Adversarially Constrained Autoencoder Interpolation using Wasserstein Autoencoder

Machine Learning

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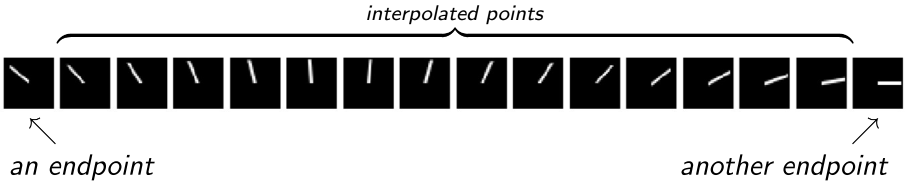
Introduction

- **Unsupervised Learning** context.
- We aim to obtain "high-quality" **interpolations**.
- An interpolation example:



- An "high-quality" interpolation point have two characteristics:
  - is indistinguishable from real data;
  - represents a semantically smooth morphing between the endpoints.

Motivation and Techniques

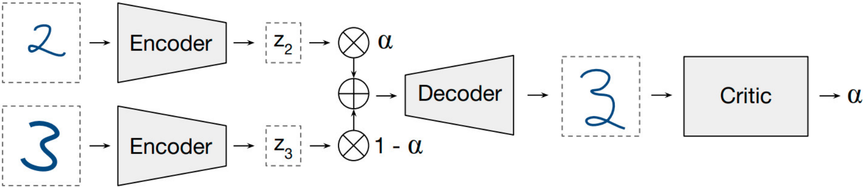
- Uncover underlying structure of dataset.
- Better representations $\rightarrow$ better results in other tasks.

- Implemented[1] frameworks:
  - Adversarially Constrained Autoencoder Interpolation (ACAI) [1];
  - Wasserstein Autoencoder (WAE) [2];

[1] All the models are implemented using pytorch [4]

ACAI

- Graphical representation of ACAI structure:



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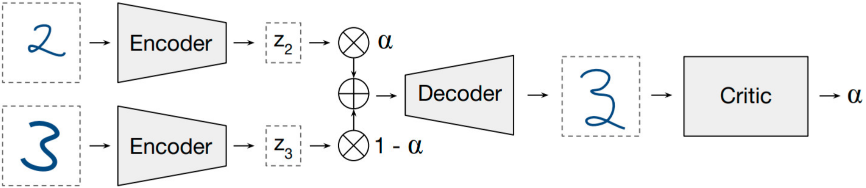

- Loss functions:

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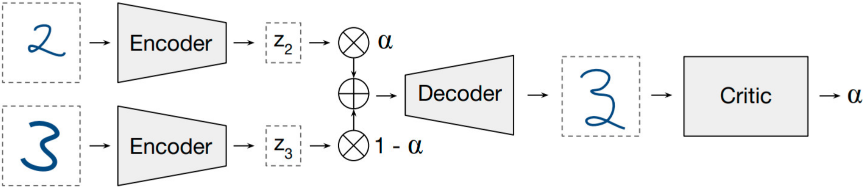

- Loss functions:

$$\mathcal{L}_d := \left\|d_\omega\left(\hat{x}_\alpha\right) - \alpha\right\|^2 + \left\|d_\omega\left(\gamma x + (1-\gamma) g_\phi\left(f_\theta(x)\right)\right)\right\|^2$$

ACAI

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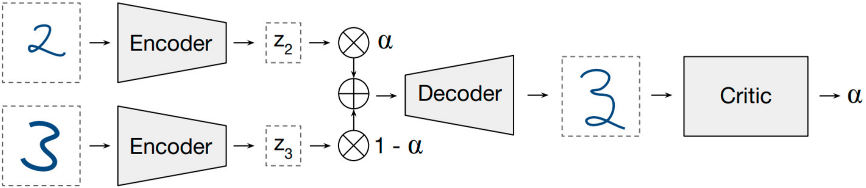

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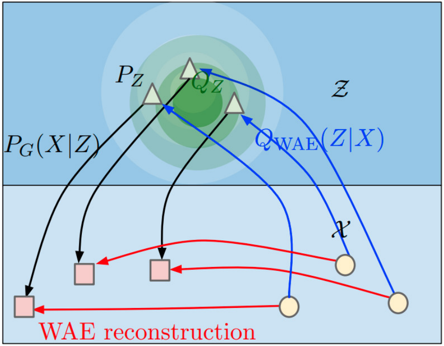
$$\mathcal{L}_{f,g} := \|x - g_\phi(f_\theta(x))\|^2 + \lambda \cdot \|d_\omega(\hat{x}_\alpha)\|^2$$

WAE

- Graphical representation of WAE structure:



- Loss function:

$$D_{WAE}(P_X, P_G) := \inf_{Q(Z|X)\in\mathcal{Q}} \mathbb{E}_{P_X}\mathbb{E}_{Q(Z|X)}[c(X, G(Z))] + \lambda \cdot \mathcal{D}_Z(Q_Z, P_Z)$$

Different penalties, different WAEs (1/2)

- Setting

$$\mathcal{D}_Z(Q_Z, P_Z) = MMD_k(Q_Z, P_Z)$$

$\rightarrow$ **we obtain WAE-MMD**.

- where $MMD_k(\cdot,\cdot)$ is the Maximum Mean Discrepancy [10]:

$$MMD_k(Q_Z, P_Z) := \left\| \int_{\mathcal{Z}} k(z,\cdot) dQ_Z(z) - \int_{\mathcal{Z}} k(z,\cdot) dP_Z(z) \right\|_{\mathcal{H}_k}$$

- where k is a positive-definite kernel k: $\mathcal{Z} \times \mathcal{Z} \rightarrow \mathcal{R}$.

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  - Inverse Multiquadratic: $k(x, y) = \frac{C}{C + \|x - y\|^2}, \ C \in \mathcal{R}$

- Setting

$$\mathcal{D}_Z(Q_Z, P_Z) = \underbrace{\|\mu_{P_Z} - \mu_{Q_Z}\|^2 + \mathrm{Tr}\left(\Sigma_{P_Z} + \Sigma_{Q_Z} - 2(\Sigma_{P_Z}\Sigma_{Q_Z})^{\frac{1}{2}}\right)}_{\text{2-Wasserstein distance between two multivariate normal distributions [6]}}$$

- and assuming the prior P_Z normal distributed,
  $\rightarrow$ **we obtain WWAE** (Wasserstein-Wasserstein Autoencoder) [3].

Roadmap of hands-on approach

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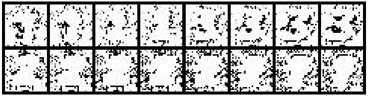

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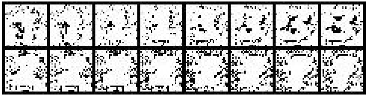

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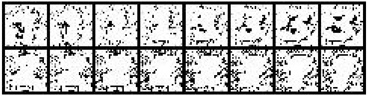

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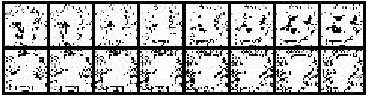
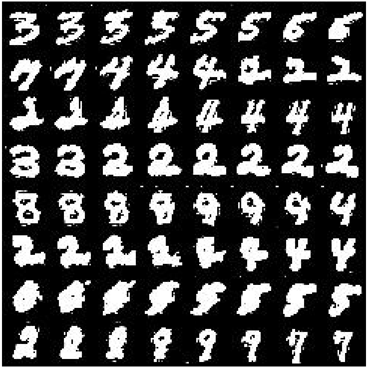


2. ACAI + WAE applied on NESMDB $\rightarrow$ poor results too.
3. ACAI + WWAE applied on MNIST $\rightarrow$ good results (next slide).
4. ACAI + WWAE applied on NESMDB $\rightarrow$ requires too time to collect results.

Results on MNIST

- Example interpolations on MNIST made with ACAI + WWAE:



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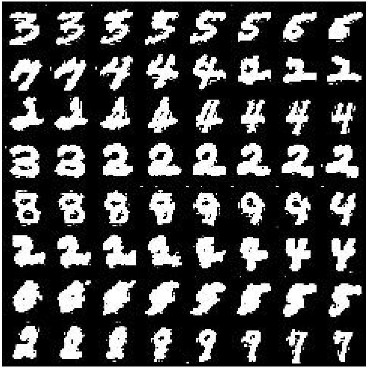

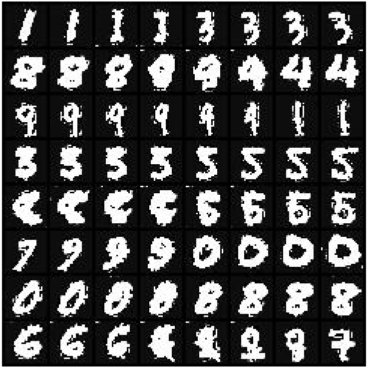

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Final discussions

- We achieve pretty good quality interpolations on image data using ACAI + WWAE.
- All models discussed so far require:
  - high computational resources for training;
  - have the lack of an objective way to assess the performance.
- Raw audio in waveform probably needs:
  - another type of representation (e.g. MIDI) [7] [8];
  - another autoencoder structure (e.g. Wavenet Autoencoder [9])

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Appendix - Wasserstein distance

- Kantorovich's formulation of the *optimal transport* problem:

$$W_c(P_X, P_G) := \inf_{\Gamma \in \mathcal{P}(X \sim P_X, Y \sim P_G)} \mathbb{E}_{(X,Y)\sim\Gamma}\left[c(X, Y)\right]$$

- with some weak constrains it could be written as the following:

$$W_1(P_X, P_G) = \sup_{f \in \mathcal{F}_{L_1}} \mathbb{E}_{X \sim P_X}\left[f(X)\right] - \mathbb{E}_{Y \sim P_G}\left[f(Y)\right]$$

- Moreover defining P_G in two steps:
  1. $Z \sim P_Z$
  2. $G : \mathcal{Z} \to \mathcal{R}^d$
- we have:

$$\inf_{\Gamma \in \mathcal{P}(X \sim P_X, Y \sim P_G)} \mathbb{E}_{(X,Y)\sim\Gamma}\left[c(X, Y)\right] = \inf_{Q : Q_Z = P_Z} \mathbb{E}_{Q(X|Z)}\left[c(X, G(Z))\right]$$