What is a Hypothesis?

- A hypothesis is a claim (assertion) about a population parameter:

  - population mean

Example: The mean monthly cell phone bill in this city is μ = Rs 42

  - population proportion

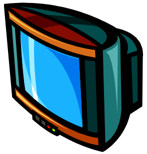
Example: The proportion of adults in this city with cell phones is $\pi = 0.68$

The Null Hypothesis, H_0

- States the claim or assertion to be tested

Example: The average number of TV sets in Indian Homes is equal to three ($H_0 : \mu = 3$)

- Is always about a population parameter, not about a sample statistic



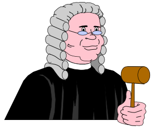
$$H_0 : \mu = 3$$

$$\cancel{H_0 : \bar{X} = 3}$$

The Null Hypothesis, H_0

(continued)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty



- **Refers to the status quo or historical value**
- Always contains "=" , "≤" or "≥" sign
- May or may not be rejected

The Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis

- e.g., The average number of TV sets in Indian homes is not equal to 3($H_1: \mu \neq 3$)

- **Challenges the status quo**

- May or may not be proven

- **Is generally the hypothesis that the <u>researcher is trying to prove</u>**

Ex: H0: $\mu = 100$ (the null hypothesis is that the population mean is 100)

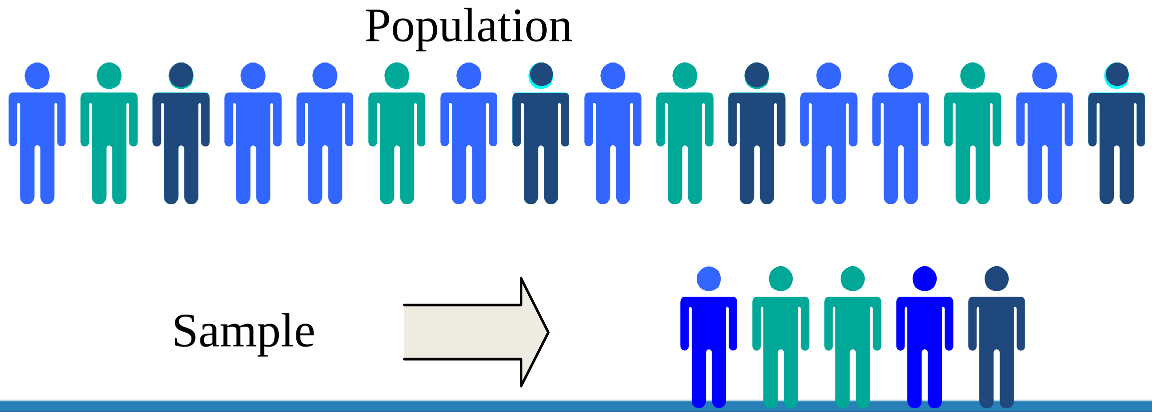
H1: $\mu \neq 100$

H1: $\mu > 100$

H1: $\mu < 100$

The Hypothesis Testing Process

- Claim: The population mean age is 50.
  - $H_0: \mu = 50, \quad H_1: \mu \neq 50$
- Sample the population and find sample mean.



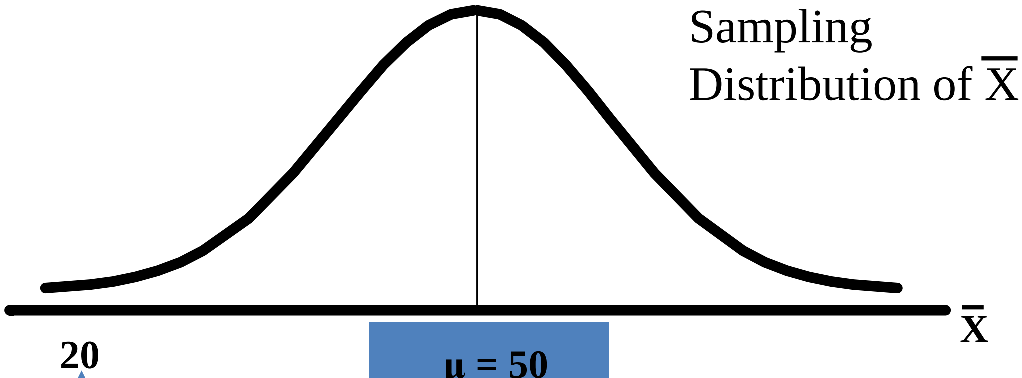
The Hypothesis Testing Process

(continued)

- Suppose the sample mean age was $\overline{X} = 20$.
- This is significantly lower than the claimed mean population age of 50.
- If the null hypothesis were true, the probability of getting such a different sample mean would be very small, so you reject the null hypothesis .
- In other words, getting a sample mean of 20 is so unlikely if the population mean was 50, you conclude that the population mean must not be 50.

The Hypothesis Testing Process

(continued)



μ = 50
If H_0 is true

If it is unlikely that you would get a sample mean of 20.

When in fact 50 was the population mean…

... then you reject the null hypothesis that μ = 50.

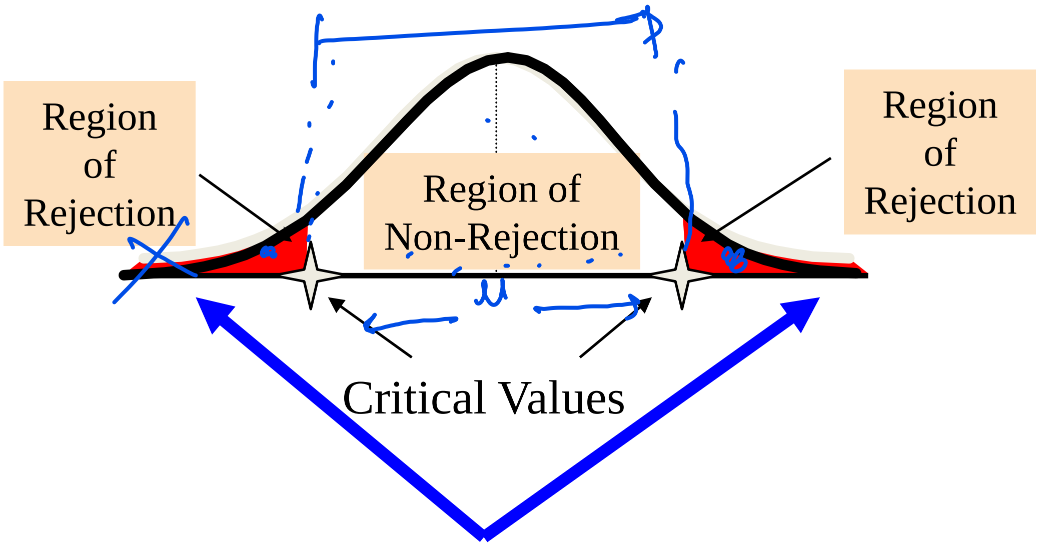
The Test Statistic and Critical Values

$\mu = 50$ $\bar{x} = 70$ $\bar{x} = 45$ $\bar{x} = 35$ ±5 ±2

- If the **sample mean** is **close** to the assumed **population mean**, the null hypothesis is **not rejected**.
- If the sample mean is **far from** the assumed population mean, the null hypothesis is rejected.
- **How far is "far enough" to reject H_0?**
- The critical value of a test statistic creates a "line in the sand" for decision making - it answers the question of **how far is far enough**.

The Test Statistic and Critical Values

Sampling Distribution of the test statistic



"Too Far Away" From Mean of Sampling Distribution

Possible Errors in Hypothesis Testing

Type I and Type II Errors:

Because the hypothesis testing process uses **sample statistics** calculated from random data to reach conclusions about population parameters, it is possible to make an incorrect decision about the null hypothesis.

In particular, two types of errors can be made in testing hypotheses: Type I error and Type II error.

A **Type I error** is committed by ***<u>rejecting a true null hypothesis</u>***. With a Type I error, the null hypothesis is true, but the business researcher decides that it is not.

$n = 100$

As an example, suppose the flour-packaging process actually is **"in control"** and is **averaging 40 ounces** of flour per package. Suppose also that a business researcher randomly selects 100 packages, weighs the contents of each, and computes a sample mean. It is possible, **by chance**, to randomly select 100 of the more extreme packages (mostly heavy weighted or mostly light weighted) resulting in **a mean** that falls in the **rejection region**. **<u>The decision is to reject the null hypothesis even though the population mean is actually 40 ounces.</u>** In this case, the business researcher has **committed a Type I error**.

For example, if a **manager fires** an employee because some evidence indicates that she is stealing from the company and if she really is not stealing from the company, then the manager has committed a **Type I error**.

accepted

Ho = not true . we have

As another example, suppose a worker on the assembly line of a large manufacturer hears an **unusual sound** and decides to **shut the line down** (reject the null hypothesis). If the sound turns **out not to be** related to the assembly line and no problems are occurring with the assembly line, then the worker has committed a Type I error.

$\alpha \leftarrow$

The probability of committing a Type I error is called **alpha (α)** or level of significance. Alpha equals the **area** under the curve that is in the **rejection region** beyond the critical value(s).

A **Type II error** is committed when a business researcher ***fails to reject a false null hypothesis***. In this case, the null hypothesis is false, but a decision is made to not reject it.

Suppose in the case of the flour problem that the packaging process is actually **producing a population mean of 41** ounces even though the null hypothesis is 40 ounces. A **sample of 100** packages yields a sample mean **of 40.2 ounces**, **which falls in the non-rejection region.**

The business decision maker decides not to reject the null hypothesis. A Type II error has been committed. The packaging procedure is **out of control** and the hypothesis testing process **does not identify** it.

Suppose in the business world an employee is stealing from the company. A manager sees some evidence that the stealing is occurring but lacks enough evidence to conclude that the employee is stealing from the company.

The manager decides not to fire the employee based on theft. The manager has committed a Type II error.

Consider the manufacturing line with the noise. Suppose the worker decides not enough noise is heard to shut the line down, but in actuality, one of the cords on the line is unraveling, creating a dangerous situation. The worker is committing a Type II error

Which is better : TI or TII??

Possible Errors in Hypothesis Testing

- **Type I Error**
  - **Reject a true null hypothesis**
  - Considered a serious type of error
  - The probability of a Type I Error is α
    - Called level of significance of the test
    - Set by researcher in advance
- **Type II Error**
  - **Failure to reject false null hypothesis**
  - The probability of a Type II Error is β

Possible Errors in Hypothesis Test Decision Making *(continued)*

Possible Hypothesis Test Outcomes		
	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	No Error Probability 1 - α	Type II Error Probability β
Reject H_0	Type I Error Probability α	No Error Probability 1 - β

Power, which is equal to 1 - α, is *the probability of a statistical test rejecting the null hypothesis when the null hypothesis is false.* Table shows the relationship between α , β, and power.

Possible Results in Hypothesis Test Decision Making

H_0 ✓ ✗

(continued)

- The confidence coefficient (1-α) is the probability of not rejecting H_0 when it is true.

- The confidence level of a hypothesis test is (1-α)*100%.

- The power of a statistical test (1-β) is the probability of rejecting H_0 when it is false.

Type I & II Error Relationship

Type I and Type II errors cannot happen at the same time

- A Type I error can only occur if H_0 is true
- A Type II error can only occur if H_0 is false

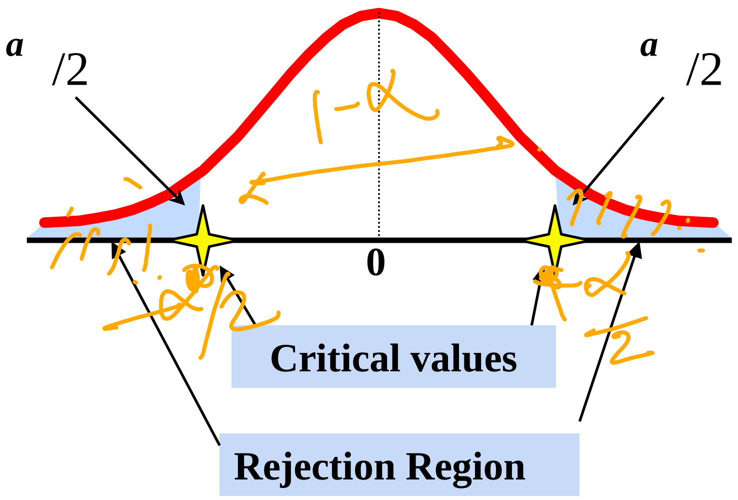
mutually exclusive

Level of Significance and the Rejection Region

H_0: $\mu = 3$ H_1: $\mu \neq 3$

Level of significance = α

α type I error



$1-\alpha = \omega$

two tail test

Le it has 2 regions.

This is a two-tail test because there is a rejection region in both tails

Hypothesis Tests for the Mean

Hypothesis Tests for μ

- **σ Known (Z test)**
- **σ Unknown (t test)**

Z Test of Hypothesis for the Mean (σ Known)

- Convert sample statistic ($\bar{X}$) to a Z_{STAT} test statistic

→ table

Hypothesis Tests for μ

σ Known (Z test)

σ Unknown (t test)

The test statistic is:

$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

error →

$\bar{x}$ = Sample mean

μ = population mean

Critical Value Approach to Testing

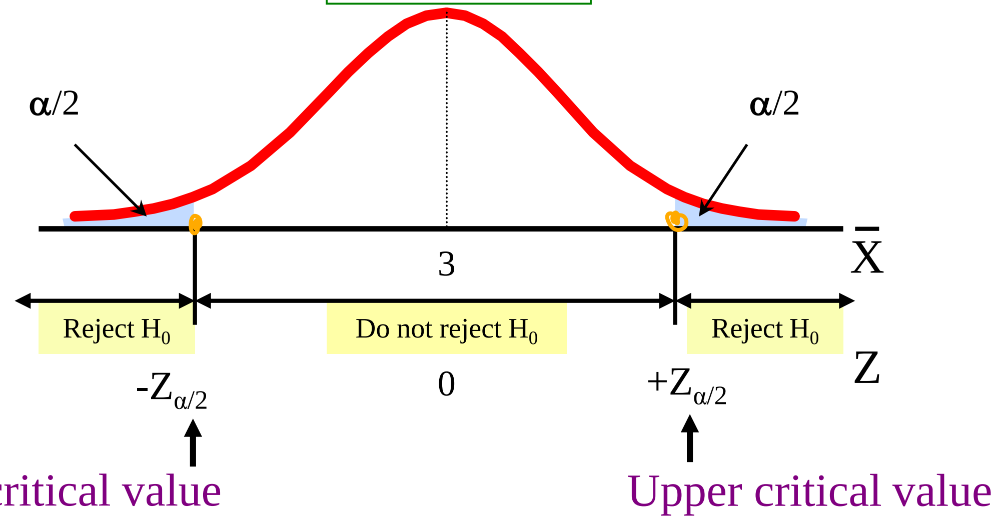
- For a two-tail test for the mean, σ known:
- Convert sample statistic ($\bar{x}$) to test statistic (Z_{STAT})
- Determine **the critical Z** values for a specified level of significance α from **a table** or computer
- Decision Rule: If the test statistic falls in the rejection region, reject H_0 ; otherwise do not reject H_0

Two-Tail Tests

- There are two cutoff values (critical values), defining the regions of rejection

$H_0: \mu = 3$

$H_1: \mu \neq 3$

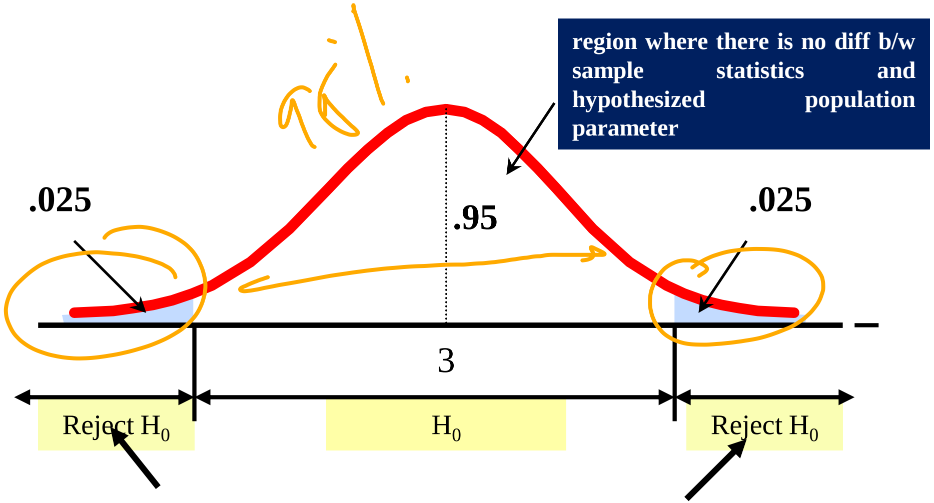


Steps in Hypothesis Testing

1. State the null hypothesis, H_0 and the alternative hypothesis, H_1
2. Choose the level of significance, α, and the sample size.

5% ← Rejection

What if we test a hypothesis at 5 % level of significance? This means that we will reject the null hypothesis if the difference between the sample statistics and the hypothesized population parameters so large that it or a larger difference would occur, on the average, only five or fewer times in every 100 samples when hypothesized population parameters is correct.



95%

$\frac{5}{2} = 2.5\%$

0.025

Selecting a significance level : Generally at .01,.05, .10 or 99,95,90%: The higher the significance level we use for testing a hypothesis, the <u>higher the probability of rejecting a null hypothesis when it is true.</u>

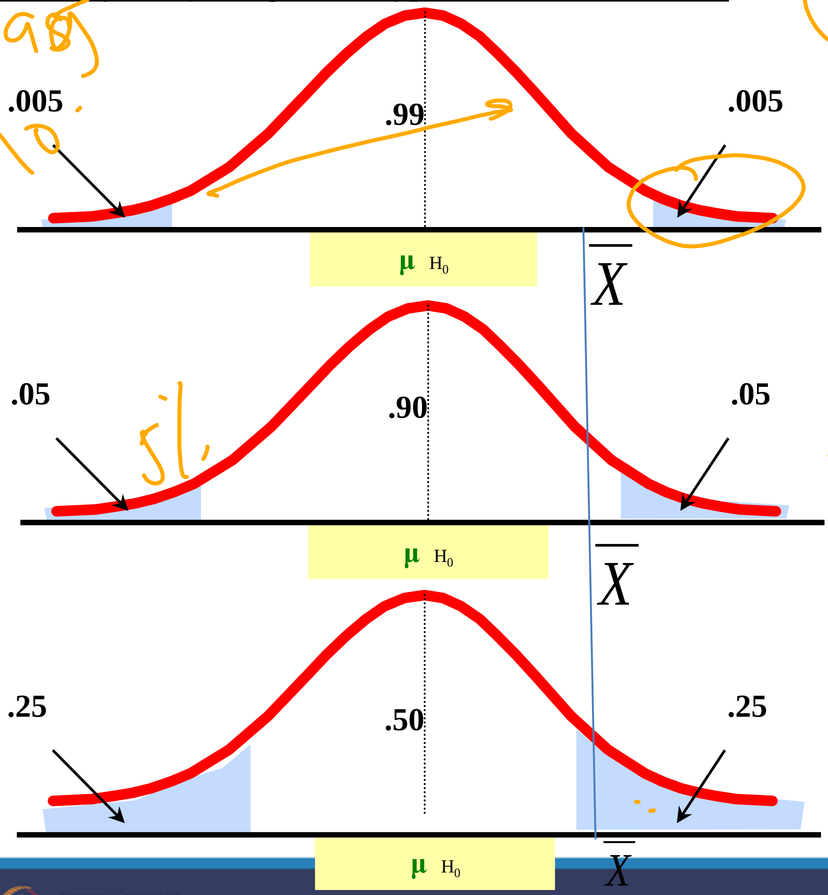
$1-\alpha = 99$

$1-\alpha = 90$

$\frac{1}{2} = 0.5\%$

0.01, 0.05, 0.10

0.005



Significance level of .01

10

5%

Significance level of .10

Significance level of .50

Steps in Hypothesis Testing

3. Determine the appropriate **test statistic** and sampling distribution: t or z test

4. Determine the **critical values** that divide the rejection and nonrejection regions

- In two tail – we have two rejection regions, it is appropriate **when null hypothesis is** $\mu = \mu H_0$ **and the alternate hypothesis** $\mu \neq \mu H_0$.

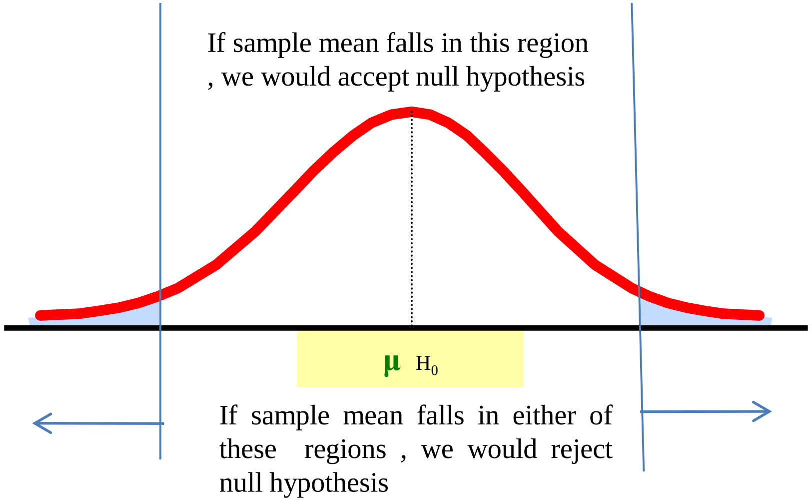
One tail $>$ $<$

$\mu \neq$ two tail

Let the mean life of bulb $\mu = \mu H_o = 1000$ (NH)

$\mu H1 \neq 1000$ (AH)

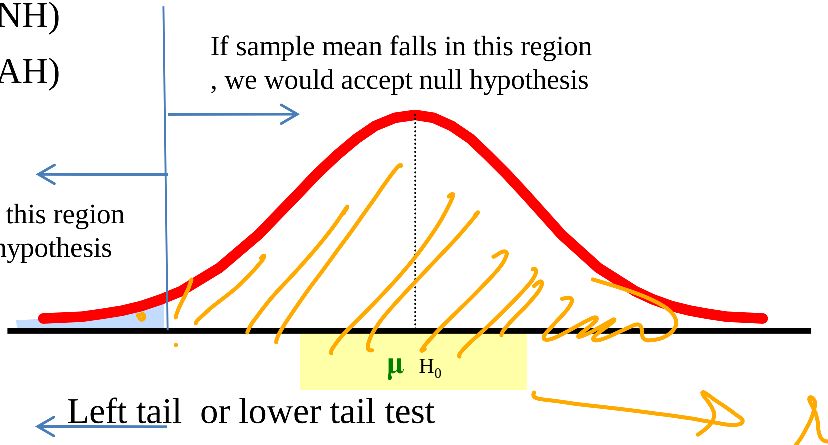
$\bar{x} = 500$



One tail test- A wholesaler that buys bulb would not accept if life is less than 1000.

Ho: $\mu = 1000$ (NH)

H1: $\mu < 1000$ (AH)



greater than

right side

$\mu >$

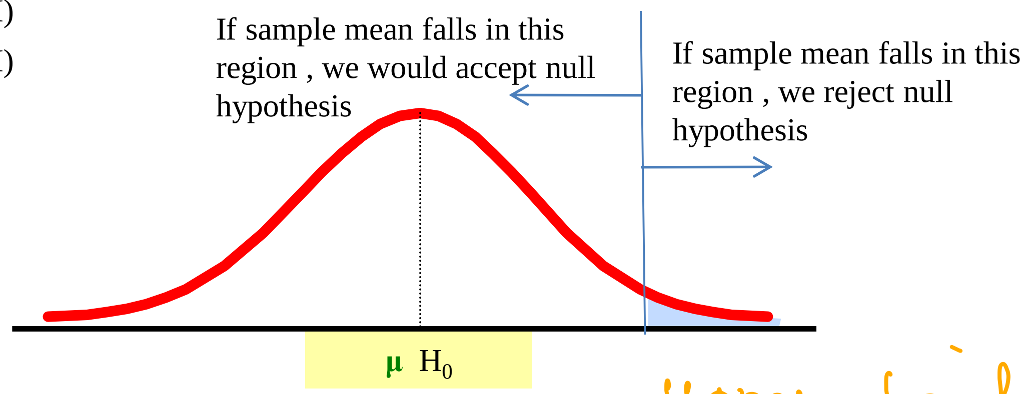
$\mu <$

left side

One tail test- Monthly expenditure should be kept at 100 on an average.

Ho: $\mu = 100$ (NH)

H1: $\mu > 100$ (AH)



upper tail

one tail