

* Double Dibble Method

00

01

10

11

100

101

110

111

Base \rightarrow Radix \rightarrow (0 to r-1)

* Decimal Number System

- In this number system, 10 digits are used to form a number. These digits are 0 to 9. As there are 10 digits the base of the decimal number is 10. The base is also called as radix.
- Number system is a positional system in which the position of a digit specifies the weighted significance attached to it.

Ex. i] 2432

ii] 2432.55

$$2 \times 1000 = 2 \times 10^3$$

$$4 \times 100 = 4 \times 10^2$$

$$3 \times 10 = 3 \times 10^1$$

$$2 \times 1 = 2 \times 10^0$$

$$2 \times 1000 = 2 \times 10^3$$

$$4 \times 100 = 4 \times 10^2$$

$$3 \times 10 = 3 \times 10^1$$

$$2 \times 1 = 2 \times 10^0$$

$$5 \times 0.1 = 5 \times 10^{-1}$$

$$5 \times 0.01 = 5 \times 10^{-2}$$

* Binary Number System

It has two digits i.e. 0 and 1. Therefore, the base is 2.

$$\text{Ex. 1}] (101)_2$$

$$1 \times 2^2 = 4$$

$$0 \times 2^1 = 0$$

$$1 \times 2^0 = 1$$

$$(5)_{10}$$

$$\text{Ex. 2}] (101011)_2$$

$$1 \times 2^6 = 64$$

$$0 \times 2^5 = 0$$

$$1 \times 2^4 = 16$$

$$0 \times 2^3 = 0$$

$$1 \times 2^2 = 4$$

$$1 \times 2^1 = 2$$

$$0 \times 2^0 = 0$$

$$(86)_{10}$$

$$\text{Ex. 3}] (101.10)_2$$

$$1 \times 2^{-2} = 4$$

$$0 \times 2^{-1} = 0$$

$$1 \times 2^0 = 1$$

$$1 \times 2^{-1} = 0.5$$

$$0 \times 2^{-2} = 0$$

$$(5.5)_{10}$$

0 and 1 are called a bit

4 bits are called nibble

8 bits are called byte

16 bits are called word

32 bits are called doubleword

$$1024 \text{ bytes} = 1 \text{ KB} = 2^{10}$$

$$2 \text{ KB} = 2^10$$

$$64 \text{ KB} = 2^{16}$$

$$1 \text{ MB} = 1024 \text{ KB} = 2^{20}$$

$$1 \text{ GB} = 1024 \text{ MB} = 2^{30}$$

$$1 \text{ TB} = 1024 \text{ GB} = 2^{40}$$

* Octal Number System

It contains 8 digits i.e. 0 to 7. Every number is formed using this digit. 8 and 9 is not used in this system.
Base of octal number system is 8.

0	10	20	- - -	70
1	11	21	31	!
2	12	22	32	8
3	13	23	33	16
4	14	24	34	24
5	15	25	35	32
6	16	26	36	40
7	17	27	37	48

Eg. [] $(357)_8$

$$3 \times 8^2 = 192$$

$$5 \times 8^1 = 40$$

$$7 \times 8^0 = 7$$

$$(239)_{10}$$

Hexadecimal Number System

Base is 16. Total digits are also 16 i.e. 0 to 9 and A, B, C, D, E, F.

0	10	F0
1	11	F1
2	12	F2
3	13	
4	14	
5	15	
6	16	
7	17	
8	18	
9	19	
A	1A	
B	1B	
C	1C	
D	1D	
E	1E	
F	1F	FF

Ex. 7) (201)₁₀

$$2 \times 16^2 = 512$$

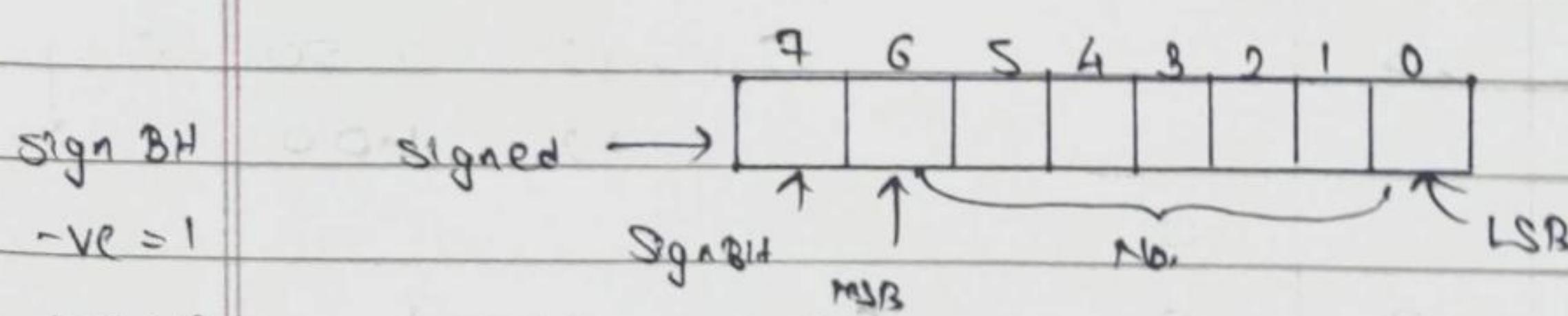
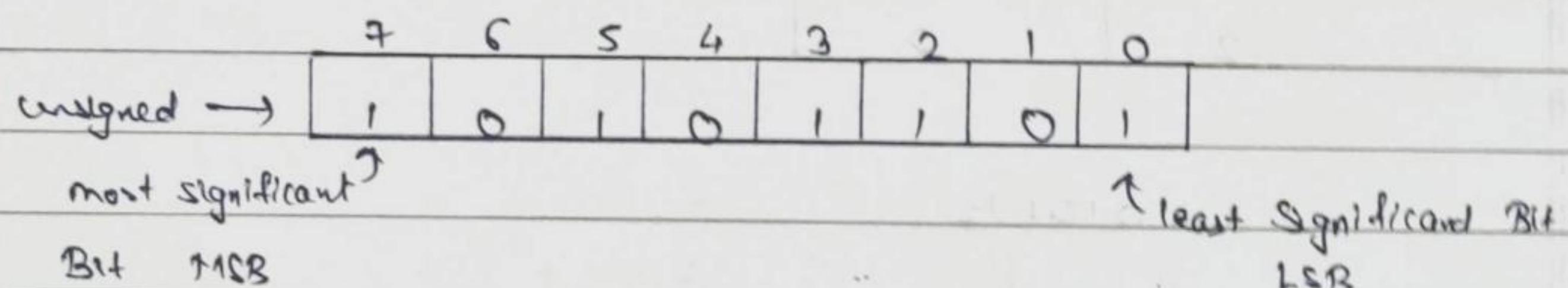
$$0 \times 16^1 = 0$$

$$1 \times 16^0 = 1$$

$$(513)_{10}$$

Decimal	Hexadecimal	Octal	Binary
0	0	0	0 000
1	1	1	0 001
2	2	2	0 010
3	3	3	0 011
4	4	4	0 100
5	5	5	0 101
6	6	6	0 110
7	7	7	0 111
8	8	10	1 000
9	9	11	1 001
10	A	12	1 010
11	B	13	1 011
12	C	14	1 100
13	D	15	1 101
14	E	16	1 110
15	F	17	1 111

3 bit number:



3. Converting Decimal numbers to binary number

Eg. i) $(25)_{10}$

	25		
2	12	1	↑ ← LSB
2	6	0	
2	3	0	
2	1	1	
2	0	1	← MSB

ii) $(173)_{10}$

	173		
2	86	1	
2	43	0	
2	21	1	
2	10	1	
2	5	0	
2	2	1	↓ add 1 to 8
2	1	0	
2	0	1	

$(1011.0101)_2$

iii) $(25.25)_{10}$

	25		$0.25 \times 2 = 0.50$	0	↓
2	12	1	$0.50 \times 2 = 1.00$	1	↓
2	6	0			
2	3	0			
2	1	1			
2	0	1			

$(11001.0100)_2$

↳ Binary point

Q] $(0.765)_{10}$

$$0.765 \times 2 = 1.53$$

$$0.53 \times 2 = 1.06$$

$$0.06 \times 2 = 0.12$$

$$0.12 \times 2 = 0.24$$

$$0.24 \times 2 = 0.48$$

$$0.48 \times 2 = 0.96$$

$$0.96 \times 2 = 1.92$$

$$0.92 \times 2 = 1.84$$

$(0.11000011)_2$

2. Converting Decimal number to octal number.

Q] $(80)_{10}$

	80		
8	10	0	↑
8	1	2	$(120)_8$
8	0	1	

Q] $(100)_{10}$

	100		
8	12	4	
8	1	4	$(144)_8$
8	0	1	

(100.88)₁₀

$$0.88 \times 8 = 7.04$$

7

$$0.04 \times 8 = 0.32$$

0

$$0.32 \times 8 = 2.56$$

2

$$0.56 \times 8 = 4.48$$

4

(144.7024)₈

3. Converting Decimal number to Hexadecimal number.

i] (100)₁₀

	100		
16	6	4	(64) ₁₆
16	0	6	

ii] (175)₁₀

	175		
16	10	15 (F)	(AF) ₁₆
16	0	10 (A)	

iii) (175.99)₁₀

$$0.99 \times 16 = 15.84$$

15 (F)

$$0.84 \times 16 = 13.44$$

13 (D)

$$0.44 \times 16 = 7.04$$

7

(AF.FD70A3).

$$0.04 \times 16 = 0.64$$

0

$$0.64 \times 16 = 10.24$$

10 (A)

$$0.24 \times 16 = 3.84$$

3

4. Converting Octal Number to Decimal Number

Q] $(88)_8$

$$\begin{aligned} & 8 \times 8^1 + 8 \times 8^0 \\ & = 64 + 8 \\ & = (72)_{10} \end{aligned}$$

Q] $(127)_8 = (?)_{10}$

$$\begin{aligned} M-1] & 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 \\ & = 64 + 16 + 7 \\ & = (87)_{10} \end{aligned}$$

	87	
16	5	7
16	0	5

M-2] $(127)_8$

Write in binary by taking 3 bit equivalent (For octant)

~~(001 010 111)₂~~

For hexadecimal, take 4 bit equivalent

~~(001 010 111)₂~~

~~= (57)₁₆~~

$$2] (756)_8 = (111\ 101\ 110)_2$$

$$(756)_{16} = (0111\ 0101\ 0110)_2$$

$$3] (8AEF)_{16} = (?)_8$$

$$(8AEF)_{16} = (1000\ 1010\ 1110\ 1111)_2$$

$$(1000\ 1010\ 1110\ 1111)_2 = (105357)_8$$

$$4] (7E.2A)_{16} = (?)_8$$

$$(7E.2A)_{16} = (0111\ 1110\ 0010\ 1010)_2$$

$$(01\ 11\ 110.\ 001\ 010\ 100)_2 = (176.124)_8$$

NOTE: For number before decimal point grouping goes from right to left, while for number after decimal point, grouping goes from left to right.

* Digital Codes

1] BCD (Binary Coded Decimal)

- It is representation of decimal number in binary form.
In BCD, each individual decimal digit is converted into a 4 bit binary equivalent.
- By combining them all, the BCD code is generated.

Ex. 1] Find BCD code of decimal number 358.

$$(358)_{10} = (0011 \ 0101 \ 1000)_{BCD}$$

In BCD code, only the decimal digits 0-9 are used.

2] Excess Three code

- In excess three code we need to add (0011) to each 4 bit group in binary coded decimal number to get desired excess three equivalent.
- It is an extension to BCD.

Decimal

BCD

ES 3

0

0000

0011

1

0001

0100

2

0010

0101

3

0011

0110

4

0100

0111

5

0101

1000

6

0110

1001

7

0111

1010

8

1000

1011

9

1001

1100

Ex. 4] Find the excess three code for $(45)_{10}$

$$(45)_{10} = (0111\ 1000)$$

$$\text{ii] } (89)_{10} = (1011\ 1100)$$

3] Gray Code

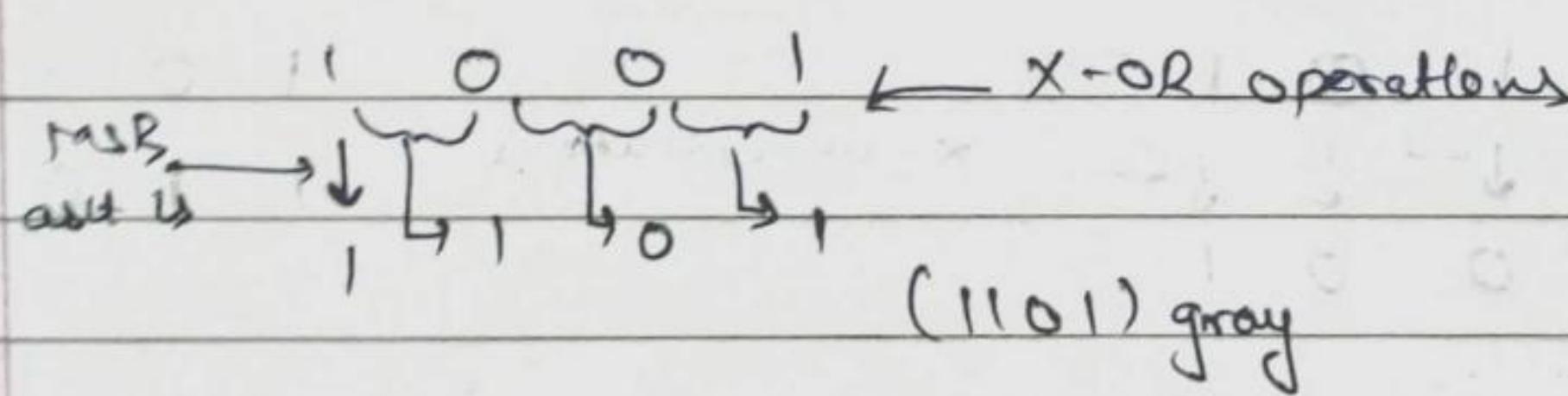
- Gray code system is a binary number system in which every successive pair of number differs in only one bit.
- Gray codes are very useful in the normal sequence of binary number generated by the hardware that may cause error or ambiguity during the transmission from one number to another.
- In finding the gray code, The X-OR operation is useful.

X - OR

a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

Ex. 1] $(1001)_2$

$$\begin{array}{r}
 & 1110 \\
 & 0001 \\
 101001 + & 1001 \\
 \hline
 & 1101
 \end{array}$$



v] $(0100)_2$

$$\begin{array}{r}
 0100 \\
 0110 \quad (0110) \text{ gray}
 \end{array}$$

w] $(0111)_2$

$$\begin{array}{r}
 0111 \\
 0100 \quad (0100) \text{ gray}
 \end{array}$$

x] $(1010)_2$

$$\begin{array}{r}
 1010 \\
 1111 \quad (1111) \text{ gray}
 \end{array}$$

Decimal	BCD	Gray code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101

Ex. 9] (1101) gray

(1010) gray

1 1 0 1 1 0 1 0
 ↓ ↓ ↓ ↓ ← x-or operations ↓ ↓
 mSB as it is 1 1 0 0
 1 0 0 1
 (1001)₂ (1100)₂.

Ex. 10] (0101) gray

0 1 0 1

0 1 1 0

(0110)₂

4] ASCII Code

- It is the short form of American Standard Code for Information Interchange.
- It is the standard that assigns letters, numbers and other characters on 8 bit code.
- ASCII Table is divided into 3 different sections:-

4] Non printable System code from 0 to 31.

ii) Lower ASCII between 32 to 127. It works on 7 bit character tables.

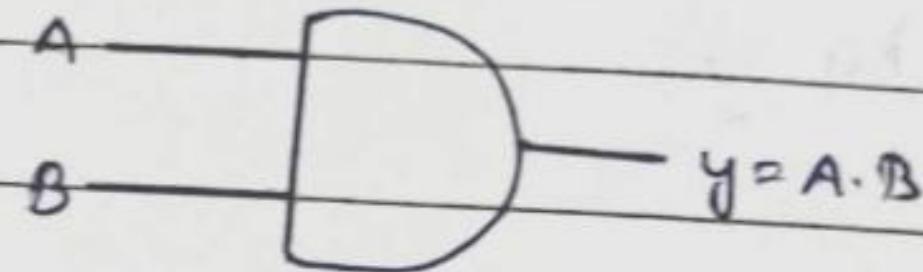
iii) Higher ASCII from 128 to 255. This portion is programmable. Characters are based on language of your operating system or program you are using.

Character	ASCII
0	48
1	49
2	50
:	59
A	65
B	66
Z	90
a	97
b	98
z	122

* LOGIC GATES

1] AND

$$y = A \cdot B$$



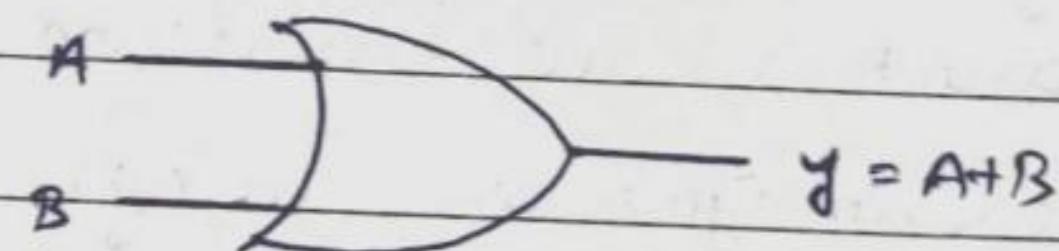
Truth Table

A	B	y
0	0	0
0	1	0
1	0	0
1	1	1

OUTPUT is high only when all INPUT is high

2] OR

$$y = A + B$$



Truth Table

A	B	y
0	0	0
0	1	1
1	0	1
1	1	1

OUTPUT is low only when all INPUT is low.

3] NOT

$$y = \bar{A}$$

Truth Table

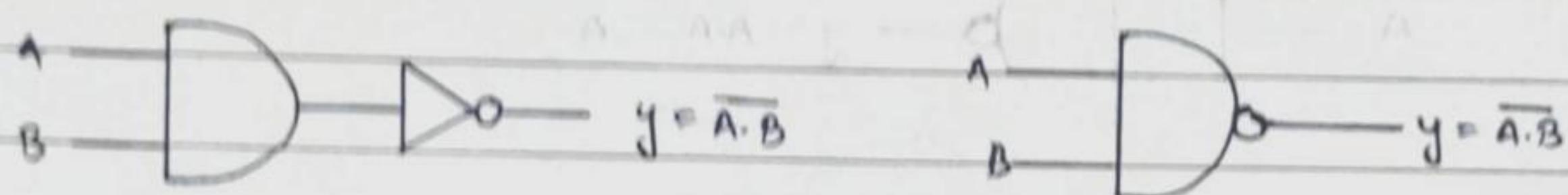


A	y
0	1
1	0

OUTPUT is the complement of the INPUT

* UNIVERSAL GATES

1] NAND

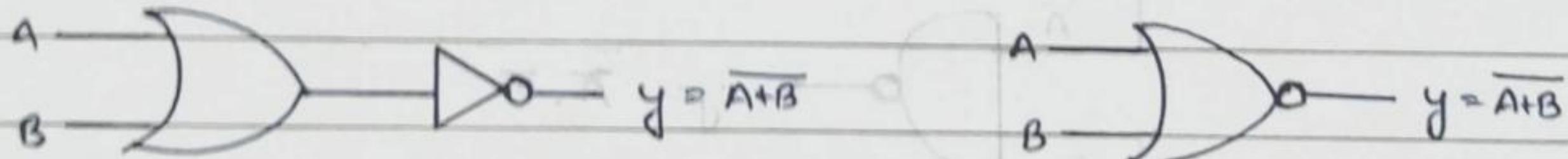


Truth Table

A	B	y
0	0	1
0	1	1
1	0	1
1	1	0

OUTPUT is low when all INPUT are high

2] NOR



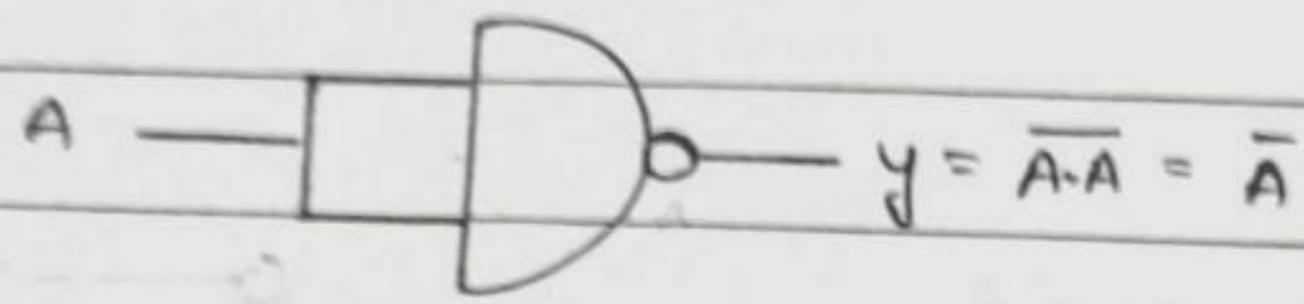
Truth Table

A	B	y
0	0	1
0	1	0
1	0	0
1	1	0

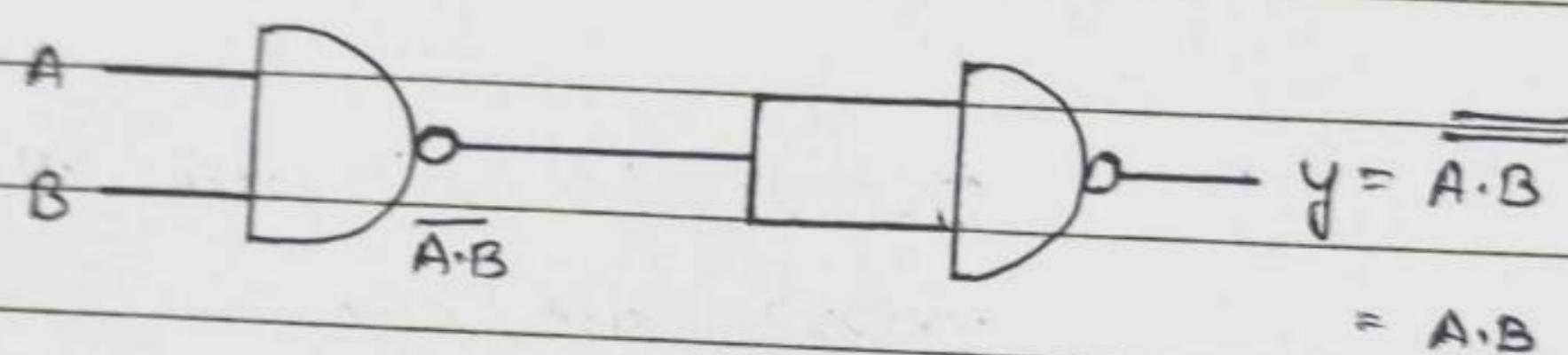
OUTPUT is high only when INPUT is low

- NAND and NOR Gates called as universal building blocks because the basic gates AND, OR, NOT can be designed using NAND and NOR.

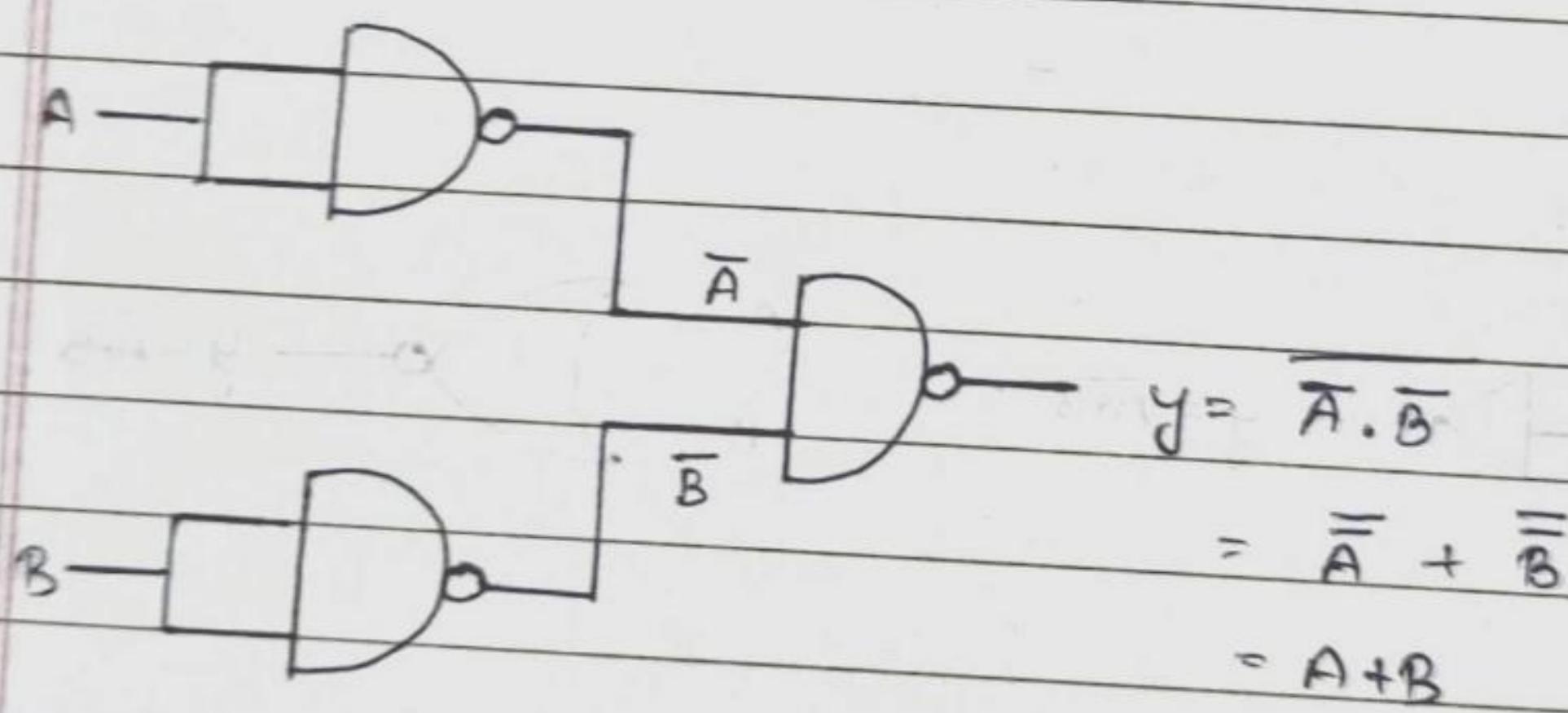
1] Implementation of NOT using NAND



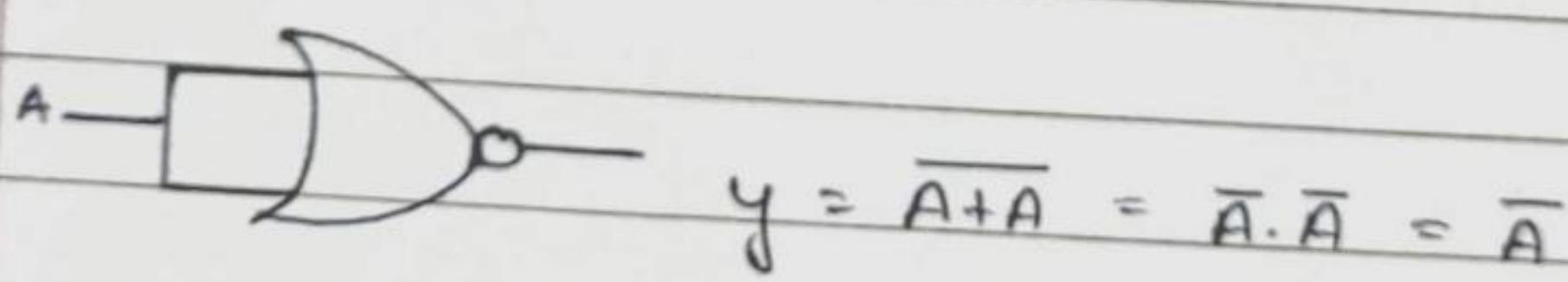
2] Implementation of AND using NAND



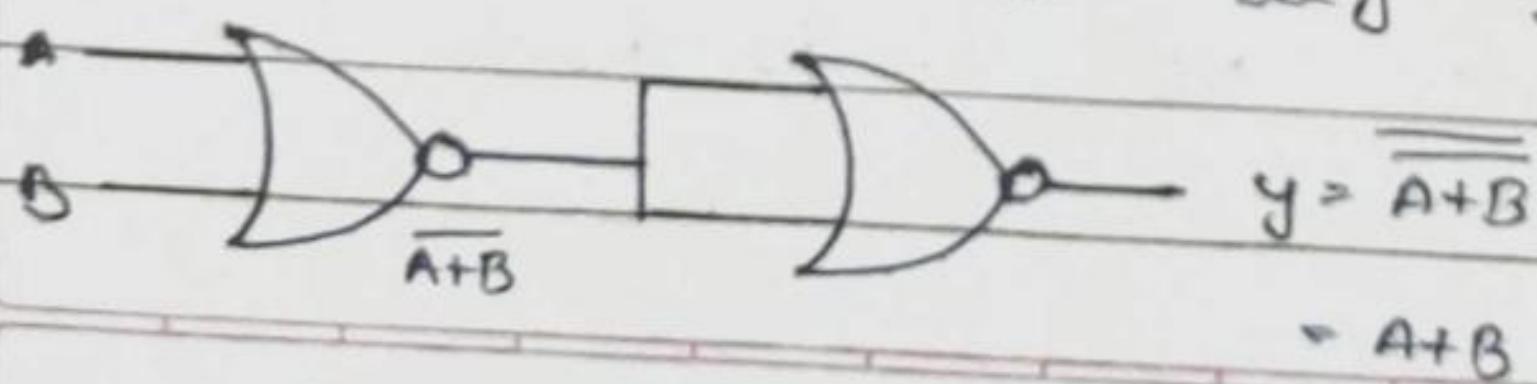
3] Implementation of OR using NAND



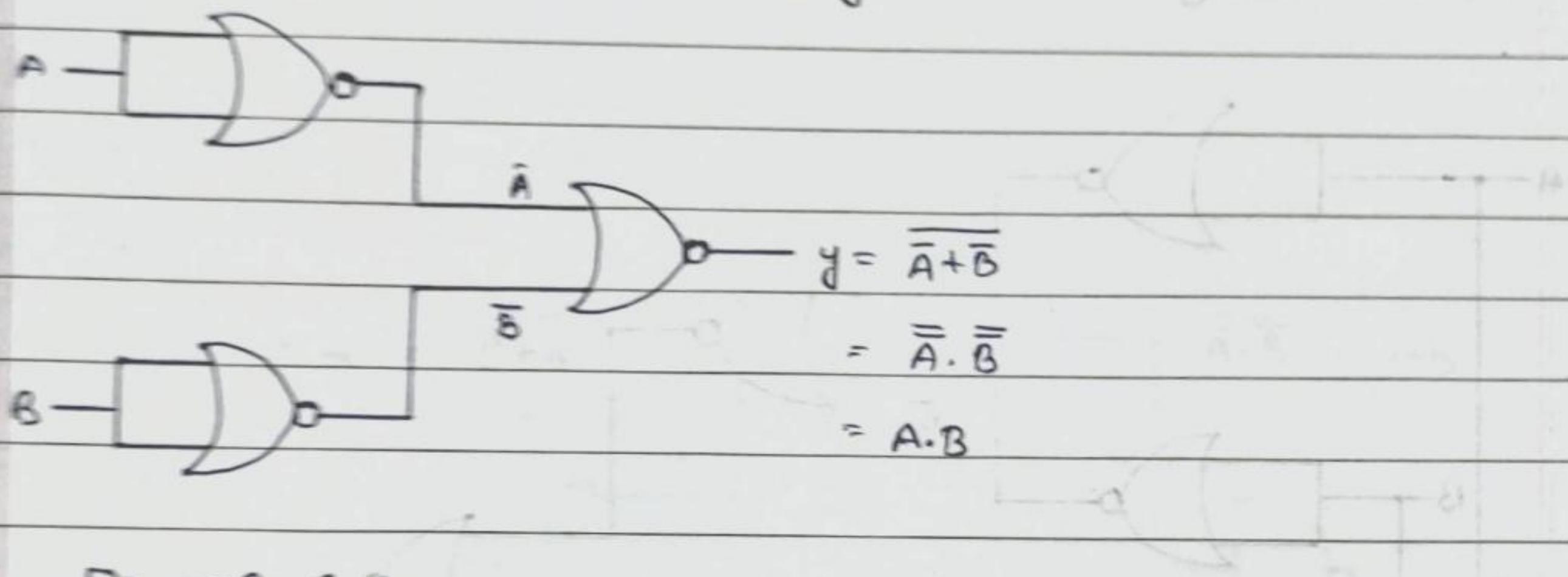
4] Implementation of NOT using NOR



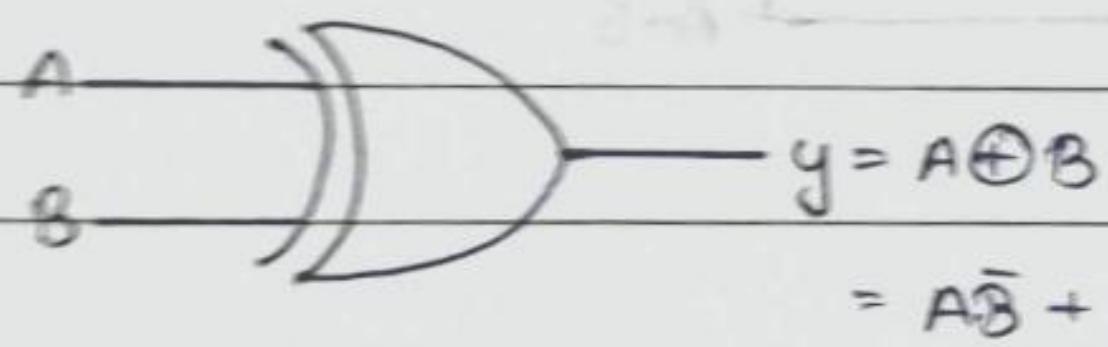
5] Implementation of OR using NOR



6] Implementation of AND using NOR



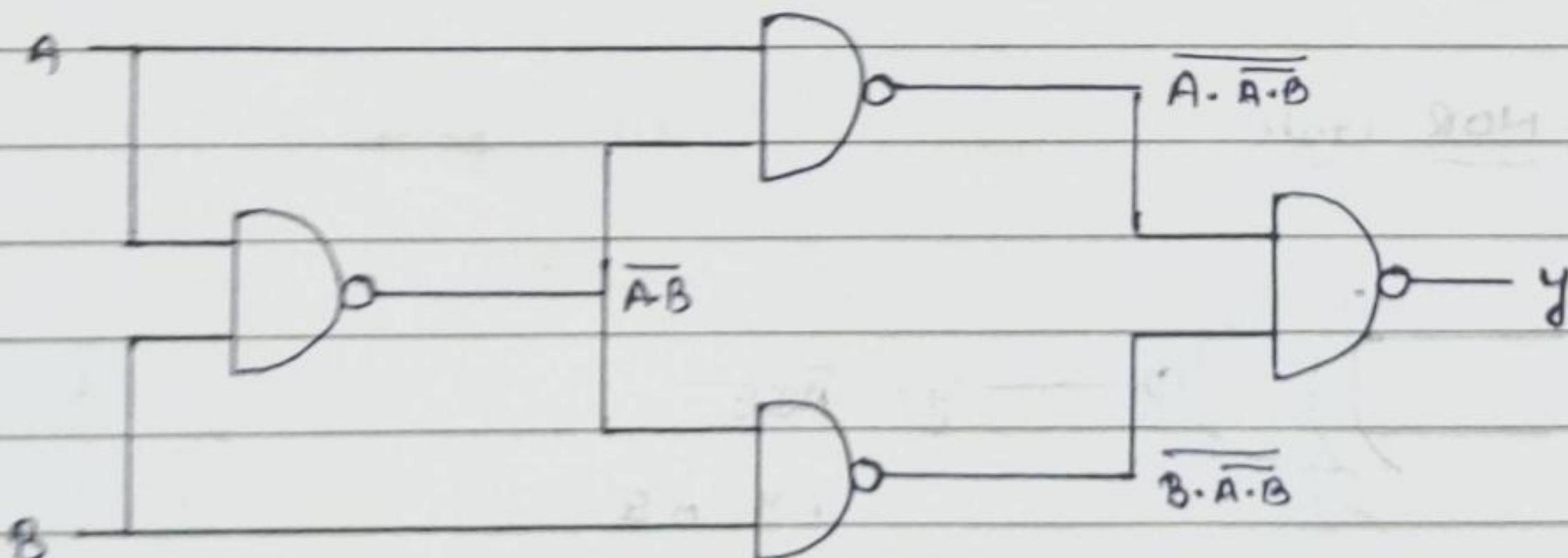
* Ex-OR Gate



Truth Table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

i) Ex-OR using NAND



$$y = (\overline{A \cdot \overline{A \cdot B}}) \cdot (\overline{B \cdot \overline{A \cdot B}})$$

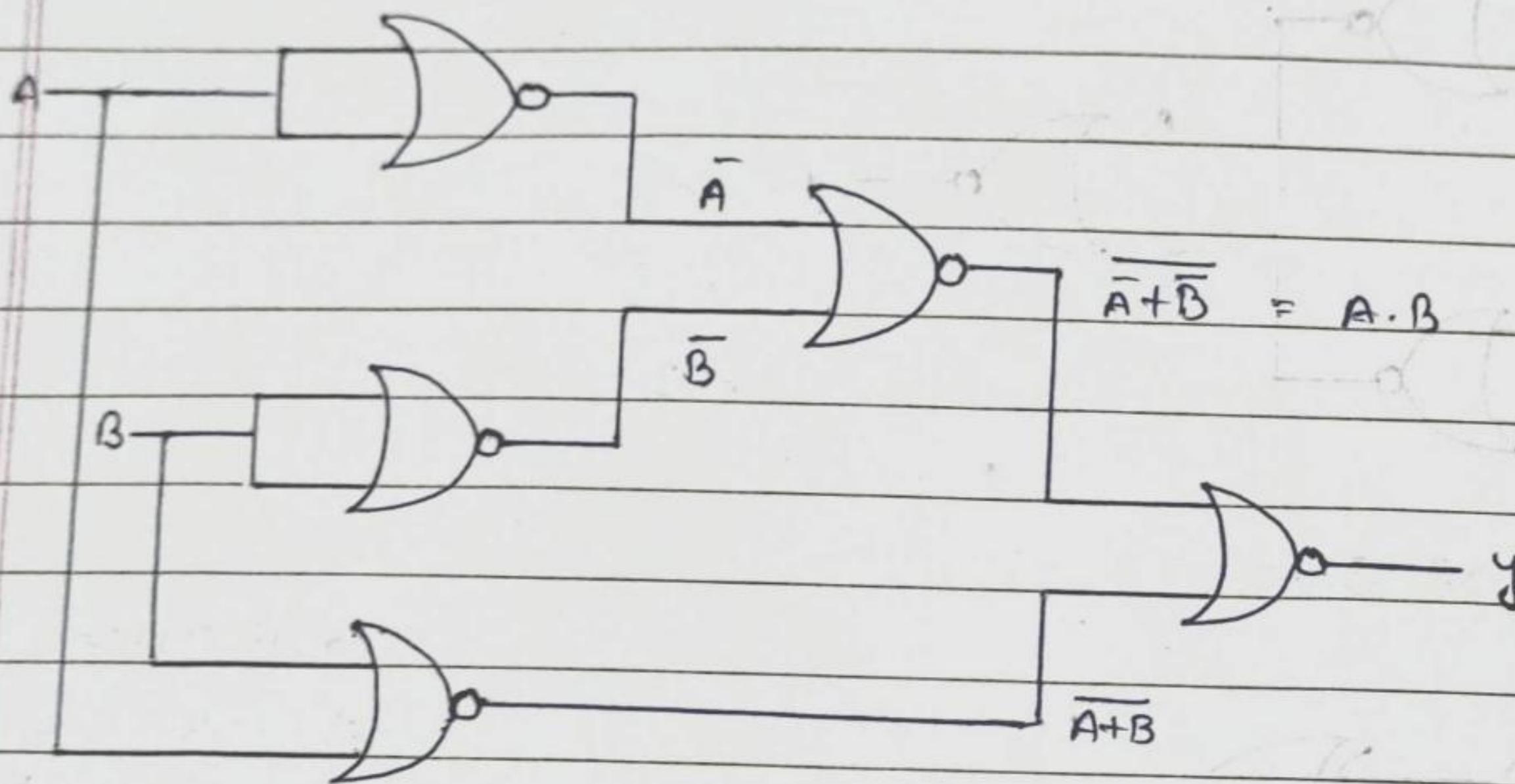
$$= (\overline{A \cdot \overline{A \cdot B}}) + (\overline{B \cdot \overline{A \cdot B}})$$

$$= A \cdot \overline{A \cdot B} + B \cdot \overline{A \cdot B}$$

$$= A \cdot \overline{A} + A \cdot \overline{B} + B \cdot \overline{A} + B \cdot \overline{B}$$

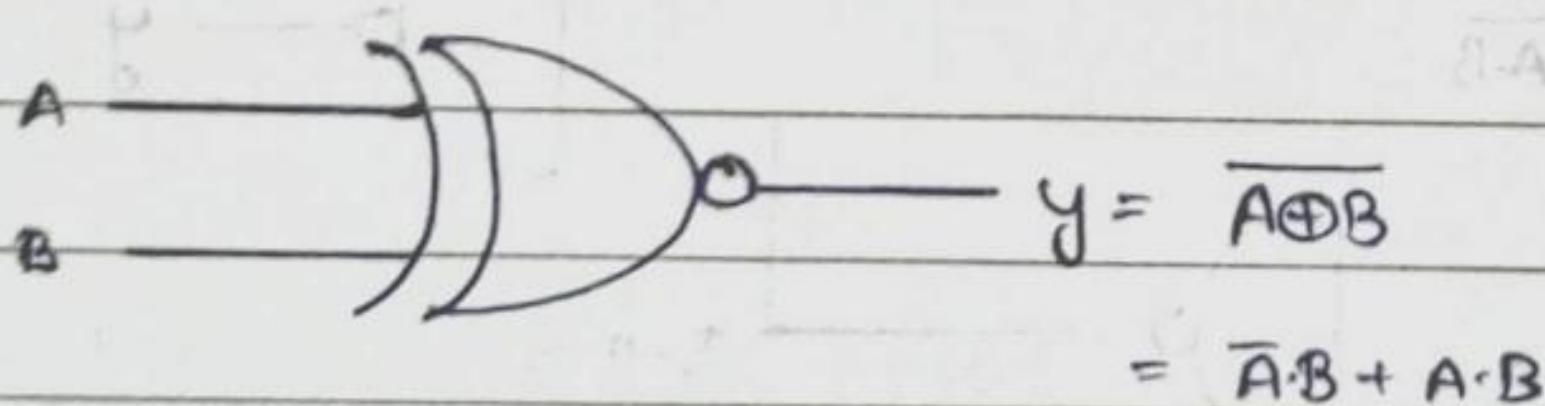
$$= A \cdot \overline{B} + \overline{A} \cdot B$$

2] EX-OR using NOR



$$\begin{aligned}
 y &= (\overline{A \cdot B}) + (\overline{\overline{A} + \overline{B}}) \\
 &= (\overline{A} \cdot \overline{B}) \cdot (\overline{\overline{A} + \overline{B}}) \\
 &= (\overline{A} + \overline{B}) \cdot (A + B) \\
 &= \overline{A} \cdot \overline{A} + \overline{A} \cdot B + \overline{B} \cdot A + \overline{B} \cdot \overline{B} \\
 &= \overline{A} \cdot B + \overline{B} \cdot A
 \end{aligned}$$

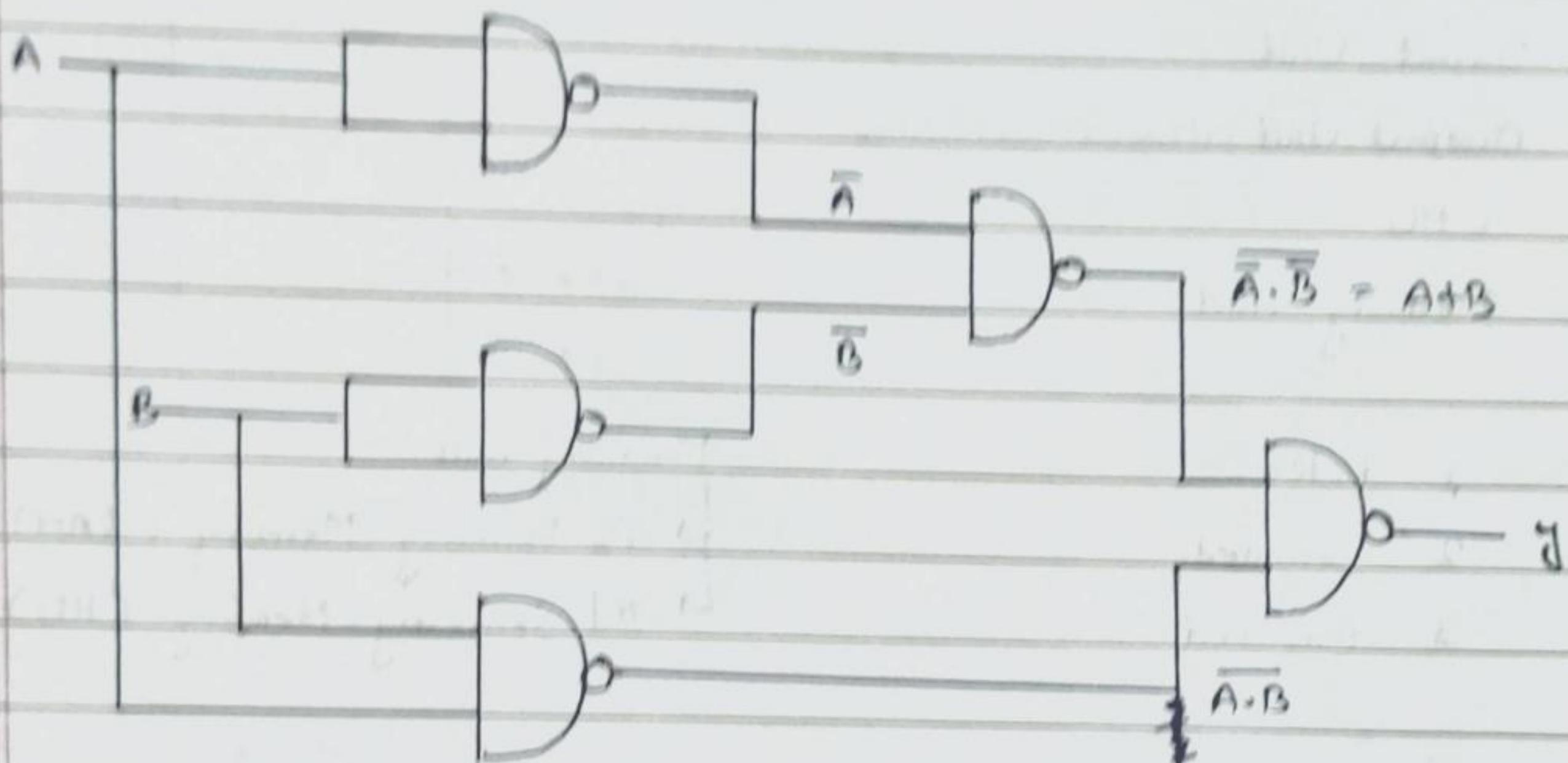
* Ex-NOR Gate



Truth Table

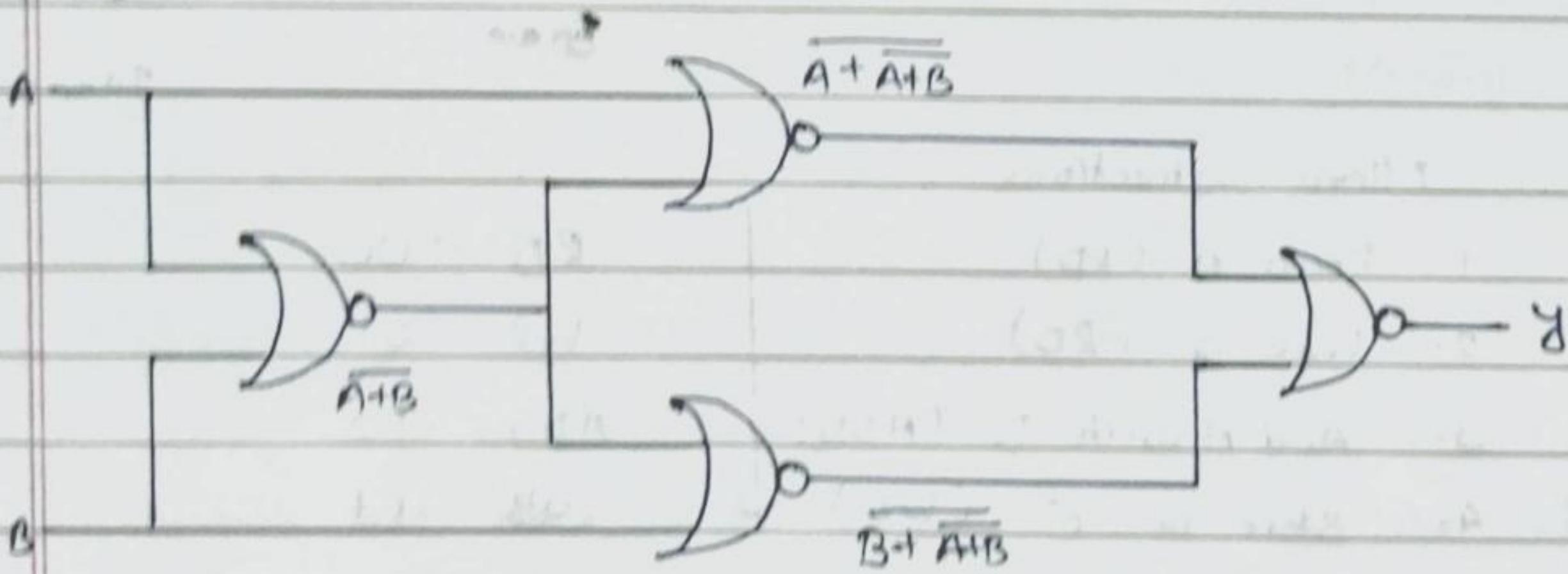
A	B	y
0	0	1
0	1	0
1	0	0
1	1	1

1] Ex- NOR using NAND



$$\begin{aligned}
 Y &= (\overline{A+B}) \cdot (\overline{\overline{A} \cdot \overline{B}}) \\
 &= (\overline{\overline{A} \cdot \overline{B}}) \cdot (\overline{A \cdot \overline{B}}) \\
 &= (\overline{\overline{A} \cdot \overline{B}}) + (\overline{A \cdot \overline{B}}) \\
 Y &= \overline{A \cdot \overline{B}} + A \cdot \overline{B}
 \end{aligned}$$

2] Ex- NOR using NOR

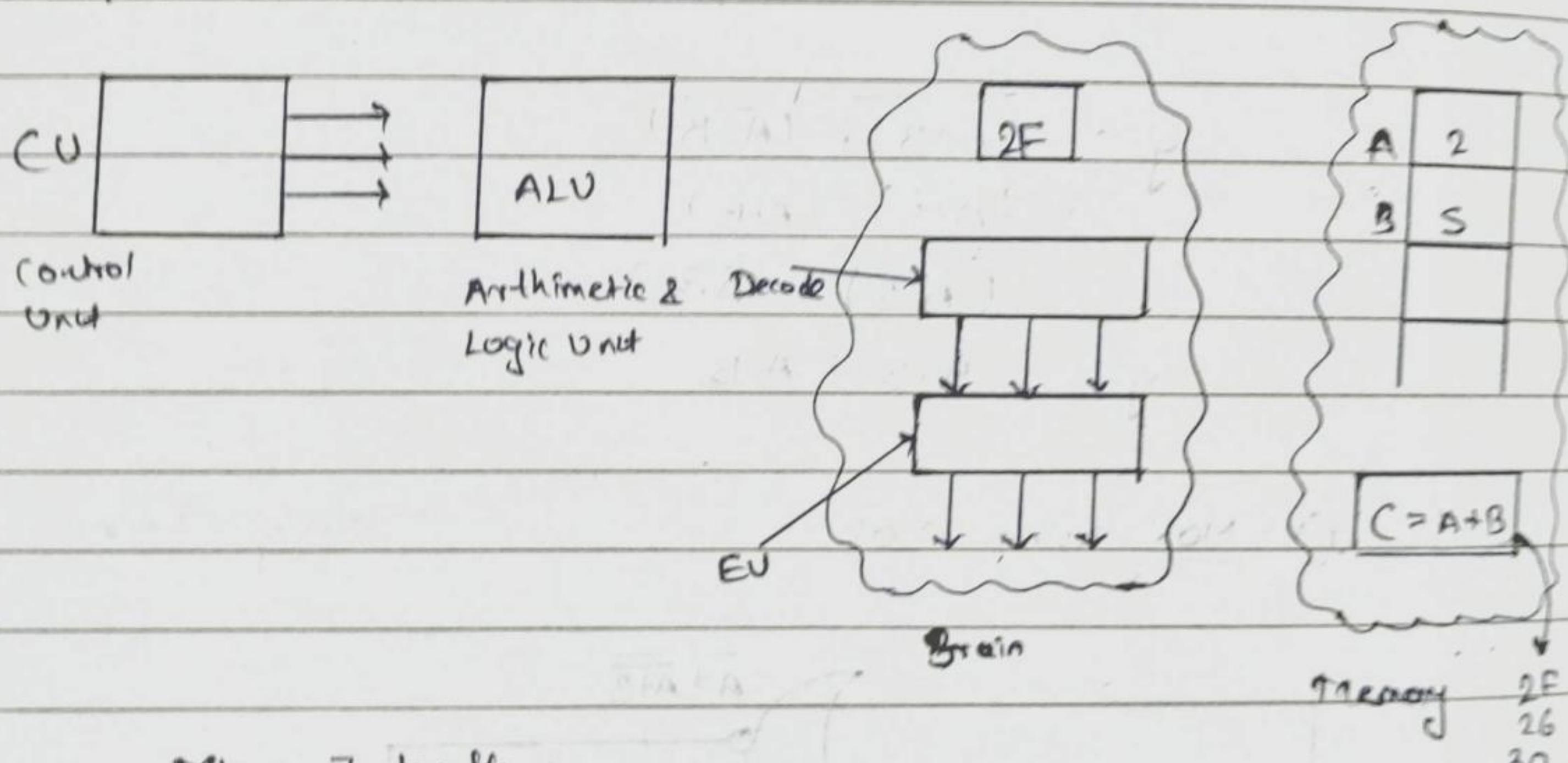
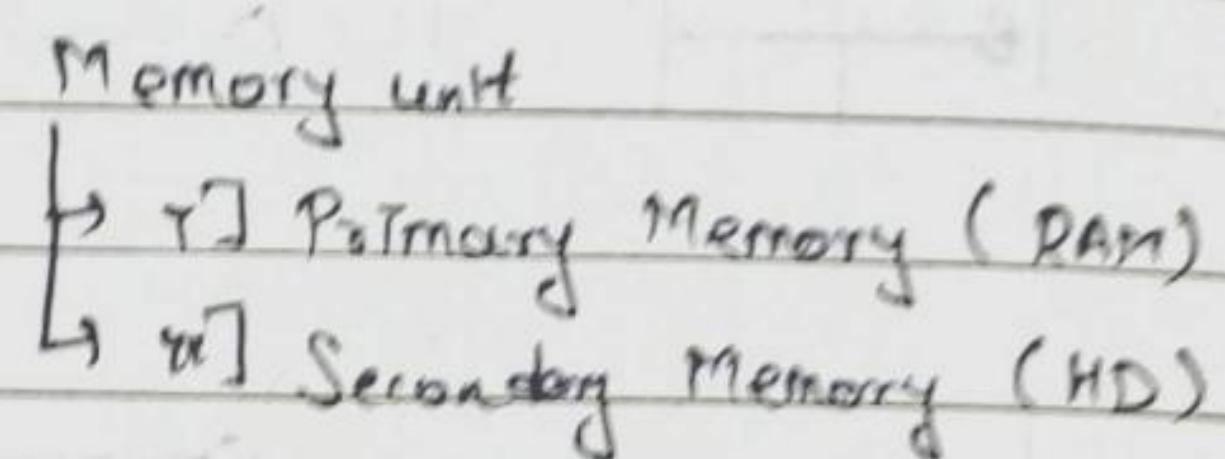


$$\begin{aligned}
 Y &= (\overline{A \cdot \overline{A+B}}) + (\overline{B \cdot \overline{A+B}}) \\
 &= (\overline{A} + \overline{A+B}) \cdot (\overline{B} + \overline{A+B}) \\
 &= (\overline{A} + \overline{A} \cdot \overline{B}) \cdot (\overline{B} + \overline{A} \cdot \overline{B}) \\
 &= A \cdot B + A \cdot \overline{A} \cdot \overline{B} + \overline{A} \cdot \overline{B} \cdot B + \overline{A} \cdot \overline{B} \cdot \overline{A} \cdot \overline{B} \\
 &= A \cdot B + \overline{A} \cdot \overline{B}
 \end{aligned}$$

* Basic Units of Computer System

- 1] Input Unit
- 2] Output Unit
- 3] CPU
- 4] Memory Unit

- 1 Fetch
- 2 Decoded
- 3 Executed



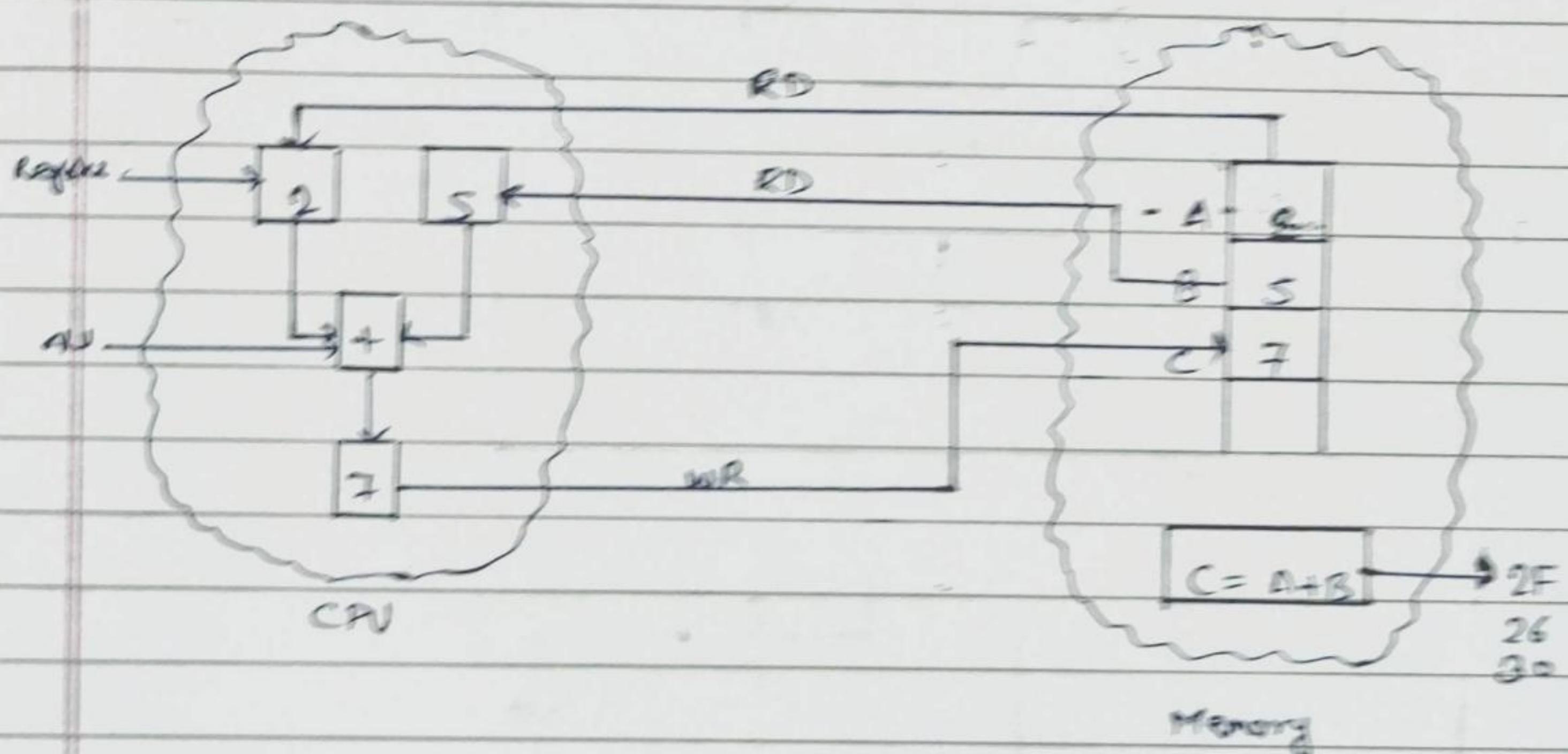
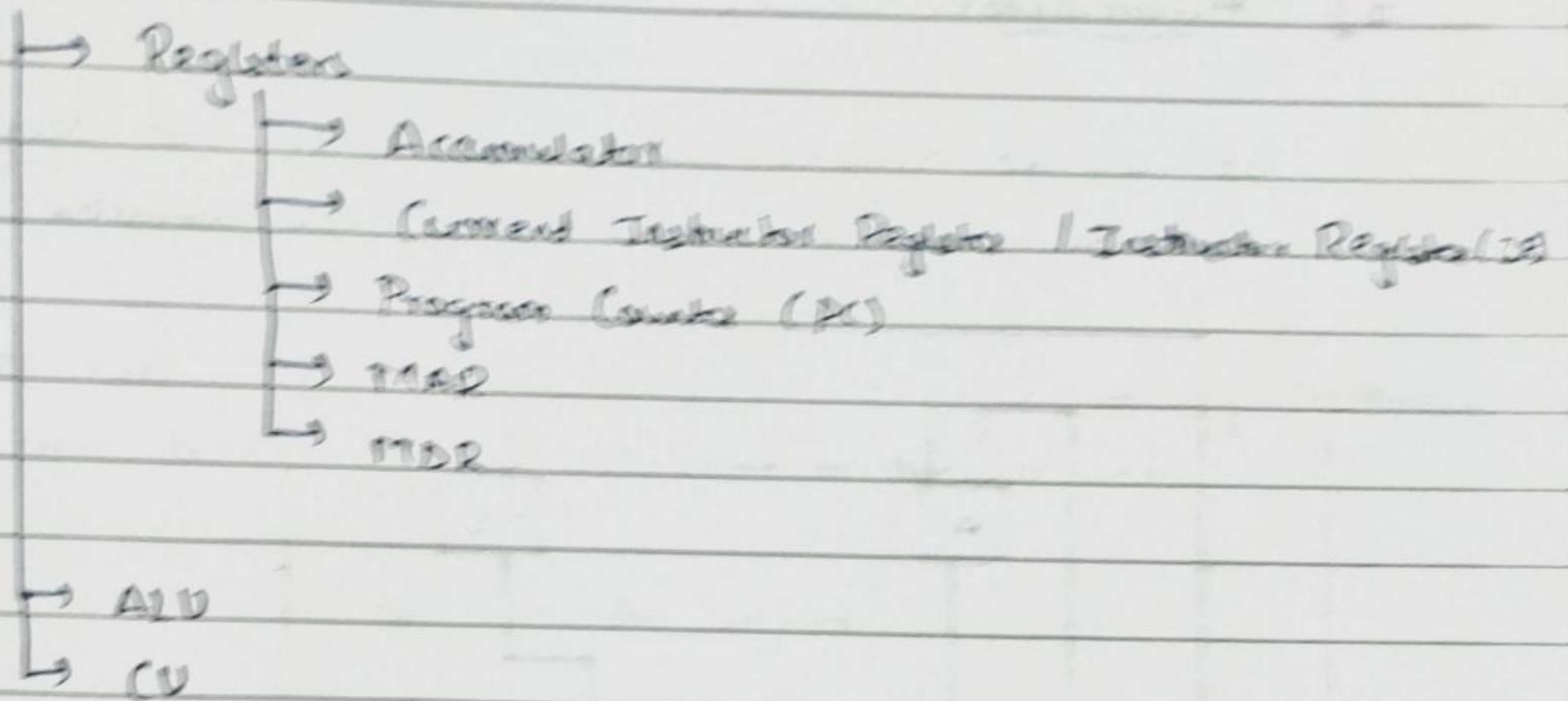
* Micro Instructions

1. Read A (RD) RD = 00
2. Read B (RD) WR = 01
3. Add A with B (ADD) ADD = 10
4. Store in C (WR) SOB = 11

Decoder:-

2:4	
0	→ RD
1	→ WR
2	→ ADD
3	→ SOB

CPU



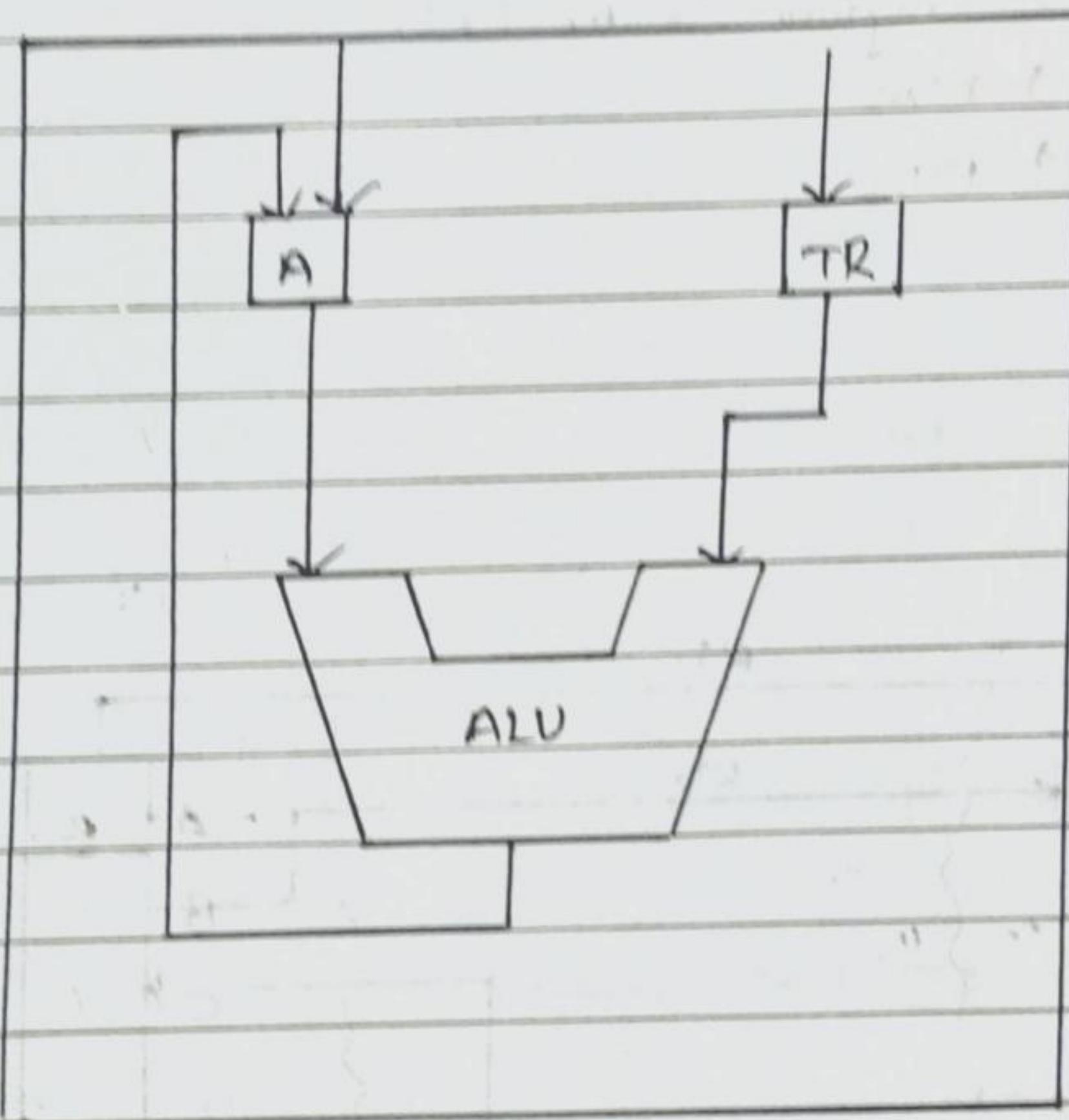
* Registers

Registers are the small memories inside the CPU which stores the data temporary before & after computation.

- Registers → i] Accumulator (A)
- v] Temp Register

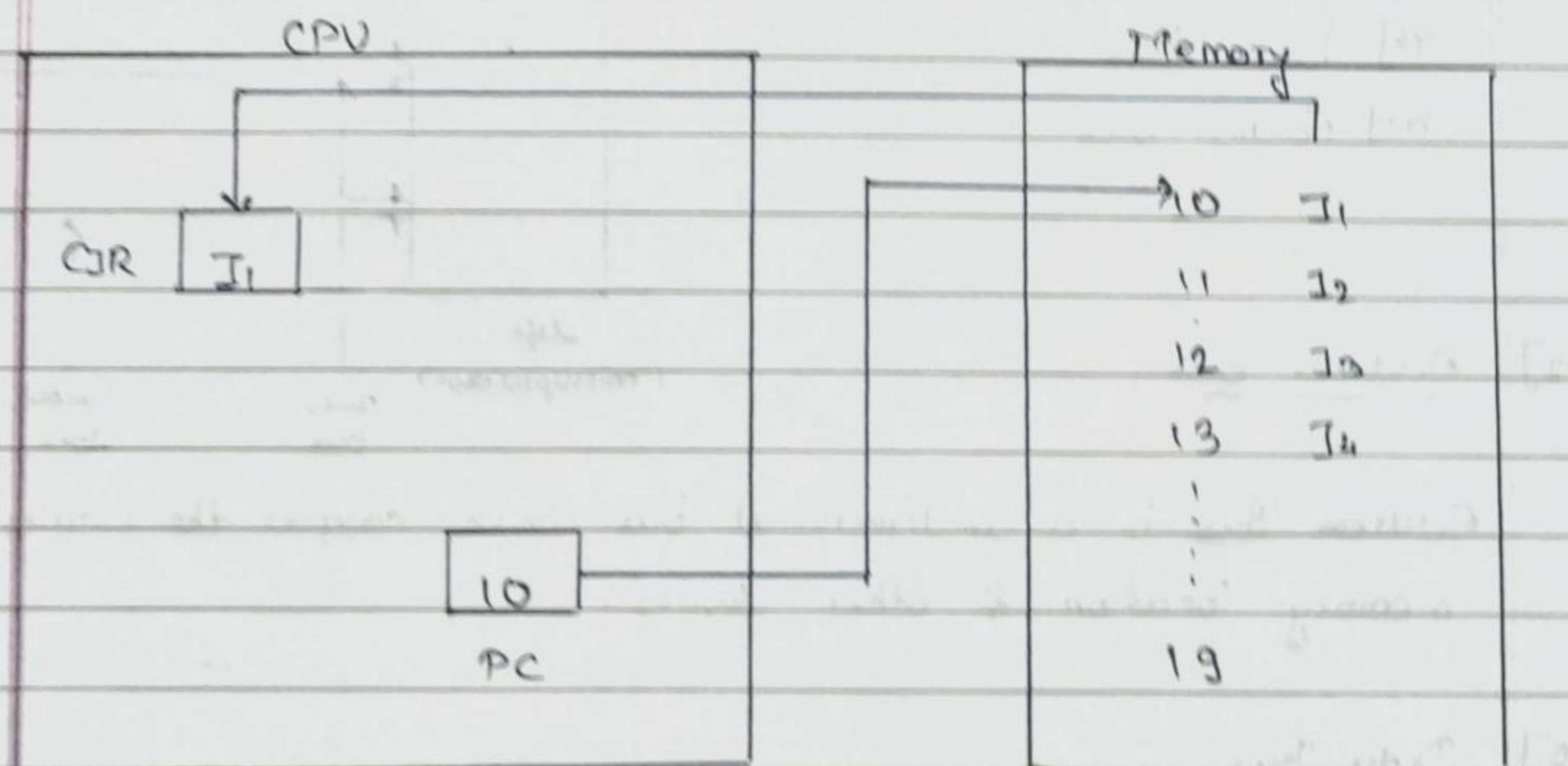
2] Accumulator

It is the register which stores the first operand as well as immediate result after ALU operation.



1] Program Counter (PC)

- Program Counter keeps the address of next instruction to be fetched from the memory.
- After execution of current instruction PC automatically incremented to point to the next to instruction to be executed.
- It is also called as instruction pointer.

2] Current Instruction Register / Instruction Register (IR)

IR holds the current instruction to be executed.

4] Memory Address Register (MAR)

This register holds the address of memory location from where the instruction or data to be accessed from the memory.

5] Memory Data Register (MDR)

It holds the data that is being transferred to or from the memory.

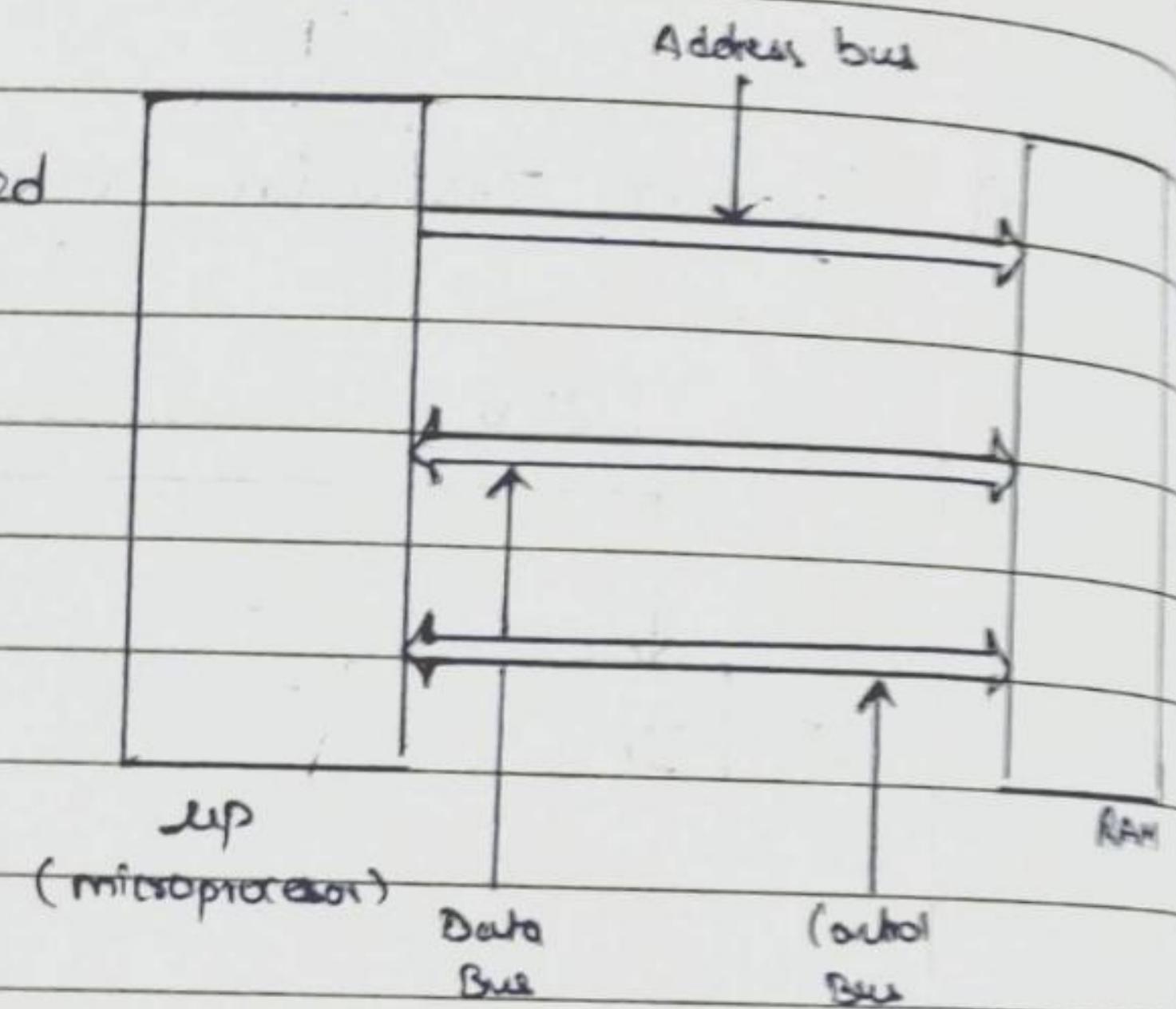
* Busses

Bus is a group of wire which carries electric signals from one device to another device.

3 types of Buses are identified

- 1] Address bus
- 2] Data bus
- 3] Control bus

1] Address Bus



Address Bus is a unidirectional bus which carries the address for memory location & other devices.

2] Data Bus

It is a bidirectional bus which carries the data transferred to and from up and other devices.

3] Control Bus

It is a bidirectional bus which carries control signals. Control signals are transferred in both the directions.

The examples of control signals are read, write, ready, etc.

* DATA REPRESENTATION & ALGORITHMS *

* Binary Integer Representation

1] Unsigned Integers

- can represent 0 and +ve numbers

2] Signed Integers

- can represent 0, +ve & -ve numbers.

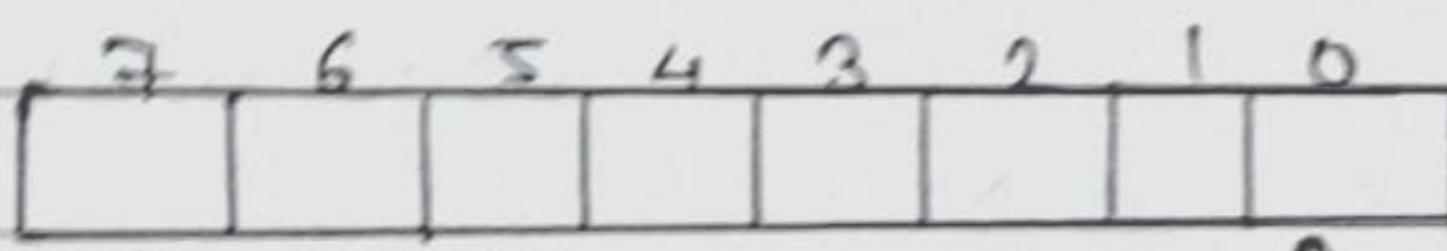
a] Signed Magnitude Representation.

b] 1st Complement Representation

c] 2nd complement Representation

3] Unsigned Integers

Ex. 8 bit unsigned binary integers



↑
MSB

↑
LSB

Magnitude of number.

Ex. 0100 0001 = 65D

$\downarrow_{1x2^6} \downarrow_{1x2^0}$

Ex. 1100 0000

$\downarrow_{1x2^7} \downarrow_{1x2^0} = 192D$

Ex. 0000 0000

\downarrow_{0D}

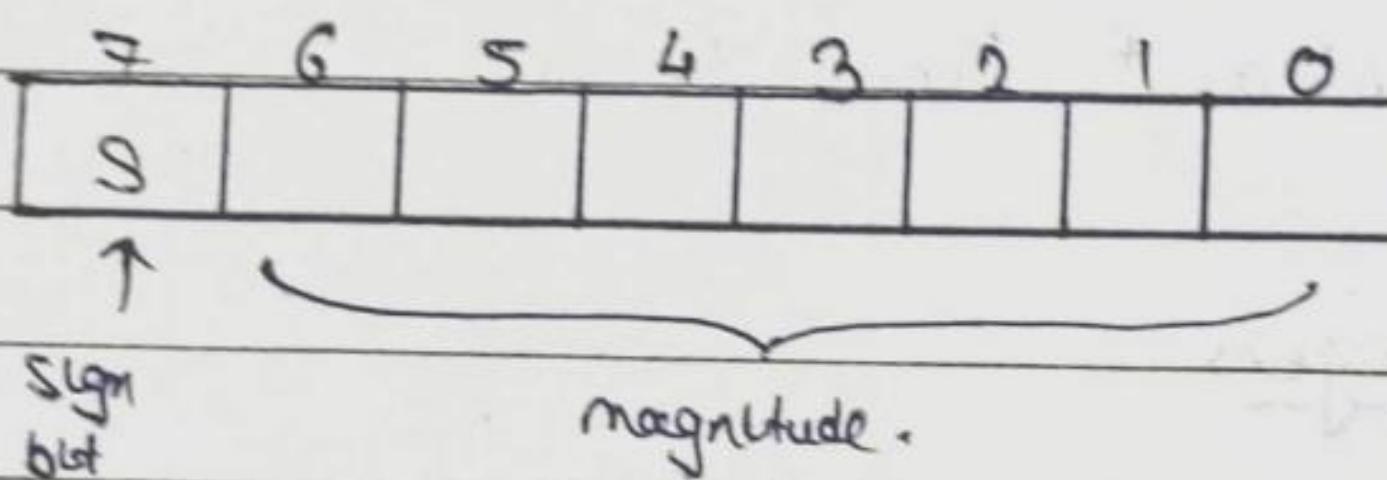
a) Signed Integer

In signed integer representation most significant bit (MSB) is reserved as a sign bit.

If MSB = 0 ; integer is +ve.

If MSB = 1 ; Integer is -ve.

a) Signed Magnitude Representation



$$\text{Ex. } 0100\ 0001 = +65 \text{ D} \quad (\text{Range})$$

$$\begin{array}{r} 0000\ 0000 \\ +0 \\ 0000\ 0001 \\ +1 \end{array}$$

$$\text{Ex. } 1100\ 0001 = -65 \text{ D}$$

$$\begin{array}{r} 0111\ 1111 \\ +127 \end{array}$$

$$\text{Ex. } 0000\ 0000 = +0 \text{ D}$$

$$\begin{array}{r} 1000\ 0000 \\ -0 \end{array}$$

$$\text{Ex. } 1000\ 0000 = -0 \text{ D}$$

$$\begin{array}{r} 1000\ 0001 \\ -1 \end{array}$$

$$1111\ 1111 \quad -127$$

$$(-127 \dots -0 +0 \dots +127)$$

- Drawback of signed magnitude representation is there are two representations for the number '0', which could lead to inefficiency and confusion, and also +ve and -ve numbers to be processed separately.

b] Is complement representation

The Is complement of a number is formed by changing all 1 to 0 and all 0 to 1.

$$\text{Ex. } \begin{array}{r} 0110 \\ 1^s \\ \hline 1001 \end{array} = +6$$

$$\text{Ex. } \begin{array}{r} 01001 \\ 1^s \\ \hline 10110 \end{array} = +9$$

$$\begin{array}{r} 11110 \\ 1^s \\ \hline 01110 \end{array} = -1$$

$$\begin{array}{r} 10110 \\ 1^s \\ \hline 01001 \end{array} = -9$$

$$\begin{array}{r} 00110 \\ 1^s \\ \hline 11001 \end{array} = +6$$

$$\begin{array}{r} 11001 \\ 1^s \\ \hline 00110 \end{array} = -6$$

begin
but

(Range)

$$\begin{array}{r} 00000 \ 00000 \\ 1^s \\ \hline 11111 \ 11111 \end{array} = +0$$

$$\begin{array}{r} 1000 \ 0000 \\ 1^s \\ \hline 1111 \ 1111 \end{array} = -127$$

$$\begin{array}{r} 1111 \ 1111 \\ 1^s \\ \hline 0000 \ 0000 \end{array} = -0$$

$$\begin{array}{r} 0000 \ 0000 \\ 1^s \\ \hline 1111 \ 1111 \end{array} = +0$$

$$\begin{array}{r} 0000 \ 0001 \\ 1^s \\ \hline 1111 \ 1110 \end{array} = +1$$

$$\begin{array}{r} 0000 \ 0001 \\ 1^s \\ \hline 1111 \ 1111 \end{array} = +1$$

$$\begin{array}{r} 0111 \ 1111 \\ 1^s \\ \hline 1000 \ 0000 \end{array} = +127$$

$$\begin{array}{r} 1000 \ 0000 \\ 1^s \\ \hline 1111 \ 1111 \end{array} = -127$$

$$\begin{array}{r} 1000 \ 0001 \\ 1^s \\ \hline 1111 \ 1110 \end{array} = -126$$

$$\begin{array}{r} 1111 \ 1110 \\ 1^s \\ \hline 0000 \ 0001 \end{array} = -1$$

$$\begin{array}{r} 1111 \ 1111 \\ 1^s \\ \hline 0000 \ 0000 \end{array} = 0$$

$$(-127 \rightarrow -0 \rightarrow +127)$$

c) 2's complement representation

2's complement of a number is obtained by taking 1's complement of a number and adding 1 to the LSB.

$$2's \text{ complement} = 1's \text{ complement} + 1$$

Ex. 2's complement of 6 is 0110

$$\begin{array}{r}
 0110 \\
 1001 \\
 + \quad 1 \\
 \hline
 1010 \rightarrow 2's \text{ complement.}
 \end{array}$$

Ex. 0100 0001 → 65D

$$\begin{array}{r}
 + 1001 \ 1110 \\
 \hline
 1011 \ 1111 \rightarrow 2's \text{ complement}
 \end{array}$$

$$\begin{array}{r}
 1000 \ 0001 \\
 + 0111 \ 1110 \\
 \hline
 0111 \ 1111 \Rightarrow -127D
 \end{array}$$

(Range)

0000 0000 - 0

0000 0001 + 1

0111 1111 + 127

1000 0000 - 128

Ex. 1111 1111 ⇒ -1D

$$\begin{array}{r}
 0000 \ 0000 \\
 + \quad 1 \\
 \hline
 0000 \ 0001
 \end{array}$$

1000 0001 - 127

1111 1111 - 1

-128 → 0 → (27)

Ex. 1000 0000

$$\begin{array}{r}
 + 0111 \ 1111 \\
 \hline
 1000 \ 0000 \rightarrow
 \end{array}$$

ASSIGNMENT NO. 1

- 1] What are the ranges of 8, 16, 32, 64 bit Integers in unsigned and signed representation.
- 2] Use the word Give the value of 88, 0, 127, 255, 1 in 8 bit unsigned representation.
- 3] Give the value of +88, -88, -1, 0, +1, -128, +127 in 8 bit 1's complement & 2's complement representation.

* Binary Arithmetic1] Binary addition

Rules:-

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

— Perform the addition of following numbers:-

$$\begin{array}{r}
 + 10101 \\
 11001 \\
 \hline
 101110
 \end{array}$$

1) Sub. Binary Subtraction

Rules :-

A	B	Difference	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$\begin{array}{r} 1011 \\ - 0110 \\ \hline 0101 \end{array}$$

$$\begin{array}{r} 0110 \\ + 1011 \\ \hline 11011 \end{array}$$

min1	min2	A	B
0	0	0	0
0	1	1	0
0	1	0	1
1	0	1	1

$$\begin{array}{r} 10011 \\ - 01101 \\ \hline \end{array}$$

DATA REPRESENTATION & ARITHMETIC, ALGORITHMS

3] Binary Multiplication

Rules 1-

A	B	Multiplication
0	0	0
0	1	0
1	0	0
1	1	1

Ex.

$$\begin{array}{r} + 1110 \\ \hline 10110 \\ + 1110 \\ \hline 1110 \end{array}$$

$$(1110) * (1011) = 1001\ 1010$$

If we multiply two 4-bit number, result is 8-bit number.

4] Binary Division

Ex. 4]

$$\begin{array}{r}
 1001 \quad | \quad 1110101 \\
 -1001 \downarrow \\
 \hline
 01011 \\
 -1001 \downarrow \\
 \hline
 00100 \\
 -0 \quad | \\
 \hline
 1001 \\
 -1001 \\
 \hline
 0000
 \end{array}$$

If divisor is 4 bit,

Quotient & Remainder by 4-bit.

A Dividend is 8-bit &

Division is called 4-bit division.

a) 15-103 15/3

$$\begin{array}{r}
 101 \\
 -1111 \\
 \hline
 11 \downarrow \downarrow \\
 0011 \\
 -11 \\
 \hline
 00 \\
 111 \\
 -10 \\
 \hline
 011 \\
 -10 \\
 \hline
 011 \\
 -10 \\
 \hline
 01
 \end{array}
 \quad \text{15/2}$$

s] Is Complement Arithmetic

Ex. 7] -7 + 8

$$\begin{array}{r}
 (0011) \\
 -11000 \\
 \hline
 (01000)
 \end{array}
 = (1101) * (0111)$$

If carry is generated

ans is true,

add to the rest answer.

$$\begin{array}{r}
 11000 \\
 -100000 \\
 \hline
 \xrightarrow{1} 00001
 \end{array}$$

\Rightarrow final answer

$$+1 = +(0001)$$

$$11010$$

$$11001$$

$$00100$$

$$0$$

$$1001$$

$$1001$$

$$v] 7 + (-8)$$

$$(00111) \quad (01000), \\ 10111$$

$$+ 00111$$

10111

$$\hline 11110$$

00001 \Rightarrow Final answer

Since carry is not generated
take its 1's complement of
the answer.

$$(-1) = - (0001)$$

$$vi] 13 + (-9)$$

\times

$$(1101) + -(1001)$$

$$1s (01101 \\ \downarrow 10010)$$

$$1s (01001 \\ \downarrow 10110)$$

$$vii] 13 + (-9)$$

$$(1101) + (-1001)$$

$$1001)_{1s}$$

0110

$$+ 1101$$

$$0110$$

$$\hline 10011$$

0100 \Rightarrow final answer (4)

vii] $(-1101) + (1001)$

$$(-13) + (9)$$

$$\begin{array}{r} 1101 \\ \text{is } G \\ 0010 \end{array}$$

$$+ 0010$$

$$\underline{1001}$$

$$\begin{array}{r} 1011 \\ \text{is } G \\ 0100 \end{array}$$

(2nd part of v)

number 1 has 4 bits
number 2 has 3 bits

1100

1100

1100

0111

0111

0111

size of both is 4 bits

$(1000) \rightarrow -1$

Final answer $- (0100)$

viii] $100 + (-11)$

$$(1001) + (-1011)$$

$$\begin{array}{r} 11 \rightarrow 01011 \\ \text{is } G \\ 0000 \end{array}$$

size of numbers should
be same.

$$+ 100$$

$$100$$

$$\underline{1000}$$

1

$$\underline{001}$$

(P-A) + S1 [Ans]

$(1001) + (-1011)$

(1001)

0110

vii] $(-100) + (-101)$

$$\begin{array}{r} 0100 \\ \text{is } G \\ 0011 \end{array}$$

$$\begin{array}{r} -0101 \\ \text{is } G \end{array}$$

$$\begin{array}{r} 1010 \\ \text{is } G \end{array}$$

(A) number length is 4 bits

$$\begin{array}{r}
 + 1011 \\
 1010 \\
 \hline
 10101
 \end{array}$$

Is G 0110 → Final Answer
1001 ↳ (1001)

vn] $(-1101) + (-1001)$

$$\begin{array}{r}
 10010 \\
 01001 \\
 \hline
 10110
 \end{array}$$

Is G 11101 → Final Answer
1001 ↳ (1001)

$$\begin{array}{r}
 + 10010 \\
 10110 \\
 \hline
 101000
 \end{array}$$

Is G 01001 → Final Answer
10110 ↳ (-10110)

6] 2s complement Arithmetic

Ex. 7] $(1101) + (-1001)$

$$\begin{array}{r}
 1001 \\
 0111 \\
 \hline
 0100
 \end{array}$$

2s

If carry is generated, ignore
it answer is rest of the number
 $\Rightarrow + (0100) \Rightarrow$ final answer.

8] $(-1101) + (1001)$

$$\begin{array}{r}
 0011 \\
 + 0011 \\
 1001 \\
 \hline
 1100
 \end{array}$$

2s

If no carry is generated,
take 2s complement of answer
and -ve sign.
 $\therefore -(0100) \Rightarrow$ final answer

$$\text{iii) } (-100) + (-101)$$

$$\begin{array}{r} 0100 \\ + 1011 \\ \hline 1100 \end{array} \quad (2)$$

$$\begin{array}{r} 0101 \\ + 1010 \\ \hline 1011 \end{array} \quad (2)$$

$$\begin{array}{r} 1100 \\ + 1011 \\ \hline 10111 \end{array} \quad (1001-1 + 1011) \quad (2)$$

Ignore carry

$- (1001) \Rightarrow$ final answer!

$$\begin{array}{r} 01100 \\ - 00010 \\ \hline 10010 \end{array}$$

count left $\leftarrow 01101$

$(00101-1)$

intermediate transform of $(1001-1) + (1011)$

$$\begin{array}{r} 1001 \\ + 1011 \\ \hline 1110 \end{array}$$

cancel between 2, 3, 4, 5

and cancel between 2, 3, 4, 5

cancel between $(00101-1) + (00101)$

$$\begin{array}{r} 00101 \\ + 00101 \\ \hline 00100 \end{array}$$

$(00101-1) + (1011-1)$

cancel between 2, 3, 4, 5

cancel between 2, 3, 4, 5

cancel between 2, 3, 4, 5

$$\begin{array}{r} 00101 \\ + 00101 \\ \hline 00100 \end{array}$$

$(00101-1)$

* Hexadecimal Arithmetic

+ 7A

0111 1010

- BA

1011 1010

+ 134

10011 0100

(+134)₁₆

CO

1100 0000

- 7A

1000 0110

- 46

10100 0110

0111 1010)₂₃ (-7A)

(-7A)

ignore the ^

Carry and result

is +ve.

- 7A

0111 1010

CO

0100 0000

(-46)

1011 1010

1100 0000)₂₃ (10)

(-10)

No carry generated \rightarrow ans is -ve thus take 2s complement

0100 0110



BCD Arithmetic

Binary Coded Decimal

1] BCD Addition

$$2 + 2 = 4$$

$$6 + 7$$

0010

0110

0010

0111

0100

1101

0110) add 6

10011

(13)

$$\begin{array}{r}
 + 24 \\
 25 \\
 49 \\
 \hline
 49
 \end{array}
 \quad
 \begin{array}{r}
 0010 \\
 0010 \\
 0100 \\
 \hline
 4
 \end{array}
 \quad
 \begin{array}{r}
 0100 \\
 0101 \\
 1001 \\
 \hline
 9
 \end{array}$$

$$\begin{array}{r}
 + 26 \\
 28 \\
 54 \\
 \hline
 54
 \end{array}
 \quad
 \begin{array}{r}
 + 0010 \\
 0010 \\
 0100 \\
 \hline
 0000
 \end{array}
 \quad
 \begin{array}{r}
 0110 \\
 1000 \\
 1110 \\
 \hline
 0101
 \end{array}
 \quad
 \begin{array}{r}
 + 46 \\
 66 \\
 \hline
 66
 \end{array}$$

$$\begin{array}{r}
 + 62 \\
 82 \\
 \hline
 144
 \end{array}
 \quad
 \begin{array}{r}
 + 0110 \\
 - 0000 \\
 \hline
 1110
 \end{array}
 \quad
 \begin{array}{r}
 0010 \\
 0010 \\
 0100 \\
 \hline
 0000
 \end{array}
 \quad
 \begin{array}{r}
 + 64 \\
 60 \\
 \hline
 64
 \end{array}$$

$$\begin{array}{r}
 + 28 \\
 29 \\
 57 \\
 \hline
 57
 \end{array}
 \quad
 \begin{array}{r}
 + 0010 \\
 0010 \\
 + 0101 \\
 \hline
 0000
 \end{array}
 \quad
 \begin{array}{r}
 1000 \\
 1001 \\
 0001 \\
 \hline
 0101
 \end{array}
 \quad
 \begin{array}{r}
 + 1101 \\
 (011) \\
 \hline
 1101
 \end{array}$$

← because here it's a carry we have to add 6.

$$\begin{array}{r}
 + 99 \\
 88 \\
 \hline
 187
 \end{array}
 \quad
 \begin{array}{r}
 + 1001 \\
 6000 \\
 \hline
 10010
 \end{array}
 \quad
 \begin{array}{r}
 1001 \\
 1000 \\
 0001 \\
 \hline
 0110
 \end{array}
 \quad
 \begin{array}{r}
 1001 \\
 1000 \\
 0010 \\
 \hline
 0110
 \end{array}$$

8 BCD Subtraction

$$\begin{array}{r}
 41 \\
 -16 \\
 \hline
 25
 \end{array}
 \quad
 \begin{array}{r}
 + 0100\ 0001 \\
 1110\ 1001 \\
 \hline
 0010\ 1010 \\
 \downarrow \\
 0010\ 1011 \\
 \hline
 0009\ 1010 \\
 0010\ 0101 \\
 \downarrow \text{ignore} \\
 \hline
 \end{array}
 \quad
 \begin{array}{l}
 0001\ 0110 \rightarrow 15 \\
 1110\ 01001 \rightarrow (-16) \\
 \hline
 0110 \rightarrow 6 \\
 1010 \rightarrow -6
 \end{array}$$

(Add A which is -6)

$$\begin{array}{r}
 41 \\
 -12 \\
 \hline
 29
 \end{array}
 \quad
 \begin{array}{r}
 0100\ 0001 \\
 1110\ 1101 \\
 \hline
 0010\ 1110 \\
 \downarrow \\
 0010\ 1111 \\
 \hline
 0000\ 1010 \\
 \hline
 0010\ 1001
 \end{array}
 \quad
 \begin{array}{l}
 \text{(If digit } a > 9 \text{ then only add A)}
 \end{array}$$

$$\begin{array}{r}
 12 \\
 -41 \\
 \hline
 -29
 \end{array}
 \quad
 \begin{array}{r}
 0001\ 0010 \\
 1011\ 1110 \\
 \hline
 1101\ 0000 \\
 \star \quad \downarrow \text{15}^2 \\
 + 0010\ 1111 \\
 \hline
 0000\ 1010 \\
 \hline
 0010\ 1001 \\
 \downarrow \text{ignore}
 \end{array}
 \quad
 \begin{array}{l}
 \text{(add A)}
 \end{array}$$

$$\begin{array}{r}
 + 341 \\
 252 \\
 \hline
 593
 \end{array}
 \quad
 \begin{array}{r}
 + 0011 \\
 0010 \\
 - 0101 \\
 \hline
 0101
 \end{array}
 \quad
 \begin{array}{r}
 0100 \\
 0101 \\
 1001 \\
 \hline
 0011
 \end{array}
 \quad
 \begin{array}{r}
 0001 \\
 0010 \\
 1101 \\
 \hline
 1110
 \end{array}$$

$$\begin{array}{r}
 + 341 \\
 252 \\
 \hline
 089
 \end{array}
 \quad
 \begin{array}{r}
 + 0011 \\
 1101 \\
 - 10000 \\
 \hline
 0000
 \end{array}
 \quad
 \begin{array}{r}
 0100 \\
 1010 \\
 1110 \\
 \hline
 1110
 \end{array}
 \quad
 \begin{array}{r}
 0001 \\
 1101 \\
 1111 \\
 \hline
 1001
 \end{array}$$

(add A)

A binary plus without B < a high E)

1111 0100

0101 0000

1001 0100

0100 1000 carrying one more

0111 1101

0000 1011

1111 0100

(A&B) 0101 0000

1001 0100

0000 0000

0100 0001

0100 0000

0100 0000

0100 0000

* BOOTH's Algorithm:-

- Booth's algorithm is used for signed as well as unsigned multiplication.

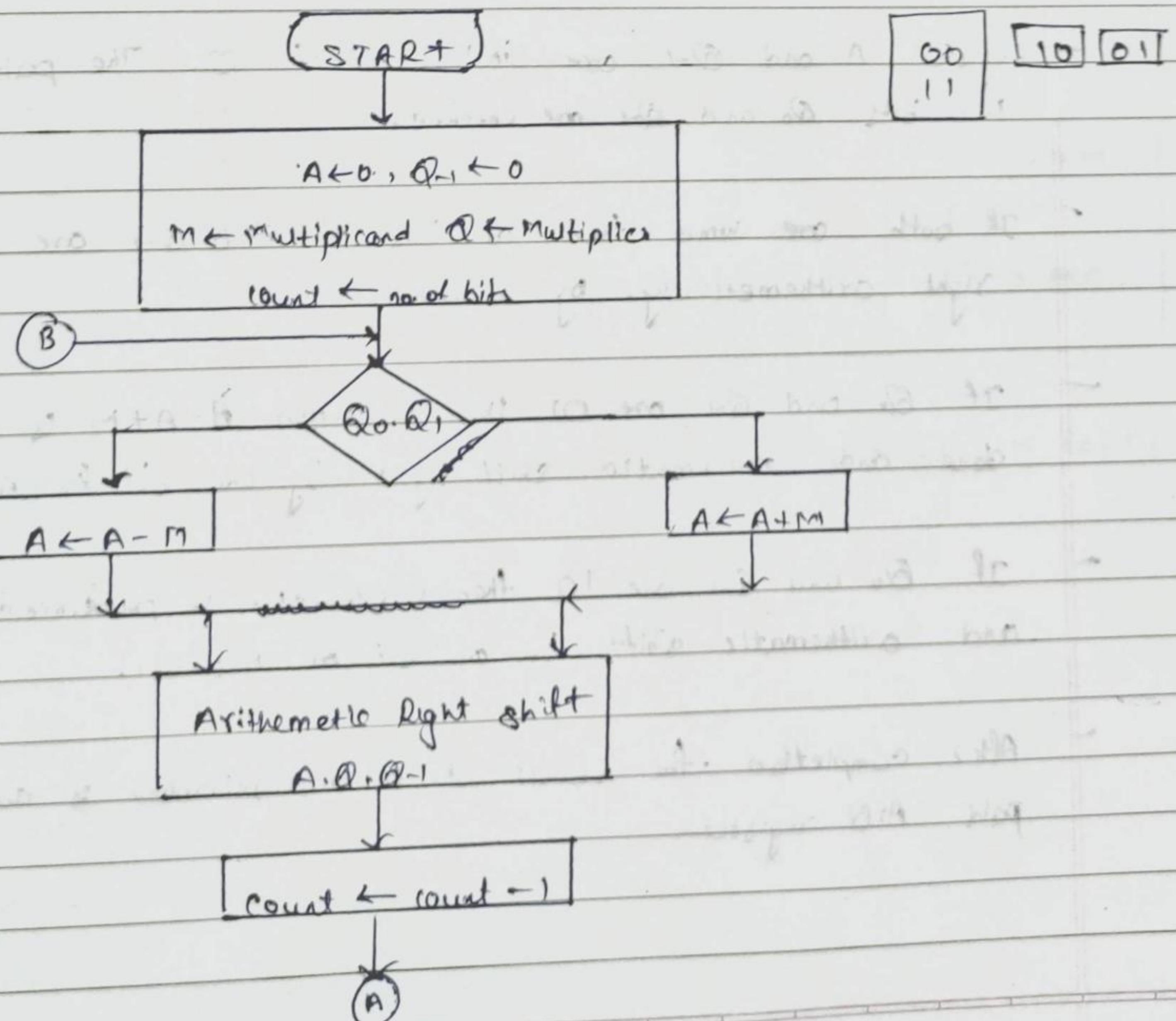
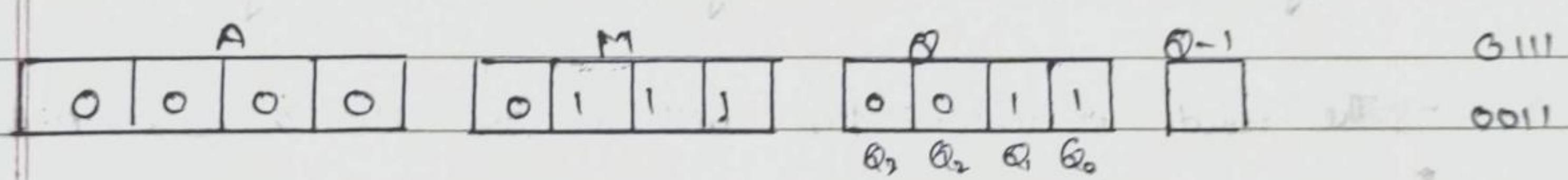
- It uses 3 registers.

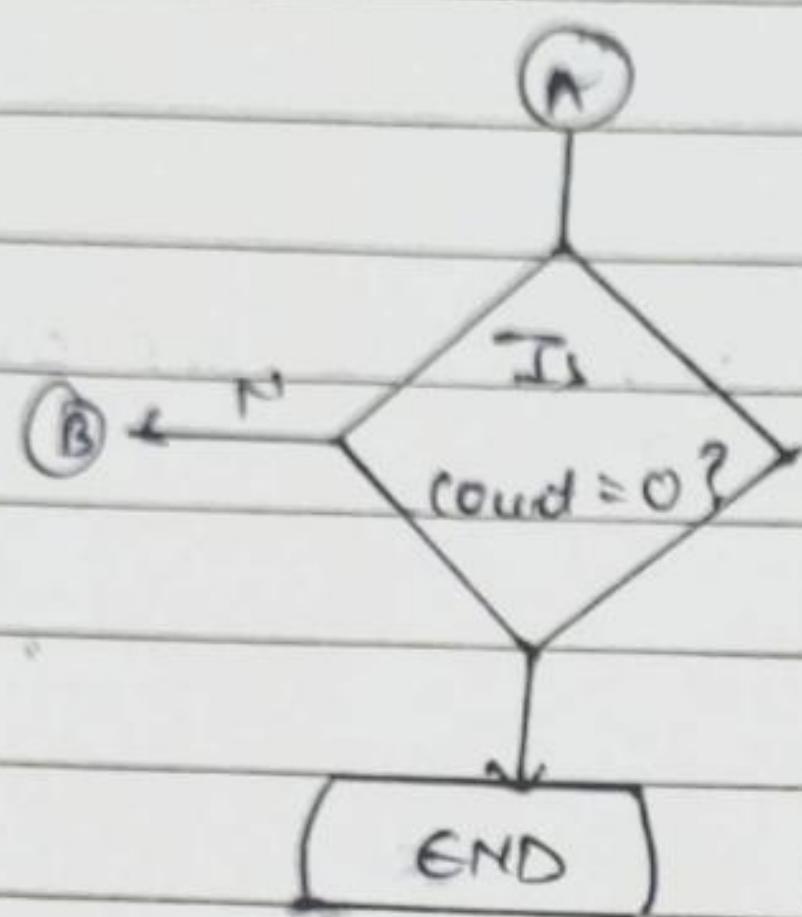
$M \rightarrow$ Multiplicand

$Q \rightarrow$ Multiplier

$A \rightarrow$ Accumulator

All the three registers are of same size, and accumulator (A) is initialized to 0.





- A one bit register placed logically to the right of the least significant bit a_0 of register A and designated as Q_1 .
- The result of multiplication will be available in pair A:Q register.
- Registers A and Q_1 are initialized to 0. The pair of both two bits a_0 and Q_1 are checked.
- If both are same then all bits of A.Q.Q₁ are shifted right arithmetically by one bit.
- If a_0 and Q_1 are 01 then addition $A = A + M$ is to be done and arithmetic shift right by one bit for A.Q.Q₁.
- If a_0 and Q_1 are 10 then subtraction is performed $A = A - M$ and arithmetic shift by one bit for A.Q.Q₁.
- After completion the result of multiplication is available in pair A:B register.

1] 7×6

$$A = 0000 ; Q = 0110 ; M = 0111 ; -M = 1001$$

$$Q_1 = 0$$

Count	A	Q	Q_1	M	-M	Action
4	0000	0110	0	0111	1001	Initially ARS AQB-1
	0000	0011	0			$Q_0 Q_1 = 10$
3	0000	0011	0			$A \leftarrow A - M$
	1001					
	1001	0011	0			
	1100	1001	1			ARS AQB-1
2	1100	1001	1			$Q_0 Q_1 = 11$
	1110	0100	1			ARS AQB-1
1	1110	0100	1			$Q_0 Q_1 = 01$
	0111					$A \leftarrow A + M$
	0101	0100	1	1000	0000	
	0010	1010	0			ARS AQB-1
0						

Product is available in AQ Register $1001010101_2 = (42)_{10}$

2] 9x8

$$A = 0000;$$

$$B = 1000;$$

$$M = 1001;$$

$$-N = 0111;$$

$$Q_4 = 0$$

Count	A	B	Q_1	$Q_2 M$	$Q_2 N$	Action
4	0000	1000	0	1001	0111	Initial $Q_{001} = 00$
	0000	0100	0	1001	0011	ARS ARS, $Q_{001} = 00$
3	0000	0100	0	0010	0111	$Q_{001} = 00$
	0000	0010	0	0010	0111	ARS ARS, $Q_{001} = 00$
2	0000	0010	0	0010	1010	$Q_{001} = 00$
	0000	0001	0	0010	1010	$ARS = ARS,$ $Q_{001} = 00$
1	0000	0001	0	0101	0100	$Q_{001} = 10$
	<u>0111</u>					$A \leftarrow A - N$
	0111	0001	0			ARS - ARS,
	0011	0000				

2] 9 x 8

$$A = 00000$$

$$@ = 01000$$

$$Q_1 = 0$$

$$M = 01001$$

$$-M = 10111$$

Count

$$5 \quad 00000 \quad 01000 \quad 0 \quad 010010 \quad 10111 \quad \text{Initially} \\ Q_0 Q_1 = 00$$

$$00000 \quad 00100 \quad 10110 \quad \dots \quad ARs \quad ARQs-1$$

$$4 \quad 00000 \quad 00100 \quad 0 \quad \dots \quad Q_0 Q_1 = 00 \\ 00000 \quad 00010 \quad 0 \quad \dots \quad ARs \quad ARQs-1$$

$$00000 \quad 00010 \quad 01000 \quad 1110 \quad Q_0 Q_1 = 00 \\ 00000 \quad 00001 \quad 01001 \quad 1100 \quad ARs \quad ARQs-1$$

$$00000 \quad 00001 \quad 01001 \quad 1100 \quad Q_0 Q_1 = 10 \\ 00000 \quad 00001 \quad 01011 \quad 1000 \quad A \leftarrow A - M$$

$$00000 \quad 00001 \quad 00111 \quad 1000 \quad ARs \quad ARQs-1 \\ 10111 \quad 00001 \quad 00111 \quad 1000 \quad Q_0 Q_1 = 01$$

$$10111 \quad 00001 \quad 00111 \quad 1000 \quad A \leftarrow A + M \\ 01011 \quad 10000 \quad 10011 \quad 0101 \\ \cancel{00101} \quad \cancel{01000} \quad \cancel{01011} \quad \cancel{10100} \quad ARs \quad ARQs-1$$

$$00100 \quad 10000 \quad 10011 \quad 1011 \quad Q_0 Q_1 = 01 \\ 100010 \quad 01000 \quad 10111 \quad 0 \quad A \leftarrow A + M$$

0

$$100010 \quad 01000 \quad 1, = (72)_{10}$$

3) -7×6

Count

$$A = 0000$$

$$Q = 0110$$

$$\leftarrow \quad M = 1001 \quad (-7)$$

$$-M = 0111 \quad (7)$$

Count

	A	Q	Q_1	M	-M	Action
4	0000	0110	0	1001	0111	Initialize $Q_0 Q_1 = 00$
	0000	0011	0	0000	0000	ARS A000,
3	0000	0011	0110	0000	0000	$Q_0 Q_1 = 10$
	<u>0111</u>			0000	0000	$A \leftarrow A - M$
	0111	0011	0	0000	0000	ARS A000,
	0011	1001	1100	0000	0000	
2	0011	1001	1100	0000	0000	$Q_0 Q_1 = 11$
	0001	1100	1	1101	0101	ARS A000,
1	+ 0001	1100	1100	1100	1101	$Q_0 Q_1 = 01$
	<u>1001</u>			0000	1101	$A \leftarrow A + M$
	1010	1100	1000	1101	1101	ARS A000,
	0101	0110	0	1101	0000	
0	1101	0110	0	0000	0000	
				0 0000	0000	

1101 0110 0

\uparrow max is 1 \therefore no is -ve.

$(1101 \ 0110 \ 1)_{23}$

0010 1010

$\boxed{(-42)_1}$

Q) $\frac{A}{B} = 6$

$$A = 0000$$

$$Q = 1010 \left(\cdot 6 \right)$$

$$H = 0111$$

$$= M = 1001$$

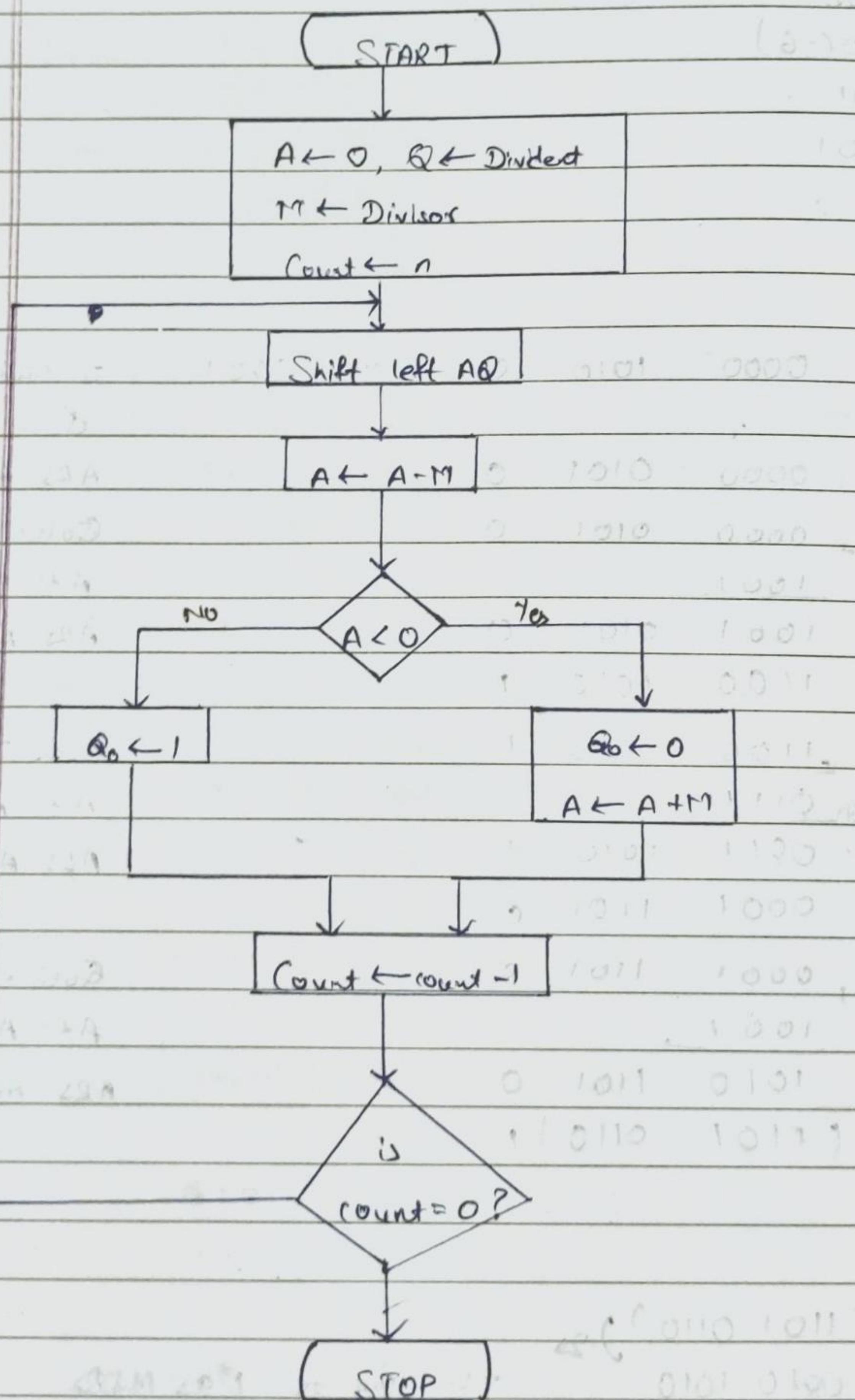
Count

4	0000	1010	0	0111	1001	Initial $Q_0 Q_1 = 00$
	0000	0101	0			All 0s initial
3	+ 0000	0101	0			$Q_2 Q_1 = 10$
	1001					$A \leftarrow A - 9A$
	1001	0101	0			All 0s after 1st
	1100	1010	1			
2	+ 1100	1010	1			$Q_0 Q_1 = 01$
	0111					$A \leftarrow A + 9A$
	0011	1010	1			All 0s after 2nd
	0001	1101	0			
1	+ 0001	1101	0			$Q_0 Q_1 = 10$
	1001					$A \leftarrow A - 9A$
	1010	1101	0			All 0s after 3rd
	(1101 0110)	1101	1			

$$(1101 \ 0110)_2 \rightarrow \\ 0010 \ 1010 \quad \text{+ve due to } 1 \text{ as MSB}$$

- 42 -

* Restoring Division



1] $13 \div 3$, $R=4$, $Q=1$

$A \leftarrow 0000$; $Q \leftarrow 1101$; $M \leftarrow 0011$; $-M \leftarrow 1101$, $n \leftarrow 4$

Count	A	Q	M	-M	Action
4	0000	1101	0011	1101	Initially
	+ 0001	101			SHL A & Q
	1101				$A \leftarrow A - M$
	+ 1110	1010	0010	1111	$A < 0$; $Q_0 \leftarrow 0$
	0011				$A \leftarrow A + M$
	00001	1010	0010	1000	
3	0001	1010	0010	1000	SHL A & Q
	+ 0011	010	0001	0100	$A \leftarrow A - M$
	1101				
	0000	0101	1001	0000	$A > 0$; $Q_0 \leftarrow 1$
2	0000	0101	1001	0000	
	+ 0000	101	0001	1000	SHL A & Q
	1101				$A \leftarrow A - M$
	1101	1010	0100	1111	$A < 0$; $Q_0 \leftarrow 0$
	0011				$A \leftarrow A + M$
	0000	1010	0100	1000	
1	0000	1010	0100	1000	
	+ 0001	010	010	0100	SHL A & Q
	1101				$A \leftarrow A - M$
	+ 1110	0100	1010	0000	$A < 0$; $Q_0 \leftarrow 0$
	0011				$A \leftarrow A + M$
	0001	0100	1010	0000	
0	0001	0100			
	(R)	(Q)			

$R = 1$, $Q = 1$

$$27 \quad 10 \div 2, \quad Q=5, \quad R=0$$

$A \leftarrow 0000; \quad Q \leftarrow 1010; \quad M \leftarrow 0010; \quad -M \leftarrow 1110; \quad n \leftarrow 4$

Count	A	M	Q	-M	Action
4	0000	1010	0010	1110	Initializer
	+ 0001	010		01	SHL AQ
	1110			1011	$A \leftarrow A-M$
	1111	0100	0101	0111	$A < 0; Q_0 \leftarrow 0$
	0010			1100	$A \leftarrow A-M$
	0001	0100	0101	1000	
3	0001	0100	0101	1000	
	+ 0010	100		010	SHL AQ
	1110			1011	$A \leftarrow A-M$
	0000	1001	1010	0000	$A \geq 0; Q_0 \leftarrow 1$
2	0000	1001	1010	0000	
	+ 0001	001		101	SHL AQ
	1110			1011	$A \leftarrow A-M$
	1111	0010	0101	1011	$A \geq 0; Q_0 \leftarrow 0$
	0010			1100	$A \leftarrow A-M$
	0001	0010	0101	0000	
1	0001	0010	0101	0000	
	+ 0010	010		010	SHL AQ
	1110			1011	$A \leftarrow A-M$
	0000	0101	0010	0111	$A \geq 0; Q_0 \leftarrow 1$
	0000	0101		1100	
	(R)	(Q)		0010	1000
				0010	1000
				101	101

$R=0, \quad Q=5.$

3] $23 \div 4$, $Q=5$, $R=3$

$A \leftarrow 000000$; $B \leftarrow 10011$; $M \leftarrow 0000100$; $-M \leftarrow 11100$; $n \leftarrow 5$

Count	A	B	M	-M	Action
-------	---	---	---	----	--------

5 00000 10111 00100 11100 ~~Indirect~~
 $+ 00001$ 0111
11100
 $+ 11101$ 01110
00100
00001 01110

4 00001 01110
 $+ 00010$ 1110
11100
 $+ 11110$ 11100
00100
00000 11100

3 00010 11100
 $+ 00101$ 1100
11100
00001 1100
1 A20; Q₀ ← 1

2 00001 11001
 $+ 00011$ 1001
11100
 $+ 11111$ 10010
00100
00011 10010

1 00011 10010
 $+ 00111$ 0010
11100
00011 00101
1 A20; Q₀ ← 1

0 00011 00101
(R) (P)

4] $g \div 8$, $Q=1$, $R=1$ (If both are 4 bits take 5 bits)

$A \leftarrow 0000$; $B \leftarrow 0000$; $C \leftarrow 0000$; $D \leftarrow 0000$; $E \leftarrow 0000$

Step 1: $M = M - Q$ (Subtract 1 from M)

Step 2: $00111 - 00100 = 00001$ (Subtract 1 from B)

Step 3: $00100 - 00100 = 00000$ (Subtract 1 from C)

Step 4: $00000 - 00100 = 00100$ (Subtract 1 from D)

Step 5: $00100 - 00100 = 00000$ (Subtract 1 from E)

Step 6: $00000 - 00100 = 00100$ (Subtract 1 from B)

Step 7: $00100 - 00100 = 00000$ (Subtract 1 from C)

Step 8: $00000 - 00100 = 00100$ (Subtract 1 from D)

Step 9: $00100 - 00100 = 00000$ (Subtract 1 from E)

Step 10: $00000 - 00100 = 00100$ (Subtract 1 from B)

Step 11: $00100 - 00100 = 00000$ (Subtract 1 from C)

Step 12: $00000 - 00100 = 00100$ (Subtract 1 from D)

Step 13: $00100 - 00100 = 00000$ (Subtract 1 from E)

Step 14: $00000 - 00100 = 00100$ (Subtract 1 from B)

Step 15: $00100 - 00100 = 00000$ (Subtract 1 from C)

Step 16: $00000 - 00100 = 00100$ (Subtract 1 from D)

Step 17: $00100 - 00100 = 00000$ (Subtract 1 from E)

Step 18: $00000 - 00100 = 00100$ (Subtract 1 from B)

Step 19: $00100 - 00100 = 00000$ (Subtract 1 from C)

Step 20: $00000 - 00100 = 00100$ (Subtract 1 from D)

Step 21: $00100 - 00100 = 00000$ (Subtract 1 from E)

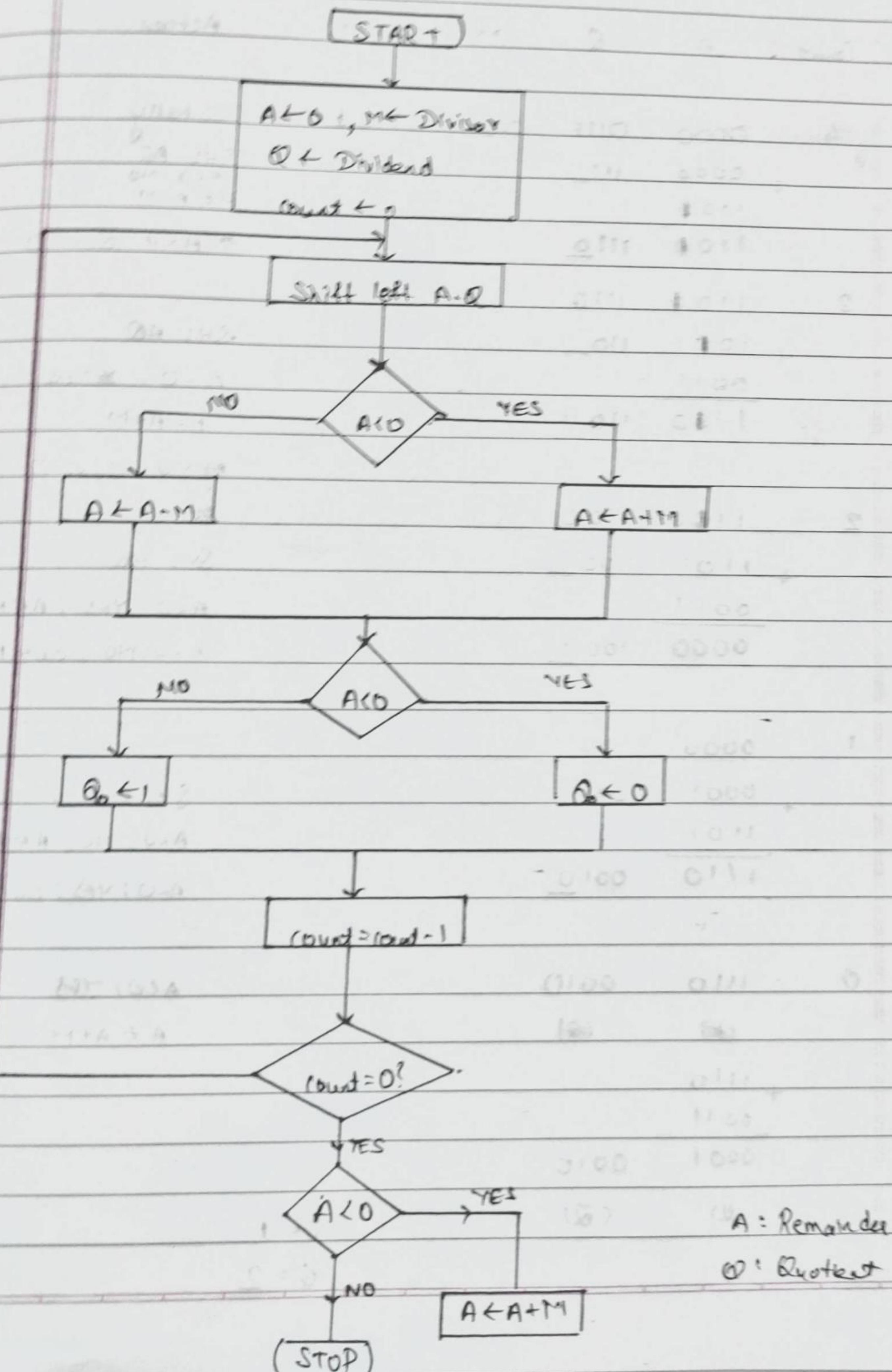
Step 22: $00000 - 00100 = 00100$ (Subtract 1 from B)

Step 23: $00100 - 00100 = 00000$ (Subtract 1 from C)

Step 24: $00000 - 00100 = 00100$ (Subtract 1 from D)

Step 25: $00100 - 00100 = 00000$ (Subtract 1 from E)

* Non Restoring Division



2] $7 \div 3$ $A \leftarrow 0000 ; Q = 0111 ; M = 0111 ; -M = 1100 ; n = 4$

Count	A	Q	M	Action
-------	---	---	---	--------

4	0000	0111	0011	1100 Initially
---	------	------	------	----------------

+ 0000	111		SHL AQ
1100			$A \leftarrow 0 ; M \leftarrow 0$
1100	1110		$A \leftarrow A - M$

3	1100	1110	
---	------	------	--

+ 1001	110		SHL AQ
0011			$A < 0 ; \text{YES}$
1100	1100		$A \leftarrow A + M$

2	1100	1100	
---	------	------	--

+ 1101	100		SHL AQ
0011			$A < 0 \text{ YES} ; A \leftarrow A + M$
0000	1001		$A < 0 ; \text{NO} ; Q_0 \leftarrow 1$

1	0000	1001	
---	------	------	--

+ 0001	001		SHL AQ
1101			$A < 0 ; \text{NO} ; A \leftarrow A - M$
1110	0010		$A < 0 ; \text{YES} ; Q_0 \leftarrow 0$

0	1110	0010	
---	------	------	--

(R)	(Q)		$A < 0 ; \text{YES}$
+ 1110			$A \leftarrow A + M$
0011			
0001	0010		
(R)	(Q)		$R = 1$
			$Q = 2$

2] $8 \div 8$

$A \leftarrow 00000 ; Q = 01000 ; M = 01000 ; -M = 01111, \neg A = 11111, N = 11000$

Count A Q M -M Action

5 00000 01000 01000 11000 Initially

Step 0 1 2 3
 Add 0 1 1 1
 DATA 0 1 0 0
 Result 1 0 0 0
 Borrow 0 0 0 0

(Right) towards addition, result 0

Step 0 1 2 3 4
 Add 0 1 1 1 1
 DATA 0 1 0 0
 Result 1 0 0 0
 Borrow 0 0 0 0

(Left) towards addition, result 0

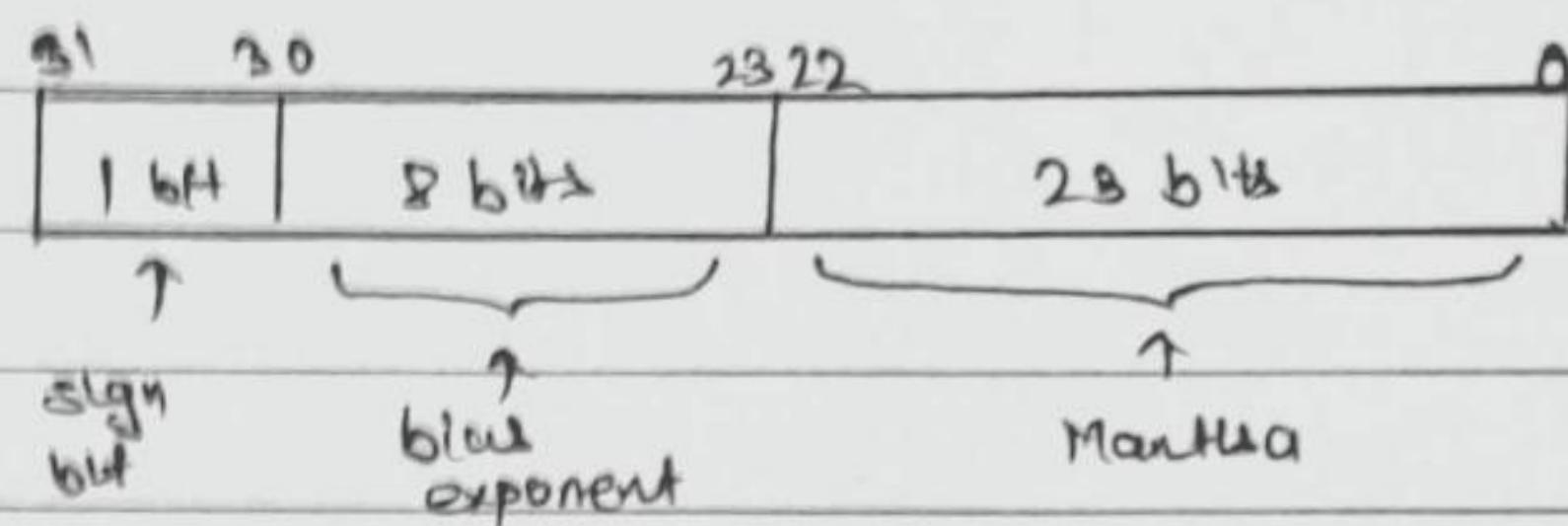
* Floating point number representation :-

IEEE 754 Format

1] 32-bit (Single Precision)

2] 64-bit (Double Precision)

1] Single precision format (32 bit)

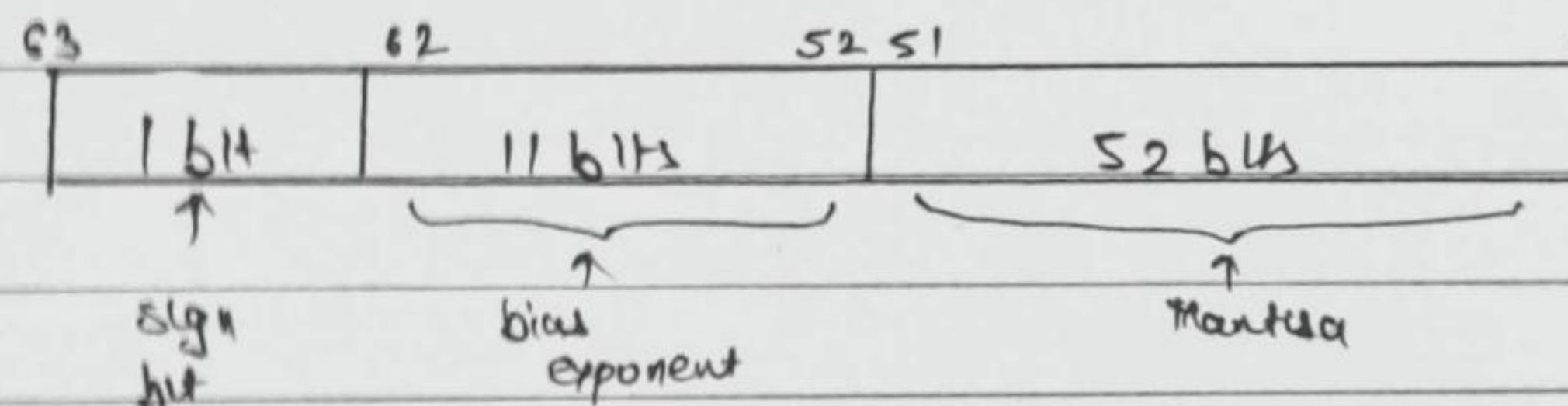


$$\text{Bias Exponent} := \text{Exponent} + 127$$

$$= \text{Exponent} + (7F)_{16}$$

$$2^{7-1} = 127$$

2] Double precision format (64 bits)



$$\text{Bias Exponent} := \text{Exponent} + 1023$$

$$= \text{Exponent} + (7FF)_{16}$$

$$2^{10-1} = 1023$$

Q. Represent the following numbers into single precision format.

$$1] (10.25)_{10}$$

$$\rightarrow (10.25)_{10} \rightarrow (1010.00100101)_2$$

$$1.025 \times 10^1 \rightarrow$$

$$0.25 \times 2 = 0.5$$

$$0.5 \times 2 = 1.0$$

$$(10.25)_{10} \rightarrow (1010.0100)_2$$

$$\begin{aligned} & 1010.0100 \\ & 1.0100 \times 2^3 \\ & 1.0 \times 2^e \end{aligned}$$

$$E = 3$$

$$\begin{aligned} BE &= 3 + 127 \\ &= (130)_{10} \end{aligned}$$

$$\begin{aligned} BE &= 3 + 127 = 130 \\ &= (82)_{16} \end{aligned}$$

$$BE = (1000\ 0010)_2$$

$$BE = 010010$$

Sign is +ve

$$\begin{array}{r} 0 \underline{1000\ 0010\ 010010\ 00} \\ \hline (41240000)_{10} \end{array}$$

2] $(10.25)_{10}$

$$0.25 \times 2 = 0.5$$

$$0.5 \times 2 = 1.0$$

$$(10.25)_{10} \rightarrow (1010, 0100)_2$$

$$\begin{array}{r} 1 \cdot 010010 \times 2^3 \\ + 1 \cdot M \times 2^0 \end{array}$$

$$E = 3$$

$$\begin{aligned} BE &= 3 + (3FF)_{16} \\ &= (402)_{16} \end{aligned}$$

$$\begin{array}{r} BE = (0\cancel{0}00 \ 0000 \ 0010)_2 \\ \text{11 bits only.} \end{array}$$

$$B.E. = (100 \ 0000 \ 0010)_2$$

$$M = 010010$$

begin is true.

0 100 0000 0010 0000 1000 --

$(402480000000000)_{16}$

3] $(-8.08)_{10}$

$$\rightarrow 8 = (1000)_2$$

$$0.08 = (000101)_2$$

$$(-8.08) = -(1000.000101)$$

$$0.08 \times 2 = 0.16$$

$$0.16 \times 2 = 0.32$$

$$0.32 \times 2 = 0.64$$

$$0.64 \times 2 = 1.28$$

$$0.28 \times 2 = 0.56$$

$$0.56 \times 2 = 1.12$$

$$1.12 \times 2^{\epsilon}$$

$$M = 1.000000101, \epsilon = 3$$

$$BE = 7F + 3$$

$$= (82)_{16}$$

$$BE = 1000\ 0010$$

$$\text{IEEE 754} = \underline{1100\ 000} \underline{000000} \underline{10100} \dots$$

$$= (C1010000)_{16}$$

4] (-35.35)

$$\rightarrow (35)_{10} = (100011)_2$$

$$0.35 \times 2 = 0.70$$

$$0.35 = (010110)_2$$

$$0.4 \times 2 = 0.8$$

$$0.8 \times 2 = 1.6$$

$$(-35.35)_{10} = (100011.010110)_2$$

$$0.6 \times 2 = 1.2$$

$$0.2 \times 2 = 0.4$$

$$1.00011010110 \times 2^5$$

$$M = 00011010110, \epsilon = 5$$

$$BE = 7F + 5$$

$$= (84)_{16}$$

$$= (1000 \ 0100)_2$$

$$\text{IEEE754} = (1100 \ 0010 \ 0000 \ 1101 \ 0110)$$

$$= (C20D6000)_{16}$$

Q. Convert the following IEEE 754 32-bit number into its equivalent decimal number.

1] $1 \ 0111 \ 1111 \ 1000 \ 0000 = (BF(00000))$

\rightarrow 1 0111 1111 1000 0000
 Signed BE

$$BE = 7F$$

$$E = 7F - 7F = 0$$

$$M = 1000 \ 0000$$

$$1 \cdot M \times 2^E$$

$$= (1.1000 \ 0000) \times 2^0$$

$$= -(1.1000 \ 0000)_2 \Rightarrow (-1.5)_{10}$$

2] (2430000)

\rightarrow 1100 0010 0100 0011 0000 ...
 BE M

$$BE = 84$$

$$E = 84 - 7F = 5$$

$$M = 1000 \ 0110 \ 0000$$

$$= (1,1000 \ 0110) \times 2^8$$

$$r = (110000, 11000)$$

$$e = (48, 75)$$

Sign

Binary
Exponent

Mantissa

Magnitude

0

0

Non-zero

10

1

0

0

-0

0

FF

0

+∞

1

FF

0

-∞

x

0

Non-zero

Normalised number

x

FF

Non-zero

Not a number (NaN)

BOOLEAN ALGEBRA

* Boolean Laws

1] AND laws

$$A \cdot 0 = 0$$

$$A \cdot A = A$$

$$A \cdot 1 = A$$

$$A \cdot \bar{A} = 0$$

2] OR laws

$$A + 0 = A$$

$$A + 1 = 1$$

$$A + A = A$$

$$A + \bar{A} = 1$$

3] Commutative laws

$$A+B = B+A$$

$$A \cdot B = B \cdot A$$

4] Associative law

$$A+B+C = (A+B)+C = A+(B+C)$$

$$A \cdot B \cdot C = (A \cdot B) \cdot C = A \cdot (B \cdot C)$$

5] Distributive law

$$A \cdot (B+C) = (A \cdot B) + (A \cdot C)$$

$$A + (B \cdot C) = (A+B) \cdot (A+C)$$

6] De Morgan's law

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

7] Other laws

$$A + AB = A$$

$$A + \bar{A} \cdot B = A+B$$

$$(A+B) \cdot (A+C) = A+B \cdot C$$

$$AB + AC = A \cdot (B+C)$$

$$\bar{\bar{A}} = A$$

$$0 = 0 \cdot A$$

$$A = A \cdot A$$

$$A = 1 \cdot A$$

$$0 = \bar{A} \cdot A$$

$$A = A \cdot 1$$

$$A = A + A$$

$$A = 1 + A$$

$$A = A \cdot A$$

$$A = A + A$$

$$A = A \cdot A$$

$$A = A + A$$

$$\begin{array}{ccccccc}
 & & & L.H.S & & & R.H.S \\
 A & B & A+B & \overline{A+B} & \bar{A} & \bar{B} & \bar{A} \cdot \bar{B}
 \end{array}$$

0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

$$\therefore \overline{A+B} = \bar{A} \cdot \bar{B}$$

$$\begin{array}{ccccccc}
 & & & L.H.S & & & R.H.S \\
 A & B & A \cdot B & \overline{A \cdot B} & \bar{A} & \bar{B} & \bar{A} + \bar{B}
 \end{array}$$

0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

$$\begin{aligned}
 7] \quad y &= A + \bar{A} \cdot B + A \cdot \bar{B} \\
 &= A + B + A\bar{B} \\
 &= A + B + A \\
 &= A + A + B \\
 &= A + B
 \end{aligned}$$

$$\begin{aligned}
 8] \quad y &= (x+y) \cdot (x+\bar{y}) \cdot (\bar{x}+z) \\
 &= ((x \cdot x) + x \cdot \bar{y} + y \cdot x + y \cdot \bar{y}) \cdot (\bar{x}+z) \\
 &= (x + x \cdot \bar{y} + x \cdot y + 0) \cdot (\bar{x}+z) \\
 &= (x + x(1)) \cdot (\bar{x}+z) \\
 &= (x + x)(\bar{x}+z) \\
 &= x(\bar{x}+z) \\
 &= x \cdot \bar{x} + xz \\
 &= 0 + xz
 \end{aligned}$$

$$w7) y = (\bar{A}\bar{B} + \bar{A} + AB)$$

$$\begin{aligned}
 &= (\bar{A} \cdot \bar{B}) \cdot (\bar{A}) \cdot (\bar{A} \cdot B) \\
 &= (\bar{A} + \bar{B}) \cdot (\bar{A}) \cdot (\bar{A} + B) \\
 &= (A + \bar{B}) \cdot (A) \cdot (\bar{A} + \bar{B}) \\
 &= (A + \bar{B}) \cdot (A \cdot \bar{A} + A \cdot B) \\
 &= (A + \bar{B}) \cdot (A \cdot \bar{A}) \\
 &= A \cdot \bar{A} \bar{B} + \bar{B} \cdot A \cdot \bar{A} \\
 &= A\bar{B} + \bar{B} \cdot \bar{B} \cdot A \\
 &= A\bar{B} + \bar{B} + A\bar{B} \\
 &= A\bar{B} \\
 &= A \cdot \bar{B}
 \end{aligned}$$

$$w7) y = \bar{A} + \bar{B}C \cdot (\bar{A}\bar{B} + \bar{A}BC)$$

$$\begin{aligned}
 &= (\bar{A} \cdot BC) (\bar{A}\bar{B} + \bar{A}B + \bar{C}) \\
 &= (\bar{A} \cdot BC) (\bar{B} + (A + \bar{B}) \cdot \bar{B} + \bar{C}) \\
 &= (\bar{A} \cdot BC) (\bar{B} + \bar{C}) \\
 &= (\bar{A} \cdot B \cdot C) \cdot (\bar{B} + \bar{C}) \\
 &= \bar{B} \cdot (\bar{A} \cdot B \cdot C) + \bar{C} (\bar{A} \cdot B \cdot C) \\
 &= \bar{B} \cdot \bar{A} \cdot B \cdot C + \bar{A} \cdot B \cdot (\bar{B} + \bar{A} \cdot B \cdot C) \\
 &= \bar{B} \cdot \bar{A} \cdot B \cdot C
 \end{aligned}$$

$$v) y = C(\bar{ABC} + A\bar{B}C)$$

$$\begin{aligned}
 &= C (\bar{A}\bar{B} + \bar{C} + A \cdot \bar{B} \cdot C) \\
 &= C (\bar{A} + \bar{B} + \bar{C} + A\bar{B} \cdot C) \rightarrow \cancel{\bar{A} \cdot C + \bar{B} \cdot C + C \cdot A \cdot \bar{B}} \\
 &= (C \cancel{\bar{A} + \bar{B} + \bar{C}} + \bar{B} (1 + A\bar{B})) \Rightarrow \cancel{(A + \bar{B}) \cdot C} + A \cdot \bar{B} \cdot C \\
 &= C (\bar{B} + \bar{C} + \bar{B}) \\
 &= C (\bar{A} \cdot \bar{B} + \bar{C}) \\
 &= \bar{A} \cdot \bar{B} \cdot C + C \cdot \bar{C} \\
 &= \bar{A} \cdot \bar{B} \cdot C \\
 &= \bar{A} \cdot \bar{B} \cdot C \\
 &= f(A + \bar{B}), \\
 &= (\bar{A} + (A \bar{B})) \cdot C \\
 &= (\bar{A} + A\bar{B}) \cdot C
 \end{aligned}$$

$$\begin{aligned}
 y &= \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{B} \cdot C \cdot \bar{D} + \bar{A} \cdot B \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot B \cdot C \cdot \bar{D} + A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} \\
 &= \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{D} (\bar{B} \cdot C + B \cdot C) + \bar{A} \cdot B \cdot \bar{C} \cdot \bar{D} + A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} \\
 &= \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{D} ((\bar{B} + B) \cdot C) + (\bar{A} \cdot B + A \cdot \bar{B}) \cdot \bar{C} \cdot \bar{D} \\
 &= \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{D} (C) + (\bar{A} \cdot B + A \cdot \bar{B}) \cdot \bar{C} \cdot \bar{D} \\
 &= (\bar{A} \cdot \bar{B} + \bar{A} \cdot B + A \cdot \bar{B}) \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{D} \cdot C \\
 &= (\bar{A} + \bar{B} + \bar{A} \cdot B + A \cdot \bar{B}) \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{D} \cdot C \\
 &= (\bar{A} + \bar{B}) \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{D} \cdot C \\
 &= \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{D} \cdot C \\
 &= (\bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{A} \cdot C) \cdot \bar{D} \\
 &= (\bar{A} \cdot \bar{B} + \bar{C} + \bar{A} \cdot C) \cdot \bar{D} \\
 &= (\bar{A} + \bar{B} + \bar{C} + \bar{A} \cdot C) \cdot \bar{D} \\
 &= (\bar{A} + \bar{B} + \bar{C}) \cdot \bar{D} \\
 &= \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} \\
 &= \dots
 \end{aligned}$$

$$\begin{aligned}
 y &= \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{B} \cdot C \cdot \bar{D} + \bar{A} \cdot B \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot B \cdot C \cdot \bar{D} + A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} \\
 &= \bar{A} \cdot \bar{B} \cdot \bar{D} (\bar{C} + C) + \bar{A} \cdot B \cdot \bar{D} (\bar{C} + C) + A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} \\
 &= \bar{A} \cdot \bar{B} \cdot \bar{D} + \bar{A} \cdot B \cdot \bar{D} + A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} \\
 &= \bar{A} \cdot \bar{D} (\bar{B} + B) + A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} \\
 &= \bar{A} \cdot \bar{D} + A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} \\
 &= \bar{D} (\bar{A} + A \cdot \bar{B} \cdot \bar{C}) \\
 &= \bar{D} (\bar{A} + \bar{B} \cdot \bar{C}) \\
 &= \bar{D} \cdot \bar{A} + \bar{D} \cdot \bar{B} \cdot \bar{C}
 \end{aligned}$$

a. $(1011011, 0010110)_2$

$\rightarrow 1011011 \underline{1011011}$

$(5B.2C)_{16}$

b. 15.4 to IEEE 754

$$\rightarrow 0.4 \times 2 = 0.8$$

$$0.8 \times 2 = 1.6$$

$$0.6 \times 2 = 1.2$$

$$0.2 \times 2 = 0.4$$

$$0.4 \times 2 = 0.8$$

$(15.4)_10 \rightarrow (111.0110)_2$

$$1.1110110 \times 2^3 = (1+3) \times 2.0$$

$$1.1110110 \times 2^2 = (1+3+1) \times 2.0$$

$$1.1110110 \times 2^1 = (1+3+1+1) \times 2.0$$

$$E = 3$$

$$BE = 3 + 7F$$

$$= (82)_{16}$$

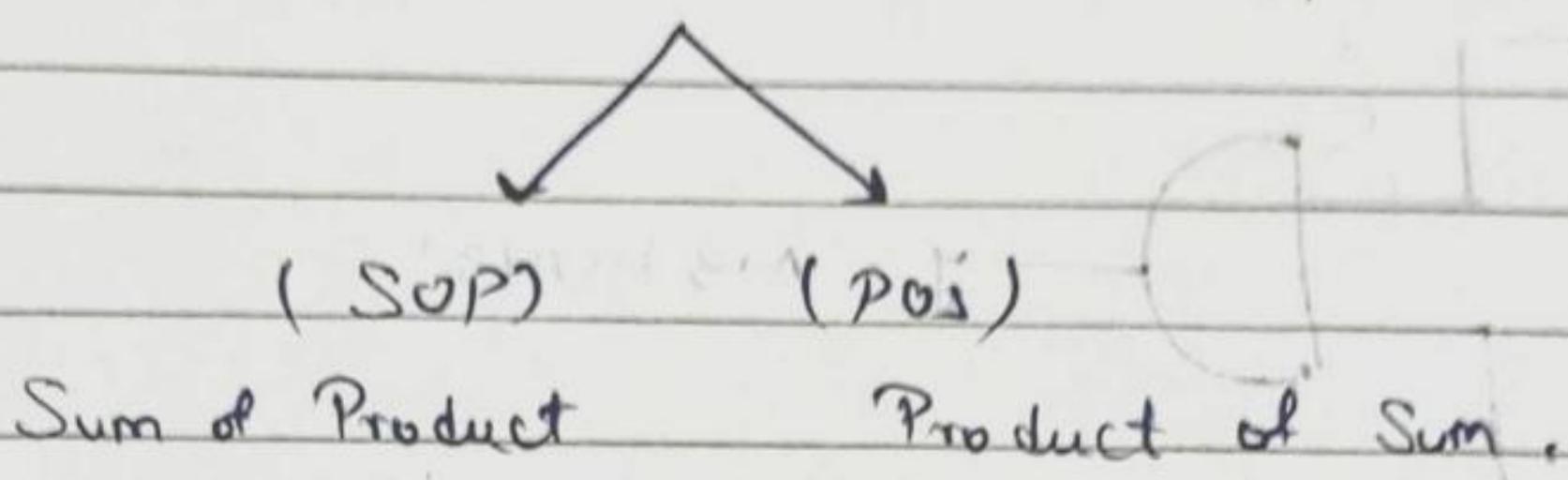
$$BE = (1000\ 0010)_2$$

$$M = 1110110$$

$$0 \underline{1000} \underline{0010} \underline{1110} \underline{1100} \dots$$

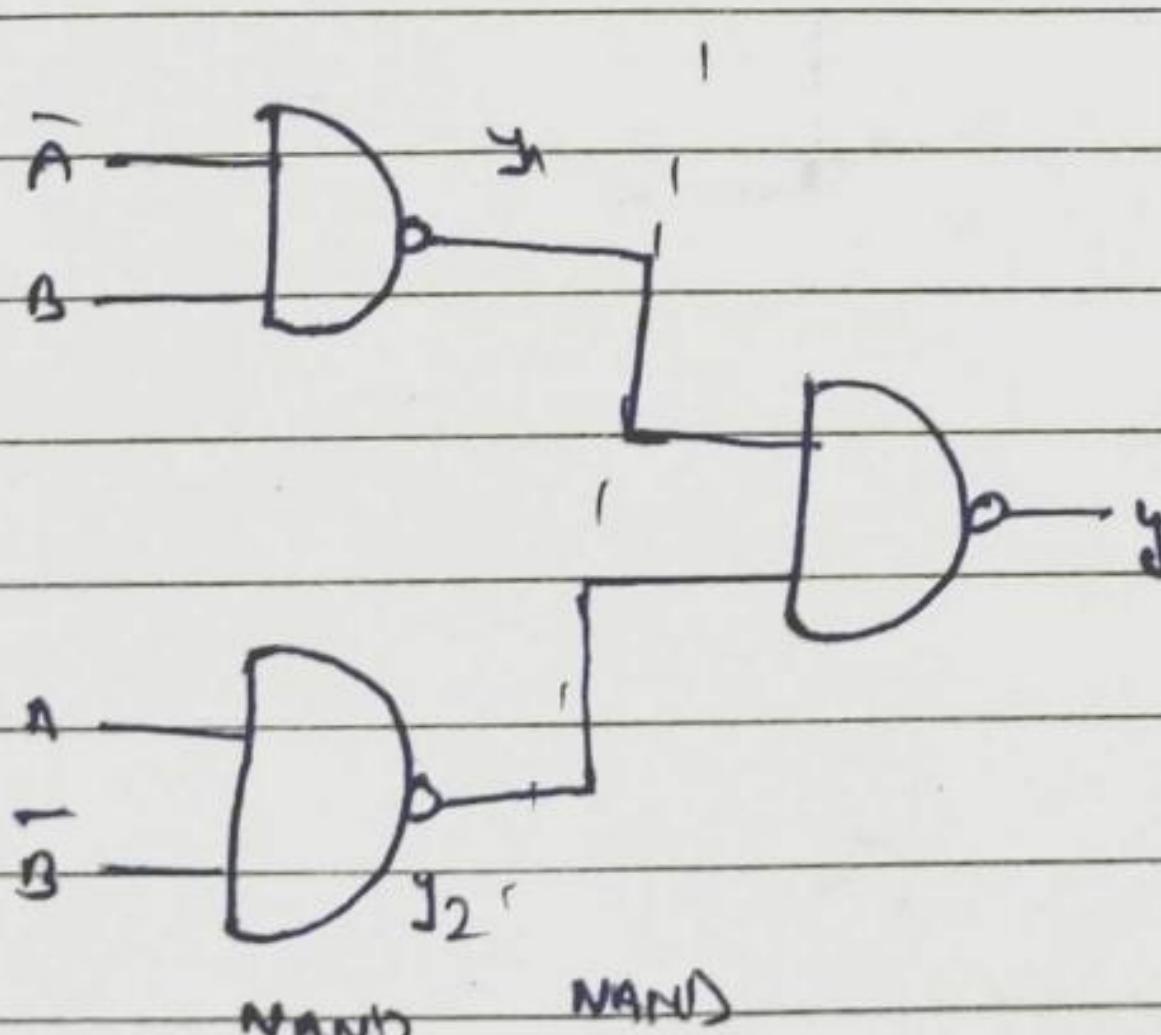
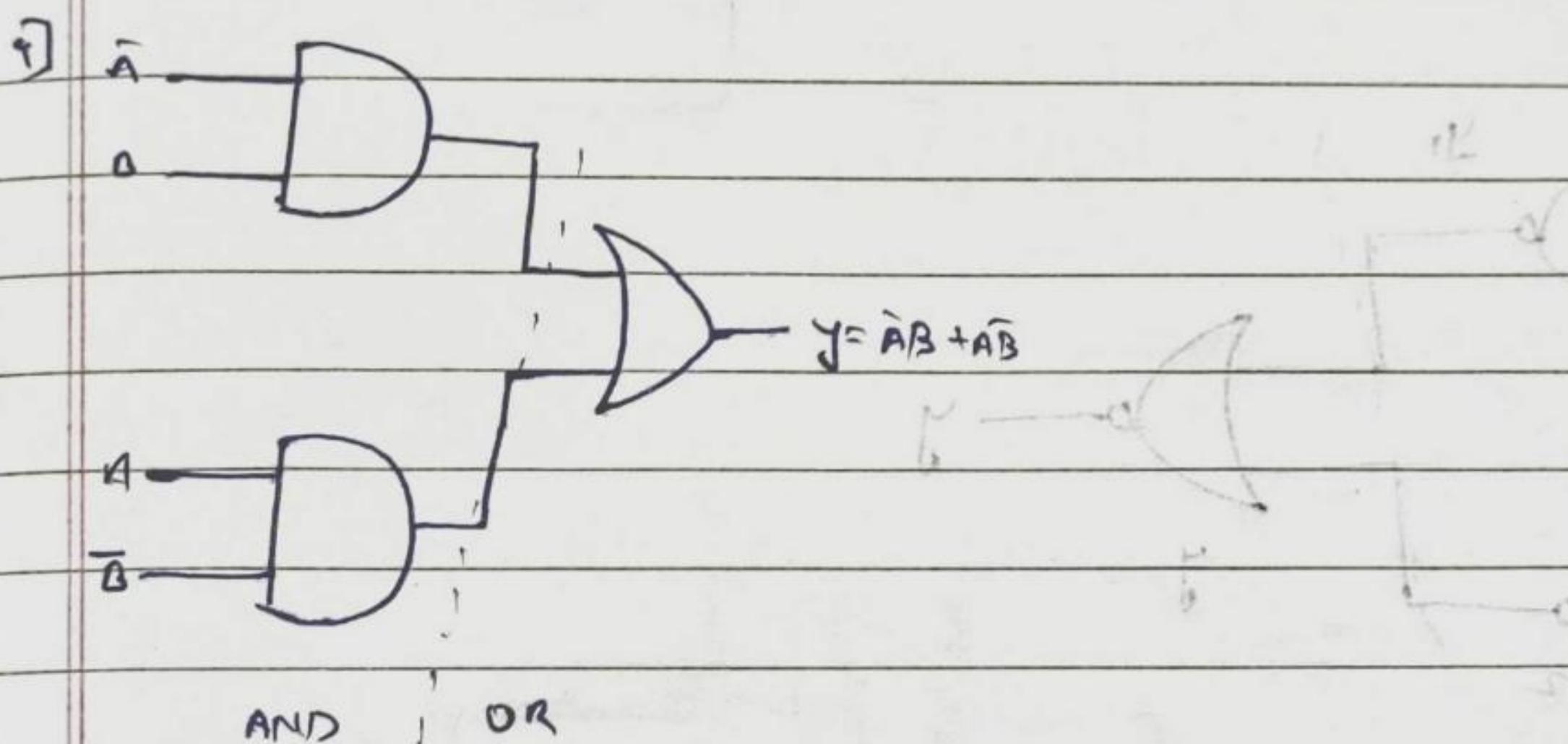
$$= (41760000)_{16}$$

* Standard Form of Boolean expression



$$\boxed{i} \quad y = \bar{A} \cdot B + A \cdot \bar{B}$$

$$\boxed{ii} \quad y = (\bar{A} + B) \cdot (A + \bar{B})$$



$$(\bar{A} \cdot B) = \overline{A} \cdot \overline{B} ; (\bar{A} + B) = \overline{\bar{A} \cdot B}$$

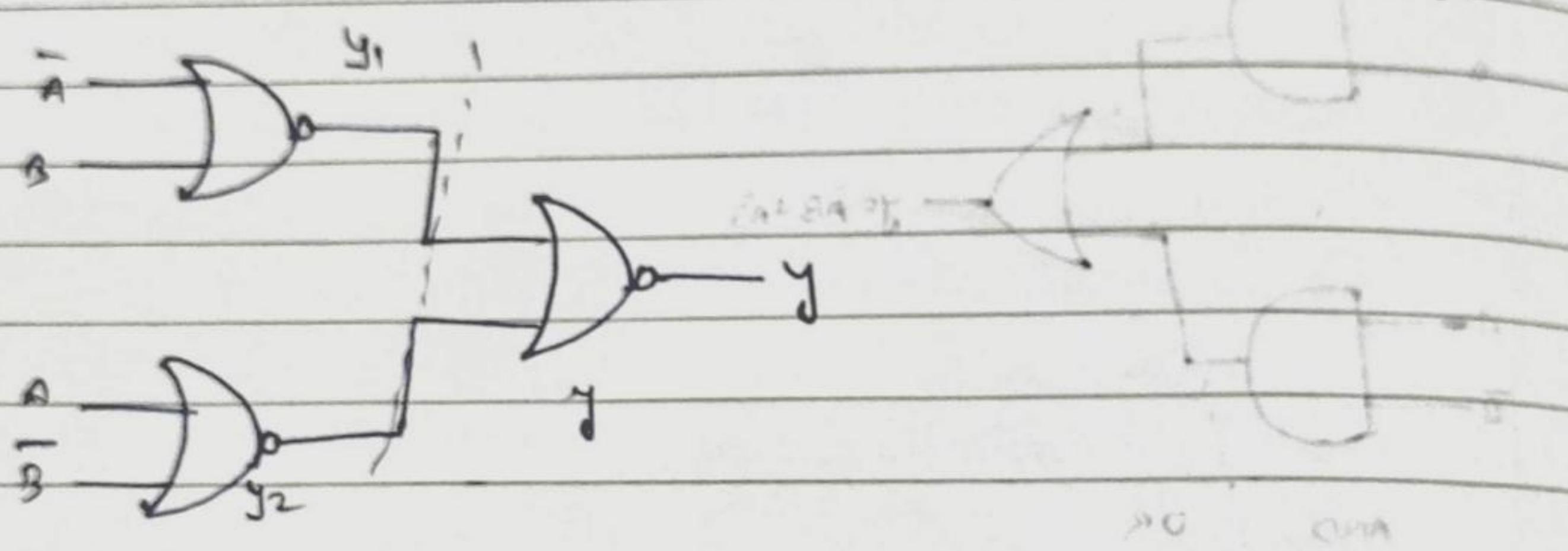
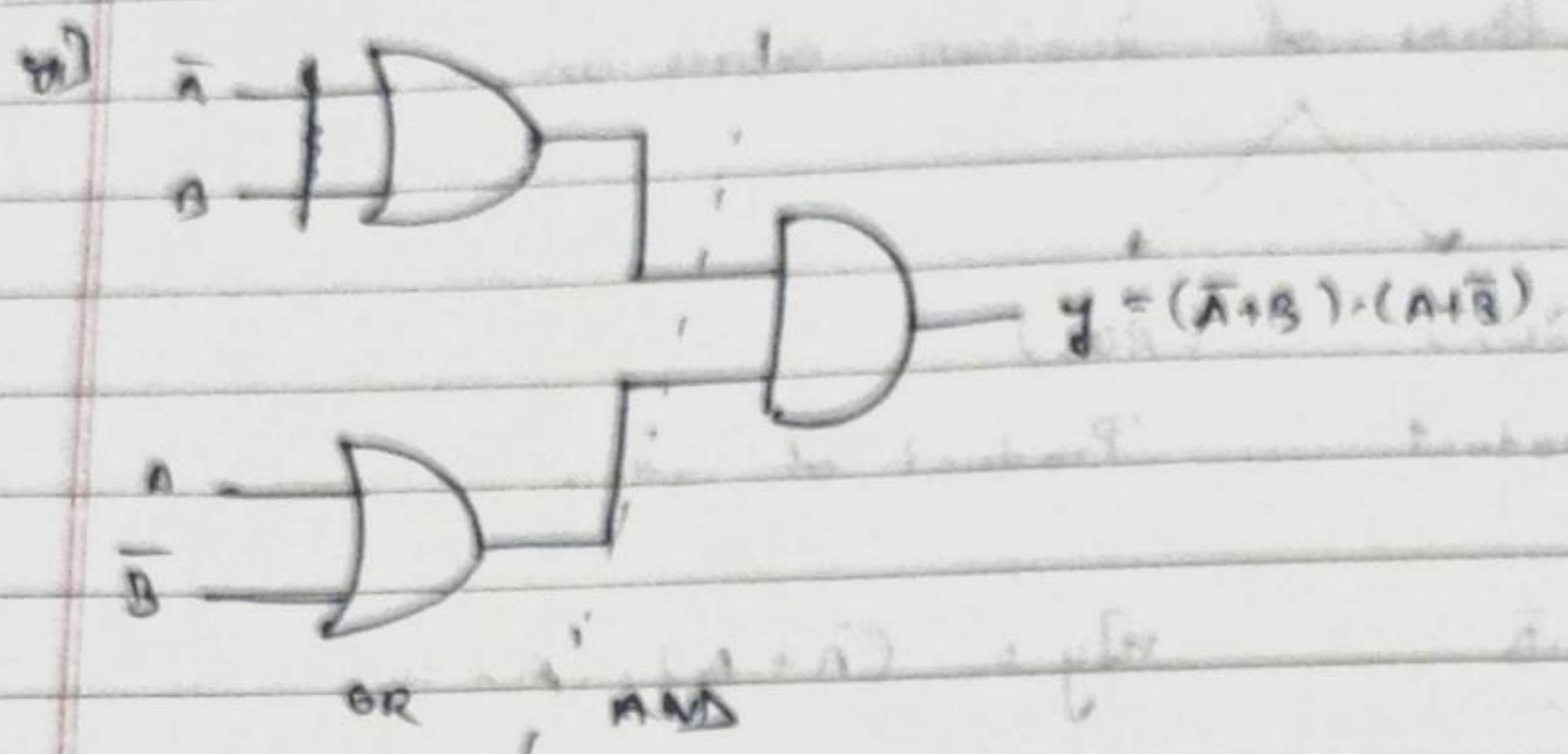
$$y_1 = \overline{\bar{A} \cdot B} ; y_2 = \overline{A \cdot \bar{B}}$$

$$y = \overline{y_1 \cdot y_2}$$

$$y = \overline{(\bar{A} \cdot B)} \cdot \overline{(A \cdot \bar{B})}$$

$$y = \overline{\bar{A} \cdot B + A \cdot \bar{B}}$$

Prove AND/OR = NAND+NAND



$$y_1 = \overline{(\bar{A} + B)} ; y_2 = \overline{(A + \bar{B})}$$

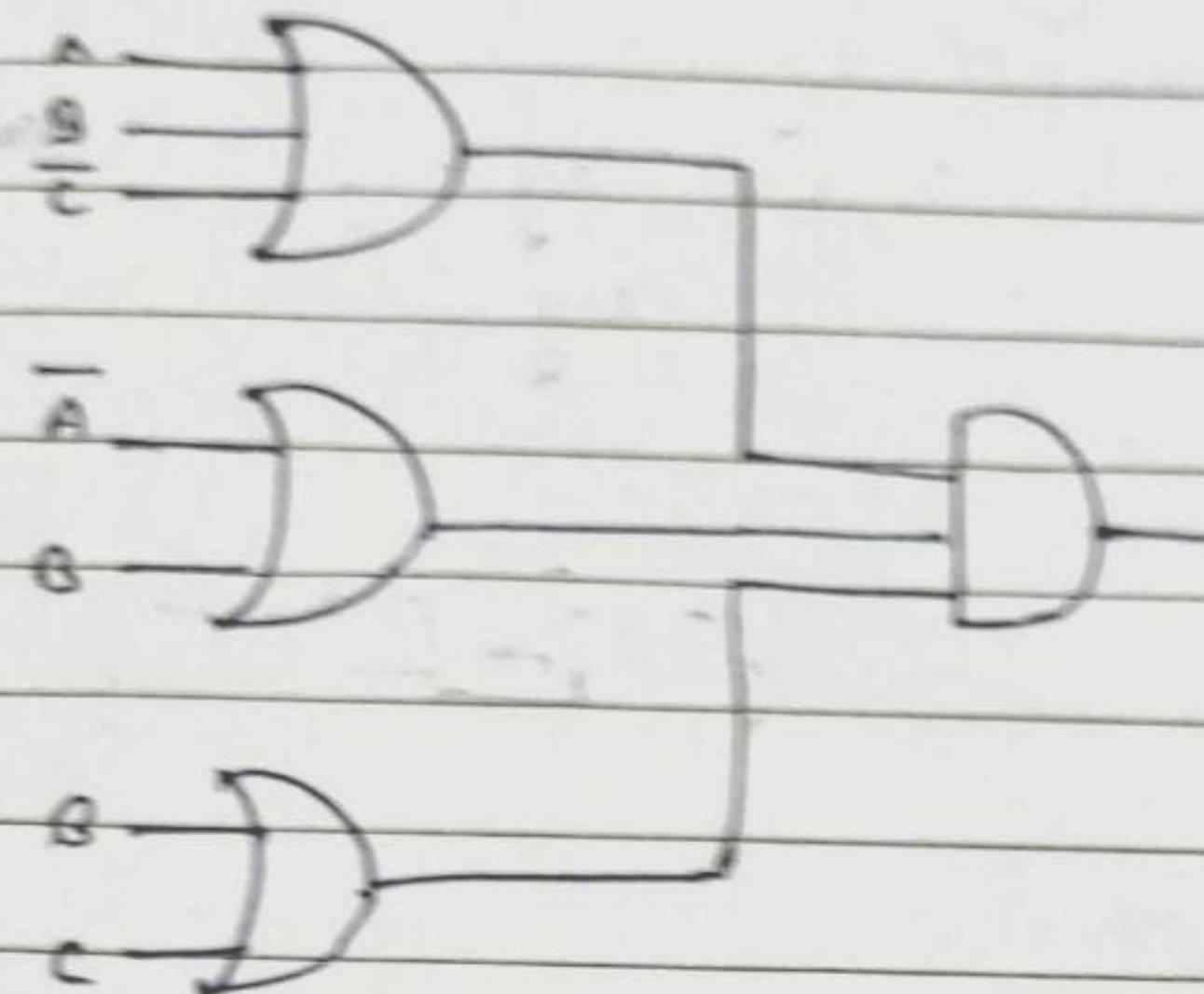
$$\begin{aligned} y &= \overline{y_1 + y_2} \\ &= \overline{\overline{(\bar{A} + B)} + \overline{(A + \bar{B})}} \end{aligned}$$

$$= (\bar{A} + B) \cdot (A + \bar{B})$$

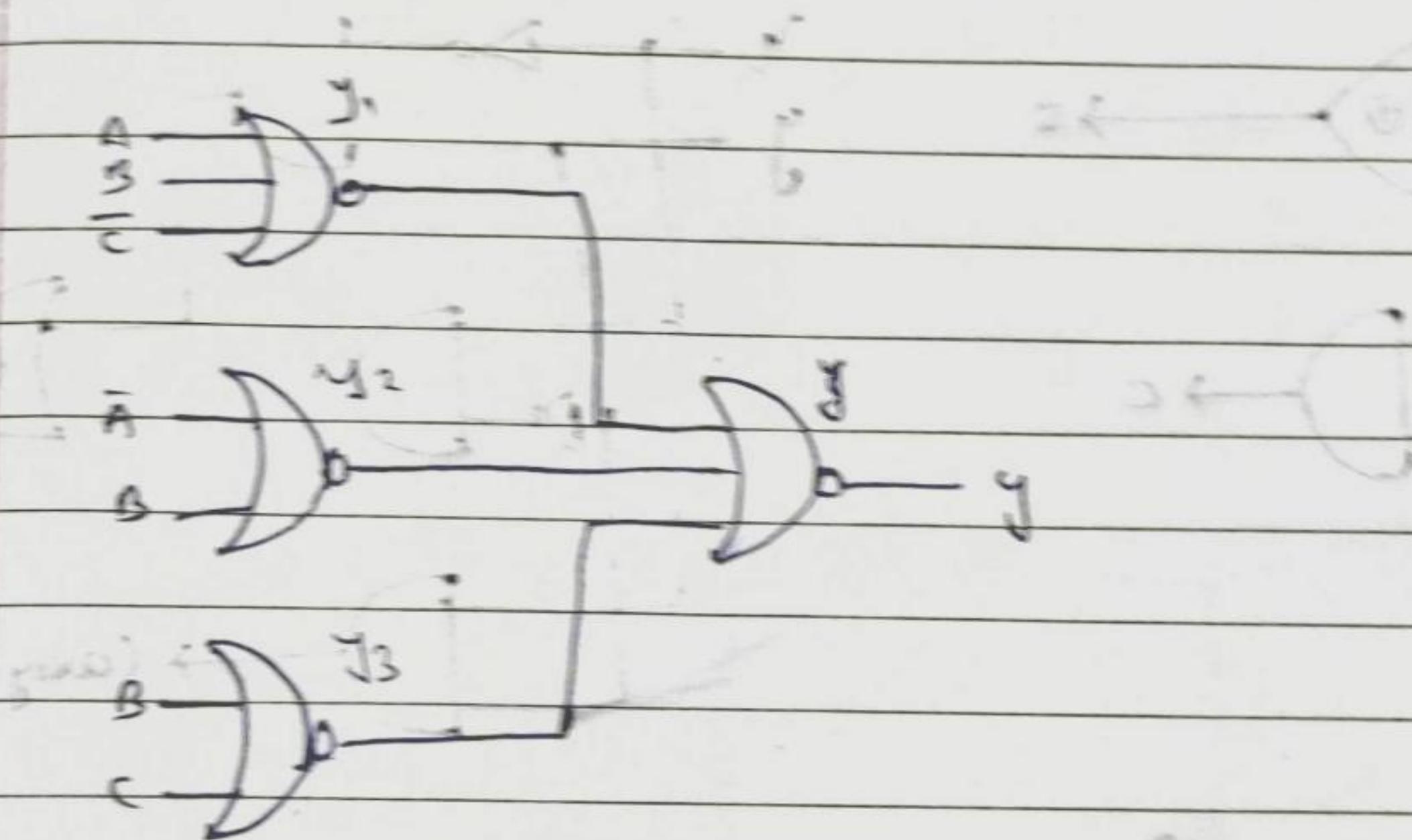
Pravne OR / AND = NOR / NOR

Umkehrung = Invertier

$$y = (A+B+\bar{C}) \cdot (\bar{A}+B) \cdot (B+C)$$



$$y = (A+B+\bar{C}) \cdot (\bar{A}+B) \cdot (B+C)$$



$$y_1 = \overline{(A+B+\bar{C})} ; y_2 = \overline{(\bar{A}+B)} ; y_3 = \overline{(B+C)}$$

$$y = \overline{y_1 + y_2 + y_3}$$

$$= \overline{(A+B+\bar{C})} + \overline{(\bar{A}+B)} + \overline{(B+C)}$$

$$= (A+B+\bar{C}) \cdot (A+\bar{B}) \cdot (B+C)$$

Prue DR/AND = NOR/NOR