Sampling Distributions

- A sampling distribution is a **distribution of all of the possible values of a sample** statistic for a given size sample selected from a population.

- For example, suppose you sample 50 students from your college regarding their mean GPA. If you obtained many **different samples of 50**, you will compute a diffe**rent mean for each sample**. We are interested in the **distribution of all potential mean GPAs** we might calculate for any given sample of 50 students.

Developing a Sampling Distribution

- **Assume there is a population …**
- Population size N=4
- Random variable, X, is age of individuals
- Values of X: 18, 20, 22, 24 (years)

Developing a Sampling Distribution

μ σ sample. ↓ Parameters $\bar{x}$ mean $\sigma_{\bar{x}}$

Summary Measures for the Population Distribution:

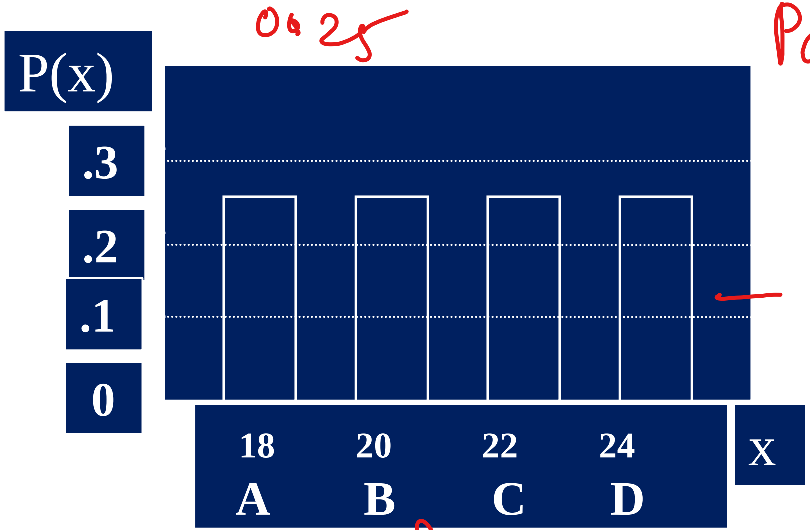
Population

$$\mu = \frac{\sum X_i}{N}$$

Mean

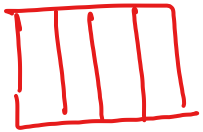
$$\mu = \mu_{\bar{x}} = \frac{18+20+22+24}{4} = 21$$

$$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{N}} = 2.236$$



uniform dist

Developing a Sampling Distribution



Now consider all possible samples of size n = 2

$4 \longrightarrow 2$

$n_1 \to x_1$

$n_2 \to x_2$

$n_3 \to x_3$

$n_4 \to x_4$

~~18~~ 20 22 24

A B C D

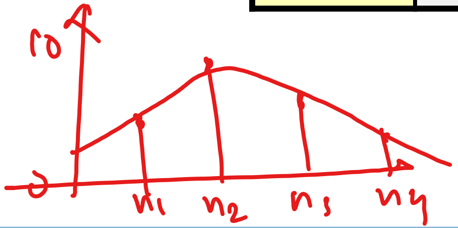
$4 \times 4 = 16$

1st Obs	2nd Observation			
	18	20	22	24
18	18,18	18,20	18,22	18,24
20	20,18	20,20	20,22	20,24
22	22,18	22,20	22,22	22,24
24	24,18	24,20	24,22	24,24

16 possible samples (sampling with replacement)

16 Sample Means

1st Obs	2nd Observation			
	18	20	22	24
18	18	19	20	21
20	19	20	21	22
22	20	21	22	23
24	21	22	23	24



Developing a Sampling Distribution

Sampling Distribution of All Sample Means

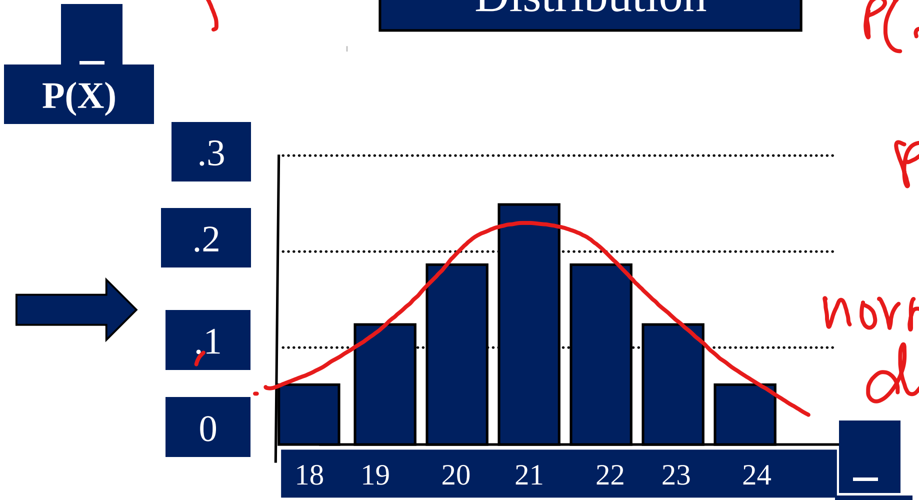
16 Sample Means

1st Obs	2nd Observation			
	18	20	22	24
18	18	19	20	21
20	19	20	21	22
22	20	21	22	23
24	21	22	23	24

p

P(x)

Sample Means Distribution



(no longer uniform)

$P(18) = \frac{1}{16}$

$P(19) = \frac{2}{16}$

$P(20) = \frac{3}{16}$

$P(21) = \frac{4}{16}$

normal distribution

Developing a Sampling Distribution

Summary Measures of this Sampling Distribution:

Population Samples

μ $\mu_{\bar{x}}$

σ $\sigma_{\bar{x}}$

$$\mu_{\bar{X}} = \frac{18+19+19+\cdots+24}{16} = 21$$

$$\sigma_{\bar{X}} = \sqrt{\frac{(18-21)^2+(19-21)^2+\cdots+(24-21)^2}{16}} = 1.58$$

$\mu = \mu_{\bar{x}}$

$\sigma \neq \sigma_{\bar{x}}$

Note: Here we divide by 16 because there are 16 different samples of size 2.

Comparing the Population Distribution to the Sample Means Distribution

Population N = 4

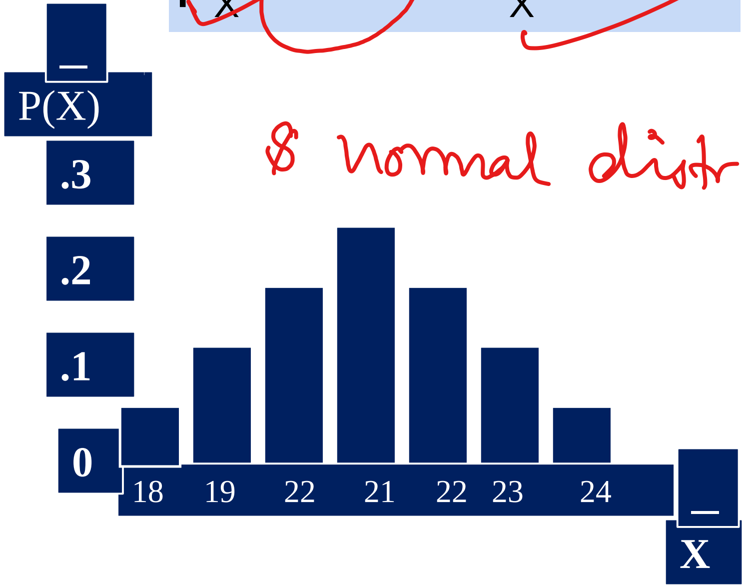
$\mu = 21 \quad \sigma = 2.236$

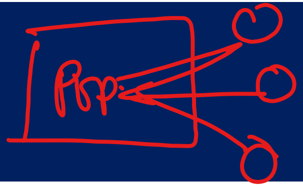
uniform

Sample Means Distribution n = 2

$\mu_{\bar{X}} = 21 \quad \sigma_{\bar{X}} = 1.58$

& normal distr.





Sample Mean Sampling Distribution: mean Standard Error of the Mean

- <u>Different samples</u> of the <u>same size</u> from the same population will yield different <u>*sample means*</u>
- A measure of the <u>variability in the mean from sample to sample</u> is given by the <u>***Standard Error of the Mean:***</u>

n = 2

(This assumes that sampling is with replacement or sampling is without replacement from an infinite population)

n = 10 - 15

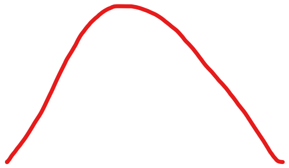
$$\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$$

$n \uparrow \quad \sigma_{\bar{x}} \downarrow$

- Note that the standard error of the mean <u>decreases</u> as the sample size <u>increases</u>

Sample Mean Sampling Distribution: If the Population is Normal

- If a population is normal with mean μ and standard deviation σ, the sampling distribution of $\boxed{\bar{X}}$ is also normally distributed with



$$\boxed{\mu_{\bar{X}} = \mu}$$

and

$$\boxed{\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}}$$

Populatn

Sample

Z-value for Sampling Distribution of the Mean

$Z = \frac{x - \mu}{\sigma}$

- Z-value for the sampling distribution of $\bar{X}$:

$$Z = \frac{(\bar{X} - \mu_{\bar{X}})}{\sigma_{\bar{X}}} = \frac{(\bar{X} - \mu)}{\frac{\sigma}{\sqrt{n}}}$$

where:

$\bar{X}$ = sample mean

μ = population mean

σ = population standard deviation

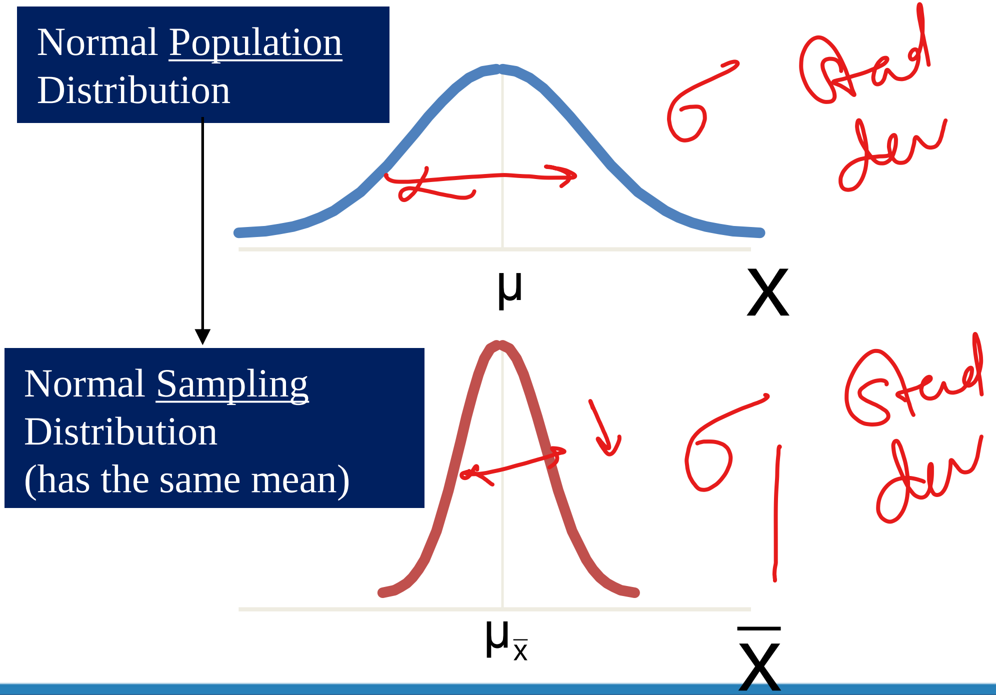
n = sample size

Sampling Distribution Properties

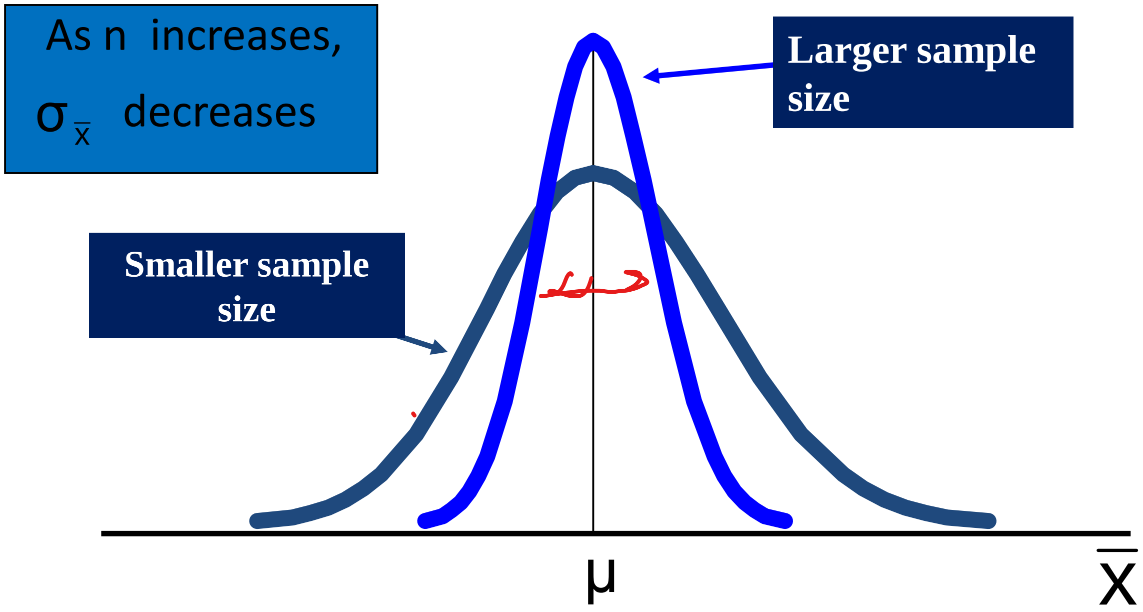
Same

- $\mu_{\bar{x}} = \mu$

(i.e. $\bar{x}$ is unbiased)



Sampling Distribution Properties



Sample Mean Sampling Distribution: If the Population is **not** Normal

- We can apply the Central Limit Theorem:
  - Even if the population is not normal,
  - …sample means from the population will be approximately normal as long as the sample size is large enough.

$n > 30$

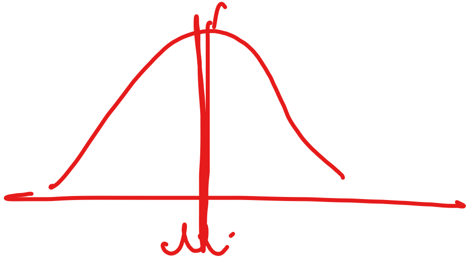
Properties of the sampling distribution:

$$\mu_{\bar{x}} = \mu$$

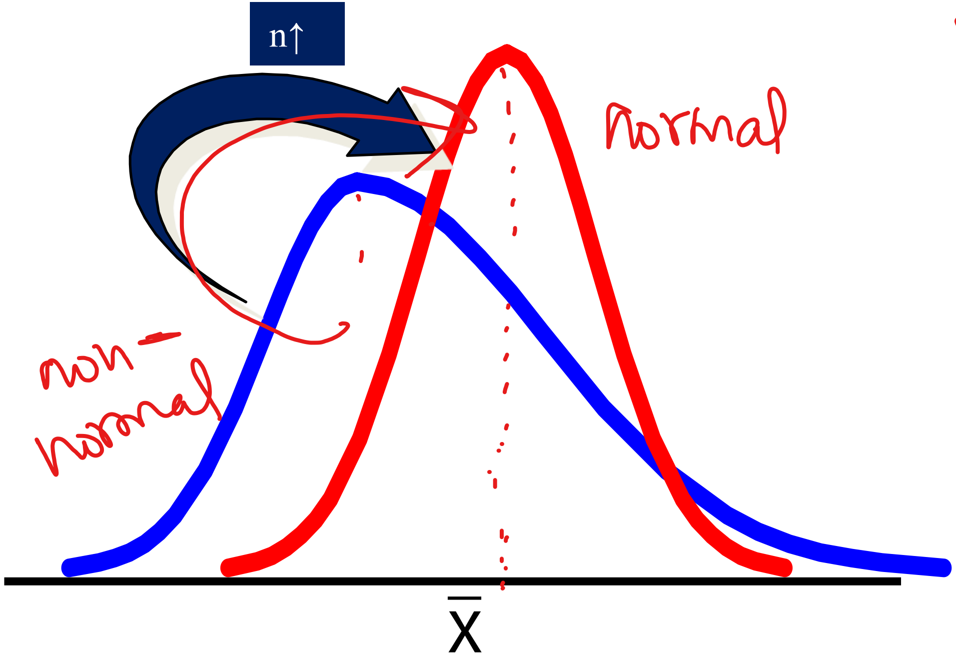
and

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Central Limit Theorem



As the sample size gets large enough…

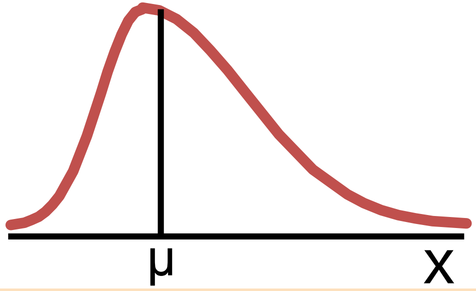


the sampling distribution becomes almost <u>normal regardless of shape of population</u>

Sample Mean Sampling Distribution: If the Population is **not** Normal

Population Distribution

Sampling distribution properties:



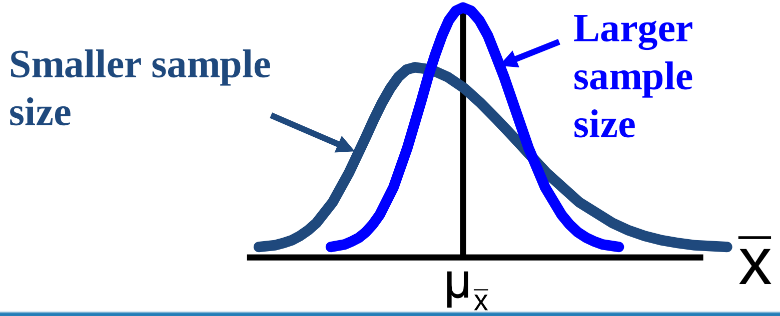
Central Tendency

$$\mu_{\bar{x}} = \mu$$

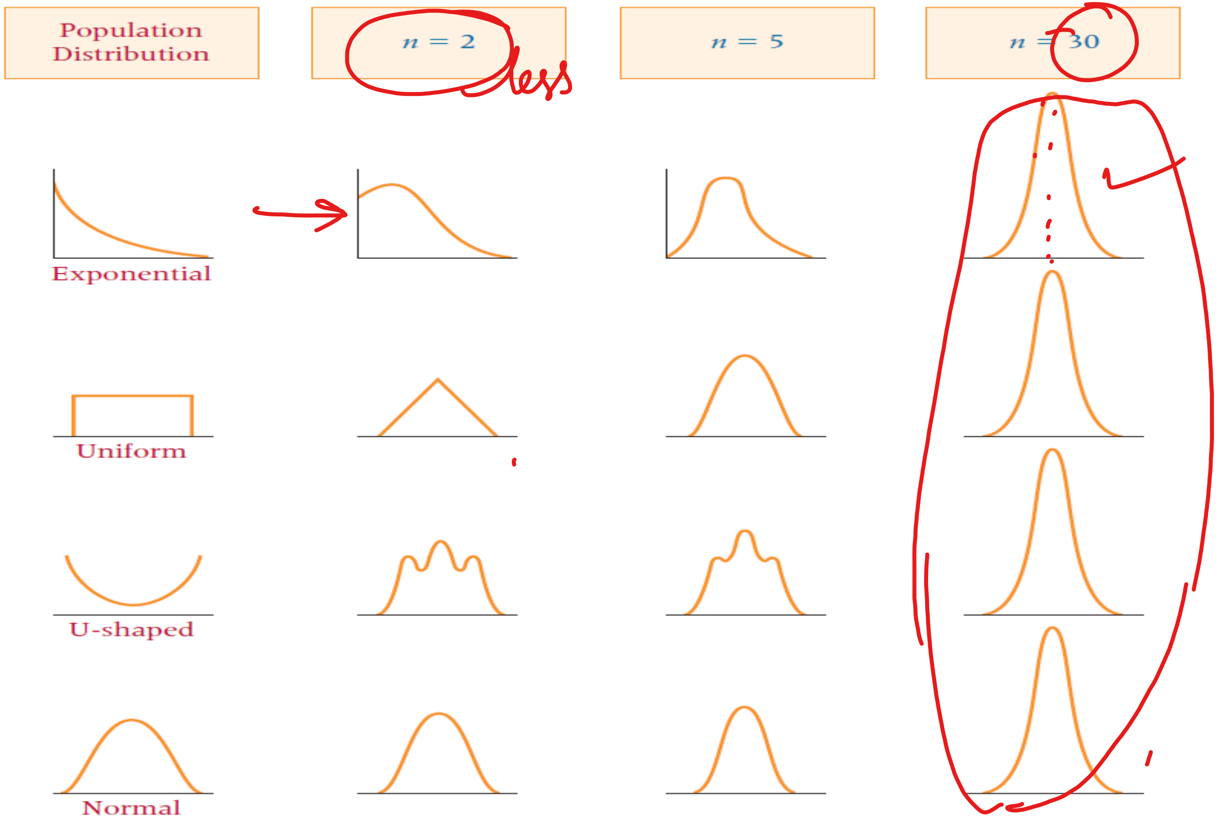
Variation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Sampling Distribution (becomes normal as "**n**" increases)



Shapes of the Distributions of Sample Means for Three Sample Sizes Drawn from Four Different Population Distributions



How Large is Large Enough?

- For most distributions, $n > 30$ will give a sampling distribution that is nearly normal
- For fairly symmetric distributions, **$n > 15$ will usually give a sampling distribution is almost normal**
- For non normal population distributions, the sampling distribution of the mean is always normally distributed

SLMS

Ex. Suppose, for example, that the mean expenditure per customer at a tire store is $85.00, with a standard deviation of $9.00. σ

μ

$n=40$

If a random sample of 40 customers is taken, what is the probability that the sample average expenditure per customer for this sample will be $87.00 or more? $\bar{X} = 87.0$

Ex. Suppose, for example, that the mean expenditure per customer at a tire store is $85.00, with a standard deviation of $9.00.

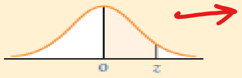
If a random sample of 40 customers is taken, what is the probability that the sample average expenditure per customer for this sample will be $87.00 or more? $P(\bar{x} > 87)$

Z-value

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\$87.00 - \$85.00}{\frac{\$9.00}{\sqrt{40}}} = \frac{\$2.00}{\$1.42} = 1.41$$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Z table



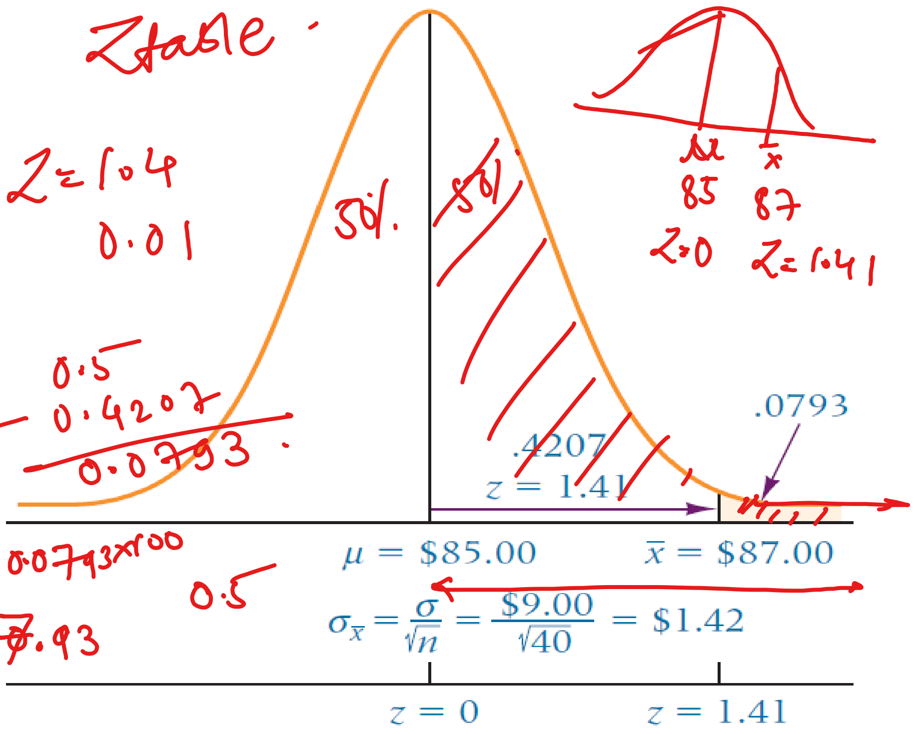
SECOND DECIMAL PLACE IN z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									
4.0	.49997									
4.5	.499997									
5.0	.4999997									
6.0	.499999999									

Z = 1.4
0.01

0.5
− 0.4207
= 0.0793

= 0.0793×100
= 7.93



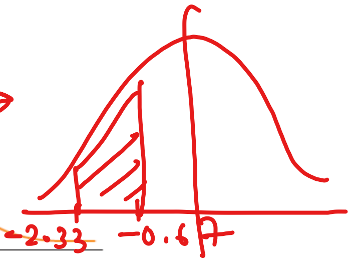
That is, 7.93% of the time, a random sample of 40 customers from this population will yield a sample mean expenditure of $87.00 or more.

8 Suppose that during any hour in a large department store, the average number of shoppers is 448, with a standard deviation of 21 shoppers. What is the probability that a random sample of 49 different shopping hours will yield a sample mean between 441 and 446 shoppers?

$\mu = 448$ $\sigma = 21$

$n = 49$ $P(441 < x < 446) = ?$

For this problem, $\mu = 448$, $\sigma = 21$, and $n = 49$. The problem is to determine $P(441 \le \bar{x} \le 446)$. The following diagram depicts the problem.



z=0

-0.67

-2.33 ← 0

Solve this problem by calculating the z scores and using Table A.5 to determine the probabilities.

$$z = \frac{441 - 448}{\frac{21}{\sqrt{49}}} = \frac{-7}{3} = -2.33$$

R → 2.3

C → 0.03

$$z = \frac{446 - 448}{\frac{21}{\sqrt{49}}} = \frac{-2}{3} = -0.67$$

R → 0.6

C → 0.07

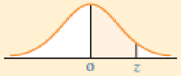
z Value	Probability
−2.33	.4901
−0.67	−.2486
	.2415

0.4901
− 0.2486

→ 0.2415 ×100

The probability of a value being between $z = -2.33$ and -0.67 is .2415; that is, there is a 24.15% chance of randomly selecting 49 hourly periods for which the sample mean is between 441 and 446 shoppers.

24.15%



SECOND DECIMAL PLACE IN z

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0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
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0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
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1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									
4.0	.49997									
4.5	.499997									
5.0	.4999997									
6.0	.499999999									