Pie & AI: Strasbourg - Autoencoders and Variational Autoencoders

Strasbourg
05.11.2020

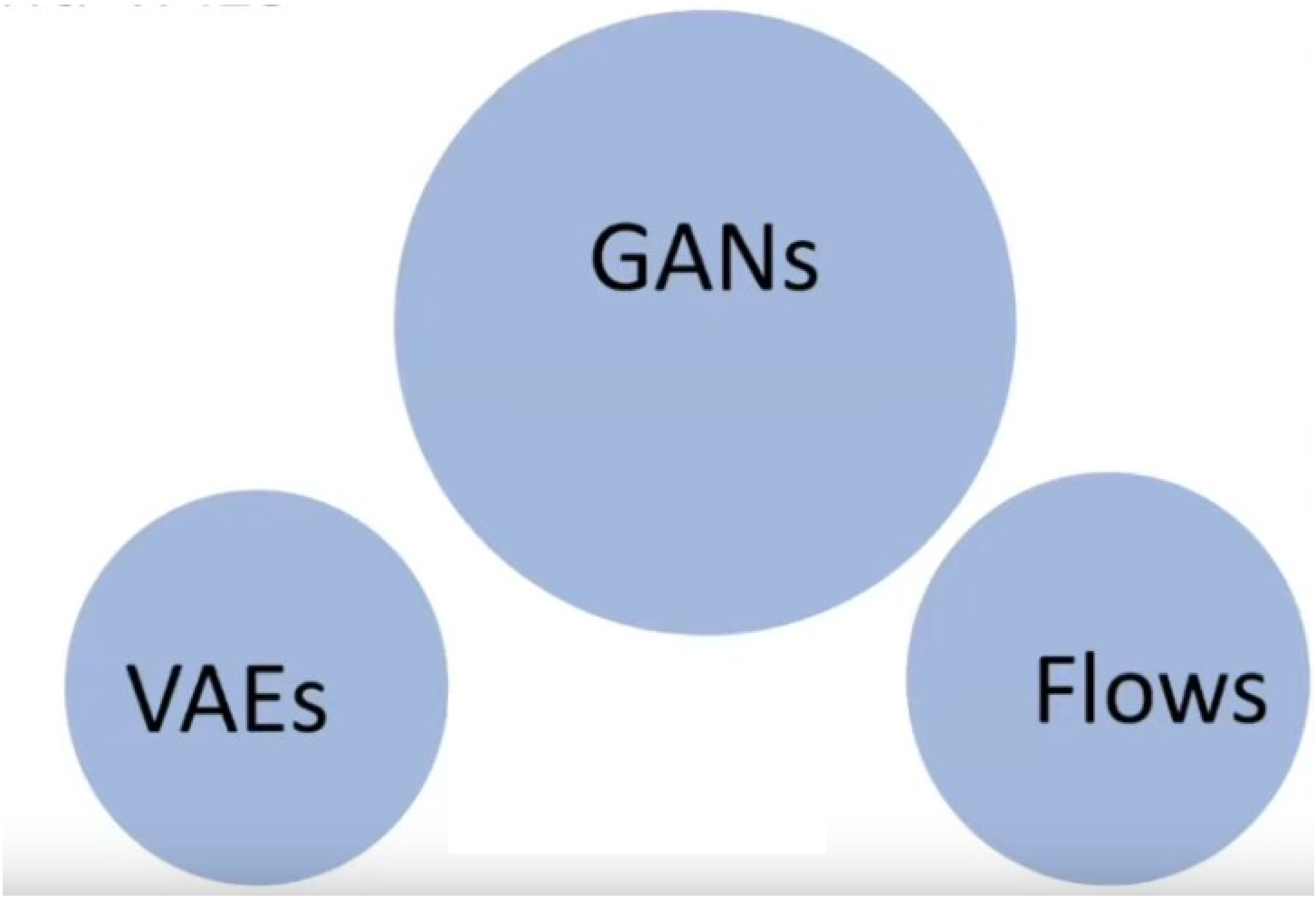
Titus Nicolae
Robert Maria

Richard Feynman: "*What I cannot create, I do not understand*"

Generative modeling: "*What I understand, I can **create***"

source

Most popular methods today



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Overview

1) Dimensionality reduction

2) Autoencoders (AE)

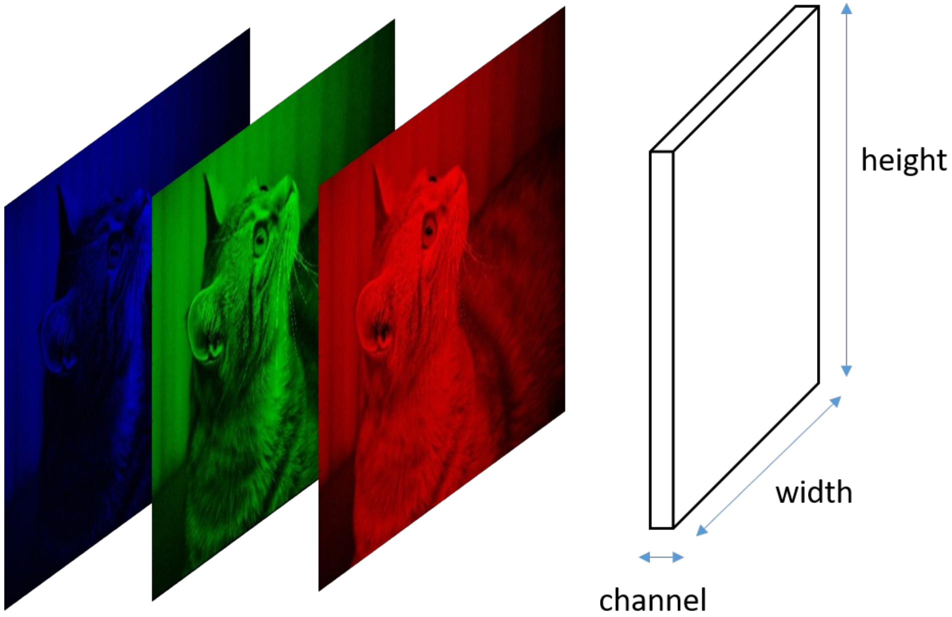
3) Variational Autoencoders (VAE)

4) Disentanglement

5) Beta-VAE

Dimensionality reduction

What is an image made of



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Dimensionality reduction

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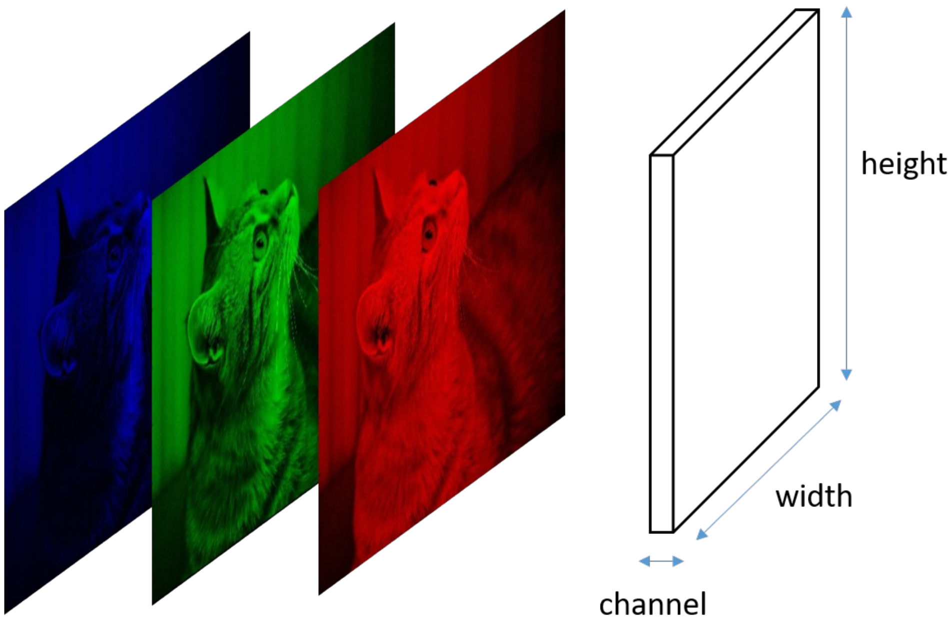

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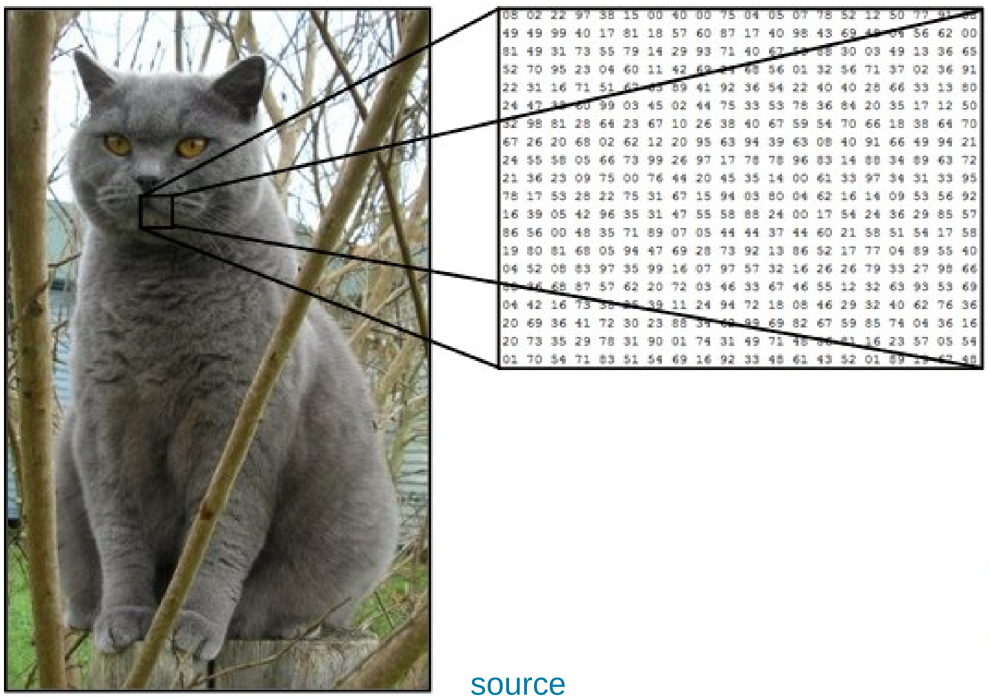
What the computer sees



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Can we reduce the dimensionality of the features (in this case pixels)?

Dimensionality reduction

Yes, we can reduce the dimension of data

by selection

Just some of the existing features are kept

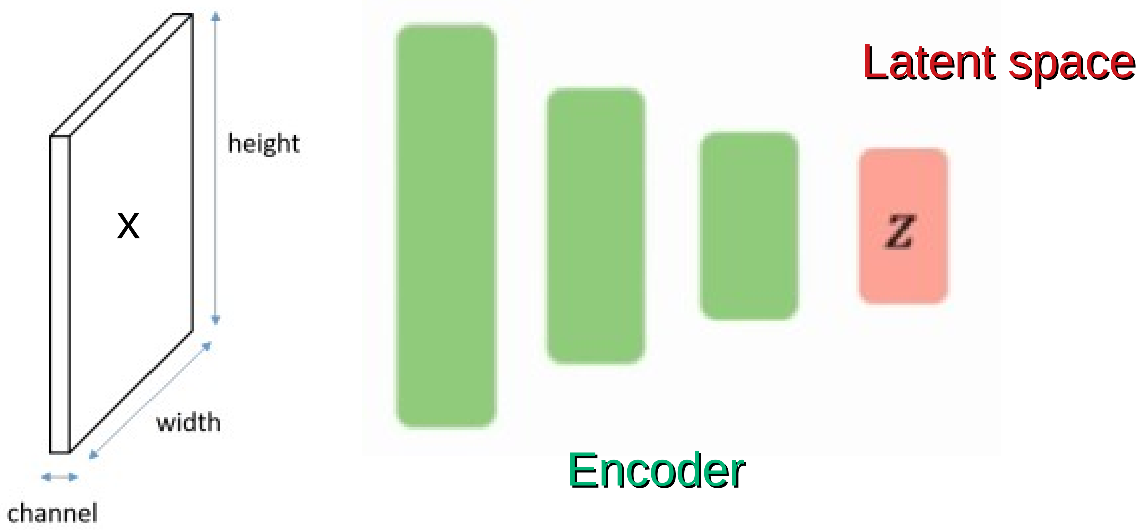
by extraction

New features are created

Reduce dimensionality in a semi-supervised manner with Autoencoders

Dimensionality reduction with Autoencoders

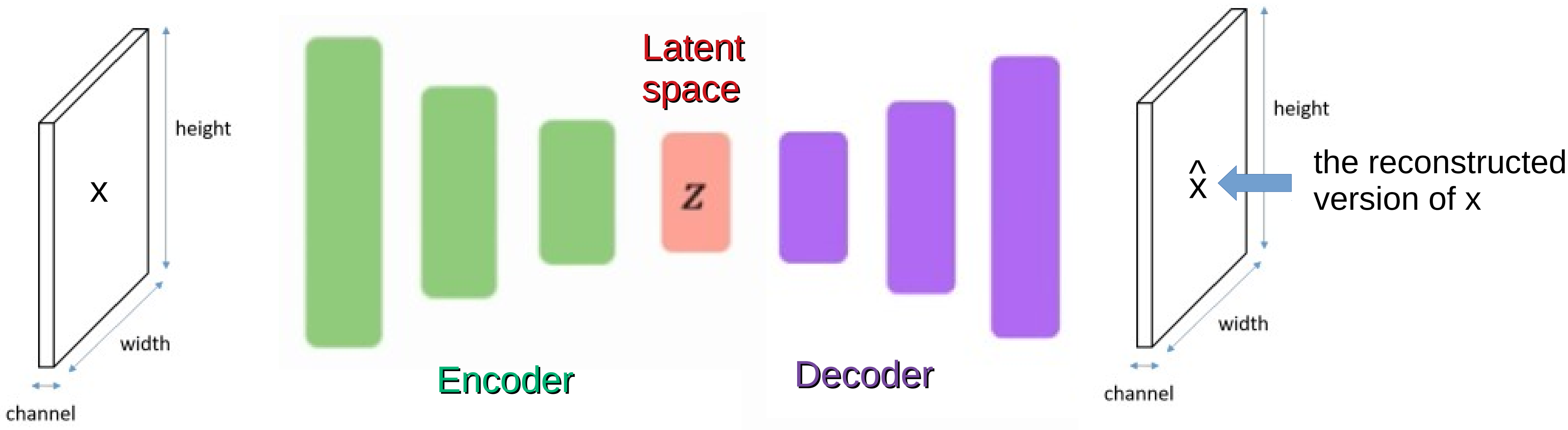
The Encoder of Autoencoder



Data reconstruction

The Encoder of Autoencoder

The Decoder of Autoencoder



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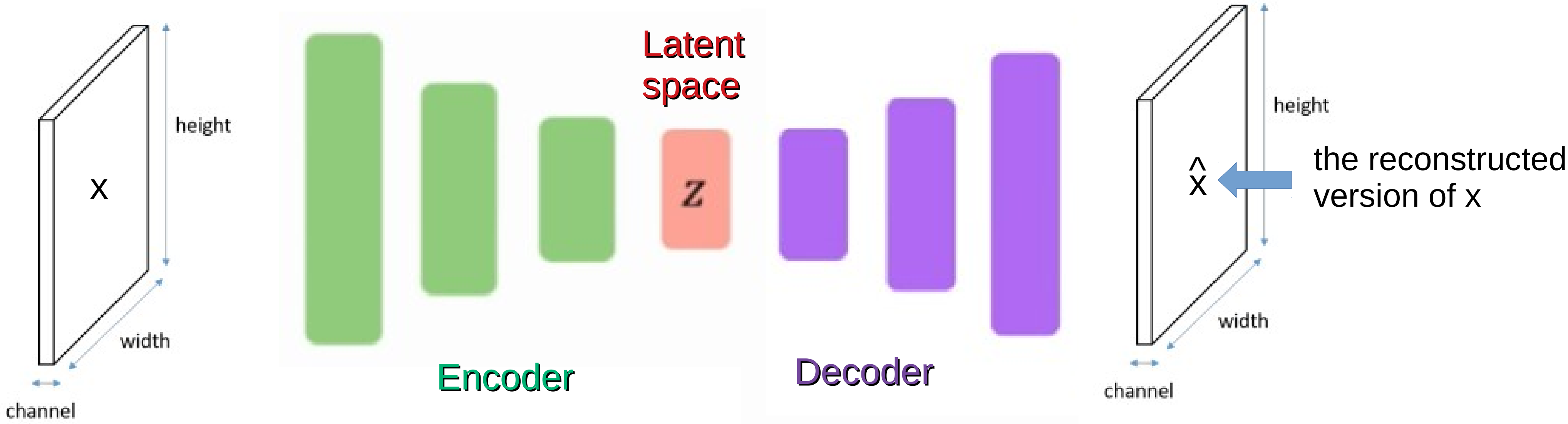
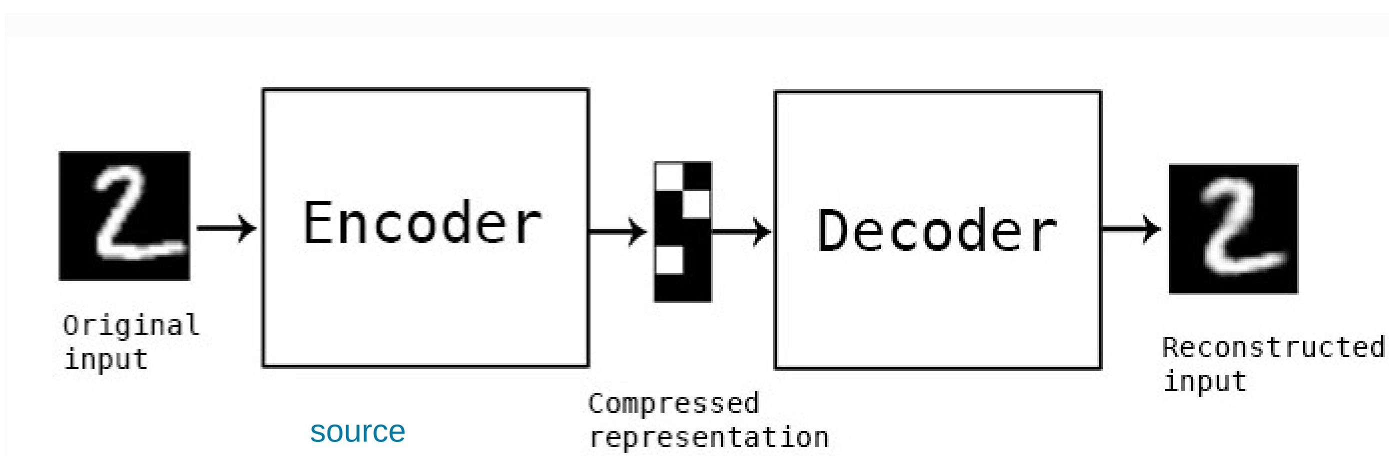


Example:



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Autoencoders

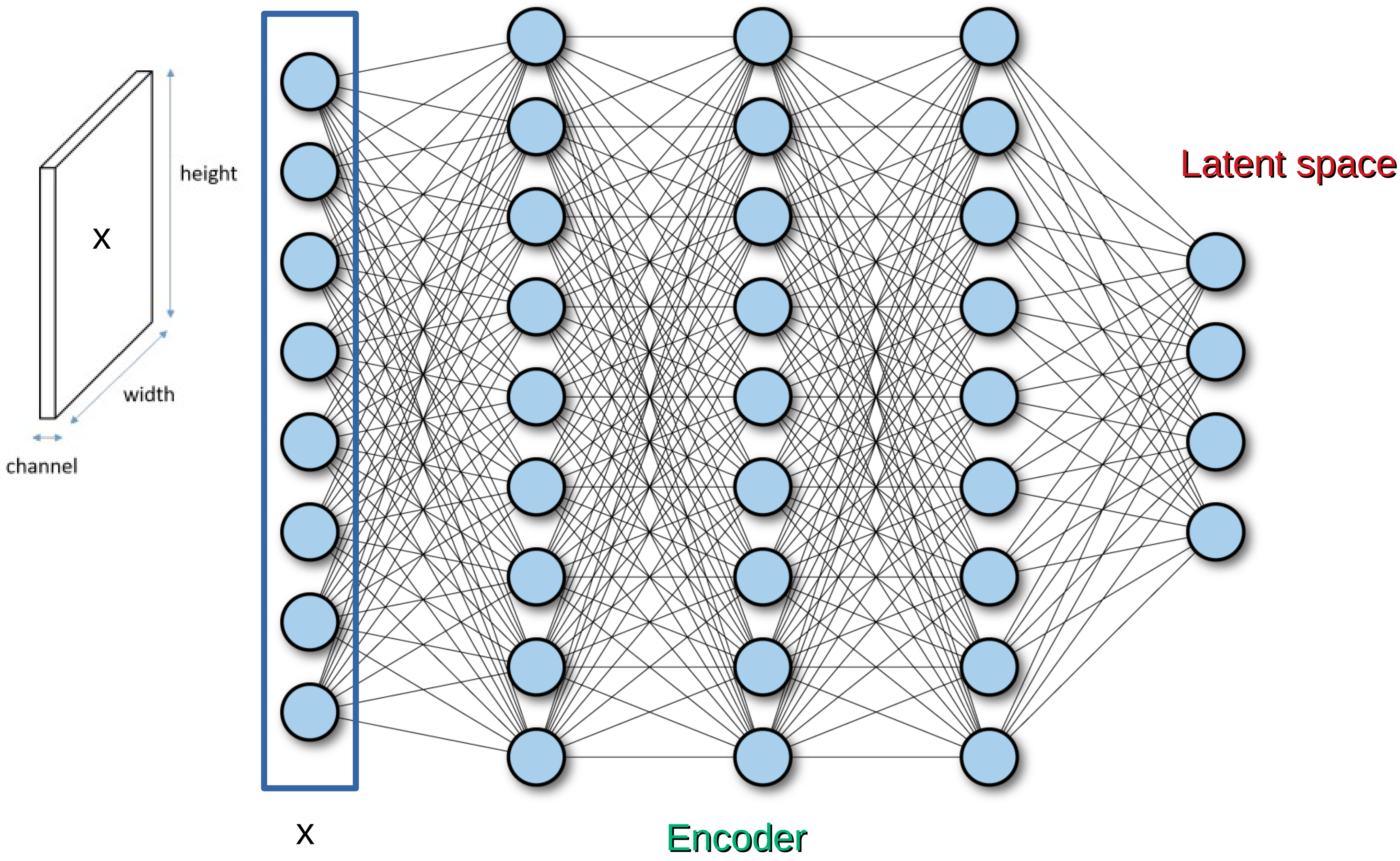
Autoencoders are a data compression algorithms where the compression and decompression functions are:

1) data-specific

2) lossy

3) learned automatically

Deep Fully Connected Autoencoder



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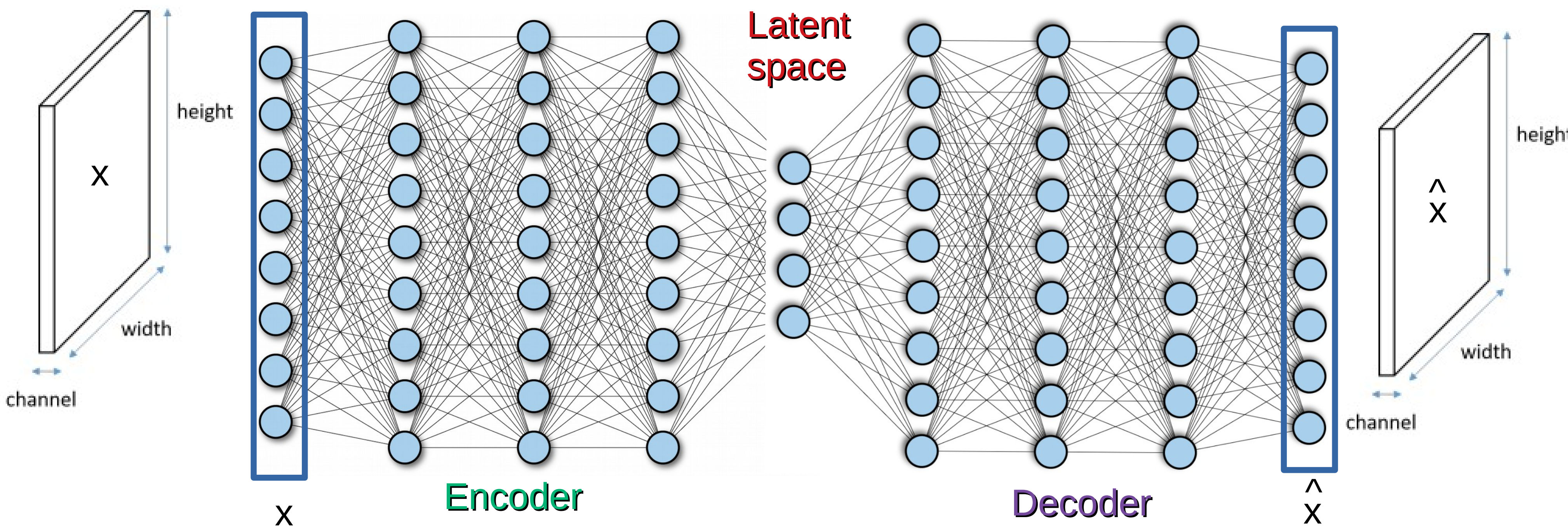
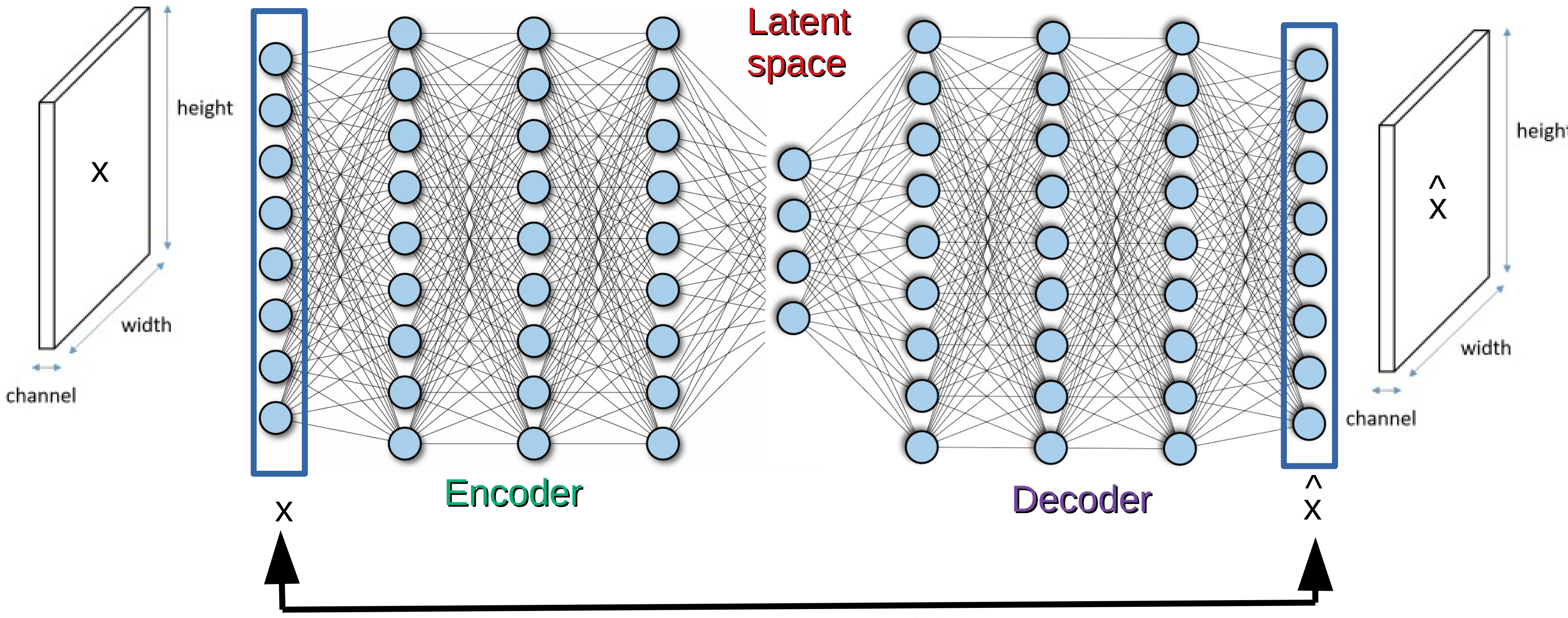


$$\hat{X} = D(E(x))$$

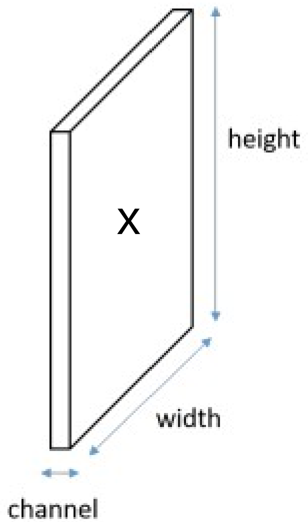
How do we train it?

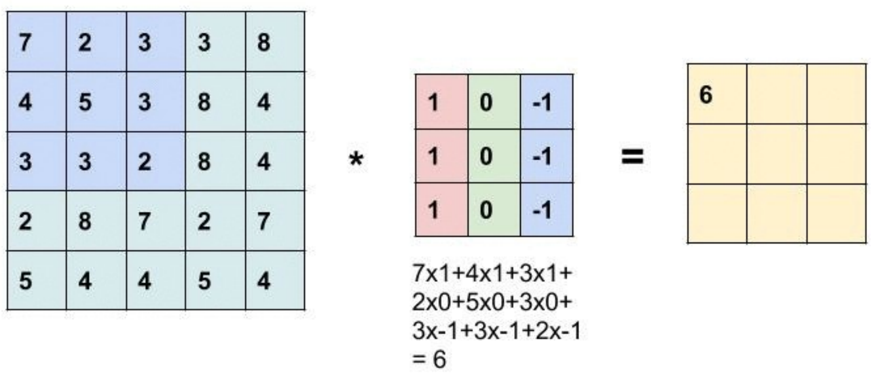


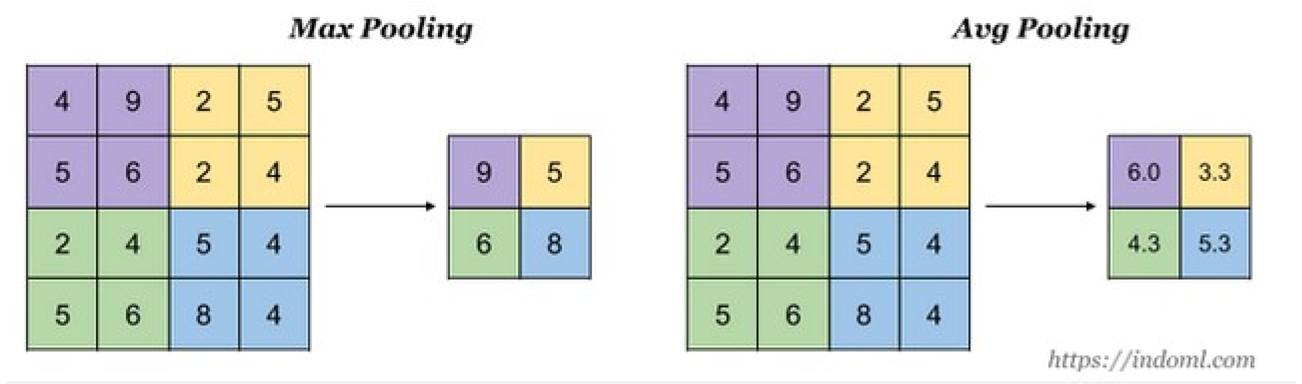
$$\mathcal{L}(x, \hat{x}) = \|x - \hat{x}\|^2$$

Deep Convolutional Autoencoder

Convolution layers







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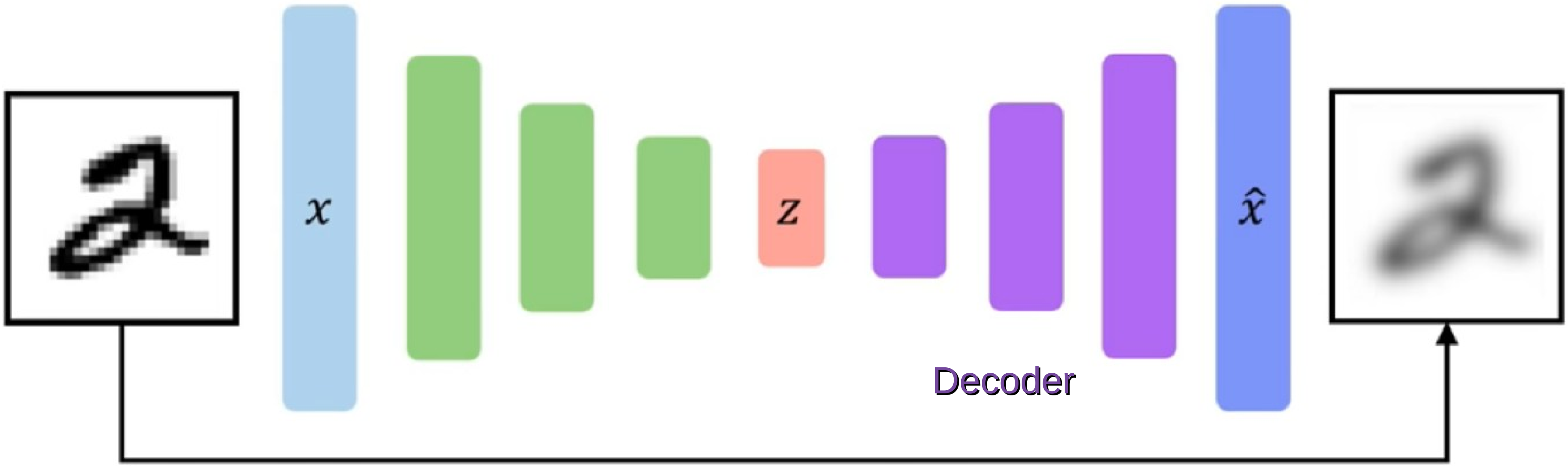
Deep Convolutional Autoencoder



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source

Encoder convolutional layers, pooling

Decoder convolutional layers, transposed convolutions or upsampling layers

What are they good for?

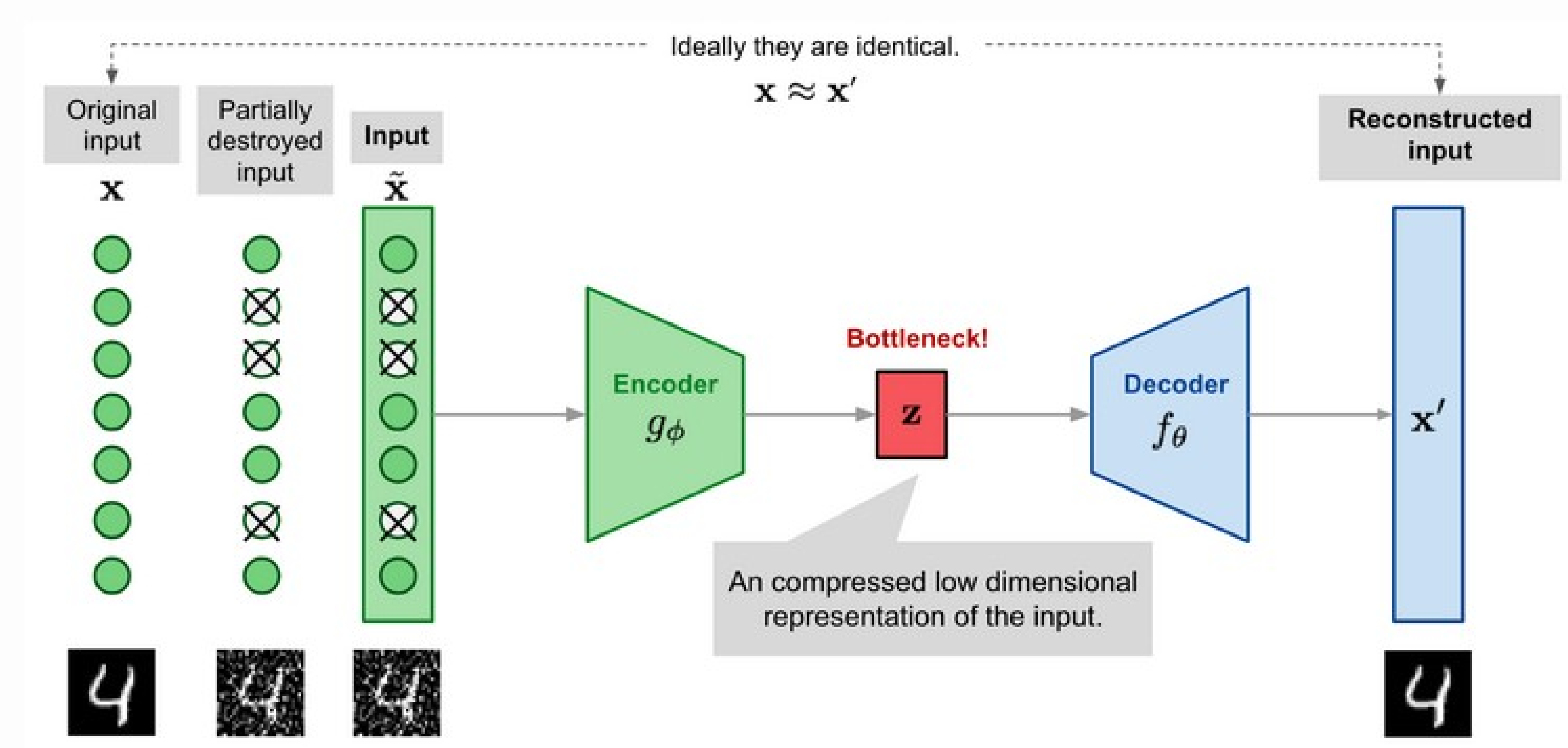
Not best choice for data compression

Data denoising (denoising autoencoders)

Dimensionality reduction for data visualization

Good starting point to understand Variational Autoencoders (VAEs)

Denoising Autoencoder



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Goal


Shutdown
You don't have permission to shut down this computer.
OK
I'm sorry Dave, I can't let you do that

Goal

Building Machines That Learn and Think Like People

Brenden M. Lake,[1] Tomer D. Ullman,[2,4] Joshua B. Tenenbaum,[2,4] and Samuel J. Gershman[3,4]
[1]Center for Data Science, New York University
[2]Department of Brain and Cognitive Sciences, MIT
[3]Department of Psychology and Center for Brain Science, Harvard University
[4]Center for Brains Minds and Machines

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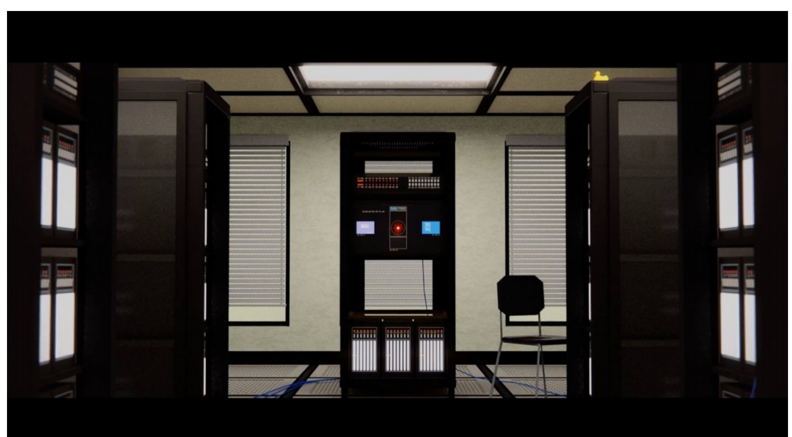

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Observe
the world

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Observe the world

Interact with the world

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Interact with the world

Internal representation of the world

Goal

Understand the laws of the world (physics, psychology, etc.)

Observe the world

Language

Interact with the world

Learn and pattern recognition

Internal representation of the world

Understand compositionality, causality

Imagination

Challenges

Generalization

Humans generalize from a single example?

Training example ("water bear")

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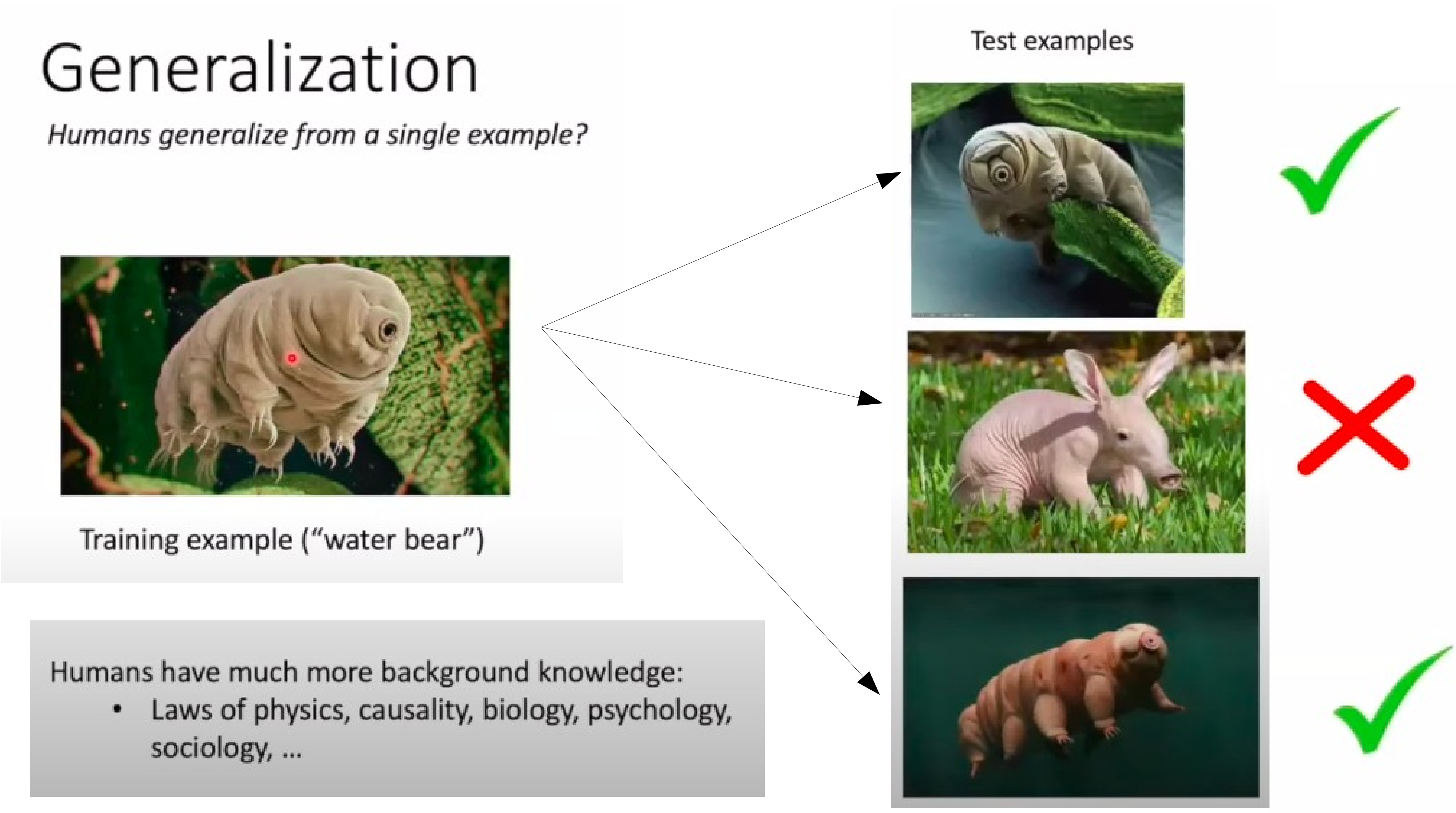
Test examples

Challenges



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Representation learning

Can you perform aritmetic on Roman numerals?

$$\boxed{XXXVII + XLII = ?}$$

Representation learning

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Or you perform faster on Arabic numerals?

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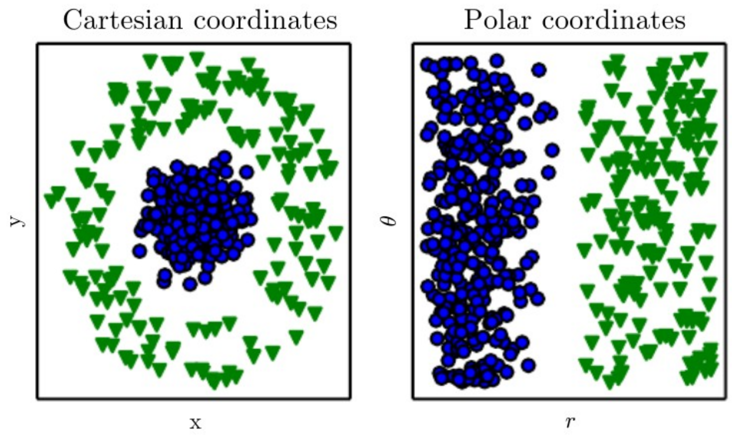
Example of different representations



Which of the two can be correctly classified by a linear classifier?

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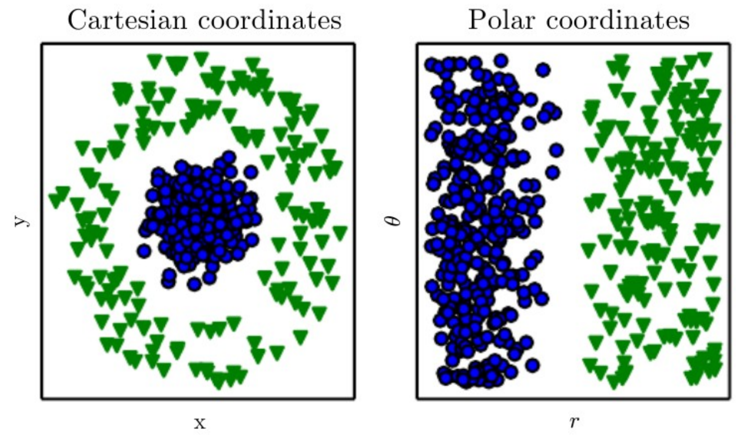

Which of the two can be correctly classified by a linear classifier?

Good representation capture posterior belief about explanatory causes, disentangle these underlying vactors of variations.

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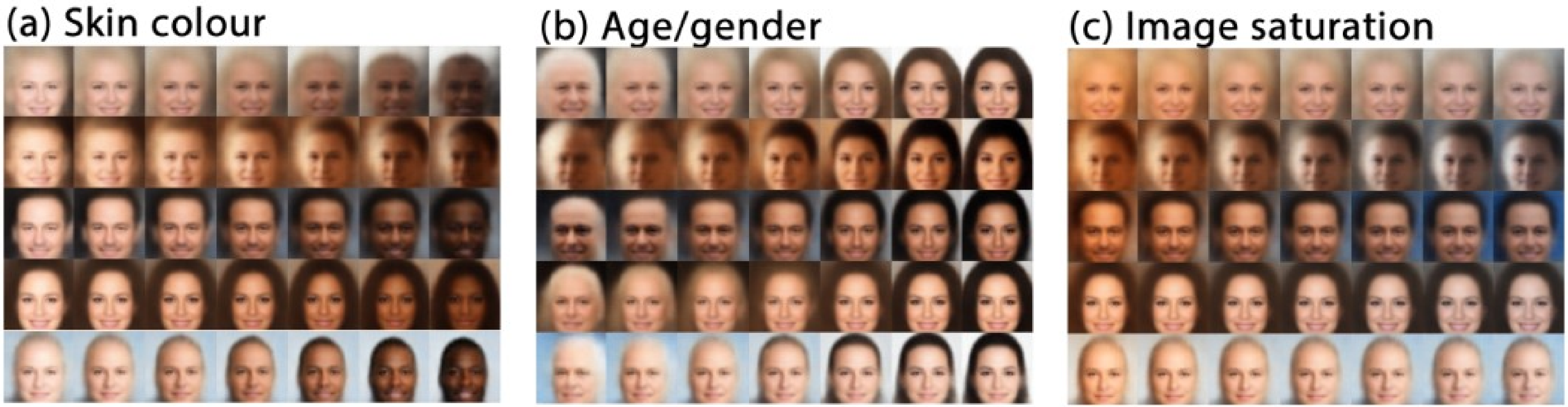
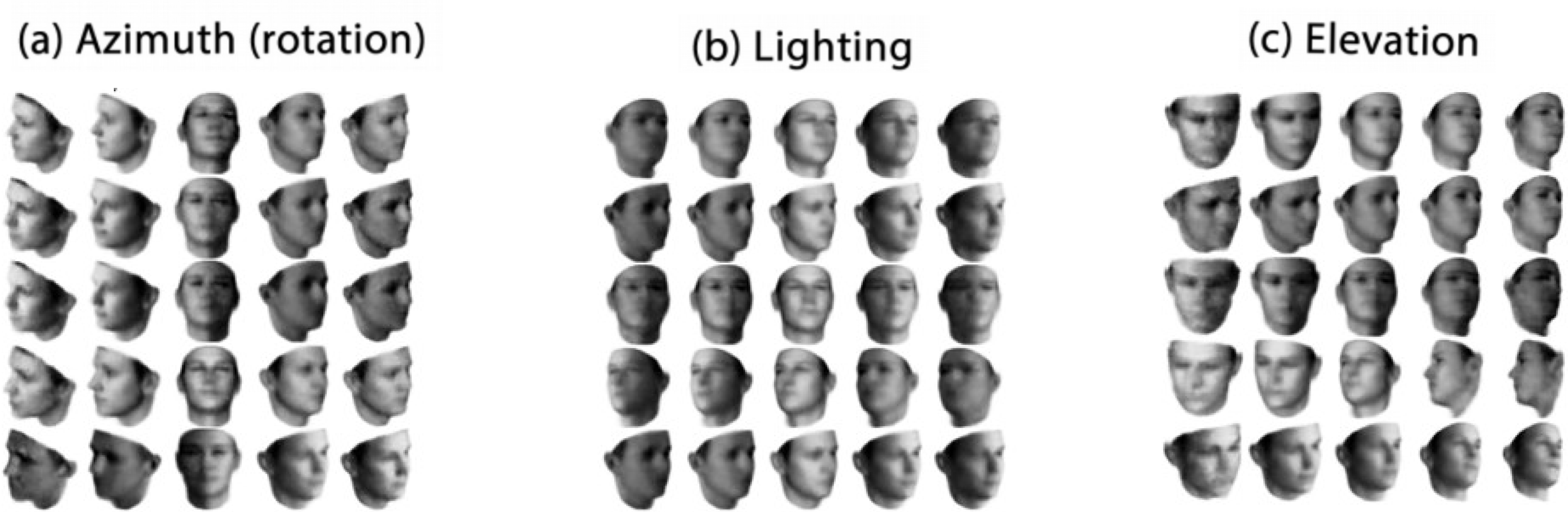
Disentanglement

Disentanglement





Challenges

Unsupervised learning of a disentangled posterior distribution over the underlying generative factors of sensory data

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Why?

Challenges

Unsupervised learning of a disentangled posterior distribution over the underlying generative factors of sensory data

Why?

Knowledge of one factor can generalize to novel configurations of other factors

Faster learning / learning from few examples

Challenges

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We may not have direct access to the generative factors

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Little or no supervision for discovering the factors

How to compare models which perform disentanglement?

β-VAE

Learning an interpretable factorised representation of the independent data generative factors of the world without supervision

β-VAE

D = {X, V, W} = a set of images x and 2 generative factors

V = conditionally independent factors

W = conditionally dependent factors

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Assume images x are generated by $p(\mathbf{x}|\mathbf{v},\mathbf{w}) = \mathbf{Sim}(\mathbf{v},\mathbf{w})$

$\mathbf{Sim}(\mathbf{v},\mathbf{w})$ = true world simulator

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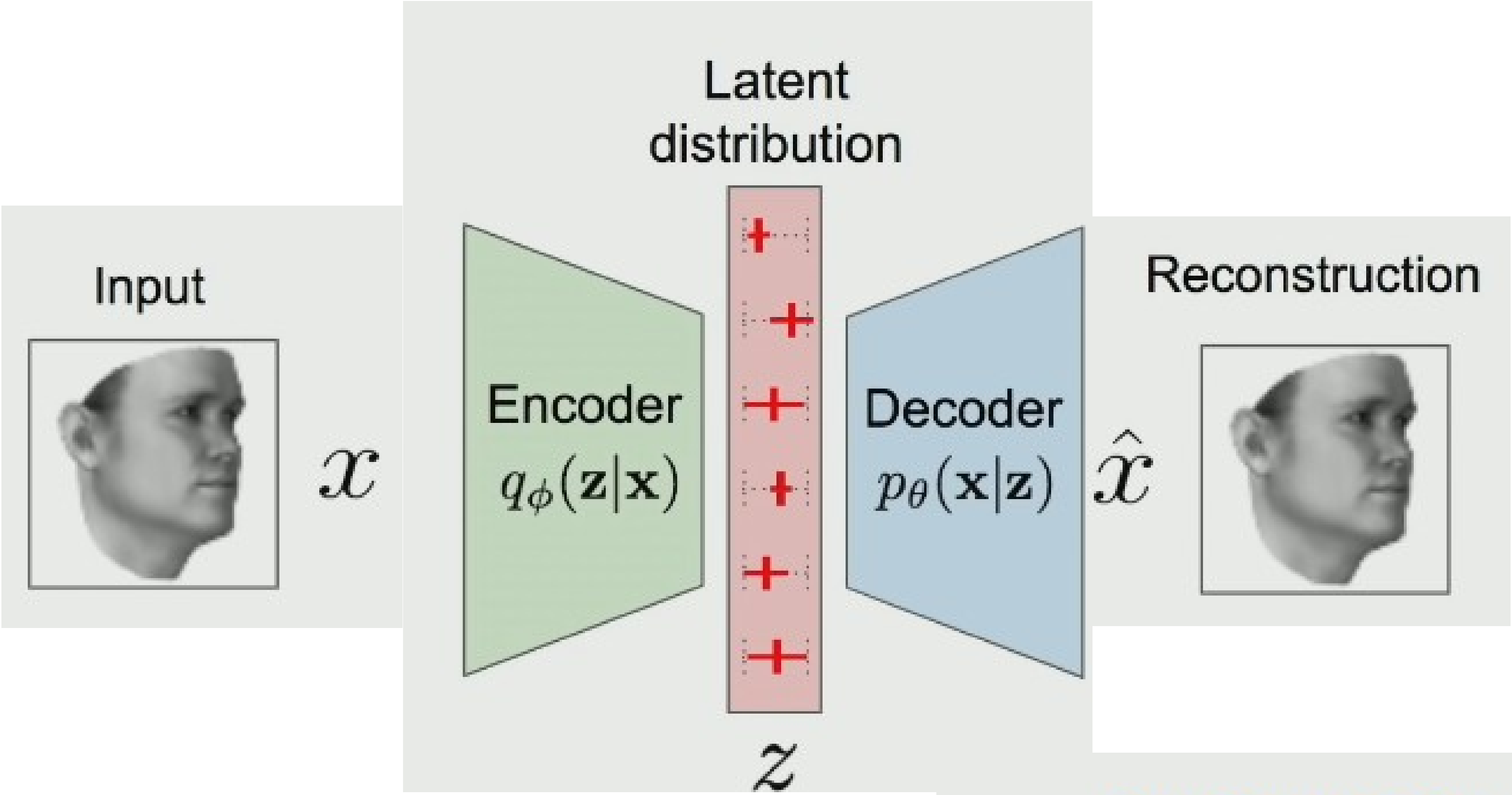
Initial objective: $\max_{\theta} \mathbb{E}_{p_\theta(\mathbf{z})}[p_\theta(\mathbf{x}|\mathbf{z})]$

β-VAE



$$\mathcal{L}_{\mathrm{VAE}}(\theta,\phi) = \underbrace{\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}\left[\log p_\theta(\mathbf{x}|\mathbf{z})\right]}_{\text{Reconstruction cost}} - \overbrace{KL\left[q_\phi(\mathbf{z}|\mathbf{x}) \;||\; p(\mathbf{z})\right]}^{\text{Capacity constraint}}$$

Second objective: $q_\phi(\mathbf{z}|\mathbf{x})$ captures **v** in a disentangled manner

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They remain entangled in a separate subset of **z** that is not used in representing **v**

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Match it to a prior $p(\mathbf{z}) = \mathcal{N}(\mathbf{0}, I)$

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Constraint optimization problem

$$\max_{\phi,\theta} \mathbb{E}_{x\sim\mathbf{D}} \left[\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}[\log p_\theta(\mathbf{x}|\mathbf{z})]\right]$$

$$\text{subject to } D_{KL}(q_\phi(\mathbf{z}|\mathbf{x})||p(\mathbf{z})) < \epsilon$$

β-VAE

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Re-write as:

$$\mathcal{F}(\theta,\phi,\beta;\mathbf{x},\mathbf{z}) \geq \mathcal{L}(\theta,\phi;\mathbf{x},\mathbf{z},\beta) = \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}[\log p_\theta(\mathbf{x}|\mathbf{z})] - \beta\, D_{KL}(q_\phi(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))$$

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$\mathcal{L}(\theta,\phi;\mathbf{x},\mathbf{z},\beta)$ = Lagrangian under KKT conditions

$\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}[\log p_\theta(\mathbf{x}|\mathbf{z})]$ = reconstruction loss

$D_{KL}(q_\phi(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))$ = regularization term

β = regularization coefficient

β-VAE

$$\mathcal{F}(\theta, \phi, \beta; \mathbf{x}, \mathbf{z}) \geq \mathcal{L}(\theta, \phi; \mathbf{x}, \mathbf{z}, \beta) = \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}[\log p_\theta(\mathbf{x}|\mathbf{z})] - \beta\, D_{KL}(q_\phi(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))$$

β
- controls the capacity of the latent
- puts independence pressure

β = 1 becomes VAE

β-VAE

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Disentanglement representation emerge when the right balance is found between information preservation and latent channel capacity restriction.

β-VAE

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Related to the information bottleneck principle:

$$\max[I(Z;Y) - \beta I(X;Z)]$$

New evaluation metric

$\beta > 1$ leads to poorer reconstructions due to the loss of high freq details

cannot use log-likelihood of the data under the learnt model

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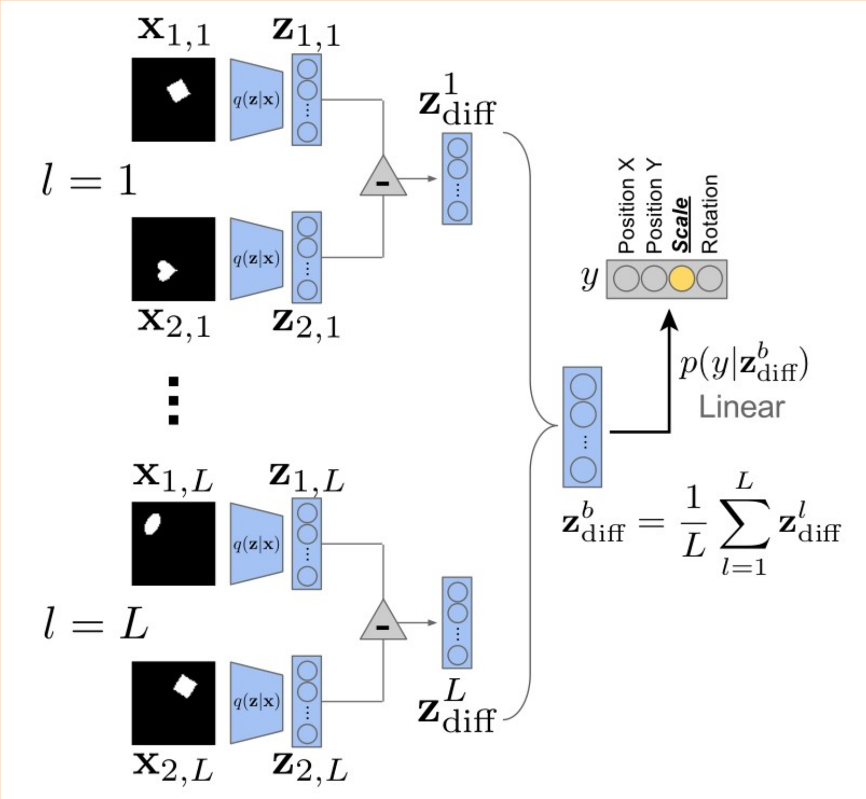
The new metric: **the accuracy of a low capacity classifier**



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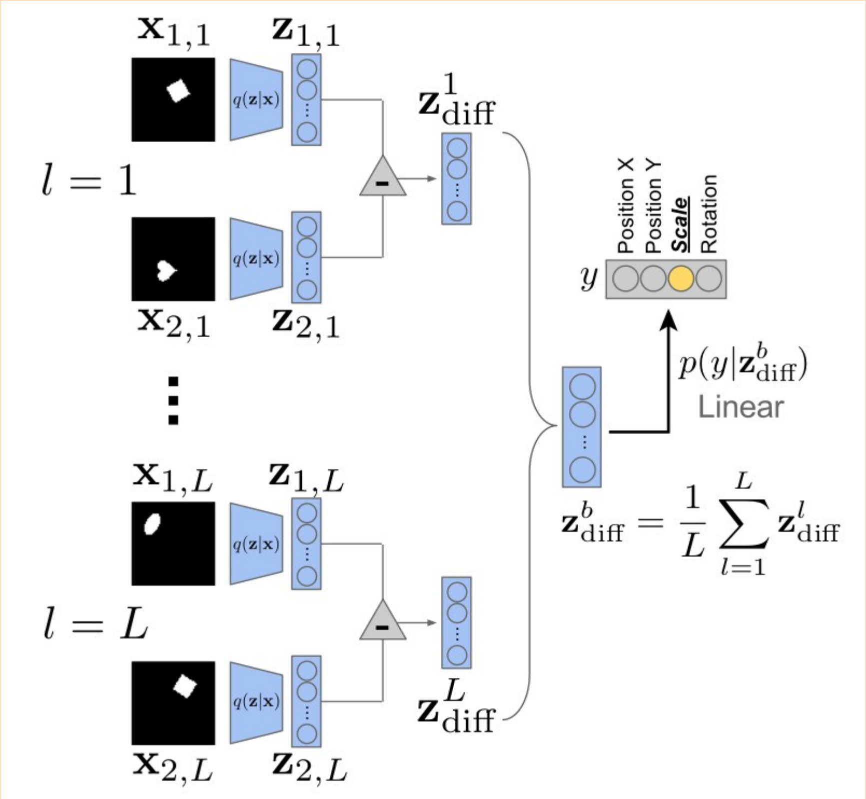


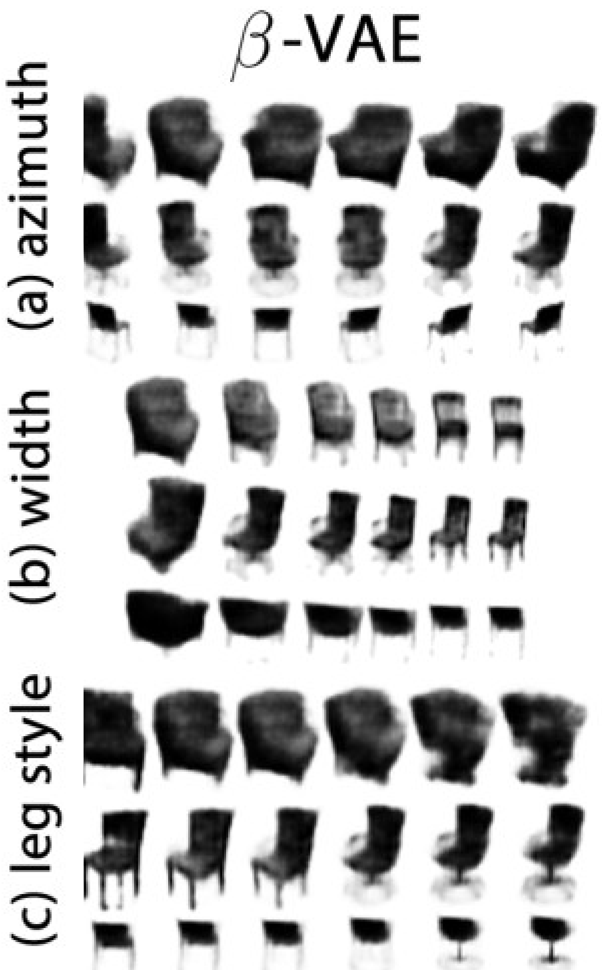
Cons: needs access to the world simulator **Sim**(**v**,**w**)

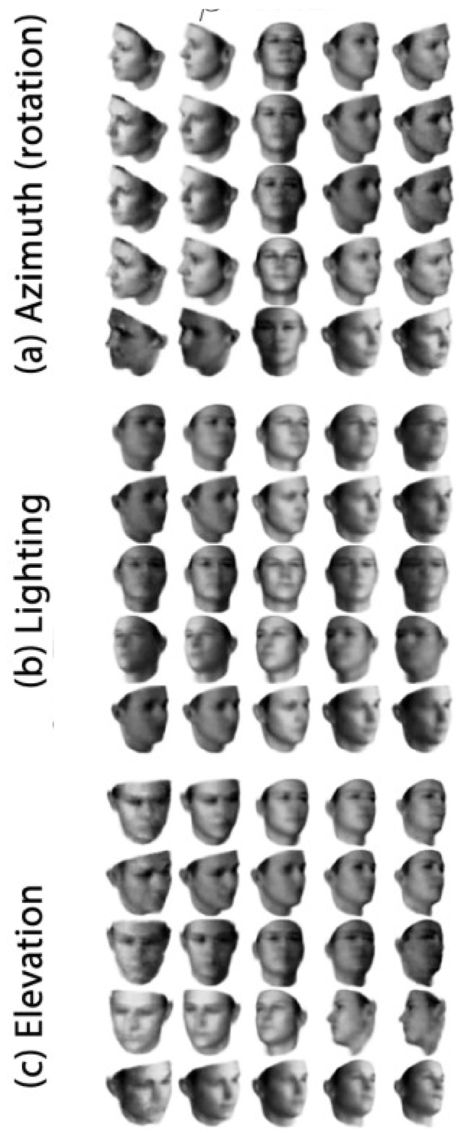
Results

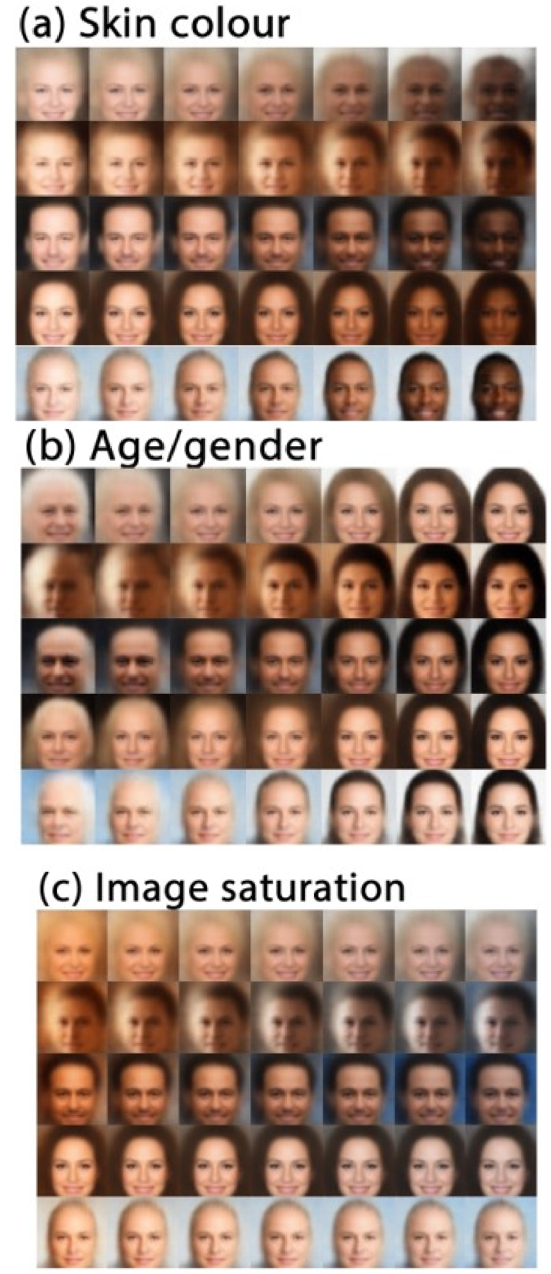
Qualitative

Results

Qualitative







Results

Quantitative

Results

Quantitative

Model	Disentanglement metric score
Ground truth	*100%*
Raw pixels	$45.75 \pm 0.8\%$
PCA	$84.9 \pm 0.4\%$
ICA	$42.03 \pm 10.6\%$
DC-IGN	$\mathbf{99.3} \pm \mathbf{0.1}\%$
InfoGAN	$73.5 \pm 0.9\%$
VAE untrained	$44.14 \pm 2.5\%$
VAE	$61.58 \pm 0.5\%$
β-VAE	$\mathbf{99.23} \pm \mathbf{0.1}\%$

On a synthetic dataset of 2D shapes (heart, oval, square)

With indep generative factors: positionX, positionY, scale and rotation