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650:350 – Mechanical Engineering Measurements
Laboratory #1

Spring 2015

First Order System Response

PURPOSE

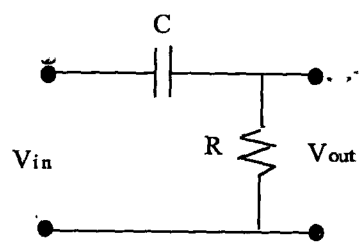
The purpose of this laboratory exercise is to examine the characteristics of first order systems with respect to different time dependent inputs. The systems considered are two simple RC circuits, which may also serve as electronic filters, but also model the response of first order measurement systems with finite time response. The exercise also includes familiarization with an oscilloscope and a signal generator.

BACKGROUND

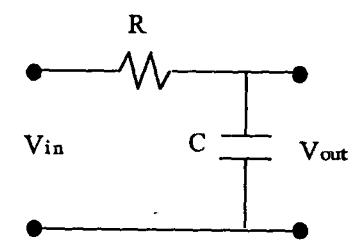
Waveshaping by R-C Circuits.

(*Refer class notes on* ***Voltage across Capacitor and Resistor*** *close/open switch*)

a) High Pass Filter.



b) Low Pass Filter



The two R-C combinations shown above are of fundamental importance. They act as frequency selective filters for time varying input signals. They also change the shape of pulse signals passing through them. The response of the R-C circuits shown is that of a first order system. The voltage across the capacitor, V_{out}, of the low pass filter is analogous to the response of a temperature sensor to ambient temperature variations similar to those of the input voltage V_{in}. The behavior of a low pass R-C circuit can therefore be used to study the behavior of thermal sensors with the same time constant.

EQUIPMENT

a. Two shielded enclosures with an 10 kΩ (approximate) resistor and a 0.01µF (approximate) capacitor for the two R-C circuits shown above with BNC cables for extracting signals from different parts of the circuits.

b. A signal generator for the generation of sinusoidal, square or triangular shaped voltage signals with adjustable frequency and amplitude.

c. A two channel digital oscilloscope for monitoring the voltages at different parts of the circuits.

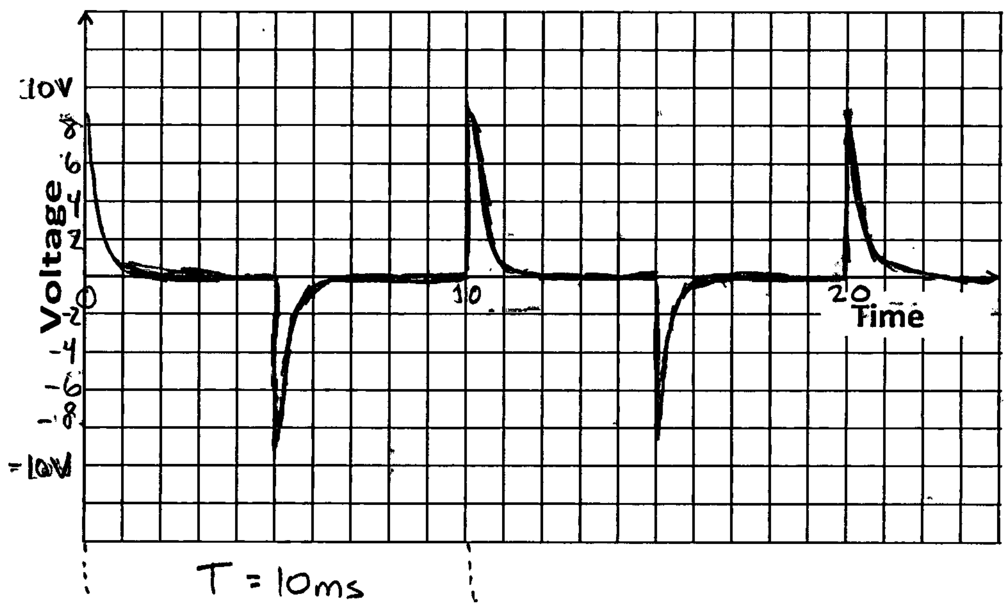
PROCEDURE

1. High pass filter

Use a splitter to split the signal from the output generator into two BNC cables. Bring the signal from the generator to the input side (V_{in}) of the capacitor. Plug the other signal from the generator into channel 1 on the oscilloscope. Plug the output signal (V_{out}) from the output side of the capacitor into channel 2 on the oscilloscope.

a. Using a *square wave input* observe, study, and sketch the output signals for input frequencies of 0.1, 1, 10, and 100 kHz. For all cases record the amplitude of the voltage levels, V_{out}, with respect to ground.

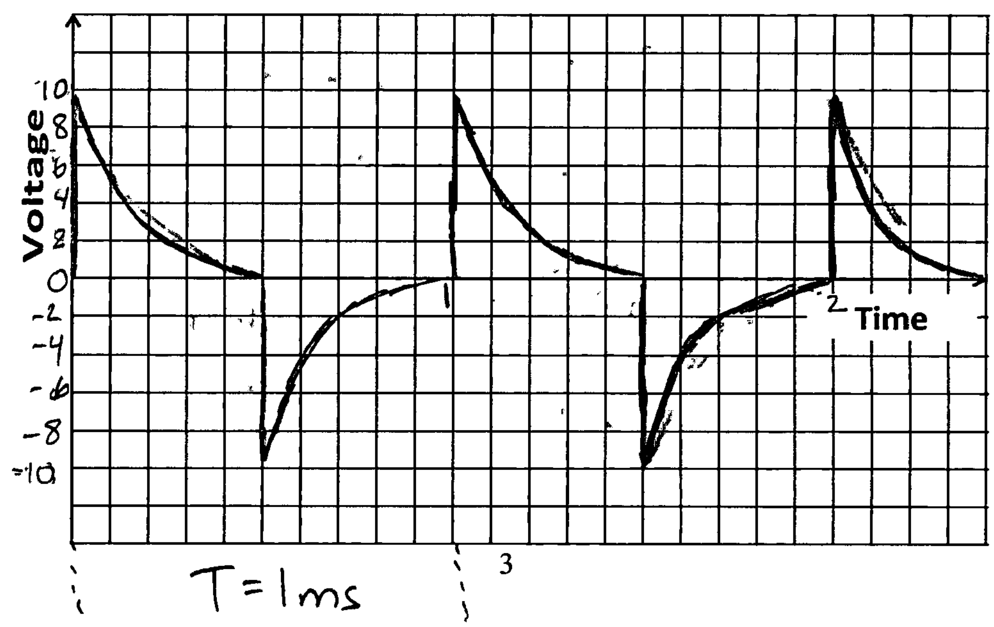
0.1 kHz



$V_{out\,p\text{-}p} = 17.0V$

$f = 99.21Hz$

1 kHz



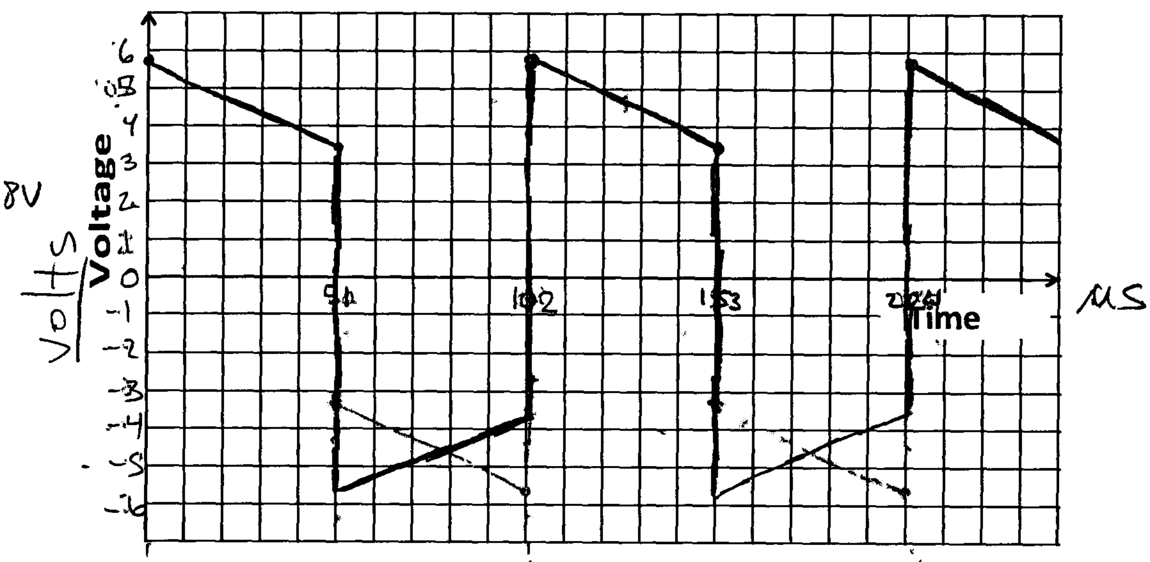
$V_{out\,p\text{-}p} = 18.44V$

$f = 975.6\,Hz$

10 kHz

$T \approx 10.2\,\mu s$

$V_{out_{p-p}} = 11.88V$

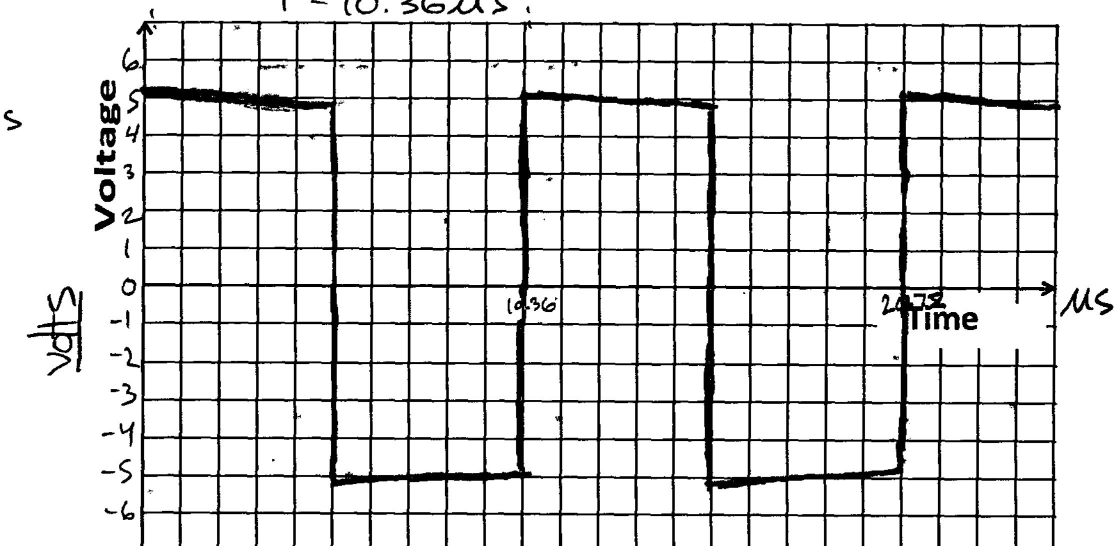


100 kHz

$T = 10.2\,\mu s$

$T = 10.36\,\mu s$

$T = 10.36\,\mu s$



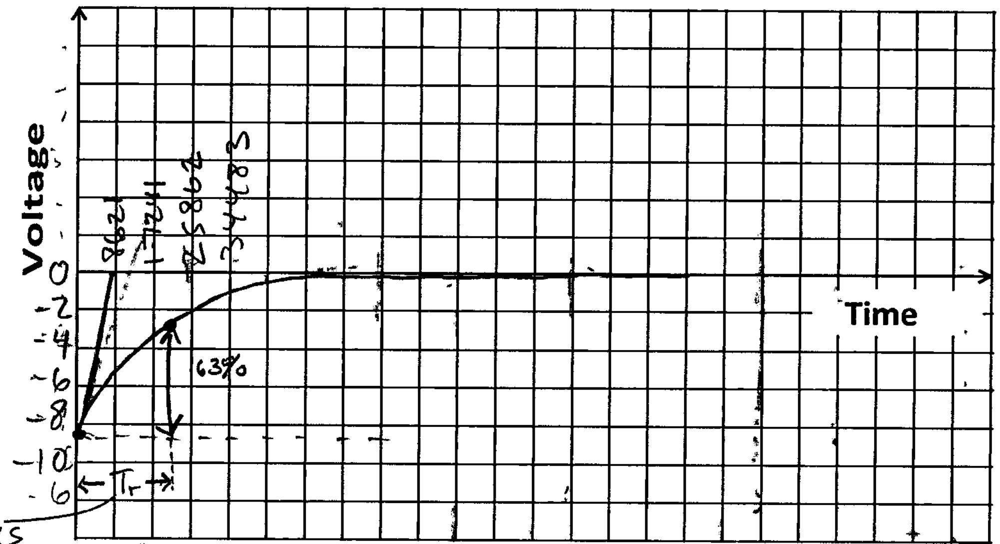
Using the measurement function of the oscilloscope, record the peak-to-peak voltage of the input (Ch1) and output (Ch2) signals for the range of frequencies in the table below.

Frequency (kHz)	V_{in} (Volts)	V_{out} (Volts)	Ratio (V_{out}/V_{in})
0.1	10.16	17.1	1.68
1	10.16	18.44	1.81
10	10	11.88	1.19
100	10.47	10.16	0.97

b. For a frequency setting of 0.1 kHz, measure the time constant of the response. Look for the location in time where the voltage changes by 63.2% of its final value. Compare this time with the beginning to the voltage jump to determine the system time constant.

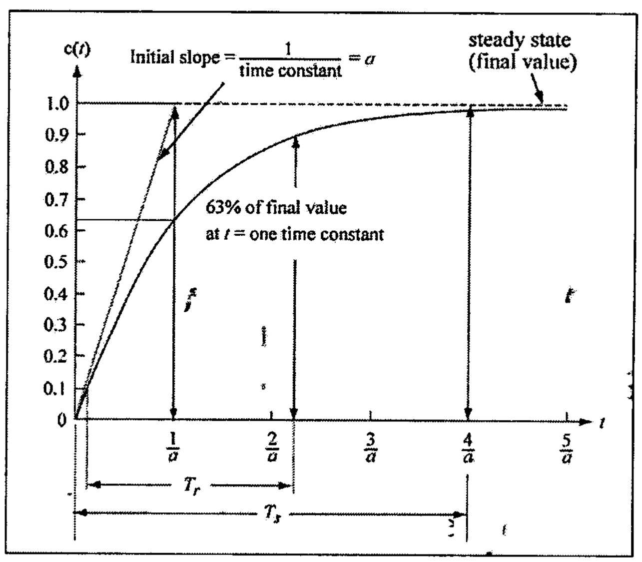
$\tau = 116.0\ \mu s$

0.1 kHz



$T_r = 1160\ \mu s$

Note: label 1/e point (where voltage change is 63.2% of the maximum ΔV)



Example of First Order System response to a step input showing the system time constant.

$$\frac{17.1}{2} = 8.55V \times 0.632 = 5.4036V$$

$$8.55V - 5.4036 = 3.1464V$$

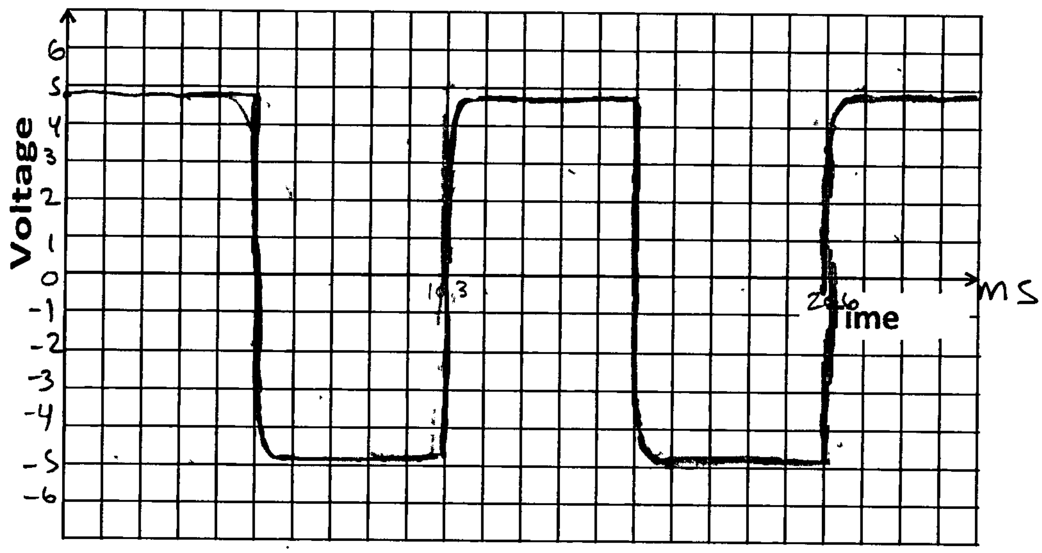
2. Low Pass Filter

The input and output voltages should now be connected as shown in the sketch of the Low Pass filter above.

a. Repeat procedure 1a using the low pass filter.

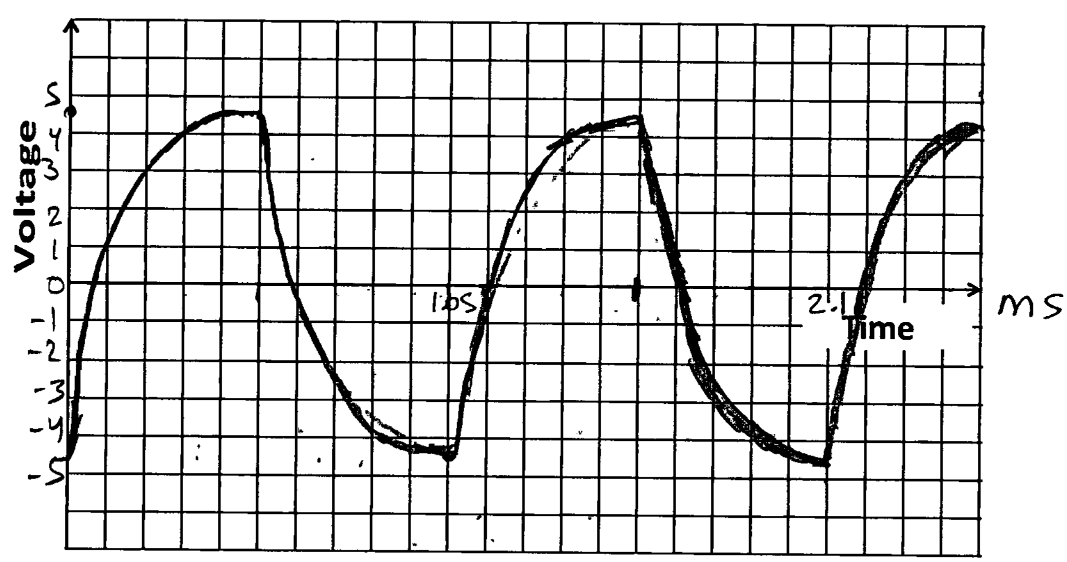
0.1 kHz

T=10.3ms



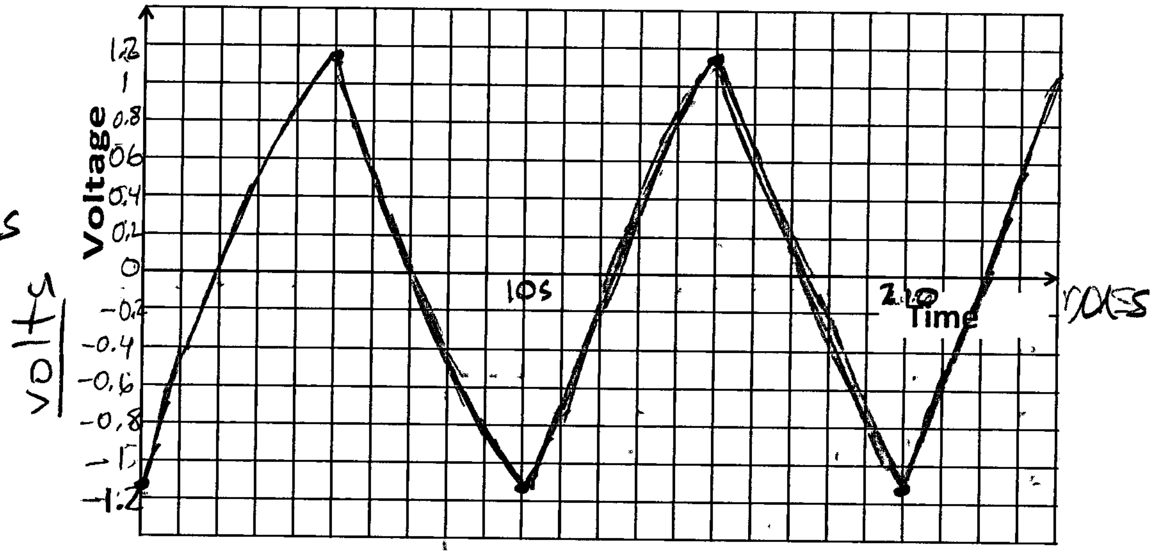
1 kHz

T=1.05ms



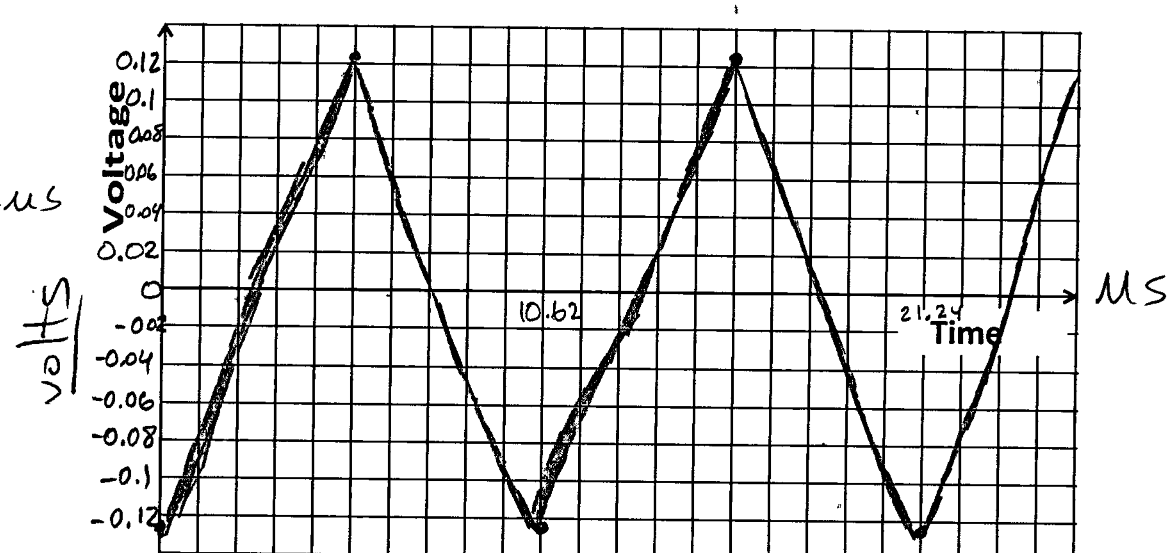
10 kHz

T=105μs



100 kHz

T=10.62μs



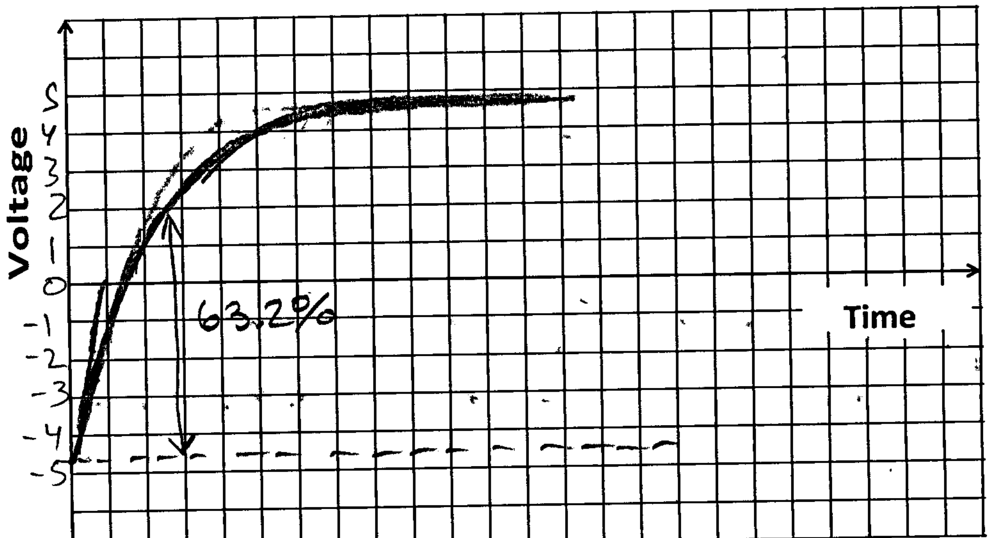
Using the measurement function of the oscilloscope, record the peak-to-peak voltage of the input (Ch1) and output (Ch2) signals for the range of frequencies in the table below.

Frequency (kHz)	V_{in} (Volts)	V_{out} (Volts)	Ratio (V_{out}/V_{in})
0.1	10	9.688	0.97
1	10	9.375	0.94
10	10	2.313	0.23
100	10.78	0.2437	0.023

b. For a frequency setting of 0.1 kHz, measure the time constant of the response. Look for the location in time where the voltage reaches 63.2% of its final value. Compare this time with the beginning to the voltage jump to determine the system time constant.

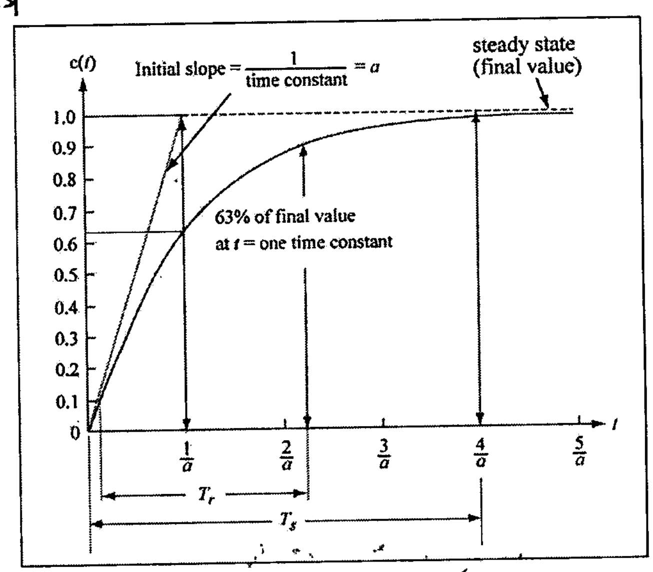
$\tau = 108.0\ \mu s$

0.1 kHz



Note: label 1/e point (where voltage change is 63.2% of the maximum ΔV)

$T = 108\ \mu s$



Example of First Order System response to a step input showing the system time constant.

$V_{p-p} = 9.688 \times 0.632 = 6.123V$

$(-9.688/2) + 6.123 = 1.279V$

White-input
Red-output

c. Using a sinusoidal input (and the labview program as the oscilloscope) with a peak to peak exactly equal to 10V vary the input frequency, carefully measure the amplitude of the output voltage for each of the following frequencies (Connect the input signal to port 1 and the output to port 0).

Frequency (kHz)	Vin (Volts)			Vout (Volts)			Ratio (Vout/Vin)			Measured Frequency (Hz)		
	S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3
0.1	10.04	9.70	10.06	10.04	9.65	10.03	1	0.9948	0.997	6.704	103.114	103.306
1	10.08	9.08	9.73	8.21	7.75	8.15	0.859	0.853	0.838	36.951	10374	1036.9
2	10.0	9.51	9.92	6.05	5.72	5.99	0.605	0.601	0.603	55.74	1944	1944.5
4	9.81	9.70	9.72	3.67	3.35	3.67	0.374	0.345	0.378	326.6	3671	3671
5	9.65	9.54	9.59	2.94	2.89	2.94	0.305	0.302	0.307	393.5	393.5	393.5

Use three different sampling rates to acquire the amplitude data and compare the amplitude data and the shapes of the response at these sampling rates (Sampling rate 1 (S1) = same as the frequency, Sampling rate 2 (S2) = 5 x frequency, Sampling rate 3 (S3) = 10 x frequency).

d. For the 1 kHz sinusoidal input, measure the phase lag between input and output. Use the cursor function of the Labview to measure the time delay (Δt) between the two signals. Also, remember that phase lag can also be expressed in degrees ($\varphi = 180\omega\Delta t/\pi = 360 f \Delta t$)

Phase Lag (φ) = 89 μsec which is equal to 32.04 degrees at 1 kHz.

of $\frac{V_{out}}{V_{in}}$ This is why the ratio increases as frequency increases for the high pass filter & for the low pass, as frequency increases, the ratio decreases because the higher frequency is being filtered out more.

CALCULATIONS AND RESULTS

For all the plots requested, please attach them to this report. If you need more space to answer any question, please do so on another sheet of paper and attach them to this report. You have to calculate the percentage error for all your measurements.

1. Using the nominal values of R and C, compute the time constant of the circuits (τ) and compare with that measured for the low and high pass filters.

$$\tau = RC = (10k\Omega)(0.01\mu F) = (1\times10^{4})(1\times10^{-8}) = 10^{-4}s = \boxed{100\mu s}$$

It is pretty close to our calculated values of 116 μs & 108 μs. High pass $\frac{116-100}{100} = 16\%$

% Error $\frac{108-100}{100} = 8\%$ low pass

2. For the square wave voltage input, describe why your output signals take that shape that they have. Use the data from the tables in part 1a and 2a from the procedure to support your description.

For the high pass filter, lower frequencies are filtered out, so the shape of the output signal is quite pointy, whereas the shape of high frequency output is more square, because the circuit does not filter out that signal. The lowpass filter does the opposite, so low frequency output signal graphs are more boxy and as frequency increases, the signal gets narrow until it is almost triangle shaped.

3. For the sinusoidal voltage input and in **the case of S3**:

a) Plot the measured values of $\frac{V_{out}}{V_{in}}$ for the low pass filter (from part 2c in the procedure) Vs. $\omega\tau$ (using τ from part 2b and $\omega = 2\pi f$)

b) Plot the same data above vs. ω this time and compare graphically with that computed from first order system response theory (i.e. plot first order magnitude response curve M(ω) vs. ω as well as your data). Select a value for the time constant (τ) when calculating M(ω) which best fits your experimental data, $\frac{V_{out}}{V_{in}}$. What value for the first-order time constant (τ) fits your experimental data the best? How does that value compare with the theoretical time constant of the filter (from question a)?

A value of $\tau = 102\,\mu s$ fits the best with our experimental data.

This value almost the same (very close to) the theoretical time constant of the filter.

c) Comment on the change in the amplitude and shape of input and output responses with the change in the sampling rate from S1 to S3 in 2c. What is the Nyquist Frequency? Calculate the Nyquist frequency for all the frequencies listed in 2c.

The Nyquist frequency is the minimum rate at which a signal can be sampled w/o introducing errors, which is twice the highest frequency present in the signal.
The amplitude is more accurate as you go from S1 to S3, and the shape is more defined an consistent. For S1, the shape troughs and crest do not match horizontally, but for S3 they do.
Nyquist frequencys in kHz: 0.2, 2, 2, 4, 8, 10 for frequencies of 0.1, 1, 1, 2, 4, 5.

d) Compare the measured phase lag with the predicted value ($\varphi = -\tan^{-1}(\omega\tau)$) using the time constant computed using the RC values and that from part b above.

τ using RC values $= 100\mu s$ τ from part b $= 102\mu s$ using $f = 1$ kHz

$\varphi = -\tan^{-1}(2\pi \times 1000 \times 100 \times 10^{-6})$ $\varphi = -\tan^{-1}(2\pi \times 1000 \times 102 \times 10^{-6})$

$\varphi = -32.14°$ $\varphi = -32.66°$

The phase lag is greater for the time constant found in part b by a small amount.

4) Please comment on the accuracy of your data. How well does your data match up with theoretical predictions. What sources of error may exist which would cause deviations between the expected and actual results?

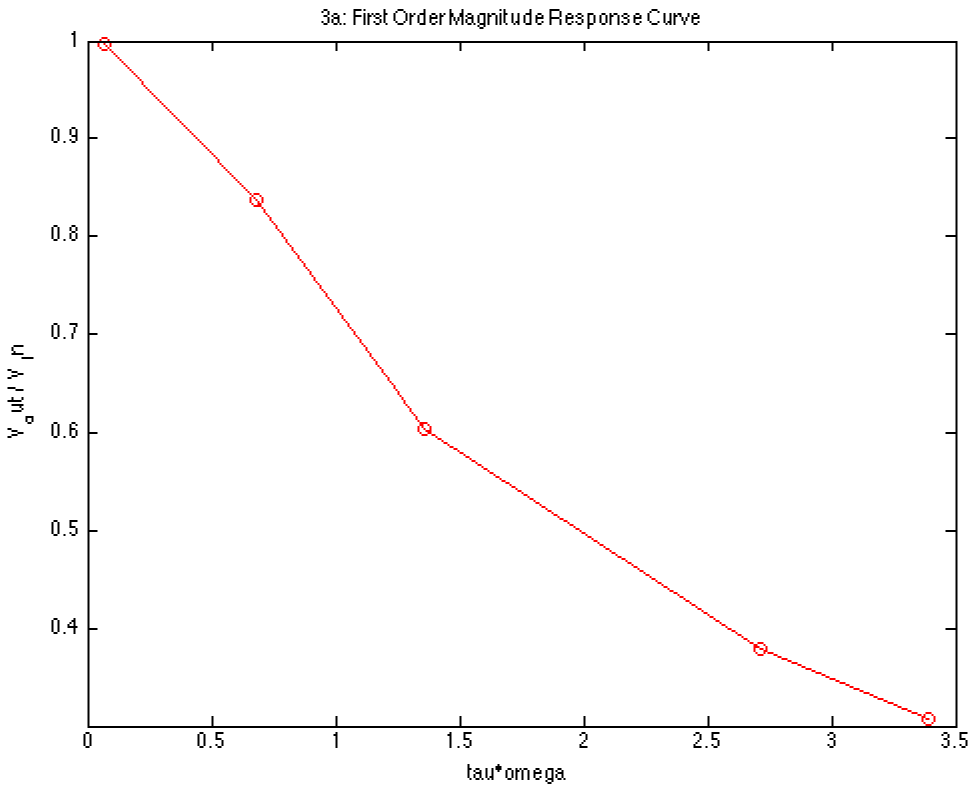
Our data was quite accurate, but there were some sources of error that did not make it perfect. Some sources could be instrumental uncertainty, bias in reading values, errors in calculations, roundoff errors, etc. Nonetheless, our actual results were pretty close to expected. Also, the resistors & capacitors may not have been exactly precise in their value.

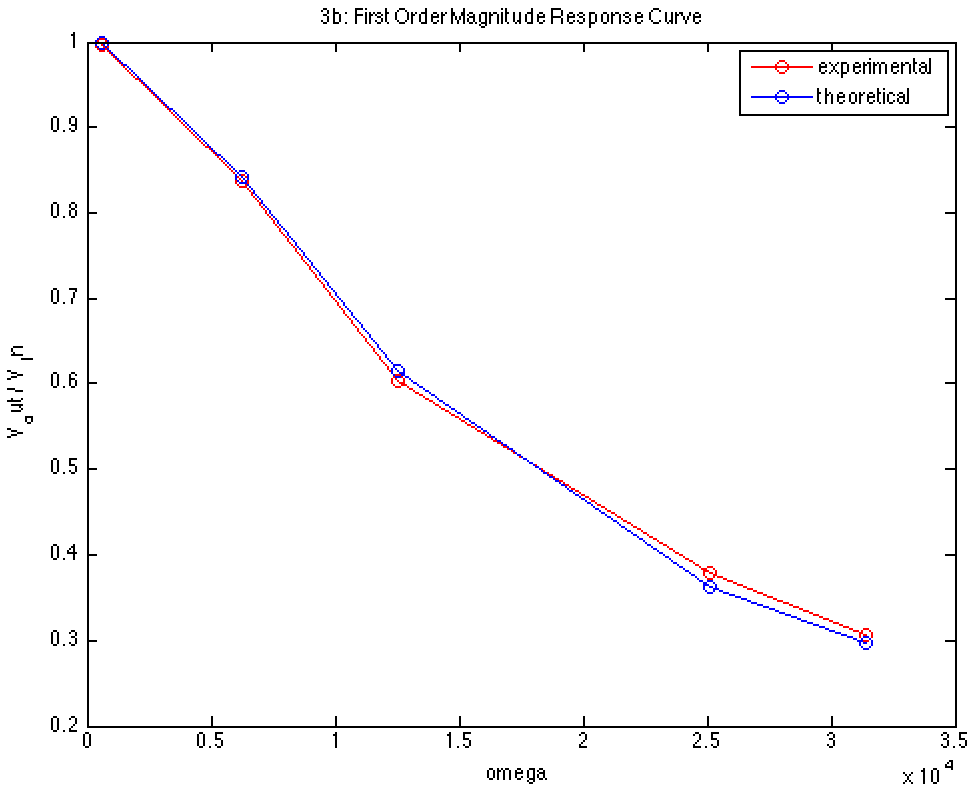
Deep Patel Measurements Lab 1

```
clear all, close all, clc

% Part a
f = [100, 1000, 2000, 4000, 5000]
w = f.*(2*pi)
tau = 108/(10.^6)
V_ratio = [0.997, 0.838, 0.603, 0.378, 0.307]
x = w*tau

figure;
plot(x, V_ratio, 'r-o')
xlabel('tau*omega'); ylabel('V_out / V_in');
title('3a: First Order Magnitude Response Curve')

% Part b
tau2 = 102/(10.^6)
M = 1./((1 + (w.*tau2).^2).^(0.5))
figure;
plot(w, V_ratio, 'r-o',w,M,'b-o')
xlabel('omega'); ylabel('V_out / V_in');
legend('experimental','theoretical');
title('3b: First Order Magnitude Response Curve')
```

```
f =

         100        1000        2000        4000        5000


w =

   1.0e+04 *

    0.0628    0.6283    1.2566    2.5133    3.1416


tau =

   1.0800e-04


V_ratio =

    0.9970    0.8380    0.6030    0.3780    0.3070


x =

    0.0679    0.6786    1.3572    2.7143    3.3929
```

```
tau2 =

   1.0200e-04


M =

    0.9980    0.8419    0.6151    0.3634    0.2979
```





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