Module 8:- Algebric structure

5

(i) 01010 & 01010

$|01010 \oplus 01010| = 00000 = 0$

(ii) 10101 & 01110

$|10101 \oplus 01110| = 11011 = 4$

6.

Here, Minimum distance for encoding function is 3.

$\boxed{\text{case-I}}$:- Error can be detect are,

$k+1 = 3$

$\therefore k = 2$

$\boxed{\text{case-II}}$:- Error can be corrected are,

$2k+1 = 3$

$2k = 2$

$k = 1$

Hence,

2 errors can be detected.

1 error can be corrected.

7. $e(00) = 00000$

$e(10) = 10111$

$e(01) = 01110$

$e(11) = 11111$

$|x_0 \oplus x_1| = |00000 \oplus 10111| = 4$

$|x_0 \oplus x_2| = |00000 \oplus 01110| = 3$

$|x_0 \oplus x_3| = |00000 \oplus 11111| = 5$

$|x_1 \oplus x_2| = |10111 \oplus 01110| = 3$

$|x_1 \oplus x_3| = |10111 \oplus 11111| = 1$

$|x_2 \oplus x_3| = |01110 \oplus 11111| = 2$

Hence, Minimum distance is 1.

$e: B^2 \to B^5$

$e(00) = 00000 = x_0$

$e(10) = 10101 = x_1$

$e(01) = 01110 = x_2$

$e(11) = 11011 = x_3$

(i) closure.

$\oplus$	x_0	x_1	x_2	x_3
x_0	x_0	x_1	x_2	x_3
x_1	x_1	x_0	x_3	x_2
x_2	x_2	x_3	x_0	x_1
x_3	x_3	x_2	x_1	x_0

It is closed.

~~x_0 is identity element~~.

(ii) Associativity.

$x_1 \oplus (x_2 \oplus x_3) = x_0$

$x_2 \oplus (x_1 \oplus x_3) = x_0$

It is Associative.

(iii) identity.

$x_0 \oplus x_0 = x_0$

$x_1 \oplus x_0 = x_1$

$x_2 \oplus x_0 = x_2$

$x_3 \oplus x_0 = x_3$

x_0 is identity element.

(iv) Inverse.

$\cancel{x_0 \oplus x_0} =$ $(x_0)^{-1} = x_0$

$(x_1)^{-1} = x_1$

$\cancel{x_1 \oplus}$

$(x_2)^{-1} = x_2$

$(x_3)^{-1} = x_3$

Hence, it is group code.

14.

Ans. $e : B^3 \to B^7$

$e(000) = 0000000 = x_0$

$e(001) = 0010110 = x_1$

$e(010) = 0101000 = x_2$

$e(011) = 0111110 = x_3$

$e(100) = 1000010 = x_4$

$e(101) = 1010011 = x_5$

$e(110) = 1101101 = x_6$

$e(111) = 1111011 = x_7$

~~(i) closure~~

$\oplus$	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_0	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_1	x_1	x_0	x_3	x_2	x_5	x_4	x_7	x_6
x_2	x_2	x_3	x_0	x_1	x_6	x_7	x_4	x_5
x_3	x_3	x_2	x_1	x_0	x_7	x_6	x_5	x_4
x_4	x_4	x_5	x_6	x_7	x_0	x_1	x_2	x_3
x_5	x_5	x_4	x_7	x_6	x_1	x_0	x_3	x_2
x_6	x_6	x_7	x_4	x_5	x_2	x_3	x_0	x_1
x_7	x_7	x_6	x_5	x_4	x_3	x_2	x_1	x_0

~~It is closed~~

~~(ii) Associative:~~

$x_1 \oplus$

Here, we find that range of e is group.

range e is subset of $B^7 \Rightarrow$ range of e is subgroup of B^7.

15.

Ans.

$m=2 \quad n=5$

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Here, $B^2 = \{00, 01, 10, 11\}$

$e(00) = 00 \quad x_1 \; x_2 \; x_3$

$$[x_1 \; x_2 \; x_3] = [0 \; 0] \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$x_1 = 0, \; x_2 = 0, \; x_3 = 0$

$e(01) = 01 \quad x_1 \; x_2 \; x_3$

$$[x_1 \; x_2 \; x_3] = [0 \; 1] \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$x_1 = 0, \; x_2 = 1, \; x_3 = 1$

$e(10) = 10 \quad x_1 \; x_2 \; x_3$

$$[x_1 \; x_2 \; x_3] = [1 \; 0] \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$x_1 = 1, \; x_2 = 1, \; x_3 = 0$

$e(11) = 11 \quad x_1 \; x_2 \; x_3$

$$[x_1 \; x_2 \; x_3] = [1 \; 1] \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$x_1 = 1, \; x_2 = 0, \; x = 1$

Hence, group code of $e: B^2 \to B^5$ is defined as

$e(00) = 00\ 000$

$e(01) = 01\ 011$

$e(10) = 10\ 110$

$e(11) = 11\ 101$

16. $n = 6, \quad m = 3$

$r = n - m = 3$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$B^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$

$e(000) = 000\ x_1\ x_2\ x_3$

$$[x_1\ x_2\ x_3] = [0\ 0\ 0] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$x_1 = 0 \qquad x_2 = 0 \qquad x_3 = 0$

$e(001) = 001\ x_1\ x_2\ x_3$

$$[x_1\ x_2\ x_3] = [0\ 0\ 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$x_1 = 1 \qquad x_2 = 1 \qquad x_3 = 1$

$e(010) = 010\ x_1\ x_2\ x_3$

$$[x_1\ x_2\ x_3] = [0\ 1\ 0] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$x_1 = 0 \qquad x_2 = 1 \qquad x_3 = 1$

$e(011) = 011 \quad x_1 \; x_2 \; x_3$

$$[x_1 \; x_2 \; x_3] = [0 \; 1 \; 1] \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$x_1 = 1 \qquad x_2 = 0 \qquad x_3 = 0$

$e(100) = 100 \quad x_1 \; x_2 \; x_3$

$x_1 = 1 \qquad x_2 = 0 \qquad x_3 \; 0$

$e(101) = 101 \quad x_1 \; x_2 \; x_3$

$x_1 = 0 \qquad x_2 = 1 \qquad x_3 = 1$

$e(110) = 110 \quad x_1 \; x_2 \; x_3$

$x_1 = 0 \qquad x_2 = 1 \qquad x_3 = 1$

$e(111) = 111 \quad x_1 \; x_2 \; x_3$

$x_1 = 0 \qquad x_2 = 0 \qquad x_3 = 0$

Hence, group code of $e : B^3 \to B^6$ is defined as

$e(000) = 000000$

$e(001) = 001111$

$e(010) = 010011$

$e(011) = 011100$

$e(100) = 100100$

$e(101) = 101011$

$e(110) = 110011$

$e(111) = 111000$

17.

Ans. $e_H : B^2 \longrightarrow B^5$

$e(00) = 00000 = x_0$

$e(10) = 10101 = x_2$

$e(01) = 01110 = x_1$

$e(11) = 11011 = x_3$

(i) $x_t = 11110$

$|x_t \oplus x_0| = |11110 \oplus 00000| = 4$

$|x_t \oplus x_2| = |11110 \oplus 10101| = 3$

$|x_t \oplus x_1| = |11110 \oplus 01110| = 1$

The encoding function $e(01)$ will be used for decoding 11110

(ii) $x_t = 10011$

$|x_t \oplus x_0| = |10011 \oplus 00000| = 3$

$|x_t \oplus x_1| = |10011 \oplus 01110| = 4$

$|x_t \oplus x_2| = |10011 \oplus 10101| = 2$

$|x_t \oplus x_3| = |10011 \oplus 11011| = 1$

The encoding function $e(11)$ will be used for decoding 10011

$e: B^3 \to B^5$

18. $e(000) = 00000$

$e(001) = 00110$

$e(010) = 01001$

$e(011) = 01111$

$e(100) = 10011$

$e(101) = 10101$

$e(110) = 11010$

$e(111) = 11100$

(i) $x_t = 11001$

$|x_t \oplus x_0| = 3$

$|x_t \oplus x_1| = 5$

$|x_t \oplus x_2| = 1$

The encoding function $e(010)$ will be used for decoding 11001.

(ii) $x_t = 01010$

$|x_t \oplus x_0| = 2$

$|x_t \oplus x_1| = 2$

$|x_t \oplus x_2| = 2$

$|x_t \oplus x_3| = 2$

$|x_t \oplus x_4| = 3$

$|x_t \oplus x_5| = 5$

$|x_t \oplus x_6| = 1$

The encoding Function $e(110)$ will be used for decoding 01010.

9 Q.3.

Ans. $G = \{1, -1, i, -i\}$

*	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

(i) closure.

It is clear that all elements of composition belongs to G.

Hence, it is closed.

(ii) Associativity.

$\forall\ a, b, c \in G$.

~~a·(b·c) =~~

$\therefore\ a\cdot(b\cdot c) = (a\cdot b)\cdot c$

It is Associative.

(iii) Identity.

$\forall\ a, b, c \in G$.

$a\cdot 1 = 1\cdot a$

'1' is an identity element.

(iv) Inverse

$(1)^{-1} = 1$

$(-1)^{-1} = -1$

$(i)^{-1} = -i$

$(-i)^{-1} = i$

~~(v) commutative~~

Hence, $G = \{1, -1, i, -i\}$ is a group.

(v) commutative

$\forall a, b, c \in G$

$a \cdot b = b \cdot a$

Hence, G is an abelian group.

(vi) cycle group.

$i^1 = i$ or $(-i)^1 = -i$

$i^2 = -1$ $(-i)^2 = -1$

$i^3 = -i$ $(-i)^3 = i$

$i^4 = 1$ $(-i)^4 = 1$

Hence, G is cyclic group.

i or $-i$ is generator of G.

$G = \{1, \omega, \omega^2\}$

$*$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

from above composition table:

(i) closure

Here, It is clear that all element in composition table belongs to G.

(ii) Associativity

$\forall a, b, c \in G$

$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

It is Associative.

(iii) identity

$\forall a, b, c \in G$

$a \cdot 1 = 1 \cdot a$.

'1' is identity element.

(iv) inverse :-

$1^2 = 1$

$(\omega)^2 = \omega^2$

$(\omega^2)^2 = \omega$

Inverse exists.

(v) commutativity

$\forall a,b,c \in G$

$a \cdot b = b \cdot a$

(vi) Cyclic group:

$(\omega)^1 = \omega$

$(\omega)^2 = \omega^2$

$(\omega^3) = 1$

ω is generator of G.

G is cyclic group.

1.

Ans. Group:-

The algebric structure $(G, *)$ is said to be group if is satisfies following:- e.g ⇒ $(I, +)$ is group

(i) closure

(ii) Associativity

(iii) Existence of identity element

(iv) existence of inverse

Abelian group:

The group is said to be abelian group if it follows commutative law.

$\forall a,b \in G$.

$a * b = b * a$.

e.g:- $(R, +)$ is abelian group.

Let e_1 & e_2 be two identity element in G
By defination,

$a * e_1 = a = e_1 * a$ —(i)

$a * e_2 = a = e_2 * a$ —(ii)

from equation (i) and (ii),

$\therefore a * e_1 = a * e_2$

$\therefore e_1 = e_2$ ....{By using left cancellation law}

In a group $(G, *)$, identity element is unique.

$G = \{0, 1, 2, 3, 4, 5\}$

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
~~4~~	~~3~~	~~5~~	~~05~~	~~0~~	~~12~~	~~3~~
45	4	5	0	1	2	3
5	5	0	1	2	3	4

(i) closure

It is clear that all elements in the composition table are belongs to G.

(ii) Associativity.

$\forall a, b, c \in G$

$a +_6 (b +_6 c) = (a +_6 b) +_6 c$

It is Associative.

(iii) identity.

$\forall a, b, c \in G$.

'0' is identity element.

(iv) inverse.

$(0)^{-1} = 0$

$(1)^{-1} = 5$

$(2)^{-1} = 4$

$(3)^{-1} = 3$

$(4)^{-1} = 2$

$(5)^{-1} = 1$

3%.6

(v) cycle :-

$(1)^1 = 1$

$(1)^2 = 1 +_6 1 = 2$

$(1)^3 = 1 +_6 1 +_6 1 = 3$

$(1)^4 = 1 +_6 1 +_6 1 +_6 1 = 4$

$(1)^5 = 1 +_6 1 +_6 1 +_6 1 +_6 1 = 5.$

$(1)^6 = 0$

Here, '1' is a generator

Hence, it is cyclic group.

~~Q.5.~~

~~Ans $G = \{1, 5, 7, 11, 13, 17\}$.~~

2.

Ans: $a * e = e * a = a$

$\therefore a + b - ab = a$

$\therefore a + e - ae = a$

$\therefore e(1-a) = 0$

$\therefore \boxed{e = 0}$

'0' is identity element.

$(a \wedge (a \vee b))$