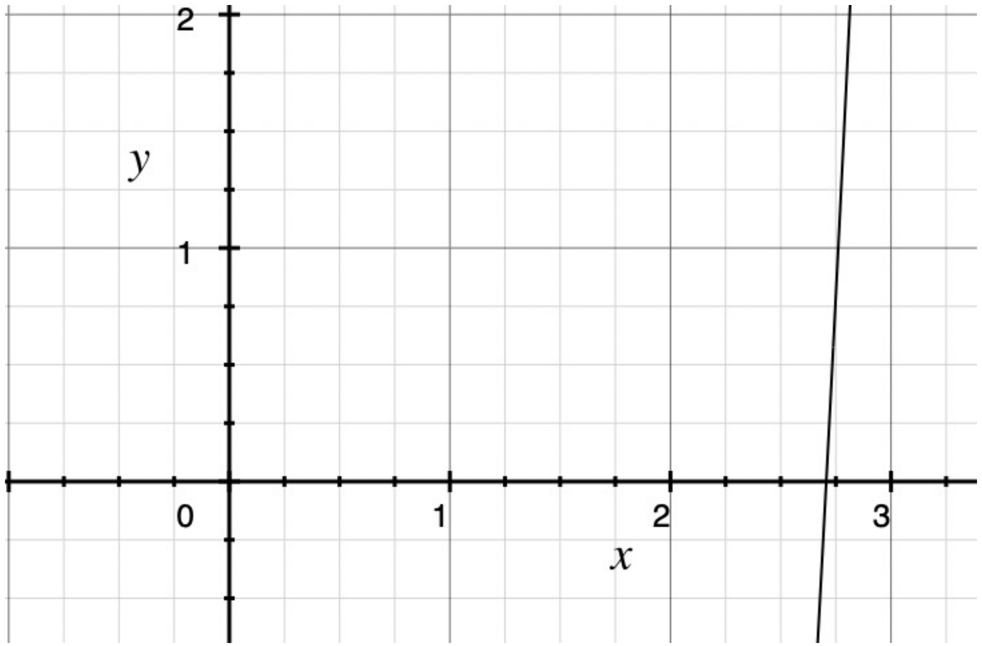
Bisection Method

Eg 1. *Find the real root of the equation* $x^3 - 4x - 9 = 0$ *by Bisection method correct. Take* $a = 2.706$, $b = 2.707$, $\epsilon = 0.0001$



Bisection Method

Eg 1. *Find the real root of the equation $x^3 - 4x - 9 = 0$ by Bisection method correct. Take $a = 2.706$, $b = 2.707$,* $\epsilon = 0.0001$

$f(x) = x^3 - 4x - 9$

$f(2.706) = -0.009488$ *i.e.,* (–)ve

x	0	1	2	3
f(x)	-9	-12	-9	6

and

$f(2.707) = 0.008487$ *i.e.,* (+)ve

Hence, the root lies between 2.706 and 2.707.

a	f(a)	b	f(b)
2.706	– 0.009488	2.707	0.008487

Bisection Method

First approximation to the root is

$$c = \frac{(2.706 + 2.707)}{2}$$

c = 2.7065

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
2.706	– 0.009488	2.707	0.008487	2.7065	– 0.0005025	-

Now $f(c)$ = – 0.0005025 *i.e.,* (–)ve and

f(b) = 0.008487 *i.e.,* (+)ve

Hence, the root lies between 2.7065 and 2.707.

Bisection Method

Second approximation to the root is

$$c = \frac{(2.7065 + 2.707)}{2}$$

= 2.70675

Now $f(c)$ = 0.003992 *i.e.,* (+)ve and $f(a)$ = – 0.0005025 *i.e.,* (–)ve

Hence, the root lies between 2.7065 and 2.70675.

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\|c_k - c_{k-1}\|$
2.706	– 0.009488	2.707	0.008487	2.7065	– 0.0005025	-
2.7065	– 0.0005025	2.707	0.008487	2.70675	0.003992	0.00025

Bisection Method

Third approximation to the root is

$$c = \frac{(2.7065 + 2.70675)}{2}$$

= 2.706625

Now $f(c)$ = 0.001744 *i.e.,* (+)ve and $f(a)$ = – 0.0005025 *i.e.,* (–)ve

Hence, the root lies between 2.7065 and 2.706625.

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
2.706	– 0.009488	2.707	0.008487	2.7065	– 0.0005025	-
2.7065	– 0.0005025	2.707	0.008487	2.70675	0.003992	0.00025
2.7065	– 0.0005025	2.70675	0.003992	2.706625	0.001744	0.000125

Bisection Method

Fourth approximation to the root is

$c = \frac{(2.7065+2.7406625)}{2}$ = 2.7065625

ϵ = 0.0001, $|c_k - c_{k-1}|$ = 0.0000625 < ϵ

Hence, the root is 2.7065625, correct to three decimal places.

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
2.706	– 0.009488	2.707	0.008487	2.7065	– 0.0005025	-
2.7065	– 0.0005025	2.707	0.008487	2.70675	0.003992	0.00025
2.7065	– 0.0005025	2.70675	0.003992	2.706625	0.001744	0.000125
2.7065	– 0.0005025	2.706625	0.001744	2.7065625		0.0000625

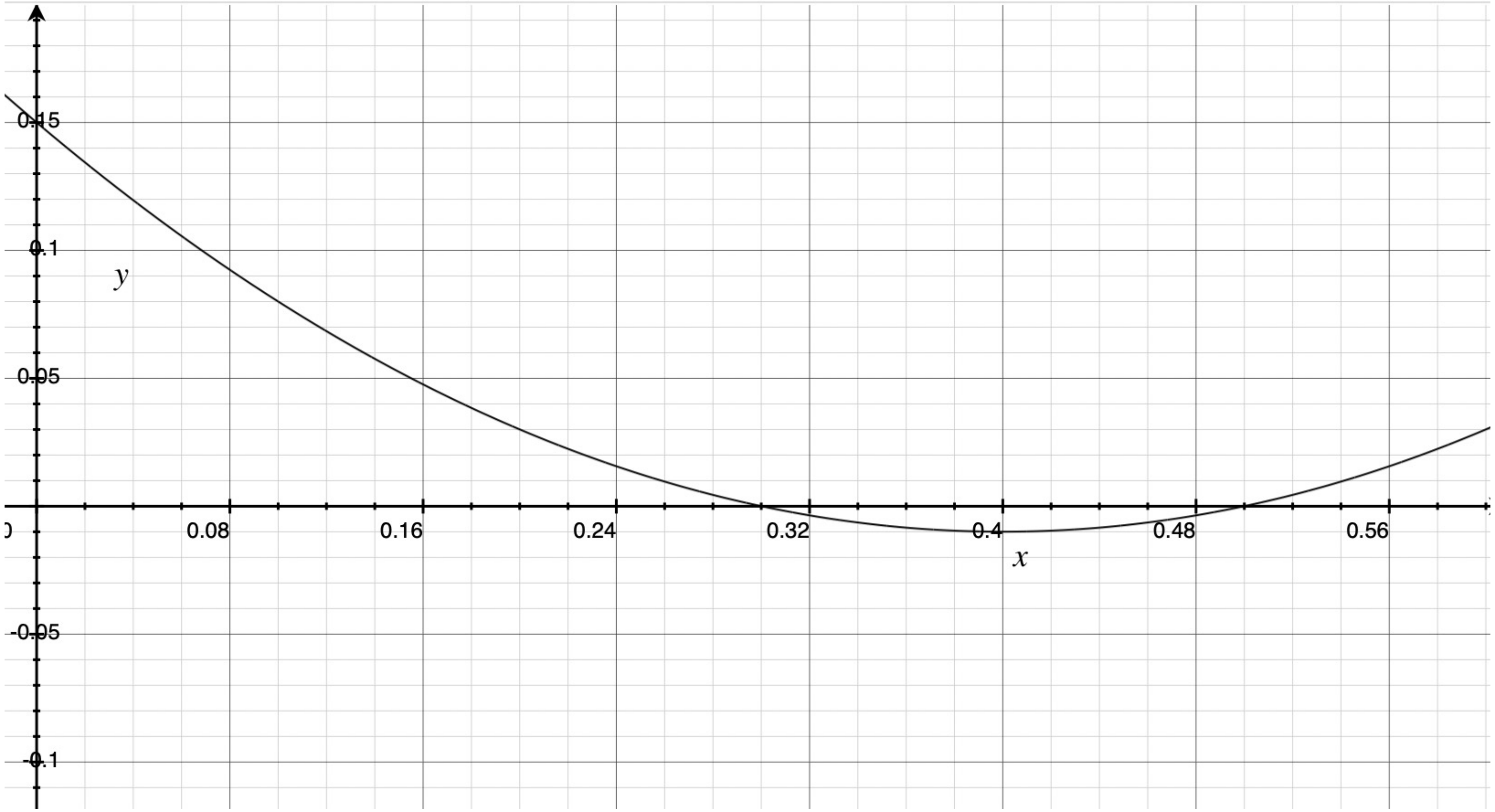
Example 2

Eg. 2 The quadratic $(x - 0.3)(x - 0.5)$ obviously has zeros at 0.3 and 0.5.

a. Why is the interval [0.1, 0.6] not a satisfactory starting interval for bisection?

b. What are good starting intervals for each root?
c. If you start with [0,0.491] which root is reached with bisection? Which root is reached from [0.31, 1.0]?

Solution 2



Solution 2

Eg. 2 The quadratic (x - 0.3)(x - 0.5) obviously has zeros at 0.3 and 0.5.

a. Why is the interval [0.1, 0.6] not a satisfactory starting interval for bisection?

Ans. Both a and b have same sign

b. What are good starting intervals for each root?

Ans. [0.2,0.4] for 0.3 and [0.4, 0.6] for 0.5

c. If you start with [0,0.491] which root is reached with bisection? Which root is reached from [0.31, 1.0]?

Ans. 0.3, 0.5

Example 3

Eg. 3 This polynomial obviously has roots at x = 2 and at x = 4; one is a double root, the other a triple root:

$f(x) = (x-2)^3(x-4)^2$

$= x^5 - 14x^4 + 76x^3 - 200x^2 + 256x - 128$

a. Which root can you get with bisection? Which root can't you get?

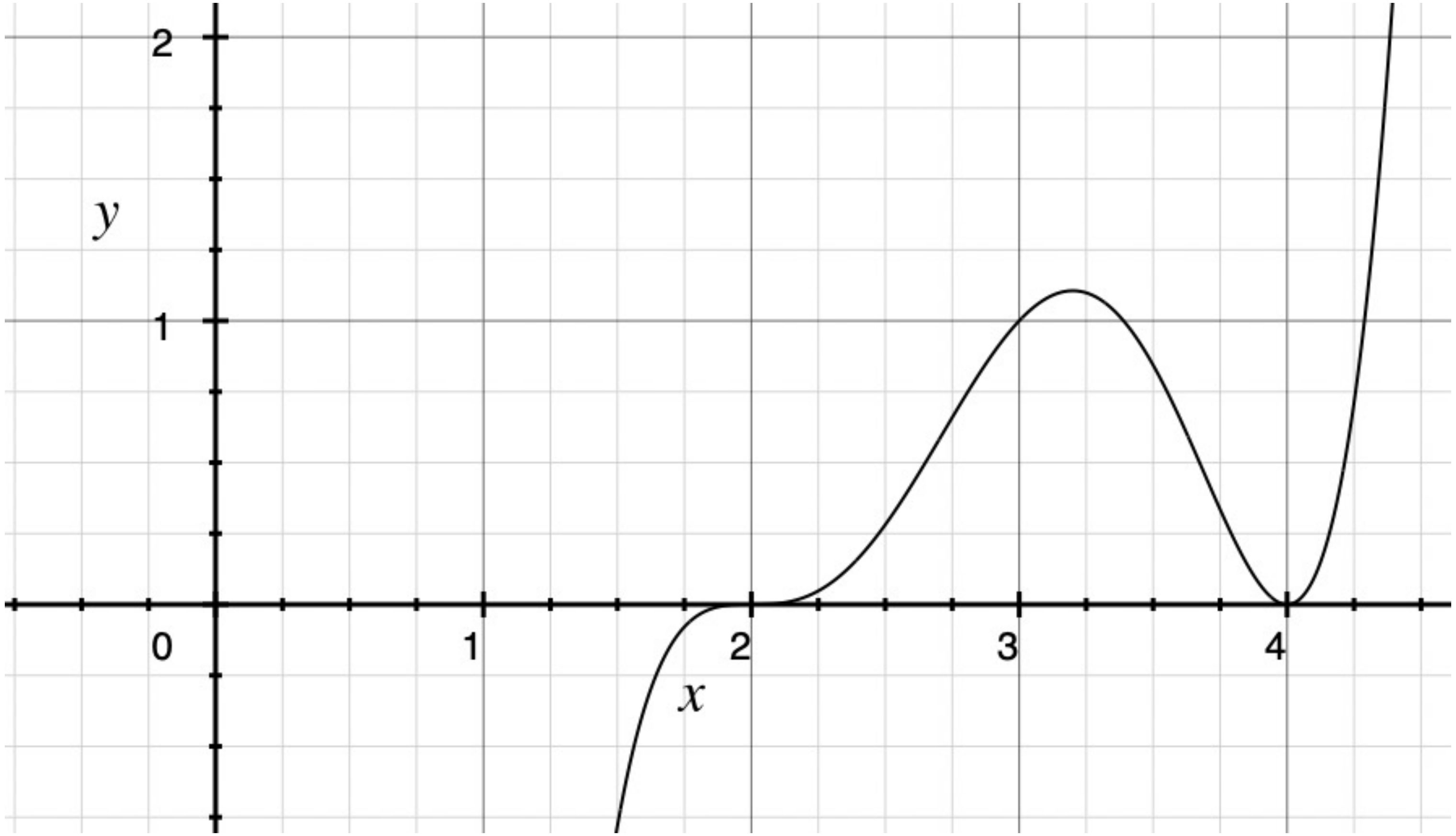
b. Repeat part (a) with the secant method.

c. If you begin with the interval [1,5], which root will you get with

(1) bisection, (2) the secant method, (3) false position?

d. Use Newton's method with x_0 = 3. Does it converge? To which root?

Solution 3



Solution 3

This polynomial obviously has roots at x = 2 and at x = 4; one is a double root, the other a triple root:

$f(x) = \ (x-2)^3(x-4)^2$

$= x^5 - 14x^4 \ {+76}x^3 - 200x^2 + 256x - 128$

a. we can get the root 2. Getting Interval for 4 is not possible

b.

c. If you begin with the interval [1,5],

(1) Bisection: Converge to 2

d.

Examples

Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places.*

ϵ = 0.01, *Take a = 1.25, b = 1.5*

Eg 5. *Find the real root of the equation*f(x) = 3x - e^x *by Bisection method correct to two decimal places.*

ϵ = 0.01, *Take a = 0, b = 1*

Example 4

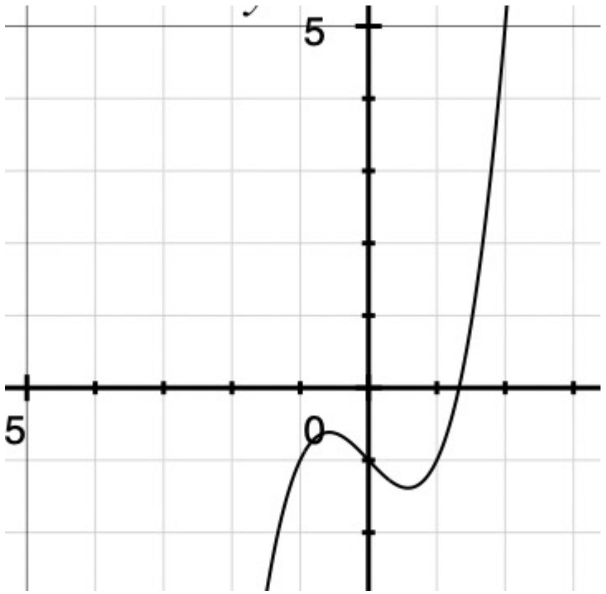
Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places.*

ϵ = 0.01, *Take a = 1.25, b = 1.5*

x	0	1	2
f(x)	-1	-1	5

Solution 4

Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places. Take a = 1.25, b = 1.5,*



Solution 4

Eg 4. *Find the real root of the equation* $x^3 - x - 1$ *by Bisection method correct to two decimal places. Take a = 1.25, b = 1.5,*

ϵ = 0.01

$f(x) = x^3 - x - 1$

$f(1.25) = -0.2969$ (–)ve

$f(1.5) = 0.875$ (+)ve

Hence, the root lies between 1.25 and 1.5.

a	f(a)	b	f(b)
1.25	– 0.2969	1.5	0.875

Solution 4

∴ First approximation to the root is

$$c = \frac{(1.25 + 1.5)}{2}$$

c = 1.375

a	f(a)	b	f(b)	c = (a+b)/2	f(c)
1.25	– 0.2969	1.5	0.875	1.375	0.2246

Now $f\,(c) = f(1.375)$ = 0.2246 (+)ve

and f(a) = – 0.2969 (–)ve

Hence, the root lies between 1.25 and 1.375

Solution 4

Second approximation to the root is

$$c = \frac{(1.25 + 1.375)}{2}$$

= 1.3125

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625

Now *f*(c) = *f*(1.3125) = -0.0515 (-)ve and *f*(b) = 0.2246 (+)ve

Hence, the root lies between 1.3125 and 1.375.

Solution 4

Third approximation to the root is

$c = \frac{(1.3125+1.375)}{2}$ = 1.3438

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625
1.3125	-0.0515	1.375	0.2246	1.3438	0.0826	0.0313

Now $f(c)$ = f(1.3438) = 0.0826 *i.e.,* (+)ve

and f(a) = -0.0515 (–)ve

Hence, the root lies between 1.3125 and 1.3438 .

Solution 4

Fourth approximation to the root is

$$c = \frac{(1.3125+1.3438)}{2} = 1.3281$$

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\lvert c_k - c_{k-1} \rvert$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625
1.3125	-0.0515	1.375	0.2246	1.3438	0.0826	0.0313
1.3125	-0.0515	1.3438	0.0826	1.3281	0.0146	0.0157

Now $f(c)$ = f(1.3281) = 0.0146 *i.e.,* (+)ve and

$f(b)$ = f(1.3125) = -0.0515 *i.e.,* (-)ve

Hence, the root lies between 1.3125 and 1.3281

Solution 4

Fifth approximation to the root is

c = $\frac{(1.3125+1.3281)}{2}$ = 1.3203

ϵ = 0.01 $|c_k - c_{k-1}|$ = 0.0078 < ϵ

The approximate real root is 1.3203

a	f(a)	b	f(b)	c = (a+b)/2	f(c)	$\|c_k - c_{k-1}\|$
1.25	– 0.2969	1.5	0.875	1.375	0.2246	-
1.25	– 0.2969	1.375	0.2246	1.3125	-0.0515	0.0625
1.3125	-0.0515	1.375	0.2246	1.3438	0.0826	0.0313
1.3125	-0.0515	1.3438	0.0826	1.3281	0.0146	0.0157
1.3125	-0.0515	1.3281	0.0146	1.3203		0.0078

Solution 5

- f(x) = 3x - e^x

a	$f(a)$	b	$f(b)$	$c = \frac{a+b}{2}$	$f(c)$
0	-1	1	0.2817	0.5	-0.1487
0.5	-0.1487	1	0.2817	0.75	0.133
0.5	-0.1487	0.75	0.133	0.625	0.0068
0.5	-0.1487	0.625	0.0068	0.5625	-0.0676
0.5625	-0.0676	0.625	0.0068	0.5938	-0.0295
0.5938	-0.0295	0.625	0.0068	0.6094	-0.0112
0.6094	-0.0112	0.625	0.0068	0.6172	-0.0021
0.6172	-0.0021	0.625	0.0068	0.6211	0.0023
0.6172	-0.0021	0.6211	0.0023	0.6191	0.0001