Exam3 : April 28.

Last lecture : Mon. April 24.

Knapsack.

Input : n items : $1, 2, 3, \cdots, n$.

item i

- wt w_i
- profit p_i

Knapsack of capacity W.

Objective : To pack items in the knapsack so that the total wt of all items in the knapsack is at most W and the total profit is maximized.

Scratch work.

$OPT[n] = OPT[n-1]$, if nth item is not in the knapsack.

$= \boxed{OPT[n-1]} + \boxed{p_n}$, if n^{th} item is in the knapsack.

$OPT[n] = \max\{ OPT[n-1], \boxed{OPT[n-1] + p_n} \}$

Subproblems

$P[j, C]$: maximum profit obtained by considering items $1, 2, \cdots, j$ to be packed in a knapsack of capacity C.

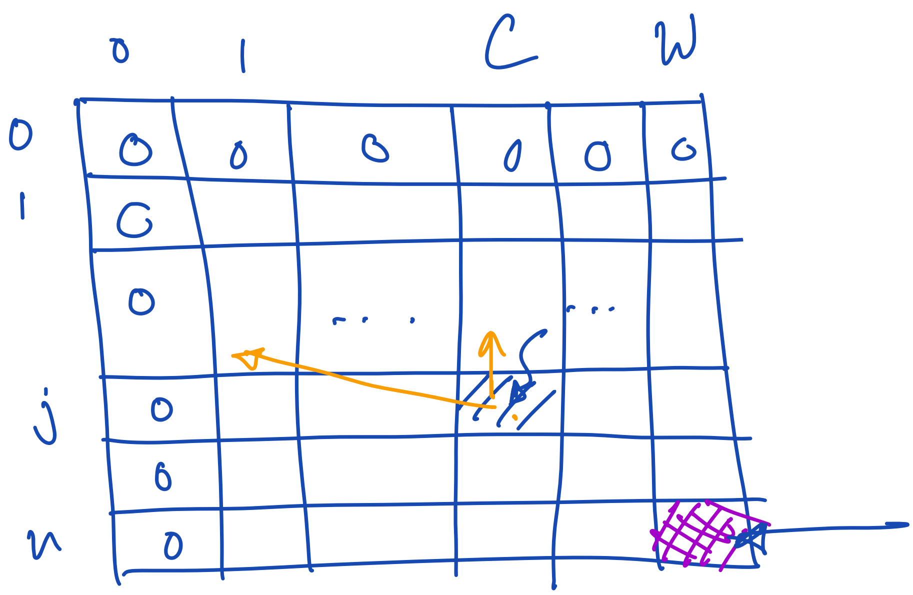
Our answer

: $P[n, W]$

Recurrence

$$P[j, C] = \begin{cases} 0, & \text{if } j = 0 \text{ or } C = 0 \\ \max\{ P[j-1, C], \; P[j-1, C - w_j] + p_j \}, & \text{o.w.} \end{cases}$$

We have to fill up the table of size $O(nW)$.




```
for i ← 0 to n do
    P[i, 0] ← 0

for C ← 0 to W do
    P[0, C] ← 0
```

for each $j \leftarrow 1$ to n do

for each $C \leftarrow 1$ to W do

if $(wt(j) \leq c)$ then

$P(j, C) \leftarrow \max\{P[j-1, C],$

$P[j-1, C-w_j]$

$+ p_j\}$

else $p[j, C] \leftarrow p[j-1, C]$

return $P[n, W]$

Running time: $O(nW)$

ItemsKnapsack (n, P)

$S \leftarrow \phi$

$i \leftarrow n$, $C \leftarrow w$.

while $i > 0$ and $C > 0$ **do**

  if $P[i, C] > P[i-1, C]$ then

    add i to S.

    $C \leftarrow C - w_i$

    $i \leftarrow i-1$

  else

    $i \leftarrow i-1$

Example :

item	wt	profit
1	$\boxed{3}$	12
2	1	5

3	2	10
4	1	2

$W = 4.$

$$P[j, C] = \begin{cases} 0, & \text{if } j = 0 \text{ or } C = 0 \\ \max\{P[j-1, C],\ P[j-1, C - w_j] + p_j\}, & \text{o.w} \end{cases}$$

P:

	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	12	12
2	0	5	5	12	17
3	0	5	10	15	17
4	0	5	10	15	17

$S = \{2, 1\}$

$$P[1,1] = P[0,1]$$

$$P[1,2] = P[0,2]$$

$$P[1,3] = \max\{P[0,3],\ P[0,0]+12\}$$

$$= \max\{0,\ 12\}$$

$$P[1,4] = \max\{P[0,4],\ P[0,1]+12\}$$

$$P[2,1] = \max\{P[1,1],\ \underline{P[1,0]} + 5\}$$

$$= \max\{0, 5\}$$

$$P[2,2] = \max\{P[1,2],\ P[1,1] + 5\}$$

$$P[2,3] = \max\{P(1,3),\ P[1,2] + 5\}$$

$$P[2,4] = \max\{P[1,4],\ P[1,3]+5\}$$

$$P[3,1] = P[2,1] = 5$$

$$P(3,2) = \max\{P[2,2],\ P[2,0]+10\}$$

$$= \max\{5, 10\} = \boxed{10}.$$

$$P(3,3) = \max\{P(2,3),\ P(2,1)+10)$$

$$= \max\{12, 15\}$$

$$= 15$$

$$P[3,4] = \max\{P[2,4], P[2,2] + 10\}$$

$$= \max\{17, 15\}$$

$$= 17$$

$$P(4,1) = \max\{P(3,1),\ P(3,0)+2\}$$

$$= 5$$

$$P(4,2) = \max\{P(3,2),\ P(3,1)+2$$

$$= \max\{10,\ 7\}$$

$$= 10$$

$$P[4,3] = \max\{P[3,3],\ P[3,2]+2\}$$

$$= \max\{15,\ 12\}$$

$$P[4,4] = \max\{P[3,4],\ P[3,3]+2\}$$

$$= \max\{17,\ 17\}$$

Longest Common Subsequence.

Input: Two sequences : X & Y.

$X = \langle x_1, x_2, \ldots, x_n \rangle$

$Y = \langle y_1, y_2, \ldots, y_m \rangle$

Obj: To find the longest common subsequence of X, Y.

X : A T T T A G C C C G A

Y : T A T A C G C C G T A