

Total given Datapoints

Consider  $\rightarrow$

height

$\bullet$   $\rightarrow$  Test set  
 $\bullet$   $\rightarrow$  Train Set

weight  $\uparrow$  bias  
 $\downarrow$  variance

model be  
consider  
Straight line

Underfitting

Bias  $\left\{ \begin{array}{l} \text{diff bet}^n \\ \text{predicted \& actual} \\ \text{in Train.} \end{array} \right.$

Diff bet<sup>n</sup>  
 predicted & actual in Test.  
 predicted value

Bias  $\rightarrow$  Inability of ML methods  
 (linear reg) to capture true  
 relationship

Train set  
 Here any Simple Straight line Cannot match all data points in  
Training Set

bias  $\downarrow$  variance  $\uparrow$

Overfitting

height

weight

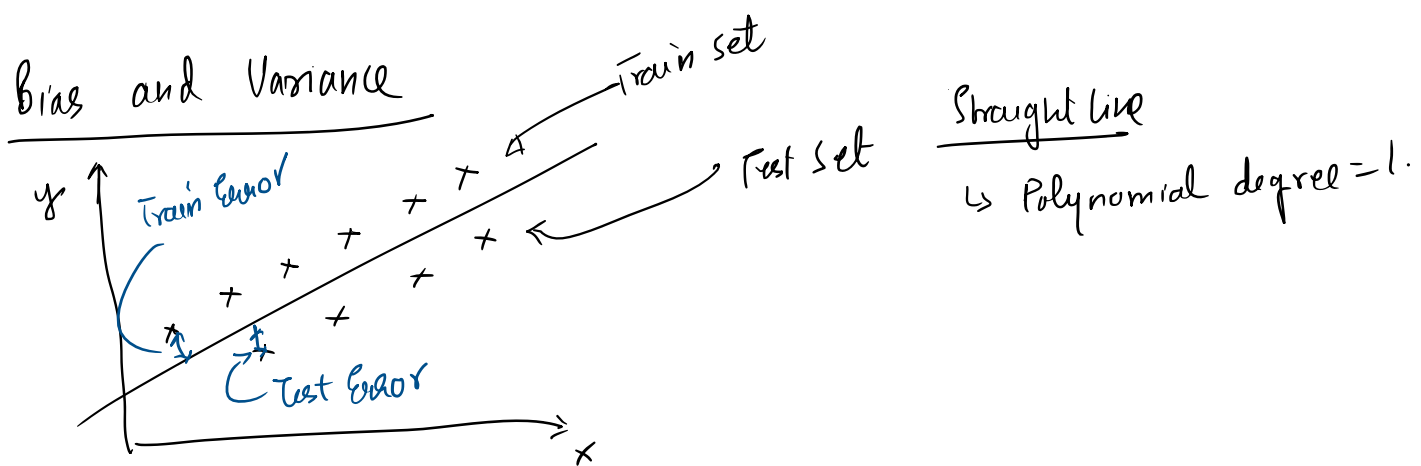
Swiggly line

Here the Swiggly line  
 handles the true  
 relationship bet<sup>n</sup>  
 height & weight  
 So Since the line  
 almost passes through  
 every training data  
 points we say  $\Rightarrow$  Little/  
Zero

... Data not

Variance  $\Rightarrow$  Difference in fits bet<sup>n</sup>  
the Datasets (Train & Test Data Sets).

points we say  $\Rightarrow$  Zero  
Bias.



\* Underfitting  $\Rightarrow$  When the model is simple (lower degree polynomial), the model might not fit the train set data points.

So here the difference bet<sup>n</sup> Actual and predicted value for Train set is higher

This is bias.

So In Underfitting bias  $\uparrow$

There will error for predicted value and actual value for Test set

So the diff in Error for Test and Train set is low

This is Variance

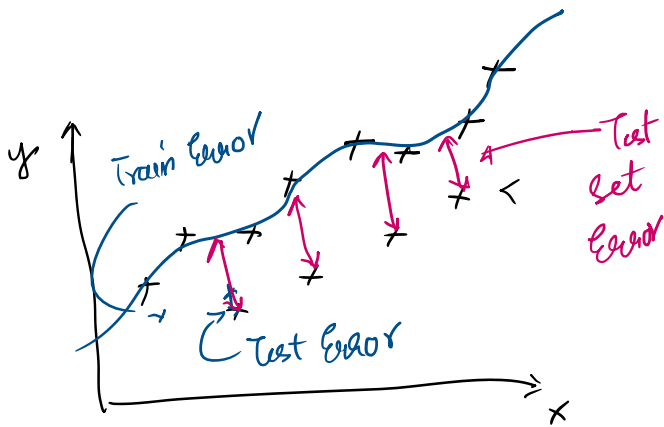
So In Underfitting Variance  $\downarrow$

\* Underfitting

For Simple Model (lower order Polynomial)

Bias  $\uparrow$  Variance  $\downarrow$

Bias  $\uparrow$  Variance  $\downarrow$



Here model is complex  
(higher order polynomial)

→ So error with Train Set is very less

∴ Bias is  $\downarrow$

Since the model perfectly fits the train set

It is case of Overfitting

and there is significant diff (error) with Test Set

∴ Variance is  $\uparrow$

\* Overfitting →

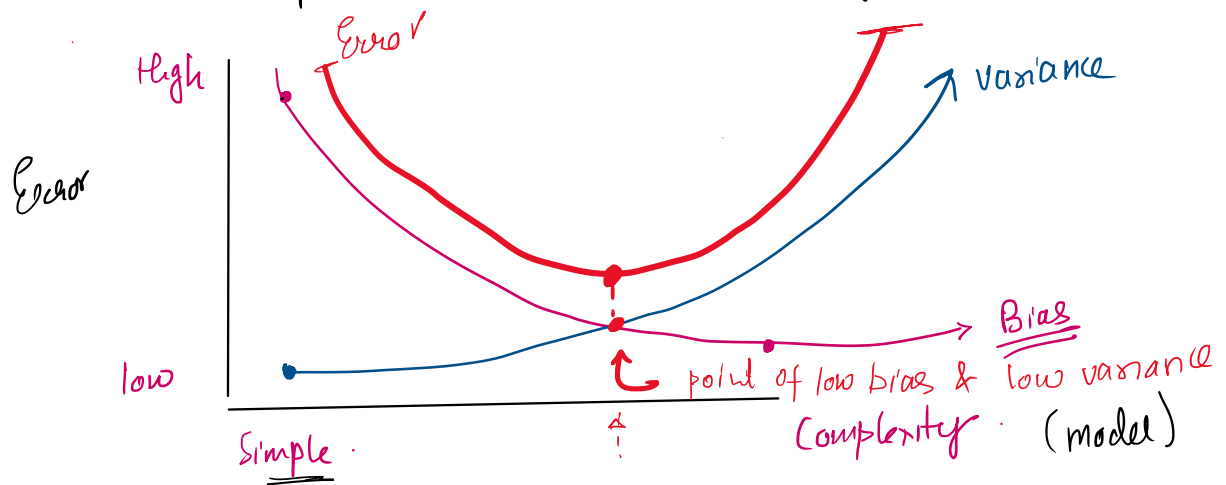
For Complex model (higher order polynomial)

bias  $\downarrow$  variance  $\uparrow$

## Imp Bias-Variance Tradeoff

① If model is too simple and very few parameter  $\rightarrow$  Underfitting  
 $\rightarrow$  High bias  
 $\rightarrow$  Low variance

② If model is complex and has large no of parameters  $\rightarrow$  Overfitting  
 $\rightarrow$  Low Bias  
 $\rightarrow$  High Variance.



Bias Variance Tradeoff says

we need a model that gives

① low bias

② low variance.

i.e. we need to find point of low bias and low variance.

The methods to achieve this  $\Rightarrow$

\* Regularization (penalize 'Q' parameter)

\* Boosting

\* Bagging

\* How to Minimize the Total Error:  $\text{Bias}^2 + \text{Variance} + \text{Irreducible Error}$

\* . \* ~~0 0 0~~  
We need to Minimize the Total Error: Bias + Variance + irreducible Error.

$$\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

$\downarrow \quad \downarrow$   
 $\quad \quad 1 \quad \quad 1$

$\hat{y}$   
 $\uparrow$

## \* Performance Metric →

Consider a Skewed Data Set (One Sided).

Ex  
1000 MRI Scan  
990 No cancer  
10 cancer

Here only 1% scan shows cancer

It is very less -

Normally Allowed.

→ might be ignored

→ But this is incorrect.

In Such Case to Evaluate Performance We may use

Confusion Matrix →

		Actual	
		True	False
Predicted	True	True +ve (TP)	False +ve
	False	False -ve	True -ve

The above Confusion matrix is OK for 2 classes.

Question →  
Cancer?

		Actual	
		Has Cancer	Do not have Cancer
Predicted	(True) Has Cancer	TP	FP
	Do not have Cancer (-ve)	FN	TN

If we have More than Two classes.

Actual

↓

↓

↓

Question →

Person watched  
Chakde?

if person  
watched +ve  
chakde

else -ve

		Actual		
		chakde	KGF	DDLJ
Predicted	chakde	TP	FP	FP
	KGF	FN	TN	TN
	DDLJ	FN	TN	TN

Predicted that  
These people  
have not  
watched  
Chakde so  
N;

Person has not  
watched Chakde &

Predicted also not  
watching Chakde.

Question → Watched Chakde?   
 Yes +ve  
 No -ve

For 4 cases

		Actual			
		chakde	KGF	DDLJ	Gadar
Predicted	chakde	TP	FP	FP	FP
	KGF	FN	TN	TN	TN
	DDLJ	FN	TN	TN	TN
	Gadar	FN	TN	TN	TN

Chakde Nahi  
dekhia phir  
bhi predicted dekha

Chakde dekha  
But predicted Nahi  
dekha

Chakde Nahi dekha  
And predicted Nahi dekha.

\*

TP (True +ve) = Correctly Identified +ve (True Hai → +ve Predicted)  
 FP (False +ve) = Incorrectly Identified +ve (True Nahi Hai → +ve Predicted)  
 TN (True -ve) = Correctly Identified -ve → (-ve Hai → -ve Predicted)  
 FN (False -ve) = Incorrectly Identified -ve → (-ve Nahi Hai → -ve Predicted)



$$1) \text{ Accuracy} = \frac{\text{Total Correct Prediction}}{\text{Total Prediction}} = \frac{TP + TN}{TP + TN + FP + FN}$$

$$2) \text{ Error Rate} = 1 - \text{Accuracy}$$

$$3) \text{ True +ve Rate (TPR)} = \frac{\text{Correctly Identified +ve}}{\text{Total Actual +ve}} = \frac{TP}{TP + FN}$$

[Sensitivity / Recall]

$$4) \text{ False Negative Rate (FNR)} = \frac{\text{Incorrectly Identified +ve}}{\text{Total Actual +ve}} = \frac{FN}{TP + FN}$$

(Aisa +ve jisko -ve predict kiya)

$$5) \text{ True Negative Rate (TNR)} = \frac{\text{Correctly Identified -ve}}{\text{Total Actual -ve}} = \frac{TN}{TN + FP}$$

[Specificity]

$$6) \text{ False Positive Rate (FPR)} = \frac{\text{Incorrect Identified -ve}}{\text{Total Actual -ve}} = \frac{FP}{TN + FP}$$

$$\left[ \begin{array}{l} \text{TPR} = \frac{\text{Correct +ve}}{\text{Total +ve}} \\ \text{FNR} = \frac{\text{Incorrect +ve}}{\text{Total +ve}} \end{array} \right] \quad \left[ \begin{array}{l} \text{TNR} = \frac{\text{Correct -ve}}{\text{Total -ve}} \\ \text{FPR} = \frac{\text{Incorrect -ve}}{\text{Total -ve}} \end{array} \right]$$

Note To Identify Heart Disease

$$\text{Sensitivity} = \text{TPR} = \frac{\text{Correct Identified +ve}}{\text{Total +ve}}$$

What percentage of patient with heart disease are correctly

$$\text{Sensitivity} = \frac{\text{TP}}{\text{TP} + \text{FN}}$$

Total +ve

patients with disease are correctly identified

$$\text{Specificity} = \text{TN}R = \frac{\text{Correct Identified -ve}}{\text{Total -ve}} = \text{What percentage of patients without heart disease are correctly identified.}$$

$$\boxed{\text{FPR} = 1 - \text{Specificity}} = 1 - \frac{\text{TN}}{\text{TN} + \text{FP}} = \frac{\text{TN} + \text{FP} - \text{TN}}{\text{TN} + \text{FP}} = \frac{\text{FP}}{\text{TN} + \text{FP}}$$

## Training Error →

It is prediction error that we get when we apply the model on same data from where it is trained.

$$E_{\text{Train}} = \frac{1}{n} \sum_{i=1}^n \text{Error} \left( \underbrace{f_0(x_i)}_{\substack{\uparrow \\ \text{prediction} \\ \text{of } x_i}}, \underbrace{y_i}_{\substack{\uparrow \\ \text{actual value}}} \right)$$

(for all the sample)

(2) Test Error: It is prediction error we get when we apply model on altogether different data set (Test set) and not on the data on which it is trained.

$$E_{\text{Test}} = \frac{1}{n} \sum_{i=1}^n \text{Error} \left( \underbrace{f_0(x_i)}_{\substack{\uparrow \\ \text{for Test set}}}, y_i \right)$$

(3) Generalization Error → also known as Out of Sample Error

→ Measure of how accurately an algorithm is able to predict outcome values for previously unseen data.

→ We want to know how the model will perform

→ We want to know how the model will perform on future data (we do not have today)

→ For Future we do not have  $x_i$  (input)  
 $y_i$  (output)

$$E_{\text{gen}} = \int \text{Error} \left( \underbrace{f_0(x_i)}_{\substack{\uparrow \\ \text{Predicted}}}, \underbrace{y_i}_{\substack{\uparrow \\ \text{actual}}} \right) \underbrace{P(y, x)}_{\substack{\uparrow \\ \text{How often} \\ \text{we expect} \\ \text{such } x \text{ \& } y}} dx.$$

overall possible value of  $x$  &  $y$

Usually

$$\underline{E_{\text{train}} \leq E_{\text{gen}}}$$

as we do not have value of future  $P(y, x)$   
So we do not compute generaliz<sup>n</sup> Error,  
we approximate it with Testing Error.