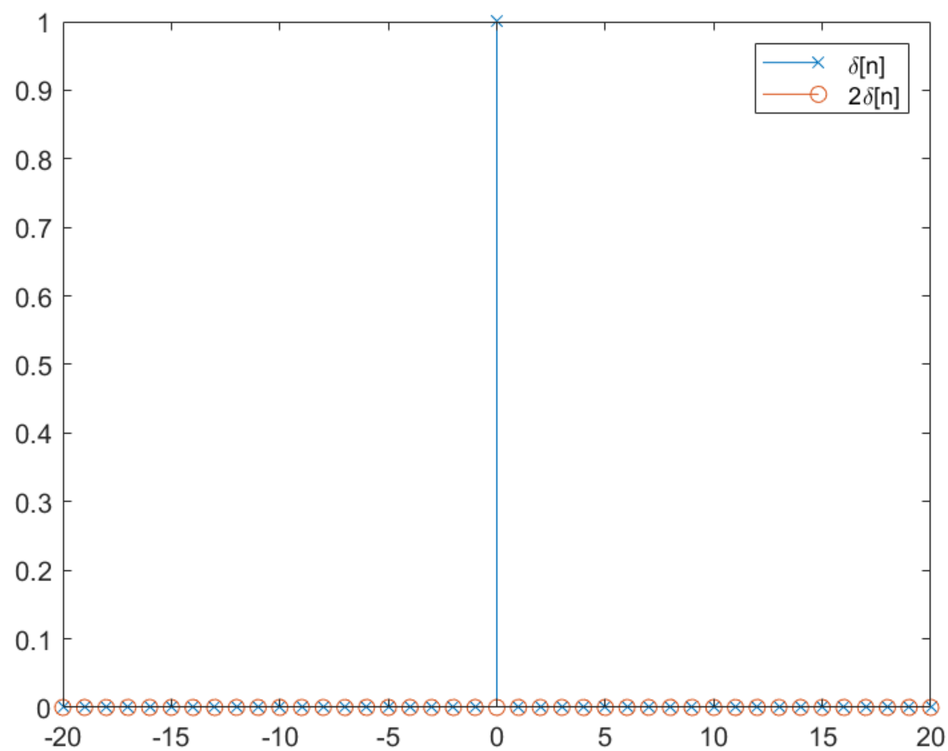
(a). The system $y[n] = \sin((\pi/2)x[n])$ is not linear. Use the signals $x_1[n] = \delta[n]$ and $x_2[n] = 2\delta[n]$ to demonstrate how the system violates linearity.

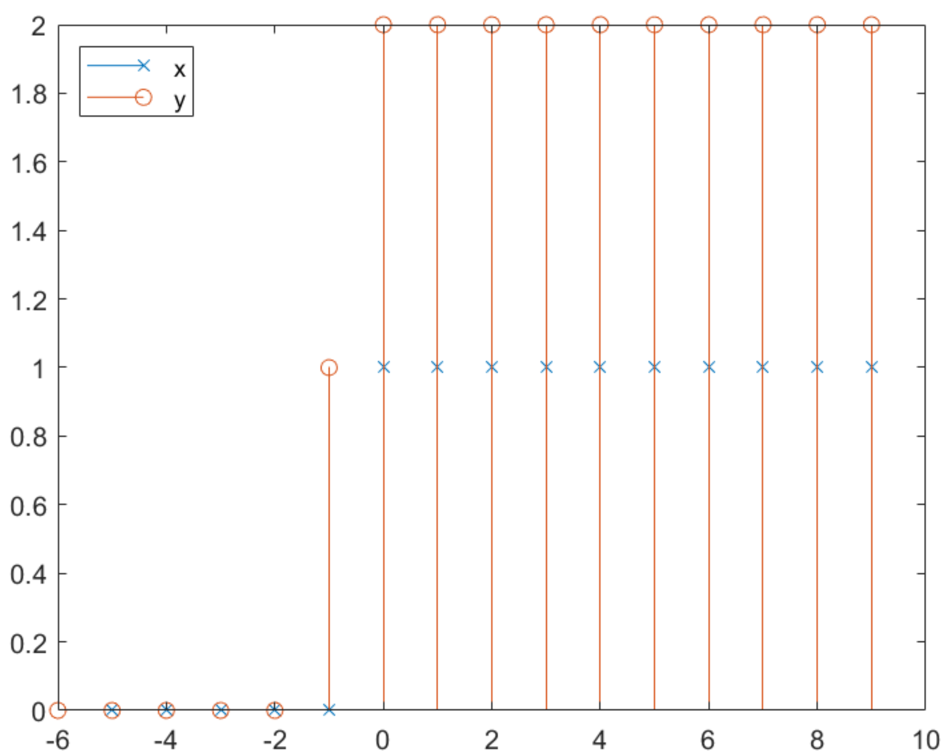
```
n = -20:20;
x1 = impu(n);
x2 = 2.*impu(n);
y1 = sys_pa(x1);
y2 = sys_pa(x2);
fa = figure;
figure(fa);
stem(n, y1, 'x');
hold on
stem(n, y2);
hold on
legend('\delta[n]', '2\delta[n]');
hold off;
```



The system is not linear. Because x_2 = 2 * x_1 but at n=0, $S(x_2)$ which is 0 is not two times of $S(x_1)$ which is 1.

(b). The system $y[n] = x[n] + x[n+1]$ is not causal. Use the signal $x[n] = u[n]$ to demonstrate this. Define the MATLAB vectors **x** and **y** to represent the input on the interval $-5 \le n \le 9$, and the output on the interval $-6 \le n \le 9$, respectively.

```
fb = figure;
figure(fb);
n1 = -5:9;
n2 = -6:10;
x = step(n1);
x2 = step(n2);
y = sys_pb(x2);
stem(-5:9, x, 'x');
hold on
stem(-6:9, y);
hold on
legend('x', 'y', 'Location','northwest');
hold off
```



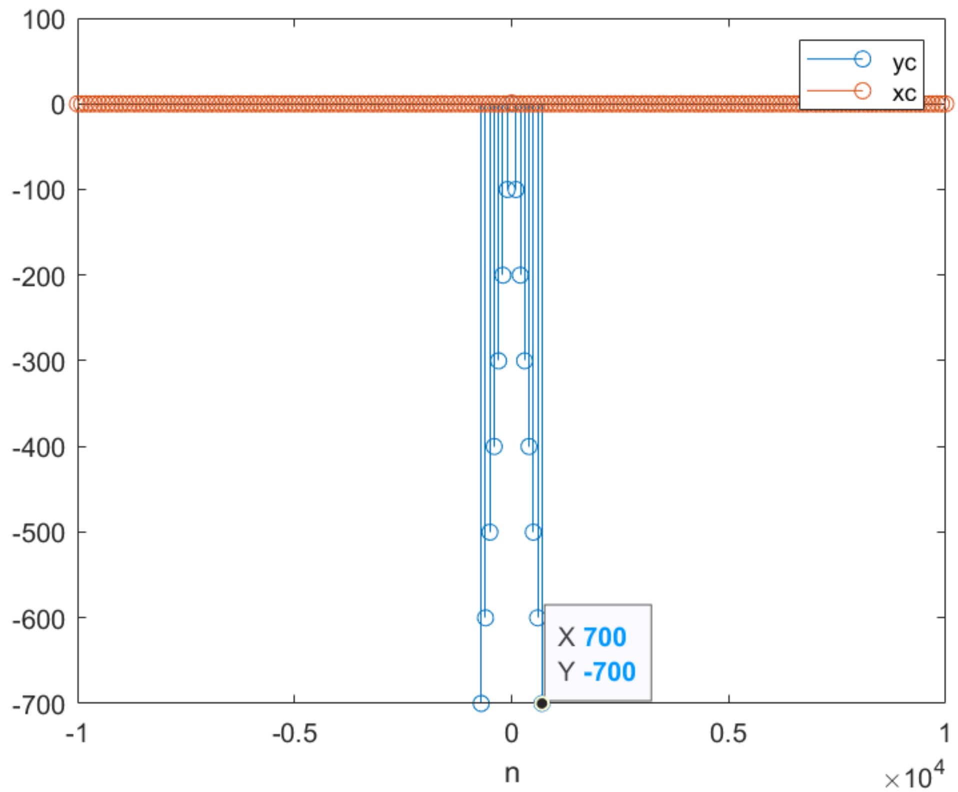
Differential equation of y only has linear constant coefficient and no additional constant. And y[-1] is not 0. So, the system is not causal.

For these problems, you will be given a system and a property that the system does not satisfy, but must discover for yourself an input or pair of input signals to base your argument upon. Again, create MATLAB vectors to represent the inputs and outputs of the system and generate appropriate plots with these vectors. Use your plots to make a clear and concise argument about why the system does not satisfy the specified property.

(c). The system $y[n] = \log(x[n])$ is not stable.

let $x[n] = \begin{cases} 1/e^{n} & x \geq 0 \\ 1/e^{-n} & x < 0 \end{cases}$. x[n] has bound. its value is always larger than 0 and always less than 1 for every n. But when abs(n) becomes larger to infinite, y[n] = log(x[n]) = -abs(n) will be minus infinite. So, y[n] has no bound. So, the system is not stable.

```
n = -10000:100:10000;
xc = arrayfun(@(x) input_c(x), n);
yc = sys_pc(xc);
fc = figure;
figure(fc);
stem(n,yc);
hold on
stem(n, xc);
hold on
legend('yc', 'xc');
xlabel('n')
xlim([-10000 10000])
ylim([-700 100])
ax = gca;
chart = ax.Children(2);
datatip(chart,700,-700);
hold off
```

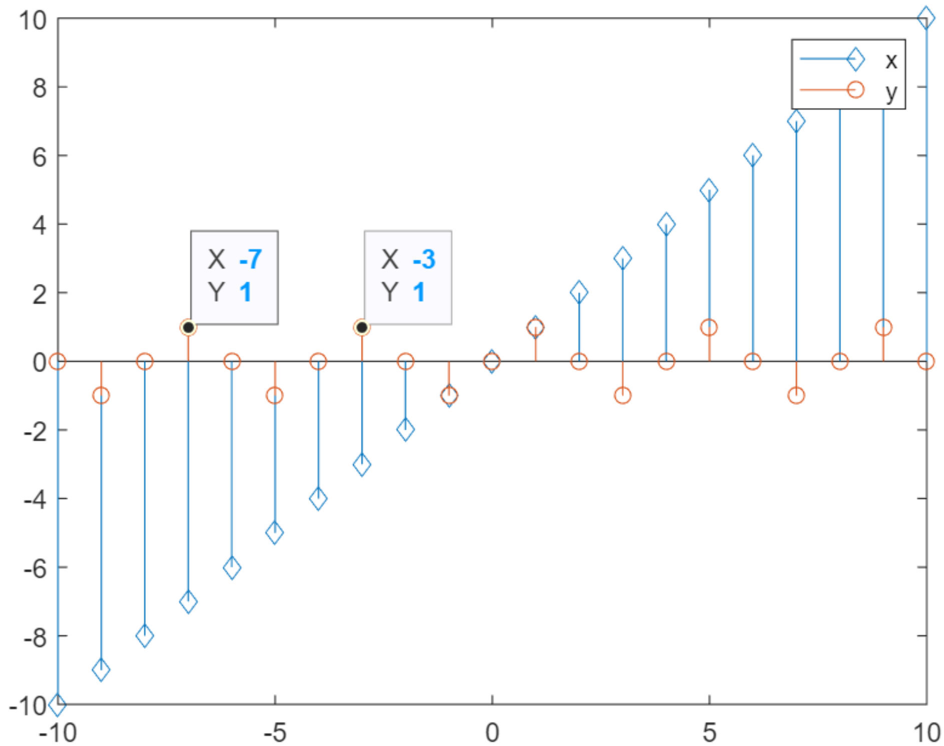


According to figure, xc has bound, and yc = -abs(n) which has no bound. So, the system is not stable.

(d). The system given in Part (a) is not invertible.

let x[n] = n. Then x[n] is distinct. Get the output of the system and see whether the output is distinct.

```
n = -10:10;
yd = sys_pa(n);
fd = figure;
figure(fd);
stem(n, n, 'd');
hold on
stem(n, yd);
hold on
legend('x', 'y');
ax2 = gca;
chart2 = ax2.Children(1);
datatip(chart2,-3,1);
datatip(chart2,-7,1);
hold off
```



The input x is distinct but the output y is not distinct for example y[-7] = y[-3]. So, the system is not invertible.

Advanced Problems

For each of the following systems, state whether or not the system is linear, time-invariant, causal, stable, and invertible. For each property you claim the system does not possess,

Sec. 1.5. Implementing a First-Order Difference Equation 11

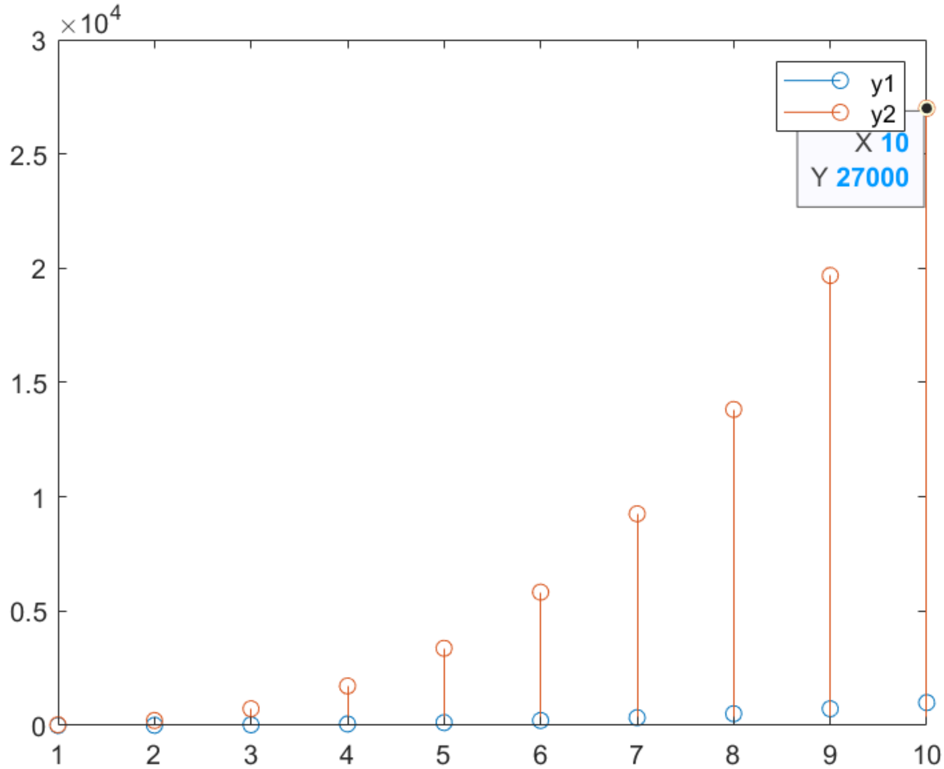
construct a counter-argument using MATLAB to demonstrate how the system violates the property in question.

(e). $y[n] = x^3[n]$

(e) not linear

let x[n] = n;

```
n = 1:10;
x1 = n;
x2 = 3.*n;
y1 = x1.^3;
y2 = x2.^3;
fe = figure;
figure(fe);
stem(n, y1);
hold on
stem(n, y2);
hold on
legend('y1', 'y2');
hold off
```



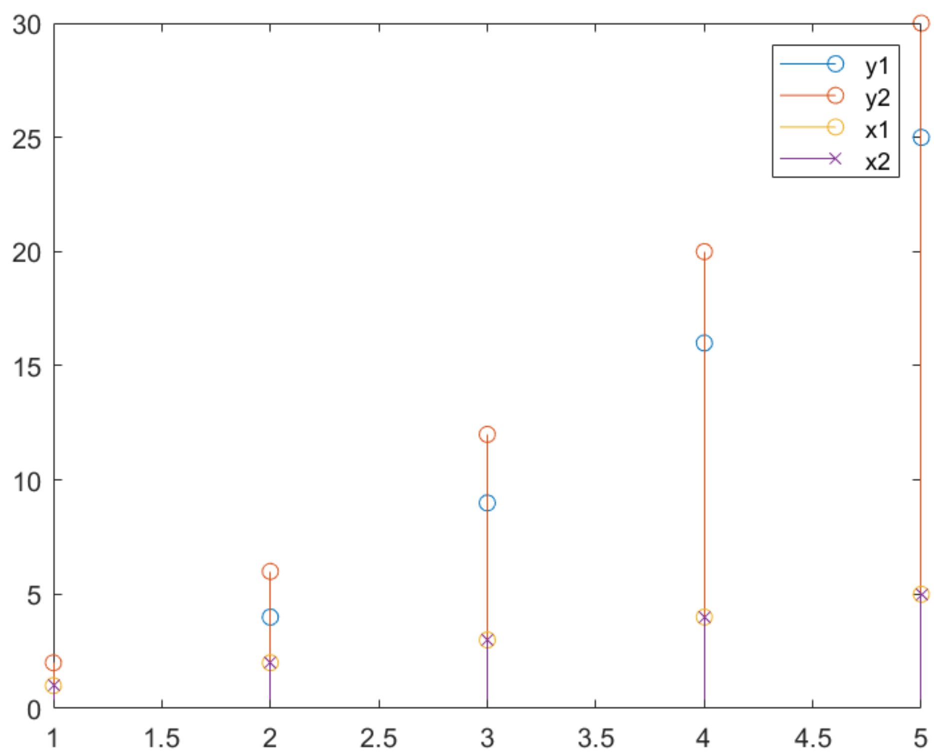
According to the stem, value of y2 is not 3 times of y1, so the system is not linear.

$$(\mathrm{f}).\ \ y[n] = n\,x[n]$$

(f) not time-invariant. let x[n] = n;

```
n = 1:6;
```

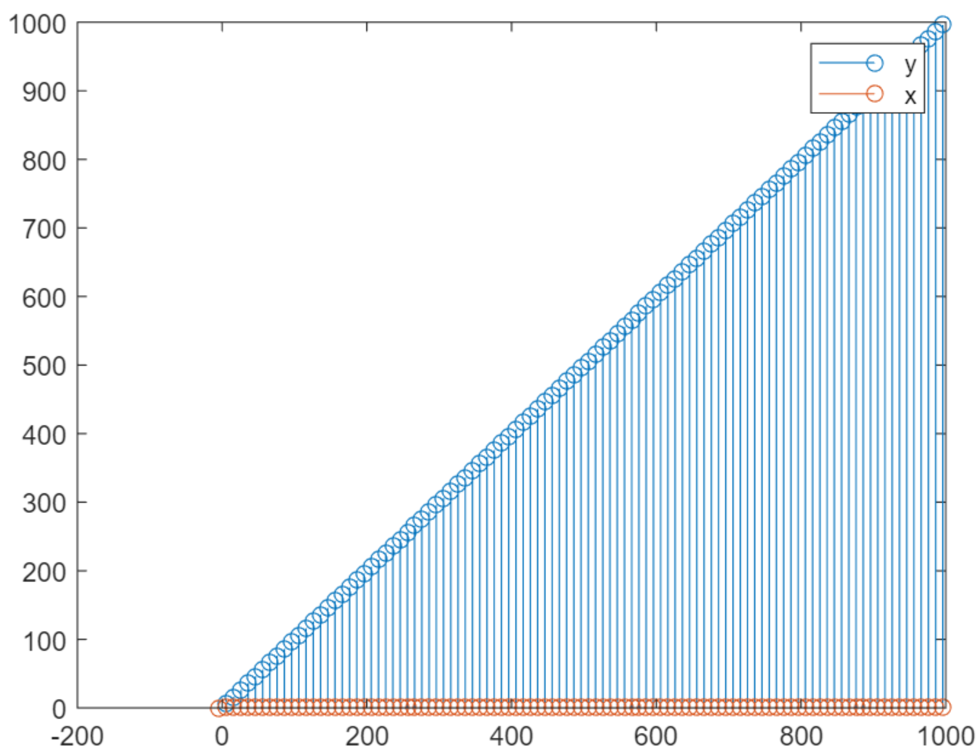
```
x1 = n;
x2 = n-1;
y1 = n.*x1;
y2 = n.*x2;
y1_t = y1(1:length(y1)-1);
x1_t = x1(1:length(x1)-1);
y2_t = y2(2:length(y2));
x2_t = x2(2:length(x2));
ff = figure;
figure(ff);
stem(1:5, y1_t);
hold on
stem(1:5, y2_t);
hold on
stem(1:5, x1_t);
hold on
stem(1:5, x2_t, 'x');
legend('y1', 'y2', 'x1', 'x2');
hold off
```



$x_2[n] = x_1[n-1]$, but $y_2[n]$ is not equal to $y_1[n-1]$. So, the system is not time-invariant.

(f) not stable. let x[n] = u(n) (step). x[n] has bound, it's absolute value is not biiger than 1. But y[n] = n when n>=0 which has no bound.

```
n = -4:10:1000;
x = step(n);
y = n.*x;
ff_s = figure;
figure(ff_s);
stem(n, y);
hold on
stem(n, x);
hold on
legend('y', 'x');
hold off
```

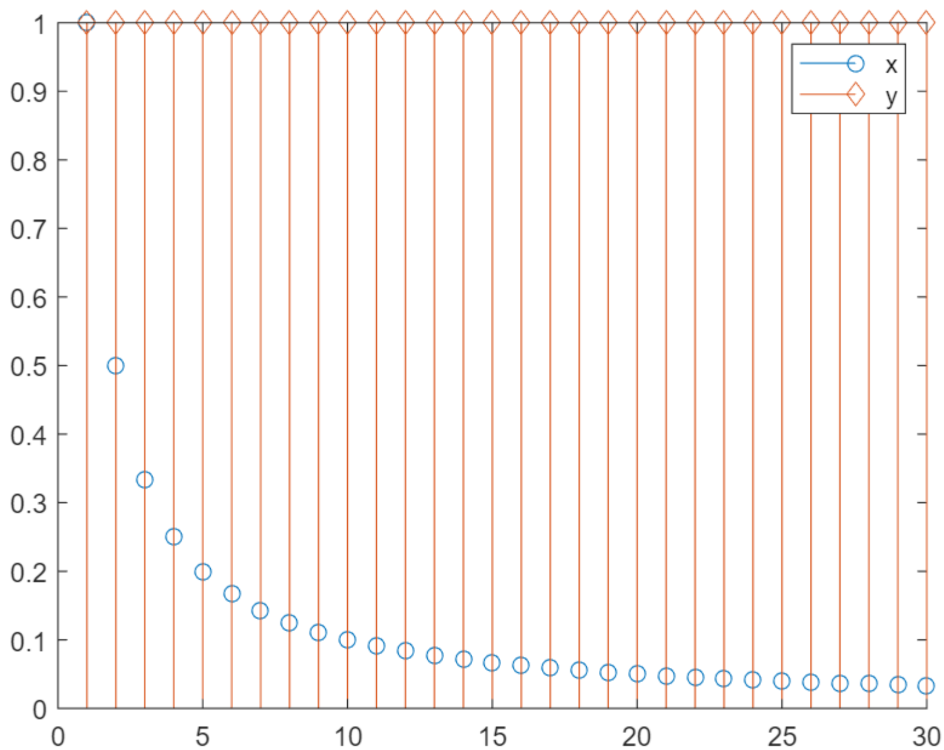


y has no bound, so the system is not stable.

(f) not invertible. let x[n] = $1/n$(n>0) then y = 1 for every n. So, the system is not invertible.

```
n = 1:30;
x = 1./n;
y = n.*x;
ff_i = figure;
figure(ff_i);
stem(n, x);
```

```
hold on
stem(n, y, 'd');
hold on
legend('x', 'y');
hold off
```



x is distinct, but y is not. So, the system is not invertible.

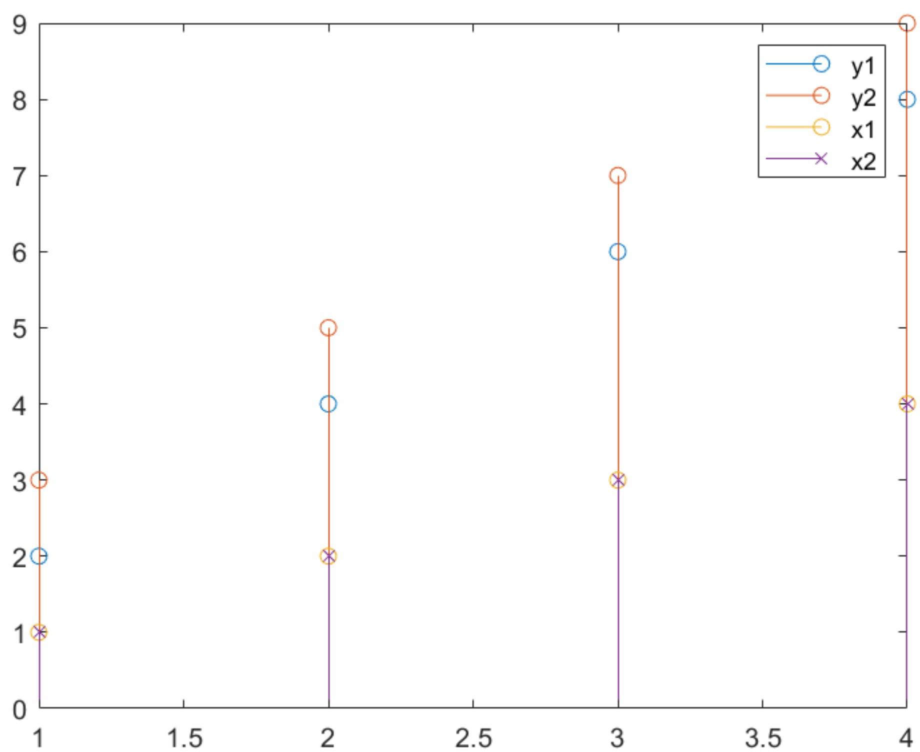
(g). $y[n] = x[2n]$.

(g) not time-invariant.

let $x_1[n]$ = n, $x_2[n]$ = $x_1[n-1]$ = n-1

```
true_n = 1:5;
n =1:11;
x1 = n;
x2 = n-1;
y1 = x1(2.*true_n);
y2 = x2(2.*true_n);
y1_t = y1(1:length(y1)-1);
y2_t = y2(2:length(y2));
x1_t = x1(1:length(true_n)-1);
```

```
x2_t = x2(2:length(true_n));
fg_ti = figure;
figure(fg_ti);
true_n = true_n(1:length(true_n)-1);
stem(true_n, y1_t);
hold on
stem(true_n, y2_t);
hold on
stem(true_n, x1_t);
hold on
stem(true_n, x2_t, 'x');
hold on
legend('y1', 'y2', 'x1', 'x2');
hold off
```



$x_1[n-1]$ = $x_2[n]$ but $y_1[n-1]$!= $y_2[n]$. So, the system is not time-invariant.

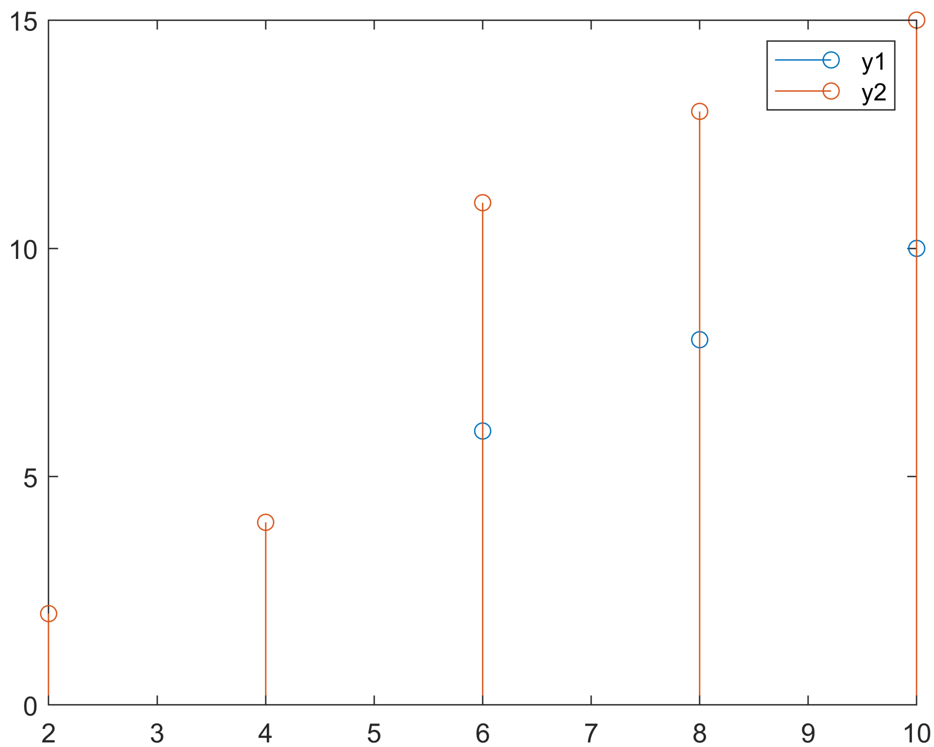
(g) not causal.

let $x_1[n]$ = [1 2 3 4 5 6 7 8 9 10]

let $x_2[n]$ = [1 2 3 4 5 11 12 13 14 15]

for n<=5 $x_1[n]$ = $x_2[n]$, if the system is causal, $y_1[n]$ should be equals to $y_2[n]$ in this interval.

```
x1 = [1 2 3 4 5 6 7 8 9 10];
x2 = [1 2 3 4 5 11 12 13 14 15];
n_t = 1:5;
n = 2.*n_t;
y1 = x1(n);
y2 = x2(n);
fg_c = figure;
figure(fg_c);
stem(n, y1);
hold on
stem(n, y2);
hold on;
legend('y1', 'y2');
hold off
```



y1 != y2. So, the system is not causal.

```
function [y] = sys_pa(x)
    y = sin((pi/2) .* x);
end

function [y] = sys_pb(x)
    xt = [x(2:length(x)) 0];
    y = x + xt;
    y = y(1:length(y)-1);
```

```
end

function [y] = impu(n)
    y = (n==0);
end

function [y] = step(n)
    y = (n>=0);
end

function [y] = sys_pc(x)
    y = log(x);
end

function [x] = input_c(n)
    if(n>=0)
        x = 1./exp(n);
    else
        x = 1./exp(-n);
    end
end
```