

DIGITAL LOGIC AND COMPUTER ARCHITECTURE

TOPIC NO 4

BOOLEAN ALGEBRA

BOOLEAN ALGEBRA

- ✓ It is the set of rules used to simplify given logic expression without changing its functionality
- ✓ It is used when number of variables are less (like 1, 2, 3)



George Boole
1815 -1864

CYCLE

Boolean
Expression

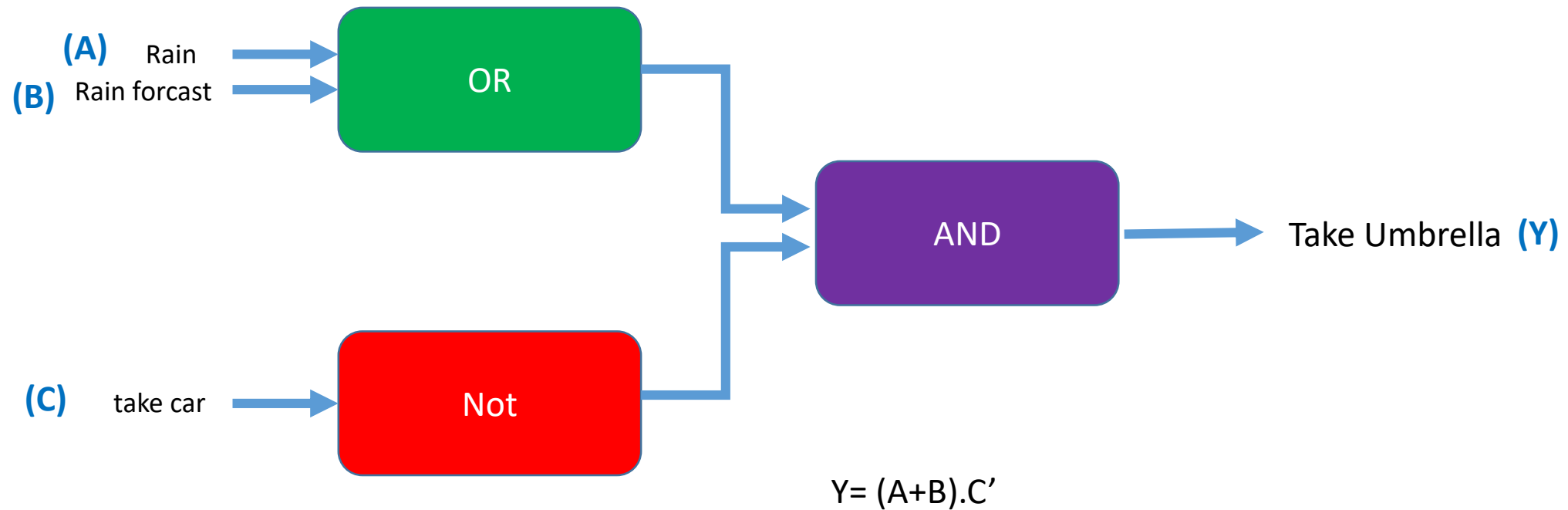


Logic Circuits



Truth Tables

Real life example converted into Logic circuit



CYCLE

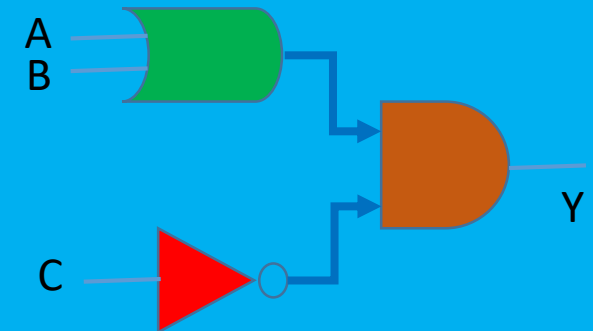
Boolean Expression

$$Y = (A + B) \cdot C'$$

Truth Table

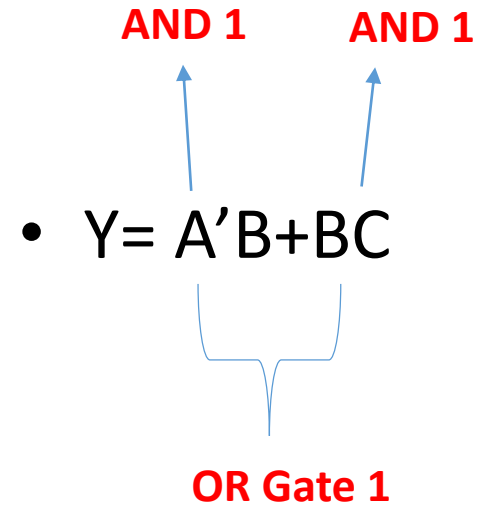
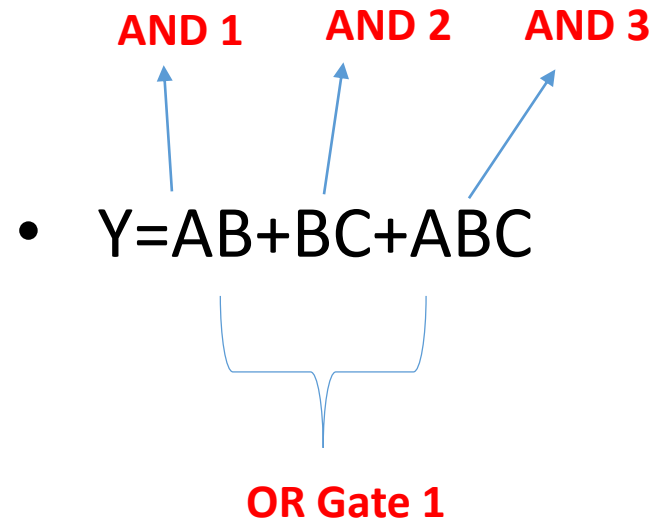
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Logic Circuit



Reduction for of Expressions

- Example:



Introduction to Reduction techniques

1. Using Boolean Laws
2. K- Maps
3. Quine Mc-Cluskey

Know George Boole



Suvarna Bhat

BOOLEAN ALGEBRA:

- George Boole introduce a new Algebra called Boolean Algebra in 1854
- Ordinary Algebra deals with a real numbers where as Boolean Algebra deals with a set two elements 0 and 1
- Operators : '+' and '•'
- Rules are same as AND ,OR, and NOT

What is Boolean Algebra ?

- Boolean Algebra may be defined with
 - A set of elements
 - A set of operators
 - A number of postulates
- A set of elements is set of any collection of objects having a common property
 - $S = \{a, b, c, d\}$ $a \in S$, $e \notin S$
- A binary operators defined on a set S of elements is a rule that assigns each pair of elements from S to a unique element from S
 - $a * b = c$
 - +
 - .
 - -
- A postulates are used to reduced the rules theorems and properties

Boolean Laws:

AND LAW:

$$A.A = A$$

$$A.0 = 0$$

$$A.1 = 1$$

$$A.\overline{A} = 0$$

OR LAW:

$$A+A = A$$

$$A+0 = A$$

$$A+1 = 1$$

$$A+\overline{A} = 1$$

NOT LAW:

$$\overline{\overline{1}} = 0$$

$$\overline{\overline{0}} = 1$$

$\overline{\overline{A}}$

$$A = A$$

COMMULATIVE LAW

$$A.B = B.A$$

$$A+B = B+A$$

ASSOCIATIVE LAW

$$A+(B+C) = (A+B)+C$$

DISTRUBUTIVE LAW

$$A(B+C) = AB+AC$$

Boolean Laws:

Other LAWs:

$$A+BC=(A+B)(A+C)$$

$$A+A'B=A+B$$

$$A(A'+B)=AB$$

$$AB+AB' = A$$

$$(A+B)(A+B')=A$$

$$AB+A'C=(A+C(A'+B))$$

$$A+AB=A$$

$$(A+B)(A'+C)=AC+A'B$$

Class work

- Verify any Boolean law using truth table method

DE Morgan's theorem

- $\overline{A \cdot B} = \overline{A} + \overline{B}$

- $\overline{A + B} = \overline{A} \cdot \overline{B}$



1	2	3	4	5	6	7	8
A	B	\overline{A}	\overline{B}	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$	$\overline{A + B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1	1	1	1
0	1	1	0	1	1	0	0
1	0	0	1	1	1	0	0
1	1	0	0	0	0	0	0

Canonical Form of Expression

- Sum of Product (SOP)
- Product of sum (POS)