25 March 2021 10:18

28/10-12m

Probability Distribution

Probability Distribution → Discrete probability Distribution (poisson dist.)

Probability Distribution → Continuous prob. distr. (Normal Distr.)

Discrete Prob. Distributions

① Discrete Uniform (equal) Distribution.

– Jab koi bhi expt perform karne ke baad har ek sample o/p ka probability same aata hai toh use discrete uniform distribution

ex: Tossing a coin. $S = \{H, T\}$

$P(H) = P(T) = \frac{1}{2}$

Throwing a ~~die~~ die ⇒ $S = \{1,2,3,4,5,6\}$

$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$

② The prob. mass function (pmf)

$P(X=x) = \frac{1}{n}$ $x = 0, 1, 2, \ldots n$

$= 0$ otherwise

③ Mean $= E(X) = \frac{n+1}{2}$

④ Var $= V(X) = \frac{n^2-1}{12}$

⑤ $SD = \sigma = \sqrt{var}$

② Bernouli's Distribution.

– Jab koi bhi expt sirf 1 bar perform kiya hoga (n=1) then we use Bernoulis distribution.

Conditions –

– pass / fail
– success / failure
– hit / miss
– defective / nondefective

② Prob mass function (pmf)

$P(X=x) = P^x (1-P)^{1-x}$

x = no of success

P = prob. of success

q = prob of failure

$P + q = 1$, $P = 1 - q$, $q = 1 - P$

③ Mean $= E(X) = P$

④ Var $= V(X) = Pq$

⑤ $SD = \sigma = \sqrt{var}$

③ Binomial Distribution [n – small]

– Jab koi bhi expt ka n-trials kiya hoga with terms / condn.

– pass / fail
– success / failure

② Prob mass function (pmf)

$P(X=x) = {}^nC_x \, p^x q^{n-x}$

x = no of success

p = prob of success

q = prob of failure

$P + q = 1$, $P = 1 - q$, $q = 1 - P$

③ mean $= E(X) = nP$

④ var $= V(X) = npq$

⑤ $SD = \sigma = \sqrt{var}$

④ Poisson Distribution –

Conditions –

ⓐ Value of trials (n) is very large say $n \to \infty$

ⓑ Value of prob of success (p) is very small as compared to n ie $P \to 0$

$n \gg p$ (n: no of trials; p: prob of success)

The pmf for poisson distribution is

$$P(X=x) = \frac{e^{-m} \cdot m^x}{x!}$$

where $m = np$ = mean

mean = var = $m = np$

$SD = \sigma = \sqrt{var}$

$${}^nC_x = \frac{n!}{x!\,(n-x)!}$$

$${}^5C_3 = \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} = 10$$

$${}^{10}C_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} = 210$$

$${}^9C_5 = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} =$$

* A Fair coin is tossed 5 times.

Find the prob. of getting

a) 3 Heads & 2 tails

b) Exactly 5 heads

c) Atleast 3 heads

Soln A fair coin is tossed 5 times

$n = 5$

The pmf for Binomial Distr

$P(X=x) = {}^nC_x \, p^x q^{n-x}$

P = prob of success = prob of heads $= \frac{1}{2}$

q = prob of failure = prob of tails $= \frac{1}{2}$

$${}^nC_0 = 1, \quad {}^nC_n = 1, \quad {}^nC_1 = n$$

① Prob of 3 heads & 2 tails

x = no of success = 3

$$P(X=3) = {}^5C_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^{5-3}$$

$$= \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} \left(\tfrac{1}{8}\right)\left(\tfrac{1}{4}\right)$$

$$\boxed{P(X=3) = \frac{5}{16}}$$

prob of failure $= 1 - P(X=3)$

$= 1 - \frac{5}{16} = \frac{11}{16}$

② Prob of exactly 5 heads

$$P(X=5) = {}^5C_5 \left(\tfrac{1}{2}\right)^5 \left(\tfrac{1}{2}\right)^{5-5}$$

$$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \left(\tfrac{1}{32}\right)(1)$$

$$\boxed{P(X=5) = \frac{1}{32}}$$

③ prob of atleast 3 heads

$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$$= {}^5C_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^2 + {}^5C_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^1 + {}^5C_5 \left(\tfrac{1}{2}\right)^5 \left(\tfrac{1}{2}\right)^0$$

$$= \frac{5}{16} + \frac{5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4}\left(\tfrac{1}{16}\right)\left(\tfrac{1}{2}\right) + \frac{1}{32}$$

$$= \frac{5}{16} + \frac{5}{32} + \frac{1}{32}$$

$$= \frac{10 + 5 + 1}{32} = \frac{1}{2}$$

* The mean and variance of random variable X follows binomial distribution are 4 and $4/3$ respectively. Find probability of $(x \geq 1)$

Soln Mean $= E(x) = np = 4$ — (1)

Var $= V(x) = npq = \frac{4}{3}$ — (2)

The pmf for Binomial Distr. is -

$P(X=x) = {}^nC_x\, p^x q^{n-x}$ — (3)

From eqn (1) + (2) we get

$\frac{\cancel{np}}{\cancel{np}q} = \frac{4}{4/3}$

$\frac{1}{q} = \frac{12}{4}$

$\frac{1}{q} = 3$

$\boxed{q = 1/3}$

$p + q = 1$

$p = 1 - q$

$= 1 - \frac{1}{3}$

$\boxed{P = \frac{2}{3}}$

Mean $= np = 4$

$n = \frac{4}{p} = \frac{4}{2/3}$

$n = \frac{12}{2}$

$\boxed{n = 6}$

Find prob of $(x \geq 1)$

$P(x \geq 1) = P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)$

OR

$P(x \geq 1) = 1 - [P(x=0)]$

$= 1 - \left[{}^6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6\right]$

$= 1 - \left[(1)(1)\frac{1}{3^6}\right]$

$= 1 - \frac{1}{3^6} = 0.9999$

$P(x \geq 1) = {}^6C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^5 + {}^6C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + {}^6C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 + {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1 + {}^6C_6\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^6$

$= 0.9999$

* The prob. of man hitting a target is $\frac{1}{3}$ of the fires 6 times. Find the prob of hitting the target atleast twice

Soln No of trials = 6

p = prob of success = prob of man hitting a target $= \frac{1}{3}$

q = prob of failure $= 1 - p$

$\boxed{q = \frac{2}{3}}$

The pmf for Binomial Dist—

$P(X=x) = {}^nC_x\, p^x q^{n-x}$ — (1)

Find prob of hitting the target atleast twice

$P(x \geq 2) = P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6)$

OR

$P(x \geq 2) = 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[{}^6C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^6 + {}^6C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^5\right]$

$= 1 - \left[(1)(1)\left(\frac{2^6}{3^6}\right) + (6)\left(\frac{1}{3}\right)\left(\frac{2^5}{3^5}\right)\right]$

$= 1 - \left[\frac{2^6}{3^6} + \frac{6 \times 2^5}{3^6}\right] = 1 - \left[\frac{2^6 + 6 \times 2^5}{3^6}\right]$

$= 1 - \left[\frac{64 + 192}{3^6}\right] = 0.6488$

* At a printing press 3% of the books are found to have defective binding. Find the prob that out of 250 books bound at the printing press, exactly 4 books will have defective binding.

Soln - There are 250 books bounds at the printing press

$n = 250$

Out of that 3% books are found to have defective binding $p = \frac{3}{100} = 0.03$

Poisson

$n >> p$

$250 >> 0.03$

The prob mass function for poisson distr

$P(X=x) = \frac{e^{-m} \cdot m^x}{x!}$ — (1) Here $m = np = 25\cancel{0} \times \frac{3}{10\cancel{0}} = \frac{75}{10} = 7.5$

$\boxed{m = 7.5}$

Calculator

e^x ^

$e^{\square}$ $e(-7.5) \times (7.5 \wedge 4) \div 24 =$

Prob of exactly 4 books have defective binding

$P(x=4) = \frac{e^{-7.5} \cdot (7.5)^4}{4!} = \cancel{0.7291}$ 0.07291

m19

M19 Q6) b) 6m

$n = 3$

$S = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$

Sample o/p $S = 36$

$P(x=x) = {}^nC_x (p)^x q^{n-x}$

p = prob of success = prob of getting sum as 9 $= \frac{4}{36}$

$(3,6)\ (4,5)\ (5,4)\ (6,3)$

$p = 4/36$

$q = 32/36$

$q = 1 - p = 1 - \frac{4}{36} = \frac{32}{36}$

① $P(x=1) = {}^3C_1 \left(\frac{4}{36}\right)^1 \left(\frac{32}{36}\right)^{3-1} \Rightarrow 0.26$

② $P(x=2) = {}^3C_2 \left(\frac{4}{36}\right)^2 \left(\frac{32}{36}\right)^{3-2} \Rightarrow 0.032$

D-18 Q3) a) 6m

Mean $= m = 3 = np$

$n = 1000$

Pmf of Poisson distribution

$P(x=x) = \frac{e^{-m} \cdot m^x}{x!}$

① Prob of no accident in a yr.

$P(x=0) = \frac{e^{-3} \cdot 3^0}{0!} = \frac{e^{-3} \cdot (1)}{1} = e^{-3}$

$\boxed{P(x=0) = 0.0498}$

② Prob of more than 3 accident in year

$P(X>3) = 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$

$= 1 - \left[0.0498 + \frac{e^{-3} \cdot 3^1}{1!} + \frac{e^{-3} \cdot 3^2}{2!} + \frac{e^{-3} \cdot 3^3}{3!}\right]$

$= 1 - \left[\frac{e^{-3}}{\cancel{0.0498}} + \frac{3e^{-3}}{1} + \frac{9e^{-3}}{2} + \frac{\cancel{27}\,9\, e^{-3}}{\cancel{6}\,2}\right]$

$= 1 - \left[e^{-3}\left(1 + 3 + \frac{9}{2} + \frac{9}{2}\right)\right]$

$= 1 - \left[\frac{2+6+9+9}{2}\right] e^{-3} = 1 - \frac{26 \cdot e^{-3}}{2}$

$= 1 - 13 e^{-3} = 1 - 13(0.0498)$

$P(x>3) = 0.3526$

M18 Q4) b) 6m

p = prob of error $= 0.01 = \frac{1}{100}$

$q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$

n = no of received digits $= 10$

① Binomial Distr.

$P(x=x) = {}^nC_x p^x q^{n-x}$

$P(x \geq 1) = 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[{}^{10}C_0 \left(\frac{1}{100}\right)^0 \left(\frac{99}{100}\right)^{10} + {}^{10}C_1 \left(\frac{1}{100}\right)^1 \left(\frac{99}{100}\right)^9\right]$

$\boxed{P(x \geq 1) = 0.0951}$

$m = n \times p = 10 \cdot \left(\frac{1}{100}\right) = 1/10$

$\boxed{m = 0.1}$

② Poisson dist

$P(x=x) = \frac{e^{-m} \cdot m^x}{x!}$

$P(x \geq 1) = 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[\frac{e^{-0.1} (0.1)^0}{0!} + \frac{e^{-0.1} \cdot (0.1)^1}{1!}\right]$

$= 1 - \left[e^{-0.1} + 0.1\, e^{-0.1}\right]$

$= 1 - 1.1 e^{-0.1}$

$\boxed{P(x \geq 1) = 0.0951}$

* For a poisson distribution $P(X=2)$ is 9 times $P(X=4)$ and 90 times $P(X=6)$. Find mean, var & SD.

Soln: The pmf for PD is

$$P(X=x) = \frac{e^{-m} \cdot m^x}{x!}$$

$$P(X=2) = 9\, P(X=4) + 90\, P(X=6)$$

$$\frac{\cancel{e^{-m}} \cdot m^2}{2!} = 9 \cdot \frac{\cancel{e^{-m}} \cdot m^4}{4!} + 90 \cdot \frac{\cancel{e^{-m}} \cdot m^6}{6!}$$

$$\frac{m^2}{2} = \frac{9 m^4}{24} + \frac{90 m^6}{720}$$

$$\frac{m^2}{2} = \frac{3m^4}{8} + \frac{m^6}{8}$$

$$\frac{m^2}{2} = \frac{3m^4 + m^6}{8}$$

$$8m^2 = 6m^4 + 2m^6$$

$$2m^6 + 6m^4 - 8m^2 = 0$$

Div by 2

$$\boxed{m^6 + 3m^4 - 4m^2 = 0}$$

Div by m^2

$$m^4 + 3m^2 - 4 = 0$$

1	1	0	3	0	-4
	↓	1	1	4	4
	1	1	4	4	$\boxed{0}$
	m^3	m^2	m^1	m^0	

$\boxed{m=1}$ or

$$m^3 + m^2 + 4m + 4 = 0$$

-1	1	1	4	4
	↓	-1	0	-4
	1	0	4	$\boxed{0}$

$\boxed{m=-1}$

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$m = 1, -1, +2i, -2i$$

Mean = var = m = 1

$SD = \sqrt{var} = 1$

Continuous Distribution – Normal Distribution.

– The prob density function (pdf) for normal distribution is –

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where

Mean $= E(X) = \mu$

Var $= V(X) = \sigma^2$

$SD = \sqrt{var} = \sigma$

NOTE – Mean (μ) and SD (σ) is always given in question.

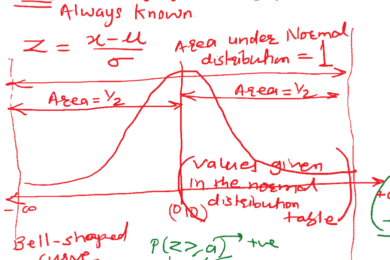
Imp. formula :-

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

Area of Normal Distribution –

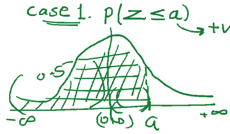
Note – Mean (μ) & SD (σ) Always Known

$$Z = \frac{x-\mu}{\sigma}$$



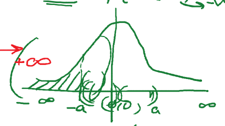
$P(Z \geq a)$ +ve

case 1. $P(Z \leq a)$ +ve



$= 0.5 + P(0 < z < a)$

case 2 – $P(Z \leq a)$ -ve



$= 0.5 - P(0 < z < a)$

case 3 - $P(Z \geq a)$ +ve

$= 0.5 - P(0 < z < a)$

case 4 – $P(Z \geq a)$ -ve

$= 0.5 + P(0 < z < a)$

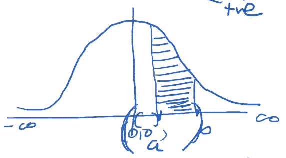
$= 0.5 - P(0<z<a)$

$= 0.5 + P(0<z<a)$

Case 5a) $P(a \le z \le b)$ +ve

$P(2 \le z \le 5)$

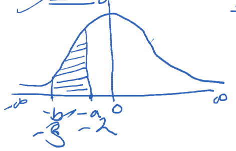
$5 - 2 = 3$



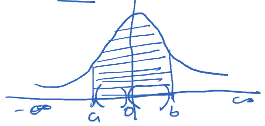
$\leftarrow P(0<z<5) - P(0<z<2)$

$P(0<z<b) - P(0<z<a)$

Case 5b) − $P(0<z<b) - P(0<z<a)$



Case 5c) $P(0<z<a) + P(0<z<b)$



M19 Q4c) 8m

10,000 items are normally distributed

Mean = $\mu = 20$

SD = $\sigma = 4$

$\boxed{Z = \frac{x-\mu}{\sigma}}$

① Find prob that items selected at random will have size betn 18 to 23 cm

$P(18 \le x \le 23)$

$P\left(\frac{x-\mu}{\sigma} \le Z < \frac{x-\mu}{\sigma}\right)$

$P\left(\frac{18-20}{4} < Z < \frac{23-20}{4}\right)$

$P(-0.5 < Z < 0.75)$

$= P(0<z<0.5) + P(0<z<0.75)$

$= 0.1915 + 0.2734 = 0.4649$

Agar question aaye hota ki find how many items are there having size betn $\boxed{18 \text{ to } 23 \text{ cm}}$

$= 10000 \times 0.4649 \Rightarrow 4649$

② Find prob of items will have size above 26 cm

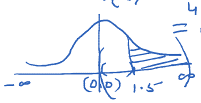
$P(x > 26) = P\left(z > \frac{x-\mu}{\sigma}\right)$

$P\left(z > \frac{26-20}{4}\right) = P(z > 1.5)$

$= 0.5 - P(0<z<1.5)$

$= 0.5 - 0.4332$

$= 0.0668$



D18 Q5c) 8m

$\mu_m = 51$ $\quad \sigma_m = 15$

$\mu_p = 53$ $\quad \sigma_p = 12$

$\mu_c = 46$ $\quad \sigma_c = 16$

By additive prop of mean and SD

$\mu = \mu_m + \mu_p + \mu_c$

$\mu = 51 + 53 + 46$

$\boxed{\mu = 150}$ Actual mean

$\sigma = \sigma_m + \sigma_p + \sigma_c$

$\sigma = 15 + 12 + 16$

$\boxed{\sigma = 43}$ Actual SD.

① Prob of marks 180 or above

$P(x \ge 180) = P\left(z > \frac{x-\mu}{\sigma}\right)$

② Prob of marks 90 or below

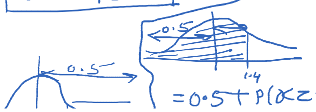
$P(x \le 90)$

$P\left(z < \frac{x-\mu}{\sigma}\right) = P\left(z < \frac{90-150}{43}\right)$

$= P\left(z < \frac{-60}{43}\right)$

$P(z < 1.3953)$

$P(z < 1.40)$



$= 0.5 + P(0<z<1.4)$

$\boxed{\mu = 150}$ mean

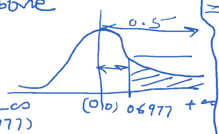
① Prob of marks 180 or above

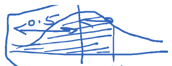
$P(x \geq 180) = P\left(Z > \frac{x-\mu}{\sigma}\right)$

$\therefore P\left(Z > \frac{180-150}{43}\right) = P\left(Z > \frac{30}{43}\right)$

$P(Z > 0.6977) = 0.5 - P(0 < Z < 0.6977)$

$= 0.5 - 0.2549 = \underline{\underline{0.2451}}$





$= 0.5 + P(0 < Z < 1.4)$

$= 0.5 + 0.4192$

$= 0.\underline{\underline{9192}}$

M18 Q5(c) 8m — Total 1000 students in an exam are normally distributed

$n = 1000$

$\mu = 70, \quad \sigma = 5$

① Estimate no of students whose marks betn 60 and 75

$P(60 < x < 75)$

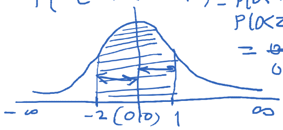
$P\left(\frac{x-\mu}{\sigma} < Z < \frac{x-\mu}{\sigma}\right)$

$P\left(\frac{60-70}{5} < Z < \frac{75-70}{5}\right)$

$P(-2 < Z < 1) = P(0 < Z < 2) + P(0 < Z < 1)$

$= \cancel{0.3413} + 0.4772 + 0.3413$

$= \underline{\underline{0.8185}}$



$\therefore$ No of students whose marks betn 60 and 75 $= 1000 \times 0.8185$

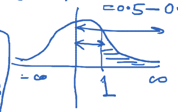
$= 818.5 \approx 819$ students

② Estimate no. of stud whose marks above 75

$P(x > 75) = P\left(Z > \frac{x-\mu}{\sigma}\right) = P\left(Z > \frac{75-70}{5}\right)$

$P(Z > 1) = 0.5 - P(0 < Z < 1)$

$= 0.5 - 0.3413 = \underline{\underline{0.1587}}$



$\therefore$ No of stud having marks above 75 is $= 1000 \times 0.1587$

$= 158.7$ ~~500~~

$= 159$

* The mean and SD of ND is 30 and 7 respectively

find ① $P(x < 20)$

(ii) $P(33 < x < 45)$

(iii) $P(15 < x < 25)$

(iv) $P(x > 25)$

Complex Integration – 20-24m

① Line Integral — 1Q/6m — Q1. Compulsory

② Cauchy's Integral formula – 1Q/6m

③ Cauchy's Residue thm (Appln) – 1-2Q/14m

④ Taylor's & Laurent series – 1Q/6m

$N = \{1, 2, 3 \dots \infty\}$

$W = \{0, 1, 2, 3 \dots \infty\}$

$I = \{-\infty \dots -2, -1, 0, 1, 2, \dots +\infty\}$

Real no. $\underline{\underline{R}} = \frac{p}{q}$ $\because q \neq 0$

$-16 \quad 1 \quad 9 \quad \frac{2}{3} \quad \frac{6}{7} \quad \frac{9}{5}$

Introduction of complex No.

Why complex No???

$\sqrt{4} = \pm 2$, $\quad \sqrt{16} = \pm 4$

$-2 \times -2 = 4$

$2 \times 2 = 4$

$\sqrt{25} = \pm 5$

$\sqrt{-9} = \sqrt{-1} \times \sqrt{9}$

$= \pm 3i$

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$Z = \underline{x} + i\underline{y}$

real part — imag part

ex. $1+2i$, $2-3i$, $-5i$, 9

Representation of complex No –

① Cartesian form – $Z = x + iy$

② Polar form = $Z = r\cos\theta + ir\sin\theta$

③ Exponential form $z = re^{i\theta}$

$Z = r(\cos\theta + i\sin\theta)$

$\cancel{r}e^{i\theta} = \cancel{r}(\cos\theta + i\sin\theta)$

$e^{\pm i\theta} = \cos\theta \pm i\sin\theta$

put $\theta = n\pi$

$e^{\pm in\pi} = \cos n\pi \pm i\sin n\pi$

$\cos n\pi = (-1)^n$

$\sin n\pi = 0$

$\boxed{e^{\pm in\pi} = (-1)^n}$

put $\theta = 2n\pi$