7 $y = wx + b$

$w = 0.5 \quad b = 0.1$

for $x = 1$ $\qquad \eta = 0.01$

$y = wx$

$\hat{y} = (0.5 \times 1) + 0.1 = 0.6$

gradient (weight) $= \frac{\partial L}{\partial w} = -(y - \hat{y})x$

$= -(3 - 0.6) \times 1$

$= -2.4$

(gradient bais) $\frac{\partial L}{\partial b} = -(3 - 0.6)$

$= -2.4$

Updated weight $= w - \alpha \frac{\partial L}{\partial w} = 0.5 - (0.01)(-2.4) = 0.524$

Updated bais $= b - \alpha \frac{\partial b}{\partial w} = 0.1 - (0.01)(-2.4) = 0.124$

for $x = 2$

$\hat{y} = wx + b$

$\hat{y} = 0.524 \times 2 + 0.124$

$\hat{y} = 1.172$

gradient (weight) $= \frac{\partial L}{\partial w} = -(y - \hat{y})x = -7.656$

$= -(5 - 1.172) \times 2$

gradient bias $= \frac{\partial L}{\partial b} = -(5 - 1.172) = -3.828$

Updated weight $= w - \alpha \frac{\partial L}{\partial w} = 0.524 - 0.01(-7.656)$

$= 0.600$

Updated bias $= b - \alpha \frac{\partial L}{\partial b}$

$= 0.124 - 0.01(-3.828)$

$= 0.162$

for $x = 3$

$\hat{y} = (0.6 \times 3) + 0.162 = 1.962$

gradient weight $= -(7 - 1.962) \times 3 = -15.114$

gradient bias $= -(7 - 1.962) = -5.038$

Updated weight $= 0.600 - 0.01(-15.114)$

$= 0.7512$

Updated bias $= 0.162 - 0.01(-5.038)$

$= 0.212$

$x = 4$

Updated weight $= 0.9827$

Updated bais $= 0.2703$

$x = 5$

Updated weight $= 1.2737$

Updated bias $= 0.3287$

final weights after epoch

MSE after applying SGD optimization Algorithm

x	y	$\hat{y}$	
$x=1$	3	$\hat{y} = 1 \times 1.2737 + 0.3287$	$(3 - 1.602)^2$
$x=2$	5	$\hat{y} = 2 \times 1.2737 + 0.3287$	$(5 - 2.8733)^2$
$x=3$	7	$\hat{y} = 3 \times 1.2737 + 0.3287$	$(7 - 4.146)^2$
$x=4$	9	$\hat{y} = 4 \times 1.2737 + 0.3287$	$(9 - 5.4111)^2$
$x=5$	11	$\hat{y} = 5 \times 1.2737 + 0.3287$	$(11 - 6.6924)^2$

f

MSE after SGD $= \frac{1}{2 \times 5}(1.954 + 4.524 + 8.145 + 12.880 + 18.558)$

$= \frac{1}{10}(46.005)$

MSE $= 4.6005$