5 :- Logic

1. Ans:

I	II	III	IV	V
a	b	$\sim a$	$\sim a \vee b$	$a \to b$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Here, column IV and column V are ~~equal~~ same.
Hence, $\sim a \vee b$ and $a \to b$ are logically equivalent.

2. Ans

$$\sim(p \vee (\sim p \wedge q)) = \sim p \wedge q .$$

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$p \vee (\sim p \wedge q)$	$\sim(p \vee (\sim p \wedge q))$	$\sim p \wedge q$
T	T	F	F	F	T	F	F
T	F	F	T	F	T	F	F
F	T	T	F	T	T	F	T
F	F	T	T	F	F	T	F

$$\equiv \sim((p \vee \sim p) \wedge (p \vee q))$$
$$\equiv \sim(T \wedge (p \vee q))$$
$$\equiv \sim(p \vee q)$$
$$\equiv \sim p \wedge \sim q$$

3. Ans.

$$\equiv ((a \to b) \wedge a) \to b$$
$$\equiv ((\sim a \vee b) \wedge a) \to b$$
$$\equiv ((a \wedge \sim a) \vee (a \wedge b)) \to b$$
$$\equiv (F \vee (a \wedge b)) \to b$$
$$\equiv (a \wedge b) \to b$$
$$\equiv (\sim a \vee \sim b) \vee b$$
$$\equiv (\sim a \vee b) \vee (b \vee \sim b)$$
$$(\sim a \vee b) \vee T = T$$
$$\equiv \cancel{(\sim a \vee b)}$$

$$((a \to b) \wedge a) \to b$$
$$\sim((\sim a \vee b) \wedge a) \vee b$$
$$((a \wedge \sim b) \vee \sim a) \vee b$$
$$((a \vee \sim a) \wedge (\sim b \vee \sim a)) \vee b$$
$$(\sim b \vee \sim a) \vee b$$
$$(\sim b \vee b) \vee (\sim a \vee b)$$
$$T \vee (\sim a \vee b)$$
$$T$$

3.

Ans:

a	b	$a \to b$	$(a \to b) \wedge a$	$((a \to b) \wedge a) \to b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

4.

Ans:

$((a \to b) \wedge (b \to c)) \to (a \to c)$

$((\neg a \vee b) \wedge (\neg b \vee c)) \to (\neg a \vee c)$

$\neg((\neg a \vee b) \wedge (\neg b \vee c)) \vee (\neg a \vee c)$

$\neg((\neg a \wedge \neg b) \vee (\neg a \wedge c) \vee (b \wedge \neg b) \vee (b \wedge c)) \vee (\neg a \vee c)$

$\cancel{\neg(\neg a \wedge (\neg b \vee c) \vee b \wedge (\neg b \vee c))}$

$\neg((\neg a \wedge \neg b) \vee (\neg a \wedge c) \vee (b \wedge c)) \vee (\neg a \vee c)$

4.

Ans:

a	b	c	$a \to b$	$b \to c$	$a \to c$	$(a \to b) \wedge (b \to c)$	$((a \to b) \wedge (b \to c)) \to (a \to c)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Here, proved that $((a \to b) \wedge (b \to c)) \to (a \to c)$ is tautology.

converse $q \to p$.
inverse $\sim p \to \sim q$.
contrapositive $\sim q \to \sim p$.

5. Ans: converse :- $q \to p$

if Narendra Modi is prime minister then India is the country.

~~inverse :- $\sim p \to \sim q$~~

~~if~~

6. Ans:

p	q	$q \wedge p$	$q \wedge \sim p$	$\sim p$	$\sim q$	$(q \wedge p) \vee (q \wedge \sim p)$	$q \to (q \to p)$	$p \to (q \wedge p)$
T	T	T	F	F	F	T	T	T
T	F	F	F	F	T	F	T	F
F	T	F	T	T	F	T	F	T
F	F	F	F	T	T	F	T	T

contigency.

7. (i) $p \wedge (q \vee p)$

$= (p \wedge q) \vee (p \wedge p)$ This is DNF form.

(ii) ~~$(p \wedge q) \wedge (p$~~ $(p \vee q) \wedge (p \vee r)$

$= (p \wedge p) \vee (p \wedge r) \vee (q \wedge p) \vee (q \wedge r)$

$\therefore$ This is DNF form.

8. $(p \wedge q) \vee (\sim p \wedge q \wedge r)$

$= (p \vee \sim p) \wedge (p \vee q) \wedge (p \vee r) \wedge (q \vee \sim p) \wedge (q \vee q) \wedge (q \vee r)$

This is CNF form.

Q. $1^2+2^2+3^2+\dots+n^2 = \frac{n}{6}(n+1)(2n+1)$

Ans.

(i) $n=1$

$LHS = 1$

$RHS = \frac{(2)(3)}{6} = 1$

(ii) $n=K$

$\therefore 1^2+2^2+3^2+\dots+K^2 = \frac{K}{6}(K+1)(2K+1)$

(iii) $n=K+1$

$\therefore 1^2+2^2+3^2+\dots+K^2+(K+1)^2 = \frac{(K+1)(K+2)(2K+3)}{6}$

$\therefore LHS = \frac{K}{6}(K+1)(2K+1)+(K+1)^2$

$= (K+1)\left(\frac{K(2K+1)}{6}+(K+1)\right)$

$= (K+1)\left(\frac{2K^2+K+6K+6}{6}\right)$

$= (K+1)\left(\frac{2K^2+7K+6}{6}\right)$

$LHS = \frac{(K+1)(K+2)(2K+3)}{6} = RHS$

10.
Ans.

$$1^2+3^2+5^2+\dots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

(i) $n=1$

LHS $=1$

$$\text{RHS} = \frac{(1)\cdot(3)}{3} = 1$$

(ii) $n=K$

$$1^2+3^2+5^2+\dots+(2K-1)^2 = \frac{K(2K-1)(2K+1)}{3}$$

(iii) $n=K+1$

$$1^2+3^2+5^2+\dots+(2(K+1)-1)^2 = \frac{(K+1)(2(K+1)-1)(2(K+1)+1)}{3}$$

$$\therefore 1^2+3^2+5^2+\dots+(2K-1)^2+(2K+1)^2 = \frac{(K+1)(2K+1)(2K+3)}{3}$$

$2K+2-1$

$(2K+1)^2$

$$\text{LHS} = \frac{K(2K-1)(2K+1)}{3} + (2K+1)^2$$

$2K$

$$= (2K+1)\left[\frac{2K^2-K}{3}+(2K+1)\right]$$

$$= (2K+1)\left[\frac{2K^2-K+6K+3}{3}\right]$$

$$= (2K+1)\left[\frac{2K^2+5K+3}{3}\right]$$

$$= \frac{(2K+1)(K+1)(2K+3)}{3}$$

$=$ RHS.

11.
Ans.

11.

Ans. $5^n - 4n - 1$ is divisible by 16

$n = 1$

$5^1 - 4(1) - 1$

$5 - 5 = 0$ is divisible by 16.

$n = k$

$5^k - 4k - 1$ is divisible by 16.

$n = k+1$

$= 5^{k+1} - 4(k+1) - 1$

$= 5^k . 5 - 4k - 5$

$= 5^k . 5 - 4k . 5 - 5 . 1 + 16k.$

$= 5(5^k - 4k - 1) + 16k$

Here first part is divisible by 16 using inductive hypothesis and second part is already multiple by 16. The result is true for $n = k+1$

It is divisible by 16.

→ Hence true for all n.