CHAPTER 1

Module 1

# Diffraction

**Syllabus**

**(Prerequisites :** Wave front and Huygen's principle, Reflection and refraction, Diffraction, Fresnel diffraction and Fraunhofer diffraction).

**Diffraction :** Fraunhofer diffraction at single slit, Diffraction grating, Resolving power of a grating; Applications of diffraction grating; Determination of wavelength of light using plane transmission grating.

## 1.1 Introduction

**MU - Dec. 2015**

**University Question**

**Q.** What is meant by diffraction? **(Dec. 15, 1 Mark)**

- To all outward appearance light seems to travel in straight line path. It is a matter of common experience that the path of light entering a dark room through a hole in a window illuminated by sunlight is straight.
- Similarly if an opaque obstacle is placed in the path of light, a sharp shadow is cast on the screen. This shows that light travels in straight lines. This is called rectilinear motion of light.
- But it is also observed that when a beam of light passes through a small narrow opening or close to the edges of an obstacle, it spreads to some extent into the region of geometrical shadow also. This happens due to the bending of light waves round the corners of the obstacle or opening.
- The bending of light is very small when the dimensions of the slit or the obstacle are large as compared to the wavelength of light and it becomes much pronounced when the dimensions of obstacle or slit are comparable with the wavelength of light.
- For example, when light waves diverging from a narrow slit S (Fig. 1.1.1) which is illuminated by a monochromatic source O, pass an obstacle AB with a straight edge A parallel to the slit, the geometrical shadow on the screen is never sharp.
- A small portion of light bends around the edge into geometrical shadow. Outside the shadow, parallel to its edge, several bright and comparatively dark bands are observed.

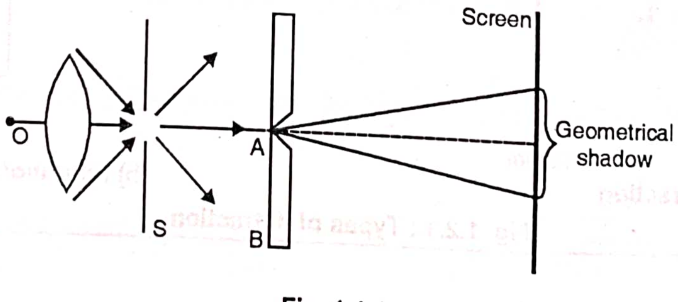



Fig. 1.1.1

- If an opaque object is placed between a point source of light and screen then, as per the rules of geometrical optics, a well defined and distinct shadow of the object should be obtained on the screen. No light should be observed in the region of shadow.
- But the pattern obtained on the screen is in the form of alternate dark and bright rings with a bright spot at the centre of the shadow.
- This spot cannot be explained on the basis of rectilinear propagation of light but can be explained due to the bending of light into the regions of shadow.
- The phenomenon of bending of light round the corners of an obstacle and spreading of light waves into the region of geometrical shadow of the obstacle is called **diffraction. Since very small obstacles are needed to create it, diffraction is not evident in daily life easily.**
- Fresnel explained the phenomenon of diffraction on the basis of Huygen's wave theory of light.
- The luminous border that surrounds the profile of a mountain just before the sun rises behind it; the light streaks that one sees while looking at a strong source of light with half shut eyes; the coloured spectra (arranged in the form of a cross) that one sees while viewing a distant source of light through a fine piece of cloth are some examples of diffraction effects.

## 1.2 Types of Diffraction

**MU - Dec. 2015**

**University Question**

Q. State diffraction types and differentiate them. **(Dec. 15, 2 Marks)**

The phenomenon of diffraction is divided into two types :

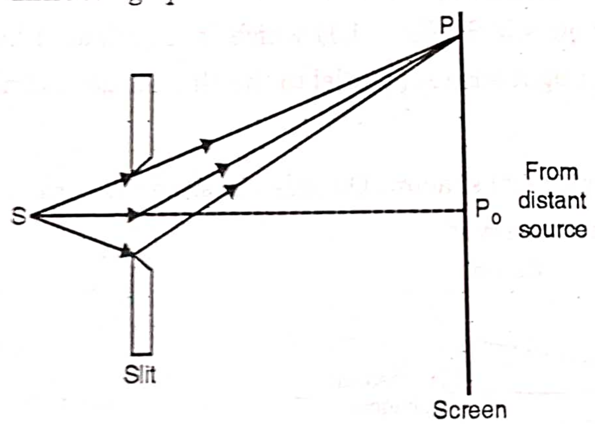
1. Fresnel diffraction
2. Fraunhofer diffraction

### 1. Fresnel diffraction

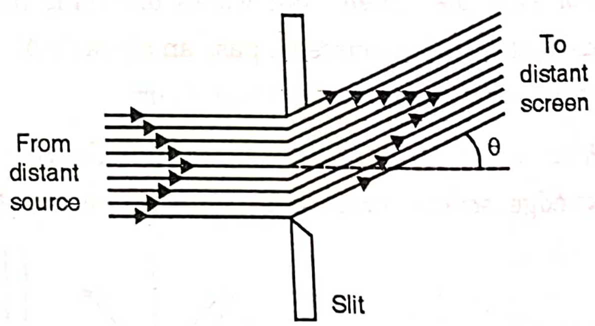
Diffraction phenomenon in which the light source and the screen are finite distance from the diffracting aperture is termed as Fresnel diffraction. This is shown in Fig. 1.2.1(a).

### 2. Fraunhofer diffraction

Diffraction phenomenon in which the light source and the screen are at infinite distance from the diffracting aperture is termed as Fraunhofer diffraction. This is shown in Fig. 1.2.1(b).




(a) Fresnel Diffraction




(b) Fraunhofer Diffraction

Fig. 1.2.1 : Types of diffraction

- The points of difference between these two types of diffraction are as given below :

**Table 1.2.1 : Difference between Fresnel diffraction and Fraunhofer diffraction**

| Sr. No. | Fresnel diffraction | Fraunhofer diffraction |
|---|---|---|
| 1. | Distance of source and screen from obstacle is finite. | Source and screen are at infinite distance from obstacle. |
| 2. | No lenses are required to study diffraction in the laboratory. | Two biconvex lenses are required to study diffraction in laboratory. |
| 3. | The wavefront incident on the aperture is either spherical or cylindrical. | The wavefront incident on aperture is plane. |
| 4. | The diffracted wavefront is either spherical or cylindrical. | The diffracted wavefront is plane. |
| 5. | The initial phase of secondary wavelets is different at different points in the plane of aperture. | The initial phase of secondary wavelets is same at all points in the plane of aperture. |
| 6. | It has no importance in optical instruments. | It is very important in optical instruments. |

## 1.3 Difference between Interference and Diffraction

**Interference**

1. Interference is the result of interaction of light coming from two different wavefronts originating from the same source.
2. Interference fringes in a particular pattern may or may not be of same width.
3. In interference all the bright fringes are of the same or uniform intensity.
4. The points of minimum intensity are perfectly dark. Hence the contrast between the fringes is good.

**Diffraction**

1. Diffraction is the result of interaction of light coming from different parts of the same source.
2. Diffraction fringes in a particular pattern are not of same width.
3. In diffraction the bright fringes are not of the same intensity.
4. The points of minimum intensity are not perfectly dark. Hence the contrast between fringes is not good.

## 1.4 Fraunhofer Diffraction at a Single Slit

**MU - May 2018**

**University Question**

**Q.** Discuss the Fraunhofer Diffraction at a single slit and obtain the condition for minima. **(May 18, 3 Marks)**

**Step I**

- Consider a single slit of width 'a' illuminated by monochromatic light of wavelength $\lambda$ as shown in Fig. 1.4.1.
- The incident plane wavefront is diffracted by the slit and is then focused on the screen by the lens L.

- Every point of the incident wavefront in the plane of the slit acts as a secondary source and sends out secondary waves in all directions.
- The secondary wavelets travelling normally to the slit are brought to focus at point $P_o$ by the lens.
- All these secondary waves travel the same distance through the lens along the direction $\theta = 0$ and hence produce maximum intensity of light.

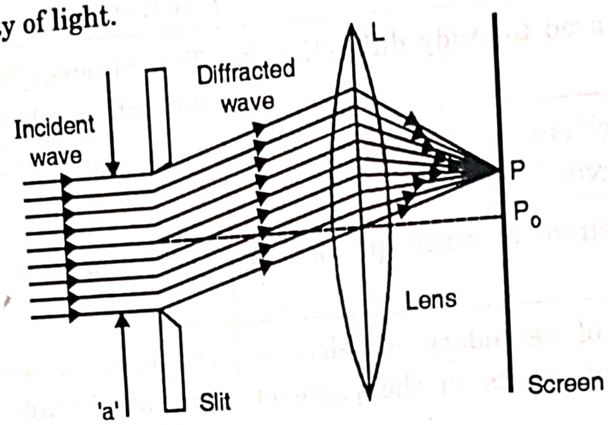



Fig. 1.4.1 : Fraunhofer Diffraction at Single Slit

- Consider a point P on the screen at which the secondary wavelets travelling at an angle θ with the normal are focussed.
- The intensity of light at point P will depend upon the path difference between the secondary wavelets originating from the corresponding points of the wavefront.

**Step II**

- Consider the given slit to be divided into N parallel slits, each of width dx. See Fig. 1.4.2(a).

$$\therefore\ a = dx_1 + dx_2 + dx_3 + \ldots + dx_N \qquad \ldots(1.4.1)$$

- One of such slits is shown in Fig. 1.4.2(b).

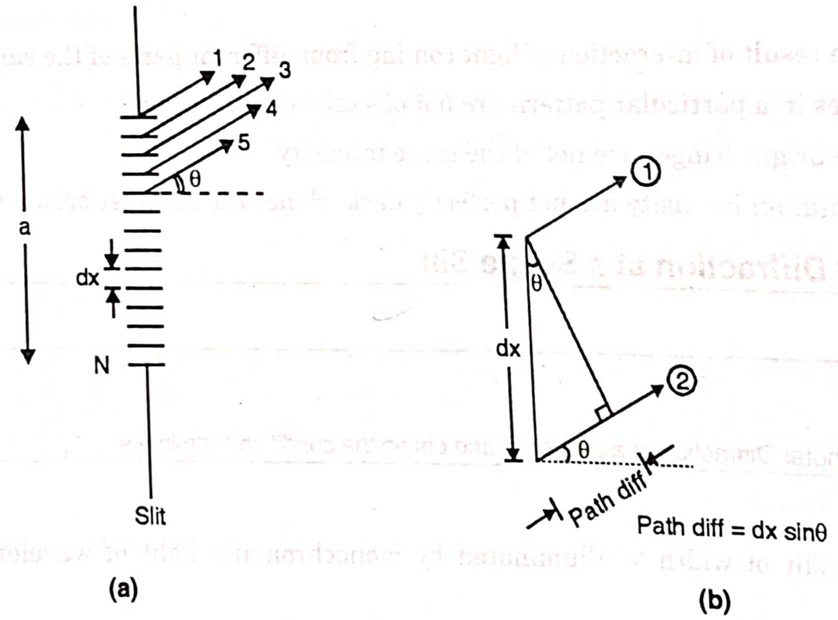



Fig. 1.4.2

- The path difference between the rays diffracted from its upper and lower edges is $dx \cdot \sin\theta$ and hen the corresponding phase difference between them is, (refer Fig. 1.4.2(b)).

$$\Delta\phi = \frac{2\pi}{\lambda} dx \sin\theta \qquad ...(1.4.2)$$

Because

| Path difference | Phase difference |
|---|---|
| $\lambda$ | $2\pi$ |
| $dx\sin\theta$ | $\Delta\phi$ |

$$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda} dx\sin\theta$$

- The total path difference between the rays diffracted from the top and bottom edges of the slit of width 'a' is $a\sin\theta$ and the corresponding phase difference between them is,

$$\phi = \frac{2\pi}{\lambda} \cdot a \cdot \sin\theta \qquad ...(1.4.3)$$

- Each infinitesimal slit acts as a radiator of Huygen's wavelets and produces a characteristic wave disturbance at point P on the screen.
- This disturbance we represent by a quantity called as phasor. (Concept of phasor is useful when we are dealing in that area where not only the magnitude but also the phase difference is significant.)
- To find out the total amplitude of disturbance at point P, we have to consider N phasors corresponding to N parallel infinitesimal slits.
- Thus at point P, N phasors with same amplitude, same frequency and same phase difference $\Delta\phi$ between the adjacent members combine to produce the resultant disturbance. Fig. 1.4.2 shows the vector addition of such N phasors.
- The resultant disturbance at P is represented by the arc AB of a circle with radius R.
- Let C be the centre of arc AB.

$$\therefore \quad AC = BC = R$$

**Step III**

- Join the chord AB and draw CD perpendicular to AB.
- $\phi$ is the phase difference between the infinitesimal vectors at the ends of the arc AB i.e. it is the phase difference between the rays coming from the top and bottom edges of the slit of width 'a'.

$$\therefore \quad \angle ACB = \phi$$

- Let us represent arc AB = $E_m$ and chord AB = $E_\theta$

  where $E_m \rightarrow$ Amplitude at the centre and $E_\theta \rightarrow$ resultant amplitude at P.
- From the geometry of Fig. 1.4.3, we have,

$$AD = DB$$

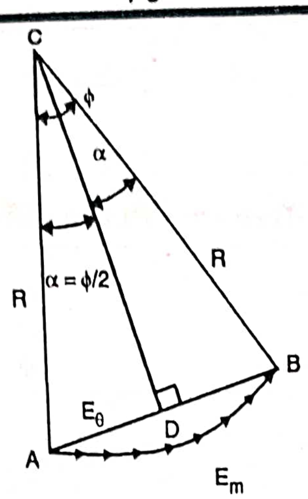


Fig. 1.4.3 : Phasor diagram

$$\angle ACD = \angle BCD$$

$$= \frac{\phi}{2} = \frac{\pi}{\lambda} \cdot a \sin\theta = \alpha \qquad \text{...(1.4.4)}$$

$$\therefore \quad AD = \frac{E_\theta}{2} = R \cdot \sin\frac{\phi}{2}$$

$$\therefore \quad E_\theta = 2R \cdot \sin\frac{\phi}{2} \qquad \text{...(1.4.5)}$$

Using the relation, $\text{angle} = \frac{\text{arc}}{\text{radius}}$, we have,

$$\phi = \frac{E_m}{R}$$

$$\therefore R = \frac{E_m}{\phi}$$

Therefore Equation (1.4.5) gives,

$$E_\theta = 2 \cdot \frac{E_m}{\phi} \cdot \sin\frac{\phi}{2} = E_m \frac{\sin \phi/2}{\phi/2}$$

$$\therefore \quad \mathbf{E_\theta = E_m \cdot \frac{\sin\alpha}{\alpha}} \qquad \text{...(1.4.6)}$$

- This gives the amplitude of wave disturbance at point P where the rays diffracted at angle θ meet.
- If $I_\theta \rightarrow$ Resultant intensity of light at P and $I_m \rightarrow$ Maximum intensity at $P_o$ then we have $I_\theta \propto E_\theta^2$ and $I_m \propto E_m^2$
- Hence squaring Equation (1.4.6) gives,

$$I_\theta = I_m \cdot \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{...(1.4.7)}$$

**Step IV : Principal Maximum**

- For $E_\theta$ to be maximum; all the phasors must be in phase i.e. α i.e. α = 0

$$\therefore \quad \alpha = \frac{\pi}{\lambda} \cdot a \sin\theta = 0$$

$$\Rightarrow \sin\theta = 0 \text{ or } \theta = 0$$

- Thus the maximum value of $E_\theta$ is $E_m$ and this principal maximum is formed at $\theta = 0$. i.e. this maximum is formed by the parts of the secondary wavelets which travel normally to the slit. Hence it is produced at $P_o$.

**Step V : Minimum Intensity**

- The intensity at point P will be zero when $\sin \alpha = 0$.
- The values of $\alpha$ which satisfy this condition are

$$\alpha = \pm m \pi \qquad \text{where} \quad m = 1, 2, 3, 4, .....$$

- Thus the condition for minimum intensity at point P is

$$\alpha = \frac{\pi}{\lambda} \cdot a \sin \theta = \pm m \pi$$

$$\text{or } a \sin \theta = \pm m \lambda$$

$$\text{where} \quad m = 1, 2, 3, .... \qquad ...(1.4.8)$$

(m = 0 is not possible because then $\theta$ becomes zero which corresponds to principal maximum).

- Thus Equation (1.4.8) shows that we have points of minimum intensity on either side of the principal maximum (as there is ± sign on RHS).

**Step VI : Secondary Maxima**

- In addition to the principal maximum at $\theta = 0$, there are weak secondary maxima on either side of it. They lie approximately halfway between the two minima.
- Hence the secondary maxima can be obtained for

$$\alpha = \pm \left(m + \frac{1}{2}\right) \pi \qquad \text{where} \quad m = 1, 2, 3, .....$$

- Putting this condition in Equation (1.4.7) we have the relative intensity of secondary maxima as

$$\frac{I_\theta}{I_m} = \left[\frac{\sin\left(m + \frac{1}{2}\right)\pi}{\left(m + \frac{1}{2}\right)\pi}\right]^2$$

Putting m = 1, 2, 3, we have

$$\frac{I_\theta}{I_m} = 0.045; 0.016; 0.008; .....$$

- Thus the successive maxima decrease in intensity rapidly.
- The relative intensity distribution in single slit diffraction pattern is shown in Fig. 1.4.4.

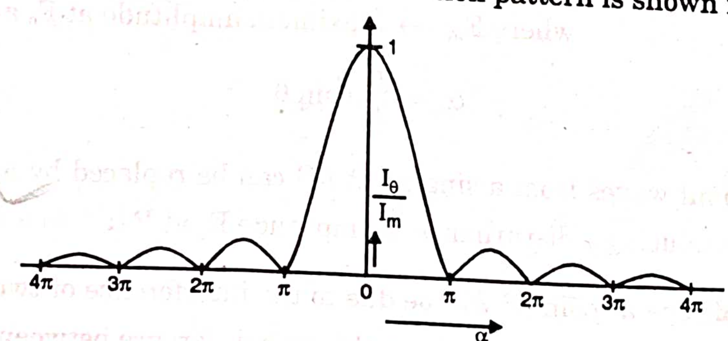



Fig. 1.4.4 : Relative intensity distribution in single slit diffraction

## 1.5 Fraunhofer Diffraction at a Double Slit

**MU - May 2019**

**University Question**

**Q.** Explain Fraunhofer's double slit diffraction experiment and obtain expression for resultant intensity of the light on the screen and derive the formula for missing orders in the double slit diffraction pattern. **(May 19, 4 Marks)**

- Consider a beam of monochromatic light of wavelength $\lambda$ incident normally on two narrow slits AB and CD as shown in Fig. 1.5.1. Let 'a' be the width of each slit and 'b' be the width of opaque space BC separating the two slits such that a and b are comparable.
- The diffracted light is focussed on the screen by a convex lens L.
- The diffraction pattern is found to consist of equally spaced interference maxima and minima in the region originally occupied by the central maximum in the single slit Fraunhofer diffraction pattern.
- The central interference maximum possesses maximum intensity while the maxima on either side of it are of gradually decreasing intensity.
- In the region originally occupied by the secondary maxima of single slit diffraction pattern, faint interference maxima and minima are observed.

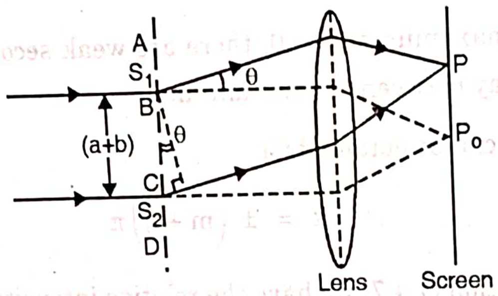



**Fig. 1.5.1 : Fraunhofer diffraction at double slit**

**Explanation**

- When the plane wavefront reaches the plane of the slits, each point in the slit (AB and CD) sends secondary wavelets in all directions.
- From the theory of diffraction at a single slit, we see that the resultant amplitude due to all wavelets diffracted from each slit at an angle $\theta$ with the normal is given by,

$$E_\theta = E_m \frac{\sin \alpha}{\alpha} \quad \text{...(1.5.1)}$$

where $E_m \rightarrow$ Maximum amplitude at $P_o$ and

$$\alpha = \frac{\pi}{\lambda} a \sin \theta \quad \text{...(1.5.2)}$$

- This disturbance due to all waves from a single slit AB can be replaced by a single wave starting from midpoint $S_1$ of AB and producing a disturbance of amplitude $E_\theta$ at P (i.e. in a direction $\theta$ to the normal).
- Thus the resultant amplitude at point P will be due to the interference of two waves of same amplitude $E_\theta$ and having a phase difference that depends on the path difference between the two.

- From Fig. 1.5.1, the path difference between the waves from $S_1$ and $S_2$ at P is,

$$S_2K_1 = S_1S_2 \sin\theta = (a+b)\sin\theta$$

$\therefore$ The corresponding phase difference is

$$\frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta \text{ (say)} \quad \text{...(1.5.3)}$$

- The resultant amplitude at P can be calculated by the method of vector addition and intensity at P is found to be proportional to square of amplitude.

**Vector addition method**

- The resultant amplitude at P can be obtained from Fig. 1.5.2.
- QR and RS represent amplitude of two waves originating from $S_1$ and $S_2$ and angle $\phi$ as phase difference between them.
- From Fig. 1.5.2, we can write,

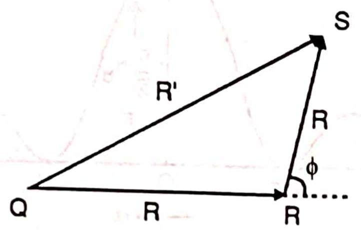



**Fig. 1.5.2**

(Using formula $|a+b| = \sqrt{a^2+b^2+2ab\cos\phi}$ )

$$QS^2 = QR^2 + RS^2 + 2\,(QR)\,(RS)\cos\phi$$

$$R'^2 = R^2 + R^2 + 2R \cdot R \cdot \cos\phi$$

$$= 2R^2(1+\cos\phi)$$

$$R'^2 = 4R^2\cos^2\phi/2$$

Substitute the following values from theory of single slit

$$R = A\frac{\sin\alpha}{\alpha} \text{ and}$$

$$\alpha = \frac{\phi}{2}$$

$$\frac{\pi(a+b)\sin\theta}{\lambda} = \beta$$

Hence,

$$R'^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

Hence, the resultant intensity at the point P is given by,

$$I_\theta = 4E_m^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta \quad \text{...(1.5.4)}$$

$$= I_m \frac{\sin^2 \alpha}{\alpha^2} \cos^2 \beta$$

$$\text{Where,} \quad A^2 = 4 E_m^2 = \text{constant} = I_m$$

- The term $\frac{I_m \cdot \sin^2 \alpha}{\alpha^2}$ gives diffraction pattern like that of a single slit. **Hence it is called diffractio component.** The factor $\cos^2\beta$ gives the interference pattern due to light waves of same amplitude fro the two slits. **Hence it is called interference component.**
- It is observed that in case of **single slit diffraction,** the factor $\frac{A^2 \sin^2 \alpha}{\alpha^2}$ gives a principal maximum centre i.e. $\theta = 0$.
- On either side of this principal maximum alternate minima and secondary maxima of diminishi intensity are observed as shown in Fig. 1.5.3(a).

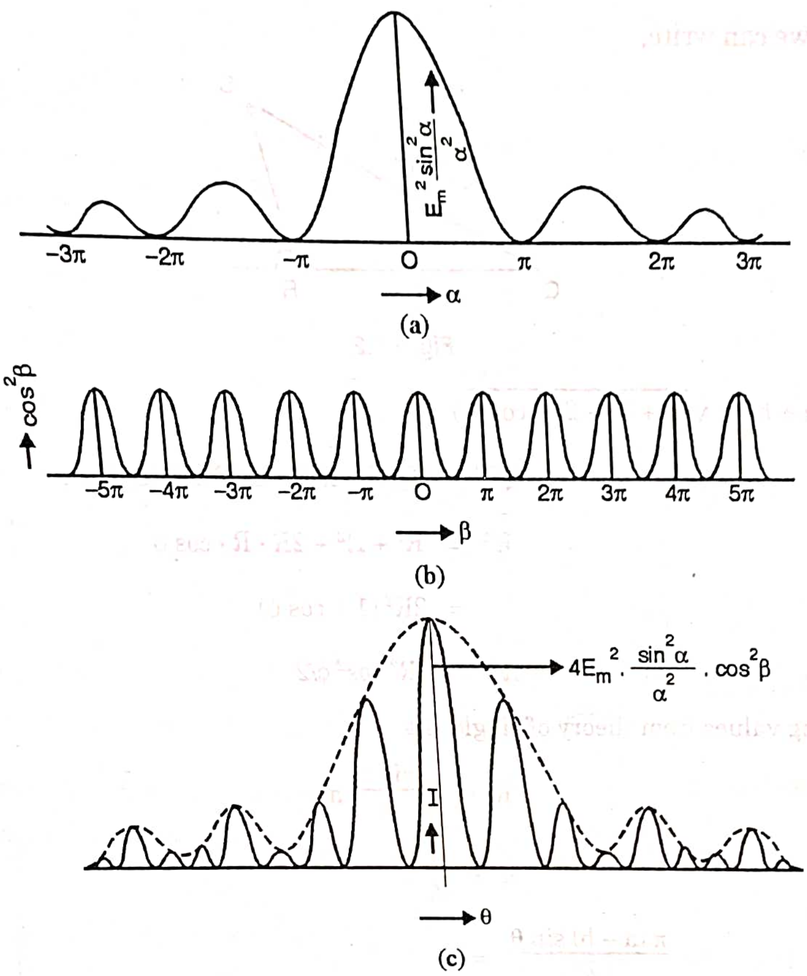



**Fig. 1.5.3**

- The angular positions of minima (for single slit) are given by,

$$\sin \alpha = 0, \qquad \text{but } \alpha \neq 0 \text{ i.e}$$

We can write,

$$\alpha = \pm m\pi, \qquad \text{where } m = 1, 2, 3, \ldots$$

$$\Rightarrow \quad \frac{\pi}{\lambda} a \sin\theta = \pm m\pi$$

$$\Rightarrow \quad a \sin\theta = m\lambda \qquad ...(1.5.5)$$

- The positions of secondary maxima for single slit due to this term ($\alpha$) may be written as

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \pm .... \text{etc.}$$

- According to the interference term, $\cos^2\beta$, the intensity will be maximum for double slit when,

$$\cos^2\beta = 1$$

i.e. bright fringes are obtained in the directions given by,

$$\beta = \pm n\pi \qquad \text{where } n = 0, 1, 2, 3, ...$$

- From Equation (1.5.3), we can write,

$$\frac{\pi(a+b)\sin\theta}{\lambda} = \pm n\pi$$

$$\Rightarrow (a+b)\sin\theta = \pm n\lambda$$

when $n = 0$, $\theta = 0$, i.e. **central maximum of interference pattern is observed along the direction of incident light.** The various maxima corresponding to $n = 0, 1, 2, 3, ...$ are the zero-order, first order, second order maxima ...

- As the central maximum of diffraction pattern due to single slit also lies in the same direction, hence, **the intensity of this central maximum is highest.**
- The intensity of interference pattern **due to double slit** will be minimum, when,

$$\cos^2\beta = 0 \qquad \text{i.e.} \quad \beta = \pm(2n+1)\pi/2$$

$$\Rightarrow \frac{\pi(a+b)\sin\theta}{\lambda} = (2n+1)\pi/2$$

$$(a+b)\sin\theta = \pm(2n+1)\lambda/2$$

where $\quad n = 0, 1, 2, 3, ...$

- The above equation represents that for interference minima the path difference between the parallel diffracted rays originating from any pair of corresponding points within the two slits be odd multiple of $\lambda/2$.
- It can be proved that for small values of $\theta$, the maxima and minima are equally spaced. So, $\cos^2\beta$ term in Equation (1.5.4) is responsible for interference fringes.
- If the width of slit 'a' kept constant and 'b' is varied, the position of maxima and minima due to diffraction remain unaffected while those due to interference undergo a change.

- The variation of the diffraction term $E_m^2 \frac{\sin^2 \alpha}{\alpha^2}$ with $\alpha$ is shown in Fig. 1.5.3(a). The variation of interference term $\cos^2 \beta$ with $\beta$ shown in Fig. 1.5.3(b). The resultant intensity distribution due to double slit is shown in Fig. 1.5.3(c).

**Conclusions**

The entire pattern due to double slit may be considered as consisting of interference fringes due to light from both slits. The intensities of these fringes being governed by diffraction occurring at the individual slit.

**Effect of increasing the number of slits**

- Instead of two slits, if we increase the number of slits to 3, 5, 6, ... to a large value, each slit being of same width 'a' and for equal separation 'b' between them, the diffraction patterns in each case can be obtained.
- It is found that with the increase in the number of slits, the narrowing of interference maxima takes place.
- With more number of slits, the sharpness of the principal maxima increases.
- **Effect of increasing the slit separation 'b' :** Keeping the slit width a constant, if the slit separation 'b' is increased, then the fringe spacing decreases and the fringes become closer together. Hence, more interference maxima lies within the central maximum.
- **Effect of increasing the wavelength 'λ' :** If the wavelength of the monochromatic light incident on the slit increases, the field of view becomes broader and the fringes move further apart.
- **Effect of increasing the slit width 'a' :** When the width of the slit 'a' is increased, the envelope of the fringe-pattern changes and central peak becomes sharper. The fringe spacing remains unaffected because it depends on slit separation 'b', which is constant in this case. So, the number of interference maxima lies within the central diffraction maximum decreases.

## 1.6 Missing Orders in a Double Slit Diffraction Pattern

**MU - May 2019**

**University Question**

**Q.** Explain Fraunhofer's double slit diffraction experiment and obtain expression for resultant intensity of the light on the screen and derive the formula for missing orders in the double slit diffraction pattern. **(May 19, 7 Marks)**

- On keeping the slit width 'a' constant, if the slit spacing 'b' is changed, the fringe width of interference maxima changes. On varying b, some orders of interference maxima are missing. The missing orders in double slit diffraction pattern depend upon the relative values of a and b.
- The direction of interference maxima is given by,

$$(a + b) \sin \theta = n\lambda \qquad ...(1.6.1)$$

And the direction of diffraction minima is given by,

$$a \sin \theta = m\lambda \qquad ...(1.6.2)$$

- For the same angle of θ, the value of a and b satisfies the Equations (1.6.2) and (1.6.1) simultaneously. Then the position of interference maxima and diffraction minima are same.
- Dividing Equation (1.6.1) by Equation (1.6.2), we get,

$$\frac{(a+b)\sin\theta}{a\sin\theta} = \frac{n\lambda}{m\lambda} \quad ...(1.6.3)$$

$$\frac{a+b}{a} = \frac{n}{m} \quad ...(1.6.4)$$

**Condition 1 :** If a = b, then,

$$\frac{n}{m} = 2$$

$$n = 2m$$

If m = 1, 2, 3, ..., then n = 2, 4, 6, ...

So, second, fourth, sixth orders of interference maxima are missing in diffraction pattern, because these maxima will coincide with $1^{st}$, $2^{nd}$, $3^{rd}$ ... order diffraction minima due to single slit.

**Condition 2 :** If 2a = b, then,

$$\frac{n}{m} = 3$$

$$n = 3m$$

If m = 1, 2, 3, ... then, n = 3, 6, 9, ...

So, $3^{rd}$, $6^{th}$, $9^{th}$ orders of interference maxima are missing in the diffraction pattern, because these maxima will coincide with $1^{st}$, $2^{nd}$, $3^{rd}$ ... order of diffraction minima.

**Condition 3 :** If a + b = a, i.e. b = 0

The two slits are joined and all orders of interference maxima are missing and the diffraction pattern obtained is similar to that of single slit of width 2a.

---

**Ex. 1.6.1 :** In a double slit Fraunhofer diffraction pattern the screen is 170 cm away from the slits. The slit widths are 0.08 mm and they are 0.4 mm apart. Calculate the wavelength of light if the fringe spacing is 0.25 cm. Also deduce the missing order.

**Soln. :** The fringe spacing in the case of interference pattern written as,

$$\beta = \frac{D\lambda}{d} \Rightarrow \lambda = \frac{\beta d}{D}$$

Where d is the separation between the slits.

**Given :** d = 0.04 cm, β = 0.25 cm, d = 170 cm

$$\lambda = \frac{0.04 \times 0.25}{170} = 5.88 \times 10^{-5} \text{ cm} = 5880 \text{ A}^\circ$$

The condition of missing order, is

$$\frac{a+b}{a} = \frac{n}{m}.$$

**Given :** b = 0.40 mm, a = 0.08 mm

$$\text{So, } \frac{0.08 + 0.40}{0.08} = \frac{n}{m}$$

$$n = 6\,m, \; m = 1, 2, 3, \ldots$$

$$\text{or, } n = 6, 12, 18, \ldots$$

**Hence, $6^{th}$, $12^{th}$ and $18^{th}$ orders are missing.** **...Ans.**

## 1.7 Fraunhofer Diffraction due to N Parallel Equidistant Slits Theory of Diffraction Grating

**MU - May 2012, May 2014, Dec. 2014, May 2017, May 2018**

**University Questions**

**Q.** Explain how the number of lines on grating decides the maximum number of orders of diffraction ? **(May 12, 5 Marks)**

**Q.** Derive condition for maximum diffraction at diffraction grating. **(May 14, 5 Marks)**

**Q.** What is grating ? **(Dec. 14, May 17, 1.5 Marks)**

**Q.** For plane transmission grating prove that $d \sin \theta = n\lambda$, $n = 1, 2, 3\ldots$ **(Dec. 14, 5 Marks)**

**Q.** What is diffraction grating ? What is the advantage of increasing the number of lines in the grating ? **(May 18, 3 Marks)**

- It is an arrangement consisting of a large number of parallel slits of same width and separated by equal opaque spaces, is called **grating.**
- It is obtained by ruling equidistant parallel lines on a glass plate with the help of a fine diamond point.
- The lines act as opaque spaces and the incident light cannot pass through them. The space between the two lines is transparent to light and acts as a slit. The number of lines in a plane transmission grating is of the order of 15,000 to 20,000 per inch.
- The grating was first devised by Fraunhofer.
- The spacing between the lines is of the order of wavelength of visible light. Hence an appreciable deviation of light is produced. For practical purposes, replicas of original grating are prepared.

### 1.7.1 Theory of Plane Transmission Grating

**MU - Dec. 2014, Dec. 2016, May 2017**

**University Question**

**Q.** What is grating element ? **(Dec. 14, Dec. 16, May 17, 2 Marks)**

**Step I**

- Consider a plane transmission grating placed perpendicular to the plane of the paper as shown in Fig. 1.7.1.
- Let N be the number of parallel slits each of width 'a' and separated by opaque space 'b'.
- Then the distance between the centres of the adjacent slits is $d = (a + b)$ and is known as the **grating element.**
- Let a plane wavefront of monochromatic light of wavelength $\lambda$ be incident normally on the grating.

- When the incident wavefront is at the plane of the slits, every point in each slit acts as a source of secondary wavelets which spread out in all directions.
- The secondary wavelets travelling in the same direction as that of the incident light are brought to focus at point $P_o$ on the screen by the lens L.

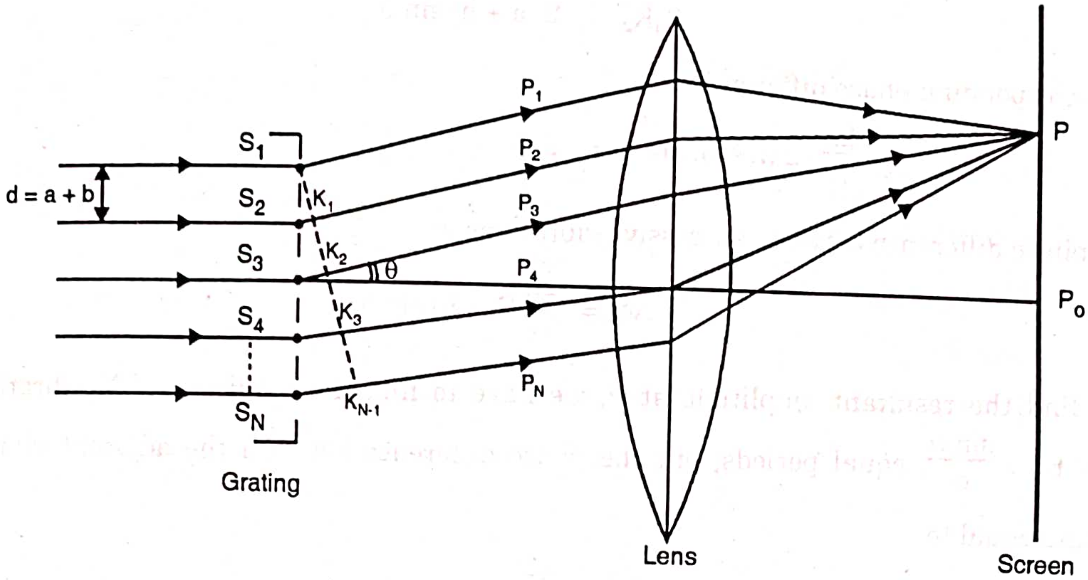



Fig. 1.7.1 : Plane Diffraction Grating

- The screen is placed at the focal plane of the lens.
- The secondary waves travelling in the direction θ with the incident light are focussed at point P on the screen.
- We have to find the resultant intensity of light at P.

**Step II**

- We have seen that the resultant amplitude of light from a single slit of width 'a' in a direction making an angle θ with the normal is given by,

$$E_\theta = E_m \cdot \frac{\sin \alpha}{\alpha} \quad ...(1.7.1)$$

$$\text{Where} \quad \alpha = \frac{\pi}{\lambda} \cdot a \sin \theta \quad ...(1.7.2)$$

$E_m \rightarrow$ Maximum amplitude

- All the secondary wavelets in each slit can be replaced by a single wave of amplitude $E_m \cdot \frac{\sin \alpha}{\alpha}$ starting from the mid - point of the slit and travelling at an angle θ with the normal.
- Let $S_1, S_2, S_3, .....S_N$ be the midpoints of the N number of slits in the grating.
- We want to find the resultant effect of these N vibrations at P.

**Step III**

- From $S_1$, draw $S_1K_{N-1}$, perpendicular on the parallel paths of the diffracted rays.
- Then the path difference between $S_1P_1$ and $S_2P_2$ is given by,

$$S_2K_1 = (a + b) \sin \theta$$

and the corresponding phase difference is,

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot (a + b) \sin \theta$$

– Similarly, the path difference between $S_3P_3$ and $S_1P_1$ is given by,

$$S_3K_2 = 2 (a + b) \sin \theta$$

and the corresponding phase difference is,

$$\frac{2\pi}{\lambda} \cdot 2 (a + b) \sin \theta = 2\Delta\phi$$

– Thus the phase difference between successive vibrations is

$$\Delta\phi = \frac{2\pi}{\lambda} (a + b) \sin \theta$$

– Hence to find the resultant amplitude at P, we have to find the resultant of N vibrations of equal amplitude $E_m \cdot \frac{\sin \alpha}{\alpha}$, equal periods; but the phase difference between the adjacent vibrations being constant and equal to

$$\phi = \frac{2\pi}{\lambda} (a + b) \sin \theta = 2\beta \qquad ...(1.7.3)$$

**Step IV**

– The resultant amplitude of these N waves can be found out by vector addition method and is given by

$$E_\theta = E_m \frac{\sin \alpha}{\alpha} \frac{\sin N\beta}{\sin\beta} \qquad ...(1.7.4)$$

– The corresponding resultant intensity of light at P is given by,

$$I_\theta = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2 \frac{\sin^2 N\beta}{\sin^2\beta} \qquad ...(1.7.5)$$

– The first factor $I_m \left(\frac{\sin \alpha}{\alpha}\right)^2$ in Equation (1.7.5) gives the intensity distribution in the diffraction pattern due to single slit.

– The second factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives intensity pattern due to N slits.

– Reader can try this take N = 1

$$I_\theta = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2$$

– This is the formula for single slit Equation (1.4.7)

– For N = 2

$$I_\theta = 4E_m^2 \frac{\sin^2 \alpha}{\alpha^2} \cos^2 \beta \qquad (\text{As } \sin 2\beta = 2\cos\beta.\sin\beta)$$

This the formula for double slit (Refer Equation (1.4.7))

- Thus every slit gives rise to a diffracted beam whose intensity depends upon the slit width and these diffracted beams then interfere to produce the final diffraction pattern. This is shown in Fig. 1.7.2.

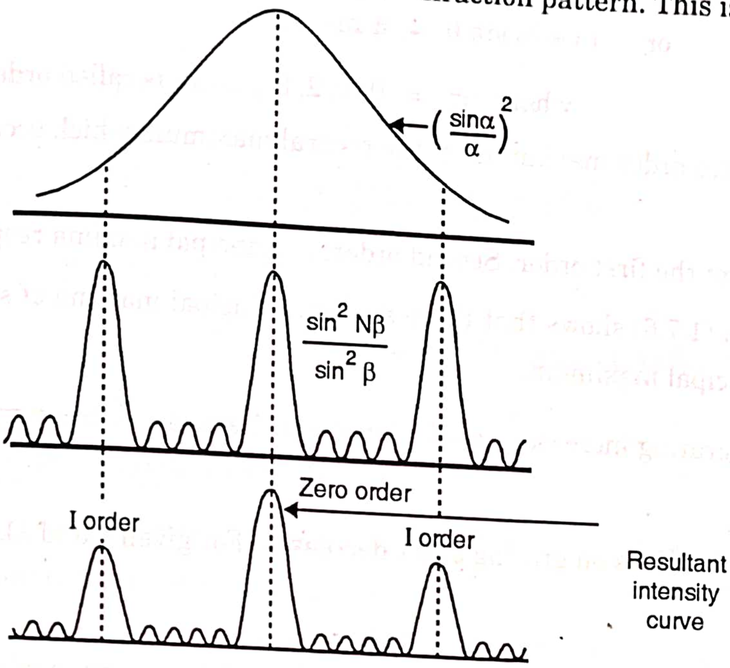



Fig. 1.7.2

**Step V : Principal maxima**

- The resultant intensity of light at P is given by Equation (1.7.5).
- The intensity is maximum when

$$\sin \beta = 0$$

or

$$\beta = \pm m\pi \qquad \text{where } m = 0, 1, 2, 3, \ldots$$

- But for these values of β, the term $\frac{\sin N\beta}{\sin\beta}$ in Equation (1.7.5) becomes indeterminate. We find its value as,

$$\lim_{\beta \to \pm m\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm m\pi} \frac{d(\sin N\beta)}{d(\sin\beta)}$$

$$= \lim_{\beta \to \pm m\pi} \frac{N\cos N\beta}{\cos\beta}$$

$$= \frac{N\cos N \cdot m\pi}{\cos m\pi} = N$$

$$\therefore \lim_{\beta \to \pm m\pi} \left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2$$

- Thus for $\beta = \pm m\pi$ the intensity of the principal maxima is,

$$I_\theta = N^2 \cdot I_m \left(\frac{\sin\alpha}{\alpha}\right)^2$$

which increases with increasing N.

- The intensity of the central principal maximum is greatest while on either side of it, the intensities of other maxima go on decreasing.

Thus the condition for principal maxima is $\beta = \pm m\pi$ or $\frac{\pi}{\lambda}(a + b) \sin \theta = \pm m\pi$

$$\text{or} \quad (a + b) \sin \theta = \pm m\lambda \quad \ldots \qquad \ldots(1.7.6)$$

where $m = 0, 1, 2, 3, \ldots$..m is called order number.

$m = 0$ corresponds to zero order maximum i.e. the central maximum which occurs at $P_o$ i.e. in a direction $\theta = 0$.

- For m = 1, 2, 3, ... we get the first order. Second order, ... principal maxima respectively.
- The ± sign in Equation (1.7.6) shows that there are two principal maxima of same order lying on either side of the central principal maximum.
- As number of lines on grating increases, (a + b) decreases. Because $a + b = \frac{1}{\text{Number of lines per cm}}$

  Also $(a + b) \sin \theta = m\lambda$.

∴ With increase in number of lines on grating order decreases (For given θ and λ).

**Step VI : Minima**

- From Equation (1.7.5) it is seen that the intensity of light at P is minimum when

$$\sin N\beta = 0 \qquad \text{but } \sin \beta \neq 0.$$

$$N\beta = \pm m\pi$$

where m can have all integral values except 0, N, 2N, .....nN; because for these values of m, sin β becomes zero and we get principal maxima.

- Hence putting for β from Equation (1.7.3) we have the condition for minima as

$$N\frac{\pi}{\lambda}(a + b) \sin \theta = \pm m\pi$$

$$\text{or} \quad N(a + b) \sin \theta = \pm m\lambda \qquad \ldots..(1.7.7)$$

where m can have all integral values except 0, N, 2N, .....nN.

**Important Observation**

- **Effect of width of the ruled surface :** When the width of the ruled surface, i.e., N (a + b) increases the width of principal maxima decreases i.e. it becomes sharper.
- **Effect of closeness of ruling :** When (a + b) is small, i.e., the lines on the grating are close together the dispersive power will be large i.e., the angular spacing between maxima increases.
- **Effect of increasing the number of rulings on a grating :** When the number of rulings in a grating increases, the principal maxima become intense and sharp i.e., secondary maxima becomes weaker.

**Ex. 1.7.1 :** Monochromatic light of wavelength 6560 Å falls normally on a grating 2 cm. wide. The first order spectrum produced at an angle of 18° 14′ from the normal. Calculate the total number of lines on the grating.

**Soln. :**

**Given :** $\lambda = 6560$ Å $= 6560 \times 10^{-8}$ cm, $m = 1$, $\theta = 18° 14'$, Width $W = 2$ cm

**Formula :** $(a + b) \sin \theta = m\lambda$

$$\therefore (a+b) = \frac{m\lambda}{\sin\theta} = \frac{1 \times 6560 \times 10^{-8}}{\sin(18^\circ\ 14')}$$

$$= 2.097 \times 10^{-4} \text{ cm}$$

$$\therefore \text{Number of lines per cm} = \frac{1}{a+b}$$

$$= \frac{1}{2.097 \times 10^{-4}} = 4770$$

$\therefore$ Total number of lines on the grating.

$$= \text{Number of lines/cm} \times \text{width of grating}$$

$$= 4770 \times 2 = 9540$$

...Ans.

## 1.8 Resolving Power of an Optical Instrument

- When the two objects are very near to each other and they are at very large distance from our eye, the eye may not be able to see them as separate.
- If we want to see them as separate and optical instruments such as telescope, microscope (for close objects) and instruments like prism, grating etc. for spectral lines are employed.
- When light from an object passes through an optical instrument we obtain a diffraction pattern with a bright central maximum and the other secondary maxima having minima in between.
- An optical instrument is said to be able to resolve two point objects if the corresponding diffraction patterns are distinguishable from each other.
- The ability of the instrument to produce just separate diffraction patterns of two close objects is known as its **resolving power.**

## 1.9 Rayleigh's Criterion of Resolution

**MU - May 2014, May 2015, Dec. 2016, Dec. 2017**

**University Question**

**Q.** What is Rayleigh's criteria of resolution? **(May 14, May 15, Dec. 16, Dec. 17, 2 Marks)**

- According to Rayleigh's criterion, two closely spaced point sources of light are said to be just resolved by an optical instrument only if the central maximum in the diffraction pattern of one falls over the first minimum in the diffraction pattern of the other and vice versa.
- In order to illustrate the criterion consider the resolution of two wavelengths $\lambda_1$ and $\lambda_2$ by an optical instrument. Fig. 1.9.1 shows the intensity curves of the diffraction patterns of the two wavelengths.
- As shown in Fig. 1.9.1(a) the principal maxima of the two wavelengths are widely separated. Hence the two wavelengths are well resolved.


Fig. 1.9.1(a) : Objects Well Resolved

- Fig. 1.9.1(b) shows the intensity curves of two wavelengths whose difference is such that the princip maximum of one coincides with the first minimum of the other and vice versa.
- The resultant intensity curve shows a distinct dip in the middle indicating the presence of tw wavelengths. This is the limiting case when the two wavelengths are just resolved.

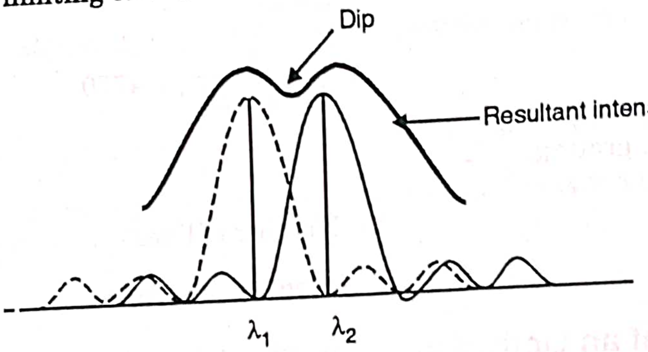



Fig. 1.9.1(b) : Objects Just Resolved

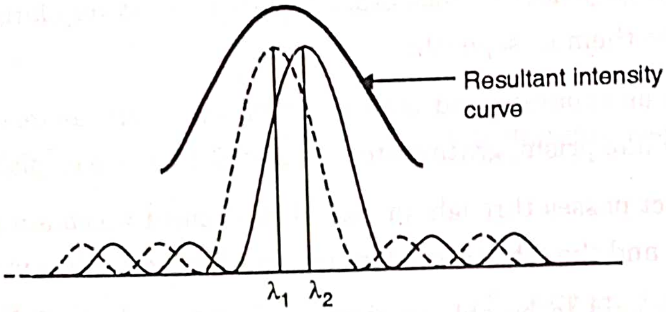



Fig. 1.9.1(c) : Objects not Resolved

- It however; the two wavelengths are still closer, their principal maxima will be still nearer as showr Fig. 1.9.1(c).
- The resultant intensity curve shows only one maximum at the centre. The two wavelengths cannot seen as separate and hence are not resolved.
- Thus the spectral lines can be resolved only upto the limit expressed Rayleigh's Criterion.

## 1.10 Resolving Power of a Grating

**MU - Dec. 2013, May 2014, May 2015, Dec. 2**

**University Questions**

**Q.** How do you increase the resolving power of a diffraction grating. **(Dec. 13, 3 Mar**

**Q.** What is resolving power of diffraction grating ? **(May 14, May 15, Dec. 17, 2 Mar**

- The resolving power of a grating is defined as the ratio of the wavelength of any spectral line to difference in the wavelength between this line and a neighbouring line such that the two lines appea be just resolved.

$$\therefore \quad R.P = \frac{\lambda}{d\lambda}$$

where $\lambda \rightarrow$ wavelength of a line

and $\lambda + d\lambda \rightarrow$ wavelength of the next line that can just be seen as separate.

**Step I**

- Consider a grating on which light consisting of two close wavelengths $\lambda$ and $(\lambda + d\lambda)$ is incident normally.
- Let $(a + b) \rightarrow$ grating element and $N \rightarrow$ Total number of lines on the grating.
- These two wavelengths will be just resolved if the principal maximum of $(\lambda + d\lambda)$ falls over the first minimum of $\lambda$.
- As per the Rayleigh's criterion, the first minimum of $\lambda$ adjacent of $m^{th}$ principal maximum should be along $(\theta_m + d\,\theta_m)$ for just resolution.

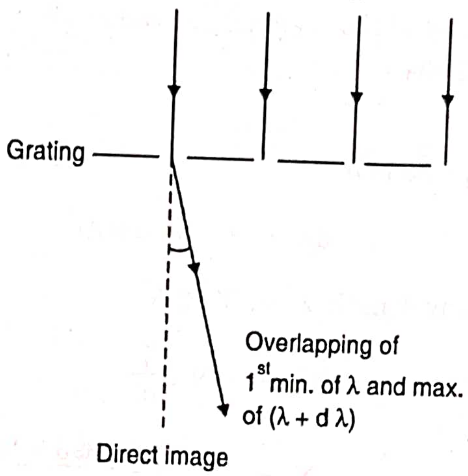



Fig. 1.10.1

**Step II**

- The $m^{th}$ principal maximum of $(\lambda + d\lambda)$ along $(\theta_m + d\,\theta_m)$ is given by the equation.

$$(a + b) \sin (\theta_m + d\,\theta_m) = m (\lambda + d\lambda) \qquad \text{...(1.10.1)}$$

- The equation for minima for $\lambda$ is

$$N (a + b) \sin \theta_m = m\lambda \qquad \text{.....(1.10.2)}$$

where m has all integral values except 0, N, 2N, 3N, .....nN.

- Thus the first minimum of $\lambda$ adjacent to $m^{th}$ principal maximum can occur along $(\theta_m + d\,\theta_m)$ by putting m as $(m \cdot N + 1)$ in Equation (1.10.2).
- Hence the equation for first minimum of $\lambda$ in the direction $(\theta_m + d\,\theta_m)$ will be given by,

$$N (a + b) \sin (\theta_m + d\,\theta_m) = (m \cdot N + 1) \cdot \lambda \qquad \text{...(1.10.3)}$$

**Step III**

- Multiply Equation (1.10.1) by N

$$\therefore\ N (a + b) \sin (\theta_m + d\,\theta_m) = m \cdot N (\lambda + d\lambda) \qquad \text{...(1.10.4)}$$

- Therefore from Equations (1.10.3) and (1.10.4) we have,

$$(m \cdot N + 1) \cdot \lambda = m \cdot N (\lambda + d\lambda)$$

$$\therefore\ \lambda = m \cdot N \cdot d\lambda$$

$$\therefore \quad \frac{\lambda}{d\lambda} = m \cdot N$$

$$\therefore \quad \text{R.P. of grating} = \frac{\lambda}{d\lambda} = m \cdot N \quad \text{.....(1.10.5)}$$

Thus from Equation (1.10.5) we see that :

(i) The R.P. of grating increases with the order of spectrum.

(ii) The R.P. increases with the increase in total number of lines on the grating.

(iii) The R.P. is independent of the grating element.

**Ex. 1.10.1 :** Calculate the minimum number of lines in a grating which will just resolve in the first order the lines whose wavelengths are 5890 Å and 5896 Å .

**MU - Dec. 2014, 5 Marks**

**Soln. :**

**Given :** $m = 1, \quad \lambda_1 = 5890\ \text{Å}, \quad \lambda_2 = 5896\ \text{Å}$

$$\therefore \quad d\lambda = \lambda_2 - \lambda_1 = 6\ \text{Å}$$

$$\therefore \text{Mean wavelength } \lambda = 5893\ \text{Å}$$

**Formula :**

$$\text{R.P.} = mN = \frac{\lambda}{d\lambda}$$

$$N = \frac{\lambda}{d\lambda} \cdot \frac{1}{m} = \frac{5893 \times 10^{-10}}{6 \times 10^{-10}} \times \frac{1}{1} = 983$$

...Ans.

## 1.11 Application of Diffraction, Determination of Wavelength of Light

**MU - Dec. 2012, May 2016, May 2017, Dec. 2017, Dec. 20**

**University Question**

**Q.** Explain the experimental method to determine the wavelength of spectral line using diffraction grating ?

**(Dec. 12, May 16, May 17, Dec. 17, Dec. 18, 5 Mark**

- The diffraction grating is often used in the laboratories for the determination of wavelength of light.
- The grating spectrum of the given source of monochromatic light is obtained by using a spectromet The arrangement is as shown in Fig. 1.11.1.

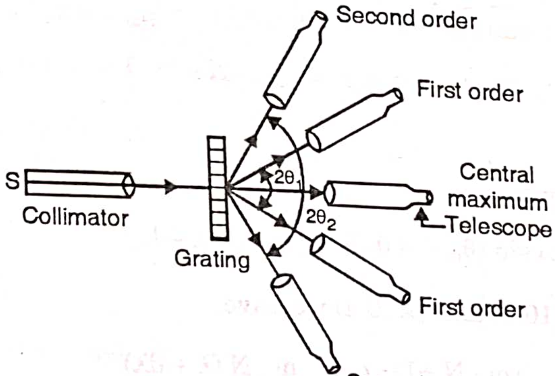



Fig. 1.11.1

- The spectrometer is first adjusted for parallel rays. The grating is then placed on the prism table and adjusted for normal incidence.
- In the same direction as that of the incident light, the direct image of the slit or the zero order spectrum can be seen in the telescope.
- On either side of this direct image a symmetrical diffraction pattern consisting of different orders can be seen.
- The angle of diffraction θ for a particular order m of the spectrum is measured.
- The number of lines per inch of grating are written over it by the manufacturers.
- Hence the grating element is,

$$(a+b) = \frac{1}{\text{Number of lines/cm}} = \frac{2.54}{\text{Number of lines/inch}}$$

- Thus using the equation

$$(a+b)\sin\theta = m\lambda$$

- The unknown wavelength λ can be calculated by putting the values of the grating element (a + b), the order m and the angle of diffraction θ.

## 1.12 Solved Problems

### Problems on Single Slit

**Ex. 1.12.1 :** A slit of width 'a' is illuminated by white light. For what value of 'a' will the first minimum for red light fall at an angle 30° ? Wavelength of red light is 6500 Å.

**Soln. :**

**Given :** $\theta = 30°$, $m = 1$, $\lambda = 6500 \times 10^{-8}$ cm

**To find :** a = ?

**Formula :** For minima in single slit diffraction pattern.

$$a \sin\theta = m\lambda$$

$$\text{For first minimum, } m = 1$$

$$\therefore \quad a = \frac{\lambda}{\sin\theta} = \frac{6500 \times 10^{-8}}{\sin 30°} = \frac{6500 \times 10^{-8}}{0.5}$$

$$\therefore \quad a = 1.3 \times 10^{4} \times 10^{-8} \text{ cm}$$

$$\therefore \quad a = \mathbf{1.3 \times 10^{-4}\ cm.}$$

**...Ans.**

**Ex. 1.12.2 :** Find the half angular width of the central maximum in the Fraunhofer diffraction pattern of a slit of width $12 \times 10^{-5}$ cm, when illuminated by light of wavelength 6000 Å.

**Soln. :**

**Given :** $a = 12 \times 10^{-5}$ cm,

$\lambda = 6000$ Å $= 6000 \times 10^{-8}$ cm

**Formula :** Half angular width of the first maxima is the angle made by the first minima with the normal to the slit.