(e)

$$\cos^2(2\pi f_1 t) = \frac{\cos(4\pi f_1 t) + 1}{2}$$

$$= \frac{1}{2}\cos(4\pi f_1 t) + \frac{1}{2}$$

$$= \frac{1}{4}e^{-j4\pi f_1 t} + \frac{1}{4}e^{j4\pi f_1 t} + \frac{1}{2}$$

$$\xleftrightarrow{CTFT} \frac{\pi}{2}\delta(w+4\pi f_1) + \frac{\pi}{2}\delta(w-4\pi f_1)$$

$$m(t)\cos^2(2\pi f_1 t) \xleftrightarrow{CTFT} \frac{1}{4}M(j(w+4\pi f_1)) + \frac{1}{4}M(j(w-4\pi f_1))$$

$$+ \frac{1}{2}M(jw)$$

$$\cos(2\pi f_1 t)\sin(2\pi f_1 t)$$

$$= \frac{1}{2}\sin(4\pi f_1 t)$$

$$= \frac{1}{4j}e^{j4\pi f_1 t} - \frac{1}{4j}e^{j4\pi f_1 t}$$

$$\xrightarrow{\text{CTFT}} \frac{\pi}{2j}\delta(\omega - 4\pi f_1) - \frac{\pi}{2j}\delta(\omega + 4\pi f_1)$$

$$m(t)\cos(2\pi f_1 t)\sin(2\pi f_1 t)$$

$$\longrightarrow \frac{1}{4j}M(j(\omega - 4\pi f_1)) - \frac{1}{4j}M(j(\omega + 4\pi f_1))$$

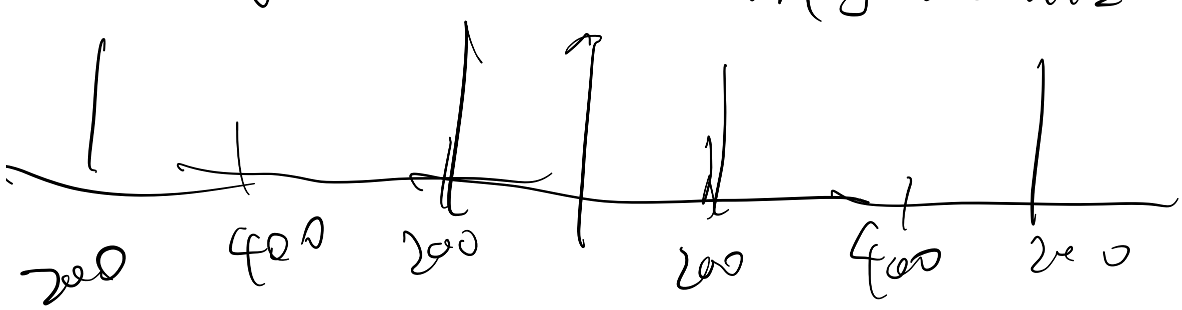
$$\cos(2\pi f_1 t)\cos(2\pi f_2 t)$$

$$= \left(\frac{1}{2}e^{j2\pi f_1 t} + \frac{1}{2}e^{-j2\pi f_1 t}\right)\left(\frac{1}{2}e^{j2\pi f_2 t} + \frac{1}{2}e^{-j2\pi f_2 t}\right)$$

$$= \frac{1}{4}e^{j2\pi(f_1+f_2)t} + \frac{1}{4}e^{j2\pi(f_1-f_2)t} + \frac{1}{4}e^{j2\pi(f_2-f_1)t} + \frac{1}{4}e^{-j2\pi(f_1+f_2)t}$$

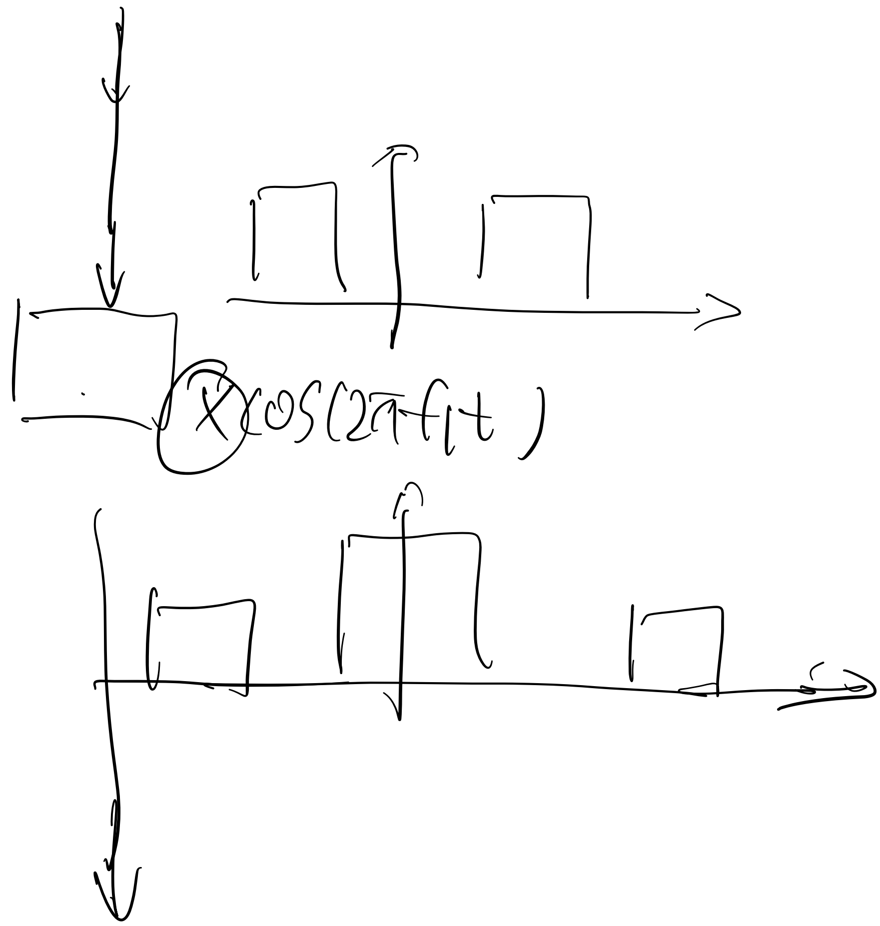
$$\xrightarrow{CTFT} = \frac{\pi}{2}\big(\delta(\omega-2\pi(f_1+f_2)) + \delta(\omega+2\pi(f_1+f_2)) + \delta(\omega-2\pi(f_1-f_2)) + \delta(\omega-2\pi(f_2-f_1))\big)$$

$$\frac{1}{4}\big(M(j(\omega-2\pi(f_1+f_2))) + M(j(\omega+2\pi(f_1+f_2))) + M(j(\omega-2\pi(f_1-f_2))) + M(j(\omega-2\pi(f_2-f_1)))\big)$$



$$x(t) = m_1(t)\cos(2\pi f_1 t) + m_2(t)\cos(2\pi f_2 t) + m_3(t)\sin(2\pi f_1 t)$$

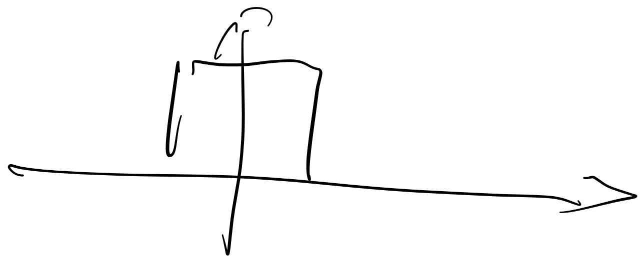

$x(t) \rightarrow \boxed{BPF}$ $*\ \mathrm{sinc}(x)\cos(2\pi f_1 t)$



$\boxed{LPF}$ $*\ sinc(x)$

$\downarrow$

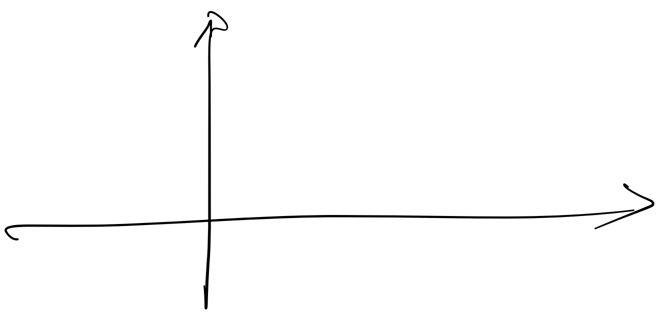
$m(t)$



$m(t)\sin 2\pi f_c t$

$M(j(\omega+2\pi f_c)) \sim$

$M(j\omega$



sin