Example: - A binary communication system transmits and also receives data as '0' and '1'. Due to noise a transmitted '0' is sometimes received as '1' and a transmitted '1' is some times received as '0'. The probability that a transmitted '0' is correctly received as 0 is 0.95 and probability that a transmitted '1' is correctly received as '1' is 0.90, probability of transmitting 0 is 0.45. If a signal is sent find,

(i) probability that a '1' is received (ii) probability that a '0' is received
(iii) Probability that a '1' is transmitted, given that a '1' is received
(iv) Probability that a '0' is transmitted, given that a '0' is received

Solution

Let T_0 be the event of transmitting zero
Let T_1 " " " " " one
Let R_0 be the event of receiving zero
Let R_1 " " " " " one

By given $P(R_0/T_0) = 0.95, \therefore P(R_1/T_0) = 0.05$ & $P(T_0) = 0.45$
$P(R_1/T_1) = 0.90, \quad P(R_0/T_1) = 0.10 \quad P(T_1) = 0.55$

$P(R_1) = P[R_1 \cap (T_0 \cup T_1)] = P[(R_1 \cap T_0) \cup (R_1 \cap T_1)] = P(R_1 \cap T_0) + P(R_1 \cap T_1)$

$\therefore P(R_1) = P(T_0)\, P(R_1/T_0) + P(T_1) \cdot P(R_1/T_1) = (0.45 \times 0.05) + (0.55 \times 0.90) =$

$P(R_1) = \frac{207}{400} = 0.5175$

$P(R_0) = P[R_0 \cap (T_0 \cup T_1)] = P[(R_0 \cap T_0) \cup (R_0 \cap T_1)] = P(R_0 \cap T_0) + P(R_0 \cap T_1)$

$= P(T_0)\, P(R_0/T_0) + P(T_1) P(R_0/T_1) = (0.45 \times 0.95) + (0.55 \times 0.10)$

$\therefore P(R_0) = \frac{193}{400} = 0.4825$

$$P(T_1/R_1) = \frac{P(T_1 \cap R_1)}{P(R_1)} = \frac{P(T_1) \cdot P(R_1/T_1)}{P(R_1)} = \frac{0.55 \times 0.90}{0.5175} = \frac{22}{23} = 0.9565$$

$$P(T_0/R_0) = \frac{P(T_0 \cap R_0)}{P(R_0)} = \frac{P(T_0) \cdot P(R_0/T_0)}{P(R_0)} = \frac{(0.45 \times 0.95)}{0.4825} = \frac{171}{193} = 0.8860$$

Example ②: A given lot of I.C. chips contains 3% defective chips. Each chip is tested before delivery. The tester itself is not totally reliable, such that p (tester shows the chip is good / chip is actually good) = 0.96 and p(tester shows chip is defective / chip is actually defective) = 0.94. If a tested chip is shown to be defective, what is the probability that it is actually defective

Solution Let D be the event that chip is defective

$\therefore P(D) = 3\% = 0.03, \quad P(\bar{D}) = 0.97$

Let T be the event that tester shows chip is good

$\therefore \bar{T}$ " " " " " " is not good

By given $P[T/\bar{D}] = 0.96, \quad P(\bar{T}/D) = 0.94$

$\therefore P(\bar{T}/\bar{D}) = 0.04 \quad P(T/D) = 0.06$

$P(\bar{T}) = P[\bar{T} \cap (D \cup \bar{D})] = P[(\bar{T} \cap D) \cup (\bar{T} \cap \bar{D})] = P(\bar{T} \cap D) + P(\bar{T} \cap \bar{D})$

$= P(D)P(\bar{T}/D) + P(\bar{D})P(\bar{T}/\bar{D}) = (0.03 \times 0.94) + (0.97 \times 0.04)$

$P(\bar{T}) = \frac{67}{1000} = 0.067$

$$P(D/\bar{T}) = \frac{P(D \cap \bar{T})}{P(\bar{T})} = \frac{P(D)P(\bar{T}/D)}{P(\bar{T})} = \frac{0.03 \times 0.94}{0.067} = \frac{141}{335} = 0.4209$$

Example ③: A manufacturing firm produces steel pipes in three plants with daily production of 500, 1000, & 2000 units. According to past experience it is known that the fraction of defective output produced by the three plants are respectively 0.005, 0.008 and 0.010. If pipe is selected at random from a day's output and was found to be defective what is the probability that it came from the first plant

Solution: Let A be the event of getting production from first plant

Let B " " " " " " second "

Let C " " " " " " third "

$\therefore P(A) = \frac{500}{3500} = \frac{1}{7}, P(B) = \frac{1000}{3500} = \frac{2}{7}, P(C) = \frac{2000}{3500} = \frac{4}{7}$

Let H be the of getting defective output

By given $P(H/A) = 0.005 \quad P(H/B) = 0.008, \; P(H/C) = 0.010$

$\therefore P(H) = P[H \cap (A \cup B \cup C)] = P[(H \cap A) \cup (H \cap B) \cup (H \cap C)] = P[H \cap A] + P[H \cap B] + P[H \cap C]$

$\therefore P(H) = P(A)P(H/A) + P(B)P(H/B) + P(C)P(H/C) = \cancel{0.005 \times}$

$P(H) = (\frac{1}{7} \times 0.005) + (\frac{2}{7} \times 0.008) + (\frac{4}{7} \times 0.010) = \frac{1}{7}(0.005 + 0.016 + 0.040)$

$\therefore P(H) = \frac{61}{7000} = 0.00871429$

$$\therefore P(A/H) = \frac{P(A \cap H)}{P(H)} = \frac{P(A) \cdot P(H/A)}{P(H)} = \frac{(\frac{1}{7} \times 0.005)}{61/7000} = \frac{5}{61} = 0.081967$$

$\therefore P(H/A) = 0.081967$

Example (4) In a bolt factory, machines A, B, C produce respectively 25%, 35% and 40% of their output 5%, 4% and 2% are defective. A bolt is drawn at random from a days production and is found to be defective, what is the probability that it was produced by machines A, B, C ?

Solution

Let A be the that bolt is produced by machine A

Let B " " " " " " " B

Let C " " " " " " " C

$\therefore P(A) = 25\% = 0.25,\ P(B) = 35\% = 0.35,\ P(C) = 40\% = 0.40$

Let H be the event that bolt produced by machine is defective

$\therefore$ By given $P(H/A) = 5\% = 0.05,\ P(H/B) = 4\% = 0.04,\ P(H/C) = 2\% = 0.02$

$\therefore P(H) = P[H \cap (A \cup B \cup C)] = P[(H\cap A) \cup (H \cap B) \cup (H \cap C)]$

$\therefore P(H) = P[H \cap A] + P[H\cap B] + P[H \cap C] = P(A)P(H/A) + P(B)P(H/B) + P(C)P(H/C)$

$\therefore P(H) = (0.25 \times 0.05) + (0.35 \times 0.04) + P(0.40 \times 0.02) = \frac{69}{2000} = 0.0345$

$\therefore P(A/H) = \frac{P(A\cap H)}{P(H)} = \frac{P(A)\,P(H/A)}{P(H)} = \frac{(0.25\times 0.05)}{0.0345} = \frac{25}{69} = 0.362319$

$\therefore P(B/H) = \frac{P(B \cap H)}{P(H)} = \frac{P(B)\,P(H/B)}{P(H)} = \frac{(0.35 \times 0.04)}{0.0345} = \frac{28}{69} = 0.405797$

$\therefore P(C/H) = \frac{P(C\cap H)}{P(H)} = \frac{P(C)\,P(H/C)}{P(H)} = \frac{(0.40\times 0.02)}{0.0345} = \frac{16}{69} = 0.231884$

Example (5) A bag contains two dice, one of which is regular and fair and other is false with number 6 on all its faces. A dice was drawn from the bag and tossed. it gave 6, what is the probability that the dice obtained was the false one ?

Solution: Let A be event of getting fair die &

Let B " " " false die

Let H be event of getting 6 on any die

$\therefore P(A) = \frac{1}{2},\ P(B) = \frac{1}{2}\ P(H/A) = \frac{1}{6},\ P(H/B) = 1$

$\therefore P(H) = P[H \cap (A \cup B)] = P[(H\cap A) \cup (H \cap B)] = P(H\cap A) + P(H \cap B)]$

$\therefore P(H) = P(A)\cdot P(H/A) + P(B)\,P(H/B) = \frac{1}{2}\cdot\frac{1}{6} + \frac{1}{2}\cdot 1 = \frac{1}{2}\left(\frac{1}{6}+1\right) = \frac{7}{12}$

$\therefore P(B/H) = \frac{P(B\cap H)}{P(H)} = \frac{P(B)\,P(H/B)}{P(H)} = \frac{\frac{1}{2}\cdot 1}{7/12} = \frac{1}{2}\,\frac{12}{7} = \frac{6}{7}$

Example ⑥ A bag contain two coines one of which is a false coin with head on both side and the other is true coin. when a coin taken at random from the bag is tossed, it gave a head. what is the probability that the true coin was take & tossed

Solution Let A be the event of getting ~~coin~~ true coin

Let B " " " " " false coin

$\therefore p(A) = 1/2, \quad p(B) = \frac{1}{2}$

Let H be the event of getting head

$\therefore p(H/A) = \frac{1}{2}, \quad p(H/B) = 1$

~~$\therefore p(H \cap A) = p(A) \cdot p(H/A) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$~~

$p(H) = p[H \cap (A \cup B)] = p[(H \cap A) \cup (H \cap B)] = p(H \cap A) + p(H \cap B)$

$\therefore p(H) = p(A) \cdot p(H/A) + p(B)\, p(H/B) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot 1 = \frac{1}{4}(1+2) = 3/4$

Now $p(A/H) = \frac{p(H \cap A)}{p(H)} = \frac{p(A)\, p(H/A)}{p(H)} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{3/4} = \frac{1/4}{3/4} = \frac{1}{3}$

Example ⑦ : if A_1, A_2 are two mutually exclusive and exhaustive events of Sample space S and if B is another event defined on S, Such that $p(A_1) = 0.6, p(A_2) = 0.4$ and $p(B/A_1) = 0.4, p(B/A_2) = 0.5$ find $p(B) = ?$

Solution: -

$p(B) = p(\cancel{B \cap A_1} = p[B \cap (A_1 \cup A_2)] = p[(B \cap A_1) \cup (B \cap A_2)]$

$\therefore p(B) = p(B \cap A_1) + p(B \cap A_2) = p(A_1)\, p(B/A_1) + p(A_2)\, p(B/A_2)$

$\therefore p(B) = (0.6 \times 0.4) + (0.4 \times 0.5) = 0.24 + 0.20 = 0.44$

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Example 1 There are in a bag three true coins and one false coin with head on both sides. A coin is chosen is chosen at random and tossed four times. If head occures all the four times, what is the probability that the false coin was chosen and used?

Solution: Let F be the event of getting false coin

T be the event of getting true coin

H be the event of getting Head in four throws

By given there are four coins of which three are true & one is false

$P(F) = \frac{1}{4}$. $P(T) = \frac{3}{4}$

Now $P(H/F) = P($getting 4 head due to false coin$)$

$= 1 \cdot 1 \cdot 1 \cdot 1 = 1$

$P(H/T) = P($getting 4-head due to true coin$)$

$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

Now $P(H) = P[H \cap (T \cup F)] = P[(H \cap T) \cup (H \cap F)] = P(H \cap T) + P(H \cap F)$

$\therefore P(H) = P(T)P(H/T) + P(F)P(H/F) = \frac{3}{4} \cdot \frac{1}{16} + \frac{1}{4} \cdot 1 = \frac{3}{64} + \frac{1}{4}$

$P(H) = \frac{3}{64} + \frac{16}{64} = \frac{19}{64}$

Now $P(F/H) = \frac{P(F \cap H)}{P(H)} = \frac{P(F) \cdot P(H/F)}{P(H)} = \frac{\frac{1}{4} \cdot 1}{19/64}$

$= \frac{1}{4} \times \frac{64}{19} = \frac{16}{19}$

Example-2 A coin is tossed. If it turns up head two balls are drawn from urn A otherwise two balls are drawn from urn B. Urn A contain 3 Black and 5 white balls urn B contains 7 black and one white ball. What is the probability that urn A was used, given that both balls drawn are black?

Solution let U_1 be the event of selecting urn A

let U_2 be the event of selecting urn B

$\therefore P(U_1) = \frac{1}{2}$ $P(U_2) = \frac{1}{2}$

$U_1 = A$

3 - Black
5 - white

$U_2 = B$

7 - Black
1 - white

let B be the event of selecting two black balls

$\therefore P(B/U_1) = \frac{{}^3C_2}{{}^8C_2} = \frac{3\times 2}{8\times 7} = \frac{6}{56}$

$P(B/U_2) = \frac{{}^7C_2}{{}^8C_2} = \frac{7\times 6}{8\times 7} = \frac{42}{56}$

$P(B) = P[B\cap(U_1\cup U_2)] = P[(B\cap U_1)\cup(B\cap U_2)] = P[B\cap U_1] + P[B\cap U_2]$

$P(B) = P(U_1)\cdot P(B/U_1) + P(U_2)\cdot P(B/U_2) = \frac{1}{2}\cdot\frac{6}{56} + \frac{1}{2}\cdot\frac{42}{56} = \frac{6+42}{112}$

$P(B) = \frac{48}{112}$

$\therefore P(U_1/B) = \frac{P(U_1\cap B)}{P(B)} = \frac{P(U_1)\cdot P(B/U_1)}{P(B)} = \frac{\frac{1}{2}\cdot\frac{6}{56}}{\frac{48}{112}} = \frac{\frac{6}{112}}{\frac{48}{112}}$

$\therefore P(U_1/B) = \frac{6}{48} = \frac{1}{8}$

Example (3) In a certain test there are multiple choice questions. there are four possible answers to each question and one of them is correct. An intelligent student can solve 90% questions correctly by reasoning and for the remaining 10% question he gives answer by guessing. A weak student can solve 90% questions correctly by reasoning and for the remaining 80% question he gives answer by guessing. An intelligent student gets the correct answer, what is the probability that he was guessing

Solution Let R be the event that answer with reasoning

$$\therefore p(R) = \frac{90}{100} = \frac{9}{10}$$

G be the event that answer with guessing

$$p(G) = \frac{10}{100} = \frac{1}{10}$$

Let C be the event that answer is correct

$$p(C/R) = 1$$

$$p(C/G) = \frac{1}{4}$$

$$p(C) = p[C \cap (R \cup G)] = p[(C \cap R) \cup (C \cap G)] = p(C \cap R) + p(C \cap G)$$

$$= p(R)\,p(C/R) + p(G)\,p(C/G) = \frac{9}{10} \cdot 1 + \frac{1}{10} \cdot \frac{1}{4}$$

$$= \frac{9}{10} + \frac{1}{40} = \frac{36+1}{40} = \frac{37}{40}$$

$$p(G/C) = \frac{p(G \cap C)}{p(C)} = \frac{p(G)\,p(C/G)}{p(C)} = \frac{\frac{1}{10} \cdot \frac{1}{4}}{\frac{37}{40}} = \frac{\frac{1}{40}}{\frac{37}{40}}$$

$$\therefore p(G/C) = \frac{1}{37}$$

Example 4:- A certain test for a particular cancer is known to be 95% accurate. A person submits to the ~~test~~ test and the result is positive. Suppose that a person comes from a population of 1,00,000 where 2000 people suffer from that disease. What can we conclude about the probability that the person under test has that particular cancer?

Solution: A be the event that person has cancer

$\bar{A}$ be the event that person has no cancer

$\cancel{P(T}$ $P(A) = \frac{2000}{1000000} = \frac{2}{1000} = \cancel{0.002 : P(\bar{A}) = 0.998}$

$\therefore 0.02 \quad P(\bar{A}) = 0.98$

$\because$ T be the event of test is correct

By given

$P(T/A) = 95\% = 0.95 \quad \therefore P(T/\bar{A}) = 0.05$

$$P(T) = P[T \cap (A \cup \bar{A})] = P[(T \cap A) \cup (T \cap \bar{A})]$$

$$= P(T \cap A) \cup P(T \cap \bar{A}) = P(A)\,P(T/A) + P(\bar{A})\,P(T/A)$$

$$= (0.02)(0.95) + (0.98)(0.05)$$

$$=$$

$$P(A/T) = \frac{P(A \cap T)}{P(T)} = \frac{P(A)\,P(T/A)}{P(T)} = \frac{0.02 \times 0.95}{(0.02 \times 0.95) + (0.9}$$

$$= \frac{\frac{2}{100} \times \frac{95}{100}}{\left(\frac{2}{100} \times \frac{95}{100}\right) + \left(\frac{98}{100} \times \frac{5}{100}\right)}$$

$$= \frac{190}{190 + 490} = \frac{190}{680}$$

$$= \frac{19}{68}$$

Example 5: A bag contain 7-red and 3-Black balls and another bag contains 4-red and 5 black balls, one ball is transferred from the first bag to the second bag and then a ball is drawn from the second bag. If this ball happen to be red, find the probability that a black ball was transferred.

Solution:

Bag-B_1: 7-R, 3-B

Bag-B_2: 4-R, 5-B

let T_R be the event of transferring Red ball from bag B_1 to bag B_2

T_B be the event of transferring Black ball from bag B_1 to bag B_2

$R \to$ be the event of selecting a red ball from bag B_2

$B \to$ be the event of selecting a black ball from bag B_2

$$P(T_R) = \frac{{}^7C_1}{{}^{10}C_1} = \frac{7}{10} \qquad P(B/T_R) = \frac{{}^5C_1}{{}^{10}C_1} = \frac{5}{10}, \quad P(R/T_R) = \frac{{}^5C_1}{{}^{10}C_1} = \frac{5}{10}$$

$$P(T_B) = \frac{{}^3C_1}{{}^{10}C_1} = \frac{3}{10} \qquad P(B/T_B) = \frac{{}^6C_1}{{}^{10}C_1} = \frac{6}{10}, \quad P(R/T_B) = \frac{{}^4C_1}{{}^{10}C_1} = \frac{4}{10}$$

$$P(R) = P[R \cap (T_R \cup T_B)] = P[(R \cap T_R) \cup (R \cap T_B)] = P(R \cap T_R) + P(R \cap T_B)$$

$$P(R) = P(T_R)\,P(R/T_R) + P(T_B).P(R/T_B) = \left(\frac{7}{10} \times \frac{5}{10}\right) + \left(\frac{3}{10} \cdot \frac{4}{10}\right)$$

$$P(R) = \frac{35}{100} + \frac{12}{100} = \frac{47}{100}$$

$$P(T_B/R) = \frac{P(T_B \cap R)}{P(R)} = \frac{P(T_B)\,P(R/T_B)}{P(R)} = \frac{\frac{3}{10} \cdot \frac{4}{10}}{\frac{47}{100}} = \frac{12/100}{47/100}$$

$$P(T_B/R) = \frac{12}{47}$$

Example-6: A man speaks truth 3-times out of 5. when a die is tossed he states that it gives an ace. what is the probability that this event has actually happned

Solution Let T be the event that man speak truth

$\therefore p(T) = \frac{3}{5} \quad \therefore p(\bar{T}) = 2/5$

Let A be the event of getting ace if he speak true

$p(A/T) = \frac{1}{6}, \quad p(A/\bar{T}) = 5/6$

$P(A) = P[A \cap (T \cup \bar{T})] = P[(A \cap T) \cup (A \cap \bar{T})]$

$= P[A \cap T] + P[A \cap \bar{T}] = P[T]\,P(A/T) + P[\bar{T}]\,P(A/\bar{T})$

$P(A) = \left(\frac{3}{5} \cdot \frac{1}{6}\right) + \left(\frac{2}{5} \cdot \frac{5}{6}\right) = \frac{3}{30} + \frac{10}{30} = \frac{13}{30}$

$P(T/A) = \frac{P(T \cap A)}{P(A)} = \frac{P(T)\,P(A/T)}{P(A)} = \frac{\frac{3}{5} \cdot \frac{1}{6}}{\frac{13}{30}} = \frac{\frac{3}{30}}{\frac{13}{30}}$

$P(T/A) = \frac{3}{13}$

E

Example 7 A lot of IC chips is known to contain 3% defective chips. Each chip is tested before delivery but the tester is not completely reliable. it is known that

p(Tester Says the chip is good / The chip is actually good) = 0.95

& P(Tester Says the chip is defective / the chip is actually ~~det~~ defectively) = 0.96

if a tested chip is declared defective by the tester. what is the probability that it is actually defective

Solution

Let D be the defective chip

$\bar{D}$ be the non-defective chip

T be the event that tester says chip is good

$\bar{T}$ be the event that tester say ship is not good

By given $P(D) = 3\% = 0.03$, $P(\bar{D}) = 0.97$

$P(T/\bar{D}) = 0.95$, $\cancel{P(\bar{T}/\bar{D}) = 0.96}$ $P(\bar{T}/D) = 0.96$

$\cancel{P(D) = P[D \cap (T \cup \bar{T})]} = P[(D \cap T) \cup (D \cap \bar{T})] = P[D \cap T] + P[D \cap \bar{T}]$

$P(\bar{T}) = P[\bar{T} \cap (D \cup \bar{D})] = P[(\bar{T} \cap D) \cup (\bar{T} \cap \bar{D})] = P(\bar{T} \cap D) + P(\bar{T} \cap \bar{D})$

$P(\bar{T}) = P(D)\,P(\bar{T}/D) + P(\bar{D})\,P(\bar{T}/\bar{D})$

$= (0.03 \times 0.96) + (0.97)(1 - P(T/\bar{D}))$

$= (0.03 \times 0.96) + (0.97 \times (1 - 0.95))$

$= (0.03 \times 0.96) + (0.97 \times 0.05) = 0.\cancel{3365}$

$= (0.03 \times 0.96) + (0.97 \times 0.05) = 0.\cancel{3365}$

$$P(D/\bar{T}) = \frac{P(D \cap \bar{T})}{P(\bar{T})} = \frac{P(D)\,P(\bar{T}/D)}{P(\bar{T})} = \cancel{0.3365}\ 0.0773$$

$$P(D/\bar{T}) = \frac{0.03 \times 0.96}{0.3365} = 0.3726$$

Example 8: A certain test for a particular cancer is know to be 95% accurate. A person submits to the test and the result is positive. Suppose that a person comes from a population of

Example :- A binary communication transmitter sends data as one of two types of signals denoted by 0 or 1. Due to noise. Sometimes a transmitted 1 is received as 0 & vice versa.

If the probability that a 0 is correctly received as 0 is 0.9 and the probability that a transmitted 1 is correctly received as 1 is 0.8 and if the probability of transmitting 0 is 0.45, find the probability that (i) a 1 is received (ii) 0 is received (iii) a 1 was transmitted given that 1 was received (iv) a 0 was transmitted given that a 0 was Received (v) the error has occured

Solution let T_0 be the event that 0 is transmitted

T_1 be " " " 1 is transmitted

R_0 be the event that 0 was Received

R_1 be the " " 1 was Received

By given $P(R_0/T_0)=0.9$, $P(R_1/T_0)=0.1$, $P(R_1/T_1)=0.8$, $P(R_0/T_1)=0.2$

$P(T_0)=0.45$, $P(T_1)=0.55$

$P(R_0)=P[R_0\cap(T_0\cup T_1)]=P[(R_0\cap T_0)\cup(R_0\cap T_1)]$

$=P(R_0\cap T_0)+P(R_0\cap T_1)=P(T_0)\,P(R_0/T_0)+P(T_1)\,P(R_0/T_1)$

$P(R_0)=[0.45\times0.9]+[0.55\times0.2]=0.515$

$P(R_1)=P[R_1\cap(T_0\cup T_1)]=P[(R_1\cap T_0)\cup(R_1\cap T_1)]=P(R_1\cap T_0)+P(R_1\cap T_1)$

$=P(T_0)\,P(R_1/T_0)+P(T_1)\,P(R_1/T_1)=(0.45\times0.1)+(0.55\times0.8)$

$P(R_1)=0.485$

(iii) $P(T_1/R_1)=\dfrac{P(T_1\cap R_1)}{P(R_1)}=\dfrac{P(T_1)\cdot P(R_1/T_1)}{P(R_1)}=\dfrac{0.55\times0.8}{0.485}=0.907216$

(iv) $P(T_0/R_0)=\dfrac{P(T_0\cap R_0)}{P(R_0)}=\dfrac{P(T_0)\cdot P(R_0/T_0)}{P(R_0)}=\dfrac{0.45\times0.9}{0.515}=0.786408$

(v) $P(\text{error occures})=P[(R_0\cap T_1)\cup(R_1\cap T_0)]=P(R_0\cap T_1)+P(R_1\cap T_0)$

$=P(T_1)\,P(R_0/T_1)+P(T_0)\,P(R_1/T_0)$

$=(0.55\times0.2)+(0.45\times0.1)=0.155$

Example 9 :- A box contains three biased coins A, B & C. The probability that a head will result when A is tossed is $\frac{1}{3}$ when B is tossed, it is 2/3 and when C is tossed, it is 3/4

(a) If one of the coins is chosen at random and is tossed 3 times head resulted twice and tail once. what is the probability that the coin chosen was A?

(b) what is the probability of getting head when a coin selected at random is tossed once.

(c) what is the probability that we would get two heads in the first three tosses and a head again in the fourth toss with the same coin?

Solution (a)

Let A be event of ~~getting~~ selecting coin A $\therefore P(A) = \frac{1}{3}$

B " " " " " B $\therefore P(B) = \frac{1}{3}$

C " " " " " C $\therefore P(C) = \frac{1}{3}$

Let H be the event of getting two head & one tail if we toss coin A or B or C in three tosses

$\therefore P(H/A) = {}^3C_2 \left(\frac{1}{3}\right)^2 (2/3)^1 = \frac{2}{9}$

$P(H/B) = {}^3C_2 \cdot \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^1 = \frac{4}{9}$

$P(H/C) = {}^3C_2 (3/4)^2 (1/4)^1 = \frac{27}{64}$

$\therefore P(H) = P[H \cap (A \cup B \cup C)] = P[(H \cap A) \cup (H \cap B) \cup (H \cap C)] = P(A)P(H/A) + P(B)P(H/B) + P(C)P(H/C)$

$= P(H \cap A) + P(H \cap B) + P(H \cap C) =$

$P(H) = \frac{1}{3}\left(\frac{2}{9}\right) + \frac{1}{3}\left(\frac{4}{9}\right) + \frac{1}{3}\left(\frac{27}{64}\right) = \frac{209}{576}$

Now ~~P(H/A) = P(H∩A)~~

$P(A/H) = \frac{P(A \cap H)}{P(H)} = \frac{P(A)\,P(H/A)}{P(H)} = \frac{\frac{1}{3}\left(\frac{2}{9}\right)}{209/576} = \frac{126}{627} = 0.204147$

(b) Let H be the event of getting head

$\therefore P(H/A) = \frac{1}{3}$ $P(H/B) = 2/3$, $P(H/C) = 3/4$

$\therefore P(H) = P(H \cap (A \cup B \cup C)) = P[(H \cap A) \cup (H \cap B) \cup (H \cap C)] = P(H \cap A) + P(H \cap B) + P(H \cap C)$

$P(H) = P(A)P(H/A) + P(B)P(H/B) + P(C)P(H/C) = \frac{1}{3}\frac{1}{3} + \frac{1}{3}\frac{2}{3} + \frac{1}{3}\frac{3}{4} = \frac{7}{12} = 0.5833$

(c) Let H be the event of getting two head in first three tosses & head in 4th toss

~~$\therefore P(H/A) = {}^3C_2 (1/3)^2 (2/3)^1 \times \frac{1}{3} = \frac{2}{27}$~~ ~~$P(H/B) = {}^3C_2 (2/3)^2 (1/3) \cdot \frac{2}{3} = \frac{8}{27}$~~

~~$P(H/C) = {}^3C_2 (3/4)^2 (1/4)^1 (3/4) = 81/256$~~

$P(H \cap A) = P(A)P(H/A) = \frac{1}{3} \cdot {}^3C_2 \left(\frac{1}{3}\right)^2 (2/3)^1 (1/3)$

$P(H \cap B) = P(B)P(H/B) = \frac{1}{3} \cdot {}^3C_2 (2/3)^2 (1/3)(2/3)$

$P(H \cap C) = P(C)P(H/C) = \frac{1}{3} \, {}^3C_2 (3/4)^2 (1/4)(3/4)$

Example: 10 A bag contains five balls, the colour of which are not known. Two balls were drawn from the bag and they were found to be white. what is the probability that all balls are white

Solution: $\because$ bag contain 5 -balls and we want two white balls

$\therefore$ bag may contain 2, 3, 4, or 5 white balls

Let A_1 be the event that bag contain 2 white balls $\therefore P(A_1) = \frac{1}{4}$

Let A_2 " " " " " " 3 " " $P(A_2) = \frac{1}{4}$

Let A_3 " " " " " " 4 " " $P(A_3) = \frac{1}{4}$

Let A_4 " " " " " " 5 " " $P(A_4) = \frac{1}{4}$

Let w be the event of selecting 2 white balls

$\therefore P(w/A_1) = \frac{2c_2}{5c_2}$, $P(w/A_2) = \frac{3c_2}{5c_2}$, $P(w/A_3) = \frac{4c_2}{5c_2}$, $P(w/A_4) = \frac{5c_2}{5c_2}$

$$P(w) = P[w \cap (A_1 \cup A_2 \cup A_3 \cup A_4)]$$

$$= P[(w \cap A_1) \cup (w \cap A_2) \cup (w \cap A_3) \cup (w \cap A_4)]$$

$$= P[w \cap A_1] + P[w \cap A_2] + P[w \cap A_3] + P[w \cap A_4]$$

$$= P[A_1] P(w/A_1) + P(A_2) P(w/A_2) + P(A_3) P(w/A_3) + P(A_4) P(w/A_4)$$

$$= \frac{1}{3} \cdot \frac{2c_2}{5c_2} + \frac{1}{3} \cdot \frac{3c_2}{5c_2} + \frac{1}{3} \frac{4c_2}{5c_2} + \frac{1}{3} \frac{5c_2}{5c_2}$$

$$= \frac{1}{3\, 5c_2} [2c_2 + 3c_2 + 4c_2 + 5c_2] = \frac{2}{3}$$

$$\therefore P(A_4/w) = \frac{P(A_4 \cap w)}{P(w)} = \frac{P(A_4) P(w/A_4)}{P(w)} = \frac{\frac{1}{3} \cdot \frac{5c_2}{5c_2}}{2/3} = \frac{1/3}{2/3} = \frac{1}{2}$$

Example-12 There are three boxes containing respectively 1-white, 2-red 3-black balls, 2-white, 3-red, 1-black; 3-white, 1-Red, 2-black balls. A box is chosen at random and two Balls are drawn from it. The two balls are found to be one red and one whit. Find the probability that these have come from box 1, box 2 and box 3

Solution

Box - B_1	Box - B_2	Box - B_3
1-W	2-W	3-W
2-R	3-R	1-R
3-B	1-B	2-B

$\therefore P(B_1) = \frac{1}{3}$

$P(B_2) = \frac{1}{3}$

$P(B_3) = \frac{1}{3}$

let ~~to~~ D_{rw} be the event of drawing one-red & one-white ball

$$P(D_{rw}/B_1) = \frac{{}^1C_1 \times {}^2C_1}{{}^6C_2} = \frac{2\times}{6\times5/2} = \frac{4}{6\times5} = \frac{2}{15}$$

$$P(D_{rw}/B_2) = \frac{{}^2C_1 \times {}^3C_1}{{}^6C_2} = \frac{2\times3}{6\times5/2!} = \frac{2}{5} = \frac{6}{15}$$

$$P(D_{rw}/B_3) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{3}{6\times5/2} = \frac{1}{5} = \frac{3}{15}$$

$$\therefore P(D_{rw}) = P[D_{rw} \cap (B_1 \cup B_2 \cup B_3)] = P[(D_{rw}\cap B_1) \cup (D_{rw}\cap B_2) \cup (D_{rw} \cap B_3)]$$

$$= P(D_{rw}\cap B_1) + P(D_{rw}\cap B_2) + P(D_{rw}\cap B_3)$$

$$= P(B_1)P(D_{rw}/B_1) + P(B_2)P(D_{rw}/B_2) + P(B_3)P(D_{rw}/B_3)$$

$$= \frac{1}{3}\left(\frac{2}{15}\right) + \frac{1}{3}\left(\frac{6}{15}\right) + \frac{1}{3}\left(\frac{3}{15}\right) = \frac{1}{3}\left(\frac{11}{15}\right) = \frac{11}{45}$$

$$P(B_1/D_{rw}) = \frac{P(B_1 \cap D_{rw})}{P(D_{rw})} = \frac{P(B_1)P(D_{rw}/B_1)}{P(D_{rw})} = \frac{\frac{1}{3}\cdot\frac{2}{15}}{11/45} = \frac{2}{11}$$

$$P(B_2/D_{rw}) = \frac{P(B_2 \cap D_{rw})}{P(D_{rw})} = \frac{P(B_2)P(D_{rw}/B_2)}{P(D_{rw})} = \frac{\frac{1}{3}\cdot\frac{6}{15}}{11/15} = \frac{6}{11}$$

$$P(B_3/D_{rw}) = \frac{P(B_3 \cap D_{rw})}{P(D_{rw})} = \frac{P(B_3)P(D_{rw}/B_3)}{P(D_{rw})} = \frac{\frac{1}{3}\cdot\frac{3}{15}}{11/15} = \frac{3}{11}$$

Example 13 Three factories A, B, & C produce 30%, 50% and 20% of the total production of an item. out of their production 80%, 50% and 10% are defective. An item is chosen at random and found to be defective. find the probability that it was produced by the factor A

Solution

Let A be the event that item is produced by machine A

Let B " " " " " " B

Let C " " " " " " C

$\therefore P(A) = 30\% = \frac{30}{100}$, $P(B) = 50\% = \frac{50}{100}$ & $P(C) = 20\% = \frac{20}{100}$

Let D be the event that it produced by machine is defective

$\therefore P(D/A) = 80\% = \frac{80}{100}$, $P(D/B) = 50\% = \frac{50}{100}$, $P(D/C) = 10\% = \frac{10}{100}$

$\therefore P(D) = P[D \cap (A \cup B \cup C)] = P[(D\cap A) \cup (D \cap B) \cup (D \cap C)]$

$P(D) = P[D \cap A] + P[D \cap B] + P[D \cap C]$

$P(D) = P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)$

$= \frac{30}{100} \cdot \frac{80}{100} + \frac{50}{100} \cdot \frac{50}{100} + \frac{20}{100} \cdot \frac{10}{100} = \frac{\cancel{43}\ 51}{100}$

$\therefore P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{P(A)P(D/A)}{P(D)} = \cancel{\frac{\frac{30}{100} \cdot \frac{80}{100}}{43/100}} = \cancel{\frac{16/100}{43/100}}$

$\therefore \cancel{P(A/D) = \frac{16}{43}} = \frac{\frac{30}{100} \cdot \frac{80}{100}}{51/100} = \frac{24}{51} = 0.4706$

Example 14 For a certain binary communication channel, the probability that a transmitted '0' is received as '0' is 0.95 while the probability that a transmitted '1' is received as '1' is 0.90. If the probability of transmitting a '0' is 0.4. find the probability that (i) a '1' is received (ii) a '1' was transmitted given that 1 was Received (iii) (0) is recieved. (iv) (0) is transmitted given that (0) is recieved. (v) Getting error.

Solution:

Let R_0 be the event that '0' is received

Let R_1 " " " " 1 " "

Let T_0 be the event that 0 is transmitted

Let T_1 " " " " 1 is "

By given $P(R_0/T_0) = 0.95$ ∴ $P(R_1/T_0) = 0.05$

$P(R_1/T_1) = 0.90$ ∴ $P(R_0/T_1) = 0.10$

$P(T_0) = 0.4$ ∴ $P(T_1) = 0.6$

$$P(R_1) = P[R_1 \cap (T_0 \cup T_1)] = P[(R_1 \cap T_0) \cup (R_1 \cap T_1)]$$

$$= P(R_1 \cap T_0) + P(R_1 \cap T_1) = P(T_0)\,P(R_1/T_0) + P(T_1)\,P(R_1/T_1)$$

$$P(R_1) = (0.4 \times 0.05) + (0.6 \times 0.90) = \frac{14}{25} = 0.56$$

$$P(T_1/R_1) = \frac{P(T_1 \cap R_1)}{P(R_1)} = \frac{P(T_1)\,P(R_1/T_1)}{P(R_1)} = \frac{0.6 \times 0.90}{0.56}$$

$$= \frac{0.54}{0.56} = \frac{27}{28} = 0.9643$$

Example 15 In a bolt factory, machines A, B, C produce 25%, 35%, 40% of the total output and 5%, 4% & 2% of the output is defective respectively. A bolt is drawn at random. A bolt drawn at random and is found to be defective. what is the probability that it was ~~drawn~~ produced on machine B

Solution:

Let A be the event that bolt is produced by machine A

Let B " " " " " " " " B

Let C " " " " " " " " C

$\therefore P(A) = 25\% = \frac{25}{100} = 0.25$, $P(B) = 35\% = \frac{35}{100} = 0.35$

$P(C) = 40\% = \frac{40}{100} = 0.40$

Let D be the event that bolt produced by machine is defective

$\therefore P(D/A) = 5\% = 0.05$, $P(D/B) = 4\% = 0.04$, $P(D/C) = 2\% = 0.02$

$= 0.05$

$\therefore P(D) = P[D \cap (A \cup B \cup C)] = P[(D \cap A) \cup (D \cap B) \cup (D \cap C)]$

$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C) = P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)$

$\therefore P(D) = (0.25 \times 0.05) + (0.35 \times 0.04) + (0.40 \times 0.02) = \frac{69}{2000} = 0.0345$

$\therefore P(B/D) = \frac{P(B \cap D)}{P(D)} = \frac{P(B)P(D/B)}{P(D)} = \frac{0.35 \times 0.04}{0.0345} = \frac{28}{69} = 0.4058$

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