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Tutorial 4 : Module 5 ⇒ Complex Variables

1] Show that given u or v is harmonic function and find corresponding analytic function $f(z) = u+iv$ and their harmonic conjugate functions.

i] $u = e^{-x}[y \cdot \sin(y) + x \cdot \cos(y)]$ .... ①

Sol.⇒ Differentiating eqⁿ ① w.r.t x

$\Rightarrow \partial u/\partial x = e^{-x}[\cos(y)] - e^{-x}[y \cdot \sin(y) + x \cdot \cos(y)]$
$\Rightarrow \partial u/\partial x = e^{-x}\{\cos(y) - [y \cdot \sin(y) + x \cdot \cos(y)]\}$
$\Rightarrow \partial u/\partial x = e^{-x}[\cos(y) - y \cdot \sin(y) - x \cdot \cos(y)]$ .... ②

Differentiating eqⁿ ② w.r.t x

$\Rightarrow \partial^2 u/\partial x^2 = e^{-x}[-\cos(y)] - e^{-x}[\cos(y) - y \cdot \sin(y) - x \cdot \cos(y)]$
$\Rightarrow \partial^2 u/\partial x^2 = e^{-x}[-\cos(y) - \cos(y) + y \cdot \sin(y) + x \cdot \cos(y)]$
$\Rightarrow \partial^2 u/\partial x^2 = e^{-x}[-2\cos(y) + y \cdot \sin(y) + x \cdot \cos(y)]$ .... Ⓐ

Differentiating eqⁿ ① w.r.t y

$\Rightarrow \partial u/\partial y = e^{-x}[y \cdot \cos(y) + \sin(y) - x \cdot \sin(y)]$ .... ③

Differentiating eqⁿ ③ w.r.t y

$\Rightarrow \partial^2 u/\partial y^2 = e^{-x}[-y \cdot \sin(y) + \cos(y) + \cos(y) - x \cdot \cos(y)]$
$\Rightarrow \partial^2 u/\partial y^2 = e^{-x}[-y \sin(y) + 2\cos(y) - x \cdot \cos(y)]$ .... Ⓑ

From eqⁿs (A) and (B),

$\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = e^{-x}[-2\cos(y) + y\cdot\sin(y) + x\cdot\cos(y)] + e^{-x}[-y\cdot\sin(y) + 2\cos(y) - x\cdot\cos(y)]$

$\therefore \boxed{\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = 0}$ $\therefore$ u is a harmonic function.

Now, $f(z) = u + iv$

$\Rightarrow f'(z) = \partial u/\partial x + i\cdot\partial v/\partial x$ ..... (4)

By CR eqⁿ (II), $\partial v/\partial x = -\partial u/\partial y$

eqⁿ (4) $\Rightarrow f'(z) = \partial u/\partial x - i\cdot\partial u/\partial y$ .... (5)

$\Rightarrow \partial u/\partial x = e^{-x}[\cos(y) - y\cdot\sin(y) - x\cdot\cos(y)]$ $\Rightarrow \partial u/\partial y = e^{-x}[y\cdot\cos(y) + \sin(y) - x\cdot\sin(y)]$

$\Rightarrow (\partial u/\partial x)_{\substack{x=z\\y=0}} = e^{-z}[1 - 0 - z]$ $\Rightarrow (\partial u/\partial y)_{\substack{x=z\\y=0}}\ e^{-z}[0 + 0 - 0] = 0$

eqⁿ (5) $\Rightarrow f'(z) = e^{-z}(1-z)$

Integrating w.r.t z

$\Rightarrow f(z) = \int (1-z)\cdot e^{-z}\cdot dz + c$ $\Rightarrow f(z) = \{(1-z)[-e^{-z}] - (-1)(e^{-z})\} + c$

$\Rightarrow f(z) = e^{-z}[z - 1 + 1] + c$ $\Rightarrow f(z) = z\cdot e^{-z} + c$

$\Rightarrow f(z) = (x+iy)\cdot e^{-(x+iy)} + c$ $\Rightarrow f(z) = (x+iy)\cdot e^{-x}\cdot e^{i(-y)} + c$

$\Rightarrow f(z) = e^{-x}\cdot(x+iy)[\cos(y) - i\cdot\sin(y)] + c$

$\Rightarrow f(z) = e^{-x}\{[x\cdot\cos(y) - ix\cdot\sin(y)] + i[y\cdot\cos(y) - iy\cdot\sin(y)]\} + c$

$\Rightarrow f(z) = e^{-x}[x\cdot\cos(y) + y\cdot\sin(y)] + i\cdot e^{-x}[y\cdot\cos(y) - x\cdot\sin(y)] + ic_1$

$\Rightarrow f(z) = e^{-x}[x\cdot\cos(y) + y\cdot\sin(y)] + i\{e^{-x}[y\cdot\cos(y) - x\cdot\sin(y)] + c_1\}$

$\therefore \boxed{f(z) = e^{-x}[x\cdot\cos(y) + y\cdot\sin(y)] + i\{e^{-x}[y\cdot\cos(y) - x\cdot\sin(y)] + c_1\}}$

This is the analytic function. $f(z) = u + iv$.

The harmonic conjugate of $u = e^{-x}[y\cdot\sin(y) + x\cdot\cos(y)]$ is

$\boxed{v = e^{-x}[y\cdot\cos(y) - x\cdot\sin(y)] + c_1}$

2] Find an analytic function $f(z) = u+iv$ such that

$$u+v = \frac{2\sin(2x)}{e^{2y}+e^{-2y}-2\cos(2x)}.$$

Sol^n $\Rightarrow$ $f(z) = u+iv \Rightarrow$ Analytic function .... ①

Multiply by i to both sides

$\Rightarrow$ $i.f(z) = iu - v$ .... ②

Adding eq^n's ① and ② $\Rightarrow$ $(1+i).f(z) = (u-v)+i(u+v)$ $\therefore F(z) = U+iV$

Now, $V = u+v = \dfrac{2.\sin(2x)}{e^{2y}+e^{-2y}-2\cos(2x)} = \dfrac{2\sin(2x)}{2\cosh(2y)-2\cos(2x)}$ $\therefore V = \dfrac{\sin(2x)}{\cosh(2y)-\cos(2x)}$

$\Rightarrow$ $F'(z) = \partial U/\partial x + i.\partial V/\partial x$

By CR eq^n (I) $\Rightarrow$ $\partial U/\partial x = \partial V/\partial y$

$\therefore$ $F'(z) = \partial V/\partial y + i.\partial V/\partial x$ .... ③

$$\Rightarrow \partial V/\partial x = \frac{[\cosh(2y)-\cos(2x)]2.\cos(2x) - \sin(2x)[0+2.\sin(2x)]}{[\cosh(2y)-\cos(2x)]^2}$$

By Milne-Thomson theorem,

$$\left(\partial V/\partial x\right)_{\substack{x=z\\y=0}} = \frac{[1-\cos(2z)].2\cos(2z) - 2\sin^2(2z)}{[1-\cos(2z)]^2} = \frac{2\cos(2z)-2\cos^2(2z)-2\sin^2(2z)}{[1-\cos(2z)]^2}$$

$$\Rightarrow \left(\partial V/\partial x\right)_{\substack{x=z\\y=0}} = \frac{2\{\cos(2z)-[\cos^2(2z)+\sin^2(2z)]\}}{[1-\cos(2z)]^2} = \frac{2[\cos(2z)-1]}{[1-\cos(2z)]^2}$$

$$\Rightarrow \left(\partial V/\partial x\right)_{\substack{x=z\\y=0}} = \frac{-2[1-\cos(2z)]}{[1-\cos(2z)]^2} = \frac{-2}{[1-\cos(2z)]} = \frac{-2}{2\sin^2(z)} = \frac{-1}{\sin^2(z)}$$

$$\therefore \boxed{\left(\partial V/\partial x\right)_{\substack{x=z\\y=0}} = -\operatorname{cosec}^2(z)} \quad \ldots Ⓐ$$

$$\Rightarrow \partial V/\partial y = \frac{[\cosh(2y)-\cos(2x)](0) - \sin(2x)[2.\sinh(2y)]}{[\cosh(2y)-\cos(2x)]^2} = \frac{-2.\sin(2x).\sinh(2y)}{[\cosh(2y)-\cos(2x)]^2}$$

By Milne-Thomson theorem,

$$\left(\partial V/\partial y\right)_{\substack{x=z\\y=0}} = \frac{-2\sin(2z)\,(0)}{[1-\cos(2z)]^2} \quad \therefore \boxed{\left(\partial V/\partial y\right)_{\substack{x=z\\y=0}} = 0} \quad \ldots \text{(B)}$$

Putting eq^{n}s (A) and (B) in eq^n (3)

$$\Rightarrow F'(z) = 0 - i.\operatorname{cosec}^2(z)$$

Integrating w.r.t z

$$\Rightarrow \int F'(z).dz = F(z) = -i\int \operatorname{cosec}^2(z).dz + c_1$$

$$\Rightarrow F(z) = i.\cot(z) + c_1$$

$$\Rightarrow (1+i).f(z) = i.\cot(z) + c_1$$

$$\Rightarrow f(z) = \frac{i.\cot(z)}{(1+i)} + \frac{c_1}{(1+i)} \Rightarrow f(z) = \frac{i(1-i).\cot(z)}{2} + c$$

$$\Rightarrow f(z) = \frac{(1+i)}{2}.\cot(z) + c$$

$$\therefore \boxed{f(z) = \frac{(1+i)}{2}.\cot(x+iy) + c}$$

This is the required analytic function $f(z) = u+iv$ such that

$$u+v = \frac{2\sin(2x)}{e^{2y}+e^{-2y}-2\cos(2x)}.$$

3] Find an analytic function $f(z) = u + iv$ such that

$$u + v = \frac{x - y}{x^2 + y^2} + e^x[\cos(y) + \sin(y)]$$

Sol^n. $\Rightarrow$ $f(z) = u + iv \Rightarrow$ Analytic function.... ①

Multiply by i to both sides

$\Rightarrow$ $i.f(z) = iu - v$ ... ②

Adding eq^{n}s ① and ② $\Rightarrow$ $(1+i).f(z) = (u-v) + i(u+v)$

$\therefore$ $F(z) = U + iV$

Now, $V = u + v = \dfrac{x-y}{x^2+y^2} + e^x[\cos(y) + \sin(y)]$ is given.

$\Rightarrow$ $F'(z) = \partial U/\partial x + i.\partial V/\partial x$ ... ③

By CR eq^n (I) $\Rightarrow$ $\partial U/\partial x = \partial V/\partial y$

eq^n ③ $\Rightarrow$ $F'(z) = \partial V/\partial y + i.\partial V/\partial x$ .... ④

$$\Rightarrow \partial V/\partial x = \frac{(x^2+y^2)(1) - (x-y)(2x)}{(x^2+y^2)^2} + e^x[\cos(y) + \sin(y)]$$

$$\Rightarrow \partial V/\partial x = \frac{x^2+y^2 - 2x^2 + 2xy}{(x^2+y^2)^2} + e^x[\cos(y) + \sin(y)]$$

$$\Rightarrow \partial V/\partial x = \frac{y^2 - x^2 + 2xy}{(x^2+y^2)^2} + e^x[\cos(y) + \sin(y)]$$

By Milne-Thomson theorem,

$$(\partial V/\partial x)_{\substack{x=z\\y=0}} = \frac{0 - z^2 + 0}{z^4} + e^z[1+0] \quad \therefore \boxed{(\partial V/\partial x)_{\substack{x=z\\y=0}} = e^z - \frac{1}{z^2}} \; ..$$

$$\Rightarrow \partial V/\partial y = \frac{(x^2+y^2)(-1)-(x-y)(2y)}{(x^2+y^2)^2} + e^x[-\sin(y)+\cos(y)]$$

$$\Rightarrow \partial V/\partial y = \frac{-x^2-y^2-2xy+2y^2}{(x^2+y^2)^2} + e^x[\cos(y)-\sin(y)]$$

$$\Rightarrow \partial V/\partial y = \frac{y^2-x^2-2xy}{(x^2+y^2)^2} + e^x[\cos(y)-\sin(y)]$$

By Milne - Thomson theorem,

$$(\partial V/\partial y)_{\substack{x=z\\y=0}} = \frac{0-z^2-0}{z^4} + e^z[1-0] \qquad \therefore \boxed{(\partial V/\partial y)_{\substack{x=z\\y=0}} = e^z - \frac{1}{z^2}} \ \dots (B)$$

Putting eqn's (A) and (B) in eqn (4)

$$\Rightarrow F'(z) = (e^z - 1/z^2) + i(e^z - 1/z^2)$$

Integrating w.r.t z

$$\Rightarrow \int F'(z).dz = F(z) = \int(e^z - 1/z^2).dz + i\int(e^z-1/z^2).dz + C_1$$

$$\Rightarrow F(z) = [e^z + 1/z] + i[e^z + 1/z] + C_1$$

$$\Rightarrow (1+i).f(z) = (1+i)(e^z+1/z) + C_1$$

$$\Rightarrow f(z) = e^z + 1/z + \frac{C_1}{(1+i)} \Rightarrow f(z) = (e^z + 1/z) + iC_2$$

$$\Rightarrow f(z) = e^{(x+iy)} + \frac{1}{(x+iy)} + c \Rightarrow f(z) = e^x . e^{i(y)} + \frac{(x-iy)}{(x^2+y^2)} + iC_2$$

$$\Rightarrow f(z) = e^x[\cos(y) + i\cdot\sin(y)] + \frac{(x-iy)}{(x^2+y^2)} + iC_2$$

$$\Rightarrow f(z) = e^x.\cos(y) + e^x.i.\sin(y) + \frac{x}{(x^2+y^2)} - \frac{iy}{(x^2+y^2)} + iC_2$$

$$\Rightarrow \boxed{f(z) = \left[e^x.\cos(y) + \frac{x}{(x^2+y^2)}\right] + i\left[e^x.\sin(y) - \frac{y}{(x^2+y^2)} + C_2\right]}$$

This is the required analytic function $f(z) = u+iv$ such that $u+v = \frac{x-y}{(x^2+y^2)} + e^x[\cos(y)+\sin(y)]$.

4) Find the orthogonal trajectories of the family of curves
$u = 2x - x^3 + 3xy^2 = c$.

Sol^n. ⇒ $f(z) = u + iv$

$f'(z) = \partial u/\partial x + i.\partial v/\partial x$

By C.R. eq^n (II), $\partial v/\partial x = -\partial u/\partial y$ ....

$\therefore f'(z) = (\partial u/\partial x) - i(\partial u/\partial y)$ .... (1)

$\partial u/\partial x = 2 - 3x^2 + 3y^2$ $\qquad$ $\partial u/\partial y = 6xy$

$\Rightarrow \left(\partial u/\partial x\right)_{\substack{x=z\\y=0}} = 2 - 3z^2$ $\qquad$ $\left(\partial u/\partial y\right)_{\substack{x=z\\y=0}} = 0$

eq^n (1) ⇒ $f'(z) = 2 - 3z^2$

Integrating w.r.t z

$\Rightarrow \int f'(z).dz = f(z) = \int (2-3z^2).dz + c_1$

$\Rightarrow f(z) = 2z - \frac{3z^3}{3} + c_1$

$\Rightarrow f(z) = 2z - z^3 + c_1$

$\Rightarrow f(z) = 2(x+iy) - (x+iy)^3 + c_1$

$\Rightarrow f(z) = 2x + 2iy - x^3 - 3x^2iy + 3xy^2 + iy^3 + c_1$

$\Rightarrow f(z) = [2x - x^3 + 3xy^2 + c_1] + i[2y - 3x^2y + y^3]$

$\Rightarrow f(z) = [(2x - x^3 + 3xy^2) - c] + i[2y - 3x^2y + y^3]$

$\therefore$ $\boxed{\text{Orthogonal trajectory of } u \text{ is } v = 2y - 3x^2y + y^3}$.

5] Find the orthogonal trajectories of the family of curves $u = x^3y - xy^3 = c$.

Sol.ⁿ ⇒ $f(z) = u + iv$

$f'(z) = \partial u/\partial x + i\,\partial v/\partial x$

By C.R. eqⁿ (II), $\partial v/\partial x = -\partial u/\partial y$

$\therefore f'(z) = (\partial u/\partial x) - i(\partial u/\partial y) \ldots$ (1)

$\partial u/\partial x = 3x^2y - y^3$ $\qquad$ $\partial u/\partial y = x^3 - 3xy^2$

$\Rightarrow (\partial u/\partial x)_{\substack{x=z\\y=0}} = 0$ $\qquad$ $(\partial u/\partial y)_{\substack{x=z\\y=0}} = z^3$

eqⁿ (1) $\Rightarrow f'(z) = 0 - iz^3$

Integrating w.r.t z

$\Rightarrow \int f'(z).dz = f(z) = i\int -z^3.dz + c_1$

$\Rightarrow f(z) = \dfrac{-iz^4}{4} + c_1$

$\Rightarrow f(z) = -\dfrac{i}{4}(x+iy)^4 + c_1$

$\Rightarrow f(z) = -\dfrac{i}{4}\left[x^4 + 4x^3(iy) + 6x^2(iy)^2 + 4x(iy)^3 + (iy)^4\right] + c_1$

$\Rightarrow f(z) = -i/4\,[x^4 + 4ix^3y - 6x^2y^2 - 4ixy^3 + y^4] + c_1$

$\Rightarrow f(z) = -i/4\,[x^4 - 6x^2y^2 + y^4] + [x^3y - xy^3] + c_1$

$\Rightarrow f(z) = [x^3y - xy^3 + c_1] + i\left[\dfrac{-x^4 + 6x^2y^2 - y^4}{4}\right]$

$\Rightarrow f(z) = [(x^3y - xy^3) - C] + i\left[\dfrac{-x^4 + 6x^2y^2 - y^4}{4}\right]$

$\therefore$ $\boxed{\text{Orthogonal trajectory of } u \text{ is } v = \dfrac{-x^4 + 6x^2y^2 - y^4}{4}}$

6/ Find the Constants a, b, c, d, e if $f(z) = (ax^4 + bx^2y^2 + dx^2 + cy^4 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is an analytic function.

Solⁿ ⇒ $f(z) = u + iv$ is an analytic function.

∴ CR equations exist.

$u = ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2$ $\qquad$ $v = 4x^3y - exy^3 + 4xy$

⇒ $\partial u/\partial x = 4ax^3 + 2bxy^2 + 2dx$ $\qquad$ ⇒ $\partial v/\partial y = 4x^3 - 3exy^2 + 4x$

By CR. eqⁿ ①, $\partial u/\partial x = \partial v/\partial y$

⇒ $4ax^3 + 2bxy^2 + 2dx = 4x^3 - 3exy^2 + 4x$

Comparing both sides

⇒ $4ax^3 = 4x^3$ $\qquad$ $2bxy^2 = -3exy^2$ $\qquad$ $2dx = 4x \Rightarrow 2d = 4$

∴ $\boxed{a = 1}$ $\qquad$ $2b = -3e$ $\qquad$ ∴ $\boxed{d = 2}$

$\partial u/\partial y = 2bx^2y + 4cy^3 - 4y$ $\qquad$ $\partial v/\partial x = 12x^2y - ey^3 + 4y$

By CR eqⁿ ②, $\partial u/\partial y = -\partial v/\partial x$

⇒ $2bx^2y + 4cy^3 - 4y = -12x^2y + ey^3 - 4y$

Comparing both sides

$2bx^2y = -12x^2y$ $\qquad$ $4cy^3 = +ey^3$ $\qquad$ $2b = -3e$

$2b = -12$ $\qquad$ $4c = e$ $\qquad$ $2(-6) = -3e$

$\boxed{b = -6}$ $\qquad$ $4c = 4$ ∴ $\boxed{c = 1}$ $\qquad$ $12 = 3e$ ∴ $\boxed{e = 4}$

Ans ⇒ The values of the Constants are $a = 1, b = -6, c = 1, d = 2, e = 4$.

7) Find a, b, c, d if $f(z) = (x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$ is an analytic function.

Sol^n ⇒ $f(z) = u + iv$ is an analytic function.
∴ CR equations exist.

$u = x^2 + 2axy + by^2$ $\quad\quad$ $v = cx^2 + 2dxy + y^2$

⇒ $\partial u/\partial x = 2x + 2ay + 0$ $\quad\quad$ ⇒ $\partial v/\partial y = 0 + 2dx + 2y$

⇒ $\partial u/\partial y = 0 + 2ax + 2by$ $\quad\quad$ ⇒ $\partial v/\partial x = 2cx + 2dy + 0$

By CR eq^n (I), $\partial u/\partial x = \partial v/\partial y$
⇒ $2x + 2ay = 2dx + 2y$

By CR eq^n (II), $\partial u/\partial y = -\partial v/\partial x$
⇒ $2ax + 2by = -2cx - 2dy$

Comparing both sides

⇒ $2x = 2dx$
$\boxed{d = 1}$

$2ay = 2y$
$\boxed{a = 1}$

Comparing both sides

$2ax = -2cx$
$a = -c$
$c = -a$
$\boxed{c = -1}$

$2by = -2dy$
$b = -d$
$\boxed{b = -1}$

Ans ⇒ The values of the constant are $a = 1, b = -1, c = -1, d = 1$.

8] Show that $f(z) = e^{2z} - z$ is an analytic function.

Sol^n. ⇒ $f(z) = e^{2z} - z \quad = e^{2(x+iy)} - (x+iy)$

$f(z) = e^{2x} \cdot e^{i(2y)} - x - iy$

$f(z) = e^{2x}[\cos(2y) + i\sin(2y)] - x - iy$

$f(z) = \{e^{2x}[\cos(2y)] - x\} + i[e^{2x} \cdot \sin(2y) - y]$

$f(z) = U + iV$

$U = e^{2x}[\cos(2y)] - x \qquad V = e^{2x} \cdot \sin(2y) - y$

⇒ $\partial U/\partial x = 2 \cdot e^{2x} \cdot \cos(2y) - 1 \quad \ldots ①$ ⇒ $\partial V/\partial y = 2 \cdot e^{2x} \cdot \cos(2y) - 1 \quad \ldots ②$

From ① and ②, $\partial U/\partial x = \partial V/\partial y$ ⇒ CR eq^n (I) ... (A)

⇒ $\partial U/\partial y = -e^{2x} \cdot 2 \cdot \sin(2y) \quad \ldots ③$ ⇒ $\partial V/\partial x = e^{2x} \cdot \sin(2y) \quad \ldots ④$

From ③ and ④, $\partial U/\partial y = -\partial V/\partial x$ ⇒ CR eq^n (II) ... (B)

From (A) and (B), we can say that both CR equations exist.

∴ $\boxed{f(z) = e^{2z} - z \text{ is an analytic function.}}$

Hence proved.

9) Find the constant K if $f(z) = r^2 \cdot \cos(2\theta) + i r^2 \sin(K\theta)$ is an analytic function.

Solⁿ ⇒ $f(z) = u + iv$ is an analytic function.

$f(z) = r^2 \cdot \cos(2\theta) + i \cdot r^2 \cdot \sin(K\theta)$

$\therefore \quad u = r^2 \cdot \cos(2\theta) \qquad v = r^2 \cdot \sin(K\theta)$

By C.R. eqⁿ (I), $\left(\partial u/\partial r\right) = 1/r \cdot \left(\partial v/\partial \theta\right)$ .... (1)

$\Rightarrow \left(\partial u/\partial r\right) = 2r \cdot \cos(2\theta) \qquad \Rightarrow \left(\partial v/\partial \theta\right) = r^2 \cdot \cos(K\theta) \cdot K$

eqⁿ (1) $\Rightarrow 2r \cdot \cos(2\theta) = \frac{1}{r} \cdot r^2 \cdot \cos(K\theta) \cdot K$

$\Rightarrow 2 \cdot \cos(2\theta) = K \cdot \cos(K\theta)$

Comparing both sides

$\boxed{K = 2}$

Ans ⇒ The value of the constant is $K = 2$.

10] Prove that there does not exist an analytic function whose real part is $u = x^2 + 3x - 4y + y^2 + 7$.

Sol.ⁿ ⇒ For any function $f(z) = u + iv$ to be an analytic function, real part (u) and imaginary part (v) both must be Harmonic functions. ... (A)

Here, $u = x^2 + 3x - 4y + y^2 + 7$

$\therefore \partial u/\partial x = 2x + 3$

$\partial^2 u/\partial x^2 = 2 \quad \ldots \text{(1)}$

$\therefore \partial u/\partial y = -4 + 2y$

$\partial^2 u/\partial y^2 = 2 \quad \ldots \text{(2)}$

From eqⁿ (1) and (2),

$\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = 2 + 2 = 4$

$\therefore \boxed{\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 \neq 0} \quad \ldots \text{(B)}$

eqⁿ (B) ⇒ The function u is not Harmonic .... (C)

From eqⁿ (A) and (C), we can say that there does not exist an analytic function whose real part is $u = x^2 + 3x - 4y + y^2 + 7$ as the real part is not a Harmonic function.