| Binary Number | Decimal Value | Binary Number | Decimal Value |
|---------------|---------------|---------------|---------------|
| 0 | 0 | 110 | 6 |
| 1 | 1 | 111 | 7 |
| 10 | 2 | 1000 | 8 |
| 11 | 3 | 1001 | 9 |
| 100 | 4 | 1010 | 10 |
| 101 | 5 | 1011 | 11 |

Table (1.2)

1.2 BINARY TO DECIMAL CONVERSION

To convert a binary numbers into decimal number follow the given procedure

Write the given binary Number

Step 2: Write the binary weightage below each number

Decimal point

| | | | | | | | | | Ÿ | | |
|------------------|-----|----------------|----------------|----------------|----------------|----------------|----|----|-----|------|-------|
| Binary Weightage | 27 | 2 ⁶ | 2 ⁵ | 2 ⁴ | 2 ³ | 2 ² | 21 | 20 | 2-1 | 2-2 | 2-3 |
| Decimal Value | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | 0.5 | 0.25 | 0.125 |

Table (1.3)

Step 3: Cancel the weightage, which is placed below zero because any number multiplied by zero is zero.

Step 4: Add the remaining numbers.

Examples:

1) Convert (1001)₂ in to decimal no.

Step 2:
$$8 + 4 + 2 + 1$$

Step 3:
$$8 + 4 + 2 + 1$$

$$(1001)_2 = (9)_{10}$$

(Verification)

Refer table (1.2) 1001 is equivalent to decimal nine.

2) Convert (110011) 2 into decimal

5) If
$$(100001)_2 = (X)_{10}$$
 find X
1 0 0 0 0 1
32 + 16 + 8 + 4 + 2 + 1
 $X = 33$
 $\therefore (100001)_2 = (33)_{10}$

Fractional binary numbers

To convert the mixed binary numbers containing integers and fractions follow the same procedure and make use of the table (1.3). Let us solve two examples

1) Convert
$$(10101. \ 101)_2$$
 into decimal
1 0 1 0 1 . 1 0 1
16 + $\frac{8}{4}$ + 4 + $\frac{7}{2}$ + 1 . + 0.5 + 0.25 + 0.125
= $(21.625)_{10}$
2) Convert $(1001.011)_2$ into decimal
1 0 0 1 . 0 1 1
8 + $\frac{4}{4}$ + $\frac{7}{2}$ + 1 . 0.5 + 0.25 + 0.125
= $(9.375)_{10}$

1.3 DECIMAL TO BINARY CONVERSION

Let us see now, how a decimal number is converted into its binary equivalent number. Dividing the given number by two and taking only remainders do this. This method of divide by two is known as "double - dabble method". Before that; let us see what is MSB and LSB. MSB represents "Most Significant Bit" that represents the first number in given number, which has got maximum weightage. LSB means the "Least Significant Bit" which represents the last number which has got lowest weightage.

For Example; 10010

MSB (1
$$\times$$
 2⁴)

MSB (1 \times 2⁴)

MSB (1 \times 2⁴)

LSB (0 \times 2⁰)

LSB (1 \times 2⁰)

Double Dabble Method

To convert decimal number into binary number divide the given number by '2' till you get '1' as shown in the following examples

1.
$$29 \div 2$$
 remainder $29 - 28 = 1$ LSB
 $14 \div 2$ remainder $14 - 14 = 0$
 $7 \div 2$ remainder $7 - 6 = 1$
 $3 \div 2$ remainder $3 - 2 = 1$
1 MSB

$$(29)_{10} = (11101)_2$$

It can be verified by reverse process

2. Convert (35) 10 into binary

| LSB | THE POST | |
|-----|----------|--------------------|
| 1 1 | 35 - 34 | $(35 \div 2 = 17)$ |
| 1 | 17 - 16 | $(17 \div 2 = 8)$ |
| 0 | 8 - 8 | $(8 \div 2 = 4)$ |
| 0 | 4-4 | $(4 \div 2 = 2)$ |
| 0 | 2-2 | $(2 \div 2 = 1)$ |
| * | -1 - | |
| | Maria 1 | MSB |

$$\therefore (35)_{10} = (100011)_2$$

3. Convert (102) 10 into binary.

| A | |
|-------------|---------|
| 0 | 102-102 |
| 13.01 | 51 - 50 |
| ing teminal | 25 - 24 |
| 0 | 12 - 12 |
| 0 | 6-6 |
| 1 | 3 - 2 |
| 4 | +1 |

$$(102)_{10} = (1\ 100110)_2$$

4) Convert (88) 10 into binary

| 10 | 88 - 88 |
|----|---------|
| 0 | 44 - 44 |
| 0 | 22 – 22 |
| 1 | 11 - 10 |
| 1 | 5-4 |
| 0 | 2-2 |
| | 1 |

$$(88)_{10} = (1011000)_2$$

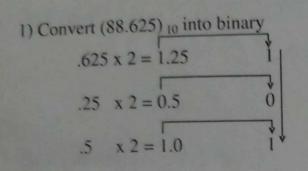
5) Convert (69) 10 into binary

| 1 1 | 69 - 68 |
|------|-----------|
| 0 | 34 - 34 |
| 1 | 17 - 16 |
| 0 | 8 - 8 |
| 0 | 4-4 |
| 0 | 2-2 |
| - | -1 |
| ((0) | (1000101) |

$$\therefore (69)_{10} = (1000101)_2$$

Fractional numbers

To convert a fractional numbers in to decimal multiply each bit by 2 and take integer value out as shown in the following examples. In these examples integers can be converted by double-dabble method (not shown) refer the above examples.



$$\therefore (88.625)_{10} = (1011000, 101)_2$$

2) Convert (69. 375) 10 into binary
$$.375 \times 2 = 0.75$$

$$.75 \times 2 = 1.50$$

$$.5 \times 2 = 1.0$$

$$\therefore (69.375)_{10} = (1000101.011)_2$$

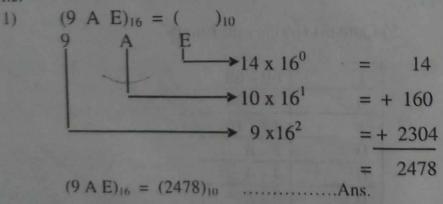
(Note that in case of odd fractional members find the binary numbers up to 4 digits or 5 digits to use approximation)

1.4 HEXADECIMAL NUMBER SYSTEM

In this system hex means six and decimal indicates ten, total sixteen symbols involves from 0 to 9 and A to F. Its radix is 16, each number is expressed in the power of 16. The following symbols indicate their decimal values.

Hex to decimal conversion

To convert a given hexadecimal number into decimal each number is multiplied by 16ⁿ. i.e.



Decimal to Hexadecimal conversion

 $= (685.1679)_{10}$

Convert the given decimal number into hexadecimal by using hex-dabble method, like double dabble method where the given number is divided by 16.

$$(928)_{10} = (3A0)_{16}$$
 $(2478)_{10} = (9A E)_{16}$
Where $14 = E$ and $10 = A$

Hex to Binary and Binary to Hex conversion

To convert a hexadecimal number in to binary number, convert each hex digit in to a 4bit binary code just like 8421 BCD. On the other hand, to convert a binary number in to Hexadecimal number make the group of 4-bits and find their Hex values refer the following examples

- Convert (5 A D)₁₆ in to binary A D = 0101 1010 1101
- 2) Convert (C9F.2A)₁₆ in to binary = C 9 F. 2 A= 1100 1001 1111, 0010 1010
- Convert (110100011111)₂ in to Hex 3) = 1101 0001 1111 = D
- 4)Convert(11110010.1110)2 in to Hex = 1111 0010 . 1110 F 2 . E

OCTAL NUMBER SYSTEM 1.5

In this system eight symbols from 0 to 7 are used and its radix is 8. To convert a octal number in to decimal follow the given table.

Decimal point 80 Octal 8-2 81 84 03 22 Weightage **Decimal Value** 4096 512 64 8 0.125 0.015625 1

FIRS

ry

Decimal to Octal conversion

Convert the given decimal number into Octal, like double dabble method where the given integer number is divided by 8 and for fractional number multiply by 8.

$$(2470)_{10} = (4 6 46)_8...$$
Ans.

Octal to Binary and Binary to Octal Conversion

To convert a Octal number in to binary number, convert each hex digit in to a 3-bit binary code. On the other hand, to convert a binary number in to Octal number make the group of 4-bits and find their decimal values refer the following examples

1) Convert
$$(5\ 2\ 3)_8$$
 in to binary 2) Convert $(7\ 0\ 4.01\)_8$ in to binary = 5 2 3 = 7 0 4. 0 1 = 111 000 100.000 001

2) Convert
$$(7 \ 0 \ 4.01)_8$$
 in to binary
= 7 0 4.0 1
= 111 000 100.000 001

3)Convert
$$(1101000111111)_2$$
 in to Octal
= $110 100 011 111$
= $6 4 3 7$

Hex to Octal and Octal to Hex Conversion

To convert a Hex number in to Octal first convert Hex in to Binary and then make groups of 3-bits from LSB add zeros on left and right side if required. After making the groups convert each 3-bit binary in to Octal equivalent.

1)
$$(5 \text{ C A})_{16} = (?)_8$$

= 5 C A
= $(0101 \ 1100 \ 1010)_2$

2)
$$(3B.2E)_{16} = (?)_8$$

= 3 B . 2 E
= $(0011 \ 1011 \ .0010 \ 1110)_2$

$$= 010 \ 111 \ 001 \ 010$$

$$= 2 \ 7 \ 1 \ 2$$

$$= 000 \ 111 \ 011 . \ 001 \ 011 \ 100$$

$$= 0 \ 7 \ 3 . \ 1 \ 3 \ 4$$

$$(5 \ C \ A)_{16} = (2712)_{8}$$

$$(3B.2E)_{16} = (073.134)_{8}$$

To convert a Octal number in to Hex first convert Octal in to Binary and then make groups of 4-bits from LSB add zeros on left and right side if required. After making the groups convert each 4-bit binary in to Hex equivalent refer the following examples

3)
$$(537)_8 = (?)_{16}$$

= 5 3 7
= $(101\ 011\ 111)_2$
= 0001 0101 1111
= 1 5 F
 $(5\ 3\ 7)_8 = (15F)_{16}$

2)
$$(53.21)_8 = (?)_{16}$$

= 5 3 . 2 1
= $(101\ 011\ .010\ 001)_2$
= 0010 1011. 0100 0100
= 2 D . 4 4
 $(53.21)_8 = (2D.44)_{16}$

1.6 BCD, ASCII and EBCDIC CODES

In binary, for higher decimal numbers it becomes a long chain of 0 & 1, to avoid this, different BCD codes are used to represent decimal numbers. In computer arithmetic circuits 8421 BCD is a most common code.

In BCD code, each decimal number is represented by a 4-bit code.

The zeros on left of 8421 BCD can be omitted. To convert a BCD number into decimal back, the process is exactly opposite, by making group of 4 bits from the left and converting each 4 - bit number into decimal number.

e. g.
$$100100111001_{BCD}$$

= $1001 0011 1001$
= $9 3 9$

There are some other codes like 5421 BCD, 5311 BCD etc. But the most common method is 8421 BCD.

Advantages of BCD

- 1 The length of number is short.
- 2 Easy to convert to and from BCD
- 3. Suitable for hexadecimal addition.

ASCII and EBCDIC CODES

These codes are normally used for computer keyboard; ASCII code uses either 7- bit code or 8-bit code to represent the numbers and the characters. ASCII stands for American Standard Code for Information Interchange. EBCDIC code is a similar 8- bit code it is developed by IBM which is obtained by extending six bit BCD code. It is Extended BCD Interchange Code, e.g. In ASCII the code for CAT is easily obtained by writing the binary code for C,A and T

C=11 0011 A = 11 0001 T= 01 0011 (Refer ASCII 7 -bit Chart) CAT =(11 0001 11 0001 01 0011)

BINARY ADDITION AND SUBTRACTION 1.7

In computer circuits arithmetic logic unit performs basic arithmetic operations by performing addition only like subtraction by addition, multiplication by addition etc.

Let us see basic addition and subtraction rules in binary.

| | Rules | Addition | Subtraction |
|-------|----------------------------|--|--|
| | 1. 2. 3. 4. 5. | 0+0=0 0+1=1 1+0=1 1+1=10 (Carry1 sum 0) 1+1+1=11 (Carry1 sum1) | 0-0=0 0-1=11 (difference1 and borrow 1) 1-0=1 1-1=0 |
| e. g. | 1 2 +1 4 | 11 0 0 +1110 Carry -> 11 | (Verification) 1 1 0 1 0 = 16 8 4 2 1 |
| | 26 | 11010 | = 16 + 8 + 2 = 26 |

The way to verify the binary addition and subtraction, first find the decimal equivalent of the given binary number and then check the addition with the final binary number.

e.g. 13 1101 First column
$$1-0=1$$

$$\frac{-10}{3} \frac{-1010}{0011}$$
 Second column $0-1=1$
i.e. difference 1 and borrow 1
Third column $1-(0+1)=0$
Fourth column $1-1=0$

The above method discussed is not suitable for signed numbers, I's or 2's complement method is used for signed numbers.

1.8 1'S AND 2'S COMPLEMENT FOR SUBTRACTION

These two methods can perform subtraction by addition. Similarly, they can generate signed numbers like + 2, - 5 etc. After subtraction one can easily verify the difference with sign.

Before we learn subtraction let us find 1's and 2's complement of a given number (A) 'A 'denotes 1's complement and it is obtained by replacing 0 by 1 and 1 by 0.

e. g. If A = 1010 then A = 01012's complement is the number that results when we add 1 to the 1's complement. It is denoted by A' 2's complement = (1's complement) + 1

$$A' = \overline{A} + 1$$

Examples1: Find 1's complement of 101011 and 100110

Solution: i) A = 101011 A = 010100 (1's complement)

ii) A = 100110 A = 011001 (1's complement)

Examples2: Find 2's complement of 111010 and 110100

Solution: i) A = 111010 $\overline{A} = 000101$ (1's complement) A' = 000101 + 1 = 000110 (2's complement)

i) A = 110100 $\overline{A} = 001011$ (1's complement)

A = 110100 A = 001011 (1's complement) A' = 001011 +1 = 001100 (2's complement)

1's complement subtraction 2's complement subtraction

The procedure to perform subtraction A - B, follow the following procedure.

| | | = 5 complement subtraction |
|--------|--------------------------------------|-----------------------------|
| Step 1 | Find binary of A | Find binary of A |
| Step 2 | Find 1's complement of B | Find 2's complement of B |
| Step 3 | Add step 1 with step 2 | Add step 1 with step 2 |
| Step 4 | i) If carry 1 is present | If carry 1 is present |
| * | answer is +ve make | answer is +ve |
| | End around carry. | Delete the carry. |
| | ii) If carry is absent answer is -ve | If carry is absent answer |
| | find 1's complement | is -ve find 2's complement. |

Examples:

1. Perform 12 – 9 and 9 – 12 by using 1's complement method.

Solution : (12 - 9)

Step 1 Binary of 12 is 1 1 0 0

Step 2 Binary of 9 is 1001

1's complement is 0 1 1 0

Step 3 1 1 0 0 + 0 1 1 0

Carry present→ 1 0 0 1 0

Step 4 Answer is +ve end around carry

10010 +1 (EAC)

Ans. $0\ 0\ 1\ 1 = (+3)$

(9 - 12)

Binary of 9 is 1 0 0 1

Binary of 12 is 1 1 0 0

1's complement is 0 0 1 1

1001

 $\begin{array}{c} + & \underline{0011} \\ \text{Carry absent} \longrightarrow & \underline{1100} \end{array}$

Answer is -ve

1's complement of 1 1 0 0 is

0011 = (-3)

2. Perform 14 - 9 and 9 - 14 by using 2's complement method.

Solution: (14-9)

Step 1 Binary of 14 is 1 1 1 0

Step 2 Binary of 9 is 1001

2's complement is 0 1 1 1

Step 3

1110

+ 0111

Carry present 10101

Step 4 Answer is +ve delete the carry

10101

Ans. $0\ 1\ 0\ 1 = (+5)$

(9 - 14)

Binary of 9 is 1 0 0 1

Binary of 14 is 1 1 1 0

2's complement is 0 0 1 0

1001

+ 0010

Carry absent → 1011

Answer is -ve

2's complement of 1 0 1 1 is

0101 = (-5)

1.9 BINARY MULTIPLICATION AND DIVISION

These two Arithmetic operations in binary are easily performed just like simple decimal multiplication and division. Consider the following illustrative examples, i) $(101 \times 110)_2 = (5 \times 6)_{10}$ ii) $(1101 \times 111)_2 = (13 \times 7)_{10}$

iii)
$$(1100 \div 10)_{2} = (12 \div 2)_{10}$$

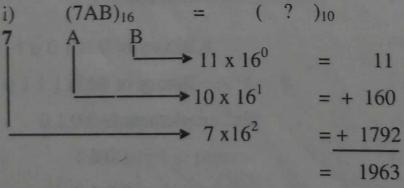
iv)(
$$11001 \div 101$$
)_{2 =} ($25 \div 5$)₁₀

$$\begin{array}{c|c}
 & 110 \\
 & 1100 \\
 & -10 \\
\hline
 & 10 \\
 & -10 \\
\hline
 & 00 \quad \text{Ans: } (110)_2 = (6)_{10}
\end{array}$$

$$\begin{array}{r}
101 \\
1001 \\
- 101 \\
\hline
00101 \\
-101 \\
\hline
000 \text{ Ans: } (101)_2 = (5)_{10}
\end{array}$$

SOLVED EXAMPLES

1. Convert the following.



ii) $(1001\ 10000\ 0001)_{BCD} = (?)_{10}$ making group of 4 bits $(1001) = (9)_{10}$ $(1000) = (8)_{10}$ $(0001) = (1)_{10}$ $(1001\ 1000\ 0001)_{BCD} = (981)_{10}$ Ans 2, Convert the following:

i)
$$[11001.101]_2 = []_{10}$$

=1 1 0 0 1 . 1 0 1
=16 8 4/2 1 . 0.5 -0.25 0.125
=16 +8+1 . 0.5 +0.125
=[25.625]_{10}

ii) $[110101.110001]_2 = []_{16}$ =110101.110001 =0011 0101. 1100 0100 = 3 5 .C =[35.C4]₁₆

iii)[2AF]16 =[]2 Convert it into BCD

 $\therefore [2AF]_{16} = [0010\ 1010\ 1111]_2$

Perform the following operation using 2's complement method use 8-bit representation of numbers $(52)_{10} - (65)_{10} = (?)_2$

8-bit binary equivalent of 52 = 00110100 (i) 8-bit binary equivalent of 65 = 010000011^S complement of 65 = 1011111102^S complement of 65 = 101111110 + 1= 10111111 (ii)

52-65 = 00110100+101111111

No carry __ 11110011

.. Answer is negative

Find 2's complement of the result

A = 11110011

A = 00001100

 $A' = 00001100 + 1 = (00001101)_2 = (-13)_{10}$ Ans:

OUESTIONS

- Select the correct answer. 1.
 - i) The BCD equivalent of a decimal number 88 is......
 - a) 0010
- b) 0001
- 0) 1000 1000

ii) The decimal equivalent of a hexadecimal number 2 A is......

- (a) 42
- b) 62
- c) 82 d) 17

iii) The Radix of the octal system is......

| | Access to the second |
|--|----------------------|
| a) 2 b) 16 c) 10 d) 8 | |
| (v) The binary equivalent of hexadecimal number (B3) ₁₆ is | |
| a) 10110011 b) 11011011 c) 0011 d) 00010001 | |
| v) In positive logic system '1' represents | |
| a) The more negative of two voltage levels. b) zero voltage. | |
| The more positive of two voltage levels. d) Negative one volt | |
| vi) 2's complement of (12) ₁₀ is | |
| a) 0001, b) 0010 c) 0011 d) none of these | |
| vii) In binary numbers if the last bit is 1 then it is anumber | |
| a) odd b) even c) none of these | |
| viii) $(11)_2 + (11)_2 + (11)_2 = \dots$ | |
| a) (111) ₂ b) (1011) ₂ g) (1001) ₂ d) (1111) ₂ | |
| ix)The equivalent decimal number of maximum highest binary number one byte is | of length |
| a)128 b) 127 c) 255 d) 256 | |

Ans. i) 1000 1000 ii) 42 iii) 8 iv) 1011 0011 v) The more +ve of two voltage levels vi) none of these vii) odd viii) (1001)2 ix)255

Problems For Practice

| I. | Convert the | following | decimal numbers into binary |
|----|-------------|-------------|-----------------------------|
| | a) 38 | b) 25.5 | c)10.625 |
| 2. | Encode the | following : | numbers into BCD code |

- - a) 128 b) 579 c) ABC

- Convert the following binary numbers into decimal number 3.

4.

a) 11001.101 b) 0.110 c) 101100.001

Convert the following Hex into Decimal.

a) 2E9

b) 09F7 c) 12C × 16

Convert the following Hex into Binary 5.

a) 3A5E

b) 0A3.D9 c) 1B5.C

Convert the following Hex into Octal. 6.

a) 2F7

b) BAB c) ABC.4D

Convert the following octal numbers into binary

a) 736

b) 574.321 c)116.54

Ans: 1) a) (100110)₂ b)(11001.1)₂ c) (1010.101)₂

2) a) (0001 0010 1000) b) (0101 0111 1001) c)(1010 1011 1100)

3) a) (25.625) 10 b) (.75) 10 c) (44.125) 10

4) a) $(745)_{10}$ b) $(2551)_{10}$ c) $(300)_{10}$

5) a) (0011 1010 0101 1110) 2 b) (0000 1010 0011.1101 1001) 2

c)(0001 1011 0101.1101)₂

6) a) (1367) 8 b) (5653) 8 c) (5274.232) 8

7) a) (111 011 110)₂ b) (101 111 100 . 011 010 001)₂ c) (001 001 110 . 101 100

Solve the following:

1. What do you mean by

a) 1's complement b) 2's complement explain with suitable examples.

2. Perform the following using binary arithmetic.