Set Theory.

(1) $U = \{1, 2, 3, \dots 10\}$.
$A = \{2, 4, 7, 9\}$, $B \{1, 4, 6, 7, 10\}$, $C = \{3, 5, 7, 9\}$

→ $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
$= \{4, 7\}$

→ $A \cup B = \{1, 2, 4, 6, 7, 9, 10\}$

→ $A \oplus B = \{1, 2, 6, 9, 10\}$

(2) (ii) $\pi_2 = \{\{1, 2\}, \{3, 4, 5\}\}$

(3) $A = \{a, b, c, d, e, f, g, h\}$
$A_1 = \{a, b, c, d\}$ $\quad$ $A_3 = \{a, c, e, g\}$
$A_2 = \{a, c, e, g, h\}$ $\quad$ $A_4 = \{b, d\}$ $\quad$ $A_5 = \{f, h\}$

(i) $\{A_1, A_2\} = \{\{a, b, c, d\}, \{a, c, e, g, h\}\}$
Not a partition of set A.
⇒ B'coz $\forall \pi$; $\pi_i \wedge \pi_j \neq \emptyset$, i.e. they have common or repeated elements.

(ii) $\{A_3, A_4, A_5\} = \{\{a, c, e, g\}, \{b, d\}, \{f, h\}\}$
Partition of set A.
⇒ $\forall \pi$; $\pi_i \neq \emptyset$
$\forall \pi$; $\pi_i \wedge \pi_j = \emptyset$ and $\bigcup_{i=1}^{n} \pi_i = A$.

(5) $X = (E \cap F) - (F \cap G)$

$= E \cap F - G \cap F$

$= (E - G) \cap F$

$= (E - E \cap G) \cap F$

$= E \cap F - E \cap F \cap G$

$Y = (E - (E \cap G)) - (E - F)$.

$= \cancel{E - G - (E - F)}$

$= E - (E \cap G) - (E - (E \cap F))$

$= E \cap F - E \cap G$

$= E \cap (F - G)$

$= E \cap (F - F \cap G)$

$= E \cap F - E \cap F \cap G$

$\therefore X = Y$

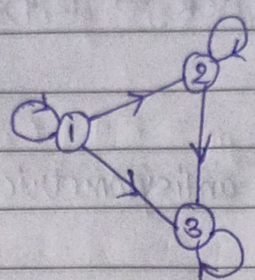
Relation

(1) $R = \{(x,y) \mid x \le y\}$, $A = \{1,2,3\}$

∴ $R = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$

Adjancy matrix : $R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 \end{array}$

Diagraph.



(2) $R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

$A = \{1,2,3,4\}$

→ Reflexive : $\forall a \in A$, aRa.

But $(1,1), (2,2), (3,3) (4,4) \notin R$

∴ Not Reflexive.

→ symmetric : if $(a,b) \in R$ then $(b,a) \in R$.

$(1,2) \in R$ but $(2,1) \notin R$

∴ Not symmetric.

→ Asymmetric : if $(a,b) \in R$ Then $(b,a) \notin R$.

$(1,2) \in R$ but $(2,1) \notin R$

$(1,3) \in R$ but $(3,1) \notin R$

$(1,4) \in R$ but $(4,1) \notin R$

⋮

∴ Asymmetric

→ Antisymmetric : $(a,a) \in R$

but $(a,b) \in R$ then $(b,a) \notin R$.

∴ Antisymmetric.

→ Transitive : if $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$.

$(1,2) \in R$, $(2,3) \in R$ → $(1,3) \in R$

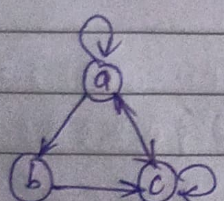
$(1,2) \in R$ $(2,4) \in R$ → $(1,4) \in R$

$(1,3) \in R$ $(3,4) \in R$ → $(1,4) \in R$

$(2,3) \in R$ $(3,4) \in R$ → $(2,4) \in R$

∴ Transitive.

∴ R is asymmetric, antisymmetric, & transitive.

③ 

$R = \{(a,a), (a,b), (a,c), (b,c), (c,c), (c,a)\}$

→ Not Reflexive

→ Not symmetric [$(a,b) \in R$, $(b,a) \notin R$]

→ Not asymmetric [$(a,c) \in R$, $(c,a) \in R$]

→ Not antisymmetric.

→ Transitive [$(a,b) \in R$, $(b,c) \in R$, $(a,c) \in R$]

④ $R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4), (5,5)\}$

$A = \{1,2,3,4,5\}$ $A/R = ?$

→ Reflexive : $\forall a \in A$ aRa

$(1,1), (2,2), (3,3), (4,4), (5,5) \in R$.

∴ it is Reflexive.

→ symmetric : $(a,b) \in R$ then $(b,a) \in R$.

$(1,2) \in R$ → $(2,1) \in R$.

$(3,4) \in R$ → $(4,3) \in R$.

∴ it is symmetric.

→ Transitive : $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$.

~~It is transitive~~

$(3,4)\ (4,3) \rightarrow (3,3)$

Transitive.

$\therefore$ R is equivalence relation.

→ Equivalence classes :

$[1] = \{1,2\}$

$[2] = \{1,2\}$

$[3] = \{3,4\}$

$[4] = \{3,4\}$

$[5] = \{5\}$

$A/R = \{\{1,2\},\{3,4\},\{5\}\}$

(5) $R = \{(x,y) \mid x-y$ is divisible by 3 $\}$.

→ Reflexive : $(a,a) \in R$.

$x - x = 0 \rightarrow$ divisible by 3.

$\Rightarrow$ Reflexive

→ symmetric : $(a,b) \in R \rightarrow (b,a) \in R$.

$(x-y) = 3n \rightarrow (y-x) = -3n \leftarrow$ divisible by 3

$\Rightarrow$ symmetric.

→ Transitive : $(a,b) \in R$, $(b,c) \in R \rightarrow (a,c) \in R$.

$(x-y) = 3n$, $(y-z) = 3m$,

$x - z = (x-y) + (y-z) = 3(n+m) \leftarrow$ divisible by 3.

$\therefore$ Transitive.

$\therefore$ equivalence relation.

→ Equivalence classes :

$[0] = \{0,3,6,9\ldots\} = 3k$

$[1] = \{1,4,7,10\ldots\} = 3k+1$

$[2] = \{2,5,8,11\ldots\} = 3k+2$

$[3] = [0]$

$[4] = [1]$

$Z/R = \{\{3k_1\},\{3k_2+1\},\{3k_3+2\}\}$

~~6. $a \equiv m \% 5$~~

7. $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (1,4), (2,4), (3,1), (3,2), (4,2), (4,3), (4,4)\}$.

→ Reflexive closure, $R^{\infty}_{ref} =$

$\{(1,1), (1,2), (1,4), (2,4), (3,1), (3,2), (4,2), (4,3), (4,4), (2,2), (3,3)\}$

$R^{\infty}_{Ref} = R + \{(2,2), (3,3)\}$

→ Symmetric closure, $R^{\infty}_{sym} = R + \{(2,1), (4,1), (1,3), (2,3), (3,4)\}$

8. $R = \{(1,1), (1,4), (2,2), (2,3), (3,1), (4,3), (4,4)\}$

$A = \{1, 2, 3, 4\}$.

Acc. to warshall's Algorithm.

$$R = \begin{array}{c|c} & 1\quad 2\quad 3\quad 4 \\ \hline \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

R: $\{1, 4\}$ C: $\{1, 3\}$

$R \times C : \{(1,1), (1,3), (4,1), (4,3)\}$

$$W = \begin{array}{c|c} & 1\quad 2\quad 3\quad 4 \\ \hline \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

R: $\{2, 3\}$ C: $\{2\}$

$R \times C : \{(2,2), (3,2)\}$

$$W = \begin{array}{c|c} & 1\quad 2\quad 3\quad 4 \\ \hline \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

R: $\{1, 2\}$ C: $\{1, 2, 4\}$

$R \times C : \{(1,1) (1,2) (1,4) (2,1) (2,2) (2,4)\}$

$$W = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

R: $\{1,3,4\}$ C: $\{1,2,4\}$

$R \times C : \{(1,1)\,(1,2)\,(1,4)\,(3,1)\,(3,2)\,(3,4)\,(4,1)\,(4,2)\,(4,4)\}$

$$W = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$ → Transitive closure of R.

$\therefore R^{\infty}_{trans} = \{(1,1)\,(1,2)\,(1,3)\,(1,4)\,(2,1)\,(2,2)\,(2,3)\,(2,4)\,(3,1)\,(3,2)\,(3,4)\,(4,1)\,(4,2)\,(4,4)\}$

9. $S = \{1,2,3,4\}$

$R = \{(4,3)\,(2,2)\,(2,1)\,(3,1)\,(1,2)\}$

(i) For transitivity, $(a,b) \in R, (b,c) \in R \rightarrow (a,c) \in R$

$(4,3), (3,1) \in R \rightarrow (4,1) \notin R$

$\therefore$ R is not transitive.

(ii) Acc. to Warshell's Algorithm.

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

R: $\{2\}$ C: $\{2,3\}$

$R \times C : \{(2,2)\,(2,3)\}$.

$$W = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

R: $\{1,2,3\}$ C: $\{1,2\}$

$R \times C : \{(1,1)\,(1,2)\,(2,1)\,(2,2)\,(3,1)\,(3,2)\}$

$$W = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

R $\{1,2\}$ C $\{2,4\}$

$R \times C : \{(1,2)\ (1,4)\ (2,2)\ (2,4)\}$

$$W = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

R $\{3\}$ C $\{1,2\}$

$R \times C : \{(3,1)\ (3,2)\}$

$$\therefore R^{t}_{\cancel{trans}} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

$\therefore R^t = \{(1,1)\ (1,2)\ (1,4)\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (3,1)\ (3,2)\ (4,3)\}$

Funct^n.

① $f : R \to R$.

$f(x) = x^2 - 4x$.

→ For injective :

let $f(x_1) = f(x_2)$

$x_1^2 - 4x_1 = x_2^2 - 4x_2$

$(x_1 - x_2)(x_1 + x_2) - 4(x_1 - x_2) = 0$

$(x_1 - x_2)[x_1 + x_2 - 4] = 0$

$\therefore x_1 = x_2$ or $x_1 = 4 - x_2$

$x_1 \neq x_2$.

$\therefore$ Not injective.

For surjective :

let $y = 4x^2 - 4x$

$4x^2 - 4x - y = 0$

$x = \frac{+4 \pm \sqrt{16 + 16y}}{8} = \frac{4 \pm 4\sqrt{1+y}}{8}$

$\sqrt{1+y} \rightarrow$ can be complex.

$\therefore$ But $f : R \rightarrow R$.

$\therefore$ Not surjective.

2. $f : R - \{2\} \rightarrow R$.

$f(x) = \frac{1}{x-2}$.

$f(x_1) = f(x_2)$

$\frac{1}{x_1 - 2} = \frac{1}{x_2 - 2}$

$x_1 - 2 = x_2 - 2$

$x_1 = x_2$

$\Rightarrow$ injective

$y = \frac{1}{x - \cancel{y}\,2}$

$\cancel{y}\, x - 2 = \frac{1}{y}$

$g : x = \frac{1}{y} + 2$.

$\Rightarrow g : R - \{0\} \rightarrow R - \{2\}$

but $f : R - \{2\} \rightarrow R$.

$\therefore$ Not surjective.

③ nb.

(2)

Q. $f: R - \left\{\frac{2}{5}\right\} \to R - \left\{\frac{4}{5}\right\}$ P.T. Bijective & find f^{-1}

$$f(x) = \frac{4x+3}{5x-2}$$

$\rightarrow$ $f(x_1) = f(x_2)$

$$\frac{4x_1+3}{5x_1-2} = \frac{4x_2+3}{5x_2-2}$$

$$\cancel{20x_1x_2} - 8x_1 + 15x_2 - \cancel{6} = \cancel{20x_1x_2} + 15x_1 - 8x_2 - \cancel{6}$$

$$15(x_1 - x_2) + 8(x_1 - x_2) = 0$$

$$x_1 = x_2$$

Injective

$$y = \frac{4x+3}{5x-2}$$

$$5xy - 2y = 4x + 3$$

$$x(5y-4) = 3 + 2y$$

$$x = \frac{3+2y}{5y-4} \qquad 5y-4 \neq 0$$

$\therefore f: R - \left\{\frac{4}{5}\right\} \to R - \left\{\frac{2}{5}\right\}$

Surjective

$\Rightarrow$ Bijective.

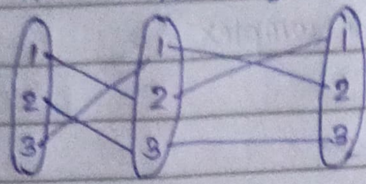
$$f^{-1}(x) = \frac{3+2x}{5x-4}$$

4. $A = \{1, 2, 3\}$

$f = \{(1,2)\ (2,3)\ (3,1)\}$

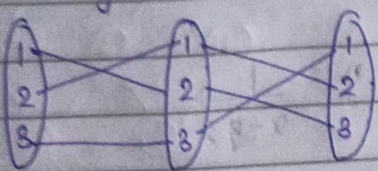
$g = \{(1,2)\ (2,1)\ (3,3)\}$

gof : $A \xrightarrow{f} A \xrightarrow{g} A$



$\therefore\ gof = \{(1,1), (2,3), (3,2)\}$

fog : $A \xrightarrow{g} A \xrightarrow{f} A$



$\therefore\ fog = \{(1,3), (2,2)\ (3,1)\}$.

5.

6. $f: R \to R \quad f(x) = 2x + 3$ | $g: R \to R,\ g(x) = 3x - 4$.

~~fog~~ f^{-1}: $y = 2x + 3$

$x = \dfrac{y-3}{2}$

$f^{-1} = \dfrac{x-3}{2}$

g^{-1} : $y = 3x - 4$

$x = \dfrac{y+4}{3}$

$g^{-1} = \dfrac{x+4}{3}$.

$\therefore\ fog = 2(3x-4)+3 = 6x - 5$

$(fog)^{-1} = \quad y = 6x - 5$

$x = \dfrac{y+5}{6}$

$LHS = (fog)^{-1} = \cancel{y}\ \dfrac{x+5}{6}$

RHS $g^{-1} o f^{-1} = \dfrac{\frac{(x-3)}{2} + 4}{3}$

$= \dfrac{x+5}{6}$

$= LHS$

(5) Q. $f: R \to R$ $\quad f(x) = x^3$ $\quad$ | gof, fog, fofof ?

$g: R \to R$ $\quad g(x) = 4x^2 + 1$

$h: R \to R$ $\quad h(x) = 7x - 2$.

Sol^n.

$$gof(x) = g[f(x)]$$
$$gof(x) = 4x^6 + 1$$

$$fog(x) = f[g(x)]$$
$$fog(x) = (4x^2 + 1)^3$$

$$fofof(x) = f[f[f(x)]]$$
$$= f[(x^3)^3]$$
$$= x^{3 \times 3 \times 3}$$
$$fofof(x) = x^{27}$$

$$(goh)of$$
$$[4(7x-2)^2 + 1]\,of$$
$$= 4(7x^3 - 2)^2 + 1$$

Date: __/__/____

7. $R : \{(a,b) \mid a|b\}$

$Z^+ = \{1, 2, 3, \dots\}$

→ Reflexive : aRa

~~a+b a++ a|a~~ $a|a$ ← a is divisible by a.

∴ Reflexive.

→ Antisymmetric : aRb & bRa. Then $a=b$

$\frac{b}{a}$ & $\frac{a}{b}$ ⇒ $a=b$.

∴ Antisymmetric.

→ Transitive : aRb, bRc then aRc.

$\frac{b}{a}$ & $\frac{c}{b}$ then $\frac{c}{a}$.

∴ Transitive.

⇒ It is Poset.

8. $S = \{a,b,c\}$ $(P(S), \subseteq)$ → Poset.

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$.

→ Reflexive : aRa.

$a \subseteq a$ → True

∴ Reflexive

→ Antisymmetric : aRb & bRa then $a=b$.

$a \subseteq b$, $b \subseteq a$ ⇒ $a=b$

∴ Antisymmetric

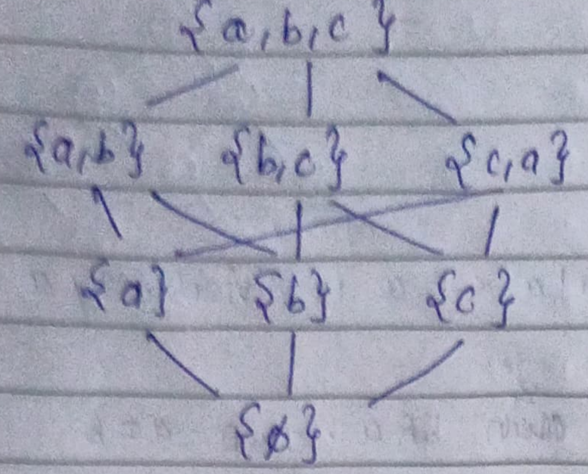
→ Transitive : aRb, bRc ⇒ aRc

$a \subseteq b$, $b \subseteq c$ ⇒ $a \subseteq c$ → True

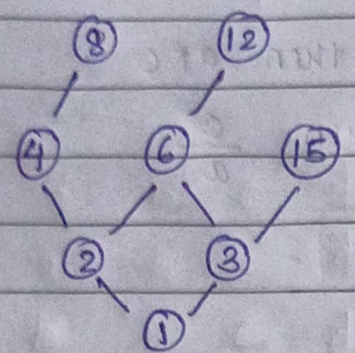
∴ Transitive

⇒ It is Poset

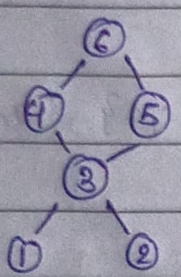
Hasse diagram :



9. poset (A, |) , A = { 1, 2, 3, 4, 6, 8, 12, 15 } .



10.



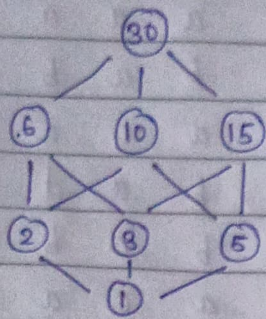
LUB (1, 2, 3)

a∨b	1	2	3
1	1	2	3
2	2	2	6
3	3	6	3

GLB (1, 2, 3)

a∧b	1	2	3
1	1	1	1
2	1	2	1
3	1	1	3

11. $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$



~~GLB(a,b)~~
~~a∧b~~

GLB (a,b)

$a \wedge b$	1	2	3	5	6	10	15	30
1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2
3	1	1	3	1	3	1	3	3
5	1	1	1	5	1	5	5	5
6	1	2	3	1	6	2	3	6
10	1	2	1	5	2	10	5	10
15	1	1	3	5	3	5	15	15
30	1	2	3	5	6	10	15	30

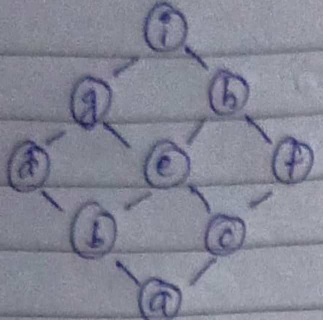
LUB (a,b)

$a \vee b$	1	2	3	5	6	10	15	30
1	1	2	3	5	6	10	15	30
2	2	2	6	10	6	10	30	30
3	3	6	3	15	6	30	15	30
5	5	10	15	5	30	10	15	30
6	6	6	6	30	6	30	30	30
10	10	10	30	10	30	10	30	30
15	15	30	15	15	30	30	15	30
30	30	30	30	30	30	30	30	30

As $\forall (a,b)$, $a \wedge b \neq 0$ & $a \vee b \neq 0$

$\therefore$ D_{30} is lattice.

12.



glb

$a \wedge b$	a	b	c	d	e	f	g	h	i
a	a	a	a	a	a	a	a	a	a
b	a	b	a	b	b	a	b	b	b
c	a	a	c	a	c	c	c	c	c
d	a	b	a	d	b	a	d	b	d
e	a	b	c	b	e	c	e	e	e
f	a	a	c	a	c	f	c	f	f
g	a	b	c	d	e	c	g	e	g
h	a	b	c	b	e	f	e	h	h
i	a	b	c	d	e	f	g	h	i

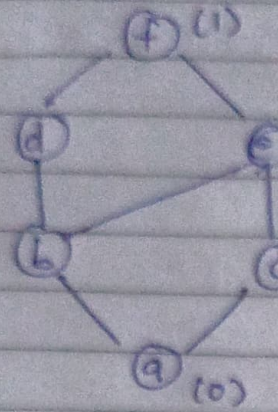
lub

$a \vee b$	a	b	c	d	e	f	g	h	i
a	a	b	c	d	e	f	g	h	i
b	b	b	e	d	e	h	g	h	i
c	c	e	c	g	e	f	g	h	i
d	d	d	g	d	g	i	g	i	i
e	e	e	e	g	e	h	g	h	i
f	f	h	f	i	h	f	i	h	i
g	g	g	g	g	g	i	g	i	i
h	h	h	h	i	h	h	i	h	i
i	i	i	i	i	i	i	i	i	i

As $\forall (a,b)$, $a \vee b \neq \phi$ & $a \wedge b \neq \phi$

$\therefore$ it is lattice.

13.



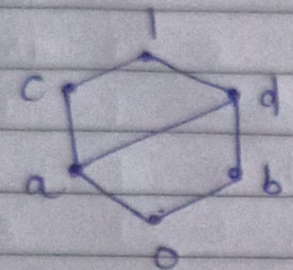
$a^c = f$

~~$b^c = e$~~ $b^c = 0$, $e^c = 0$.

~~$e^c = b$.~~ $d^c = c$, $c^c = d$.

Not compliment lattice.

14.



$c^c = b$, $b^c = c$.

$a^c = 0$, $d^c = 0$.