Linear Algebra and Numerical Methods

Course Contents

- Introduction to Matrices
- Introduction to Numerical Methods
- Linear Equations
- Eigen Values and Eigen Vectors
- Computational Complexity

Linear Algebra

- Linear algebra is the branch of mathematics concerning vector spaces and linear mappings between such spaces. It includes the study of lines, planes and subspaces, but is also concerned with properties common to all vector spaces.

Linear Algebra

- Provides a way to compactly represent and operate on sets of linear equations
- In machine learning, we represent data as matrices

Matrices

Linear Algebra and its Applications , David C. Lay

Linear Algebra and its Applications ,Gilbert Strang

Linear Algebra and Probability for Computer Science Applications – Ernest Davis

Matrix Notation

- A Matrix consists of a rectangular array of elements represented by a single symbol. [A] is the notation for the matrix and a_{ij} designates individual element of the matrix.

$$\begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \dots & a_{ij} & \dots \\ a_{n1} & \cdots & a_{nm} \end{bmatrix}$$

Matrix Notation

$$\begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \dots & a_{ij} & \dots \\ a_{n1} & \cdots & a_{nm} \end{bmatrix}$$

- A horizontal set of elements is called a row and a vertical set is called a column. The first subscript *i* always designates the number of the row in which the element lies. The second subscript *j* designates the column. For example, element a_{23} is in row 2 and column 3.
- The matrix has n rows and m columns and is said to have a dimension of n by m (or n x m).

Special Types of Matrices

- A *zero matrix* is the a square matrix where all elements are equal to zero, as in

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- An *identity matrix* is a diagonal matrix where all elements on the main diagonal are equal to 1, as in

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Special Types of Matrices

- An *upper triangular matrix* is the one where all the elements below the main diagonal are zero

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

- A *lower triangular matrix* is the one where all elements above the main diagonal are zero

$$\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Diagonal Matrix

- A *diagonal matrix* is the a square matrix where all elements off the main diagonal are equal to zero, as in

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

Diagonal of a Matrix

- Let A= [a_{ij}] be a n-square matrix. The diagonal or main diagonal of A consists of the elements with the same subscripts i.e. a_{11},a_{22},…..a_{nn}. It is denoted by diag(A)

- Eg. $diag\left(\begin{bmatrix}1 & 4 & 5\\ 4 & 2 & 7\\ 5 & 2 & 6\end{bmatrix}\right) = [1, 2, 6]$

Trace of a Matrix

- Trace of matrix A, denoted by trA, is the sum of the elements along the diagonal of A
- i.e. $trA = a_{11} + a_{22} + \ ....+a_{\mathrm{nn}}$
- eg. For $\left(A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 2 & 7 \\ 5 & 2 & 6 \end{bmatrix} \right)$

  trA= 1 + 2 + 6 = 9

Block or Partitioned Matrix

- A key feature of our work with matrices has been the ability to regard a matrix A as a list of column vectors rather than just a rectangular array of numbers. This point of view has been so useful that we wish to consider other partitions of A, indicated by horizontal and vertical dividing rules, as in Example below.
- $\left[\begin{array}{cc|cc|cc} 3 & 0 & -1 & 5 & 9 & -2 \\ -5 & 2 & 4 & 0 & -3 & 1 \end{array}\right]$

Augmented Matrix

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{2n} \\ a_{m1} & a_{m2} & a_{m3} & a_{mn} \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \\ b_m \end{bmatrix}$

Then, the system of equations

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + \ \ldots \ldots . + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ \ldots \ldots . + a_{2n}x_n = b_2 \\ a_{m1}x_1 + a_{m2}x_2 + \ \ldots \ldots . + a_{mn}x_n = b_m \end{gathered}$$

can be rewritten as Ax = b, where A is called the coefficient matrix and the matrix [A | b] is called the augmented matrix.

Eg. Given the system

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 - x_3 &= 10 \end{aligned}$$

The matrix $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 8 \\ 5 & 0 & -1 \end{bmatrix}$ is called the coefficient matrix

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -1 & 10 \end{array}\right]$ is called the augmented matrix of the system.

Scalar Multiplication

- The product of the matrix A by a scalar k, written k.a or simply kA, is the matrix obtained by multiplying each element of A by k.

$$kA = \begin{bmatrix} ka_{11} & \cdots & ka_{1n} \\ \vdots & \ddots & \vdots \\ ka_{m1} & \cdots & ka_{mn} \end{bmatrix}$$

Example: Scalar Multiplication

A = $\begin{bmatrix} 2 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, k = 3, find 3.A

Solution

3.A = $\begin{bmatrix} 3(2) & 3(2) & 3(0) \\ 3(1) & 3(0) & 3(0) \\ 3(0) & 3(0) & 3(3) \end{bmatrix}$

= $\begin{bmatrix} 6 & 6 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

Matrix Addition

Addition of two matrices, [A] and [B], is accomplished by adding corresponding terms in each matrix. The elements of the resulting matrix [C] are computed,

$c_{ij} = a_{ij} + b_{ij}$ for i = 1, 2, . . ., n and j = 1, 2, . . ., m.

Example: Addition

A = $\begin{bmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}$, find A + B

Solution

A + B = $\begin{bmatrix} 4+1 & 0+1 & 5+1 \\ -1+3 & 3+5 & 2+7 \end{bmatrix}$

= $\begin{bmatrix} 5 & 1 & 6 \\ 2 & 8 & 9 \end{bmatrix}$

Matrix Subtraction

Similarly, the subtraction of two matrices, [A] minus [B], is obtained by subtraction corresponding terms,

c_{ij} = a_{ij} - b_{ij} for i = 1, 2, . . ., n and j = 1, 2, . . ., m.

- It follows directly from the above definition that addition and subtraction can be performed only between matrices having the same dimensions

Properties of Matrix Addition and Scalar Multiplication

Let A, B, and C be matrices of the same size, and let r and s be scalars.

- $A + B = B + A$
- $(A + B) + C = A + (B + C)$
- $A + 0 = A$
- $r(A + B) = rA + rB$
- $(r + s)A = rA + sA$
- $r(sA) = (rs)A$

Multiplication of Matrices

- The product AB (or $A \cdot B$) of two matrices A and B is defined only when:
  - The number of columns in A is equal to the number of rows in B.

Multiplication of Matrices

- This means that, if we write their dimensions side by side, the two inner numbers must match:

Matrices	A	B
Dimensions	$m \times \boxed{n}$	$\boxed{n} \times k$

Columns in A Rows in B

If the dimensions of A and B match in this fashion, then the product AB is a matrix of dimension $m \times k$.

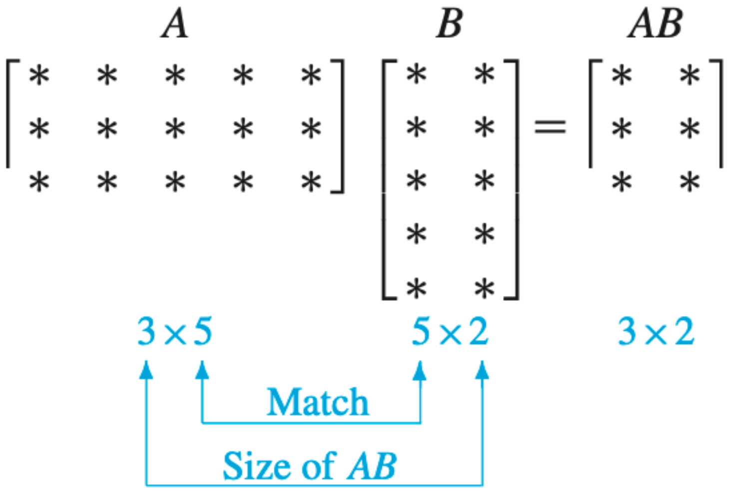
Example Multiplication of Matrices

If A is a 3×5 matrix and B is a 5×2 matrix, what are the sizes of AB and BA, if they are defined?

Example Multiplication of Matrices

If A is a 3×5 matrix and B is a 5×2 matrix, what are the sizes of AB and BA, if they are defined?

- Since A has 5 columns and B has 5 rows, the product AB is defined and is a 3×2 matrix:



- The product BA is *not* defined because the 2 columns of B do not match the 3 rows of A.

Matrix Operating Rules

- The product of two matrices is represented a [C] = [A][B], where the elements of [C] are defined as

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

where n= the column dimension of [A] and the row dimension of [B]. That is, the c_{ij} element is obtained by adding the product of individual elements from the i^{th} row of the first matrix, in this case [A], by the j^{th} column of the second matrix [B]

Multiplying Matrices

If we define $C = AB = [c_{ij}]$, the entry c_{11} is the inner product of the first row of A and the first column of B:

$c_{11} = a_{11}.b_{11} + a_{12}.b_{21}$

Entry	Product	Value	Product Matrix
c_{11}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$1 \cdot (-1) + 3 \cdot 0 = -1$	$\begin{bmatrix} -1 & & \\ & & \end{bmatrix}$

Multiplying Matrices

Entry	Product	Value	Product Matrix
c_{12}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$1 \cdot 5 + 3 \cdot 4 = 17$	$\begin{bmatrix} -1 & 17 & \\ & & \end{bmatrix}$
c_{13}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$1 \cdot 2 + 3 \cdot 7 = 23$	$\begin{bmatrix} -1 & 17 & 23 \\ & & \end{bmatrix}$

Multiplying Matrices

Entry	Product	Value	Product Matrix
c_{21}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$(-1) \cdot (-1) + 0 \cdot 0 = 1$	$\begin{bmatrix} -1 & 17 & 23 \\ 1 & & \end{bmatrix}$
c_{22}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$(-1) \cdot 5 + 0 \cdot 4 = -5$	$\begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & \end{bmatrix}$
c_{23}	$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$	$(-1) \cdot 2 + 0 \cdot 7 = -2$	$\begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$

Multiplying Matrices

- However, the product *BA* is not defined—because the dimensions of *B* and *A* are 2 x 3 and 2 x 2.
- The inner two numbers are not the same.
- So, the rows and columns won't match up when we try to calculate the product.

Properties of Matrix Multiplication

- Associative : Matrix Multiplication is Associative

  $(AB)C = A(BC)$
- Distributive : Matrix Multiplication is Distributive

  $A(B + C) = AB + BC$
- Commutative : Matrix Multiplication is not commutative.

  $A B \neq BA$
- $r(AB) = (rA)B = A(Br)$, where r is a scalar
- $IA = A = AI$

Property of matrix multiplication

- Suppose A =$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} B = \begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix}$, find AB and BA

Solution

AB = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix}$ =

and

BA = $\begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ =

Property of matrix multiplication

- Suppose A =$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} B = \begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix}$

Solution

AB = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} . \begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 5+0 & 6-4 \\ 15+0 & 18-8 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 15 & 10 \end{bmatrix}$

and

BA = $\begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix} . \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5+18 & 10+24 \\ 0-6 & 0-8 \end{bmatrix} = \begin{bmatrix} 23 & 34 \\ -6 & -8 \end{bmatrix}$

The above example shows that matrix multiplication is not commutative.

i.e. A B $\neq$ BA

Powers of a Matrix

- If A is an n×n matrix and if k is a positive integer, then A^k denotes the product of k copies of A:
- $A^k = A \dots \dots A(k \; times)$
- If A is nonzero and if x is in R^n, then $A^k \; x$ is the result of left multiplying x by A repeatedly k times. If k = 0, then $A^0 \; x$ should be x itself. Thus A^0 is interpreted as the identity matrix.

Hadamard Product

- Hadamard product or element wise product is denoted $\odot$ or O
- The Hadamard product is only defined over matrices of equal size and returns a matrix of the same size
- Eg.

$$\begin{bmatrix} 5 & 10 \\ -2 & 0 \\ 1 & -1 \end{bmatrix} \odot \begin{bmatrix} 2 & -1 \\ -1 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5\times 2 & 10\times -1 \\ -2\times -1 & 0\times 5 \\ 1\times 0 & -1\times 1 \end{bmatrix} = \begin{bmatrix} 10 & -10 \\ 2 & 0 \\ 0 & -1 \end{bmatrix}$$

Hadamard Product

- Find the Hadamard Product

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \odot \begin{bmatrix} 1 & 4 & 7 \\ 8 & 20 & 5 \\ 2 & 8 & 3 \end{bmatrix}$$

Hadamard Product

- Find the Hadamard Product

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \odot \begin{bmatrix} 1 & 4 & 7 \\ 8 & 20 & 5 \\ 2 & 8 & 3 \end{bmatrix}$$

Solution

$$= \begin{bmatrix} 1 & 8 & 21 \\ 32 & 100 & 30 \\ 14 & 64 & 27 \end{bmatrix}$$

Properties of Hadamard product

- Associative : Hadamard product is Associative

  $(AB)C = A(BC)$

- Distributive : Hadamard product is Distributive

  $A(B + C) = AB + BC$

- Commutative : Hadamard product is commutative.

  $A\,B = BA$

- Hadamard product is used in image compression techniques such as JPEG.
- It is also used in LSTM(Long Short-Term Memory) cells of Recurrent Neural Networks(RNNs)
- Used in Tensors

Kronecker Product

Definition Let A be a $K \times L$ matrix and B an $M \times N$ matrix. Then, the Kronecker product between A and B is the $(KM \times LN)$ block matrix

$$A \otimes B = \begin{bmatrix} A_{11}B & \dots & A_{1L}B \\ \vdots & \ddots & \vdots \\ A_{K1}B & \dots & A_{KL}B \end{bmatrix}$$

where A_{kl} denotes the (k,l)-th entry of A.

In other words, the Kronecker product $A \otimes B$ is a block matrix whose (k,l)-th block is equal to the (k,l)-th entry of A multiplied by the matrix B.

Note that, unlike the ordinary product between two matrices, the Kronecker product is defined regardless of the dimensions of the two matrices A and B.

Kronecker Product

- Find the Kronecker product of the following matrices

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & -1 \\ -1 & 4 \end{bmatrix}$$

- Solution

$$A \otimes B = \begin{bmatrix} 2B & 0B \\ 1B & 3B \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -2 & 0 & 0 \\ -2 & 8 & 0 & 0 \\ 5 & -1 & 15 & -3 \\ -1 & 4 & -3 & 12 \end{bmatrix}$$

Kronecker Product

- Find the Kronecker product of the following matrices

a) $A = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 0 \\ 2 & 7 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & -1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 0 & 5 \end{bmatrix}$

c) $A = \begin{bmatrix} 3 & 6 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Kronecker Product

a) Find the Kronecker product of the following matrices

$$A = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 2 & 7 \end{bmatrix}$$

Solution

$$A \otimes B = \begin{bmatrix} 3B \\ 2B \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 6 & 21 \\ 2 & 0 \\ 4 & 14 \end{bmatrix}$$

Kronecker Product

b) Find the Kronecker product of the following matrices

$$A = \begin{bmatrix} 1 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 5 \end{bmatrix}$$

Solution

$$A \otimes B = \begin{bmatrix} B & -B \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 5 & -1 & 0 & -5 \end{bmatrix}$$

Kronecker Product

c) Find the Kronecker product of the following matrices

$$A = \begin{bmatrix} 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Soluiton

$$A \otimes B = \begin{bmatrix} 3B & 6B \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 0 & 0 \\ 3 & 6 \end{bmatrix}$$

Properties of Kronecker product

- Associative: Kronecker product is Associative
  $$(A \otimes B) \otimes C = A \otimes (B \otimes C)$$
- Distributive : Kronecker product is Distributive
  $$A \otimes (B + C) = A \otimes B + B \otimes C$$
  $$(A + B) \otimes C = A \otimes C + B \otimes C$$
- Commutative : Kronecker product is not commutative.
  $$(A \otimes B) \neq (B \otimes A)$$
- $r \otimes B = rB$ where r is a scalar
- $(rA) \otimes (sB) = (rs)(A \otimes B)$ where r,s are scalars
- $A \otimes 0 = 0$ and $0 \otimes B = 0$

Transpose of Matrices

- Transpose of a matrix is obtained by switching the row elements with the column elements. We denote the transpose of a matrix A by A^T

  For example,

  A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ then $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

Properties of Transpose

- $(A^T)^T = A$
- $(A + B)^T = A^T + B^T$
- For any scalar r, $(rA)^T = rA^T$
- $(AB)^T = B^T A^T$

Examples

A = $\begin{bmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}$, C = $\begin{bmatrix} 3 & 1 \\ 8 & 6 \\ 0 & 4 \end{bmatrix}$, D = $\begin{bmatrix} 5 & 9 \\ 7 & 2 \end{bmatrix}$

Compute:

a) 2B

b) A- 2B

c) CD

d) C^T

e) diag(D)

f) tr(D)

Solution

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}$$

a) $$2B = \begin{bmatrix} 2 & 2 & 2 \\ 6 & 10 & 14 \end{bmatrix}$$

Solution

A = $\begin{bmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}$

b) A - 2B = $\begin{bmatrix} 4-2 & 0-2 & 5-2 \\ -1-6 & 3-10 & 2-14 \end{bmatrix}$ = $\begin{bmatrix} 2 & -2 & 3 \\ -7 & -7 & -12 \end{bmatrix}$

Solution

C = $\begin{bmatrix} 3 & 1 \\ 8 & 6 \\ 0 & 4 \end{bmatrix}$, D = $\begin{bmatrix} 5 & 9 \\ 7 & 2 \end{bmatrix}$

c) CD = $\begin{bmatrix} 22 & 29 \\ 82 & 84 \\ 28 & 8 \end{bmatrix}$

Solution

$$C = \begin{bmatrix} 3 & 1 \\ 8 & 6 \\ 0 & 4 \end{bmatrix}$$

d) $C^T = \begin{bmatrix} 3 & 8 & 0 \\ 1 & 6 & 4 \end{bmatrix}$

Solution

D = $\begin{bmatrix} 5 & 9 \\ 7 & 2 \end{bmatrix}$

e) diag(D) =$\begin{bmatrix} 5 & 2 \end{bmatrix}$

Solution

D = $\begin{bmatrix} 5 & 9 \\ 7 & 2 \end{bmatrix}$

f) tr(D) = $5 + 2 = 7$

Example

a) If a matrix A is 5×3 and the product AB is 5×7, what is the size of B ?

b) How many rows does B have if BC is a 3×4 matrix?

c) Let A= $\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any, will make AB = BA?

d) Compute AB and BA,

where A = $\begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}$

Solution

a) If a matrix A is 5×3 and the product AB is 5×7, what is the size of B ?

- Since A has 3 columns, B must match with 3 rows. Otherwise, AB is undefined. Since AB has 7 columns, so does B. Thus, B is 3×7.

Solution

b) How many rows does B have if BC is a 3×4 matrix?

The number of rows of B matches the number of rows of BC, so B has 3 rows.

Solution

c) Let A= $\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any, will make AB = BA?

AB = $\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$ = $\begin{bmatrix} 23 & -10+5k \\ -9 & 15+k \end{bmatrix}$

BA = $\begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix} \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ = $\begin{bmatrix} 23 & 15 \\ 6-3k & 15+k \end{bmatrix}$

Solution

c) Let A= $\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any, will make AB = BA?

- Then AB = BA

if and only if –10 + 5k = 15 and –9 = 6 – 3k,

which happens if and only if k = 5.

Solution

d) Compute AB and BA,

where A = $\begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}$

AB = $\begin{bmatrix} 11 & 0 & 21 \\ -1 & 13 & -9 \end{bmatrix}$

Solution

d)

- However, the product *BA* is not defined—because the dimensions of *B* and *A* are 2 x 3 and 2 x 2.

- The inner two numbers are not the same.

- So, the rows and columns won't match up when we try to calculate the product.