# Probabilistic Classification

MACHINE LEARNING UNIT 11

# Predict user’s ratings on a product

- Predict the ratings by a particular user to a given restaurant!
- Possible indicators:

  - The user’s ratings of other restaurants

    - most frequently, (s)he rates 4.0!

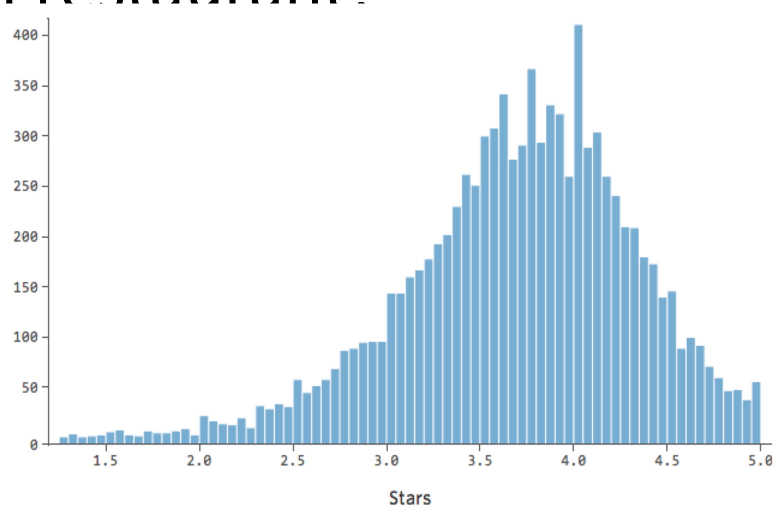


Image source: Google Images

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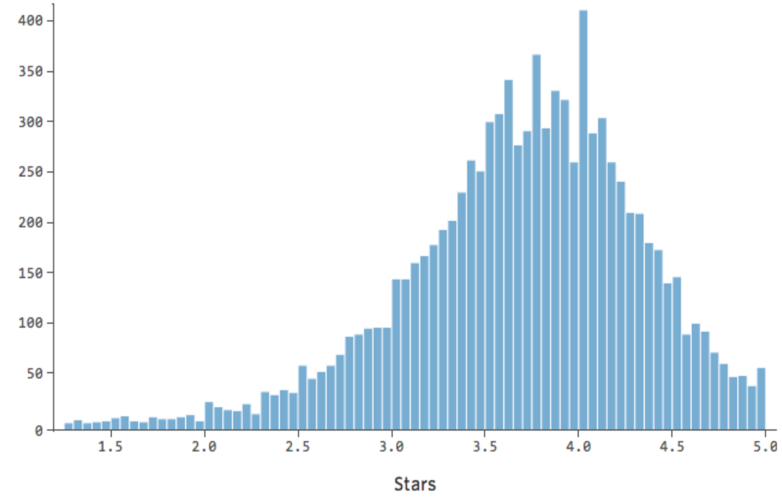
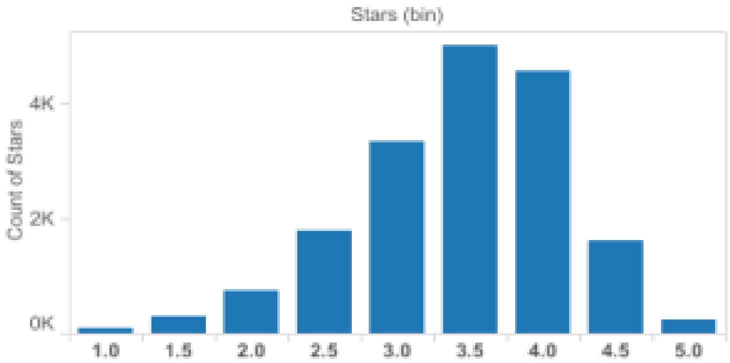
  - Other users’ ratings of that restaurant
    - most frequently, others rate 3.5!

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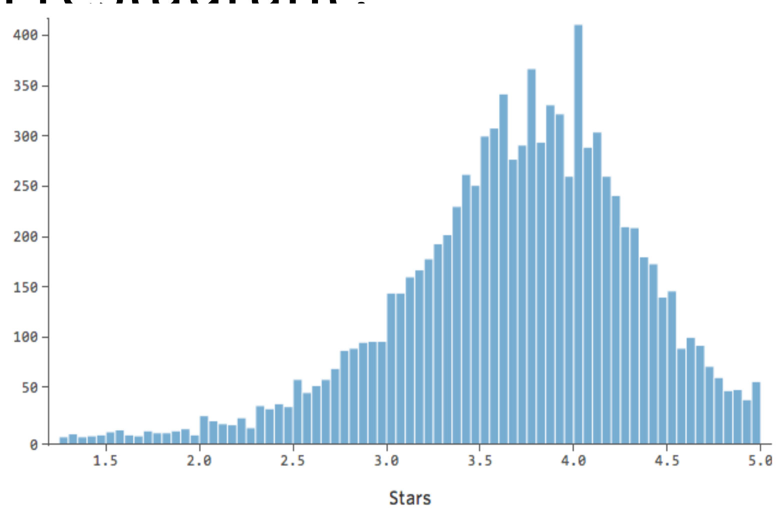


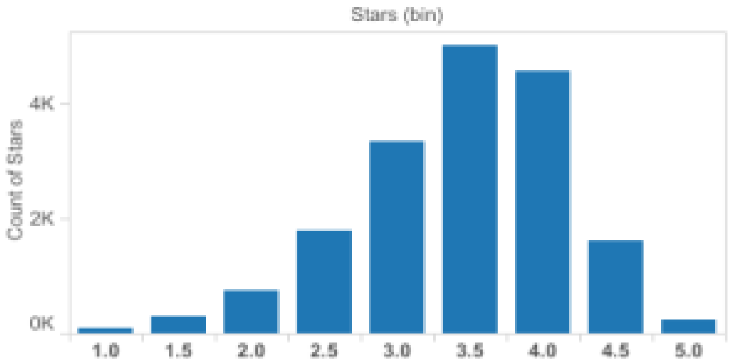


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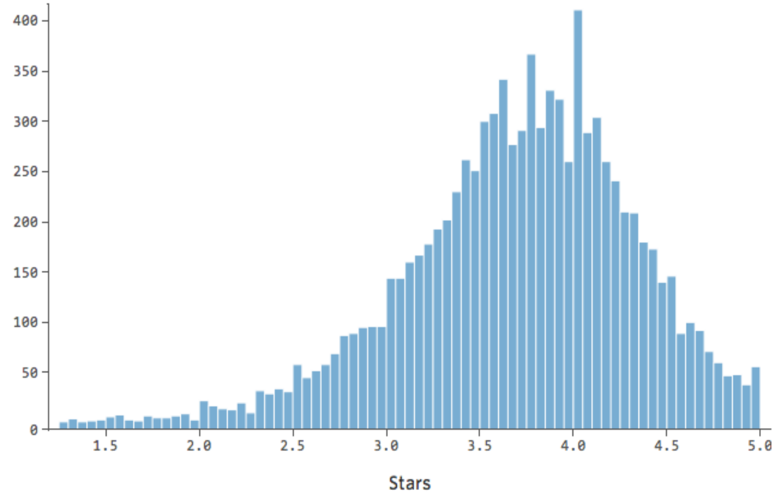
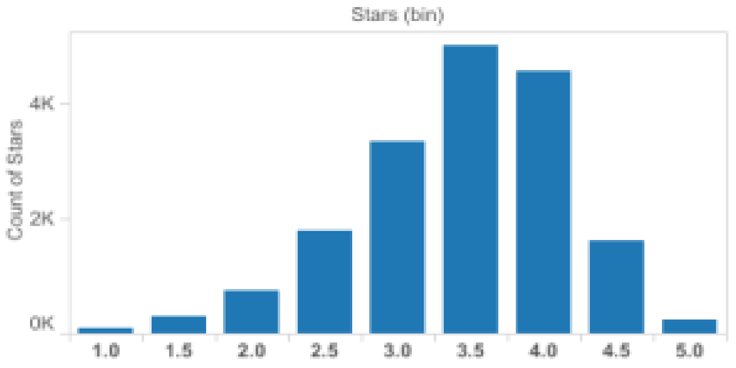
- But we can never be sure!
- Can we attach a probability to each possible rating?

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# Generative Model

- A story about how the observed data were "born"
- Story in the language of probability!

- Treat labels Y and features X as random variables
- Story outline:

    - Y created first

    - X created based on Y

# Generative Model

- A story about how the observed data were "born"
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- Treat labels Y and features X as random variables
- Story outline:
- - Y created first $p(Y)$: prior distribution
- - X created based on Y $p(X|Y)$: class-conditional distribution
- Probabilistic Classification: $p(Y|X)$: posterior distribution

# Prior Distribution

- Prior: information before observing X

| | Y =1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| | 80 | 50 | 40 | 30 |

- P(Y = k) = ?
- Frequentist approach: just relative frequencies!

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| | 0.4 | 0.25 | 0.20 | 0.15 |

# Posterior Distribution

- Posterior: information after observing X
- $P(Y = k \mid X) = ?$

- Bayes Theorem:
- $P(Y = k \mid X) = (p(X \mid Y = k) * p(Y = k)) / p(X)$

$$= K * p(X \mid Y = k) * p(Y = k)$$

# Class-conditional Distribution

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| X < 15 | 40 | 45 | 10 | 5 |
| X > 15 | 40 | 5 | 30 | 25 |

- $P(X \mid Y = k) = ??$
- $P(X<15 \mid Y=1) = 40/(40+40) = 0.5$
- $P(X>15 \mid Y=3) = 30/(30+10) = 0.75$

- $P(Y = k \mid X) = ???$

# Frequentist Approach: Direct estimation

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 | |
|---|---|---|---|---|---|
| X < 15 | 40 | 45 | 10 | 5 | 100 |
| X > 15 | 40 | 5 | 30 | 25 | 100 |

- $P(Y = 1 \mid X<15) = 40 / 100 = 0.4$

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 | |
|---|---|---|---|---|---|
| $p(Y \mid X < 15)$ | 0.4 | 0.45 | 0.10 | 0.05 | 1.0 |
| $p(Y \mid X > 15)$ | 0.4 | 0.05 | 0.30 | 0.25 | 1.0 |

- Similar to Decision Trees

# Bayesian Approach: Posterior Distribution

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| X < 15 | 40 | 45 | 10 | 5 |
| X > 15 | 40 | 5 | 30 | 25 |

- P(Y = 1 | X<15) = K * p(X<15 | Y=1) * p(Y=1) = K*(40/80)*(80/200)
- P(Y = 2 | X<15) = K * p(X<15 | Y=2) * p(Y=2) = K*(45/50)*(50/200)
- P(Y = 3 | X<15) = K * p(X<15 | Y=3) * p(Y=3) = K*(10/40)*(40/200)
- P(Y = 4 | X<15) = K * p(X<15 | Y=4) * p(Y=4) = K* (5/30)*(30/200)
- K = ???

# Posterior Distribution

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| X < 15 | 40 | 45 | 10 | 5 |
| X > 15 | 40 | 5 | 30 | 25 |

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| Prior p(Y) | 0.40 | 0.25 | 0.20 | 0.15 |
| p(Y \| X < 15) | 0.40 | 0.45 | 0.10 | 0.05 |
| P(Y \| X > 15) | 0.40 | 0.05 | 0.30 | 0.25 |

# Probabilistic Classifier

- Predicted label : mode of the posterior distribution!
- $Y_{pred} = \text{argmax}_k \, p\,(Y = k \mid X)$
- Confidence of the prediction = $p(Y = Y_{pred} \mid X)$

- If Bayesian approach used for $p(Y \mid X)$: Bayesian Classifier!

# Posterior Distribution

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| X < 15 | 40 | 45 | 10 | 5 |
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| | Y = 1 | Y = 2 | Y = 3 | Y = 4 | Ypred |
|---|---|---|---|---|---|
| Prior | 0.40 | 0.25 | 0.20 | 0.15 | 1 |
| X < 15 | 0.40 | 0.45 | 0.10 | 0.05 | 2 |
| X > 15 | 0.40 | 0.05 | 0.30 | 0.25 | 1 |

# Multi-dimensional Features

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| X1<15 | 40 | 45 | 10 | 5 |
| X1>15 | 40 | 5 | 30 | 25 |
| X2 = a | 40 | 30 | 15 | 30 |
| X2 = b | 40 | 20 | 25 | 0 |

- $P(Y = k \mid X1=12, X2=a) =$ ????
- We need Joint Distribution of the features!!!

# Multi-dimensional Features

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| X1<15, X2=a | 30 | 25 | 0 | 5 |
| X1<15, X2=b | 10 | 20 | 10 | 0 |
| X1>15, X2=a | 10 | 5 | 15 | 25 |
| X1>15, X2=b | 30 | 0 | 15 | 0 |

- $P(Y = 3 \mid X1=12, X2 =b) = K * P(X1<15, X2=b \mid Y = 3) *P(Y=3)$

  $= ?$

# Multi-dimensional Features

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| X1<15, X2=a | 0.375 | 0.50 | 0 | 0.166 |
| X1<15, X2=b | 0.125 | 0.40 | 0.25 | 0 |
| X1>15, X2=a | 0.125 | 0.10 | 0.375 | 0.837 |
| X1>15, X2=b | 0.375 | 0 | 0.375 | 0 |

- $P(Y = 3 \mid X1=12, X2 = b) = K * P(X1<15, X2=b \mid Y = 3) * P(Y=3)$

$$= K * 0.25 * 0.20$$

# Multi-dimensional Features

| | Y = 1 | Y = 2 | Y = 3 | Y = 4 |
|---|---|---|---|---|
| X1<15, X2=a | 0.375 * 0.4 * K1 | 0.50 * 0.25 * K1 | 0 * 0.2 * K1 | 0.166 * 0.15 * K1 |
| X1<15, X2=b | 0.125 * 0.4 * K2 | 0.40 * 0.25 * K2 | 0.25 * 0.2 * K2 | 0 * 0.15 * K2 |
| X1>15, X2=a | 0.125 * 0.4 * K3 | 0.10 * 0.25 * K3 | 0.375 * 0.2 * K3 | 0.837 * 0.15 * K3 |
| X1>15, X2=b | 0.375 * 0.4 * K4 | 0 * 0.25 * K4 | 0.375 * 0.2 * K4 | 0 * 0.15 * K4 |

- $P(Y = 3 \mid X1=12, X2 =b) = K2 * P(X1<15, X2=b \mid Y = 3) *P(Y=3)$

  $= K2 * 0.25 * 0.20$

  $= ??$

# Naïve Bayes Classifier

- D-dimensional feature vector, M values each
- Rows of table = M**D

- Assumption: all features are independent (Naïve!)
- $P(X1<15, X2 =b) = p(X1<15) * p(X2=b)$

- D tables, rows of each table = M
- Naïve, but computationally efficient!

# Naïve Bayes Classification

- $P(Y = k \mid X1<15, X2=b) = K * p(X1<15, X2=b \mid Y = k) * p(Y = k)$
  $= K * p(X1<15 \mid Y = k) * p(X2=b \mid Y = k) * p(Y = k)$

Final prediction $= \mathrm{argmax}_k\ K*p(X_1|Y=k)\ p(X_2|Y=k)*\ldots\ldots\ p(X_D|Y=k)*p(Y=k)$

Confidence $= \max_k\ K*p(X_1|Y=k)\ p(X_2|Y=k)*\ldots\ldots\ p(X_D|Y=k)*p(Y=k)$

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- If Bayesian approach used for $p(Y \mid X)$: Bayesian Classifier!

# Error in Bayes Classifier

- Bayes error probability = total probability of the non-mode classes!
- Risk of prediction = 1 – confidence of prediction
- Bayes error = expected risk (expectation over all X and Y)

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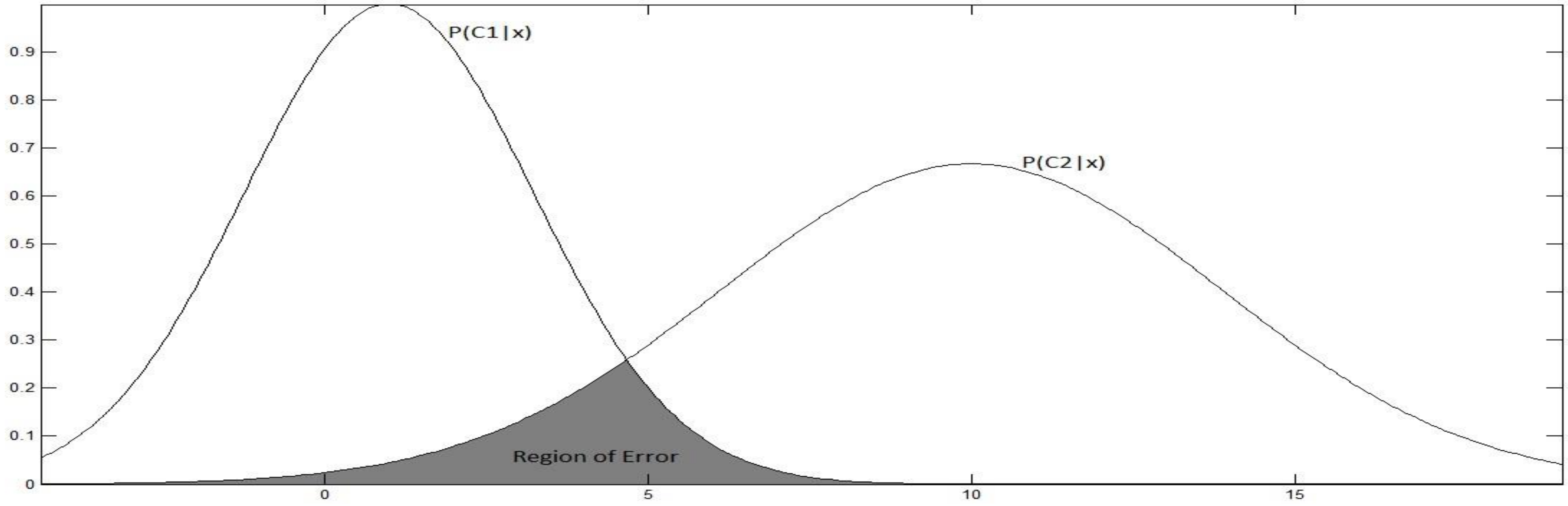


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```python
import numpy as np
```

```python
def fit(X_train,Y_train):
    result ={}
    class_values = set(Y_train)
    for current_class in class_values:
        result[current_class] = {}
        result["total_data"] = len(Y_train)
        current_class_rows = (Y_train == current_class)
        X_train_current = X_train[current_class_rows]
        Y_train_current = Y_train[current_class_rows]
        num_features = X_train.shape[1]
        result[current_class]["total_count"] = len(Y_train_current)
        for j in range(1,num_features+1):
            result[current_class][j] ={}
            all_possible_values = set(X_train[:,j-1])
            for current_value in all_possible_values:
                result[current_class][j][current_value] = (X_train_current[:,j-1] == current_value).sum()
    return result
```

```python
def probablity(dictionary,x,current_class):
    output= np.log(dictionary[current_class]["total_count"])-np.log(dictionary["total_data"])
    num_features = len(dictionary[current_class].keys())-1;
    for j in range(1,num_features+1):
        xj = x[j-1]
        count_current_class_with_value_xj = dictionary[current_class][j][xj] + 1
        count_current_class = dictionary[current_class]["total_count"] + len(dictionary[current_class][j].keys())
        current_xj_prob = np.log(count_current_class_with_value_xj) -np.log(count_current_class)
        output = output + current_xj_prob
    return output
```

```python
def predictSinglePoint(dictionary,x):
    classes = dictionary.keys()
    best_p  = -1000
    best_class = -1
    first_run = True
    for current_class in classes:
        if(current_class == "total_data"):
            continue
        p_current_class = probablity(dictionary,x,current_class)
        if(first_run or p_current_class > best_p):
            best_p = p_current_class
            best_class = current_class
        first_run = False
    return best_class
```

```python
def predict(dictonary, X_test):
    y_pred = []
    for x in X_test:
        x_class = predictSinglePoint(dictionary,x)
        y_pred.append(x_class)
    return y_pred
```