

SECTION - I

- Q. 1 Attempt the following: [18]
 a) Explain the followings terms with the help of an example of each : [6]
 i. Diagonal Matrix
 ii. Transpose of a Matrix
 iii. Orthogonal matrix
- b) When are two matrices said to be similar? Show that similar matrices have the same eigen values [6]
- c) What are the properties of Determinants of Matrices? [6]
- Q. 2 Attempt any two: [16]
 a) Compute the Characteristic equation and the eigen values of the following matrix: [8]
- $$\begin{bmatrix} 3 & 6 & 5 \\ 6 & -5 & 3 \\ -24 & 38 & 2 \end{bmatrix}$$
- b) Diagonalize the following matrix: [8]
- $$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$
- c) Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k , given that $A = PDP^{-1}$ where [8]
 $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$
- Q. 3 Attempt the following : [16]
 a) Explain the followings terms with the help of an example of each [6]
 i. Square Matrix
 ii. Inverse of a Matrix
 iii. Symmetric matrix
- b) What is singular value decomposition? Perform Singular Value decomposition for the matrix $\begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$ [10]
 OR
- Q. 3 Attempt the following : [16]
 a) Prove that eigen vectors corresponding to distinct eigen values are orthogonal. [8]
- b) Explain Principal Component Analysis , its method and its two applications. [8]

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SECTION - II

- Q. 4** **Attempt Any Three:** [18]
 a) Give advantage of using Gauss elimination with partial pivoting method [6]
 over Gauss elimination method.
 b) Explain graphically Bisection method to find a root of equation $f(x) = 0$. [6]
 c) Explain the difference between the Secant method and Regular False Position method. [6]
 d) Explain the situations, when Newton Raphson method may fail to converge? [6]
- Q. 5** **Attempt any two:** [16]
 a) Solve the following system with Gauss Elimination with partial pivoting. [8]

$$\begin{aligned} 2x + y + z &= 5 \\ 4x - 6y &= -2 \\ -2x + 7y + 2z &= 9 \end{aligned}$$

 b) Using Gauss Seidel iteration, solve the following system of equations [8]
 [perform four iterations] starting from $[1, 1, 1]$.

$$\begin{aligned} 10x + y + z &= 6 \\ x + 10y + z &= 6 \\ x + y + 10z &= 6 \end{aligned}$$

 c) Solve the following system using the inverse of a matrix [8]

$$\begin{aligned} 5x + 15y + 56z &= 35 \\ -4x - 11y - 41z &= -26 \\ -x - 3y - 11z &= -7 \end{aligned}$$
- Q. 6** **Attempt the following :** [16]
 a) Find a root of equation $x^3 - x - \sin x = 1$ by using the Regula Falsi method. [8]
 b) Solve $x^4 - x - 10 = 0$ correct up to four decimal places by Secant method. [8]
- OR**
- Q. 6** **Attempt the following :** [16]
 a) Solve $x^3 - 2x^2 + 3x - 1 = 0$ correct up to four decimal places by [8]
 Newton Raphson method. (Take initial $x_0 = 4$).
 b) Explain inherent errors, truncation errors and errors due to rounding . [8]
 Illustrate by an example of each.
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M.SC. (AI & ML) Semester I

Problem Solving with Python

Time: 3 Hours

[Max. Marks : 100]

SECTION I

Q:1 Answer the following (Any Two): [20]

1. Explain built-in data types of python.
2. What is List in python? Explain list bounds and list slicing with example.
3. Explain Recursive functions in python with example of Fibonacci series and also explain anonymous function lambda.

Q:2 Answer the following (Any Two): [16]

1. Write a difference between function and method and write how to define a function in Python.
2. Explain list assignment and its equivalence with example. Also explain list methods.
3. Write a python program that takes a number from user and calculate factorial, check whether it is prime or not and print its multiplication table.

Q:3 Answer the following (Any Two): [14]

1. Explain set in python. Also explain set Quantification with all and any.
2. Explain Prime number generation with Python.
3. Write a python program to arrange the characters of a given string "mountain" in an alphabetical order and counts number of vowels.

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SECTION II

Q:4 Answer the following(Any THREE): [18]

1. What are various file positions methods? Explain with example
2. What is class and Object?? Explain Encapsulation and Information hiding in Python.
3. How to implement method overriding in Python? Demonstrate with an example.
4. Demonstrate any six operation on strings

Q:5 Answer the following(Any TWO): [20]

1. What is dictionary in Python? What are the advantages of Python dictionary type? How do you get a list of all the keys in a dictionary? Explain with example.
2. Write a python program to read contents of the text file and write into another.
3. Discuss with an example, exception with arguments in Python.

Q:6 Answer the following(Any SIX): [12]

1. What is class instantiation?
2. What is Pickling and how does it differ from Unpickling?
3. Explain Inheritance in Python in brief.
4. Explain seek() and tell() Methods in file operations of Python
5. Explain if-elif condition with example
6. Write a python code to generate two 4X4 matrices and add and subtract them in Python.(Take any elements in array)
7. "With" statement in Python.

Note: (1) Make necessary assumptions wherever necessary.

(2) Write precise and to the point answers.

SECTION - I

Q:1 Do as directed (Any Seven)

[14]

- (a) What gets printed by the following expressions?
 - (i) `print(map(len, ['ab', 'cd']))`
 - (ii) `print(divmod(10, 3))`
- (b) What is purpose of `type()` function in Python.
- (c) List with example Boolean operators in Python.
- (d) What is the difference between `==` and `is` operator in Python?
- (e) What are the different methods to format string in Python?
- (f) Write the syntax for list comprehension.
- (g) What is the difference between a `for` loop and `while` loop?
- (h) What is the difference between `pass` and `continue` in Python?

Q:2 Answer the following (Any Two)

[16]

- (a) Consider a string `my_str = "GU is the best University"`. Give the output of the following commands.
 - (i) `my_str.rfind('i')`
 - (ii) `my_str.replace('t', '**', 2)`
 - (iii) `my_str.casfold()`
 - (iv) `my_str.split('the')`
- (b) Explain with example built-in function `range()` and `enumerate()`.
- (c) Explain any four built-in list methods with example.

Q:3 Answer the following (Any Two)

[20]

- (a) What is Python? What are the key features of Python?

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- (b) What is "any" and "all" in python? How they can be used in python? Explain with example.
- (c) Write in brief about Dictionary in python. Explain properties of dictionary keys with example.

SECTION - II

Q:4 Do as directed (Any Six)

[18]

- (a) What is class instantiation?
- (b) Differentiate between class variables and instance variables.
- (c) What does single-level inheritance mean?
- (d) Let `a` and `b` be objects of class `Foo`. Which functions or methods are called when `print(a + b)` is executed?
- (e) What is Pickling and how does it differ from Unpickling?
- (f) Differentiate between `TypeError` and `ValueError`.
- (g) Discuss about `assert` statement with example.

Q:5 Answer the following (Any Two)

[12]

- (a) How to open a file in Python? What are the different techniques for reading text file?
- (b) What is Method Resolution Order (MRO)? Explain the principles followed by MRO with example.
- (c) What are the benefits of using exception handling? Discuss with an example exception with arguments in Python.

Q:6 Answer the following (Any Two)

[20]

- (a) Explain Class methods, Instance methods and Static methods used in Python class. Also, state with an example when they can be applied.
- (b) How to implement method overriding in Python? Demonstrate with an example.
- (c) What is property decorator? Why it is used in Python class? Discuss with an example.

Instructions:

1. Figures to the right indicate full marks
2. Each section should be written in a separate answer book
3. Be precise and to the point in your answer

SECTION-I

1. All the statements given below are false. Correct the statements with proper [20]

justification. (Any Ten)

- a) The characteristics of the computer system capable of thinking, reasoning and learning is known as Machine Intelligence.
- b) Depth First Search method takes more memory than Breadth First Search.
- c) The production rule comprises of a single rule.
- d) A knowledge representation scheme in which knowledge is represented using objects, their attributes and corresponding value of the attributes is called Relational knowledge.
- e) A heuristic function guarantees to reach to best solution.
- f) Semantic Nets are better in representing facts than Conceptual Dependency.
- g) Commutative production system always gives different results.
- h) Game Playing is not an application area of AI.
- i) Neural Networks can solve only linear problems.
- j) Syntactic Analysis identifies every token in the sentence.
- k) Fuzzy Logic cannot deal with Uncertainty.

2. Answer the following: (Any Four) [20]

- A. What are the problems of Hill Climbing methods? What are their solutions?
- B. What role does State Space Representation play in searching? What is the importance of Generator and Tester functions in State Space Search?
- C. Heuristic functions improve the search process. Give an example of Heuristic function for blocks world problem.
- D. Analyze the Seven problem characteristics for 3-Water Jug Problem.
- E. Differentiate Declarative Knowledge from Procedural Knowledge.

3. Consider the following sentences: [10]

1. Ram is an employee and he works for a company.
 2. All employees are people.
 3. Sudhir is Ram's boss.
 4. All employees either consider the boss a friend or dislike him.
 5. A person only criticizes someone who is not his friend.
 6. Ram criticizes Sudhir.
- a) Translate these sentences into formulas in predicate logic.
 - b) Find "Does Ram like Sudhir?" using Backward Chaining.

SECTION-II

4.

Do as Directed: (Any Two)

- A. What do you understand by Reasoning? What role does it play in Knowledge Representation? Distinguish between:
 a) Forward Vs. Backward Reasoning
 b) Monotonic Vs. Non-monotonic Reasoning [10]
- B. What are Hopfield Networks? Explain the Network Architecture. Explain the steps to reach to stable state in a Hopfield Network with the help of appropriate diagrams. [10]
- C. Attempt the following:
 a) Three candidates standing in an election to be mayor in a city have 0.25, 0.35 und 0.40 chance of winning according to a public opinion poll. The chances that they will build a bridge after they have been elected are 0.60, 0.90 und 0.80. What is the probability that the bridge will be built in city after the elections? [05]
 b) Explain the Certainty Factor Theory. [05]

5.

Answer the following with the help of appropriate examples: (Any Two)

[20]

- A. Take an example of your choice to discuss Natural Language Processing as a prominent application area of AI.
- B. Explain any one Probabilistic Graphical Model.
- C. What is Multilayer Perceptron? Which type of problems can it solve? Draw a labeled diagram to show a 3 layered fully connected Multilayer Perceptron. Consider input vector X and show the calculations performed at each layer to obtain the output y.

6.

Explain the following:

[10]

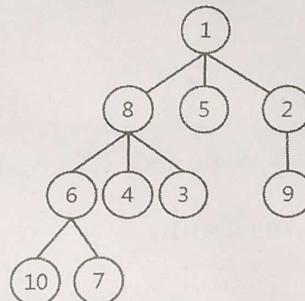
- A. Truth Maintenance System
 B. Fuzzy Logic

Instructions:

1. Figures to the right indicate full marks
2. Each section should be written in a separate answer book
3. Be precise and to the point in your answer

SECTION-I

- 1.** **Answer the following: (Any Two)** [20]
- A. a) Write any 2 definitions of Artificial Intelligence. [03]
 b) Discuss an AI technique for solving tic-tac-toe problem for a non-magic square board. [03]
 c) Describe all environment types by which Intelligent Agents get affected. [04]
- B. Analyze 7-problem characteristics of Tower of Hanoi Problem. Give proper justification for each characteristic. [10]
- C. a) What is State Space Search? Draw an example State Space for tic-tac-toe. [03]
 b) What role does Heuristics play in searching? [03]
 c) Show both, a BFS and a DFS traversal for the following tree considering node 1 as initial state and node 7 as goal state. [04]



- 2.** **Answer the following: (Any Four)** [20]
- A. Explain Steepest Ascent Hill Climbing technique with appropriate diagram of search trees.
- B. What are Partitioned Semantic Networks? What are their advantages? Give any 2 examples to show the behavior of Semantic Networks.
- C. Represent the following knowledge using Conceptual Dependency:
 a) Jack punched Tim with a broken nose.
 b) The bank manager ate pasta with a fork.
- D. Represent the following using Semantic Nets:
 a) Jack punched Tim with a broken nose.
 b) The bank manager ate pasta with a fork.
- E. Write PEAS description for an Automated Taxi Driver. [10]

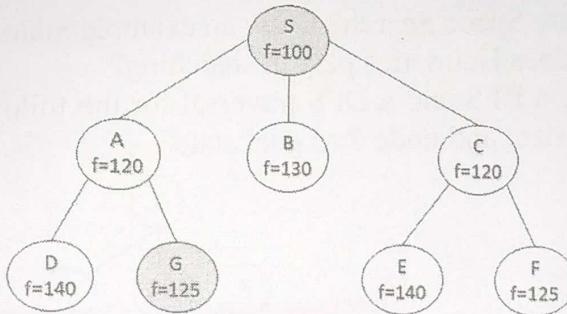
- 3.** **Consider the following sentences:**

1. Lucy is an employee and he works for a company.
2. All employees are people.
3. Fred is the boss.
4. Bosses give instructions to employees.
5. All employees do consider the boss a friend or dislike him.
6. Everyone is a friend of someone.
7. People only criticize people that are not their friends.

8. Lucy criticized Fred.
- Translate these sentences into formulas in predicate logic.
 - Find "Does Lucy like Fred?" using Backward Chaining.

SECTION-II

- 4. Answer the following: (Any Two)** [20]
- How are Expert Systems build? Explain the steps involved in the development process.
 - Discuss the additional refinements of Game Playing.
 - Which are the various issues encountered in Knowledge Representation?
- 5. Answer the following with the help of appropriate examples: (Any Two)** [20]
- Explain Mini-Max Procedure of Game playing.
 - Who is a Knowledge Engineer? What are the stages involved in designing a Knowledge Base?
 - What do you understand by Iterative Deepening? Why is it said to be Admissible and Complete? Apply IDA* algorithm on the following tree and explain each step.



Node S indicates start state while node G indicates the goal state.

- 6. Explain the following (Any Two):** [10]
- Truth Maintenance System
 - Decision Theory
 - Types of Uncertainties

1712E624 - 3 Candidate's Seat No : 11005

M.Sc. (AI & ML) (Sem.-1) (New) Examination

MSCAI 114

Object Oriented Concepts & Programming Using C++

Time : 3-00 Hours]

December 2019

[Max. Marks : 100

Note: (1) Write both the sections in different answer books.
(2) Figure to the right indicates full marks.
(3) Make necessary assumptions wherever necessary.

SECTION - I

Q-1.

Answer the Following.

(18)

- (1) What is polymorphism? What is the difference between compile time and runtime polymorphism? How C++ compiler does achieve run time polymorphism?
(2) What do you mean by object oriented programming? Explain in detail.
(3) Explain Friend function with its merits and demerits.

Q-2.

Answer the Following.

(16)

- (1) What is constructor? Explain different types of constructor.
(2) Why we need friend function to overload operator? Explain it using example.

OR

- (1) Explain rules to overload the operator in C++.
(2) Explain copy constructor with example.

Q-3.

Answer the Following. (Any Two)

(16)

- (1) Explain following terms :
 i) pure virtual function
 ii) this pointer
(2) What is stream? Explain advantages of using C++ I/O over C I/O.
(3) Write a C++ program to implement template class STACK with all operations.

SECTION - II

Q-4.

Answer the Following.

(18)

- (1) Explain difference between Manipulators and IOS member functions.
(2) What are the issues one must consider while dealing with multiple inheritance? How to avoid it. Explain it using example.
(3) Explain four different cases where user defined conversion are needed.

[P.T.O]

Q-5. Answer the Following.

- (1) Explain derivation using different access modifier.
- (2) Define a class **student** with appropriate data members and store five subject marks and make mark sheet like total, percentage, grade, etc. using appropriate member function. Write a C++ program to test a class that can store more than one student information.

OR.

- (1) Explain MIL with example
- (2) Define a class **employee** with appropriate data members and store salary detail like basic salary, HRA, DA, Gross, etc. using appropriate member function. Write a C++ program to test a class that can store more than one employee information.

Q-6. Write a short note on the following. (Any Two)

- (1) IO modes
 - (2) Standard Template Library (STL)
 - (3) Namespaces
-
-

Note : (1) Write both the sections in the separate answer books
 (2) Figures to the right indicate full marks.
 (3) Make necessary assumptions wherever necessary.

SECTION-I

- Q.1** Answer the following [16]
 (a) Compare and Contrast Inline function and normal function
 (b) Describe the following terms with Example
 i) this pointer ii) Dynamic binding
 (c) Explain Scope resolution operator.
 (d) Explain different data types in C++.
- Q.2** Answer the following (Any four) [16]
 (a) Explain function template and class template with examples.
 (b) What do you mean by object oriented programming? Explain in detail.
 (c) What is Member Initialization List (MIL)? Explain giving suitable example.
 (d) What is the use of private member function?
 (e) Explain importance of destructor?
- Q.3** Answer the following (Any three) [18]
 (a) Difference between : i) Aggregation and Generalization
 ii) Manipulators and ios member function
 (b) What are abstract classes? Give few examples of abstract classes.
 (c) Explain static variable and static function with example.
 (d) Explain namespace in detail.

SECTION-II

- Q.4** Explain the following: [16]
 (a) What is Friend Function? What are the merits and demerits of using friend function?
 (b) What is an operator function? Write a program to overload binary + operator as a member function for a class of your choice.
 (c) What is stream? Describe various stream classes for console I/O operations.
 (d) What is polymorphism? What is the difference between compile time and runtime polymorphism?
- P.T.O.

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Q.5

Answer the following:

[16]

- (a) What do you mean by implicit and explicit conversion? Explain with suitable example.
- (b) Explain copy constructor giving suitable C++ programming example.

OR

Q.5

Answer the following:

[16]

- (a) What is Virtual Function? What is its need? Explain with example.
- (b) What is an Exception? Explain Exception Handling Mechanism. Give example with multiple catch blocks.

Q.6

Write short note on the following (Any Three)

[18]

- (a) tellg and seekp
- (b) Inheritance types
- (c) IO modes
- (d) Standard Template Library (STL)

M.Sc. (Sem.-1) (AI & ML) (Old) Examination

Mathematical Foundation

December 2019

[Max. Marks : 100]

Time : 3-00 Hours]

- Instructions :**
- (1) Figures to the right indicate Full Marks.
 - (2) All questions are compulsory.
 - (3) Write both sections in the separate answer books.
 - (4) Make necessary assumptions wherever necessary.

SECTION-I

- 1 Attempt any three : 18
- (a) Define consist system of linear equations. Determine if the following system is consistent :

$$2x - 3y = -2$$

$$2x + y = 1$$

$$3x + 2y = 1$$

- (b) What is an echelon form of a matrix? Row reduce the matrix :

$$\begin{bmatrix} 1 & -5 & 7 \\ 6 & 9 & 7 \\ -9 & 6 & 1 \end{bmatrix}$$

- (c) Find the inverse of the matrix : $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$. Using elementary row transformation.

- (d) Apply Gauss elimination method to solve the following system of equations :

$$x + y + z = 1$$

$$3x + 5y + 6z = 4$$

$$9x + 2y - 36z = 17$$

- 2 Attempt any two : 16

- (a) Diagonalize a matrix $A = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix}$ if possible.

- (b) Define eigen values and eigen vectors of a square matrix. Determine eigen value of the matrix :

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

- (c) Define rank of a matrix. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

[P.T.O.]

3 Attempt any two from the following :

- (a) A, B and C toss a coin in succession on the undering that first one to throw a head wins.
What are their respective chances of winning?
- (b) Three cards are drawn from a pack of 52. Find the probability that they are of the same colour.
- (c) Three light bulbs are chosen at random from fifteen bulbs of which five are defective.
Find the probability that :
(i) none is defective
(ii) exactly one is defective
(iii) at least one is defective.

SECTION II

4 Attempt any three from the following :

- (a) Is the set V of all pairs of real numbers (x, y) with the operations.
Addition : $(x_1, y_1) + (x_2, y_2) = (x_1 + y_2 + 5, y_1 + y_2 + 5)$
Scalar multiplication : $k(x, y) = (kx, ky)$
a vector space ? If yes, then prove it, if no then identify all axioms that fail to hold.
- (b) Show that the set $w = \{[x, y, z] | y = x + z\}$ is a subspace of R^3 under usual addition and scalar multiplication.
- (c) Define linear combination of vectors $\{v_1, v_2, \dots, v_n\}$. Is the vector $[0, 5, 4]$ liner combination of $v_1 = [0, -2, 2]$ and $v_2 = [1, 3, -1]$?

(d) Show that $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, M_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, M_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $M_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

is a basis for the vector space M_{22}

- (e) Find the coordinate vector of $v = [2, -1, 1]$ relative to the basis
 $s = \{[1, 1, 0], [1, 0, 1], [0, 1, 1]\}$

5 Attempt any two from the following :

- (a) Find the row space and column space of the matrix

$$A = \begin{bmatrix} 1 & 5 & -1 \\ 2 & -3 & -6 \\ -1 & 0 & 2 \\ 4 & 1 & 1 \end{bmatrix}$$

- (b) Find a basis for the null space of the matrix.

$$A = \begin{bmatrix} -1 & 1 & 1 \\ 3 & -1 & 0 \\ 2 & -4 & -5 \end{bmatrix}$$

- (c) Show that the transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ as under $T(x, y, z) = x, y, -z$ is a linear operator on \mathbb{R}^3 .

OR

- (c) Determine the linear transformation T of the following whose images of basis vectors are given below :

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2, \text{ basis } B = \{[1, 0], [0, 1]\}, T[1, 0] = [1, 1] \text{ & } T[0, 1] = [2, -1]$$

Also compute $T(5, -2)$

- 6 Attempt any two from the following :

16

- (a) Find the associated matrix of the following linear transformation.

$$T : \mathbb{R}^3 \rightarrow \mathbb{R}^2, T(x_1, x_2, x_3) = 2x_1 = x_2 + x_3, x_2 - 4x_3$$

with bases $B = \{[1, 0, 0], [1, 1, 0], [1, 1, 1]\}$ and

$B = \{[1, 1] [1, -1]\}$ for the domain and co-domain of T respectively.

- (b) Compute $\langle u, v \rangle // u //$ and $\langle v, v \rangle$ for the given pair of vectors and inner product.
 $u = x, v = x^2$ in $C[0, 1]$ using inner product

$$\langle u, v \rangle = \int_0^1 uv dx.$$

OR

- (b) Verify the Cauchy-Schwarz inequality for the vectors.

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

respect to inner product $\langle A, B \rangle = \text{tr}(A^T B)$

- (c) Determine which of the following pair of vectors are orthogonal with respect to Euclidean inner product.

$$(i) \quad u = [2, 1], v = [-1, 2]$$

$$(ii) \quad u = [1, 2, -2, 1], r = [4, 0, 4, 0]$$

Note: (1) Figures to the right indicates full marks of the respective question.

(2) Intermediate calculation steps and results are to be shown.

SECTION - I

Q:1 Answer the following (Any two)

[16]

- (a) (i) Define power set. Find the power set of $\{\{a\}, \{b, c\}, \{a, d\}\}$.
 (ii) Let $A = \{-3, -2, -1, 0, 1, 2\}$. Find the range of the function $f: A \rightarrow \mathbb{R}$, defined by $f(x) = x^2 - 5x + 6$ for all $x \in A$.
- (b) Find an equation of tangent lines to the hyperbola $x^2 - y^2 = 16$ that passes through point $(2, -2)$.
- (c) Find $\frac{dy}{dx}$ where

$$(i) \quad y = \frac{x}{\sqrt{x^2 - 1}} \quad (ii) \quad y = \cos\left(\frac{e^{x^2}}{1-x^2}\right)$$

Q:2 Answer the following (Any Two)

[14]

- (a) Using Taylor series expansion, obtain the approximate value of $\sqrt{1.05}$. Assume $f(x) = \sqrt{1+x}$.
- (b) Each set X_r contains 5 elements and each set Y_r contains 2 elements and $\bigcup_{r=1}^{20} X_r = S = \bigcup_{r=1}^n Y_r$. If each element of S belongs to exactly 10 of the X_r 's and to exactly 4 of the Y_r 's, then find n .
- (c) Find local extreme values of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

Q:3 Answer the following (Any Two)

[20]

- (a) Given that $z = 4e^x \ln(y)$, $x = \ln(u \cos v)$, $y = u \sin v$. Find $\partial z / \partial u$ and $\partial z / \partial v$ using the chain rule. Also, evaluate them at $(u, v) = (2, \pi/4)$.

- (b) If $Y = \{1, 2, \dots, 10\}$, $L = \{a \in Y \mid a^2 \notin Y\}$, $M = \{a \in Y \mid a+1=6\}$, and $N = \{a \in Y \mid a \text{ is not more than } 6\}$. Draw Venn diagram showing the relation of Y, L, M and N . Is the statement $L - (M - N) = (L - M) - N$ is true? Justify your answer.
- (c) Find the directional derivative of $f(x, y) = e^x \cos(y)$ at $(1, \pi/4)$ in the direction of the unit vector that makes an angle of $\pi/3$ with the positive x -axis.

SECTION - II**Q:4** Answer the following (Any Two)**[16]**

- (a) Consider the two sets of vectors $S_1 = \{(1, 2, -1, 3), (2, 4, 1, -2), (3, 6, 3, -7)\}$ and $S_2 = \{(1, 2, -4, 11), (2, 4, -5, 14)\}$ in \mathbb{R}^4 . Let $U = \text{Span}\{S_1\}$, and $W = \text{Span}\{S_2\}$. Show that $U = W$.
- (b) (i) Find the matrix for 45° counterclockwise rotation of the plane about the origin.
(ii) Obtain the projection of $u = (1, -2, 3, -4)$ on $v = (1, 2, 1, 2)$.
- (c) Let $u = (1, 3, -4, 2)$, $v = (4, -2, 2, 1)$, $w = (5, -1, -2, 6)$ in \mathbb{R}^4 . (i) Show that $\langle 3u - 2v, w \rangle = 3\langle u, w \rangle - 2\langle v, w \rangle$, (ii) normalize u and v .

Q:5 Answer the following (Any Two)**[14]**

- (a) Express M as a linear combination of matrices A, B, C where
 $M = \begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix}$, and $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$
- (b) Check whether $(3, 7, 4)$ in $\text{Span}\{(1, 2, 3), (2, 3, 7), (3, 5, 6)\}$.
- (c) Obtain orthonormal basis from the given basis $\{(1, 1, -1), (0, 3, 5), (1, 2, 1)\}$.

Q:6 Answer the following (Any Two)**[20]**

- (a) Define linearly independent vectors and linearly dependent sets. Are the vectors $u_1 = (1, 1, 2)$, $u_2 = (1, 2, 5)$, $u_3 = (5, 3, 4)$. If so, find the relation between them. Also, extend it as a basis of the vector space \mathbb{R}^3 .
- (b) Contrast between $\text{Nul } A$ and $\text{Col } A$ for $m \times n$ matrix A .
- (c) Define orthonormal set of a vector space. Explain Gram-Schmidt process to construct orthonormal set.
