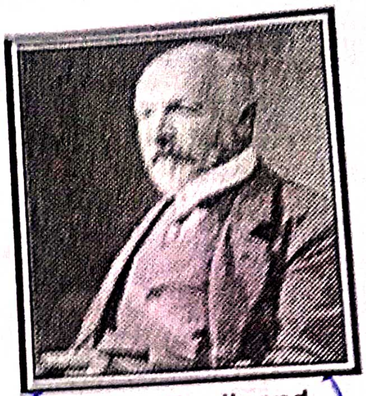
Chap[illegible] 1 Set Theory

1.1 Introduction



Georg Ferdinand Ludwig Phillipp Cantor

The concept of set is basic in all branches of mathematics. The study of sets has many applications in logic, philosophy etc. It is used in the foundation of relations and functions, sequences, geometry, probability theory etc.

The theory of sets was developed by German mathematician Georg Ferdinand Ludwig Philipp Cantor (1845-1918).

In this chapter, we will discuss some basic definitions of set and operations on it.

1.2 Set and Representation of a set

Set is undefined term. Often we speak a collection of a particular kind such as a crowd of people, cricket team, rivers of India etc. In mathematics also, we come across various collections for example, collection of natural numbers, prime numbers.

We note that each of the above collection is a well defined collection of objects in the sense that we can definitely decide whether a given object belongs to a given collection or not.

We say that a set is a well defined collection of objects.

Objects we mean members or elements of the set.

Sets are usually denoted by capital letters A, B, C, X, Y, Z, ...

The elements of a set are represented by small letters a, b, c, x, y, z, ...

If 'x' is an element of a set A, we say that 'x' belongs to A. The Greek symobl $\in$ is used to denote 'belongs to'. Thus, we write $x \in A$.

If 'y' is not an element of a set A, we say that 'y' does not belong to A and we write $y \notin A$.

For example; in the set P of prime numbers. $3 \in P$ but $4 \notin P$.

Methods of representing a set

(1) Listing (Roster) method

(2) Property (Set-builder) method

(1) Listing Method.

In this method, all the elements of a set are listed, the elements being seperated by commas and are enclosed within braces { }.

For example,

The set of all even positive integers less than 10 is described in listing method as {2, 4, 6, 8}.

Note that, the order in which the elements are listed is immaterial, the set {1, 2, 3, 6, 7}can be represented as {1, 3, 6, 2, 7}.

It may be noted that while writing the set in listing method an element is not generally repeated, i.e. all the elements are taken as distinct.

For example, the set of letters forming the word 'SCHOOL' is {S, C, H, O, L}.

(2) Property Method

In this method, a set is expressed with the help of common property of its elements.

For example,

(a) In the set {a, e, i, o, u}, all the elements posses a common property, each of them is a vowel in the English alphabet. We write {x | x is a Vowel in the English alphabet}.

(b) Set {1, 3, 5, 7, ...} can be written as {x | x is an odd natural number}.

EXAMPLE 1 The set of positive integers less than 100 can be denoted by {1, 2, 3, ..., 99}.

EXAMPLE 2 Write the set {2, 4, 6, 8, 10, 12, 14, 16, 18} in the property form.

SOLUTION Each member of the set is positive even integers less than 20. Hence, the set is {x | x is even positive integer; $x < 20$}.

EXAMPLE 3 Write the set $\left\{\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}\right\}$ in the property form.

SOLUTION Given set is $\left\{\frac{n+1}{n} \mid n \text{ is a natural number and } 1 \leq n \leq 4\right\}$.

EXAMPLE 4 Write the set of letters of the word 'SMALL'.

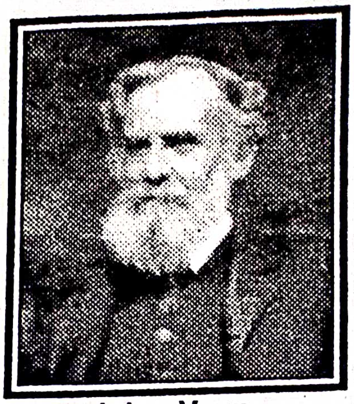
SOLUTION The set is {S, M, A, L}.

EXAMPLE 5 Write the set {x | x is a positive integer and $x^2 < 30$} using listing method.

SOLUTION The required numbers are 1, 2, 3, 4, 5.

The set is {1, 2, 3, 4, 5}.

Venn – Diagrams



John Venn

Relationship between sets can be represented by means of diagrams, are called Venn – diagrams named after the English mathematician John Venn (1834–1923).

In Venn diagrams the universal set U is represented by a rectangle. Inside this rectangle, circles or other geometrical figures are used to represent sets.

For example,

Set U = {1, 2, 3, ..., 10}, A = {4, 6, 8, 10} and B = {6, 8}

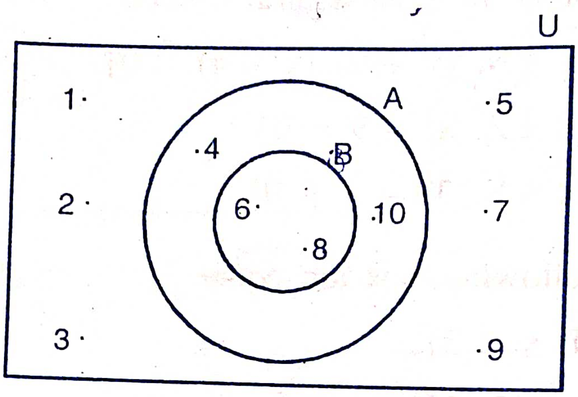


Fig. 1.1 Venn diagram for the set A and B

The extensive use of the Venn – diagrams will be discussed in the operations on sets.

EXERCISES 1.1

1. Which of the following statements are true ? Justify your answer.
   - (i) The sets A = {a} and B = {{a}} are equal.
   - (ii) The sets A = {1, 3, 3, 3, 5, 5} and B = {1, 3, 5} are equal.
   - (iii) $\{a\} \subset \{a\}$
   - (iv) $\{a\} \in \{a\}$
   - (v) $\varnothing \subset \{a\}$
   - (vi) $\varnothing \in \{a\}$
   - (vii) $\{3, 4\} \subset \{1, 2, 3, \{3, 4\}, 5\}$
   - (viii) $\{3, 4\} \in \{1, 2, \{3, 4\}, 5\}$
   - (ix) $\{a, b\} \not\subset \{b, c, a\}$
   - (x) $\{1, 2, 3\} \subset \{1, 2, 3\}$
2. List the members of the following sets.
   - (i) A = {x | x is a real number such that $x^2 = 4$}

$B = \{x \mid x$ is a prime number less than $20\}$

(ii) $B = \{x \mid x$ is a square of a positive integer and $x < 50\}$

(iii) $C = \{x \mid x$ is a letter is the word 'India'$\}$

(iv) $D = \{x \mid x$ is a divisior of $20\}$

(v) $E = \{x \mid x$ is a multiple of $4\}$

(vi) $F = \{x \mid x$ is a multiple of $4\}$

3. State which of the following sets are finite and which are infinite.

(i) $A = \{x \mid x$ is a prime$\}$

(ii) $B = \{x \mid x$ is an even natural number$\}$

(iii) $C = \{x \mid x \in N, (x + 2)(x - 4) = 0\}$

(iv) $D = \{x \mid x \in Z, x^2 - 9 = 0\}$

(v) $E = \{x \mid x \in N, 3x - 1 = 0\}$

4. Which of the following sets are equal ?

$A = \{x \mid x \in N, x < 3\}$,

$B = \{1, 2\}$, $C = \{3, 1\}$

$D = \{x \mid x \in N, x$ is odd, $x < 5\}$

$E = \{1, 2, 1\}$

5. Can you say that $A = B$, if A and B have same number of the elements ?

6. Find two sets X and Y such that $X \in Y$ and $X \subseteq Y$.

7. Find the power set of each of the following sets. Also write no. of elements in each of these power set.

(i) $\{a\}$ (ii) $\{\varnothing\}$

(iii) $\{a, b\}$ (iv) $\{a, \{a\}\}$

(v) $\{\varnothing, \{\varnothing\}\}$ (vi) $\{1, 2, 3\}$

8. If $A = \varnothing$, the find the number of elements of $P(P(P(A)))$.

9. Write down all possible proper subsets of each of the following sets.

(i) $\{1, 2\}$ (ii) $\{a, b, c\}$ (iii) $\{1\}$

10. What is the total number of proper subsets of a set consisting of

(i) 2 (ii) 3 (iii) n-elements ?

1.3 Operations on Sets

Introduction

Two sets can be combined in many different ways. We shall now define certain operations on sets and examine their properties. Hence forth, we shall refer all our sets as subsets of some universal set.

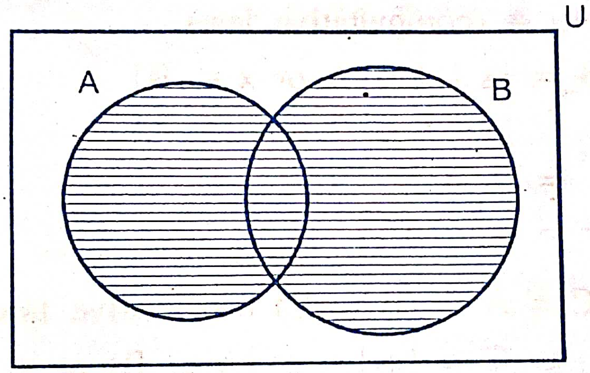
Definition 1

Let A and B be sets. The union of sets A and B is the set that contains those elements that are either in A or in B or in both.

The symbol '$\cup$' is used to denote the union.

Symbolically, we write $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

The union of two sets can be represented by a Venn–diagram as shown in the Fig. 1.2.



$A \cup B$ is shaded

Fig. 1.2

EXAMPLE 1 The union of $\{1, 3, 5\}$ and $\{1, 2, 3\}$ is the set $\{1, 2, 3, 5\}$.

EXAMPLE 2 Let $A = \{2, 4, 6, 8\}$
$B = \{6, 8, 10, 12, 14\}$. Find $A \cup B$.

SOLUTION We have, $A \cup B = \{2, 4, 6, 8, 10, 12, 14\}$

EXAMPLE 3 Let $A = N$, $B = Z$. Find $A \cup B$.

SOLUTION We have,
$$\begin{aligned} A \cup B &= N \cup Z \\ &= \{1, 2, 3, \ldots\} \cup \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\} \\ &= \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\} \\ &= Z \end{aligned}$$

Properties of the union

Let A, B and C be any three sets and U be the universal set.

(i) $A \subset (A \cup B)$, $B \subset (A \cup B)$ are obvious.

(ii) $A \cup A = A$ (idempotent law)

Proof : $A \cup A = \{x \mid x \in A \text{ or } x \in A\}$

$= \{x \mid x \in A\}$

$= A$

$\therefore A \cup A = A$

(iii) If $A \subset B$, $C \subset D$ then $(A \cup C) \subset (B \cup D)$

Proof : Let $x \in (A \cup C)$

$\Rightarrow x \in A$ or $x \in C$

$\Rightarrow x \in B$ or $x \in D$ $(\because A \subset B, C \subset D)$

$\Rightarrow x \in (B \cup D)$

Thus, $\forall x;\ x \in (A \cup C) \Rightarrow x \in (B \cup D)$

$\therefore (A \cup C) \subset (B \cup D)$ is proved.

(iv) $A \cup B = B \cup A$ (commutative law)

Proof : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$= \{x \mid x \in B \text{ or } x \in A\}$

$= B \cup A$

$\therefore A \cup B = B \cup A$

(v) $(A \cup B) \cup C = A \cup (B \cup C)$ (associative law)

Proof : $(A \cup B) \cup C = \{x \mid x \in (A \cup B) \text{ or } x \in C\}$

$= \{x \mid (x \in A \text{ or } x \in B) \text{ or } x \in C\}$

$= \{x \mid x \in A \text{ or } (x \in B \text{ or } x \in C)\}$

$= \{x \mid x \in A \text{ or } x \in (B \cup C)\}$

$= A \cup (B \cup C)$

$\therefore (A \cup B) \cup C = A \cup (B \cup C)$

(vi) $A \cup \varnothing = A$ is obvious

(vii) $A \cup U = U$

Proof : $A \cup U = \{x \mid x \in A \text{ or } x \in U\}$

$= \{x \mid x \in U\}$

$= U$

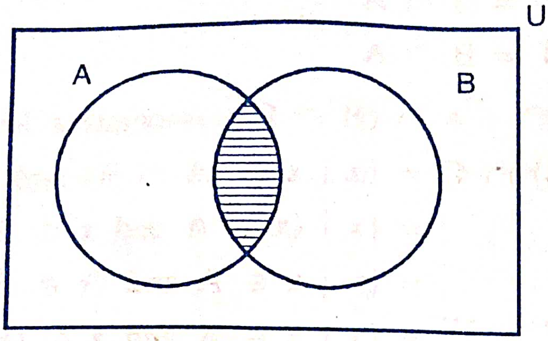
$\therefore A \cup U = U$ is proved

Definition 2

Let A and B be sets. The intersection of sets A and B denoted by $A \cap B$, is defined as the set containing those elements which are in both A and B.

Symoblically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$are read as 'A intersection B'.

The intersection of two sets can be represented by a Venn–diagram as shown in the Fig. 1.3.



$A \cap B$ is shaded

Fig. 1.3

EXAMPLE 4 Let $A = \{2, 4, 6, 8\}$, $B = \{2, 4, 10, 12\}$

then $A \cap B = \{2, 4\}$.

EXAMPLE 5 Let X = {Ram, Shyam, Akbar} and

Y = {Shyam, David, Ashok}

then $X \cap Y$ = {Shyam}

EXAMPLE 6 Let $A = \{1, 2, 3\}$; $B = \{6, 8, 10\}$ then $A \cap B = \varnothing$.

Properties of intersection

Let A, B and C be any three sets and U be the universal set.

(i) $(A \cap B) \subset A$, $(A \cap B) \subset B$ are obvious

(ii) $A \cap A = A$ (idempotent law)

Proof : $A \cap A = \{x \mid x \in A \text{ and } x \in A\}$

$= \{x \mid x \in A\}$

$= A$

$\therefore A \cap A = A$

(iii) If $A \subset B$, $C \subset D$ then $(A \cap C) \subset (B \cap D)$

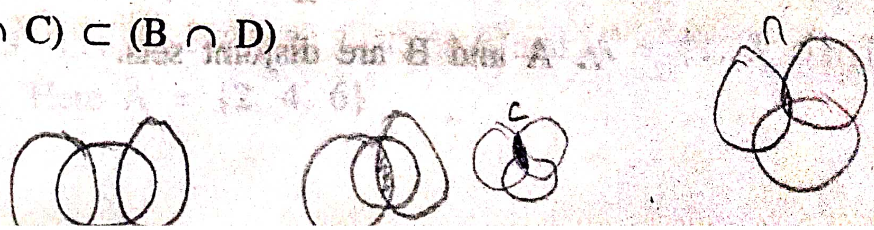
Proof : First, suppose that $x \in (A \cap C)$

$\Rightarrow x \in A$ and $x \in C$

$\Rightarrow x \in B$ and $x \in D$ $\quad (\because A \subset B \text{ and } C \subset D)$

$\Rightarrow x \in (B \cap D)$

$\therefore (A \cap C) \subset (B \cap D)$



(iv) $A \cap B = B \cap A$ (commutative law)

Proof : $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$= \{x \mid x \in B \text{ and } x \in A\}$

$= B \cap A$

$\therefore A \cap B = B \cap A$

(v) $(A \cap B) \cap C = A \cap (B \cap C)$ (associative law)

Proof : $(A \cap B) \cap C = \{x \mid x \in (A \cap B) \text{ and } x \in C\}$

$= \{x \mid (x \in A \text{ and } x \in B) \text{ and } x \in C\}$

$= \{x \mid x \in A \text{ and } (x \in B \text{ and } x \in C)\}$

$= \{x \mid x \in A \text{ and } x \in (B \cap C)\}$

$= A \cap (B \cap C)$

(vi) $A \cap \varnothing = \varnothing$ is obvious

(vii) $A \cap U = A$

Proof : $A \cap U = \{x \mid x \in A \text{ and } x \in U\}$

$= \{x \mid x \in A\}$

$= A$

$\therefore A \cap U = A$ is proved.

Definition 3

Two sets A and B are said to be disjoint, if $A \cap B = \varnothing$.
Venn–diagram of disjoint sets is represented by the Fig. 1.4

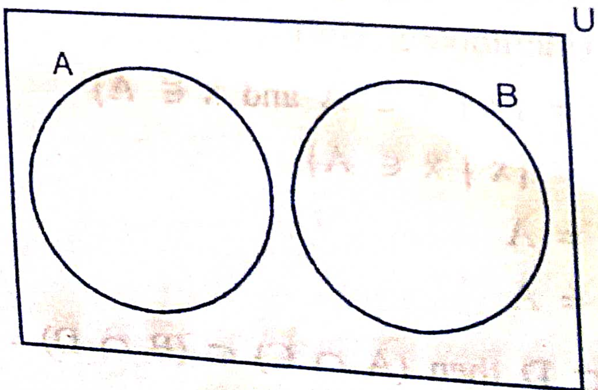


Fig. 1.4

EXAMPLE 7 Let $A = \{-4, 4, 0\}$ and $B = \{x \in Z \mid x^2 = 1\}$
then $A \cap B = \{-4, 4, 0\} \cap \{-1, 1\}$

$= \varnothing$

$\therefore$ A and B are disjoint sets.

EXAMPLE 8 Let $A = \{x \mid x \in Z^+\}$

$B = \{x : x \text{ is a multiple of } 2,\ x \in Z\}$

$C = \{x : x \text{ is a negative integer}\}$

$D = \{x : x \text{ is an odd integer}\}$

Find (i) $A \cap B$ (ii) $A \cap C$ (iii) $A \cap D$

(iv) $B \cap C$ (v) $B \cap D$ (vi) $C \cap D$

SOLUTION Here $A = \{1, 2, 3, 4, ...\}$

$B = \{..., -4, -2, 0, 2, 4, 6, 8, ...\}$

$C = \{-1, -2, -3, ...\}$

And $D = \{..., -5, -3, -1, 1, 3, 5, 7, ...\}$

(i) $A \cap B = \{2, 4, 6, 8, ...\}$

$= \{2n \mid n \in Z\}$

(ii) $A \cap C = \{\ \} = \varnothing$

(iii) $A \cap D = \{1, 3, 5, 7, ...\}$

(iv) $B \cap C = \{-2, -4, -6, -8, ...\}$

(v) $B \cap D = \varnothing$

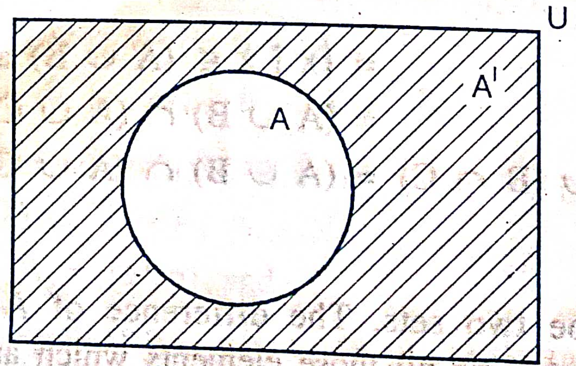
(vi) $C \cap D = \{-1, -3, -5, ...\}$

Definition 4

Let U be the universal set. The complement of a set A, denoted by A' is defined as all those elements which are in U but not in A.

Thus, $A' = \{x \mid x \in U \text{ and } x \notin A\}$.

It can be represented by Venn–diagram as in Fig. 1.5.



Shaded portions is A'

Fig. 1.5

EXAMPLE 9 Let $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{1, 3, 5\}$. Find A'.

SOLUTION Here $A' = \{2, 4, 6\}$

EXAMPLE 10 Let $U = Z$ and $A = N$ then $A' = \{0, -1, -2, -3, \ldots\}$

Properties of complement of a set

For any set A and U be the universal set.

We have,

(i) $A \cap A' = \varnothing$ and $A \cup A' = U$ are obvious

(ii) $\varnothing' = U$, $U' = \varnothing$

(iii) $(A')' = A$

THEOREM 1 **Distributive laws**

For any three sets A, B and C.

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof :

(i) We have,
$$\begin{aligned} A \cap (B \cup C) &= \{x \mid x \in A \text{ and } x \in (B \cup C)\} \\ &= \{x \mid x \in A \text{ and } (x \in B \text{ or } x \in C)\} \\ &= \{x \mid (x \in A \text{ and } x \in B) \text{ or } \\ &\qquad (x \in A \text{ and } x \in C)\} \\ &= \{x \mid x \in (A \cap B) \text{ or } x \in (A \cap C)\} \\ &= (A \cap B) \cup (A \cap C) \end{aligned}$$

$\therefore$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is proved.

(ii) Again,
$$\begin{aligned} A \cup (B \cap C) &= \{x \mid x \in A \text{ or } x \in (B \cap C)\} \\ &= \{x \mid x \in A \text{ or } (x \in B \text{ and } x \in C)\} \\ &= \{x \mid (x \in A \text{ or } x \in B) \text{ and } \\ &\qquad (x \in A \text{ or } x \in C)\} \\ &= \{x \mid x \in (A \cup B) \text{ and } x \in (A \cup C)\} \\ &= (A \cup B) \cap (A \cup C) \end{aligned}$$

$\therefore$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

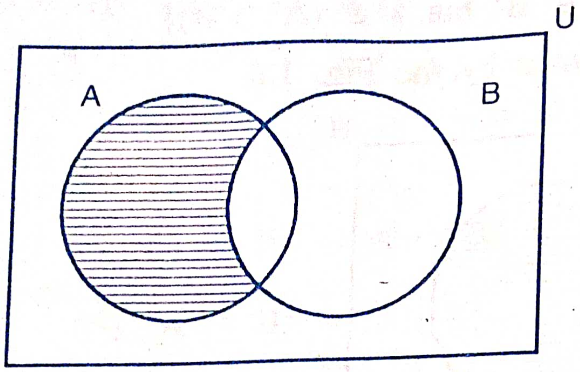
Definition 5

Let A and B be two sets. The difference of A and B, denoted by $A - B$, is the set containing all those elements which are in A but not in B. (The difference $A - B$ is also called complement in B with respect to A).

Thus, $A - B = \{x \mid x \in A \text{ but } x \notin B\}$

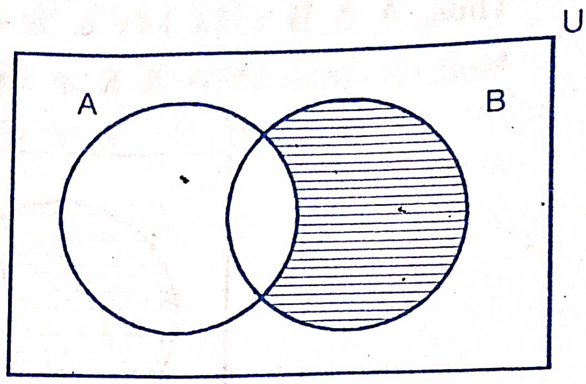
Similarly, $B - A = \{x \mid x \in B \text{ but } x \notin A\}$

Venn–diagrams of A – B and B – A are given in Fig. 1.6 and Fig. 1.7.



A - B is shaded

Fig. 1.6



B - A is shaded

Fig. 1.7

Properties of difference operation

(i) $A - B = A \cap B'$

Proof : $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

$= \{x \mid x \in A \text{ and } x \in B'\}$

$= A \cap B'$

(ii) $B - A = B \cap A'$

Proof : As above

(iii) $(A - B) \subset A$, $(B - A) \subset B$ are obvious from venn-diagrams.

(iv) If $A \subset B$ then $A - B = \varnothing$.

Proof : $A - B = \{x \mid x \in A \text{ but } x \notin B\}$

$= \{x \mid x \in B \text{ but } x \in B'\}$ $(\because A \subset B)$

$A - B = \varnothing$

(v) $A - \varnothing = A$ is obvious

EXAMPLE 11 Let $A = \{1, 2, 3, 4\}$, $B = \{4, 1, 3, 6\}$.
Find $A - B$, $B - A$.

SOLUTION $A - B = \{2\}$; $B - A = \{6\}$

EXAMPLE 12 Let $V = \{a, e, i, o, u\}$, $A = \{a, i, k, u\}$.
Find $V - A$, $A - V$.

SOLUTION $V - A = \{e, o\}$ and

$A - V = \{k\}$

We observe that $A - V \neq V - A$

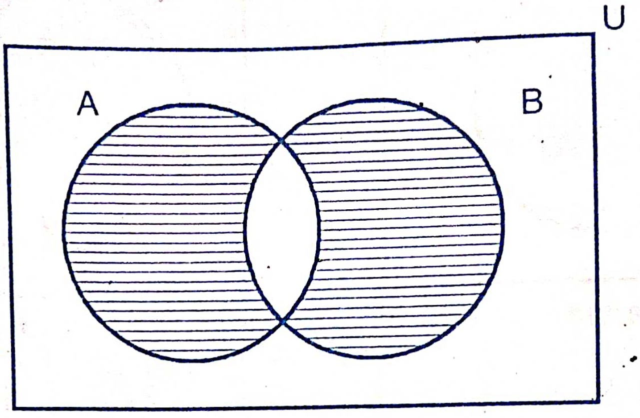
Definition 6

Let A and B be two sets. The set of all elements which are in A or in B but not in both is called the **symmetric difference set**.

It is denoted by A Δ B.

Thus, $A \,\Delta\, B = \{x \mid (x \in A \text{ or } x \in B) \text{ but } x \notin (A \cap B)\}$

Venn–diagram of A Δ B is represented by the Fig. 1.8.



A Δ B is shaded

Fig. 1.8

From the definition, we observe that $A \,\Delta\, B = (A - B) \cup (B - A)$

$= (A \cup B) - (A \cap B)$

EXAMPLE 13 Let $A = \{x \mid x \in R, x^2 - 2 = 0\}$ and $B = \{1, \sqrt{2}, 2\}$. Find A Δ B.

SOLUTION Here $A = \{\sqrt{2}, -\sqrt{2}\}$

And $B = \{1, \sqrt{2}, 2\}$

$\therefore A \,\Delta\, B = (A \cup B) - (A \cap B)$

$= \{1, \sqrt{2}, -\sqrt{2}, 2\} - \{\sqrt{2}\}$

$A \,\Delta\, B = \{1, -\sqrt{2}, 2\}$

EXAMPLE 14 Let $A = \{1, 2, 3\}$, find A Δ A.

SOLUTION We have, $A \,\Delta\, A = (A \cup A) - (A \cap A)$

$= \{1, 2, 3\} - \{1, 2, 3\}$

$\therefore A \,\Delta\, A = \varnothing$

THEOREM 2 De Morgan's laws

For any sets A, B and U be the universal set.

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof :

(i) $(A \cup B)' = \{x \mid x \in U \text{ and } x \notin (A \cup B)\}$

$= \{x \mid x \in U \text{ and } (x \notin A \text{ and } x \notin B)\}$

$= \{x \mid x \in U \text{ and } (x \in A' \text{ and } x \in B')\}$

$= A' \cap B'$

$\therefore \quad (A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = \{x \mid x \in U \text{ and } x \notin (A \cap B)\}$

$= \{x \mid x \notin A \text{ or } x \notin B\}$

$= \{x \mid x \notin A' \text{ or } x \notin B'\}$

$= A' \cup B'$

$\therefore \quad (A \cap B)' = A' \cup B'$

TABLE OF SET IDENTITIES

Name	Identity
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$A \cup \varnothing = A$ $A \cap U = A$	Identity laws
$A \cup U = U$ $A \cap \varnothing = \varnothing$	Domination laws
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws
$(A \cup B)' = A' \cap B'$ $(A \cap B)' = A' \cup B'$	De' Morgan's law
$(A')' = A$	Complementation law

EXAMPLE 15 If $A = \{2, 4, 6, 8\}$, $B = \{1, 2, 3, 4\}$ and $C = \{2, 3, 7\}$ and $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, verify the following results

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $(A \cup B)' = A' \cap B'$

(iii) $A - B = A - (A \cap B)$

SOLUTION

(i) $B \cap C = \{2, 3\}$

$\therefore \quad A \cup (B \cap C) = \{2, 4, 6, 8\} \cup \{2, 3\}$

$= \{2, 3, 4, 6, 8\}$ ...(i)

and $A \cup B = \{1, 2, 3, 4, 6, 8\}$

and $A \cup C = \{2, 3, 4, 6, 7, 8\}$

$\therefore (A \cup B) \cap (A \cup C) = \{2, 3, 4, 6, 8\}$ ...(ii)

From (i) and (ii),

We obtain,

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

$\therefore \quad (A \cup B)' = \{5, 7\}$ ...(i)

Also, $A' = \{1, 3, 5, 7\}$; $B' = \{5, 6, 7, 8\}$

$\therefore \quad A' \cap B' = \{5, 7\}$ ...(ii)

From (i) and (ii),

We obtain,

$(A \cup B)' = A' \cup B'$

(iii) $A - B = \{2, 4, 6, 8\} - \{1, 2, 3, 4\}$

$\therefore \quad A - B = \{6, 8\}$ ...(i)

Now, $A \cap B = \{2, 4\}$

$\therefore \quad A - (A \cap B) = \{2, 4, 6, 8\} - \{2, 4\}$

$A - (A \cap B) = \{6, 8\}$

From (i) and (ii), ...(ii)

We obtain,

$\therefore A - B = A - (A \cap B)$

EXAMPLE 16 Let $U = \{1, 2, ..., 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$ verify that

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

SOLUTION

(i) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

$\therefore \quad (A \cup B)' = \{1, 9\}$ ...(i)

Also, $A' = \{1, 3, 5, 7, 9\}$

and $B' = \{1, 4, 6, 8, 9\}$

$\therefore \quad A' \cap B' = \{1, 9\}$ ...(ii)

From (i) and (ii),

We obtain,

$$(A \cup B)' = A' \cap B'$$

$U - (A \cap B)$

(ii) $A \cap B = \{2\}$

$\therefore \quad (A \cap B)' = \{1, 3, 4, ..., 9\}$ ...(i)

Also, $A' = \{1, 3, 5, 7, 9\}$, $B' = \{1, 4, 6, 8, 9\}$

$\therefore \quad A' \cup B' = \{1, 3, 4, 5, ..., 9\}$ ...(ii)

From (i) and (ii),

We obtain,

$$(A \cap B)' = A' \cup B'$$

EXAMPLE 17 Let $A = \{1, 2, 5, 6\}$, $B = \{2, 5, 7\}$ and $C = \{2, 5, 8, 9\}$. Verify the following identifies :

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iii) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(iv) $A - (B \cup C) = (A - B) \cap (A - C)$

(v) $A - (B \cap C) = (A - B) \cup (A - C)$

(vi) $A \cap (B \,\Delta\, C) = (A \cap B) \,\Delta\, (A \cap C)$

SOLUTION Left to the reader.

EXAMPLE 18 If A and B be sets such that $A \subset B$, then prove that $A \cup B = B$.

SOLUTION Let $x \in (A \cup B)$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in B$ $(\because A \subset B)$

$\Rightarrow x \in B$

Thus, $\forall x;\ x \in (A \cup B) \Rightarrow x \in B$

$\therefore \quad (A \cup B) \subset B$ ...(i)

Also, by definition $B \subset (A \cup B)$ ...(ii)

From (i) and (ii), We obtain

$$A \cup B = B$$

EXAMPLE 19 Using properties of sets, show that

$$(A \cup B) \cap (A \cup B') = A.$$

SOLUTION LHS $= (A \cup B) \cap (A \cup B')$

$= A \cup (B \cap B')$ ($\because$ Distributive law)

$= A \cup \varnothing$ ($\because B \cap B' = \varnothing$)

$= A$

$=$ RHS

$\therefore \quad (A \cup B) \cap (A \cup B') = A.$

EXAMPLE 20 Using properties of set, prove that

$$(A - B) \cup (A \cup B) = A.$$

SOLUTION LHS $= (A - B) \cup (A \cap B)$

$= (A \cap B') \cup (A \cap B)$ ($\because A - B = A \cap B'$)

$= A \cap (B' \cup B)$ ($\because$ Distributive law)

$= A \cap U$ ($\because B' \cup B = U$)

$= A$

$=$ RHS

$\therefore \quad (A - B) \cup (A \cap B) = A$

EXAMPLE 21 If $A \cap B = \varnothing$, prove that $A \subset B'$.

SOLUTION Suppose that $x \in A$

$\Rightarrow x \notin B$ ($\because A \cap B = \varnothing$)

$\Rightarrow x \in B'$

Thus, $\forall x;\ x \in A \Rightarrow x \in B'$

$\therefore \quad A \subset B'$ is proved.

EXAMPLE 22 If $A \subset B$ show that $B' \subset A'$

SOLUTION Suppose that $x \in B'$

$\Rightarrow x \notin B$

$\Rightarrow x \notin A$ ($\because A \subset B$)

$\Rightarrow x \in A'$

Thus, $\forall x;\ x \in B' \Rightarrow x \in A'$

$\therefore \quad B' \subset A'$ is proved.

EXAMPLE 23 Let A and B be any two sets. Using properties of sets prove that $(A' \cup B')' \cup (A' \cup B)' = A$

SOLUTION

$$\begin{aligned}
\text{LHS} &= (A' \cup B')' \cup (A' \cup B)' \\
&= [(A')' \cap (B')'] \cup [(A')' \cap B'] && (\because \text{De' Morgan's laws}) \\
&= (A \cap B) \cup (A \cap B') && (\because (A')' = A) \\
&= A \cap (B \cup B') && (\because \text{Distributive law}) \\
&= A \cap U \\
&= A \\
&= \text{RHS}
\end{aligned}$$

$\therefore$ Proved.

EXERCISES 1.2

1. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{0, 2, 3, 6\}$.
   Find (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $B - A$.

2. Find the sets A and B, if $A - B = \{1, 5, 7, 8\}$ and $B - A = \{2, 10\}$ and $A \cap B = \{3, 6, 9\}$.

3. Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$.

4. Let A be a set, prove that $(A')' = A$.

5. Let A and B be sets, using properties of sets show that
   (i) $A \cap (B - A) = \varnothing$
   (ii) $A \cup (B - A) = A \cup B$
   (iii) $(A - B) \cap (B - A) = \varnothing$

6. Can you conclude that $A = B$, if
   (i) $A \cup C = B \cup C$?
   (ii) $A \cap C = B \cap C$? for sets A, B and C. Justify your answer.

7. Shade the following sets in the Venn–diagram.
   (i) $A' \cap (B \cup C)$
   (ii) $A \cap (B \cap C)$
   (iii) $A - (B \cap C)$
   (iv) $A' \cap (C - B)$
   (v) $(A \cup B) - A$

1.4 Cartesian Product of Sets

Let A and B be two sets.

If $a \in A$, $b \in B$ then (a, b) denotes an ordered pair whose first component is 'a' and second component is 'b'.

Two ordered pairs (a, b) and (c, d) are equal. If and only if a = c and b = d.

Ordered pairs (a, b) and (b, a) are two distinct ordered pairs.

Also, ordered pair (a, b) is not same as {a, b}.

Definition 1

Let A and B be any two non-empty sets. The set of all ordered pairs (a, b), where $a \in A$, $b \in B$ is called the **Cartesian product** of the sets A and B and is denoted by $A \times B$.

Thus, $A \times B = \{(a, b) \mid a \in A, b \in B\}$

For example,

Let $A = \{a_1, a_2\}$ and $B = \{b_1, b_2, b_3\}$

We find $A \times B$ as follows :

Take $a_1 \in A$, write all elements of B with a_1 that is (a_1, b_1), (a_1, b_2), (a_1, b_3).

Now take $a_2 \in A$, write all elements of B with a_2 that is (a_2, b_1), (a_2, b_2), (a_2, b_3).

Therefore, $A \times B$ will be

$A \times B = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_2, b_1), (a_2, b_2), (a_2, b_3)\}$

Remarks

(i) If $A = \emptyset$ or $B = \emptyset$ then $A \times B = \emptyset$.

(ii) In general, $A \times B \neq B \times A$.

(iii) If the set A has m–elements and B has n–elements, then $A \times B$ has mn elements.

(iv) If A = B, then $A \times B = A \times A = A^2$.

We can define, ordered triplets, if A, B and C are three sets; then (a, b, c). $a \in A$, $b \in B$ and $c \in C$ is called **an ordered triplet**.

The Cartesian product of sets A, B and C is defined as

$$A \times B \times C = \{(a, b, c) \mid a \in A, b \in B, c \in C\}$$

EXAMPLE 1 Find a and b, if (a + 2, 4) = (5, 2a + b).

SOLUTION By definition, $a + 2 = 5 \Rightarrow a = 3$

$$2a + b = 4 \Rightarrow b = -2$$

EXAMPLE 2 If $A = \{1, 2\}$ and $B = \{4, 5\}$, find $A \times B$, $B \times A$, $A \times A$, $B \times B$ and show that $A \times B \neq B \times A$.

SOLUTION $A \times B = \{1, 2\} \times \{4, 5\}$

$= \{(1, 4), (1, 5), (2, 4), (2, 5)\}$

And $B \times A = \{4, 5\} \times \{1, 2\}$

$= \{(4, 1), (4, 2), (5, 1), (5, 2)\}$

Also, we observe that $A \times B \neq B \times A$.

Now, $A^2 = A \times A = \{1, 2\} \times \{1, 2\}$

$\therefore$ $A^2 = A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

and $B \times B = B^2 = \{4, 5\} \times \{4, 5\}$

$\therefore$ $B^2 = \{(4, 4), (4, 5), (5, 4), (5, 5)\}$

EXAMPLE 3 Let $A = \{1, 2, 3\}$, $B = \{3, 5\}$ and $C = \{2, 3, 5\}$. Find

(i) $A \times (B \cap C)$ (ii) $(A \times B) \cap (A \times C)$

(iii) $A \times (B \cup C)$ (iv) $(A \times B) \cup (A \times C)$

SOLUTION

(i) $B \cap C = \{3, 5\}$

$\therefore$ $A \times (B \cap C) = \{1, 2, 3\} \times \{3, 5\}$

$\therefore$ $A \times (B \cap C) = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$

(ii) $A \times B = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$

$A \times C = \{(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5) (3, 2), (3, 3), (3, 5)\}$

$\therefore$ $(A \times B) \cap (A \times C) = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$

(iii) $B \cup C = \{2, 3, 5\}$

$\therefore$ $A \times (B \cup C) = \{1, 2, 3\} \times \{2, 3, 5\}$

$= \{(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2),$
$(3, 3), (3, 5)\}$

(iv) $(A \times B) \cup (A \times C) = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5),$
$(1, 2), (2, 2), (3, 2)\}$

EXAMPLE 4 Let $P = \{1, 2, x\}$, $Q = \{a, x, y\}$ and $R = \{x, y, z\}$
Find $P \times Q$, $P \times R$, $(P \times Q) \cap (P \times R)$.
Also, find complement of P in R and complement of P in Q.

SOLUTION

$$P \times Q = \{1, 2, x\} \times \{a, x, y\}$$
$$= \{(1, a), (1, x), (1, y)\ (2, a), (2, x), (2, y)\ (x, a), (x, x), (x, y)\}$$
$$P \times R = \{1, 2, x\} \times \{x, y, z\}$$
$$= \{(1, x), (1, y), (1, z)\ (2, x), (2, y), (2, z)\ (x, x), (x, y), (x, z)\}$$
$$\therefore (P \times Q) \cap (P \times R) = \{(1, x), (1, y), (2, x), (2, y), (x, x), (x, y)\}$$

Complement of P in R $= R - P$

$= \{y, z\}$

Complement of P in Q $= Q - P$

$= \{a, y\}$

EXAMPLE 5 Let A and B be two sets such that $n(A) = 5$, $n(B) = 2$ and if $(a_1, 2)$, $(a_2, 3)$, $(a_3, 2)$, $(a_4, 3)$, $(a_5, 2)$ are in $A \times B$. Find A and B.

SOLUTION Since, $a_1, a_2, a_3, a_4, a_5 \in A$

$\therefore \quad A = \{a_1, a_2, a_3, a_4, a_5\}$

Also, $2, 3 \in B$

$\therefore \quad B = \{2, 3\}$

EXAMPLE 6 Let $A = \{1, 2, 3, 4\}$ and $S = \{(a, b) \mid a, b \in A, a \text{ divides } b\}$. Write S explicitly.

SOLUTION Here, $(a, b) \in S \Leftrightarrow a$ divides b

$\therefore \quad (1, 1) \in S \qquad \because$ 1 divides 1

$(2, 2) \in S \qquad \because$ 2 divides 2

$(3, 3) \in S$, $(4, 4) \in S$, $(1, 2) \in S$, $(1, 3)$, $(1, 4)$

$(2, 4) \in S$.

$\therefore \quad S = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)\}$

EXAMPLE 7 If A and B are two non–empty sets such that $A \times B = B \times A$, show that $A = B$.

SOLUTION W need to prove : (i) $A \subseteq B$

(ii) $B \subseteq A$

Suppose $x \in A$

$\Rightarrow (x, y) \in A \times B$; for any $y \in B$

$\Rightarrow (x, y) \in B \times A \qquad (\because$ given that $A \times B = B \times A)$

$\Rightarrow x \in B$

Thus, $\forall x;\ x \in A \Rightarrow x \in B$

$\therefore \quad A \subseteq B \qquad \text{...(i)}$

Also, suppose $y \in B$

$\Rightarrow (y, x) \in B \times A;\ \forall x \in A$

$\Rightarrow (y, x) \in A \times B \qquad (\because A \times B = B \times A)$

$\Rightarrow y \in A$

Thus, $\forall y;\ y \in B \Rightarrow y \in A$

$\therefore \quad B \subseteq A \qquad \text{...(ii)}$

From (i) and (ii), we get

$A = B.$

EXAMPLE 8 If $A = \{1\}$ and $B = \{2, 3\}$, write all the subsets of $A \times B$.

SOLUTION Here, $A \times B = \{(1, 2), (1, 3)\}$

$\therefore$ Subsets of $A \times B$ are $\emptyset$, $\{(1, 2)\}$, $\{(1, 3)\}$ and $\{(1, 2), (1, 3)\}$.

EXAMPLE 9 If $n(A) = 2$ and $n(B) = 3$ what will be the no. of element in $P(A \times B)$?

SOLUTION We have, $n(A) = 2$ and $n(B) = 3$

$\therefore n(A \times B) = 2 \times 3 = 6$

$\therefore$ No. of elements in power set of $A \times B$ that is $P(A \times B) = 2^6 = 64$.

EXAMPLE 10 If $n(A) : n(B) = 3 : 2$ and $n(A \times B) = 54$, find $n(A)$.

SOLUTION We have, $n(A) : n(B) = 3 : 2$

$$\therefore \quad \frac{n(A)}{n(B)} = \frac{3}{2}$$

Also, $n(A \times B) = 54$

$$\therefore n(A) \cdot n(B) = 54$$

$$\therefore n(A) \cdot \frac{2}{3} n(A) = 54 \qquad \left(\because n(B) = \frac{2}{3} n(A)\right)$$

$$\therefore [n(A)]^2 = 27 \times 3$$

$$\therefore [n(A)]^2 = 81$$

$$\therefore n(A) = 9$$

1.5 Applications of Set Theory

Let A be a finite set,

n(A) : denotes the number of elements in A. We are often interested in finding the number of elements **(This is called Cardinality)** in the union of sets.

If A and B are disjoint sets, then we have

$$n(A \cup B) = n(A) + n(B)$$

Similarly, if $A \cap B = B \cap C = C \cap A = \varnothing$

Then, $n(A \cup B \cup C) = n(A) + n(B) + n(C)$.

For example,

Let $A = \{a, b\}$ and $B = \{c, d, e\}$ then $A \cup B = \{a, b, c, d, e\}$

$\therefore \quad n(A \cup B) = 5$

Also, $n(A) + n(B) = 2 + 3 = 5$

$\therefore \quad n(A \cup B) = n(A) + n(B)$.

Remark : (i) $n(A - B) = n(A) - n(A \cap B)$

(ii) $n(B - A) = n(B) - n(A \cap B)$

Result 1 : If A and B are finite sets, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

This is called addition rule.

Proof :

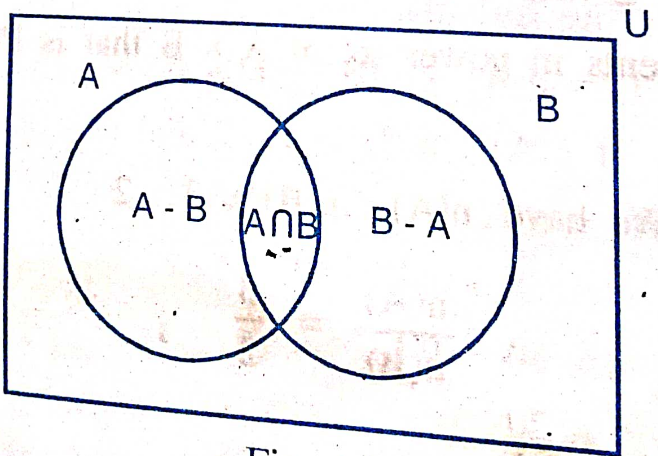


Fig. 1.9

We observe that,

$A - B$, $A \cup B$ and $B - A$ are disjoint sets.

Also, $A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$

$$\therefore \quad n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$$

$$= n(A) - n(A \cap B) + n(A \cap B) + n(B) - n(A \cap B)$$

$$= n(A) + n(B) - n(A \cap B)$$

$$\therefore \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Result 2 : If A, B and C are finite sets, then prove that

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C).$$

Proof :

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B \cup C) - n(A \cap (B \cup C)) \quad (\because \text{Result } 1) \\ &= n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap (B \cup C)) \\ &\qquad (\because \text{Result } 1) \\ &= n(A) + n(B) + n(C) - n(B \cap C) \\ &\qquad - n((A \cap B) \cup (A \cap C)) \\ &\qquad (\because \text{Distributive Law}) \\ &= n(A) + n(B) + n(C) - n(B \cap C) \\ &\qquad - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap A \cap C)] \\ &\qquad (\because \text{Result } 1) \\ &= n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) \\ &\qquad - n(A \cap C) + n(A \cap B \cap C) \end{aligned}$$

$$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C).$$

EXAMPLE 1 If $n(A \cup B) = 60$, $n(A) = 38$ and $n(B) = 42$, find $n(A \cap B)$.

SOLUTION By the formula,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

We find,

$$\begin{aligned} n(A \cap B) &= n(A) + n(B) - n(A \cup B) \\ &= 38 + 42 - 60 \\ &= 80 - 60 \\ n(A \cap B) &= 20 \end{aligned}$$

EXAMPLE 2 If $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$; find $n(X \cap Y)$, $n(X - Y)$ and $n(Y - X)$.

SOLUTION By the formula,

$$\begin{aligned} n(X \cup Y) &= n(X) + n(Y) - n(X \cap Y) \\ \therefore n(X \cap Y) &= n(X) + n(Y) - n(X \cup Y) \\ &= 17 + 23 - 38 \\ &= 40 - 38 \end{aligned}$$

$\therefore \quad n(X \cap Y) = 2$

Now, $\quad n(X - Y) = n(X) - n(X \cap Y)$

$= 17 - 2$

$\therefore \quad n(X - Y) = 15$

Again, $\quad n(Y - X) = n(Y) - n(X \cap Y)$

$= 23 - 2$

$\therefore \quad n(Y - X) = 21$

EXAMPLE 3 Sets A and B such that $n(A \cup B) = 60$, $n(A) = 30$ and $n(B) = 45$, find $n(A \cap B)$, $n(B - A)$ and $n(A - B)$.

SOLUTION We have, $n(A \cup B) = 60$, $n(A) = 30$ and $n(B) = 45$

Also, we have $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 60 = 30 + 45 - n(A \cap B)$

$\Rightarrow 15 = n(A \cap B)$

Now, $\quad n(B - A) = n(B) - n(A \cap B)$

$= 45 - 15 \qquad (\because n(A \cap B) = 15)$

$= 30$

and $\quad n(A - B) = n(A) - n(A \cap B)$

$= 30 - 15$

$= 15$

EXAMPLE 4 Let A and B be two finite sets such that $n(A) = 20$, $n(A \cup B) = 42$ and $n(A \cap B) = 4$. Find (i) $n(B)$ (ii) $n(B - A)$.

SOLUTION We are given

$n(A) = 20$, $n(A \cup B) = 42$ and $n(A \cap B) = 4$

We have, $\quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$42 = 20 + n(B) - 4$

$\therefore 26 = n(B)$

(ii) $\quad n(B - A) = n(B) - n(A \cap B)$

$= 26 - 4$

$\therefore n(B - A) = 22$

EXAMPLE 5 In a computer science department of a college; there are 20 faculties who teach computer or mathematics of these, 12 teach computer and 4 teach computer and mathematics. How many teach mathematics ?

SOLUTION Let M : Set of teachers who teach mathematics.

C : Set of teachers who teach computer science.

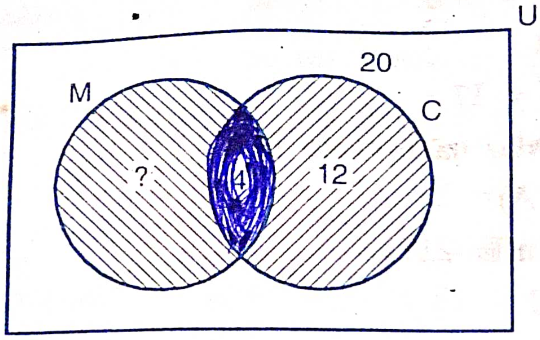


Fig. 1.10

$\therefore$ We are given that

$$n(M \cup C) = 20; \; n(C) = 12 \text{ and } n(C \cap M) = 4$$

We find : n(M)

By the formula

$$n(M \cup C) = n(M) + n(C) - n(C \cap M)$$

$\Rightarrow$ $20 = n(M) + 12 - 4$

$\Rightarrow$ $20 - 8 = n(M)$

$\Rightarrow$ $n(M) = 12$

Hence, 12 teachers teach mathematics.

EXAMPLE 6 In a class of 100 students, 48 have taken e–commerce, 32 have taken e–commerce but not ADBMS. Find the numbers of students who have taken both e–commerce and ADBMS. Each student has taken either e–commerce or ADBMS or both.

SOLUTION Let E : Set of the students who have taken e–commerce

A : Set of the students who have taken ADBMS.

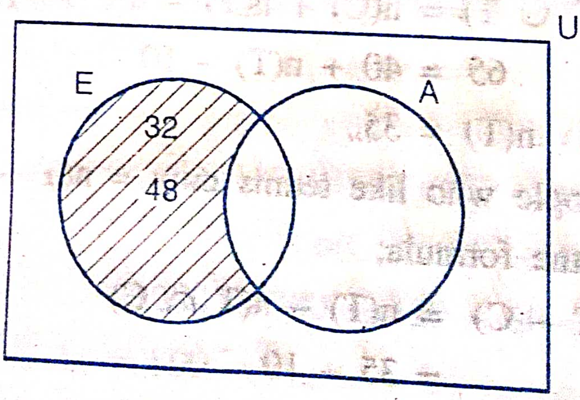


Fig. 1.11

We are given that, each student has taken either e–commerce or ADBMS or both

$\therefore \quad n(E \cup A) = 100$

Also, $n(E) = 48$

and $n(E - A) = 32$

No. of students who have taken both

$= n(E \cap A)$

$= n(E) - n(E - A)$

$= 48 - 32$

$= 16$

EXAMPLE 7 In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis ? How many like tennis only ?

SOLUTION Let C : Set of people who like cricket.

T : Set of people who like tennis.

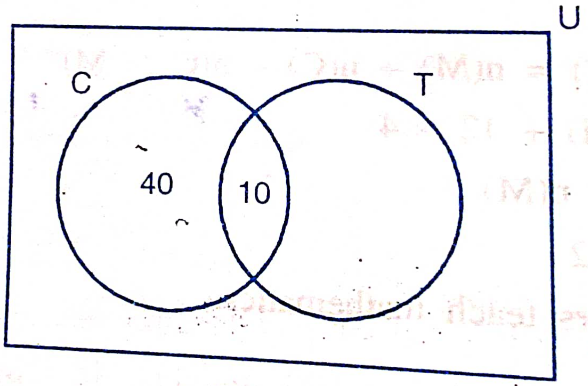


Fig. 1.12

$\therefore \quad n(C \cup T) = 65,\ n(C) = 40,\ n(C \cap T) = 10$

We find (i) $n(T)$ (ii) $n(T - C)$

(i) No. of people who like tennis $= n(T)$

We use,

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

$$\Rightarrow \quad 65 = 40 + n(T) - 10$$

$$\Rightarrow \quad n(T) = 35$$

(ii) No. of people who like tennis only $= n(T - C)$

By using the formula;

$$n(T - C) = n(T) - n(T \cap C)$$

$$= 35 - 10$$

$$n(T - C) = 25$$

EXAMPLE 8 In a committee 50 people speak French, 20 speak Spanish and 10 speak both. How many speak at least one of the two languages ?

SOLUTION Let F : Set of people speaking French.

S : Set of people speaking Spanish.

We are given, $n(F) = 50$, $n(S) = 20$ and $n(F \cap S) = 10$

We find : $n(F \cup S)$

By the formula,

$$n(F \cup S) = n(F) + n(S) - n(F \cap S)$$

$$\Rightarrow \quad n(F \cup S) = 50 + 20 - 10$$

$$\Rightarrow \quad n(F \cup S) = 60$$

EXAMPLE 9 In a survey of 1000 persons, it was found that 280 read magazine A, 300 read magazine B, 420 read magazine C, 80 read magazines A and B, 100 read magazines A and C, 50 read magazines B and C and 30 read all three magazines. Find

(i) How many read at least one of these magazines ?

(ii) How many read none of three magazines ?

(iii) How many read magazine A only ?

(iv) How many read magazine A and B but not C ?

SOLUTION Let A : Set of people who read magazine A

B : Set of people who read magazine B

C : Set of people who read magazine C

We are given $n(A) = 280$, $n(B) = 300$, $n(C) = 420$

$n(A \cap B) = 80$, $n(A \cap C) = 100$, $n(B \cap C) = 50$

and $n(A \cap B \cap C) = 30$, $n(U) = 1000$

(i) No. of people who read at least one of three magazines

$= n(A \cup B \cup C)$.

By the formula,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

$$= 280 + 300 + 420 - 80 - 100 - 50 + 30$$

$$= 800$$

$\therefore n(A \cup B \cup C) = 800.$

(ii) No. of people who read none of these three magazines

$= n(U) - n(A \cup B \cup C)$

$= 1000 - 800$

$= 200$

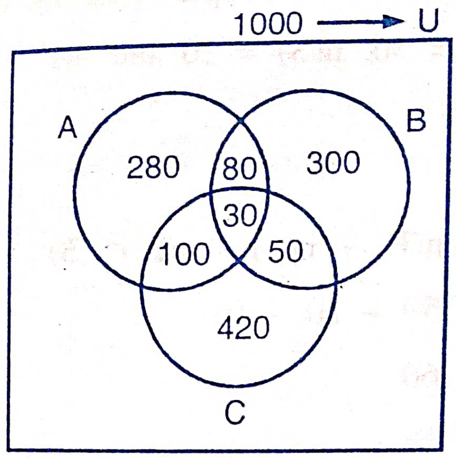


Fig. 1.13

(iii) From the Venn–diagram.

No. of persons who read magazine A only.

$= 280 - \{(80 - 30) + (100 - 30) + 30\}$

$= 280 - \{50 + 70 + 30\}$

$= 280 - 150$

$= 130$

(iv) From the Venn–diagram.

No. of persons who read magazines A and B but not C.

$= 80 - 30$

$= 50$

EXERCISES 1

1. In each of the following determine, whether the statement is true or false. If it is true, prove it. If it is false give an example.
   (i) If $a \in A$ and $A \in B$, then $a \in B$.
   (ii) Every set has at least one proper subset.
   (iii) If $A \subset B$, $B \subset C$ then $A \subset C$.
   (iv) If $A \cup B = A \cup C$ then $B = C$.
   (v) If $A \cap C = A \cap B$ then $B = C$.
   (vi) The collection of most talented writers of India is a set.
   (vii) The collection of all natural numbers less than 50 is a set.

(viii) $\{1, 2, 1, 2, 1, 2, \ldots\}$ is an infinite set.

(ix) The set $\{x \mid x + 9 = 9\}$ is the null set.

(x) For any two sets A and B either $A \subseteq B$ or $B \subseteq A$.

2. Consider the following sets.

$\varnothing$, $A = \{1, 2, 3\}$, $B = \{1, 5, 6, 9\}$ and $C = \{1, 2, 3, 4\}$

Insert the correct symbol $\subset$ or $\not\subset$ between each pair of sets

(i) $\varnothing$ ..... B (ii) A .... B

(iii) A .... C (iv) B C

3. Decide, among the following sets, which are the subsets of which ?

$A = \{x \in R \mid x^2 - 8x + 12 = 0\}$ $B = \{2, 4, 6\}$

$C = \{2, 4, 6, 8, 10, \ldots\}$ $D = \{6\}$.

4. If $U = \{x \mid 1 \le x \le 10, x \in N\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$; then obtain

(i) $A \cup B$ (ii) $A \cup C$ (iii) $B \cup C$

(iv) $A \cap B$ (v) $B \cap C$ (vi) $B - C$

(vii) $(A \cup B)'$ (viii) A' (ix) $A \Delta C$

(x) $C - A$

5. For $A = \{3k \mid k \in N\}$, $B = \{3k - 1 \mid k \in N\}$ and $C = \{3k - 2 \mid k \in N\}$. Obtain

(i) $A \cup B \cup C$ (ii) $A \cap B \cap C$

6. If $\{4k \mid k \in Z\} \cap \{6k \mid k \in Z\} = \{kn \mid n \in Z\}$, then find k.

7. If $a \in N$ such that $aN = \{ax \mid x \in N\}$. Describe the sets 3N, 7N and also find $3N \cap 7N$.

8. Let $A = \{1, 2, 4, 5\}$, $B = \{2, 3, 5, 6\}$ and $C = \{4, 5, 6, 7\}$. Verify the results.

(i) $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $A - (B \cup C) = (A - B) \cap (A - C)$

(iii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

9. Find sets A, B and C such that $A \cap B$, $A \cap C$ and $B \cap C$ are non-empty sets but $A \cap B \cap C = \varnothing$.

10. Let A, B and C be three sets, if $A \in B$ and $B \subset C$, is it true that $A \subset C$? If not, give an example.

11. Taking the set of natural numbers as the universal set; write down the complement of the following sets.

(i) $X = \{x : x \in N, x + 5 = 8\}$

(ii) $Y = \{x : x \text{ is divisible by } 3 \text{ and } 5\}$

12. Find the smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$.

13. If $A = \{x \in N \mid x^3 = x\}$ and $B = \{x \in Z \mid x^2 = x\}$. Find $P(A) \cap P(B)$.

14. If $A = \{1, 2, 3\}$, $B = \{4\}$ and $C = \{6\}$, then verify that

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(iii) $A \times (B - C) = (A \times B) - (A \times C)$

15. If $A \times B = \{(a, 1), (b, 3), (a, 3), (b, 1), (b, 2), (a, 2)\}$. Find A and B.

16. Verify the following by Venn–diagrams.

(i) $(A \cup B')' = A' \cap B$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(iii) $(A \cap B) \cup (A \cap B') = A$

17. Using properties of sets, prove that

(i) $A \cap (A \cup B) = A$

(ii) $A \cup (A \cap B) = A$

18. Let A and B such that $n(A) = 20$, $n(A \cup B) = 42$ and $n(A \cap B) = 4$. Find (i) $n(B)$ (ii) $n(A - B)$ (iii) $n(B - A)$

19. If A, B and U be two sets with universal set U, such that $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Find $n(A' \cap B')$.

20. A survey shows that 76% of indians like oranges, whereas 62% like bananas. What percentage of indians like both oranges and bananas ?

21. In a competitive examination, out of 1000 candidates 750 passed in mathematics and 400 passed in english.

(i) How many passed both the subjects ?

(ii) How many passed in mathematics only ?

(iii) How many passed in english only ?

22. A school has 800 students. Of these, 400 students drink milk. 300 students drink coffee. 150 students drink both milk and coffee. How many students drink neither milk or coffee ? How many students drink only milk ?

23. In a class of a school there are 40 students, 10 students of this class have no liking to play any game and they do not take part in any game. If 21 students of this class play cricket and 12 students play volleyball, draw Venn–diagram and answer the following questions.

   (i) How many students both the games ?

   (ii) How many students play only cricket ?

   (iii) How many students play only volley ball ?

24. In a survey of 100 students, it was found that 60 had taken mathematics, 48 had taken physics and 44 had taken chemistry, 20 had taken mathematics and chemistry, 36 had taken mathematics and physics, 16 had taken physics and chemistry and 12 had taken all three subjects. Find the numbers of students that had

   (i) Only Mathematics

   (ii) Only Physics

   (iii) Only Chemistry

   (iv) At least one of the three sets

   (v) None of the subjects.

25. Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct ?

26. A company studies that product preferences of 20,000 consumers, it was found that each of the product A, B and C was liked by 7020, 6230 and 5980 respectively and all the product were liked by 1500, product A and B were liked by 2580, product A and C were liked by 1200, product B and C were liked by 1950, prove that study result are inconsistent.

27. **Each of the following question has four alternatives of which one and only one is correct. Find out the correct alternative.**

(1) Which of the following is correct ?

   (a) $\{x \mid x + 2 = 2\} = \phi$ (b) $a \subset \{a, b, c\}$

   (c) $\{a\} \subset \{a, b, c\}$ (d) $a \in \{\{a\}, b\}$

(2) What is the total number of proper subsets of a set consisting of n-elements?

   (a) 2^n (b) 2^{n-2}

   (c) $2^n - 2$ (d) none

(3) Which of the following statements is/are true ?

   (i) For any two sets A and B either $A \subset B$ or $B \subset A$

   (ii) Every subset of a finite set is finite.

(iii) {a, b, a, b, a, b, ...} is an infinite set.

(iv) Every set has a proper subet

(a) (i), (ii) are false; (iii), (iv) are true (b) (i), (ii), (iii) are false; (iv) is true

(c) (ii), (iv) are false; (i), (iii) are true (d) (i), (iii), (iv) are false; (ii) is true

(4) $\{x \in N / 5 < x < 6\}$ = ...

(a) (5, 6) (b) [5, 6]

(c) ϕ (d) none

(5) Set of all lines in a plane is ....

(a) finite set (b) singleton set

(c) infinite set (d) none

(6) Which of the following is/are true ?

(i) $1 \in \{1, 2, 3\}$

(ii) $\{a\} \in \{a, b, c, d\}$

(iii) $\{a, b\} = \{a, a, a, b, a, b\}$

(iv) The set $\{x \mid x + 9 = 9\}$ is a singleton set.

(a) (i), (ii) are true; (iii), (iv) are false (b) (i), (iii), (v) are true; (ii) is false

(c) (ii), (iii) are true; (i), (iv) are false (d) none

(7) If $A = \{x \mid x = 3n, n \in N\}$; $B = \{x \mid x = 4n, n \in N\}$ then $A \cap B$ = ...

(a) $\{x \mid x = 7n, n \in N\}$ (b) $\{x \mid x = 6n, n \in N\}$

(c) $\{x \mid x = 12n, n \in N\}$ (d) none

(8) $(A - B) \cup B$ = ...

(a) A (b) B

(c) $A \cup B$ (d) none

(9) $(A - B) \cap B$ = ...

(a) ϕ (b) $A \cup B$

(c) B (d) none

(10) $A - (A - B)$ = ...

(a) A (b) $A \cup B$

(c) $A \cap B$ (d) none

(11) $A \cup B = A \cup C,\ A \cap B = A \cap C \Rightarrow$ ....

(a) $A = B$

(b) $B = C$

(c) $A = C$

(d) none

(12) $A \cap (A' \cup B) =$ ....

(a) $A \cap B$

(b) U

(c) $A \cup B$

(d) none

(13) $A \cap (A \cup B)' =$ ....

(a) $A \cap B$

(b) U

(c) ϕ

(d) none

(14) $A \cap B = \phi \Rightarrow$ ....

(a) $A = B \neq \phi$

(b) $A \subseteq B'$

(c) $B' \subseteq A'$

(d) none

(15) $n(A - B) =$ ....

(a) $n(A) - n(B)$

(b) $n(A) - n(A \cap B)$

(c) $n(A)$

(d) none

(16) Let $n(A) = 20$, $n(A \cup B) = 42$, $n(A \cap B) = 4$, then $n(B) =$ ...., $n(A - B) =$ ....

(a) 16, 26

(b) 26, 16

(c) 26, 10

(d) none

(17) If $A \cap B = \phi$ then $A - B =$ ...., $B - A =$ ....

(a) A, B

(c) ϕ, B

(c) A, ϕ

(d) none

(18) $A = \{x \mid x = 4n + 1, n \leq 5, n \in N\}$, $B = \{3n \mid n \leq 8, n \in N\}$, then

$A - (A - B) =$ ...., $A - (A \cap B) =$ ....

(a) {9, 21}, {5, 13, 21}

(b) {9, 17}, {5, 13, 17}

(c) {9, 21}, {5, 13, 17}

(d) none

(19) Which of the following is/are true ?

(i) $x \notin (A \cup B) \Rightarrow x \notin A$ and $x \notin B$

(ii) $x \notin (A \cup B) \Rightarrow x \notin A$ or $x \notin B$

(iii) $x \notin (A \cap B) \Rightarrow x \notin A$ or $x \notin B$

(iv) $x \notin (A \cap B) \Rightarrow x \notin A$ and $x \notin B$

(a) (i), (iii) are true; remaining are false (b) (i), (ii) are true; remaining are false

(c) (ii), (iv) are true; remaining are false (d) none

(20) $A \times (B \cup C) =$ ....

(a) $(A \times B) \times (A \times C)$ (b) $(A \times B) \cup (A \times C)$

(c) $(A \times B) \cap (A \times C)$ (d) none

(21) For sets $n(A) = 115$, $n(B) = 326$ and $n(A - B) = 47$, then $n(A \cap B) =$ ....

(a) 162 (b) 68

(c) 115 (d) none

(22) A and B are disjoint sets, $n(A \cup B) = 480$ and $n(A) = 440$; then $n(B) =$ ....

(a) 80 (b) 40

(c) 920 (d) none

(23) There are 210 members in a club, 100 of them drink tea and 165 drink coffee how many of them drink both ?

(a) 465 (b) 65

(c) 55 (d) none

(24) If A and B be two sets containing 20 and 29 elements respectively, what can be the maximum number elements in $A \cup B$?

(a) 29 (b) 20

(c) 49 (d) none

(25) $(A - B) \cup (B - A) \cup (A \cap B) =$ ....

(a) A (b) B

(c) $A \cup B$ (d) A'

(26) $A - (B \cup C)$ is equal to

(a) $(A - B) \cup (A - C)$ (b) $(A - B) \cap (A - C)$

(d) $A \cup B$ (d) none

: ANSWERS :

EXERCISES 1.1

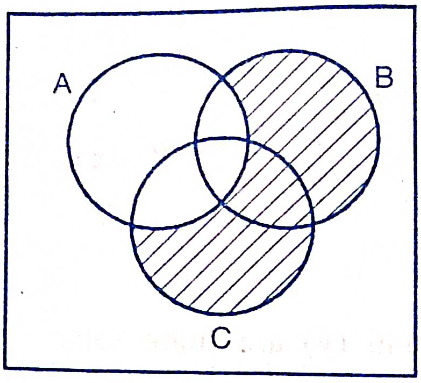
1. (ii), (iii), (v), (viii), (x) are true, remaining false.
2. (i) $A = \{-2, 2\}$
   (ii) $B = \{2, 3, 5, 7, 11, 13, 19\}$
   (iii) $C = \{1, 4, 9, 16, 25, 36, 49\}$
   (iv) $D = \{I, N, D, A\}$
   (v) $E = \{1, 2, 4, 5, 10, 20\}$
   (vi) $F = \{4, 8, 12, 16, 20, ...\}$
3. (i), (ii) are infinite sets; (iii), (iv) and (v) are finite sets.
4. $A = B = E$, $C = D$
5. No. For instance take $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$.
6. We many take $X = \{a, b\}$ and $Y = \{a, b, \{a, b\}\}$
7. (i) $P(\{a\}) = \{\varnothing, \{a\}\}$
   (ii) $P(\{\varnothing\}) = \{\varnothing, \{\varnothing\}\}$
   (iii) $P(\{a, b\}) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$
   (iv) $P(\{\{a\}, a\}) = \{\varnothing, \{a\}, \{\{a\}\}, \{a, \{a\}\}\}$
   (v) $P(\{\varnothing, \{\varnothing\}\}) = \{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$
   (vi) $P(\{1, 2, 3\}) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}\}$
8. 4
9. (i) $\{1\}, \{2\}$
   (ii) $\{a\}, \{b\}, \{c\}$
   $\{a, b\}, \{b, c\}, \{a, c\}$
   (iii) No proper subset
10. (i) 2 (ii) 6 (iii) $2^n - 2$

EXERCISES 1.2

1. (i) $A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$
   (ii) $A \cap B = \{2, 3, 6\}$
   (iii) $A - B = \{1, 4, 5\}$
   (iv) $B - A = \{0\}$
2. $A = \{1, 3, 5, 6, 7, 8, 9\}$; $B = \{2, 3, 6, 9, 10\}$
3. $\{2, 5\}$

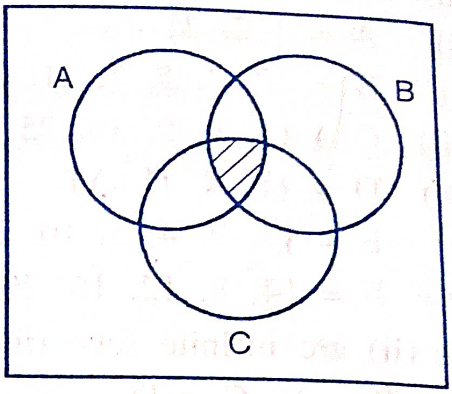
6. No, in both the cases
   (i) Take $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{1, 3, 4\}$
   (ii) Take $A = \{2, 3\}$, $B = \{2, 4\}$ and $C = \{2, 6\}$

7. (i)



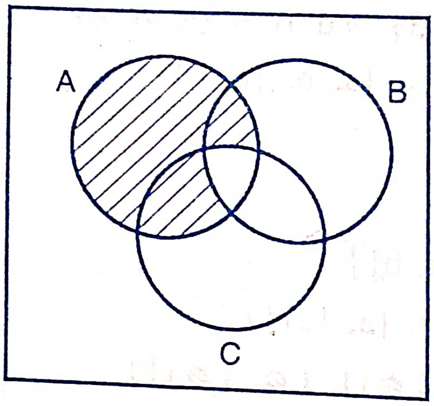
A'∩(BUC)

(ii)



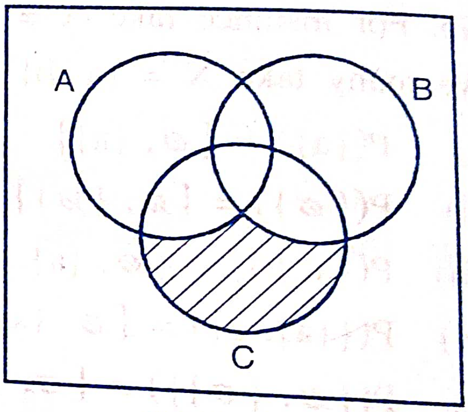
A∩(B∩C)

(iii)



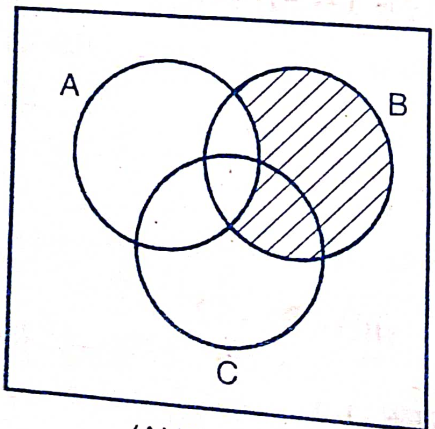
A - (B ∩C)

(iv)



A'∩(C - B)

(v)



(AUB) - A

EXERCISES 1

1. (i) False, Take $A = \{a, b\}$, $B = \{\{a, b\}, c\}$
   (ii) False, $\varnothing$ has no proper subset
   (iii) True, it is obvious from the definition of union.
   (iv) False, We may take $A = \{1, 2, 3\}$, $B = \{2, 3\}$ and $C = \{1, 2\}$

(v) False,

(vi) False, It is not well defined collection.

(vii) True

(viii) False, it is a finite set $\{1, 2\}$.

(ix) False, it is the singleton set $\{0\}$.

(x) False, if we take $A = \{1, 2\}$ and $B = \{2, 3\}$ then $A \not\subseteq B$ and $B \not\subseteq A$.

2. (i) $\subset$ (ii) $\not\subset$ (iii) $\subset$ (iv) $\not\subset$

3. $D \subset A \subset B \subset C$

4. (i) $\{1, 2, 3, 4, 6, 8\}$

(ii) $\{1, 2, 3, 4, 5, 6\}$

(iii) $\{2, 3, 4, 5, 6, 8\}$

(iv) $\{2, 4\}$

(v) $\{4, 6\}$

(vi) $\{2, 8\}$

(vii) $\{5, 7, 9, 10\}$

(viii) $\{5, 6, ..., 10\}$

(ix) $\{1, 2, 5, 6\}$

(x) $\{5, 6\}$

5. (i) N (ii) $\varnothing$

6. $k = 12$

7. (i) $\{3, 6, 9, 12, ...\}$ (ii) $\{7, 14, 21, 28, ...\}$ (iii) $\{21, 42, 63, ...\}$

9. Take $A = \{a, b\}$, $B = \{b, c\}$ and $C = \{a, c\}$

10. No, We may take $A = \{a\}$, $B = \{\{a\}, b\}$, $C = \{\{a\}, b, c\}$

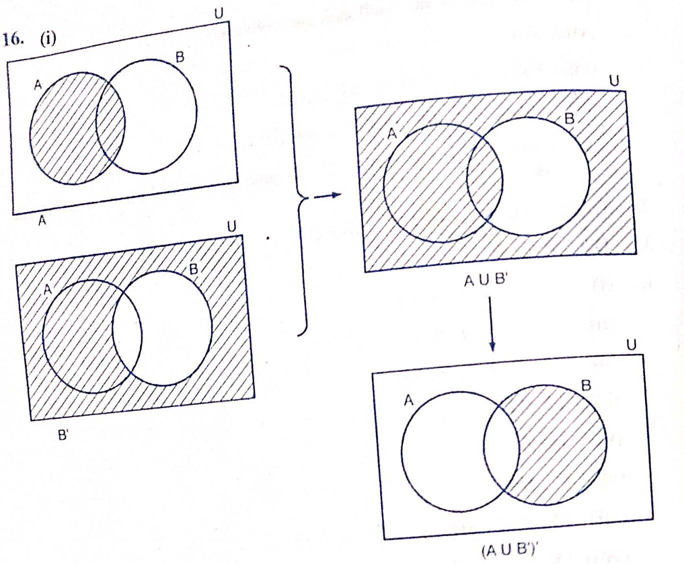
11. (i) $N - \{3\}$ (ii) $N - \{15, 30, 45, ...\}$

12. $A = \{3, 5, 9\}$

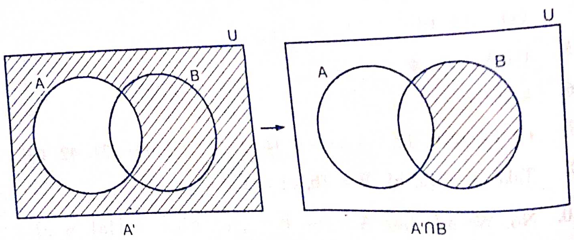
13. $P(A) \cap P(B) = \{\varnothing, \{1\}\}$

15. $A = \{a, b\}$, $B = \{1, 2, 3\}$

16. (i)



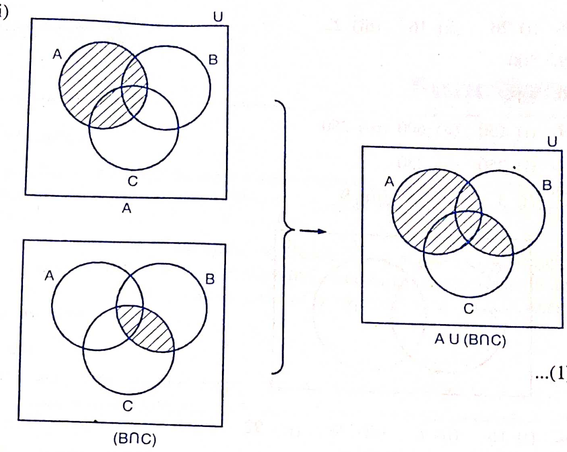
...(1)



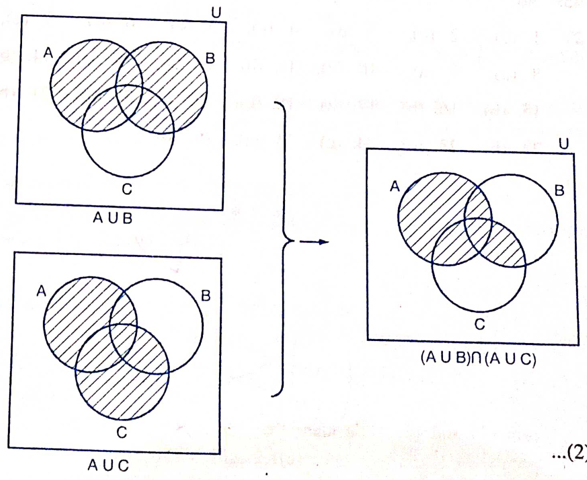
...(2)

From (1) and (2), it is clear that $(A \cup B')' = A' \cap B$.

(ii)



...(1)



...(2)

From (1) and (2), it is clear that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

18. (i) 26 (ii) 16 (iii) 22

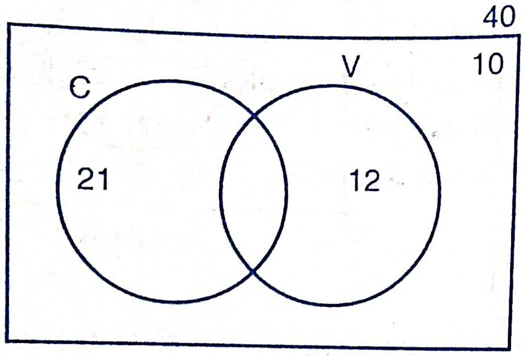
19. 300

20. 38%

21. (i) 150 (ii) 600 (iii) 250

22. (i) 250 (ii) 250

23. (i) 3 (ii) 18 (iii) 9



24. (i) 16 (ii) 8 (iii) 20 (iv) 92 (v) 8

25. No

27. 1. (c), 2. (c), 3. (d), 4. (c), 5. (c), 6. (d), 7. (c),
8. (c), 9. (a), 10. (c), 11. (b), 12. (a), 13. (c), 14. (b),
15. (b), 16. (b), 17. (a), 18. (c), 19. (a), 20. (b), 21. (b),
22. (b), 23. (c), 24. (c), 25. (c), 26. (b).

* * *