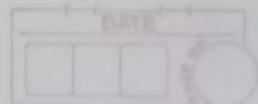


LECTURE 03: Markov chains



SOLVED EXAMPLES OF THE MARKOV CHAINS USING TRANSITION PROBABILITY MATRIX (PART 2):
(How to find the probabilities).

TYPE 2: How to calculate the probability after n-steps.

For example, say you want to calculate

$$P(X_3 = 3 \mid X_1 = 1) = P_{13}^{(2)}$$

↑
it means first row, 3rd element of P^2

say $P^2 = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$ — our answer.

for P^3 do $P^2 P$, $P^4 = P^2 \cdot P^2$

eg1. Lec 02: eg 4 cont.

Also, find the probability that he changes from going by train to driving exactly in 4 days.

→

$$P = \begin{matrix} & \begin{matrix} T & C \end{matrix} \\ \begin{matrix} T \\ C \end{matrix} & \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} \end{matrix}$$

$$P(X_4 = C \mid X_0 = T) = P_{TC}^{(4)}$$

$$P^2 = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix} \quad \therefore P^4 = \begin{matrix} & \begin{matrix} T & C \end{matrix} \\ \begin{matrix} T \\ C \end{matrix} & \begin{bmatrix} 3/8 & 5/8 \\ 5/16 & 11/16 \end{bmatrix} \end{matrix}$$

$$P_{TC}^{(4)} = \frac{5}{8}$$

eq2 lecture 1 : Q1 cont.

The present market share of the three brands A, B and C is 60%, 30% and 10% resp. using this info, find (i) the p. that a cust. who is currently a brand A purchaser will purchase brand B after two time periods.

$$\rightarrow P(X_2 = B | X_0 = A) = ? = P_{AB}^{(2)}$$

$$P = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.5 & 0.4 & 0.1 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$$

$$\therefore P^2 = \begin{array}{c} \begin{matrix} & A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 0.65 & 0.24 & 0.11 \\ 0.61 & 0.28 & 0.11 \\ 0.64 & 0.24 & 0.12 \end{bmatrix} \end{array}$$

$$\boxed{P(X_2 = B | X_0 = A) = 0.24}$$

(ii) The prob. that a cust. who is currently a purchaser of brand A will purchase brand C after two time periods?

$$\rightarrow P(X_2 = C | X_0 = A) = ? = P_{AC}^{(2)} = 0.11$$

$$\boxed{P(X_2 = C | X_0 = A) = 0.11}$$

(iii) The prob. that brand C will be able to retain its customers after two time periods?

$$\rightarrow P(X_2 = C | X_0 = C) = ? \quad P_{CC}^{(2)} = ?$$

$$\boxed{P(X_2 = C | X_0 = C) = 0.12}$$

(v) The proba that brand B purchaser will purchase brand A three period from now.

→ $P(X_3 = A | X_0 = B) = ?$ $P_{BA}^{(3)}$

$$P^3 = P^2 \cdot P = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0.64 & 0.25 & 0.11 \\ \boxed{0.63} & 0.26 & 0.11 \\ 0.64 & 0.25 & 0.11 \end{bmatrix} \end{matrix}$$

$$\boxed{P(X_3 = A | X_0 = B) = 0.63}$$

(vi) The prob. that a cust. will purchase brands A, B, C three periods from now. — lecture 2

→ $P(X_3 = A) = ?$ $q_3(A)$
 $P(X_3 = B) = ?$ $q_3(B)$
 $P(X_3 = C) = ?$ $q_3(C)$

$$q_3 = q_0 P^3$$

$$q_0 = \begin{bmatrix} 0.6 & 0.3 & 0.1 \end{bmatrix} \quad P^3 = \begin{bmatrix} 0.64 & 0.25 & 0.11 \\ 0.63 & 0.26 & 0.11 \\ 0.64 & 0.25 & 0.11 \end{bmatrix}$$

$$q_0 P^3 = \begin{bmatrix} 0.637 & 0.253 & 0.11 \end{bmatrix}$$

Hence, market shares for brands A, B and C are 63.7%, 25.3% and 11% respectively.

eq3 lec 2 eq5 cont

(i) $P(X_3 = F | X_2 = N)$

(ii) $P(X_2 = N | X_1 = H)$

(iii) $P(X_2 = H | X_2 = F)$

→

		F	H	N
P =	F	0.30	0.25	0.45
	H	0.45	0.40	0.15
	N	0.65	0.25	0.10

(i) $P_{NF}^{(1)} = 0.65$

(ii) $P_{HN}^{(1)} = 0.15$

(iii) $P_{FH}^{(2)} = 0.2875$

		F	H	N
P ² =	F	0.4950	0.2875	0.2175
	H	0.4125	0.3100	0.2775
	N	0.3725	0.2875	0.3400