Ex: Retirement policy is to be presented to top management. To know the support of the policy a manger conducts a poll.

	Machinists	inspector
Strongly support	9	10
Mildly support	11	3
Undecided	2	2
Mildly oppose	4	8
Strongly oppose	4	7
	30	30

a) What is the prob that a **machinist** randomly selected from the polled group **mildly supports** the package.
b) What is the prob that an **inspector** randomly selected from the polled group **is undecided**.
c) What is the prob that a worker (machinist or inspector) randomly selected from the polled group **strongly or mildly supports** the package.
d) What prob **estimates** are these.

	Machinists	inspector
Strongly support	9	10
Mildly support	11	3
Undecided	2	2
Mildly oppose	4	8
Strongly oppose	4	7
	30	30

a) What is the prob that a **machinist** randomly selected from the polled group **mildly supports** the package =11/30
b)) What is the prob that an **inspector** randomly selected from the polled group **is undecided**=2/30
c)) What is the prob that a worker (machinist or inspector) randomly selected from the polled group **strongly or mildly supports** the package = 9+11+10+3 / 60 = 33/60=11/20
d) What prob **estimates** are these = **Relative frequency**.

Explain different types of sampling methods with the help of examples.

Selecting a sample is **less time-consuming** than selecting every item in the population (census).

time presentative – mean var σ.

Selecting a sample is **less costly** than selecting every item in the population. → cost effective

An **analysis** of a sample is less **cumbersome** and more practical than an analysis of the entire population.

A Sampling Process Begins With A Sampling Frame

- The sampling frame is a **listing** of items that make up the population
- Frames are data sources such as population **lists, directories, or maps**
- Inaccurate or biased results can result if a frame **excludes** certain portions of the population
- Using different frames to generate data can lead to dissimilar conclusions

Classification of Sampling Techniques

Sampling Techniques

1. Nonprobability Sampling Techniques
   1. Convenience Sampling
   2. Judgmental Sampling
   3. Quota Sampling
   4. Snowball Sampling
2. Probability Sampling Techniques
   1. Simple Random Sampling
   2. Systematic Sampling
   3. Stratified Sampling
   4. Cluster Sampling
   5. Other Sampling Techniques

Types of Samples: Nonprobability Sample

- In a nonprobability sample, items included are chosen without regard to their probability of occurrence.
  - In **convenience sampling**, items are selected based only on the fact that they **are easy, inexpensive, or convenient** to sample.
  - In a **judgment sample,** you get the opinions of pre-selected experts in the subject matter. biased
  - **Quota Sampling.** UG, PG, PhD 1000 800 500 150 50
  - **Snowball.** Health Age 60 65

$$\frac{800}{1000} \times 100 = x \qquad \frac{150}{1000} \times 100 = y \qquad \frac{50}{1000} \times 100 = \frac{1000}{2}$$

2) Probability Sample: Simple Random Sample

- Every individual or item from the frame has an **equal chance of being selected**
  100 → 10 → 10 groups 1 student 1/10 → 1, 2 ... 10 & 1/10
- Selection may be **with** replacement (selected individual is returned to frame for possible reselection) or **without** replacement (selected individual isn't returned to the frame). 1, 2, 3, ... 10 ⇒ 1st time 1/10 1/10
- Samples obtained from table of random numbers or computer random number generators.

1) Selecting a Simple Random Sample Using A Random Number Table

SRS

492 808 ~~892~~

Sampling Frame For Population With 850 Items

Item Name	Item #
Bev R.	001
Ulan X.	002
.	.
.	435
. Age	492
. abc	808
Joann P.	849
Paul F.	850

Portion of A Random Number Table

49280	88924	35779	00283	81163	07275
11100	02340	12860	74697	96644	89439
09893	23997	20048	49420	88872	08401

The First 5 Items in a simple random sample

Item # 492
Item # 808
Item # 892 -- does not exist so ignore
Item # 435
Item # 779
Item # 002

2)

Probability Sample: Systematic Sample

- Decide on sample size: n
- Divide frame of N individuals into groups of k individuals: k=N/n
- Randomly select one individual from the 1st group
- Select every k^{th} individual thereafter

(1,2,3,4)

40

56

sample → 8

56/8 = 7

N = 56
n = 8
k = 7

First Group →

8
8
8
8

3)

Probability Sample: Stratified Sample

2 stage Sampling

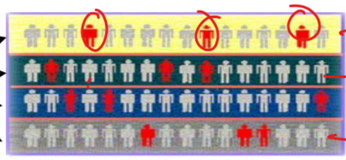
- Divide population into two or more subgroups (called *strata*) according to some common characteristic
- A **simple random sample is selected** from each subgroup, with sample sizes proportional to strata sizes
- Samples from subgroups are combined into one
- This is a common technique when sampling population of voters, stratifying across racial or socio-economic lines.

Stage 1 → Subgroups

Stage 2 → Select Sample

SE → INFT Cmpn Biom EXTC → UIT

Population divided into 4 strata



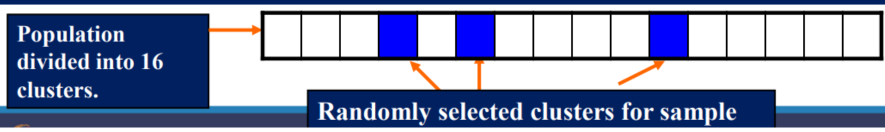
3
3
3

4

Probability Sample : Cluster Sample

2 stage

- Population is divided into several "clusters," each representative of the population
  Cluster
  ① → Cluster
- A simple random sample of clusters is selected
  ② select → SRS
- All items in the selected clusters can **be used, or items can be chosen** from a cluster using another probability sampling technique
- A common application of cluster sampling involves election exit polls, where certain election districts are selected and sampled.



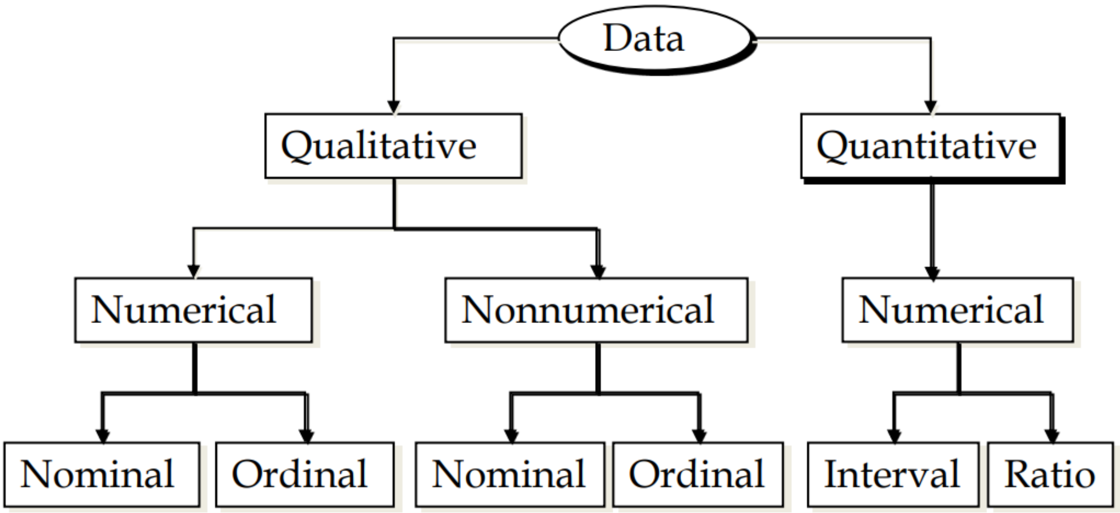
Probability Sample: Comparing Sampling Methods

Single stage sampling method

- SRS and Systematic sample
  - Simple to use ↔
    characteristics
  - May not be a good representation of the population's underlying characteristics
- Stratified sample
  Strata costly efficiency
  - **Ensures representation of individuals across the entire population**
- Cluster sample
  2 stage sampling
  Homogeneity within group
  Heterogeneity across group
  - **More cost effective**
  - Less efficient (need larger sample to acquire the same level of precision)

Short note on various Measures of variations.

Scales of Measurement



Scales of Measurement

- Nominal

Example:
Students of a university are classified by the school in which they are enrolled using a nonnumeric label such as Business, Humanities, Education, and so on.

Alternatively, a numeric code could be used for the school variable (e.g. 1 denotes Business,2 denotes Humanities, 3 denotes Education, and so on).

- Ordinal
  - The data have the properties of nominal data and the order or rank of the data is meaningful.
  - A nonnumeric label or numeric code may be used.

Example: Students of a university are classified by their class standing using a nonnumeric label such as Freshman, Sophomore, Junior, or Senior. Alternatively, a numeric code could be used for the class standing variable (e.g. 1 denotes Freshman, 2 denotes Sophomore, 3 denotes Junior, 4 denotes Senior)

- Interval

  - The data have the properties of ordinal data, and the interval between observations is expressed in terms of a **fixed unit of** measure.
  - Interval data are always numeric.

Interval

Example:
Melissa has an SAT score of 1205, while Kevin has an SAT score of 1090. Melissa scored 115 points more than Kevin.

- Ratio

  - The data have all the properties of interval data and the ratio of two values is meaningful.
  - Variables such as distance, height, weight, and time use the ratio scale.
  - This scale must contain a zero value that indicates that **nothing exists** for the variable at the zero point.

Example: Melissa's college record shows 36 credit hours earned, while Kevin's record shows 72 credit hours earned. Kevin has twice as many credit hours earned as Melissa.

Use the data given below to construct relative frequency using 13 equal intervals.

83 51 66 61 82 65 54 56 92 60 65 87 68 64 51 70 75 66 74 68 44 55 78 69 98 67 82 77 79 62 38 88 76 99 84 47 60 42 66 74 91 71 83 80 68 65 51 56 73 55

---

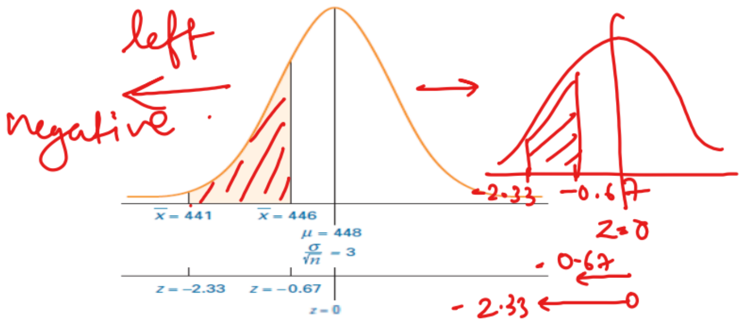
13 Intervals

Class	Relative Frequency	Class	Relative Frequency
35-39	0.02	70-74	0.10
40-44	0.04	75-79	0.10
45-49	**0.02**	80-84	0.12
50-54	0.08	85-89	0.04
55-59	0.08	90-94	0.04
60-64	0.10	95-99	0.04
65-69	0.22		**1.00**

$Z = (x-\mu)/\sigma_x$

$\sigma_x = \sigma/\sqrt{n}$

For this problem, $\mu = 448$, $\sigma = 21$, and $n = 49$. The problem is to determine $P(441 \le \bar{x} \le 446)$. The following diagram depicts the problem.



Solve this problem by calculating the z scores and using Table A.5 to determine the probabilities.

$$z = \frac{441 - 448}{\frac{21}{\sqrt{49}}} = \frac{-7}{3} = -2.33$$

R→2.3

C→0.03

$$z = \frac{446 - 448}{\frac{21}{\sqrt{49}}} = \frac{-2}{3} = -0.67$$

R→0.6

C→0.07

z Value	Probability
−2.33	.4901
−0.67	−.2486
	.2415

0.4901

− 0.2486

⇒ 0.2415 ×100

The probability of a value being between $z = -2.33$ and -0.67 is .2415; that is, there is a 24.15% chance of randomly selecting 49 hourly periods for which the sample mean is between 441 and 446 shoppers.

24.15%

SECOND DECIMAL PLACE IN z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									
4.0	.49997									
4.5	.499997									
5.0	.4999997									
6.0	.499999999									

Here even though curve is left side, for finding Z score we use right side curve plot. Bcz left ad right oth are symmetrical oly their sig will chage hece values are writte as -0.4901 & -0.2496

-0.2496-(-0.4901) = 0.2415

0.2415*100

24.15%

What is a Hypothesis?

- A hypothesis is a claim (assertion) about a population parameter:

  – population mean

Example: The mean monthly cell phone bill in this city is $\mu = \text{Rs } 42$

  – population proportion

Example: The proportion of adults in this city with cell phones is $\pi = 0.68$

The Null Hypothesis, H_0

- States the claim or assertion to be tested

Example: The average number of TV sets in Indian Homes is equal to three ($H_0 : \mu = 3$)

- Is always about a population parameter, not about a sample statistic

$H_0 : \mu = 3$ $\qquad$ $\cancel{H_0 : \bar{X} = 3}$

The Null Hypothesis, H_0

(continued)

Begin with the assumption that the null hypothesis is true

– Similar to the notion of innocent until proven guilty

Refers to the status quo or historical value

Always contains "=" , "≤" or "≥" sign

May or may not be rejected

The Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis

- e.g., The average number of TV sets in Indian homes is not equal to 3(H_1: $\mu \neq 3$)

- **Challenges the status quo**

- May or may not be proven

- **Is generally the hypothesis that the <u>researcher is trying to prove</u>**

Ex: H0: $\mu = 100$ (the null hypothesis is that the population mean is 100)

H1: $\mu \neq 100$

H1: $\mu > 100$

H1: $\mu < 100$

Possible Errors in Hypothesis Testing

Type I and Type II Errors:

Because the hypothesis testing process uses **sample statistics** calculated from random data to reach conclusions about population parameters, it is possible to make an incorrect decision about the null hypothesis.

In particular, two types of errors can be made in testing hypotheses: Type I error and Type II error.

A **Type I error** is committed by ***rejecting a true null hypothesis***. With a Type I error, the null hypothesis is true, but the business researcher decides that it is not.

n = 100

As an example, suppose the flour-packaging process actually is **"in control"** and is **averaging 40 ounces** of flour per package. Suppose also that a business researcher randomly selects 100 packages, weighs the contents of each, and computes a sample mean. It is possible, **by chance**, to randomly select 100 of the more extreme packages (mostly heavy weighted or mostly light weighted) resulting in **a mean** that falls in the **rejection region**. **The decision is to reject the null hypothesis even though the population mean is actually 40 ounces.** In this case, the business researcher has **committed a Type I error**.

For example, if a **manager fires** an employee because some evidence indicates that she is stealing from the company and if she really is not stealing from the company, then the manager has committed a **Type I error**.

accepted
Ho = not true . we have

As another example, suppose a worker on the assembly line of a large manufacturer hears an **unusual sound** and decides to **shut the line down** (reject the null hypothesis). If the sound turns **out not to be** related to the assembly line and no problems are occurring with the assembly line, then the worker has committed a Type I error.

$\alpha \uparrow$

The probability of committing a Type I error is called **alpha (α)** or level of significance. Alpha equals the **area** under the curve that is in the **rejection region** beyond the critical value(s).

A **Type II error** is committed when a business researcher ***fails to reject a false null hypothesis***. In this case, the null hypothesis is false, but a decision is made to not reject it.

Suppose in the case of the flour problem that the packaging process is actually **producing a population mean of 41** ounces even though the null hypothesis is 40 ounces. A **sample of 100** packages yields a sample mean **of 40.2 ounces**, **which falls in the non-rejection region.**

The business decision maker decides not to reject the null hypothesis. A Type II error has been committed. The packaging procedure is **out of control** and the hypothesis testing process **does not identify** it.

Suppose in the business world an employee is stealing from the company. A manager sees some evidence that the stealing is occurring but lacks enough evidence to conclude that the employee is stealing from the company.

The manager decides not to fire the employee based on theft. The manager has committed a Type II error.

Consider the manufacturing line with the noise. Suppose the worker decides not enough noise is heard to shut the line down, but in actuality, one of the cords on the line is unraveling, creating a dangerous situation. The worker is committing a Type II error

- **Type I Error**
  - **Reject a true null hypothesis**
  - Considered a serious type of error
  - The probability of a Type I Error is **α**
    - Called level of significance of the test
    - Set by researcher in advance
- **Type II Error**
  - **Failure to reject false null hypothesis**
  - The probability of a Type II Error is **β**

Possible Errors in Hypothesis Test Decision Making *(continued)*

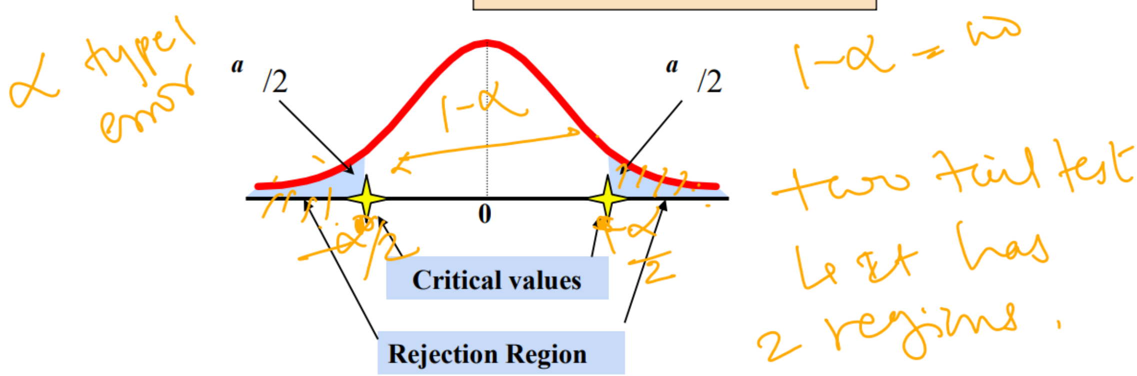
Possible Hypothesis Test Outcomes		
	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	No Error Probability $1 - \alpha$	Type II Error Probability β
Reject H_0	Type I Error Probability α	No Error Probability $1 - \beta$

Power, which is equal to $1 - \alpha$, is *the probability of a statistical test rejecting the null hypothesis when the null hypothesis is false.* Table shows the relationship between α, β, and power.

Level of Significance and the Rejection Region

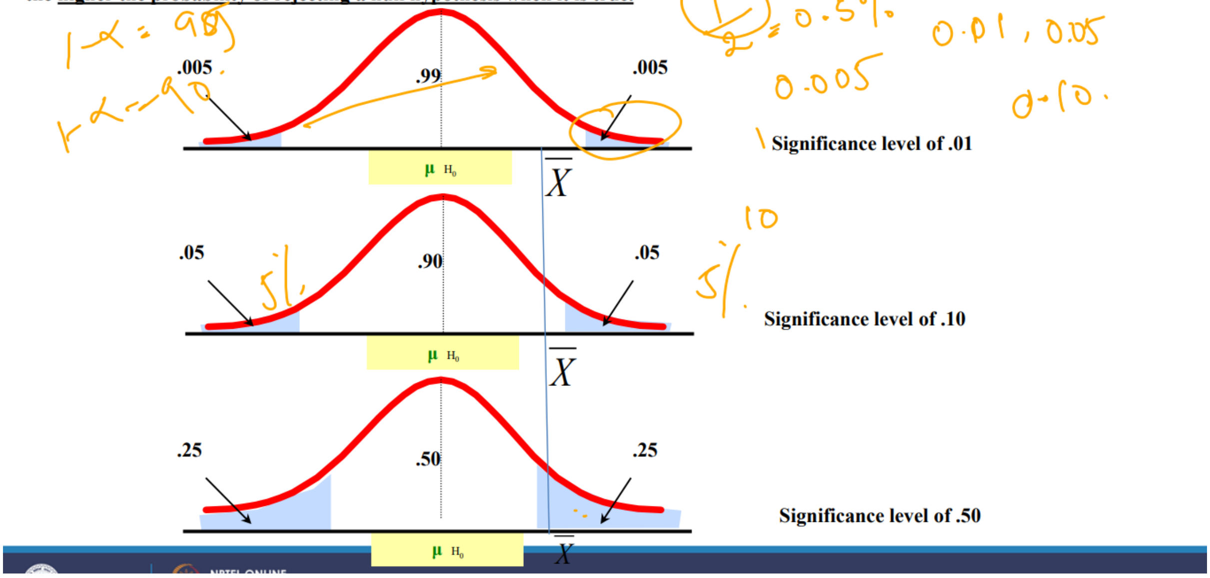
H_0: $\mu = 3$ H_1: $\mu \neq 3$

Level of significance = $\boldsymbol{\alpha}$



This is a two-tail test because there is a rejection region in both tails

Selecting a significance level : Generally at .01,.05, .10 or 99,95,90%: The higher the significance level we use for testing a hypothesis, the <u>higher the probability of rejecting a null hypothesis when it is true.</u>



Explain Continuous Probability Distribution & its Types. Find the Normal Probabilities for following cases:

a)Suppose X is normal with mean 18.0 and standard deviation 5.0. Find P(X > 18.6).

b) Suppose X is normal with mean 18.0 and standard deviation 5.0. Find P(18 < X < 18.6).

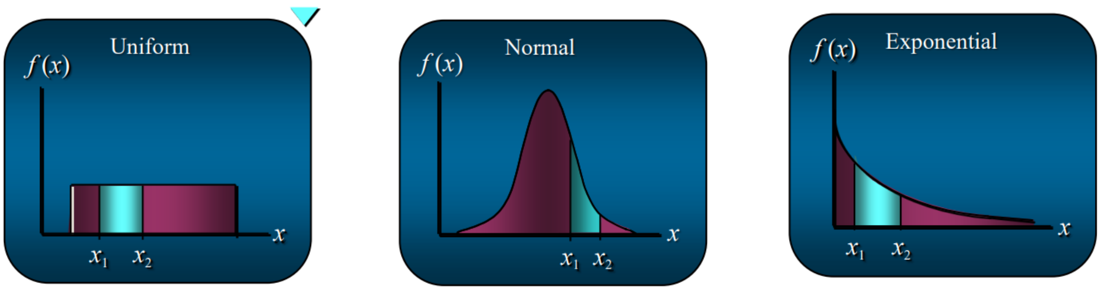
c) Suppose X is normal with mean 18.0 and standard deviation 5.0. • Now Find P(17.4 < X < 18).

➔Continuous Probability Distributions ν A continuous random variable can assume any value in an interval on the real line or in a collection of intervals.

It is not possible to talk about the probability of the random variable assuming a particular value. Instead, we talk about the probability of the random variable assuming a value within a given interval.

Continuous Probability Distributions

- The probability of the random variable assuming a value within some given ***interval*** from x_1 to x_2 is **defined** to be the **area under the graph** of the **probability density function** between x_1 and x_2.



Uniform Probability Distribution

- A random variable is uniformly distributed whenever the probability is proportional to the *interval's length*.
- The uniform probability density function is:

$$f(x) = 1/(b - a) \quad \text{for } a \le x \le b$$
$$= 0 \quad \text{elsewhere}$$

where: a = smallest value the variable can assume
b = largest value the variable can assume

Expected Value of x

$$E(x) = (a + b)/2$$

Variance of x

$$Var(x) = (b - a)^2/12$$

Example: Slater's Buffet Slater customers are charged for the amount of salad they take. Sampling suggests that the amount of salad taken is uniformly distributed between 5 ounces and 15 ounces. Find out mean and S.D.

Uniform Probability Density Function

$$f(x) = 1/10 \quad \text{for } 5 \le x \le 15$$
$$= 0 \quad \text{elsewhere}$$

where:

x = salad plate filling weight

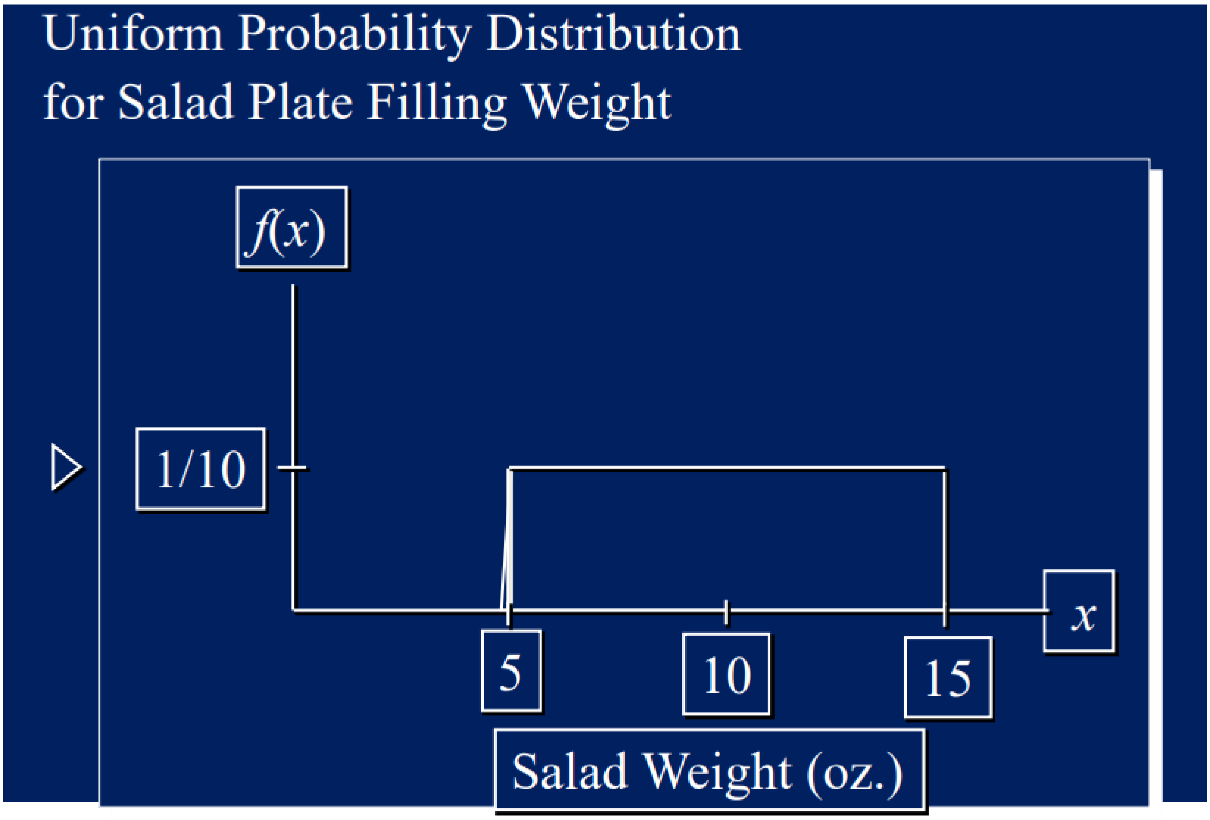
Expected Value of x

$$E(x) = (a + b)/2$$
$$= (5 + 15)/2$$
$$= 10$$

Variance of x

$$Var(x) = (b - a)^2/12$$
$$= (15 - 5)^2/12$$
$$= 8.33$$

$$SD = \sqrt{8.33}$$



2) Normal Probability Distribution

The normal probability distribution is the most important distribution for describing a continuous random variable.

It is widely used in statistical inference.

Probably the most widely known and used of all distributions is the normal distribution. It fits many human characteristics, such as height, weight, length, speed, IQ, scholastic achievement, and years of life expectancy, among others. Like their human counterparts, living things in nature, such as trees, animals, insects, and others, have many characteristics that are normally distributed.

Many variables in business and industry also are normally distributed. Some examples of variables that could produce normally distributed measurements include the annual cost of household insurance, the cost per square foot of renting warehouse space, and managers' satisfaction with support from ownership on a five-point scale.

Normal Probability Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

where:

μ = mean

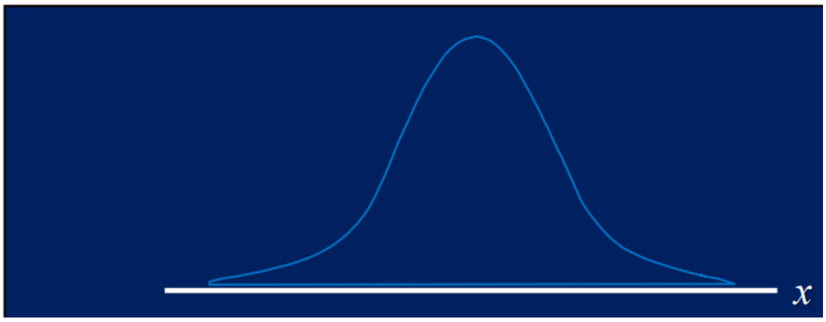
σ = standard deviation

π = 3.14159

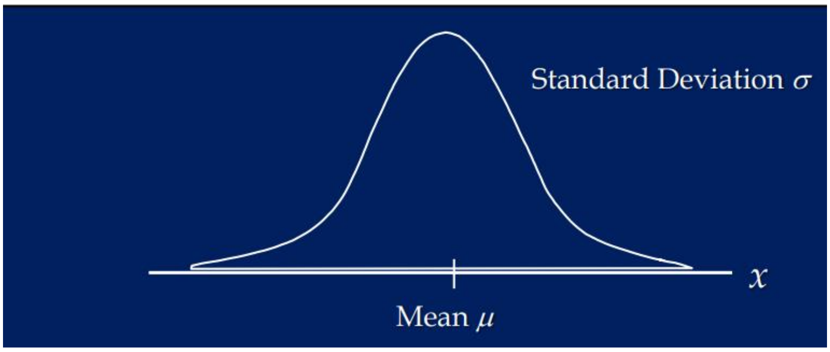
e = 2.71828

Characteristics

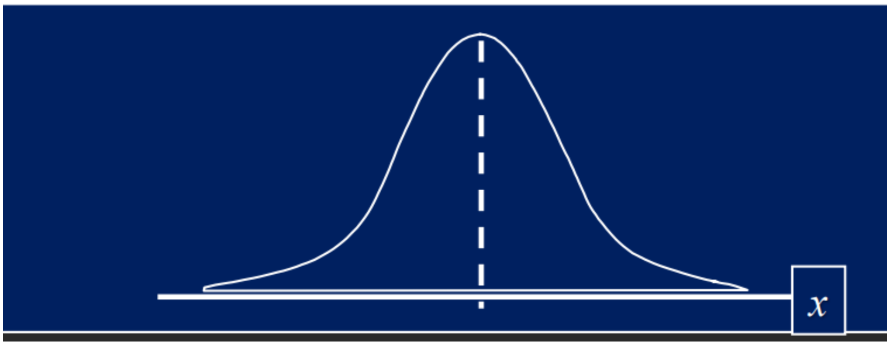
The distribution is symmetric; its skewness measure is zero.



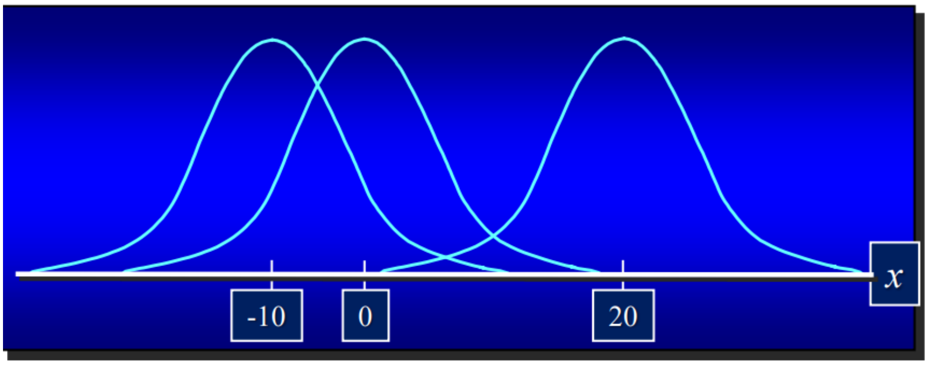
The entire family of normal probability distributions is defined by its mean μ and its standard deviation σ.



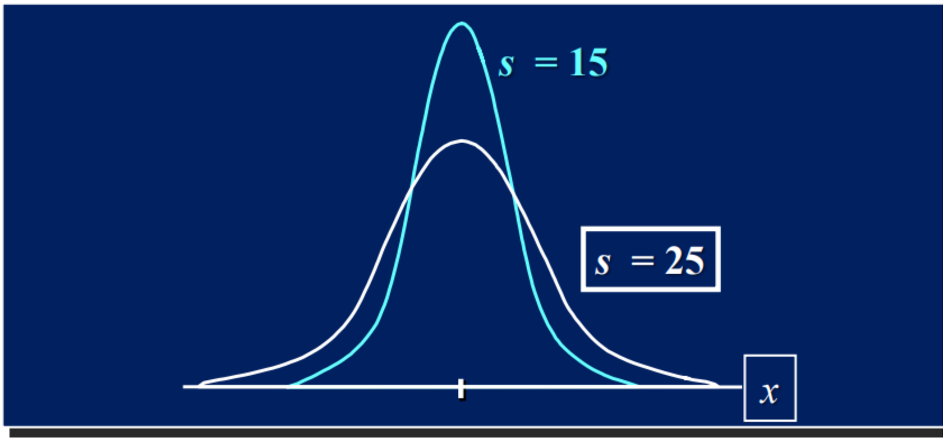
The highest point on the normal curve is at the mean, which is also the median and mode.



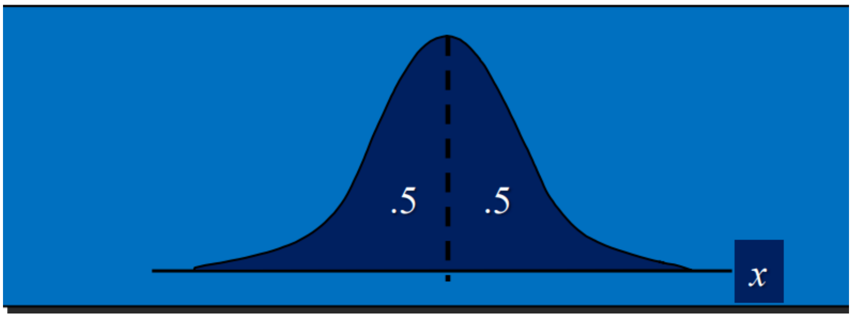
The mean can be any numerical value: negative, zero, or positive.



The standard deviation determines the width of the curve: **larger values result in wider, flatter curves**.



Probabilities for the normal random variable are given by <u>areas under the curve</u>. The total area under the curve is 1 (.5 to the left of the mean and .5 to the right).



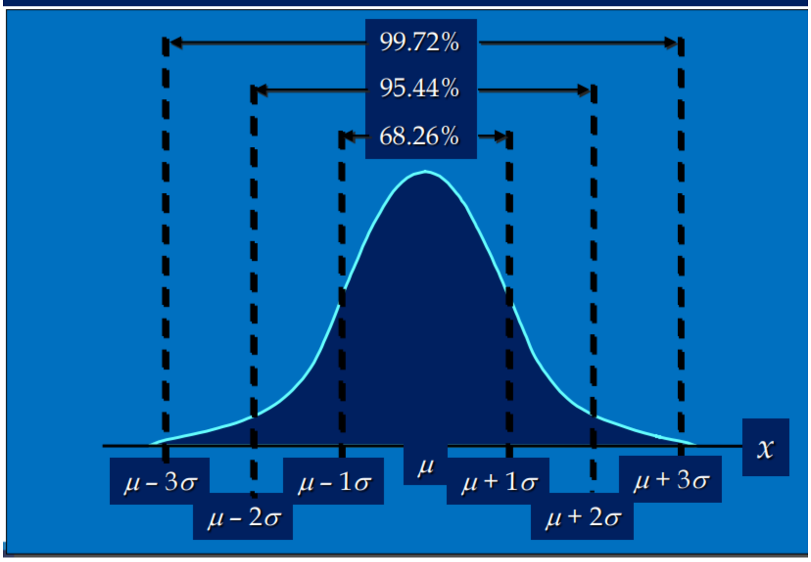
Characteristics

68.26% of values of a normal random variable are within +/- 1 standard deviation of its mean.

95.44% of values of a normal random variable are within +/- 2 standard deviations of its mean.

99.72% of values of a normal random variable are within +/- 3 standard deviations of its mean.

Characteristics



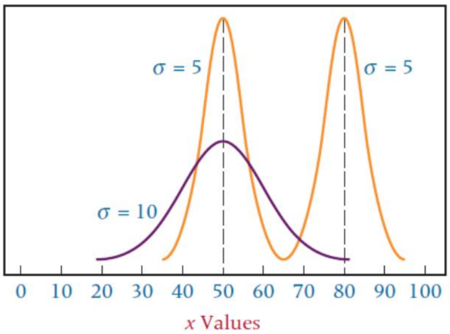
Standardized Normal Distribution

Every unique pair of and values defines a different normal distribution.

1. = 50 and = 5

2. = 80 and = 5

3. = 50 and = 10



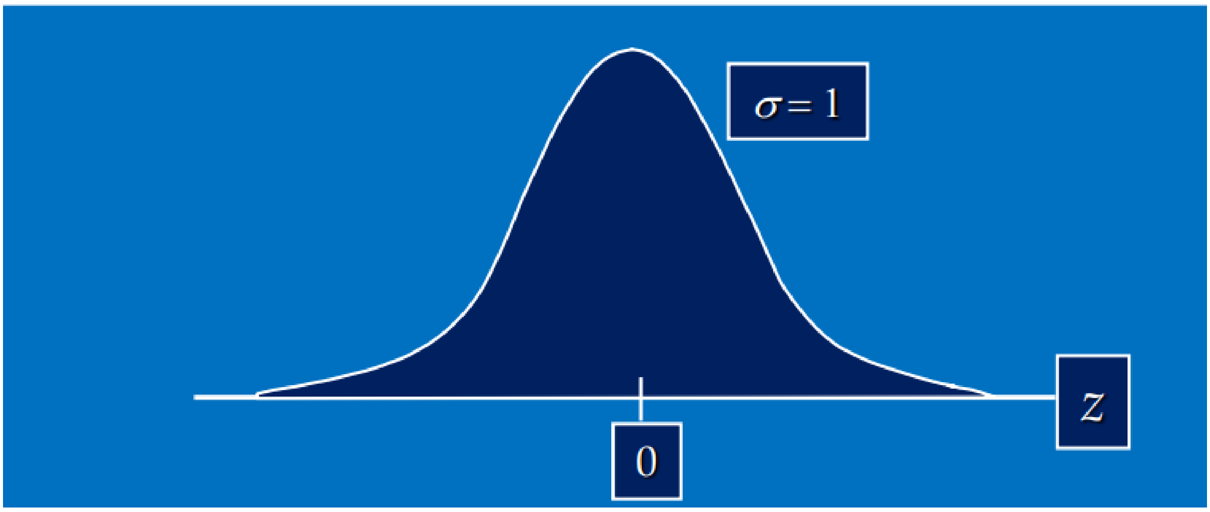
Note that every change in a parameter (Mean or SD) determines a different normal distribution. This characteristic of the normal curve (a family of curves) could make analysis by the normal distribution tedious because volumes of normal curve tables-one for each different combination of mean and SD - would be required. Fortunately, a mechanism was developed by which all normal distributions can be converted into a single distribution: the z distribution. This process yields the standardized normal distribution (or curve). The conversion formula for any x value of a given normal distribution follows.

$$z = \frac{x - \mu}{\sigma}$$

Standard Normal Probability Distribution

A random variable having a normal distribution with a mean of 0 and a standard deviation of 1 is said to have a standard normal probability distribution.

The letter *z* is used to designate the standard normal random variable.

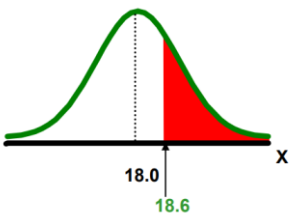


Converting to the Standard Normal Distribution

$$z = \frac{x - \mu}{\sigma}$$

We can think of *z* as a measure of the **number of standard deviations *x* is from *μ*.**

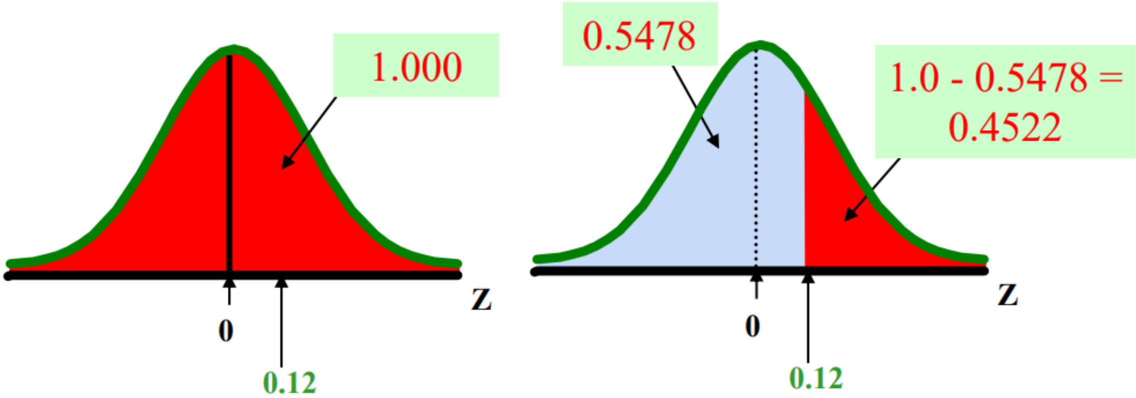
a)Suppose X is normal with mean 18.0 and standard deviation 5.0. Find P(X > 18.6).



Finding Normal
Upper Tail Probabilities

- Now Find P(X > 18.6)…

$$P(X > 18.6) = P(Z > 0.12) = 1.0 - P(Z \le 0.12)$$
$$= 1.0 - 0.5478 = \boxed{0.4522}$$



Use formula of z=> $z = \dfrac{x-\mu}{\sigma}$

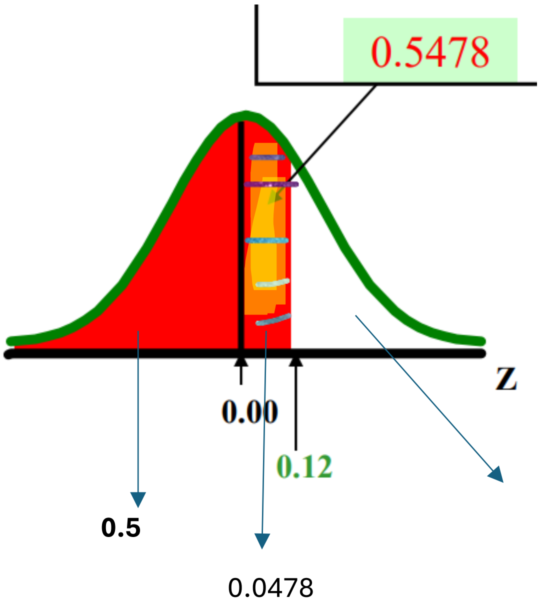
$$Z = \frac{X-\mu}{\sigma} = \frac{18.6-18.0}{5.0} = 0.12$$

Use z table to calculate z score

STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998



Row=0.1 & Colum=0.02 ➔0.0478

Left half curve value is 0.5 hence total shaded portion value is 0.5+0.0478=0.5478

We want area P(x>18.6)

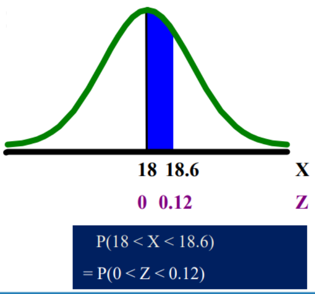
P(x>18.6) hence ➔ 1-0.5478= 0.4522.

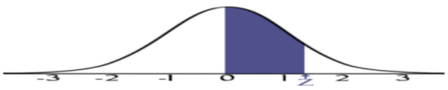
Suppose X is normal with mean 18.0 and standard deviation 5.0. Find P(18 < X < 18.6).

Calculate Z-values:

$$Z = \frac{X-\mu}{\sigma} = \frac{18-18}{5} = 0$$

$$Z = \frac{X-\mu}{\sigma} = \frac{18.6-18}{5} = 0.12$$





STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

Solution: Finding $P(0 < Z < 0.12)$

Standardized Normal Probability Table (Portion)

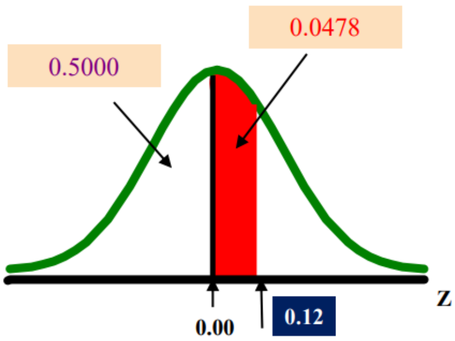
Z	.00	.01	.02
0.0	.5000	.5040	.5080
0.1	.5398	.5438	.5478
0.2	.5793	.5832	.5871
0.3	.6179	.6217	.6255

$P(18 < X < 18.6)$

$= P(0 < Z < 0.12)$

$= P(Z < 0.12) - P(Z \le 0)$

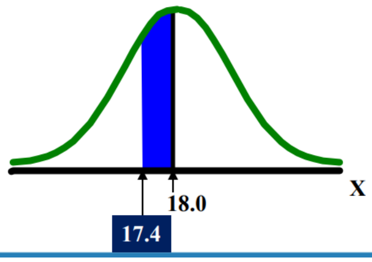
$= 0.5478 - 0.5000 = 0.0478$



Probabilities in the *Lower Tail*

Suppose X is normal with mean 18.0 and standard deviation 5.0.

Now Find P(17.4 < X < 18)



$$P(17.4 < X < 18)$$
$$= P(-0.12 < Z < 0)$$
$$= P(Z < 0) - P(Z \le -0.12)$$
$$= 0.5000 - 0.4522 = \boxed{0.0478}$$

The Normal distribution is symmetric, so this probability is the same as P(0 < Z < 0.12)

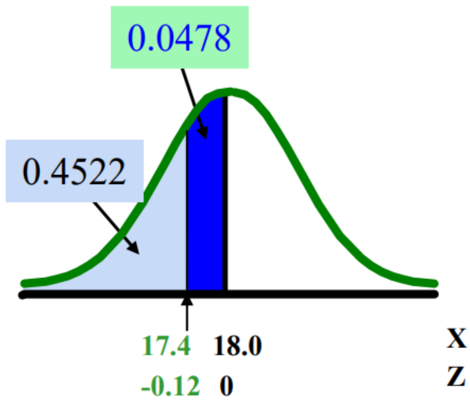
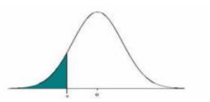
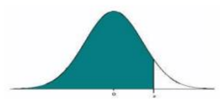


Table of Standard Normal Probabilities for Negative Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Note that the probabilities given in this table represent the area to the LEFT of the z-score.
The area to the RIGHT of a z-score = 1 – the area to the LEFT of the z-score

Refer these chart for checking z values