A

APPENDIX

Solved University Question Paper of April 2021

April 2021

Program: First Year Engineering

Curriculum Scheme: Rev 2019 C Scheme

Examination: FE Semester-I : Engineering Mathematics – I

Time: 2 hour **Max. Marks: 80**

Dr. Uday Kashid ; (PhD)

Q. 1 Choose the correct option for following questions. All the Questions are compulsory and carry equal marks

1. The value of tanh (log x), if $x = \sqrt{2}$,will be given by

   (A) $\sqrt{2}$ (B) $\frac{1}{4}$ (C) 2 (D) $\frac{1}{3}$

☑ **Ans. : D**

2. If $z = e^{i\theta}$, the value of $z^6, -\frac{1}{z^6}$ will be given by

   (A) 2i sin 6θ (B) 2 sin 6θ

   (C) 2 cos 6θ (D) – 2i sin 6θ

☑ **Ans. : A**

3. The real part of $z = \sqrt{i}$ will be given by

   (A) 1 B : – 1 (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

☑ **Ans. : D**

4. Find x, if 5 sinh x – cosh x = 5

   (A) x = log 3 (B) $x = e^3$

   (C) x = – log 3 (D) x = – 3

☑ **Ans. : A**

5. Roots of $x^3 - i = 0$ are

   (A) $e^{\frac{i(2k\pi+\pi)}{6}}$, k = 0, 1, 2

   (B) $e^{\frac{i(2k\pi+\pi)}{6}}$, k = 0, 1, 2

   (C) $e^{\frac{i(2k\pi+\pi)}{6}}$, k = 0, 1, 2

   (D) $e^{\frac{i(2k\pi+\pi)}{6}}$, k = 0, 1, 2

☑ **Ans. : C**

6. What is the value of $\sinh^{-1}(\tan\theta)$

   (A) $\log\left(\sec\frac{\theta}{2} + \tan\frac{\theta}{2}\right)$

   (B) log (sec θ + tan θ)

   (C) log (sec θ)

   (D) log (cot θ + tan θ)

☑ **Ans. : B**

7. If tan (x + iy) = 1, then the value of y is

   (A) log 2 (B) $\frac{1}{4}$ log 2

   (C) indeterminate (D) ∞

☑ **Ans. : D**

8. Imaginary part of Log (3 +4i) is

   (A) $\tan^{-1}\left(\frac{1}{4}\right)$ (B) log 5

   (C) $\tan^{-1}\left(\frac{4}{3}\right) + 2n\pi$ (D) $\tan^{-1}\left(\frac{4}{3}\right) + 2\pi$

☑ **Ans. : C**

9. If PAQ is in the normal form of A, where A is a non -singular square matrix of order 3, then A^{-1}, will be ,

   (A) PQ (B) QP

   (C) $Q^{-1}P^{-1}$ (D) $P^{-1}Q^{-1}$

☑ **Ans. : B**

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10. The rank of a Unitary matrix of order n is

(A) $n-1$ (B) $n+1$
(C) n (D) $n+2$

☑ **Ans. : C**

11. Find for which value of λ and μ the simultaneous equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have infinite number of solution

(A) $\lambda=3, \mu=10$
(B) $\lambda\neq 3, \mu=10$
(C) $\lambda=3$, μ can take any value
(D) $\lambda=3, \mu\neq 10$

☑ **Ans. : A**

12. For which value of λ the following system of equations $3x+y-\lambda z=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$ have non-trivial solution ?

(A) $\lambda\neq -9$ and $\lambda=1$
(B) $\lambda=-9$ and $\lambda=1$
(C) $\lambda=-9$ and $\lambda\neq 1$
(D) $\lambda=9$ and $\lambda=1$

☑ **Ans. : B**

13. If $u=e^{\frac{x}{y}}$, then find the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ is

(A) 1 (B) $\frac{1}{2}$ (C) -1 (D) 0

☑ **Ans. : D**

14. If $z=f(x,y)$ and $x=uv$, y, then the value of $\frac{\partial z}{\partial u}$ will be given by

(A) $v\frac{\partial z}{\partial x}-\frac{1}{v}\frac{\partial z}{\partial y}$ (B) $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}$
(C) $v\frac{\partial z}{\partial x}+\frac{1}{v}\frac{\partial z}{\partial y}$ (D) $v\frac{\partial z}{\partial x}+u\frac{\partial z}{\partial y}$

☑ **Ans. : C**

15. If $z=\log r$, $r=x^2+y^2$ then find the value of $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$

(A) -2 (B) 2
(C) $2r$ (D) $\frac{1}{r}$

☑ **Ans. : B**

16. If $z=\frac{x}{y}+\frac{y}{x}$, then the value of $\frac{\partial^2 z}{\partial x\,\partial y}$ is

(A) $-\frac{1}{x^2}-\frac{1}{y^2}$ (B) $-\frac{1}{x^2}$
(C) $-\frac{1}{y^2}$ (D) $\frac{1}{x^2}+\frac{1}{y^2}$

☑ **Ans. : A**

17. If $y=\sin^2 x$, find y_{10}

(A) $-2^9\cos 2x$ (B) $2^9\cos 2x$
(C) $2^9\sin 2x$ (D) $-2^9\sin 2x$

☑ **Ans. : B**

18. If $y=x^n\log x$, then y_{n+1} is

(A) $n!\,x$ (B) $n!\log x$
(C) $\frac{n!}{x}$ (D) $n!$

☑ **Ans. : C**

19. If $(1+x^2)\,y_2=1$, then choose the correct option

(A) $(1+x^2)\,y_{2+1}+2nx\,y_{n+1}+n(n-1)\,y_n=0$
(B) $y_{n+2}+2nx\,y_{n-1}-n(n-1)\,y_n=0$
(C) $y_{n+2}-2nx\,y_{n-1}+n(n-1)\,y_n=0$
(D) $y_{n+2}+2nx\,y_{n+1}-n^2\,y_n=0$

☑ **Ans. : A**

20. The stationary values for $f(x,y)=xy(3-x-y)$ are

(A) (0,0), (3,0), (1,1), (1,-1)
(B) (0,0), (0,3), (3,0), (1,1)
(C) (0,0), (0,-3), (3,3), (1,1)
(D) (0,0), (0,-3), (3,0), (1,1)

☑ **Ans. : B**

Q. 2 Solve any Four out of Six (20 Marks)

(a) If $\cos 6\theta = a\cos^6\theta + b\cos^4\theta\sin^2\theta + c\cos^2\theta\sin^4\theta + d\sin^6\theta$. Find a, b, c, d

☑ **Ans. : a = 1, b = – 15, c = 15, d = – 1**

Please refer UEx. 1.9.3.

(b) If $\log\sin(x+iy)=a+ib$, prove that

(i) $2e^{2a}=\cosh 2y-\cos 2x$

(ii) $\tan b=\cot x\tan hy$

☑ **Ans. : Please refer UEx. 3.2.9.**

(c) Find the non singular matrices P and Q such that PAQ is in the normal form and hence find Rank of the following matrix

$$A = \begin{bmatrix} 2 & 1 & 13 \\ 1 & 0 & 12 \\ 3 & 1 & 25 \end{bmatrix}$$

☑ **Ans. : Please refer UEx. 8.5.1.**

(d) Find a, b, c and A^{-1} if $A = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$ is orthogonal.

☑ **Ans. : Please refer UEx. 8.7.3.**

(e) Divide 24 into 3 parts such that the continued product of the first, square of second and cube of the third is maximum using Lagrange's method.

☑ **Ans. : Please refer UEx. 6.4.3.**

(f) If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$, then prove that

$$\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$$

☑ **Ans. : Please refer UEx. 4.5.16.**

Q. 3 Solve any Four out of Six (20 Marks)

(a) Find the continued product of the roots of $x^4 = 1 + i$.

☑ **Ans. : Please refer UEx. 1.7.31.**

(b) Prove that $2e^{2x} = \cosh 2v - \cos 2u - \cos 2u$, where $e^z = \sin(u + iv)$ and $z = x + iy$

☑ **Ans. : Please refer UEx. 3.2.9.**

(c) Express the matrix $\begin{bmatrix} 1+2i & 2 & 3-i \\ 2+3i & 2i & 1-2i \\ 1+i & 0 & 3+2i \end{bmatrix}$ as P + iQ, where both P and Q are Hermitian.

☑ **Ans. : Please refer UEx.8.8.10.**

(d) If $x = \cosh\left(\frac{1}{m}\log y\right)$, then prove that

$$(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

☑ **Ans. : Please refer UEx. 7.4.41.**

(e) If $u = \log r$, and $r^2 = x^2 + y^2$, then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + 1 = 0$$

☑ **Ans. : Please refer UEx.5.4.22.**

(f) If $u = \log\frac{x+y}{\sqrt{x^2+y^2}} + \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 v}{\partial y^2} = -\frac{\sin w \cos 2w}{4\cos^3 w}$$ where $w = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$

☑ **Ans. : Please refer UEx. 5.4.37.**

Dr. Uday Kashid ; (PhD)

...Chapter Ends