

Exam3 : April 28.

Last lecture: Mon. April 24.

Knapsack.

Input : n items : $1, 2, 3, \dots, n$.

item i

- wt w_i

- profit P_i

Knapsack of capacity W .

Objective : To pack items in the knapsack

so that the total wt of all items in the knapsack is at most W and the total profit is maximized.

Scratch work.

$$\text{OPT}(n) = \text{OPT}(n-1) \quad , \quad \text{if } n\text{th item is not in the knapsack.}$$

~~$$= \text{OPT}[n-1] + p_n, \text{ if } n^{\text{th}} \text{ item is in the knapsack.}$$~~

$$\text{OPT}[n] = \max \{ \text{OPT}[n-1], \text{OPT}[n-1] + p_n \}$$

Subproblems

$P[j, C]$: maximum profit obtained by considering items $1, 2, \dots, j$ to be packed in a knapsack of capacity C .

Our answer : $P[n, W]$

Recurrence

$$P[j, C] = \begin{cases} 0, & \text{if } j=0 \text{ or } C=0 \\ \max \begin{cases} P[j-1, C], \\ P[j-1, C-w_j] + p_j, \end{cases} & \text{otherwise} \end{cases}$$

We have to fill up the table of

Size $O(nW)$.

	0	1	C		W	
0	0	0	0	0	0	0
1	C					
	0		
j	0					
	0					
n	0					

Diagram illustrating a table structure for dynamic programming. The table has rows indexed 0 to n and columns indexed 0 to W. The first row (i=0) is filled with 0s. The first column (j=0) is filled with 0s. The cell at (j, C) is highlighted with a blue box and a blue arrow pointing to it from the cell at (j, C-1). The cell at (n, W) is highlighted with a pink box and a blue arrow pointing to it from the cell at (n, W-1).

for $i \leftarrow 0$ to n do
 $P[i, 0] \leftarrow 0$

for $C \leftarrow 0$ to W do
 $P[0, C] \leftarrow 0$

for each $j \leftarrow 1$ to n do

for each $C \leftarrow 1$ to W do

if $(wt(j) \leq C)$ then

$\rightarrow P(j, C) \leftarrow \max \{ P(j-1, C),$
 $P(j-1, C - w_j)$

$+ p_j \}$
else $P(j, C) \leftarrow P(j-1, C)$

return $P(n, W)$

Running time: $O(nW)$

Items Knapsack (n, P)

$S \leftarrow \emptyset$

$i \leftarrow n$, $C \leftarrow W$.

while $i > 0$ and $C > 0$ do
if $P(i, C) > P(i-1, C)$ then
add i to S .
 $C \leftarrow C - w_i$
 $i \leftarrow i-1$

else.

$i \leftarrow i-1$

Example :

item	wt	Profit
1	<u>3</u>	12
2	<u>1</u>	5

3		2		10
4		1		2

$$W = 4.$$

$$P(j, C) = \begin{cases} 0, & \text{if } j = 0 \text{ or } C = 0 \\ \max \{ P(j-1, C), \\ P(j-1, C - w_j) + P_j \} \end{cases}$$

	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	12	12
2	0	5	5	12	17
3	0	5	10	15	17
4	0	5	10	15	17

$S = \{2, 1\}$

$$P[1,1] = P[0,1]$$

$$P[1,2] = P[0,2]$$

$$P[1,3] = \max \{ P[0,3], \\ P[0,0] + 12 \}$$

$$= \max \{ 0, 12 \}$$

$$P[1,4] = \max \{ P[0,4], \\ P[0,1] + 12 \}$$

$$P(2,1) = \max \{ P(1,1), \\ P(\underline{1,0}) + 5 \}$$

$$= \max \{ 0, 5 \}$$

$$P(2,2) = \max \{ P(1,2), \\ P(1,1) + \underline{5} \}$$

$$P(2,3) = \max \{ P(1,3), \\ P(1,2) + 5 \}$$

$$P(2,4) = \max \{ P(1,4), \\ P(1,3) + 5 \}$$

$$P(3,1) = P(2,1) = 5$$

$$P(3,2) = \max \{ P(2,2), \\ P(2,0) + 10 \} \\ = \max \{ 5, 10 \} = \boxed{10}.$$

$$P(3,3) = \max \{ P(2,3), \\ P(2,1) + 10 \}$$

$$= \max \{ 12, 15 \}$$

$$= 15$$

$$P[3,4] = \max \{ P[2,4], \\ P[2,2] + 10 \}$$

$$= \max \{ 17, 15 \}$$

$$= 17$$

$$\begin{aligned}
 P(4,1) &= \max \{ P(3,1), \\
 &\quad P(3,0) + 2 \} \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 P(4,2) &= \max \{ P(3,2), \\
 &\quad P(3,1) + 2 \\
 &= \max \{ 10, 7 \} \\
 &= 10
 \end{aligned}$$

$$P(4,3) = \max \{ P(3,3), \\ P(3,2) + 2 \}$$

$$= \max \{ 15, 12 \}$$

$$P(4,4) = \max \{ P(3,4), \\ P(3,3) + 2 \}$$

$$= \max \{ 17, 17 \}$$

Longest Common Subsequence.

Input: Two sequences : X & Y .

$$X = \langle x_1, x_2, \dots, x_n \rangle$$

$$Y = \langle y_1, y_2, \dots, y_m \rangle$$

Obj: To find the longest common subsequence of X, Y .

