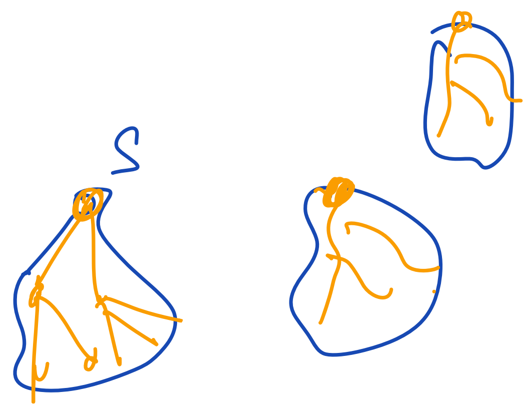
- Exam 2 - next Friday (March 24)
  - Stable matching
  - Divide & Conquer
  - Asymptotic notation.
- HW 4

↳ SaeePatil → never attended school.
Rajas Wande → Grade 9 in London.

## Breadth First Search (BFS)

- BFS
- properties of BFS
- Graph representation



BFS (G, s)

for each $u \in V$ do
    disc[u] ← false

$O(n)$

→ disc[s] ← True
→ $T \leftarrow \phi$
$i \leftarrow 0$
→ $L_i \leftarrow \{s\}$
→ while $L_i \neq \phi$ do ← ✓
→ $L_{i+1} \leftarrow \phi$

for each $u \in L_i$ do

  for each $v \in N(u)$ do

    if $disc[v] = false$

      $L_{i+1} \leftarrow L_{i+1} + v$

      Add $(u,v)$ to $T$

      $disc[v] \leftarrow True$

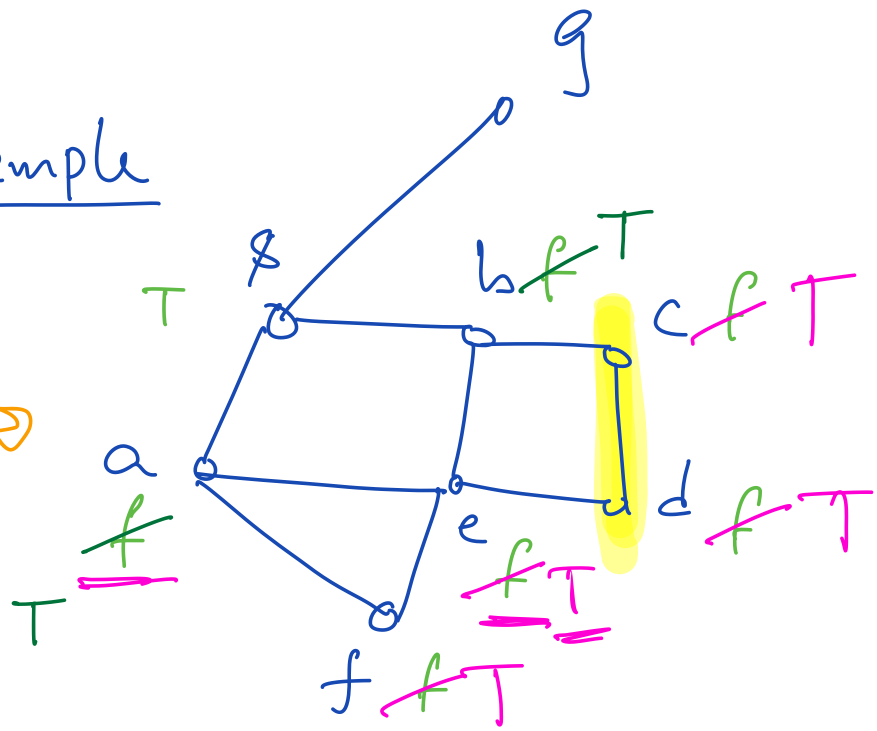
$O(1)$

"$h_i$"

$i \leftarrow i+1$

return $T$

Example



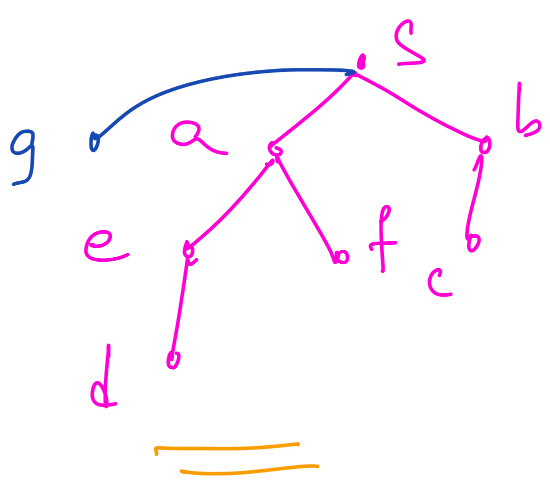
$L_0 \leftarrow \{s\}$

$i \leftarrow \cancel{1}\ 2$

$L_1 = \{a, \underline{b}\}$

$L_2 = \{e, f, c\}$

$L_3 = \{d\}$




$\leftarrow L_0$

$\leftarrow L_1$ $\leftarrow$ output.

$\leftarrow L_2$

$\leftarrow L_3$

Running time. $\underline{O(n^3)}$. ✓

$O(n+m)$. ✓

#vertices ($n$) #edges ($m$).

- each vertex belongs to exactly one layer.

- for each vertex in $G$, we go through each of its neighbors and say "hi".

$$\therefore \text{Total time}: O\left(n + \sum_u \deg(u)\right)$$

$$= O(n + 2m)$$

$$= \underline{\Theta(n+m)}$$

Application of BFS.

## Testing Bipartiteness.

**Input**: Undirected graph $G = (V, E)$

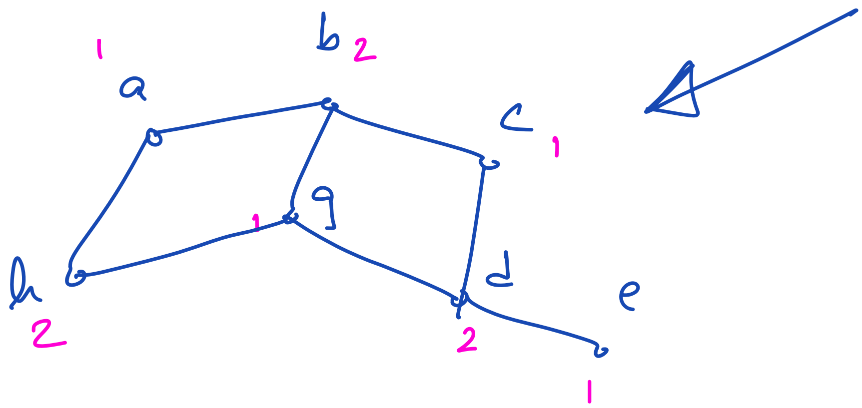
**Obj**: Output YES, if $G$ is bipartite

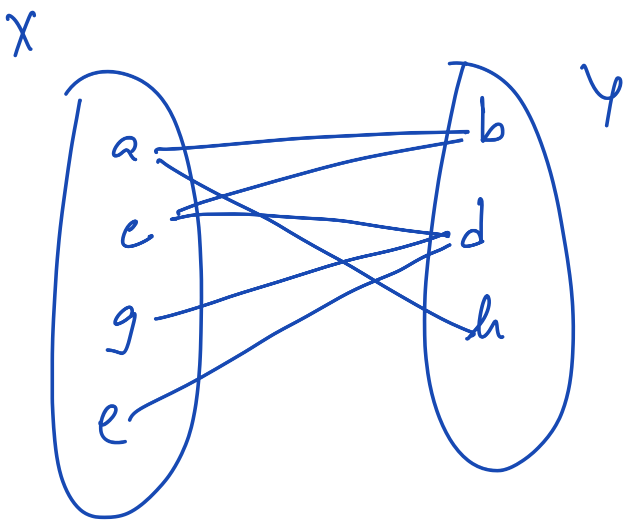
NO, o.w.

Bipartite graphs are 2-colorable graphs.

In other words, we can partition the vertices of $G$ into two sets $X, Y$ s.t.

all edges have one end pt in X and the other in Y.




<u>Lemma</u> : G is bipartite iff G has no odd cycles.

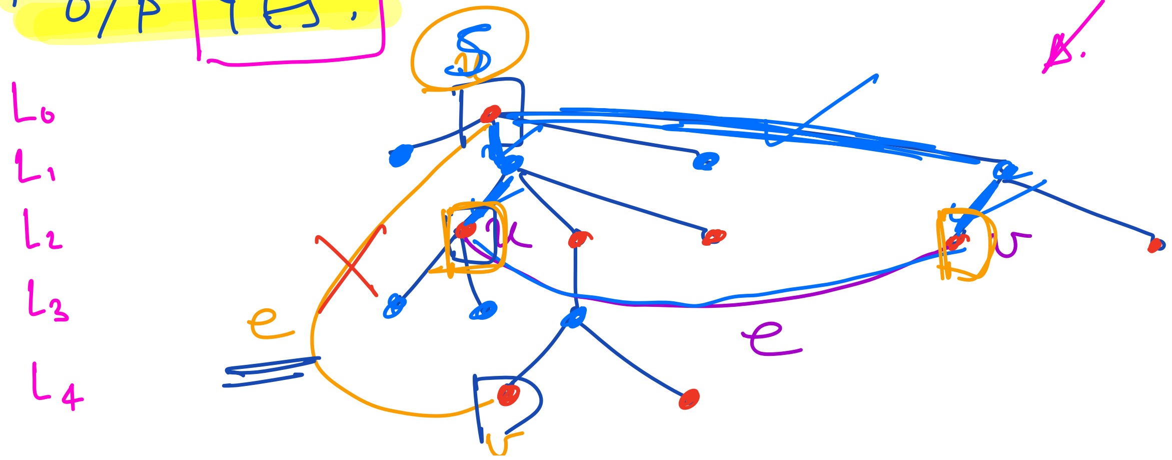
# Algorithm

$\Theta(n+m)$

1. $T \leftarrow BFS(G, s)$ // $s$ is an arbitrary vertex

$O(n)$
2. Color vertices in even layers RED &
   color " " odd " BLUE.

$O(m)$
3. for each $e = (u,v) \in E(G) \setminus E(T)$ do
4. if $u$ & $v$ are colored the same then
5. o/p $\boxed{\text{NO}}$
6. o/p $\boxed{\text{YES.}}$



Theorem: Our alg. outputs the correct answer.

Proof: If our answer is YES then we know that our answer is correct because we have checked the coloring of endpts of every single edge.

It remains to show that if our algorithm outputs NO then G is not bipartite.

Why would our algorithm output "NO"?

Because there is an edge $e = (u, v)$ in $G$ s.t. $u$ & $v$ are both colored RED or both colored BLUE.

Note that by Property 3 of BFS, $u$ & $v$ must be in the same level.

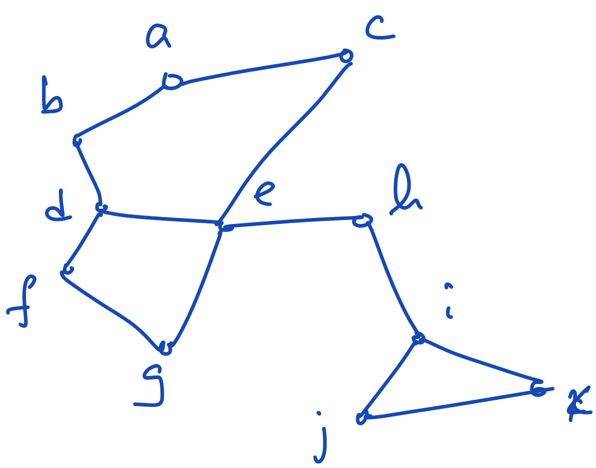
Now let $w$ be the lowest common ancestor of $u$ & $v$ in the BFS tree $T$.

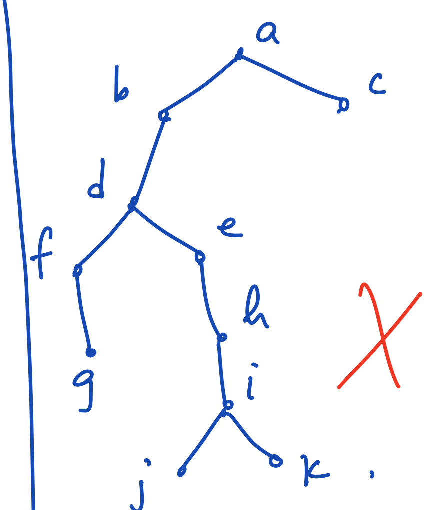
Consider the cycle $w \sim u — v \sim w$.

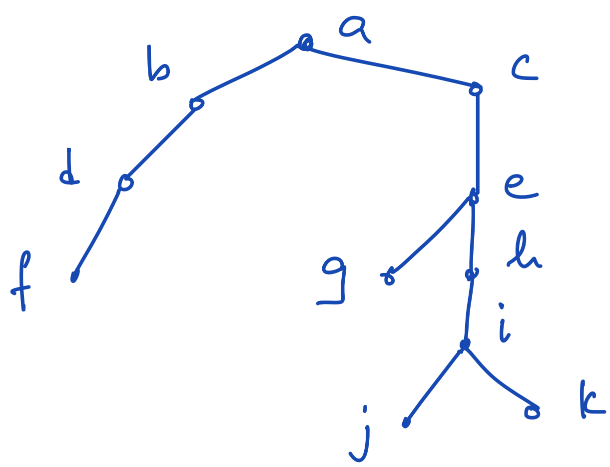
Since $u$ & $v$ are at the same distance 'h' from $w$, the length of the cycle is of $2h+1$, which is odd. Thus $G$ has an odd length cycle & hence $G$ is not bipartite.

Running time: $O(n+m)$

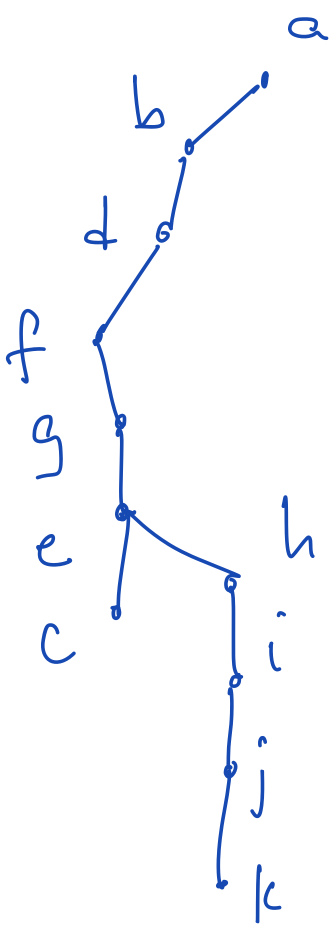
Depth first Search.






BFS tree




DFS tree

DFS (G)

for each $u \in V$ do

color [u] ← white

π [u] ← NIL

time ← 0

for each $u \in V$ do

if color [u] is white then

DFS_VISIT (u)

each vertex

- color
  - white, Gray, Black
- disc. time, finish time.
- π : parent

DFS_VISIT (u)

color (u) ← Gray

time ← time + 1

d [u] ← time

for each $v \in N(u)$ do

if color(v) is white then

$\pi(v) \leftarrow u$

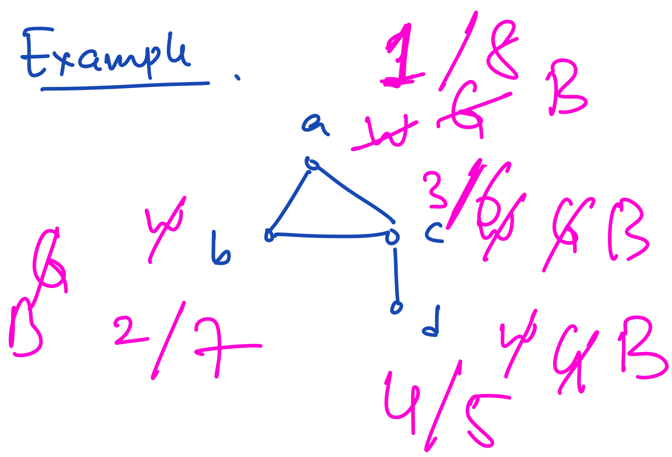
DFS-VISIT(v)

→ color(u) ← Black

time ← time + 1

f[u] ← time.

Example.



time ← ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ 8