DIGITAL LOGIC AND COMPUETR AECHITECTURE

TOPIC NO : 5

BINARY ARITHMATIC

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BINARY SIGNED AND UNSIGNED NUMBER REPRESENTATION

- SIGN Magnitude
- 1’s Complement
- 2’s Complement

What is Signed Magnitude?

- Add 1 bit at MSB
- As per Sign of the Number
  - \+ ⟶ 0
  - \- ⟶ 1

Examples:

What is 1's complement

- $(-6)_{10}$ Represent number in 1's complement form.
- $(-12)_{10}$ Represent number in 1's complement form.

What is 2's complement

- $(-6)_{10}$ Represent number in 2's complement form.
- $(-12)_{10}$ Represent number in 2's complement form.

Signed Binary Number Representation

No	SIGN MAGNITUDE	1'S COMPLEMENT	2'S COMPLEMENT
3	011	011	011
2	010	010	010
1	001	001	001
0	000	000	000
0	100	111	000
-1	101	110	111
-2	110	101	110
-3	111	100	101

Binary ADDITION Rule

A	B	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Binary SUBTRACTION Rules

A	B	RESULT	BORROW
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

EXAMPLES

(Based on Addition/ Subtraction Rules)

1) 1 0 1 1

\+

0 1 0 0

---

2) 1 1 1 0

\-

0 0 1 0

---

3) 1 1 0 0

\+

1 1 1 1

---

4) 1 0 0 0

\-

0 1 1 1

SUBTRACTION USING 1's COMPLEMENT

- $(7)_{10} - (5)_{10}$
- 7 → $(0111)_2$
- 5 → $(0101)_2$

→ • (-5)

→ $(1010)_2$

1's complement

$(0\ 1\ 1\ 1)_2$

$+ (1\ 0\ 1\ 0)_2$

Carry $(1\ 0\ 0\ 0\ 1)_2$

$+ \quad 1$

$(0\ 0\ 1\ 0)_2$

Carry is generated ans is positive add back carry

SUBTRACTION USING 1's COMPLEMENT

- $(5)_{10} - (7)_{10}$
- 5 → $(0101)_2$
- 7 → $(0111)_2$
  - (-7)
  - $(1000)_2$
  - 1's complement

$$(0\ 1\ 0\ 1)_2$$
$$+\ (1\ 0\ 0\ 0)_2$$
$$(1\ 1\ 0\ 1)_2$$

Carry not generated ans is in 1's complement form

Carry not generated ans is in 1's complement form

1's complement of ans:

$(1\ 1\ 0\ 1)_2 \rightarrow (0\ 0\ 1\ 0)_2$

Rules for Subtraction 1's Complement

- After addition if carry is generated then add back Carry and answer is positive

- After addition if carry is not generated then answer is negative and its in 1's complement form

SUBTRACTION USING 2's COMPLEMENT

Carry is generated ans is positive discard carry

- $(7)_{10} - (5)_{10}$
- 7 → $(0111)_2$
- 5 → $(0101)_2$

• (-5)

$(1010)_2$

\+ 1

2's Complement → $(1011)_2$

ADD numbers

$(0\ 1\ 1\ 1)_2$

\+ $(1\ 0\ 1\ 1)_2$

Carry $(\mathbf{1}\ 0\ 0\ 1\ 0)_2$

Discard carry

SUBTRACTION USING 2's COMPLEMENT

- $(5)_{10} - (7)_{10}$
- 5 → $(0101)_2$
- 7 → $(0111)_2$

• (-7)

$(1000)_2$

$+ \quad 1$

2's Complement → $(1001)_2$

$(0\ 1\ 0\ 1)_2$

$+(1\ 0\ 0\ 1)_2$

$(1\ 1\ 10)_2$

Carry not generated ans is in 2's complement form

Carry not generated ans is in 1's complement form

2's complement of ans:

$(1\ 1\ 1\ 0)_2 \rightarrow (0\ 0\ 0\ 1)_2$

$+ \quad 1$

$(0\ 0\ 1\ 0)_2$

Rules for Subtraction 2's Complement

- After addition if carry is generated then **Discard** Carry and answer is **positive**

- After addition if carry is not generated then answer is **negative** and its in 2's complement form

EXERCISE PROBLEMS

- $(10)_{10} - (3)_{10}$ using 1's and 2's complement
- $(3)_{10} - (10)_{10}$ using 1's and 2's complement

BINARY MULTIPICATION

- EXAMPLE
  - 101111.011* 1011.01

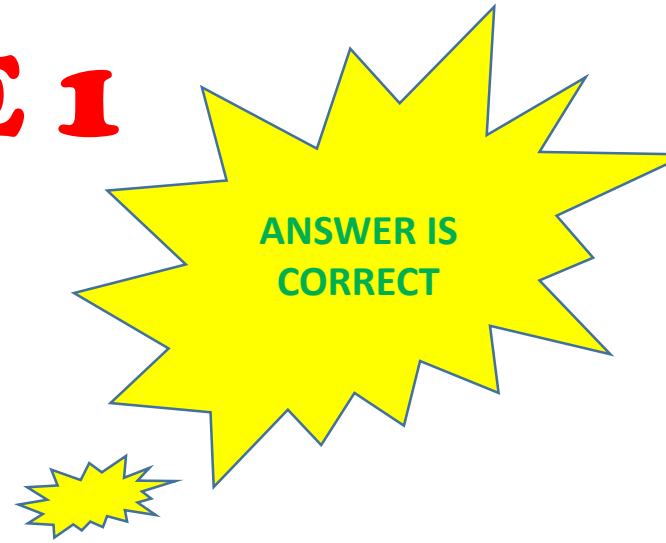
BINARY DIVISION

HEXA DECIMAL ARITHMETIC

BCD ADDITION

- RULES:
  1. SUM < 9, Final Carry = 0 → **ANSWER IS CORRECT**
  2. SUM < 9, Final Carry = 1 → TAKE CORRECTIVE MEASURE **ADD 0110**
  3. SUM > 9, Final Carry = 0 → TAKE CORRECTIVE MEASURE **ADD 0110**

BCD ADDITION : CASE 1



$(2)_{10} + (6)_{10}$

- 2 → $(0010)_{BCD}$
- 6 → $(0110)_{BCD}$

$$\begin{array}{r} (0\ 0\ 1\ 0)_{BCD} \\ + \ (0\ 1\ 1\ 0)_{BCD} \\ \hline (1\ 0\ 0\ 0)_{BCD} \end{array}$$

ANSWER < 9

Carry not generated

BCD ADDITION : CASE 2

$(3)_{10} + (7)_{10}$

- 3 → $(0011)_{BCD}$
- 7 → $(0111)_{BCD}$

$$\begin{array}{r} (0\ 0\ 1\ 1)_{BCD} \\ +\ (0\ 1\ 1\ 1)_{BCD} \\ \hline (1\ 0\ 1\ 0)_{BCD} \end{array}$$

ANSWER IS NOT CORRECT

Carry not generated but

ANSWER > 9

Take Corrective Measure ADD 0110

$$\begin{array}{r} (1\ 0\ 1\ 0)_{BCD} \\ +\ (0\ 1\ 1\ 0)_{BCD} \\ \hline (0\ 0\ 0\ \mathbf{1}\ 0\ 0\ 0\ 0)_{BCD} \end{array}$$

$(1\quad 0)_{10}$

BCD ADDITION : CASE 3

$(8)_{10} + (9)_{10}$

- 8 → $(1000)_{BCD}$
- 9 → $(1001)_{BCD}$

$$\begin{array}{r} (1\ 0\ 0\ 0)_{BCD} \\ +\ (1\ 0\ 0\ 1)_{BCD} \\ \hline (1\ 0\ 0\ 0\ 1)_{BCD} \end{array}$$

ANSWER IS NOT CORRECT

ANSWER < 9 but

Carry is generated

Take Corrective Measure ADD 0110

$$\begin{array}{r} (1\ 0\ 0\ 0\ 1)_{BCD} \\ +\ (0\ 1\ 1\ 0)_{BCD} \\ \hline (0\ 0\ 0\ 1\ 0\ 1\ 1\ 1)_{BCD} \end{array}$$

$(1\quad 7)_{10}$

EXAMPLES:

- $(57)_{10} + (26)_{10}$
- $(77)_{10} + (77)_{10}$

BCD SUBTRACTION

- Steps:
  - 1. Take 9’s complement for negative number
  - Perform BCD Addition
  - If result is invalid BCD, Then take corrective measure by adding 0110
  - Shift the carry to next
  - If the end carry generated then add it to result (Rule of 9’s complement)
  - If the end carry not generated answer is negative and it is in it’s 9’s complement form

BCD SUBTRACTION USING 9's COMPLEMENT

- $(7)_{10} - (5)_{10}$
- 7 → $(0111)_{BCD}$
- 5 → $(0101)_{BCD}$
  - (-5)
  - 9's complement → $(0100)_{BCD}$

$(0\ 1\ 1\ 1)_{BCD}$

$+ (0\ 1\ 0\ 0)_{BCD}$

$(1\ 0\ 1\ 1)_{BCD}$ INVALID BCD

$+0\ 1\ 1\ 0$

$(1\ 0\ 0\ 0\ 1)_{BCD}$

$+$

1

$(0\ 0\ 1\ 0)_{BCD}$

EXAMPLES:

- $(52)_{10}$ - $(68)_{10}$
- $(98.3)_{10}$ - $(81.2)_{10}$

BCD SUBTRACTION USING 10's COMPLEMENT

- $(7)_{10} - (5)_{10}$
- 7 → $(0111)_{BCD}$
- 5 → $(0101)_{BCD}$

→ • (-5)

→ $(0101)_{BCD}$

↑ 10's complement

$(0\ 1\ 1\ 1)_{BCD}$

$+ (0\ 1\ 0\ 1)_{BCD}$

$(1\ 1\ 0\ 0)_{BCD}$

$+0\ 1\ 1\ 0$

$(1\ 0\ 0\ 1\ 0)_{BCD}$

INVALID BCD

→ Discard carry ANSWER is positive