** scroll bar to None, because random error alone can erroneously suggest a seasonal pattern.

### 6. True Trend and True Seasonality

Deselect the **Fitted Trend and Seasonality** check box. Select the **True Trend** check box to see the true trend displayed as a blue line on the graph. Select the **True Trend and Seasonality** check box, to see the true trend with its seasonality as a cyan line. The effect of seasonality is easily seen. In real data, these two lines are unknown. Being able to visualize these lines is one of the advantages of a simulation. Deselect **True Trend**. Select **Fitted Trend and Seasonality** to see how well your fitted model reflects the components of the true model. This is illustrated to the right.



### 7. Copying a Display

Click on the display you wish to copy. Its window title will be highlighted. Select **Copy** from the **Edit** menu (on the menu bar at the top of the screen) or Ctrl-C to copy the display. It can then be pasted into other applications, such as Word or WordPerfect, so it can be printed.

### 8. Help

Click on **Help** on the menu bar at the top of the screen. **Search for Help** lets you search an index for this module, **Contents** shows a table of contents for this module, **Using Help** gives instructions on how to use Help, and **About** gives licensing and copyright information.

### 9. Exit

Close the module by selecting **Exit** in the **File** menu (or click  in the upper right-hand corner of the window). You will be returned to the **Visual Statistics** main menu.

## Orientation to Additional Features

### 1. Additive or Multiplicative Model

Two types of seasonality can be modeled (**Additive**, **Multiplicative**). Select the desired check box and click the **New Sample** button to update the display.

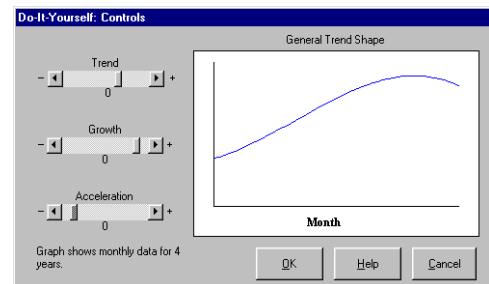
### 2. Periodicity and Number of Periods

Click on the **Periodicity** combo box to choose between yearly, quarterly, and monthly data. The **Number of Periods** spin button specifies the number of periods to sample.

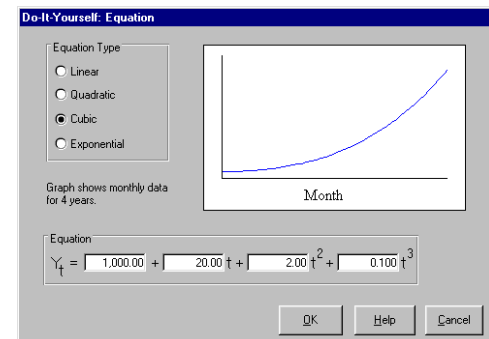
### 3. Do-It-Yourself Trend

There are two ways you can create your own trend:

- a. You can create your own trend by opening the Notebook, selecting the **Do-It-Yourself** tab, clicking on **Scroll Bar Controls**, and pressing **OK**. A screen like the one to the right will appear. The scroll bars control the trend, growth, and acceleration parameters. Your trend is shown on the accompanying graph. Click **Help** to get more information, **OK** to use the trend created, or **Cancel** to return to the Notebook. Once a trend has been created, the **Change Scenario** button becomes a **Change Trend** button, providing easy access to the scroll bar controls.

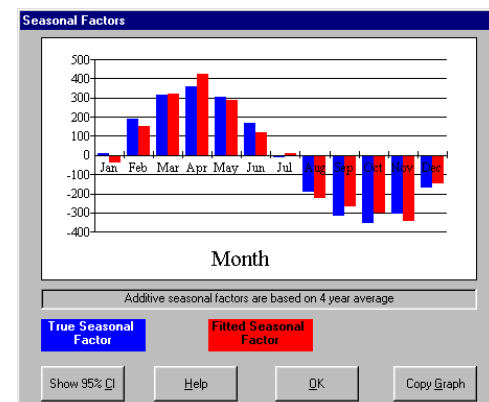


- b. You can create your own equation by opening the Notebook, selecting the **Do-It-Yourself** tab, clicking **Enter Equation Parameters**, and pressing **OK**. A screen like the one to the right will appear. Select an equation type and enter its parameter values. Your trend is shown on the small graph. Click **Help** to get more information, **OK** to use the trend created, or **Cancel** to return to the Notebook. Once you have created a trend, the **Change Scenario** button becomes a **Change Trend** button, providing easy access to the equation editor.



### 4. Seasonal Factors

Click on **Seasonal Factors** to reveal a bar graph of the true seasonal factors and fitted seasonal factors. An example is to the right. Click on **Show 95% CI** to display a confidence interval about the fitted seasonal factors. Press Ctrl-C to copy the seasonal factors graph to the clipboard.



## Basic Learning Exercises

Name \_\_\_\_\_

### Trend Component

Press the **Show Notebook** button, select the **Scenarios** tab, click on **Monthly Data**, and select **General Aviation Shipments**. Read the scenario and click **OK**.

1. Click on **Options** on the menu bar (top of screen) and select **Display True Trend**. Why would general aviation shipments have this type of trend (Peak then Trough)?

2. Under **Fitted Trend**, select **Cubic**. Use the check boxes to deselect **True Trend** and select **Fitted Trend**. Look at the fit statistics. Record the smallest and largest value of  $R^2$ , MSE, and MAPE from 10 different samples. If you extrapolated the fitted cubic trend for a few periods, would it give reasonable forecasts?

$R^2$  \_\_\_\_\_ MSE \_\_\_\_\_ MAPE \_\_\_\_\_

3. Use the scroll bar to increase **Random Error** to Lots (all the way to the right). Take 10 samples and again record the smallest and largest value of  $R^2$ , MSE, and MAPE. Note any irregularities in the behavior of these statistics.

$R^2$  \_\_\_\_\_ MSE \_\_\_\_\_ MAPE \_\_\_\_\_

4. When you increased the random error, what happened to the fit? Does a larger or smaller value for  $R^2$ , MSE, and MAPE signify a “better fit”?

5. Change **Fitted Trend** to **Quadratic**. Take 10 samples and record the smallest and largest value of  $R^2$ , MSE, and MAPE. What happened to the fit statistics, compared with the cubic trend you estimated in exercise 3? If you extrapolated the fitted quadratic trend for a few periods, would it give reasonable forecasts? Explain.

$R^2$  \_\_\_\_\_ MSE \_\_\_\_\_ MAPE \_\_\_\_\_

6. Click **Show Notebook**, select the **Templates** tab and click **OK**. Select the **Peak then Decline Template** (third row, middle button). Click the **Describe** button. Read the description, click **OK**, and click **OK** again. Set **Fitted Trend** to **Linear**. Take 10 samples and record the smallest and largest values of  $R^2$ , MSE, and MAPE. Would forecasts based on the linear model be too low, too high, or about right? Explain. Give an example of when this trend could occur.

$R^2$  \_\_\_\_\_ MSE \_\_\_\_\_ MAPE \_\_\_\_\_

7. Choose a quadratic fitted model. Take 10 samples and record the smallest and largest values of  $R^2$ , MSE, and MAPE. Then choose a cubic fitted model and do the same. Compared to the linear, do the quadratic and cubic models offer improved fit? All things considered, which of the three fitted trend models is preferable? Explain.

Quadratic:  $R^2$  \_\_\_\_\_ MSE \_\_\_\_\_ MAPE \_\_\_\_\_  
 Cubic:  $R^2$  \_\_\_\_\_ MSE \_\_\_\_\_ MAPE \_\_\_\_\_

8. Explain why the cubic model cannot give a worse fit than a linear or quadratic model. Since this is so, why wouldn't a cubic trend always be preferable? Explain.

9. Change **Fitted Trend** to **Quadratic**. Increase the number of years to 8. Take 10 samples and record the smallest and largest value of  $R^2$ , MSE, and MAPE. Then reduce the number of years to 3 and do the same. How does a smaller sample size affect the fit statistics?

8 years:  $R^2$  \_\_\_\_\_ MSE \_\_\_\_\_ MAPE \_\_\_\_\_  
 3 years:  $R^2$  \_\_\_\_\_ MSE \_\_\_\_\_ MAPE \_\_\_\_\_

10. In general, are larger samples preferred? If you had data over 8 years, but during the first four years the country was at war, would using all 8 years of data be a good idea? Explain.

## Intermediate Learning Exercises

Name \_\_\_\_\_

### Seasonal Component

Press the **Change Scenario** button, select **Hospital Emergency Visits**, read the scenario, and click **OK**. Set **Seasonality** to Lots and **Random Error** to None. Select **Display True Trend**.

11. What is the shape of the true trend? Use the combo box to select a fitted trend. Then use the check boxes to select both **Fitted Trend** and **True Trend** so that both are displayed (true trend in blue, fitted trend in red). Which fitted trend works the best? The worst? Explain.
12. Choose an exponential fitted trend. Uncheck both **Fitted Trend** and **True Trend** (to reduce screen clutter). Then check both **True Trend and True Seasonality** and **Fitted Trend and Fitted Seasonality**. Notice that the estimated model (magenta line) and true model (cyan line) are almost identical. Click the **Seasonal Factors** button. Click **Show 95% CI**. Why is seasonality so accurately estimated? Is this situation realistic?
13. Increase the error on the **Random Error** scroll bar by eight clicks (the middle of the scroll bar). Take a new sample and compare the true model (cyan line) and estimated model (magenta line). Click **Seasonal Factors**. Click **Show 95% CI**. Is this what you expected? How many of the twelve confidence intervals contain 0? What does this mean?
14. Increase the error to its maximum on the **Random Error** scroll bar. Take a new sample and visually compare the true model (cyan line) and estimated model (magenta line). Click **Seasonal Factors**. Click **Show 95% CI**. a) What do you notice about the length of the confidence intervals? b) How many of the 12 intervals now contain 0? c) Interpret this result. Why do more of the intervals contain 0?
15. What role does random error play in estimating seasonality?

**Periodicity**

Click **Change Scenario** and click on **Next page** in the lower right corner of the current page to display scenarios using quarterly data. Click on **Maternal Hospital Stay**. Read the scenario and click **OK**. Select a cubic fitted trend. Select **True Trend and True Seasonality** and **Fitted Trend and Fitted Seasonality**. Change the number of years to 5.

16. Set the **Seasonality** scroll bar to Lots and click **New Sample**. Why is the fitted model (magenta line) jagged? Does it resemble the true model (cyan)?
17. Change the amount of random error to Lots. Take a new sample. Compare the fitted model (magenta line) with the true model (cyan). Click on **Seasonal Factors**. Compare the red bars and the blue bars. Click **Show 95% CI**. Do this experiment two or three times. Evaluate the quality of your seasonal estimates.
18. Triple the number of years to 15 (you now have 60 quarterly observations) and repeat the process in exercise 17. Evaluate the quality of your seasonal model and the estimated seasonal factors and their confidence intervals relative to those with five years of data.
19. Return the number of years to 5 but change the periodicity to monthly (you now have 60 monthly observations). Repeat the process in exercises 16 and 17. Evaluate the quality of your seasonal estimates and confidence intervals relative to those in exercises 16 and 17.
20. Why do you think that using 60 quarterly estimates gives more accurate estimates of seasonality than does using 60 monthly observations?

## Advanced Learning Exercises

Name \_\_\_\_\_

### Exploring Multiplicative Time Series Models

Click **Show Notebook**, select the **Scenarios** tab, click on **Monthly Data**, and select **Residential Security Systems**. Read the scenario and click **OK**. Select a **Cubic** trend. Use the check boxes to display the **True Trend and True Seasonality** and **Fitted Trend and Fitted Seasonality**.

21. Set **Random Error** to None. Press the **New Sample** button. How does the fitted model (magenta) compare with the true model (cyan)? What do you notice about the magnitude of the seasonality of the true time series displayed on the graph? Why?
22. Press the **Change Scenario** button. Select the **General Aviation Shipments** scenario. Read the scenario and click **OK**. Set **Random Error** to None and set **Seasonality** to the midpoint of its scroll bar. Select a **Cubic** trend. Press the **New Sample** button. What do you notice about the magnitude of the seasonality?
23. Press the **Show Notebook** button. Click on the **Templates** tab and click **OK**. Select the **No Growth** template (3rd button, 1st row) and click **OK**. Select a **Linear** trend. What do you notice about seasonality?
24. Interpret what multiplicative seasonality means. Give an example of this type of seasonality. Do you think this type of seasonality is rare or common?
25. Press the **Show Notebook** button, select the **Scenarios** tab, click on **Monthly Data**, select the **General Aviation Shipments** scenario, and click **OK**. Set **Random Error** to Lots, set **Seasonality** to the midpoint of its scroll bar, and set **Fitted Trend** to **Cubic**. Press the **New Sample** button. Press the **Seasonal Factors** button and click **Show 95% CI**. Is multiplicative seasonality more easily estimated than additive seasonality (look at exercise 13 above)? Why or why not?

**Exploring the Do-It-Yourself Methods of Creating a Time Series**

26. Set **Periodicity** to **Annual**, choose 15 years of data, press **Show Notebook**, select the **Do-It-Yourself** tab, click on **Scroll Bar Controls**, and click **OK**. Play with each control one at a time, setting it back to zero before going on to the next control. Explain what each control does.
27. How would you get a U-shaped curve? An upside down U-shaped curve? A curve with a peak, followed by a trough?
28. Press **Show Notebook**, click on **Next page** to bring up the **Enter Equation Parameters** option. Click **OK**. Select **Exponential**. Try various values for the second term between 0.7 and 1.3. Describe what happens to the function.
29. Recall that in exercise 26 you set the **Periodicity** to **Annual**. What does a coefficient of 1.05 for the second term mean? If this were quarterly or monthly data, what would it mean? Is this a likely growth rate for someone's annual salary? Someone's monthly salary?
30. Select **Cubic Equation**. How would you get a U-shaped curve? An upside-down U-shaped curve? A curve with a peak followed by a trough?
31. Under what circumstances would you prefer to use the Do-It-Yourself methods to create an equation rather than the buttons on the template pallet?

## Individual Learning Projects

Write a report on one of the three topics listed below. Use the cut-and-paste facilities of the module to place the appropriate graphs in your report. For each project, generate data by using the **Scenario**, **Template** or **Do-It-Yourself** tabs. If the data comes from a scenario, discuss the reason for its apparent trend, type of seasonality (multiplicative or additive), amount of seasonality, and amount of random error. Otherwise, give an example of the type of situation that would have the trend, type of seasonality (multiplicative or additive), amount of seasonality, and amount of random error you selected.

1. The amount of seasonality or random error in a data set affects the reliability of estimation. However, the two components have very different meanings. Use monthly data. Estimate the model with different amounts of random error and seasonality. Explain and illustrate the role each factor plays in reliably estimating the data's trend and seasonality.
2. Fit statistics show how well a fitted trend explains the data. Evaluate their effectiveness in leading a statistician to use the best fitted trend. Choose the **Templates** tab in the Notebook, and click the **Random** button. Generate 60 observations of data using an additive model with no seasonality and lots of random error. Fit three different trends to the data before you click **Reveal Random Trend**. How effective were the fit statistics in guiding you to the correct fitted trend? Repeat this process two more times.
3. Larger sample sizes are generally preferred. Suppose you have five years of quarterly sales data and are given ten more. Alternately, imagine that you replace five years of quarterly sales data with monthly sales data covering the same five years. Either would result in a threefold increase in your sample size. Explain and illustrate the different issues you would encounter in estimating trend and seasonality with these two larger data sets.

## Team Learning Projects

Select one of the three projects listed below. In each case, produce a team project that is suitable for an oral presentation. Use presentation software or large poster boards to display your results. Graphs should be large enough for your audience to see. Each team member should be responsible for producing some of the graphs. Include in your report a copy of all graphs and statistics that you evaluated. Ask your instructor if a written report is also expected. If the data comes from a scenario, discuss the reason for its apparent trend, type of seasonality (multiplicative or additive), amount of seasonality and amount of random error. Otherwise, give an example of the type of situation that would have the trend, type of seasonality (multiplicative or additive), amount of seasonality, and amount of random error you selected.

1. This is a project for a team of three or more. Investigate the role sample size plays in estimating models with different amounts of error. Generate data by using the **Template** or **Scenario** tab. The team should agree on an amount of seasonality, the true trend, the periodicity, three sample sizes (number of years), and three fitted trends (one should be the correct trend). Each team member should select a different amount of error (the team should cover the entire error range). Using that error, each team member should take five samples for each sample size and fitted trend combination. For each sample record the estimated model, its fit statistics, and its fitted seasonal factors. After you have finished estimation, analyze the statistics you recorded. Illustrate and explain how sample size aids in estimating the correct trend and seasonality.
2. This is a project for a team of three or more. Investigate the role sample size plays in estimating trend models with different amounts of seasonality. Generate data by using the **Template** or **Scenario** tab. The team should agree on an amount of error, the true trend, the periodicity, three sample sizes (number of years), and three fitted trends (one should be the correct trend). Each team member should select a different amount of seasonality (the team should cover the entire seasonality range). Using that seasonality, each team member should take five samples for each sample size and fitted trend combination. For each sample record the estimated model, its fit statistics, and its fitted seasonal factors. After you have finished estimation, analyze the statistics you recorded. Illustrate and explain how sample size aids in estimating the correct trend and seasonality.
3. This is a project for a team of three. Each team member should use the **Do-It-Yourself Equation** method to generate a realistic time series exhibiting appropriate amounts of seasonality and random error. The team should interact in planning each member's contribution. The first team member should create a time series representing monthly measurements of ozone concentrations in a city over a 10-year period during which successful government efforts are being made to reduce this pollutant (use 280 parts per billion as the starting point). The second team member should create a time series representing monthly earnings by a free-lance computer systems consultant during the first 60 months of acting as a solo consultant. The third team member should create a time series representing quarterly sales of a new product over a 5-year life cycle, during which sales rapidly increase, and then drop off sharply (use any starting point that works). In each case, explain how the levels of seasonality and randomness were chosen, and determine whether the seasonal factors can be estimated accurately from the generated data.

## Self-Evaluation Quiz

1. Which is *not* a component of a time series?
  - a. Trend.
  - b. Error.
  - c. Cycle.
  - d. Amplitude.
  - e. Seasonality.
2. Cycles often are ignored in time series decomposition because
  - a. there is no agreed-upon theory of cycles.
  - b. their periodicity is irregular.
  - c. their interpretation is not primarily a statistical question.
  - d. their underlying causes are varied.
  - e. all of the above.
3. Which statements are *not* correct concerning fitted trend?
  - a. The quadratic model allows a single peak or trough.
  - b. The linear model allows no peak or trough.
  - c. The cubic model allows three peaks or troughs.
  - d. The exponential model allows no peak or trough.
  - e. The cubic model is particularly dangerous if extrapolated.
4. Which model allows a constant percent rate of growth?
  - a. Linear.
  - b. Quadratic.
  - c. Cubic.
  - d. Exponential.
  - e. None of the above.
5. Which statistic(s) of fit will be unit-free?
  - a. MAPE.
  - b. MSE.
  - c. R-squared.
  - d. Only a and c.
  - e. Only a and b.
6. If a cubic trend is fitted instead of a quadratic trend
  - a. the R-squared cannot fall.
  - b. the R-squared cannot rise.
  - c. the R-squared can be higher or lower.
  - d. the R-squared is irrelevant.
  - e. none of the above is correct.

7. If the true trend is cubic and we fit a quadratic model
  - a. the MAPE will be too low.
  - b. the seasonal factors may be poorly estimated.
  - c. the R-squared will be low.
  - d. all of the above.
  - e. both b and c.
8. If we fit a linear trend to data that are growing exponentially
  - a. the fitted trend will be too low at the beginning and end.
  - b. the fitted trend will be too high in the middle.
  - c. the forecasts (if extrapolated) will be too low.
  - d. the fit could probably be improved by fitting a quadratic model.
  - e. all of the above.
9. Seasonal factors
  - a. always form a smooth, sinusoidal monthly cycle.
  - b. are irrelevant for annual data.
  - c. are calculated before the trend is estimated.
  - d. are ratios between 0 and 1 in the additive model.
  - e. are irrelevant for most business data.
10. Which is *not* true of confidence intervals for seasonal factors?
  - a. They are wider if there is little seasonality.
  - b. They are wider if the number of observed years is small.
  - c. They are wider if the random error is high.
  - d. They are wider if an inappropriate trend model is fitted.
  - e. They are wider if outliers are present.
11. If random error is large but the correct functional form is specified, which is *most likely* to be estimated well in monthly data?
  - a. Mean seasonal factors.
  - b. Fitted trend.
  - c. Estimated cycles.
  - d. Confidence intervals for seasonality.
  - e. Forecasts for the next period.
12. Which statement is *not* correct in comparing additive with multiplicative models?
  - a. The additive model may be too simplistic for financial data that reflect inflation.
  - b. The multiplicative model may be harder to explain to the average manager.
  - c. Relative growth is best handled by a multiplicative model.
  - d. Additive models may fail when long time periods are covered.
  - e. Monthly data works best with additive models due to seasonality.

## Glossary of Terms

**Additive model** Model of time series decomposition of the form  $Y = T + C + S + E$  where  $Y$  is the time series,  $T$  is the trend,  $C$  is the cycle,  $S$  is the seasonal, and  $E$  is the error. Generally, the linear model is appropriate for a series with little trend or over short periods of time. Since financial data usually has trend (e.g., due to inflation) the additive model is used less often.

**Cubic trend** Polynomial trend model of the form  $Y = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$  which is used to fit a time series trend with two turning points. The disadvantage of a cubic trend (or any higher-order polynomial) is that they are harder to interpret or explain, and may give strange forecasts when extrapolated more than a few periods.

**Cycle** Component of time series data that represents repetitive movement about a trend over several years. Cycles are difficult to analyze unless the theoretical basis for their existence can be described in a formal model (e.g., a capital goods replacement cycle based on life of the good). For this reason, they are not considered in this module.

**Decomposition** Attempt to divide a time series into four components: trend (long-term tendency), cycle (fluctuation over several years), seasonal (fluctuation within a year), and error (random “noise” that is not systematic). See **Additive model** and **Multiplicative model**.

**Error** Irregular component of a time series which follows no detectable pattern. The error may be assumed additive or multiplicative.

**Exponential Trend** Trend model of the form  $Y = \beta_0 t^{\beta_1}$  which is used to fit a non-linear time series trend with constant percent growth ( $\beta_1 > 1$ ) or decline ( $\beta_1 < 1$ ) each period. It is an attractive choice for modeling growth in economic data or biological populations. It is considered intrinsically linear because its logarithmic form is a straight line.

**Fit statistics** Collective name given to certain statistics such as  $R^2$ , MAPE, and MSE. These statistics measure the correspondence between observed data and a proposed model.

**Fitted trend** Mathematical equation based on historical data for a time series that attempts to define the long-term pattern of change (e.g., linear, quadratic, cubic, exponential).

**Linear trend** Trend model of the form  $Y = \beta_0 + \beta_1 t$  which is used to fit a time series trend with no turning points (constant change each period). It is the most common trend model because it is simple and easy to explain.

**MAPE** Mean absolute percent error of a fitted time series model. Because it is a percent, it is unit-free, allowing us to compare MAPE between time series variables that are measured in different units (e.g., dollars and yen). MAPE is easy to understand, but is sensitive to division by small data values, and cannot be used with negative data. Other things equal, smaller MAPE signifies a better fit.

**MSE** Mean squared error, obtained by summing the squared residuals from a fitted time series model and dividing by the number of time periods. It is not unit-free, so it cannot be used to compare time series variables that are measured in different units (e.g., dollars and yen). Its meaning is unintuitive, but it works with any kind of data and has useful mathematical properties. Other things equal, smaller MSE signifies a better fit. The MSE is sometimes called MSD (mean squared deviation).

**Multiplicative model** Model of time series decomposition of the form  $Y = T \times C \times S \times E$  where  $Y$  is the time series,  $T$  is the trend,  $C$  is the cycle,  $S$  is the seasonal, and  $E$  is the error. Generally, this model is preferred for strongly trended data observed over many time periods, because it expressed variation in *relative* terms. It is useful, for example, for financial data whose growth over time reflects inflation (e.g., sales).

**Periodicity** Number of subperiods over which a regular cycle or seasonal pattern is observed (e.g., months or quarters). If the data are annual, no seasonality can be observed.

**Quadratic trend** Polynomial trend model of the form  $Y = \beta_0 + \beta_1 t + \beta_2 t^2$  which is used to fit a time series trend with one turning point. It is harder to explain and interpret than a linear model. However, it may produce useful forecasts in the very short run.

**R-squared** Ratio of the fitted model's sum of squares to the total sum of squares. It is unit-free, so it can be used to compare variables that are measured in different units (e.g., dollars and yen).  $R^2$  near 0 indicates the fit is poor, while  $R^2$  near 1 indicates a good fit.

**Seasonal** Repetitive cyclical movement about a trend within a calendar year (e.g., over 12 months or 4 quarters). Also, a reference to the seasonal component of time series data. The concept can be generalized to include other kinds of sub-period variation, such as weekly, daily, hourly, and so on. However, this module considers only monthly and quarterly seasonality.

**Time series** Any observed data that is recorded over time.

**Trend** General pattern over many periods. Generally considered the most important component of time series data. Removal of trend is the customary starting point for the statistical task of time series decomposition.

## Solutions to Self-Evaluation Quiz

1. d Read the Overview of Concepts. Consult the Glossary.
2. e Consult the Glossary. Read the Overview of Concepts.
3. c Do Exercises 1–10. Read the Overview of Concepts.
4. d Read the Overview of Concepts. Consult the Glossary.
5. d Consult the Glossary.
6. a Do Exercises 7–8. Consult the Glossary. Read the Overview of Concepts.
7. e Do Exercises 1–5. Read the Overview of Concepts.
8. e Do Exercise 11.
9. b Do Exercises 11–15. Read the Overview of Concepts. Consult the Glossary.
10. a Do Exercises 11–20. Consult the Glossary.
11. b Do Exercises 1–3 and 11–15. Read the Overview of Concepts.
12. e Do Exercises 21–25. Read the Overview of Concepts. Consult the Glossary.