HW2 due on Fri @ 11:59pm IST.

Ex: Let $x_1, x_2, \dots, x_n$ be n distinct real nos. Prove that no matter how the parenthesis are inserted into the product of the n nos, the # multiplications $= n-1$.

Soln:

$$\underbrace{(x_1(x_2\,x_3))(x_4(x_5\,x_6))}_{n-1}\underbrace{(x_7)}_{1}$$

$$= \boxed{n}\,.$$

$$((x_1)(x_2)(x_3))((x_4)(x_5)(x_6))$$

We will prove the claim by doing

induction on n.

IH: Let $k \geq 1$ be an integer. Assume that the claim holds when $\cancel{n=k}$ $n=i$, $1 \leq i \leq k$.

That is, given $x_1 x_2 \cdots x_i$ then any parenthesization of the product of $x_1 x_2 \cdots x_i$ will always yield $\cancel{k-1}$ $i-1$ multiplications.

BC: (x_1) 0 multiplications. ✓

IS: We want to prove the claim when $n = k+1$.

$$\Big(\big(x_1\ (x_2\ x_3)\ (\cdots\ x_i)\big)\Big)\Big(\big(x_{i+1}\cdots\big)\ (x_k\ x_{k+1})\Big)$$

For some i, $1 \le i \le n-1$, let the last product given by the parentheses structure be between

$$\underbrace{\big((x_1\ (x_2 \cdots)\cdot x_i)\big)}_{i-1}\ \underbrace{\big(x_{i+1} \cdots\cdots x_{k+1}\big)}_{k-i}$$

$$+1$$

$$\text{Total \# mult} = i-1+k-i+1$$

$= \cancel{c}[k]$. ✓.

Graphs :

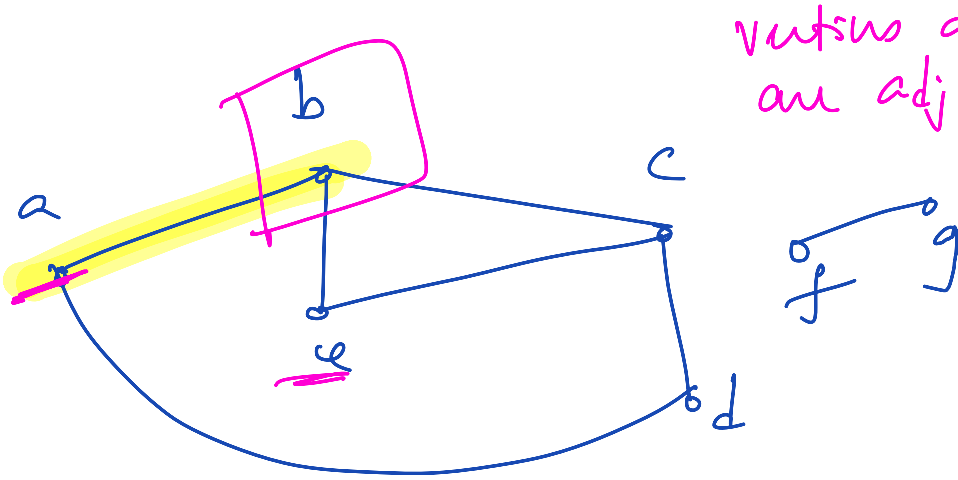
An **undirected graph** $G = (V, E)$

is a collection of vertices & edges.

V : set of vertices. $|V| = n$

E : set of edges. $|E| = m$

vertices a&b are adjacent.



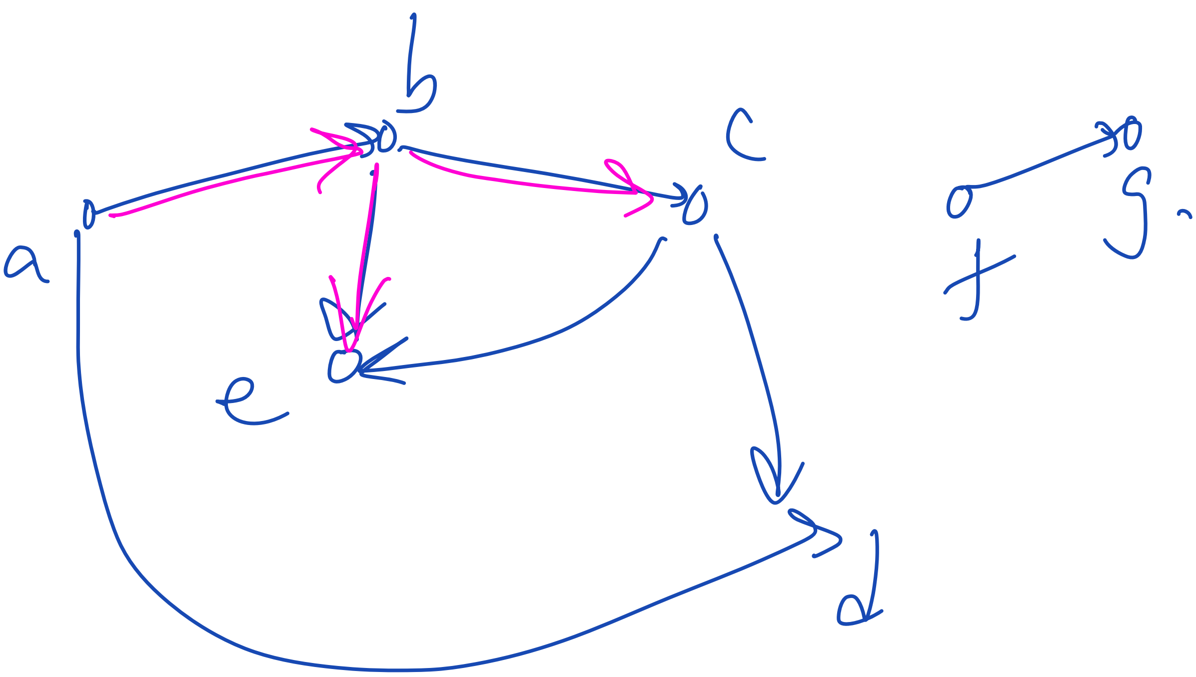
$N(b) = \{a, c, e\}$

$V = \{a, b, c, d, e, f, g\}$

$E = \{\{a,b\}, \{b,c\}, \{b,e\}, \{c,e\}, \{a,d\}, \{f,g\}\}$

$E = \{(a,b), (b,c), (b,e), (c,e), (a,d), (c,d), (f,g)\}$

(b,a)



$N(u)$: neighbors of a vertex u

$$N(u) = \{ v \mid (u,v) \in E \}$$

$$\deg(u) = |N(u)|.$$

$\deg(b) = 3$

outdeg$(b) = 2$

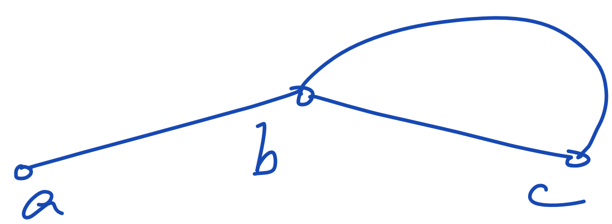
indeg$(b) = 1$

Vertices u & v are adjacent if $(u,v) \in E$.

<u>Simple graphs.</u>

- at most one edge between

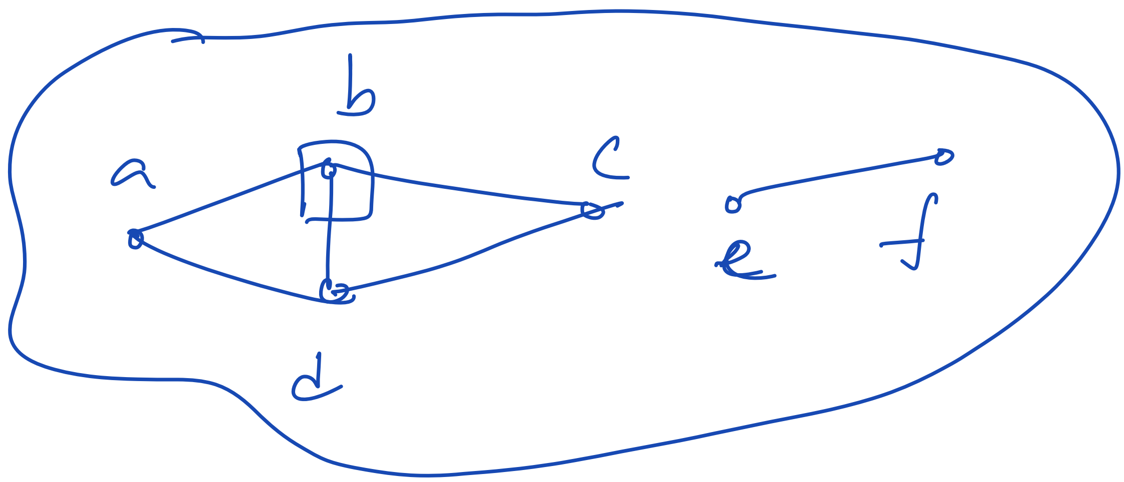
any two vertices.



$\longrightarrow$ multigraph.

$$\delta(G) = \min_{u \in V} \{\deg(u)\}$$

G:



$$\delta(G) = 1$$

$$\Delta(G) = \max_{u \in V} \{\deg(u)\}$$

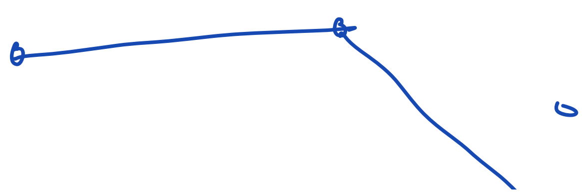
$\Delta(G) = 3$

<u>Ex</u>: Let $G = (V, E)$ be a simple, undirected graph. Then

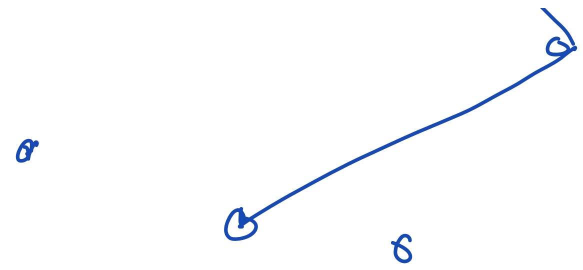
$$\sum_{u \in V} \deg(u) = \underline{2|E|}$$

function of the # edges

<u>Proof</u>: "Each edge contributes degree of 1 to each of its end pts".



Total deg
0
2

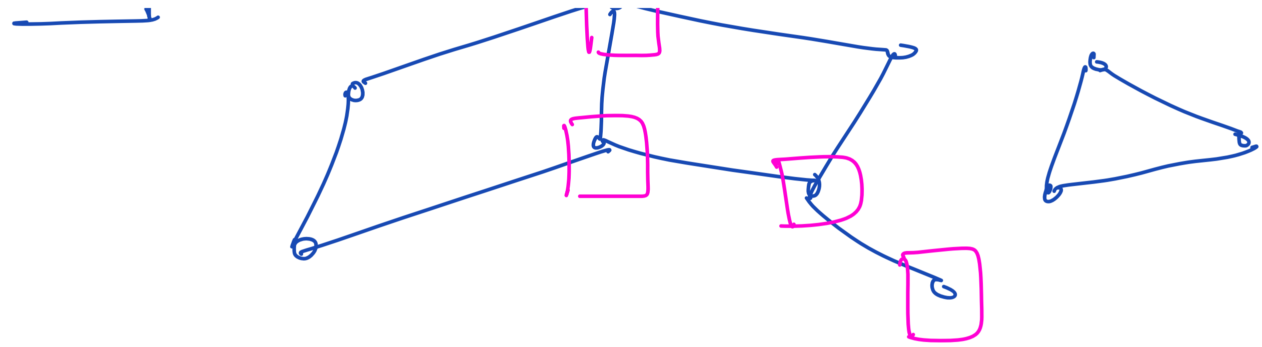


4
6

$$\sum_{u \in V} dg(u) = 2|E|.$$

Ex: Prove that in a simple undirected graph G, there is an even no. of odd degree vertices.

Proof :



We know from the previous

Claim that

$$\sum_{u \in V} dg(u) = 2|E|.$$

$$\therefore \sum_{u:\text{ odd deg. vertex in } G} dg(u) + \sum_{u:\text{ even deg vertex in } G} dg(u) = \text{even}$$

total deg

even

$$\therefore \boxed{\sum_{\substack{u:\text{ odd}\\ \text{deg vertex}}} \deg(u) = \underline{\text{even}}.}$$

Let $v_1, v_2, \dots, v_k$ be the odd degree vertices. We want to prove that $\boxed{k \text{ is even}}$

$\deg(v_1) + \deg(v_2) + \dots + \deg(v_k)$ is even.

v_1 is odd deg   odd deg.

$$(2k_1+1)+(2k_2+1)+\cdots+(2k_h+1) = \boxed{2\cdot l},$$

where $k_1, k_2, \ldots, k_h, l$ are int.

$$\therefore\ 2(k_1+k_2+\cdots+k_h) + \overbrace{\underbrace{(1+1+\cdots+1)}_{h \text{ times}}}^{=h} = \boxed{2l}$$

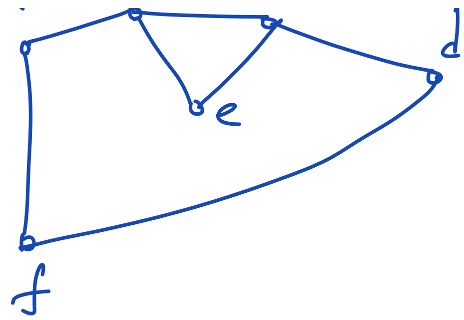
$$\therefore\ h = 2\underbrace{(l-k_1-k_2-\cdots-k_h)}_{\text{int.}}$$

$\therefore$ $\boxed{h \text{ is even.}}$ ✓

Definitions.

① Walk : sequence of adj vertices in the graph.

a b c .



a b c b a b c e b a
b c d

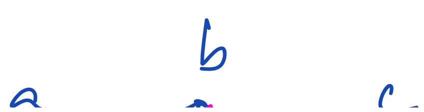
② **Path** : walk in which all vertices are distinct.

a b c d

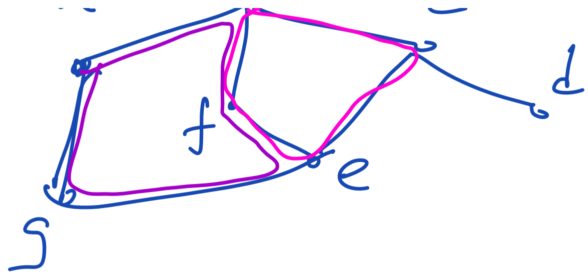
a b c b a

③ **Cycle** : "closed path".

- walk in which
  - no vertex except the first & the last vertex is repeated.
- first & the last vertex are the same.



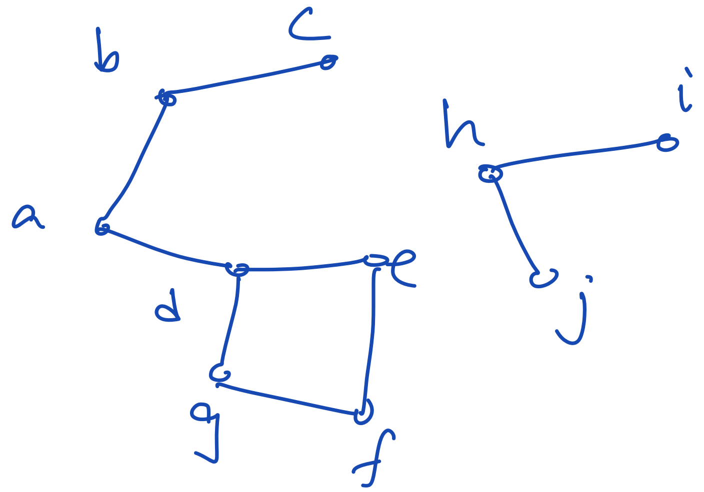
b c e f b → cycle

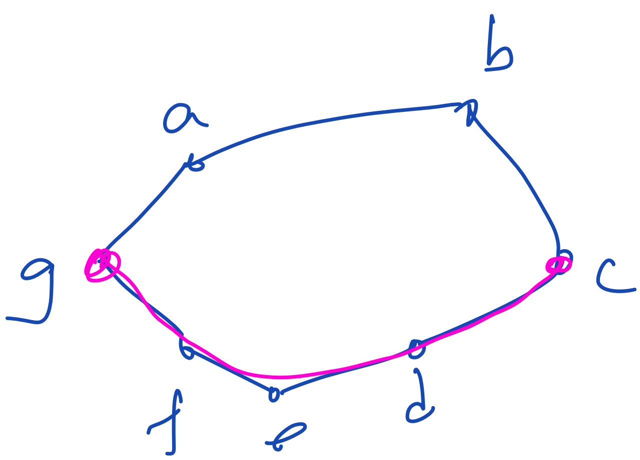


a b f e g a → cycle

④ G is connected if there is a path between every two vertices in G.

G:

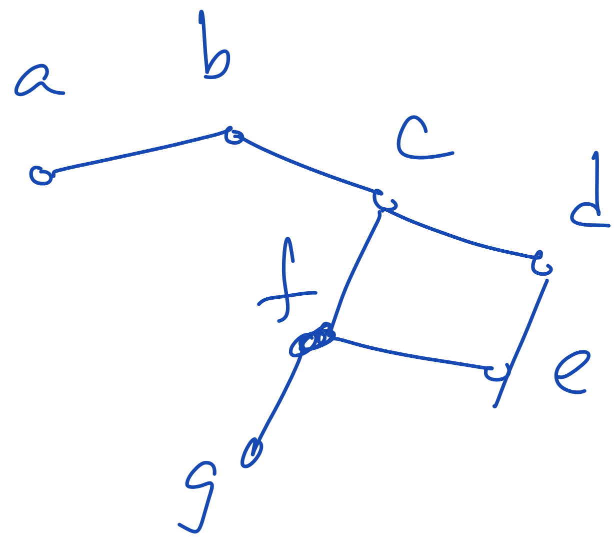




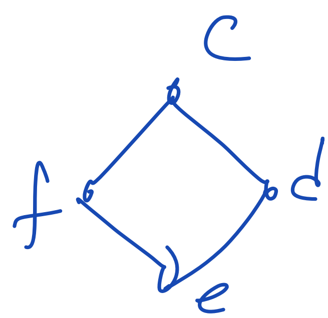
(5) $H=(V_H, E_H)$ is a subgraph of $G=(V,E)$ if

- $V_H \subseteq V$
- $E_H \subseteq E$.

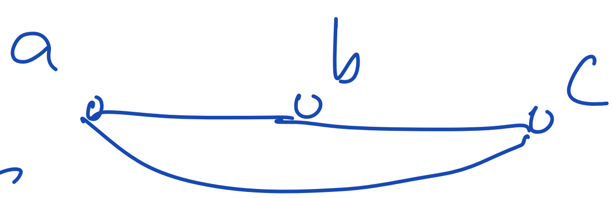
G:



H:



H':



F:

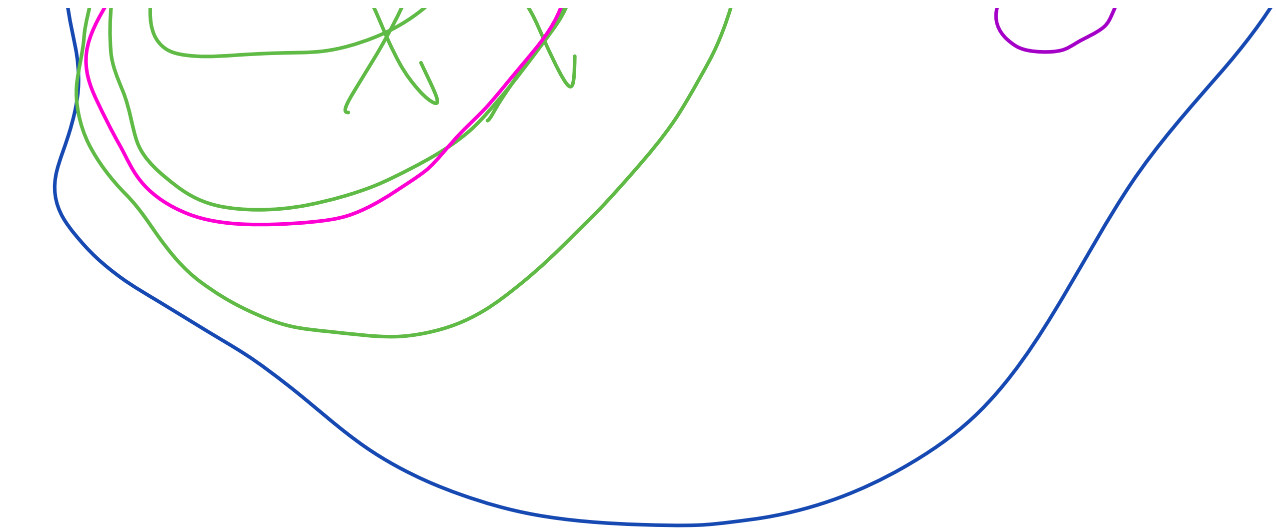
F is not a subgraph of G.

(6) H is a connected component of G if

- ✓ H is connected.
- ✓ H is a subgraph of G
- ✗ H is maximal

there is no larger CC containing H

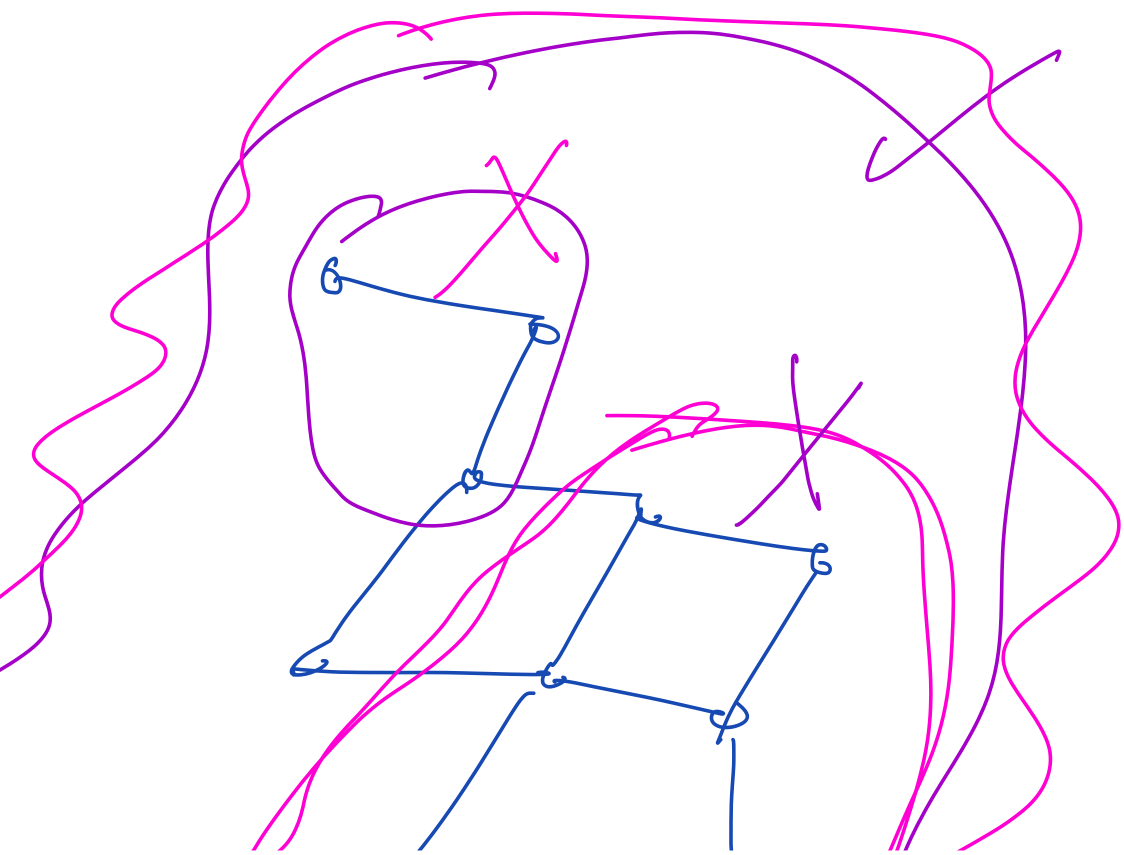
G :

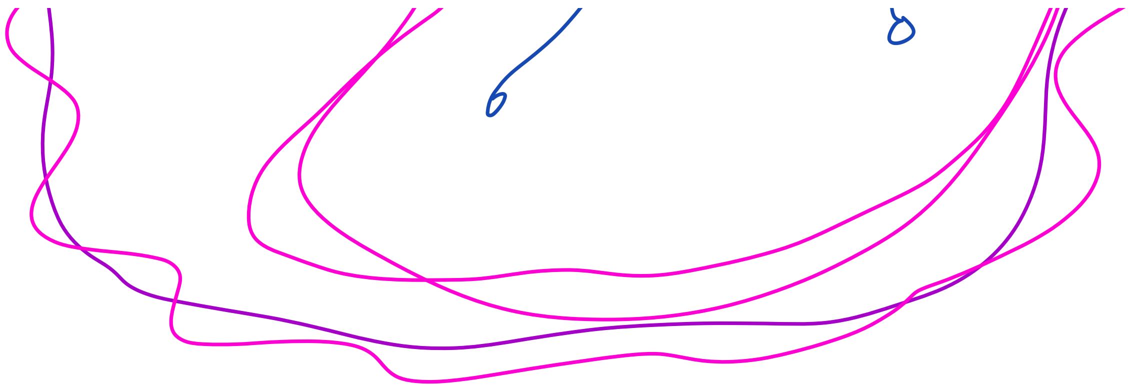


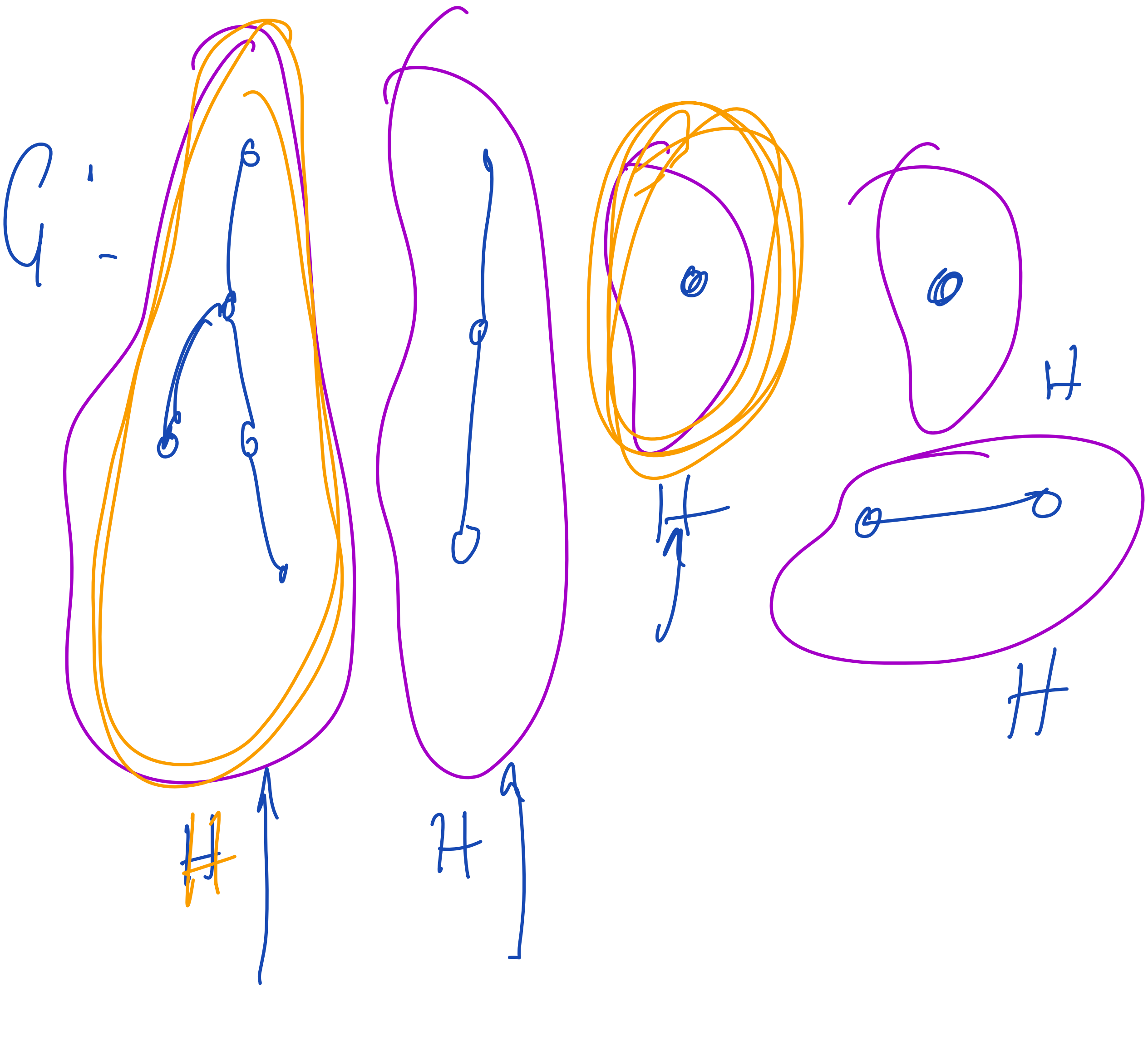
$\#$ connected components of G

$= 5$

G :






G :
H
H
2
H
H
H

Induced subgraph

$H = (V_H, E_H)$ is an induced subgraph of $G = (V, E)$ if

- H is a subgraph of G
- $\forall\ a \in V_H$ and $b \in V_H$, $a \neq b$, if $(a, b) \in E$ then $(a, b) \in E_H$.