

Computational Imaging Course

Inverse problems & Computational Imaging

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Objectives

A word cloud of mathematical and computational objectives arranged in a circular pattern. The words are: Bayesian model (light blue), Variational Method (blue), Inverse Problem (purple), MRF (yellow-green), Euler-Lagrange (light green), Minimisation (orange), PDE (green), Regularisation (dark red), Level-set (dark blue), Non-local (red), A priori (dark blue), and Total Variation (teal). The words are rotated at various angles to fit the circular arrangement.

Bayesian model

Variational Method

Inverse Problem

MRF

Euler-Lagrange

Minimisation

PDE

Regularisation

Level-set

Non-local

A priori

Total Variation

Course sequence

11/02	Inverse imaging & PDE
18/02	Inverse imaging & MRF
03/03	Flipped Classroom 1-2
10/03	Flipped Classroom 3-4
17/03	Flipped Classroom 5-6

Flipped classroom & project:

- **2 groups max for each topic**
- **Topics:** see [link](#)
 - #1: Sparsity & Patches, #2 Learning & PDEs
 - #3: Graph cuts, #4 Optimal transport & imaging
 - #4: Texture synthesis, #6 Motion magnification
- **Flipped classroom:**
 - **30' course** (~ 15/20 slides) **prepared jointly by the 2 groups**
 - 4 main aspects to be addressed: considered issue, proposed model/scheme, numerical implementation, experiments
 - **30' for questions/discussions** (students not presenting shall prepare and send in advance two questions per paper)
 - **15' synthesis** (Prof.)
- **Goal of the project:**
 1. Implement and evaluate the proposed framework (Python)
 2. Benchmarking experiments between the two groups (same experimental setup)
- **Targeted skills:**
 1. Understanding and reformulation of comp. Imaging pbs/models
 2. Implement & benchmark algorithms from scientific papers
- **Evaluation:** Graded lab session / Flipped classroom / Questions / Final presentation

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Flipped Classrooms & Projects

- **Evaluated skills:**

- In-depth understanding and presentation of a state-of-the-art framework from a research paper (flipped classroom)
- Ability to question a topic/method based on a quick screening of a paper (30' max per paper) (flipped classroom)
- Ability to implement an algorithm described in a research paper (project/notebook)
- Ability to define and perform benchmarking experiments to evaluate and demonstrate the relevance of different algorithms (project/final-presentation)
- Ability to perform a short presentation of an problem/algorithm and associaed experiments w.r.t. selected key messages (max 10') (project/final-presentation)

Synthesis : Variational vs. Bayesian

• **Formulation générale :** $\hat{x} = \arg \min_x U_1(x, y) + U_2(x)$

Observation term

Regularisation/prior
term

Variational methods

x,y «continuous» fonctions from \mathbb{R}^2 to \mathbb{R}
Gradient descent algorithm using Euler-Lagrange equation
Typical energy terms:

$$U_1(x, y) = \|x - y\|^2, \|g(x) - y\|^2$$

$$U_2(x) = \|\nabla x\|^2, \|\nabla x\|$$

Pros : theoretical analysis of the existence and unicity of the solution, region-based modeling (level-set), relationship to physical models (fluid dynamics)

Bayesian models/Markov Random Fields

x,y « discrete », matrices $\mathbb{R}^{N \times M}$
Statistical criterion (MAP, MPM,...)
Different types of minimisation scheme solution
(gradient descent, graph cut, MCMC methods)

$$U_1(x, y) = -\log P(x|y)$$

$$U_2(x) = -\log P(x)$$

Pros : Model parameter estimation, variety of optimisations scheme, existence of the solution

Synthesis : patch, non-local, dictionary & sparsity

$$\hat{x} = \arg \min_x U_1(x, y) + U_2(x)$$

• Problem statement:

$$U_i(x, y) = \sum_k V_i(x(P_k), y(P_k))$$

Non-local model

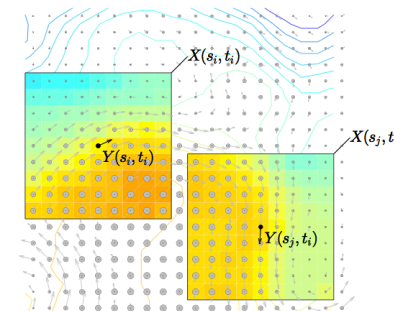
Key idea: self-similarity in images

$$U_i(x, y) = \sum_{k,l} w(k, l) V_i(x(P_k), y(P_k), x(P_l), y(P_l))$$

Similarity between patches k and l

Interest: using self-similarity in images so that a given image is its own model

k^{th} patch of the image
(KxK area)



Dictionary / Sparsity

$$\forall k, x(P_k) = \sum_i \alpha_{i,k} D_i = D \cdot \alpha_k$$

Dictionary of patches

Projection of patch k onto dictionary D

Examples of dictionaries : Fourier, Wavelet, learning (PCA, NMF, K-SVD,...)

Sparsity prior/constraint on coefficients α_k :

$$\forall k, U_2(x) = U(\alpha), \text{ par ex. } \sum_k \|\alpha_k\|$$