- HW1 grades are not finalized yet. When the grades are finalized, the TAs will make a Piazza post. You can expect this post by the end of Saturday.

- HW2 : Selecting pages
  - deadline has passed, but we will give you until 12:30 pm IST on Friday (2.5 hours after our session ends) to do the needful.

- Starting HW3 no such exceptions will be made.

Exam 1 : Feb, 20

- re-create lectures, hws, recitations.
- memorization will not help much.
- 8 questions, 90 minutes

- You may use any result proven in lecture, hws, recitations without justification
for e.g. you may assume $\sqrt{2}$ is irrational,
but you may not assume π is irrational.

Stable Matching.

Input : . n people , n pets.

- each person ranks the pets in descending order of their preferences

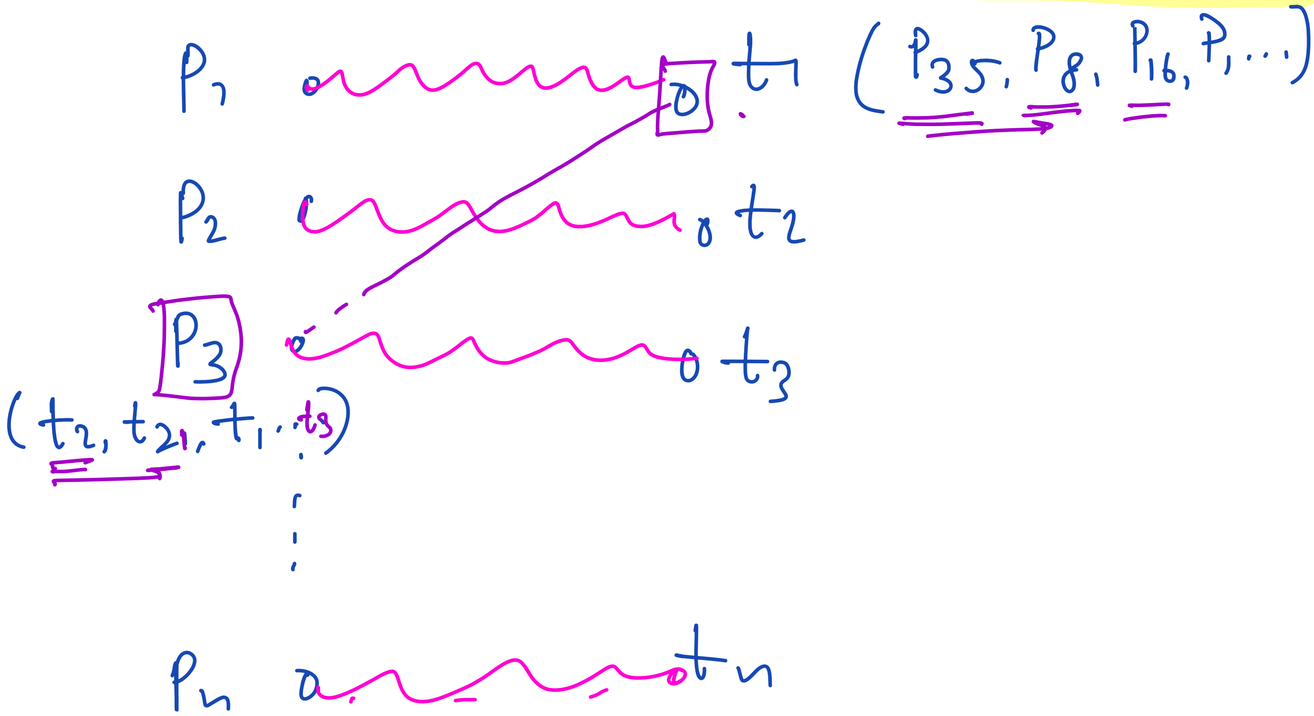
- each pet ranks the people in descending order of their preferences

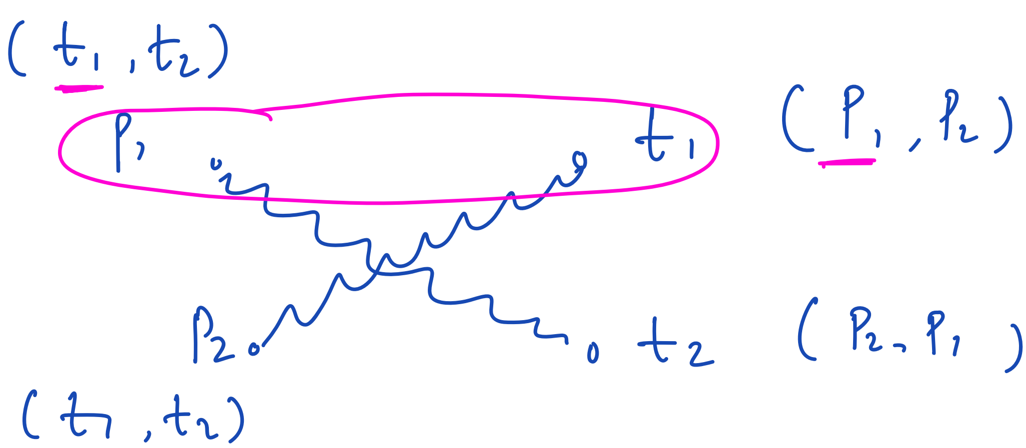
- no ties

perfect matching.

Objective : (To pair each person with exactly one pet and each pet with exactly one person,) such that no instability occurs.

no person & no pet must prefer each other over their existing partners.

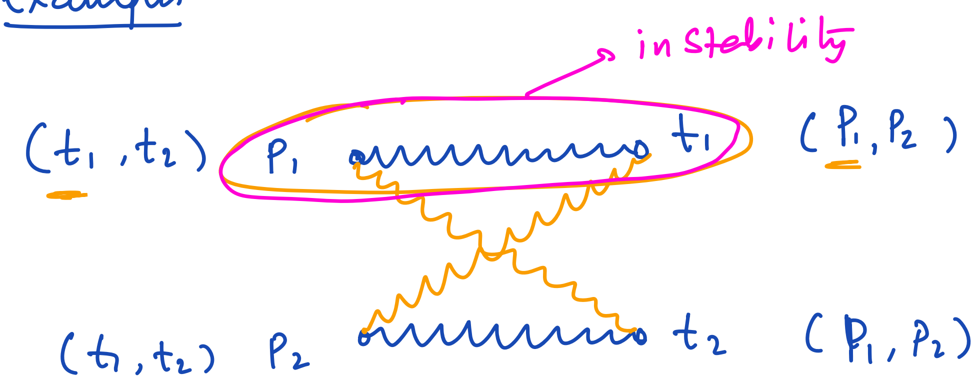


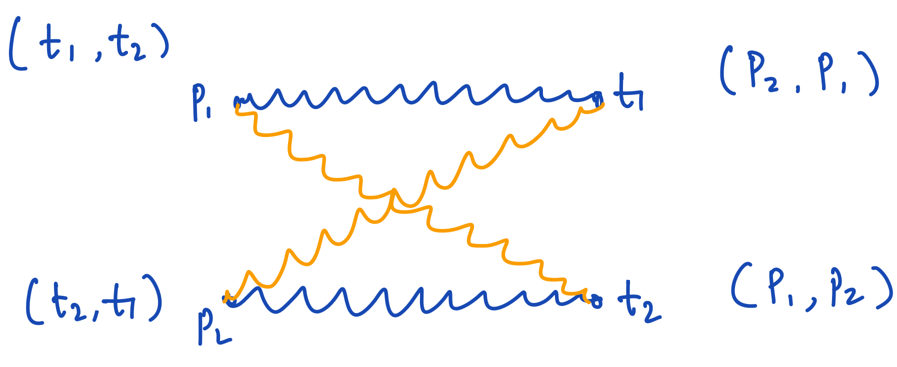


Q1: Does every input have a stable matching?

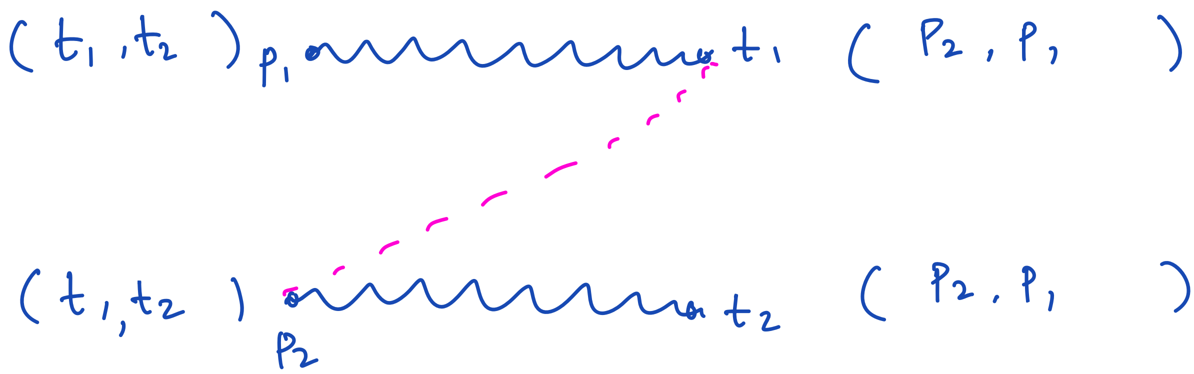
Q2: Can an input have more than one stable matching?

Examples





example with multiple stable matchings



<u>Algorithm</u>.

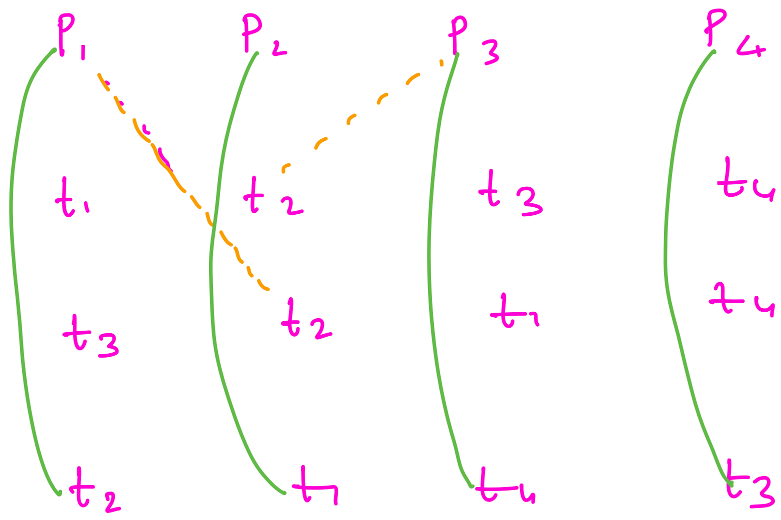
1. Line all the people in a row.

2. for each permutation/ordering of the pets :

   - pair person i with pet in the ith posn in the ordering.

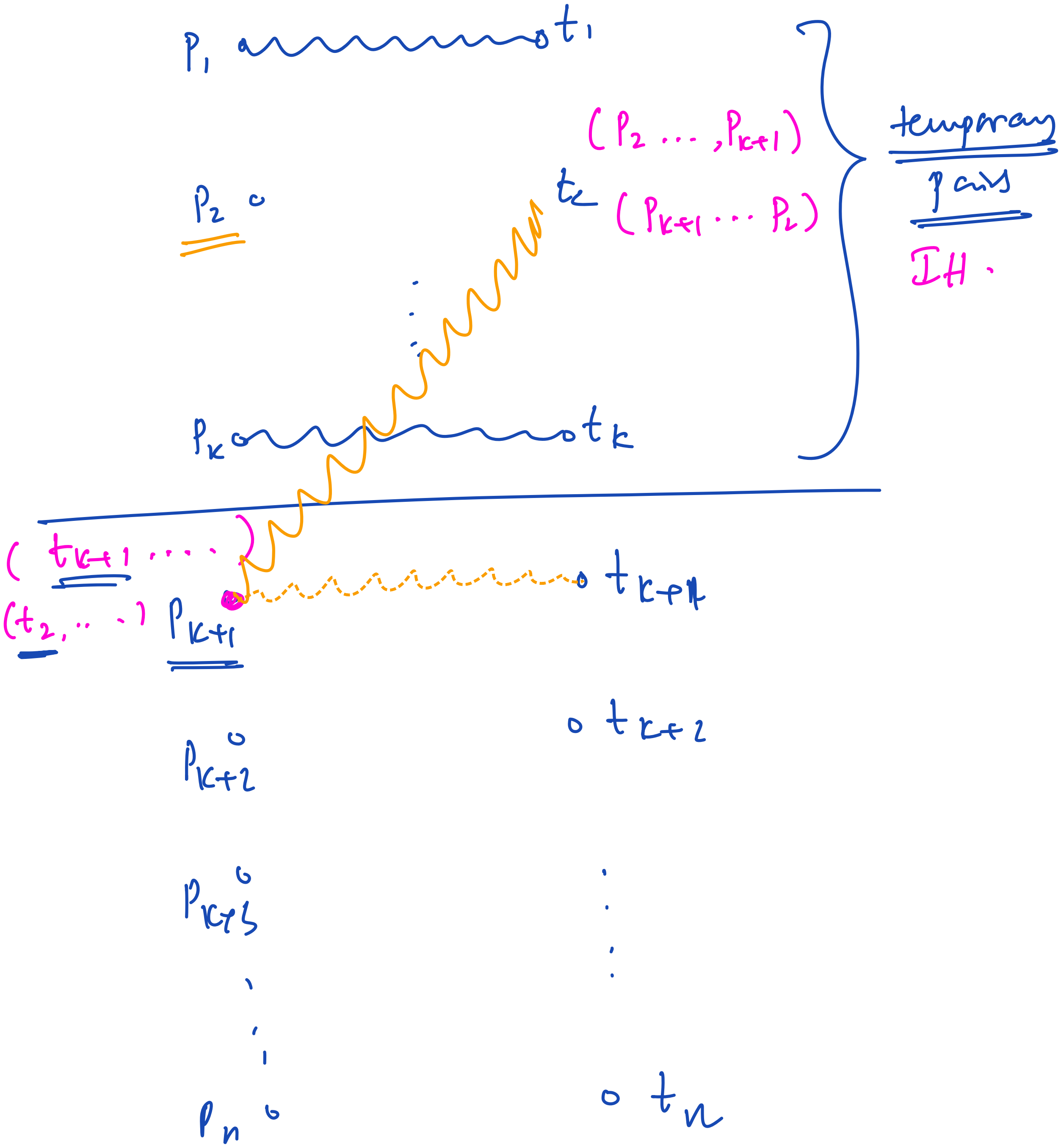
   . if there is no instability then o/p the matching

3. return false (no stable matching)



The above algorithm will always output a correct answer, but it is very inefficient.

The above algorithm takes atleast ___n!___ steps.



p1
t1
(p2 ... , pk+1)
t2
(pk+1 ... p2)
temporary pairs
IH.
p2
pk
tk
(tk+1 , ...)
(t2 , .. .)
pk+1
tk+1
pk+2
tk+2
pk+3
pn
tn

Gale-Shapley algorithm. (GS alg)

1. Initially all people & all pets are free.

2. while there is a free person p who has not yet proposed to all pets do

3. t ← highest ranked pet whom p has not yet proposed to.

4. if t is free then

5. (p, t) becomes a pair

6. else if (p', t) is a pair then

7. if t prefers p over p' then

8. (p, t) becomes a pair

9. p' becomes free

10. return all matched pairs.

Q. Does this algorithm always terminate?

Yes.

Q. # iterations of the while loop $\leq$ ~~n^n~~.

n^2

- each person proposes to a pet at most once. There are n people. Thus n^2 proposals in the maximum.

Lemma: Once a pet receives their first proposal, they always remain engaged and as the algorithm progresses, their partners can only get better.

* Question: Does the GS alg favor the people or the pets?

Lemma: The GS alg. outputs a perfect matching.

↳ matching in which each person is paired with exactly one pet & each pet is paired with exactly one person.

Proof : Assume for contradiction that the o/p of the GS alg is not a perfect matching.

Clearly, each person is paired with at most one pet & each pet is paired with at most one person.

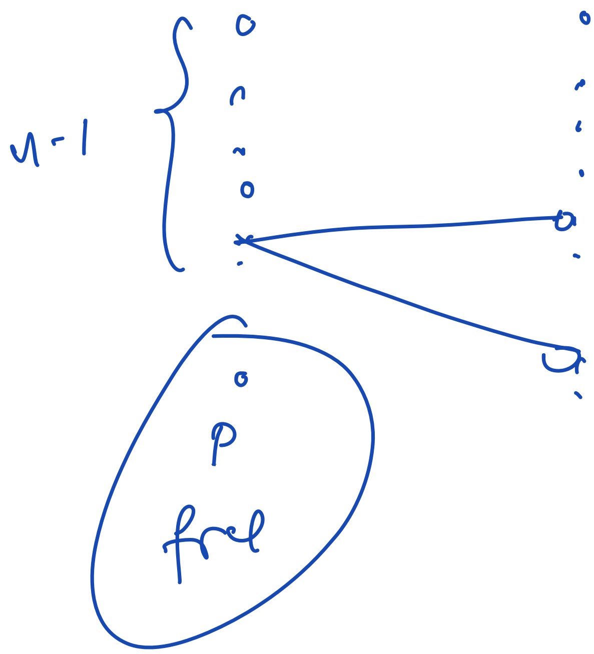
Case I : at the end of the GS alg, person p is free.

- Since the alg. has ended, it must be that p has proposed to every single pet.

- By the previous lemma, every

pet must be engaged.

- This means that two pets are paired with the same person., a <u>contradiction</u>.



<u>Case II</u> : a pet is free.