① Find Eigen value and Eigen vector of the matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

<u>Solution</u>: Since A be a square matrix of order 3

$\therefore$ it's characteristic equation is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad - ①$$

where $S_1 = 1+2-1 = 2$

$$S_2 = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} = (-3) + (-1) + (3) = -1$$

$|A| = -2$

$\therefore$ $\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$

$\lambda^2(\lambda - 2) - 1(\lambda - 2) = 0$

$(\lambda - 2)(\lambda^2 - 1) = 0$

$(\lambda - 2)(\lambda - 1)(\lambda + 1) = 0$

$\therefore$ $\lambda = \lambda_1 = 2,\ \lambda = \lambda_2 = +1,\ \lambda = \lambda_3 = -1$ be the Eigen values

To find Eigen vectors consider $(A - \lambda I)X = 0$

i.e.

$$\begin{bmatrix} 1-\lambda & 1 & -2 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & -1-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

<u>Case 1</u> If $\lambda = \lambda_1 = 2$

$$\begin{bmatrix} -1 & 1 & -2 \\ -1 & 0 & 1 \\ 0 & 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

~~Using~~

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ 1 & -3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 1 \\ 0 & -3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}}$$

$$\frac{x_1}{-1} = \frac{-x_2}{3} = \frac{x_3}{-1} = k = -1$$

$\therefore$ $x_1 = 1,\ x_2 = 3,\ x_3 = 1$

Thus for $\lambda = \lambda_1 = 2$ Eigen vector $X_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

<u>Case-2</u>: If $\lambda = \lambda_2 = 1$

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 1 & 1 \\ 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 1 \\ 0 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix}}$$

$$\frac{x_1}{-3} = \frac{-x_2}{2} = \frac{x_3}{-1} = k = -1$$

$\therefore$ $x_1 = 3,\ x_2 = 2,\ x_3 = 1$

Thus for Eigen value $\lambda = \lambda_2 = 1$, Eigen vector $X_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Case - 3

For $\lambda = \lambda_3 = -1$

$$\begin{bmatrix} 2 & 1 & -2 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 3 \\ 0 & 1 \end{vmatrix}}$$

$$\frac{x_1}{-1} = \frac{-x_2}{0} = \frac{x_3}{-1} = k = -1$$

$\therefore x_1 = 1, \quad x_2 = 0, \quad x_3 = 1$

Thus For Eigenvalue $\lambda = \lambda_3 = -1$, $X_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

② Find Eigen value and Eigen vector of matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

Solution $\because$ A be a square matrix of order 3

$\therefore$ it's characteristic equation is

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$, where

$S_1 = 8 + (-3) + 1 = 6$

$S_2 = \begin{vmatrix} -3 & -2 \\ -4 & 1 \end{vmatrix} + \begin{vmatrix} 8 & -2 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 8 & -8 \\ 4 & -3 \end{vmatrix} = (-3-8) + (8+6) + (-24+32) = 11$

$|A| = 6$

$\therefore \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$(\lambda - 1)(\lambda^2 - 5\lambda + 6) = 0$

$\Rightarrow (\lambda-1)(\lambda-2)(\lambda-3) = 0$

$\therefore \lambda = \lambda_1 = 1,\ \lambda = \lambda_2 = 2,\ \lambda = \lambda_3 = 3$ be Eigen values of a matrix A

To find Eigen vectors consider $(A - \lambda I)X = 0$

$$\begin{bmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Case 1 For $\lambda = \lambda_1 = 1$

$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} -4 & -2 \\ -4 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 4 & -2 \\ 3 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & -4 \\ 3 & -4 \end{vmatrix}}$$

$$\frac{x_1}{-8} = \frac{-x_2}{+6} = \frac{x_3}{-4}$$

ie $\dfrac{x_1}{-4} = \dfrac{x_2}{-3} = \dfrac{x_3}{-2} = k = -1$

$\therefore x_1 = 4,\ x_2 = 3,\ x_3 = 2$

Thus For Eigen value $\lambda = \lambda_1 = 1$, Eigen vector $X_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$

Case-2 For $\lambda = \lambda_2 = 2$

$$\begin{bmatrix} 6 & -8 & -2 \\ 4 & -5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} -5 & -2 \\ -4 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 4 & -2 \\ 3 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & -5 \\ 3 & -4 \end{vmatrix}}$$

$$\frac{x_1}{-3} = \frac{-x_2}{2} = \frac{x_3}{-1} = k = -1$$

$$x_3 = 3, \quad x_2 = 2, \quad x_3 = 1$$

Thus For Eigen value $\lambda = \lambda_2 = 2$, Eigen vector $X_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Case 3 For $\lambda = \lambda_3 = 3$

$$\begin{bmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} -6 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 4 & -2 \\ 3 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & -6 \\ 3 & -4 \end{vmatrix}}$$

$$\frac{x_1}{4} = \frac{-x_2}{-2} = \frac{x_3}{2}$$

ie $\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{1} = k = 1$

$\therefore \quad x_1 = 2, \quad x_2 = 1, \quad x_3 = 1$

Thus for $\lambda = \lambda_1 = 3$, $X_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

③ Find Eigen value and Eigen vector of the matrix $A = \begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix}$

Solution $\because$ A be a square matrix of order 3

$\therefore$ it's characteristic equation is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad \text{— (1)}$$

where $S_1 = 9 + (-1) + (-7) = 1$

$$S_2 = \begin{vmatrix} -1 & 3 \\ 1 & -7 \end{vmatrix} + \begin{vmatrix} 9 & 9 \\ -7 & -7 \end{vmatrix} + \begin{vmatrix} 9 & -1 \\ 3 & -1 \end{vmatrix} = (7-3) + (-63+63) + (-9+3) = 4+0-6 = -2$$

$|A| = 0$

$$\therefore \lambda^3 - 1\lambda^2 - 2\lambda - 0 = 0$$

$$\therefore \lambda(\lambda^2 - \lambda - 2) = 0 \Rightarrow \lambda(\lambda-2)(\lambda+1) = 0$$

$\Rightarrow \lambda = \lambda_1 = 0,\ \lambda = \lambda_2 = 2,\ \lambda = \lambda_3 = -1$ be the Eigen values of given matrix

To find Eigen vector consider $(A - \lambda I)X = 0$

$$\begin{bmatrix} 9-\lambda & -1 & 9 \\ 3 & -1-\lambda & 3 \\ -7 & 1 & -7-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{— (2)}$$

Case 1 : If $\lambda = \lambda_1 = 0$

$$\begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} -1 & 3 \\ 1 & -7 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 3 & 3 \\ -7 & -7 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 3 & -1 \\ -7 & 1 \end{vmatrix}}$$

$$\frac{x_1}{4} = \frac{-x_2}{0} = \frac{x_3}{-4}$$

ie $$\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{-1} = k = 1$$

$\therefore x_1 = 1,\ x_2 = 0,\ x_3 = -1$

Thus for Eigen value $\lambda = \lambda_1 = 0$, $X_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Case-2 If $\lambda = \lambda_2 = 2$

$$\begin{bmatrix} 7 & -1 & 9 \\ 3 & -3 & 3 \\ -7 & 1 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} -3 & 3 \\ 1 & -9 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 3 & 3 \\ -7 & -9 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 3 & -3 \\ -7 & 1 \end{vmatrix}}$$

$$\therefore \frac{x_1}{24} = \frac{-x_2}{-6} = \frac{x_3}{-18}$$

$$\therefore \frac{x_1}{4} = \frac{x_2}{1} = \frac{x_3}{-3} = k = 1$$

$\therefore x_1 = 4, x_2 = 1, x_3 = -3$

Thus for Eigen Value $\lambda = \lambda_2 = 2$, Eigen vector $X_2 = \begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix}$

Case-3 If $\lambda = \lambda_3 = -1$,

$$\begin{bmatrix} 10 & -1 & 9 \\ 3 & 0 & 3 \\ -7 & 1 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 0 & 3 \\ 1 & -6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 3 & 3 \\ -7 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 3 & 0 \\ -7 & 1 \end{vmatrix}}$$

$$\frac{x_1}{-3} = \frac{-x_2}{3} = \frac{x_3}{3}$$

$$\therefore \frac{x_1}{-1} = \frac{x_2}{-1} = \frac{x_3}{1} = k = -1$$

$\therefore x_1 = 1, x_2 = 1, x_3 = -1$

$\therefore$ For Eigen value $\lambda = \lambda_1 = -1$, Eigen vector $X_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

④ Find Eigen value and Eigen vector of matrix $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

<u>Solution</u> $\because$ A be a square matrix of order 3

$\therefore$ it's characteristic equation is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad ①$$

where $S_1 = 4+3+1 = 8$

$$S_2 = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} + \begin{vmatrix} 4 & -2 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 4 & 2 \\ -5 & 3 \end{vmatrix} = -5 + 0 + 22 = 17$$

$|A| = 10$

$\therefore \lambda^3 - 8\lambda^2 + 17\lambda - 10 = 0$

$$\begin{array}{c|cccc} 1 & 1 & -8 & 17 & -10 \\ & & 1 & -7 & 10 \\ \hline & 1 & -7 & 10 & \boxed{0} \end{array}$$

$\therefore (\lambda-1)(\lambda^2 - 7\lambda + 10) = 0$

$\therefore (\lambda-1)(\lambda-2)(\lambda-5) = 0$

$\therefore \lambda = \lambda_1 = 1, \lambda = \lambda_2 = 2, \lambda = \lambda_3 = 5$ be the Eigen values of a matrix A

To find Eigen vector consider $(A - \lambda I)X = 0$

i.e $$\begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad - ②$$

<u>Case-1</u> If $\lambda = \lambda_1 = 1$

$$\begin{bmatrix} 3 & 2 & -2 \\ -5 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 2 \\ 4 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -5 & 2 \\ -2 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -5 & 2 \\ -2 & 4 \end{vmatrix}}$$

$$\frac{x_1}{-8} = \frac{-x_2}{4} = \frac{x_3}{-16}$$

$$\frac{x_1}{-2} = \frac{x_2}{-1} = \frac{x_3}{-4} = k = -1$$

$x_1 = 2, \; x_2 = 1, \; x_3 = 4$

For Eigen value $\lambda = \lambda_1 = 1$, Eigen vector $X_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$

<u>Case-2</u> If $\lambda = \lambda_2 = 2$

$$\begin{bmatrix} 2 & 2 & -2 \\ -5 & 1 & 2 \\ -2 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 2 \\ 4 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -5 & 2 \\ -2 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -5 & 1 \\ -2 & 4 \end{vmatrix}}$$

$$\frac{x_1}{-9} = \frac{-x_2}{9} = \frac{x_3}{-18}$$

$$\therefore \frac{x_1}{-1} = \frac{x_2}{-1} = \frac{x_3}{-2} = k = -1$$

$$\therefore x_1 = 1,\ x_2 = 1,\ \&\ x_3 = 2$$

Thus for ~~...~~ Eigen value $\lambda = \lambda_1 = 2$, Eigen vector $X_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

Case-3 $\lambda = \lambda_3 = 5$, $\begin{bmatrix} -1 & 2 & -2 \\ -5 & -2 & 2 \\ -2 & 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\frac{x_1}{\begin{vmatrix} -2 & 2 \\ 4 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -5 & 2 \\ -2 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -5 & -2 \\ -2 & 4 \end{vmatrix}}$$

$$\frac{x_1}{0} = \frac{-x_2}{24} = \frac{x_3}{-24}$$

$$\therefore \frac{x_1}{0} = \frac{x_2}{-1} = \frac{x_3}{-1} = k = -1$$

$$\therefore x_1 = 0,\ x_2 = 1,\ x_3 = 1$$

$\therefore$ For Eigen value $\lambda = \lambda_1 = 5$, Eigen vector $X_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

⑤ Find Eigen value and Eigen vector of a matrix $A=\begin{bmatrix}2 & -2 & 3\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}$

Solution. ∵ A be a square matrix of order 3

∴ it's characteristic equation is

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$ — (1)

$S_1 = 2 + 1 + (-1) = 2$

$S_2 = \begin{vmatrix}1 & 1\\ 3 & -1\end{vmatrix} + \begin{vmatrix}2 & 3\\ 1 & -1\end{vmatrix} + \begin{vmatrix}2 & -2\\ 1 & 1\end{vmatrix} = -4 + (-5) + 4 = -5$

$|A| = -6$

$\therefore \lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0$

$$\begin{array}{c|cccc} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$\therefore (\lambda - 1)(\lambda^2 - 1\lambda - 6) = 0$

$\therefore (\lambda - 1)(\lambda - 3)(\lambda + 2) = 0$

$\therefore \lambda = \lambda_1 = +1,\ \lambda = \lambda_2 = 3\ \ \lambda = \lambda_3 = -2$ be the Eigen values of matrix A

To find Eigen vector consider $(A - \lambda I)X = 0$

i.e. $\begin{bmatrix}2-\lambda & -2 & 3\\ 1 & 1-\lambda & 1\\ 1 & 3 & -1-\lambda\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$

Case-1 If $\lambda = \lambda_1 = 1$, $\begin{bmatrix}1 & -2 & 3\\ 1 & 0 & 1\\ 1 & 3 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$

$$\frac{x_1}{\begin{vmatrix}0 & 1\\ 3 & -2\end{vmatrix}} = \frac{-x_2}{\begin{vmatrix}1 & 1\\ 1 & -2\end{vmatrix}} = \frac{x_3}{\begin{vmatrix}1 & 0\\ 1 & 3\end{vmatrix}}$$

$$\frac{x_1}{-3} = \frac{-x_2}{-3} = \frac{x_3}{3}$$

$$\therefore \frac{x_1}{-1} = \frac{x_2}{1} = \frac{x_3}{1} = t = 1$$

$\therefore x_1 = -1,\ x_2 = 1,\ x_3 = 1$

Thus for $\lambda = \lambda_1 = 1$ Eigen vector $X_1 = \begin{bmatrix}-1\\ 1\\ 1\end{bmatrix}$

Case-2 If $\lambda = \lambda_2 = 3$, $\begin{bmatrix}-1 & -2 & 3\\ 1 & -2 & 1\\ 1 & 3 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$

$$\frac{x_1}{\begin{vmatrix}-2 & 1\\ 3 & -4\end{vmatrix}} = \frac{-x_2}{\begin{vmatrix}1 & 1\\ 1 & -4\end{vmatrix}} = \frac{x_3}{\begin{vmatrix}1 & -2\\ 1 & 3\end{vmatrix}}$$

$$\frac{x_1}{5} = \frac{-x_2}{-5} = \frac{x_3}{5}$$

$$\therefore \frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{1} = k = 1$$

$$\therefore x_1 = 1,\ x_2 = 1,\ x_3 = 1$$

$\therefore$ For Eigen value $\lambda = \lambda_2 = 3$, Eigen vector $X_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

<u>case-3</u> If $\lambda = \lambda_3 = -2$

$$\begin{bmatrix} 4 & -2 & 3 \\ 1 & 3 & 1 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & -2 \\ 1 & 3 \end{vmatrix}}$$

$$\frac{x_1}{-11} = \frac{-x_2}{1} = \frac{x_3}{14}$$

$$\therefore \frac{x_1}{-11} = \frac{x_2}{-1} = \frac{x_3}{14} = t = 1$$

$$x_1 = -11,\ x_2 = -1,\ x_3 = 14$$

$\therefore$ For Eigen value $\lambda = \lambda_3 = -2$, Eigen vector $X_3 = \begin{bmatrix} -11 \\ -1 \\ 14 \end{bmatrix}$

⑥ Find Eigen value and Eigen vector of a matrix $A = \begin{bmatrix} -9 & 2 & 6 \\ 5 & 0 & -3 \\ -16 & 4 & 11 \end{bmatrix}$

<u>Solution</u>: $\because$ A be a square matrix of order 3

$\therefore$ it's characteristic equation is

$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$ — ①

where $S_1 = -9+0+11 = 2$

$S_2 = \begin{vmatrix} 0 & -3 \\ 4 & 11 \end{vmatrix} + \begin{vmatrix} -9 & 6 \\ -16 & 11 \end{vmatrix} + \begin{vmatrix} -9 & 2 \\ 5 & 0 \end{vmatrix} = 12+(-3)+(-10) = -1$

$|A| = -2$

$\therefore \lambda^3 - 2\lambda^2 - 1\lambda + 2 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -1 & 2 \\ & & 1 & -1 & -2 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array}$$

$(\lambda-1)(\lambda^2-\lambda-2) = 0$

$(\lambda-1)(\lambda-2)(\lambda+1) = 0$

$\therefore \lambda = \lambda_1 = 1,\ \lambda = \lambda_2 = 2,\ \lambda = \lambda_3 = -1$, be the Eigen values of matrix A

To find Eigen vector consider $(A-\lambda I)X = 0$

ie $\begin{bmatrix} -9-\lambda & 2 & 6 \\ 5 & 0-\lambda & -3 \\ -16 & 4 & 11-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

<u>case-1</u> If $\lambda = \lambda_1 = 1$, $\begin{bmatrix} -10 & 2 & 6 \\ 5 & -1 & -3 \\ -16 & 4 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\frac{x_1}{\begin{vmatrix} -1 & -3 \\ 4 & 10 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & -3 \\ -16 & 10 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -1 \\ -16 & 4 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{-x_2}{2} = \frac{x_3}{4}$$

$$\therefore \frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{2} = k = 1$$

$\therefore x_1 = 1,\ x_2 = -1,\ x_3 = 2$

Thus For Eigen value $\lambda = \lambda_1 = 1$, Eigen vector $X_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

<u>case-2</u> If $\lambda = \lambda_2 = 2$ $\begin{bmatrix} -11 & 2 & 6 \\ 5 & -2 & -3 \\ -16 & 4 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\frac{x_1}{\begin{vmatrix} -2 & -3 \\ 4 & 9 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & -3 \\ -16 & 9 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -2 \\ -16 & 4 \end{vmatrix}}$$

$$\frac{x_1}{-6} = \frac{-x_2}{-3} = \frac{x_3}{-12}$$

$$\therefore \frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{-4} = k = -1$$

$$\therefore x_1 = 2, \quad x_2 = -1, \quad x_3 = 4$$

Thus For Eigen value $\lambda = \lambda_2 = 2$, Eigen vector $X_2 = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

<u>Case-3</u> If $\lambda = \lambda_3 = -1$,

$$\begin{bmatrix} -8 & 2 & 6 \\ 5 & 1 & -3 \\ -16 & 4 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 6 \\ 1 & -3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -8 & 6 \\ 5 & -3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -8 & 2 \\ 5 & 1 \end{vmatrix}}$$

$$\frac{x_1}{-12} = \frac{-x_2}{-6} = \frac{x_3}{-18}$$

$$\frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{-3} = k = -1$$

$$\therefore x_1 = 2, \quad x_2 = -1, \quad x_3 = 3$$

Thus for Eigen value $\lambda = \lambda_3 = -1$, Eigen vector $X_3 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

⑦ Find Eigen value and Eigen vector of a matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Solution: $\because$ A be a square matrix of order 3

$\therefore$ it's characteristic equation is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad \text{--- ①}$$

where $S_1 = 6$

$$S_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 3 + 3 + 5 = 11$$

$|A| = 6$

$$\therefore \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$$\therefore (\lambda - 1)(\lambda^2 - 5\lambda + 6) = 0$$

$$\therefore (\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$$

$\therefore \lambda = \lambda_1 = 1,\ \lambda = \lambda_2 = 2$ and $\lambda = \lambda_3 = 3$ be the Eigen values of a matrix A

To find Eigen vector consider $(A - \lambda I)X = 0$

ie
$$\begin{bmatrix} 2-\lambda & -1 & 1 \\ 1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Case - 1 If $\lambda = \lambda_1 = 1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{x_1}{0} = \frac{-x_2}{2} = \frac{x_3}{-2}$$

$$\therefore \frac{x_1}{0} = \frac{x_2}{-1} = \frac{x_3}{-1} = k = -1$$

$$\therefore x_1 = 0,\ x_2 = 1,\ x_3 = 1$$

$\therefore$ For Eigen value $\lambda = \lambda_1 = 1$, Eigen vector $X_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Case - 2 If $\lambda = \lambda_2 = 2$

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{x_1}{-1} = \frac{-x_2}{1} = \frac{x_3}{-1} =$$

$$\therefore \frac{x_1}{-1} = \frac{x_2}{-1} = \frac{x_3}{-1} = k = -1$$

$\therefore\ x_1 = 1,\ x_2 = 1,\ x_3 = 1$

Thus for Eigen value $\lambda = \lambda_2 = 2$, Eigen vector $X_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Case-3 If $\lambda = \lambda_3 = 3$

$$\begin{bmatrix} -1 & -1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} -1 & 1 \\ -1 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & -1 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{-x_2}{0} = \frac{x_3}{2}$$

$$\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{1} = k = 1$$

$\therefore\ x_1 = 1,\quad x_2 = 0,\quad x_3 = 1$

Thus For Eigen value $\lambda = \lambda_3 = 3$ Eigen vector $X_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(8) Find the Eigen value and Eigen vector of a matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

Solution : $\because$ A be a square matrix of order 3

$\therefore$ it's characteristic equation is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad \text{— (1)}$$

where $S_1 = 4$

$$S_2 = \begin{vmatrix} 3 & 2 \\ -4 & -3 \end{vmatrix} + \begin{vmatrix} 4 & 6 \\ -1 & -3 \end{vmatrix} + \begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix} = -1 + (-6) + (6) = -1$$

$|A| = -4$

$$\therefore \lambda^3 - 4\lambda^2 - 1\lambda + 4 = 0$$

$$\begin{array}{c|cccc} 1 & 1 & -4 & -1 & 4 \\ & & 1 & -3 & -4 \\ \hline & 1 & -3 & -4 & \boxed{0} \end{array}$$

$\therefore (\lambda - 1)(\lambda^2 - 3\lambda - 4) = 0$

$\therefore (\lambda - 1)(\lambda - 4)(\lambda + 1) = 0$

$\therefore \lambda = \lambda_1 = 1,\ \lambda = \lambda_2 = 4,\ \lambda = \lambda_3 = -1$ be the Eigen values of a matrix A

To find Eigen vectors consider $(A - \lambda I)X = 0$

$$\begin{bmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -4 & -3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{— (2)}$$

<u>case-1</u> : If $\lambda = \lambda_1 = 1$

$$\begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ -1 & -4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 2 & 2 \\ -4 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 2 \\ -1 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ -1 & -4 \end{vmatrix}}$$

$$\frac{x_1}{0} = \frac{-x_2}{-2} = \frac{x_3}{-2}$$

$$\therefore \frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{-1} = k = 1$$

$\therefore x_1 = 0,\ x_2 = 1,\ x_3 = -1$

Thus for Eigen value $\lambda = \lambda_1 = 1$, Eigen vector $X_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

<u>Case-2</u> : If $\lambda = \lambda_2 = 4$,

$$\begin{bmatrix} 0 & 6 & 6 \\ 1 & -1 & 2 \\ -1 & -4 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ -1 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 6 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{x_1}{18} = \frac{-x_2}{-6} = \frac{x_3}{-6}$$

$$\frac{x_1}{3} = \frac{x_2}{1} = \frac{x_3}{-1} = k = 1$$

$\therefore x_1 = 3,\ x_2 = 1,\ x_3 = -1$

Thus for Eigen value $\lambda = \lambda_2 = 4$ Eigen vector $X_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$

Case-3 If $\lambda = \lambda_3 = -1$

$$\begin{bmatrix} 5 & 6 & 6 \\ 1 & 4 & 2 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ 4 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 6 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & 6 \\ 1 & 4 \end{vmatrix}}$$

$$\frac{x_1}{-12} = \frac{-x_2}{4} = \frac{x_3}{14}$$

$$\frac{x_1}{-6} = \frac{x_2}{-2} = \frac{x_3}{7} = k = -1$$

$$x_1 = 6, \ x_2 = 2, \ x_3 = -7$$

Thus for Eigen value $\lambda = \lambda_3 = -1$, Eigen vector $X_3 = \begin{bmatrix} 6 \\ 2 \\ -7 \end{bmatrix}$