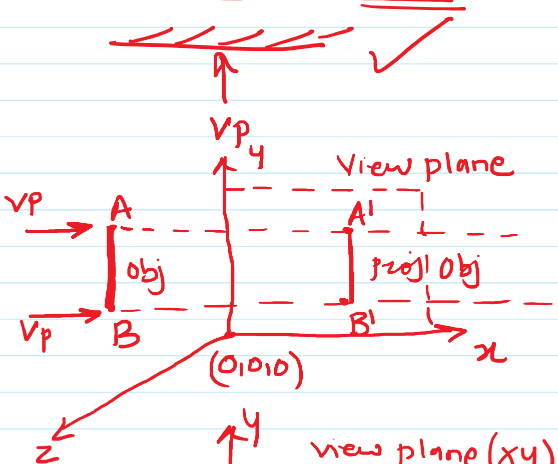
① 3D-projections → Parallel projection, Perspective projection } 10

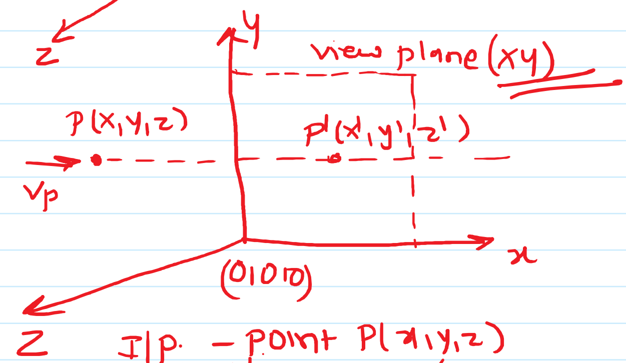
② Curves → Bezier curve [parametric eqn / properties / Num] 10, → B Spline curve

③ Fractals | Hilbert & koch curve ⇐ 5M / 5M

* 3D-Projections -

① Parallel Projection:-





I/P. - point P(x,y,z)
View plane (xy)

O/P. - P'(x',y',z')

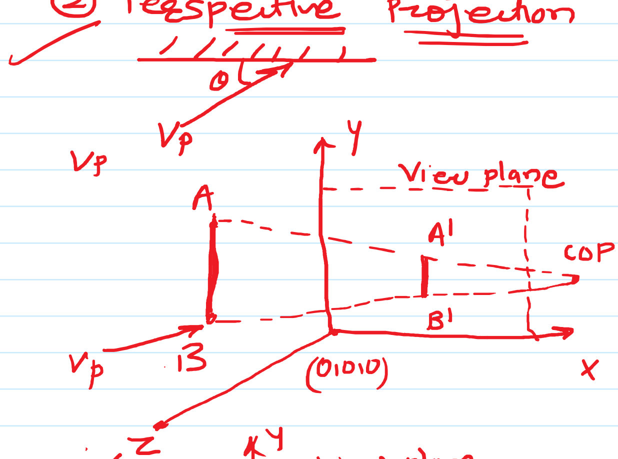
$$\begin{cases} x' = x & \text{—— ①} \\ y' = y & \text{—— ②} \\ z' = 0 & \end{cases}$$

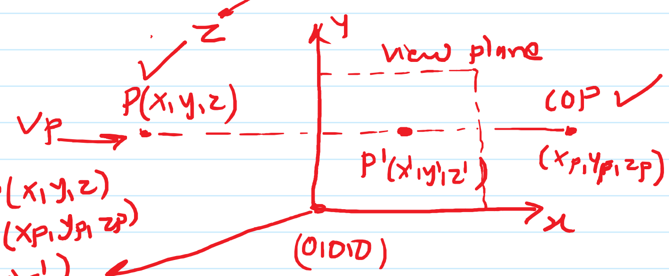
∵ View plane is xy plane

$$P' = M_{ORTHO} \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

② Perspective Projection





I/P. ⇒ P(x,y,z)
COP(xp,yp,zp)

O/P. ⇒ P'(x',y',z') z

$$\frac{x'-x}{xp-x} = \frac{y'-y}{}$$

$$\frac{}{zp-z} = \cancel{\ \ }$$

[∵ View plane is xy plane ∴ z' = 0]

$$\frac{-z}{zp-z} = u \Rightarrow \boxed{u = \frac{-z}{zp-z}}$$

$$\frac{x'-x}{xp-x} = u$$

$$\boxed{x' = x + u(xp - x)}$$

$$\frac{y'-y}{yp-y} = u \Rightarrow \boxed{y' = y + u(yp - y)}$$

$$\begin{bmatrix} \vdots \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \vdots \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \vdots \\ z \\ 1 \end{bmatrix}$$

Matrix representation for orthographic/parallel projection

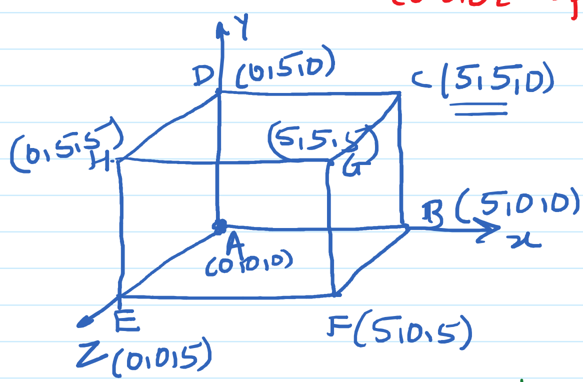
$$\frac{y'-y}{y_p-y} = u \Rightarrow \boxed{y' = y + u(y_p - y)}$$

$$P' = M_{Persp.} \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & u(x_p-x) \\ 0 & 1 & 0 & u(y_p-y) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

* Consider a cube of sides 5 unit each resting on the origin. The sides of a cube are along the real axis. Find the perspective projection of a cube. Assume XY plane as view plane and center of projection is $(0,0,-5)$.

Soln



$$COP \Rightarrow (x_p, y_p, z_p) = (0, 0, -5)$$

XY plane is view plane $[\therefore z' = 0]$

$$\frac{x'-x}{x_p-x} = \frac{y'-y}{y_p-y} = \frac{z'-z}{z_p-z} = u$$

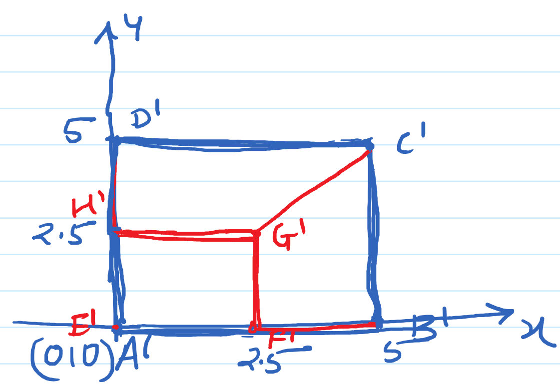
$$u = \frac{-z}{z_p-z} = \frac{-z}{-5-z} = \boxed{\frac{z}{5+z}}$$

$$\frac{x'-x}{x_p-x} = u \Rightarrow x' = x + u(x_p - x)$$

$$\boxed{x' = x - ux}$$

$$\frac{y'-y}{y_p-y} = u \Rightarrow \boxed{y' = y - uy}$$

Points $P(x,y,z)$	$u = \frac{z}{z+5}$	$x' = x - ux$	$y' = y - uy$	$z' = 0$
A(0,0,0)	0	0	0	0
B(5,0,0)	0	5	0	0
C(5,5,0)	0	5	5	0
D(0,5,0)	0	0	5	0
E(0,0,5)	1/2	0	0	0
F(5,0,5)	1/2	2.5	0	0
G(5,5,5)	1/2	2.5	2.5	0
H(0,5,5)	1/2	0	2.5	0



① Bezier Curve →

① For any set of control points an approximate curve is formed by adding a sequence of polynomial function from the coordinates of control point is called as Bezier Curve

② The parametric eqn of Bezier curve is defined as,

$$P(u) = \sum_{k=0}^{n} P_k \cdot B_{k,n}(u) \qquad 0 \le u \le 1$$

where $B_{k,n}(u)$ is called as Blending function

$$B_{k,n}(u) = {}^{n}C_k \, u^k (1-u)^{n-k}$$

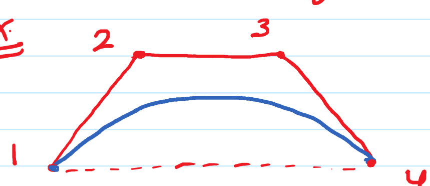
where $${}^{n}C_k = \frac{n!}{k!\,(n-k)!}$$

$n \Rightarrow$ deg. of Bezier curve }
$n+1 \Rightarrow$ no. of control points }

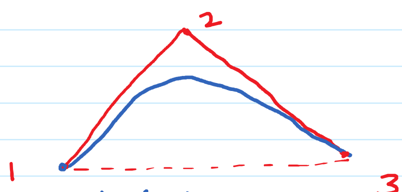
③ The deg. of Bezier curve is one less than the no. of control points. Say no. of control pts $\Rightarrow n+1$, deg $\Rightarrow n$ || no. of control pts $\Rightarrow n$, deg $\Rightarrow n-1$

If the deg of Bezier curve is 3 $\Rightarrow$ Cubic Bezier curve

ex.



No. of control pts $= n+1 = 4$
deg $= n = 3$
cubic bezier



Control pts $= n+1 = 3$
deg $= n = 2$

④ Appln of Bezier curve $\Rightarrow$ ⓐ painting/Drawing packages like CAD/CAM, engg drawing, architectural drawing

ⓑ Game theory to design various games.

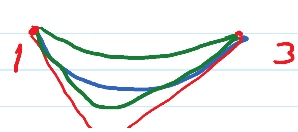
ⓒ

⑤ Properties of Bezier curve :- ⓐ Bezier curve always passes through the 1st & last control points

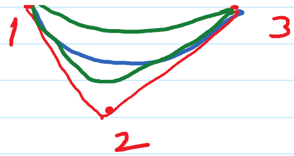
$P(0) = P_0$ and $P(1) = P_n$

ⓑ Bezier curve is indep. of coordinate geometry

ⓒ Bezier curve always lies within the polygon boundary formed by the control points.



ⓓ Bezier curve do not provide localized control ie changing any control points will change the entire



(d) Bezier curve do not provide localized control ie changing any control points will change the entire shape of bezier curve

(e) Bezier curve is a smooth curve to draw against the given set of control points

(6) Drawbacks of Bezier curve → (1) Since it is indep. of coordinate geometry, it always gives approx curve formed by control points

(2) Bezier curve do not provide localized control

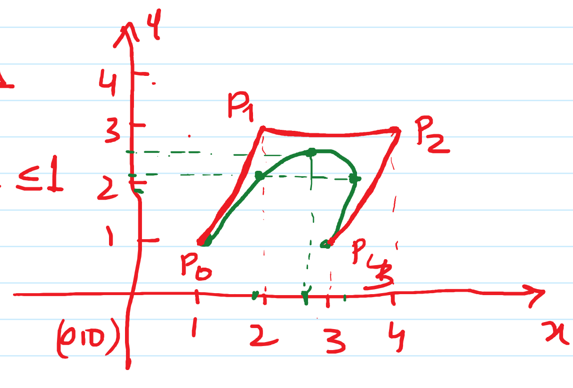
(3) At joints, complex mathematical cal. are involved.
∴ Bezier curve is less smooth at joints

* Give the mathematical eqn and properties of Bezier curve.
Given the vertices of bezier polygon as (1,1), (2,3), (4,3) and (3,1). Determine 5 points on bezier curve.

Soln

$$P(u) = \sum_{k=0}^{n} P_k \cdot B_{k,n}(u) \qquad 0 \le u \le 1$$

$$P(u) = \sum_{k=0}^{n} P_k \cdot {}^nC_k\, u^k (1-u)^{n-k} \qquad 0 \le u \le 1$$



Here no. of control points ⇒ $n+1=4$
deg ⇒ $n=3$

$$P(u) = \sum_{k=0}^{3} P_k \cdot {}^3C_k\, u^k (1-u)^{3-k}$$

$$P(u) = P_0\, {}^3C_0\, u^0 (1-u)^3 + P_1\, {}^3C_1\, u^1 (1-u)^2 + P_2\, {}^3C_2\, u^2 (1-u)^1 + P_3\, {}^3C_3\, u^3 (1-u)^0$$

$$\boxed{P(u) = P_0 (1-u)^3 + P_1 (3) u (1-u)^2 + P_2 (3) u^2 (1-u) + P_3 u^3} \quad \text{—— (I)}$$

Determine 5 pts on curve $\quad u = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$

At $u=0$ in eqn (1)

$$P(0) = [1,1](1)^3 + 0 + 0 + 0$$

$$\boxed{P(0) = (1,1)}$$

At $u = \frac{1}{4}$ in eqn (1)

$$P(1/4) = [1,1]\left(\tfrac{3}{4}\right)^3 + [2,3](3)\left(\tfrac{1}{4}\right)\left(\tfrac{3}{4}\right)^2 + [4,3](3)\left(\tfrac{1}{4}\right)^2\left(\tfrac{3}{4}\right) + [3,1]\left(\tfrac{1}{4}\right)^3$$

$$= \left[\frac{27}{64}+\frac{54}{64}+\frac{36}{64}+\frac{3}{64},\ \frac{27}{64}+\frac{81}{64}+\frac{27}{64}+\frac{1}{64}\right]$$

$$\boxed{P(1/4) = (1.875, 2.125)}$$

At $u = 1/2$ in eqn ②

$$\boxed{P(1/2) = (2.75, 2.5)}$$

At $u = 3/4$ eqn ①

$$\boxed{P(3/4) = (3.25, 2.125)}$$

At $u = 1$ in eqn ①

$P(1) = P_3(1)^3$

$$\boxed{P(1) = (3,1)}$$

* **B-Spline curve** → Local control.

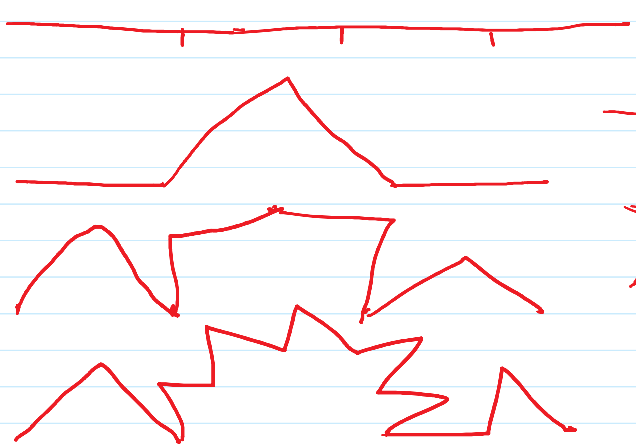
A general expression / mathematical eqn of B-spline curve is –

$$P(u) = \sum_{k=0}^{n} P_k \cdot \underbrace{B_{k,d}(u)}_{\text{Blending fn}} \qquad 0 \le u \le 1$$

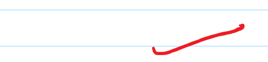
$$2 \le d \le n+1$$

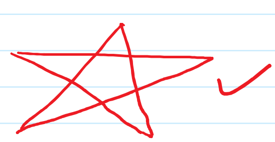
* **Fractals** / Approximation of fractals → Koch curve, Hilbert curve

Koch Curve – Approximation of fractals



line $y = mx + c$





Hilbert curve