Gauss –Jordan Elimination Process

Eg2.

$$3x_1 + 18x_2 + 9x_3 = 18$$
$$2x_1 + 3x_2 + 3x_3 = 117$$
$$4x_1 + x_2 + 2x_3 = 283$$

Eg3.

$$2x_1 + x_2 + 2x_3 = 10$$
$$x_1 + 2x_2 + x_3 = 8$$
$$3x_1 + x_2 - x_3 = 2$$

Gauss –Jordan Elimination Process

Eg2.

$$3x_1 + 18x_2 + 9x_3 = 18$$
$$2x_1 + 3x_2 + 3x_3 = 117$$
$$4x_1 + x_2 + 2x_3 = 283$$

The augmented matrix can be written as

$$\left[\begin{array}{ccc|c} 3 & 18 & 9 & 18 \\ 2 & 3 & 3 & 117 \\ 4 & 1 & 2 & 283 \end{array}\right]$$

Gauss –Jordan Elimination Process

$$\left[\begin{array}{ccc|c} 3 & 18 & 9 & 18 \\ 2 & 3 & 3 & 117 \\ 4 & 1 & 2 & 283 \end{array}\right]$$

The first step is to divide the first row by 3 gives us

Replace R_1 by (1/3) R_1

$$\left[\begin{array}{ccc|c} 1 & 6 & 3 & 6 \\ 2 & 3 & 3 & 117 \\ 4 & 1 & 2 & 283 \end{array}\right]$$

Gauss –Jordan Elimination Process

$$\begin{bmatrix} 1 & 6 & 3 & \bigg| & 6 \\ 2 & 3 & 3 & \bigg| & 117 \\ 4 & 1 & 2 & \bigg| & 283 \end{bmatrix}$$

- Subtract the two times reduced first row from the second row and also multiply the first row by 4 and then subtract from the third, gives us

Replace R_2 by $R_2 - 2R_1$ and R_3 by $R_3 - 4R_1$ to get

- $\begin{bmatrix} 1 & 6 & 3 & \bigg| & 6 \\ 0 & -9 & -3 & \bigg| & 105 \\ 0 & -23 & -10 & \bigg| & 259 \end{bmatrix}$

Gauss –Jordan Elimination Process

- $\left[\begin{array}{ccc|c} 1 & 6 & 3 & 6 \\ 0 & -9 & -3 & 105 \\ 0 & -23 & -10 & 259 \end{array}\right]$

The second step is to divide the second row by -9 gives us

Replace R_2 by (-1/9) R_2

- $\left[\begin{array}{ccc|c} 1 & 6 & 3 & 6 \\ 0 & 1 & {}^{1}/_{3} & {}^{-35}/_{3} \\ 0 & -23 & -10 & 259 \end{array}\right]$

Gauss –Jordan Elimination Process

- $\left[\begin{array}{ccc|c} 1 & 6 & 3 & 6 \\ 0 & 1 & {}^{1}/_{3} & {}^{-35}/_{3} \\ 0 & -23 & -10 & 259 \end{array}\right]$
- We reduce the second column to [0,1,0] by row operations

Replace R_1 by $R_1 - 6R_2$ and R_3 by $R_3 + 23R_2$ to get

- $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 76 \\ 0 & 1 & {}^{1}/_{3} & {}^{-35}/_{3} \\ 0 & 0 & {}^{-7}/_{3} & {}^{28}/_{3} \end{array}\right]$

Gauss –Jordan Elimination Process

- $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 76 \\ 0 & 1 & {}^{1}/_{3} & {}^{-35}/_{3} \\ 0 & 0 & {}^{-7}/_{3} & {}^{28}/_{3} \end{array}\right]$
- The third step is to divide the third row by -7/3 gives us

Replace R_3 by (-7/3) R_3

- $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 76 \\ 0 & 1 & {}^{1}/_{3} & {}^{-35}/_{3} \\ 0 & 0 & 1 & 4 \end{array}\right]$

Gauss –Jordan Elimination Process

- $\begin{bmatrix} 1 & 0 & 1 & \big| & 76 \\ 0 & 1 & {}^{1}/_{3} & \big| & {}^{-35}/_{3} \\ 0 & 0 & 1 & \big| & 4 \end{bmatrix}$

Replace R_1 by $R_1 - R_3$, R_3 by $R_2 - (1/3)R_3$

- $\begin{bmatrix} 1 & 0 & 0 & \big| & 72 \\ 0 & 1 & 0 & \big| & -13 \\ 0 & 0 & 1 & \big| & 4 \end{bmatrix}$
- The solution is $x_1 = 72,\ x_2 = -13,\ x_3 = 4$

Gauss –Jordan Elimination Process

Eg3.

$$\begin{aligned} 2x_1 + x_2 + 2x_3 &= 10 \\ x_1 + 2x_2 + x_3 &= 8 \\ 3x_1 + x_2 - x_3 &= 2 \end{aligned}$$

The augmented matrix can be written as

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 10 \\ 1 & 2 & 1 & 8 \\ 3 & 1 & -1 & 2 \end{array}\right]$$

Gauss –Jordan Elimination Process

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 10 \\ 1 & 2 & 1 & 8 \\ 3 & 1 & -1 & 2 \end{array}\right]$$

We want a 1 in row one, column one. This can be obtained by dividing the first row by 2 or interchanging the second row with the first. Interchanging the rows is a better choice because that way we avoid fractions.

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 8 \\ 2 & 1 & 2 & 10 \\ 3 & 1 & -1 & 2 \end{array}\right]$$

Gauss –Jordan Elimination Process

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 8 \\ 2 & 1 & 2 & 10 \\ 3 & 1 & -1 & 2 \end{array}\right]$$

Subtract the two times reduced first row from the second row and also multiply the first row by 3 and then subtract from the third, gives us

Replace R_2 by $R_2 - 2R_1$ and R_3 by R_3 $-3R_1$ to get

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 8 \\ 0 & -3 & 0 & -6 \\ 0 & -5 & -4 & -22 \end{array}\right]$$

Gauss –Jordan Elimination Process

- $\begin{bmatrix} 1 & 2 & 1 & | & 8 \\ 0 & -3 & 0 & | & -6 \\ 0 & -5 & -4 & | & -22 \end{bmatrix}$

The second step is to divide the second row by -3 gives us

Replace R_2 by (-1/3) R_2

- $\begin{bmatrix} 1 & 2 & 1 & | & 8 \\ 0 & 1 & 0 & | & 2 \\ 0 & -5 & -4 & | & -22 \end{bmatrix}$

Gauss –Jordan Elimination Process

- $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 8 \\ 0 & 1 & 0 & 2 \\ 0 & -5 & -4 & -22 \end{array}\right]$
- We reduce the second column to [0,1,0] by row operations

Replace R_1 by $R_1 - 2R_2$ and R_3 by $R_3 + 5R_2$ to get

- $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -4 & -12 \end{array}\right]$

Gauss –Jordan Elimination Process

- $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -4 & -12 \end{array}\right]$
- The third step is to divide the third row by -4 gives us

Replace R_3 by (-1/4) R_3

- $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Gauss –Jordan Elimination Process

- $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

Replace R_1 by $R_1 - R_3$

- $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$
- The solution is $x_1 = 1, \; x_2 = 2, \; x_3 = 3$