Fourier series

① Drichlet's conditions to exist F.S.

1) $f(x)$ is integrable & have single values.

2) $f(x)$ have finite max & min.

$f(x)$ not having F.S. are, $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\sinh^{-1}x$, $\tanh^{-1}x$, $\tan x$.

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos nx + b_n \sin nx\right]$$

(In radian)

$[0, 2\pi]$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos nx + b_n \sin nx\right]$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$[-\pi, \pi]$

$f(x)$ even

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = 0$$

$f(x)$ odd

$$a_0 = 0$$

$$a_n = 0$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

$$a_0 = \frac{1}{2L}\int_0^{2L} f(x)\,dx \qquad a_n = \frac{1}{L}\int_0^{2L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$b_n = \frac{1}{L}\int_0^{2L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

Similar for $[-L, L]$ //

parsevel's Identity

$$\frac{1}{2\pi}\int_0^{2\pi} (f(x))^2\,dx = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

Similar for $(-\pi, \pi)$

Half Range sine / cosine series.

① For cosine series.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) \qquad \begin{aligned} &\pi \to L \\ &nx \to \frac{n\pi x}{L} \end{aligned}$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx \qquad a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)\,dx$$

Complex Variables.

- $z = x + iy = re^{i\theta}$
- $w = f(z) = u(x,y) + i\,v(x,y) = u + iv$
- $|f(z)|^2 = f(z) \cdot f^*(\bar{z}) = u^2 + v^2$

$f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}$

C.R. equations (if $f(z)$ is analytic function)

cartesian form

- $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$
- $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

polar form

- $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$
- $\frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$

- if $f(z)$ is analytic then,

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] = \frac{4\partial^2}{\partial z \partial \bar{z}}$$

- If $f(z)$ is analytic then u & v are $\boxed{\text{Harmonic}}$ functions. $\nabla^2 = 0$ (also called as Laplace eqn)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

$$\nabla^2 = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}\right)\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}\right)$$

In $f(z)$ if u is given we can find v, as v is u's Harmonic conjugate.
can be found using Milne Thompson

Milne Thomson theorem.

- If u given.

$$f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y} \quad \underline{\text{put } x=z,\ y=0}$$

then Integrate wrt z,

$$f(z) = \phi_1 + i\,\underbrace{\phi_2}_{\text{Harmonic conjugate of } u.}$$

- If v given

$$f'(z) = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x} \qquad \text{put } x=z,\ y=0$$

then Integrate w.r.t. z,

$$f(z) = \phi_1 + i\,\underbrace{\phi_2}_{\text{Harmonic conjugate of } v.}$$

- ~~If u+v given~~.

$$f(z) = u + iv \qquad if(z) = iu - v$$

$$(1+i)f(z) = (u-v) + i(u+v)$$

$$F(z) = U + iV$$

If $u+v$ is given then $V = u+v$

if $u-v$ is given then $U = u-v$

then solve accordingly by M.T.T.

* Statistics

1) Karl pearson's coeff of correlation (r)

$$r = \frac{\operatorname{cov}(x,y)}{\sigma_x \sigma_y} \qquad -1 \le r \le 1$$

$$= \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

Assumed mean method.

2) Spearman's coeff of Rank correlation (R)

$$R = 1 - 6\left[\frac{\Sigma(R_1-R_2)^2}{N^3-N}\right] \quad \text{— non repeated}$$

* For repeated.

$$R = 1 - \left\{\frac{6\left[\Sigma(R_1-R_2)^2 + \frac{1}{12}(m_1^3-m_1) + \frac{1}{12}(m_2^3-m_2) + \cdots\right]}{N^3-N}\right\}$$

Type I: Regression line y on x

$$y = ax + b$$

$$(y-\bar{y}) = b_{yx}(x-\bar{x})$$

$$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} = \frac{\Sigma(y-\bar{y})(x-\bar{x})}{\Sigma(x-\bar{x})^2}$$

slope $= b_{yx}$

Regression line x on y.

$$x = ay + b$$

$$(x-\bar{x}) = b_{xy}(y-\bar{y}) \qquad b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y} = \frac{\Sigma(y-\bar{y})(x-\bar{x})}{\Sigma(y-\bar{y})^2}$$

$$r^2 = b_{xy}\, b_{yx}$$

- Fitting of straight line.

① y on x.

$y = ax + b$

$\Sigma y = a\Sigma x + bN$ — ① $a = b_{yx}$

$\Sigma xy = a\Sigma x^2 + b\Sigma x$ — ②

find a & b

② x on y

$x = ay + b$

$\Sigma x = a\Sigma y + bN$ — ①

$\Sigma xy = a\Sigma y^2 + b\Sigma y$ — ②

$\frac{1}{a} = b_{xy}$

- Fitting of 2 degree curve.

$y = ax^2 + bx + c$

$\Sigma y = a\Sigma x^2 + b\Sigma x + cN$ — ①

$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$ — ②

$\Sigma x^2y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$ — ③

find a, b, c values.