Deep Patel
Intro to AI - HW 4
11/14/18

Problem 1

Let r = rain, f = forecast of rain

P(r) = 0.33
P(¬r) = 1 - P(r) = 1 - 0.333333 = 0.67
P(f|r) = 0.8
P(¬f|r) = 1 - P(r|f) = 1 - 0.8 = 0.2
P(f|¬r) = 0.2
P(¬f|¬r) = 1 - P(f|¬r) 1 - 0.8 = 0.2
P(f) = P(f|r)P(r) + P(f|¬r)P(¬r) = (0.8)(0.33) + (0.2)(0.67) = 0.398

P(r|f) = P(f|r) P(r) / P(f) = (0.8) * (0.33) / (0.398) = ***0.6633***

The probability of it raining on her wedding day is 0.6633 or 63.33%.

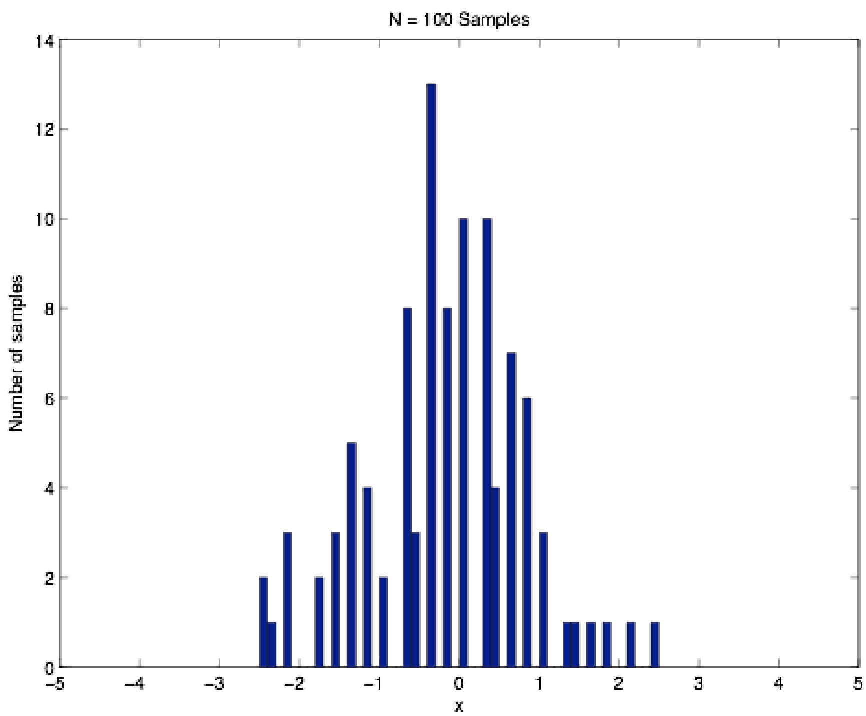
Problem 2

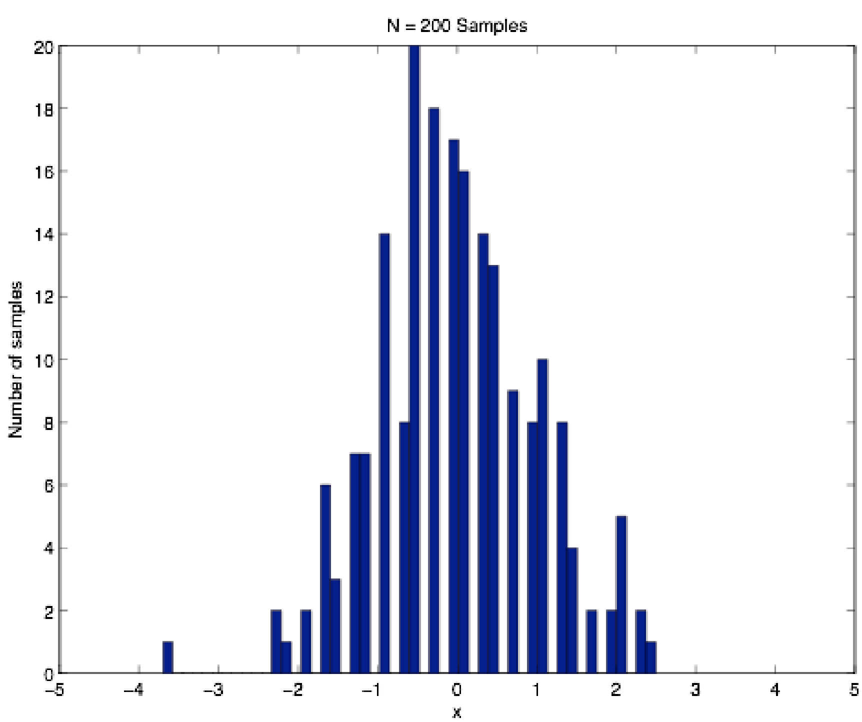
Originally:
Let
D_i = be the ith door with $1 \le i \le 4$,
G_1 = a door with one goat behind it,
G_2 = a door with another goat behind it,
P = prize behind door,
N = nothing behind door.

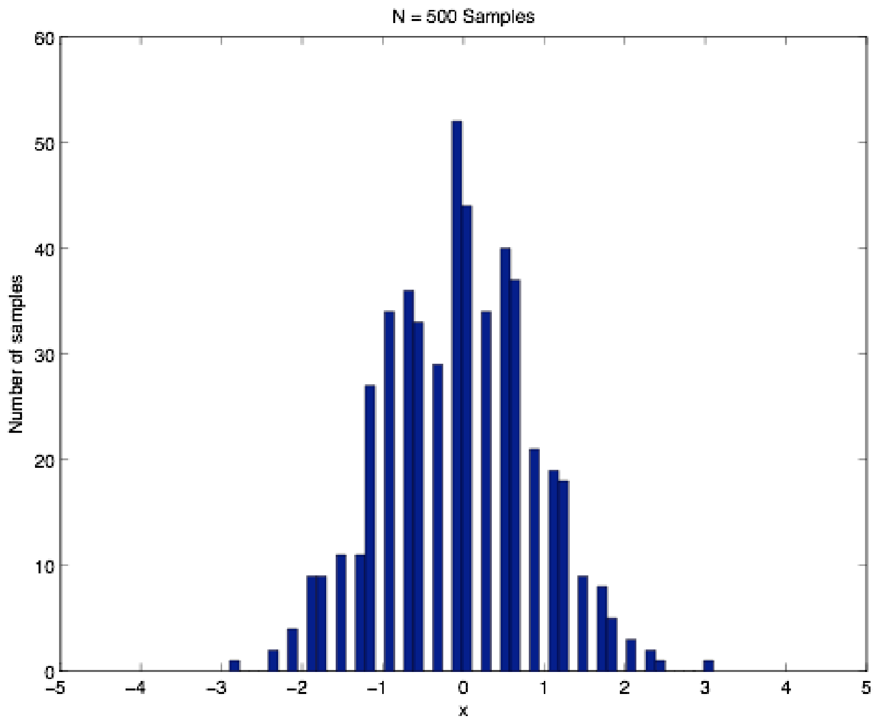
Let D_1 be the door that we choose. The same argument can be generalized for any other door.
$P(D_1 = P) = 0.25$
After revealing one of the doors, D_2 with a goat behind it, say G_1, (again the same argument can be generalized by switching G_1 with G_2), we get
$P(D_1 = P \mid D_2 = G_1) = 0.25$, which is still the same.
$P(D_1 \neq P \mid D_2 = G_1) = 1 - P(D_1 = P \mid D_2 = G_1) = 1 - 0.25 = 0.75$
Because $P(D_1 \neq P \mid D_2 = G_1) = P(D_3 = P \mid D_2 = G_1) + P(D_4 = P \mid D_2 = G_1) = 0.75$
Also, because the probability it equal, $P(D_3 = P \mid D_2 = G_1) = P(D_4 = P \mid D_2 = G_1)$, and
$2P(D_4 = P \mid D_2 = G_1) = 0.75$
$P(D_4 = P \mid D_2 = G_1) = 0.75/2 =$ ***0.375***

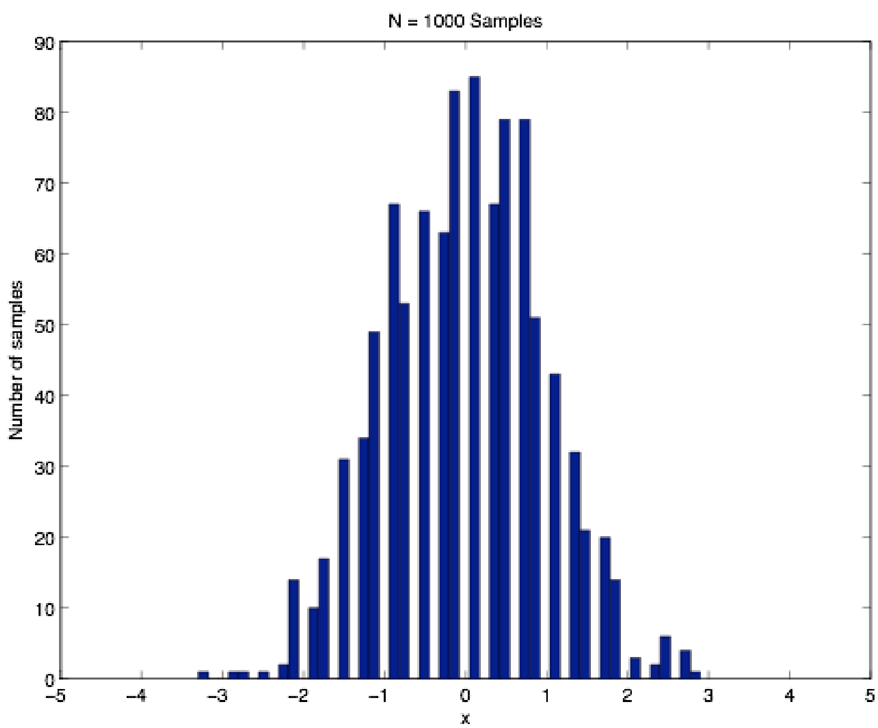
Clearly, the probability of switching to either of the other two remaining doors increases the probability of getting the prize. ***Thus, the guest should switch their choice to one of the other 2 remaining doors. Their probability of winning if they switch will be 0.375 or 37.5%.***

Problem 3









Problem 4

Part a)

P(A=true, B=true, C=true, D=true, E=true) =

P(A=true)P(B=true)P(D=true|A=true,B=true)P(C=true)P(E=true|B=true,C=true) =

$0.2 * 0.5 * 0.1 * 0.8 * 0.3 =$ ***0.0024***

Part b)

P(A,E|B) = P(A|B) * P(E|B) (because A and E and independent)
= P(A) * P(E|B) (because A is independent of B)
= P(A) * (P(E|B, C = true)P(C=true) + P(E|B,C = false)P(C=false)) (Sum over all possible C)
= 0.2 * (0.3*0.8 + 0.8*0.3) = ***0.096***

Part c)

P(A=true | B=false, C=false, D=false, E=false)
= P(A=true) because A is independent of all the other variables
= ***0.2***

Part d)

P(C=true, D=false | B=false)
= P(C=true|B=false)*P(D=false|B=false) (Because C and D are independent)
= P(C=true)*P(D=false|B=false) (Because C and B are independent)
= P(C=true)*[P(D=false|B=false,A=true)P(A=true) + P(D=false|B=false,A=false)P(A=false)]
(Marginalization - sum over all possible values of A)
= P(C=true)*[P(D=false|B=false,A=true)P(A=true) + P(D=false|B=false,A=false)(1- P(A=true))]
(Law of total probability to get P(A=false))
= 0.8 * [0.5*0.2 + 0.9*(1-0.2)]
= ***0.656***

Part e)

P(B=true, C=false, D=true | A=false)
= P(B=true|A=false)*P(C=false|A=false)*P(D=true|A=false)
= P(B=true) *P(C=false)*P(D=true|A=false) (because B and A are independent and C and A are independent)
= P(B=true) *P(C=false)*[P(D=true|A=false,B=true)P(B=true) + P(D=true|A=false,B=false)P(B=false)]
(Marginalization - sum over all possible values of B)
= P(B=true) *(1-P(C=true))*[P(D=true|A=false,B=true)P(B=true) + P(D=true|A=false,B=false)(1-P(B=true))]
(Law of total probability to get P(C=false))
= 0.5 * (1-0.8) * [0.6*0.5 + 0.9*(1-0.5)]
= ***0.075***

Problem 5

Part a)

P(Burglary | JohnCalls=true, MaryCalls=true)
$= \alpha\, P(B)\, \Sigma_e\, P(e)\, \Sigma_a\, P(a|B,e)\, P(j|a)\, P(m|a)$
$= \alpha\, f_1(B) \times \Sigma_e\, f_2(e) \times \Sigma_a\, f_3(a,B,e) \times f_4(a) \times f_5(a)$

Let $f_6(B,e) \quad = \Sigma_a\, f_3(a,B,e) \times f_4(A) \times f_5(A)$
$\qquad = (f_3(a,B,e) \times f_4(a) \times f_5(a)) + (f_3(\neg a,B,e) \times f_4(\neg a) \times f_5(\neg a))$

Now,
P(Burglary | JohnCalls=true, MaryCalls=true)
$= \alpha\, f_1(B) \times \Sigma_e\, f_2(e) \times f_6(B,e)$

Let $f_7(B) \quad = \Sigma_e\, f_2(e) \times f_6(B,e)$
$\qquad = f_2(e) \times f_6(B,e) \times f_2(\neg e) \times f_6(B,\neg e)$

Now,
P(Burglary | JohnCalls=true, MaryCalls=true)
$= \alpha\, f_1(B) \times f_7(B)$

Part b)

The number of operations for variable elimination consists of computing f_5 which takes 2 sets of 3 operations (2 multiplications and 1 addition) and computing P(A) which takes 5 operations (4 multiplications and an addition). This gives a total of **11 operations** for variable elimination. For enumeration, there are 4 values of E, A and for each set of values, 5 terms are multiplied (4 multiplications) giving 16 multiplications. These 4 terms are then added (3 additions). This makes a total of **19 operations** for enumeration.

Part c)

Enumeration:

We have $P(X_1|X_n = \text{true}) = \alpha\, \Sigma_{X2\ldots Xn\text{-}1}\, P(X_n\text{=true}|X_{n\text{-}1})P(X_{n\text{-}1}|X_{n\text{-}2})\ldots P(X_3|X_2)P(X_2|X_1)P(X_1)$
There are $n - 2$ variables to sum over and each term needs $n - 1$ multiplications. Thus enumeration will take $\mathbf{O(n2^{n-2})}$ time.

Variable Elimination:

$P(X_1|X_n = \text{true}) = \alpha\, \Sigma_{Xn\text{-}1}\, P(X_n = \text{true}|X_{n-1})\, \Sigma_{Xn-2}\, P(X_{n-1}|X_{n-2})\ldots\Sigma_{X2}\, [P(X_3|X_2)\, P(X_2|X_1)\, P(X_1)]$
$\qquad = \alpha\, \Sigma_{Xn\text{-}1}\, P(X_n = \text{true}|X_{n-1})\, \Sigma_{Xn-2}\, P(X_{n-1}|X_{n-2})\ldots\, \Sigma_{X3}\, P(X_4|X_3)\, f_{X2}(X_3)$
$\qquad = \ldots$

There are n-2 summations, each of which take 2 additions, each of 2 terms. Thus the time complexity is **O(n)**.