WEIGHTED AVERAGE (WEIGHTED MEAN)

The average obtained by giving the importance to the quantities of a distribution is called **WEIGHTED AVERAGE**.

It is denoted by $\overline{X}_w$ and weight is denoted by **'w'**

FORMULA:- $\overline{X}_w = \dfrac{\sum WX}{\sum W}$

QU -Find weighted mean of the following distribution

Subject	Marks	Weight
Maths	20	5
English	25	8
physics	35	6

Solution:-

Subject	Marks (x)	Weight(w)	w*x
Math's	20	5	100
English	25	8	200
physics	35	6	210
		$\sum w = 19$	$\sum w\,x = 510$

$$\overline{X_w} = \frac{\sum WX}{\sum W} = 510/19 = 26.8$$

Example: The performance of a student – manager in a business school was evaluated as follows:

Basis of Evaluation	Maximum Marks	Marks obtained	Weight
Class Tests	50	38	10
Presentations	50	36	15
Attendance	20	15	5
Class Participation	30	20	10
Final Examination	70	55	60

Calculate simple mean, weighted mean and conclude which of them is the appropriate answer.

Solution:

Basis	Marks Obtained (x)	Weights (W)	Wx
Class Tests	38	10	380
Presentations	36	15	540
Attendance	15	5	75
Class Participation	20	10	200
Final Examination	55	60	3300
n = 5	Σx= 164	Σw = 100	ΣWx = 4495

Simple Mean $\bar{x} = \frac{\Sigma x}{n} = \frac{164}{5} = 32.8$

Weighted Mean $= \frac{\Sigma Wx}{\Sigma W} = \frac{4495}{100} = 44.95$

where W = weight of individual items

Weighted mean is the real index of performance

GEOMETRIC MEAN (G.M.)

Definition:-

This mean meant by multiplying numbers together and then finding the n^{th} root of the numbers such that the n^{th} root is equal to the amount of numbers you multiplied

"The n^{th} root of the product of n values is called geometric mean of the variable and is denoted by 'G'

$$G = \sqrt[n]{X_1 * X_2 * X_3 * \ldots\ldots\ldots\ldots X_N}$$

$$= (x_1 * x_2 * x_3 * x_4 * \ldots\ldots\ldots\ldots x_n)^{1/n}$$

$$\log m^n = n \log m$$

taking logarithm of both sides

$$\log m*n = \log m + \log n$$

$$\log G = 1/n \log (x_1 * x_2 * x_3 * x_4 * \ldots\ldots\ldots\ldots x_n)$$

$$\text{Log } G = 1/n(\log x_1 + \log x_2 + \log x_3 + \ldots\ldots\ldots\ldots \log x_n)$$

$$= 1/n \sum \log x$$

$$G = \text{Antilog}(1/n \sum \log x)$$

QU:- find G.M. for the following data-

2,3,5,3,10,8

Solution- $G=\sqrt[6]{2*3*5*3*10*8}$

$=(2*3*5*3*10*8)^{1/6}$

$= 7200^{1/6}$

G =4.39

QU:- find geometric mean

0.5,1,2,16

Solution:- $G =(0.5*1*2*16)^{1/4}$

$=(16)^{1/4}$

$=(2^4)^{1/4}$

G.M. = 2

Qu:- find **G.M**. for the following----- 42.7, 37.2, 23, 39.7 , 45.3

Solution:-

n=5

applying formula-

$$\log G = 1/n \sum \log x$$

$= 1/5 (\log 42.7 + \log 37.2 + \log 23 + \log 39.7 + \log 45.3)$

$=1/5(1.6304+1.5705+1.3617+1.5988+1.6561)$

(from log table)

$=1/5(7.8175)$

$=1.5\ 635$

G= Antilog 1.5635

G =36.60 (From antilog table)

QU:- determine G.M. for the following distribution-

X	**:**	**135**	**231**	**352**	**430**
f	**:**	**2**	**3**	**4**	**3**

Solution:-

X	f	log x	f log x
135	2	2.1303	4.2606
231	3	2.3636	7.0908
352	4	2.5465	10.1860
430	3	2.6335	7.9005
	N = 12		∑ f log x = 29.4379

log G = 1/N ∑f log x

= 1/12 x29.4379

=2.4532

G = antilog 2.4532

=**283.9**

Merits of G.M.:-

- It is rigidly defined
- it is based on all items
- it is suitable for further algebraic calculations
- it is not too much affected by sampling fluctuations

Demerits of G.M.:-

- it is difficult to calculate
- unfit when any value is zero or negative

Uses of G.M.:-

- it is used in averaging ratios ,rates and percentages
- it is used in economics
- it is used in financial transaction
- it is used in interest rates
- it is used in personal finances

EXAMPLE- **Log 42.7**

we read 42 → 7

=.6304

Before decimal =2

so 2-1=1

Answer =1.6304

EXAMPLE- Log 135

we read 13 → 5

=.1303

Before decimal = 3

so 3-1=2

Answer =2.1303

rules for read to Log table –

1. characteristic
2. mantissa

Before decimal → characteristic (C)

After decimal → mantissa (M)

Example:- ANTILOG (2.6452)

C =2

M=6452

Then read on table 64 → 5 =4416

mean difference =2

So 4416+2=4418

decimal - c+1 =2+1=3

Answer = 441.8

HARMONIC MEAN

This mean is the reciprocal of the arithmetic mean of the variables. It is denoted by H.M.

FORMULA:- **1.- individual series** H.M. = $\dfrac{n}{\dfrac{1}{X_1}+\dfrac{1}{X_2}+\dfrac{1}{X_3}+\ldots\ldots\dfrac{1}{X_n}}$

2. –Discrete series $\longrightarrow$ H.M. = $\dfrac{n}{\dfrac{f_1}{x_1}+\dfrac{f_2}{x_2}+\ldots\ldots\ldots\ldots+\dfrac{f_n}{x_n}}$

Calculate H.M. from the following data —

10, 13, 8, 4, 5

Solution —

$$H.M. = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots \frac{1}{x_n}}$$

$$= \frac{5}{\frac{1}{10} + \frac{1}{13} + \frac{1}{8} + \frac{1}{4} + \frac{1}{5}}$$

$$= \frac{5}{\frac{52 + 40 + 65 + 130 + 104}{520}}$$

$$= \frac{5 \times 520}{391}$$

$$\boxed{H.M. = 6.65}$$

Qu - Find Harmonic Mean -

x	10	20	30	40	50
f	6	11	16	10	7

Solution -

$$H.M. = \frac{n}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots \frac{f_n}{x_n}}$$

$\boxed{n = \Sigma f}$

x	f	$\frac{1}{x}$	f/x
10	6	1/10	6/10
20	11	1/20	11/20
30	16	1/30	16/30
40	10	1/40	10/40
50	7	1/50	7/50
	N=50		

$$H.M. = \frac{50}{\frac{6}{10} + \frac{11}{20} + \frac{16}{30} + \frac{10}{40} + \frac{7}{50}}$$

$$= \frac{50}{\frac{360 + 330 + 320 + 150 + 84}{600}}$$

$$= \frac{50 \times 600}{1244} = \boxed{24.12}$$

Merits of Harmonic mean:-

1. it is based on all items.
2. it is rigidly defined.
3. it is least affected by sampling fluctuations
4. it is suitable for further mathematical treatment.

Demerits of Harmonic mean:-

1. it is difficult to calculate and understand.
2. it is unfit in case of zero or negative value.
3. it gives more weights to smaller value.

Uses of Harmonic mean:-

1. To determine the average speed which travel different time different distance and different speed.
2. To find mileage of a car.

RELATION BETWEEN A.M ,G.M. & H.M.

$$\text{A.M.} \geq \text{G.M.} \geq \text{H.M.}$$

$$\text{G.M.} = \sqrt{\text{A.M.} * \text{H.M.}}$$

Qu:- The A.M. and G.M. of two numbers are 15 and 10 respectively. Find the H.M. of the numbers.

Solution-

$$(\text{G.M.})^2 = \text{A.M.} \times \text{H.M.}$$

$$(10)^2 = 15 \times \text{H.M.}$$

$$100 = 15 \times \text{H.M.}$$

$$\text{H.M.} = 100/15$$

$$= 20/3$$

$$= 6.66$$

QU -1. calculate G.M.-

salary in lacs	:	1	2	3	4	5	6
no. of employees	:	10	8	17	7	5	3

(ANS = 2.600)

QU-2. find G.M & H.M.-

5, 10,15,20, 25,30 **(ANS – 14.63 & 12.24)**

QU -3. if A.M. & G.M. of two values are 5 & 4 respectively then find H.M**.** **(ANS = 3.2)**

QU -4. Determine H.M. of the following distribution-

class	:	10-14	15-19	20-24	25-29	30-34
frequency	:	4	6	8	2	2

(ANS = 18.54)

Qu-5. Find G.M.-

X	:	14	23	37	68	70
f	:	3	9	16	10	2

(ANS = 37.15)

If a and b are postive numbers, then

$$\text{Arithmetic Mean (AM)} = \frac{a + b}{2}$$

$$\text{Geometric Mean (GM)} = \sqrt{ab}$$

$$\text{Harmonic Mean (HM)} = \frac{2ab}{a + b} = \frac{(GM)^2}{AM}$$