5.3.1. Class work Problems

① Theory predicts that the proporation of beans in 4 groups A, B, C and D should 9:3:3:1. In an experiment among 1600 beans the number in the 4-groups in the 4 groups. where 882, 313, 287 and 118 respectively. Does the experiment support the theory ?

<u>Solution</u> H_0: Theory predicts that the proporations of bens in 4 groups A, B, C and D should be 9:3:3:1

let $\frac{A}{9} = \frac{B}{3} = \frac{C}{3} = \frac{D}{1} = k$ say $\therefore A = 9k,\ B = 3k,\ C = 3k,\ D = 1k$

$\because A+B+C+D = 1600 \Rightarrow 9k+3k+3k+1k = 1600 \Rightarrow 16k = 1600 \therefore \boxed{k = 100}$

$A = 900,\ B = 300,\ C = 300,\ \&\ D = 100$

O_i	E_i	$(O_i-E_i)^2/E_i$
882	900	$(882-900)^2/900$
313	300	$(313-300)^2/300$
287	300	$(287-300)^2/300$
118	100	$(118-100)^2/100$

$\therefore \chi^2_{cal} = \sum_i \frac{(O_i-E_i)^2}{E_i}$

$\therefore \chi^2_{cal} = 4.7267,\quad \nu = n-1 = 3$

$\therefore \chi^2_{\nu}(\alpha) = \chi^2_3(5\%) = 7.815$

$\because \chi^2_{cal} < \chi^2_{\nu}(\alpha)$ $\therefore$ we can conclude that the proporations of beans in 4 groups A, B, C, D should be 9:3:3:1

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② The following table gives the no. of accidents in a district during a week. Test whether the accidents are uniformerly distributed over the week

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents (O_i)	13	12	11	9	15	10	14

H_0: accidents are uniformely distributed over a week

<u>Solution</u>

O_i	13	12	11	9	15	10	14
E_i	12	12	12	12	12	12	12

$\therefore$

O_i :	13	12	20	15	10	14
E_i :	12	12	24	12	12	12
$\frac{(O_i-E_i)^2}{E_i}$	$\frac{(1)^2}{12}$	$\frac{0^2}{12}$	$\frac{(-4)^2}{24}$	$\frac{3^2}{12}$	$\frac{(-2)^2}{12}$	$\frac{2^2}{12}$

$\chi^2 = \sum_i \frac{(O_i-E_i)^2}{E_i} = 2.1667$

$\nu = n-1 = 5 \quad \therefore \chi^2_{\nu}(\alpha) = \chi^2_5(5\%) = 11.070$

$\because \chi^2_{cal} < \chi^2_{\nu}(\alpha)$ $\therefore H_0$ is accepted

ie accidents are uniformely distributed over a week

③ Fit a poisson distribution for the following data and also test the goodness of fit

x	0	1	2	3	4	5
f	142	156	69	27	5	1

Solution H_0: Fitting of poisson distribution good for the following data

x	0	1	2	3	4	5	
$f(x)$	142	156	69	27	5	1	$= N = \Sigma f_i = 400$
$x_i f_i$	0	156	138	81	20	05	$= \Sigma x_i f_i = 400$
$E_i = N\frac{e^{-\lambda}\lambda^x}{x!} = 400\frac{e^{-1}1^x}{x!}$	147.15 = 147	147.15 = 147	73.575 = 74	24.525 = 25	6.1317 = 6	1.2262 = 1	

$\therefore \bar{x} = \frac{\Sigma x_i f_i}{N} = \frac{400}{400}$, $\bar{x} = 1$

O_i	142	156	69	33
E_i	147	147	74	32
$\frac{(O_i-E_i)^2}{E_i}$	$\frac{(142-147)^2}{147}$	$\frac{(156-147)^2}{147}$	$\frac{(69-74)^2}{74}$	$\frac{(33-32)^2}{32}$

$\therefore \chi^2_{cal} = \sum_i \frac{(O_i-E_i)^2}{E_i} = 1.0902$

$\nu = n-1 = 3$ $\chi^2_\nu(\alpha) = \chi^2_3(5\%) = 7.815$

$\because \chi^2_{cal} < \chi^2_\nu(\alpha)$ $\therefore H_0$ is accepted

$\therefore$ Poisson distribution is good for the given data

④ A die was thrown 132 times and following frequencies were obtained

Number obtained	1	2	3	4	5	6
Frequency	15	20	25	15	29	28

Test whether die is unbiased?

Solution H_0: die is unbiased

$E_i = \cancel{N \cdot {}^nC_x p^x q^{n-x} = 132\, {}^6C_x \left(\tfrac{1}{6}\right)^x\left(\tfrac{5}{6}\right)^{6-x} = 132\, {}^6C_x \left(\tfrac{1}{2}\right)^6}$

Number obtained	1	2	3	4	5	6
Frequency (O_i)	15	20	25	15	29	28
~~$E_i = N\, {}^nC_x p^x q^{n-x} = 132\, {}^6C_x (2^{-6})$~~	~~12.375~~ ≈ 12	~~30.937~~ ≈ 31	~~41.25~~ ≈ 41	~~30.937~~ ≈ 31	~~12.375~~ ≈ 12	~~2.0625~~ ≈ 2
$E_i = \cancel{N\, {}^nC_x \left(\tfrac{1}{6}\right)^x \left(\tfrac{5}{6}\right)^{n-x}} = \cancel{132\, {}^6C_x \left(\tfrac{1}{6}\right)^6}$	22	22	22	22	22	22
$\frac{(O_i-E_i)^2}{E_i} =$	$\frac{(15-22)^2}{22}$	$\frac{(20-22)^2}{22}$	$\frac{(25-22)^2}{22}$	$\frac{(15-22)^2}{22}$	$\frac{(29-22)^2}{22}$	$\frac{(28-22)^2}{22}$

$\therefore \chi^2_{cal} = \sum\left[\frac{(O_i-E_i)^2}{E_i}\right] = 8.9091$, $\nu = n-1 = 5$ $\therefore \chi^2_\nu(\alpha) = \chi^2_5(5\%) = 11.707$

$\therefore \chi^2_{cal} > \chi^2_\nu(\alpha)$ $\therefore H_0$ is rejected

$\therefore$ we can not say that the die is unbiased

5. Sample of two types of electric bulbs were tested for length of life and following data were obtained.

	Sample Size	mean of Sample	Standard deviation
Sample - 1	8	1134 hrs	36 hrs
Sample - 2	7	1024 hrs	40 hrs

Test at 5% LOS whether the difference in the sample means is significant

Solution: $n_1 = 8$, $n_2 = 7$, $\bar{x}_1 = 1134$, $\bar{x}_2 = 1024$, $s_1 = 36$, $s_2 = 40$

$H_0: \bar{x}_1 = \bar{x}_2$

$H_1: \bar{x}_1 \neq \bar{x}_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{1134 - 1024}{\sqrt{\frac{(8 \times 36^2) + (7 \times 40^2)}{8 + 7 - 2}\left(\frac{1}{8} + \frac{1}{7}\right)}} = 5.480$$

$\nu = n_1 + n_2 - 2 = 13 \quad \therefore t_\nu(\alpha) = t_{13}(5\%) = 2.160$

$\because t > t_\nu(\alpha) \therefore H_0$ is rejected ie H_1 is accepted $\therefore \bar{x}_1 \neq \bar{x}_2$

$\therefore$ we can conclude that the difference in the sample mean is significant

6. Fitting of binomial distribution for the following data and testing goodness of fit:

x	0	1	2	3	4	5	6
f	5	18	28	12	7	6	4

Solution: H_0: Fitting of binomial distribution is good for given data

x	0	1	2	3	4	5	6	
$f(O_i)$	5	18	28	12	7	6	4	$N = \sum f_i = 80$
$x_i f_i$	00	18	56	36	28	30	24	$\sum x_i f_i = 192$

$\therefore \bar{x} = \frac{1}{N}\sum x_i f_i = \frac{192}{80} = 2.4$ But in binomial distribution mean $= np$

$\therefore np = 2.4 \Rightarrow 6p = 2.4 \quad p = \frac{2.4}{6} = 0.4, \; q = 0.6$

$\therefore E_i = N \cdot {}^nC_x \, p^x q^{n-x} = 80 \; {}^6C_x (0.4)^x (0.6)^{6-x}$

x	0	1	2	3	4	5	6
E_i	3.7324 ≈ 4	14.929 ≈ 15	24.883 ≈ 25	22.118 ≈ 22	11.059 ≈ 11	2.9491 ≈ 3	0.3296 ≈ 0

O_i	23	28	12	17
E_i	19	25	22	14
$\frac{(O_i - E_i)^2}{E_i}$	$\frac{(23-19)^2}{19}$	$\frac{(28-25)^2}{25}$	$\frac{(12-22)^2}{22}$	$\frac{(17-14)^2}{14}$

$\therefore \chi^2_{cal} = \sum_i \frac{(O_i - E_i)^2}{E_i} = 6.3904$

$\nu = n - k = 4 - (1+1) = 2$

$\therefore \chi^2_\nu(\alpha) = \chi^2_2(5\%) = 5.991$

$\because \chi^2_{cal} > \chi^2_\nu(\alpha)$

$\therefore H_0$ is rejected

$\therefore$ Fitting of binomial is not good for the given data.

5.3.1.: Home Work Problems

① Four coins are tossed and the number of heads is noted. The experiment is repeated 100 times and the following distribution is obtained

No. of heads	0	1	2	3	4
Frequency	7	18	40	31	4

Do these results supports the assumption of a binomial distribution?

Solution $E_i = N \cdot P(X=x) = N\, {}^{n}c_{x}\, p^{x} q^{n-x} = 100\, {}^{4}c_{x} (\tfrac{1}{2})^{x} (\tfrac{1}{2})^{4-x} = 100\, {}^{4}c_{x} (\tfrac{1}{2})^{4}$

x	0	1	2	3	4
$\therefore E_i$	6.25 ≈ 6	25	37.5 ≈ 38	25	6.25 ≈ 6
O_i	7	18	40	31	4

H_0: binomial distribution is good for given data

$\therefore O_i$	25	40	35
E_i	31	38	31
$\frac{(O_i-E_i)^2}{E_i}$	$\frac{(25-31)^2}{31}$	$\frac{(40-38)^2}{38}$	$\frac{(35-31)^2}{31}$

$\therefore \chi^2_{cal} = \sum_i \frac{(O_i-E_i)^2}{E_i} = 1.7827$

$\nu = n-k = 3-1 = 2$

$\chi^2_\nu(\alpha) = \chi^2_2(5\%) = 5.991$ $\therefore \chi^2_{cal} < \chi^2_\nu(\alpha)$ $\therefore H_0$ is accepted

$\therefore$ Binomial distribution is good for the given data

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② Fit a binomial distribution to the following data and test goodness of fit

$x =$	0	1	2	3	4	5	6
$p(x=x)$	5	18	28	12	7	6	4

H_0: ~~Poisson~~ Binomial distribution is good for the given data

Solution mean $= np = \frac{1}{N}\sum x_i f_i = \frac{192}{80} = 2.4$ $\therefore$ ~~6p~~ $6p = 2.4$ $\therefore p = 0.4,\ q = 0.6$

$E_i = N\, {}^{n}c_{x} p^{x} q^{n-x}$
$= 80\, {}^{6}c_{x} (0.4)^{x} (0.6)^{6-x}$

E_i	3.7324	14.929	24.883	22.118	11.059	2.9491	0.3276
	≈ 4	≈ 15	≈ 25	≈ 22	≈ 11	≈ 3	0

$\therefore O_i$	13	28	12	17
E_i	19	25	22	14
$\frac{(O_i-E_i)^2}{E_i}$	$\frac{(13-19)^2}{19}$	$\frac{(28-25)^2}{25}$	$\frac{(12-22)^2}{22}$	$\frac{(17-14)^2}{14}$

$\therefore \chi^2_{cal} = \sum_i \frac{(O_i-E_i)^2}{E_i} = 7.4430$

$\nu = n-k = 4-(1+1) = 2$

$\therefore \chi^2_\nu(\alpha) = \chi^2_2(5\%) = 5.991$

$\therefore \chi^2_{cal} > \chi^2_\nu(\alpha)$

$\therefore H_0$ is ~~accepted~~ rejected

i.e Fitting of binomial distribution is not good for the given data

③ Fit a poisson distribution to the following data and test goodness of fit

x	0	1	2	3	4	5
Frequency	223	142	48	15	4	0

Solution: H_0: Fitting of poisson distribution is good

mean $= \lambda = \frac{1}{N}\sum x_i f_i = \frac{299}{432} = 0.6921296296$

$E_i = N \frac{e^{-\lambda}\lambda^x}{x!} = 432\ \cancel{e^{-0.69}}\ e^{-\frac{299}{432}} \frac{\left(\frac{299}{432}\right)^x}{x!}$ $x = 0, 1, 2, 3 \ldots$

E_i: 216.4 149.65 51.789 11.948 2.0675 0.286

≈ 217 ≈ 150 ≈ 52 ≈ 12 ≈ 2 ≈ 0

Thus

O_i : 223 142 48 19

E_i : 217 150 52 13

$\frac{(O_i-E_i)^2}{E_i}$ $\frac{(223-217)^2}{217}$ $\frac{(142-150)^2}{150}$, $\frac{(48-52)^2}{52}$ $\frac{(19-13)^2}{13}$

$\therefore \chi^2_{cal} = \sum_i \frac{(O_i-E_i)^2}{E_i} = 3.6695$, $\nu = n-k = 4-(1+1) = 2$ $\therefore \chi^2_\nu(\alpha) = \chi^2_2(5\%) = 5.991$

$\therefore \chi^2_{cal} < \chi^2_\nu(\alpha)$ $\therefore$ H_0 is accepted $\therefore$ Fitting of poission distribution is good for given data

④ 300 digits where chosen at random from a table of random numbers. The frequency of digits are as follows

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	28	29	33	31	26	35	32	30	31	25

Using χ^2-test examine the hypothesis that the digits were distributed equally in the table

Solution: H_0: digits are distributed equally

$O_i \rightarrow$ 28 29 33 31 26 35 32 30 31 25

$E_i \rightarrow$ 30 30 30 30 30 30 30 30 30 25

$\frac{(O_i-E_i)^2}{E_i} \rightarrow \frac{(28-30)^2}{30}, \frac{(29-30)^2}{30}, \frac{(33-30)^2}{30} \frac{(31-30)^2}{30} \frac{(26-30)^2}{30} \frac{(35-30)^2}{30} \frac{(32-30)^2}{30} \frac{(30-30)^2}{30} \frac{(31-30)^2}{30} \frac{(25-25)^2}{25}$

$\therefore \chi^2_{cal} = \sum_i \frac{(O_i-E_i)^2}{E_i} = 2.0333$

$\nu = n-k = 10-1 = 9$ $\therefore \chi^2_\nu(\alpha) = \chi^2_9(5\%) = 16.919$

$\therefore \chi^2_{cal} < \chi^2_\nu(\alpha)$ H_0 is accepted

i.e. Digits are equally distributed

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⑤ Fit a binomial distribution for the following data and test the goodness of fit.

No. of ~~days~~ boys	0	1	2	3	4	5
No. of families	8	40	88	110	56	18

Solution,

H_0: Fitting of poisson distribution is good for the given data

By given $n=5$, we know for Binomial distribution $\bar{x} = np = 5p$

$\therefore 5p = \frac{1}{N}\sum x f_i = \frac{1}{320}(860) = \frac{43}{16} = 2.6875$ $\therefore p = 0.5375,\ q = 0.4625$

$E_i = N P(x=x) = 320 \cdot {}^5C_x (0.5375)^x (0.4625)^{5-x}$, $x = 0,1,2,3,4,5$

$E_i \to$ 6.7718 39.35 91.462 106.29 61.765 14.355

≈ 7 ≈ 39 ≈ 92 ≈ 106 62 14

O_i	48	88	110	56	18
E_i	36	92	106	62	14
$\frac{(O_i-E_i)^2}{E_i}$	$\frac{(48-36)^2}{36}$	$\frac{(88-92)^2}{92}$	$\frac{(110-106)^2}{106}$	$\frac{(56-62)^2}{62}$	$\frac{(18-14)^2}{14}$

$\therefore \chi^2_{cal} = \sum_i \frac{(O_i-E_i)^2}{E_i} = 1.1429$

$\nu = n-k = 5-(1+1) = 3$ $\chi^2_\nu(\alpha) = \chi^2_3(5\%) = 7.815$

$\because \chi^2_{cal} < \chi^2_{\nu}(\alpha)$ $\therefore H_0$ is accepted

$\therefore$ Fitting of poisson distribution is good for the given data

5.3.2 Test of independence of attributes

5.3.2 : class work Problems

① Table below show the performance of students in mathematics and physics. Test the hypothesis that performances in Mathematics is independent of the performance in physics

Grade in physics		Grade in mathematics		
		high	medium	Low
	High	56	71	12
	medium	47	163	38
	Low	14	42	85

Solution: Ho: Performance in physics and mathematics is independent

		Grades in Mathematics			
		High	medium	Low	Total
Grades in physics	High	$O_{11}=56$, $E_{11}=30.80 \approx 31$	$O_{12}=71$, $E_{12}=72.66 \approx 72$	$O_{13}=12$, $E_{13}=35.54 \approx 36$	139
	medium	$O_{21}=47$, $E_{21}=54.95 \approx 55$	$O_{22}=163$, $E_{22}=129.64 \approx 130$	$O_{23}=38$, $E_{23}=63.40 \approx 63$	248
	Low	$O_{31}=14$, $E_{31}=31.24 \approx 31$	$O_{32}=42$, $E_{32}=73.70=74$	$O_{33}=85$, $E_{33}=36.05 \approx 36$	141
	Total	117	276	135	528

$$\chi^2_{cal} = \sum_{j=1}^{n}\sum_{i=1}^{m} \frac{(O_{ij}-E_{ij})^2}{E_{ij}} = \frac{(56-31)^2}{31} + \frac{(71-72)^2}{72} + \frac{(12-36)^2}{36} + \frac{(47-55)^2}{55} + \frac{(163-130)^2}{130}$$
$$+ \frac{(38-63)^2}{63} + \frac{(14-31)^2}{31} + \frac{(42-74)^2}{74} + \frac{(85-36)^2}{36}$$

$$\chi^2_{cal} = 145.49$$

$v = (m-1)\times(n-1) = (3-1)\times(3-1) = 4$

$\chi^2_v(\alpha) = \chi^2_4(5\%) = 9.488$

$\therefore \chi^2_{cal} > \chi^2_v(\alpha)$ $\therefore$ Ho is rejected

$\therefore$ Performance in physics and Mathematics is dependent

② A random Sample of 220 Students in a college were asked to give opinion in terms of year or no about the winning of their College cricket team in a tournment. The following data are collected

	Class in college		
	I^{st} year	II^{nd} year	III^{rd} year
Yes	43	20	37
no	23	57	40

Test whether there is any association between opinion and class in college

Solution H₀: There is no association between opinion and class in college

	Class in college			
	I^{st} year	II^{nd} year	III^{rd} year	Total
yes	$O_{11}=43$, $E_{11}=30$	$O_{12}=20$, $E_{12}=35$	$O_{13}=37$, $E_{13}=35$	160
No	$O_{21}=23$, $E_{21}=36$	$O_{22}=57$, $E_{22}=42$	$O_{23}=40$, $E_{23}=42$	120
Total	66	77	77	220

$$\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{(O_{ij}-E_{ij})^2}{E_{ij}} = \frac{(43-30)^2}{30} + \frac{(20-35)^2}{35} + \frac{(37-35)^2}{35} + \frac{(23-36)^2}{36} + \frac{(57-42)^2}{42} + \frac{(40-42)^2}{42} = 22.32$$

$\nu = (m-1)(n-1) = 1 \times 2 = 2$, $\chi^2_\nu(\alpha) = \chi^2_2(5\%) = 5.991$, $\chi^2_{cal} > \chi^2_\nu(\alpha)$ ∴ H_0 is rejected

i.e. There is some association between opinion and class in college

③ Based on data below determine if there is relation between literacy and smoking

	Smokers	Non-smokers
literate	83	57
illiterate	45	68

Solution: H₀: There is no relation between literacy and smoking

	Smokers	Non-smokers	Total
literate	83 = a	57 = b	140 = a+b
illiterate	45 = c	68 = d	113 = c+d
Total	128 = a+c	125 = b+d	253 = N = a+b+c+d

$$\therefore \chi^2_{cal} = \frac{N[ad-bc]^2}{(a+b)(c+d)(a+c)(b+d)} = \frac{253[(83\times68)-(57\times45)]^2}{140\times113\times128\times125} = 9.4757$$

$\nu = (m-1)\times(n-1) = 1$

$\therefore \chi^2_\nu(\alpha) = \chi^2_1(5\%) = 3.841$

$\therefore \chi^2_{cal} > \chi^2_\nu(\alpha)$ ∴ H_0 is rejected

∴ There is some relation between Literacy and smoking

④ Investigate the association between the darkness of eyes colour in father and son from the following data.

	Dark	Not Dark	Total
Dark	48	90	138
Not-Dark	80	782	862
Total	128	872	1000

H_0: There is no association between the darkness of eyes colour in father and son

	Dark	Not-Dark	Total
Dark	48 = a	90 = b	138 = a+b
Not-Dark	80 = c	782 = d	862 = c+d
Total	128 = a+c	872 = b+d	1000 = N = a+b+c+d

$$\chi^2 = \frac{N \times [ad-bc]^2}{(a+b)(c+d)(a+c)(b+d)} = \frac{1000 \times [(48 \times 782) - (90 \times 80)]^2}{138 \times 862 \times 128 \times 872} = 69.2112$$

$\nu = (m-1) \times (n-1) = 1$ $\therefore \chi^2_\nu(\alpha) = \chi^2_1(5\%) = 3.841$ $\therefore \chi^2_{cal} > \chi^2_\nu(\alpha)$ $\therefore H_0$ is rejected

$\therefore$ There is some association between the darkness of eyes colour in father and son

⑤ Two batches of 12 animals each are taken for test of innoculation. one batch was innoculated and other was not innoculated. the number of dead and surviving animials are given in the table for both cases. Can the innoculation be regarded as effective against the disease at 5% LOS

~~Soln~~

	Dead	surviving	Total
innoculated	02	10	12
Non-innoculated	08	04	12
Total	10	14	24

Solution :- H_0: There is no association between innoculation and survival

	Dead	surviving	Total
innoculation	02 = a	10 = b	12 = a+b
Non-innoculation	08 = c	04 = d	12 = c+d
Total	10 = a+c	14 = b+d	24 = N = a+b+c+d

$$\chi^2_{cal} = \frac{N\left[|ad-bc| - \frac{N}{2}\right]^2}{(a+b)(c+d)(a+c)(b+d)} = \frac{24\left[|(2 \times 4) - (10 \times 8)| - \frac{24}{2}\right]^2}{12 \times 12 \times 10 \times 14} = 4.29$$

$\nu = (m-1)(n-1) = 1$ $\chi^2_\nu(\alpha) = \chi^2_1(5\%) = 3.841$

$\therefore \chi^2_{cal} > \chi^2_\nu(\alpha)$ $\therefore H_0$ is rejected

$\therefore$ There is some association between innoculation and survival

(7) The following data is collected on two ~~charteristic~~ characters. Based on this can you say that there is no relation between smoking and literacy

	Smoking	Non-smoking
Literates	83	57
illiterates	45	68

Solution Ho: There is no association between Literacy & smoking

	Smoking	Non-smoking	Total
Literates	83 = a	57 = b	140 = a+b
illiterates	45 = c	68 = d	113 = c+d
Total	128 = a+c	125 = b+d	253 = N = a+b+c+d

$$\chi^2_{cal} = \frac{N \times [ad-bc]^2}{(a+b)(c+d)(a+c)(b+d)} = \frac{253\,[(83\times 68)-(57\times 45)]^2}{140\times 113\times 128\times 125} = 9.4757$$

$v = (m-1)\times(n-1) = 1 \quad \therefore \chi^2_v(\alpha) = \chi^2_1(5\%) = 3.841$

$\because \chi^2_{cal} > \chi^2_v(\alpha) \therefore$ Ho is rejected ie There is association between literacy & smoking

(8) The number of car accidents in a metropolitan city was found to be 20, 17, 12, 6, 7, 15, 8, 5, 16 and 14 per month respectively. use χ^2 test to check whether these frequencies are in agreement with the belief that occurance of accidents was the same during 10 months period. Test at 5% LoS.

Solution : Ho: occurance of accidents was ~~the~~ same during 10 months

O_i	20	17	12	6	7	15	8	5	16	14
E_i	12	12	12	12	12	12	12	12	12	12

$\therefore O_i$	20	17	12	13	15	13	16	14
E_i	12	12	12	24	12	24	12	12

$$\frac{(O_i-E_i)^2}{E_i} \quad \frac{(20-12)^2}{12}, \frac{(17-12)^2}{12}, \frac{(12-12)^2}{12}, \frac{(13-24)^2}{24}, \frac{(15-12)^2}{12}, \frac{(13-24)^2}{24}, \frac{(16-12)^2}{12}, \frac{(14-12)^2}{12}$$

$$\therefore \chi^2_{cal} = \sum_i \frac{(O_i-E_i)^2}{E_i} = 19.92$$

$v = n-1 = 7 \quad \therefore \chi^2_v(\alpha) = \chi^2_7(5\%) = 14.067$

$\therefore \chi^2_{cal} > \chi^2_v(\alpha\%) \quad \therefore$ Ho is rejected

$\therefore$ occurance of accidents are not same during 10 months

5.3.2 : Home Work Problems

① A certain drug is claimed to be effective in curing cold in an experiment on 500 person's with cold. Half of them were given drug and half of them were given the sugar Pills. The patients reaction to the treatment are recorded in the following table

	Helped	Harmed	No effect	Total
Drug	150	30	70	250
Sugar pills	130	40	80	250
Total	280	70	150	500

on the basis of this data can it be concluded that the drug and sugar pills differ significantly in curing cold.

Solution :- H_0: we can not conclude that the drug and sugar pills differe significantly in curing cold

	Helped	Harmed	No effected	Total
Drug	$O_{11}=150$, $E_{11}=140$	$O_{12}=30$, $E_{12}=31$	$O_{13}=70$, $E_{13}=75$	250
Sugar pills	$O_{21}=130$, $E_{21}=140$	$O_{22}=40$, $E_{22}=35$	$O_{23}=80$, $E_{23}=75$	250
Total	280	70	150	$N=500$

$$\therefore \chi^2_{cal} = \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{(O_{ij}-E_{ij})^2}{E_{ij}} = \frac{(150-140)^2}{140}+\frac{(30-31)^2}{31}+\frac{(70-75)^2}{75}+\frac{(130-140)^2}{140}+\frac{(40-35)^2}{35}+\frac{(80-75)^2}{75}$$

$\therefore \chi^2_{cal} = 2.8419$, $\nu=(m-1)\times(n-1)=(2-1)\times(3-1)=3$ $\therefore \chi^2_{\nu}(\alpha)=\chi^2_3(5\%)=7.815$

$\therefore \chi^2_{cal} < \chi^2_{\nu}(\alpha)$ $\therefore H_0$ is accepted

i.e. we can not conclude that the drug and sugar pills differ significantly in curing cold

② Justify, if there is any relationship between sex & colour for the following data

Colour	male	Female
Red	10	40
white	70	30
Green	30	20

H_0: There is no relationship between sex and colour

Colour	Male	Female	Total
Red	$O_{11}=10$, $E_{11}=27.5$	$O_{12}=40$, $E_{12}=22.5$	50
white	$O_{21}=70$, $E_{21}=55$	$O_{22}=30$, $E_{22}=45$	100
Green	$O_{31}=30$, $E_{31}=27.5$	$O_{32}=20$, $E_{32}=22.5$	50
Total	110	90	200

$\therefore \chi^2_{cal} = \sum_{j=1}^{} \sum_{i=1}^{m} \frac{(O_{ij}-E_{ij})^2}{E_{ij}}$

$\chi^2_{cal} = \frac{(10-27.5)^2}{27.5} + \frac{(40-22.5)^2}{22.5} + \frac{(70-55)^2}{55} + \frac{(30-45)^2}{45} + \frac{(30-27.5)^2}{27.5} + \frac{(20-22.5)^2}{22.5}$

$\therefore \chi^2_{cal} = 34.34$

$\nu = (m-1)\times(n-1) = 2\times1 = 2 \therefore \chi^2_{\nu}(\alpha) = \chi^2_{2}(5\%) = 5.991 \therefore \chi^2_{cal} > \chi^2_{\nu}(\alpha)$

$\therefore$ H_0 is rejected i.e There is some relation between sex and colour

(3) The manager of a chain of restaurents wants to know whether the customer satisfication is related to the ~~waiter~~ waiter. He take a random sample of 100 customers asking the name of the waiter and whether the service was excellent, good or poor. He then categorizes the salaries of the waiters as low, medium and high. His results are shown below. Test at 0.05 LOS of significance whether the quality of service is independent of waiter's salary

	Low	Medium	High
Excellent	9	10	7
Good	11	9	31
poor	12	8	3

Solution:

H_0: customers satisfication is not related to the waiter

	C_1	C_2	Total
R_1	$O_{11}=9$, $E_{11}=8.32$	$O_{12}=17$, $E_{12}=17.68$	26
R_2	$O_{21}=11$, $E_{21}=16.32$	$O_{22}=40$, $E_{22}=34.68$	51
R_3	$O_{31}=12$, $E_{31}=7.36$	$O_{32}=11$, $E_{32}=15.64$	23
Total	32	68	100

$\therefore \chi^2_{cal} = \frac{(9-8.32)^2}{8.32} + \frac{(17-17.68)^2}{17.68} + \frac{(11-16.32)^2}{16.32} + \frac{(40-34.68)^2}{34.68} + \frac{(12-7.36)^2}{7.36} + \frac{(11-15.64)^2}{15.64}$

$\therefore \chi^2_{cal} = 6.9338$, $\nu = (m-1)\times(n-1) = 2$; $\chi^2_{\nu}(\alpha) = \chi^2_{2}(5\%) = 5.991$

$\therefore \chi^2_{cal} > \chi^2_{\nu}(\alpha)$ $\therefore$ H_0 is rejected

$\therefore$ customers satisfication is related to waiter

④ The ~~manager~~ of a chain of ~~restaurants wants~~ to know whether the customer's satisfaction is related to the wai

④ Explain two application of χ^2 distribution. To test two methods of instruction, 50 students are selected at random from each of the two groups. At the end of the instruction period each student is assigned a grade (A, B, C, D, or F) by an evaluating team. The data is recorded as follows

	Grade					
	A	B	C	D	F	Total
Group 1	8	13	16	10	3	50
Group-2	4	9	14	16	7	50

Does the data indicate that there is relation between grades and method of instructions?

Solution:- H_0: There is no relation between grades and method of instructions

	Grades			
	A and B	C	D and F	Total
Group-1	$O_{11} = 21$, $E_{11} = 17$	$O_{12} = 16$, $E_{12} = 15$	$O_{13} = 13$, $E_{13} = 18$	50
Group-2	$O_{21} = 13$, $E_{21} = 17$	$O_{22} = 14$, $E_{22} = 15$	$O_{23} = 23$, $E_{23} = 18$	50
Total	34	30	36	100

$$\chi^2_{cal} = \frac{(21-17)^2}{17} + \frac{(16-15)^2}{15} + \frac{(13-18)^2}{18} + \frac{(13-17)^2}{17} + \frac{(14-15)^2}{15} + \frac{(23-18)^2}{18}$$

$\therefore \chi^2_{cal} = 4.7935$, $\nu = (m-1)\times(n-1) = 1\times 2 = 2$

$\therefore \chi^2_{\nu}(\alpha) = \chi^2_2(5\%) = 5.991$

$\therefore \chi^2_{cal} < \chi^2_{\nu}(\alpha)$ $\therefore$ H_0 is accepted

$\therefore$ There is no relation between grades and method of instructions

(5) In a survey of ~~boys~~ 200 boys of which 75 are intelligents, 40 had educated fathers while 85 of the unintelligent boy had uneducated fathers. Do these figures support the hypothesis that educated fathers having intelligent boy at 5% LOS

Solution: H_0 There is no relation between educated father and intelligent boy

	intelligent boy	unintelligent boy	Total
Educated Father	40 = a	40 = b	80 = a+b
uneducated Fater	35 = c	85 = d	120 = c+d
Total	75 = a+c	125 = b+d	N = 200 = a+b+c+d

$$\chi^2_{cal} = \frac{N \times (ad-bc)^2}{(a+b)\times(c+d)\times(a+c)(b+d)} = \frac{200[(40\times85)-(40\times35)]^2}{80\times120\times75\times125}$$

$\therefore \chi^2_{cal} = 8.89$

$\nu = (m-1)\times(n-1) = 1$

$\therefore \chi^2_\nu(\alpha) = \chi^2_1(5\%) = 3.841$

$\therefore \chi^2_{cal} > \chi^2_\nu(\alpha)$ $\therefore$ H_0 is rejected

$\therefore$ There is some relation between educated father and intelligent boy