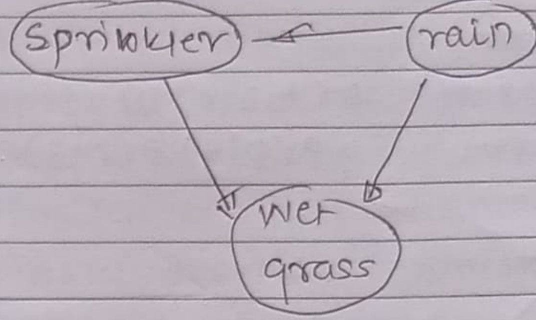
Bayesian Network Inference

11/9/23

(1). Bayesian Model :

- Bayesian Network Model also known as Bayesian Network or a Probabilistic Graphical Model which is a statistical model that represents probabilistic relationship among a set of variables. It is based on Bayesian probability theory which provides a mathematical framework for reasoning under uncertainity.
- In Bayesian model, variables are represented as nodes and the relationships between variables are represented as directed edge. This graphical representation allows for a visual picturisation of the dependencies and conditional relationships among variables.

Q1. let's consider a scenario where we want to understand the relationships between 3 variables: the sprinkler (it's states are whether it is 'ON' or 'OFF'), the occurence of rain and the wetness of the grass. It's important to note that the grass can become wet due to two factors, ie. that an active sprinkler and or rain. Additionally, rain can directly influence the use of the sprinkler, meaning when it rains the sprinkler is usually inactive. To represent and analyse this situation, we can utilize a 'Bayesian Network'. Each variable in the network has two possible values, ie, true and false.



rain	T	F
	0.2	0.8

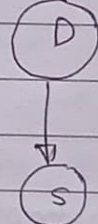
rain	sprinkler T	sprinkler F
F	0.4	0.6
T	01	0.99

sprinkler	rain	wet grass T	wet grass F
F	F	0.0	1.0
F	T	0.99	0.01
T	F	0.45	0.75
T	T	0.99	0.00

$P(G, S, R) = P(G|S, R) \cdot P(S|R) \cdot P(R)$.

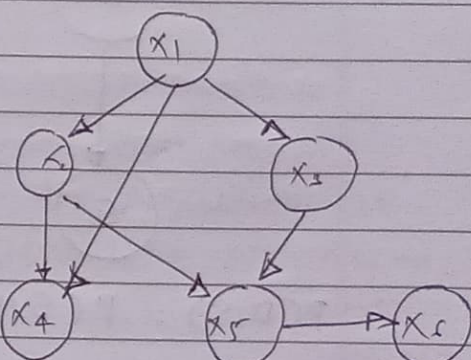
<u>Note:</u> Method to write joint conditional probability function from the given graph.

Q1.



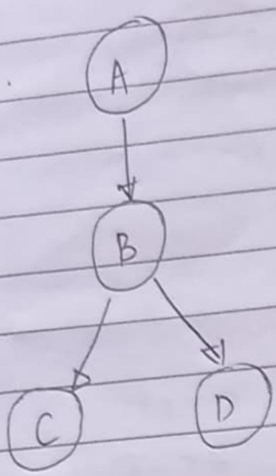
$P(S, D)$
$= P(S|D) \cdot P(D)$

Q2.



$P(X_1, X_2, X_3, X_4, X_5, X_6)$
$= P(X_6|X_5) \cdot P(X_5|X_2, X_3) \cdot$
$P(X_4|X_2, X_1) \cdot P(X_3|X_1) \cdot$
$P(X_2|X_1) \cdot P(X_1)$

Q3.



$P(A, B, C, D) =$
$P(D|B) \cdot P(C|B) \cdot P(B|A) \cdot P(A)$

Q. Suppose we have two variables: drugs and symptoms. The 'drug' variable represents whether a patient has taken a particular drug and 'symptom' variable represents whether the patients exhibits certain symptoms. The Bayesian Network will have two nodes: D&S. The drug will have two status: present(T) or absent(F). Symptom will also have present and absent as a state. Create Bayesian N/W and write conditional probabilities between the ~~AB~~ nodes.

	Drug
T	0.05
F	0.015

$P(D, S) = P(S/D) \cdot P(D)$

Drug	Symptom T	Symptom F
T	0.8	0.2
F	0.1	0.9

12/10/23

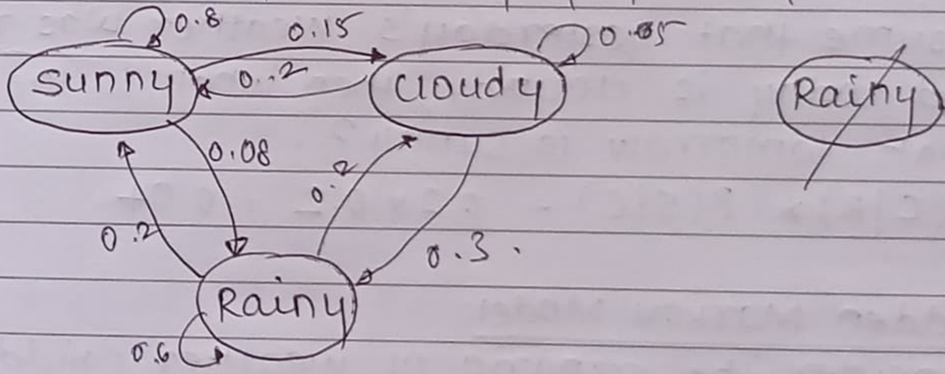
- Markov chain Method:

It describes the stocastic/random process with Markov Property where the propability of random process transitioning to the next state is only dependent on the current state & is independent of the states that precede the current state.

The assumptions of Markov Model are:

(1) Finite no. of states
(2) Mutually exclusive states
(3) The sum of outgoing probability of each state in Markov Model was 1.
(4) The transition probability from one state to another is constant over time.

Q. Consider the weather seauence day-wise with assumption one weather for a day. The traditional probability graph is given as



Now, consider the sequence of weather as
Sunny → Sunny → Rainy → Cloudy → Cloudy → Sunny → Sunny → Rainy

From the above given transitional probability of graph we can write conditional probability as:

tom. today

$P(\text{sunny} \mid \text{sunny}) = 0.8$
$P(\text{Rainy} \mid \text{Sunny}) = 0.15$
$P(\text{cloudy} \mid \text{sunny}) = \underline{0.05}$
1

$P(\text{Sunny} \mid \text{cloudy}) = 0.2$
$P(\text{Rainy} \mid \text{cloudy}) = 0.3$
$P(\text{cloudy} \mid \text{cloudy}) = 0.5$

$P(\text{Sunny} \mid \text{Rainy}) = 0.2$
$P(\text{Rainy} \mid \text{Rainy}) = 0.6$
$P(\text{cloudy} \mid \text{Rainy}) = 0.2$

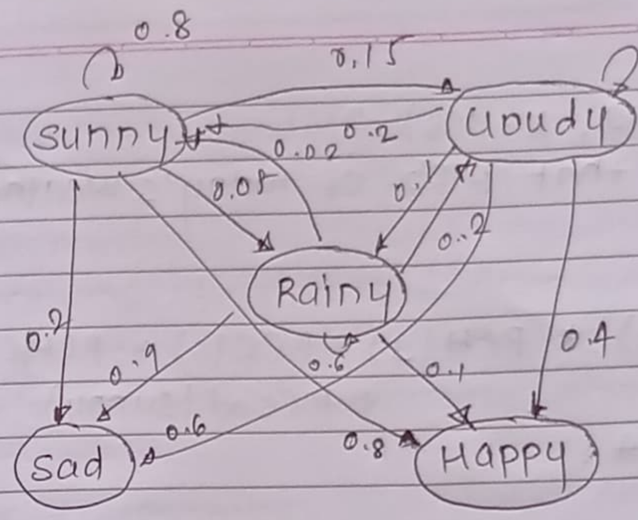
(1) Given that today is sunny, what is the probability that tommorow is sunny and next day is rainy?

→ $P(S, R|S) = P(S|S) * P(R/S)$
$= 0.8 \times 0.05$
$= 0.040$

(2) Assume that yesterday's weather was rainy and today is cloudy, then what is the P. that tomorrow is sunny?

→ $P(C|R) * P(S|C) = 0.2 \times 0.2 = 0.04$

- <u>Hidden Markov Model:</u>
Consider the scenario of weather (hidden states) and moods ~~and~~ as observed variables



In the above scenario, suppose, we ~~can~~ are not aware of the weather, so the ~~t~~ weathe states are hidden from us. However, from the above eq. we have the known probability of mood depending on that day's weather. So in hidden markov model = hidden markov chain + observed variable.

Emmission matrix for the probability of observed variable.

(1) Transition Matrix.

	R	C	S
R	0.6	0.2	0.2
C	0.2	0.5	0.3
S	0.05	0.15	0.8

(2) Emmission matrix

	Sad	Happy
R	0.9	0.1
C	0.6	0.4
S	0.2	0.8

Consider the sequence of weather and mood

	Today	Tommorow	Day after-tomorrow.
Weather	S	C	S
mood	H	H	S

What is the prob. that (joint prob.) if weather seq. is SCS then the mood seq. is HHS.

→ $P(y = HHS, x = SCS) = ?$

Assume that prob. of today's weather is sunny is 0.5

$P(\text{sunny}) * P(H|S) * P(C|S) * P(H|C) * P(S|C) * P(\text{sad}|\text{sunny})$

$= 0.0384$

For a given mood sequence as HHS, what is the most likely weather sequence?

→

26/10/23

① Baysian N/W model (BNM)
② Markov chain " (MCM)
③ Hidden markov " (HMM)
④ Applications of probabilistic graphical model

Q1. Q M 5/7 × 2 = 10 → convert the prob. statement into Bayesian N/W.

eq → d-separation
→ finding conditional prob. from the graph.
→ representation of transitional probab. matrix.
→ finding conditional probab. of markov model network.
→ finding one step two step transition probab.
→ completion of TPM w.r.t. its properties.
→ state any two applications of Bayesian model
→ " " " " " Markov
→ " " " " " Hidden Markov.

Q2. → BNM ke probs.
(Rain sprinkle, Burgler alarm)

Q3. MCM (sunny, cloudy, rainy vala prob. for finding conditional probability, n-state transition prob., MC probability for given problem domain)
(car, bus, train)

Q4. (HMM) (Hidden states and visible states valu num, applications of HMM in speech recog. and AV processing

Q5. purely theory → applications of PGM.
(of all three mod...