Graph theory.

- A graph is collection of object with relationship object is called vertices or node and relationship is called edge.

- A graph is represented by a ordered pair $G = (V, E)$ where,

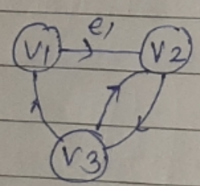
  V = Non-empty set of vertices

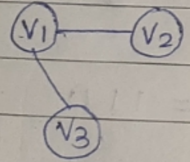
  E = Set of edges

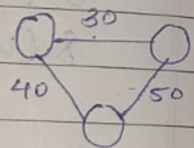
Graph Classification

According direction

weighted



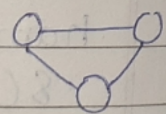




directed graph

- unweighted

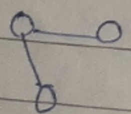
↓



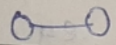
$e_1 = (v_1, v_2)$ $\neq (v_2, v_1)$

$e_1 = \{v_1, v_2\}$ $= \{v_2, v_1\}$

connected



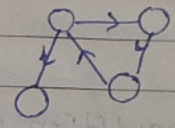
disconnected

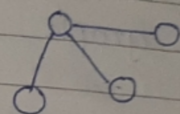


- every subgraph which are connencted is called as component

connected graph has 1 component

disconnected graph has more than 1 component





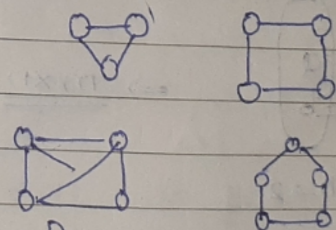
cyclic graph . Acyclic group

- Special graph

G_1 O ⟶ trival graph

G_2 O O O ⟶ NULL graph

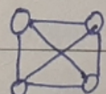
G_3



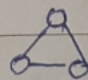
D3

⟶ Regular graph degree ~~for~~ of each vertex are same

- complete graph

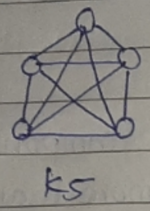


K4



K3

every vertex is connected to all other vertices



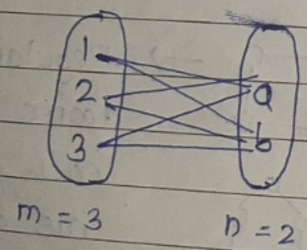
degree of each vertices should be '$n-1$'

- Bipartite graph

if group vertices are partition into two set such that all vertices of same partition are disconnected.

-

- Complete bipartite graph



$\Rightarrow$ $m \times n$

- Graph traversal

Euler path | ~~Hamitton~~ Hamiltanian path

- a line subgraph in which every edge visited only once is called Euler path

- a line subgraph in which every vertex only once is called Hamitton path.

- Theorem
if graph have exactly the odd degree vertex then graph euler path start & end at odd vertices

- Theorem
- If in graph sum of degree of each adjacent pair of vertices is < n-1 then graph never have hamitton path.

Euler graph

- a closed euler path is called euler circuit ~~in~~ and graph in which exists Euler circuit is called Euler graph.

- Theorem :
If degree of each vertex is even more than graph is called Euler graph

~~Hamilton~~

Hamiltonian graph :

- A closet hamiltonian path is called Hamitonian circuit and graph is called hamiltanian graph.

• theorem.

if degree of each vertex is $< \frac{n}{2}$ then graph never have hamiltinian circuit

20/10/2022

• logic

- proportional logic

p : Ram is good boy

q : Shyam is good boy

Ram is good boy $\underset{\wedge}{\underline{\text{and}}}$ shyam is good boy

p ... ∧ ... q

$= p \wedge q$.

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logical operator.

i] And ∧

- binary operator.

Truth table :

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2] OR ∨

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

3] Not ~

~~p q p∧q~~

~~T F~~

p	~p
T	F
F	T

implication

$p \to q \equiv$ if p than q

$\equiv \sim p \vee q = \sim q \to \sim p$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Q. $(p \to q) \vee q \to p$

p	q	$p \to q$	$q \to p$	$p \to q \vee q \to p$
T	T	T	~~T~~	~~T~~
T	F	F	~~T~~	~~T~~
F	T	T	~~F~~	~~T~~
F	F	T	~~T~~	~~T~~

$(p \to q) \vee q$	$(p \to q) \vee q \to p$
T	T
F	T
T	F
T	F

Q. $(p \to q) \wedge p \to q$

p	q	$p \to q$	$(p \to q) \wedge p$	$(p \to q) \wedge p \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

1] Idempotent law

$\to$

~~$p \wedge q \Leftrightarrow p$~~ $\quad p \wedge p \to p$

~~$p \vee p \Leftrightarrow p$~~ $\quad p \vee p \to p$

2] Commutative law

$p \wedge q \to q \wedge p$

$p \vee q \to q \vee p$

3] Associative law

$(p \vee q) \vee r = p \vee (q \vee r)$

$(p \wedge q) \wedge r = p \wedge (q \wedge r)$

4] De-morgan law

$\sim (p \wedge q) \to \sim p \vee \sim q$

$\sim (p \vee q) \to \sim p \, \cancel{\vee} \wedge \sim q$

$\to \sim p \vee q$

$\wedge$ – and – conjunction
$\vee$ → or – disjunction

08/11/22

- Mathematical Induction (MI)

- Induction base
check the given statements / series is true or not for some value of n.
$n=0$
$n=1$ $\quad f(0) =$ True (check)

- Induction Hypothesis

Assume that statement / series is true upto ~~→~~ $n=k$

$f(k) =$ True (assume)

- Induction step :

check for $n = k+1$
$f(k+1) = ?$

Q. P. T. by mathematical induction

$$1^2+2^2+3^2+4^2+\dots = \frac{n}{6}(n+1)(2n+1)$$

→

$$f(n) = 1^2+2^2+3^2+\dots = \frac{n}{6}(n+1)(2n+1)$$

1] Induction base

$n=1$ $\quad f(1) = 1^2 = 1$ — L.H.S
$f(1) = \frac{1}{6}(1+1)(2+1)$ — R.H.S

for $n=1$ — True

2] Induction hypothesis.

assume true for $n=k$

$$f(k) = 1^2+2^2+\dots+k^2 = \frac{k}{6}(k+1)(2k+1) \text{ — I}$$

3] Induction step.
check for $n=k+1$

$$f(k+1) = \underbrace{1^2+2^2+3^2+\dots+k^2}+(k+1)^2$$

$$= \frac{k}{6}(k+1)(2k+1)+(k+1)^2$$

$$= (k+1)\left[\frac{k(2k+1)}{6}+(k+1)\right]$$

$$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$$

$$= \frac{k+1}{6}\,(2k^2+7k+6)$$

$$= \frac{k+1}{6}\,(k+2)\,(2k+3)$$

$$= \frac{k+1}{6}\,((k+1)+1)\,(2(k+1)+1)$$

True for $n = k+1$

$\therefore$ by MI. gives. series is true $n \geq 1$

Q. P.T.

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

$\rightarrow$ Induction base

$n=1 \quad f(1) = \frac{1}{1 \times 3} = \frac{1}{3}$

$f(1) = \frac{1}{2+1} = \frac{1}{3}$

for $n=1$ — True

Induction hyposis

assume true for $n=k$

$$f(k) = \frac{1}{1 \times 3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2k-1)(2k+1)}$$

$$= \frac{k}{2k+1} \quad \text{— I}$$

Induction step.

check for $n=k+1$

~~f(k+1) =~~

$f(k+1)$

$$= \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k-1)(2k+3)}$$

$$= \frac{k}{2k+1} + \frac{1}{(2k-1)(2k+3)}$$

$$= \frac{k[(2k-1)(2k+3)] + 2k+1}{(2k+1)(2k-1)(2k+3)}$$

$$= \frac{k[2k^2+6k-2k-3] + 2k+1}{(2k+1)(2k-1)(2k+3)}$$

$$= \frac{2k^3+4k^2-3k+2k+1}{(2k+1)(2k-1)(2k+3)}$$

$$= \frac{2k^3+4k^2-k+1}{(2k+1)(2k-1)(2k+3)}$$

$=$

Q. $11^{n+2} + 12^{2n+1}$ is divisible by 133, $n \geq 0$

Induction base :

$f(n) = 11^{n+2} + 12^{2n+1}$

~~f(0) =~~

$n = 0$

$f(0) = 11^2 + 12$

$= 121 + 12$

$= 133$

$\therefore f(0)$ is divisible by 133 → True.

Induction hypothesis

True for $n = k$

$f(k) = 11^{k+2} + 12^{2k+1}$

$= 133x$, $x \in I$

Induction step

check for $n = k+1$

$f(k+1) = 11^{k+3} + \cancel{12^{2k+3}}\ 12^{2k+3}$

$= 11^{k+2} \cdot 11 + 12^{2k+1} \cdot 12^2$

$= 11^{k+2} \cdot 11 + 11 \cdot 12^{2k+1} + 133 \cdot 12^{k+1}$

$= 11(11^{k+2} + 12^{2k+1}) + 133 \cdot 12^{k+1}$

$= 11 \cdot (133x) + 133 \cdot 12^{k+1}$

$= 133(11x + 12^{k+1})$

$= 133y$ $\quad y \in I$

P.T. $7^n - 1$ is divisible by 6.

$f(k+1) = 133y$ $\quad y \in I$

Induction base

$f(n) = 7^n - 1$

$n = 1$

$f(1) = 7 - 1$

$= 6$

$f(1)$ is divisible by 6.

Induction hyposis

True for $n = k$

$f(k) = 7^k - 1$

$= 6x$

Induction step

check for $n = k+1$

$f(k+1) = \cancel{7}\ 7^{k+1} - 1$

$= 7^k \cdot 7 - 1$

$= 7^k \cdot 7 - 7 \cdot 1 + 6 \cdot 1$

$= 7(7^k - 1) + 6$

$= 7 \cdot 6x + 6$

$= 6(7x + 1)$

- permutation & combination

Q. If 13 are selected from class prove that atleast two of them must have their birthday on the same month of the year.

Q. P.T. if 8 people are assemble in room then two ro same day of week.

Q How many friends must have to guarantee 5 of them will have birthday in same month.

Q. P.T seven colours are used to paint 15 bicycles then 8 of them must have same colour.