Q 4

Let vanishing points of mutually perpendicular directions l_1, l_2 and l_3 in I_P be $P_1(p_{1x}, p_{1y})$, $P_2(p_{2x}, p_{2y})$ and $P_3(p_{3x}, p_{3y})$. Corresponding vanishing points in I_Q be Q_1, Q_2 and Q_3 respectively.

Let $O_P(o_{px}, o_{py})$ be the optical center of first camera in its pixel coordinate system. From the properties of geometry, O_P is the orthocenter of P_1, P_2 and P_3. Thus using basic coordinate geometry (product of slopes of perpendicular line is -1 and intersection of line) we can obtain pixel coordinates of O_P and same way coordinates of optical center O_Q of second camera in its pixel coordinate system.

Assume that we know s_p, s_q and f_p, f_q (How to obtain it will be mentioned later). In the coordinate system of first camera, direction vector of l_1 can be given by $u_1 = (s_p(p_{1x} - o_{px}), s_p(p_{1y} - o_{py}), f_p)$. Dividing u_1 by $\|u_1\|$ we get normalized u_1 denoted by $\mathbf{u_1}$. Similarly we can obtain normalized direction vector for l_2, l_3 in the coordinate system of first camera as $\mathbf{u_2}, \mathbf{u_3}$ respectively and normalized direction vector for l_1, l_2, l_3 as $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ in the coordinate system of second camera.

The orientation and position of the two cameras are related by $\mathbf{R}$ and $\mathbf{T}$ respectively. We **can not obtain T** because only translation of parallel lines does not change vanishing point in the direction of the line. However, We can recover $\mathbf{R}$ by exploiting the relation between $\mathbf{u_i}$ with $\mathbf{v_i}$ as following :

$$\mathbf{u_i} = \mathbf{R}(\mathbf{v_i}) \qquad \forall i \in \{1, 2, 3\}$$

$$\Rightarrow \frac{1}{\|u_1\|} \begin{bmatrix} s_p(p_{1x} - o_{px}) \\ s_p(p_{1y} - o_{py}) \\ f_p \end{bmatrix} = \frac{1}{\|v_1\|} \mathbf{R} \begin{bmatrix} s_q(q_{1x} - o_{qx}) \\ s_q(q_{1y} - o_{qy}) \\ f_q \end{bmatrix}$$

$$\Rightarrow \frac{f_p}{\sqrt{f_p^2 * \{(\frac{s_p}{f_p}(p_{1x} - o_{px}))^2 + (\frac{s_p}{f_p}(p_{1y} - o_{py}))^2 + 1^2\}}} \begin{bmatrix} \frac{s_p}{f_p}(p_{1x} - o_{px}) \\ \frac{s_p}{f_p}(p_{1y} - o_{py}) \\ 1 \end{bmatrix}$$

$$= \frac{f_q}{\sqrt{f_q^2 * \{(\frac{s_q}{f_q}(q_{1x} - o_{qx}))^2 + (\frac{s_q}{f_q}(q_{1y} - o_{qy}))^2 + 1^2\}}} \mathbf{R} \begin{bmatrix} \frac{s_q}{f_q}(q_{1x} - o_{qx}) \\ \frac{s_q}{f_q}(q_{1y} - o_{qy}) \\ 1 \end{bmatrix}$$

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$\mathbf{R}$ has 3 degrees of freedom. Thus by solving above system of equations, we can recover $\mathbf{R}$ if $\frac{s_p}{f_p}$ and $\frac{s_q}{f_q}$ is known.

In order to obtain the ratio of s_p to f_p, we can use the perpendicular nature of the direction l_1, l_2, l_3. Since any two direction is mutually perpendicular, for first camera we have $\mathbf{u_1}.\mathbf{u_2} = 0 \Rightarrow s_p^2(p_{1x} - o_{px})(p_{2x} - o_{px}) + s_p^2(p_{1y} - o_{py})(p_{2y} - o_{py}) + f_p^2 = 0$. Similarly we can obtain $\frac{s_q}{f_q}$ by solving $\mathbf{v_1}.\mathbf{v_2} = 0$. Other equations obtained from dot product of $\mathbf{u_3}$ with $\mathbf{u_1}$ and $\mathbf{u_2}$ does not give any more information other than the ratio $\frac{s_p}{f_p}$. Thus we can not recover exact value of s_p or f_p. Note that in above system of equations, we only need $\frac{s_p}{f_p}$ and not the exact value of s_p or f_p.

Conclusion : We can infer $\mathbf{R}$ but not $\mathbf{T}$. s_p and f_p can be determined up to the ratio of s_p to f_p. Same way, only $\frac{s_q}{f_q}$ can be obtained.