Mathematical Foundations of Computer Science
Homework Assignment 1

Given: January 20, 2023 **Due:** January 27, 2023

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1. Let p, q, and r be the following propositions.

p: You get an A on the final exam.

q: You do every exercise in the text book

r: You get an A in the course.

Express the following propositions using p, q, r and logical operators.

(a) You get an A in the course but you do not do every exercise in the text book.

(b) You get an A on the final exam, you do every exercise in the text book, and you get an A in the course.

(c) To get an A in the course it is necessary for you to get an A on the final exam.

(d) You get an A on the final, but you don't do every exercise in the text book; nevertheless, you get an A in the course.

(e) Getting an A on the final exam and doing every exercise in the text book is sufficient for getting an A in the course.

(f) You will get an A in the course if and only if you either do every exercise in the text book or you get an A on the final exam.

2. Rewrite the following formally using quantifiers and variables, and write a negation for the statement.

(a) Everybody loves somebody.

(b) Somebody loves everybody.

(c) Any even integer equals twice some other integer.

(d) There is a program that gives the correct answer to every question that is posed to it.

(e) There is a prime number between every integer and its double.

3. Decide if the following proposition forms are a tautology using a truth table.

(a) $(p \vee q) \vee (\neg p \vee \neg q)$

(b) $(p \wedge q) \rightarrow (p \rightarrow q)$

4. Prove or disprove the following.

(a) For every prime p, $p + 2$ is a prime.
(b) For all integers m and n, $m+n$ and $m-n$ are either both odd or both even.
(c) For any positive real numbers x and $y \leq x$, $\lfloor x - y \rfloor = \lfloor x \rfloor - \lfloor y \rfloor$.
(d) For all natural numbers x, $x^2 - x + 3$ is odd.
(e) For all natural numbers m, if m is even then m^7 is even

5. Suppose $a, b, x,$ and y are integers. Prove that if $d|a$ and $d|b$, then $d|(ax + by)$.

6. Given any numbers x, y and z, if $x - y$ is odd and $y - z$ is even, is $x - z$ odd or even? Prove your claim.

7. Let t be a positive integer. Prove the following statement by proving its contrapositive.

$$\text{if } r \text{ is irrational, then } r^{1/t} \text{ is irrational.}$$

Be sure to state the contrapositive explicitly.

8. Prove that for all integers n, if $n - 3$ is divisible by 4 then $n^2 - 1$ is divisible by 8.

9. Prove or disprove the following.

(a) For all integers n, $n^3 - n$ is divisible by 3.
(b) For all real numbers x, $2x^2 - 4x + 3 > 0$.

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