Singular Value Decomposition:

Theorem: A rectangular Matrix $A_{m\times n}$ can be decomposed into the product of three matrices on orthogonal matrix $U_{m\times m}$ a diagonal matrix $D_{m\times n}$ and transpose of another orthogonal matrix $V_{m\times m}$

i.e. we can write $A_{m\times n} = U_{m\times m} \times D_{m\times n} \times \; V_{n\times n}^T$

Since U and V are orthogonal matrices, we have $UU^T = I, and\ VV^T = I$

Further,

The columns of U are the orthonormal vectors of AA^T, and

The columns of V are the orthonormal vectors of $A^T A$

D is a diagonal matrix whose elements are square roots of Eigen values $U\ or\ V$ arranged in decreasing order.

Finding the singular value decomposition consistent of finding eigen values and eigen vectors of AA^T and $A^T A$.

The normalized eigen vector of AA^T are the columns of U

The normalized eigen vector of $A^T A$ are the columns of V