Ex: Mean thickness of aluminum sheet ($\mu$) =.04, **$\sigma$ = 0.004**

n= 100, $\overline{X}$ = 0.0408,

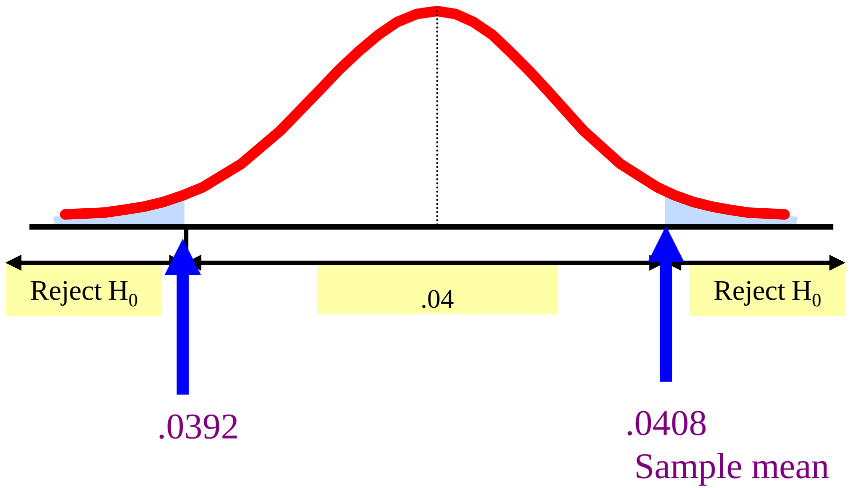
$$Z_{STAT} = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Z = (0.0408-0.04)/0.0004 = 2

0.04+- (2)* (0.0004)

=0.0392 and 0.0408

Reject company's claim that population mean is 0.04



Ex: Mean thickness of aluminum sheet ($\mu$) =.04, **$\sigma$ = 0.004**

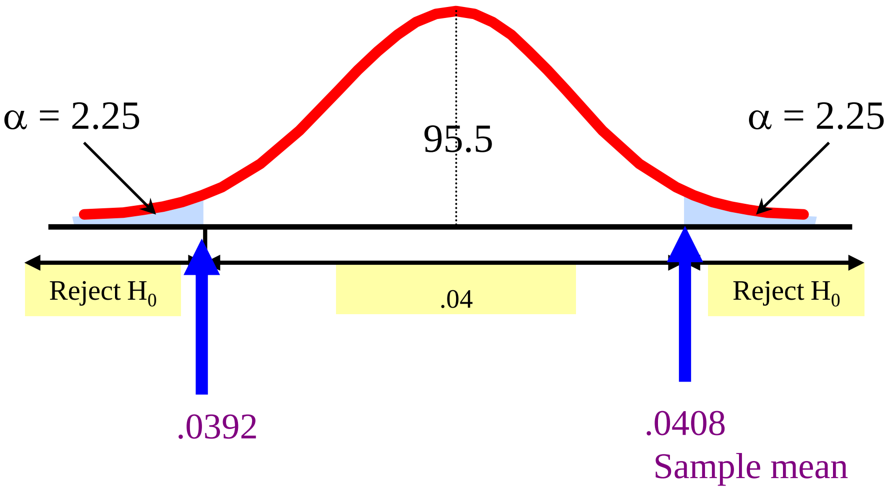
n= 100, $\overline{X}$ = 0.0408,

$$Z_{STAT} = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Z = (0.0408-0.04)/0.0004 = 2

0.04+- (2)* (0.0004)

=0.0392 and 0.0408




Reject company's claim that population mean is 0.04

Ex: Axel strength is 80000 pounds per square inch

$\mu$ $H_0$ = 80000 (Hypothesized value of population mean)

$\sigma$ = 4000

n=100

$\bar{X}$ = 79600

Significance level=.05

Ex: Axel strength is 80000 pounds per square inch

$\mu$ $H_0$ = 80000 (Hypothesized value of population mean)

$\sigma$ = 4000

n=100

$\bar{X}$ = 79600

Significance level=.05

Soltn: $\mu$= 80000 (NH)

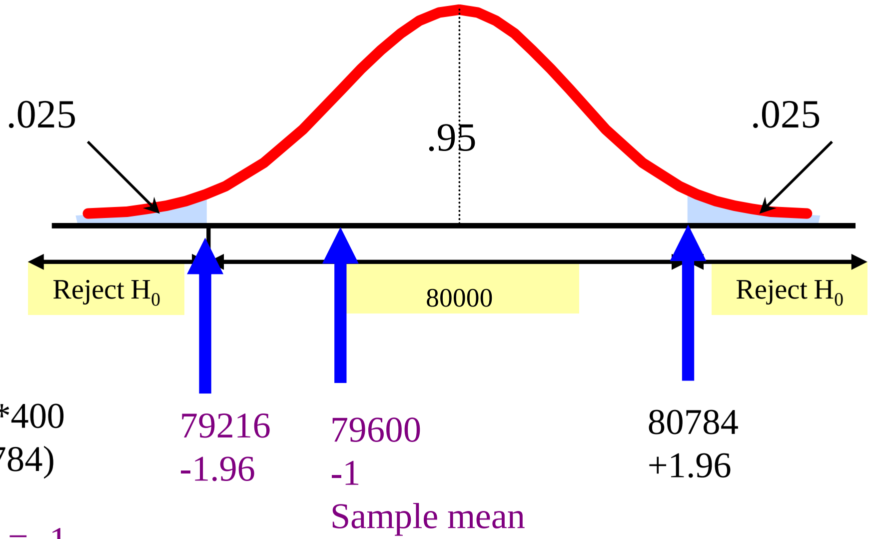
$\mu \neq$ 80000 (AH)

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

= 400

=80000+-1.96*400

(79216 to 80784)




Z = (79600-80000)/400 = -1

Accept (Do not reject ) the null hypothesis.

Ex: Drug dose of 100cc, excess dose is not harmful but insufficient dose does not produce results.

$\mu_{H_0} = 100$ (Hypothesized value of population mean)

$\sigma = 2$

$n=50$

$\bar{X} = 99.75$

Significance level=.10

Soltn: $\mu = 100$ (NH)

$\mu < 100$ (AH)

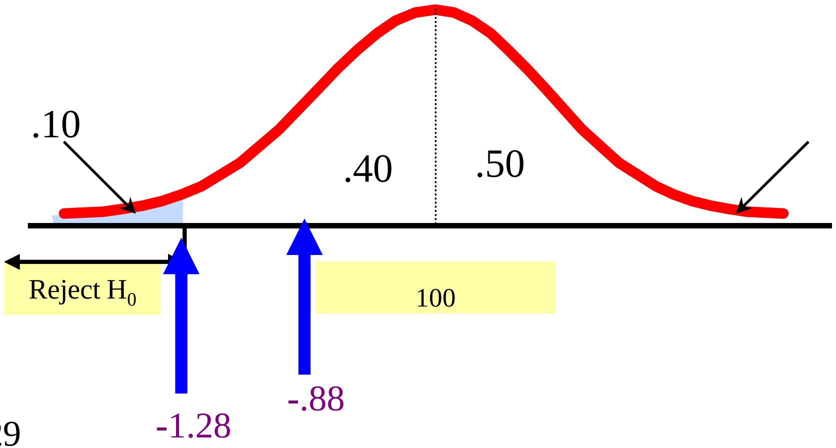
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$=2/7.07$

$= .2829$

$100 \pm 1.28 * .2829$

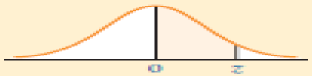
100.36 and 99.63




Look in z table where area is .40

Z = (99.75-100)/.2829= -.88

So, accept (Do not reject ) null hypothesis.




SECOND DECIMAL PLACE IN $z$

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 3.1 | .4990 | .4991 | .4991 | .4991 | .4992 | .4992 | .4992 | .4992 | .4993 | .4993 |
| 3.2 | .4993 | .4993 | .4994 | .4994 | .4994 | .4994 | .4994 | .4995 | .4995 | .4995 |
| 3.3 | .4995 | .4995 | .4995 | .4996 | .4996 | .4996 | .4996 | .4996 | .4996 | .4997 |
| 3.4 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4998 |
| 3.5 | .4998 | | | | | | | | | |
| 4.0 | .49997 | | | | | | | | | |
| 4.5 | .499997 | | | | | | | | | |
| 5.0 | .4999997 | | | | | | | | | |
| 6.0 | .499999999 | | | | | | | | | |

3. Hinton press hypothesizes that the average life of its largest web press is 14,500 hours. They know that the standard deviation of press life is 2,100 hours. From a sample of 25 presses, the company finds a sample mean of 13,000 hours. At a 0.01 significance level, should the company conclude that the average life of the presses is less than the hypothesized 14,500 hours?

## Solution:

σ = 2100 n = 25 $\bar{x} = 13000$

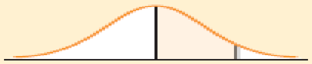
Ho: μ = 14500 H1: μ < 14500 α = 0.01

The lower limit of the acceptance region is z = -2.33, or

$\bar{x} = \mu_{Ho} - z\frac{\sigma}{\sqrt{n}}$ = 14500 - 2.33 $\frac{2100}{\sqrt{25}}$ = 13521.4 hours

Because the observed z value = $\frac{\bar{x} - \mu_{Ho}}{\frac{\sigma}{\sqrt{n}}} = \frac{13000 - 14500}{\frac{2100}{\sqrt{25}}}$ = -3.57 < -2.33

(or $\bar{x}$ < 13521.4), we **should reject Ho**. The average life is significantly less than the hypothesized value.)




SECOND DECIMAL PLACE IN z

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 3.1 | .4990 | .4991 | .4991 | .4991 | .4992 | .4992 | .4992 | .4992 | .4993 | .4993 |
| 3.2 | .4993 | .4993 | .4994 | .4994 | .4994 | .4994 | .4994 | .4995 | .4995 | .4995 |
| 3.3 | .4995 | .4995 | .4995 | .4996 | .4996 | .4996 | .4996 | .4996 | .4996 | .4997 |
| 3.4 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4998 |
| 3.5 | .4998 | | | | | | | | | |
| 4.0 | .49997 | | | | | | | | | |
| 4.5 | .499997 | | | | | | | | | |
| 5.0 | .4999997 | | | | | | | | | |
| 6.0 | .499999999 | | | | | | | | | |

# p-Value Approach to Testing

- p-value: Probability of obtaining a **test statistic equal to or more extreme** than the **observed sample value** given $H_0$ is true

  Z → Ztable

  – The p-value is also called the **observed** level of significance

  – **It is the smallest value of $\alpha$ for which $H_0$ can be rejected**

# p-Value Approach to Testing: Interpreting the p-value

- Compare the p-value with $\alpha$
  - If p-value $<$ $\alpha$, reject $H_0$
  - If p-value $\geq$ $\alpha$, do not reject $H_0$

- Remember

  - If the p-value is low then $H_0$ must go

# The p-value approach to Hypothesis Testing

1. State the null hypothesis, $H_0$ and the alternative hypothesis, $H_1$

2. Choose the level of significance, $\alpha$, and the sample size, n

3. Determine the appropriate test statistic and sampling distribution

4. Collect data and compute the value of the test statistic and the p-value

5. Make the statistical decision and state the managerial conclusion. **If the p-value is < $\alpha$ then reject $H_0$, otherwise do not reject $H_0$.** State the managerial conclusion in the context of the problem

# p-value Hypothesis Testing Example

**Test the claim that the true mean # of TV sets in Indian homes is equal to 3.**

**(Assume $\sigma = 0.8$)**

1. State the appropriate null and alternative hypotheses
   - $H_0: \mu = 3$ $H_1: \mu \neq 3$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   - Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test

# p-value Hypothesis Testing Example

*(continued)*

3.Determine the appropriate technique

- σ is assumed known so this is a Z test.

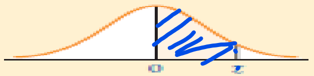
4. Collect the data, compute the test statistic and the p-value

- Suppose the sample results are

  $n = 100, \quad \bar{X} = 2.84 \quad (\sigma = 0.8 \text{ is assumed known})$

So the test statistic is:

$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$




SECOND DECIMAL PLACE IN $z$

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
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| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 3.1 | .4990 | .4991 | .4991 | .4991 | .4992 | .4992 | .4992 | .4992 | .4993 | .4993 |
| 3.2 | .4993 | .4993 | .4994 | .4994 | .4994 | .4994 | .4994 | .4995 | .4995 | .4995 |
| 3.3 | .4995 | .4995 | .4995 | .4996 | .4996 | .4996 | .4996 | .4996 | .4996 | .4997 |
| 3.4 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4997 | .4998 |
| 3.5 | .4998 | | | | | | | | | |
| 4.0 | .49997 | | | | | | | | | |
| 4.5 | .499997 | | | | | | | | | |
| 5.0 | .4999997 | | | | | | | | | |
| 6.0 | .499999999 | | | | | | | | | |

# p-Value Hypothesis Testing Example: Calculating the p-value

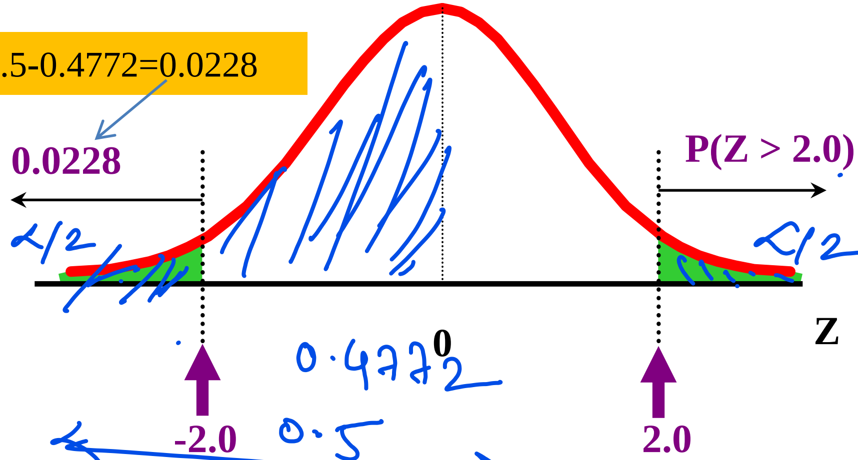
4. (continued) Calculate the p-value.

- How likely is it to get a $Z_{STAT}$ of -2 (or something further from the mean (0), in either direction) if $H_0$ is true?

0.5-0.4772=0.0228

P(Z < -2.0) = 0.0228

P(Z > 2.0) = 0.0228




$\frac{\alpha}{2} + \frac{\alpha}{2}$

$= \alpha$

$= 0.0228 + 0.0228$

$0.0456$

$0.0228$
$0.0228$

$\alpha = 0.0456.$

**p-value = 0.0228 + 0.0228 = 0.0456**

# p-value Hypothesis Testing Example

*(continued)*

Computed    assume value

- 5. Is the p-value < α?
  - Since p-value = 0.0456 < α = **0.05 Reject $H_0$**
- 5. (continued) State the managerial conclusion in the context of the situation.
  - **There is sufficient evidence to conclude the average number of TVs in Indian homes is not equal to 3.**

# Connection Between Two Tail Tests and Confidence Intervals

$\boxed{\bar{x} = \mu}$

$= \bar{x} \pm Z\sigma_{\bar{x}}$.

$\mu$

- For $\bar{X} = 2.84$, $\sigma = 0.8$ and $n = 100$, the 95% confidence interval is:

$$\boxed{2.84 - (1.96)\frac{0.8}{\sqrt{100}} \quad \text{to} \quad 2.84 + (1.96)\frac{0.8}{\sqrt{100}}}$$

$$\boxed{2.6832 \le \mu \le 2.9968}$$

$\boxed{\mu = 3}$

- Since this interval does not contain the **hypothesized mean (3.0),** we reject the null hypothesis at $\alpha = 0.05$

# Do You Ever Truly Know σ?

- Probably not!
- In virtually all real world business situations, σ is not known.
- If there is a situation where σ is known then μ is also known (since to calculate σ you need to know μ.)
- If you truly know μ there would be no need to gather a sample to estimate it.

# Hypothesis Testing: σ Unknown

- If the population standard deviation is unknown, you instead **use the sample standard deviation S.**
- Because of this change, you **use the t distribution** instead of the Z distribution to test the null hypothesis about the mean.
- When using the t distribution you must assume the population you are sampling from follows a **normal distribution**.
- All other steps, concepts, and conclusions are the same.

# t Test of Hypothesis for the Mean (σ Unknown)

- Convert sample statistic ( $\bar{X}$ ) to a $t_{STAT}$ test statistic

Hypothesis Tests for μ

- σ Known (Z test)
- σ Unknown (t test)

The test statistic is: t-values

$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

σ → Population std. dev

Z-value

α → level of significance

sample std dev

# Example: Two-Tail Test
# ($\sigma$ Unknown)

The average cost of a hotel room in New Delhi is said to be Rs168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in an $\overline{X}$ of Rs 172.50 and an S of Rs.15.40. Test the appropriate hypotheses at $\alpha$ = 0.05.

(Assume the population distribution is normal)

$H_0: \mu = 168 \quad H_1: \mu \neq 168$

# Example: Two-Tail Test ($\sigma$ Unknown)

The average cost of a hotel room in New Delhi is said to be Rs168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in an $\bar{X}$ of Rs 172.50 and an S of Rs.15.40. Test the appropriate hypotheses at $\alpha = 0.05$.

(Assume the population distribution is normal)

**$H_0$: ????**

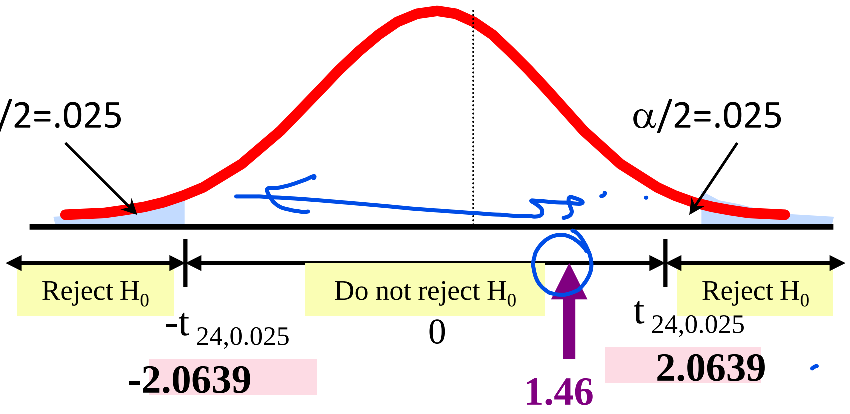
**$H_1$: ????**

# Example Solution: Two-Tail t Test

$H_0: \mu = 168 \quad H_1: \mu \neq 168$

t test

σ is unknown




- $\alpha = 0.05$
- n = 25, df = 25-1=24 (n-1)
- σ is unknown, so use a **t statistic**
- Critical Value: table

$\pm t_{24,0.025} = \pm 2.0639$

df = 24

α = 0.025

$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

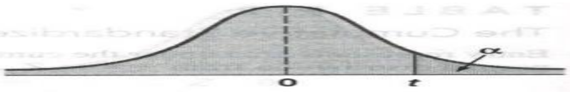
Accept H0

**Do not reject $H_0$:** insufficient evidence that true mean cost is different than Rs.168

**TABLE E.3**

Critical Values of $t$

For a particular number of degrees of freedom, entry represents the critical value of $t$ corresponding to the cumulative probability $(1 - \alpha)$ and a specified upper-tail area $(\alpha)$.




| Degrees of Freedom | Cumulative Probabilities 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 |
|---|---|---|---|---|---|---|
| | Upper-Tail Areas 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 1 | 1.0000 | 3.0777 | 6.3138 | 12.7062 | 31.8207 | 63.6574 |
| 2 | 0.8165 | 1.8856 | 2.9200 | 4.3027 | 6.9646 | 9.9248 |
| 3 | 0.7649 | 1.6377 | 2.3534 | 3.1824 | 4.5407 | 5.8409 |
| 4 | 0.7407 | 1.5332 | 2.1318 | 2.7764 | 3.7469 | 4.6041 |
| 5 | 0.7267 | 1.4759 | 2.0150 | 2.5706 | 3.3649 | 4.0322 |
| 6 | 0.7176 | 1.4398 | 1.9432 | 2.4469 | 3.1427 | 3.7074 |
| 7 | 0.7111 | 1.4149 | 1.8946 | 2.3646 | 2.9980 | 3.4995 |
| 8 | 0.7064 | 1.3968 | 1.8595 | 2.3060 | 2.8965 | 3.3554 |
| 9 | 0.7027 | 1.3830 | 1.8331 | 2.2622 | 2.8214 | 3.2498 |
| 10 | 0.6998 | 1.3722 | 1.8125 | 2.2281 | 2.7638 | 3.1693 |
| 11 | 0.6974 | 1.3634 | 1.7959 | 2.2010 | 2.7181 | 3.1058 |
| 12 | 0.6955 | 1.3562 | 1.7823 | 2.1788 | 2.6810 | 3.0545 |
| 13 | 0.6938 | 1.3502 | 1.7709 | 2.1604 | 2.6503 | 3.0123 |
| 14 | 0.6924 | 1.3450 | 1.7613 | 2.1448 | 2.6245 | 2.9768 |
| 15 | 0.6912 | 1.3406 | 1.7531 | 2.1315 | 2.6025 | 2.9467 |
| 16 | 0.6901 | 1.3368 | 1.7459 | 2.1199 | 2.5835 | 2.9208 |
| 17 | 0.6892 | 1.3334 | 1.7396 | 2.1098 | 2.5669 | 2.8982 |
| 18 | 0.6884 | 1.3304 | 1.7341 | 2.1009 | 2.5524 | 2.8784 |
| 19 | 0.6876 | 1.3277 | 1.7291 | 2.0930 | 2.5395 | 2.8609 |
| 20 | 0.6870 | 1.3253 | 1.7247 | 2.0860 | 2.5280 | 2.8453 |
| 21 | 0.6864 | 1.3232 | 1.7207 | 2.0796 | 2.5177 | 2.8314 |
| 22 | 0.6858 | 1.3212 | 1.7171 | 2.0739 | 2.5083 | 2.8188 |
| 23 | 0.6853 | 1.3195 | 1.7139 | 2.0687 | 2.4999 | 2.8073 |
| 24 | 0.6848 | 1.3178 | 1.7109 | 2.0639 | 2.4922 | 2.7969 |
| 25 | 0.6844 | 1.3163 | 1.7081 | 2.0595 | 2.4851 | 2.7874 |
| 26 | 0.6840 | 1.3150 | 1.7056 | 2.0555 | 2.4786 | 2.7787 |
| 27 | 0.6837 | 1.3137 | 1.7033 | 2.0518 | 2.4727 | 2.7707 |
| 28 | 0.6834 | 1.3125 | 1.7011 | 2.0484 | 2.4671 | 2.7633 |
| 29 | 0.6830 | 1.3114 | 1.6991 | 2.0452 | 2.4620 | 2.7564 |
| 30 | 0.6828 | 1.3104 | 1.6973 | 2.0423 | 2.4573 | 2.7500 |
| 31 | 0.6825 | 1.3095 | 1.6955 | 2.0395 | 2.4528 | 2.7440 |
| 32 | 0.6822 | 1.3086 | 1.6939 | 2.0369 | 2.4487 | 2.7385 |
| 33 | 0.6820 | 1.3077 | 1.6924 | 2.0345 | 2.4448 | 2.7333 |
| 34 | 0.6818 | 1.3070 | 1.6909 | 2.0322 | 2.4411 | 2.7284 |
| 35 | 0.6816 | 1.3062 | 1.6896 | 2.0301 | 2.4377 | 2.7238 |
| 36 | 0.6814 | 1.3055 | 1.6883 | 2.0281 | 2.4345 | 2.7195 |
| 37 | 0.6812 | 1.3049 | 1.6871 | 2.0262 | 2.4314 | 2.7154 |
| 38 | 0.6810 | 1.3042 | 1.6860 | 2.0244 | 2.4286 | 2.7116 |
| 39 | 0.6808 | 1.3036 | 1.6849 | 2.0227 | 2.4258 | 2.7079 |
| 40 | 0.6807 | 1.3031 | 1.6839 | 2.0211 | 2.4233 | 2.7045 |
| 41 | 0.6805 | 1.3025 | 1.6829 | 2.0195 | 2.4208 | 2.7012 |
| 42 | 0.6804 | 1.3020 | 1.6820 | 2.0181 | 2.4185 | 2.6981 |
| 43 | 0.6802 | 1.3016 | 1.6811 | 2.0167 | 2.4163 | 2.6951 |
| 44 | 0.6801 | 1.3011 | 1.6802 | 2.0154 | 2.4141 | 2.6923 |
| 45 | 0.6800 | 1.3006 | 1.6794 | 2.0141 | 2.4121 | 2.6896 |
| 46 | 0.6799 | 1.3002 | 1.6787 | 2.0129 | 2.4102 | 2.6870 |
| 47 | 0.6797 | 1.2998 | 1.6779 | 2.0117 | 2.4083 | 2.6846 |
| 48 | 0.6796 | 1.2994 | 1.6772 | 2.0106 | 2.4066 | 2.6822 |
| 49 | 0.6795 | 1.2991 | 1.6766 | 2.0096 | 2.4049 | 2.6800 |
| 50 | 0.6794 | 1.2987 | 1.6759 | 2.0086 | 2.4033 | 2.6778 |

# Example Two-Tail t Test Using A p-value from Excel

- Since this is a t-test we cannot calculate the p-value without some calculation aid.
- The Excel output below does this:

**t Test for the Hypothesis of the Mean**

| Data | | |
|---|---|---|
| **Null Hypothesis μ=** | **$** | **168.00** |
| **Level of Significance** | | **0.05** |
| **Sample Size** | | **25** |
| **Sample Mean** | **$** | **172.50** |
| **Sample Standard Deviation** | **$** | **15.40** |

| Intermediate Calculations | | | |
|---|---|---|---|
| Standard Error of the Mean | $ | 3.08 | =B8/SQRT(B6) |
| Degrees of Freedom | | 24 | =B6-1 |
| **t test statistic** | | **1.46** | =(B7-B4)/B11 |

| Two-Tail Test | | |
|---|---|---|
| Lower Critical Value | -2.0639 | =-TINV(B5,B12) |
| Upper Critical Value | 2.0639 | =TINV(B5,B12) |
| p-value | 0.157 | =TDIST(ABS(B13),B12,2) |
| Do Not Reject Null Hypothesis | | =IF(B18<B5, "Reject null hypothesis", "Do not reject null hypothesis") |

p-value > α
So do not reject $H_0$