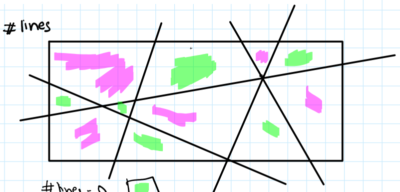
Recitation: 11.02

11 February 2023 21:12

1. Suppose you take a piece of paper and draw a bunch of straight lines, no one exactly on top of another, that completely cross the paper. This divides the paper up into polygonal regions. Prove by induction that you can always color the various regions using only *two* colors, so that any two regions that share a boundary line have different colors. Regions that share only a boundary point are permitted to have the same color.

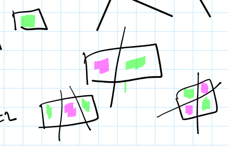
lines



#lines = 0

Base Case. #lines = 1

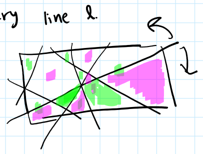
lines = 2



Induction Hypothesis #lines = k, claim holds.

Induction Step # lines = k+1

Chose an arbitrary line l
Remove it.



2. There are n cities in a OneWayCountry (country in which every road is a One-Way road). Every pair of cities is connected by exactly one direct one-way road. Show that there exists a city which can be reached from every other city either directly or via at most one other city.

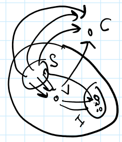
We will prove by induction on n.

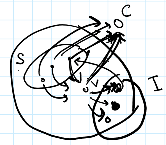
Base Case $n=2$ ✓

I.H. $n=k$ assume

I.S. $n=k+1$

chose an arbitrary city c
remore it.





5. Prove or disprove the follow
at least two people who have th
friend of a person q then q is al

→ Assume for
person has
Thus, the nu
0, 1, 2, .., n -

ving. In any group of two or more people, there are always
ne same number of friends. (Assume that if a person p is a
so a friend of p.)

contradiction that each
different # friends.
mber of friends must be
t.

n people
$\nearrow$ 0, 1, 2, ..., n-1