Basics of Matrices

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MSE Module 1 → Linear algebra — Matrices & vectors

ESE Module 2 → Probability & Statistics

MSE Module 3 → Introduction to graphs → Types of data, Types of plots

ESE Module 4 → Exploratory Data analytics

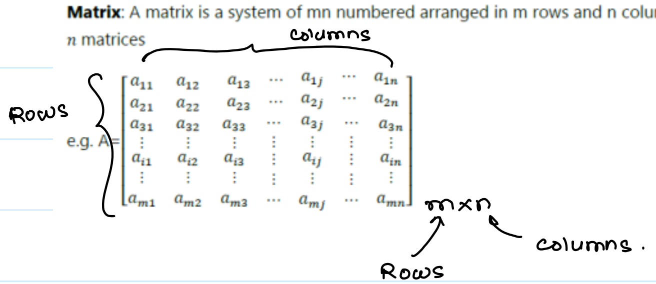
ESE Module 5 → optimization Techniques. (methods)

ESE Module 6 → Dimension reduction Algorithms.

Module1: Linear Algebra

Basics of matrices

Matrix: A matrix is a system of mn numbered arranged in m rows and n columns it is called and m× n matrices



Types of Matrices

Row Matrix ⇒ 1) Matrix which contain only one rows is called row matrix

e.g. A=[2 3 21 63 21] 1×5 1 → row, 5 → col

Column matrix ⇒ 2) Matrix which contain only one column is called column matrix

e.g. A= $\begin{bmatrix} 5 \\ 23 \\ 15 \\ 8 \\ 10 \end{bmatrix}$ 5×1 5 ⇒ Row, 1 ⇒ Col.

3) Matrix in which number of rows and columns are same is called square matrix → 4×4, 2×2, 3×3, ⋮, n×n

e.g. A= $\begin{bmatrix} 4 & 2 & 6 & 8 \\ 8 & 5 & 5 & 9 \\ 4 & 2 & 7 & 8 \\ 5 & 3 & 9 & 8 \end{bmatrix}$ 4×4 Row ⇒ 4, col ⇒ 4

4) Diagonal elements: In square matrix the elements lying along diagonal of a matrix are called diagonal elements

Diagonal element

e.g. A= $\begin{bmatrix} 4 & 2 & 6 & 8 \\ 8 & 5 & 5 & 9 \\ 4 & 2 & 7 & 8 \\ 5 & 3 & 9 & 8 \end{bmatrix}$ 4×4 Here **4, 5, 7, 8** are the diagonal elements

5) Diagonal Matrix: a square matrix in which all the non-diagonal elements are equal to zero are called diagonal matrix.

Diagonal elements

e.g. A=$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}$ Here 4, 5, 7, 8 are the diagonal elements

Non-diagonal elements

6) Scalar matrix: a diagonal matrix whose all diagonal elements are equal is called scalar matrix. e.g.

A=$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ Diagonal element are equal & non-diagonal elements are zero.

7) Unit matrix: a diagonal matrix whose all diagonal elements equal to one is called unit matrix.

I

e.g. A=$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Diagonal elements = 1
Non diagonal elements = 0

8) Trace of a matrix: The sum of all the diagonal elements of a matrix is called trace of a matrix

If A=$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}_{4\times 4}$ then Trace of matrix A=$\sum_{i=1}^{n} a_{ii}=a_{11}+a_{22}+a_{33}+\cdots+a_{nn}$

a_{11}, a_{22}, a_{33}, a_{44}

Trace of matrix $A = a_{11}+a_{22}+a_{33}+a_{44}$
$= 4+5+7+8 = 24$

9) Determinant of a square matrix A is denoted by $|A|$ ← Determinant of matrix A

square matrix

$|\text{matrix}| = 0$

10) Singular matrix: A square matrix A is said to be singular matrix if $|A| = 0$

n×n

e.g. , $C = [0]$, $|C| = 0$,A=$\begin{bmatrix} 3 & 2 \\ 18 & 12 \end{bmatrix}$ then $|A| = 0$, $B = \begin{bmatrix} 1 & 3 & 5 \\ 8 & 4 & 3 \\ 2 & 6 & 10 \end{bmatrix}$ then $|B| = 0$

1×1, 2×2, 3×3

square matrix

So matrices C, A, and B are singular matrices.

11) Non-singular matrix: A square matrix A is said to be non- singular matrix if $|A| \neq 0$

$|A| \neq 0$

n×n

e.g. $A = [5]$, $|A| = 5 \neq 0$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$,$|B| = -1 \neq 0$ $C = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$,$|C| = 27 \neq 0$

1×1, 2×2, 3×3

12) Upper triangular matrix: A matrix $A = [a_{ij}]$ is upper triangular matrix if $a_{ij} = 0$ for $i > j$

$\neq 0$

e.g. . A=$\begin{bmatrix} 4 & 2 & 6 & 8 \\ 0 & 5 & 5 & 9 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 8 \end{bmatrix}$ i.e. all the elements below the diagonal elements are zero.

Lower = 0 Part

Diagonal element

13) Lower triangular matrix: A matrix $A = [a_{ij}]$ is lower triangular matrix if $a_{ij} = 0$ for $i < j$

upper part = 0

. A=$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 8 & 5 & 0 & 0 \\ 4 & 2 & 7 & 0 \\ 5 & 3 & 9 & 8 \end{bmatrix}$ i.e. all the elements above the diagonal elements are zero.

lower Part ≠ 0

Diagonal elements.

14) Transpose of a matrix: Matrix obtained by interchange of rows and column of a matrix A is called Transpose of a matrix and is denoted by A^T

Row, Col

If . A=$\begin{bmatrix} 4 & 2 & 6 & 8 \\ 8 & 5 & 5 & 9 \\ 4 & 2 & 7 & 8 \\ 5 & 3 & 9 & 8 \end{bmatrix}_{4\times 4}$ then A^T= $\begin{bmatrix} 4 & 8 & 4 & 5 \\ 2 & 5 & 2 & 3 \\ 6 & 5 & 7 & 9 \\ 8 & 9 & 8 & 8 \end{bmatrix}$

Transpose of matrix A

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 9 & 11 \end{bmatrix}_{3\times 3}$ $A^T = \begin{bmatrix} 1 & 4 & 8 \\ 2 & 5 & 9 \\ 3 & 6 & 11 \end{bmatrix}_{3\times 3}$

15) Conjugate of a matrix: The matrix obtained from a given matrix by replacing each element by it's complex conjugate is called the conjugate of the given matrix and is denoted by $\bar{A}$

Complex no ⇒ $a + jb$

Real, Img

Thus if $A = \begin{bmatrix} -2+3i & 3-2i & -6i \\ 4i & 1-2i & -5+2i \\ 3 & -4-6i & 8+2i \end{bmatrix}$ then $\bar{A} = \begin{bmatrix} -2-3i & 3+2i & +6i \\ -4i & 1+2i & -5-2i \\ 3 & -4+6i & 8-2i \end{bmatrix}$

Complex conjugate ⇒ $a - jb$

Complex no $(a \pm jb)$

16) Transposed conjugate of a matrix: The transpose of a complex conjugate of a given is called the transposed conjugate of a matrix A and is denoted by A^{θ}

i.e. $A^{\theta} = \bar{A}' = \bar{A}'$

step ② → conjugate

step①

e.g. if if $A = \begin{bmatrix} -2+3i & 3-2i & -6i \\ 4i & 1-2i & -5+2i \\ 3 & -4-6i & 8+2i \end{bmatrix}$ then $\bar{A} = \begin{bmatrix} -2-3i & 3+2i & 6i \\ -4i & 1+2i & -5-2i \\ 3 & -4+6i & 8-2i \end{bmatrix}$

Given matrix

complex conjugate

and $A^{\theta} = \bar{A}' = \begin{bmatrix} -2-3i & -4i & 3 \\ 3+2i & 1+2i & -4+6i \\ 6i & -5-2i & 8-2i \end{bmatrix}$

Step ③ → Transpose

17) Symmetric matrix: A matrix $A = [a_{ij}]$ is said to be symmetric matrix if $a_{ij} = a_{ji}\ \forall\ i, j$

$a_{ij} = a_{ji}$

$a_{12} = a_{21}$

$a_{23} = a_{32}$

Or $A^T = A$

a_{12} a_{13}

$A = \begin{bmatrix} 4 & -2 & 6 & 8 \\ -2 & 5 & -1 & -3 \\ 6 & -1 & 7 & 9 \\ 8 & -3 & 9 & 8 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 5 & -7 \\ 3 & -7 & 8 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$

a_{21}

a_{31}

diagonal element

18)Skew- symmetric matrix: A matrix $A = [a_{ij}]$ is said to be skew-symmetric matrix if $a_{ij} = -a_{ji}\ \forall\ i, j$

Or $A^T = -A$

$a_{12} = -a_{21}$

a_{12} a_{13}

$A = \begin{bmatrix} 0 & 2 & -6 & -8 \\ -2 & 0 & 1 & 3 \\ 6 & -1 & 0 & -9 \\ 8 & -3 & 9 & 0 \end{bmatrix}$ $A = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & 7 \\ 3 & -7 & 0 \end{bmatrix}$, $A = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

$-a_{21}$

a_{31}

Diagonal elements = 0

Note: In skew-symmetric matrix all the diagonal elements equal to zero

complex conjugate

19) Hermitian matrix: Matrix $A = [a_{ij}]$ is said to be Hermitian matrix if $\overline{a_{ij}} = a_{ji}\ \forall\ i, j$

$\overline{a_{12}} = a_{21}$

$\overline{a_{13}} = a_{31}$

Or $A^{\theta} = A$

$\overline{a_{12}}$ $\overline{a_{13}}$ $\overline{a_{12}}$

$A = \begin{bmatrix} 1 & 2+3i & 5+6i \\ 2-3i & 3 & 8-2i \\ 5-6i & 8+2i & 6 \end{bmatrix}$, $A = \begin{bmatrix} 2 & 3-5i \\ 3+5i & 7 \end{bmatrix}$

a_{21} a_{31} a_{21}

Purely Real

Note: In Hermitian matrix all the diagonal elements are purely real.

20) Skew-Hermitian matrix: Matrix $A = [a_{ij}]$ is said to be Skew-Hermitian matrix if

$\overline{a_{ij}} = -a_{ji}\ \forall\ i, j$ Or $A^{\theta} = -A$

$\overline{a_{12}}$

$A = \begin{bmatrix} 0 & 2-3i & 5+6i \\ -2-3i & 3i & 8-2i \\ -5+6i & -8-2i & -6i \end{bmatrix}$, $A = \begin{bmatrix} 2i & 3-5i \\ -3-5i & 0 \end{bmatrix}$

a_{21}

diagonal elements are either 0 or complex no

Theorem: Every matrix A can be expressed as sum of symmetric and skew-symmetric matrices. i.e. $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

Where $\frac{1}{2}(A + A^T)$ *is symmetric matrix, and* $\frac{1}{2}(A - A^T)$ *is Skew − symmetric matrix*

e.g. Let $A = \begin{bmatrix} 6 & 10 & 16 \\ 20 & 26 & 30 \\ 40 & 50 & 60 \end{bmatrix}$ $\therefore$ $A^T = \begin{bmatrix} 6 & 20 & 40 \\ 10 & 26 & 50 \\ 16 & 30 & 60 \end{bmatrix}$

Now $A + A^T = \begin{bmatrix} 12 & 30 & 56 \\ 30 & 52 & 80 \\ 56 & 80 & 120 \end{bmatrix}$, and $A - A^T = \begin{bmatrix} 0 & -10 & -24 \\ 10 & 0 & -20 \\ 24 & 20 & 0 \end{bmatrix}$

$\therefore \frac{1}{2}(A + A^T) = \begin{bmatrix} 6 & 15 & 28 \\ 15 & 26 & 40 \\ 28 & 40 & 60 \end{bmatrix} = B$ *is symmetric, and* $|B| = 0$

$\frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -5 & -12 \\ 5 & 0 & -10 \\ 12 & 10 & 0 \end{bmatrix}$ *is skew − symmetric*

$a_{ij} = -a_{ji}$

Diagonal element = 0

And $\begin{bmatrix} 6 & 10 & 16 \\ 20 & 26 & 30 \\ 40 & 50 & 60 \end{bmatrix} = \begin{bmatrix} 6 & 15 & 28 \\ 15 & 26 & 40 \\ 28 & 40 & 60 \end{bmatrix} + \begin{bmatrix} 0 & -5 & -12 \\ 5 & 0 & -10 \\ 12 & 10 & 0 \end{bmatrix}$ i.e. $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

given matrix

Theorem: Every matrix A can be expressed as sum of Hermitian and skew-Hermitian matrices. i.e. $A = \frac{1}{2}(A + A^\theta) + \frac{1}{2}(A - A^\theta)$

Where $\frac{1}{2}(A + A^\theta)$ is Hermitian *matrix, and* $\frac{1}{2}(A - A^\theta)$ *is Skew* − Hermitian *matrix*

① Complex conjugate

e.g. Let $A = \begin{bmatrix} 2+4i & 4-6i & 6+8i \\ 8-10i & 10+12i & 12-14i \\ 14+16i & 16-18i & 18+20i \end{bmatrix}$ $\therefore$ $\bar{A} = \begin{bmatrix} 2-4i & 4+6i & 6-8i \\ 8+10i & 10-12i & 12+14i \\ 14-16i & 16+18i & 18-20i \end{bmatrix}$

$A^\theta = \bar{A}^T = \begin{bmatrix} 2-4i & 8+10i & 14-16i \\ 4+6i & 10-12i & 16+18i \\ 6-8i & 12+14i & 18-20i \end{bmatrix}$

② Transpose of complex conjugate

Now

$A + A^\theta = \begin{bmatrix} 4 & 12+4i & 20-8i \\ 12-4i & 20 & 28+4i \\ 20+8i & 28-4i & 36 \end{bmatrix}$, and $A - A^\theta = \begin{bmatrix} 8i & -4-16i & -8+24i \\ 4-16i & 24i & -4-32i \\ 8+24i & 4-32i & 40i \end{bmatrix}$

Transpose of cc

Term 1 $\therefore \frac{1}{2}(A + A^\theta) = \begin{bmatrix} 2 & 6+2i & 10-4i \\ 6-2i & 10 & 14+2i \\ 10+4i & 14-2i & 18 \end{bmatrix}$ *is Hermitian matrix, and*

Real no.

Term 2 $\frac{1}{2}(A - A^\theta) = \begin{bmatrix} 4i & -2-8i & -4+12i \\ 2-8i & 12i & -2-16i \\ 4+12i & 2-16i & 20i \end{bmatrix}$ *is skew − Hermitain matrix*

complex no.

And $\begin{bmatrix} 2+4i & 4-6i & 6+8i \\ 8-10i & 10+12i & 12-14i \\ 14+16i & 16-18i & 18+20i \end{bmatrix} = \begin{bmatrix} 2 & 6+2i & 10-4i \\ 6-2i & 10 & 14+2i \\ 10+4i & 14-2i & 18 \end{bmatrix} + \begin{bmatrix} 4i & -2-8i & -4+12i \\ 2-8i & 12i & -2-16i \\ 4+12i & 2-16i & 20i \end{bmatrix}$

given matrix

i.e. $A = \frac{1}{2}(A + A^\theta) + \frac{1}{2}(A - A^\theta)$

Eigen Value and Eigen vector

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Q1. Find eigen value & Eigen vector of matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}_{3\times 3}$

Soln :- Since matrix A is of order 3.

$\therefore$ characteristic equation is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad \text{——} \quad (1)$$

where $S_1 = 1 + 2 + (-1) = 2$

$$S_2 = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix}$$

$$= (-3) + (-1) + (3) = -1$$

$$|A| = \begin{vmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & -1 & 1 \end{vmatrix} = -2$$

Substitute values in eqn (1)

$$\lambda^3 - 2\lambda^2 - 1\lambda + 2 = 0$$

$$\boxed{\begin{aligned} \lambda_1 &= 2 \\ \lambda_2 &= +1 \\ \lambda_3 &= -1 \end{aligned}}$$

To find eigen vectors $(A - \lambda I)x = 0$

$$\begin{bmatrix} 1-\lambda & 1 & -2 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & -1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Case (I) :- $\lambda = \lambda_1 = 2$

$$\begin{bmatrix} -1 & 1 & -2 \\ -1 & 0 & 1 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ 1 & -3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 1 \\ 0 & -3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}}$$

$$\therefore \frac{x_1}{-1} = \frac{-x_2}{3} = \frac{x_3}{-1} = -1$$

$$\therefore x_1 = 1, \quad x_2 = +3, \quad x_3 = 1$$

for $\lambda = \lambda_1 = 2$ Eigen vector $X_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

Case (II) If $\lambda = \lambda_2 = 1$

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 1 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 1 \\ 0 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix}}$$

$$\therefore \quad \frac{x_1}{-3} = \frac{-x_2}{2} = \frac{x_3}{-1} = \lambda = -1$$

$$x_1 = 3, \quad x_2 = 2, \quad x_3 = 1.$$

Thus for eigen value $\lambda = \lambda_2 = 1$, Eigen vector $x_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Case ③ If $\lambda = \lambda_3 = -1$.

$$\begin{bmatrix} 2 & 1 & -2 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 3 \\ 0 & 1 \end{vmatrix}}$$

$$\frac{x_1}{-1} = \frac{-x_2}{0} = \frac{x_3}{-1} = \lambda = -1$$

$$\therefore \quad x_1 = 1, \quad x_2 = 0, \quad x_3 = 1$$

$\therefore$ Thus for eigen value $\lambda = \lambda_3 = -1$ eigen vector $x_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

② Find Eigen value and Eigen vector of matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}_{3\times 3}$

Solution

$\because$ A is a square matrix of order 3

$\therefore$ The characteristic equation of 3X3 matrix.

$$\lambda^3 - s_1\lambda^2 + s_2\lambda - |A| = 0.$$

$$s_1 = 8 + (-3) + 1 = 6$$

$$s_2 = \begin{vmatrix} -3 & -2 \\ -4 & 1 \end{vmatrix} + \begin{vmatrix} 8 & -2 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 8 & -8 \\ 4 & -3 \end{vmatrix}$$

$$s_2 = (-3-6) + (8+6) + (-24+32)$$

$$s_2 = 11$$

$$|A| = 6.$$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$
$$(\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$$

$\lambda = \lambda_1 = 1$

$\lambda = \lambda_2 = 2$

$\lambda = \lambda_3 = 3$ are the eigen values of matrix A

To find Eigen vectors

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Case I :- $\lambda = \lambda_1 = 1$

$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \frac{x_1}{\begin{vmatrix} -4 & -2 \\ -4 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 4 & -2 \\ 3 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & -4 \\ 3 & -4 \end{vmatrix}}$$

$$\therefore \frac{x_1}{-8} = \frac{-x_2}{6} = \frac{x_3}{-4}$$

$$\therefore \frac{x_1}{-4} = \frac{-x_2}{3} = \frac{x_3}{-2} = k = -1$$

$$x_1 = 4, \quad x_3 = 3, \quad x_2 = 2$$

Case II :- $\lambda = \lambda_2 = 2$

$$\begin{bmatrix} 6 & -8 & -2 \\ 4 & -5 & -2 \\ -3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} -5 & -2 \\ -4 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 4 & -2 \\ 3 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & -5 \\ 3 & -4 \end{vmatrix}}$$

$$\frac{x_1}{-3} = \frac{-x_2}{2} = \frac{x_3}{-1}$$

$$\frac{x_1}{-3} = \frac{-x_2}{2} = \frac{x_3}{-1} = k = -1$$

$$x_1 = 3, \quad x_2 = 2, \quad x_3 = 1$$

case ③ :- for $\lambda = \lambda_3 = 3$.

Case 3 For $\lambda = \lambda_3 = 3$ $\begin{bmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\frac{x_1}{\begin{vmatrix} -6 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 4 & -2 \\ 3 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & -6 \\ 3 & -4 \end{vmatrix}}$$

$$\frac{x_1}{4} = \frac{-x_2}{-2} = \frac{x_3}{2}$$

ie $\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{1} = k = 1$

$\therefore$ $\boxed{x_1 = 2,\ x_2 = 1,\ x_3 = 1}$

Thus for $\lambda = \lambda_1 = 3$, $X_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

Rank of Matrix

Rank of a Matrix: Let A be a non-zero matrix. Then the integer r is called the rank of a matrix A if,

i) There exists at least one non-zero minor order r of a matrix A. and

ii) Every minor of order greater than r is zero of a matrix A.

Or Order of any highest order non-zero minor of a matrix A is called order of a matrix.

Note: i) rank of matrix A and A^T are same.

ii) Any row or column transformation will not change the rank of the matrix

Rank of Matrix

→ Echelon form

→ Normal or Canonical form.

Echelon form

Example-1 Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 8 & 5 & 14 & 17 \\ 1 & 5 & 5 & 7 \end{bmatrix}$

Given ⇒ $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 8 & 5 & 14 & 17 \\ 1 & 5 & 5 & 7 \end{bmatrix}_{4\times 4}$

① $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 8R_1$, $R_4 \to R_4 - R_1$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & -11 & -10 & -15 \\ 0 & 3 & 2 & 3 \end{bmatrix}$$

② $R_4 \to R_4 + R_2$, $R_3 \to R_3 - 4R_2$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

③ $R_2 \leftrightarrow R_3$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & -3 \\ 0 & -3 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

④ $R_3 \to R_3 + 3R_2$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -8 & -12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ ← Required Echelon form of matrix A which contains 3 non zero rows ($\because$ order of matrix = 3)

$\therefore$ Rank of matrix = 3.

$\rho(A) = 3$.

Example-3 Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

① Given $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}_{4\times 4}$

① $R_4 \to R_4 - (R_1 + R_2 + R_3), R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -2 & 2 \\ \ldots \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -2 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

② $R_2 \longleftrightarrow R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & -3 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow$$

This is required echelon form of matrix which Contains 3 non zero rows

$\therefore$ Rank of matrix $A = 3$

$\therefore \rho(A) = 3$.

Example-5 Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}$

Solution :- $R_3 \to R_3 - 3(R_1 + R_2), R_3 \to R_3 - (2R_1 + R_2), R_3 \to R_3 - 4R_1$

$A \sim \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -6 & -12 & -18 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Which is the required Echelon form of a matrix A, contain ~~3~~ 2 non-zero rows

Therefore rank of a matrix A is 2 i.e.$\rho(A) = 2$

Normal or canonical form

Example-1 Find the rank of the matrix using normal form or canonical form of matrix

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{bmatrix}$$

① $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 6R_1$

$$A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

② $C_2 \to C_2 + C_1$, $C_3 \to C_3 + 2C_1$, $C_4 \to C_4 + 4C_1$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

③ $R_4 \to R_4 - (R_2 + R_3)$, $R_2 \to R_2 - R_3$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

④ $R_3 \to R_3 - 4R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(5) $C_3 \to C_3 + 6C_2$, $C_4 \to C_4 + 3C_2$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(6) $C_3 \to \frac{1}{33} C_3$, $C_4 \to \frac{1}{22} C_4$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(6) $C_4 \to C_4 - C_3$

I_3 unit matrix

$$A \sim \begin{bmatrix} \boxed{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 0 \end{bmatrix}$$ Which is of the form $A \sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$ 3x3

Therefore rank of matrix A is 3 i.e. i.e.$\rho(A) = 3$

Example-2 Find the rank of the matrix using normal form or canonical form of matrix

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

(1) $R_4 \to R_4 - (R_1 + R_2 + R_3)$, $R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_1$.

$$A \sim \begin{bmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & -2 & -4 & -6 \end{bmatrix}$$

(2) $R_1 \leftrightarrow R_2$, $R_4 \to \frac{-1}{2} R_4$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

(3) $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

(4) $R_4 \to R_4 - R_2$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(5) $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1, C_4 \to C_4 - C_1$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5 $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1, C_4 \to C_4 - C_1$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

6 $C_3 \to C_3 - 2C_2, C_4 \to C_4 - 3C_2$

unit matrix

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ This is of the form $\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$ 2×2

Therefore rank of matrix A is 2 i.e. i.e. $\rho(A) = 2$

Example-3 Find the rank of the matrix using normal form or canonical form of matrix

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$$

Singular value decomposition

12 August 2023 14:35

Singular Value Decomposition:

Theorem: A rectangular Matrix $A_{m\times n}$ can be decomposed into the product of three matrices on orthogonal matrix $U_{m\times m}$ a diagonal matrix $D_{m\times n}$ and transpose of another orthogonal matrix $V_{m\times m}$

i.e. we can write $\boxed{A_{m\times n} = U_{m\times m} \times D_{m\times n} \times V_{n\times n}^T}$

Since U and V are orthogonal matrices, we have $UU^T = I, and\ VV^T = I$

Further,

1. The columns of U are the orthonormal vectors of AA^T, and
2. The columns of V are the orthonormal vectors of A^TA
3. D is a diagonal matrix whose elements are square roots of Eigen values $U\ or\ V$ arranged in decreasing order.

Finding the singular value decomposition consistent of finding eigen values and eigen vectors of AA^T and A^TA

The normalized eigen vector of AA^T are the columns of U

The normalized eigen vector of A^TA are the columns of V

Rectangular matrix $A_{m\times n}$

1. orthogonal $(U_{m\times m})$
2. Diagonal matrix $(D_{m\times n})$
3. transpose of another orthogonal matrix $(V_{m\times m})$

Q. Find singular value decomposition of matrix

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}_{2\times 2}$$

Given:- $A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$, $A^T = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}$

$$AA^T = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 13 & 6 \\ 6 & 4 \end{bmatrix}_{2\times 2} = M \text{ (say)}$$

It's characteristic equation

$$\lambda^2 - S_1\lambda + |A| = 0$$

$$S_1 = 17 \ \& \ |A| = 16$$

$$\lambda^2 - 17\lambda + 16 = 0$$

$$(\lambda-16)(\lambda-1) = 0$$

$$\therefore \lambda = 16, \lambda = 1.$$

$$\therefore \lambda = \lambda_1 = 16, \lambda = \lambda_2 = 1$$

To find eigen vector

$$(AA^T - \lambda)x = 0$$

$$\begin{bmatrix} 13-\lambda & 6 \\ 6 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Case ① $\lambda = \lambda_1 = 16$

$$A^TA = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 4 & 6 \\ 6 & 13 \end{bmatrix}_{2\times 2} = M' \text{ (say)}$$

It's characteristic equation

$$\lambda^2 - S_1\lambda + |A| = 0.$$

$$S_1 = 17 \ \& \ |A| = 16$$

$$\lambda^2 - 17\lambda + 16 = 0$$

$$(\lambda-16)(\lambda-1) = 0$$

$$\lambda = 16, \lambda = 1.$$

$$\lambda = \lambda_1 = 16, \lambda = \lambda_2 = 1.$$

To find eigen vector

$$(A^TA - \lambda)x = 0$$

$$\begin{bmatrix} 4-\lambda & 6 \\ 6 & 13-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

case ① :- $\lambda = \lambda_1 = 16.$

$$\begin{bmatrix} -3 & 6 \\ 6 & -12 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-3x_1 + 6x_2 = 0$
& $6x_1 - 12x_2 = 0$

$x_1 = 2 \quad x_2 = 1.$

$\therefore X_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \|X_1\| = \sqrt{4+1} = \sqrt{5} \quad X_1' = \frac{X_1}{\|X_1\|} = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

Case (2) :- $\lambda = \lambda_2 = 1$

$$\begin{bmatrix} 12 & 6 \\ 6 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 2R_2, \; R_2 = \frac{1}{3}R_2$

$$\begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2x_1 + 1x_2 = 0$

$2x_1 = -1x_2$

$\frac{x_1}{-1} = \frac{x_2}{2} = k = 1$

$x_1 = -1, \quad x_2 = 2$

$X_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \|X_2\| = \sqrt{1+4} = \sqrt{5}$

$X_2' = \frac{X_2}{\|X_2\|} = \begin{bmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$

$$U = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}$$

$$\begin{bmatrix} -12 & 6 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-12x_1 + 6x_2 = 0$

$6x_1 - 3x_2 = 0$

$x_1 = 1, \quad x_2 = 2$

$X_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\|X_1\| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$X_1' = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$

Case (2) :- $\lambda = \lambda_2 = 1$

$$\begin{bmatrix} 3 & 6 \\ 6 & 12 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$3x_1 + 6x_2 = 0$
$6x_1 + 12x_2 = 0.$

$x_1 = 2$
$x_2 = -1$

$X_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \|X_2\| = \sqrt{4+1} = \sqrt{5}$

$X_2' = \frac{X_2}{\|X_2\|} = \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \end{bmatrix}$

$$V = \begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ 2/\sqrt{5} & -1/\sqrt{5} \end{bmatrix}$$

$$\therefore V^T = \begin{bmatrix} \frac{1}{\sqrt{5}} & 2/\sqrt{5} \\ 2/\sqrt{5} & -1/\sqrt{5} \end{bmatrix}$$

Note :- σ_1 & σ_2 are roots of eigen values

$$D = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = U D V^T$$

$$= \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{5}} & 2/\sqrt{5} \\ 2/\sqrt{5} & -1/\sqrt{5} \end{bmatrix}$$

Q2. Find singular value decomposition of $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$

Ans:- $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}_{2\times 2}$ $\quad A^T = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}_{2\times 2}$

$$AA^T = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 12 \\ 12 & 41 \end{bmatrix}_{2\times 2}$$

Here $|A| = 225$, $S_1 = 50$

It's characteristic equation

$$\lambda^2 - S_1\lambda + |A| = 0$$

$$\lambda^2 - 50\lambda + 225 = 0$$

$$(\lambda - 45)(\lambda - 5) = 0$$

$\therefore \lambda = \lambda_1 = 45$ & $\lambda = \lambda_2 = 5$

$$\sigma_1 = \sqrt{\lambda_1} = 3\sqrt{5}$$

$$\sigma_2 = \sqrt{\lambda_2} = \sqrt{5}$$

To find Eigen vectors

$$(AA^T - \lambda I)x = 0$$

$$\begin{bmatrix} 9-\lambda & 12 \\ 12 & 41-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Case I If $\lambda = \lambda_1 = 45$

$$\begin{bmatrix} -36 & 12 \\ 12 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 = 1$, $x_2 = 3$.

For $\lambda = \lambda_1 = 45$

$x_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\quad \|x_1\| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$$x_1' = \frac{x_1}{\|x_1\|} = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$$

Case II $\lambda = \lambda_2 = 5$

$$\begin{bmatrix} 4 & 12 \\ 12 & 36 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 = -3$, $x_2 = 1$

$\therefore \frac{x_1}{-3} = \frac{x_2}{1} = k = 1$

$\therefore x_1 = -3$, $x_2 = 1$.

For $\lambda = \lambda_2 = 5$

$$A^TA = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 25 & 20 \\ 20 & 25 \end{bmatrix}_{2\times 2}$$

Here $|A| = 225$, $S_1 = 50$

It's characteristic equation,

$$\lambda^2 - S_1\lambda + |A| = 0$$

$$\lambda^2 - 50\lambda + 225 = 0$$

$$(\lambda - 45)(\lambda - 5) = 0$$

$\lambda = \lambda_1 = 45$, $\lambda = \lambda_2 = 5$

$$\sigma_1 = \sqrt{\lambda_1} = \sqrt{45} = 3\sqrt{5}$$

$$\sigma_2 = \sqrt{\lambda_2} = \sqrt{5}$$

To find Eigen vector

$$(A^TA - \lambda I)x = 0.$$

$$\begin{bmatrix} 25-\lambda & 20 \\ 20 & 25-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Case I $\lambda = \lambda_1 = 45$

$$\begin{bmatrix} -20 & 20 \\ 20 & -20 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 = 1$, $x_2 = 1$

$\therefore \frac{x_1}{1} = \frac{x_2}{1} = k = 1$

$\therefore x_1 = 1$, $x_2 = 1$.

for $\lambda = \lambda_1 = 45$

$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \|x_1\| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$$x_1' = \frac{x_1}{\|x_1\|} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

Case II $\lambda = \lambda_2 = 5$

$$\begin{bmatrix} 20 & 20 \\ 20 & 20 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 = -1$, $x_2 = 1$.

$\therefore \frac{x_1}{} = \frac{x_2}{} = k = 1$

$\therefore\ x_1 = -3,\ x_2 = 1.$

For $\lambda = \lambda_2 = 5$

$$X_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \quad \|X_2\| = \sqrt{(-3)^2 + (1)^2} = \sqrt{10}.$$

$$\therefore X_2' = \frac{X_2}{\|X_2\|} = \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$$

$$U = \begin{bmatrix} 1/\sqrt{10} & -3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}$$

$x_1 = -1,\ x_2 = 1.$

$$\therefore \frac{x_1}{-1} = \frac{x_2}{1} = t = 1$$

$\therefore\ x_1 = -1,\ x_2 = 1$

For $\lambda = \lambda_2 = 5$

$$X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \|X_2\| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

$$X_2' = \frac{X_2}{\|X_2\|} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$V = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$A = UDV^T = \begin{bmatrix} & & \\ & U & \\ & & \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} & & \\ & V & \\ & & \end{bmatrix}$$

$$A = \begin{bmatrix} 1/\sqrt{10} & -3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix} \begin{bmatrix} 3\sqrt{5} & 0 \\ 0 & \sqrt{5} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Q. Find singular value decomposition of matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}_{3\times 2}$

<u>Ans:-</u> $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}_{3\times 2} \quad A^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}_{2\times 3}.$

$$AA^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix}_{3\times 3}$$

As A is square matrix of order 3.

It's characteristic equation is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$$

where $S_1 = 5$

$S_2 = 1 + 4 + 1 = 6$

$|A| = 0.$

$$\therefore \lambda^3 - 5\lambda^2 + 6\lambda - 0 = 0.$$

$$\lambda(\lambda^2 - 5\lambda + 6) = 0$$

$$\lambda(\lambda - 2)(\lambda - 3) = 0$$

$\lambda = \lambda_1 = 3,\ \lambda = \lambda_2 = 2,\ \lambda = \lambda_3 = 0$ These are eigen values of matrix

$\sigma_1 = \sqrt{\lambda_1} = \sqrt{3}$

$\sigma_2 = \sqrt{\lambda_2} = \sqrt{2}$

$\sigma_3 = \sqrt{\lambda_3} = \sqrt{0} = 0$

<u>To find eigen vector</u>

$$(AA^T - \lambda I)X = 0$$

$$\begin{bmatrix} 2-\lambda & 1 & 0 \\ 1 & 1-\lambda & 1 \\ 0 & -1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 1 \quad , \quad x_2 = -1 \quad , \quad x_3 = -1$

$\therefore \quad \dfrac{x_1}{1} = \dfrac{-x_2}{-1} = \dfrac{x_3}{-1} = k = 1$

$\therefore \quad x_1 = 1, \quad x_2 = 1, \quad x_3 = -1$

For $\lambda = \lambda_1 = 3$ $\quad X_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $\quad \|X_1\| = \sqrt{(1)^2 + (1)^2 + (-1)^2} = \sqrt{3}$

$$X_1' = \frac{X_1}{\|X_1\|} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix}$$

<u>case ② :-</u> If $\lambda = \lambda_2 = 2$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = -1, \quad x_2 = 0, \quad x_3 = -1.$

$\dfrac{x_1}{-1} = \dfrac{-x_2}{0} = \dfrac{x_3}{-1} = k = -1$

$x_1 = 1, \quad x_2 = 0, \quad x_3 = 1.$

For $\lambda = \lambda_2 = 2$

$\therefore \quad X_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\quad \|X_2\| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$

$$\therefore \quad X_2' = \frac{X_2}{\|X_2\|} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

<u>Case (III) :-</u> For $\lambda = \lambda_3 = 0.$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 = 2 \quad , \quad x_2 = 4 \quad , \quad x_3 = -2$

$$\frac{x_1}{2} = \frac{-x_2}{4} = \frac{x_3}{-2} = k = 1$$

$$\frac{x_1}{1} = \frac{-x_2}{2} = \frac{x_3}{-1} = k = 1.$$

$\therefore \; x_1 = 1 \;,\; x_2 = -2 \;,\; x_3 = -1.$

for $\lambda = \lambda_3 = 0$ $\quad x_3 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$ $\quad \|x_3\| = \sqrt{1^2 + (-2)^2 + (-1)^2}$

$\|x_3\| = \sqrt{6}$

$$x_3' = \frac{x_3}{\|x_3\|} = \begin{bmatrix} 1/\sqrt{6} \\ -2/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix}$$

$$U = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ -1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6}. \end{bmatrix}$$

Now $A^TA = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

$\because A^TA$ is diagonal matrix & $\lambda = \lambda_1 = 3$ & $\lambda = \lambda_2 = 2$ be the Eigen value of matrix A^TA

$\therefore \; \sigma_1 = \sqrt{\lambda_1} = \sqrt{3} \quad \sigma_2 = \sqrt{\lambda_2} = \sqrt{2}$

To find Eigen vector consider $(A^TA - \lambda I)X = 0$

$$\therefore \begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad - (6)$$

<u>Case-1</u> If $\lambda = \lambda_1 = 3$, $\quad \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\therefore \; -1x_1 + 0x_2 = 0 \Rightarrow x_1 = 0, \; x_2 = 1$ say

$\therefore$ For $\lambda = \lambda_1 = 3$ $\quad X_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad \|x_1\| = \sqrt{1}$, $\quad x_1' = \frac{x_1}{\|x_1\|} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

If $\lambda = \lambda_2 = 2$, $\quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$0x_1 + 1x_2 = 0 \Rightarrow x_2 = 0, \; x_1 = 1$ say

$\therefore$ For $\lambda = \lambda_2 = 2$, $X_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\|X_2\| = \sqrt{1}$, $X_2' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\therefore V = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Thus $U = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ -1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6} \end{bmatrix}$, $V = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\because$ A is a matrix of order 3×2

$$\therefore D = \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{2} \\ 0 & 0 \end{bmatrix}_{3 \times 2}$$

$$\therefore A = UDV^T = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ -1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6} \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

System of linear equations

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System of Non-Homogeneous linear equations

A system of m linear algebraic equations in n unknowns $x_1, x_2, x_3, \cdots\cdots, x_n$ is a set of equations of the form

system of linear Algebraic equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots\cdots + a_{1n}x_n = b_1$$ → equation 1

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots\cdots + a_{2n}x_n = b_2$$ → equation 2

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots\cdots + a_{3n}x_n = b_3$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots\cdots + a_{mn}x_n = b_m$$

The above system of equation can be written as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix} \cdots\cdots(1)$$ ← Linear Matrix form of equations

The above system of equation can be written as

$$AX = D \cdots\cdots(2)$$

variables

$$\text{Where } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}, D = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}$$

matrix of coefficients

$$[A/D] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} & b_3 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & b_m \end{bmatrix}$$

ρ(matrix) = Rank of matrix

Note:-

condition

i) If $\rho(A) = \rho(A|D)$= n, the number of unknowns, then the system of equation (1) has unique solution

ii) If $\rho(A) = \rho(A|D)$< n, the number of unknowns, then the system of equation (1) has infinitely many solution

iii) If $\rho(A) \neq \rho(A|D)$,then the system of equation (1) is inconsistent i.e. it has no solution.

Examples for linear equations

Type-1

Example-1

1 Is the following system of equations is consistent if consistent find it's solution

$$1x + 2y + 3z = 14, 3x + 1y + 2z = 11, 2x + 3y + 1z = 11$$

Solution: Given system of equations is

$1x + 2y + 3z = 14,$

$3x + 1y + 2z = 11,$

$2x + 3y + 1z = 11 \cdots(1)$

The above system of equations can be written as

$$[A/D] = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix} \begin{matrix} 14 \\ 11 \\ 11 \end{matrix}\Bigg]$$

A

Coefficient

RHS

Coefficient

step 1 $R_2 \to R_2 - 3R_1, R_3 \to R_3 - 2R_1$

$$[A/D] \sim \begin{bmatrix} 1 & 2 & 3 & 14 \\ 0 & -5 & -7 & -31 \\ 0 & -1 & -5 & -17 \end{bmatrix}$$

step 2 $R_2 \leftrightarrow R_3$

$$[A/D] \sim \begin{bmatrix} 1 & 2 & 3 & 14 \\ 0 & -1 & -5 & -17 \\ 0 & -5 & -7 & -31 \end{bmatrix}$$

step 3 $R_3 \to R_3 - 5R_2, R_2 \to -1R_2$

$$[A/D] \sim \begin{bmatrix} 1 & 2 & 3 & 14 \\ 0 & 1 & 5 & 17 \\ 0 & 0 & 18 & 54 \end{bmatrix}$$

$$R_3 \to \frac{1}{18} R_3$$

$$[A/D] \sim \begin{bmatrix} 1 & 2 & 3 & 14 \\ 0 & 1 & 5 & 17 \\ 0 & 0 & 1 & 3 \end{bmatrix} \cdots (2)$$ ⇒ Rank of matrix = 3

Since $\rho(A) = \rho(A|D) = 3 = No.of\ unknowns$ ← condition 1 in description.

n=3

Rank of matrix → Rank of matrix (A|D)

The system of equation is consistent and it has unique solution

Equation (2) can be written as

$1x + 2y + 3z = 14, \cdots (3)$

$1y + 5z = 17, \cdots (4)$

1z=3⇒$\boxed{z = 3}$

Now put z=3 in equation (4) we get

$1y + (5 \times 3) = 17$

$y + 15 = 17 \ \therefore \boxed{y = 2}$

Now put $y = 2, and\ z = 3$ in equation (3) we get

$1x + (2 \times 2) + (3 \times 3) = 14$

$1x + 4 + 9 = 14$

$1x = 1 \Rightarrow \boxed{x = 1}$

$Thus\ x = 1, y = 2, and\ z = 3\ be\ the\ required\ solution$

Q 2. Example-2

Is the following system of equations is consistent if consistent find it's solution

$2x + 1y + 1z = 4, 1x - 1y + 3z = 3, 4x - 1y - 1z = 2$

Solution: Given system of equations is

$2x + 1y + 1z = 4,$

$1x - 1y + 3z = 3,$

$4x - 1y - 1z = 2 \cdots (1)$

The above system of equations can be written as

$$[A/D] = \begin{bmatrix} 2 & 1 & 1 & 4 \\ 1 & -1 & 3 & 3 \\ 4 & -1 & -1 & 2 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$[A/D] \sim \begin{bmatrix} 1 & -1 & 3 & 3 \\ 2 & 1 & 1 & 4 \\ 4 & -1 & -1 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 4R_1$

$$[A/D] \sim \begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & 3 & -5 & -2 \\ 0 & 3 & -13 & -10 \end{bmatrix}$$

$R_3 \to R_3 - 5R_2$

$$[A/D] \sim \begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & 3 & -5 & -2 \\ 0 & 0 & -8 & -8 \end{bmatrix}$$

$R_3 \to \frac{-1}{8} R_3$

$$[A/D] \sim \begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & 3 & -5 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \cdots (2)$$

Since $\rho(A) = \rho(A|D) = 3 = No. of\ unknowns$

The system of equation is consistent and it has unique solution

Equation (2) can be written as

$1x - 1y + 3z = 3, \cdots (3)$

$3y - 5z = -2, \cdots (4)$

$1z=1 \Rightarrow z = 1$

Now put z=1 in equation (4) we get

$3y - (5 \times 1) = -2$

$3y - 5 = -2 \therefore 3y = 3 \Rightarrow y = 1$

Now put $y = 1, and\ z = 1$ in equation (3) we get

$1x - (1 \times 1) + (3 \times 1) = 3$

$1x - 1 + 3 = 3$

$1x = 1 \Rightarrow x = 1$

$Thus\ \boxed{x = 1, y = 1, and\ z = 1}\ be\ the\ required\ solution$

Vector subspaces & basis function

26 August 2023 13:42

Module 3: Introduction to graphs

26 August 2023 13:44

Quantitative vs. Qualitative data

When it comes to conducting data research, you'll need different collection, hypotheses and analysis methods, so it's important to understand the key differences between quantitative and qualitative data:

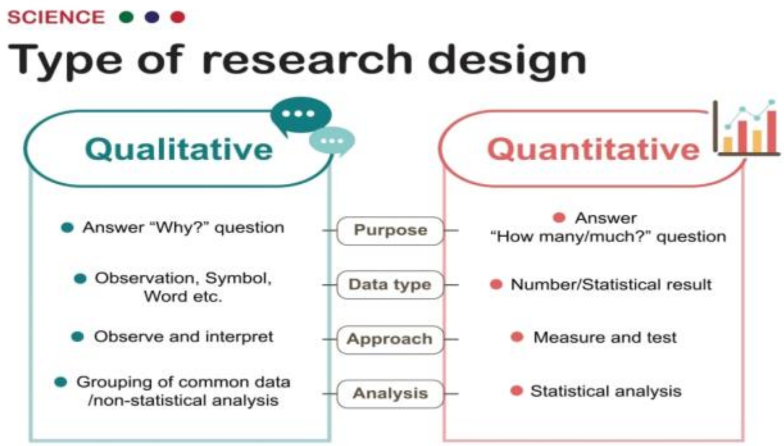
- Quantitative data is numbers-based, countable, or measurable. Qualitative data is interpretation-based, descriptive, and relating to language.
- Quantitative data tells us how many, how much, or how often in calculations. Qualitative data can help us to understand why, how, or what happened behind certain behaviors.
- Quantitative data is fixed and universal. Qualitative data is subjective and unique.
- Quantitative research methods are measuring and counting. Qualitative research methods are interviewing and observing.
- Quantitative data is analyzed using statistical analysis. Qualitative data is analyzed by grouping the data into categories the data into categories and themes.

(Non-numerical type of data)

Examples

observations, words, sound

Images



Examples

→ Numbers, graphs, Statistical Results

>

Examples

Here are some examples of qualitative data:

1. **Interview transcripts**: Verbatim records of what participants said during an interview or focus group. They allow researchers to identify common themes and patterns, and draw conclusions based on the data. Interview transcripts can also be useful in providing direct quotes and examples to support research findings.
2. **Observations**: The researcher typically takes detailed notes on what they observe, including any contextual information, nonverbal cues, or other relevant details. The resulting observational data can be analyzed to gain insights into social phenomena, such as human behavior, social interactions, and cultural practices.
3. **Unstructured interviews:** generate qualitative data through the use of open questions. This allows the respondent to talk in some depth, choosing their own words. This helps the researcher develop a real sense of a person's understanding of a situation.
4. **Diaries or journals**: Written accounts of personal experiences or reflections.
5. Notice that qualitative data could be much more than just words or text. Photographs, videos, sound recordings, and so on, can be considered qualitative data. Visual data can be used to understand behaviors, environments, and social interactions.

Limitations of Qualitative Research

- Because of the time and costs involved, qualitative designs do not generally draw samples from large-scale data sets.
- The problem of adequate validity or reliability is a major criticism. Because of the subjective nature of qualitative data and its origin in single contexts, it is difficult to apply conventional standards of reliability and

validity. For example, because of the central role played by the researcher in the generation of data, it is not possible to replicate qualitative studies.

- Also, contexts, situations, events, conditions, and interactions cannot be replicated to any extent, nor can generalizations be made to a wider context than the one studied with confidence.
- The time required for data collection, analysis, and interpretation is lengthy. Analysis of qualitative data is difficult, and expert knowledge of an area is necessary to interpret qualitative data. Great care must be taken when doing so, for example, looking for mental illness symptoms.

Advantages of Qualitative Research

- Because of close researcher involvement, the researcher gains an insider's view of the field. This allows the researcher to find issues that are often missed (such as subtleties and complexities) by the scientific, more positivistic inquiries.
- Qualitative descriptions can be important in suggesting possible relationships, causes, effects, and dynamic processes.
- Qualitative analysis allows for ambiguities/contradictions in the data, which reflect social reality (Denscombe, 2010).
- Qualitative research uses a descriptive, narrative style; this research might be of particular benefit to the practitioner as she or he could turn to qualitative reports to examine forms of knowledge that might otherwise be unavailable, thereby gaining new insight.

What Is Quantitative Research?

Quantitative research involves the process of objectively collecting and analyzing numerical data to describe, predict, or control variables of interest.

The goals of quantitative research are to test causal relationships between variables, make predictions, and generalize results to wider populations.

Quantitative researchers aim to establish general laws of behavior and phenomenon across different settings/contexts. Research is used to test a theory and ultimately support or reject it.

Quantitative Methods

Experiments typically yield quantitative data, as they are concerned with measuring things. However, other research methods, such as controlled observations and questionnaires, can produce both quantitative information.

For example, a rating scale or closed questions on a questionnaire would generate quantitative data as these produce either numerical data or data that can be put into categories (e.g., "yes," "no" answers).

Experimental methods limit how a research participant can react to and express appropriate social behavior.

Findings are, therefore, likely to be context-bound and simply a reflection of the assumptions that the researcher brings to the investigation.

Examples

There are numerous examples of quantitative data in psychological research, including mental health. Here are a few examples:

1 Standardized psychological assessments: One example of a standardized psychological assessment of IQ that uses quantitative data is the Wechsler Adult Intelligence Scale (WAIS).
Another example is the Experience in Close Relationships Scale (ECR), a self-report questionnaire widely used to assess adult attachment styles.
The ECR provides quantitative data that can be used to assess attachment styles and predict relationship outcomes.

2 Neuroimaging data: Neuroimaging techniques, such as MRI and fMRI, provide quantitative data on brain structure and function.
This data can be analyzed to identify brain regions involved in specific mental processes or disorders.

3 Clinical outcome measures: The use of clinical outcome measures provides objective, standardized data that can be used to assess treatment effectiveness and monitor symptoms over time, helping mental health professionals make informed decisions about treatment and care.
For example, the Beck Depression Inventory (BDI) is a clinician-administered questionnaire widely used to assess the severity of depressive symptoms in individuals.
The BDI consists of 21 questions, each scored on a scale of 0 to 3, with higher scores indicating more severe depressive symptoms.

What are the advantages and disadvantages of quantitative data?

Each type of data set has its own pros and cons.

Advantages of quantitative data (Benefits of quantitative data)

- It's relatively quick and easy to collect and it's easier to draw conclusions from.
- When you collect quantitative data, the type of results will tell you which statistical tests are appropriate to use.
- As a result, interpreting your data and presenting those findings is straightforward and less open to error and subjectivity.

Another advantage is that you can replicate it. Replicating a study is possible because your data collection is measurable and tangible for further applications.

Disadvantages of quantitative data (Limitations of quantitative data)

- Quantitative data doesn't always tell you the full story (no matter what the perspective).
- With choppy information, it can be inconclusive.
- Quantitative research can be limited, which can lead to overlooking broader themes and relationships.
- By focusing solely on numbers, there is a risk of missing larger focus information that can be beneficial.

Types of Quantitative data

Quantitative data is split into two types of data: discrete one, which represents countable items. And continuous data, which outlines data measurement. The continuous numerical data is further subdivided into interval and ratio data, known for measuring certain items.

The discrete data fundamentals

Discrete data is a count that involves integers — only a limited number of values is possible. This type of data cannot be subdivided into different parts. Discrete data includes discrete variables that are finite, numeric, countable, and non-negative integers. In many cases, discrete data can be prefixed with "the number of". For example:

o/p ? Integer values

- The number of students who have attended the class; o/p → Number of students = 25 Integers
- The number of customers who have bought different products;
- The number of groceries people are purchasing every day;

This data type is mainly used for simple statistical analysis because it's easy to summarize and compute. In most of the practices, discrete data is displayed by bar graphs, stem-and-leaf-plot and pie charts.

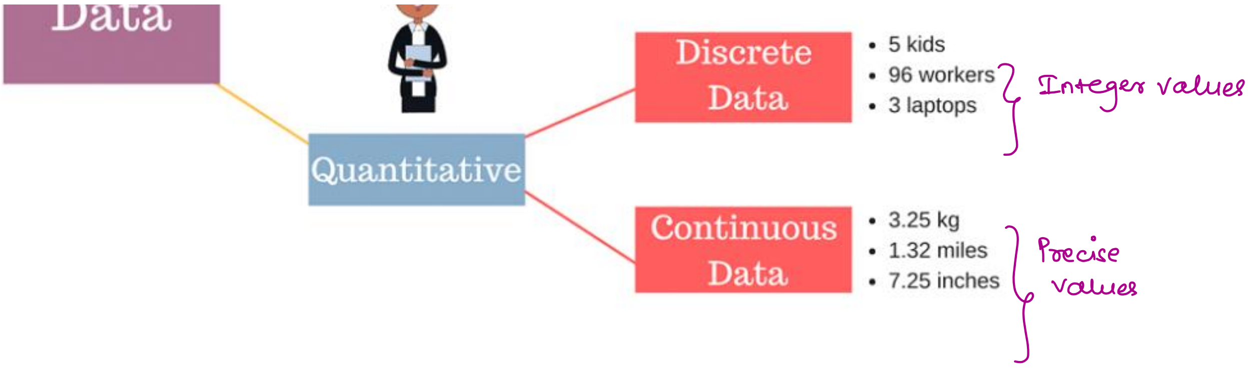
Continuous data — it's all about accuracy

Continuous data is considered the complete opposite of discrete data. It's the type of numerical data that refers to the unspecified number of possible measurements between two presumed points.
The numbers of continuous data are not always clean and integers, as they are usually collected from very precise measurements. Measuring a particular subject is allowing for creating a defined range to collect more data. Variables in continuous data sets often carry decimal points, with the number stretching out as far as possible. Typically, it changes over time. It can have completely different values at different time intervals, which might not always be whole numbers. Here are some examples:

- The weather temperature;
- The wind speed;
- The weight of the kids;

- 

? Integer values



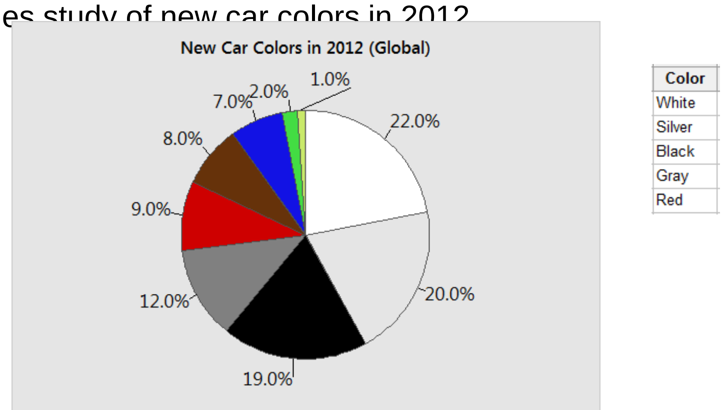
Types of Qualitative data: Categorical data, Binary data, Ordinary data

Qualitative Data: Categorical, Binary, and Ordinal

When you record information that categorizes your observations, you are collecting qualitative data. There are three types of qualitative variables—**categorical, binary, and ordinal**. With these data types, you're often interested in the proportions of each category. Consequently, bar charts and pie charts are conventional methods for graphing qualitative variables because they are useful for displaying the relative percentage of each group out of the entire sample.

Categorical data

Categorical data have values that you can put into a countable number of distinct groups based on a characteristic. For a categorical variable, you can assign categories, but the categories have no natural order. Analysts also refer to categorical data as both **attribute and nominal variables**. For example, college major is a categorical variable that can have values such as psychology, political science, engineering, biology, etc. Categorical data is also known as nominal data.The categorical data in the pie chart are the results of a PPG Industries study of new car colors in 2012.



Binary data

Binary data can have only two values. If you can place an observation into only two categories, you have a binary variable. Statisticians also refer to binary data as both dichotomous and indicator variables. For example, pass/fail, male/female, and the presence/absence of a characteristic are all binary data.

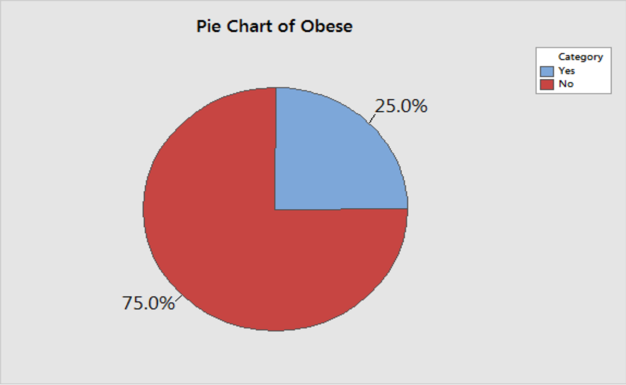
Binary variables are helpful for calculating proportions or percentages, such as the proportion of defective products in a sample. You just take the number of faulty products and divide by the sample size.

→ 2 categories — Binary data

and divide by the sample size. → 2 categories → binary data

The binary yes/no data for the pie chart are based on the continuous body fat percentage data in the histogram above. Compare how much we learn from the continuous data that the histogram displays as a distribution compared to the simple proportion that the binary version of the data provides in the pie chart below.

Obese
Yes
No
No
Yes
No



Ordinal data

ordinal data (Natural order)

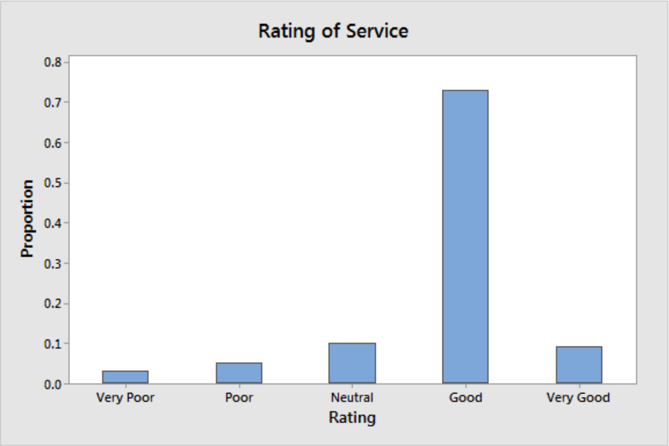
Rating	
Very Poor	0
Poor	1
Neutral	2
Good	3
Very Good	5

Ordinal data have at least three categories, and the categories have a natural order. Examples of ordinal variables include overall status (poor to excellent), agreement (strongly disagree to strongly agree), and rank (such as sporting teams).

Analysts often consider ordinal variables to have a combination of qualitative and quantitative properties. Analysts often represent ordinal variables using numbers, such as a 5-point Likert scale that measures satisfaction. In number form, you can calculate average scores as with quantitative variables. However, the numbers have limited usefulness because the differences between ranks might not be constant. Learn more in-depth about Ordinal Data: Definition, Examples & Analysis.

For example, first, second, and third in a race are ordinal data. The difference in time between first and second place might not be the same the difference between second and third place.

The bar chart below displays the proportion of each service rating category in their natural order.


Rating of Service
Proportion
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Very Poor
Poor
Neutral
Good
Very Good
Rating