* If you are still not able to access Gradescope, please let us know

* HW2 is released, please get started on it!

* We expect you to participate in the recitations and answer questions

Problems:

1) Prove using Induction that for any positive integer n and for any integers $d_0, d_1, \dots d_{n-1} \in [0 \dots 9]$ we have:

$$\sum_{j=0}^{n-1} d_j \, 10^j < \underline{\underline{10^n}}$$

Proof:

Base case: $n = 1$

$$\sum_{j=0}^{0} d_j \, 10^j = \underline{d_0 \, 10^0} = d_0 < \underline{10}$$

Induction hypothesis: $n = k \qquad k \geq 1$

$$\sum_{j=0}^{n-1} d_j \, 10^j < 10^n$$

$$\boxed{\sum_{j=0}^{k-1} d_j \, 10^j < 10^k} \leftarrow \text{True}$$

Induction step: $\boxed{n = k+1}$

To prove: $\displaystyle\sum_{j=0}^{k} d_j \, 10^j < 10^{k+1} \leftarrow$

$$\sum_{j=0}^{k} d_j 10^j = \underline{d_k 10^k + \sum_{j=0}^{k-1} d_j 10^j}$$

$$< d_k 10^k + 10^k$$

$[0 \ldots\ldots 9]$

$$< 9 \cdot 10^k + 10^k \cdot 1$$

$$< 10^k (9 + 1) \qquad 10^k \times 10$$

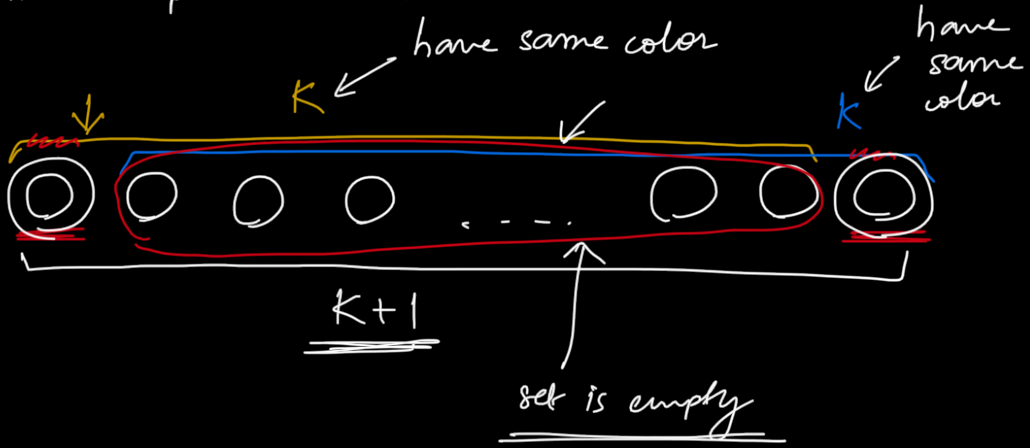
$$\boxed{\sum_{j=0}^{k} d_j 10^j < 10^{k+1}}$$

2) All the sheep in Bethany's flock have the same color!

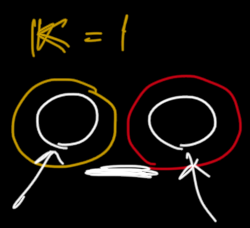
Base case: $n = 1$ Trivally true

→ Induction hypothesis: $\underline{n = k}$ $k \geq 1$

Inductive step: $n = k + 1$



$n = 2$

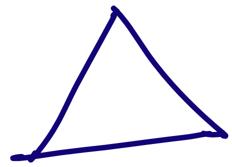


BOGUS!

Q) Prove by induction on n that the number of diagonals in a convex polygon is $\frac{n(n-3)}{2}$ where $n = \#$ sides of the polygon.

Proof:

Base Case: $n = 3$



$$\text{no. of diagonals} = \frac{3(3-3)}{2}$$
$$= 0$$

IH: $n = k$: Assume that the claim holds true when $n = k$

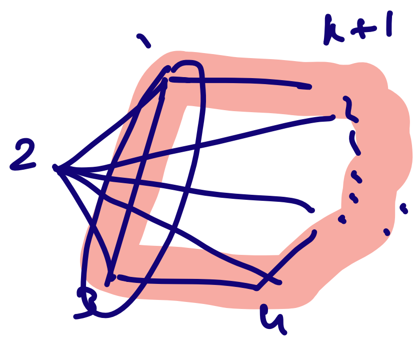
I.O.W:

$$\text{No. of diagonals} = \frac{k(k-3)}{2}$$

IS : Prove claim when $n = k+1$, i.e. when we have $k+1$ sides in a polygon, no. of diagonals $= \frac{(k+1)(k-2)}{2}$

Consider a polygon with $k+1$ sides, equivalently, $k+1$ vertices. ✓

Name the vertices $1, 2, 3 \ldots \; k+1$



No. of diagonals $: \frac{k(k-3)}{2} + k-2+1$

$$= \frac{k(k-3)}{2} + k-1$$

$$= \frac{k^2 - 3k + 2k - 2}{2}$$

$$= \frac{k^2 - k - 2}{2}$$

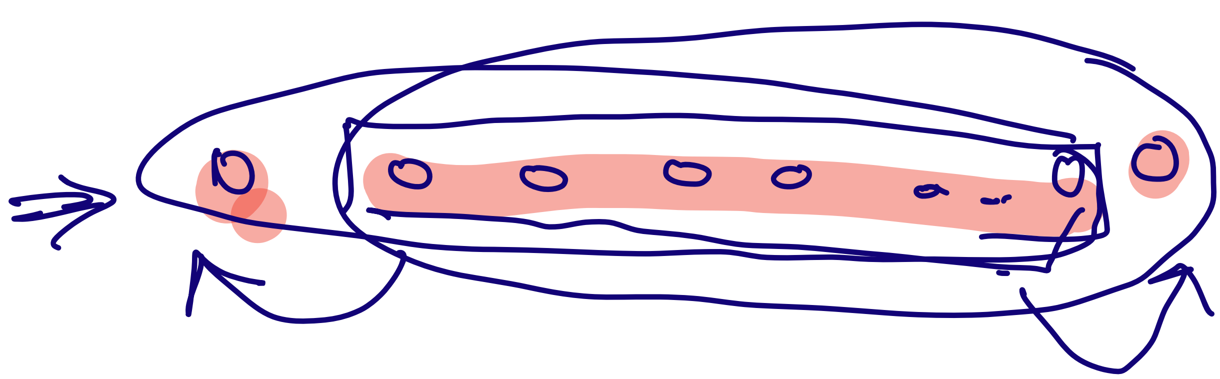
$$= \frac{(k+1)(k-2)}{2}$$

Q) To prove that all sheep are of the same colour.

A) BC ✓ $n = 1$ ✓

IH: $n = k$ claim holds. ✓

IS: <u>$k+1$ sheep.</u>



$k + 1 = 2$