Instructions :

* Please use Piazza to ask questions and for announcements regarding lectures and recitations.

* You will be using Gradescope for submitting homeworks.
  - HW 1 is released

* For Recitations,
  - Please use the raise hand feature if you have any question or wish to answer.
  - Do not hesitate to ask questions
  - Your participation is encouraged!

Lecture Review

★ LOGIC

What is a proposition?

Let p and q be arbitary propositions

Fill the following table

Connectives	Symbols
Negation	$\sim p$
Conjunction	$p \wedge q$
Disjunction	$p \vee q$
Exclusive Or	$p \oplus q$
Implication	$p \rightarrow q$
Biconditional	$p \leftrightarrow q$

What is a Tautology? Stat always True

What is a Contradiction? False

Quick Exercise

① $p : T$ $\sim p :$ F

② $p : T$ $q : F$ $p \rightarrow q :$ F

③ $p : F$ $q : T$ $p \oplus q :$ T

④ $p : F$ $q : T$ $p \leftrightarrow q :$ F

⑤ $p : F$ $q : F$ $p \vee q :$ F

⑥ $p : T$ $q : F$ $p \wedge q :$ F

⑦ $p : T$ $q : F$ $\sim p \vee q :$ F

Are these statements propositions?

① $x^3 + 1$ is composite X $\exists\, x \in \mathbb{N}$

$x = 2$ True

$x = 3 \cdot 3$

② $x^2 > x$ X $\exists\, x \in \mathbb{R}$

$x = 1$

Quantified Statements

'$\forall$' stands for for all

'$\exists$' stands for there exists

Problem 3: Prove that the product of a non-zero rational and irrational number is irrational.

Proof:

Let a be an irrational number and b be a rational number then ab is irrational.

If a is irrational and b is rational then ab is rational.

$$b = \frac{p}{q} \quad , \quad p, q \in I \quad q \neq 0$$

$$\rightarrow ab = \frac{m}{n} \quad , \quad m, n \in I \quad n \neq 0$$

$$a \, b = \frac{m}{n}$$

$$\Rightarrow a \frac{p}{q} = \frac{m}{n}$$

$$\boxed{a = \frac{mq}{np} \quad mq, np \in I \text{ and } np \neq 0}$$

$a \rightarrow$ rational

contradiction !!

Recitation

Proofs

- $\rightarrow$ n is even iff $\exists$ an integer k s.t. $n = 2k$
- $\rightarrow$ n is odd iff $\exists$ an integer k s.t. $n = 2k+1$
- $\rightarrow$ prime, composite
- $\rightarrow$ rational, irrational
- $\rightarrow$ floor and ceil of a real number.
- $\rightarrow$ $p \rightarrow q \equiv \bar{p} \vee q \equiv \bar{q} \rightarrow \bar{p}$ (contrapositive)
- $\rightarrow$ Proof by contradiction.

- Prove that $\sqrt{2}$ irrational
  - by contradiction (2 ways)

- Other proofs.

Lemma : For two integers x & y, if xy is odd, then x & y both are odd.

Proof : $p \to q \equiv \sim q \to \sim p$

Proof by contrapositive.

If x or y is even, then xy is even.

WLOG: Assume x is even : i.e. $x = 2k$ for some int k

$$\begin{aligned} xy &= (2k)y \\ &= 2(ky) \\ &= 2m \quad \text{for some int } m \end{aligned}$$

xy is even.

Problems

Q) Let m & n be two integers.
Prove that $mn+m$ is odd **iff** m is odd and n is even.

A) $\Rightarrow$ If $mn+m$ is odd, then m is odd & n is even.

$$mn+m = \underbrace{m(n+1)}_{\text{odd}}$$

m has to be odd by Lemma

$n+1$ has to be odd by Lemma.

$\downarrow$

n is even.

$\Leftarrow$ If m is odd and n is even then $mn+m$ is odd.

$\therefore$ $m = 2k+1$ for some int k.
$\therefore$ $n = 2l$ for some int l.

$$mn + m = (2k+1)2l + 2k+1$$
$$= 4kl + 2l + 2k + 1$$
$$= 2(2kl + l + k) + 1$$
$$= 2m + 1$$

where $m = 2kl + l + k$ which is an integer.

Hence $mn + m$ is odd.

u

Problem 2: Suppose $x, y \in \mathbb{R}$. Prove that if $y^3 + yx^2 \leq x^3 + xy^2$, then $y \leq x$

Proof: Solve by contrapositive.

If $y > x$ then $y^3 + yx^2 > x^3 + xy^2$

$$y > x$$

$$y - x > 0$$

$$\Rightarrow (y - x)(x^2 + y^2) > 0(x^2 + y^2)$$

$$yx^2 + y^3 - x^3 - xy^2 > 0$$

$$yx^2 + y^3 > x^3 + xy^2$$

Hence proved!

$$2 = \frac{a^2}{b^2}$$

$S(a^2) = 2 \cdot S(a)$ By dfn

$\rightarrow$ even

$$a^2 = 2 \times b^2$$

$S(a^2) = 1 + 2 \cdot S(b)$

$\rightarrow$ odd

Contradiction