Adversarially Constrained Autoencoder Interpolation using Wasserstein Autoencoder

Machine Learning

Lorenzo Palloni

University of Florence

lorenzo.palloni@stud.unifi.it

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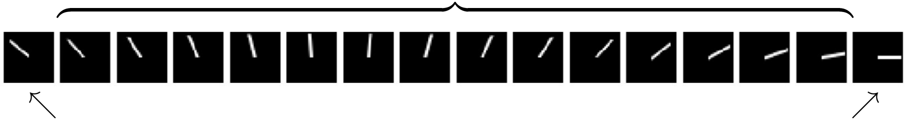
Introduction

- **Unsupervised Learning** context
- We aim to obtain "high-quality" **interpolations**
- An interpolation example:



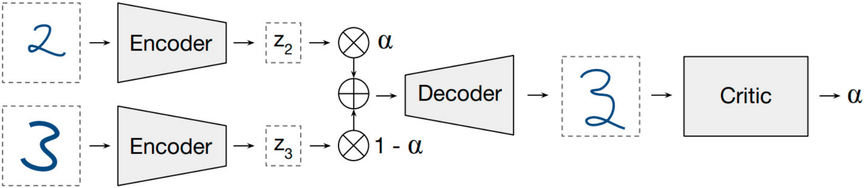
- An "high-quality" interpolation point have two characteristics:
  - is indistinguishable from real data
  - represent a semantically smooth morphing between the endpoints

Motivation and Techniques

- Uncover underlying structure of dataset
- Better representations $\rightarrow$ better results in other tasks

- Implemented techniques (using pytorch [1]):
    - Adversarially Constrained Autoencoder Interpolation (ACAI) [2]
    - Wasserstein Autoencoder (WAE) [3]
    - Wasserstein-Wassertein Autoencoder (WWAE) [4]

- Graphical representation of ACAI structure:



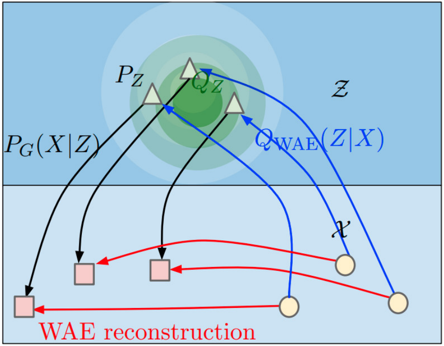
- Loss functions (discriminator and autoencoder respectively):

$$\mathcal{L}_d := \left\| d_\omega\left(\hat{x}_\alpha\right) - \alpha \right\|^2 + \left\| d_\omega\left(\gamma x + (1-\gamma) g_\phi\left(f_\theta(x)\right)\right) \right\|^2 \tag{1}$$

$$\mathcal{L}_{f,g} := \left\| x - g_\phi\left(f_\theta(x)\right) \right\|^2 + \lambda \cdot \left\| d_\omega\left(\hat{x}_\alpha\right) \right\|^2 \tag{2}$$

WAE

- Graphical representation of WAE structure:



- Loss function:

$$D_{WAE}(P_X, P_G) := \inf_{Q(Z|X)\in\mathcal{Q}} \mathbb{E}_{P_X}\mathbb{E}_{Q(Z|X)}[c(X, G(Z))] + \lambda \cdot \mathcal{D}_Z(Q_Z, P_Z)$$

Different penalties, different WAEs (1/3)

- Setting
$$\mathcal{D}_Z(Q_Z, P_Z) = D_{JS}(Q_Z, P_Z)$$
$\rightarrow$ we obtain WAE-GAN
(WAE using a GAN[1]-based penalty)

- where $D_{JS}(\cdot, \cdot)$ is the Jensen-Shannon Divergence:
$$D_{JS}(Q_Z, P_Z) := \frac{1}{2} D_{KL}(Q_Z, \frac{Q_Z + P_Z}{2}) + \frac{1}{2} D_{KL}(P_Z, \frac{Q_Z + P_Z}{2}) \quad (3)$$
- and D_{KL} is the Kullback-Leibler Divergence:
$$D_{KL}(Q_Z, P_Z) := \int Q_Z \log \left(\frac{Q_Z}{P_Z} \right) \quad (4)$$

[1] Generative Adversarial Network [5]

Different penalties, different WAEs (2/3)

- Setting
$$\mathcal{D}_Z(Q_Z, P_Z) = MMD_k(Q_Z, P_Z)$$
$\rightarrow$ we obtain WAE-MMD
(WAE with Maximum Mean Discrepancy as penalty)

- where $MMD_k(\cdot, \cdot)$ is the Maximum Mean Discrepancy with *characteristic* kernel k:
$$MMD_k(Q_Z, P_Z) := \left\| \int_{\mathcal{Z}} k(z, \cdot) dQ_Z(z) - \int_{\mathcal{Z}} k(z, \cdot) dP_Z(z) \right\|_{\mathcal{H}_k}$$

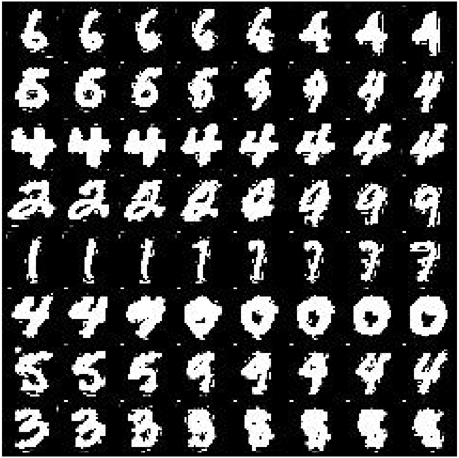
- Setting

$$\mathcal{D}_Z(Q_Z, P_Z) = \underbrace{\|\mu_{P_Z} - \mu_{Q_Z}\|^2 + \mathrm{Tr}\left(\Sigma_{P_Z} + \Sigma_{Q_Z} - 2(\Sigma_{P_Z}\Sigma_{Q_Z})^{\frac{1}{2}}\right)}_{\text{2-Wasserstein distance between two multivariate normal distributions [6]}}$$

$\rightarrow$ we obtain WWAE

(WAE with 2-Wasserstein penalty)

- Example interpolations on MNIST with ACAI + WWAE:



Conclusion

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Appendix - Wasserstein distance

References

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