Mathematical model of GISC spectral camera

GISC spectral camera is an imaging technique which can observed the 3D spectral image data-cube by a two-dimensional detector in a single exposure. The image recovery process of GISC spectral camera can be seen as solving the optimization problem as shown below, including a loss term, a TV term and a nuclear norm term representing the spatial correlation and spectral correlation of spectral images:

$$\min_{\mathbf{x}} \|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2 + \mu_1 \|\tilde{\mathbf{X}}\|_{\mathrm{TV}} + \mu_2 \|\mathbf{X}\|_*, \quad s.t. \quad \mathbf{x} \geq \mathbf{0}$$

where the scalar parameters $\mu_1, \mu_2 \geq 0$ are the weights of regularizations. $\mathbf{y} \in \mathbb{R}^m$ is the observed signal , $\mathbf{A} \in \mathbb{R}^{m \times n} \left(m << n\right)$ is the measurement matrix which is known. $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{X} \in \mathbb{R}^{n_1 n_2 \times n_3}$, $\tilde{\mathbf{X}} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$ are the vector form, matrix form and cube form of unknown spectral images, respectively. $n_1 \times n_2$ is the size of the image of each wavelength, and n_3 is the number of wavelengths, $n = n_1 n_2 n_3$.

Regularization terms

TV term $\|\tilde{\mathbf{X}}\|_{\mathrm{TV}}$ is the sum of 2D tv norm of images of different wavelengths, i.e.

$$\|\tilde{\mathbf{X}}\|_{\mathrm{TV}} = \sum_{\lambda=1}^{n_3} \|\tilde{\mathbf{X}}(:,:,\lambda)\|_{\mathrm{tv}} = \sum_{\lambda=1}^{n_3} \sum_{i=1}^{n_2} \sum_{i=1}^{n_1} \sqrt{\left(\tilde{\mathbf{X}}_{i+1,j,\lambda} - \tilde{\mathbf{X}}_{i,j,\lambda}\right)^2 + \left(\tilde{\mathbf{X}}_{i,j+1,\lambda} - \tilde{\mathbf{X}}_{i,j,\lambda}\right)^2}$$

$\|\mathbf{X}\|_*$ is the nuclear norm of spectral image matrix, $\|\mathbf{X}\|_* = \sum_{i=1}^{n_3} \sigma_i$, where σ_i is the singular value of $\mathbf{X}$.

References: *Spectral Camera based on Ghost Imaging via Sparsity Constraints*

http://www.nature.com/articles/srep25718