Vidyalankar Institute of Technology
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University of Mumbai

Vidyalankar Institute of technology

Department of Computer Engineering

Two D Transformations

Course : Computer Graphics (CG)

Sem-III

by

Sachin Deshpande

Obtain a composite x^n for rotating a pt. $p(x,y)$ w.r.t. some pivot (reference point) $P_r(x_r, y_r)$ by given angle θ in anticlockwise manner.

Sol^n

1.) Transl^n

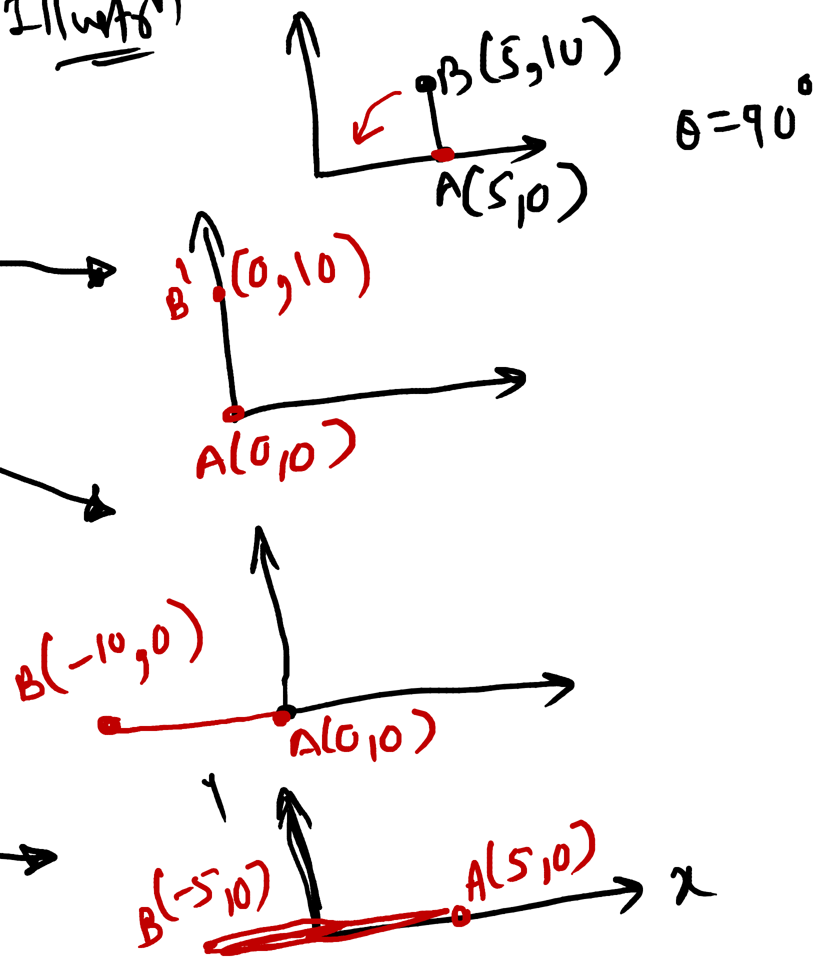
$T_x = -x_r \quad T_y = -y_r$

2.) Rot^n (θ)

3.) Transl^n

$T_x = x_r \quad T_y = y_r$

Illustr^n



To obtain composite x^n

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\overset{3}{\begin{bmatrix} 1 & 0 & +x_r \\ 0 & 1 & +y_r \\ 0 & 0 & 1 \end{bmatrix}} \overset{2}{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}} \overset{1}{\begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & x_r \\ \sin\theta & \cos\theta & y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & -x_r\cos\theta + y_r\sin\theta + x_r \\ \sin\theta & \cos\theta & -x_r\sin\theta - y_r\cos\theta + y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x\cos\theta - y\sin\theta - x_r\cos\theta + y_r\sin\theta + x_r = (x - x_r)\cos\theta - (y - y_r)\sin\theta + x_r$$

$$y' = (x - x_r)\sin\theta + (y - y_r)\cos\theta + y_r$$

Properties of Composite X^n

1.) Associative (i.e $(A \cdot B) \cdot C = A \cdot (B \cdot C)$)

2.) Sequence of X^n_s is important, if changed the result may be different

Means : May not be commutative

(i.e $A \cdot B \neq B \cdot A$)

Prove that rotaⁿ & scaling x^ns are commutative if & only if $\theta = 90°$ (i.e $\pi/2$ radians) & scaling is uniform (i.e $S_x = S_y = S$)

Solⁿ: To prove $S(S_x, S_y) \cdot R(\theta) = R(\theta) \cdot S(S_x, S_y)$

LHS

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_x\cos\theta & -S_x\sin\theta & 0 \\ S_y\sin\theta & S_y\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

RHS

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_x\cos\theta & -S_y\sin\theta & 0 \\ S_x\sin\theta & S_y\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

LHS ≠ RHS ∴ Rotⁿ & Scaling x^n are not commutative

However if $\theta = \pi/2 \Rightarrow \sin\theta = 1$ & $\cos\theta = 0$

& if Scaling is Uniform $\Rightarrow S_x = S_y = S$

substituting

$$\begin{bmatrix} 0 & -S & 0 \\ S & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & -S & 0 \\ S & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

Hence proved

To prove that 2 successive Translations are additive in nature.

Sol^n: To prove $T_1(tx_1, ty_1) \cdot T_2(tx_2, ty_2) = T(tx_1+tx_2, ty_1+ty_2)$

LHS

$$\begin{bmatrix} 1 & 0 & tx_1 \\ 0 & 1 & ty_1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & tx_2 \\ 0 & 1 & ty_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & tx_2+tx_1 \\ 0 & 1 & ty_2+ty_1 \\ 0 & 0 & 1 \end{bmatrix}$$

RHS

$$\begin{bmatrix} 1 & 0 & tx_1+tx_2 \\ 0 & 1 & ty_1+ty_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$\because$ LHS = RHS Hence proved

To prove that 2 successive Rotn x^n_s are additive in nature.

Soln: To prove

$$R_1(\theta_1) \cdot R_2(\theta_2) = R(\theta_1+\theta_2)$$

LHS RHS

$$\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2) & 0 \\ \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2 & (-\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ LHS = RHS

Hence proved

To prove that 2 Successive Scaling $X^n s$ are multiplicative in nature

Sol^n: To prove $S_1(sx_1, sy_1) \cdot S(sx_2, sy_2) = S(sx_1 \cdot sx_2, sy_1 \cdot sy_2)$

Ex: Find the sequence of X^ns to convert a circle centered at (x_c, y_c) & with radius $= r$, into an ellipse centered at (x_e, y_e), with semimajor $= r_x$ & semiminor $= r_y$

sol^n: Sequence

1.) To translate, so that center of circle coincides with origin.

$T(-x_c, -y_c)$

2) $S\left(S_x = \frac{r_x}{r},\ S_y = \frac{r_y}{r}\right)$

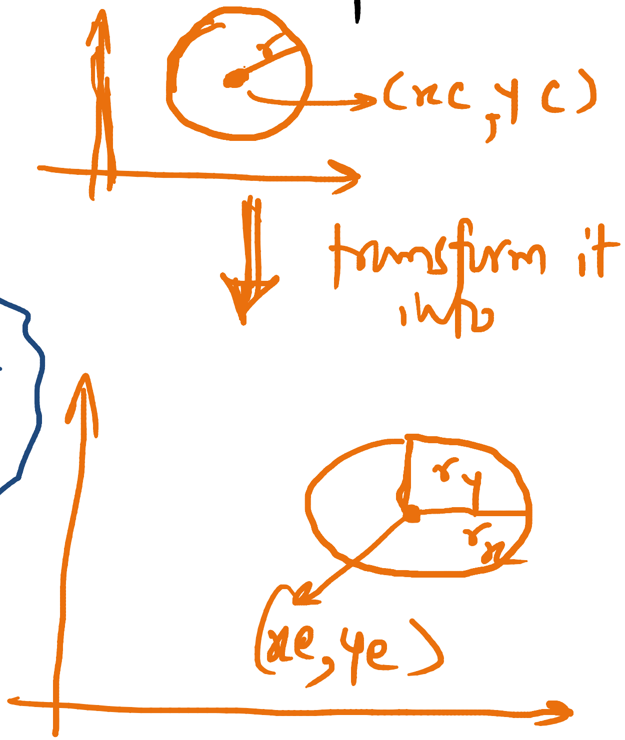
$r * \frac{r_x}{r} = r_x$

$r * \frac{r_y}{r} = r_y$

3.) Translation

$T(T_x = x_e,\ T_y = y_e)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \overset{(3)}{\begin{bmatrix} 1 & 0 & x_e \\ 0 & 1 & y_e \\ 0 & 0 & 1 \end{bmatrix}} \overset{(2)}{\begin{bmatrix} \frac{r_x}{r} & 0 & 0 \\ 0 & \frac{r_y}{r} & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{(1)}{\begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



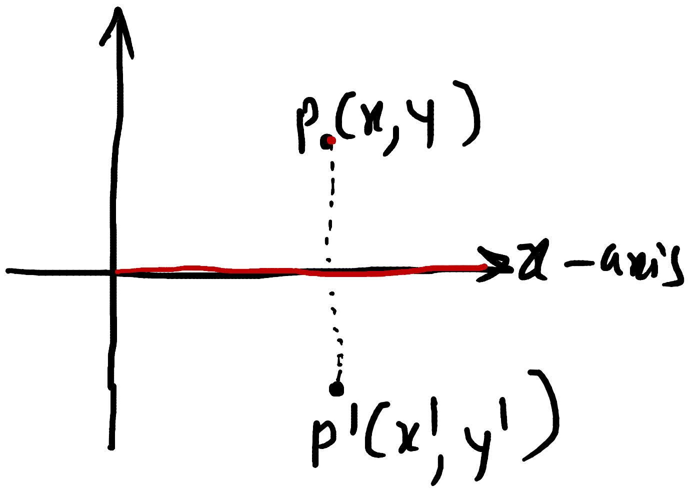
Reflection

purpose : To obtain mirror image

In 2-D, reflection is taken w.r.t a reflection axis

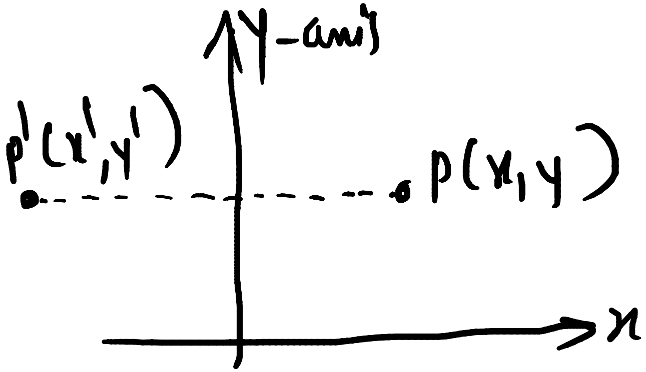
I.) Refl^n w.r.t. x-axis

$$x' = x$$
$$y' = -y$$



$$Refl^n_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

II.) Refl^n w.r.t. y-axis

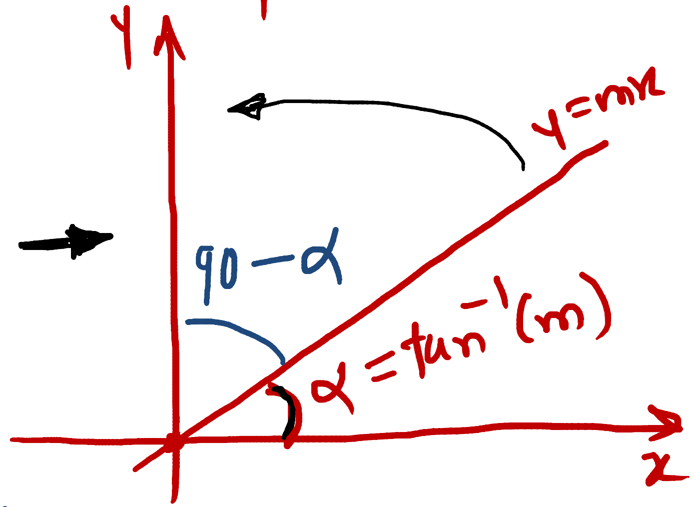


$$x' = -x$$
$$y' = y$$

$$Refl^n_y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find sequence of x^n's to reflect a point w.r.t. a line $\boxed{y = mx}$

(i.e $y = mx$ is reflec^n axis)



$\underline{\underline{sol^n}}$: Sequence of x^n

① Rot^n $(90 - \alpha)$ where $\alpha = \tan^{-1}(m)$

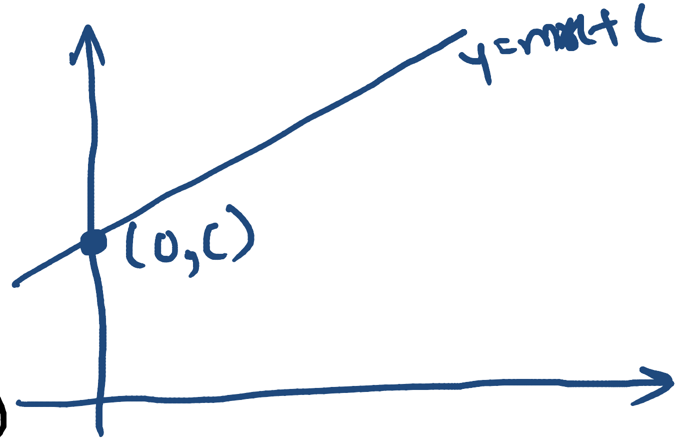
→ To make $y = mx$ to allign with y-axis

② Refl^n_y (Apply refl^n w.r.t. y-axis)

③ Rot$^n(-(90 - \alpha))$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{pmatrix} \cos(-(90-\alpha)) & -\sin(-(90-\alpha)) & 0 \\ \sin(-(90-\alpha)) & \cos(-(90-\alpha)) & 0 \\ 0 & 0 & 1 \end{pmatrix} \overset{②}{\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \overset{①}{\begin{bmatrix} \cos(90-\alpha) & -\sin(90-\alpha) & 0 \\ \sin(90-\alpha) & \cos(90-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

To find sequence of X^ns to reflect a 2D pt. w.r.t. $y = mx + c$



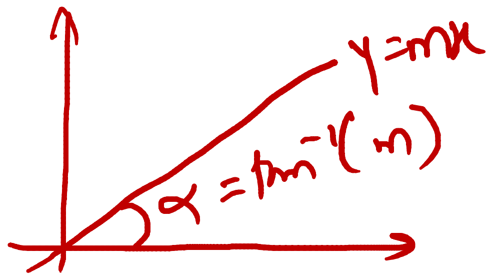
Sol^n :

1.) Transl^n $(T_x = 0, T_y = -c)$

2.) Rot^n $(-\alpha)$ where $\alpha = \tan^{-1}(m)$

To make reference axis to fall on x-axis

After 1.



3. Refl^n w.r.t. x-axis

4. Rot^n (α)

5. Transl^n $(T_x = 0, T_y = c)$

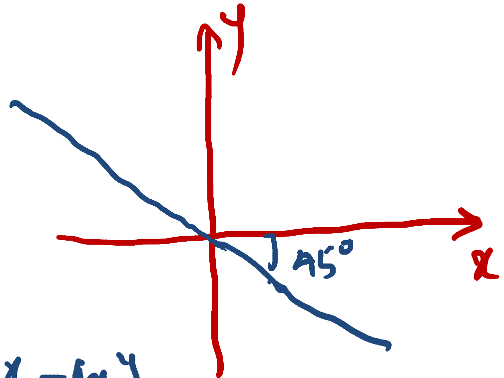
① To reflect a point $p(x,y)$ w.r.t. line $y=-x$

Sol^n

1.) Rot^n $(\pi/4)$

2.) Refl^n w.r.t. x-axis

3.) Rot^n $(-\pi/4)$



② To reflect a point $p(x,y)$ w.r.t. line $y=x$

Shear Transformⁿ

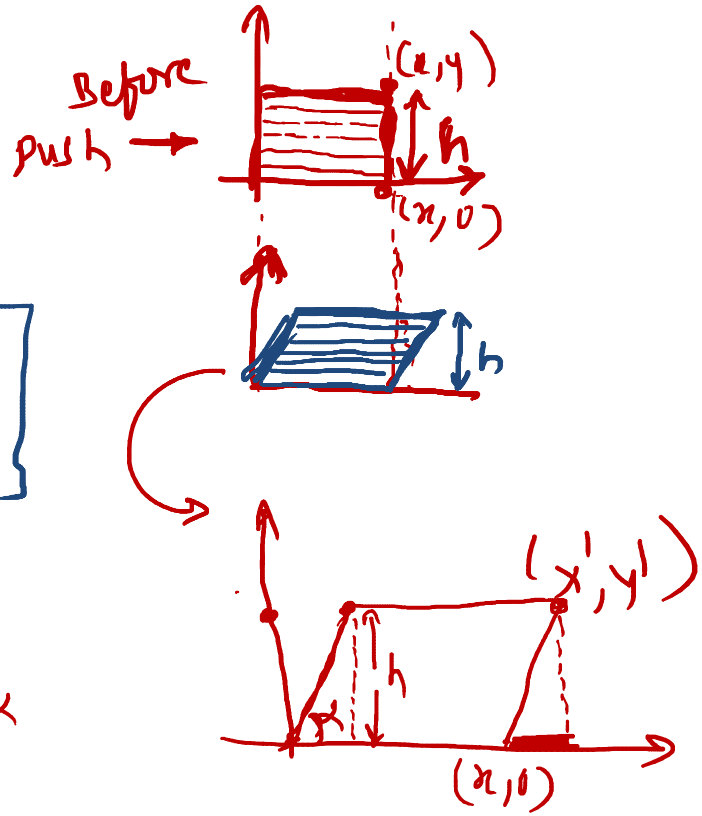
Purpose : To apply shearing effect
(Assuming that the object is composed of layers alligned in the dirⁿ of shear, & we make these layers to slide over each other)

x-dirⁿ shear Transformⁿ

sh_x is x-dirⁿ shear factor

$$\boxed{\begin{aligned} x' &= x + sh_x \cdot y \\ y' &= y \end{aligned}}$$

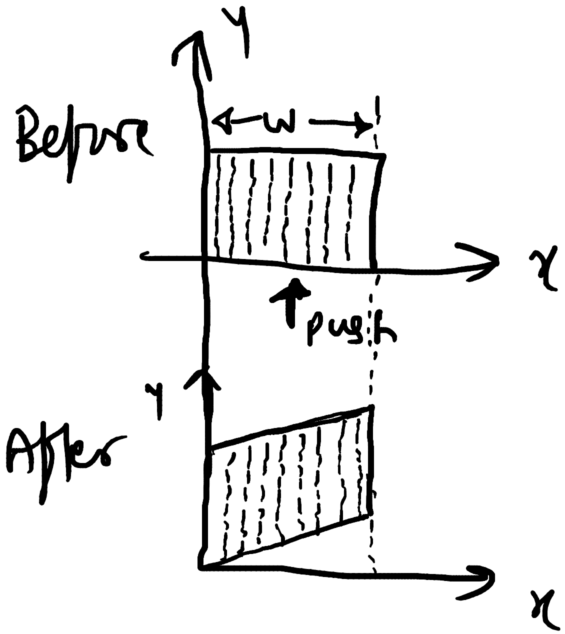
$$SHR = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\tan\alpha = \frac{y}{x'-x} \Rightarrow x'-x = \frac{y}{\tan\alpha}$$

$$x' = x + \frac{1}{\tan\alpha} y$$

y-dir^n Shear Transform



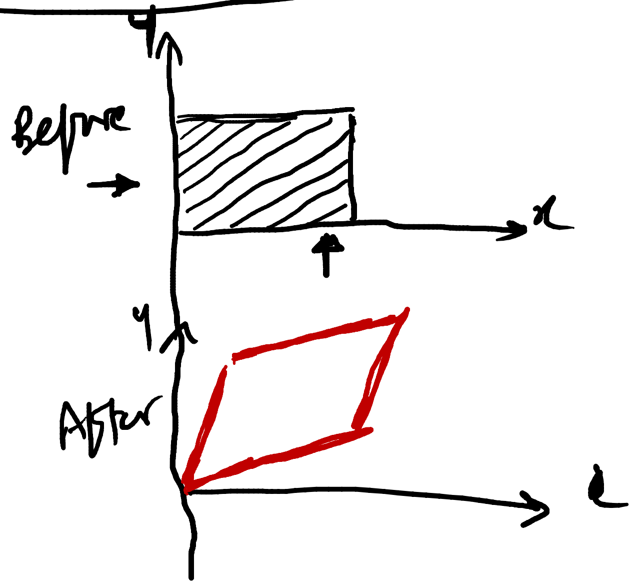
$x' = x$

$y' = sh_y \cdot x + y$

where sh_y : y-dir^n shear factor

$$SHR_y = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

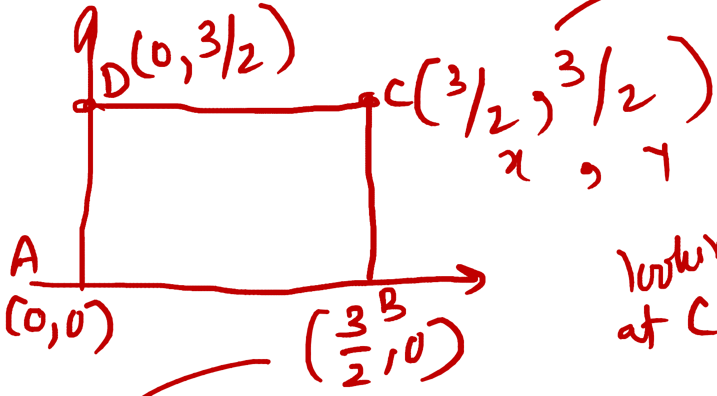
xy dir^n shear Transform



$x' = x + sh_x \cdot y$

$y' = sh_y \cdot x + y$

$$SHR_{xy} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Target coords for C(2,2)

x', y'

looking at C

$x' = x + sh_x \cdot y$

$x' = \frac{3}{2} + sh_x \cdot 3/2$

should become $1/2$

$\therefore sh_x = 1/3$

$y' = sh_y \cdot x + y$

$y' = sh_y \cdot \frac{3}{2} + 3/2$

should become $1/2$

$\therefore sh_y = 1/3$

with $sh_x = sh_y = 1/3$

$x' = x + sh_x \cdot y = 3/2$

$y' = sh_y \cdot x + y = 1/2$

3D Transformations

1.) 3D Transl^n :

$x' = x + T_x$

$y' = y + T_y$

$z' = z + T_z$

⇒ The size of Homogeneous Matrix for 3D is 4×4

$(x, y, z) \xrightarrow[\text{In homogeneous coord system}]{} (x, y, z, 1)$

$$T = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.) 3D Scaling :

$x' = x \cdot S_x$

$y' = y \cdot S_y$

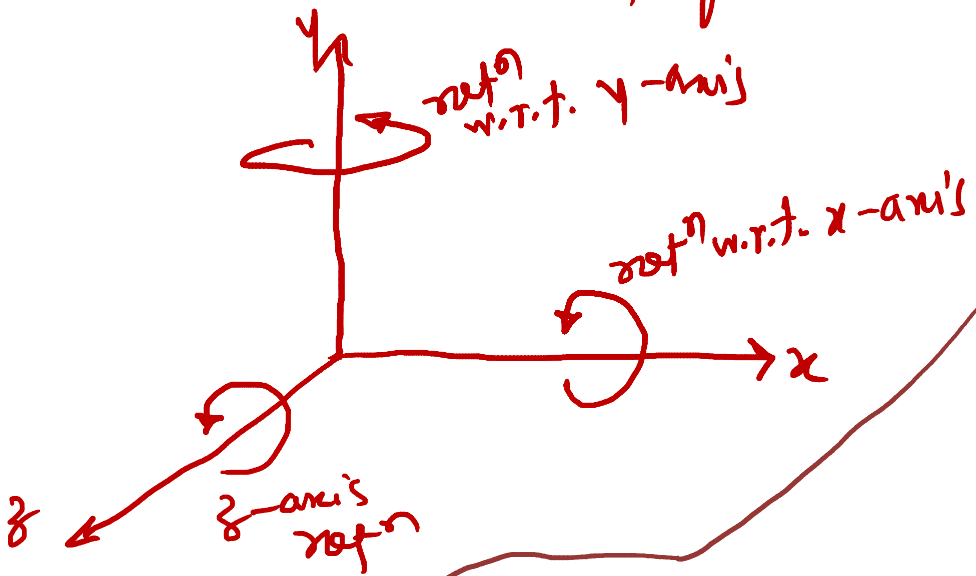
$z' = z \cdot S_z$

$1 = 1$

$$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Rot^n:

3D Rot^n is performed w.r.t. reference-axis



(*) If we apply rot^n w.r.t. z-axis, x & y coordinates changes but z remains the same.

In similar lines

(*) If we apply rot^n w.r.t. x-axis, y & z coord^{n}s changes but x remains the same

Same is applicable for y

Rot^n w.r.t. z-axis

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$$\therefore \quad R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rot^n w.r.t. z-axis

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$z' = z$

reference

To find eq^{n}s for rot^n w.r.t. x & y axis resp., we can use cyclic permutation

$$x \leftarrow y \leftarrow z$$

∴ Rot^n w.r.t. y-axis

$z' = z\cos\theta - x\sin\theta$

$y' = y$

$x' = z\sin\theta + x\cos\theta$

$$\therefore R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rot^n w.r.t. z-axis

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$z' = z$

reference

for rot^n w.r.t. x-axis

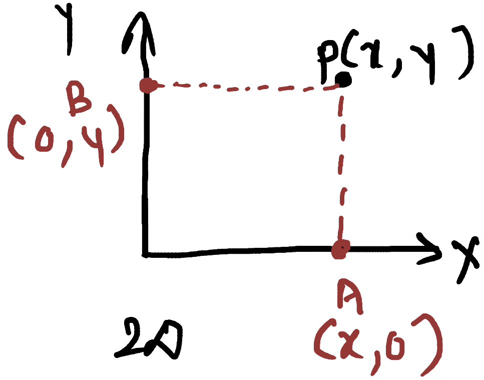
$$x \rightarrow y \rightarrow z$$

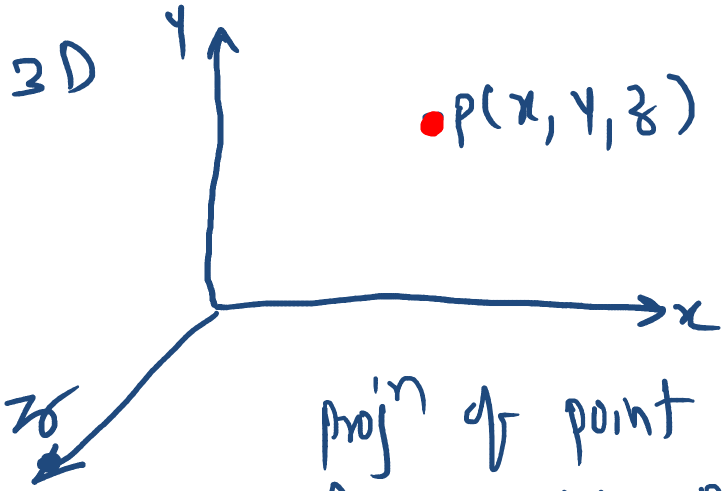
$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

$x' = x$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$





proj^n of point P

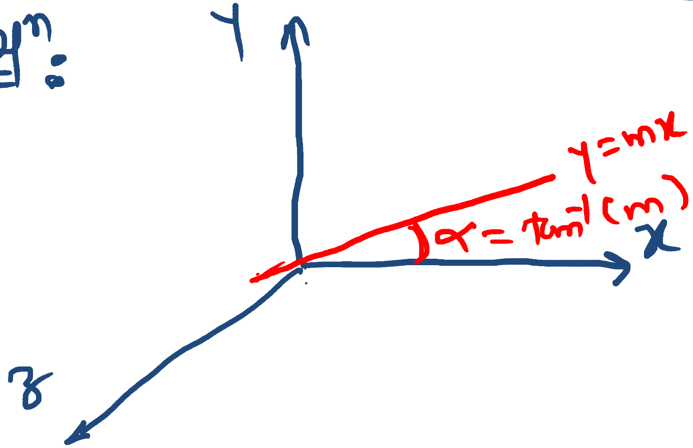
1) on xy plane
$P'(x, y, 0)$

2) on yz plane
$P'(0, y, z)$

3) on xz plane
$P'(x, 0, z)$

Find sequence of X^ns to rotate a solid object (3D) w.r.t. a line $\boxed{y = mx}$ by angle θ in anticlockwise manner.

Sol^n:



Obser^n: The given reference line is in xy plane

1.) To make ref line to fall on x-axis.

$$R_z(-\alpha)$$

2.) Apply Rot^n w.r.t. x-axis

$$R_x(\theta)$$

3.) To make ref. line to retain its original inclination

$$R_z(\alpha)$$

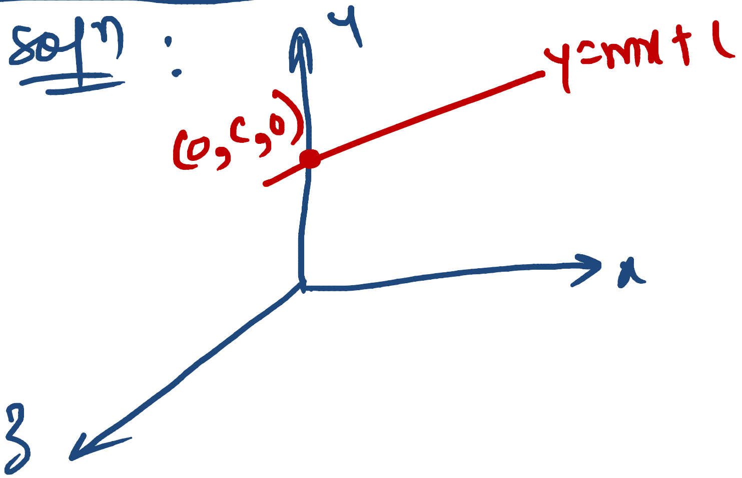
---

3 2 1

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Find the composite X^n to perform rotn of a point w.r.t. $y = mx + c$ by angle θ in anticlockwise manner.

Soln :



1) Transl^n $(T_x = 0, T_y = -c, T_z = 0)$

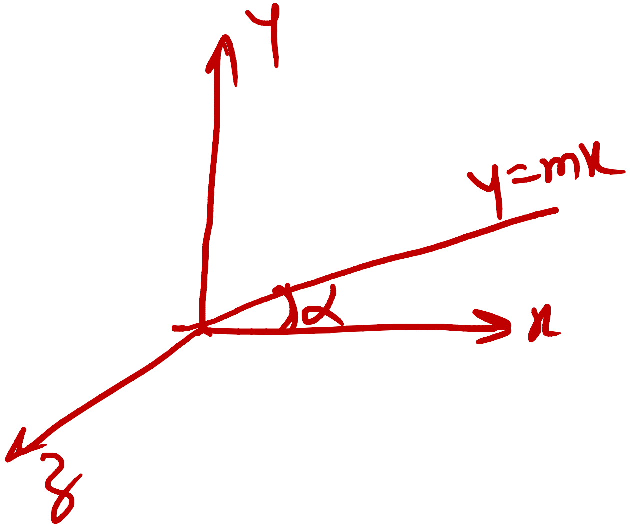
2) $R_z(-\alpha)$ where $\alpha = \tan^{-1}(m)$

3) $R_x(\theta)$

4) $R_z(\alpha)$

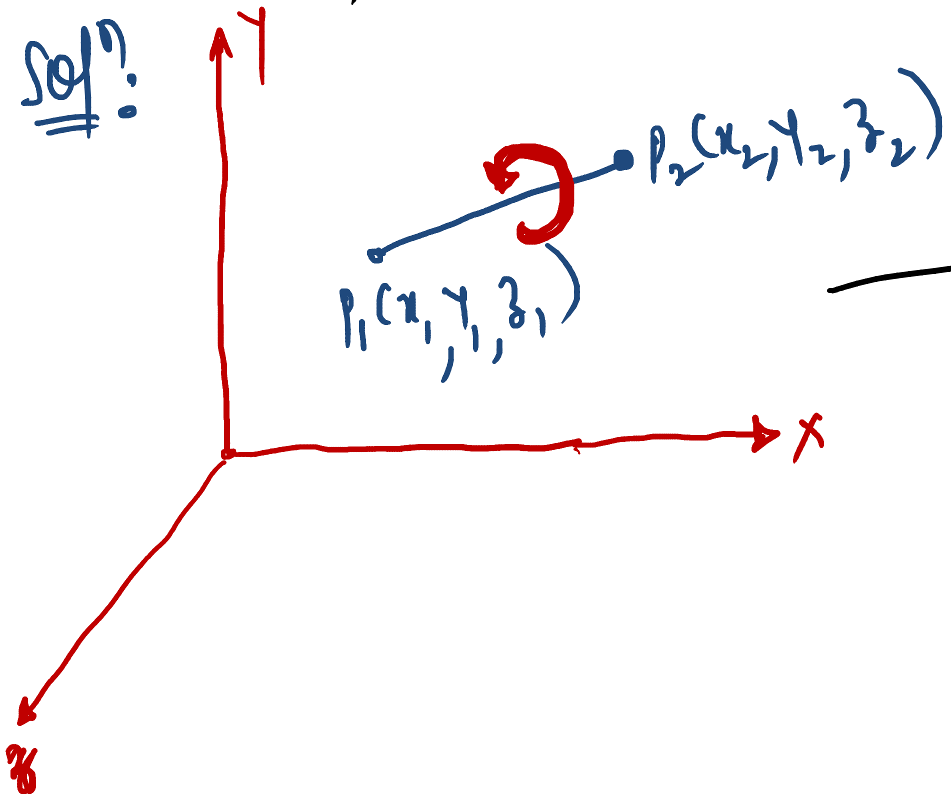
5) Transl^n $(T_x = 0, T_y = c, T_z = 0)$

After 1.



Find the sequence of X^n to rotate a point w.r.t. a line passing through $P_1(x_1, y_1, z_1)$ & $P_2(x_2, y_2, z_2)$ (i.e arbitrary axis) in counter clockwise manner by angle θ.

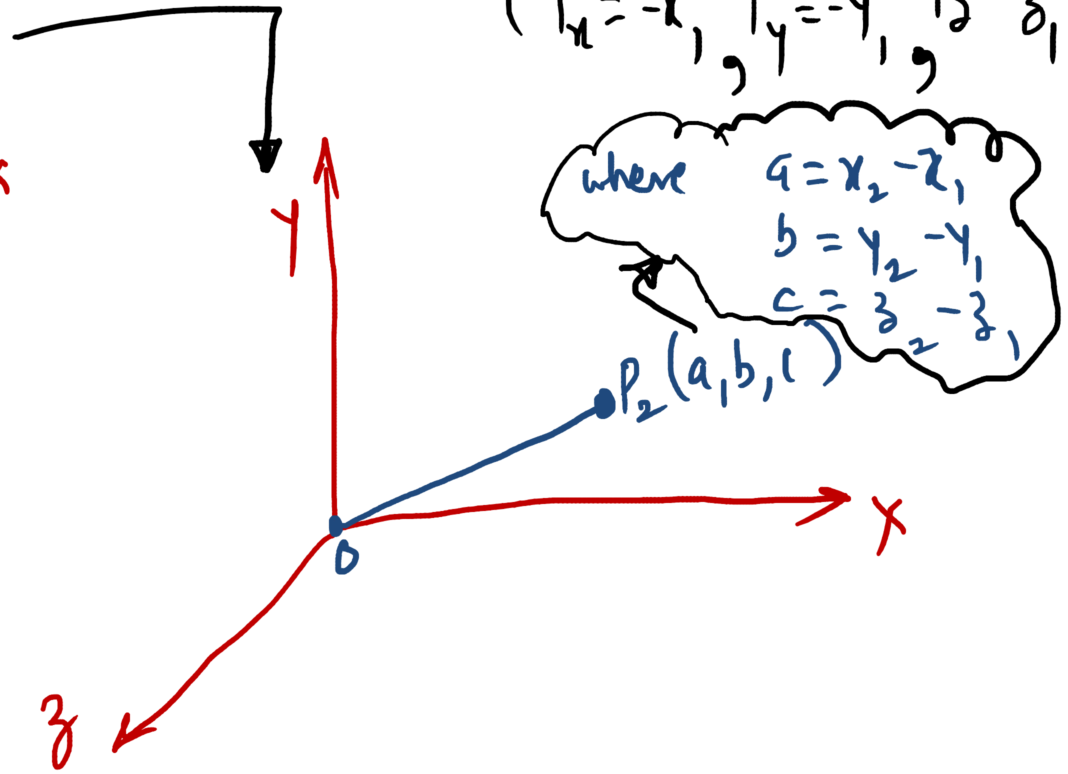
Sol^n:

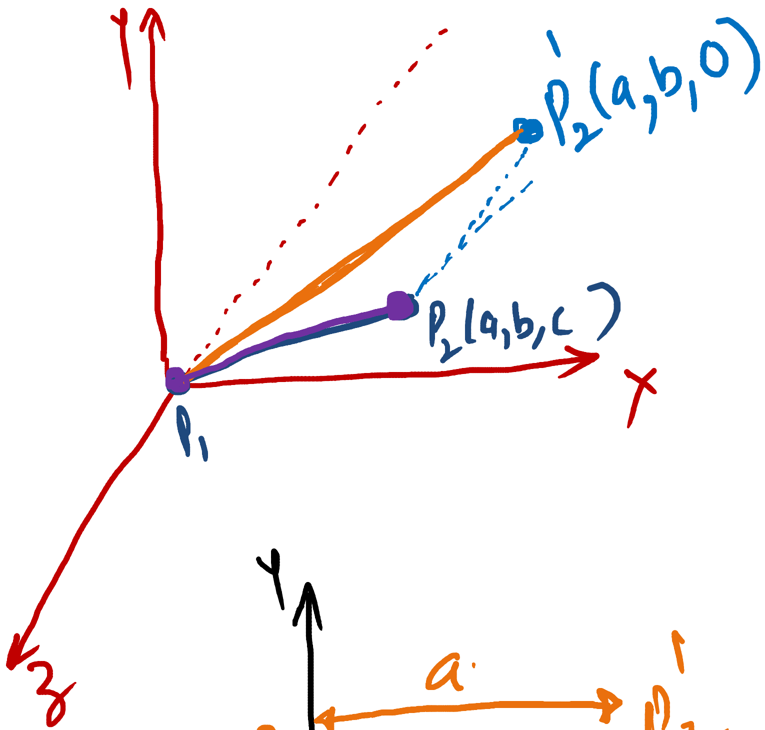


Steps (To translate P_1 to Origin)

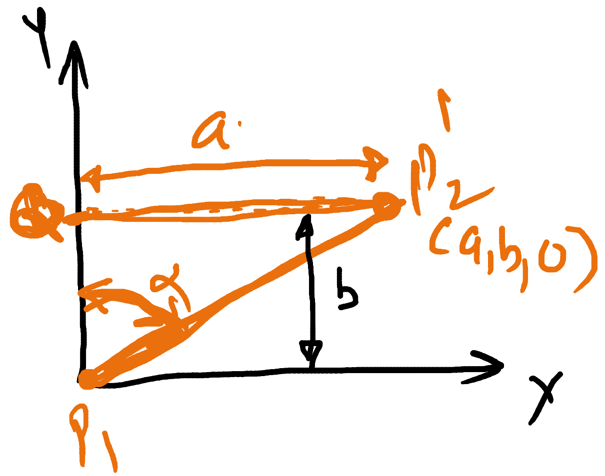
I) Translation

$(T_x = -x_1, \; T_y = -y_1, \; T_z = -z_1)$





Now To make this line $P_1 P_2$ to fall in YZ plane, let us think about projection of P_2 on x-y plane as P_2'



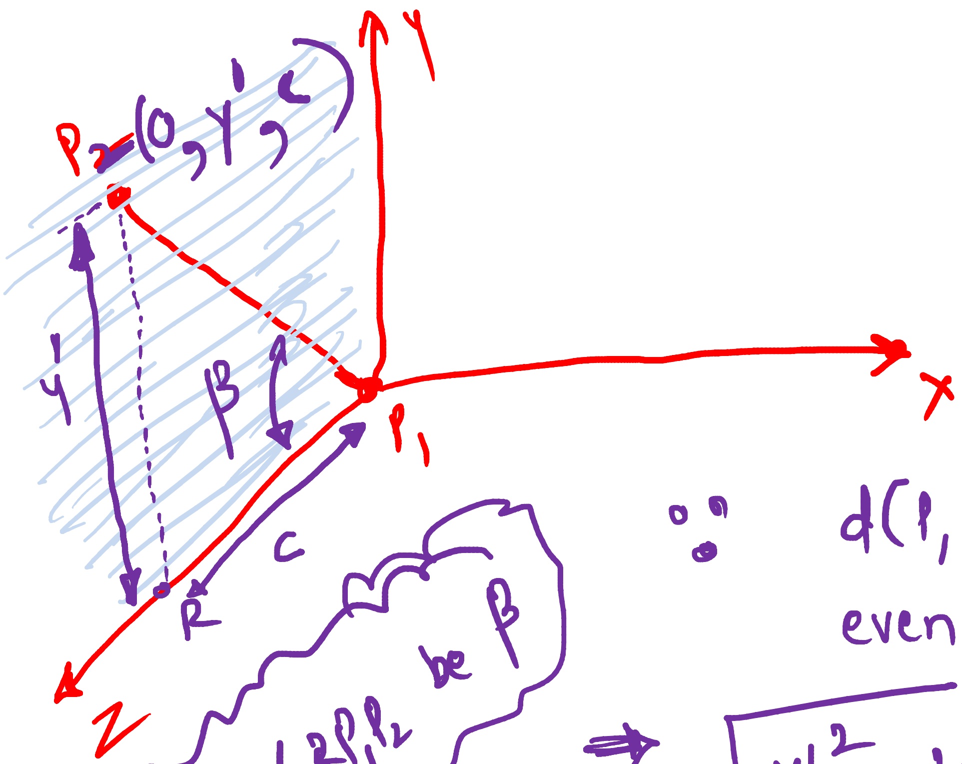
$dist(P_1 P_2') = \sqrt{a^2+b^2}$

∴ we should rotate $P_1 P_2$ in anticlockwise manner w.r.t. z-axis by some angle (say α)

& $\sin\alpha = \dfrac{a}{\sqrt{a^2+b^2}}$ & $\cos\alpha = \dfrac{b}{\sqrt{a^2+b^2}}$

what is dist. of $P_1 P_2 = \sqrt{a^2+b^2+c^2}$

II.) $Rot_z(\alpha)$ where $\sin\alpha = \frac{a}{\sqrt{a^2+b^2}}$ & $\cos\alpha = \frac{b}{\sqrt{a^2+b^2}}$



let angle $\angle RP_1P_2$ be β

As we know, in rot^n dist. of point from origin, before rot^n & after rot^n remains the same

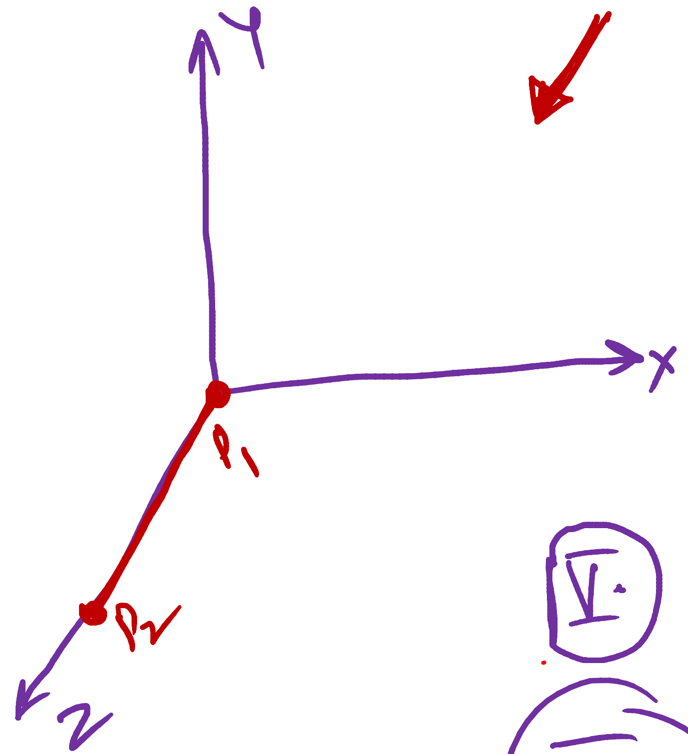
$\therefore$ $d(P_1, P_2)$ remains same even after rot^n

$$\Rightarrow \sqrt{y'^2 + c^2} = \sqrt{a^2 + b^2 + c^2}$$

$$\therefore \quad y'^2 = a^2 + b^2 \Rightarrow y' = \sqrt{a^2+b^2}$$

III. $R_x(\beta)$ where: $\sin\beta = \dfrac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2+c^2}}$

& $\cos\beta = \dfrac{c}{\sqrt{a^2+b^2+c^2}}$



IV $\boxed{Rot^n_z(\theta) \quad \theta \text{ is given}}$

V. $R_x(-\beta)$

VI $R_z(-\alpha)$

VII. $Trans^n(T_x = x_1, T_y = y_1, T_z = z_1)$

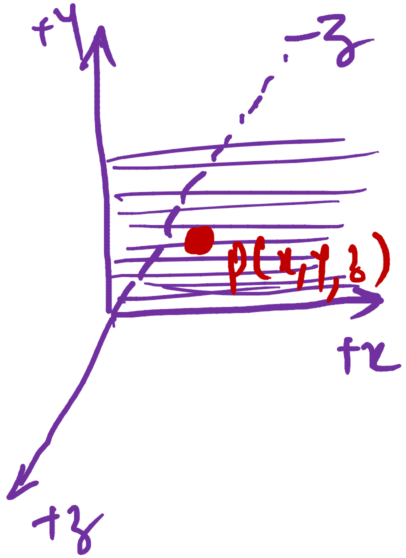
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \overset{7}{\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix}} \overset{6}{\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix}} \overset{5}{\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix}} \overset{4}{\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix}} \overset{3}{\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix}} \overset{2}{\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix}} \overset{1}{\begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\boxed{\text{3D Refl}^n}$ → 3D refl^n happens w.r.t. plane

$\boxed{\text{w.r.t. } xy \text{ plane}}$

$Refl^n_{xy}$: $x' = x \quad y' = y \quad z' = -z$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Similarly

$$Refl^n_{yz} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } Refl^n_{zx} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find sequence of X^n's required to reflect a point w.r.t. a plane parallel to xz plane & with $y = -b$.

Sol^n:

I.) $T(T_x = 0,\ T_y = b,\ T_z = 0)$

II.) Refl^n_{xz}

III.) $T(T_x = 0,\ T_y = -b,\ T_z = 0)$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

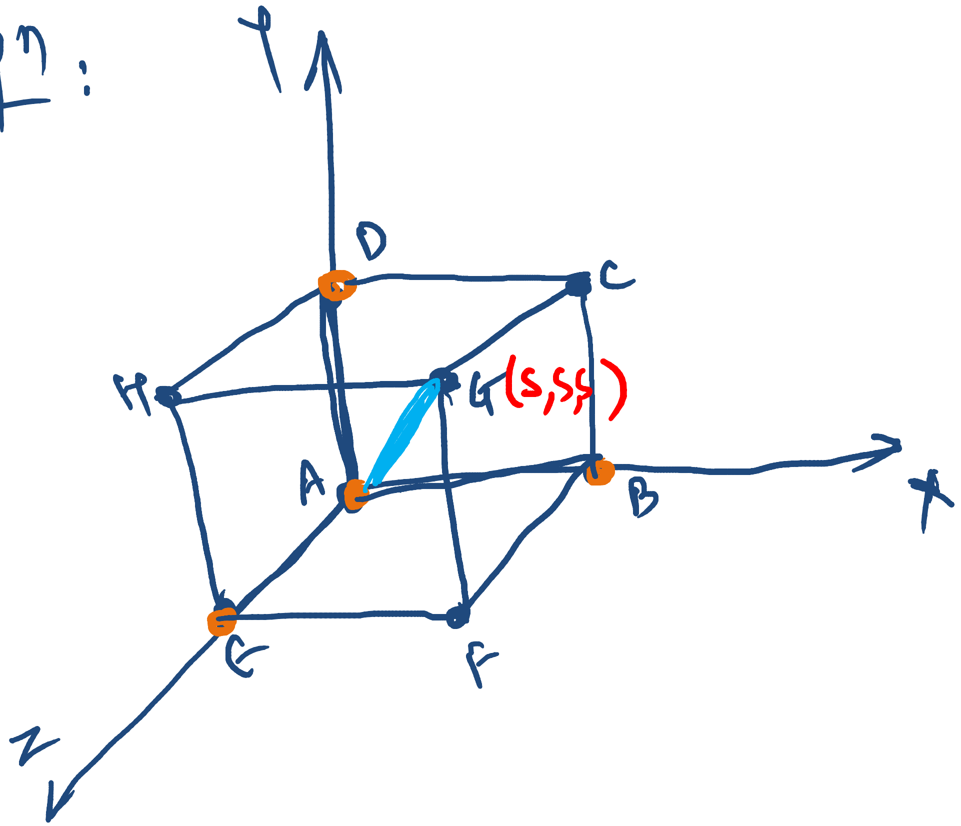
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -b-b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$-2b$

$$\boxed{x' = x,\quad z' = z \qquad y' = -y - 2b}$$

Find the sequence of X^n's to rotate a cube with side dimension $= s$, with one vertex at Origin & one vertex at each of the principal axis, by angle θ in clockwise manner w.r.t. a line passing through origin & diagonally opposite vertex.

Sol^n :



Take Home Test