LECTURE 03: Markov Chains

SOLVED EXAMPLES OF THE MARKOV CHAINS USING TRANSITION PROBABILITY MATRIX (Part 2): (How to find the probabilities).

Type 2: How to calculate the probability after n-steps

For example; say you want to calculate

$P(X_3 = 3 \mid X_1 = 1) = P_{13}^{(2)}$

↑ it means first row, 3rd element of P^2

say $P^2 = \begin{bmatrix} P_{11} & P_{12} & (P_{13}) \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$ ← our answer.

for P^3 do $P^2 P$. , $P^4 = P^2 P^2$

eg1. lec 02: eg 4 cont.
Also, find the probability that he changes from going by train to driving exactly in 4 days.

→

$$P = \begin{array}{c} \\ T \\ C \end{array} \begin{array}{c} \begin{array}{cc} T & C \end{array} \\ \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} \end{array}$$

$P(X_4 = C \mid X_0 = T) = P_{TC}^{(4)}$

$$P^2 = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix} \quad \therefore P^4 = \begin{array}{c} \\ T \\ C \end{array} \begin{array}{c} \begin{array}{cc} T & C \end{array} \\ \begin{bmatrix} 3/8 & (5/8) \\ 5/16 & 11/16 \end{bmatrix} \end{array}$$

$$\boxed{P_{TC}^{(4)} = \frac{5}{8}}$$

eg2 lecture 1 : Q1 cont.

The present market share of the three brands A, B and C is 60%, 30% and 10% resp. Using this info, find (i) the p. that a cust. who is currently a brand A purchaser will purchase brand B after two time periods.

→ $P(X_2 = B \mid X_0 = A) = ? = P_{AB}^{(2)}$

$$P = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.5 & 0.4 & 0.1 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$$

$$\therefore P^2 = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.65 & \boxed{0.24} & 0.11 \\ 0.61 & 0.28 & 0.11 \\ 0.64 & 0.24 & 0.12 \end{bmatrix} \end{array}$$

$$\boxed{P(X_2 = B \mid X_0 = A) = 0.24}$$

(ii) The prob. that a cust. who is currently a purchaser of brand A will purchase brand C after two time periods ?

→ $P(X_2 = C \mid X_0 = A) = ? = P_{AC}^{(2)} = 0.11$

$$\therefore \boxed{P(X_2 = C \mid X_0 = A) = 0.11}$$

(iii) The prob that brand C will be able to retain its customers after two time periods?

→ $P(X_2 = C \mid X_0 = C) = ? \quad P_{CC}^{(2)} = ?$

$$\boxed{P(X_2 = C \mid X_0 = C) = 0.12}$$

(v) The proba that brand B purchaser will purchase brand A three period from now.

→ $P(X_3 = A \mid X_0 = B) = ?$ $\quad p^{(3)}_{BA}$

$p^3 = p^2 \cdot p =$

	A	B	C
A	0.64	0.25	0.11
B	**0.63**	0.26	0.11
C	0.64	0.25	0.11

$$\boxed{P(X_3 = A \mid X_0 = B) = 0.63}$$

(vi) The prob. that a cust. will purchase brands A, B, C three periods from now. — lecture 2

→ $P(X_3 = A) = ?$ $\quad q_3(A)$

$P(X_3 = B) = ?$ $\quad q_3(B)$

$P(X_3 = C) = ?$ $\quad q_3(C)$

$$q_3 = q_0 P^3$$

$$q_0 = \begin{bmatrix} 0.6 & 0.3 & 0.1 \end{bmatrix} \qquad P^3 = \begin{bmatrix} 0.64 & 0.25 & 0.11 \\ 0.63 & 0.26 & 0.11 \\ 0.64 & 0.25 & 0.11 \end{bmatrix}$$

$$q_0 P^3 = \boxed{\begin{bmatrix} 0.637 & 0.253 & 0.11 \end{bmatrix}}$$

Hence, market shares for brands A, B and C are 63.7%, 25.3% and 11% respectively.

eg3 lec 2 eg5 cont

(i) $P(X_3 = F | X_2 = N)$

(ii) $P(X_2 = N | X_1 = H)$

(iii) $P(X_4 = H | X_2 = F)$

→

$$P = \begin{array}{c|ccc} & F & H & N \\ F & 0.30 & 0.25 & 0.45 \\ H & 0.45 & 0.40 & 0.15 \\ N & 0.65 & 0.25 & 0.10 \end{array}$$

(i) $P_{NF}^{(1)} = 0.65$

(ii) $P_{HN}^{(1)} = 0.15$

(iii) $P_{FH}^{(2)} = 0.2875$

$$P^2 = \begin{array}{c|ccc} & F & H & N \\ F & 0.4950 & 0.2875 & 0.2175 \\ H & 0.4125 & 0.3100 & 0.2775 \\ N & 0.3725 & 0.2875 & 0.3400 \end{array}$$