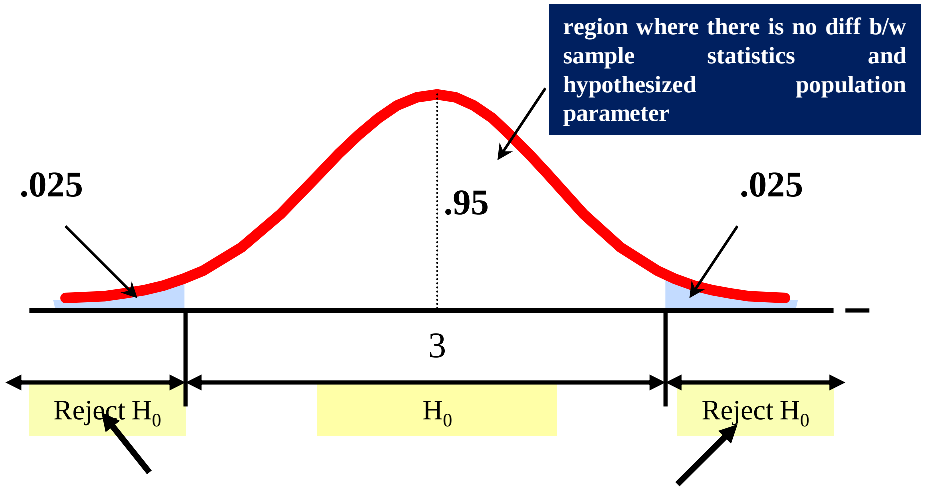
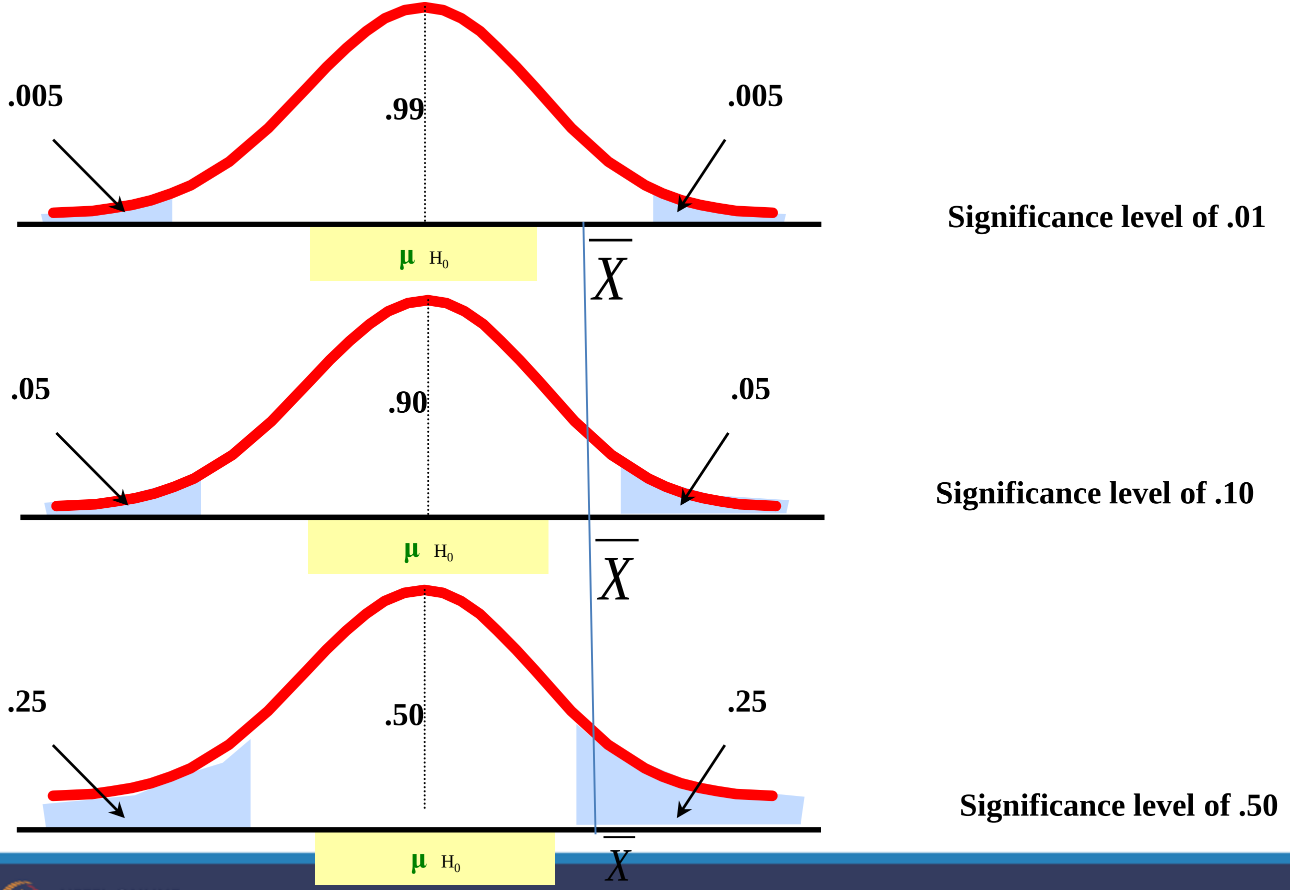
Steps in Hypothesis Testing

1. State the null hypothesis, H_0 and the alternative hypothesis, H_1
2. Choose the level of significance, α, and the sample size.

What if we test a hypothesis at 5 % level of significance? This means that we will reject the null hypothesis if the difference between the sample statistics and the hypothesized population parameters so large that it or a larger difference would occur, on the average, only five or fewer times in every 100 samples when hypothesized population parameters is correct.



Selecting a significance level : Generally at .01,.05, .10 or 99,95,90%: The higher the significance level we use for testing a hypothesis, the <u>higher the probability of rejecting a null hypothesis when it is true.</u>



Steps in Hypothesis Testing

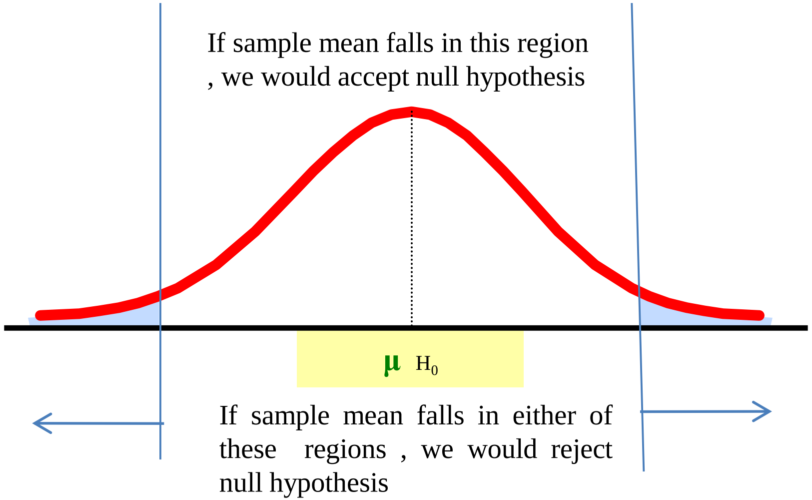
3. Determine the appropriate **test statistic** and sampling distribution: t or z test

4. Determine the **critical values** that divide the rejection and nonrejection regions

- In two tail – we have two rejection regions, it is appropriate **when null hypothesis is $\mu = \mu H_0$ and the alternate hypothesis $\mu = \mu H_0$.**

Let the mean life of bulb $\mu = \mu Ho = 1000$ (NH)

$\mu H1 \neq 1000$ (AH)



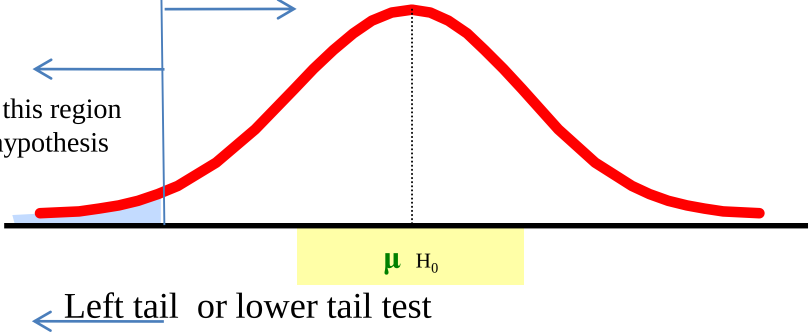
One tail test- A wholesaler that buys bulb would not accept if life is less than 1000.

Ho: $\mu = 1000$ (NH)

H1: $\mu < 1000$ (AH)

If sample mean falls in this region , we would accept null hypothesis

If sample mean falls in this region , we would reject null hypothesis

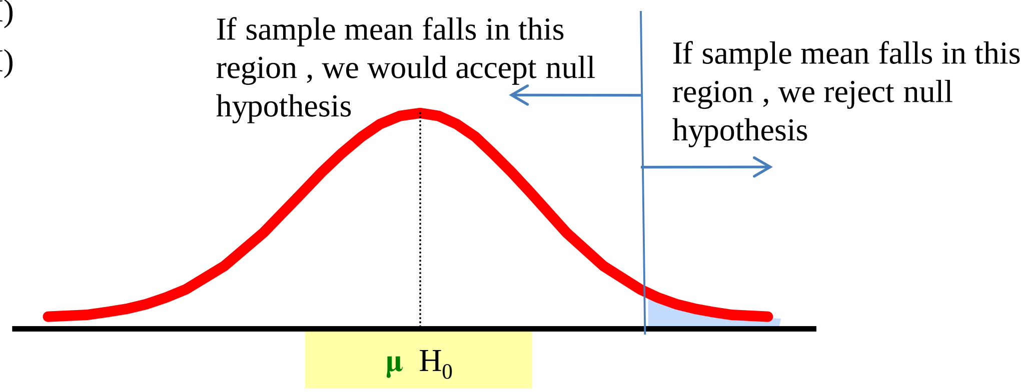


Left tail or lower tail test

One tail test- Monthly expenditure should be kept at 100 on an average.

Ho: $\mu = 100$ (NH)

H1: $\mu > 100$ (AH)



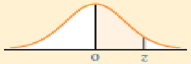
Steps in Hypothesis Testing

(continued)

5. Collect data and compute the value of the **test statistic**

6. Make the statistical decision and state the managerial conclusion. If the test **statistic** falls into the **non rejection** region, **do not reject the null hypothesis** H_0. If the test statistic falls into the rejection region, reject the null hypothesis. Express the managerial conclusion in the context of the problem

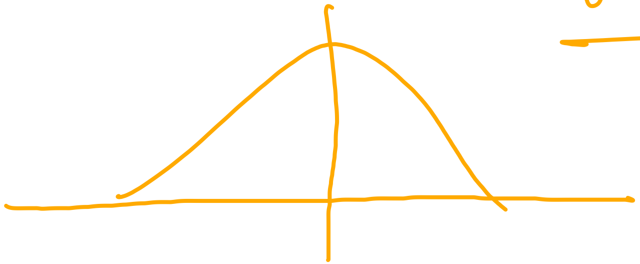
How many standard errors around the hypothesized value should we use to be 99.44 percent certain that we accept the hypothesis when it is true?



SECOND DECIMAL PLACE IN z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									
4.0	.49997									
4.5	.499997									
5.0	.4999997									
6.0	.499999999									

$$\frac{0.9944}{2}$$



$= 0.0056$

Solution:

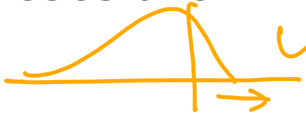
To leave a probability of 1 – 0.9944 = 0.0056 in the tails, the absolute value of z must be greater than or equal to 2.77, so the interval should be $\pm$2.77 standard errors about the hypothesized value.

Martha Inman, a highway safety engineer, decides to test the load-bearing capacity of a bridge that is 20 years old. Considerable data are available from similar tests on the same type of bridge. Which is appropriate, a one-tailed or a two-tailed test? If the minimum load-bearing capacity of this bridge must be 10 tons, what are the null and alternative hypotheses?

$10 \geq$

Solution:

The engineer would be interested in whether a bridge of this age could withstand minimum load bearing capacities necessary for safety purposes. She therefore wants its capacity to be above a certain minimum level, so a one tailed test (specifically an upper-tailed or right-tailed test) would be used. The hypotheses are



upper tail

one tail test

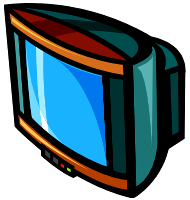
$$H_o: \mu = 10 \text{ tons} \qquad H_1: \mu > 10 \text{ tons}$$

Hypothesis Testing Example

Test the claim that the true mean # of TV sets in Indian homes is equal to 3.

(Assume $\sigma = 0.8$)

1. State the appropriate null and alternative hypotheses
   - $H_0: \mu = 3$ $H_1: \mu \neq 3$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   - Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test



Hypothesis Testing Example

(continued)

3. Determine the appropriate technique
   - **σ is assumed known so this is a Z test.**
4. Determine the critical values
   - For α = 0.05 the critical Z values are ±1.96
5. Collect the data and compute the test statistic
   - Suppose the sample results are

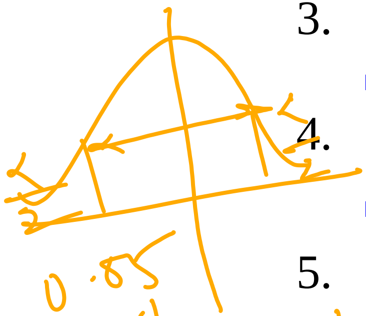
     n = 100, $\bar{X}$ = 2.84 (σ = 0.8 is assumed known)

So the test statistic is:

$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

±1.5

1-α



0.05

5%

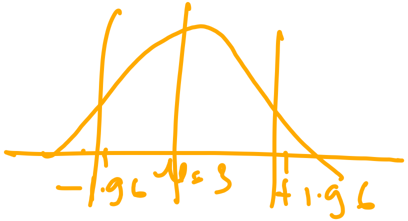
100-5 ⇒ 95%

$\frac{0.95}{2} = 0.475$

from z table
z = 1.96

if σ is unknown then use t test

← z table



Hypothesis Testing Example

(continued)-+-

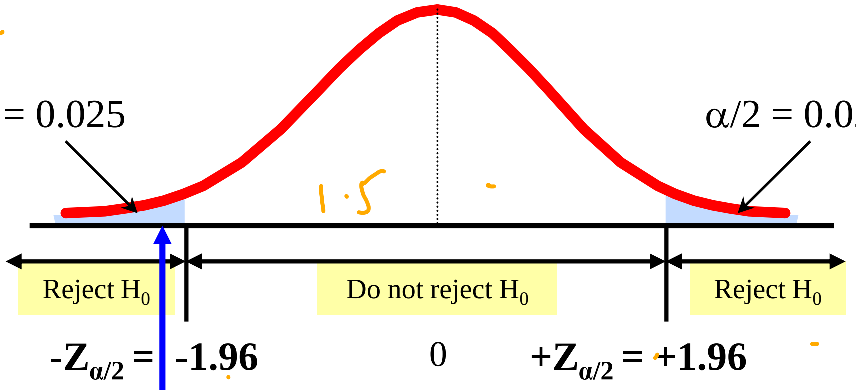
value of z from table

compare value of z

Calculated from formula

H_0 A R

- 6. Is the test statistic in the rejection region?



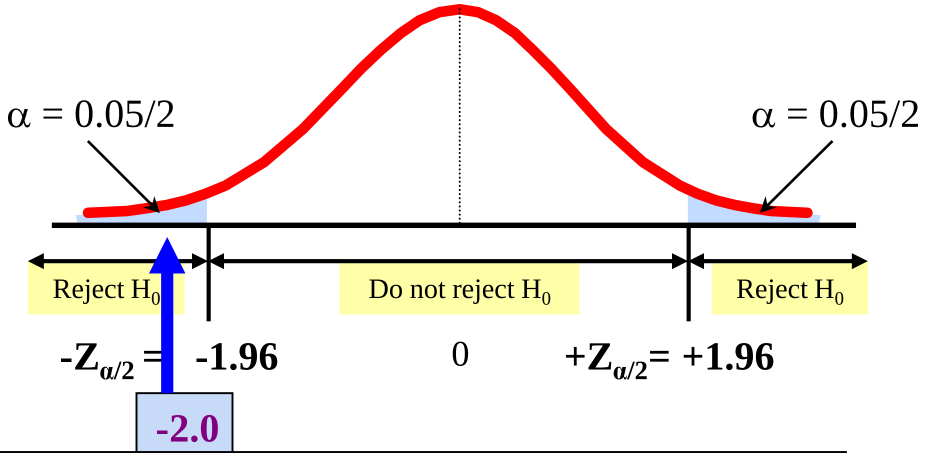
Reject H_0 if $Z_{STAT} < -1.96$ or $Z_{STAT} > 1.96$; otherwise do not reject H_0

Here, $Z_{STAT} = -2.0 < -1.96$, so the test statistic is in the rejection region

Hypothesis Testing Example

(continued)

6 (continued). Reach a decision and interpret the result



Since $Z_{STAT} = -2.0 < -1.96$, reject the null hypothesis and conclude there is sufficient evidence that the mean number of TVs in Indian homes **is not equal to 3**

2) Ex: Mean thickness of aluminum sheet (μ) =.04, **σ = 0.004**

n= 100, $\bar{X}$ = 0.0408,

level of significance is not given

$H_0: \mu = 0.04$
$H_a: \mu \neq 0.04$ — 2 tail

$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

→ 2nd method
sample mean

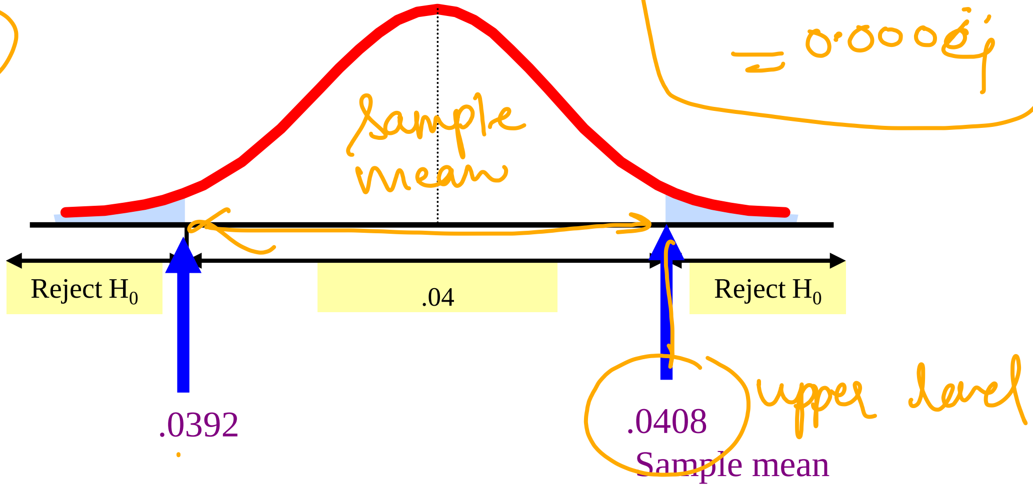
$$\bar{X} = \mu \pm Z\sigma_{\bar{x}}$$

$$\sigma_{\bar{x}} = \frac{0.004}{100} = 0.0004$$

Z = (0.0408-0.04)/0.0004 = 2

$\bar{X}$ = 0.04 ± (2)* (0.0004)

=0.0392 and 0.0408



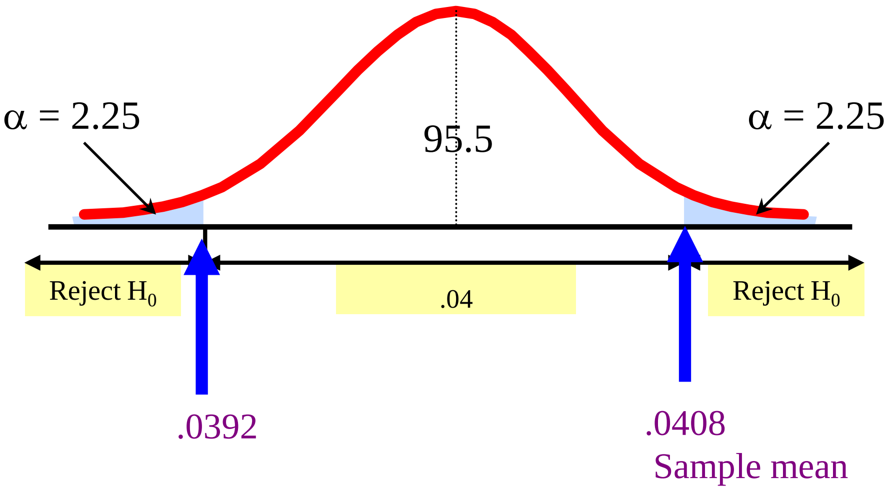
Reject company's claim that population mean is 0.04

Ex: Mean thickness of aluminum sheet (μ) =.04, **σ = 0.004**
n= 100, $\overline{X}$ = 0.0408,

$$Z_{STAT} = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Z = (0.0408-0.04)/0.0004 = 2
0.04+- (2)* (0.0004)
=0.0392 and 0.0408

Reject company's claim that population mean is 0.04



Ex: Axel strength is 80000 pounds per square inch

μ H_0 = 80000 (Hypothesized value of population mean)

σ = 4000

n=100

$\bar{X}$ = 79600

Significance level=.05

Sample mean

Z test

$H_0 \Rightarrow \mu = 80000$

$H_a \Rightarrow \mu \neq 80000$

$\alpha \longrightarrow Z = \pm 1.96$

Ex: Axel strength is 80000 pounds per square inch

two tail method

μ H_0 = 80000 (Hypothesized value of population mean)

σ = 4000

n=100

$\bar{X}$ = 79600

Significance level=.05 $= \frac{\alpha}{2}$

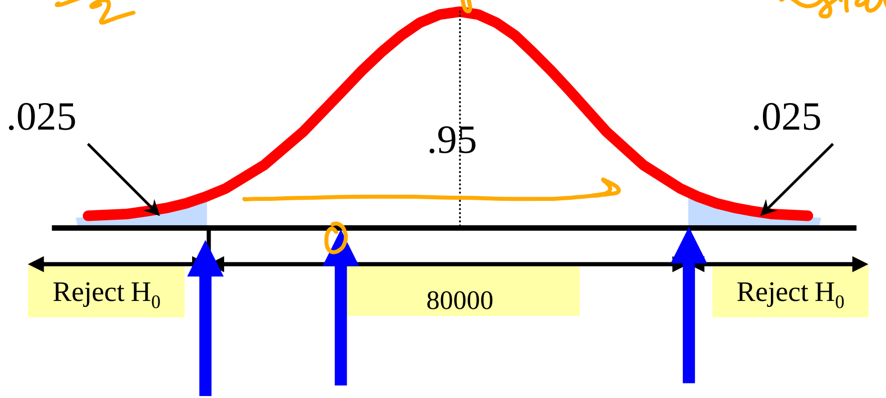
$1 - 0.5 = 0.95$

Soltn: μ= 80000 (NH)

$\mu \neq$ 80000 (AH)

acceptance

$Z_{stat} = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$



$= \dfrac{79600 - 80000}{400}$

$= ?$

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = 400$

$\bar{x} = \mu \pm Z\sigma_{\bar{x}}$ ← = 400

=80000+-1.96*400

(79216 to 80784)

79216
-1.96

79600
-1
Sample mean

80784
+1.96

Z-table

Z = (79600-80000)/400 = -1

Accept (Do not reject) the null hypothesis.

Ex: Drug dose of 100cc, excess dose is not harmful but insufficient dose does not produce results.

$\mu\ H_0 = 100$ (Hypothesized value of population mean)

$\sigma = 2$ → z test

$n=50$

$\bar{X} = 99.75$

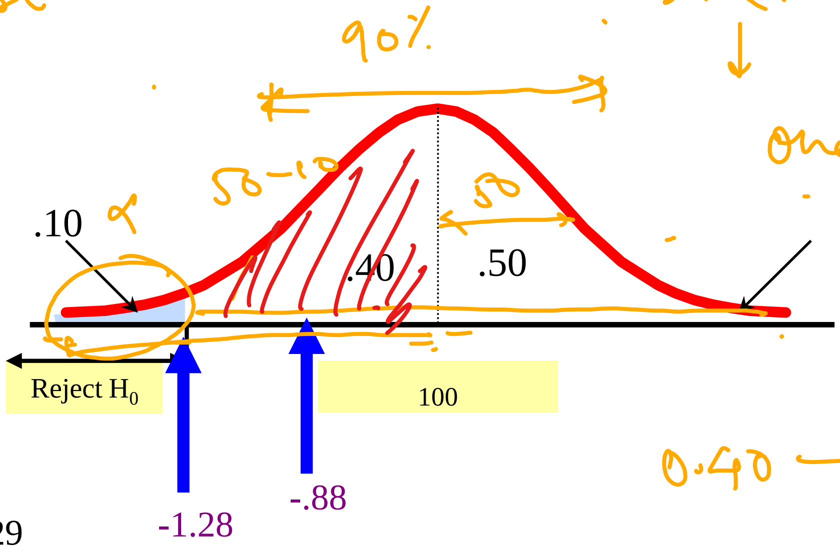
Significance level=.10

$H_0 \ni \mu_0 = 100$ cc

$\mu < 100$ cc

↓

one tail test



Soltn: $\mu = 100$ (NH)

$\mu < 100$ (AH)

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$\bar{x} = \mu \pm z\sigma_{\bar{x}}$

$$\sigma_{\bar{x}} = 2/7.07 = .2829$$

$\bar{x} =$ 100+-1.28*.2829

100.36 and 99.63

$0.40 \rightarrow -1.28$

Look in z table where area is .40

$Z = (99.75-100)/.2829 = -.88$

So, accept (Do not reject) null hypothesis.