Curriculum Scheme: Rev2019

Examination: FE Semester I

Course Code: FEC101 and Course Name: Engineering Mathematics I

Time: 2 Hours 30 Minutes Max. Marks: 80

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Note: All questions are compulsory.

Q No/ Options	Questions	Marks
Question 1	**Choose the correct option for the following examples.**	**20**
1.	$The\ roots\ of\ \ x^4 - x^3 + x^2 - x + 1 = 0\ are$	2
Option A:	$cos\frac{7\pi}{5} \mp i\,sin\frac{7\pi}{5}\ \ and\ \ cos\frac{3\pi}{5} \mp i\,sin\frac{3\pi}{5}$	
Option B:	$cos\frac{\pi}{5} \mp i\,sin\frac{\pi}{5}\ \ and\ \ cos\frac{3\pi}{5} \mp i\,sin\frac{3\pi}{5}$	
Option C:	$cos\frac{\pi}{5} \mp i\,sin\frac{\pi}{5}\ \ and\ \ cos\frac{5\pi}{5} \mp i\,sin\frac{5\pi}{5}$	
Option D:	$cos\frac{-3\pi}{5} \mp i\,sin\frac{-3\pi}{5}\ \ and\ \ cos\frac{3\pi}{5} \mp i\,sin\frac{3\pi}{5}$	
2.	$If\ \ a = cos\,\alpha + i\,sin\,\alpha\ and\ b = cos\,\beta + i\,sin\,\beta,$ $then\ the\ value\ of\ \ \frac{(a+b)(ab-1)}{(a-b)(ab+1)}\ \ is$	2
Option A:	$\frac{cos\,\alpha + cos\,\beta}{cos\,\alpha - cos\,\beta}$	
Option B:	$\frac{sin\,\beta + sin\,\alpha}{sin\,\beta - sin\,\alpha}$	
Option C:	$\frac{sin\,\alpha - sin\,\beta}{sin\,\alpha + sin\,\beta}$	
Option D:	$\frac{sin\,\alpha + sin\,\beta}{sin\,\alpha - sin\,\beta}$	
3.	$The\ value\ of\ \ sin^{-1}(cosec\vartheta)\ is$	2

Option A:	$\frac{\pi}{2} + i\,log\,cot\left(\frac{\vartheta}{2}\right)$	
Option B:	$\frac{\pi}{2} - i\,log\,cot\left(\frac{\vartheta}{2}\right)$	
Option C:	$\frac{\pi}{2} + i\,log\,cos\left(\frac{\vartheta}{2}\right)$	
Option D:	$\frac{\pi}{2} - i\,log\,cos\left(\frac{\vartheta}{2}\right)$	
4.	$If\ \tan(x+iy) = \alpha + i\beta\,, then\ the\ value\ of\ \frac{1-\alpha^2-\beta^2}{1+\alpha^2+\beta^2}$	2
Option A:	$\frac{\tan 2x}{\tanh 2y}$	
Option B:	$\frac{\cos 2x}{\sinh 2y}$	
Option C:	$\frac{\cosh 2y}{\sinh 2x}$	
Option D:	$\frac{\cos 2x}{\cosh 2y}$	
5.	$If\ u = \ sinh^{-1}\left(\frac{x^3+y^3}{x^2+y^2}\right), then\ the\ value\ of$ $\left[x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}\right]\ is$	2
Option A:	$(\tanh u)^3$	
Option B:	$-(\tanh u)^3$	
Option C:	$(\coth u)^3$	
Option D:	$-(\coth u)^3$	
6.	$If\ u = 3(ax+by+cz)^2 - (x^2+y^2+z^2) and\ a^2+b^2+c^2 = 1,$ $then\ the\ value\ of\ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ is	2

Option A:	1	
Option B:	-10	
Option C:	0	
Option D:	10	
7.	$If\ y = x^n logx, then\ value\ of\ y_{n+1} is$	2
Option A:	$\frac{n!}{x^2}$	
Option B:	$\frac{(n-1)!}{x}$	
Option C:	$\frac{(n+1)!}{x}$	
Option D:	$\frac{n!}{x}$	
8.	$If\ a > 0, u = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right), then$	2
Option A:	$u\ shall\ have\ maximum\ at\ (a, a)$	
Option B:	$u\ shall\ have\ minimum\ at\ (a, a)$	
Option C:	$u\ shall\ have\ maximum\ at\ (a, -a)$	
Option D:	$u\ shall\ have\ neither\ maxima\ nor\ minima\ at\ (a, a)$	

9.	$Rank\ of\ the\ matrix\ A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\ 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 \\ 28 & 29 & 30 & 31 & 32 & 33 & 34 & 35 & 36 \\ 37 & 38 & 39 & 40 & 41 & 42 & 43 & 44 & 45 \\ 46 & 47 & 48 & 49 & 50 & 51 & 52 & 53 & 54 \\ 55 & 56 & 57 & 58 & 59 & 60 & 61 & 62 & 63 \\ 64 & 65 & 66 & 67 & 68 & 69 & 70 & 71 & 72 \\ 73 & 74 & 75 & 76 & 77 & 78 & 79 & 80 & 81 \end{bmatrix} is$	2
Option A:	1	
Option B:	2	
Option C:	3	
Option D:	9	
10.	$If\ r\ is\ rank\ and\ n\ is\ number\ of\ independent\ variables\ then$ $for\ non\ homogeneous\ linear\ equations\ AX = B,$ $which\ of\ the\ following\ statement\ is\ correct?$	2
Option A:	$If\ \rho(A) = \rho(A{:}B), \Rightarrow\ system\ is\ inconsistent\ \&\ has\ no\ solution,$ $If\ \rho(A) = \rho(A{:}B) = r = n, \Rightarrow\ system\ is\ consistent\ \&\ has\ unique\ solution,$ $If\ \rho(A) < \rho(A{:}B) = r < n, \Rightarrow\ system\ is\ consistent\ \&\ has\ infinite\ solution$	
Option B:	$If\ \rho(A) < \rho(A{:}B), \Rightarrow\ system\ is\ inconsistent\ \&\ has\ no\ solution,$ $If\ \rho(A) \neq \rho(A{:}B) = r = n, \Rightarrow\ system\ is\ consistent\ \&\ has\ unique\ solution,$ $If\ \rho(A) = \rho(A{:}B) = r < n, \Rightarrow\ system\ is\ consistent\ \&\ has\ infinite\ solution$	
Option C:	$If\ \rho(A) < \rho(A{:}B), \Rightarrow\ system\ is\ inconsistent\ \&\ has\ no\ solution,$ $If\ \rho(A) = \rho(A{:}B) = r = n, \Rightarrow\ system\ is\ consistent\ \&\ has\ unique\ solution,$ $If\ \rho(A) = \rho(A{:}B) = r > n, \Rightarrow\ system\ is\ consistent\ \&\ has\ infinite\ solution$	
Option D:	$If\ \rho(A) < \rho(A{:}B), \Rightarrow\ system\ is\ inconsistent\ \&\ has\ no\ solution,$ $If\ \rho(A) = \rho(A{:}B) = r = n, \Rightarrow\ system\ is\ consistent\ \&\ has\ unique\ solution,$ $If\ \rho(A) = \rho(A{:}B) = r < n, \Rightarrow\ system\ is\ consistent\ \&\ has\ infinite\ solution$	
Question 2	**Answer any four questions out of the following examples.**	**20**
A	$Expand\ \sin^7\theta\ in\ the\ series\ of\ sines\ of\ multiple\ of\ \theta$	5
B	$If\ cosec\left(\frac{\pi}{4} + ix\right) = u + iv$ $where\ x, y, u, v\ are\ real, then\ show\ that\ (u^2 + v^2)^2 = 2(u^2 - v^2)$	5

C	$If\ u = x^2 - y^2, v = 2xy\ and\ z = f(x,y), then\ prove\ that$ $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 4\sqrt{u^2+v^2}\left[\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2\right]$	5
D	$If\ y = \frac{1}{(3x-2)(x-3)^2}, then\ find\ y_n$	5
E	$If\ N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix},$ $then\ show\ that\ (I-N)(I+N)^{-1} is\ a\ unitary\ matrix$	5
F	$Determine\ the\ value\ of\ k\ for\ which\ the\ following\ system\ of\ equations$ $has\ non-trivial\ solutions\ and\ find\ them\ in\ each\ case$ $(k-1)x + (4k-2)y + (k+3)z = 0$ $(k-1)x + (3k+1)y + (2k)z = 0$ $(2)x + (3k+1)y + (3k-3)z = 0$	5
Question 3	**Answer any four questions out of the following examples.**	**20**
A	$Show\ that\ the\ roots\ of\ z^7 - 1 = 0\ are\ 1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6,$ $hence\ show\ that\ (1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4)(1-\alpha^5)(1-\alpha^6) = 7$	5
B	$Show\ that\ \tan^{-1}(\cos\vartheta + i\sin\vartheta) = \left(\frac{n\pi}{2} + \frac{\pi}{4}\right) - \frac{i}{2}\log\left(\tan\left(\frac{\pi}{4} - \frac{\vartheta}{2}\right)\right)$	5
C	$If\ \log(x^3 + y^3 - xy^2 - yx^2), prove\ that\ \frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = \frac{-4}{(x+y)^2}$	5
D	$If\ y = (\sin^{-1}x)^2, then\ show\ that$ $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0\ hence\ find\ y_n(0)$	5
E	$Find\ two\ non\ singular\ matrices\ P\ and\ Q\ such\ that\ PAQ\ is\ in\ Normal\ Form,$ $hence\ find\ rank\ of\ A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$	5
F	$Prove\ that\ e^{2ai\cot^{-1}b}\left[\frac{bi-1}{bi+1}\right]^{-a} = 1$	5
Question 4	**Answer any four questions out of the following examples.**	**20**
A	$If\ \cos\alpha + 2\cos\beta + 3\cos\gamma = \sin\alpha + 2\sin\beta + 3\sin\gamma = 0,$ $Prove\ that\ \cos3\alpha + 8\cos3\beta + 27\cos3\gamma = 18\cos(\alpha+\beta+\gamma)$	5
B	$If\ \cosh x = \sec\theta, then\ prove\ that$ $a)\ x = \log(\sec\theta + \tan\theta), b)\ \tanh\left(\frac{x}{2}\right) = \tan\left(\frac{\theta}{2}\right)$	5
C	$If\ u = \sin^{-1}\left[\frac{x^3 - y^3}{7x - 9y}\right] then, find\ the\ value\ of$ $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$	5

D	Find the extreme values of $x^3 + xy^2 - 12x^2 - 2y^2 + 21x + 10$	5
E	Test the sysyem of equatios for consistency and slove if consistent. $x_1 - 2x_2 + x_3 - x_4 = 2;\ x_1 + 2x_2 + 2x_4 = 1;\ 4x_2 - x_3 + 3x_4 = -1$	5
F	Using Lagrange's Multipliers Method, find minimum distance of a point lying on the plane $x + 2y + 3z = 14$ from origin $O(0,0,0)$	5