Recitation

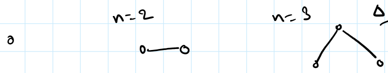
04 February 2023 20:19

Q: Let T be a tree

- max degree $= \Delta$
- prove: T has at least Δ leaves

S: Induction on n.

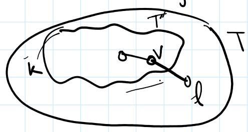
Base Case. $n=1$ $n=2$ $n=3$ $\underline{\underline{\Delta=2}}$



$\to$ tree, k vertices, max deg $=\Delta$

$\geq \Delta$ leaves

I.S. $k+1$ vertices, max deg $=\Delta$ $T=(V,E)$



$T' = (V', E')$

$V' = V / \{\ell\}$

$E' = E / \{v, \ell\}$

$|V'| = k$

Case 1. max deg $T' = \Delta$ $\geq\Delta$

By I.H., T' has $\geq\Delta$ leaves

$\geq\Delta$ $\geq\Delta$

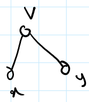
Case 2 max deg $T' = \Delta - 1$

by I.H, T' has $\geq \Delta - 1$ leaves

if v not a leaf in T', the #leaves in $T \geq \Delta - 1 + 1 = \underline{\underline{\Delta}}$

if v is a leaf in T'

$T =$



Q: $G=(V,E)$

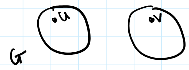
- compliment of G:

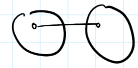
$\bar{G} = (V, \bar{E})$

$\bar{E} = \{\{x,y\} \mid x \neq y, \{x,y\} \notin E\}$

prove: either G or $\bar{G}$ (or both) are connected.

$\hookrightarrow$ Let's assume that G is not connected.





i.e. it has ≥ 2 connected components