1. Find Eigen value and Eigen vector of a matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

Solution: ∵ A be a square matrix of order 3

∴ it's characteristic eq is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad -(1)$$

where $S_1 = 5$

$$S_2 = \begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 + 4 + 2 = 8$$

$|A| = 4$

$$\therefore \lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$$

$$\begin{array}{c|cccc} 1 & 1 & -5 & 8 & -4 \\ & & 1 & -4 & 4 \\ \hline & 1 & -4 & 4 & \boxed{0} \end{array}$$

$$(\lambda - 1)(\lambda^2 - 4\lambda + 4) = 0$$

$$\therefore (\lambda-1)(\lambda-2)(\lambda-2) = 0$$

∴ $\lambda = \lambda_1 = 1$, $\lambda = \lambda_2 = 2$, $\lambda = \lambda_2 = 2$ be the Eigen value of matrix A

To Find Eigen vector consider $(A - \lambda I)X = 0$

$$\therefore \begin{bmatrix} 1-\lambda & 2 & 2 \\ 0 & 2-\lambda & 1 \\ -1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad -(2)$$

Case 1: If $\lambda = \lambda_1 = 1$

$$\begin{bmatrix} 0 & 2 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix}}$$

$$\frac{x_1}{-1} = \frac{-x_2}{1} = \frac{x_3}{1} = k = -1$$

$$\therefore x_1 = 1, \; x_2 = 1, \; x_3 = -1$$

Thus For Eigen value $\lambda = \lambda_1 = 1$, Eigen vector $X_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

Case-2: If $\lambda = \lambda_2 = 2$

$$\begin{bmatrix} -1 & 2 & 2 \\ 0 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} -1 & 2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} -1 & 2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad — ③$$

$O(A-\lambda I) = 3$ & $\rho(A-\lambda I) = 2$

$\therefore$ For $\lambda = \lambda_2 = 2$, G.M. $= O(A-\lambda I) - \rho(A-\lambda I) = 3-2 = 1$

$\therefore$ For $\lambda = \lambda_2 = 2$, only one Eigen vector exist

Now From ③ $$\frac{x_1}{\begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 2 \\ 0 & 0 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{-x_2}{-1} = \frac{x_3}{0} = k = 1$$

$\therefore x_1 = 2, \; x_2 = 1, \; x_3 = 0$

$\therefore$ For $\lambda = \lambda_2 = 2$, $X_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

② Find Eigen value and Eigen vector of a Matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$

Solution: $\because$ A be a square matrix of order 3

$\therefore$ it's characteristic equation is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad - ②$$

where $S_1 = 5$

$$S_2 = \begin{vmatrix} 3 & 2 \\ -5 & -2 \end{vmatrix} + \begin{vmatrix} 4 & 6 \\ -1 & -2 \end{vmatrix} + \begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix} = 4 + 2 + 6 = 8$$

$|A| = 4$

$$\therefore \lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$$

$$\begin{array}{c|cccc} 1 & 1 & -5 & 8 & -4 \\ & & 1 & -4 & 4 \\ \hline & 1 & -4 & 4 & \boxed{0} \end{array}$$

$\therefore \lambda = \lambda_1 = +1 \qquad \lambda = \lambda_2 = 2$, and $\lambda = \lambda_3 = 2$

To find Eigen vectors consider $(A - \lambda I)X = 0$

$$\therefore \begin{bmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -5 & -2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad - ②$$

Case-1 If $\lambda = \lambda_1 = +1$

$$\begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ -1 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 2 \\ -5 & -3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 2 \\ -1 & -3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ -1 & -5 \end{vmatrix}}$$

$$\frac{x_1}{4} = \frac{-x_2}{-1} = \frac{x_3}{-3} \quad k = 1$$

$$x_1 = 4, \quad x_2 = 1, \quad x_3 = -3$$

$\therefore$ For $\lambda = \lambda_1 = 1, \quad X_1 = \begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix}$

Case-2: If $\lambda = \lambda_2 = 2$

$$\begin{bmatrix} 2 & 6 & 6 \\ 1 & 1 & 2 \\ -1 & -5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2, \quad R_1 \to R_1 - 2R_1$

$$\begin{bmatrix} 0 & 4 & 2 \\ 1 & 1 & 2 \\ 0 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_1$ & $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \; \rho(A-\lambda I) = 2, \quad O(A-\lambda I) = 3$

$\therefore$ For $\lambda = \lambda_2 = 2$, G.M. $= O(A-\lambda I) - \rho(A-\lambda I) = 3-2 = 1$

$\therefore$ For $\lambda = \lambda_2 = 2$, only one Eigen vector exist and is given by

$$\frac{x_1}{\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 0 & 4 \end{vmatrix}}$$

$$\therefore \; \frac{x_1}{-6} = \frac{-x_2}{2} = \frac{x_3}{4}$$

$$\therefore \; \frac{x_1}{-3} = \frac{x_2}{-1} = \frac{x_3}{2} = k = -1$$

$$\therefore \; x_1 = 3, \quad x_2 = 1, \quad x_3 = -2$$

$\therefore$ For Eigen value $\lambda = \lambda_2 = 2$, and corresponding Eigen vector $X_2 = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$

③ Find the Eigen value and Eigen vector of a Matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

Solution: ∵ A be a square matrix of order 3

∴ it's characteristic equation is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad -①$$

$S_1 = 7$

$$S_2 = \begin{vmatrix} -3 & -4 \\ 5 & 7 \end{vmatrix} + \begin{vmatrix} 3 & 5 \\ 3 & 7 \end{vmatrix} + \begin{vmatrix} 3 & 10 \\ -2 & -3 \end{vmatrix} = -1 + 6 + 11 = 16$$

$|A| = 12$

∴ $\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$

∴ $\lambda = \lambda_1 = 3$, $\lambda = \lambda_2 = 2$, $\lambda = \lambda_3 = 2$ be the Eigen values of a matrix A

To find Eigen vector consider $(A - \lambda I)X = 0$

i.e
$$\begin{bmatrix} 3-\lambda & 10 & 5 \\ -2 & -3-\lambda & -4 \\ 3 & 5 & 7-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad -②$$

Case-1: If $\lambda = \lambda_1 = 3$
$$\begin{bmatrix} 0 & 10 & 5 \\ -2 & -6 & -4 \\ 3 & 5 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 10 & 5 \\ -6 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 5 \\ -2 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 10 \\ -2 & -6 \end{vmatrix}}$$

$$\frac{x_1}{-10} = \frac{-x_2}{10} = \frac{x_3}{20}$$

$$∴ \frac{x_1}{-1} = \frac{x_2}{1} = \frac{x_3}{2} = t = -1$$

$x_1 = 1,\ x_2 = 1,\ x_3 = -2$

Thus For Eigen value $\lambda = \lambda_1 = 3$ corresponding Eigen vector $X_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

Case-2: If $\lambda = \lambda_2 = 2$
$$\begin{bmatrix} 1 & 10 & 5 \\ -2 & -5 & -4 \\ 3 & 5 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$, and $R_2 \to R_2 + 2R_1$
$$\begin{bmatrix} 1 & 10 & 5 \\ 0 & 15 & 6 \\ 0 & -25 & -10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \end{bmatrix}$$

$R_2 \to \frac{1}{3}R_2$

$R_3 \to \frac{1}{5}R_3$

$$\begin{bmatrix} 1 & 10 & 5 \\ 0 & 5 & 2 \\ 0 & -5 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 10 & 5 \\ 0 & 5 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \rho(A-\lambda I) = 2, \quad O(A-\lambda I) = 3$

$\therefore$ For $\lambda = \lambda_2 = 2$, G.M. $= O(A-\lambda I) - \rho(A-\lambda I) = 3-2 = 1$

$\therefore$ For Eigen value $\lambda = \lambda_2 = 2$, only one Eigen vector exist

and
$$\frac{x_1}{\begin{vmatrix} 10 & 5 \\ 5 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 5 \\ 0 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 10 \\ 0 & 5 \end{vmatrix}}$$

$$\frac{x_1}{-5} = \frac{-x_2}{2} = \frac{x_3}{5} = k = -1$$

$\therefore x_1 = 5, \quad x_2 = 2, \quad x_3 = -5$

Thus For Eigen value $\lambda = \lambda_2 = 2$, Eigen vector $X_2 = \begin{bmatrix} 5 \\ 2 \\ -5 \end{bmatrix}$

④ Find Eigen value and Eigen vector of a Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

Solution :- A be a upper triangular matrix

$\therefore$ it's diagonal elements are the eigen values

$\therefore \lambda = \lambda_1 = 1,\ \lambda = \lambda_2 = 2$, and $\lambda = \lambda_3 = 2$ be the Eigen values of matrix A

To Find Eigen vectors consider $(A - \lambda I)X = 0$

i.e
$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 2-\lambda & 3 \\ 0 & 0 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow ①$$

Case-1 If $\lambda = \lambda_1 = 1$
$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 3 \\ 0 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}}$$

$$\frac{x_1}{1} = \frac{-x_2}{0} = \frac{x_3}{0} = k = 1$$

$\therefore x_1 = 1,\ x_2 = 0,\ x_3 = 0$

Thus For Eigen value $\lambda = \lambda_1 = 1$, Eigen vector $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Case-2 : If $\lambda = \lambda_2 = 2$
$$\begin{bmatrix} -1 & 2 & 3 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \frac{x_1}{\begin{vmatrix} 2 & 3 \\ 0 & 3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 3 \\ 0 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 2 \\ 0 & 0 \end{vmatrix}}$$

$$\frac{x_1}{6} = \frac{-x_2}{-3} = \frac{x_3}{0}$$

$$\therefore \frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{0} = k = 1$$

$\therefore x_1 = 2,\ x_2 = 1,\ x_3 = 0$

Thus For Eigen value $\lambda = \lambda_2 = 2$, corresponding Eigen vector $X_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

① Find Eigen value and Eigen vector of a matrix $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

Solution : $\because$ A be a square matrix of order 3

$\therefore$ it's characteristic equation is

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$ — ①

where $S_1 = 3$

$S_2 = \begin{vmatrix} 4 & 3 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} -3 & -5 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} -3 & -7 \\ 2 & 4 \end{vmatrix} = 2 + (-1) + (2) = 3$

$|A| = 1$

$\therefore \lambda^3 - \lambda^2 + 3\lambda - 1 = 0$

$\therefore (\lambda+1)^3 = 0$

$\therefore \lambda = \lambda_1 = 1,\ \lambda = \lambda_2 = 1,\ \lambda = \lambda_3 = 1$ be the Eigen values of a matrix A

To find Eigen vectors, consider $(A - \lambda I)X = 0$

$$\therefore \begin{bmatrix} -3-\lambda & -7 & -5 \\ 2 & 4-\lambda & 3 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Case-1 if $\lambda = \lambda_1 = 1$,
$$\begin{bmatrix} -4 & -7 & -5 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \to R_1 + 4R_3,\ R_2 \to R_2 - 2R_3$

$$\begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \to R_1 + R_2$
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$
$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A - \lambda I) = 2,\ O(A - \lambda I) = 3$

For $\lambda = \lambda_1 = 1$, G.M. $= O(A - \lambda I) - \rho(A - \lambda I) = 3 - 2 = 1$

$\therefore$ For Eigen value $\lambda = \lambda_1 = 1$, only one Eigen vector exist

and is
$$\frac{x_1}{\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix}}$$

$$\frac{x_1}{3} = \frac{-x_2}{1} = \frac{x_3}{-1} = k = -1$$

$x_3 = -3,\ x_2 = 1,\ x_3 = 1$

Thus For Eigen value $\lambda = \lambda_1 = 1$, Eigen vector $X_1 = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$

③ Find Eigen value and Eigen vector of a matrix $A=\begin{bmatrix}2&1&0\\0&2&1\\0&0&2\end{bmatrix}$

Solution : A be a upper diagonal matrix

$\therefore$ diagonal elements are the Eigen values

$\therefore \lambda=\lambda_1=2, \lambda=\lambda_2=2, \lambda=\lambda_3=2$ be the Eigen values of a matrix A

To find Eigen vectors consider $(A-\lambda I)X=0$

$$\begin{bmatrix}2-\lambda&1&0\\0&2-\lambda&1\\0&0&2-\lambda\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \quad (1)$$

Case-1 If $\lambda=\lambda_1=2$

$$\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\therefore \rho(A-\lambda I)=2, \quad O(A-\lambda I)=3$

$\therefore$ For $\lambda=\lambda_1=2$, G.M. $=O(A-\lambda I)-\rho(A-\lambda I)=3-2=1$

$\therefore$ For Eigen value $\lambda=\lambda_1=2$, only one Eigen vector exist and is given by

$$\frac{x_1}{\begin{vmatrix}1&0\\0&1\end{vmatrix}}=\frac{-x_2}{\begin{vmatrix}0&0\\0&1\end{vmatrix}}=\frac{x_3}{\begin{vmatrix}0&1\\0&0\end{vmatrix}}=$$

$$\frac{x_1}{1}=\frac{-x_2}{0}=\frac{x_3}{0}=k=1$$

$\therefore x_1=1, \; x_2=0, \; x_3=0$

Thus For Eigen value $\lambda=\lambda_1=2$, Eigen vector $X_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$