HW #1

Deep Patel

① $P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3)$

② $(\exists x \forall y (P(x,y) \rightarrow Q(x,y))) \rightarrow (\forall x \forall y \exists z (P(x,y) \rightarrow P(y,z)))$.

$\neg(\exists x \forall y (P(x,y) \rightarrow Q(x,y))) \vee (\forall x \forall y \exists z (P(x,y) \rightarrow P(y,z)))$

$\neg(\exists x \forall y (\neg P(x,y) \vee Q(x,y))) \vee (\forall x \forall y \exists z (\neg P(x,y) \vee P(y,z)))$

Negate now $\neg($ ~~~~~~ $)$

$\neg\neg(\exists x \forall y (\neg P(x,y) \vee Q(x,y))) \wedge \neg(\forall x \forall y \exists z (\neg P(x,y) \vee P(y,z)))$

$\boxed{(\exists x \forall y (\neg P(x,y) \vee Q(x,y))) \wedge (\exists x \exists y \forall z (P(x,y) \wedge \neg P(y,z)))}$

③ gcd(123,46)

Using the Euclidean Algorithm,

$A = 123 \neq 0, B = 46 \neq 0$

$123 = 46 * 2 + 31$

$A = 46, B = 31$

$46 = 31 \times 1 + 15$

$A = 31, B = 15$

$31 = 15 \times 2 + 1$

$A = 15, B = 1$

$15 = 1 \times 15 + 0$

$A = 1, B = 0$

$\boxed{GCD(123,46) = 1}$

④ Proof by Induction

Base: $n=1$ vertex $\rightarrow$ 0 edges b/c no other vertex to connect to.

$|E| = \frac{n(n-1)}{2} = \frac{1(1-1)}{2} = \frac{1(0)}{2} = \frac{0}{2} = 0$

So, the given statement is true for $n=1$ vertices

Inductive Hypothesis: Suppose that the given statement that for an undirected simple graph with n verticies, there can be at most $|E| = \frac{n(n-1)}{2}$ edges is true for $n=k$ verticies

In other words, $|E| = \frac{k(k-1)}{2}$

Inductive step: we must prove $|E| = \frac{n(n-1)}{2}$ is true for $n=k+1$ verticies as well.

If we add a vertex, v, to the existing graph of k verticies, then we can connect the vertex v to all k verticies using k edges. Thus, the total # of edges in the graph w/ k+1 verticies are

$$|E| = \frac{k(k-1)}{2} + k = \frac{k(k-1)+2k}{2} = \frac{k^2+k}{2} = \frac{k(k+1)}{2} = \frac{(k+1)((k+1)-1)}{2}$$

substituting $n = k+1$ gives $|E| = \frac{n(n-1)}{2}$. So, by the principle of mathematical induction, it can be concluded that for an undirected simple graph, there can be at most $|E| = \frac{n(n-1)}{2}$ edges.

(5) # of states = $\boxed{9 \times 8 \times 7 \times 6} = 3{,}024$ (state space size)

possible positions for empty cell | possible positions for #1 | possible positions for #2 | possible positions for #3

& the rest of the cells must have a *.

(6) Complete Graphs that are planar

$n=1$ (K_1) 

$n=2$ (K_2) 

$n=3$ (K_3) 

$n=4$ (K_4) 

K_5 is not planar

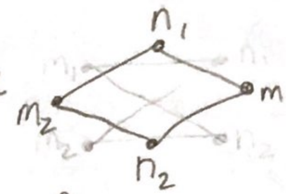
Bipartite graph K_{mn} is planar if $m \leq 2$ OR $n \leq n$

$m=1, n=1$ 

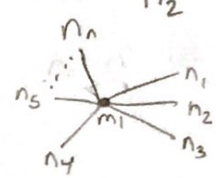
(K_{11})

$m=2, n=1$ 

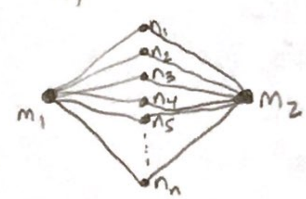
(K_{21})

$m=2, n=2$ 

(K_{22})

$m=1, n=n$ 

($K_{1,n}$)

$m=2, n=n$ 

($K_{2,n}$)

(7) K_5 is the smallest possible complete graph. We see that K_4 is planar in Problem 6. The next smallest complete graph is K_5.

__Proof:__ Assume K_5 is planar. Then it must obey Euler's relationship. $F+V-E=2$ where F=faces, V=vertices, & E=edges. For K_5, $V=5$, $E=10$, so $F=7$. For any planar graph, $E \geq \frac{3}{2}F$, so in this case, there must be atleast $\frac{3}{2}(7)=10.5$ edges. However, K_5 has only 10 edges, so it cannot be a planar graph and it is the smallest such graph because we show that $K_4, K_3 K_2$ & K_1 are all planar in Problem 6 by showing that no edge intersects another.

HW#1 (continued)

Deep Patel

8 (1)

# of bars	configuration	
n :	[1] [2] [3] . . . [n]	Size 1
$n-1$:	[1 | 2] [2 | 3] . . . [n-1 | n]	Size 2
$n-2$:	[1 | 2 | 3] [2 | 3 | 4] . . . [n-2 | n-1 | n]	Size 3
⋮		
2 :	[1 . . . n-1] [2 . . . n]	Size n-1
1 :	[1 . . . n]	Size n

Answer: $\boxed{\sum_{i=1}^{n} i}$

$$= \boxed{\frac{n(n+1)}{2} = \frac{n^2+n}{2} = \binom{n+1}{2}}$$

(2) n-1 possible divisions. 2 possibilities: divided or not, so Total # of possible ways to divide = $\boxed{2^{n-1}}$.

(3) $$\boxed{\left(\sum_{i=1}^{n} i\right)\left(\sum_{k=1}^{m} k\right) = \binom{n+1}{2}\binom{m+1}{2} = \frac{nm(n+1)(m+1)}{2}}$$