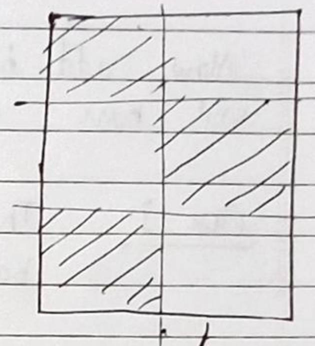
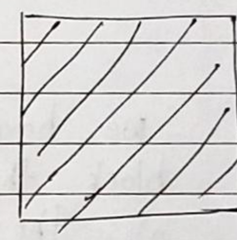
H.W

1] Induction Hypothesis:

If we use n=k lines to create polygonal region then it is possibly that no two adjacent polygons have some colour and we can colour the entire page using just two colour. / max 2 colour are needed



Base Case: For $\boxed{n=0}$

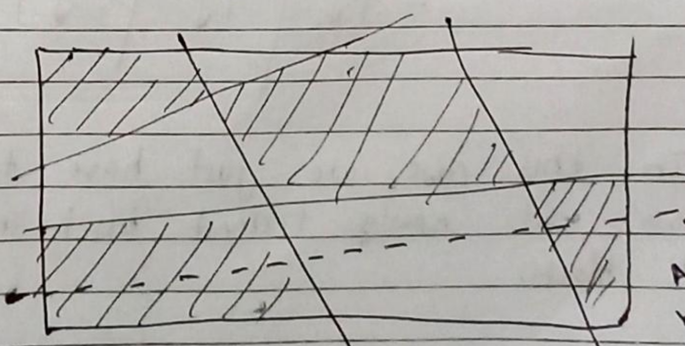


The Entire page can be coloured using just one colour, and all other condition are also satisfied

Induction step

For $n = k+1$

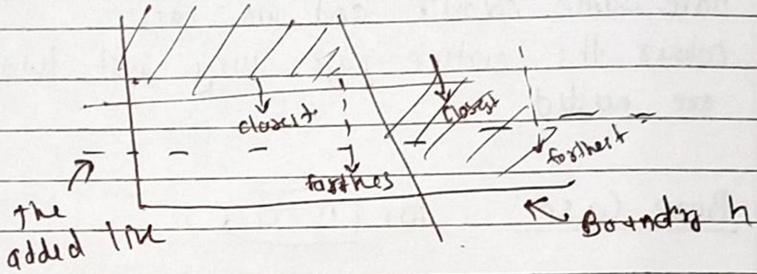
let there be $k+1$ lines that divides the page in the polygonal region,




Now, remove any arbitry, but particular lines
Hence, now we have n line which satisfies all the condition from the hypothesis

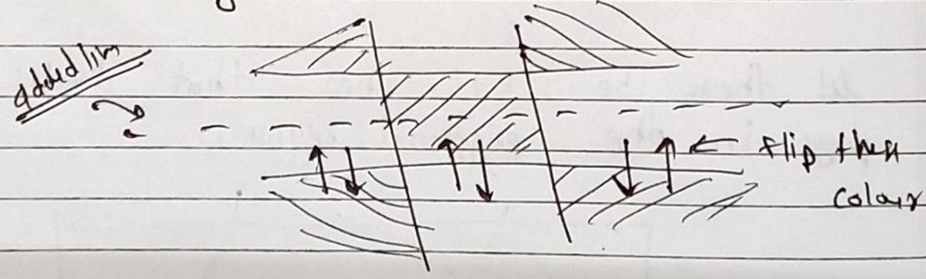
Now add back the removed line. we will have two cases

Case I: The formed section is near boundry




we have to check. the adjacent of the block that got divided and give the same colour as the adjacent to the one who is most. farthes and the opposite to ~~the~~ ~~the~~ the closest.

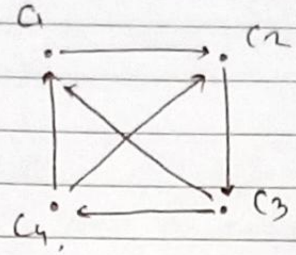
Case II: The formed section is surrounded by other section




In this case we just have to flip the colours of newly formed block and it's adjacent old block.

Hence proved

2]




Induction : Hypothesis.

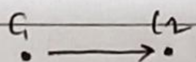
Let the given statmunt be true for $n=k$ cities.

Base case

For $n=0$
It is true as there are no cities to travel

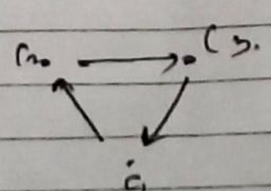
For $n=1$
It is true as there is only one city

for $n=2$,




Here $c_2$ is that city.

for $n=3$




Here all the cities satisfy the given condition

3) Base case

Every positive integer $n$ can be expressed as distinct sum of power of 2.

For $n = 1$

$\boxed{1 = 2^0}$ Hence true

For $n = 2$

$\boxed{2 = 2^1}$ Hence true

Induction Hypothesis

The the statement stand true for $n = m$ st $1 \leq m \leq k$

Induction step

let $n = k+1$

Case I : $(k+1)$ is even

$\therefore \left(\frac{k+1}{2}\right)$ is a integer.

$\therefore 1 \leq \frac{k+1}{2} \leq k.$

$\therefore \frac{k+1}{2}$ is ~~tru~~ satisfies the condition

$2\left(\frac{k+1}{2}\right)$ ~~&~~ also satis-fies the condition and has the ~~#~~ distinct power of two as ~~1~~ is added to even power

Induction step

For $n = k+1$

Now lets ~~ignore~~ remove one city

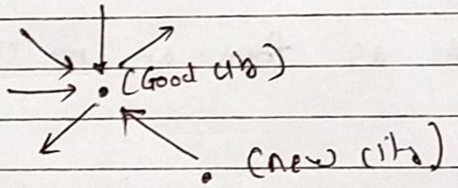
$\therefore$ Number of cities $= k$

By Induction Hypothesis

For $n = k$

there exist a city which satisfies all the give condition

Let that city be "Good city"




Now build a new city and build a road which goes from 'new city' to 'good city',

$\therefore$ $n = k+1$

and 'Good city' satisfies all conditions

Hence proved

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set which was used to represent $\left(\frac{k+1}{2}\right)$.

Case II : $(k+1)$ is odd

$\therefore$ $k$ is even

$\therefore$ $k$ does not have $2^0$ and is represented using distinct power of two

$\therefore$ $k+1$ = power set of $k$ $+ 2^0$

Hence proved

4] We know, for the graph with n vertices max No. edge in simple graph

$$= \frac{n(n-1)}{2}.$$

$\therefore$ Max value of $m = \frac{n(n-1)}{2}$.

But it is possible that all the edges may not be present.

$\therefore$ $m \leq \frac{n(n-1)}{2}$

$\therefore$ $2m \leq n(n-1)$

$\boxed{2m \leq n^2 - 1}$

Q5]. Let us assume that every one in the group has different No. of friends.

So for n people in group:
Following set represent the No. of friend each person have

$0, 1, 2, 3 \ldots, n-1$

But according to this assumption one person is friends with every other person but at the same time there is a person with No friends.
Hence our assumption is false
$\therefore$ The given statement is true.

Q8) $r \rightarrow$ Degree of each vertex
$n \rightarrow$ No. of vertex
$m \rightarrow$ No. of edges

As $r$ is Degree of each vert
and $n$ is No. of vertx

$r \times n \Rightarrow$ summation of degree's of all vertices

we know
If we consider one edge then it add 2 to the summation of degrees

$\therefore \quad 2m = r \times n$

$$\therefore \boxed{m = \frac{r \times n}{2}}$$

Q7] <u>Base case:</u>

$n = 2$.

i.e 

$\delta(G) = 1$

$\frac{2}{2} = 1$

$\therefore \quad \delta(G) \geq n/2$.

<u>Induction Hypothesis</u>

let us assume that the statement stands true for $n = k$.

$\therefore \quad \delta(G) \geq \frac{k}{2}$ and it is connected

<u>Induction Step</u>

let $n = k+1$

~~ign lets ignore the re.~~

To create this graph let us use the the graph ($G'$) with $n = k$.

$\therefore \quad \delta(G') \geq \frac{k}{2}$

Now add a vertex and draw any number of edges to, to that new vertex from the remaining vertex of $G'$

$\therefore$ minimum value of
$\delta(G) = \delta(G')$

Case I : If $k$ is even

the $\frac{k+1}{2} = \frac{k}{2}$

as they both are integers

$\therefore \delta(G) \geq \frac{k+1}{2}$.

Case II : If $k$ is odd

then $\Rightarrow$
max of $\frac{k+1}{2} = \delta(G)$

$\boxed{\text{Hence proved}}$

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Q8] Let us assume there is a point P with odd degree

Let this be a part of simple connected graph.

By, by previous ~~l~~amma we know that the No. of ~~even~~ odd ordered vertices is always even

∴ There must be one more vertex with odd order,

And as this graph is connect, then there always exist a path from P to every other vertex including the vertex with odd order

Examples: