Module-3: Algebraic Structure

1. Define group, Abelian group.
2. What is identity element for group $(Q,*)$ where * defined as $a * b = a + b - ab;\ a, b \in Q$ and Q is rational number.
3. Create composition table for abelian group $G = \{1, -1, i, -i\}$ under multiplication.
4. What is/are generator for cyclic group of cube roots of unity.
5. What is Hamming distance between the words i) 01010 & 01010 ii) 10101 & 01110.
6. For encoding function $e: B^2 \to B^6$ if the minimum distance for encoding function is 3, then How many errors can be detect & corrected
7. Consider encoding function e : B2 → B5 defined as

   e(00) = 00000; e(10) = 10111; e(01) = 01110 ; e(11) = 11111

   Find the minimum distance for encoding function.
8. Prove that in a group (G, *), identity element is unique.
9. Prove that $G = \{1, -1, i, -i\}$ is a group under usual multiplication of complex numbers.
10. Let G be a set of all square matrices of type $\begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix}$ where $m \in Z$. Prove that G is a group under multiplication. Is it Abelian group?
11. Prove that the Group G = {0, 1, 2, 3, 4, 5} is a cyclic group under addition modulo 6.
12. Consider G = {1, 5, 7, 11, 13, 17} a reduced system modulo 18 (i.e., the set of integers between 1 and 18 which are relatively prime to 18). Prepare composition table and prove that G is a cyclic group.
13. Show that encoding function e: B2 → B5 defined below is group code.

    e(00) = 00000 e(10) = 10101 e(01) = 01110 e(11) = 11011

14. Show that (3, 7) encoding function e : B3 → B7 below is group code

    e(000) = 0000000; e(001) = 0010110; e(010) = 0101000; e(011) = 0111110

    e(100) = 100010; e(101) = 1010011; e(110) = 1101101; e(111) = 1111011

15. Let $H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ be a parity check matrix. Determine the group code $e_H: B^2 \to B^5$

16. Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ be a parity check matrix. Determine the group code $e_H: B^3 \rightarrow B^6$

17. Consider (2, 5) group encoding function $e_H: B^2 \rightarrow B^5$ defined by

e(00) = 00000; e(10) = 10101; e(01) = 01110; e(11) = 11011

Decode the following words relative to maximum likelihood decoding function.

i) 11110 ii) 10011

18. Consider (3, 5) group encoding function $e_H: B^3 \rightarrow B^5$ defined by

e(000) = 00000; e(001) = 00110; e(010) = 01001; e(011) = 01111

e(100) = 10011; e(101) = 10101; e(110) = 11010; e(111) = 11100

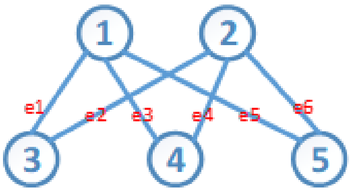
Decode the following words relative to maximum likelihood decoding function.

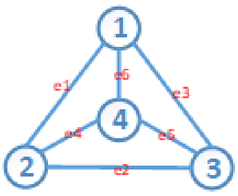
i) 11001 ii) 01010

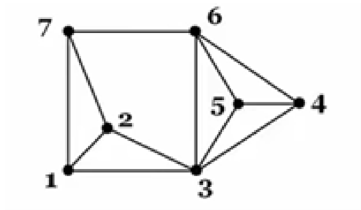
Module-4: Graph Theory Algebraic Structure

Q.1 Short Answer (Each for 2 Marks)

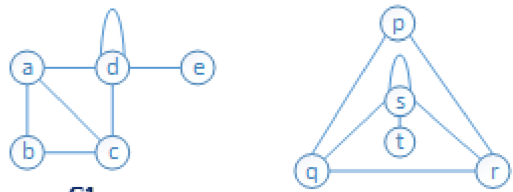
1. Draw graph K5.
2. Explain the terms with example i) Regular Graph ii) Complete Bipartite Graph
3. Define with example Euler path and Euler circuit.
4. Define with example Hamiltonian path and Hamiltonian circuit.
5. What is necessary and sufficient condition for Euler graph and Euler path.
6. What is necessary condition for Hamiltonian graph and Hamiltonian path.
7. Determine which of the following graph contains Euler path, Euler circuit, Hamiltonian path and Hamiltonian circuit. State the path/circuit.





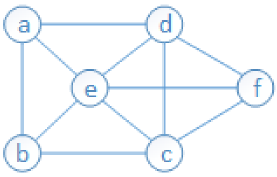


8. Define isomorphic graph.
9. Write any four necessary conditions for isomorphic graph.
10. Check, whether the following graphs are isomorphic.

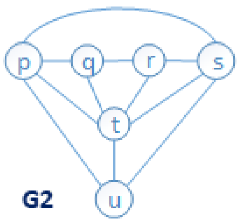


G1 G2

11. Check, whether the following graphs are isomorphic.



G1



G2

12. Define isomorphic graph. Draw K_6 and $K_{3,3}$ graphs. Find whether they are Isomorphic or not?

Module-5: Logic

1. Using truth table prove $a \rightarrow b$ and $\sim a \vee b$ are logically equivalent
2. Show that $\sim\big(p \vee (\sim p \wedge q)\big) = \sim p \wedge q$
3. Prove that $\big((a \rightarrow b) \wedge a\big) \rightarrow b$ is a tautology.
4. Prove that $\big((a \rightarrow b) \wedge (b \rightarrow c)\big) \rightarrow (a \rightarrow c)$ is a tautology.
5. What is converse of statement "If India is a country then Narendra Modi is prime minister"
6. Construct truth table to determine whether each of the following is tautology or contingency i) $(q \wedge p) \vee (q \wedge \sim p)$ ii) $q \rightarrow (q \rightarrow p)$ iii) $p \rightarrow (q \wedge p)$
7. Convert to DNF i) $p \wedge (q \vee p)$ ii) $(p \vee q) \wedge (p \vee r)$
8. Obtain the CNF of the form $(p \wedge q) \vee (\sim p \wedge q \wedge r)$
9. Prove by mathematical induction $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n}{6}(n+1)(2n+1)$
10. Prove by mathematical induction $1^2 + 3^2 + 5^2 + \ldots \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$
11. Show that $5^n - 4n - 1$ *is divisible by* 15 *for* $n \geq 1$

Module-6: Counting

1. State the extended pigeonhole principle.
2. If seven colors are used to paint 50 bicycles show that atleast 8 of them will be of the same color.
3. How many friends must you have to guarantee that atleast five of them have their birthday in the same month?
4. What is the minimum number of students required in a DSGT class to be sure that atleast six will receive the same grade, if there are five possible grades A, B, C, D and E.
5. Solve $a_{r+2} + 2a_{r+1} - 3a_r = 0$ that satisfies $a_0 = 1,\ a_1 = 2$
6. Solve the recurrence relation $a_n = 4a_{n-1} + 5a_{n-2}$ with the condition $a_1 = 2,\ a_2 = 6.$
7. Solve the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ subject to the conditions $a_1 = 1.5,$ $a_2 = 3.$
8. Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 6 + 8n$ with $a_0 = 1,\ a_1 = 2$
9. A box contains 6 white balls and 5 red balls. In how many ways can 4 balls be drawn from the box i) if they are to be of the same colour. ii) if they are to be of any colour.
10. In a group of 6 boys and 4 girls, 4 children are to be selected. In how many ways can they be selected such that atleast one boy should be there.
11. In how many ways can 8 different books be divided among three students Ram, Mohan and Sohan if Ram gets 4 books, Mohan and Sohan get 2 each?
12. State and prove mutual inclusion – exclusion for three sets
13. Find generating function for the finite sequence {2,2,2,2,2} and infinite sequence {2,2,2,2,2...}