- Welcome to Algorithm Design.
- Class times : M, F - 8:30am - 10:00am IST.
- Instructors : Sanjeev Dwivedi, Rajiv Gandhi
- Teaching assistants : Mona Gandhi, Srushti Nandu
- Piazza
- Recipe for Success

Proofs.

Algorithms.

<u>Mathematical logic.</u>

Proposition : Stmt that is either True or false.

$2+2=5$ ✓

p, q : simple propositions

Compound propositions

$\overline{p}$: is a proposition that is False if
p is True & it is True o.w.

conjunction

$p \wedge q$: True, if both p & q are True
false, o.w.

disjunction

$p \vee q$: True, if at least one of p or q is
true & false, o.w.

exclusive-or

$p \oplus q$: True, if exactly one of p or q is
True & false, o.w.

$p \Rightarrow q$: "If p then q".

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

If $\overbrace{\text{Bob comes at 6am}}^{p}$ $\underbrace{\text{he will get the job}}_{q}$.

<u>Biconditional</u> "if and only if".

$$p \Longleftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$$

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \Leftrightarrow q$
T	T	T	T	$\boxed{T}$
T	F	F	T	F
F	T	T	F	F
F	F	T	T	$\boxed{T}$

Logical Equivalence.

$$p \Rightarrow q \equiv \bar{q} \Rightarrow \bar{p}.$$

Contrapositive of $p \Rightarrow q$.

p	q	$\bar{p}$	$\bar{q}$	$p \Rightarrow q$	$\bar{q} \Rightarrow \bar{p}$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$p \Rightarrow q \equiv \bar{p} \vee q$.

Tautology: stmt that is always true.

Contradiction: stmt that is always false.

Eg: $\boxed{p} \equiv \bar{p} \Rightarrow C$.

p	$\bar{p}$	C	$\bar{p} \Rightarrow C$
T	f	f	T
f	T	f	f

Quantifiers.

$x + 5 \leq 25$

$$\boxed{\exists\, x \in \mathbb{Z} \text{ s.t. } x + 5 \leq 25.}$$

$\hookrightarrow$ existential quantifier.

$\forall\, x \in \mathbb{Z}^{+}, \quad x + 5 \leq 25 \rightarrow$ proposition that is false.

$\hookrightarrow$ universal quantifier.

Ex: $\exists\, x \in \mathbb{Z}$ s.t. $2 \mid x$ and $2 \mid x+1$

$\hookrightarrow$ evenly divides

$7 \mid 35, \quad 7 \nmid 40.$

Negation:

$\forall x \in \mathbb{Z}$, $2 \nmid x$ or $2 \nmid x+1$.

<u>Proofs.</u>

n is even iff $\exists$ an integer k s.t.
$n = 2k$.

n is odd iff $\exists$ an integer k s.t.
$n = 2k+1$.

An integer n is <u>prime</u> iff $n > 1$ and

for all positive integers r & s if $n = r \cdot s$, then $r=1$ or $s=1$. Otherwise, n is composite.

$\lceil x \rceil = n \iff n \leq x < n+1$, where n is an int.

$\lceil 6.1 \rceil = 7$

$\lceil 7 \rceil = 7$

$\lceil 6.9 \rceil = 7$

$\lfloor x \rfloor$: largest integer that is $\leq x$.

A real no. is <u>rational</u> iff it can be expressed as a ratio of two integers s.t. the denominator is non-zero.

$$r = \frac{p}{q}, \quad p, q \in \mathbb{Z}, \; q \neq 0.$$

<u>Ex</u>: Prove: if $\underbrace{\text{the sum of two integers is even}}_{p}$ then $\underbrace{\text{so is their difference.}}_{q}$

<u>Proof</u> :

$$\left.\begin{aligned} 18 + 6 &= 24 \\ 18 - 6 &= 12 \end{aligned}\right] \checkmark$$

bogus!

Let x & y be arbitrary, but particular integers, s.t. $x+y$ is even.

<u>Can I</u> : $x+y$ is even.

By def^n

$x+y = 2k$, for ~~all~~ Some integers k.

$$x + y - 2y = 2k - 2y$$

$$x - y = 2(k - y)$$

$x - y$ is even because

k & j are integers &

difference of two integers

is an integer.

<u>Case II</u> $x + y$ is odd.

vacuously true.

Ex: Prove that for all integers

n, if n is odd then

n^2+n+1 is odd.

Proof: Let n be arbitrary

but particular integer s.t.

n is odd.

By def^n,

$n = 2k+1$, for some int k.

$n^2+n+1 =$

$(2k+1)^2 + (2k+1) + 1$

$(2k+1) + (2k+1) + 1$

$$= (4k^2 + 4k + 1) + (2k+1) + 1$$

$$= 2(2k^2 + 3k + 1) + 1$$

$n^2 + n + 1 = 2\ell + 1$, where

$$\ell = \underline{2k^2 + 3k + 1}$$

is an int.

$\therefore n^2+n+1$ is odd.