APPLIED MATHEMATICS-III

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- Laplace Transform (20 Marks)
- Inverse Laplace Transform (20 Marks)
- Fourier Series (20 Marks)
- Complex Variable (20 Marks)
- Linear Algebra: Matrix Theory (20 Marks)
- Vector Differentiation & Integral (20 Marks)

WHAT IS LAPLACE TRANSFORM ?

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- French Mathematician **Pierre-Simon Laplace** in 19th century
- **Definition LT** : If f(t)→Real valued function, s → Complex parameter

$$L[f(t)] = f(s)$$

Bilateral $(-\infty \le t \le \infty)$ → $$\int_{t=-\infty}^{t=\infty} e^{-st}\, f(t)\, dt$$

Unilateral $(0 \le t \le \infty)$ → $$\int_{t=0}^{t=\infty} e^{-st}\, f(t)\, dt$$

- **Definition Inverse LT:**

$$L^{-1}[f(s)] = f(t) = \frac{1}{2\pi i}\int_{s=\sigma-i\omega}^{s=\sigma+i\omega} f(s)\, e^{st} ds$$

THEOERM FOR EXISTENCE OF L. TRANSFORM

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Sufficient conditions for Existence of Laplace transform:

If $f(t)$, $t \geq 0$ be Piecewise continuous on $[0, \infty)$ and of Exponential order a , then $L[f(t)] = f(s)$ exists for $s > a \geq 0$

- **Ex.** $e^{\pm at}$, $Sin(at)$, $Cos(at)$, $Sinh(at)$, $Cosh(at)$ $\forall\ a \in \Re$
  t^n $\forall\ n > -1$

SUMMARY OF LECTURE

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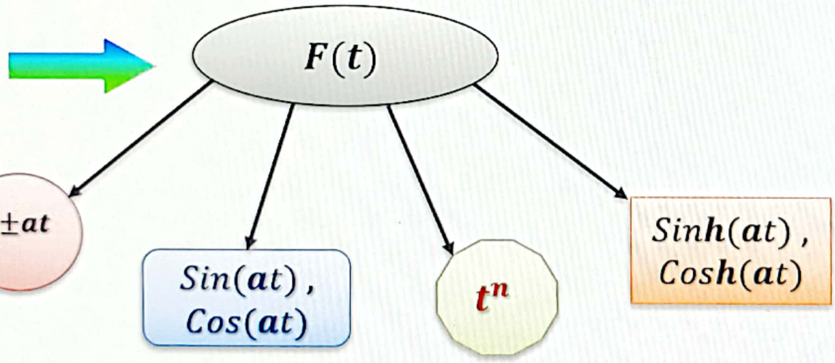
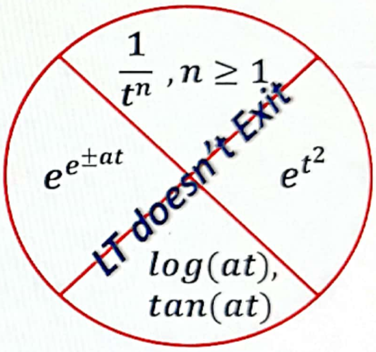
- ✓ Mathematical Operator and denoted by $L[f(t)]$
- ✓ Three main aspects→ Transformation , Evaluation of integral, Differential Equations.
- ✓ Definitions of LT → Bilateral, Unilateral
- ✓ Existence conditions → Piecewise cont. , Exponential order
- ✓ Four types of functions:





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LAPLACE TRANSFORM OF STANDARD FUNCTIONS

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- **Linear Property of LT:**

$$L[k_1 f_1(t) \pm k_2 f_2(t)] = k_1\, L[f_1(t)] \pm k_2\, L[f_2(t)]$$

1. Let $f(t) = e^{at}$

$$\boxed{L[f(t)] = L[e^{at}] = f(s) = \int_{t=0}^{t=\infty} e^{-st} e^{at}\, dt}$$

$$\boxed{L[e^{at}] = \int_{t=0}^{t=\infty} e^{-(s-a)t}\, dt}$$

$$\boxed{L[e^{at}] = \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_{t=0}^{t=\infty} = \left[o - \frac{1}{-(s-a)}\right] = \frac{1}{(s-a)},\ s > a}$$

$$\boxed{L[e^{-at}] = \frac{1}{(s+a)}}$$

$$\boxed{L[e^{-0t}] = L[1] = \frac{1}{(s)},\ s > 0}$$

- Let $f(t) = t^n$, Then $L[t^n] = f(s) = \int_{t=0}^{t=\infty} e^{-st}\, t^n\, dt$
- Put st $= u \Rightarrow dt = \frac{du}{s}$, Then $L[t^n] = \int_{u=0}^{u=\infty} e^{-u}\left(\frac{u^n}{s^n}\right)\frac{du}{s}$
- $L[t^n] = \frac{1}{s^{n+1}}\int_{u=0}^{u=\infty} e^{-u}\, u^{(n+1)-1} du$

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$$\boxed{L[t^n] = \frac{1}{s^{n+1}}\Gamma(n+1) = \frac{n!}{s^{n+1}},\quad n > -1}$$

Ex. $L\left[\, t^{\frac{-1}{2}}\right] = \boxed{\frac{1}{s^{\frac{-1}{2}+1}}\Gamma\left(-\frac{1}{2}+1\right) = \frac{1}{s^{\frac{1}{2}}}\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{\sqrt{s}}}$

- Let $f(t) = Sin(at)\ or\ \ Cos(at)$

LT OF STANDARD FUNCTIONS...

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- $Cos(at) + i\,Sin(at) = e^{i(at)}$
- $L[Cos(at) + i\,Sin(at)] = L[e^{(ia)t}]$
- $L[Cos(at)] + i\,L[\,Sin(at)] = \frac{1}{(s-ia)} = \frac{(s+ia)}{(s-ia)(s+ia)} = \frac{(s+ia)}{(s^2+a^2)}$
- $L[Cos(at)] = \frac{(s)}{(s^2+a^2)}$ and $L[\,Sin(at)] = \frac{(a)}{(s^2+a^2)}$
- $L[Cosh(at)] = L\left[\frac{e^{at}+e^{-at}}{2}\right] = \frac{1}{2}\left[\frac{1}{(s-a)} + \frac{1}{(s+a)}\right] = \frac{1}{2}\left[\frac{(s+a+s-a)}{(s-a)(s+a)}\right] = \frac{(s)}{(s^2-a^2)}$
- $L[Sinh(at)] = L\left[\frac{e^{at}-e^{-at}}{2}\right] = \frac{1}{2}\left[\frac{1}{(s-a)} - \frac{1}{(s+a)}\right] = \frac{(a)}{(s^2-a^2)}$

SUMMARY OF LECTURE

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$f(t)$	$L[f(t)] = f(s)$	$f(t)$	$L[f(t)] = f(s)$
e^{at}	$\frac{1}{(s-a)}$	e^{-at}	$\frac{1}{(s+a)}$
1	$\frac{1}{(s)}$	t^n	$\frac{1}{s^{n+1}}\Gamma(n+1) = \frac{n!}{s^{n+1}}$
$Sin(at)$	$\frac{(a)}{(s^2+a^2)}$	$Cos(at)$	$\frac{(s)}{(s^2+a^2)}$
$Sinh(at)$	$\frac{(a)}{(s^2-a^2)}$	$Cosh(at)$	$\frac{(s)}{(s^2-a^2)}$