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Recurrent Inference Machines for Solving Inverse Problems

UNIVERSITY OF AMSTERDAM

Recurrent Inference Machines in Practice

Inverse Problems

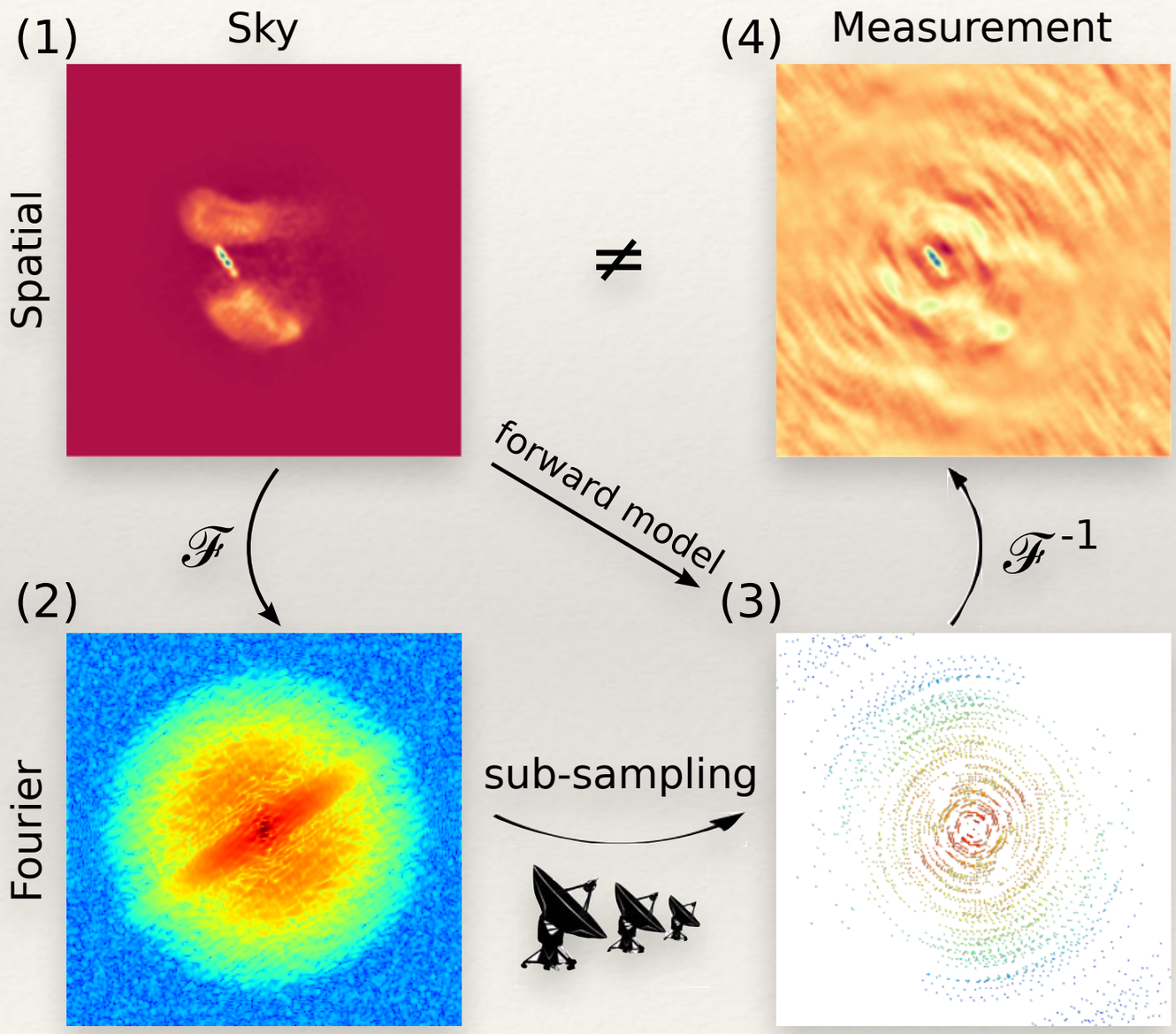
Quantity of interest

Measurement

Forward Model $\mathbf{y} = g(\mathbf{x}) + n$

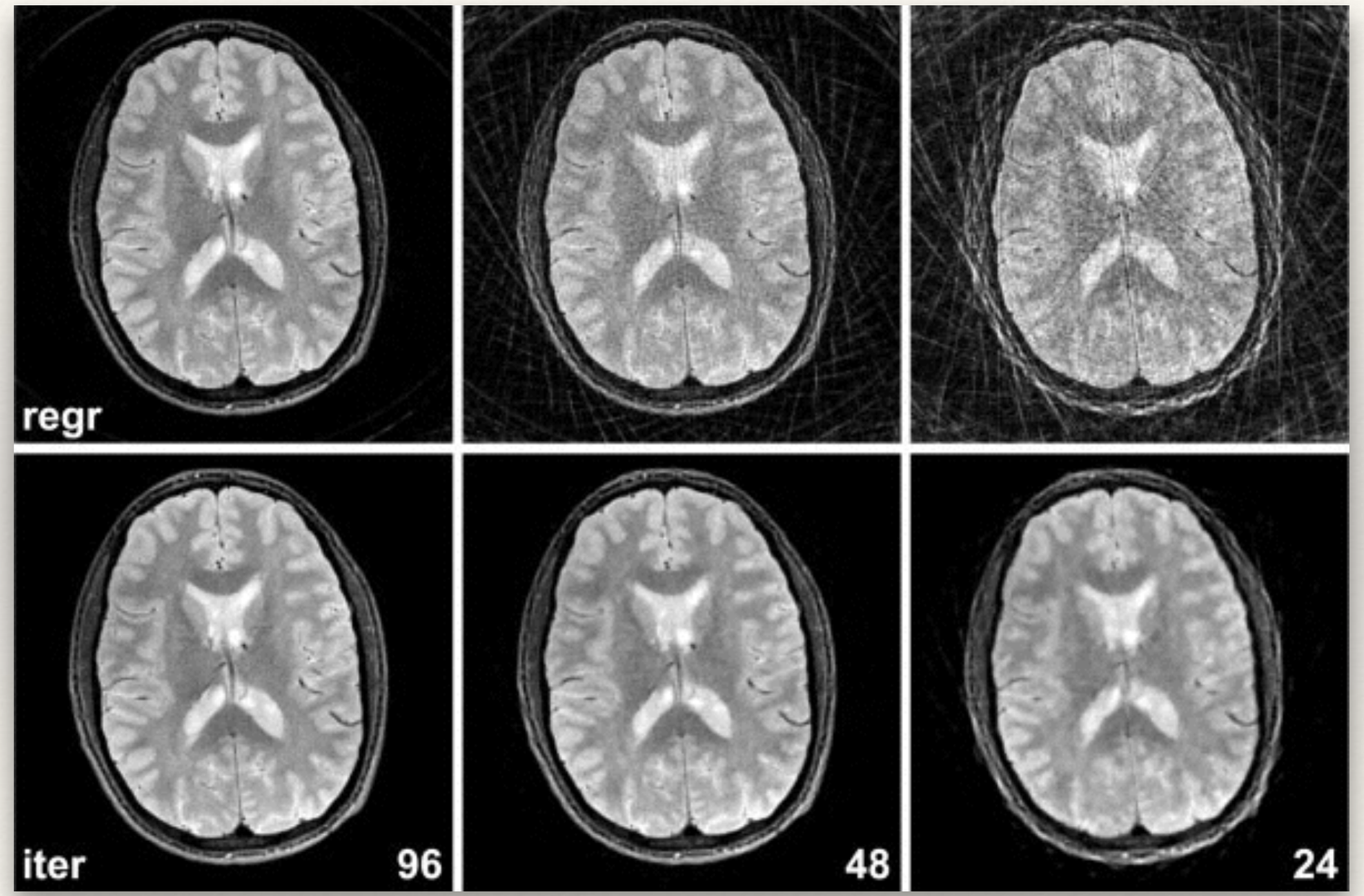
Inverse Model $\hat{\mathbf{x}} = h(\mathbf{y})$

Inverse Problems - Examples



Up to 14.4 Gigapixels
With thousands of Channels

Inverse Problems - Examples

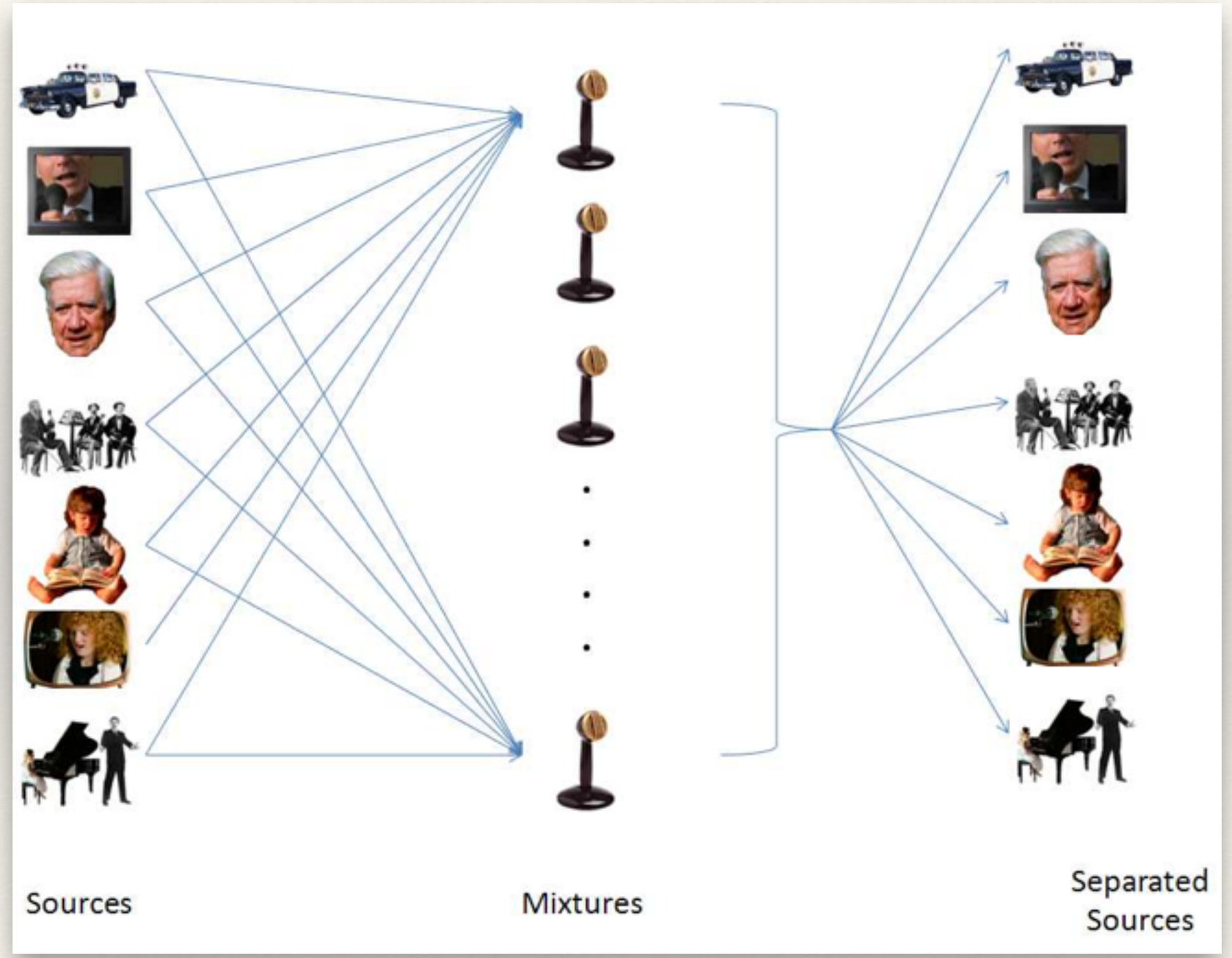


[Block et.al, 2007]

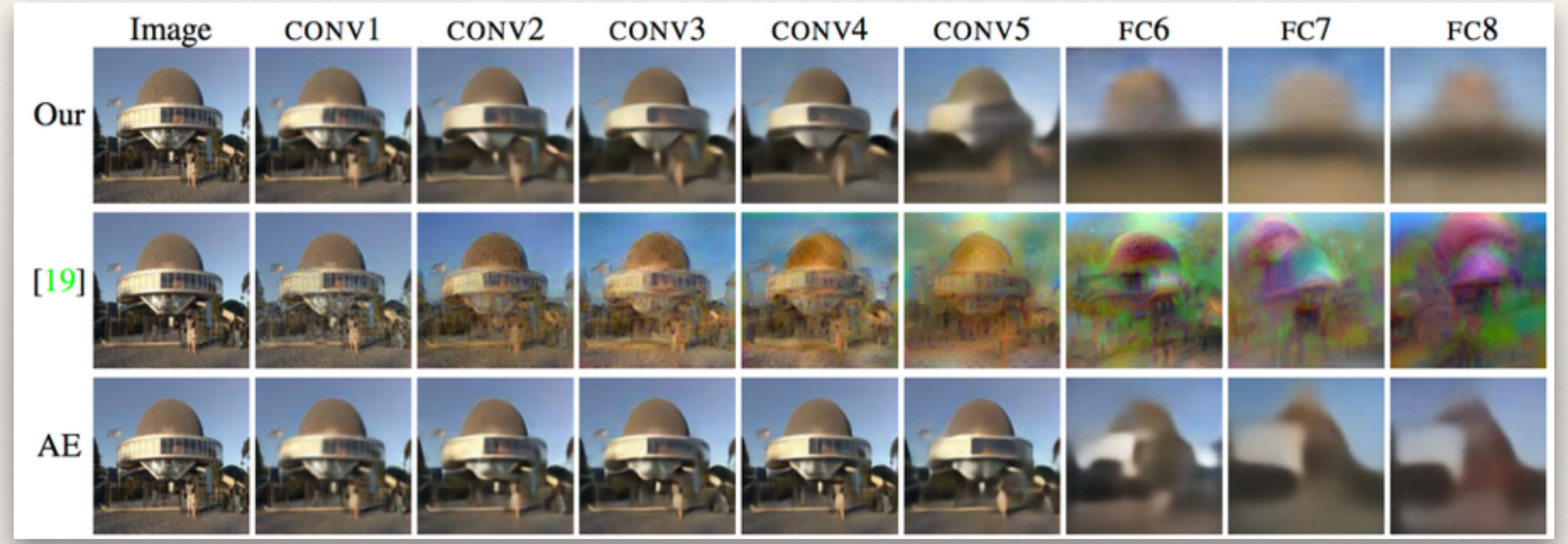
Inverse Problems - Examples

[Xu et al., 2014]

Inverse Problems - Examples



Inverse Problems - Examples

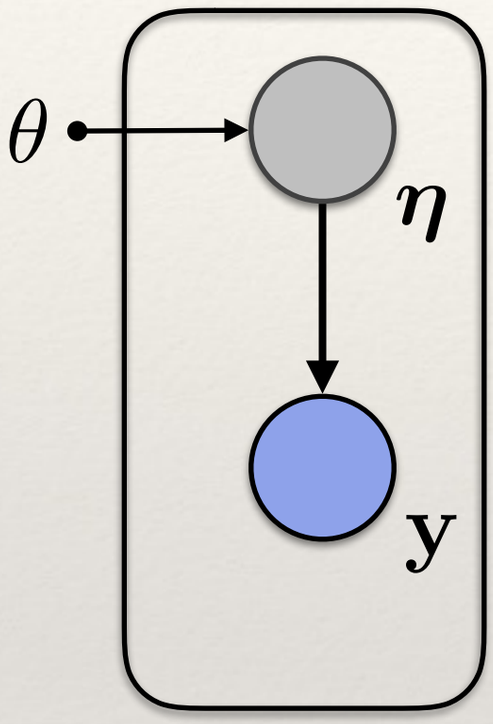


[Dosovitskiy & Brox, 2016]

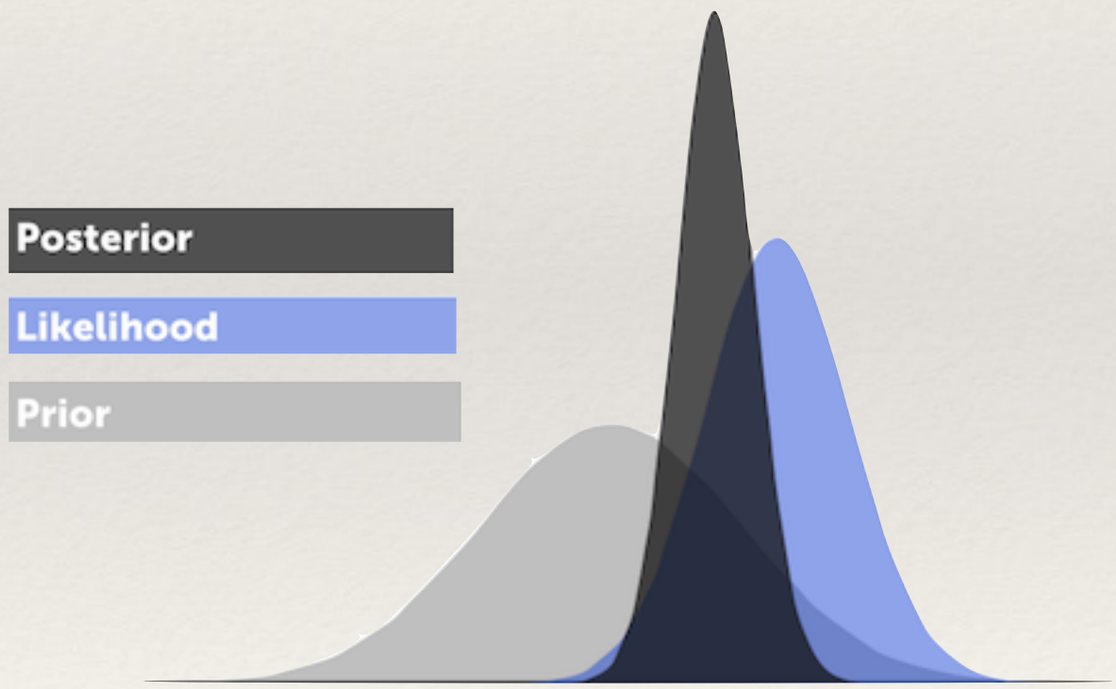
Inverse Problems - Examples

And many more…

Bayesian Inference



$$p_\theta(\boldsymbol{\eta}|\mathbf{y}) = \frac{p(\mathbf{y}|\boldsymbol{\eta})p_\theta(\boldsymbol{\eta})}{p(\mathbf{y})}$$



Iterative Bayesian Inference

$$p_\theta(\boldsymbol{\eta}|\mathbf{y}) = \frac{p(\mathbf{y}|\boldsymbol{\eta})p_\theta(\boldsymbol{\eta})}{p(\mathbf{y})}$$

Choose/learn a prior $p_\theta(\boldsymbol{\eta})$

For likelihood $p(\mathbf{y}|\boldsymbol{\eta})$

Choose inference method $\boldsymbol{\Gamma}$

Iterate

Iterative Bayesian Inference

$$p_\theta(\boldsymbol{\eta}|\mathbf{y}) = \frac{p(\mathbf{y}|\boldsymbol{\eta})p_\theta(\boldsymbol{\eta})}{p(\mathbf{y})}$$

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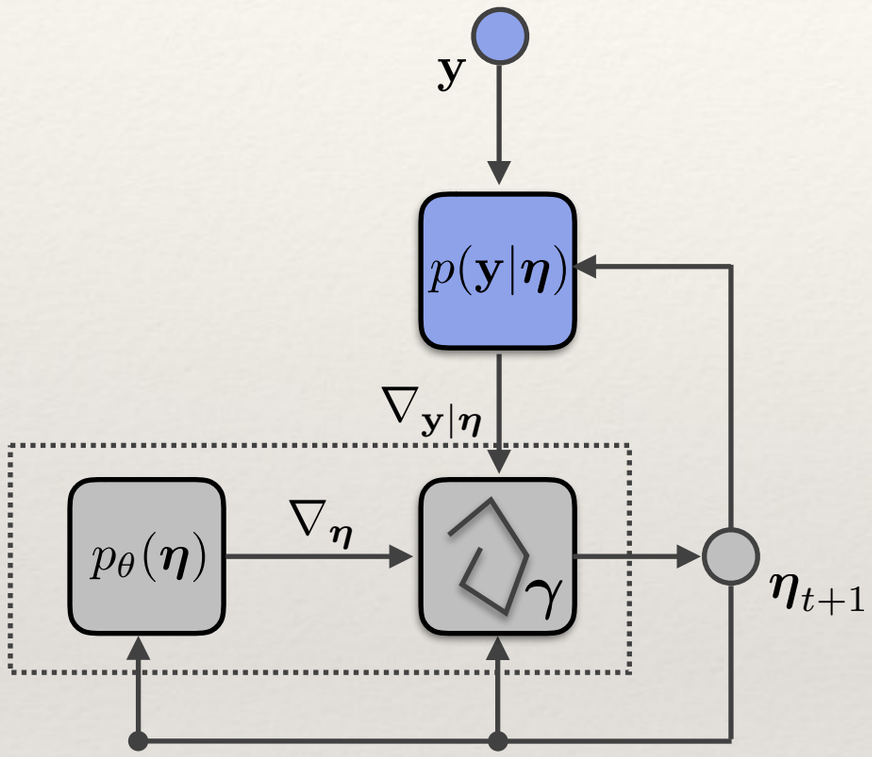
Iterative Inference



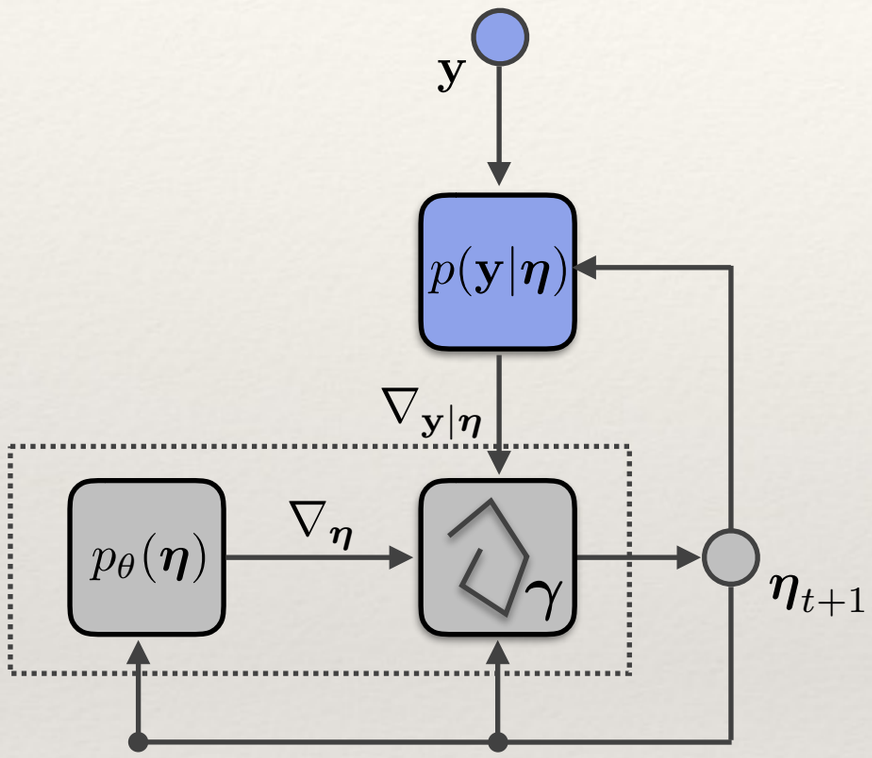
Maximum a posteriori (MAP) inference

$$\hat{\boldsymbol{\eta}} = \arg\max_{\boldsymbol{\eta}} p(\mathbf{y}|\boldsymbol{\eta}) p_\theta(\boldsymbol{\eta})$$

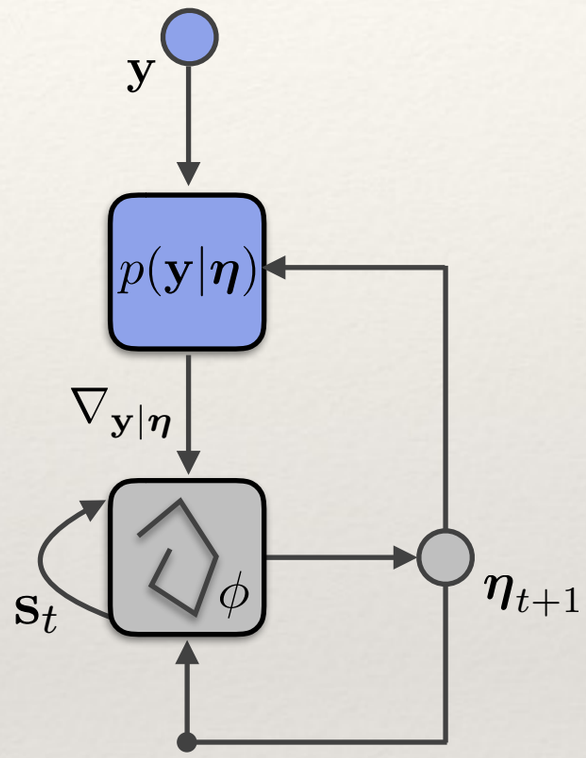
Gradient ascent

$$\begin{aligned}
\boldsymbol{\eta}_{t+1} &= \boldsymbol{\eta}_t + \gamma_t \nabla \log p(\boldsymbol{\eta}|\mathbf{y}) \\
&= \boldsymbol{\eta}_t + \gamma_t (\nabla \log p(\mathbf{y}|\boldsymbol{\eta}) + \nabla \log p(\boldsymbol{\eta})) \\
&= \boldsymbol{\eta}_t + \gamma_t (\nabla_{\mathbf{y}|\boldsymbol{\eta}} + \nabla_{\boldsymbol{\eta}})
\end{aligned}$$

Recurrent Inference Machine

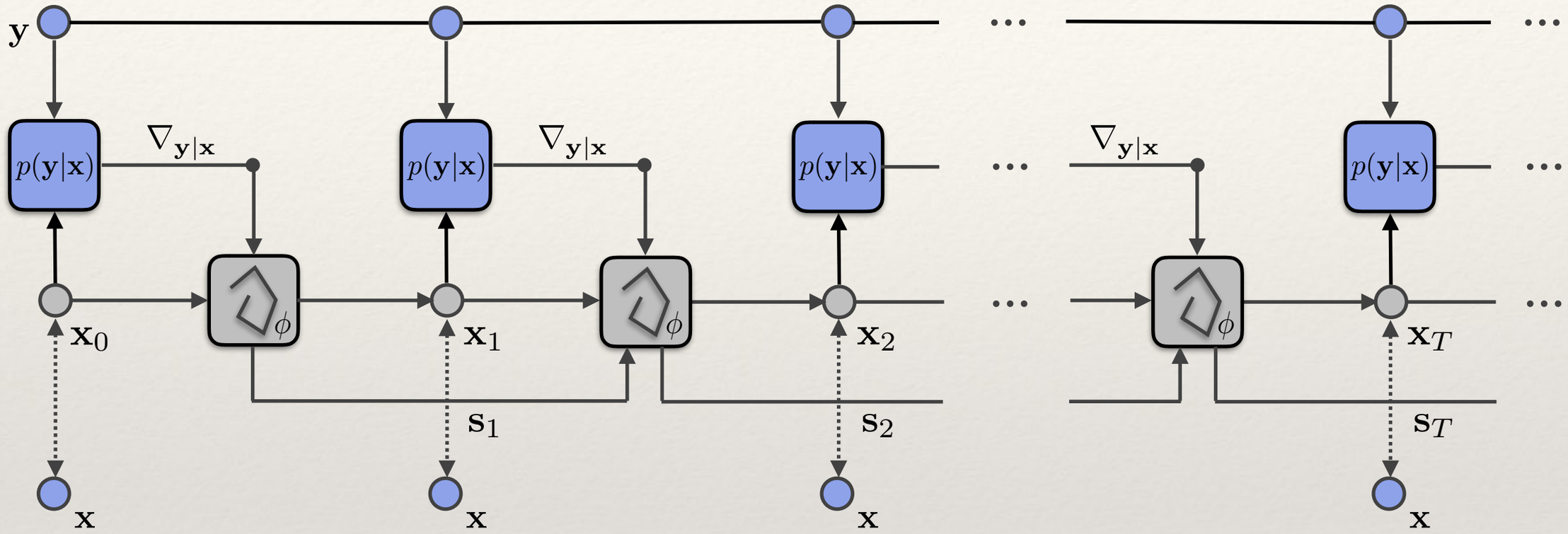


$$\boldsymbol{\eta}_{t+1} = \boldsymbol{\eta}_t + \gamma_t(\nabla_{\mathbf{y}|\boldsymbol{\eta}} + \nabla_{\boldsymbol{\eta}})$$



$$\boldsymbol{\eta}_{t+1} = \boldsymbol{\eta}_t + h_\phi(\nabla_{\mathbf{y}|\boldsymbol{\eta}}, \boldsymbol{\eta}_t, \mathbf{s}_t)$$

Recurrent Inference Machines in Time



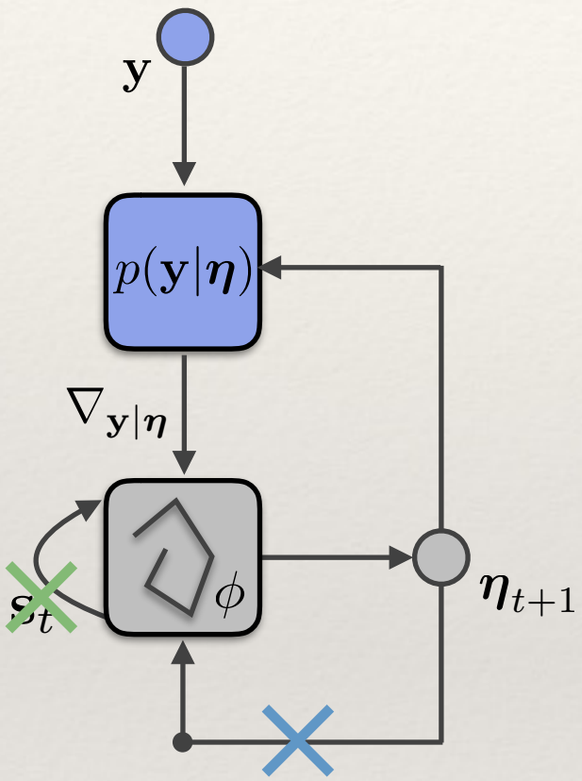
Objective $$g(\phi) = \frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{x}^{(i)} - \hat{\mathbf{x}}_t^{(i)})$$

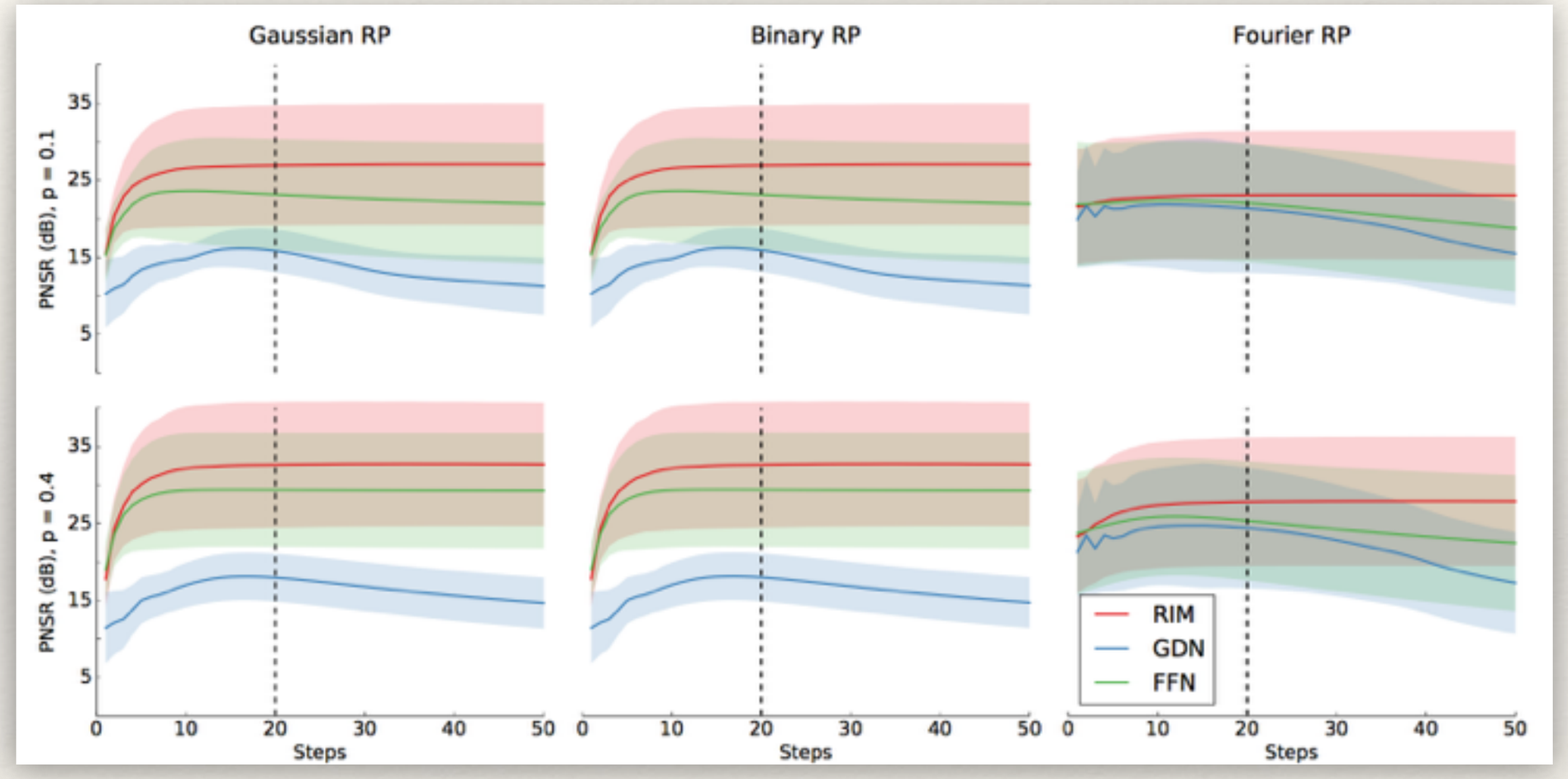
Simple Super-Resolution

Natural Images

200 training images, 481 x 321 pixel each, ~30 Megapixel

Reconstruction from Random Projections

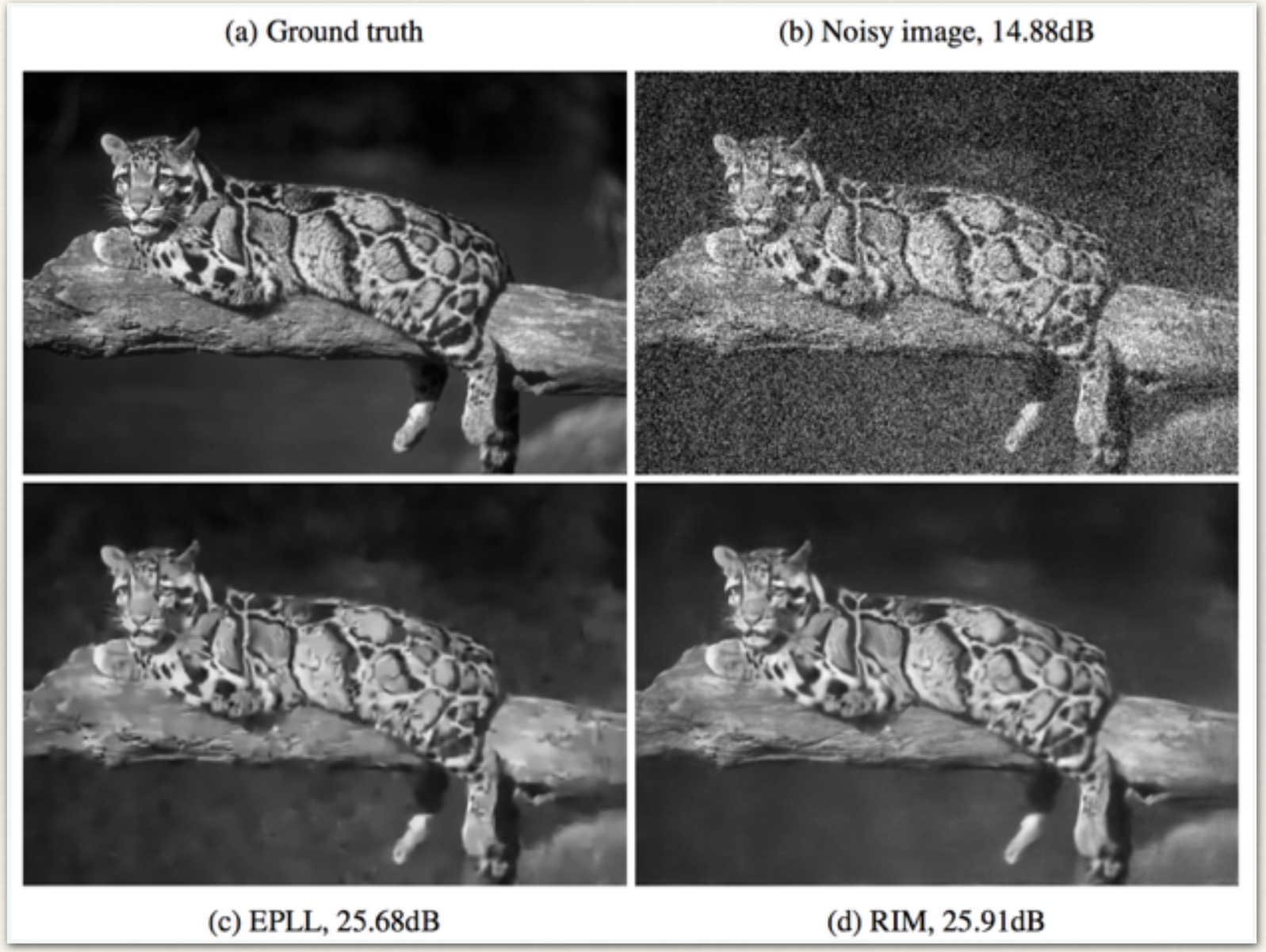




32 x 32 pixel image patches

Fast Convergence on all tasks

Image Denoising



Denoising trained on small image patches, generalises to full-sized images

Image Denoising

Grayscale

	Not Quantized		
σ	15	25	50
KSVD	30.87	28.28	25.17
5x5 FoE	30.99	28.40	25.35
BM3D	31.08	28.56(28.35)	25.62(25.45)
LSSC	31.27	28.70	25.72
EPLL	31.19	28.68(28.47)	25.67(25.50)
opt-MRF	31.18	28.66	25.70
MLP		28.85(28.75)	(25.83)
RTF-5		28.75	
RIM-3task	31.19(30.98)	28.67(28.45)	25.78(25.59)
RIM-denoise	**31.31**(31.10)	**28.91**(28.72)	**26.06**(25.88)

RGB

Method	PSNR
CBM3D	30.18
RTF-5	30.57
RIM (ours)	**30.84**(30.67)

Super-resolution

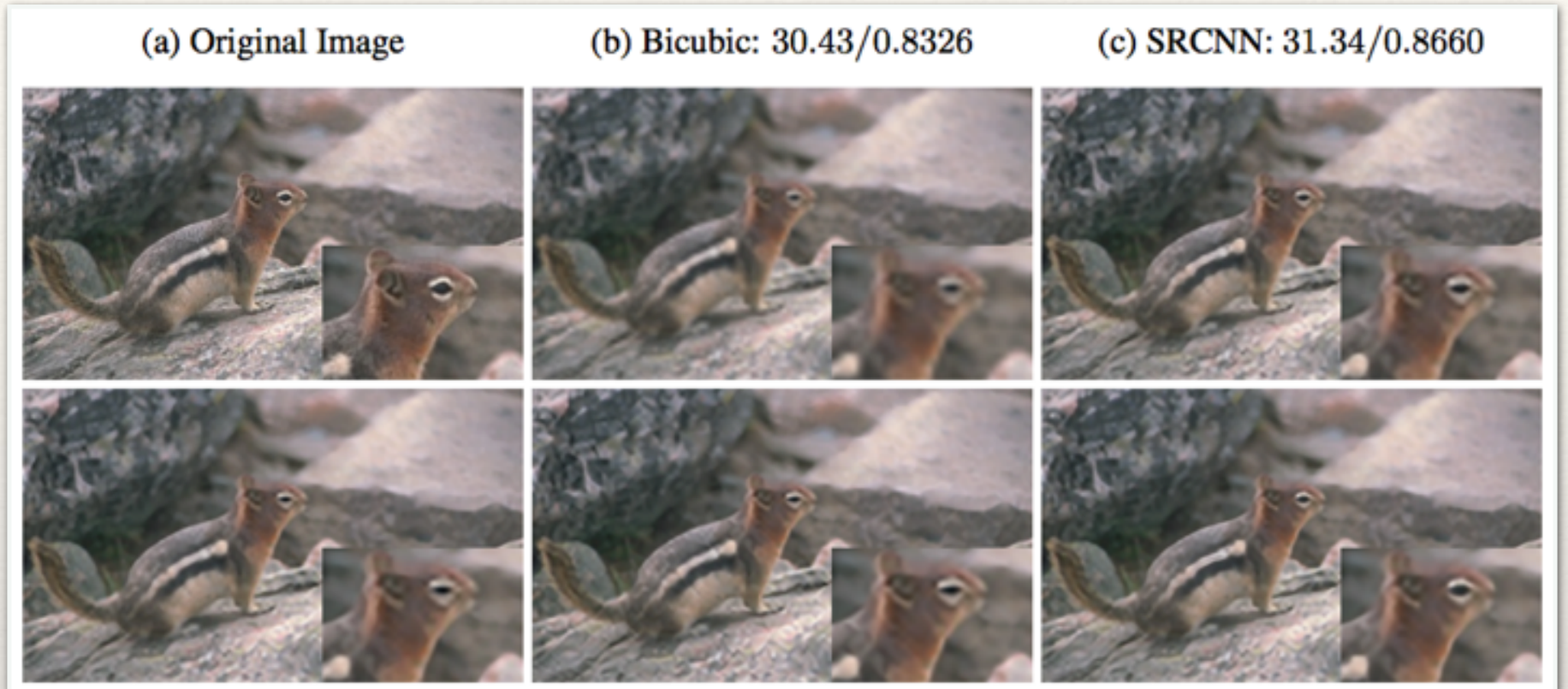
LR

HR

Bicubic Interpolation

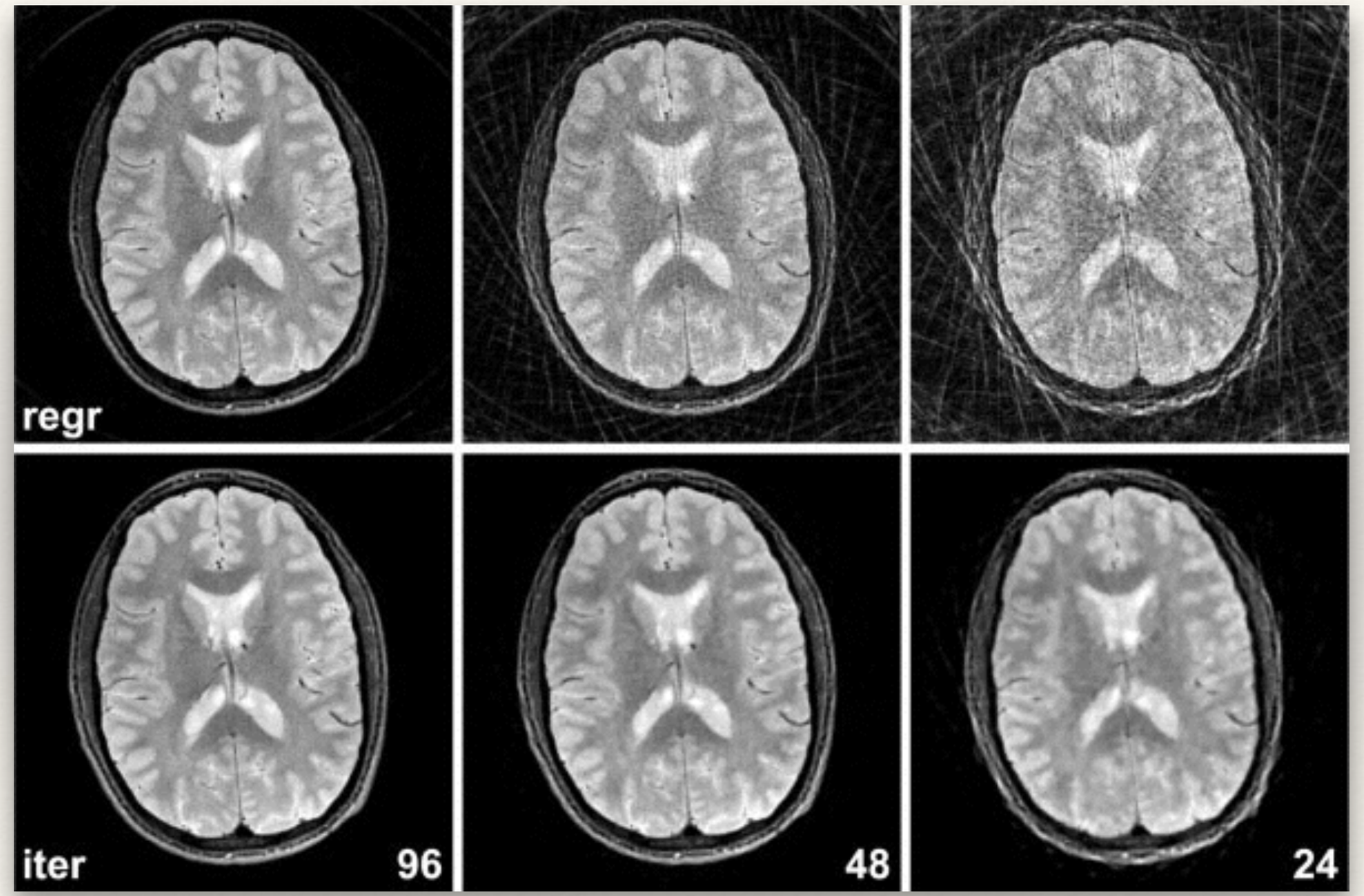
RIM

Super-resolution



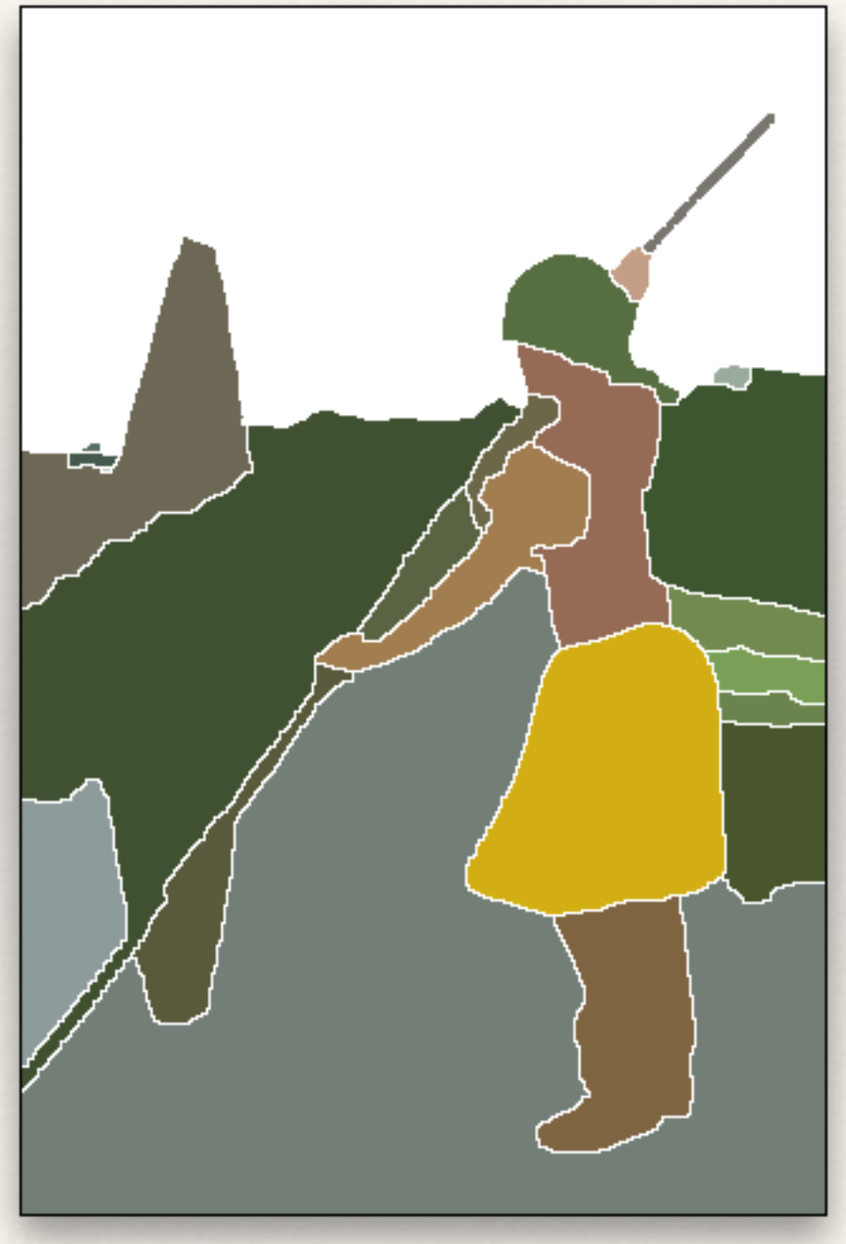
Metric	Scale	Bicubic	SRCNN	A+	SelfExSR	**RIM (Ours)**
PSNR	2x	29.55 ± 0.35	31.11 ± 0.39	31.22 ± 0.40	31.18 ± 0.39	**31.39** ± 0.39
	3x	27.20 ± 0.33	28.20 ± 0.36	28.30 ± 0.37	28.30 ± 0.37	**28.51** ± 0.37
	4x	25.96 ± 0.33	26.70 ± 0.34	26.82 ± 0.35	26.85 ± 0.36	**27.01** ± 0.35
SSIM	2x	0.8425 ± 0.0078	0.8835 ± 0.0062	0.8862 ± 0.0063	0.8855 ± 0.0064	**0.8885** ± 0.0062
	3x	0.7382 ± 0.0114	0.7794 ± 0.0102	0.7836 ± 0.0104	0.7843 ± 0.0104	**0.7888** ± 0.0101
	4x	0.6672 ± 0.0131	0.7018 ± 0.0125	0.7089 ± 0.0125	0.7108 ± 0.0124	**0.7156** ± 0.0125

Projects: MRI

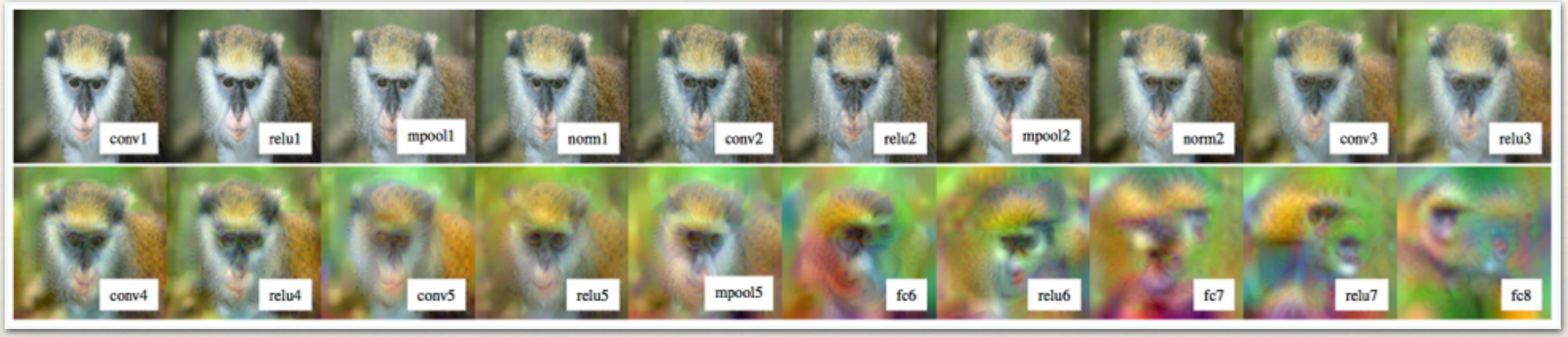


[Block et.al, 2007]

Projects: Content-Aware Image Restoration



Projects: Deep Visualisation



[Mahendran & Vedaldi, 2014]

[Yosinski et al., 2015]

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OpenReview: https://openreview.net/forum?id=HkSOlP9lg