ex $u = (2, 4, -3, 6)$ find Euclidean distn betn u & v

$v = (-3, 2, -1, 4)$

$$d(u,v) = \|u - v\| = \sqrt{(2+3)^2 + (4-2)^2 + (-3+1)^2 + (6-4)^2}$$

$$= \sqrt{25 + 4 + 4 + 4} = \sqrt{37}$$

3. Probability Distributions -

x	~~0~~	~~1~~	~~2~~	~~3~~	~~4~~
$p(x)$	~~1/2~~	~~1/4~~	~~1/8~~		

Random Variables

① Discrete Random Variable

- A random variable is said to be discrete (distinct) values of x_i & p_i

* The probability density function or prob distribution function (pdf) for discrete Random variable is

$$\sum P(x_i) = 1$$

* Mean $= E(X) = \sum x_i \cdot P(x_i)$

* Variance $= E(X^2) - [E(x)]^2$

$$E(X^2) = \sum x^2 \cdot P(x)$$

* Std Deviation $= SD = \sqrt{\text{Variance}}$

② Continuous Random Variables

* The probability distribution function for continuous random variable within the interval $[a, b]$ is defined as

$$\int_a^b f(x)\, dx = 1$$

* Mean $= E(X) = \int_a^b x \cdot f(x)\, dx$

* Variance $= E(x^2) - (E(x))^2$

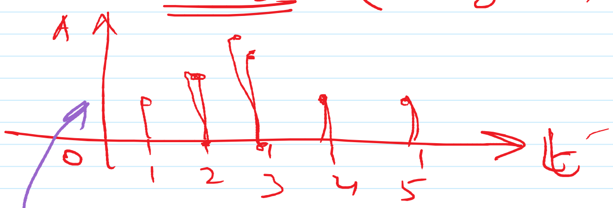
$$E(x^2) = \int_a^b x^2 \cdot f(x)\, dx$$

* $SD = \sqrt{\text{var}}$

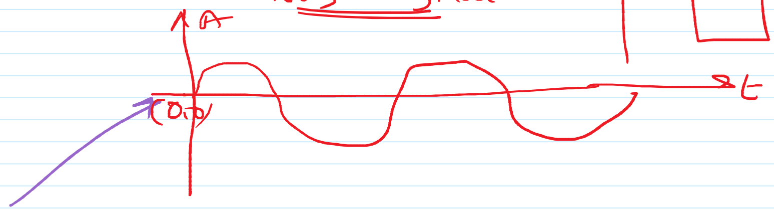
* Std Deviation $= SD = \sqrt{\text{Variance}}$

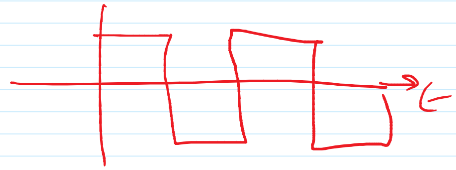
* $SD = \sqrt{\text{var}}$

* Discrete Signal (Histogram)



Continuous Signal





* A random variable x has the following probability distributions.

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

(i) Find K

(ii) Find mean, var & SD

(iii) Eval $P(x<6)$, $P(x \geq 6)$, $P(0<x<5)$, $P(2 \leq x < 5)$

(iv) Determine distribution function of x.

$\dfrac{1}{100}+\dfrac{1}{10}$

$\dfrac{17+2+1+30+20+20+10}{10} = 100$

Sol^n – The prob dist. function (pdf) for discrete random variable is – $\sum P(x) = 1$

$0 \leq p(x) \leq 1$

$P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)+P(x=7) = 1$

$0+K+2K+2K+3K+K^2+2K^2+7K^2+K = 1$

$10K^2+9K = 1$

$10K^2+9K-1 = 0$ $\qquad$ -10; $10 \;\; -1$

$10K^2+10K-K-1 = 0$

$10K(K+1)-1(K+1) = 0$

$(K+1)(10K-1) = 0$

$K+1 = 0$ or $10K-1 = 0$

$K = -1$ ✗ $\qquad \boxed{K = 1/10}$

x	$P(x)$	$x \cdot P(x)$	$E(x^2) = \sum x^2 \cdot P(x)$
0	0	0	0
1	1/10	1/10	1/10
2	2/10	4/10	8/10
3	2/10	6/10	18/10
4	3/10	12/10	48/10
5	1/100	5/100	25/100
6	2/100	12/100	72/100
7	17/100	119/100	833/100
	$\sum P = 1$	3.66	16.8

(2) Mean $= E(x) = \sum x \cdot P(x)$

$= 3.66$

(3) $\text{Var} = E(x^2) - [E(x)]^2 = 16.8 - (3.66)^2 = 3.4044$

$SD = \sqrt{\text{var}}$

$= \sqrt{3.4044}$

$\boxed{SD = 1.8451}$

$P(x<6) = P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)$

$P(x<6) = 0+\dfrac{1}{10}+\dfrac{2}{10}+\dfrac{2}{10}+\dfrac{3}{10}+\dfrac{1}{100}$

$P(x<6) = 0.81$

$P(x \geq 6) = P(x=6)+P(x=7)$

$= \dfrac{2}{100}+\dfrac{17}{100} = \dfrac{19}{100} = 0.19$

0.19

D18

x	10	11	12	13	14	15
$P(x)$	0.08	3K	6K	4K	4K	0.07

$$\sum P(x_i) = 1$$

$$0.08 + 3k + 6k + 4k + 4k + 0.07 = 1$$

$$17k + 0.15 = 1$$

$$17k = 1 - 0.15$$

$$17k = 0.85$$

$$k = \frac{0.85}{17}$$

$$\boxed{k = 0.05}$$

x	$P(x)$	$x \cdot P(x)$
10	0.08	
11	0.15	
12	0.3	
13	0.2	
14	0.2	
15	0.07	

$$\sum = 1$$

Mean $= E(x) = \sum x \cdot P(x)$

* If the mean of follo. distribution is 16. Find m, n, var & S.D

x | 8 12 16 20 24

* If the mean of follo. distribution is 16. find m, n, var & S.D

x	8	12	16	20	24
$p(x)$	$1/8$	m	n	$1/4$	$1/12$

Sol^n The pdf is $\sum P(X_i) = 1$

$$P(X=8) + P(X=12) + P(X=16) + P(X=20) + P(X=24) = 1$$

$$\frac{1}{8} + m + n + \frac{1}{4} + \frac{1}{12} = 1$$

$$m + n + \frac{3+6+2}{24} = 1$$

$$m + n + \frac{11}{24} = 1$$

$$m + n = 1 - \frac{11}{24}$$

$$\boxed{m + n = \frac{13}{24}} \quad \text{--- (1)}$$

$$\text{Mean} = E(X) = \sum x \cdot p(x) = 16$$

$$8\left(\frac{1}{8}\right) + 12(m) + 16(n) + 20\left(\frac{1}{4}\right) + 24\left(\frac{1}{12}\right) = 16$$

$$1 + 12m + 16n + 5 + 2 = 16$$

$$12m + 16n = 8 \quad \text{--- (2)}$$

$$\boxed{\begin{aligned} m &= 1/6 \\ n &= 3/8 \end{aligned}}$$

x	$p(x)$
8	$1/8$
12	$1/6$
16	$3/8$
20	$1/4$
24	$1/12$
	1

* Suppose that in a certain region the daily rainfall (in inches) is a continuous random variable x with p.d.f is given by

$f(x) = \frac{k}{}(2x - x^2)$ $\quad$ $0 \le x \le 2$ $\quad$ (i) find k (ii) Mean, var & S.D.

$< L$

is a continuous random variable x with p.d.f is given by

$f(x) = \frac{k}{4}(2x - x^2)$ $\quad 0 \le x < 2$

$= 0$ otherwise

(i) find k (ii) Mean, var & S.D.

(iii) Find prob that on a given day in this region, the rainfall is

(a) not more than 1 inch

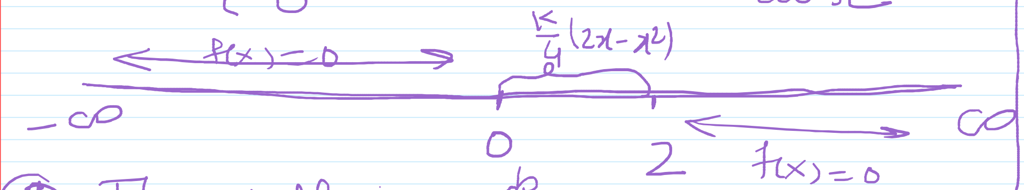
(b) greater than 1.5 inch

(c) betn 1 to 1.5 inch

(d) betn 1 to 4 inch

Soln The pdf for continuous random variable

$$f(x) = \begin{cases} \frac{k}{4}(2x - x^2) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$



(1) The pdf is $\int_a^b f(x)\,dx = 1$

$\int_0^2 f(x)\,dx = 1 \Rightarrow \int_0^2 \frac{k}{4}(2x - x^2)\,dx = 1$

$\frac{k}{4}\left[2\left(\frac{x^2}{2}\right) - \frac{x^3}{3}\right]_0^2 = 1$

$\frac{k}{4}\left[4 - \frac{8}{3}\right] = 1$

$\frac{k}{4}\left(\frac{12 - 8}{3}\right) = 1$

$\frac{k}{4}\left(\frac{4}{3}\right) = 1$

$\frac{k}{3} = 1$

$\boxed{k = 3}$

∴ The pdf for rainfall is

$$\boxed{f(x) = \frac{3}{4}(2x - x^2) \quad 0 < x < 2 \qquad = 0 \quad \text{otherwise}}$$

(2) Mean $= E(x) = \int_a^b x \cdot f(x)\,dx = \int_0^2 x \cdot \frac{3}{4}(2x - x^2)\,dx$

$= \frac{3}{4}\int_0^2 2x^2 - x^3\,dx = \frac{3}{4}\left[2\frac{x^3}{3} - \frac{x^4}{4}\right]_0^2$

$= \frac{3}{4}\left[\frac{16}{3} - \frac{16}{4}\right] = \frac{3}{4}\left(\frac{64 - 48}{12}\right) = \frac{3}{4}\left(\frac{16}{12}\right) = 1$

(3) Var $= E(x^2) - [E(x)]^2$

$E(x^2) = \int_a^b x^2 \cdot f(x)\,dx$

$E(x^2) = \int_0^2 x^2 \cdot \frac{3}{4}(2x - x^2)\,dx$

$= \frac{3}{4}\left[2\frac{x^4}{4} - \frac{x^5}{5}\right]_0^2 = \frac{3}{4}\left[8 - \frac{32}{5}\right]$

$= \frac{3}{4}\left[\frac{8}{5}\right] = 6/5 = E(x^2)$

$\frac{6}{5} - 1^2$

$= 1/5$

$= 0.2$ var

(a) prob. of ... than 1 ...

$-\infty$

$P(x < $ ...

$-\infty$

< 1

rainfall not more
inch $p(x<1)$

$f(x)$

0 1 2 ∞

$$p(x<1) = \int_{-\infty}^{1} f(x)\,dx = \underbrace{\int_{-\infty}^{0} f(x)\,dx}_{\text{zero}} + \int_{0}^{1} f(x)\,dx$$

$$= \int_{0}^{1} \frac{3}{4}(2x - x^2)\,dx$$

$$= \frac{3}{4}\left[\cancel{2}\frac{x^2}{\cancel{2}} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{3}{4}\left[1 - \frac{1}{3}\right] = \frac{\cancel{3}}{4}\left(\frac{2}{\cancel{3}}\right)$$

$$= 0.5$$

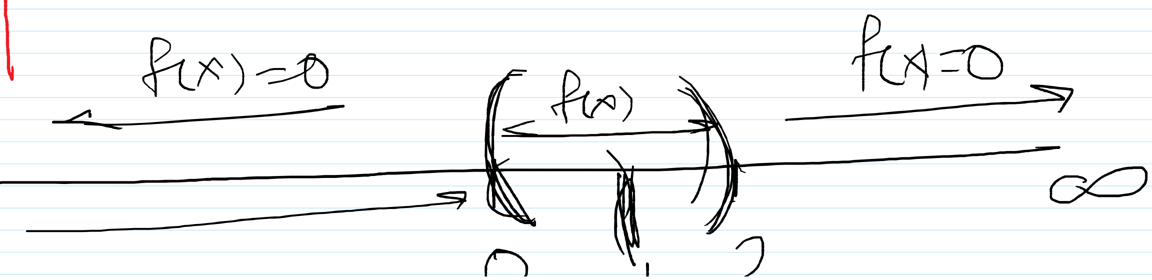
(2) prob of rainfall greater than 1.5 inch

$-\infty$ 0 $f(x)$ 2 ∞

$f(x)$

$-\infty$ 0 2 1.5 ∞

$$\int_{1.5}^{\infty} f(x)\,dx$$



< 1 $-\infty$

D-19

Q5 a) 6m

$$f(x) = \begin{cases} \frac{x}{6} + K & 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

The pdf $\int_a^b f(x)\,dx = 1$

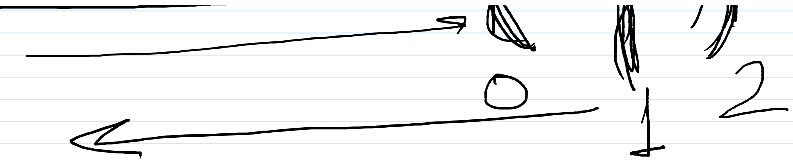
$$\int_0^3 \frac{x}{6} + K \, dx = 1$$

$$P(1 \le x \le 2)$$

$$\left[\frac{x^2}{12} + Kx\right]_0^3 = 1$$

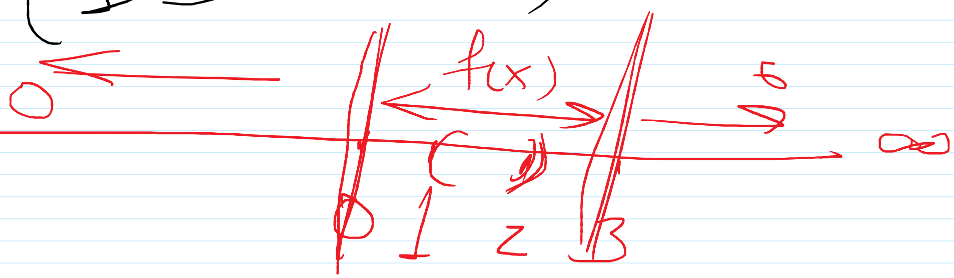
$$\left[\frac{9}{12} + 3K\right] = 1$$

$$= \left[\frac{x^2}{12} + \right.$$



d K

$p(1 \le x \le 2)$



2

$f(x)\,dx = \int_1^2 \frac{x}{6} + \frac{1}{12}\,dx$

$\frac{x}{12}\Big]^2 = \left[\frac{4}{12} + \frac{2}{12} - \frac{1}{12} - \frac{1}{12}\right]$

$$\left[\frac{ }{12} + 3k\right] = 1$$

$$\left[\frac{ }{12} + \right.$$

$$3k = 1 - \frac{9}{12}$$

$$=$$

$$\cancel{3}k = \frac{\cancel{3}}{12}$$

$$\boxed{k = 1/12}$$

~~M19~~ Q5 a) 6m

$$f(x) = \begin{cases} k(1-x^2) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(i) $P($

(ii) $P($

Sol^n The pdf is $\int_a^b f(x)\,dx = 1$

$* \; P(0$

$$\int^1 k(1-x^2)\,dx = 1$$

$0 \longleftarrow$

$$12 \int_{0.1} \left[\frac{}{12} + \frac{}{12} - \frac{1}{12} - \frac{1}{12} \right]$$

$$\frac{4+2-1-1}{12} = \frac{4}{12} = \frac{1}{3}$$

$(0.1 < x < 0.2)$

$(x > 0.5)$

$$(0.1 < x < 0.2) = \int_{0.1}^{0.2} f(x)\,dx$$

0

$f(x)$

∞

$$\int_0^1 K(1-x^2)\,dx = 1$$

$$K\left[x - \frac{x^3}{3}\right]_0^1 = 1$$

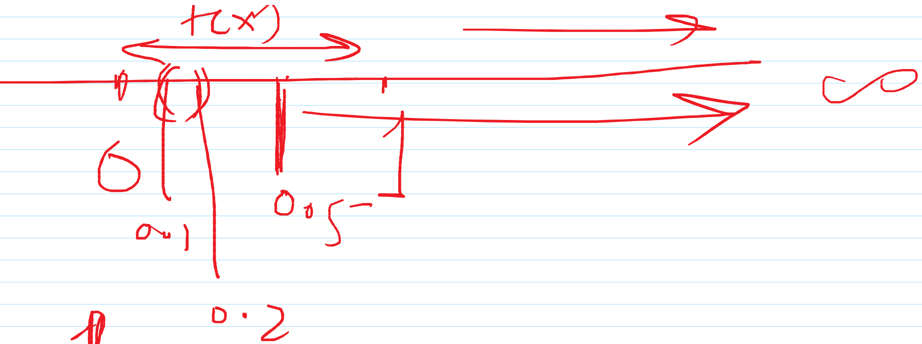
$$K\left[1 - \frac{1}{3}\right] = 1$$

$$K\left(\frac{2}{3}\right) = 1$$

$$\boxed{K = 3/2}$$

$-\infty$ 0

* $P(x > 0.5) =$



$$\int_{0.5}^{1} f(x)\,dx$$

M18 / Q6) b) 6m

$$f(x) = Kx^2 e^{-x} \quad \text{for } x > 0$$
$$= 0 \quad \text{otherwise}$$

$e^0 = 1$

$e^{\infty} = 0 \qquad e^{-\infty} = \frac{1}{e^{\infty}}$

Sol^n The pdf for CRV is $\int_a^b f(x)\,dx = 1$

$$\int_0^\infty K x^2 e^{-x}\,dx = 1$$

$$K\int_0^\infty \underset{u}{x^2} \cdot \underset{v}{e^{-x}}\,dx = 1$$

$$K\left[(x^2)\left(\frac{e^{-x}}{-1}\right) - (2x)\left(\frac{e^{-x}}{-1\times-1}\right) + (2)\left(\frac{e^{-x}}{-1\times-1\times-1}\right)\right]_0^\infty = 1$$

$$K\left\{[0-0+0] - [0-0-2]\right\} = 1$$

$$2K = 1$$

$$\boxed{K = 1/2}$$

Probability Distributions — ① Poisson Distribution
② Normal Distribution