07 September 2021 17:31

Key points - $T = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow$ Translation matrix

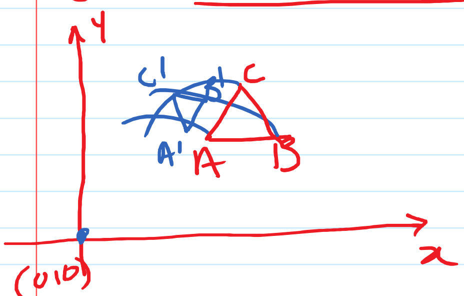
$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$ Rotation of an object by angle θ in anticlockwise wrt origin.

$R^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$ Inverse rotation

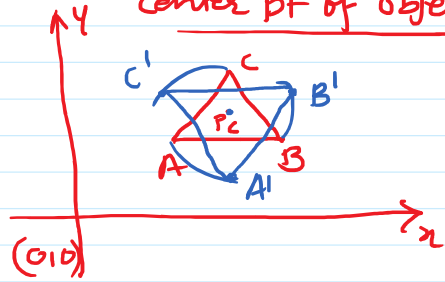
* Types of Rotation -

Arbitary pt

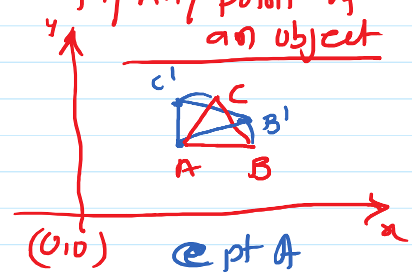
① Rotation @ origin



② Rotation @ pivot pt / center pt of object



③ Rotation @ arbitary pt / Any point of an object



Algorithm for arbitary point rotation -

① Translate an arbitary point to origin. The reqd translation transformation matrix is $T1 = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}$

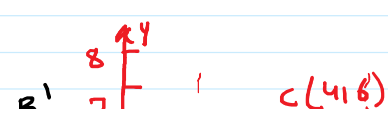
② Now rotate an object about origin. The reqd rotation transformation matrix is $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

③ Now retranslate an arbitary point to its original position. The reqd translation matrix is $T2 = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}$

④ Composite matrix $M_T = T2 \cdot R \cdot T1$

$\boxed{A'B'C' = M_T \cdot ABC}$

* Consider an object ABCD as shown

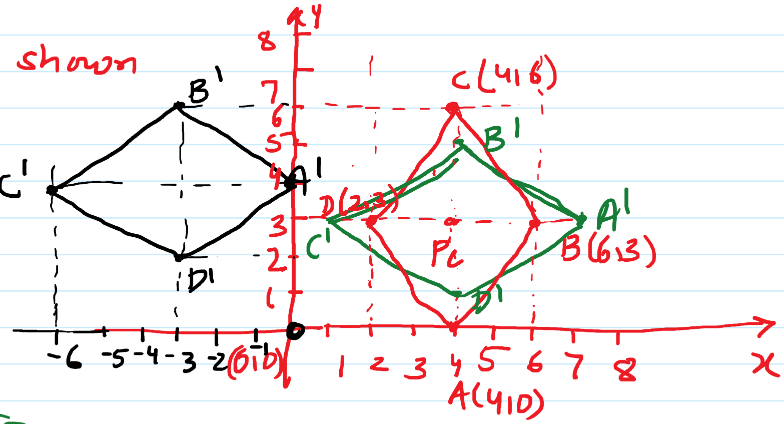


* Consider an object ABCD as shown in fig. Rotate ABCD by $\theta = 90^\circ$

(i) wrt origin

(ii) wrt pt A

(iii) wrt pivot pt $P_c(4,3)$



$x' = \overset{y}{x} + \overset{-y}{tx}$

$y' = y + ty$

Sol^n

(i) Rotate ABCD wrt origin.

$$A'B'C'D' = R \cdot ABCD$$

$$= \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 6 & 4 & 2 \\ 0 & 3 & 6 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 6 & 4 & 2 \\ 0 & 3 & 6 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 0 & -3 & -6 & -3 \\ 4 & 6 & 4 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ \begin{matrix} A' & B' & C' & D' \end{matrix} \end{array}$$

(iii) Rotate obj ABCD @ pivot pt $P_c(4,3)$ ⇓ (0,0)

step1 - Translate pivot pt to origin.

$$T_1 = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

step2 - Now rotate an obj ABCD by $\theta = 90^\circ$ in ↶

$$R = \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

step3 - Now retranslate pivot pt to its original pos.

$$T_2 = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

step 4. Composite matrix.

$$M_T = T_2 \cdot R \cdot T_1$$

$$= \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 4 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_T = \begin{bmatrix} 0 & -1 & 7 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \underset{P_c}{\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}} = \underset{P_c}{\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}}$$

Teacher kaise chk karega

$$A'B'C'D' = M_T \cdot ABCD$$

$$= \begin{bmatrix} 0 & -1 & 7 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 6 & 4 & 2 \\ 0 & 3 & 6 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{array}{c} \begin{bmatrix} 7 & 4 & 1 & 4 \\ 3 & 5 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ \begin{matrix} A' & B' & C' & D' \end{matrix} \end{array}$$

(3) 2D-Scaling :- Scaling transformation alters/changes (shrink/swell) the size of an objects.

Input parameters - End point coordinates of an object

S_x - Scaling factor along x-axis (width scaling)

S_x – Scaling factor along x-axis (width scaling)
S_y – Scaling factor along y-axis (Height scaling)

Point Scaling ⇒ $x' = S_x \cdot x$ — ①
$y' = S_y \cdot y$ — ②

$$P' = S \cdot P$$

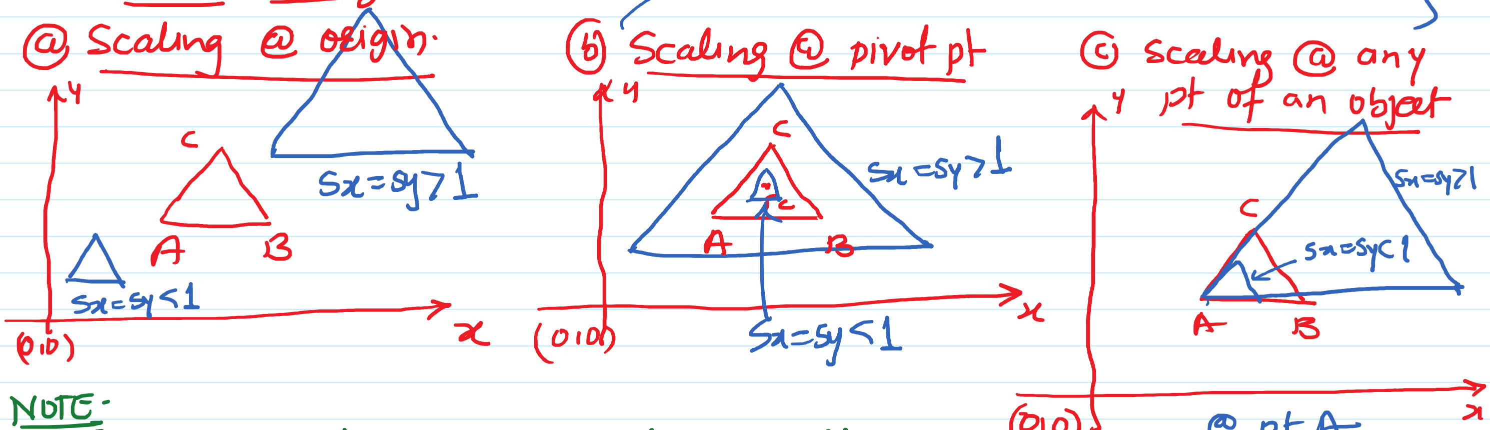
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

NOTE- Here $S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is known as basic 2D-scaling transformation matrix which scales the objects @ origin only.

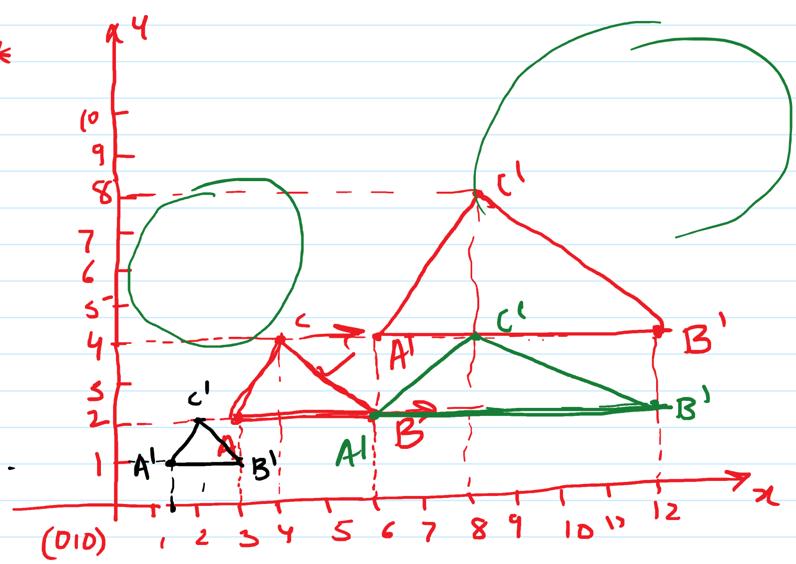
Scaling @ arbitrary point

Types of scaling -



NOTE-
① If $S_x = S_y > 1$ then object size will increase and object moves away from origin
② If $S_x = S_y < 1$ ⇒ object size will decrease and object moves towards origin
③ If $S_x = S_y = 1$ ⇒ No change in the size of an object as well as position will remain unchanged

*

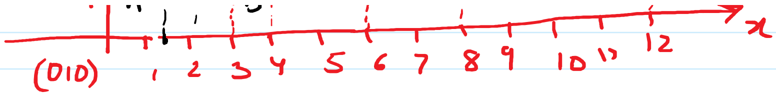


Consider an object ABC with A(3,2), B(6,2) & C(4,4). Scale obj ABC
① $S_x = S_y = 2$ ④ $S_x = 2, S_y = 1$
② $S_x = S_y = 1/2$ ⑤ $S_x = 1/2, S_y = 2$
③ $S_x = S_y = 1$ ⑥ $S_x = 2, S_y = 3$

① $S_x = S_y = 2$

$$A'B'C' = S \cdot ABC$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 6 & 4 \\ 2 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix} =$$



② $S_x = S_y = 1/2$

$A'B'C' = S \cdot ABC$

$$= \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 6 & 4 \\ 2 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 3/2 & 3 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \\ \begin{matrix} A' & B' & C' \end{matrix} \end{array}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 6 & 4 \\ 2 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix} =$$

$$A'B'C' = \begin{bmatrix} 6 & 12 & 8 \\ 4 & 4 & 8 \\ 1 & 1 & 1 \end{bmatrix}$$

$\underline{A \cdot I = I \cdot A = A}$

③ $S_x = S_y = 1$

$A'B'C' = S \cdot ABC$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 6 & 4 \\ 2 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 4 \\ 2 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

④ $S_x = 2, S_y = 1$

$A'B'C' = S \cdot ABC$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 6 & 4 \\ 2 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 12 & 8 \\ 2 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

Algorithm for arbitary point scaling :-

① Translate arbitary point to origin. The reqd translation transformation matrix is $T1 = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}$

② Now scale an object about origin. The reqd scaling matrix is

$$S = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

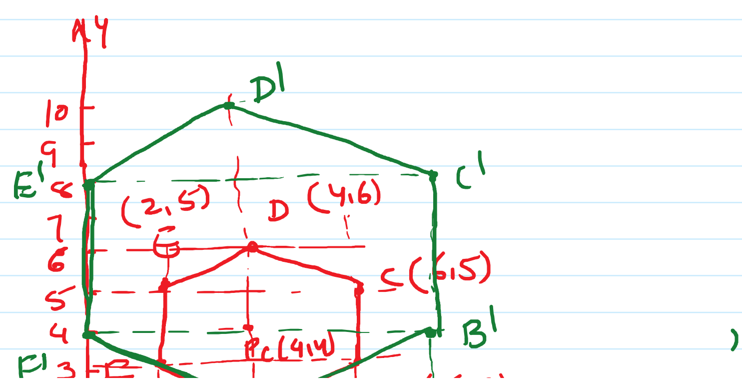
③ Now retranslate an arbitary point to its original position. Reqd translation matrix is – $T2 = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}$

④ Composite matrix – $M_T = T2 \cdot S \cdot T1$

$$A'B'C' = M_T \, \underline{ABC}$$

* Scale an object ABCDEF with $S_x = S_y = 2$
① @ pivot pt $P_c(4,4)$
② @ pt $A(4,2)$

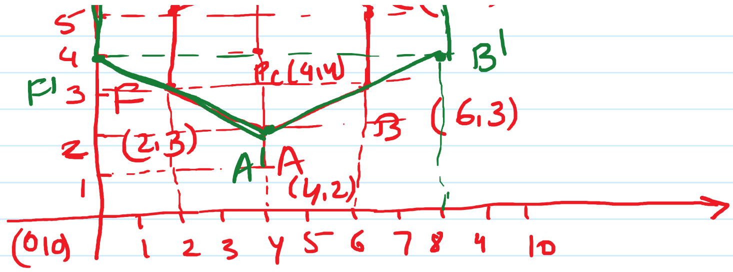
Soln → Scale obj ABCDEF @ pt A



Soln → Scale obj ABCDEF @ pt A

① Translate pt A (4,2) to origin.

$$T_1 = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$



② Now scale object wrt origin

$$S = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

③ Now retranslate pt A to its original pos.

$$T_2 = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

④ Composite matrix

$$M_T = T_2 \cdot S \cdot T_1$$

$$= \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_T = \begin{bmatrix} 2 & 0 & -4 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \underset{A}{\begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}} = \underset{A}{\begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}}$$

$$A'B'C'D'E'F' = M_T \cdot ABCDEF$$

$$= \begin{bmatrix} 2 & 0 & -4 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 6 & 6 & 4 & 2 & 2 \\ 2 & 3 & 5 & 6 & 5 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 & 8 & 4 & 0 & 0 \\ 2 & 4 & 8 & 10 & 8 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Theorem based questions.

① Prove that two successive rotations are additive ie

$$R(\theta_1) \cdot R(\theta_2) = R(\theta_1 + \theta_2)$$

Soln Let $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be the 2D-rotation matrix

$$LHS = R(\theta_1) \cdot R(\theta_2)$$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \sin\theta_2 & -\cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2 & 0 \\ \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2 & -\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \sin\theta_2 & -\cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2 & 0 \\ \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2 & -\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$LHS = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} = R(\theta_1+\theta_2)$$

$$R(\theta_1) \cdot R(\theta_2) = R(\theta_1+\theta_2)$$

* 2D-Rotation and Scaling commutes iff $S_x = S_y$ and $\theta = n\pi$

<u>Hint</u> $R \cdot S = S \cdot R$ iff $S_x = S_y$ & $\theta = n\pi$

<u>Sol^n</u> Let $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is basic 2D-Rotation matrix

and $S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is basic 2D-Scaling matrix

$$LHS = R \cdot S = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} S_x\cos\theta & -S_y\sin\theta & 0 \\ S_x\sin\theta & S_y\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here put $S_x = S_y$ and $\theta = n\pi$ $\begin{bmatrix} \sin n\pi = 0 \\ \cos n\pi = (-1)^n \end{bmatrix}$

$$LHS = R \cdot S = \begin{bmatrix} S_x \cdot (-1)^n & 0 & 0 \\ 0 & S_x(-1)^n & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{—— (1)}$$

$$RHS = S \cdot R = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} S_x\cos\theta & -S_x\sin\theta & 0 \\ S_y\sin\theta & S_y\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here put $S_x = S_y$ and $\theta = n\pi$

$$RHS = S \cdot R = \begin{bmatrix} S_x(-1)^n & 0 & 0 \\ 0 & S_x(-1)^n & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{—— (2)}$$

$\therefore$ From eqn (1) & (2) we get $LHS = RHS$

$S \cdot R = R \cdot S$

(4) <u>2D-Reflection Transformation</u> –

By reflection transformation we get the reflected image

By reflection transformation we get the reflected image of an object.

(a) Reflection along x-axis

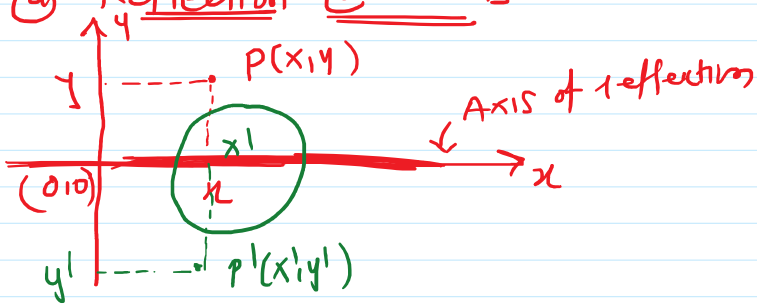
(b) Reflection along y-axis

I/P :- point $P(x, y)$

Axis of reflection (mirror)

O/P :- $P'(x', y')$

(a) Reflection @ x-axis



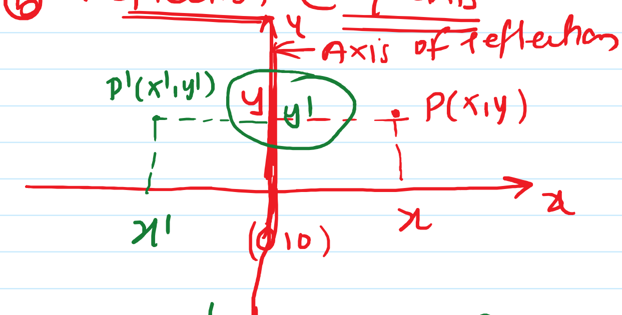
$$x' = x \quad \text{——} \; (1)$$

$$y' = -y \quad \text{——} \; (2)$$

$$P' = Ref_x \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(b) Reflection @ y-axis



$$x' = -x \quad \text{——} \; (1)$$

$$y' = y \quad \text{——} \; (2)$$

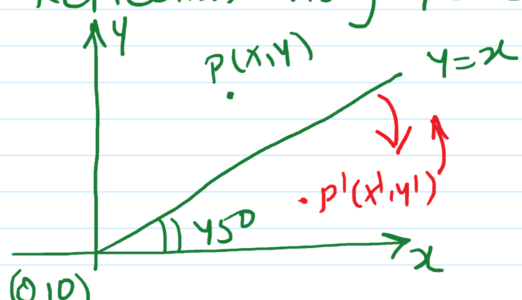
$$P' = Ref_y \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

* Give matrix representation for the following

1. Reflection along line $y = 0$ $\longrightarrow$ Reflection along x-axis
2. Reflection along line $x = 0$ $\longrightarrow$ Reflection along y-axis
3. Reflection along line $y = x$
4. Reflection @ along line $y = -x$

(3) Reflection along $y = x$



Step 1. Rotate line $y = x$ by $\theta = 45^\circ$ in clockwise direction

$$R_1 = \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 2 - Now reflect pt P along x-axis

$$Ref_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3 - Now rotate line $y = x$ by $\theta = 45^\circ$ in anticlockwise

matrix

(0 10)'

Step3 - Now rotate line y=x by −45 in anticlockwise

$$R_2 = \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step4. Composite ~~matri~~ matrix

$$M_T = R_2 \cdot Ref_x \cdot R_1$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Gaarwala method -

Find reflection about line $y = x$

$$Ref_{x=y} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$y = x' \Rightarrow x' = y$

$y' = x \Rightarrow y' = x$

④ Reflection along line $y = -x$

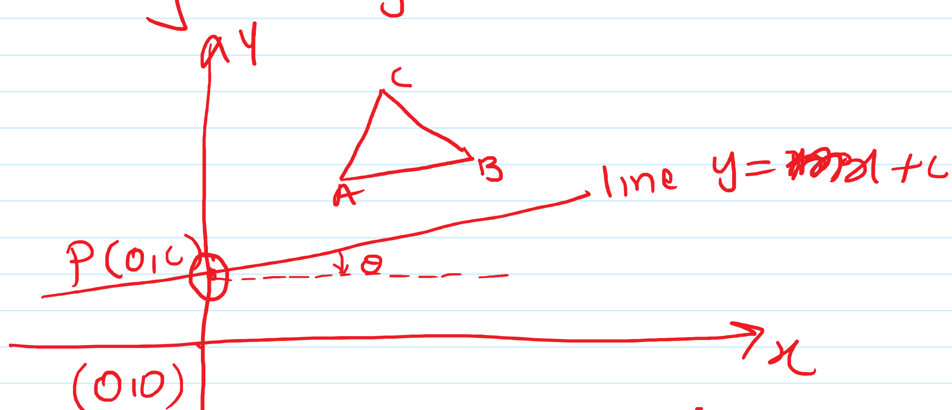
$y = -x' \Rightarrow x' = -y$

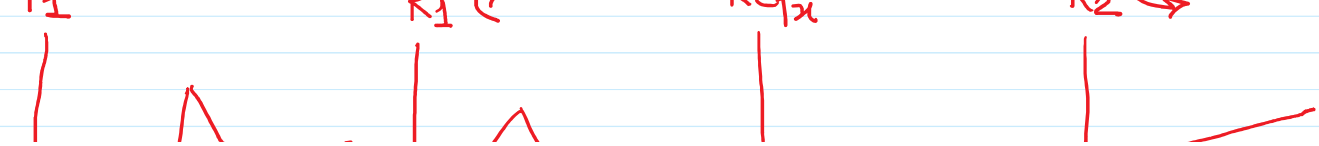
$y' = -x \Rightarrow y' = -x$

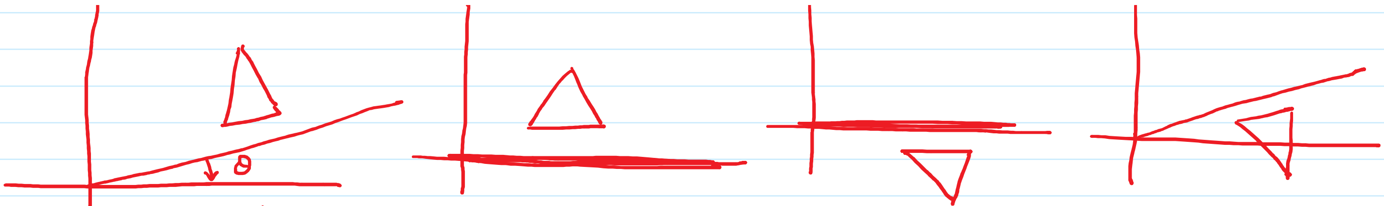
$$Ref_{y=-x} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Derive composite matrix for the following reflection

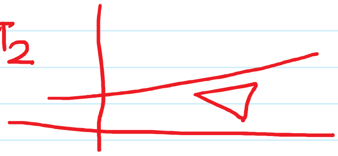
OR Write composite matrix which reflects obj ABC along orbitrary line $y = mx + c$







T_2

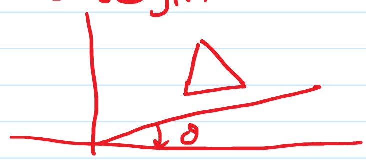


$$M_T = T_2 \cdot R_2 \, Ref_x \, R_1 \, T_1$$

$$A'B'C' = M_T \cdot ABC$$

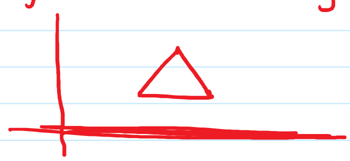
Step 1. – Translate point $P(0, c)$ to origin

$$T_1 = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$$



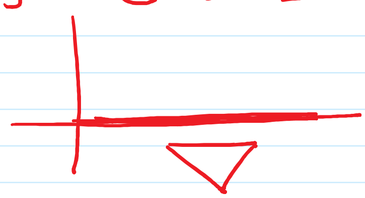
Step 2 - Now rotate line and object about origin by an angle θ in clockwise

$$R_1 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



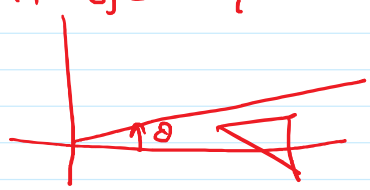
Step 3 - Now reflect an object @ x axis

$$Ref_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



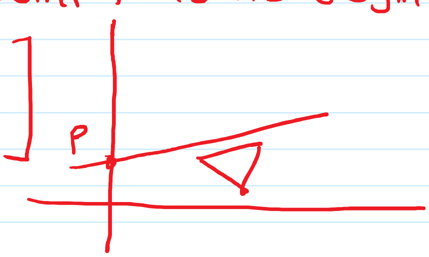
Step 4. Now rotate line and object by θ = in ↺ wrt origin.

$$R_2 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Step 5 – Now retranslate point P to its original position

$$T_2 = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$



Step 6. – Composite matrix.

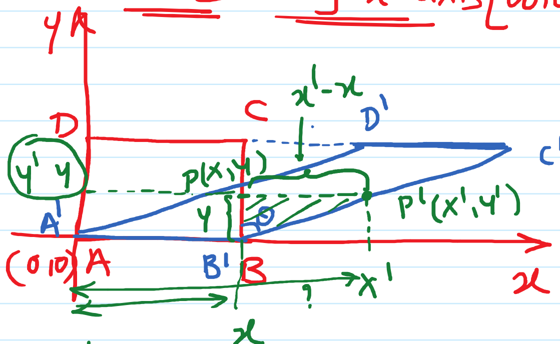
$$M_T = T_2 \cdot R_2 \cdot Ref_x \cdot R_1 \cdot T_1$$

$$A'B'C' = M_T \cdot ABC$$

(5) 2D-Shear Transformation -

Shear transformation slants the surface of an object
To perform shear transformation the i/p parameters are end pt coordinates of an object and θ - angle of shear

(a) Shear along x-axis [Width shear]



$y' = y$ —— (1) ✓

$\tan\theta = \dfrac{x'-x}{y}$

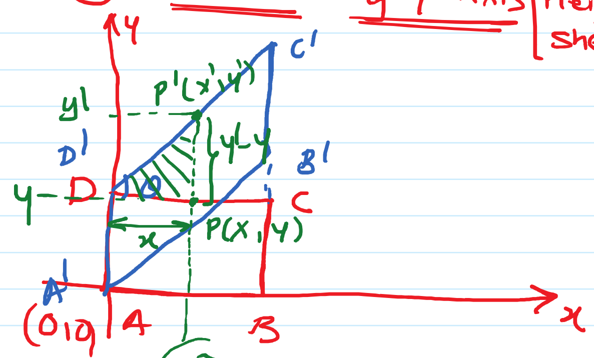
$y\tan\theta = x' - x$

$x' = x + y\tan\theta$ —— (2) ✓

$P' = Sh_x \cdot P$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(b) Shear along y-axis [Height-shear]



$x' = x$ —— (1) ✓

$\tan\theta = \dfrac{y'-y}{x}$

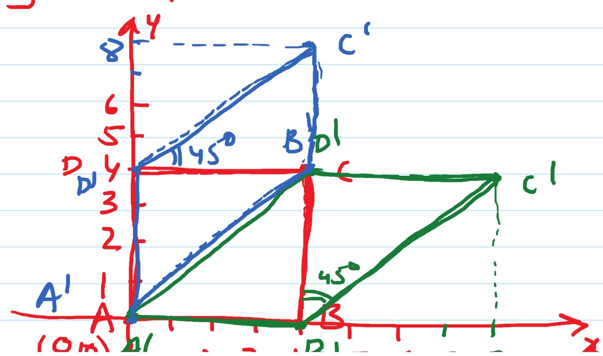
$y' = x\tan\theta + y$ —— (2) ✓

$P' = Sh_y \cdot P$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \tan\theta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ ✓

* Shear an object ABCD with end pt coordinates A(0,0), B(4,0), C(4,4) and D(0,4) along x + y direction by $\theta = 45°$

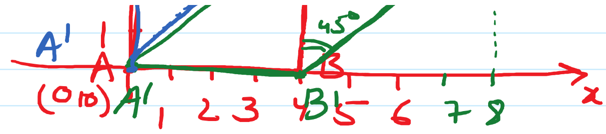
(a) Shear along x-axis.



$A'B'C'D' = Sh_x \cdot ABCD$

$$= \begin{bmatrix} 1 & \tan 45 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 4 & 4 & 0 \\ 0 & 0 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



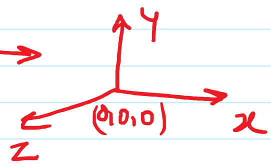
$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 4 & 4 & 0 \\ 0 & 0 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 8 & 4 \\ 0 & 0 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(b) Shear along y-axis.

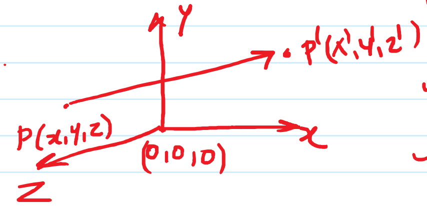
$A'B'C'D' = Sh_y \cdot ABCD$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 4 & 4 & 0 \\ 0 & 0 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 4 & 0 \\ 0 & 4 & 8 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

3D-Geometric Transformations →



(1) 3D-Translation



✓ $x' = x + t_x$

✓ $y' = y + t_y$

✓ $z' = z + t_z$

$1 = 1$

$P' = T \cdot P$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(2) 3D-Scaling – $P' = S \cdot P$

✓ $x' = S_x \cdot x$

✓ $y' = S_y \cdot y$

$z' = S_z \cdot z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(3) 3D-Reflection -

(a) Along xy-plane

$x' = x$ ✓
$y' = y$ ✓
$z' = -z$ ✓

$$Ref_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Along yz plane

$x' = -x$ ✓
$y' = y$ ✓
$z' = z$ ✓

$$Ref_{yz} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

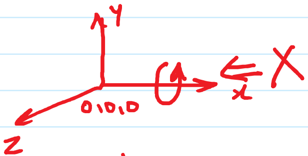
(c) Along xz plane

$x' = x$ ✓
$y' = -y$ ✓
$z' = z$ ✓

$$Ref_{xz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4) 3D-Rotation.

(a) Rotation @ x-axis -



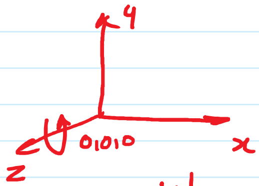
$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$x' = x$ ✓

$y' = y\cos\theta - z\sin\theta$ ✓

$z' = y\sin\theta + z\cos\theta$ ✓

(b) Rotation @ y-axis -



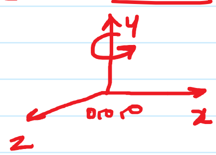
$y' = y$ ✓

$z' = z\cos\theta - x\sin\theta$ ✓

$x' = z\sin\theta + x\cos\theta$ ✓

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) Rotation @ z-axis.



$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$z' = z$ ✓

$x' = x\cos\theta - y\sin\theta$ ✓

$y' = x\sin\theta + y\cos\theta$ ✓