Total Marks of Question no.		Examiner	
		Moderator	
		Re-Assessor	

Space for Marks	Question No.	START WRITING HERE
	Q1.	choose the correct option for the following questions.
	1.1	Laplace transform of $e^{-5t}(t^2+\sin 2t)$ is
		Ans: option A
		$L\left[e^{-5t}(t^2+\sin 2t)\right] = \frac{2}{(s+3)^2} + \frac{2}{(s+5)^2+4}$
	1.2	If $L\{F(t)\} = 3s/s^2+1$, then $L\{F(2t)\}$ at $s=1$ is
		Ans: option A
		$L\{F(2t)\} = 3/5$
	1.3	Inverse Laplace transform of $1/(s^2+4)$ is
		Ans: option A
		$L^{-1}[1/(s^2+4)] = \int_0^t \cos 2u\,du$
	1.4	Inverse Laplace transform of $F(s) = \frac{6e^{-5s}}{(s+2)^4}$ is
		Ans: option A
		$L^{-1}\{F(s)\} = \begin{cases} 0, & 0<t<5 \\ e^{-2(t-5)}(t-5)^3, & t>5 \end{cases}$
	1.5	If $f(z) = u(x,y)+iv(x,y)$ is analytic then $f'(z)$ is equal to
		Ans: option B
		$f'(z) = \partial u/\partial x + i\,\partial v/\partial x$
	1.6	value of m so that $2x-x^2+my^2$ is analytic is
		Ans. option C
		$m=1$

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	1.7	The value of coefficient of correlation lies between
		Ans: Option D
		-1 to 1
	1.8	Rank correlation coefficient of the following data is

x	23	25	27	29	31	33
y	43	45	47	49	51	53

Ans: Option C

$R = 1$

1.9 Expansion of Fourier series of $f(x) = x$ in $(-1, 1)$ is

Ans: option D

$$f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(n x \pi)$$

1.10 What would be the expectation of the number of failure preceding the first success in an infinite series of independent trials with the constant probability of success p and failure q

Ans: Option B

q/p

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	Q2A	Find Laplace Transform of $e^{-3t} t\sqrt{1-\sin 2t}$

Solution:-

$$1-\sin 2t = \cos^2 t + \sin^2 t - 2\sin t\cos t$$

$$1-\sin 2t = (\cos t - \sin t)^2$$

$$\longrightarrow \sqrt{1-\sin 2t} = \cos t - \sin t$$

taking L.T. we get

$$L\left[\sqrt{1-\sin 2t}\right] = \frac{s}{s^2+1} - \frac{1}{s^2+1} = \frac{s-1}{s^2+1}$$

$\boxed{\text{Now } L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} L\{F(t)\}}$

$$\therefore L\left[t\sqrt{1-\sin 2t}\right] = -\frac{d}{ds}\frac{s-1}{s^2+1}$$

$$= -\left[\frac{(s^2+1)-(s-1)2s}{(s^2+1)^2}\right]$$

$$L\left[t\sqrt{1-\sin 2t}\right] = \frac{s^2-2s-1}{(s^2+1)^2}$$

$\boxed{\text{we know that } L\{F(t)\} = F(s) \Longrightarrow L\{e^{at}F(t)\} = F(s-a)}$

$$\therefore L\left[e^{-3t} t\sqrt{1-\sin 2t}\right] = \frac{(s+3)^2-2(s+3)-1}{[(s+3)^2+1]^2}$$

Total Marks of Question no.		Examiner	
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	Q2. B.	Find inverse Laplace Transform of $\frac{5s^2-15s-11}{(s+1)(s-2)^2}$

Solution:-

$$F(s) = \frac{5s^2-15s-11}{(s+1)(s-2)^2} = \frac{A}{(s+1)} + \frac{B}{(s-2)} + \frac{C}{(s-2)^2} \longrightarrow ①$$

$$\text{If } s=0,\ ① \Longrightarrow -11/4 = A - B/2 + C/4 \longrightarrow ②$$

$$\text{If } s=1,\ ① \Longrightarrow -21/2 = A/2 - B + C \longrightarrow ③$$

$$\text{If } s=3,\ ① \Longrightarrow -11/4 = A/4 + B + C \longrightarrow ④$$

Solving above equations we get,

$$\boxed{A=1,\ B=4,\ C=-7}$$

$$\therefore F(s) = \frac{1}{s+1} + \frac{4}{s-2} - \frac{7}{(s+2)^2}$$

taking Laplace inverse Transform we get,

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1}{s+1}\right\} + 4L^{-1}\left\{\frac{1}{s-2}\right\} - 7L^{-1}\left\{\frac{1}{(s+2)^2}\right\}$$

$$\longrightarrow f(t) = e^{-t} + 4e^{2t} - 7e^{2t} L^{-1}\{1/s^2\}$$

$$\Longrightarrow \boxed{f(t) = e^{-t} + 4e^{2t} - 7e^{2t}t}$$

Total Marks of Question no.		Examiner	
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	Q2.c.	Expand Fourier series for $f(x)=\frac{1}{2}(\pi-x)$, in $(0, 2\pi)$

Solution :- In $(0, 2\pi)$, angle $= nx$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx) \longrightarrow (1)$$

To find a_0 :-

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}(\pi - x)\,dx$$

$$= \frac{1}{4\pi}\left[\frac{(\pi-x)^2}{-2}\right]_{x=0}^{x=2\pi} = \frac{-1}{8\pi}(\pi^2 - \pi^2)$$

$$\Longrightarrow \boxed{a_0 = 0}$$

To find a_n :-

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\int_0^{2\pi} \frac{1}{2}(\pi - x)\cos(nx)\,dx$$

$$= \frac{1}{2\pi}\left\{(\pi - x)\left(\frac{\sin nx}{n}\right) - (-1)\left(\frac{-\cos nx}{n^2}\right)\right\}_{x=0}^{x=2\pi}$$

$$= \frac{1}{2\pi}\left\{\left[0 - \frac{1}{n^2}\right] - \left[0 - \frac{1}{n^2}\right]\right\}$$

$$\Longrightarrow \boxed{a_n = 0}$$

To find b_n :-

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(nx)\,dx = \frac{1}{\pi}\int_0^{2\pi} \frac{1}{2}(\pi - x)\sin(nx)\,dx$$

$$= \frac{1}{2\pi}\left\{(\pi - x)\left(\frac{-\cos nx}{n}\right) - (-1)\left(\frac{-\sin nx}{n^2}\right)\right\}_{x=0}^{x=2\pi}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{\pi}{n} + 0\right] - \left[\frac{-\pi}{n} + 0\right]\right\}$$

$$\Longrightarrow \boxed{b_n = 1/n}$$

using this in equation (1) we get,

$$\boxed{x = \sum_{n=1}^{\infty} \frac{1}{n}\sin(nx)}$$

Total Marks of Question no.		Examiner	
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Q2.D Find the constants a, b, c, d, e, if

$(ax^4+bx^2y^2+cy^4+dx^2-2y^2)+i(4x^3y-exy^3+4xy)$

is analytic.

Sol^n:- given function $F(z)=u+iv$ is analytic

$\Longrightarrow u_x = v_y \longrightarrow (1) \quad v_x = -u_y \longrightarrow (2)$

where $u = ax^4+bx^2y^2+cy^4+dx^2-2y^2$

$v = 4x^3y - exy^3+4xy$

using values of u, v in above equation we get,

$(1) \Longrightarrow 4ax^3+2bxy^2+2dx = 4x^3-3exy^2+4x$

$\Longrightarrow 4a=4, \quad 2b=-3e, \quad 2d=4$

$\Longrightarrow \boxed{a=1} \quad \boxed{d=2} \quad 2b=-3e \longrightarrow (3)$

$(2) \Longrightarrow 12x^2y - ey^3+4y = -(2bx^2y+4cy^3-4y)$

$= -2bx^2y-4cy^3+4y$

$\Longrightarrow 12=-2b, \quad -e=-4c$

$\Longrightarrow \boxed{b=-6} \quad e=4c \longrightarrow (4)$

using value of b in eq^n (3)

$-12=-3e \Longrightarrow \boxed{e=4}$

$\therefore (4) \Longrightarrow 4=4c$

$\Longrightarrow \boxed{c=1}$

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Q2.E Ten students got the following percentage of marks in Mathematics and Statistics

Maths	78	36	98	25	75	82	90	62	65	39
Stats	84	51	91	60	68	62	86	58	53	47

Calculate the coefficient of correlation

Solution :- coefficient of correlation is given by

$$r = \frac{\frac{\Sigma xy}{n} - \bar{x}\bar{y}}{\sigma_x \sigma_y} \longrightarrow (1) \quad \text{where,}$$

$$\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}, \quad \sigma_y = \sqrt{\frac{\Sigma y^2}{n} - \bar{y}^2}$$

$n = 10$, $\Sigma x = 650$ $\Sigma y = 660$

$\Sigma x^2 = 47648$ $\Sigma y^2 = 45784$

$$\boxed{\Sigma xy = 45604}$$

$\bar{x} = \Sigma x / n = 650/10 \Longrightarrow \boxed{\bar{x} = 65}$

$\bar{y} = \Sigma y / n = 660/10 \Longrightarrow \boxed{\bar{y} = 66}$

$$\sigma_x = \sqrt{\frac{47648}{10} - (65)^2} \qquad \sigma_y = \sqrt{\frac{45784}{10} - (66)^2}$$

$\Longrightarrow \boxed{\sigma_x = 23.23}$ $\boxed{\sigma_y = 14.91}$

$\therefore$ eq^n (1) gives,

$$r = \frac{\frac{45604}{10} - (65)(66)}{(23.23)(14.91)}$$

$\Longrightarrow \boxed{r = 0.78}$

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	Q 2.F	A bolt is manufactured by three machines A, B, C. A turns out twice as many times B and machine B and C produces equal number of items. 3% of bolt produced by A and B are defective and 5% of bolt produced by machine 'C' are defective.
		All bolt are put into one stock pile and one is chosen from this pile. What is the probability that it is defective.
		Solution:- Let capacity of machine B is x
		$\Longrightarrow$ capacity of machine A = $2x$ and machine C = x
		$\therefore \quad 2x+x+x=1 \Longrightarrow x=0.25$
		$\Longrightarrow \quad P(A)=0.5, \ P(B)=0.25, \ P(C)=0.25$
		Let D: defective bolt.
		given $P(D/A) = 03\% = 0.03$, $P(D/B) = 0.03$
		$P(D/C) = 0.05$
		$\therefore$ probability that bolt is defective
		$= P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)$
		$= (0.5)(0.03) + (0.25)(0.03) + (0.25)(0.05)$
		$= \boxed{0.035}$

Total Marks of Question no.		Examiner	
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	Q3.A	Using Laplace transform evaluate $\int_0^\infty \left[\frac{\sin 2t + \sin 3t}{te^t}\right] dt$

Soln :- $\int_0^\infty e^{-t}\left[\frac{\sin 2t + \sin 3t}{t}\right] dt = L\left[\frac{\sin 2t + \sin 3t}{t}\right]\Big|_{s=1} \longrightarrow (1)$

$L(\sin 2t + \sin 3t) = \frac{2}{s^2+2^2} + \frac{3}{s^2+3^2}$

$\therefore L\left[\frac{\sin 2t + \sin 3t}{t}\right] = \int_s^\infty L(\sin 2t + \sin 3t)\, dt$

$= \int_s^\infty \left(\frac{2}{s^2+2^2} + \frac{3}{s^2+3^2}\right) ds$

$= \left[\frac{2}{2}\tan^{-1}\frac{s}{2} + \frac{3}{3}\tan^{-1}\frac{s}{3}\right]_{s=s}^{s=\infty}$

$= [\tan^{-1}\infty + \tan^{-1}\infty] - [\tan^{-1}(\frac{s}{2}) + \tan^{-1}(\frac{s}{3})]$

$= (\frac{\pi}{2} + \frac{\pi}{2}) - \tan^{-1}\left[\frac{s/2 + s/3}{1 - (\frac{s}{2})(\frac{s}{3})}\right]$

$\therefore$ at $s=1$ $\quad$ Ans $= \pi - \tan^{-1}\left(\frac{1/2 + 1/3}{1 - 1/6}\right)$

$= \pi - \tan^{-1}(1)$

$= \pi - \pi/4$

$\boxed{\text{Ans} = 3\pi/4}$

Total Marks of Question no.		Examiner	
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Q3.A. Using Laplace transform evaluate $\int_0^\infty \left[\frac{\sin 2t + \sin 3t}{t e^t}\right] dt$

Soln :- $\int_0^\infty e^{-t}\left[\frac{\sin 2t + \sin 3t}{t}\right] dt = L\left[\frac{\sin 2t + \sin 3t}{t}\right]\Bigg|_{s=1} \longrightarrow (1)$

$$L(\sin 2t + \sin 3t) = \frac{2}{s^2+2^2} + \frac{3}{s^2+3^2}$$

$$\therefore L\left[\frac{\sin 2t + \sin 3t}{t}\right] = \int_s^\infty L(\sin 2t + \sin 3t)\, ds$$

$$= \int_s^\infty \left(\frac{2}{s^2+2^2} + \frac{3}{s^2+3^2}\right) ds$$

$$= \left[\frac{2}{2}\tan^{-1}\frac{s}{2} + \frac{3}{3}\tan^{-1}\frac{s}{3}\right]_{s=s}^{s=\infty}$$

$$= \left[\tan^{-1}\infty + \tan^{-1}\infty\right] - \left[\tan^{-1}\left(\frac{s}{2}\right) + \tan^{-1}\left(\frac{s}{3}\right)\right]$$

$$= \left(\frac{\pi}{2} + \frac{\pi}{2}\right) - \tan^{-1}\left[\frac{s/2 + s/3}{1 - \left(\frac{s}{2}\right)\left(\frac{s}{3}\right)}\right]$$

$\therefore$ at $s=1$ $\quad$ Ans $= \pi - \tan^{-1}\left(\frac{1/2 + 1/3}{1 - 1/6}\right)$

$$= \pi - \tan^{-1}(1)$$

$$= \pi - \pi/4$$

$\boxed{\text{Ans} = 3\pi/4}$

Total Marks of Question no.		Examiner	
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Space for Marks	Question No.	START WRITING HERE
	Q3.B.	Using convolution theorem, Find inverse Laplace Transform of $\frac{s}{(s^2+1)(s^2+4)}$

Solution:- Let $f_1(s) = \frac{1}{s^2+1^2}$, $f_2(s) = \frac{s}{s^2+2^2}$

$\Longrightarrow$ $f_1(t) = \sin t$ $f_2(t) = \cos 2t$

By convolution Thm,

$$F(t) = \int_0^t f_1(t-u)\, f_2(u)\, du$$

$$= \int_0^t \sin(t-u) \cos(2u)\, du$$

$$= \int_0^t \left[\frac{\sin(t+u) + \sin(t-3u)}{2}\right] du$$

$$= \frac{1}{2}\left[-\cos(t+u) - \frac{\cos(t-3u)}{-3}\right]_{u=0}^{u=t}$$

$$= \frac{1}{2}\left[-\cos 2t + \frac{1}{3}\cos 2t\right] - \frac{1}{2}\left[-\cos t + \frac{\cos t}{3}\right]$$

$\longrightarrow$ $\boxed{F(t) = -\frac{1}{3}\cos 2t + \frac{1}{3}\cos t}$

Total Marks of Question no.		Examiner	
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	Q3.c	Expand Fourier series for $f(x)=1-x^2$ in $(-1,1)$

Solution :- $(-1,1)=(-l,l) \Longrightarrow l=1 \Longrightarrow$ angle $= n x\pi$

$f(x)$ is even function $\Longrightarrow \boxed{b_n=0}$

$$f(x)=a_0+\sum_{n=1}^{\infty} a_n \cos(nx\pi) \longrightarrow (1)$$

To find a_n :- $a_n=\frac{1}{1}\int_{-l}^{l} f(x)\cos(nx\pi)\,dx = 2\int_0^1 (1-x^2)\cos(nx\pi)\,dx$

$$=2\left[(1-x^2)\left(\frac{\sin nx\pi}{n\pi}\right)-(-2x)\left(\frac{-\cos nx\pi}{n^2\pi^2}\right)+(-2)\left(\frac{-\sin nx\pi}{n^3\pi^3}\right)\right]_0^1$$

$$=2\left[0-\frac{2(-1)^n}{n^2\pi^2}+0\right]-\left[0+0+0\right]$$

$$\longrightarrow \boxed{a_n=\frac{-4(-1)^n}{n^2\pi^2}}$$

To find a_0 :- $a_0=\frac{1}{2}\int_{-1}^{1} f(x)\,dx = \frac{2}{2}\int_0^1 (1-x^2)\,dx$

$$=\left[x-\frac{x^3}{3}\right]_0^1$$

$$=\left(1-\frac{1}{3}\right)-(0)$$

$$\longrightarrow \boxed{a_0=2/3}$$

using values of a_0, a_n in (1) we get,

$$(1-x^2)=\frac{2}{3}+\sum_{n=1}^{\infty}\frac{(-4)(-1)^n}{n^2\pi^2}\cos(nx\pi)$$

Total Marks of Question no.		Examiner	
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	Q3.D.	Find analytic function $f(z)=u+iv$ in terms of z if $v=\sinh 2y/(\cosh 2y+\cos 2x)$

Solution:

Let $f(z)=u+iv$ be analytic $\Longrightarrow u_x=v_y,\ v_x=-u_y$

$$f'(z)=u_x+iv_x$$

$$=v_y+iv_x$$

$$\therefore f'(z)=\frac{(\cosh 2y+\cos 2x)2\cosh 2y-\sinh 2y\, 2\sinh 2y}{(\cosh 2y+\cos 2x)^2}$$

$$+i\frac{-\sinh 2y}{(\cosh 2y+\cos 2x)^2}(-2\sin 2x)$$

put $x=z,\ y=0$

$$\therefore f'(z)=\frac{(1+\cos 2z)2-0}{(1+\cos 2z)^2}+i0$$

$$=\frac{2}{1+\cos 2z}$$

$$=\frac{2}{2\cos^2 z}$$

$$f'(z)=\sec^2 z$$

integrating w.r.t. z we get

$$\boxed{f(z)=\tan z+C}$$

Total Marks of Question no.		Examiner	
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	Q3. E	obtain the equations of line of regression for the following data:

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

Soln: $n = 8$ $\Sigma x = 544$ $\Sigma y = 552$

$\Sigma xy = 37560$ $\Sigma x^2 = 37028$

$\Sigma y^2 = 38132$

line of regression of y on x is $y = ax + b \longrightarrow (1)$

$\Sigma y = a\Sigma x + nb \Longrightarrow 552 = (544)a + 8b$

$\Longrightarrow$ $\Sigma xy = a\Sigma x^2 + b\Sigma x \Longrightarrow 37560 = (37028)a + (544)b$

solving we get, $a = 0.66$ $b = 23.67$

using this in eqn (1) we get,

$$\boxed{y = (0.66)x + 23.67}$$

line of regression of x on y is $x = ay + b \longrightarrow (2)$

$\Sigma x = a\Sigma y + bn \Longrightarrow 544 = (552)a + 8b$

$\Longrightarrow$ $\Sigma xy = a\Sigma y^2 + b\Sigma y \Longrightarrow 37560 = (38132)a + (552)b$

solving we get,

$a = 0.55$, $b = 30.36$

using this in equation (1) we get,

$$\boxed{x = (0.55)y + 30.36}$$

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	Q3.F.	A random variable has following distribution

x	-2	-1	0	1	2	3
P	0.1	K	0.1	2K	0.2	3K

i) Find the constant K

ii) Find the mean and variance of X.

Solution:.

as given distribution is p.d.f.

$$\Sigma p(x) = 1$$

$$\Longrightarrow 0.1 + K + 0.1 + 2K + 0.2 + 3K = 1 \Longrightarrow 6K = 0.6$$

$$\Longrightarrow \boxed{K = 0.1}$$

$$\text{mean} = E(X) = \Sigma x p(x)$$

$$= -0.2 - K + 0 + 2K + 0.4 + 9K$$

$$= 10K + 0.2$$

$$= 10(0.1) + 0.2$$

$$\Longrightarrow \boxed{\text{mean} = E(X) = 1.2}$$

$$E(X^2) = \Sigma x^2 p(x)$$

$$= 0.4 + K + 0 + 2K + 0.8 + 27K$$

$$= 30K + 1.2$$

$$= 30(0.1) + 1.2$$

$$\Longrightarrow \boxed{E(X^2) = 4.2}$$

$$\therefore \text{variance } V = E(X^2) - [E(X)]^2$$

$$= 4.2 - (1.2)^2$$

$$\Longrightarrow \boxed{V = 2.76}$$

Total Marks of Question no.		Examiner	
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	Q4. A	Find the Laplace transform of $\int_0^t e^{-3u}\cos^2 u \, du$

Soln:-

$$\cos^2 u = \frac{(1+\cos 2u)}{2}$$

$$\longrightarrow L[\cos^2 u] = \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+4}\right]$$

we know that $L\{f(t)\} = F(s)$, then

$$L\{e^{at} f(t)\} = F(s-a)$$

$$\longrightarrow L\{e^{-3u}\cos^2 u\} = \frac{1}{2}\left[\frac{1}{s+3} + \frac{s+3}{(s+3)^2+4}\right]$$

we know that $L\{\int_0^t f(t)\,dt\} = \frac{1}{s} L\{f(t)\}$

$$\longrightarrow L\{\int_0^t e^{-3u}\cos^2 u \, du\} = \frac{1}{s} L\{e^{-3u}\cos^2 u\}$$

$$= \frac{1}{2s}\left[\frac{1}{s+3} + \frac{s+3}{(s+3)^2+4}\right]$$

Total Marks of Question no.		Examiner	
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Q4.B Find inverse Laplace transform of $\frac{1}{s}\log\sqrt{\left(\frac{s^2+9}{s^2+16}\right)}$

Soln:- Let $f_1(s)=\frac{1}{s}$, $f_2(s)=\log\left(\frac{s^2+9}{s^2+16}\right)^{1/2}$

$$f_2(s)=\frac{1}{2}\log\left(\frac{s^2+9}{s^2+16}\right)=\frac{1}{2}\left[\log(s^2+9)-\log(s^2+16)\right]$$

$$\longrightarrow f_2'(s)=\frac{2s}{2(s^2+3^2)}-\frac{2s}{2(s^2+4^2)}$$

taking L^{-1} we get,

$\boxed{F_1(t)=1}$, $L^{-1}[f_2'(s)]=\cos 3t-\cos 4t$

$$\Longrightarrow -t\,F_2(t)=\cos 3t-\cos 4t$$

$$\Longrightarrow \boxed{F_2(t)=(\cos 4t-\cos 3t)/t}$$

By convolution Thm

$$L^{-1}\{f(s)\}=F(t)=\int_0^t F_1(t-u)\,F_2(u)\,du$$

$$\longrightarrow F(t)=\int_0^t\left(\frac{\cos 4u-\cos 3u}{u}\right)du$$

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	Q4.C.	Find half range cosine series for, $F(x)=(x-1)^2, 0<x<1$

Soln:- $(0,1)=(0,l) \Longrightarrow l=1 \Longrightarrow$ angle $= n x \pi$

Half range cosine series is given by

$$F(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n x \pi) \longrightarrow (1)$$

$$a_0 = \frac{1}{1}\int_0^1 F(x)\,dx$$

$$= \int_0^1 (x-1)^2 dx = \left[\frac{(x-1)^3}{3}\right]_0^1 = 0-\left(\frac{-1}{3}\right)$$

$$\Longrightarrow \boxed{a_0 = 1/3}$$

$$a_n = \frac{1}{1/2}\int_0^1 F(x)\cos(n x\pi)\,dx = 2\int_0^1 (x-1)^2 \cos(n x \pi)\,dx$$

$$= 2\left\{(x-1)^2\left(\frac{\sin n x\pi}{n\pi}\right) - [2(x-1)]\left(\frac{-\cos n x \pi}{n^2\pi^2}\right) + (2)\left(\frac{-\sin n x \pi}{n^3\pi^3}\right)\right\}_{x=0}^{x=1}$$

$$= 2\left\{[0+0+0]-\left[0-\frac{2}{n^2\pi^2}+0\right]\right\}$$

$$\Longrightarrow \boxed{a_n = 4/n^2\pi^2}$$

using this in equation (1) we get,

$$(x-1)^2 = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2}\cos(n x \pi)$$

Total Marks of Question no.		Examiner	
		Moderator	
		Re-Assessor	

Space for Marks	Question No.	START WRITING HERE
	Q4. D	Find the family of curves orthogonal to the family of curves $x^3y - xy^3 = c$

Solution:- Let $u = x^3y - xy^3$

we know that if $f(z) = u + iv$ is analytic then u, v are orthogonal to each other.

To Find v:

$$dv = v_x\, dx + v_y dy$$

$$= -u_y\, dx + u_x\, dy \quad [\because u_x = v_y,\ v_x = -u_y]$$

$$dv = -(x^3 - 3xy^2)\, dx + (3x^2y - y^3)\, dy$$

which is exact D.E. integrating,

$$\int dv = \int_{y=\text{cont}} (-x^3 + 3xy^2)\, dx + \int_{\text{free from } x} (3x^2y - y^3)\, dy + c$$

$$\Longrightarrow \quad v = -\frac{x^3}{4} + \frac{3x^2y^2}{2} - \frac{y^4}{4} + c$$

which is required orthogonal trajectory.

Total Marks of Question no.		Examiner	
		Moderator	
		Re-Assessor	

Space for Marks	Question No.	START WRITING HERE
	Q4. E.	Fit straight line of the form $y = ax+b$ to the following data

x	1	3	5	7	8	10
y	8	12	15	17	18	20

soln :-

$y = ax+b \longrightarrow ①$

$\Longrightarrow \Sigma y = a\Sigma x + bn \longrightarrow ②$ $\Sigma xy = a\Sigma x^2 + b\Sigma x \longrightarrow ③$

$n = 6$

$\Sigma x = 1+3+\cdots+10 = 34$

$\Sigma y = 8+12+\cdots+20 = 90$

$\Sigma x^2 = 1+9+25+\cdots+100 = 248$

$\Sigma xy = 8+36+\cdots+200 = 582$

$\therefore ② \Longrightarrow 90 = (34)a + 6b$

$③ \Longrightarrow 582 = (248)a + (34)b$

Solving we get,

$\boxed{a = 1.30}$ $\boxed{b = 7.63}$

using this in ①

$\boxed{y = (1.30)x + 7.63}$

Total Marks of Question no.		Examiner	
		Moderator	
		Re-Assessor	

Space for Marks	Question No.	START WRITING HERE

Q4.F. A random variable x has probability density function,

$$f(x) = \begin{cases} Kx^2 e^{-x} & , x>0,\ K>0 \\ 0 & , \text{otherwise} \end{cases}$$

Find K and hence find mean, variance.

Sol^n:- given distribution is continuous p.d.f.

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1 \implies \int_0^{\infty} Kx^2 e^{-x}\,dx = 1$$

$$\implies K\left[(x^2)(-e^{-x}) - (2x)(e^{-x}) + (2)(-e^{-x})\right]_0^{\infty} = 1$$

$$\implies K[0-(-2)] = 1 \implies 2K = 1$$

$$\implies \boxed{K = 0.5}$$

$$\text{mean} = E(X) = \int_0^{\infty} x f(x)\,dx$$

$$= \int_0^{\infty} x\, Kx^2 e^{-x}\,dx = K\int_0^{\infty} e^{-x} x^3\,dx$$

$$= K\,\Gamma 4 \qquad \left[\because \int_0^{\infty} e^{-x} x^n\,dx = \Gamma n+1\right.$$

$$= 0.5\,(3!) \qquad \left[\because \Gamma n = (n-1)!\right.$$

$$\implies \boxed{\text{mean} = E(X) = 3}$$

$$E(X^2) = \int_0^{\infty} x^2 f(x)\,dx = \int_0^{\infty} x^2 Kx^2 e^{-x}\,dx$$

$$= K\int_0^{\infty} e^{-x} x^4\,dx$$

$$= K\,\Gamma 5 = \frac{1}{2}(4!)$$

$$\implies \boxed{E(X^2) = 12}$$

$$\text{variance } V = E(X^2) - (E(X))^2$$

$$= 12 - (3)^2$$

$$\implies \boxed{V = 3}$$