

Algorithm Design

Homework Assignment 6

Given: April 3, 2023

Due: April 14, 2023

- ✓ 1. Given a directed graph G with n vertices represented using an $n \times n$ adjacency matrix, give an algorithm that determines whether there is a node in G whose indegree is $n - 1$ and outdegree is 0.
- ✓ 2. KT Chapter 3, Problem 2 (page 107). Also, design an algorithm that takes an undirected graph G and a particular edge e in it, and determines whether G contains a cycle containing e .
- ✓ 3. KT Chapter 3, Problem 6 (page 108).
- ✓ 4. Give an efficient algorithm to find a longest path in an unrooted tree.
5. Give an efficient algorithm that takes as input a directed acyclic graph $G = (V, E)$, and two vertices $s, t \in V$, and outputs the number of different directed paths from s to t in G .
- ✓ 6. Consider a weighted, directed acyclic graph $G = (V, E)$ in which the edges that leave the source vertex s may have negative weights and all other edge weights are non-negative. Does Dijkstra's algorithm, started at s , correctly compute the shortest paths from s to every other vertex in the graph? Prove your answer.
- ✓ 7. Prof. Midas postulates that if every edge in an undirected graph has a unique positive weight, then the shortest path tree rooted at v in that graph is always the same as the minimum spanning tree found by Prim's algorithm when seeded initially with the vertex v . Is this correct? If so, prove it. If not, give a counter-example.