Discrete Mathematics

Statistical Analysis

Chapter No.	Dec. 2017	May 2018
Chapter 1	10 Marks	17 Marks
Chapter 2	05 Marks	09 Marks
Chapter 3	33 Marks	25 Marks
Chapter 4	19 Marks	12 Marks
Chapter 5	16 Marks	21 Marks
Chapter 6	09 Marks	12 Marks
Chapter 7	22 Marks	24 Marks
Repeated Questions	–	–

Dec. 2017

Chapter 1 : Set Theory [Total Marks – 10]

Q. 4(a) (i) Among 50 students in a class, 26 got an A in the first examination and 21 got an A in the second examination. If 17 students did not get an A in either examination, how many students got an A in both examinations ?

(ii) If the number of students who got an A in the first examination is equal to that in the second examination, if the total number of students who got an A in exactly one examination is 40 and if 4 students did not get an A in either examination then determine the number of students who got an A in the first examination only, who got an A in the second examination only and who got an A in both the examination. (6 Marks)

Ans. :

(i) Let 'T' be the number of students.

Let 'F' be the students who got A in first examination

Let 'S' be the students who got A in second examination

$$|T| = 50$$
$$|F| = 26$$
$$|S| = 21$$

Number of students who did not get an A in either examination=17

Number of students got an A in at least one examination is 50-17 = 33

Number of students got an A in both examinations is $|F \cap S|$

$$33 = |F| + |S| - |F \cap S|$$
$$|F \cap S| = (26 + 21) - 33$$
$$|F \cap S| = 47 - 33 = 14$$

(ii) Number of students who got an A in the first examination is equal to that in the second examination, so $|F| = |S|$

Total number of students who got an A in exactly one examination is 40

$$|F| + |S| - 2|F \cap S| = 40 \quad ...(1)$$

4 students did not get an A in either examination

So the number of students who got A in at least one examination is $50 - 4 = 46$

$$|F| + |S| - |F \cap S| = 46 \quad ...(2)$$

Using Equation (1)

$$|F| + |S| - 2|F \cap S| = 40$$
$$|F| + |S| - |F \cap S| - |F \cap S| = 40$$
$$46 - |F \cap S| = 40 \quad \text{...using Equation (2)}$$
$$|F \cap S| = 6$$

6 Students got an A in both examinations.

Using Equation (1)

$$|F| + |S| - 2|F \cap S| = 40$$
$$|F| + |S| - (2 * 6) = 40 \quad ...(3)$$
$$|F| + |S| = 52$$
$$\text{So, } |F| = |S| = 26$$

$|F| - |F \cap S| = 26 - 6 = 20$ students got an 'A' in first examination only.

$|S| - |F \cap S| = 26 - 6 = 20$ students got an 'A' in second examination only.

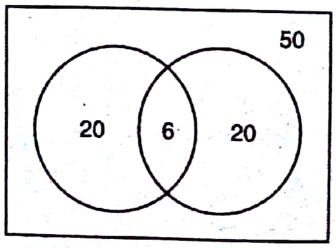


Fig. 1-Q. 4(a)

Q. 6(d) Prove the following (use laws of set theory) $A \times (X \cap Y) = (A \times X) \cap (A \times Y)$ (4 Marks)

Ans. :

Let $(a, x) \in A \times (X \cap Y)$ ...(1)

By the definition of the Cartesian product, this means that

$a \in A$ and $x \in X \cap Y$

Since $x \in X \cap Y$

$\therefore$ $x \in X$ and $x \in Y$

$\therefore$ $(a, x) \in A \times X$ and $(a, x) \in A \times Y$

$\therefore$ $(a, x) \in (A \times X) \cap (A \times Y)$

$\therefore A \times (X \cap Y) \subseteq (A \times X) \cap (A \times Y)$ ...(2)

Again, Let $(a, x) \in (A \times X) \cap (A \times Y)$

$\therefore$ $(a, x) \in (A \times X)$ and $(a, x) \in A \times Y$

$\therefore$ $a \in A, x \in X$ and $x \in Y$

$\therefore$ $a \in A$ and $x \in X \cap Y$

$(a, x) \in A \times (X \cap Y)$

$\therefore (A \times X) \cap (A \times Y) \subseteq A \times (X \cap Y)$ ...(3)

From Equations (1) and (2) we have,

$A \times (X \cap Y) = (A \times X) \cap (A \times Y)$

Chapter 2 : Logic [Total Marks – 05]

Q. 1(a) Prove that $1.1! + 2.2! + 3.3! + \ldots\ldots + n \cdot n! = (n + 1)! - 1$, where n is a positive integer. (5 Marks)

Ans. :

Let $P(n) = 1.1! + 2.2! + 3.3! + \ldots n.n! = (n + 1)! - 1$

(i) **Basis of induction :**

For $n = 1$, $1.1! = 1$ and $(1 + 1)! - 1 = 2! - 1 = 1$

$\therefore$ P(1) is true.

(ii) **Induction step :**

Assume P(k) is true, i.e,

$1.1! + 2.2! + 3.3! + \ldots k.k! = (k + 1)! - 1$ ...(i)

To prove that P(k + 1) is true,

$P(k + 1) : 1.1! + 2.2! + 3.3! + \ldots (k + 1) \cdot (k + 1)! = (k + 2)! - 1$

L.H.S $= 1.1! + 2.2! + 3.3! + \ldots k\,k! + (k + 1) \cdot (k + 1)!$

$= (k + 1)! - 1 + (k + 1) \cdot (k + 1)!$

(by induction hypothesis).

$= (k + 1)! + (k + 1) \cdot (k + 1)! - 1$

$= (k + 1)!\,(1 + k + 1) - 1$

$= (k + 1)!\,(k + 2) - 1 = (k + 2)! - 1$

L.H.S. = R.H.S.

Hence the result is proved.

Chapter 3 : Relations, Digraphs and Lattice [Total Marks – 33]

Q. 1(b) Let A = {a, b, c}. Show that (P(A), ⊆) is a poset and draw its Hasse diagram. (5 Marks)

Ans. : Set containment ⊆ is always a partial order since for any subset B of A, B ⊆ B i.e. ⊆ is reflexive.

If B ⊆ C and C ⊆ B, B = C. So ⊆ is anti-symmetry.

If B ⊆ C, C ⊆ D then B ⊆ D. So ⊆ is transitive.

Partial ordered relation of set containment on set P(A) is as follows

R = {{ϕ}, {ϕ}}, {{ϕ}, {a}}, {{ϕ}, {b}}, {{ϕ}, {c}}, {{ϕ}, {a, b}}, {{ϕ}, {a, c}}, {{ϕ}, {b, c}}, {{ϕ}, {a, b, c}}, {{a}, {a, b}}, {{a}, {a, c}}, {{b}, {a, b}}, {{b}, {b, c}}, {{c}, {a, c}}, {{c}, {b, c}}, {{a, b}, {a, b, c}}, {{a, c}, {a, b, c}}, {{b, c}, {a, b, c}}, {{a, b, c,}, {a, b, c}}

Matrix of the above relation is as follows :

$$M_R = \begin{array}{c} \\ \phi \\ \{a\} \\ \{b\} \\ \{c\} \\ \{a,b\} \\ \{a,c\} \\ \{b,c\} \end{array} \begin{array}{c} \begin{array}{cccccccc} \phi & \{a\} & \{b\} & \{c\} & \{a,b\} & \{a,c\} & \{b,c\} & \{a,b,c\} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Digraph of the abve matrix is

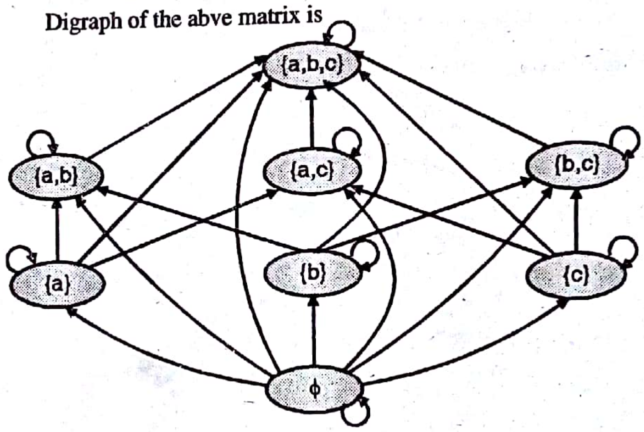


Fig. 1 - Q. 1(b)

To convert this digraph into Hasse Digraph

Step 1 : Remove Cycles.

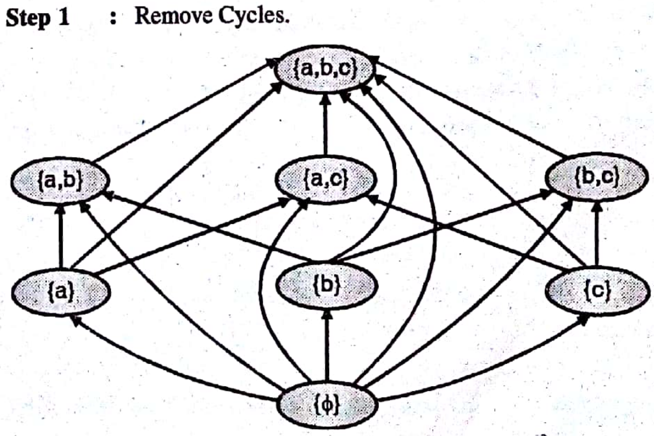


Fig. 2- Q. 1(b)

Step 2 : Remove transitive edges.

$\{\{\phi\}, \{a, b\}\}, \{\{\phi\},$
$\{a,c\}\}\ \{\{\phi\}, \{b, c\}\},$
$\{\{\phi\}, \{a, b, c\}\},$
$\{\{a\}, \{a, b, c\}\},$
$\{\{b\}, \{a, b, c\}\},$
$\{\{c\}, \{a, b, c\}\}$

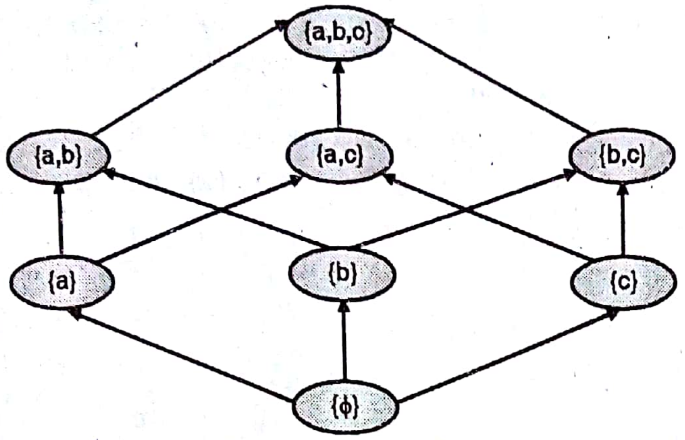


Fig. 3 - Q. 1(b)

Step 3 : All edges are pointing upwords. Now replace circles by dots and remove arrows from edges.

Hasse Diagram :

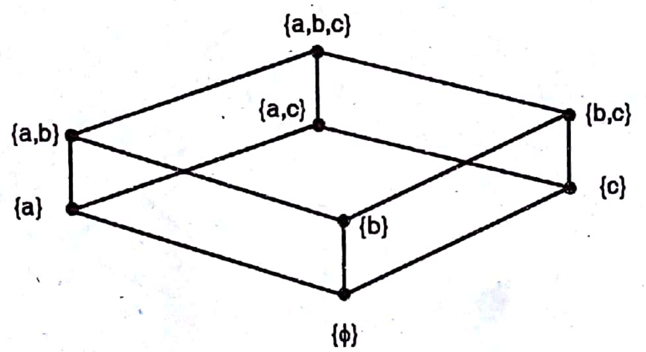


Fig. 4 - Q. 1(b)

Q. 1(c)(i) Explain the terms : Lattice (1 Mark)

Ans. : A lattice is a poset $(L, \leq)$ in which every subset $\{a, b\}$ consisting of two elements has a least upper bound and a greatest lower bound. We denote LUB $(\{a, b\})$ by $a \vee b$, and call it the join of a and b. Similarly, we denote GLB $(\{a, b\})$ by $a \wedge b$ and call it the meet of a and b.

Q. 1(c)(ii) Explain the terms : Poset (1 Mark)

Ans. : Partially ordered relation : A relation R on a set A is called **partial order** if R is reflexive, anti-symmetric and transitive poset.

The set A together with the partial order R is called a **partially ordered set** or simply a poset. It is denoted by (A, R).

Q. 2(b) Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ and let R be a relation on A whose matrix is

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find M_R * by Warshall's algorithm. (6 Marks)

Ans. :

Let $M_R = W_0,\ n = 5$

$$W_0 = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

First we compute W_1 so that $k = 1$ $\therefore$ check for column 1 and row 1 of W_0

$p_1 : (a_1, a_1), p_2 : (a_4, a_1)$

$q_1 : (a_1, a_1), q_2 : (a_1, a_4)$

To obtain W_1 we must put 1's in positions (a_1, a_1), (a_1, a_4), (a_4, a_1) and (a_4, a_4). Thus

$$W_1 = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

To obtain W_2 so that $k = 2$ $\therefore$ check column 2 and row 2 of W_1

$p_1 : (a_2, a_2), p_2 : (a_5, a_2)$

$q_1 : (a_2, a_2)$

To obtain W_2 we must put 1's in positions (a_2, a_2), (a_5, a_2). Thus

$$W_2 = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Now compute W_3 so that $k = 3$ $\therefore$ check column 3 and row 3 of W_2

$p_1 :$

$q_1 : (a_3, a_4), (a_3, a_5)$

Hence no new ordered pair. Thus $W_2 = W_3$

$$W_3 = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \end{array} \\ \left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

Now compute W_4 so that $k = 4$ $\therefore$ check column 4 and row 4 of W_3

$p_1 : (a_1, a_4), p_2 : (a_3, a_4), p_3 : (a_4, a_4)$

$q_1 : (a_4, a_1), q_2 : (a_4, a_4)$

To obtain W_4, we must put 1's in positions (a_1, a_1), (a_1, a_4), (a_3, a_1), (a_3, a_4), (a_4, a_1), (a_4, a_4).

Thus

$$W_4 = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \end{array} \\ \left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

Now we compute W_5 so that $k = 5$ $\therefore$ check column 5 and row 5 of W_4

$p_1 : (a_3, a_5), p_2 : (a_5, a_5)$,

$q_1 : (a_5, a_2), q_2 : (a_5, a_5)$

To obtain W_5, we must put all 1's in positions (a_3, a_2), (a_3, a_5), (a_5, a_2), and (a_5, a_2). Thus

$$W_4 = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \end{array} \\ \left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$\therefore M_{P^\infty} = W_5 =$ Transitive closure

Q. 3(b) Define equivalence relation with example. Let 'T' be a set of triangles in a plane and define R as the set R = {(a, b) | a, b ∈ T and a is congruent to b} then show that R is an equivalence relation. (6 Marks)

Ans. :

A relation R on a set A is called an equivalence relation if it is reflexive, symmetric, and transitive.

The following are some of the common but important examples of equivalence relations.

For Example :

(i) Let A = IR and R be 'equality' of numbers.

(ii) Consider all subsets of a universal set and R be the relation, 'equality' of sets.

(iii) A is the set of triangles and R is 'similarity' of triangles.

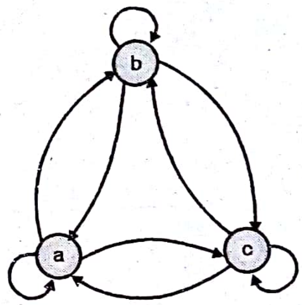


Fig. 1 - Q. 3(b)

(iv) A is a set of students and R is the relation of being in the same class or division.

(v) Let A be set of statement forms and R be the relation of 'Logical equivalence.'

(vi) A is set of lines in a plane and R is the relation of lines being 'parallel'.

The digraph of an equivalence relation will have the following characteristics.

(i) Every vertex will have a loop.

(ii) If there is an arc from a to b, there should be an arc from b to a.

(iii) If there is an arc from a to b and arc from b to c, there should be an arc from a to c

In short, the following is a typical digraph of an equivalence relation.

R is reflexive since, a triangle is congruent to itself.

R is symmetric since, if a is congruent to b, then b is congruent to a.

R is transitive since, if a is congruent to b, b is congruent c, then a is congruent to c.

The relation satisfies all the three properties of an equivalence relation.

Q. 5(b) Determine whether the Poset with the following Hasse diagrams are lattices or not. Justify your answer. (6 Marks)

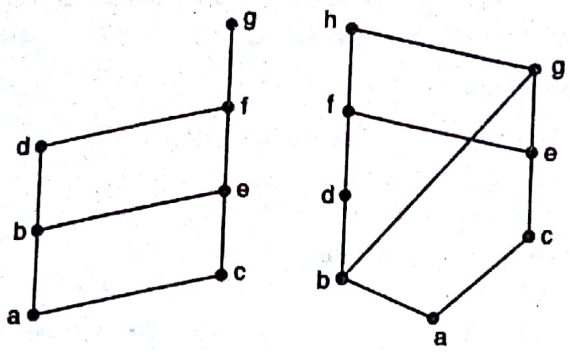


Fig. 1-Q. 5(b)

Ans. :

(i)

LUB :

∨	a	b	c	d	e	f	g
a	a	b	c	d	e	f	g
b	b	b	e	d	e	f	g
c	c	e	c	f	e	f	g
d	d	d	f	d	f	f	g
e	e	e	e	f	e	f	g
f	f	f	f	f	f	f	g
g	g	g	g	g	g	g	g

GLB :

∧	a	b	c	d	e	f	g
a	a	a	a	a	a	a	a
b	a	b	a	b	b	b	b
c	a	a	c	a	c	c	c
d	a	b	a	d	b	d	d
e	a	b	c	b	e	e	e
f	a	b	c	d	e	f	f
g	a	b	c	d	e	f	g

Fig. 1(a) - Q. 5(b) is a lattice since every pair of elements has a GLB and a LUB.

(ii)

LUB :

∨	a	b	c	d	e	f	g	h
a	a	b	c	d	e	f	g	h
b	b	b	f	d	f	f	g	h
c	c	f	c	f	e	f	g	h
d	d	d	f	d	f	f	h	h
e	e	f	e	f	e	f	g	h
f	f	f	f	f	f	f	h	h
g	g	g	g	h	g	h	g	h
h	h	h	h	h	h	h	h	h

GLB :

∧	a	b	c	d	e	f	g	h
a	a	a	a	a	a	a	a	a
b	a	b	a	b	a	b	b	b
c	a	a	c	a	c	a	c	a
d	a	b	a	d	a	d	b	d
e	a	a	c	a	e	e	e	e
f	a	b	a	d	e	f	-	f
g	a	b	c	b	e	-	g	g
h	a	b	a	d	e	f	g	h

Fig. 1(b) - Q. 5(b) is not a lattice because GLB (f, g) does not exist.

Q. 5(c) From the following diagraphs, write the relation a set of ordered pairs. Are the relations equivalence relations ? (4 Marks)

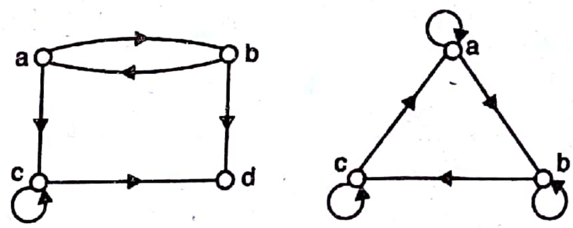


Fig. 1-Q. 5(c)

Ans. :

An equivalence relation is reflexive, symmetric and transitive.

R1 = {(a, b), (a, c), (b, a), (b, d), (c, c), (c, d)}

Relation R1 is not reflexive because (a,a), (b,b), (d,d) do not belong to the relation set.

Hence R1 is not an equivalence relation.

R2 = (a, a), (a, b), (b, b), (b, c), (c, c), (c, a)}

Relation R2 is reflexive and transitive but not symmetric because (b, a), (c, b), (a, c) do not belong to the relation set. Hence R2 is not an equivalence relation.

Q. 5(d) For the set X = {2, 3, 6, 12, 24, 36}, a relation ≤ is defined as x ≤ y if x divides y. Draw the Hasse diagram for (X, ≤). Answer the following.

(i) What are the maximal and minimal elements ?

(ii) Give one example of chain and antichain.

(iii) Is the poset a lattice ? (4 Marks)

Ans. :

R = {(2,2), (2,6), (2,12), (2,24), (2,36), (6,6), (6,12), (6,24), (6,36), (12,12), (12,24), (12,36), (24,24), (3,3), (3,6), (3,12), (3,24), (3,36)}

Final hasse diagram :

Maximal Elements : 24,36

Minimal Elements : 2,3

Chain = {2,6,12,24}

Antichain = {2,3}

This poset is not a lattice.

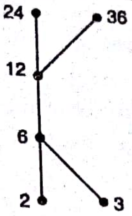


Fig. 1 - Q. 5(d)

Chapter 4 : Functions and Pigeon Hole Principle [Total Marks – 19]

Q. 1(d) Comment whether the function f is one to one or onto.

Consider function : $f : N \rightarrow N$ where N is set of natural numbers including zero.

$f(j) = j^2 + 2$ (5 Marks)

Ans. :

$$N = \{0, 1, 2, 3, 4..................\}$$
$$f(0) = 2$$
$$f(1) = 3$$
$$f(2) = 6$$
$$f(3) = 11$$
$$f(4) = 18 \quad \text{..and so on.}$$

For every number 'n' from 'N' we can find another 'n' .So the given function is one to one.

But not every element 'n' of set 'N' is image of some element 'n'. So the given function is not onto.

Q. 2(d) Let $f : R \rightarrow R$ defined as $f(x) = x^3$ and $g : R \rightarrow R$ defined as $g(x) = 4x^2 + 1$

Find out g o f, f o g, f^2, g^2. (4 Marks)

Ans. :

$$g \circ f = g(f(x)) = g(x^3) = [4x^6+1]$$
$$f \circ g = f(g(x)) = f(4x^2+1) = (4x^2+1)^3$$
$$f \circ f = f(f(x)) = f(x^3) = x^9$$
$$g \circ g = g(g(x)) = g(4x^2+1) = 4(4x^2+1)^2+1$$

Q. 3(c) Let A = B = R, the set of real numbers

Let $f : A \rightarrow B$ be given by the formula $f(x) = 2x^3 - 1$ and Let $g : B \rightarrow A$ be given by

$$g(y) = \sqrt[3]{\frac{1}{2}y + \frac{1}{2}}$$

Show that f is a bijection between A and B and g is a bijection between B and A. (4 Marks)

Ans.:

A function from A to B is Bijection if it is one to one and onto

$\therefore$ for $f(x) = 2x^3 - 1$ to be one to one and onto.

If $a, b, \in A$

such that $f(a) = f(b)$.

$$\Rightarrow \quad 2a^3 - 1 = 2b^3 - 1.$$
$$a = b.$$

$\therefore$ f is one to one

Now for
$$y = 2x^3 - 1.$$
$$1 + y = 2x^3.$$
$$x^3 = \frac{1}{2} + \frac{y}{2}.$$
$$\therefore \quad x = \sqrt[3]{\frac{1}{2} + \frac{y}{2}}.$$

$\therefore$ for each $y \in B$. There is a unique x in A such that $f(x) = y$.

$\therefore$ f is onto.

$\therefore$ f is bijective function between A and B.

Similarly for $g : B \rightarrow A$ to be one to one and onto

$$g(a) = g(b) = \sqrt[3]{\frac{1}{2} + \frac{a}{2}} = \sqrt[3]{\frac{1}{2} + \frac{b}{2}}$$
$$\therefore \quad \frac{1}{2} + \frac{a}{2} = \frac{1}{2} + \frac{b}{a}$$
$$\Rightarrow \quad a = b$$

$\therefore$ g is one to one.

Also for
$$x = \sqrt[3]{\frac{1}{2} + \frac{y}{2}}.$$
$$x^3 = \frac{1}{2} + \frac{y}{2}$$
$$2x^3 = 1 + y$$
$$y = 2x^3 - 1$$

for each x in A. There is a corresponding y in B. Such that $g(y) = x$

$\therefore$ g is onto function

so g is bijective function between B and A.

Q. 5(a) Explain Pigeonhole principle and Extended Pigeonhole principle. Show that in any room of people who have been doing some handshaking there will always be atleast two people who have shaken hands the same number of times. (6 Marks)

Ans. :

Theorem of Pigeonhole Principle :

If n pigenons are assigned to m pigeonholes, and $m < n$, then at least one pigeonhole contains two or more pigeons.

Proof : Consider labeling the m pigeonholes with the numbers 1 through m and the n pigeons with the numbers 1 through n. Now, beginning with pigeon 1, assign each pigeon in order to the pigeonhole with the same number. This assigns as many pigeons as possible to individual pigeon holes, but because $m < n$, there are $n - m$ pigeons that have not yet been assigned to a pigeonhole. At least one pigeonhole will be assigned a second pigeon.

The Extended Pigeonhole Principle :

If there are m pigeonholes and more than 2m pigeons, then three or more pigeons will have to be assigned to at least. One of the pigeonholes. In general if the number at pigeons is much larger than the number at pigeonholes, given theorem can be restated to give a stronger conclusion.

First , a word about notation. If n and m are positive integers, then $\lfloor n/m \rfloor$ stands for the largest integer less than or equal to the rational number n/m. Thus $\lfloor 3/2 \rfloor$ is 1, $\lfloor 9/4 \rfloor$ is 2 and $\lfloor 6/3 \rfloor$ is 2.

Theorem : (The extended pigeonhole principle)

If n pigeons are assigned to m, pigeonholes, then one of the pigeonholes must obtain at least $\lfloor (n-1)/m \rfloor + 1$ pigeons.

Proof : (by contradiction) :

If each pigeonhole contains no more than $\lfloor (n-1)/m \rfloor$ pigeons, then there are at most m. $\lfloor (n-1)/m \rfloor \le$ m. (n – 1) /m = n – 1 pigeons in all. This contradicts our assumption, so one of the pigeonholes must contain at least $\lfloor (n-1)/m \rfloor + 1$ pigeons.

There are n people attending the party. Obviously, this problem makes sense only when n ≥ 2. If no two people have shaken hands with equal number of people then there handshake count must differ at least by 1. So the possible choices for hand shake count would be 0,1,...,n – 1. There are exactly n choices and n people. If there exist a person with (n-1) handshake count, there can't be a person with 0 handshake count. Thus reducing the possible choices to (n-1). Now, due to pigeon hole principle, we have that at least two person will have the same number of handshake count.

Chapter 5 : Counting [Total Marks – 16]

Q. 2(c) Find the complete solution of the recurrence relation :

$a_n + 2a_{n-1} = n + 3$ for $n \ge 1$ and with $a_0 = 3$.

(4 Marks)

Ans. : The characteristic equation is

$$\alpha + 2 = 0$$

$$\therefore \quad \alpha = -2$$

Hence, Homogeneous solution is

$$a_n^{(h)} = A_1(-2)^n$$

For particular solution, since f (r) (Right hand side) is a linear polynomial, therefore, the particular solution will be of the form $(P_0 + P_{1n})$.

$$\text{i.e.} \quad a_n = P_0 + P_{1n}$$

$$a_{n-1} = P_0 + P_1(n-1)$$

Substituting these values in the given recurrence relation, we get,

$$(P_0 + P_{1n}) + 2[P_0 + P_1(n-1)] = n + 3$$

$$P_0 + P_1 n + 2P_0 + 2P_1 n - 2P_1 = n + 3$$

$$(P_0 + 2P_0 - 2P_1) + n(P_1 + 2P_1) = n + 3$$

$$(3P_0 - 2P_1) + n(3P_1) = n + 3$$

On comparing the coefficients of polynomials, we get

$$\text{and} \quad 3P_0 - 2P_1 = 3$$

$$3P_1 = 1$$

Which on solving give $P_1 = 1/3$, $P_0 = 11/9$

Hence, the particular solution is

$$a_n^{(p)} = \frac{11}{9} + \frac{1}{3}n$$

Thus the general solution is

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$\therefore \quad a_n = A_1(-2)^n + \frac{11}{9} + \frac{1}{3}$$

Using the initial conditions $a_0 = 3$ we get

$$a_0 = A_1 + \frac{11}{9}$$

$$\text{Given} \quad a_0 = 3$$

$$3 = A_1 + \frac{11}{9}$$

$$3 - \frac{11}{9} = A_1$$

$$A_1 = 1.78$$

$$\text{Therefore} \quad a_n = 1.78(-2)^n + \frac{11}{9} + \frac{1}{3}n$$

Q. 3(a) Given that a student had prepared, the probability of passing a certain entrance exam is 0.99. Given that a student did not prepare, the probability of passing the entrance exam is 0.05. Assume that the probability of preparing is 0.7. The student fails in the exam. What is the probability that he or she did not prepare ?

(6 Marks)

Ans. : Refer Fig. 1-Q. 3(a) probability tree diagram,

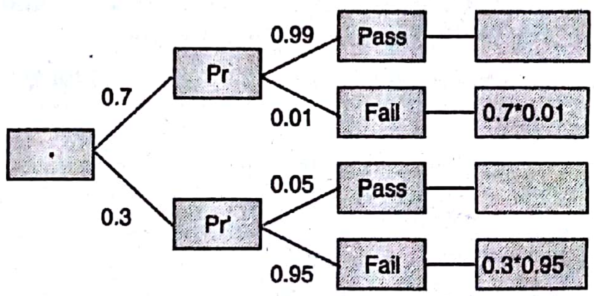


Fig. 1-Q. 3(a)

We will find :

$$P(\text{Fail} \mid Pr') = \frac{P(\text{Fail} \cap Pr')}{P(\text{Fail} \cap Pr') + P(\text{Fail} \cap Pr)}$$

$$\frac{0.3 \cdot 0.95}{0.3 \cdot 0.95 + 0.7 \cdot 0.01} = 0.976$$

Q. 6(b) Given a generating function, find out corresponding sequence.

(i) $\frac{1}{3-6x}$ (ii) $\frac{x}{1-5x+6x^2}$ (6 Marks)

Ans. :

(i) $$\frac{1}{(3-6x)} = \frac{1}{3(1-2x)}$$

The simple generating function that gives the sum of a geometric series :

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

Replace x by 2x

$$\frac{1}{1-2x} = \sum_{n=0}^{\infty} 2^n x^n$$

Multiply this by $\frac{1}{3}$

$$\frac{1}{3(1-2x)} = \frac{1}{3}\sum_{n=0}^{\infty} 2^n x^n$$

$$\frac{1}{3(1-2x)} = \sum_{n=0}^{\infty} \frac{2^n x^n}{3}$$

Thus, the associated sequence is

$$\left(0, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \frac{16}{3}, \ldots\ldots\right)$$

(ii) Given

$$\frac{x}{1-5x+6x^2}$$

We know that $1-5x+6x^2 = (1-2x)(1-3x)$

Therefore

$$\frac{x}{1-5x+6x^2} = \frac{A}{(1-2x)} + \frac{B}{(1-3x)}$$

for some suitable A and B. By multiplying this equation by x^2 we obtain

$$x = A(1-3x) + B(1-2x)$$

By equating the coefficients or by substituting $x = \frac{1}{2}$ and $x = \frac{1}{3}$, we find the solution $A = -1$ and $B = 1$, so

$$f(x) = -\frac{1}{1-2x} + \frac{1}{1-3x} = -\sum_{n=0}^{\infty} 2^n x^n + \sum_{n=0}^{\infty} 3^n x^n$$

$$= \sum_{n=0}^{\infty} (3^n - 2^n)\, x^n,$$

therefore $a_n = 3^n - 2^n$ for all integers $n \geq 0$.

Thus, the associated sequence is (0, 1, 5, 19, 65, ...)

Chapter 6 : Graphs[Total Marks -- 09]

Q. 1(c)(v) Explain the terms : Plannar Graph (1 Mark)

Ans. :

Plannar Graph :

A graph is said to be **planar** if it can be drawn on a plane in such a way that no edges cross one another, except of course at common vertices.

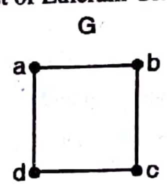
Q. 4(c) (I) Is every Eulerian graph a Hamiltonian ?
(II) Is every Hamiltonian graph a Eulerian ?
Explain with the necessary graph. (4 Marks)

Ans. :

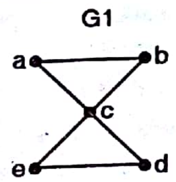
(i) Let G = (V, E) be a graph. A Eulerian Graph is a graph which passes through every edge exactly once.

Let G1 = (V1, E1) be a graph. A Hamiltonian circuit is a circuit which passes through every vertex exactly once (with only the first and last vertex being a repeat). A graph is called Hamiltonian if it possesses a Hamiltonian circuit.

In following Fig. 1(a) - Q. 4(c) Graph G consists of both Eulerian and Hamiltonian Graph. But in Fig. 1(b) - Q. 4(c) Graph G1 consist of Eulerain Graph but not Hamiltonian.



(a) Eulerian Circuit : a, b, c, d, a (covers all edges of Graph G) Hamltonian Graph : a, b, c, d, a (covers all vertices of Graph G)



(b)Eulerian Circuit : c, a, b, c, e, d, c No Hamiltonian Graph

Fig. 1 - Q. 4(c)

Hence every Eulerian Graph is not necessarily Hamiltonian

(ii) In Hamiltonian Graph, we need to visit each and every vertex only once except the last vertex which is also the first vertex. In this type of graph we don't require to visit each and every edge of the graph (which is necessary in case of Eulerian Graph) i.e. need to cover all vertices only once. Since every Hamiltonian graph not necessarily be Eulerian. Refer Fig. 2 - Q. 4(c) to better understand this statement. In following Graph G2 solid lines indicate Hamiltonian Graph, but it is not Eulerian Graph, since it is not possible to cover all edges exactly once.

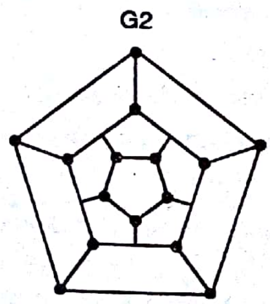


Fig. 2

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Q. 6(c) Determine whether following graphs are isomorphic or not. (4 Marks)

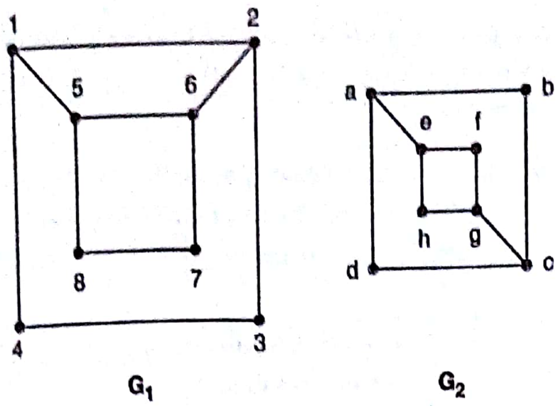


Fig. 1-Q. 6(c)

Ans. :

Hence both the graphs G_1 and G_2 contain 8 vertices and 10 edges. The number of vertices of degree 2 in both the graphs are four. Also the number of vertices of degree 3 in both the graphs are 4.

For adjacency, consider the vertex 1 of degree 3 in G_1. It is adjacent to two vertices of degree 3 and 1 vertex of degree 2. But in G_2 there does not exist any vertex of degree 3 which is adjacent to two vertices of degree. 3 and 1 vertex of degree 2. Hence adjacency is not preserved. Hence given graphs are not isomorphic.

Chapter 7 : Algebraic Structures [Total Marks – 22]

Q. 1(c)(iii) Explain the terms : Normal Subgroup. (1 Mark)

Ans. :

A subgroup H of G is said to be a **normal subgroup** of G if for every $a \in G$, $aH = Ha$.

A subgroup of an Abelian group is normal.

Q. 1(c)(iv) Explain the terms : Group. (1 Mark)

Ans. :

Let (A, $*$) be an algebraic system, where $*$ is a binary operation. (A, $*$) is called a **group** if the following conditions are satisfied.

1. $*$ is a closed operation.
2. $*$ is an associative operation
3. There is an identity.
4. Every element in A has a left inverse.

Because of associativity, a left inverse, of an element is also a right inverse of the element in a group.

Q. 3(d) Let Z_n denote the set of integers {0, 1, 2, ... n – 1}. Let O be binary operation on z_n denote such that a O b = the remainder of ab divided by n.

(i) Construct the table for the operation O for n = 4.

(ii) Show that (Z_n, O) is a semigroup for any n. (4 Marks)

Ans. : assuming O as binary operation such as $*$.

(i) The table for the operation $*$ for n = 4

$*_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

(ii) The set Z_n is closed under the operation $*$ because for any a, $b \in Z_n$, $a * b \in Z_n$ ...(1)

Now check for associativity for any a, b, $c \in Z_n$.

$$(a *_4 b) *_4 c = a *_4 (b *_4 c)$$

Let a = 1, b = 2, c = 3

$$(1 *_4 2) *_4 3 = 1 *_4 (2 *_4 3)$$

$$2 *_4 3 = 1 *_4 2$$

$$2 = 2$$

$\therefore$ '$*$' is an associative operation ...(2)

From Equations (1) and (2) we conclude that (Z_n, $*$) is a semigroup for any 'n'.

Q. 4(b) Consider the (2, 5) group encoding function $e : B^2 \rightarrow B^5$ defined by,
e (00) = 00000 e (01) = 01110
e (10) = 10101 e (11) = 11011
Decode the following words relative to a maximum likelihoods decoding function.
(i) 11110 (ii) 10011 (iii) 10100 (6 Marks)

Ans. :

Prepare decoding table :

00000	01110	10101	11011
00001	01111	10100	11010
00010	01100	10111	11001
00100	01010	10001	11111
01000	00110	11101	10011
10000	11110	00101	01011

(i) If we receive the word 1 1 1 1 0 we first locute it in 2^{nd} column of the decoding table. Where it is underlined once. The word at the top of the 2^{nd} column is 0 1 1 1 0. Since e (01) = 0 1 1 1 0. We decode 1 1 1 1 0 as 0 1.

(ii) If we receive the word 1 0 0 1 1, we first locate it in 4^{th} column. Where it is underlined twice. The word at the top of the 4^{th} column is 1 1 0 1 1. Since e (11) = 11011. We denote 1 0 0 1 1 as 11.

(iii) Similarly 1 0 1 0 0 is located in 3^{rd} column of decoding table it is underlined thrice. The word at the top of the 3rd column is 1 0 1 0 1. Since e (1 0) = 1 0 1 0 1. We decode 1 0 1 0 0 as 1 0.

Q. 4(d) Given the parity check matrix.

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find the minimum distance of the code generated by H. How many errors it can detect and correct ? (4 Marks)

Ans. : In the given parity check matrix all columns are distinct and non-zero, so d> = 3.

We can use the property that the minimum distance of a binary linear code is equal to the smallest number of columns of the parity-check matrix H that sum up to zero.

We can see that sum of first three columns is zero. So minimum distance $d_{min} = 3$.

It can correct $(d_{min} - 1) / 2 = 1$ error.

It can detect $d_{min} - 1 = 2$ errors.

Q. 6(a) Prove that the set {1, 2, 3, 4, 5, 6} is group under multiplication modulo 7. (6 Marks)

Ans. :

(a) Multiplication modulo 7 table for set A is

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	1	3	2	1

(b) Note that 1 is an identity element of the algebraic system $(A, \times_7)$

Since for any $a \in A$,

$$a \times_7 1 = a = 1 \times_7 a$$

that is,

$$1 \times_7 1 = 1 \times_7 1 = 1$$
$$2 \times_7 1 = 1 \times_7 2 = 2$$
$$3 \times_7 1 = 1 \times_7 3 = 3$$
$$4 \times_7 1 = 1 \times_7 4 = 4$$
$$5 \times_7 1 = 1 \times_7 5 = 5$$
$$6 \times_7 1 = 1 \times_7 6 = 6$$

Recall that a^{-1} is that element of G such that $a * a^{-1} = 1$

$$2 \times_7 4 = 1 = 4 \times_7 2 = 1$$
$$3 \times_7 5 = 1 = 5 \times_7 3 = 1$$
$$6 \times_7 6 = 1 = 6 \times_7 6 = 1$$

$\therefore$ Inverse of 2 is 4

Inverse of 4 is 2

Inverse of 3 is 5

Inverse of 5 is 3

Inverse of 6 is 6

(c) We have

$$2^1 = 2,$$
$$2^2 = 2 \times_7 2 = 4$$
$$2^3 = 2^2 \times_7 2 = 4 \times_7 2$$
$$= 1$$
$$2^4 = 2^3 \times_7 2 = 1 \times_7 2 = 2$$

$\therefore$ Hence $|2| = 3$

$\therefore$ 2 is not generator of this group

We have

$$3^1 = 3$$
$$3^2 = 3 \times_7 3 = 2$$
$$3^3 = 3^2 \times_7 3 = 2 \times_7 3 = 6$$
$$3^4 = 3^3 \times_7 3 = 6 \times_7 3 = 4$$
$$3^5 = 3^4 \times_7 3 = 4 \times_7 3 = 5$$
$$3^6 = 3^5 \times_7 3 = 5 \times_7 3 = 1$$

Hence $|3| = 6$

$\therefore$ 3 is generator of this group and this group is cyclic.

(d) Subgroup generated by {3, 4} is denoted by < {3, 4} > since 3, 4 are elements of this set they have to be there in < {3, 4} >

Inverse of 3 is 5, inverse of 4 is 2

$\therefore$ 3, 4, 5, 2, $\in$ < {3, 4} >

$3 \times_7 4 = 5$	$5 \times_7 4 = 6$
$3 \times_7 3 = 2$	$6 \times_7 6 = 1$
$3 \times_7 5 = 1$	$5 \times_7 1 = 5$
$4 \times_7 4 = 2$	$1 \times_7 1 = 1$
$3 \times_7 2 = 6$	$5 \times_7 2 = 3$
$5 \times_7 5 = 4$	$3 \times_7 6 = 4$
$5 \times_7 6 = 2$	$2 \times_7 2 = 4$

$\therefore$ < {3, 4} > = < 1, 2, 3, 4, 5, 6 >

$\therefore$ Subgroup generated by {3, 4} is the set A itself whose order is 6.

Subgroup generated by {2, 3} is denoted by < {2, 3} >.

Since 2, 3 are elements of this set they have to be there in < {2, 3} >.

Inverse of 2 is 4.

Inverse of 3 is 5

$\therefore$ 2, 3, 4, 5 $\in$ < {2, 3} >

$2 \times_7 3 = 6$	$3 \times_7 4 = 5$
$4 \times_7 4 = 2$	$5 \times_7 5 = 4$
$2 \times_7 4 = 1$	$3 \times_7 5 = 1$
$4 \times_7 1 = 4$	$5 \times_7 6 = 2$

$2 \times_7 5 = 3$ $\quad$ $3 \times_7 6 = 4$

$4 \times_7 5 = 6$ $\quad$ $5 \times_7 1 = 5$

$2 \times_7 6 = 5$ $\quad$ $3 \times_7 1 = 3$

$6 \times_7 6 = 1$ $\quad$ $2 \times_7 1 = 2$

$3 \times_7 3 = 2$ $\quad$ $6 \times_7 1 = 6$

$2 \times_7 2 = 4$

$\therefore < \{2, 3\} > \; = \; < 1, 2, 3, 4, 5, 6 >$

$\therefore$ Subgroup generated by $< \{2, 3\} >$ is the set A and is of order 6.

May 2018

Chapter 1 : Set Theory [Total Marks – 17]

Q. 1(b) Find the generating function for the following finite sequences. (5 Marks)

(i) 2, 2, 2, 2, 2, 2

(ii) 1, 1, 1, 1, 1, 1

Ans. :

i) 2, 2, 2, 2, 2, 2

Here 2 comes 6 times $\rightarrow$ 0 to 5

i.e. $a_n = 2$ for $n \leq 5$

$$\therefore \; G(x) = \sum_{n=0}^{5} 2\, x^n = \frac{2(1-x^6)}{1-x}$$

ii) 1, 1, 1, 1, 1, 1

Here 1 comes 6 times $\rightarrow$ 0 to 5

i.e. $a_n = 1$ for $n \leq 5$

$$\therefore \; G(x) = \sum_{n=0}^{5} x^n = \frac{1-x^6}{1-x}$$

Q. 4(c) How many friends must you have to guarantee that at least five of them will have birthdays in the same month ? (4 Marks)

Ans. :

If there are 4 friends who have birthday in the same month, then

Total friends $= 12 * 4 = 48$

The 5th friend will have birthday to be the same month as one of the others.

$\therefore$ Total friends $= 48 + 1 = 49$

Q. 5(a) Let G be a set of rational numbers other than 1. Let * be an operation on G defined by a * b = a + b – ab for all a, b € G. Prove that (G, *) is a group. (8 Marks)

Ans. :

(i) as a, b $\in$ G. hence G is closed with respect to *

(ii)
$$\begin{aligned}
a * (b * c) &= a * (b + c - bc) = a + (b + c - bc) - a(b + c - bc)\\
&= a + b + c - bc - ab - ac + abc\\
(a * b) * c &= (a + b - ab) * c = a + b - ab + c - (a + b - ab)\,c\\
&= a + b - ab + c - ac - bc + abc\\
&= a + b + c - ab - ac - bc + abc
\end{aligned}$$

Hence a * (b * c) = (a * b) * c. hence G is associative under operation *.

(iii) We know that for every $a \in G$

$$\begin{aligned}
a * e &= a\\
\therefore\; a + e - ae &= a\\
\therefore\; a + e - a &= a \qquad \therefore\; e = a
\end{aligned}$$

(iv)
$$\begin{aligned}
a * a^{-1} &= e\\
\therefore\; a + a^{-1} - aa^{-1} &= e\\
\therefore\; a + a^{-1} - e &= e\\
\therefore\; a + a^{-1} &= 2e \qquad \therefore\; a^{-1} = 2e - a = 2a - a = a
\end{aligned}$$

$\therefore$ a is the inverse of G

Hence (G, *) satisfies above conditions

Hence (G, *) is a group.

Chapter 2 : Logic [Total Marks – 09]

Q. 1(a) Prove by induction that the sum of the cubes of three consecutive numbers is divisible by 9. (5 Marks)

Ans. :

Step 1 : Basic of induction

$P(1) = 1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36$

36 is divisible by 9

Hence P(1) is ture

Step 2 : Induction Hypothesis

P(k) : For n = k is true

$P(k) = k^3 + (k+1)^3 + (k+2)^3$ is divisible by 9

$\therefore \; k^3 + (k+1)^3 + (k+2)^3 = 9\lambda$ is true where λ is a number multiple of 9.

Step 3 : Induction

P (k + 1)

$$\begin{aligned}
&(k+1)^3 + (k+2)^3 + (k+3)^3\\
&= (k+1)^3 + (k+2)^3 + (k+3)(k^2 + 6k + 9)\\
&= (k+1)^3 + (k+2)^3 + k^3 + 6k^2 + 9k + 3k^2 + 18k + 27\\
&= (k+1)^3 + (k+2)^3 + k^3 + 9k^2 + 27k + 27\\
&= (k+1)^3 + (k+2)^3 + k^3 + 9(k^2 + 3k + 3)\\
&= 9\lambda + 9(k^2 + 3k + 3)\\
&= 9(\lambda + k^2 + 3k + 3)
\end{aligned}$$

Hence P (k + 1) is multiple of 3.

Hence P (n) is true.

Q. 2(c) **Prove $\neg (p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.** **(4 Marks)**

Ans. :

		Reason
$\neg(p \vee (\neg p \wedge q))$	$\equiv \neg p \wedge \neg(\neg p \wedge q)$	1[st] Demorgan law
	$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	1[st] Demorgan law
	$\equiv \neg p \wedge (p \vee \neg q)$	Double Negation law
	$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	2[nd] distributive law
	$\equiv F \vee (\neg p \wedge \neg q)$	$\neg p \wedge p \equiv F$
	$\equiv (\neg p \wedge \neg q) \vee F$	Commutative law
	$\equiv \neg p \wedge \neg q$	Identity law for F

Hence $\neg (p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

Chapter 3 : Relations, Digraphs and Lattice [Total Marks – 25]

Q. 1(d) **Find the complement of each element in D_{30}.** **(5 Marks)**

Ans. :

(i) D_{30} means all nos. by which 30 is divisible

$\therefore D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

Let's draw the Hasse Diagram.

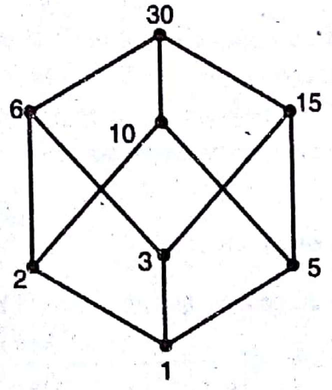


Fig. 1-Q. 1(d)

(ii)

a	b	GLB	LUB
1	30	1	30
2	15	1	30
3	10	1	30
5	6	1	30

Hence $1' = 30$ and $30' = 1$, $2' = 15$ and $15' = 2$,
$3' = 10$ and $10' = 3$, $5' = 6$ and $6' = 5$

Q. 3(b) **Let A = {1, 2, 3, 4, 5}, let R = {(1 ,1) , (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5)} and S = {(1, 1), (2, 2), (3, 3), (4, 4), (4, 5), (5, 4), (4, 5)} be the relations on A. Find the smallest equivalence relation containing the relation R and S.** **(8 Marks)**

Ans. :

Let's consider Q is relation $R \cup S$

$\therefore$ Q = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4), (4, 5) (5, 4), (5, 5)}

(i) Q is reflexive since it has {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)}

(ii) Q is symmetric since it has (1, 2), (2, 1), (3, 4), (4, 3), (4, 5), (5, 4)

(iii) By adding two pairs as follows, it becomes transitive

1. As (3, 4), (4, 5) $\rightarrow$ Add (3, 5)
2. As (5, 4), (4, 3) $\rightarrow$ Add (5, 3)

Hence smallest equivalence relation containing R and S is

$\therefore$ Q = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (3, 5), (4, 3), (4, 4) (4, 5), (5, 3), (5, 4), (5, 5)}

Q. 5(c) **Let A = {a, b, c ,d, e, f, g, h}. Consider the following subsets of A**

A_1 = {a, b, c, d} A_2 = {a, c, e, g, h}
A_3 = {a, c, e, g} A_4 = {b, d}
A_5 = {f, h}

Determine whether following is partition of A or not. Justify your answer.

(i) {A_1, A_2} (ii) {A_3, A_4, A_5} **(4 Marks)**

Ans. :

(i) {A_1, A_2} $\rightarrow$ It is not a partition. Since a is common element between A_1 and A_2. Hence A_1 and A_2 are not disjoint sets.

(ii) {A_3, A_4, A_5} $\rightarrow$ It is a partion. Since A_3, A_4 and A_5 are disjoint sets and $A_1 \cup A_2 \cup A_3 = A$.

Q. 6(a) **Draw the Hasse Diagram of the following sets under the partial order relation divides and indicate which are chains. Justify your answers.**

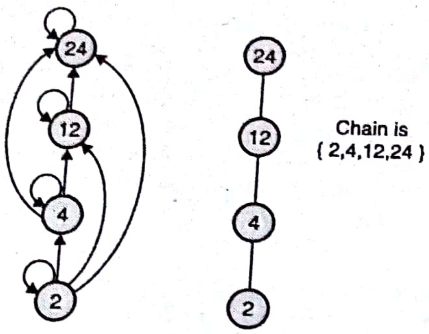
I. A = {2, 4,12, 24}
II. A = {1, 3, 5, 15, 30} **(8 Marks)**

Ans. :

(i) A = {2, 4, 12, 24}

First find out relation R and it is,

R = {(2, 2), (2, 4), (2, 12), (2, 24), (4, 4), (4, 12), (4, 24), (12, 12), (12, 24), (24, 24)}



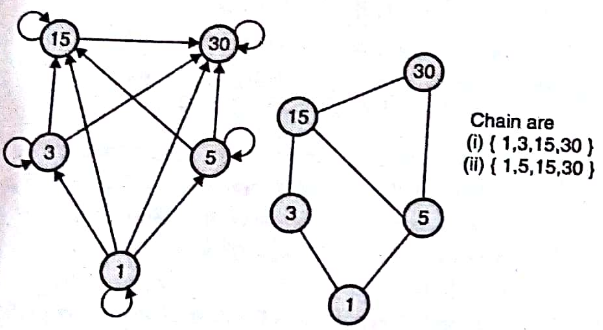
(a) : Diagraph (b) : Hasse Diagram

Fig. 1 - Q. 6(a)

(ii) A = {1, 3, 5, 15, 30}

First find out relation R and it is

R = {(1, 1), (1, 3), (1, 5), (1, 15), (1, 30), (3, 3), (3, 15), (3, 30), (5, 5), (5, 15), (5, 30), (15, 15), (15, 30), (30, 30)}



Diagraph

Fig. 2 - Q. 6(a)

Chapter 4 : Functions and Pigeon Hole Principle [Total Marks – 12]

Q. 3(c) Test whether the following function is one-to-one, onto or both. $F : Z \to Z$, $f(x) = x^2 + x + 1$. (4 Marks)

Ans. :

(i) For $x = 1$

$f(x) = 1^2 + 1 + 1 = 3$

(ii) for $x = 2$

$f(x) = 2^2 + 2 + 1 = 7$

(ii) for $x = -2$

$f(x) = (-2)^2 - 2 + 1 = 3$

(ii) for $x = -3$

$f(x) = (-3)^2 - 3 + 1 = 7$

i.e.

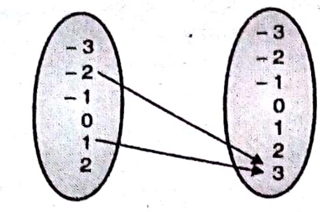


Fig. 1-Q. 3(c)

Hence it not one - to - one negative value of second set are also unassigned as f(x) is always + ve. Hence it not onto also.

Q. 6(b) Let the functions f, g, and h defined as follows :

$f : R \to R$, $f(x) = 2x + 3$

$g : R \to R$, $g(x) = 3x + 4$

$h : R \to R$, $h(x) = 4x$

Find gof, fog, foh, gofoh (8 Marks)

Ans. :

(i) $(g \circ f)(x) = g\,f(x) = g(2x + 3) = 3(2x + 3) + 4$

$= 6x + 9 + 4 = 6x + 13$

(ii) $(f \circ g)(x) = f(g(x)) = f(3x + 4) = 2(3x + 4) + 3$

$= 6x + 8 + 3 = 6x + 11$

(iii) $(f \circ h)(x) = f(h(x)) = f(4x) = 2(4x) + 3 = 8x + 3$

(iv) $(g \circ f \circ h)(x) = g(f(h(x)))$ As $(f \circ h)(x) = 8x + 3$

$g(8x + 3) = 3(8x + 3) + 4 = 24x + 9 + 4$

$= 24x + 13$

Chapter 5 : Counting [Total Marks – 21]

Q. 1(c) A box contains 6 white balls and 5 red balls. In how many ways 4 balls can be drawn from the box if, (I) they are to be of any color (II) all the balls to be of the same color. (5 Marks)

Ans. :

1. They are to be of any color

4 balls can be drawn randomly from total 11 balls.

$\therefore$ No. of ways $= {}^{11}C_4 = \dfrac{11!}{4! \times 7!} = \dfrac{11 \times 10 \times 9 \times 8 \times \cancel{7!}}{4 \times 3 \times 2 \times \cancel{7!}}$

$= 330$

2. All the balls to be of the same color

Either 4 balls are white i.e. No. of ways $= {}^6C_4$

OR 4 balls are red i.e. No. of ways $= {}^5C_4$

$\therefore$ Total no. of ways $= {}^6C_4 + {}^5C_4 = \dfrac{6!}{4! \times 2!} + \dfrac{5!}{4! \times 1!}$

$= \dfrac{6 \times 5 \times 4!}{4! \times 2} + \dfrac{5 \times 4!}{4!} = 15 + 5 = 20$

Q. 2(b) In a certain college 4% of the boys and 1% of the girls are taller than 1.8 mts. Furthermore 60% of the students are girls. If a student selected at random is taller than 1.8 mts, what is the probability that the student was a boy ? Justify your answer (8 Marks)

Ans. :

Since 60% of students are girls, 40% of students are boys.

We use Bayes theorem,

$$P(B \mid h > 1.8) = \frac{P(B \text{ and } h > 1.8)}{P(B \text{ and } h > 1.8) + P(G \text{ and } h > 1.8)}$$

Since 40% of boys and of those 4% are taller than 1.8 mts.

$P(B \text{ and } h > 1.8) = 0.4 * (0.04)$

Now,

$P(h > 1.8) = P(G \text{ and } h > 1.8) + P(B \text{ and } h > 1.6)$

$\therefore\ P(G \text{ and } h > 1.8) = 0.6 * 0.01$

$$\therefore\ P(B \mid h > 1.8) = \frac{0.4 * 0.04}{(0.4 * 0.04) + 0.6 * 0.01}$$

$$= \frac{.016}{0.016 + 0.006} = 0.7273$$

Hence probability that a student was boy and if a student selected random is taller than 1.8 mts. = 72.73%.

Q. 5(b) Solve $a_r - 7a_{r-1} + 10\, a_{r-2} = 6 + 8\, r$ given $a_0 = 1, a_1 = 2$. (8 Marks)

Ans. :

The corrosponding homogeneous equation is

$$a_r - 7a_{r-1} + 10\, a_{r-2} = 0$$

here degree = 2

thus,

$$\alpha^2 - 7\alpha - 10 = 0$$

$$\alpha^2 - 5\alpha - 2\alpha - 10 = 0$$

$$\therefore\ (\alpha - 5)(\alpha - 2) = 0$$

$$\therefore\ \alpha = 2, 5$$

The homogeneous solution is

$$a_r^{(h)} = A_1 2^r + A_2 5^r$$

Particular solution

Since R.H.S. is polynomial, the particular solution will be of the form $(P_0 + P_1 r)$

i.e. $a_r = P_0 + P_1 r$

$a_{r-1} = P_0 + P_1 (r - 1)$

$a_{r-2} = P_0 + P_1 (r - 2)$

Substituting these values in the given relation,

We get,

$$(P_0 + P_1 r) - 7[P_0 + P_1 (r-1)] + 10\,[P_0 + P_1 (r-2)] = 6 + 8r$$

$$\text{i.e. } P_0 + P_1 r - 7P_0 - 7\,P_1 (r-1) + 10\,P_0 + 10\,P_1 (r-2) = 6 + 8r$$

$$\therefore\ 4P_0 + P_1 [r - 7r + 7 + 10r - 20] = 6 + 8r$$

$$4\,P_0 - P_1 (4r - 13) = 6 + 8r$$

$$\therefore\ (4\,P_0 - 13\,P_1) + 4\,P_1\, r = 6 + 8r$$

On comparing the coefficients and polynomials,

We get

$4\,P_0 - 13\,P_1 = 6$ and $4\,P_1 = 8$

$\therefore\ P_1 = 2$

$4\,P_0 - 13\,P_1 = 6$

$\therefore\ 4\,P_0 - 23 \times 2 = 6$

$\therefore\ 4\,P_0 - 26 = 6$

$\therefore\ P_0 = 8$

Here the particular solution is

$$a_r(p) = 8 + 2r$$

Thus general solution is

$$a_r = a_r(h) + a_r(p)$$

$$\therefore\ a_r = A_1 2^r + A_2 5^r + 8 + 2r$$

Using the initial conditions $a_0 = 1$ and $a_1 = 2$

We get,

$A_1 = -9$ and $A_2 = 2$

Therefore,

$$a_r = -9\,(2)^r + 2\,(5)^r\, 8 + 2r$$

Chapter 6 : Graphs [Total Marks – 12]

Q. 2(a) Define Isomorphism of graphs. Find if the following two graphs are isomorphic . If yes, find the one-to-one correspondence between the vertices. (8 Marks)

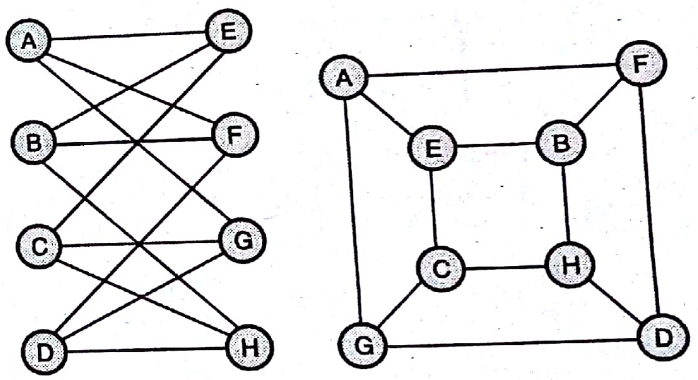


Fig. 1-Q. 2(a)

Ans. :

Isomorphism of graphs

Two graphs have exactly the same form, in the sense that there is one - to - one correspondence between their vertex sets that preserve edges. In such a case, two graphs are isomorphic.

Degree of each vertex is 3 in both graphs.

	A	B	C	D	E	F	G	H
A	0	0	0	0	1	1	1	0
B	0	0	0	0	1	1	0	1
C	0	0	0	0	1	0	1	1
D	0	0	0	0	0	1	1	1
E	1	1	1	0	0	0	0	0
F	1	1	0	1	0	0	0	0
G	1	0	1	1	0	0	0	0
H	0	1	1	1	0	0	0	0

Adjancy matrix of G_1

	A	B	C	D	E	F	G	H
A	0	0	0	0	1	1	1	0
B	0	0	0	0	1	1	0	1
C	0	0	0	0	1	0	1	1
D	0	0	0	0	0	1	1	1
E	1	1	1	0	0	0	0	0
F	1	1	0	1	0	0	0	0
G	1	0	1	1	0	0	0	0
H	0	1	1	1	0	0	0	0

Adjancy matrix of G_2

By observing G1 and G2 (Adjancy matirx), mapping is as follows :

Graph G_1	Graph G_2
A	A
B	B
C	C
D	D
E	E
F	F
G	G
H	H

Hence graph G_1 and G_2 are isomorphic

Q. 6(c) Determine Euler Cycle and path in graph shown below. (4 Marks)

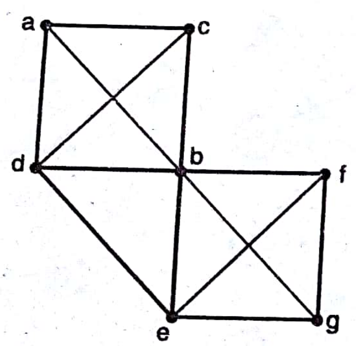


Fig. 1 - Q. 6(c)

Ans. :

Euler Cycle

Theorem : A connected multigraph with at least two vertices has an Euler cycle if and only if each of its vertices has an even degree.

Here deg (a) = 3.

Hence it doesn't have Euler cycle

Euler Path

Since given graph has not Euler Cycle.

Theorem

A Connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree. deg (a) = 3, deg (c) = 3, deg (f) = 3, deg (g) = 3.

Hence it doesn't have Euler path.

Chapter 7 : Algebraic Structures [Total Marks – 24]

Q. 3(a) Prove that set G = {1, 2, 3, 4, 5, 6} is a finite abelian group of order 6 with respect to multiplication module 7. (8 Marks)

Ans. : First draw a matrix for multiplication modulo 7.

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

(i) As $\{1, 2, 3, 4, 5, 6\} \in G$. Hence G is closed with respect to multiplication modulo 7.

(ii) $1 * (2 * 3) = 1 * 6 = 6$

$(1 * 2) * 3 = 2 * 3 = 6$

OR

$3 * (4 * 5) = 3 * 6 = 4$

$(3 * 4) * 5 = 5 * 5 = 4$

Hence G is associative with respect to multiplication modulo 7.

(iii) 1 is the identity element

(iv) 1 is inverse of 1, 2 is inverse of 4, 3 is inverse of 5, 6 is inverse of 6 and vice versa.

(v) $4 * 2 = 1$ and $2 * 4 = 1$

$4 * 6 = 3$ and $6 * 4 = 3$

Hence G is commutative under operation multiplication modulo 7.

As G satisfies above (i) to (v) conditions, hence G is a finite abelian group.

Q. 4(a) Show that the (2, 5) encoding function $e : B^2 \rightarrow B^5$ defined by e(00) = 00000 e(01) = 01110 e(10) = 10101 e(11) = 11011 is a group code. How many errors will it detect and correct? (8 Marks)

Ans. :

(i) $x, y \in S$, then $x \oplus y \in S$. Hence it is closed.

(ii) Identity element

$$00000 \oplus 00000 = 00000$$
$$00000 \oplus 01110 = 01110$$
$$00000 \oplus 10101 = 10101$$
$$00000 \oplus 11011 = 11011$$

Here (0, 0, 0, 0, 0)

(iii) $(B_6, \oplus)$ is associative

$$00000 \oplus (01110 \oplus 10101) = (00000 \oplus 01110) \oplus 10101$$

(iv) Every code is in inverse of itself.

Hence it is a group code.

Error detection :

Minimum distance is the minimum weight of non-zero codewords of a group code.

$\therefore$ Minimum distance $= 3$

$\therefore$ Number of errors can be detect $= 3 - 1 = 2$

Number of errors can be correct $= 3 - 2 = 1$

Q. 4(b) **Let H =** $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Be a parity check matrix. Determine the group code $e_H : B^3 \to B^6$ **(8 Marks)**

Ans. :

(i) $H = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

As $H = [P^T \,\vdots\, I_K]$

$$P_T = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\therefore \quad P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(ii) Find parity bits p_1, p_2, p_3 as

$$(p_1\, p_2\, p_3) = [x_1\, x_2\, x_3] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$\therefore$ $p_1 = x_1, p_2 = x_2 \oplus x_3, p_3 = x_1 \oplus x_2 \oplus x_3$

(iii) As I_K is 3×3 matrix No. of msg. bits = 3

$\therefore$ No. of codewords $= 2^3 = 8$

x_1	x_2	x_3	p_1	p_2	p_3	
0	0	0	0	0	0	e(000) = 000000
0	0	1	0	1	1	e(001) = 001011
0	1	0	0	1	1	e(010) = 010011
0	1	1	0	0	0	e(011) = 011000
1	0	0	1	0	1	e(100) = 100101
1	0	1	1	1	0	e(101) = 101110
1	1	0	1	1	0	e(110) = 110110
1	1	1	1	0	1	e(111) = 111101

Question Paper

Dec. 2017

Q. 1 (a) Prove that 1.1 ! + 2.2 ! + 3.3 ! + ... + n · n ! = (n + 1) ! – 1, where n is a positive integer. **(5 Marks)**

(b) Let A = {a, b, c}. Show that (P(A), ⊆) is a poset and draw its Hasse diagram. **(5 Marks)**

(c) Explain the terms : **(5 Marks)**

(i) Lattice

(ii) Poset

(iii) Normal Subgroup

(iv) Group

(v) Plannar Graph

(d) Comment whether the function f is one to one or onto.

Consider function : $f : N \to N$ where N is set of natural numbers including zero.

$f(j) = j^2 + 2$ **(5 Marks)**

Q. 2 (a) Find the number of the ways a person can be distributed Rs. 601 as pocket money to his three sons, so that no son should receive more than the combined total of the other two. (Assume no fraction of a rupee is allowed). **(6 Marks)**

(b) Let A = $\{a_1, a_2, a_3, a_4, a_5\}$ and let R be a relation on A whose matrix is

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find M_R* by Warshall's algorithm. **(6 Marks)**

(c) Find the complete solution of the recurrence relation :

$a_n + 2a_{n-1} = n + 3$ for $n \geq 1$ and with $a_0 = 3$.

(4 Marks)

(d) Let $f : R \to R$ defined as $f(x) = x^3$ and $g : R \to R$ defined as $g(x) = 4x^2 + 1$

Find out $g \circ f, f \circ g, f^2, g^2$. **(4 Marks)**

Q. 3 (a) Given that a student had prepared, the probability of passing a certain entrance exam is 0.99. Given that a student did not prepare, the probability of passing the entrance exam is 0.05. Assume that the probability of preparing is 0.7. The student fails in the exam. What is the probability that he or she did not prepare ? **(6 Marks)**

(b) Define equivalence relation with example. Let 'T' be a set of triangles in a plane and define R as the set $R = \{(a, b) \mid a, b \in T$ and a is congruent to b$\}$ then show that R is an equivalence relation.

(6 Marks)

(c) Let $A = B = R$, the set of real numbers

Let $f : A \to B$ be given by the formula $f(x) = 2x^3 - 1$ and Let $g : B \to A$ be given by -

$$g(y) = \sqrt[3]{\frac{1}{2}y + \frac{1}{2}}$$

Show that f is a bijection between A and B and g is a bijection between B and A. **(4 Marks)**

(d) Let Z_n denote the set of integers $\{0, 1, 2, ... n-1\}$. Let O be binary operation on z_n denote such that a O b = the remainder of ab divided by n.

(i) Construct the table for the operation O for $n = 4$.

(ii) Show that (Z_n, O) is a semigroup for any n.

(4 Marks)

Q. 4 (a) (i) Among 50 students in a class, 26 got an A in the first examination and 21 got an A in the second examination. If 17 students did not get an A in either examination, how many students got an A in both examinations ?

(ii) If the number of students who got an A in the first examination is equal to that in the second examination, if the total number of students who got an A in exactly one examination is 40 and if 4 students did not get an A in either examination then determine the number of students who got an A in the first examination only, who got an A in the second examination only and who got an A in both the examination. **(6 Marks)**

(b) Consider the (2, 5) group encoding function $e : B^2 \to B^5$ defined by,

$e(00) = 00000$ $e(01) = 01110$

$e(10) = 10101$ $e(11) = 11011$

Decode the following words relative to a maximum likelihoods decoding function.

(i) 11110 (ii) 10011 (iii) 10100 **(6 Marks)**

(c) (i) Is every Eulerian graph a Hamiltonian ?

(ii) Is every Hamiltonian graph a Eulerian ?

Explain with the necessary graph. **(4 Marks)**

(d) Given the parity check matrix. **(4 Marks)**

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find the minimum distance of the code generated by H. How many errors it can detect and correct ?

Q. 5 (a) Explain Pigeonhole principle and Extended Pigeonhole principle. Show that in any room of people who have been doing some handshaking there will always be atleast two people who have shaken hands the same number of times.

(6 Marks)

(b) Determine whether the Poset with the following Hasse diagrams are lattices or not. Justify your answer. **(6 Marks)**

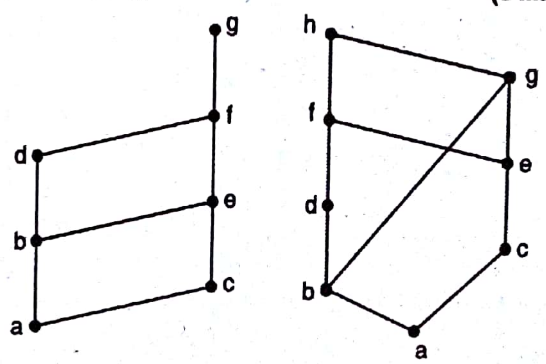


Fig. 1-Q. 5(b)

Q. 5(c) From the following diagraphs, write the relation a set of ordered pairs. Are the relations equivalence relations ? **(4 Marks)**

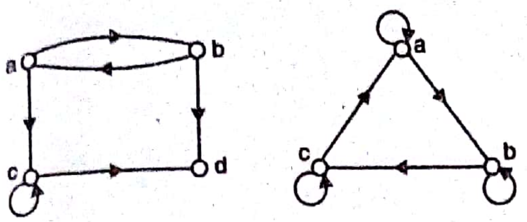


Fig. 1-Q. 5(c)

(d) For the set X = {2, 3, 6, 12, 24, 36}, a relation $\leq$ is defined as $x \leq y$ if x divides y. Draw the Hasse diagram for (X, $\leq$). Answer the following .

(i) What are the maximal and minimal elements ?

(ii) Give one example of chain and antichain.

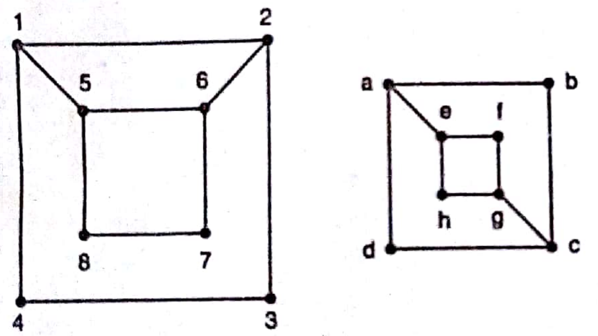
(iii) Is the poset a lattice ? **(4 Marks)**

Q. 6 (a) Prove that the set {1, 2, 3, 4, 5, 6} is group under multiplication modulo 7. **(6 Marks)**

(b) Given a generating function, find out corresponding sequence. **(6 Marks)**

(i) $\frac{1}{3-6x}$ (ii) $\frac{x}{1-5x+6x^2}$

(c) Determine whether following graphs are isomorphic or not. **(4 Marks)**



G_1 G_2

Fig. 1-Q. 6(c)

Q. 6(d) Prove the following (use laws of set theory)

$A \times (X \cap Y) = (A \times X) \cap (A \times Y)$ **(4 Marks)**

May 2018

Q. 1 (a) Prove by induction that the sum of the cubes of three consecutive numbers is divisible by 9. **(5 Marks)**

(b) Find the generating function for the following finite sequences. **(5 Marks)**

(i) 2, 2, 2, 2, 2, 2

(ii) 1, 1, 1, 1, 1, 1

(c) A box contains 6 white balls and 5 red balls. In how many ways 4 balls can be drawn from the box if, (i) they are to be of any color (ii) all the balls to be of the same color. **(5 Marks)**

(d) Find the complement of each element in D_{30}. **(5 Marks)**

Q. 2 (a) Define Isomorphism of graphs. Find if the following two graphs are isomorphic . If yes, find the one-to-one correspondence between the vertices. **(8 Marks)**

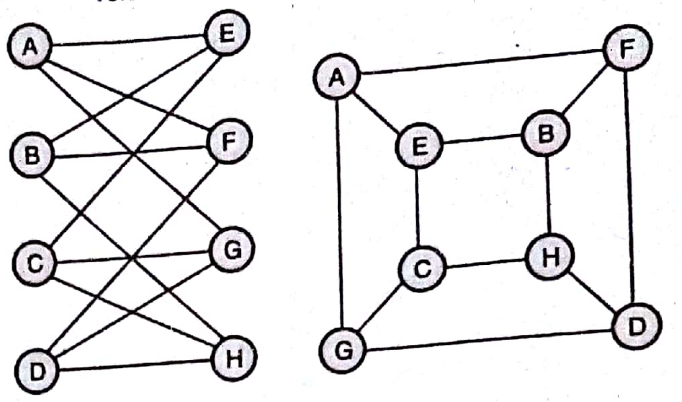


Fig. 1-Q. 2(a)

(b) In a certain college 4% of the boys and 1% of the girls are taller than 1.8 mts. Furthermore 60% of the students are girls. If a student selected at random is taller than 1.8 mts, what is the probability that the student was a boy ? Justify your answer **(8 Marks)**

(c) Prove $\neg (p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences . **(4 Marks)**

Q. 3 (a) Prove that set G = {1,2,3,4,5,6} is a finite abelian group of order 6 with respect to multiplication module 7. **(8 Marks)**

(b) Let A = {1,2,3,4,5}, let R = {(1 ,1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5)} and S = {(1, 1), (2, 2), (3, 3), (4, 4), (4, 5), (5, 4), (4,5)} be the relations on A. Find the smallest equivalence relation containing the relation R and S. **(8 Marks)**

(c) Test whether the following function is one-to-one, onto or both. $F : Z \rightarrow Z$, $f(x) = x^2 + x + 1$. **(4 Marks)**

Q. 4 (a) Show that the (2, 5) encoding function $e : B^2 \rightarrow B^5$ defined by

e(00) = 00000 e(01) = 01110

e(10) = 10101 e(11) = 11011 is a group code. How many errors will it detect and correct ? **(8 Marks)**

(b) Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Be a parity check matrix. Determine the group code $e_H : B^3 \rightarrow B^6$ **(8 Marks)**

(c) How many friends must you have to guarantee that at least five of them will have birthdays in the same month? **(4 Marks)**

Q. 5 (a) Let G be a set of rational numbers other than 1. Let * be an operation on G defined by a * b = a + b − ab for all a, b € G. Prove that (G,*) is a group. **(8 Marks)**

(b) Solve $a_r - 7a_{r-1} + 10\, a_{r-2} = 6 + 8\, r$ given $a_0 = 1$, $a_1 = 2$. **(8 Marks)**

(c) Let A = {a, b, c ,d, e, f, g, h}. Consider the following subsets of A

$A_1 = \{a, b, c, d\}$ $A_2 = \{a, c, e, g, h\}$

$A_3 = \{a, c, e, g\}$ $A_4 = \{b, d\}$

$A_5 = \{f, h\}$

Determine whether following is partition of A or not. Justify your answer.

(i) $\{A_1, A_2\}$ (ii) $\{A_3, A_4, A_5\}$ **(4 Marks)**

Q. 6 (a) Draw the Hasse Diagram of the following sets under the partial order relation divides and indicate which are chains. Justify your answers.

i. A = {2, 4,12, 24}

ii. A = {1, 3, 5, 15, 30} **(8 Marks)**

(b) Let the functions f, g, and h defined as follows :

$f : R \rightarrow R,\ f(x) = 2x + 3$

$g : R \rightarrow R,\ g(x) = 3x + 4$

$h : R \rightarrow R,\ h(x) = 4x$

Find gof, fog, foh, gofoh **(8 Marks)**

(c) Determine Euler Cycle and path in graph shown below. **(4 Marks)**

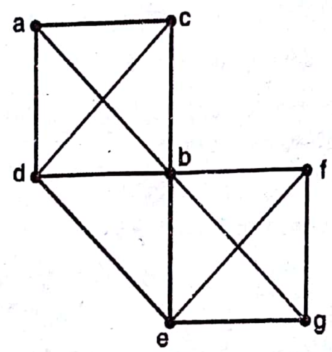


Fig. 1 – Q. 6(c)

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