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Assigument No. - 1

1] Define the following.

a) Alphabet
- Feinite set of symbols used as building blocks for constructing strings in formal language.

b) String
- A sequence of symbols from a specified alphabe one after the other. (junxtaposted).

c) Grammer
- Finite set of rules that define structure of strings in formal language.

d) Language
- Finite ~~set of~~ or Infinite set of strings from alphabet set Σ.

2] Give five tuple representation of

a) NFA with ϵ

→

$M = (Q, \Sigma, \delta, q_0, F)$

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Q: Set of states in the NFA

Σ : Finite set of input symbols

δ : Transition Function.

$$Q \times (\Sigma \cup \{\epsilon\}) \longrightarrow \cancel{Q}\ 2^Q$$

q_0: Initial State.

F : Finite set of final | Accepting states.

e.g.

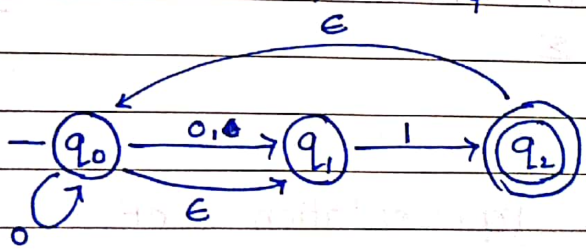
$M = (Q, \Sigma, \delta, q_0, F)$

$\Sigma = \{0, 1\}$

$Q = \{q_0, q_1, q_2\}$ $\quad$ $F = \{q_2\}$

$\delta : Q \times \Sigma \cup \{\epsilon\} \longrightarrow 2^Q$

$Q \backslash \Sigma$	0	1	ϵ
q_0	$\{q_0, q_1\}$	ϕ	$\{q_1\}$
q_1	ϕ	$\{q_2\}$	ϕ
q_2	ϕ	ϕ	$\{q_0\}$



b) NFA without ε

→

$M = (Q, \Sigma, \delta, q_0, F)$

Q : set of states in NFA

Σ : set of symbols

δ : $Q \times \Sigma \longrightarrow 2^Q$

q_0 : Initial state

F : Finite set of final | Accepting states.

e.g.

$M = (Q, \Sigma, \delta, q_0, F)$

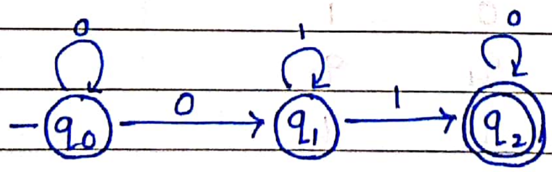
$\Sigma = \{0, 1\}$

$Q = \{q_0, q_1, q_2\}$

$F = \{q_2\}$

δ : $Q \times \Sigma \longrightarrow 2^Q$

$Q \backslash \Sigma$	0	1
q_0	$\{q_0, q_1\}$	ϕ
q_1	ϕ	$\{q_1, q_2\}$
q_2	~~ϕ~~ $\{q_2\}$	ϕ



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c) DFA

$M = (Q, \Sigma, \delta, q_0, F)$

Q : ~~finite~~ set of ~~symbols~~ states

Σ : set of symbols

δ : $Q \times \Sigma \rightarrow Q$

q_0 : Initial State

F : Finite set of final | Accepting states.

e.g.

$M = (Q, \Sigma, \delta, q_0, F)$

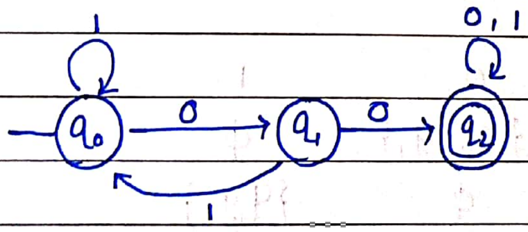
$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$\delta = Q \times \Sigma \rightarrow Q$

$q_0 = q_0$

$F = q_2$



$Q \backslash \Sigma$	0	1
q_0	q_1	q_0
q_1	q_2	q_0
q_2	q_2	q_2

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3] Prove equivalence of NFA with ϵ and NFA without ϵ (ϵ is null string).

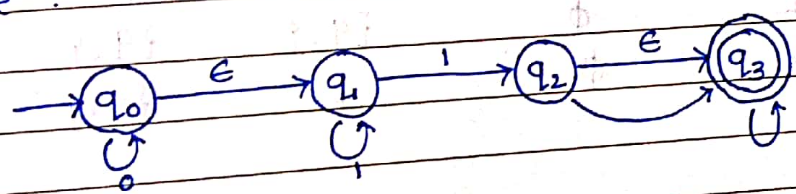
→

For every NFA with ϵ , $M = (Q, \Sigma, \delta, q_0, F)$ where δ says, $Q \times \Sigma \cup \{\epsilon\}$ to 2^Q , there exist a NFA without ϵ $M' = (Q, \Sigma, \delta', q_0, F')$ where δ' says, $Q \times \Sigma$ to 2^Q such that.

i) $\delta'(p, a) = \epsilon\text{-closure}(\delta(\delta^*(p, \epsilon), a))$
$\delta^*(p, \epsilon)$ i.e ϵ-closure (p) is set of all states reachable from p over ϵ.

ii) $F' = F \cup \{q_0\}$ If $\delta^*(q_0, \epsilon)$ contains $f \in F$
$= F$ otherwise.

Example :



$M = (Q, \Sigma, \delta, q_0, F)$
$Q = \{q_0, q_1, q_2, q_3\}$
$\Sigma = \{0, 1, 2\}$
$F = \{q_3\}$
$\delta : Q \times \Sigma \cup \{\epsilon\} \longrightarrow 2^Q$

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$\Sigma \cup \{\epsilon\}$ / Q	0	1	2	ϵ	ϵ-closure
q_0	$\{q_0\}$	ϕ	ϕ	$\{q_1\}$	$\{q_0, q_1\}$
q_1	ϕ	$\{q_1, q_2\}$	ϕ	ϕ	$\{q_1\}$
q_2	ϕ	$\{q_3\}$	ϕ	$\{q_3\}$	$\{q_2, q_3\}$
q_3	ϕ	ϕ	$\{q_3\}$	ϕ	$\{q_3\}$

$\because \; \delta^*(q_0, \epsilon) \notin F$

$\therefore \; F' = F = \{q_3\}$

$\therefore \; \delta' : Q \times \Sigma \longrightarrow 2^Q$

Σ / Q	0	1	2
q_0	$\{q_0, q_1\}$	$\{q_1, q_2, q_3\}$	ϕ
q_1	$\{q_1\}$	$\{q_1, q_2, q_3\}$	ϕ
q_2	ϕ	$\{q_3\}$	$\{q_3\}$
q_3	ϕ	ϕ	$\{q_3\}$

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4) Prove equivalence of NFA and DFA.

→ For NFA without ϵ, $M = (Q, \Sigma, \delta, q_0, F)$ where δ maps $Q \times \Sigma$ to 2^Q, there exists a DFA $M' = (Q', \Sigma, \delta', q_0', F')$ where $Q' = 2^Q$

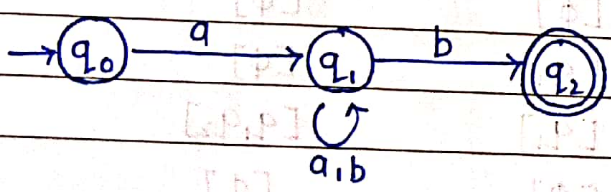
δ' maps $Q' \times \Sigma$ to Q' such that
$\forall P \in 2^Q \quad [P] \in Q'$

$\forall P \in Q$ and $a \in \Sigma \quad \delta'([P], a) = [\delta(P, a)]$

$\forall [P,q] \in Q'$ and $a \in \Sigma \quad \delta'([P,q], a) = [\delta(P,a) \cup \delta(q,a)]$

$F' = \{ [f] \mid [f]$ contains $m \in F \}$ and $q_0' = [q_0]$

Example :



$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$F = \{q_2\}$

$\delta : Q \times \Sigma \rightarrow 2^Q$

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$Q \backslash \Sigma$	a	b
q_0	$\{q_1\}$	ϕ
q_1	$\{q_1\}$	$\{q_1, q_2\}$
q_2	ϕ	ϕ

Equivalent DFA.

$M' = (Q', \Sigma, \delta', q_0', F')$

$Q' = 2^Q = \{ [\phi], [q_0], [q_1], [q_2], [q_0 q_1], [q_0 q_2],$
$[q_1 q_2], [q_0 q_1 q_2] \}$

$q_0' = [q_0]$

$F' = \{ [q_2], [q_0 q_2], [q_1 q_2], [q_0 q_1 q_2] \}$

$\delta' : Q' \times \Sigma \longrightarrow Q$

$Q \backslash \Sigma$	a	b
$[\phi]$	$[\phi]$	$[\phi]$
$[q_0]$	$[q_1]$	$[\phi]$
$[q_1]$	$[q_1]$	$[q_1 q_2]$
$[q_2]$	$[\phi]$	$[\phi]$
$[q_0 q_1]$	$[q_1]$	$[q_1 q_2]$
$[q_1 q_2]$	$[q_1]$	$[q_1 q_2]$
$[q_0 q_2]$	$[q_1]$	$[\phi]$
$[q_0 q_1 q_2]$	$[q_1]$	$[q_1 q_2]$

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5] Explain steps of minimization of a DFA.

→

1) Eliminate the states that are not reachable from start state.

2) Minimise by classical method

$$P \equiv q$$

if $((P \in F \wedge q \in F)$ OR $(P \notin F \wedge q \notin F))$ AND

$\forall a \in \Sigma \quad \delta(P, a) = \delta(q, a)$

3) Equivalence class method. (Myhill Nerode Theorem)

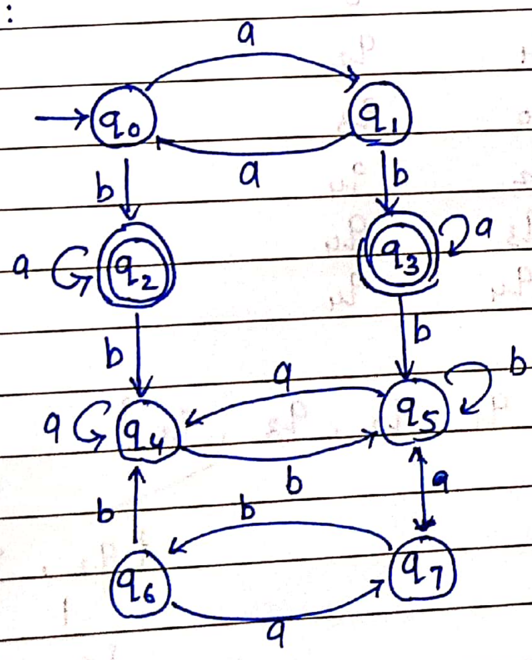
$$P \not\equiv q \quad \text{iff}$$

either $(P \in F \wedge q \notin F)$ OR $\exists a \in \Sigma$ $\delta(P, a) = r$

$\delta(q, a) = s$

$$r \not\equiv s$$

Example :



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1)

$Q \backslash \Sigma$	a	b
q_0	q_1	q_2
q_1	q_0	q_3
$*q_2$	q_2	q_4
$*q_3$	q_3	q_5
q_4	q_4	q_5
q_5	q_4	$q_{\cancel{3}5}$

$\because$ q_6 and q_7 are not reachable from q_0

2) $q_4 \equiv q_5$

$\therefore$ Discard q_5 and replace transitions to q_5 by q_4

$Q \backslash \Sigma$	a	b
q_0	q_1	q_2
q_1	q_0	q_3
$*q_2$	q_2	q_4
$*q_3$	q_3	q_4
q_4	q_4	q_4

3) $\{q_0, q_1, q_2, q_3, q_4\}$

$\{q_0, q_1, q_4\}$ $\qquad$ $\{q_2, q_3\}$

$\{q_0, q_1\}$ $\qquad$ $\{q_4\}$ $\qquad$ $\{q_2, q_3\}$

$\{q_0, q_1\}$ $\qquad$ $\{q_4\}$ $\qquad$ $\{q_2, q_3\}$

A-4 BOOK
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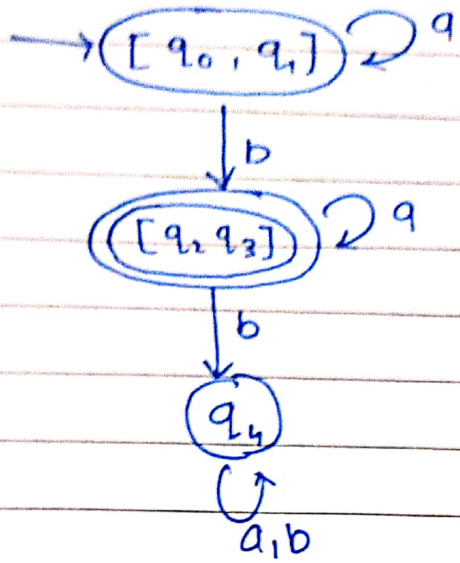

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