Guidelines. Students may discuss problems in the assignment across group boundaries. However, each group must write down the answers independently. Submit as a SINGLE PDF (either typeset or scanned) per group. IMPORTANT: name your PDF file as

NETID.PDF

where NETID is your actual netid. For groups of two students, the file name should be

NETID1-NETID2.PDF

One group should submit only a single copy. Submitting two copies through two students accounts for the same group will result in 20% penalty for both students.

Problem 1 [20 points]. Using information gain as the criteria for deciding the node to split, learn the decision tree to depth 2 (i.e., the root should have grandchildren) using the data provided in Table 1. Show your work for the first splitting decision (i.e., how do you decide which feature to use for the first split).

Table 1: Previous tennis playing decisions based on outlook, temperature, humidity, and wind

Day	Outlook	Temperature	Humidity	Wind	Played Tennis?
01	Sunny	Hot	High	Weak	No
02	Sunny	Hot	High	Strong	No
03	Overcast	Hot	High	Weak	Yes
04	Rain	Mild	High	Weak	Yes
05	Rain	Cool	Normal	Weak	Yes
06	Rain	Cool	Normal	Strong	No
07	Overcast	Cool	High	Strong	Yes
08	Sunny	Cool	High	Weak	No
09	Sunny	Mild	Normal	Weak	Yes
10	Rain	Hot	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

Problem 2 [10 points]. Using the method of Lagrange multipliers, find all potential extremal points for $f(x, y) = xy$ subject to the constraint $x^2 + 2y^2 = 6$. Which ones yield maximum? What is the maximum?

Problem 3 [20 points]. Consider building a linear classifier and then an SVM for the following two-class training data:

Positive class:

$$[-1\ 3]^T, [0\ 2]^T, [0\ 1]^T, [0\ 0]^T$$

Negative class:

$$[1\ 5]^T, [1\ 6]^T, [3\ 3]^T$$

a) [10 points]. Compute a linear classifier using $\alpha = 0.8$ and using the samples in the following

order $[-1, 3], [1, 6], [0, 1], [1, 5], [0, 2], [0, 0], [3, 3]$. Show your work for the first seven iterations and also provide the final $w = (w_0, w_1, w_2)$.

b) [3 points]. Plot the training points and, by inspection, draw a linear classifier that separates the data with maximum margin.

c) [3 points]. This linear SVM is parameterized by $h(x) = w^T x + b$. Estimate the parameters w and b for the classifier that you have drawn.

d) [4 points]. Suppose you observe an additional set of points, all from the positive class:

$$[-2\ 0]^T, [-2\ 1]^T, [-2\ 3]^T, [-1\ 0]^T, [-1\ 1]^T, [0\ 0]^T$$

What is the linear SVM (in terms of w and b) now?

Problem 4 [20 points]. What is the VC-dimension of (a) two half-planes (i.e., two linear separators)? (b) the inside of a triangle? Explain.

Problem 5 [30 points]. Carry out policy iteration over the MDP example covered in class with R given in Table 2 and $\gamma = 0.9$. For a state s, if $R(s) = \pm 1$, s is a terminal state. For the transition model, assume that the agent has 0.9 probability of going to the intended direction and 0.1 probability of moving to the left. For example, if the agent is at the lower left corner (coordinates $(1, 1)$) and intends to go right, then it will reach $(2, 1)$ with 0.9 probability and $(1, 2)$ with 0.1 probability. If a target cell is not reachable, then the corresponding probability goes back to the current cell. For example, if the agent is at $(3, 3)$ and is trying to go up, then with 0.1 probability it goes to $(2, 3)$ and with 0.9 probability it is stuck at $(3, 3)$. For your answer you should provide:

a) [15 points]. The first two iterations of your computation.

b) [15 points]. The converged rewards and the extracted policy. For this problem, you need to provide last two iterations showing that the value changes are within 0.001 for all cells.

Table 2: Reward R for a 4×3 grid world

-0.05	-0.05	-0.05	+1
-0.05	OBS	-0.05	-1
-0.05	-0.05	-0.05	-0.05

As a suggestion, you should complete the first question manually to make sure you will be able to do so, for obvious reasons :). For solving the second, it is perhaps better to do it using a program, perhaps using Python or excel.

