5. Logic

It is used to determine if a particular reasoning or argument is true or false.

→ Proposition

It is a declarative statement which can be either true or false but not both of them.

→ Compound Proposition

It consists of many propositions connected using logical connectives.

1) Conjunction → $\wedge$ (AND)

eg. $p \wedge q$

$p \cdot q$

$p \,\&\, q$

p	q	Truth value
T	T	T
T	F	F
F	T	F
F	F	F

2) Disjunction → $\vee$ (OR)

eg. $p \vee q$

$p + q$

p or q

p	q	Truth value
T	T	T
T	F	T
F	T	T
F	F	F

3) Negation $\longrightarrow \sim$

eg. $\sim p$

$\bar{p}$

$\neg p$

p	Truth value
T	F
F	T

4) Logical Implication

If p and q are statements, then the compound statement, "if p then q" denoted by $p \to q$ is called implication.

p	q	Truth Value ($p \to q \equiv \neg p \vee q$)
T	T	T
T	F	F
F	T	T
F	F	T

5) Biconditional Statement

If p & q are statements, then the statement "if and only if p then q" denoted by $p \leftrightarrow q$ is called equivalence or biconditional.

$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$

p	q	Truth value:
T	T	T
T	F	F
F	T	F
F	F	T

→ Tautology

Statement is always true for all possible values of its propositional variables.

→ Contradiction

Statement is always false for all possible values of its propositional variables.

ex.1 Verify the proposition $p \vee \cancel{q} \sim(p \wedge q)$ is a tautology

Sol^n

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
T	T	T	F	T
T	F	~~T~~ F	~~F~~ T	T
F	T	~~T~~ F	~~F~~ T	T
F	F	F	T	T

⇒ It is a tautology.

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~~ex.2~~

ex.2 Using laws of logic, show that it is a tautology.

$[(p \to q) \land \sim q] \to \sim p$

Sol^n $\equiv [(\sim p \lor q) \land \sim q)] \to \sim p$

$\equiv [(\sim p \land \sim q) \lor (q \land \sim q)] \to \sim p$ (Distributive Law

$\equiv [(\sim p \land \sim q) \lor F] \to \sim p$

$\equiv (\sim p \land \sim q) \to \sim p$

$\equiv \sim(\sim p \land \sim q) \lor \sim p$

$\equiv p \lor q \lor \sim p$

$\equiv T \lor q$

$\equiv T$

$\Rightarrow$ It is a tautology.

ex.3 Using laws of logic, show that it is a tautology.

$[p \land (p \to q)] \to q$

Sol^n $\equiv [p \land (\sim p \lor q)] \to q$

$\equiv [(p \land \sim p) \lor (p \land q)] \to q$ (Distributive Law

$\equiv [F \lor (p \land q)] \to q$

$\equiv (p \land q) \to q$

$\equiv \sim(p \land q) \lor q$

$\equiv \sim p \lor \sim q \lor q$

$\equiv \sim p \lor T$

$\equiv T$

$\Rightarrow$ It is a tautology.

ex. 14 Prove $p \wedge (q \vee r)$ & $(p \wedge q) \vee (p \wedge r)$ are logically equivalent using truth table.

Sol^n

① p	② q	③ r	④ $q \vee r$	⑤ $p \wedge (q \vee r)$	⑥ $p \wedge q$	⑦ $p \wedge r$	⑧ $(p \wedge q) \vee (p \wedge r)$
F	F	F	F	F	F	F	F
F	F	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	T	T	T	F	F	F	F
T	F	F	F	F	F	F	F
T	F	T	T	T	F	T	T
T	T	F	T	T	T	F	T
T	T	T	T	T	T	T	T

From columns ⑤ & ⑧,

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \to q$

converse $q \to p$

inverse $\bar{p} \to \bar{q}$

contrapositive $\bar{q} \to \bar{p}$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$T \to F$

converse

$q \to p$
T
T
F
T

inverse

$\bar{p}$	$\bar{q}$	$\bar{p} \to \bar{q}$
F	F	T
F	T	T
T	F	F
T	T	T

contrat

$\bar{p}$	$\bar{q} \to$	$\bar{q} \to \bar{p}$
F	F	T
F	T	F
T	F	T
T	T	T