

MEDIAN

Median is the middle value of observation, It is denoted by M_d

INDIVIDUAL SERIES:- $M_d = \left(\frac{N+1}{2} \right)$ th value (ODD)

DISCRETE SERIES:- $M_d = \frac{\left(\frac{N}{2} \right) \text{th value} + \left(\frac{N}{2} + 1 \right) \text{th value}}{2}$ (EVEN)

CONTINUOUS SERIES:- $M_d = L_1 + \frac{\frac{N}{2} - f_c}{f} * i$

where L_1 = lower limit of median class

N = total number of observation

f_c = c.f of the class preceding the median class

f = frequency of the median class

i = length of class interval

QU:- determine **MEDIAN** for the following series:-

1. 70,77, 73,72,75,79, 78

2. 94,33,86,68,32,80,48,70

solution – arranging the values of the series in ascending order

we get-- 70, 72 ,73 ,75, 77 ,78, 79

no. of observation = $N = 7$ = an **odd** number

$$M_d = \frac{(N+1)}{2} \text{ th term} = \frac{(7+1)}{2} = \frac{8}{2} = 4 \text{ th term} = \mathbf{75}$$

2. 23, 9 ,65 ,44 ,11

arranging in ascending order

9, 11, 23, 44, 65

$N = 5$ (**odd**)

MEDIAN = $(5+1)/2 = 6/2 = 3 \text{ th term} = \mathbf{23}$

IF 'N' IS EVEN :

QU:- find median- 94,33,86,68,32,80,48,70

solution – arranging in ascending order

32,33,48,,68,70,80,86,96

we get N= 8 which is **EVEN**

so median = $M_d = \frac{(8/2) \text{ th term} + (8/2) + 1 \text{ th term}}{2}$

= $\frac{4^{\text{th}} \text{ term} + 5 \text{ th term}}{2}$

= $\frac{68 + 70}{2}$

= $138/2$

$M_d = 69$

Discrete series (ungrouped data)-

QU:- determine median for the following data-

wages	:	20	21	22	23	24	25	26	27	28
No. of workers	:	8	10	11	16	20	25	19	9	6

Solution:-

$$N = 124 \text{ (EVEN)}$$

$$M_d = \frac{(\frac{124}{2})^{\text{th}} \text{ term} + (\frac{124}{2} + 1)^{\text{th}} \text{ term}}{2}$$

$$= \frac{62^{\text{th}} \text{ term} + 63^{\text{th}} \text{ term}}{2}$$

This term lies on c.f. 65

And 65 is c. f. of 24

$$M_d = \frac{24+24}{2} = \frac{48}{2} = 24$$

Then **median =24**

Wages(x)	Workers(f)	Cumulative frequency(c f)
20	8	8
21	10	18
22	11	29
23	16	45
24	20	65
25	25	90
26	19	109
27	9	118
28	6	124
	$\Sigma f = 124$	

QU:- find median –

Runs scored : 25 35 45 55 65 75 90

No. of players : 6 11 25 30 17 8 2

Solution :

N = 99 (odd)

x	f	C f
25	6	6
35	11	17
45	25	42
55	30	72
65	17	89
75	8	97
90	2	99
	$\Sigma f = 99$	

$M_d = \frac{(99+1)}{2}$ th term

= (100/2) th term

= 50

Which lies on c.f. 72

And 72 is c.f . Of 55

Then **median = 55**

Qu:- determine median-

x : 5 10 15 20 25
f : 3 4 2 5 4

Solution:-

x	f	C f
5	3	3
10	4	7
15	2	9
20	5	14
25	4	18
	$\Sigma f = 18$	

Here N= 18 (even)

$$\begin{aligned}M_d &= \frac{\text{18 th term} + (\text{18} + 1)\text{th term}}{2} \\&= \frac{\text{9 th term} + \text{10 th term}}{2} = \frac{15 + 20}{2} = 35/2 = 17.5\end{aligned}$$

MEDIAN = 17.5

CONTINUOUS SERIES (GROUPED DATA)

Determine MEDIAN for the following data

Wages	No. of workers
20-25	14
25-30	28
30-35	33
35-40	30
40-45	20
45-50	15
50-55	13
55-60	7

Solution :-

Class interval	f	C f
20-25	14	14
25-30	28	42
30-35	33	75
35-40	30	105
40-45	20	125
45-50	15	140
50-55	13	153
55-60	7	160
	$\Sigma f = 160$	

$$L_1 = 35, N = 160, i = 5$$

$$f_c = 75, f = 30$$

Now $N/2$ th term $= 160/2$
 $= 80$ th term

Which is included in 35-40

MEDIAN class $= 35-40$

applying formula

$$\begin{aligned} M_d &= L_1 + \frac{\frac{N}{2} - f_c}{f} \times i \\ &= 35 + \frac{\frac{160}{2} - 75}{30} \times 5 \\ &= 35 + \frac{80 - 75}{30} \times 5 \\ &= 35 + \frac{5}{30} \times 5 \\ &= 35 + \frac{25}{30} = \underline{\underline{35.82}} \end{aligned}$$

Find median:-

production per day :	21-22	23-24	25-26	27-28	29-30
No. of day :	7	13	22	10	8

Solution :-

Class interval	f	C f
20.5-22.5	7	7
22.5-24.5	13	20
24.5-26.5	22	42
26.5-28.5	10	52
28.5-30.5	8	60
	$\Sigma f = 60$	

Now $N/2$ th term = $60/2$
= 30 th term

Which included in c f (40)

Then MEDIAN CLASS = 24.5 - 26.5

So $L_1 = 24.5$, $i = 2$ $N = 60$

$f_c = 20$ $f = 22$

Change into exclusive

Size = 1, so $1/2 = 0.5$ then $21 - 0.5 = 20.5$ & $22 + 0.5 = 22.5$

Then class interval 20.5-22.5

$$\begin{aligned} M_d &= L_1 + \frac{N/2 - f_c}{f} * i \\ &= 24.5 + \frac{60/2 - 20}{22} * 2 \\ &= 24.5 + \frac{30-20}{22} * 2 \\ &= 24.5 + 20/22 \\ &= 24.5 + 0.909 \\ &= 25.409 = 25.41 \text{ (approx.)} \end{aligned}$$



MERITS OF MEDIAN

- It is very easy to calculate
- It is determined by graphically
- It can be used to find the average of qualitative data, like ranks ,grades.
- It can be determined simply by observation in individual & ungrouped data.



DEMARITS OF MEDIAN

- It is not based on all items.
- It is affected by sampling fluctuations.
- It is unfit for further algebraic calculations.
- It is not affected by extreme values

Find Median - UNEQUAL CLASS INTERVAL

Daily time (in minutes)	No. of executives
Less than 20	6
20 – 25	11
25 – 30	15
30 – 40	24
40 – 50	19
50 – 65	12
65 and above	7

We make same size class interval

Class interval	frequency
15-20	6
20-25	11
25-30	15
30-35	12 (24/2)
35-40	12
40-45	9.5 (19/2)
45-50	9.5
50-55	4 (12/3)
55-60	4
60-65	4
65-70	7

MODE

Mode is the value of the observation having the maximum frequency

Individual series :- maximum frequency

Grouped data:-
$$\text{Mode} = M_o = L_1 + \frac{f_1 - f_0}{2 f_1 - f_0 - f_2} * i$$

L = lower limit of model class

f_1 = frequency of model class

f_0 = frequency of the class proceeding the model class

f_2 = frequency of the class succeeding the model class

i = length of the class interval

QU:- **find mode** –

28,24,26,42,24,41,46,24

Solution – The number **24** is repeated maximum times (3 times)

hence **24** is the mode of given observation

QU :- 7,9,11,7,6,5,9,13

Solution – here 7 (2 times) & 9 (2 times) ,so there are two modes here

i.e. **7 & 9**

QU:- 3,5,6,7,9,12,3,6,5,9,12,7

Solution:- Here each of the observation is the same (2 times)

So there is **no mode** in the given observation

Determine mode-

Marks	No. of students
1-5	7
6-10	10
11-15	16
16-20	32
21-25	24
26-30	18
31-35	10
36-40	5
41-45	1

Class interval	Frequency
0.5-5.5	7
5.5-10.5	10
10.5-15.5	16 f_0
<u>15.5-20.5</u>	<u>32</u> f_1
20.5-25.5	24 f_2
25.5-30.5	18
30.5-35.5	10
35.5-40.5	5
40.5-45.5	1

Model class – 15.5-20.5

$$L_1 = 15.5, \quad f_1 = 32$$

$$i = 5, \quad f_0 = 16$$

$$f_2 = 24$$

$$\begin{aligned}
 M_d &= L_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} * i \\
 &= 15.5 + \frac{32 - 16}{2(32) - 16 - 24} * 5 \\
 &= 15.5 + \frac{16}{64 - 16 - 24} * 5 \\
 &= 15.5 + 80 / 64 - 40 \\
 &= 15.5 + 80 / 24 \\
 &= 15.5 + 3.333 \\
 &= 18.83
 \end{aligned}$$

QU – find mode –

shoe size	:	6	7	8	9	10
no. of pairs	:	3	11	128	57	4

QU: - determine mode –

collar size	:	7	8	9	10	11	12	13
no. of shirts	:	5	8	11	45	42	38	25

QU:- find mode :-

age (in years)	No. of patients
More than 10	148
More than 20	124
More than 30	109
More than 40	71
More than 50	30
More than 60	16
More than 70 and up to 80	1

Solution

Class interval	f
10-20	148-124 =24
20-30	124-109 = 15
30-40	109-71 =38 f_0
40-50	71-30 =41 f_1
50-60	30-16 = 14 f_2
60-70	16-1 =15
70-80	1

$$\text{Mode} = 40 + \frac{41-38}{2(41)-38-14} * 10$$

$$= 40 + 30/82-52$$

$$= 40 + 30 /30$$

$$= 40 + 1$$

$$\mathbf{M_d = 41}$$

UNEQUAL CLASS INTERVAL

QU – Find mode -

Daily wages	no. of workers
0-5	5
5-10	7
10-20	9
20-40	25
40-60	30
60-80	24
80-90	8
90-100	6

Class interval	F
0-20	5+7+9 =21
20-40	25 f_0
40-60	30 f_1
60-80	24 f_2
80-100	8+6 =14

$$\text{Mode} = 40 + \frac{30-25}{2(30) - 25 - 24} \times 20$$

$$= 40 + 5 \times 20 / 60 - 49$$

$$= 40 + 100 / 11$$

$$= 40 + 9.09$$

$$\mathbf{M_d = 49.09}$$

MERITS OF MODE –

- it is easy to understand
- It can be determined graphically
- It is not affected by extreme values
- It can be easily determined simply by observation in ungrouped data
- It is use for qualitative data

DEMERITS OF MODE –

- It is not based on all observation
- It is not suitable for mathematical treatment

USES OF MODE

- it is used by manufactures, businessmen and agriculturists
- It is useful for industry
- It is used in economic survey
- weather forecasts are based on mode

RELATION AMONG MEAN MODE & MEDIAN

$$\text{Mean} - \text{mode} = 3 (\text{mean} - \text{median})$$

QU- if mean = 70.2, and mode = 70.5 then find median

Solution - $70.2 - 70.5 = 3 (70.2 - \text{median})$
 $- 0.3 = 210.6 - 3 \text{ median}$
 $3 \text{ median} = 210.6 + 0.3$
 $\text{median} = 210.9 / 3$
median = 70.3

QU - if arithmetic mean is 150 and median is 140 then find mode

QU- If mode is 7 and median is 9 then find mean

QU: - determine mode – (grouping method)

collar size : 7 8 9 10 11 12 13
no. of shirts : 5 8 11 45 42 38 25

GROUPING TABLE

X	f(1)	(2)	(3)	(4)	(5)	(6)
7	5	13				
8	8		19	24		
9	11	56			64	
10	45		87			98
11	42	80		125		
12	38		63		105	
13	25					

ANALYSIS TABLE

X	7	8	9	10	11	12	13
1							
2							
3							
4							
5							
6							
TOTAL			1	4	5	3	1

Since highest total is 5 . Which is the value of 11
therefore mode is 11

Qu if the weighted mean is 9.85 ,then find the missing weight

x	Weight(w)	wx
5	20	100
8	15	120
10	30	300
12	? (let x)	12x
15	15	225
	$\sum w = 80+x$	$\sum wx = 745+ 12x$

Applying formula - $X_w = \frac{\sum w x}{\sum w}$

$$9.85 = \frac{745+12X}{80+X}$$

$$9.85(80)+ 9.85X = 745+12X$$

$$788.00+ 9.85 X=745+12X$$

$$788-745 = 12X - 9.85X$$

$$43 = 2.15X$$

$$X = 43 / 2.15$$

$$= 4300/ 215$$

$$\mathbf{X = 20}$$

The mean of the following data is 50, but frequencies f_1 and f_2 in classes 20-40 and 60-80 are missing. Find the missing frequencies:-

Class interval	frequency
0-20	17
20-40	f_1
40-60	32
60-80	f_2
80-100	19
	120

SOLUTION :-

Classes	Mid value(x)	f	d= (x-A/i	f d
0-20	10	17	-2	-34
20-40	30	f_1	-1	$-f_1$
40-60	50	32	0	0
60-80	70	f_2	+1	$+f_2$
80-100	90	19	+2	+38
		$\Sigma f = 120$		$\Sigma f d = 4 + f_2 - f_1$

$$\Sigma f = 68 + f_1 + f_2 = 120$$

$$f_1 + f_2 = 120 - 68$$

$$f_1 + f_2 = 52$$

$$X = 50 + \frac{4 + f_2 - f_1}{120} \times 20$$

$$50 = 50 + 4 + f_2 - f_1 / 6$$

$$50 - 50 = 4 + f_2 - f_1 / 6$$

$$4 + f_2 - f_1 = 0$$

$$f_1 - f_2 = 4$$

by 1 & 2 ,we get $f_1 = 28$ and $f_2 = 24$