$n=1$, head $= \frac{1}{2}$
$n=2$, two head $= \frac{1}{4}$
$n=3$, Three head $= \frac{1}{8}$
increases   decreases $\Rightarrow$ Binomial fails

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Poisson Distribution :-

Def^n:- If x is discrete random variable then

$$\boxed{p(x) = \frac{e^{-m} m^x}{x!}}, \quad x = 0, 1, 2, \cdots$$ is called Poisson distribution

where m is the mean.

Note corresponding frequencies are given by $N p(x)$

Ex Show that poisson distribution is the limiting case of Binomial distribution, state the assumption used.

Solution:-

assumption :- If number of trials is $n \to \infty$
and probability of success is very small $p \to 0$
so that np = constant then Binomial reduces to Poisson.

Proof:- $p(x) = {}^nc_x p^x q^{n-x}$, $\quad np = \text{const} \Rightarrow np = m$ say
$\Rightarrow p = \frac{m}{n}$, $q = 1 - \frac{m}{n}$

$$\therefore p(x) = \frac{n!}{(n-x)!} \frac{1}{x!} \left(\frac{m}{n}\right)^x \left(1 - \frac{m}{n}\right)^{n-x}$$

taking limit as $n \to \infty$

$$\lim_{n \to \infty} p(x) = \lim_{n \to \infty} \frac{n!}{(n-x)! x!} \frac{m^x}{n^x} \left(1 - \frac{m}{n}\right)^{n-x}$$

$$= \frac{m^x}{x!} \lim_{n \to \infty} \frac{n(n-1)(n-2)\cdots(n-x+1)(n-x)!}{(n-x)!\; n.n.n.\cdots(x \text{ times})} \left(1 - \frac{m}{n}\right)^n \left(1 - \frac{m}{n}\right)^{-x}$$

$$= \frac{m^x}{x!} \lim_{n \to \infty} 1\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\cdots\left[1 - \left(\frac{x-1}{n}\right)\right] \left(1 - \frac{m}{n}\right)^n \left(1 - \frac{m}{n}\right)^{-x}$$

$$p(x) = \frac{m^x}{x!} 1(1-0)(1-0)\cdots(1-0)\, e^{-m} (1-0)^{-x}$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e$$

$$\therefore \boxed{p(x) = \frac{e^{-m} m^x}{x!}}$$

Ex Find moment generating function for Poisson distribution.

$-\Sigma$ EI-10 M
E-09

Solution:- $p(x) = \frac{e^{-m} m^x}{x!}$, $x = 0, 1, 2, \cdots$

m.g.f. about origin $= M(t) = E[e^{tx}]$

$$= \sum_{x=0}^{\infty} e^{tx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-m} m^x}{x!} = e^{-m} \sum_{x=0}^{\infty} \frac{(e^t)^x m^x}{x!}$$

$$= e^{-m} \sum_{x=0}^{\infty} \frac{(me^t)^x}{x!}$$

$\therefore \boxed{M(t) = e^{-m} \cdot e^{me^t}}$ $\quad \because e^y = \sum_{n=0}^{\infty} \frac{y^n}{n!} = 1 + y + \frac{y^2}{2!} + \ldots$

Ex Find mean, variance, std. deviation for poisson distribution. — E1-10 (M) E-09

OR find first four moments about origin for poisson. IT-10, 09

Solution:-

m.g.f. for Poisson $M(t) = e^{-m} \cdot e^{me^t}$

mean = Ist moment about origin $= E(X) = \frac{d}{dt} M(t) \Big|_{t=0} = e^{-m} e^{me^t} \cdot me^t \Big|_{t=0}$

$$= e^{-m} e^{m} \cdot m$$

$$= m$$

$\therefore \boxed{mean = m} = E(X)$

IInd moment about origin $= \frac{d^2}{dt^2} M(t) \Big|_{t=0} = me^{-m} \left[e^{me^t} me^t \cdot e^t + e^{me^t} \cdot e^t \right] \Big|_{t=0}$

$$= me^{-m} (me^{m} + e^{m})$$

$\therefore E(X^2) = m^2 + m$

variance $= E(X^2) - [E(X)]^2 = m^2 + m - m^2 \Rightarrow \boxed{variance = m}$

$\boxed{\text{std. deviation} = \sqrt{m}}$

Simillarly IIIrd moment $= E(X^3) = \frac{d^3}{dt^3} M(t) \Big|_{t=0}$ = H.W.

4th moment $= E(X^4) = \frac{d^4}{dt^4} M(t) \Big|_{t=0}$ = H.W.

Note For Poisson distribution mean = variance.

EX Show that in a poisson distribution with unit mean, the mean deviation about mean is $\left(\frac{2}{e}\right)$ times std. deviation.

—{98, 2001, 04, 08

solution:- $p(x) = \dfrac{e^{-m} m^x}{x!}$, $x = 0, 1, 2, \dots$

given $m=1$, $\Rightarrow p(x) = \dfrac{e^{-1} 1^x}{x!} \Rightarrow p(x) = \dfrac{e^{-1}}{x!}$ — ①

mean deviation about mean $= M.D. = E(|x-m|) = \sum_{x=0}^{\infty} |x-m|\, p(x)$

$\therefore M.D. = \sum_{x=0}^{\infty} |x-1| \dfrac{e^{-1}}{x!} = e^{-1} \sum_{x=0}^{\infty} \left\{ \dfrac{|x-1|}{x!} \right\}$

$= e^{-1} \left\{ 1 + 0 + \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots \right\}$

$= e^{-1} \left\{ 1 + \sum_{n=1}^{\infty} \frac{n}{(n+1)!} \right\} = \frac{1}{e} \left\{ 1 + \sum_{n=1}^{\infty} \frac{(n+1)-1}{(n+1)!} \right\}$

$= \frac{1}{e} \left\{ 1 + \sum_{n=1}^{\infty} \frac{(n+1)}{(n+1)!} - \frac{1}{(n+1)!} \right\} = \frac{1}{e} \left\{ 1 + \sum_{n=1}^{\infty} \left[\frac{1}{n!} - \frac{1}{(n+1)!} \right] \right\}$

$= \frac{1}{e} \left\{ 1 + \frac{1}{1!} - \frac{1}{2!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} + \cdots \right\}$ $\qquad \because \frac{n+1}{(n+1)!} = \frac{(n+1)}{(n+1)n!} = \frac{1}{n!}$

$= \frac{1}{e}[1+1] = \frac{2}{e}(1)$

$\therefore M.D. = \left(\frac{2}{e}\right)$ times std. deviation $\qquad \because m = \text{variance} = 1 \Rightarrow$ std. devi. $= 1$

EX Find mode of Poisson distribution

OR

Show that mode of Poisson distribution lies bet^n $m-1$ and m.

Sol^n:

$p(x) = \dfrac{e^{-m} m^x}{x!}$

mode is the value of x at which probability is maximum.

Let x be the mode then

$p(x-1) \le p(x)$ — ① $\qquad$ and $p(x) \ge p(x+1)$ — ②

① $\Rightarrow \dfrac{e^{-m} m^{x-1}}{(x-1)!} \le \dfrac{e^{-m} m^x}{x!}$

$\therefore \dfrac{e^{-m} m^x m^{-1}}{(x-1)!} \le \dfrac{e^{-m} m^x}{x(x-1)!}$

$\Rightarrow \frac{1}{m} \le \frac{1}{x}$

$\Rightarrow x \le m$ — ③

② $\Rightarrow \dfrac{e^{-m} m^x}{x!} \ge \dfrac{e^{-m} m^{x+1}}{(x+1)!}$

$\Rightarrow \dfrac{e^{-m} m^x}{x!} \ge \dfrac{e^{-m} m^x m}{(x+1)\, x!}$

$\Rightarrow 1 \ge \dfrac{m}{x+1} \Rightarrow x+1 \ge m$

$\Rightarrow m-1 \le m$ — ④

From ③, ④, $m-1 \le x \le m$

Ex If a random variable x follows Poisson distribution such that $p(x=1) = 2p(x=2)$. Find mean, variance, std. deviation, $p(3)$ and $p(x \le 2)$

Important

Solution: For Poisson distribution $p(x) = \frac{e^{-m} m^x}{x!}$ — ①

given $p(1) = 2p(2) \Rightarrow \frac{e^{-m} m}{1!} = 2\frac{e^{-m} m^2}{2!} \Rightarrow m = m^2 \Rightarrow \boxed{m = 1 = \text{mean}}$

$\therefore \boxed{\text{variance} = m = 1}$ and std. deviation $\sigma = \sqrt{1} \Rightarrow \boxed{\sigma = 1}$

$\therefore ① \Rightarrow p(x) = \frac{e^{-1} 1^x}{x!} = \frac{e^{-1}}{x!}$

$p(3) = \frac{e^{-1}}{3!}$

and $p(x \le 2) = p(0) + p(1) + p(2)$

$= \frac{e^{-1}}{1} + \frac{e^{-1}}{1} + \frac{e^{-1}}{2} \Rightarrow p(x \le 2) = e^{-1}\frac{5}{2}$

Ex If mean 'm' of poisson is 4, find $p(m-6 < x < m+6)$ ($P(|x-m| < 6)$)

— 2007

Solution:- For Poisson $p(x) = \frac{e^{-m} m^x}{x!} = \frac{e^{-4} 4^x}{x!}$

mean = variance = 4 $\Rightarrow \sigma = 2$

$\therefore p(m-6 < x < m+6) = p(2 < x < 6) = p(3) + p(4) + p(5)$

$= \frac{e^{-4} 4^3}{3!} + \frac{e^{-4} 4^4}{4!} + \frac{e^{-4} 4^5}{5!} = e^{-4} 4^3 \left(\frac{1}{6} + \frac{4}{24} + \frac{16}{120}\right)$

$= 0.547$

Ex A transmission channel has per digit error probability $p = 0.01$. Calculate the probability of more than 1 error in 10 received digits using the binomial distribution and Poisson distribution. Find m.g.f. e in each case.

— 2004

Solution:- given $n = 10$, $p = 0.01$, $q = 0.99$

ⓘ Binomial distribution:-

$p(x) = {}^nC_x\, p^x q^{n-x} = {}^{10}C_x (0.01)^x (0.99)^{10-x}$

which is error probability of exactly 'x' digits.

$\therefore$ required probability $= p(x > 1) = 1 - p(x \le 1)$

$= 1 - p(0) - p(1)$

$= 1 - {}^{10}C_0 (0.01)^0 (0.99)^{10} - {}^{10}C_1 (0.01)^1 (0.99)^9$

$= 0.0043$

ⓘⓘ Poisson distribution:-

$p(x) = \frac{e^{-m} m^x}{x!}$, $m = np = 10(0.01) = 0.1$

$= \frac{e^{-0.1} (0.1)^x}{x!}$

required probability $= p(x > 1) = 1 - p(x \le 1)$

$= 1 - p(0) - p(1) \qquad = 1 - \frac{e^{-0.1}(0.1)^0}{0!} - \frac{e^{-0.1}(0.1)^1}{1!}$

$= 0.004678$

to find m.g.f. :-

For binomial

$m.g.f. = (pe^t + q)^n$

$M(t) = [(0.01)e^t + 0.99]^n$

For Poisson

$m.g.f. = e^{-m+me^t}$

$M(t) = e^{-0.1+(0.1)e^t}$

EX Using Poisson distribution find the approximate value of

$^{300}C_2 (0.02)^2 (0.98)^{298} + {}^{300}C_3 (0.02)^3 (0.98)^{297}$ — {2004}

Soln:- given distribution is Binomial $p(x) = {}^nC_x p^x q^{n-x}$

$\therefore {}^{300}C_2 (0.02)^2 (0.98)^{298} + {}^{300}C_3 (0.02)^3 (0.98)^{297} = p(2) + p(3)$

$\Rightarrow p = 0.02, \quad q = 0.98, \quad n = 300$

For Poisson $m = np = 300(0.02) = 6$

$p(x) = \frac{e^{-m} m^x}{x!} = \frac{e^{-6} 6^x}{x!}$

$\therefore p(2) + p(3) = \frac{e^{-6} 6^2}{2!} + \frac{e^{-6} 6^3}{3!} = 54e^{-6} = 0.1338$

EX Between 2 p.m. to 4 p.m. the average number of phone calls per minute comming into the switch board of a company is 2.5. Find the probability that during a perticular minute there will be (i) no phone calls (ii) exactly 3 calls.

Solution:- note that n, p not given ⇒ binomial distribution fails.

given average number of phone calls = mean = m = 2.5

$\therefore p(x) = \frac{e^{-m} m^x}{x!} = \frac{e^{-2.5}(2.5)^x}{x!}$ exactly x phone calls comming into switch board.

(i) no phone calls ⇒ $x = 0$

$\therefore p(0) = \frac{e^{-2.5}(2.5)^0}{0!} = e^{-2.5} = 0.082$

(ii) $p(3) = \frac{e^{-2.5}(2.5)^3}{3!} = 0.213$

EX The probability of failure in physics practical examination is 20%. If 25 batches of 6 students each take the examination in how many batches ~~4~~ or more students would pass ? — {2003}

Solution:- Let probability of failure $p = 0.2$,
Probability of success $= q = 0.8$

$n=6, \quad N=25$

Poisson distribution:- $p(x) = \dfrac{e^{-m} m^x}{x!}$, $\quad m = nq = 6(0.8) = 4.8$

note that success is Passing

$\therefore p(x) = \dfrac{e^{-4.8}(4.8)^x}{x!}$ is the probability of 'x' Passed students.

required probability $p(x \geq 4) = p(4) + p(5) + p(6)$

$$\therefore p(x \geq 4) = \frac{e^{-4.8}(4.8)^4}{4!} + \frac{e^{-4.8}(4.8)^5}{5!} + \frac{e^{-4.8}(4.8)^6}{6!}$$

$$= e^{-4.8}(4.8)^4\left[\frac{1}{24} + \frac{4.8}{120} + \frac{(4.8)^2}{720}\right]$$

$p(x \geq 4) = 0.49657$

$\therefore$ number of batches with four or more students would pass

$= 25(0.49657) = 12.414 \simeq 12$ batches.

Ex An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policy holders were randomly selected from the population, what is the probability that no more than two of its clients are involved in such accident next year?

—§ 2002,

solution:- Probability of accident $p = 0.01\% = 0.0001$, $n = 1000$

n is very large, p is very small $\Rightarrow$ Poisson.

$\therefore m = np = 1000(0.0001) = 0.1$

$\therefore$ probability of exactly 'x' accidents

$= p(x) = \dfrac{e^{-m} m^x}{x!} = \dfrac{e^{-0.1}(0.1)^x}{x!}$

$\therefore$ required probability $p(x \leq 2) = p(0) + p(1) + p(2)$

$$\therefore p(x \leq 2) = \frac{e^{-0.1}(0.1)^0}{0!} + \frac{e^{-0.1}(0.1)^1}{1!} + \frac{e^{-0.1}(0.1)^2}{2!} = e^{-0.1}\{1 + 0.1 + 0.005\}$$

$p(x \leq 2) = 0.9998$

Ex A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days and the number of days on which

(i) neither car is used (ii) some demand is refused.

—§ 96, 98

Solution:- $p(x) = \dfrac{e^{-m} m^x}{x!} = \dfrac{e^{-1.5}(1.5)^x}{x!}$ Probability of exactly 'x' cars on demand.

(i) neither car is used $\Rightarrow x = 0$

$p(0) = \dfrac{e^{-1.5}(1.5)^0}{0!} = 0.2231$

$\therefore$ number of days, $365(0.2231) = 81.44 \simeq 81$

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(ii) Probability that some demand is refused means demand is more than 2

$\therefore p(x>2) = 1 - p(x \le 2) = 1 - [p(0)+p(1)+p(2)]$

$= 1 - \frac{e^{-1.5}(1.5)^0}{0!} - \frac{e^{-1.5}(1.5)^1}{1!} - \frac{e^{-1.5}(1.5)^2}{2!} = 1 - e^{-1.5}\{1+1.5+1.125\}$

$\therefore p(x>2) = 0.191$

$\therefore$ number of days $= 365(0.191) = 69.72 \approx 70$

EX State true or false with proper justification.

(1) The mean of Poisson distribution is 2 and variance 3 —{2007}

(2) If x is Poisson variate such that $p(2) = 9p(4) + 90p(6)$ then mean of x is 1. —{97}

(i) given mean $= 2$, variance $= 3$

For Poisson distribution mean = variance

$\therefore$ given statement is False.

(ii) $p(x) = \frac{e^{-m} m^x}{x!}$,

given, $p(2) = 9p(4) + 90p(6) \Rightarrow \frac{e^{-m} m^2}{2!} = 9 \cdot \frac{e^{-m} m^4}{4!} + \frac{e^{-m} m^6}{6!} 90$

$\Rightarrow \frac{1}{2} = \frac{9}{24} m^2 + \frac{m^4}{720}(90) \Rightarrow \frac{1}{2} = \frac{3m^2}{8} + \frac{m^4}{8} \Rightarrow 4 = 3m^2 + m^4$

$\Rightarrow m^4 + 3m^2 - 4 = 0 \Rightarrow (m^2+4)(m^2-1) = 0 \Rightarrow m^2 = -4, \; m^2 = 1$

$\therefore m \ne \pm 2i, \; m = \pm 1 \Rightarrow m = 1$ $\quad$ $n>0, \; p>0 \Rightarrow m = np > 0$

$\therefore$ given statement is true.

Fitting Poisson Distribution

Note! → fitting poisson distribution means finding parameter m, and theoretical frequencies.

EX Fit Poisson distribution for the following data

No. of deaths :	0	1	2	3	4
Frequencies :	123	59	14	3	1

→ E-09 / 2007

Soln:
$$\text{mean} = m = \frac{\Sigma xf}{\Sigma f} = \frac{0+59+28+9+4}{123+59+14+3+1} = \frac{100}{200} = 0.5$$

$\therefore m = 0.5, \quad n = 4$

$$p(x) = \frac{e^{-m} m^x}{x!} = \frac{e^{-0.5}(0.5)^x}{x!}$$

Now $N = \Sigma f = 200$

$\therefore$ corresponding frequencies $N\,p(x)$, $x = 0, 1, 2, 3, 4$

$$200\,p(0) = 200\,\frac{e^{-0.5}(0.5)^0}{0!} = 121.30$$

$$200\,p(1) = 200\,\frac{e^{-0.5}(0.5)^1}{1!} = 60.65 \approx 61$$

$$200\,p(2) = 200\,\frac{e^{-0.5}(0.5)^2}{2!} = 15.16 \approx 15$$

$$200\,p(3) = 200\,\frac{e^{-0.5}(0.5)^3}{3!} = 2.52 \approx 3$$

$$200\,p(4) = 200\,\frac{e^{-0.5}(0.5)^4}{4!} = 0.3159 \approx 1$$

$\boxed{\text{EX}}$ For a Poisson distribution Prove that $\mu_{r+1} = rm\mu_{r-1} + m\frac{d\mu_r}{dm}$ where μ_r is r^{th} moment about mean

— 99, 2001 & 2006

Solution:- by defination r^{th} moment about mean is given by

$$\mu_r = E([X-m])^r = \Sigma\,[x-m]^r p(x)$$

$$\therefore \mu_r = \sum_{x=0}^{\infty}(x-m)^r\,\frac{e^{-m}m^x}{x!} \quad \text{—①} \qquad \left[\because p(x) = \frac{e^{-m}m^x}{x!}\right]$$

differentiating w.r.t. m,

$$\frac{d\mu_r}{dm} = \sum_{x=0}^{\infty} r(x-m)^{r-1}(-1)\frac{e^{-m}m^x}{x!} + \sum_{x=0}^{\infty}\frac{(x-m)^r}{x!}\left\{-e^{-m}m^x + e^{-m}x\,m^{x-1}\right\}$$

$$\therefore \frac{d\mu_r}{dm} = -r\sum_{x=0}^{\infty}(x-m)^{r-1}\frac{e^{-m}m^x}{x!} + \sum_{x=0}^{\infty}\frac{(x-m)^r}{x!}e^{-m}m^{x-1}(-m+x)$$

$$= -r\sum_{x=0}^{\infty}(x-m)^{r-1}\frac{e^{-m}m^x}{x!} + \frac{1}{m}\sum_{x=0}^{\infty}(x-m)^{r+1}\frac{e^{-m}m^x}{x!}$$

$$\frac{d\mu_r}{dm} = -r\mu_{r-1} + \frac{1}{m}\mu_{r+1} \quad \Rightarrow \mu_{r+1} = mr\mu_{r-1} + m\frac{d\mu_r}{dm}$$

$\boxed{\text{EX}}$ If X is poisson variate such that $p(x=1) = p(x=2)$ Find $E(X^2)$

— 2004

Soln:- For Poisson, $p(x) = \frac{e^{-m}m^x}{x!}$

given $p(1) = p(2) \Rightarrow \frac{e^{-m}m^1}{1!} = \frac{e^{-m}m^2}{2!} \Rightarrow 1 = \frac{m}{2} \Rightarrow \boxed{m=2}$

to find $E(X^2)$:→ $\quad$ m.g.f. $= M(t) = e^{-m+me^t}$

$$\therefore M(t) = e^{-2+2e^t}$$

$$\therefore E(X^2) = \left.\frac{d^2}{dt^2}M(t)\right|_{t=0} = \left.\frac{d}{dt}\left(e^{-2+2e^t}\cdot 2e^t\right)\right|_{t=0} = \left.\left\{2e^t\cdot e^{-2+2e^t} + 2e^t e^{-2+2e^t}2e^t\right\}\right|_{t=0}$$

$$E(X^2) = 2+4 = 6$$

For Poisson $V = m = 2$

$E(X^2) - [E(X)]^2 = 2$

$E(X^2) - [2]^2 = 2$

$E(X^2) = 6$

EX If X and Y are Poisson variates such that $P(X=1) = P(X=2)$ and $P(Y=2) = P(Y=3)$, find variance of $3X-2Y$

{2009 M}

Solution:- Let m_1 be the mean of X and m_2 be the mean of Y

then $P(x) = \frac{e^{-m_1} m_1^x}{x!}$ and $P(y) = \frac{e^{-m_2} m_2^y}{y!}$

given $P(x=1) = P(x=2)$

$\Rightarrow \frac{e^{-m_1} m_1}{1!} = \frac{e^{-m_1} m_1^2}{2!}$

$\Longrightarrow 1 = \frac{m_1}{2} \Rightarrow m_1 = 2 = \text{mean}$

given $P(y=2) = P(y=3)$

$\Longrightarrow \frac{e^{-m_2} m_2^2}{2!} = \frac{e^{-m_2} m_2^3}{3!}$

$\Longrightarrow 1 = \frac{m_2}{3} \Rightarrow m_2 = 3 = \text{mean}$

But for Poisson mean = variance

$\therefore \boxed{V(X) = m_1 = 2}$

$\therefore \boxed{V(Y) = m_2 = 3}$

Now $V(3X-2Y) = 3^2 V(X) + (-2)^2 V(Y)$

$= 9(2) + 4(3)$

$\therefore$ variance of $3X-2Y$ is $\boxed{30}$

Ex Fit Poisson distribution to the following data:-

x	0	1	2	3	4	5	6	7	8
f	56	156	132	92	37	22	4	0	1

M — Σ EL-10

Solution:-

$$\text{mean } m = \frac{\Sigma xf}{\Sigma f} = \frac{0+156+264+276+148+110+24+0+8}{56+156+132+92+37+22+4+0+1} = \frac{986}{500}$$

$\therefore \boxed{m = 1.972}$ note that $\boxed{N = \Sigma f = 500}$

$$p(x) = \frac{e^{-m} m^x}{x!} \Rightarrow p(x) = \frac{e^{-1.972}(1.972)^x}{x!} \; ; \; x = 0, 1, 2, \cdots 8$$

corresponding frequencies are given by $Np(x)$

$$x=0 \Rightarrow Np(0) = 500 \frac{e^{-1.972}(1.972)^0}{0!} = 69.59 \approx 70$$

$$x=1 \Rightarrow Np(1) = 500 \frac{e^{-1.972}(1.972)}{1!} = 137.23 \approx 137$$

$$x=2 \Rightarrow Np(2) = 500 \frac{e^{-1.972}(1.972)^2}{2!} = 135.31 \approx 135$$

$$x=3 \Rightarrow Np(3) = 500 \frac{e^{-1.972}(1.972)^3}{3!} = 88.94 \approx 90$$

$$x=4 \Rightarrow Np(4) = 500 \frac{e^{-1.972}(1.972)^4}{4!} = 43.85 \approx 44$$

$$x=5 \Rightarrow Np(5) = 500 \frac{e^{-1.972}(1.972)^5}{5!} = 17.29 \approx 17$$

$$x=6 \Rightarrow Np(6) = 500 \frac{e^{-1.972}(1.972)^6}{6!} = 5.68 \approx 6$$

$$x=7 \Rightarrow Np(7) = 500 \frac{e^{-1.972}(1.972)^7}{7!} = 1.60 \approx 2$$

$$x=8 \Rightarrow Np(8) = 500 \frac{e^{-1.972}(1.972)^8}{8!} = 0.39 \approx 1$$

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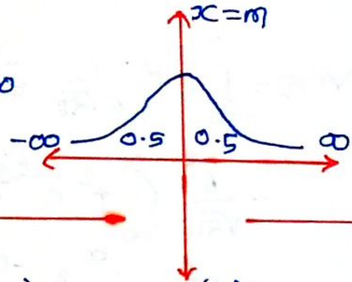
Normal Distribution

Def^n^:- A continuous random variable X is said to follow normal distribution

If $\boxed{p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}}$, $-\infty < x < \infty$

where m = mean and σ = std. deviation.



Properties of normal distribution :-

① curve is bell shaped as shown in figure which is symmetric about $x=m$ axis.

② area to the left of $x=m$ is 0.5 and to the right of $x=m$ is 0.5

③ For normal distribution

$$\boxed{\text{mean} = \text{mode} = \text{median} = m}$$

④ mean deviation about mean is $\frac{4}{5}\sigma$, σ is std. deviation.

i.e. $\boxed{M.D. = \frac{4}{5}\sigma}$

Ex Find mean, variance of normal distribution. — {2001, 2002, 2008

Solution:- $p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$ $\quad -\infty < x < \infty$

to find mean:-

$$\text{mean} = E(X) = \int_{-\infty}^{\infty} x\, p(x)\, dx = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$$

Put $\frac{x-m}{\sigma} = t \Rightarrow x = m+\sigma t$, $dx = \sigma\, dt$

and $x=-\infty \Rightarrow t=-\infty$, $x=\infty \Rightarrow t=\infty$

$$\therefore \text{mean} = \int_{-\infty}^{\infty} (m+\sigma t) \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2/2} \sigma\, dt =$$

$$= \frac{m}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2} dt + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t \cdot e^{-t^2/2} dt$$

$\because e^{-t^2/2}$ even, $te^{-t^2/2}$ odd.

$$= \frac{2m}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-t^2/2} dt + 0$$

put $\frac{t^2}{2} = p \Rightarrow t = \sqrt{2}\, p^{1/2}$

$dt = \frac{\sqrt{2}}{2} p^{-\frac{1}{2}} dp$

$t=0 \Rightarrow p=0$ and $t=\infty \Rightarrow p=\infty$

$$= \frac{2m}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-p} \frac{\sqrt{2}}{2} p^{-\frac{1}{2}} dp$$

$$= \frac{m}{\sqrt{\pi}} \int_{0}^{\infty} e^{-p} p^{-\frac{1}{2}} dp$$

$$= \frac{m}{\sqrt{\pi}} \Gamma\tfrac{1}{2} = \frac{m}{\sqrt{\pi}} \sqrt{\pi} \qquad \left\{ \because \Gamma n = \int_{0}^{\infty} e^{-x} x^{n-1} dx \right\}$$

$$\therefore \boxed{\text{mean} = m}$$

to find variance :-

$$V(X) = E(X-m)^2 = \int_{-\infty}^{\infty} (x-m)^2 p(x)\, dx$$

$$= \int_{-\infty}^{\infty} (x-m)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$$

$$V(x) = \int_{-\infty}^{\infty} (\sigma t)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2/2} \sigma\, dt$$

put $\frac{x-m}{\sigma} = t \Rightarrow x = m + \sigma t$
$dx = \sigma\, dt$, $x = -\infty \Rightarrow t = -\infty$, $x = \infty \Rightarrow t = \infty$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t^2 e^{-t^2/2} dt$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} 2 \int_{0}^{\infty} t^2 e^{-t^2/2} dt$$

$\frac{t^2}{2} = p \Rightarrow t = \sqrt{2}\, p^{\frac{1}{2}} \Rightarrow dt = \sqrt{2}\frac{1}{2} p^{-\frac{1}{2}} dp$
$t = 0 \Rightarrow p = 0$ and $t = \infty \Rightarrow p = \infty$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^{\infty} 2p \cdot e^{-p} \sqrt{2}\frac{1}{2} p^{-\frac{1}{2}} dp$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_0^{\infty} e^{-p} p^{\frac{1}{2}} dp = \frac{2\cdot\sigma}{\sqrt{\pi}} \Gamma 3/2 = \frac{2\cdot\sigma}{\sqrt{\pi}} \frac{1}{2}\Gamma 1/2 \qquad \left[\because \Gamma n = \int_0^{\infty} e^{-x} x^{n-1} dx\right]$$

$\boxed{V = \sigma^2}$ $\quad [\because \Gamma 1/2 = \sqrt{\pi}]$ $\quad \Rightarrow \boxed{\text{std. deviation} = \sigma}$

EX Find mode of normal distribution.

Solution:-

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} \quad \text{—(1)}$$

mode is that value of 'x' at which probability is maximum
i.e, at which $p'(x) = 0$, $p''(x) < 0$.

taking log on both side of (1)

$$\log p(x) = \log\left\{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}\right\} = \log\left\{\frac{1}{\sigma\sqrt{2\pi}}\right\} + \log e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

$$\therefore \log p(x) = \log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) - \frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2 \Rightarrow \log p(x) = \log\left\{\frac{1}{\sigma\sqrt{2\pi}}\right\} - \frac{1}{2\sigma^2}(x-m)^2$$

differentiating w.r.t. x,

$$\frac{1}{p(x)} p'(x) = 0 - \frac{2}{2\sigma^2}(x-m) \Rightarrow p'(x) = -\frac{1}{\sigma^2}(x-m)p(x) \quad \text{—(2)}$$

again differentiating $\quad p''(x) = -\frac{1}{\sigma^2}\{p(x) + (x-m)p'(x)\} \quad$ —(3)

$p'(x) = 0 \Rightarrow -\frac{1}{\sigma^2}(x-m)p(x) = 0 \Rightarrow \boxed{x = m} \qquad \because p(x) \neq 0$

at $x=m$
$\therefore$ (3) $\Rightarrow p''(m) = -\frac{1}{\sigma^2}[p(m) + 0]$

$$= -\frac{1}{\sigma^2}\frac{1}{\sigma\sqrt{2\pi}} \qquad [\because (1)]$$

$\therefore p''(m) < 0$

$\therefore p(x)$ is maximum at $x = m$

$\therefore \boxed{\text{mode} = m}$

EX Find median of normal distribution —S 2008

Solution:-

For normal distribution $p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$,

If M is median then $\int_{-\infty}^{M} p(x)\,dx = \frac{1}{2}$

$$\Rightarrow \int_{-\infty}^{m} p(x)\,dx + \int_{m}^{M} p(x)\,dx = \frac{1}{2} \Longrightarrow \int_{-\infty}^{m} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx + \int_{m}^{M} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx = \frac{1}{2} \quad \text{--- (1)}$$

consider $I = \int_{-\infty}^{m} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$

put $\frac{x-m}{\sigma} = t \Rightarrow x-m = \sigma t$
$dx = \sigma\,dt$
$x = -\infty \Rightarrow t = -\infty$
$x = m \Rightarrow t = 0$

$$= \int_{-\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2/2} \sigma\,dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-t^2/2} dt \quad [\because \text{by symm.}]$$

put $\frac{t^2}{2} = p \Rightarrow t = \sqrt{2}\,p^{1/2}$
$\Rightarrow dt = \sqrt{2}\,\frac{1}{2} p^{-1/2} dp$
$t = 0 \Rightarrow p = 0$
$t = \infty \Rightarrow p = \infty$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-p} \sqrt{2}\,\frac{1}{2} p^{-\frac{1}{2}} dp$$

$$= \frac{1}{2\sqrt{\pi}} \int_{0}^{\infty} e^{-p} p^{-\frac{1}{2}} dp$$

$$= \frac{1}{2\sqrt{\pi}} \Gamma\tfrac{1}{2} = \frac{1}{2} \qquad \left[\because \Gamma n = \int_{0}^{\infty} e^{-x} x^{n-1} dx \text{ and } \Gamma\tfrac{1}{2} = \sqrt{\pi}\right]$$

$$\therefore \int_{-\infty}^{m} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx = \frac{1}{2}$$

$$\therefore (1) \Rightarrow \frac{1}{2} + \int_{m}^{M} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx = \frac{1}{2} \quad \Rightarrow \int_{m}^{M} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx = 0$$

$$\Rightarrow m = M \Rightarrow \boxed{\text{median} = m} \qquad \left[\because p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} \neq 0\right]$$

EX If X is standard normal variate, show that mean deviation about mean is $\frac{4}{5}$ times std. deviation.

i.e. $M.D. = \frac{4}{5}\sigma$ —S 98,

Solution:-

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

mean deviation about mean $M.D. = E(|x-m|) = \int_{-\infty}^{\infty} |x-m| \cdot p(x)\,dx$

$$\therefore M.D. = \int_{-\infty}^{\infty} |x-m| \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} \cdot dx$$

put $\frac{x-m}{\sigma} = t \Rightarrow x-m = \sigma t$
$dx = \sigma\,dt$
$x = -\infty \Rightarrow t = -\infty$
$x = \infty \Rightarrow t = \infty$

$$= \int_{-\infty}^{\infty} |\sigma t| \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2/2} \sigma\,dt$$

$$= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |t| e^{-t^2/2} dt = \frac{2\cdot\sigma}{\sqrt{2\pi}} \int_{0}^{\infty} |t| e^{-t^2/2} dt = \frac{2\cdot\sigma}{\sqrt{2\pi}} \int_{0}^{\infty} t\,e^{-t^2/2} dt$$

(even function)

$$M.D. = \frac{2\sigma}{\sqrt{2\pi}} \int_0^\infty \sqrt{2}\, p^{\frac{1}{2}} e^{-p} \frac{\sqrt{2}}{2} p^{-\frac{1}{2}} dp$$

Put $\frac{t^2}{2} = p \Rightarrow t = \sqrt{2}\, p^{1/2}$, $dt = \frac{\sqrt{2}}{2} p^{-\frac{1}{2}} dp$

$t = 0 \Rightarrow p = 0$

$t = \infty \Rightarrow p = \infty$

$$= \frac{2\sigma}{\sqrt{2\pi}} \int_0^\infty e^{-p} p^0 \, dp = \frac{2\sigma}{\sqrt{2\pi}} \left[\frac{e^{-p}}{-1}\right]_0^\infty$$

$$= \frac{2\sigma}{\sqrt{2\pi}} (0+1) = \frac{2}{\sqrt{2\pi}} \sigma$$

$\therefore \boxed{M.D. = \frac{4\sigma}{5}}$ where σ is std. deviation.

EX Find m.g.f. for normal variate. —{2000, 2004, 2007}

Solution:-

For normal distribution, $p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$

moment generating function about origin

$$= M(t) = E\{e^{tx}\} = \int_{-\infty}^{\infty} e^{tx} p(x)\, dx = \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$$

$$M(t) = \int_{-\infty}^{\infty} e^{t(\sigma p + m)} \frac{1}{\sigma\sqrt{2\pi}} e^{-p^2/2} \sigma\, dp$$

Put $\frac{x-m}{\sigma} = p \Rightarrow x - m = \sigma p \Rightarrow dx = \sigma\, dp$

$x = -\infty \Rightarrow p = -\infty$

$x = \infty \Rightarrow p = \infty$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\sigma p} \cdot e^{tm} \cdot e^{-p^2/2} dp$$

$$= \frac{e^{tm}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\sigma p - \frac{p^2}{2}} dp = \frac{e^{tm}}{\sqrt{2m}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(p^2 - 2\sigma t p)} dp$$

$$= \frac{e^{tm}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[(p^2 - 2\sigma t p + \sigma^2 t^2) - \sigma^2 t^2]} dp = \frac{e^{tm}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(p - \sigma t)^2} \cdot e^{\sigma^2 t^2/2} dp$$

$$= \frac{e^{tm} \cdot e^{\sigma^2 t^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(p - \sigma t)^2} dp$$

Put $p - \sigma t = v$

$dp = dv$

$p = -\infty \Rightarrow v = -\infty$

$p = \infty \Rightarrow v = \infty$

$$= \frac{e^{\left(tm + \frac{\sigma^2 t^2}{2}\right)}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-v^2/2} dp$$

$\frac{v^2}{2} = y \Rightarrow v = \sqrt{2}\, y^{1/2} \Rightarrow dv = \frac{\sqrt{2}}{2} y^{-\frac{1}{2}} dy$

$$= \frac{e^{\left(tm + \frac{\sigma^2 t^2}{2}\right)}}{\sqrt{2\pi}} \cdot 2 \int_0^\infty e^{-y} \sqrt{2} \frac{1}{2} y^{-\frac{1}{2}} dy$$

$$= \frac{e^{\left(tm + \frac{\sigma^2 t^2}{2}\right)}}{\sqrt{\pi}} \int_0^\infty e^{-y} y^{-\frac{1}{2}} dp = \frac{e^{\left(tm + \frac{\sigma^2 t^2}{2}\right)}}{\sqrt{\pi}} \overline{|1/2} \qquad \left[\because \overline{|n} = \int_0^\infty e^{-x} x^{n-1} dx\right]$$

$$\therefore \boxed{M(t) = e^{\left(tm + \frac{\sigma^2 t^2}{2}\right)}} \qquad \left[\because \overline{|1/2} = \sqrt{\pi}\right]$$

EX Find m.g.f. Hence find first two moments about origin. —{2007}

Solution:- $M(t) = e^{tm + \frac{\sigma^2 t^2}{2}}$

$$1^{st} \text{ moment} = \text{mean} = E(x) = \frac{d}{dt} M(t)\Big|_{t=0} = \frac{d}{dt}\left\{e^{tm + \frac{\sigma^2 t^2}{2}}\right\}\Big|_{t=0}$$

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$$= e^{tm+\frac{\sigma^2 t^2}{2}}\left(m+\frac{2\sigma^2 t}{2}\right)\Bigg|_{t=0} = e^0(m+0)$$

$\therefore$ I^{st} moment about origin = mean = $\boxed{E(X) = m}$

II^{nd} moment about origin $= E(X^2)$

$$E(X^2) = \frac{d^2}{dt^2}M(t)\Bigg|_{t=0} = \frac{d}{dt}\left\{e^{tm+\frac{\sigma^2 t^2}{2}}(m+\sigma^2 t)\right\}\Bigg|_{t=0}$$

$$= \left\{e^{tm+\frac{\sigma^2 t^2}{2}}(m+\sigma^2 t)(m+\sigma^2 t) + e^{tm+\frac{\sigma^2 t^2}{2}}\cdot\sigma^2\right\}\Bigg|_{t=0} = m^2+\sigma^2$$

$\boxed{E(X^2) = m^2+\sigma^2}$

variance $V(X) = E(X^2) - \{E(X)\}^2 = m^2+\sigma^2 - m^2$

$\therefore$ $\boxed{\text{variance} = \sigma^2}$ $\Rightarrow$ $\boxed{\text{std. deviation} = \sigma}$

Ex Define standard normal variate.
Find m.g.f. of standard normal variate. — S 2005,
Hence find mean, variance.

Solution:- we know that $p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$

$\frac{x-m}{\sigma} = y$ is called standard normal variate.
$\Rightarrow dx = \sigma\, dy$

$\therefore$ corresponding distribution for std. normal variate is given by

$$p(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{— [note this]}$$

$$\therefore \text{m.g.f.} = M(t) = E[e^{tY}] = \int_{-\infty}^{\infty} e^{ty} p(y)\, dy = \int_{-\infty}^{\infty} e^{ty} \cdot \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(y^2-2ty)} dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[(y^2-2ty+t^2)-t^2]} dy$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(y-t)^2} \cdot e^{t^2/2} dy$$

$$= \frac{e^{t^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(y-t)^2} dy$$

$$\boxed{\begin{array}{l} \text{put } y-t = v \Rightarrow dy = dv \\ y=-\infty \Rightarrow v=-\infty \\ y=\infty \Rightarrow v=\infty \end{array}}$$

$$= \frac{e^{t^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-v^2/2} dv = \frac{2e^{t^2/2}}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-v^2/2} dv \quad \text{(even)}$$

$$\boxed{\begin{array}{l} \text{put } \frac{v^2}{2} = x \Rightarrow v = \sqrt{2}x^{\frac{1}{2}} \\ dv = \sqrt{2}\,\frac{1}{2} x^{-\frac{1}{2}} dx \\ v=0 \Rightarrow x=0 \text{ and } v=\infty \Rightarrow v=\infty \end{array}}$$

$$= \frac{2e^{t^2/2}}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-x} \frac{\sqrt{2}}{2} x^{-\frac{1}{2}} dx$$

$$= \frac{e^{t^2/2}}{\sqrt{\pi}} \sqrt{\tfrac{1}{2}} = e^{t^2/2} \quad [\because \sqrt{\tfrac{1}{2}} = \sqrt{\pi}]$$

$$\therefore \text{m.g.f. } \boxed{M(t) = e^{t^2/2}}$$

$$\text{mean} = \frac{dM(t)}{dt}\Big|_{t=0} = e^{t^2/2} \cdot \frac{2t}{2}\Big|_{t=0} = 0 \qquad \Rightarrow \boxed{\text{mean} = 0}$$

$$E(x^2) = \frac{d^2}{dt^2} M(t)\Big|_{t=0} = \frac{d}{dt}(te^{t^2/2})\Big|_{t=0} = \left(e^{t^2/2} \cdot 1 + t \cdot e^{t^2/2} \cdot \frac{2t}{2}\right)\Big|_{t=0}$$

$$E(x^2) = 1 + 0$$

$$\therefore \text{variance} = E(x^2) - (E(x))^2$$
$$= 1 - 0^2$$

$$\therefore \boxed{\text{variance} = 1}$$

Ex In a normally distributed group of 450 students with mean 42 and S.D. 8, find the percentage and the number of students scoring (i) between 48 and 52 (ii) more than 60 (iii) less than 40.

— S 2008

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solution:- consider standard normal variate

$t = \frac{x-m}{\sigma} \Rightarrow t = \frac{x-42}{8}$ where $m=42$, $\sigma=8$, $N=450$

$\therefore P(x) \Rightarrow$ probability of students getting exactly 'x' marks.

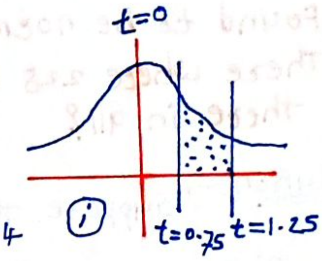
(i) probability of students getting marks between 48 to 52

$= P(48<x<52) = P\left(\frac{48-42}{8} < t < \frac{52-42}{8}\right) = P(0.75<t<1.25)$

$= P(0<t<1.25) - P(0<t<0.75)$

$= 0.3944 - 0.2734$

$P(48<x<52) = 0.121$



$\therefore$ % of above students $= 0.121 \times 100 = 12.1\%$

number of students $= (0.121)450 = 54.45 \approx 54$

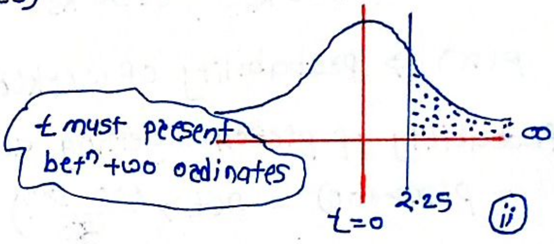
(ii) $P(x>60) = P\left(t > \frac{60-42}{8}\right) = P(t>2.25)$

$= P(2.25<t<\infty)$

$= P(0<t<\infty) - P(0<t<2.25)$

$= 0.5 - 0.4878$

$P(x>60) = 0.0122$



$\therefore$ % of students above 60 $= 0.0122\,(100) = 1.22\%$

number of students $= (0.0122)\,450 = 5.49 \approx 5$

(iii) $P(x<40) = P\left(t < \frac{40-42}{8}\right) = P(t<-0.25)$

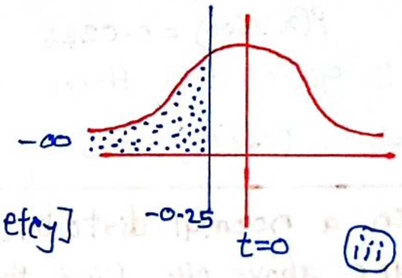
$= P(-\infty<t<-0.25)$

$= P(-\infty<t<0) - P(-0.25<t<0)$

$= 0.5 - P(0<t<0.25)$ [$\because$ symmetry]

$= 0.5 - 0.0987$

$P(x<40) = 0.4013$



$\therefore$ % of students getting less than 40 $= (0.4013)100 = 40.13\%$

number of students $= (0.4013)\,450 = 180.585 \approx 181$

Ex The life of army shoes is normally distributed with mean 8 months and std. deviation 2 months. If 5000 pairs are issued, how many pairs would be expected to need replacement after 12 months?

— S 2002

solution:- given $N = 5000$, $m = 8$ months, $\sigma = 2$ months

$t = \frac{x-m}{\sigma} \Rightarrow t = \frac{x-8}{2}$

$\therefore P(x) \Rightarrow$ Probability of shoes having life exactly 'x' months.

given condition $\Rightarrow$ If life of shoes is greater than 12 months then we need replacement.

$\therefore$ probability of shoes that needs replacement after 12 months

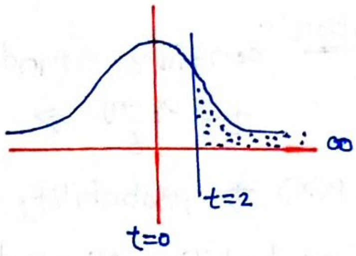
$= P(x \geq 12) = P\left(t > \frac{12-8}{2}\right) = P(t>2)$

$= P(2<t<\infty)$

$= P(0<t<\infty) - P(0<t<2)$

$= 0.5 - 0.4772$

$P(x \geq 12) = 0.0228$



$\therefore$ number of shoes $= 5000(0.0228) = 114$ shoes.

[EX] The income distribution of workers in a certain company was found to be normal with mean 500 Rs and std. deviation 50 Rs. There were 228 persons above 600 Rs. How many persons were there in all?

— [2000]

Solution:- Suppose there were N persons in all.

given $m = 500$ Rs., $\sigma = 50$ Rs $\quad t = \frac{x-m}{\sigma} \Rightarrow t = \frac{x-500}{50}$

$P(x) \Rightarrow$ probability of workers getting exactly income of 'x' Rs.

$\therefore$ Probability of persons getting above 600

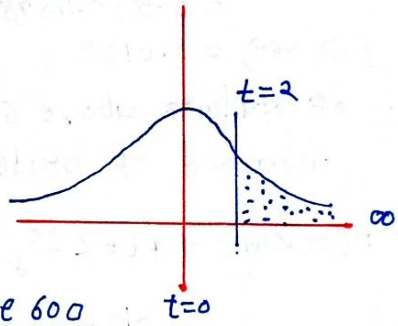
$= P(x>600) = P\left(t > \frac{600-500}{50}\right) = P(t>2)$

$= P(2<t<\infty)$

$= P(0<t<\infty) - P(0<t<2)$

$= 0.5 - 0.4772$

$P(x>600) = 0.0228$



It is given that there are 228 persons above 600

$\Rightarrow N(0.0228) = 228 \quad \Rightarrow N = 10000$

[EX] In a normal distribution 31% items are under 45 and 8% are above 64. Find the mean and std. deviation. —[2009 M] also find % of items betn 30 and 75. —[96, 98, 2003, 2008]

Solⁿ: $t = \frac{x-m}{\sigma}$

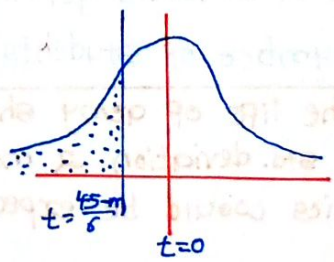
given 31% items are under 45

$\Rightarrow P(x<45) = 31\% \quad \Rightarrow P\left(t < \frac{45-m}{\sigma}\right) = 0.31$

$\Rightarrow P\left(-\infty < t < \frac{45-m}{\sigma}\right) = 0.31$

$\therefore P(-\infty<t<0) - P\left(\frac{45-m}{\sigma} < t < 0\right) = 0.31$

$\Rightarrow 0.5 - P\left(0<t<\frac{45-m}{\sigma}\right) = 0.31 \quad$ [$\because$ by symmetry]



$\therefore P\left(0<t<\frac{45-m}{\sigma}\right) = 0.19$

$\Rightarrow \frac{45-m}{\sigma} = -0.5 \quad$ ['−' sign idicates area to the left of $t=0$]

$\therefore 45 - m = -(0.5)\sigma \quad -①$

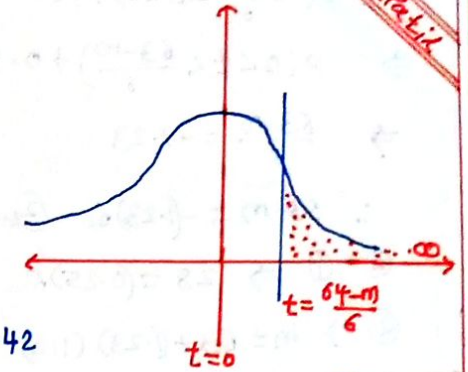
Given 8% items are above 64

$\Rightarrow P(x > 64) = 0.08 \quad \Longrightarrow P\left(t > \frac{64-m}{\sigma}\right) = 0.08$

$\Rightarrow P\left(\frac{64-m}{\sigma} < t < \infty\right) = 0.08$

$\therefore P(0 < t < \infty) - P\left(0 < t < \frac{64-m}{\sigma}\right) = 0.08$

$0.5 - P\left(0 < t < \frac{64-m}{\sigma}\right) = 0.08 \quad \Rightarrow P\left(0 < t < \frac{64-m}{\sigma}\right) = 0.42$



$\Rightarrow \frac{64-m}{\sigma} = +1.41 \quad$ {'+' denotes area to the right of t=0}

$64 - m = (1.41)\sigma \quad -②$

$② - ① \Rightarrow 19 = (1.91)\sigma \quad \Rightarrow \boxed{\sigma = 9.94}$

$② \Rightarrow m = 64 - (1.41)(9.94) \quad \Rightarrow \boxed{m = \text{mean} = 49.97}$

$\Bigg\} \Rightarrow \boxed{t = \frac{x - 49.97}{9.94}}$

Probability of items betⁿ 30 and 75

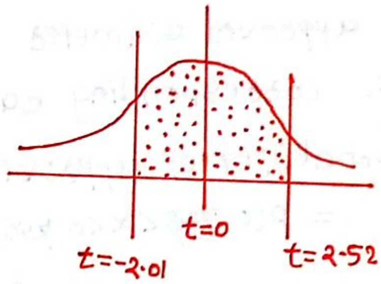
$= P(30 < x < 75) = P\left(\frac{30-49.97}{9.94} < t < \frac{75-49.97}{9.94}\right) = P(-2.01 < t < 2.52)$

$= P(-2.01 < t < 0) + P(0 < t < 2.52)$

$= P(0 < t < 2.01) + 0.4941 \quad$ [by symmetry]

$= 0.4778 + 0.4941$

$= 0.9719$



$\therefore$ % of items betⁿ 30 to 75 ; $= (0.9719)100 = 97.19\%$.

EX In a normal distribution 7% items are under 35 and 89% are above 63. What are the mean and std. deviation.

-{Jan. 2004, EL-10}

Solution:-

$t = \frac{x-m}{\sigma}$,

given 7% items are under 35

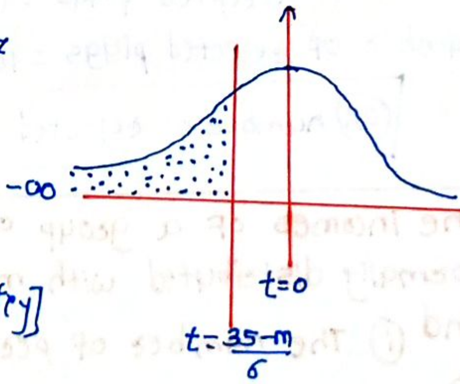
$\Rightarrow P(x < 35) = 7\% \quad \Rightarrow P\left(t < \frac{35-m}{\sigma}\right) = 0.07$

$\Rightarrow P\left(-\infty < t < \frac{35-m}{\sigma}\right) = 0.07$

$\therefore P(-\infty < t < 0) - P\left(\frac{35-m}{\sigma} < t < 0\right) = 0.07$

$\therefore 0.5 - P\left(0 < t < \frac{35-m}{\sigma}\right) = 0.07 \quad$ [$\because$ symmetry]

$P\left(0 < t < \frac{35-m}{\sigma}\right) = 0.43$



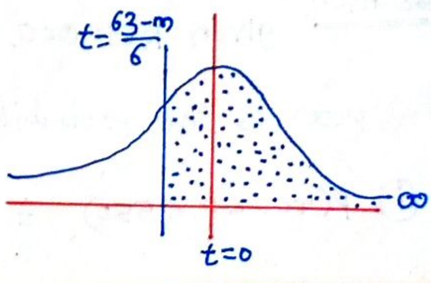
$\Rightarrow \frac{35-m}{\sigma} = -1.48 \quad$ {'-' denotes ordinate to the left of t=0}

$\therefore 35 - m = -(1.48)\sigma \quad -①$

given 89% items are above 63

$\therefore P(x > 63) = 89\% \quad \Rightarrow P\left(t > \frac{63-m}{\sigma}\right) = 0.89$

$\therefore P\left(\frac{63-m}{\sigma} < t < \infty\right) = 0.89$



$\therefore p\left(\frac{63-m}{6} < t < 0\right) + p(0 < t < \infty) = 0.89$

$\Rightarrow p\left(0 < t < \frac{63-m}{6}\right) + 0.5 = 0.89 \qquad \Rightarrow p\left(0 < t < \frac{63-m}{6}\right) = 0.39$

$\Rightarrow \frac{63-m}{6} = -1.23 \qquad$ {-ve sign denotes ordinate to the left of t=0}

$\therefore 63 - m = -(1.23)6 \quad -(2)$

$(2)-(1) \Rightarrow 28 = (0.25)6 \qquad \Rightarrow \boxed{6 = 112}$

$(2) \Rightarrow m = 63 + (1.23)(112) \qquad \Rightarrow \boxed{m = 200.76}$

EX Assuming that diameter of 1000 brass plugs taking consecutively from a normal distribution with mean 0.7515 cm and std. deviation 0.0020 cm, how many plugs are likely to be rejected if the approved diameter is 0.752 ± 0.004 cm? —{2003}

Solution:-

given $m = 0.7515$, $6 = 0.0020$, $t = \frac{x-m}{6} \Rightarrow t = \frac{x - 0.7515}{0.0020}$

$p(x) \Rightarrow$ Probability of plugs having diameter exactly 'x' cm.

approved diameter is 0.752 ± 0.004

i.e. corresponding range is 0.748 to 0.756 cm.

$\therefore$ probability of approved plugs

$= p(0.748 < x < 0.756) \quad = p\left(\frac{0.748 - 0.7515}{0.0020} < t < \frac{0.756 - 0.7515}{0.0020}\right)$

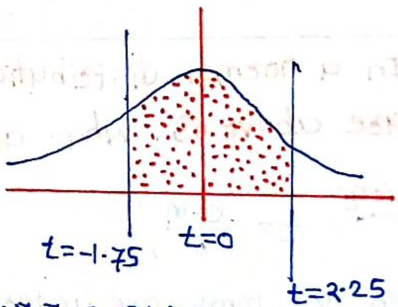
$= p(-1.75 < t < 2.25)$

$= p(-1.75 < t < 0) + p(0 < t < 2.25)$

$= p(0 < t < 1.75) + 0.4878$

$= 0.4599 + 0.4878$

$= 0.9477$



$\therefore$ number of accepted plugs $= (0.9477)1000 = 947.7 \approx 948$

$\therefore$ number of rejected plugs $= 1000 - 948 = 52$

$\boxed{\text{(OR) number of rejected plugs} = 1000\{p(x < 0.748) + p(x > 0.756)\} = 52}$

EX The incomes of a group of 10,000 persons were found to be normally distributed with mean Rs 520 and standard deviation Rs 60. Find (i) The number of persons having incomes between Rs 400 and 550 (ii) The lowest income of the richest 500. (iii) the highest income of the lowest paid 600 persons —{2007}

Solution:-

given $N = 10{,}000$, $m = 520$ Rs, $6 = 60$ Rs $\qquad t = \frac{x-m}{6} \Rightarrow t = \frac{x-520}{60}$

$p(x)$ is the probability of persons having income 'x' Rupees.

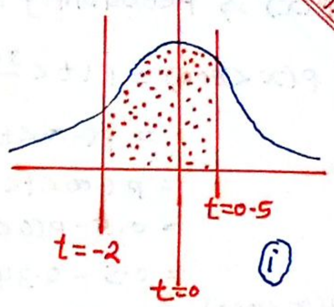
(i) $p(400 < x < 550) \quad = p\left(\frac{400-520}{60} < t < \frac{550-520}{60}\right)$

$= P(-2 < t < 0.5)$

$= P(-2 < t < 0) + P(0 < t < 0.5)$

$= P(0 < t < 2) + P(0 < t < 0.5)$

$= 0.4772 + 0.1915 \quad = 0.6687$



$\therefore$ number of persons having income between 400 and 550 $= (0.6687)\,10000$

$= 6687$

(ii) Let x_1 be the lowest income of richest 500

$\therefore$ Probability of richest persons $= P(x > x_1) = \frac{500}{10000}$

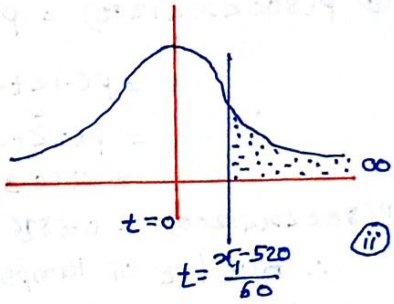
$\Rightarrow P\left(t > \frac{x_1 - 520}{60}\right) = 0.05$

$\Rightarrow P\left(\frac{x_1 - 520}{60} < t < \infty\right) = 0.05$

$\therefore P(0 < t < \infty) - P\left(0 < t < \frac{x_1 - 520}{60}\right) = 0.05$

$0.5 - P\left(0 < t < \frac{x_1 - 520}{60}\right) = 0.05$

$\therefore P\left(0 < t < \frac{x_1 - 520}{60}\right) = 0.45$



$\Rightarrow \frac{x_1 - 520}{60} = +1.67 \quad \Rightarrow \boxed{x_1 = 620.2 \text{ Rs}}$ { '+' denotes ordinate to the right of $t=0$ }

(iii) Let x_2 be the highest income of lowest 600

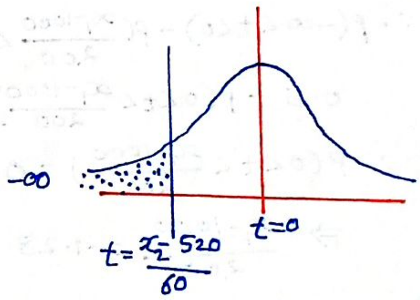
$\Rightarrow$ Prob. of lowest persons $P(x < x_2) = \frac{600}{10000}$

$\Rightarrow P\left(t < \frac{x_2 - 520}{60}\right) = 0.06$

$\therefore P\left(-\infty < t < \frac{x_2 - 520}{60}\right) = 0.06$

$P(-\infty < t < 0) - P\left(\frac{x_2 - 520}{60} < t < 0\right) = 0.06$

$0.5 - P\left(0 < t < \frac{x_2 - 520}{60}\right) = 0.06$



$\Rightarrow P\left(0 < t < \frac{x_2 - 520}{60}\right) = 0.44 \quad \Rightarrow \frac{x_2 - 520}{60} = -1.56$ { -ve denotes ordinate to the left of $t=0$ }

$\therefore \boxed{x_2 = 426.4 \text{ Rs.}}$

Ex Local Authority in a certain city install 10000 electric lamps in the streets of the city. If these lamps have average life of 1000 burning hours with standard deviation 200 hours, how many lamps might be expected to fail

(i) in the first 800 burning hours (ii) Between 800, 1200

After how many burning hours would you expect

(iii) 10% of the lamps to fail (iv) 10% of the lamps to be still burning

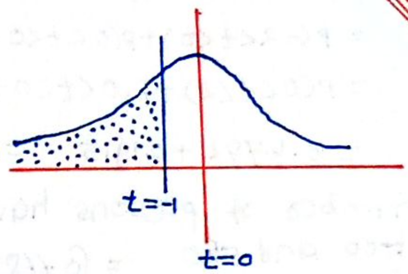
— {2003}

Sol^n:– Given $N = 10{,}000$, $m = 1000$, $\sigma = 200$

$t = \frac{x - m}{\sigma} \Rightarrow t = \frac{x - 1000}{200}$

$P(x) \Rightarrow$ probability of lamps having burning life 'x' hours.

(i) $P(x<800) = P\left(t<\frac{800-1000}{200}\right) = P(t<-1)$



$= P(-\infty<t<-1)$

$= P(-\infty<t<0) - P(-1<t<0)$

$= 0.5 - P(0<t<1)$

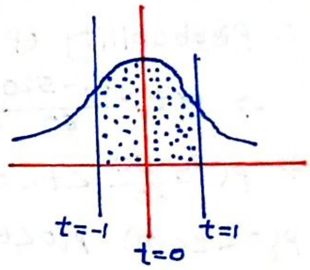
$= 0.5 - 0.3413$

$P(x<800) = 0.1587$

$\therefore$ number of lamps failing ~~in first~~ 800 hours

$= 10000(0.1587) = 1587$

(ii) $P(800<x<1200) = P\left(\frac{800-1000}{200}<t<\frac{1200-1000}{200}\right)$



$= P(-1<t<1) \qquad = P(-1<t<0) + P(0<t<1)$

$= P(0<t<1) + P(0<t<1)$

$= 2(0.3413)$

$P(800<x<1200) = 0.6826$

$\therefore$ number of lamps failing between 800 to 1200 hour

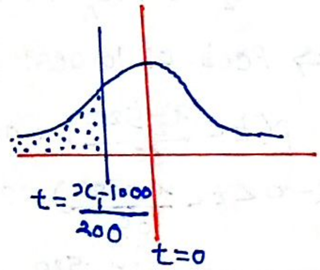
$= 10000(0.6826) = 6826$

(iii) Let x_1 denotes the number of hours,

then by condition, $P(x<x_1) = 10\%$. $\Rightarrow P\left(t<\frac{x_1-1000}{200}\right) = 0.1$

$\Rightarrow P\left(-\infty<t<\frac{x_1-1000}{200}\right) = 0.1$

$\therefore P(-\infty<t<0) - P\left(\frac{x_1-1000}{200}<t<0\right) = 0.1$



$0.5 - P\left(0<t<\frac{x_1-1000}{200}\right) = 0.1$

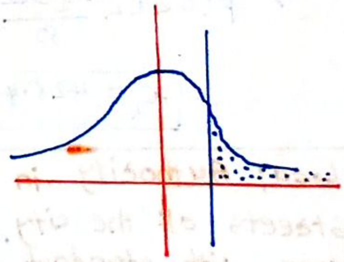
$\therefore P\left(0<t<\frac{x_1-1000}{200}\right) = 0.4$

$\Rightarrow \frac{x_1-1000}{200} = -1.28 \Rightarrow \boxed{x_1 = 744}$

(iv) Let ~~after~~ x_2 hours 10% of the lamps still burning

$\Rightarrow P(x>x_2) = 10\% \quad \Rightarrow P\left(t>\frac{x_2-1000}{200}\right) = 0.1$

$\Rightarrow P\left(\frac{x_2-1000}{200}<t<\infty\right) = 0.1$



$\therefore P(0<t<\infty) - P\left(0<t<\frac{x_2-1000}{200}\right) = 0.1$

$0.5 - P\left(0<t<\frac{x_2-1000}{200}\right) = 0.1$

$\Rightarrow P\left(0<t<\frac{x_2-1000}{200}\right) = 0.4 \quad \Rightarrow \frac{x_2-1000}{200} = +1.28$

$\Rightarrow \boxed{x_2 = 1256}$

EX Steel rods are manufactured to be 3 inches diameter but they are acceptable if they are inside the limits 2.99 and 3.01 inch. It is observed that 5% are rejected as oversize and 5% are rejected as undersize. Assuming that the diameters are normally distributed, find the std. deviation. Hence calculate what would be the proportion of rejected if the permissible limits were widened to 2.985 to 3.015 inches?

S 2004

Solution:- given $m=3$, $t=\frac{x-m}{\sigma} \Rightarrow t=\frac{x-3}{\sigma}$

$p(x) \Rightarrow$ Probability of rods having diameter 'x' inches

It is given that 5% of the rods are rejected as diameter is oversize.

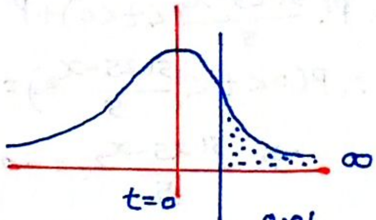
$\therefore p(x>3.01) = 5\% \Rightarrow p\left(t>\frac{3.01-3}{\sigma}\right)=0.05$

$\therefore p\left(\frac{0.01}{\sigma}<t<\infty\right)=0.05$

$\Rightarrow p(0<t<\infty)-p\left(0<t<\frac{0.01}{\sigma}\right)=0.05$

$\Rightarrow 0.5-p\left(0<t<\frac{0.01}{\sigma}\right)=0.05$

$\Rightarrow p\left(0<t<\frac{0.01}{\sigma}\right)=0.45$



$\therefore \frac{0.01}{\sigma}=1.64 \Rightarrow \boxed{\sigma=0.006}$

by taking undersize $p(x<2.99)=0.05$ we get $\sigma=0.006$

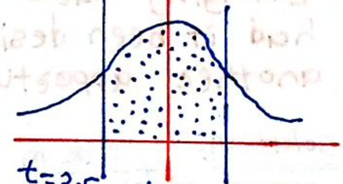
Proportion of the accepted rods if the diameter widened to the limits 2.985 to 3.015

$\Rightarrow p(2.985<x<3.015)=p\left(\frac{2.985-3}{0.006}<t<\frac{3.015-3}{0.006}\right)$

$=p(-2.5<t<2.5)$

$=2\,p(0<t<2.5)$

$=2(0.4938) \quad =0.9876$



$\therefore$ proportion of rejected rods $=1-0.9876=0.0124$

EX when mean marks was 50 and S.D. 5, then 60% of the students failed in an examination. Determine the 'grace' marks to be awarded in order to show that 70% of the students passed. Assume that the marks are normally distributed.

— 2008,

Solution:- given $m=50$, $\sigma=5$

Let x_1 denotes the minimum marks required for passing

$t=\frac{x-m}{\sigma} \Rightarrow t=\frac{x-50}{5}$

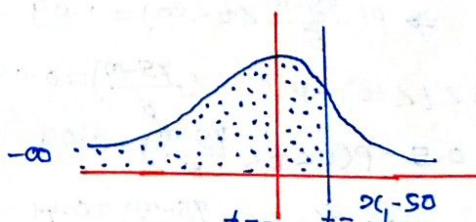
$p(x) \Rightarrow$ Prob. of students getting exactly 'x' marks.

given 60% students failed $\Rightarrow p(x<x_1)=60\%$.

$\Rightarrow p\left(t<\frac{x_1-50}{5}\right)=0.6$

$\Rightarrow p\left(-\infty<t<\frac{x_1-50}{5}\right)=0.6$

$\therefore p(-\infty<t<0)+p\left(0<t<\frac{x_1-50}{5}\right)=0.6$



$0.5+p\left(0<t<\frac{x_1-50}{5}\right)=0.6$

$\therefore p\left(0<t<\frac{x_1-50}{5}\right)=0.1$

$\Rightarrow \frac{x_1-50}{5}=+0.25 \Rightarrow \boxed{x_1=51.25}$

Let x_2 be the grace marks, $x_1 = 51.25$ min. marks for passing

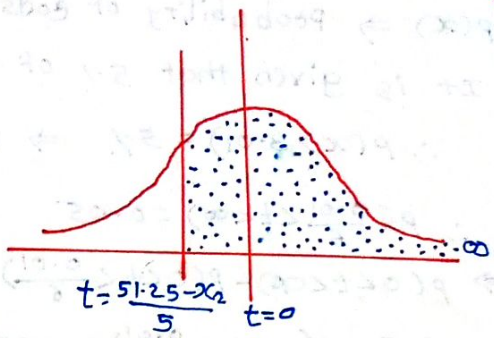
given condition $\Rightarrow p(x > x_1 - x_2) = 0.7$

$\therefore p\left(t > \frac{51.25 - x_2 - \cancel{50}}{5}\right) = 0.7$

$\Rightarrow p\left(\frac{51.25 - x_2}{5} < t < \infty\right) = 0.7$

$\therefore p\left(\frac{51.25 - x_2}{5} < t < 0\right) + p(0 < t < \infty) = 0.7$

$\therefore p\left(0 < t < \frac{51.25 - x_2}{5}\right) = 0.2$



$\Rightarrow \frac{51.25 - x_2}{5} = -0.52$

$\Rightarrow \boxed{x_2 = 3.85 \approx 4}$ grace marks

EX In a certain examination the percentage of passes and distinction were 46 and 9. Estimate the average marks obtained by candidates, the minimum pass and distinction marks being 40 and 75 respectively, assuming the distribution of marks normal.

Also determine what would have been the minimum qualifying marks for admission to re-exam of the failed candidates had it been desired that the best 25% of them would be given another opportunity of being re-examined.

-{2007

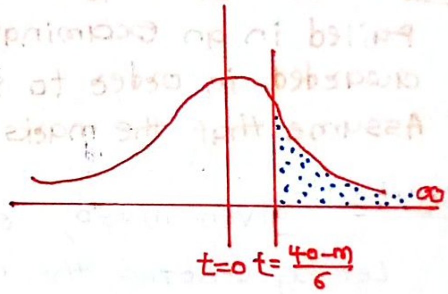
<u>solution:-</u> $t = \frac{x - m}{\sigma}$, given $p(x > 40) = 0.46$, $p(x > 75) = 0.09$

$p(x) \Rightarrow$ prob. of students getting exactly 'x' marks.

$p(x > 40) = 0.46 \Rightarrow p\left(t > \frac{40 - m}{\sigma}\right) = 0.46$

$\Rightarrow p\left(\frac{40 - m}{\sigma} < t < \infty\right) = 0.46$

$\therefore p(0 < t < \infty) - p\left(0 < t < \frac{40 - m}{\sigma}\right) = 0.46$



$0.5 - p\left(0 < t < \frac{40 - m}{\sigma}\right) = 0.46$

$\Rightarrow p\left(0 < t < \frac{40 - m}{\sigma}\right) = 0.04 \qquad \Rightarrow \frac{40 - m}{\sigma} = +0.1$

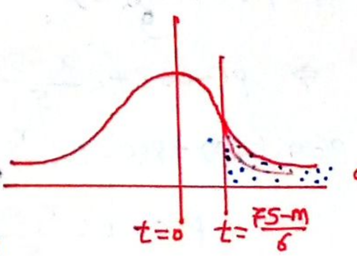
$\therefore 40 - m = (0.1)\sigma \quad —①$

Simillarly $p(x > 75) = 0.09 \Rightarrow p\left(t > \frac{75 - m}{\sigma}\right) = 0.09$

$\Rightarrow p\left(\frac{75 - m}{\sigma} < t < \infty\right) = 0.09$

$\therefore p(0 < t < \infty) - p\left(0 < t < \frac{75 - m}{\sigma}\right) = 0.09$

$0.5 - p\left(0 < t < \frac{75 - m}{\sigma}\right) = 0.09$



$\Rightarrow p\left(0 < t < \frac{75 - m}{\sigma}\right) = 0.41$

$\therefore \frac{75 - m}{\sigma} = +1.34 \qquad \Rightarrow 75 - m = (1.34)\sigma \quad —②$

② − ① $\Rightarrow 35 = (1.24)\sigma \Rightarrow \boxed{\sigma = 28.22}$

② $\Rightarrow m = 75 - 1.34(28.22) \Rightarrow \boxed{m = 37.20}$

% of passing candidate is 46% $\Rightarrow$ % of failed students = 54%

of which 25% are best $\Rightarrow$ 13.5% $\quad 54 \frac{25}{100}$

Let x_1 be the minimum marks for qualifying for re-exam

$\Rightarrow$ total % above x_1 is $(13.5+46)\% = 59.5\%$

then by given condition

$$p(x > x_1) = 0.595$$

$$\therefore p\left(t > \frac{x_1 - 37.2}{28.22}\right) = 0.595$$

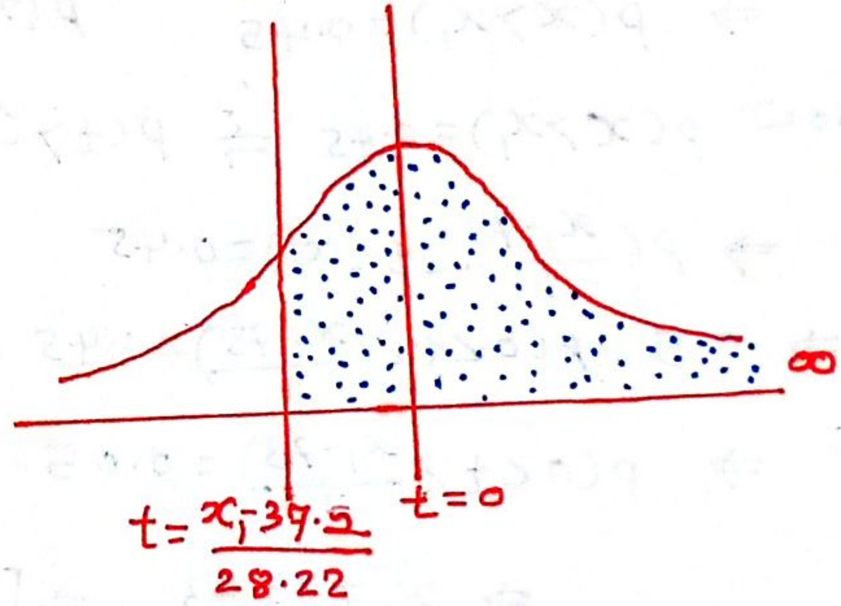
$$\Rightarrow p\left(\frac{x_1 - 37.2}{28.22} < t < \infty\right) = 0.595$$

$$\therefore p\left(\frac{x_1 - 37.2}{28.22} < t < 0\right) + p(0 < t < \infty) = 0.595$$

$$\therefore p\left(0 < t < \frac{x_1 - 37.2}{28.22}\right) + 0.5 = 0.595$$

$$\Rightarrow p\left(0 < t < \frac{x_1 - 37.2}{28.22}\right) = 0.095$$

$$\therefore \frac{x_1 - 37.2}{28.22} = -0.24 \quad \Rightarrow \boxed{x_1 = 30.42 \simeq 30}$$



[EX] If x_1, x_2 be two independant random variates with means 30 and 25 and variances 16 and 12 and if $Y = 3X_1 - 2X_2$, Find $p(60 \leq Y \leq 80)$

Solution:- Since x_1, x_2 are two normal variates with means 30 and 25 and variances 16 and 12, $Y = 3X_1 - 2X_2$ is also normal variate with

mean $m = 3m_1 - 2m_2 = 3(30) - 2(25) = 40 \Rightarrow \boxed{m = 40}$

and variance $\sigma^2 = 3^2\sigma_1^2 + (-2)^2\sigma_2^2 = 9(16) + 4(12) \Rightarrow \sigma^2 = 192 \Rightarrow \boxed{\sigma = 13.87}$

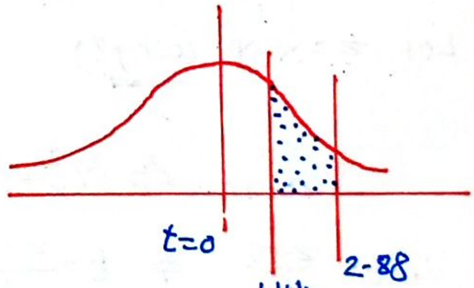
$\therefore t = \frac{y - m}{\sigma} \Rightarrow t = \frac{y - 40}{13.87}$

$\therefore P(60 < Y < 80) = p\left(\frac{60-40}{13.87} < t < \frac{80-40}{13.87}\right) = P(1.44 < t < 2.88)$

$= p(0 < t < 2.88) - p(0 < t < 1.44)$

$= 0.4980 - 0.4251$

$p(60 < Y < 80) = 0.0729$



[EX] In an examination marks obtained by students in SS, EWT, ECI are normally distributed with mean 51, 53 and 49 with std. deviations 11, 9, 6 respectively. Find the probability securing total marks (i) 181 or above (ii) 134 or below.

→ S2005

solution:- Let x_1, x_2, x_3 denotes marks in three subjects, then x_1, x_2, x_3 are normally distributed with mean $m_1 = 51$, $m_2 = 53$, $m_3 = 49$

and variances $\sigma_1^2 = 11^2 = 121$, $\sigma_2^2 = 9^2 = 81$, $\sigma_3^2 = 6^2 = 36$ $\longrightarrow$ {note this}

Let $Y = X_1 + X_2 + X_3$ clearly normal with

mean $m = m_1 + m_2 + m_3 = 51 + 53 + 49$ $\Rightarrow \boxed{m = 153}$

variance of y is. $\sigma^2 = 1^2\sigma_1^2 + 1^2\sigma_2^2 + 1^2\sigma_3^2 = 121 + 81 + 36$ $\Rightarrow \sigma^2 = 238 \Rightarrow \boxed{\sigma = 15.43}$

$$\therefore\ t = \frac{y - m}{\sigma} \Rightarrow t = \frac{y - 153}{15.43}$$

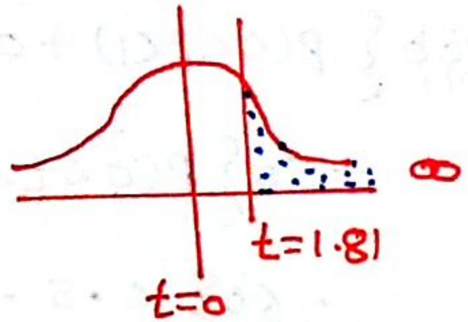
$p(y) \Rightarrow$ probability of students getting total 'y' marks.

(i) $p(y \geq 181) = p\left(t > \frac{181 - 153}{15.43}\right) = p(1.81 < t < \infty)$

$= p(0 < t < \infty) - p(0 < t < 1.81)$

$= 0.5 - 0.4649$

$\boxed{P(y \geq 181) = 0.0351}$



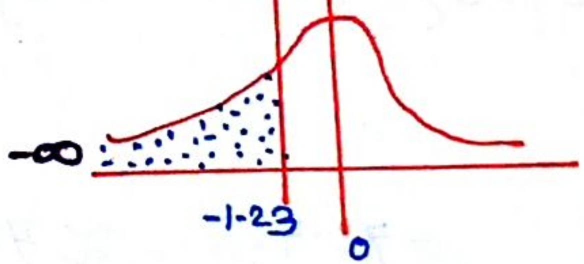
(ii) $p(y \leq 134) = p\left(t < \frac{134 - 153}{15.43}\right) = p(-\infty < t < -1.23)$

$= p(-\infty < t < 0) - p(-1.23 < t < 0)$

$= 0.5 - p(0 < t < 1.23)$

$= 0.5 - 0.3907$

$\boxed{P(y \leq 134) = 0.1093}$



V.M. Patil

Ex If X is a normal variate with mean 10 and standard deviation 4 find -

(i) $P(|x-14|<1)$ (ii) $P(5 \le x \le 18)$ (iii) $P(x \le 12)$

→ { E1-10 M

Solution:-

given $m=10$, $\sigma=4$

consider $t = \dfrac{x-m}{\sigma} = \dfrac{x-10}{4}$

$|x-14|<1$

$\Rightarrow (x-14)<1, \quad -(x-14)<1$

$\Rightarrow x<15, \quad -x<-13$

$\Rightarrow x<15, \quad x>13$

$\Rightarrow 13<x<15$

(i) $P(|x-14|<1) = P(13<x<15)$

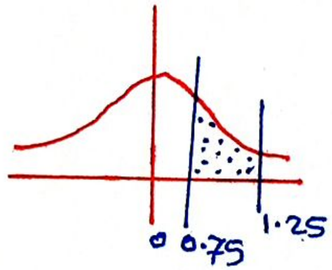
$= P\left(\dfrac{13-10}{4} < t < \dfrac{15-10}{4}\right)$

$= P(0.75<t<1.25)$

$= P(0<t<1.25) - P(0<t<0.75)$

$= 0.3944 - 0.2734$

$= 0.121$



(ii) $P(5 \le x \le 18) =$

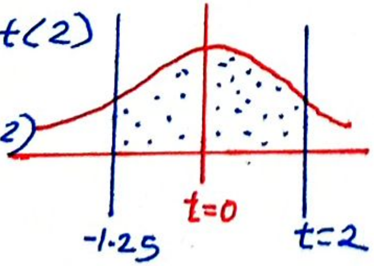
$= P\left(\dfrac{5-10}{4} < t < \dfrac{18-10}{4}\right) = P(-1.25<t<2)$

$= P(-1.25<t<0) + P(0<t<2)$

$= P(0<t<1.25) + P(0<t<2)$

$= 0.3944 + 0.4772$

$= 0.8716$



(iii) $P(x \le 12) = P\left(t \le \dfrac{12-10}{4}\right)$

$= P(-\infty<t<0.5)$

$= P(-\infty<t<0) + P(0<t<0.5)$

$= 0.5 + 0.1915$

$= 0.6915$