goa (Global Optimization Animations)

Optimization Methods - Global Optimization Project

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Introduction

What is **goa** (Global Optimization Animations)?

- goa is a Python package that implements:
 - some problems (optimization test functions)
 - Ackley function



Rastrigin function

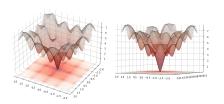


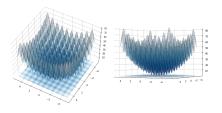
- some solutions (optimization algorithms)
 - Differential Evolution [1]
 - Memetic Differential Evolution [3]
 - Coordinate Method with Simple Descent Direction [5]
- goa aims to animate the execution of these algorithms on a given problem

Ackley and Rastrigin functions

Ackley and Rastrigin functions have common similarities:

- multinomial $(f: \mathbb{R}^n \to \mathbb{R})$
- many local minima
- unique global minimum at $f(x^*) = 0$, with $x^* = (0, ..., 0)$





Differential Evolution algorithm

Given a population of p vectors $x_i \in \mathbb{R}^n, i \in \{0, \dots, p-1\}$, a high level description of one iteration of the algorithm is the following:

- for each x_i:
 - \bullet apply to x_i a linear transformation and call it *Trial*
 - 2 restore randomly chosen dimensions of Trial
 - 3 if $f(Trial) < f(x_i)$, then set $x_i := Trial$

Memetic Differential Evolution vs Differential Evolution

The two algorithms differ only by the 10^{th} line of code.

```
Data: F \in (0, 2), CR \in [0, 1]
    foreach i \in 1, \ldots, p do
            let ii := \mathcal{U}(1, \ldots, n):
            randomly choose k_1, k_2, k_3 \in \{1, \ldots, p\} \setminus \{i\};
            let Trial := x_{k_1} + F(x_{k_2} - x_{k_2});
            for j \in 1, \ldots, n : j \neq ii do
                    if \mathcal{U}(0,1) < CR then
                            let Trial^{(j)} := x^{(j)};
                    end
            end
            let Trial := \mathcal{L}(f, Trial);
10
            if f(Trial) < f(x_i) then
11
12
                    let x_i := Trial;
            end
14 end
```

Algorithm 1: Memetic Differential Evolution

```
Data: F \in (0, 2), CR \in [0, 1]
    foreach i \in 1, \ldots, p do
            let ii := \mathcal{U}(1, \ldots, n);
            randomly choose k_1, k_2, k_3 \in \{1, \ldots, p\} \setminus \{i\};
            let Trial := x_{k_1} + F(x_{k_2} - x_{k_2});
            for j \in 1, \ldots, n : j \neq ii do
                     if \mathcal{U}(0,1) < CR then
 6
                             let Trial^{(j)} := x_{\cdot}^{(j)}:
 7
                     end
            end
            // let Trial := \mathcal{L}(f, Trial)
            if f(Trial) < f(x_i) then
11
12
                    let x_i := Trial;
13
            end
    end
```

Algorithm 2: Differential Evolution

Animations - DE vs MDE on Ackley

- Left example: DE reaches convergence in 38 iterations
- Right example: MDE reaches convergence in 3 iterations

DE on Ackley

MDE on Ackley

Animations - DE vs MDE on Quadratic

- Left example: DE reaches convergence in 19 iterations
- Right example: MDE reaches convergence in 1 iteration

DE on Quadratic

MDE on Quadratic

Command line interface (CLI)

goa provides a CLI that allows to choose:

- problem
- problem bounds (optional)
- optimization algorithm
- Iocal optimizer (required only for memetic global optimizer)
- animation filepath (optional)

```
(goa) → presentation git:(main) x python -m goa
Select a problem (Ackley, Rastrigin, quadratic) [Ackley]:
[OPTIONAL] Change the problem bounds [(-2.5, 2.5)]:
Select an optimization algorithm (MDE, DE, CM) [DE]:
[REQUIRED only with MDE] Select a local search algorithm (CM, None) [None]:
[OPTIONAL] Want an animation? Provide a filepath .gif []: 'example.gif'
Iteration:
               I RMSE: 0.91623379
Iteration:
            10 | RMSE: 0.84967181
Iteration:
               I RMSE: 0.74277798
Iteration:
               | RMSE: 0.06209373
Iteration: 25 | RMSE: 0.00837453
Iteration: 30 | RMSE: 0.00156031
Iteration: 35 | RMSE: 0.00026433
Terminated at Iteration: 37
```

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Conclusion

Who would need goa?

- anyone that would like to support an explanation
- anyone that would like to have another point of view

What could be improved?

- scalability (abstract classes for both global and local optimization algorithms)
- reproducibility (all RNGs should depend on a single random seed)

Where can the implementation be found?

• https://github.com/deeplego/goa

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Thanks for your attention!

Do you have any questions?

References

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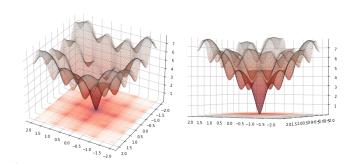
Backup slides

Definition (Ackley function)

$$f(x_1 \cdots x_n) = -20 exp(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}) - exp(\frac{1}{n} \sum_{i=1}^n cos(2\pi x_i)) + 20 + e$$

$$-32 < x_i < 32$$

minimum at $f(x^*) = 0$, with $x^* = (0, \dots, 0)$



Definition (Rastrigin function)

$$f(x_1 \cdots x_n) = 10n + \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i))$$
$$-5.12 \le x_i \le 5.12$$

minimum at $f(x^*) = 0$, with $x^* = (0, ..., 0)$

