Longest Common Subsequence.

Input : Two Sequences X, Y

$$X : \langle x_1, x_2, \ldots, x_m \rangle$$

$$Y : \langle y_1, y_2, \ldots, y_n \rangle$$

Objective : To find $LCS(X,Y)$.

X :   A A A T A C T A G A T C C A

Y :   T T A T C T C G C A A T A

Subproblems ✱

$L[i,j]$: length of the longest common subsequence of $X[1..i]$ and $Y[1..j]$.

Our solution :   L[m,n]

Recurrence : ⊛

$$
L[i,j] = \begin{cases} 0, & i = 0 \text{ or } j = 0 \\ L[i-1, j-1] + 1, & x_i = y_j \\ \max\{L[i-1,j], L[i, j-1]\}, & o.w. \end{cases}
$$

$$x_1 \quad x_2 \quad \cdots \cdots \quad x_{i-1} \quad x_i$$

$$y_1 \quad y_2 \quad \cdots \cdots \quad y_{j-1} \quad y_j$$

	0	1		j		n
0	0	0	◡	0	◡	0
1	0					
	0			⋮		
i	0					
	0			⋮		
m	0					

$O(mn)$.

LLCS (X, Y)

for i ← 0 to m do

    $L[i, 0] \leftarrow 0$

for j ← 1 to n do

    $L[0, j] \leftarrow 0$

for i ← 1 to m do

  for j ← 1 to n do

    if $x_i = y_j$ then

      $L[i, j] \leftarrow L[i-1, j-1] + 1$

    else

      $L[i, j] \leftarrow \max\{L[i, j-1], L[i-1, j]\}$

<u>return</u> $L[m,n]$

$LCS(L, X, Y)$

   $S \leftarrow \{ \}$

   $i \leftarrow m, \quad j \leftarrow n$

   <u>while</u> $i > 0$ and $j > 0$ <u>do</u>

      <u>if</u> $x_i = y_j$ <u>then</u>

        add $\underline{x_i}$ at the beginning of S.

        $i \leftarrow \underline{i-1}$, $\underline{j-1}$

     <u>else</u> if $L[i,j] > L[i-1,j]$ <u>then</u>

        $j \leftarrow j-1$

    <u>else</u>

       $i \leftarrow i-1$

  <u>return</u> S

Edit Distance

Input: Two strings $X[1 \cdots m]$ & $Y = [1 \cdots n]$

Objective: To find the edit distance of X & Y,
i.e., # inserts, # deletes & # swaps to
convert string X into string Y.

R o h a n

S o h a m

| X | X | X | X | X | ✓ | ✓ | ✓ | ✓ ✓ |

✓R ✓O h a n — — = — —

✓= — ✓ — = < — S o h a m

R	o	h	a	n
S	o	h	a	m
	v	v	v	v

Swap   Swap

R	o	h	h	a	n
S	o	h	—	a	m

Swap   delete   Swap

Subproblems

$E[i, j]$: edit distance of $X[1..i]$ & $Y[1..j]$.

Our soln : $E[m, n]$

Recurrence :