Engineering Mechanics

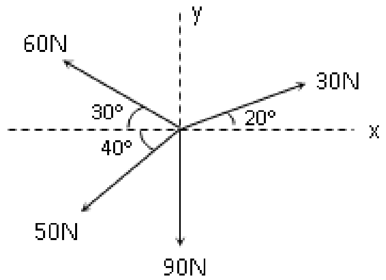
F.E. (All branches)

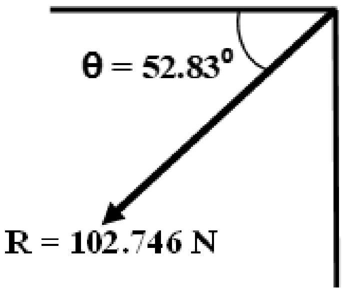
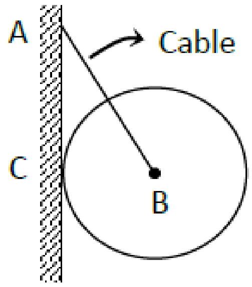
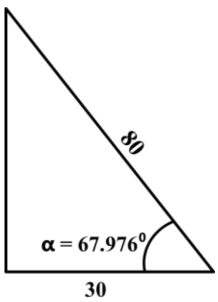
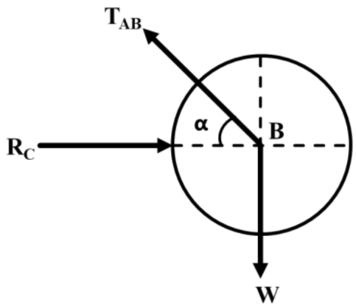
Time: 2 hr. 30 min.   Examination: Prelim Exam FE Semester I - Solution   Max. Marks: 80

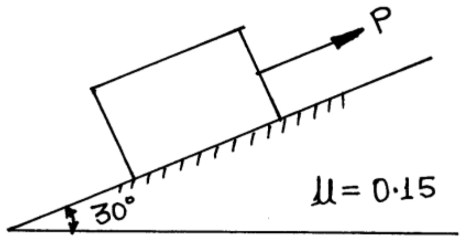
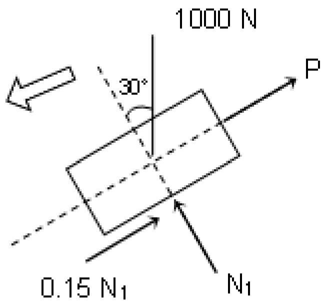
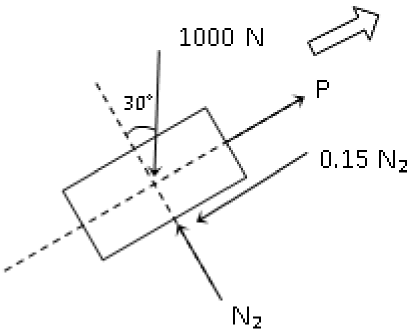
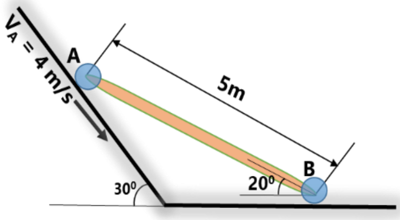
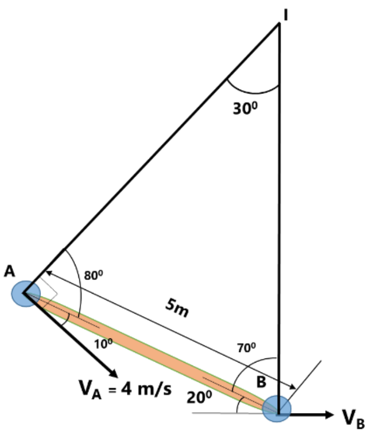
==

Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
1.	The forces, which do meet at a point but lie in a single plane, are known as
Option A:	Coplanar concurrent forces
Option B:	Coplanar non-concurrent forces
Option C:	Non-coplanar concurrent forces
Option D:	Non-coplanar non-concurrent forces
Answer	**Coplanar concurrent forces**
2.	A train enters curve of radius 600 m with a speed of 30 m/s, what will be the magnitude of tangential and normal acceleration at the instant the brakes are applied so that the train stops by covering a distance of 400 m along the curve.
Option A:	tangential acceleration = 1.125 m/s^2, normal acceleration = 1.5 m/s^2
Option B:	tangential acceleration = 1.125 m/s^2, normal acceleration = - 1.5 m/s^2
Option C:	tangential acceleration = - 1.125 m/s^2, normal acceleration = - 1.5 m/s^2
Option D:	tangential acceleration = - 1.125 m/s^2, normal acceleration = 1.5 m/s^2
Answer	**tangential acceleration = -1.125 m/s^2, normal acceleration = 1.5 m/s^2**
3.	Which of the following is not a projectile
Option A:	a bullet fired from a rifle
Option B:	a bomb dropped from an aeroplane
Option C:	hydrogen balloon floating in air
Option D:	a boy throw a ball oblique with vertical.
Answer	**hydrogen balloon floating in air**
4.	The point at which the total area of a plane figure is assumed to be concentrated is called
Option A:	Centre of gravity
Option B:	Central point
Option C:	Centroid
Option D:	Inertial point
Answer	**Centroid**
5.	Kinematics of the rigid body is
Option A:	Study of geometry of motion considering the cause of motion
Option B:	Study of external forces acting on it without considering the geometry of motion
Option C:	Study of geometry of motion without considering the cause of motion
Option D:	Finding the reaction forces and moments at the supports
Answer	**Study of geometry of motion without considering the cause of motion**
6.	A rod AB 26 m long leans against a vertical wall. The end A on the floor is drawn away from the wall at a rate of 24 m/s, when the end A of the rod is 10 m from the wall. What is the velocity of end B sliding down vertically?

Option A:	velocity of end B = 57 m/s
Option B:	velocity of end B = 10 m/s
Option C:	velocity of end B = 24 m/s
Option D:	velocity of end B = 12 m/s
Answer	**velocity of end B = 10 m/s**
7.	Lami's theorem is applicable for ________ force system.
Option A:	parallel force system
Option B:	general force system
Option C:	concurrent force system
Option D:	None of the above
Answer	**concurrent force system**
8.	A fixed support constrains
Option A:	Restrict the translation motion in one direction
Option B:	Restrict the translation motion in two mutually perpendicular directions
Option C:	Restrict rotational motion
Option D:	Restrict rotational motion and translation motion in two mutually perpendicular directions
Answer	**Restrict rotational motion and translation motion in two mutually perpendicular directions**
9.	Choose the correct statements from the following:
Option A:	Force is the product of Mass and Velocity
Option B:	Momentum is the product of Mass and Acceleration
Option C:	Torque is the product of Mass and Gravitational Acceleration
Option D:	Moment is the product of Force and perpendicular distance
Answer	**Moment is the product of Force and perpendicular distance**
10.	A 50 N force acts from point A (0,0,0) to the point B (3,0,4) then force is represented as
Option A:	10
Option B:	$10(3\vec{i}+0\vec{j}+4\vec{k})$
Option C:	$\frac{50}{\sqrt{7}}(0\vec{i}+3\vec{j}+4\vec{k})$
Option D:	$150(3\vec{j}+4\vec{k})$
Answer	$\mathbf{10(3\vec{i}+0\vec{j}+4\vec{k})}$

Q2.	**Solve any Four** **5 marks each**
i.	Find resultant of the force system.  **Figure 1**

	Solution: $\sum F_x = 30\cos 20 - 60\cos 30 - 50\cos 40 =$ $62.07\ N\ (\leftarrow)$ $\sum F_y = 30\sin 20 + 60\sin 30 - 90 - 50\sin 40 =$ $81.87\ N\ (\downarrow)$ $\therefore\ R = \sqrt{\sum {F_x}^2 + \sum {F_y}^2} = 102.746\ N$ $\therefore\ \theta = 52.83^0$ 
ii.	A cylinder B, W_B = 1000 N, dia. 60 cm, hangs by a cable AB = 80 cm rests against a smooth wall. Find out reaction at C and T_{AB}.  **Figure 2** **Solution:** $\propto = cos^{-1}\left(\frac{30}{80}\right) = 67.976°$   By Lami's theorem, $$\frac{T_{AB}}{sin\,90} = \frac{R_C}{sin\,157.976} = \frac{1000}{sin\,112.024}$$ $\therefore\ T_{AB}$ = 1078.717 N $\therefore\ R_C$ = 404.513 N
iii.	A car moves in a straight line such that for a short time its velocity is defined by v = ($9t^3$+$2t$) m/sec where 't' is time in seconds. Determine its position and acceleration when t = 3 sec. **Solution:** $a = \frac{dv}{dt} = 27t^2 + 2$ At t = 3 sec, a_3 = (27×9) + 2 = 245 m/s^2 Also, $\int ds = \int v \cdot dt$ $\therefore$ s = $2.25t^4 + t^2 + c$ At t = 0 → s = 0 m, $\therefore$ c = 0 At t = 3 sec, S_3 = (2.25 × 3^4) + 3^2 = 191.25 m

iv.	A block of weight 1000 N is kept on a rough inclined surface. Find out range of ‘P’ for which the block will be in equilibrium.  **Figure 3** **Solution:** **Case (i)**  P_{min} =? for holding the block $\sum F_Y = 0$ ∴ $N_1 - 1000 \cos 30 = 0$ $N_1 = 866.025$ N $\sum F_X = 0$ ∴ $P - 1000 \sin 30 + 0.15 N_1 = 0$ ∴ $P_{min} = 370.1$ N **Case (ii)**  P_{max} =? for pulling the block. $\sum F_y = 0$ ∴ $N_2 - 1000 \cos 30 = 0$ ∴ $N_2 = 866.025$ N $\sum F_X = 0$ ∴ $P - 1000 \sin 30 - 0.15 N_2 = 0$ ∴ $P_{max} = 629.9$ N Range of force P is $370.1 \leq P \leq 629.9$ N.
v.	Rod AB of length 5m is kept on smooth planes as shown in the figure. The velocity of the end A is 4 m/sec. along the inclined plane. Locate the ICR and find the velocity of the end B.  **Figure 4** **Solution:** The velocity of end A is downward along the inclined plane so end B will move towards right. Drawing perpendicular to both velocities, v_A & v_B we get the location of ICR which is shown by the point, I on the figure.  Applying sine rule to the ΔIAB, we get, $\frac{IA}{\sin 70} = \frac{IB}{\sin 80} = \frac{AB}{\sin 30}$ ∴ IA = 9.396 m ∴ IB = 9.848 m

$\therefore V_A = IA \times \omega_{AB}$

$\omega_{AB} = 4/9.396 = 0.425$ rad/s

$\therefore V_B = IB \times \omega_{AB} = 4.192$ m/s

vi. A force of 1200N acts along PQ, P (4, 5, –2) and Q (–3, 1, 6) m. Calculate its moment about a point A (3, 2, 0) m.

Solution:

$$\overline{F_{PQ}} = \mathbf{1200} \times \overline{e_{PQ}} = \mathbf{1200} \times \left[\frac{(-3-4)\hat{\imath}+(1-5)\hat{\jmath}+(6-(-2))\hat{k}}{\sqrt{7^2+4^2+8^2}}\right]$$

$$\overline{F_{PQ}} = \mathbf{105.65}(-7\hat{\imath} - 4\hat{\jmath} + 8\hat{k})\ N$$

$$\overline{M^F_{AP}} = \overline{r_{AP}} \times \overline{F_{PQ}} = \mathbf{105.65}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ (4-3) & (5-2) & (-2-0) \\ -7 & -4 & 8 \end{vmatrix}$$
$$= \mathbf{105.65}\ (16\hat{\imath} + 6\hat{\jmath} + 17\hat{k})\ Nm$$

Q3	**Solve any One** **10 marks each**

i. Determine the co-ordinates of the centroid of the plane area as shown in figure 5 with reference to the axes as shown.

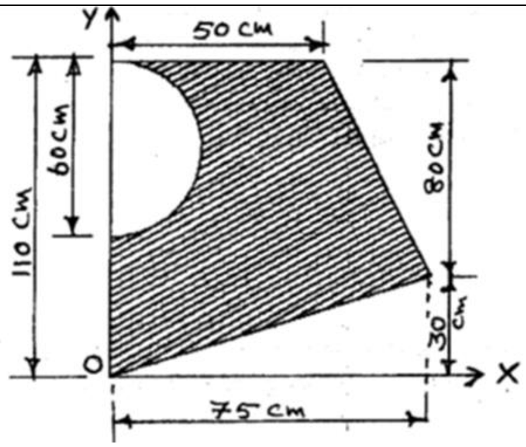


Figure 5

Solution:

Part	**A_i**	**x_i**	**y_i**	**A_ix_i**	**A_iy_i**
Rectangle	75×110 = 8250	37.5	55	309375	453750
Lower Triangle	-0.5×75×30 = -1125	$\frac{2\times 75}{3} = 50$	10	-56250	-11250
Upper Triangle	-0.5×25×80 = -1000	$50 + \frac{2\times 25}{3} = \frac{200}{3}$	$30 + \frac{2\times 80}{3} = \frac{250}{3}$	-66666.66	-83333.33
Semi-circle	$-\pi\times 30^2/2 = -450\pi$	$\frac{4\times 30}{3\pi} = 12.73$	80	-18000	-113097.33
	$\sum A_i$ = 4711.28			**$\sum A_ix_i$ = 168458.33**	**$\sum A_iy_i$ = 246069.34**

<table>
<tr><td></td><td>$$\therefore\ \bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = 35.75\ cm \quad \& \quad \bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 52.22\ cm$$</td></tr>
<tr><td>ii.</td><td>

A particle is projected with an initial velocity of 2 m/s along a straight line. The acceleration-time diagram for the linear motion is given in the figure. Construct velocity time and displacement time diagrams for the motion.

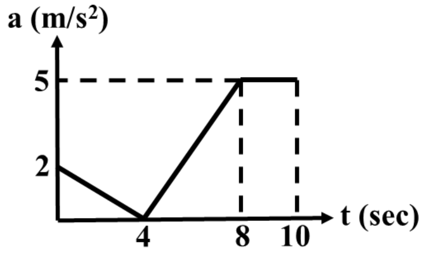


Figure 6

Solution: $V_0 = 2$ m/s; $S_0 = 0$ m ……….. (given)

From a – t curve:

From 0 – 4 sec:

Change in Velocity = Area under a – t curve

$V_4 - V_0 = 0.5\times4\times2 = 4$

$\therefore$ **V_4 = 6 m/s**

From 4 – 8 sec:

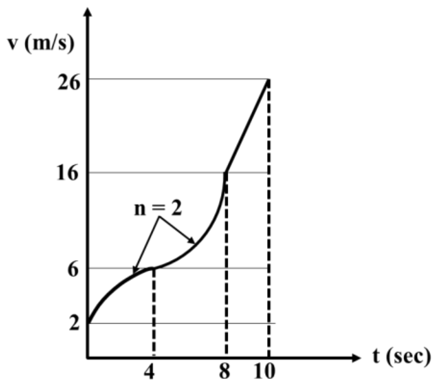
$V_8 - V_4 = 0.5\times4\times5 = 10$

$\therefore$ **V_8 = 16 m/s**

From 8 – 10 sec:

$V_{10} - V_8 = 2\times5 = 10$

$\therefore$ **V_{10} = 26 m/s**



From v – t curve:

From 0 – 4 sec:

Change in Displacement = Area under v – t curve

$S_4 - S_0 = (4\times2) + \frac{2\times4\times4}{3}$

$\therefore$ **S_4 = 18.667 m**

From 4 – 8 sec:

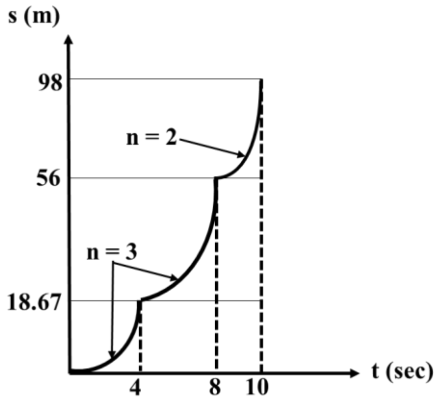
$S_8 - S_4 = (4\times6) + \frac{10\times4}{3}$

$\therefore$ **S_8 = 56 m**

From 8 – 10 sec:

$S_{10} - S_8 = 16\times2 + 0.5\times10\times2$

$\therefore$ **S_{10} = 98 m**



</td></tr>
</table>

iii.

Two cylinders are kept in a channel as shown in figure. Determine the reactions at all the contact points A, B, C and D. Assume all surfaces smooth.

(Taken W_A = 1000 N & W_B = 750 N)

(Also, r_A = 400 mm & r_B = 300 mm)

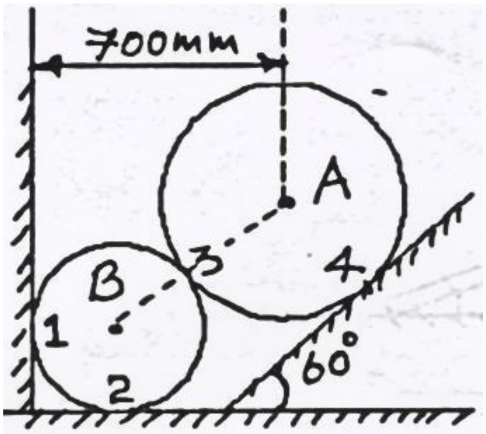
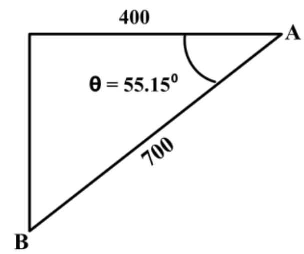
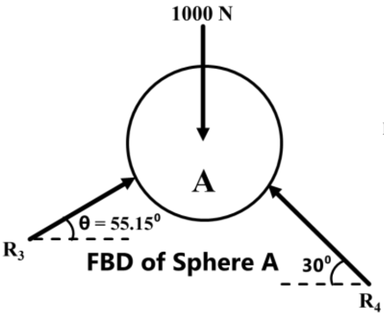


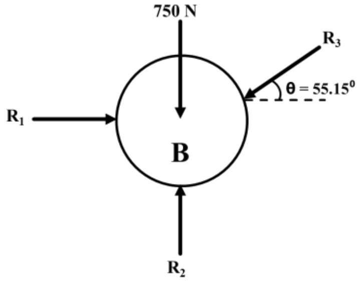
Figure 7

Solution:



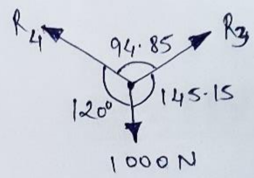


FBD of Sphere A



FBD of Sphere B

$$\frac{1000}{\sin 94.85} = \frac{R_4}{\sin 145.15} = \frac{R_3}{\sin 120}$$



$\therefore R_4 = 573.483$ N

$R_3 = 869.137$ N

From FBD of sphere B,

$\Sigma F_y = 0$

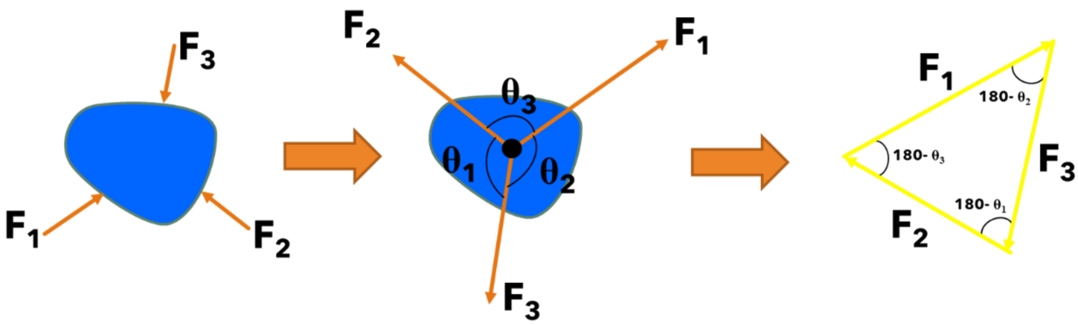
$R_2 - R_3 \sin 55.15 - 750 = 0$

$\therefore R_2 = 1463.258$ N

$\Sigma F_x = 0$

$R_1 - R_3 \cos 55.15 = 0$

$\therefore R_1 = 496.65$ N

Q4. A	**Solve any Two** **5 marks each**
i.	A ball dropped from a height of 4 m, bounces to a height of 1.5 m. Find 'e' and the height to which it would rise on the second bounce. **Solution:** Given: h_2 = 1.5 m & h = 4 m $e = (\frac{h_2}{h})^{\frac{1}{4}} = 0.7825$
ii.	State and prove Lami's Theorem. Also state its limitations. **Ans:** If a body is in equilibrium under the action of three concurrent coplanar forces, then each force is proportional to the sine of the angle between the other two forces. $\therefore \frac{F_1}{\sin\theta_1} = \frac{F_2}{sin\,\theta_2} = \frac{F_3}{sin\,\theta_3}$ **Proof:** Let us consider F_1, F_2 & F_3 be the three forces acting on the body keeping it in equilibrium.  As we know that forces are vector quantities, they can be vectorially added by head and tail connections. Thus, we get a closed triangle in interior angles as (180- θ_1), (180- θ_2) & (180- θ_3). Applying Sine rule, we get, $\frac{F_1}{\sin(180 - \theta_1)} = \frac{F_2}{sin(180 - \theta_2)} = \frac{F_3}{sin(180 - \theta_3)}$ $\therefore \frac{F_1}{\sin\theta_1} = \frac{F_2}{sin\,\theta_2} = \frac{F_3}{sin\,\theta_3}$ Limitations of Lami's Theorem: 1) Lami's theorem is applicable to only coplanar, concurrent forces. 2) Lami's theorem is not applicable for more than or less than three forces.

iii. A ball thrown with speed of 12 m/s at an angle of 60^0 with a building strikes the ground 11.3 m horizontally from the foot of the building as shown. Determine the height of the building.

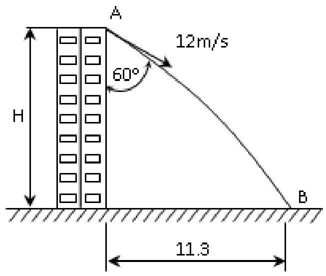


Figure 8

Solution:

Given: x = 11.3 m; v = 12 m/sec; α = - 30^0

$$y = x\tan \propto - \frac{gx^2}{2u^2}(1 + \tan^2 \propto)$$

$$h = 11.3\tan 30 - \frac{9.81\times 11.3^2}{2\times 12^2}(1 + \tan^2 30)$$

$\therefore$ h = 12.32 m (↓)

Q4. B Solve any One — 10 mark each

i. Determine the reactions at hinged support and roller support as shown in figure 9.

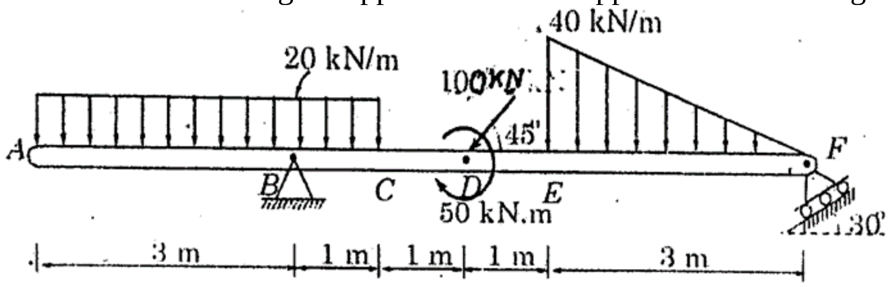
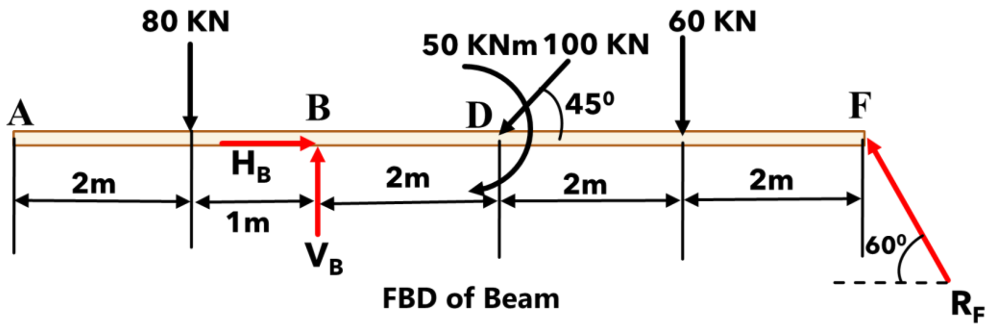


Figure 9



FBD of Beam

Solution:

ΣM_B = (80×1) – 50 – (2×100×sin45) – (60×4) + (R_F×6×sin60) = 0

$\therefore$ **R_F = 67.63 KN**

ΣF_x = 0 $\quad\quad$ $\therefore$ **H_B =** 100×cos45 + R_F ×cos60 **=104.526 KN**

ΣF_y = 0 = V_B – 80 – 100×sin45 – 60 + R_F ×sin60

$\therefore$ **V_A = 152.141 KN**

ii. Determine the force ‘P’ required to move the block ‘A’ of weight 5000 N up the inclined plane. Coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 15°.

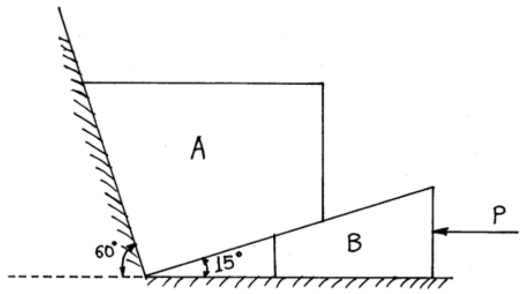
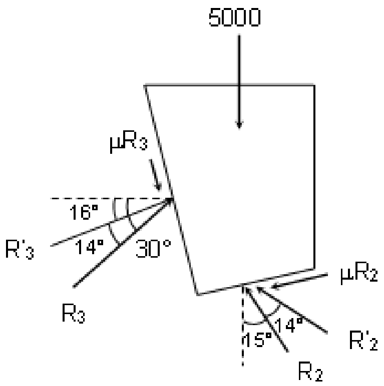
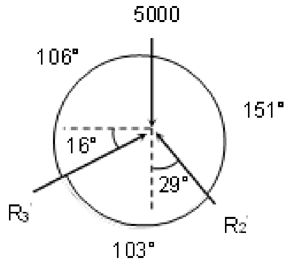
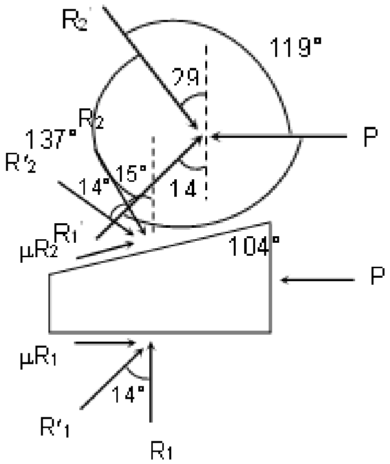


Figure 10
Solution:







$\phi \quad = \tan^{-1}(0.25) = 14°$

$$\frac{5000}{\sin 103°} = \frac{R_3'}{\sin 151°} = \frac{R_2'}{\sin 106°}$$

$\therefore \quad R_3' = 2487.8$ N

$R_2' = 4932.7$ N

$$\frac{4932.7}{\sin 104°} = \frac{P}{\sin 137°} = \frac{R_1'}{\sin 119°}$$

$\therefore \quad P = 3467.1$ N

$R_1' = 4446.3$ N