CSC 402
Practice Problems for Exam 1
February 16, 2023

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1. Prove that for any prime p, $\sqrt{p}$ is irrational.

2. For sets A, B, C, and D, suppose that $A \setminus B \subseteq C \cap D$ and $x \in A$. Prove that if $x \notin D$ then $x \in B$.

3. Prove that if for some integer a, $a \geq 3$, then $a^2 > 2a + 1$.

4. Consider an undirected graph G with minimum degree $\delta(G) \geq 2$. Prove that G has a path of length $\delta(G)$ and a cycle with at least $\delta(G) + 1$ vertices.

5. Let G be a connected graph where all vertices are of even degree. Prove that G has no *cut edges*. A *cut edge* is an edge, that if removed, would increase the number of connected components of the graph.

6. An angel tells you in a dream that every connected graph has a connected subgraph that is a tree, which retains all the vertices of the original graph (called a *spanning tree*). The angel also tells you a procedure that allows you to find that exact subgraph given any connected graph, G. The following is a procedure: We will keep adding edges to a subgraph H of G so that at the end H is a spanning tree of G. Initially H has no edges and $V(H) := V(G)$. While H has more than 1 component, find an edge in G that has endpoints in two different components of H and add it to H. Prove the following properties:

   a. If H has more than 1 component, there is some edge in G whose endpoints lie in different components of H.

   b. At all times H is an acyclic graph.

   c. When this procedure terminates, H will be a spanning tree of G.