Module 6 :- Counting

1.

Ans. Extended pigeonhole principle :-
if 'n' pigeons are assigned to 'm' pigeonholes $(km < n)$, then atleast one pigeonhole must have $\left(\frac{n-1}{m}\right)+1$ pigeons.

2.

Ans. There 7 colours to be used to paint 50 bicycles.

$\therefore\ 7 < 50$

Extended pigeon-hole principle is applicable.
By extended pigeon hole principle, one pigeon hole must have atleast $\left[\frac{n-1}{m}\right]+1$ pigeons.

Here,

$n = 50,\ m = 7$

$$= \left[\frac{n-1}{m}\right]+1$$

$$= \left[\frac{50-1}{7}\right]+1$$

$$= 7+1$$

$$= 8$$

Hence, proved that atleast 8 of them will be same color.

8.

Ans. As, there are atleast five of them have their birthday in same month.

$n = ?\qquad m = 12$

By extended pigeon hole principle,

$$\therefore \left(\frac{n-1}{m}\right)+1 = 5$$

$$\therefore \frac{n-1}{12} = 4$$

$$\therefore \boxed{n = 49}$$

4.

Ans. As, there are five possible grades A, B, C, D, E

$m = 5$

Also, there atleast six will recieve same grade.

By extended pigeon hole principle,

$\therefore \left[\frac{n-1}{m}\right] + 1 = 6$

$\therefore \left[\frac{n-1}{5}\right] = 5$

$\therefore n - 1 = 25$

$\boxed{\therefore n = 26}$

$\therefore$ 26 students will recieve same grade in DSGT subject.

5.

Ans. $\alpha^2 + 2\alpha - 3 = 0$

Here,

$\boxed{\alpha = 1, -3}$

$\therefore a_r = A_1 \alpha_1^r + A_2 \cdot \alpha_2^r$

$\therefore A_1 + A_2 = 1$

$-3A_1 + A_2 = 2$

$\boxed{A_1 = -\frac{1}{4}}$ $\boxed{A_2 = \frac{5}{4}}$

$\therefore a_r = -\frac{1}{4}(-3)^r + \frac{5}{4}(1)^r$

$$\boxed{\begin{array}{l} \therefore a_0 = 1 \\ \\ \therefore a_1 = \frac{3}{4} + \frac{5}{4} = 2 \end{array}}$$

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6.

Ans. $a_n = 4a_{n-1} + 5a_{n-2}$, $a_1 = 2$, $a_2 = 6$

$\therefore a_n - 4a_{n-1} - 5a_{n-2} = 0$

$\therefore x^2 - 4x - 5 = 0$

$\therefore x = 5, -1$

$\therefore a_n = u(-1)^n + v(5)^n$

$\underline{\underline{n=1}}$ $\underline{\underline{n=2}}$

$\therefore a_1 = -u + 5v$ $\therefore a_2 = u + 25v$

$\therefore 2 = -u + 5v$ —(i) $\therefore 6 = u + 25v$ —(ii)

$\cancel{u} + 25v = 6$

$-\cancel{u} + 5v = 2$

$30v = 8$

$v = \frac{4}{15}$

$-u + \frac{4}{3} = 2$

$u = -\frac{2}{3}$

$$\boxed{\therefore a_n = -\frac{2}{3}(-1)^n + \frac{4}{15}(5)^n}$$

7.

Ans. $a_n = 2a_{n-1} - a_{n-2}$ $a_1 = 1.5$, $a_2 = 3$

$\therefore a_n - 2a_{n-1} + a_{n-2} = 0$

$\therefore x^2 - 2x + 1 = 0$

$\therefore x = 1, 1$

$\therefore a_n = u + vn$

$\underline{\underline{n=1}}$ $\underline{\underline{n=2}}$

$1.5 = u + v$ —(i) $3 = u + 2v$ —(ii)

$\cancel{u} + v = 1.5$

$\cancel{u} + 2v = 3$

$-v = -1.5$

$v = 1.5$ & $u = 0$

$a_n = (1.5)n$

Sequence is 1.5, 3, 4, ...

4

Ans. $a_n - 7a_{n-1} + 10a_{n-2} = 6+8n$ $\quad a_0=1,\ a_1=2$

$\therefore a_n = nA_0 + A_1$

$\therefore a_{n-1} = (n-1)A_0 + A_1$

$\therefore a_{n-2} = (n-2)A_0 + A_1$

$\therefore nA_0 + A_1 - 7((n-1)A_0 + A_1) + 10((n-2)A_0 + A_1) = 6+8n$

$n=0$

$\therefore \cancel{A_0 + A_1 + 10((-1)A_0 + A_1) = 6}$

$\therefore \cancel{A_0} + A_1 - 7(-A_0 + A_1) + 10(-2A_0 + A_1) = 6$

$\therefore \cancel{A_0} + A_1 + 7A_0 - 7A_1 - 20A_0 + 10A_1 = 6$

$\therefore 4A_1 - 13A_0 = 6$ —(i)

$n=1$

$\therefore A_0 + A_1 - 7(0 + A_1) + 10(-A_0 + A_1) = 14$

$\therefore A_0 + A_1 - 7A_1 + 10A_1 - 10A_0 = 14$

$\therefore 4A_1 - 9A_0 = 14$ —(ii)

$$\begin{array}{r} 4A_1 - 13A_0 = 6 \\ 4A_1 - 9A_0 = 14 \\ - \quad + \quad - \\ \hline -4A_0 = -8 \end{array}$$

$\boxed{A_0 = 2}$

$4A_1 - 26 = 6$

$4A_1 = 32$

$\boxed{A_1 = 8}$

9.

Ans. (i) if they are to be of same colour.

$\Rightarrow \; ^6C_4 + {}^5C_4$

$\Rightarrow \; 15 + 5$

$\Rightarrow \; 20$

$\therefore$ 20 ways can 4 balls be drawn from the box if they are to be of same colour.

(ii) if they are to be of colour.

$\Rightarrow \; ^{11}C_4$

$\Rightarrow \; 330$

330 ways can 4 balls be drawn from the box if they are to be of colour.

10.

Ans.

Boys $\Rightarrow$ 6
Girls $\Rightarrow$ 4 } children $\Rightarrow$ 4 to be selected.

$\Rightarrow \; ^6C_1 \cdot {}^4C_3 + {}^6C_2 \cdot {}^4C_2 + {}^6C_3 \cdot {}^4C_1 + {}^6C_4$

$\Rightarrow \; 6 \times 4 + 15 \times 6 + 20 \times 4 + 15$

$\Rightarrow \; 24 + 90 + 80 + 15$

$\Rightarrow \; 209$

11.

Ans.

Ram
Mohan
Sohan $\Big\}$ 8 books divided.

Ram $\Rightarrow$ 4 books.
Mohan
Sohan $\Big\}$ 2 books each.

$\Rightarrow \; {}^8C_4 \times {}^4C_2 \times {}^2C_2$

$\Rightarrow \; 420.$

∴ There are 420 way that 8 different books be divided among three students.

12.

Ans. ☆ Mutual Exclusion-Inclusion principle for three sets.

let D be assumed as B∪C.

So, $(A \cup B \cup C) = (A \cup D)$

Now,

$|A \cup D| = |A| + |D| - |A \cap D| \quad \text{—(i)}$

$|D| = |B \cup C| = |B| + |C| - |B \cap C| \quad \text{—(ii)}$

~~(A∪D) = (A∪B∪C) =~~

$\because \; |A \cap D| = |A \cap (B \cup C)|$

$= |(A \cap B) \cup (A \cap C)|$

$= |A \cap B| + |A \cap C| - |A \cap B \cap C| \quad \text{—(iii)}$

Substitute eqⁿ (ii) & (iii) in eqⁿ (i),

$|A \cup B \cup C| = |A| + |B| + |C| - |B \cap C| - \cancel{|A \cap B| \; |A \cap C|}$
$- |A \cap B| - |A \cap C| + |A \cap B \cap C|$

$$\boxed{\therefore \; |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|}$$

Hence, proved Mutual Exclusion - Inclusion principle for three sets.

13.

Ans. (i) 2, 2, 2, 2, 2.

The generating function for the sequence 2, 2, 2, 2, 2 is as follows :-

$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$

$a_i = 2$, $0 \leq i \leq 5$

$\therefore$ $2 + 2x + 2x^2 + 2x^3 + 2x^4 + 2x^5$ is the required generating function.

(ii) 2, 2, 2 ....

The given set is (2, 2, 2, ...)

The generating function for sequence 2, 2, ... is

$a(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \dots + a_n z^n + \dots$

$a_0 = a_1 = a_2 = a_n = 2$

$a(z) = 2 + 2z + 2z^2 + 2z^3 + \dots + 2z^n + \dots$

$\boxed{a(z) = \left(\frac{2}{1-z}\right)}$