- Starting Monday, March 13, lectures will begin @ 7:30 am IST.

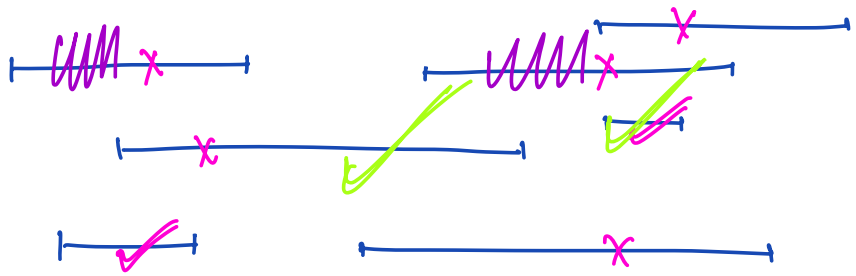
- Exam 2 : Mon. March 27 (tentative).
  - Stable matching, asymptotic notations, divide & conquer, greedy ... .

Greedy algorithms

Input : n intervals : 1, 2, 3, ..., n

interval i
- start time s_i
- finish time f_i

Objective : To find a set of non-overlapping intervals of maximum cardinality.



Alg

1. Sort intervals in ↗ order of finish times.

2. $S \leftarrow \phi$

3. Repeat until no more intervals left.

   - $I \leftarrow$ interval with the smallest finish time.
   - $S \leftarrow S \cup \{I\}$
   - Remove all intervals overlapping with I.

Running time: $\Theta(n \lg n)$

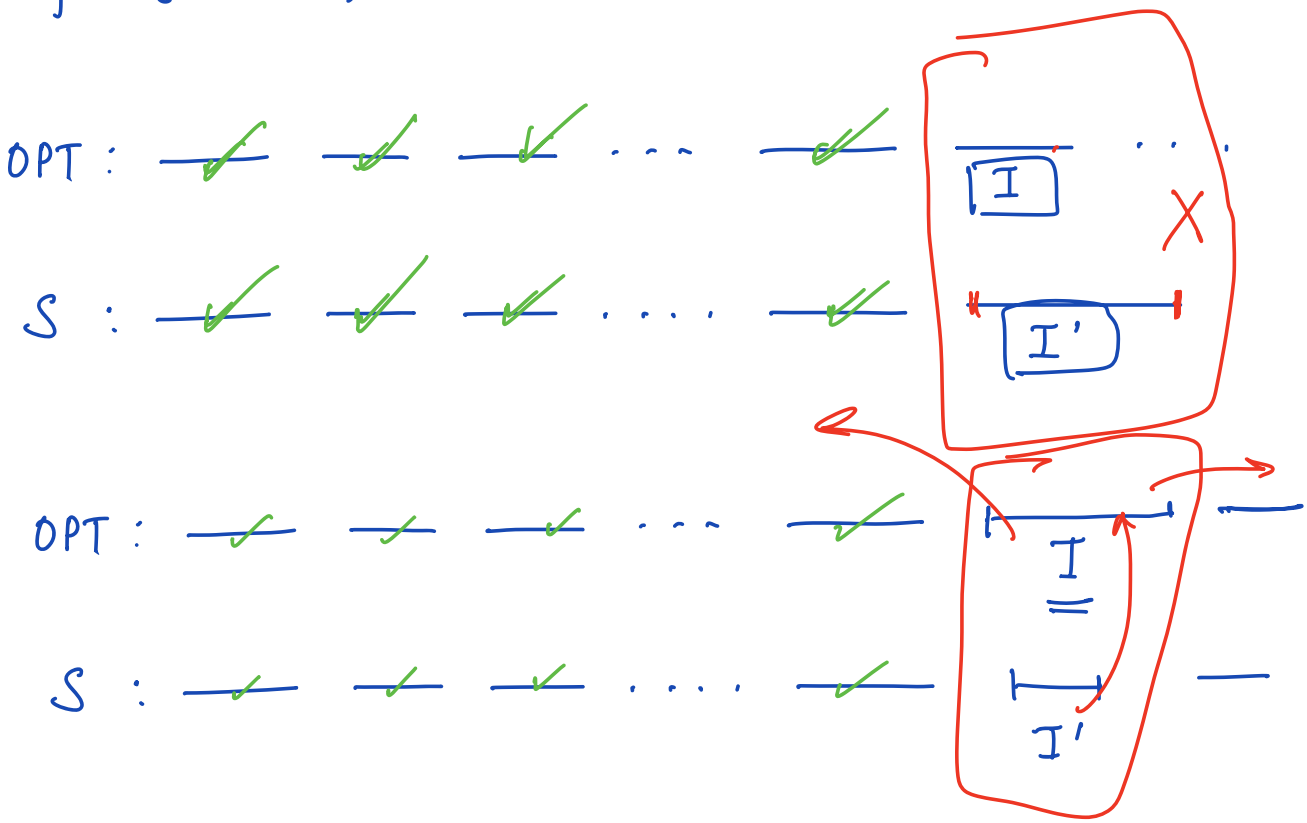
Correctness

Thm: Our algorithm yields an optimal soln.

Proof: Assume for contradiction that our alg. is suboptimal. Among all optimal solns, let OPT be an opt. soln. that contains maximum no. of intervals that are common to our soln, S.

OPT & S in

Consider the intervals in increasing order of finish times.



Consider $OPT' = OPT \setminus \{I\} \cup \{I'\}$

Clearly, OPT' is a feasible soln. ✓

- $f_{I'} < f_I \Rightarrow I'$ cannot conflict with any intervals that

begins after f_I.

Since all intervals before I & I' (& hence I') finish before s_I and $s_{I'}$.

OPT' is a feasible soln.

~~Furthermore $|OPT'| = |S|$. Thus,~~

~~S is an opt. soln~~

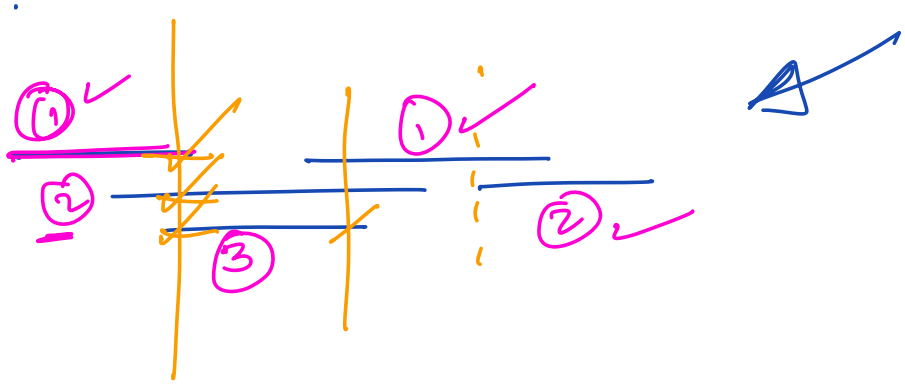
$|OPT'| = |OPT|$

OPT' clearly has more intervals that are common to S than OPT, a contradiction!

Interval Coloring.

Input: Same as before.

Output: Color the intervals using min # colors s.t no two overlapping intervals are colored the same.



Alg.

1. Sort intervals in ↗ order of their start times.

2. Process intervals in the above order & give each interval the smallest indexed room, if possible

Correctness

Lemma: Lower bound on # rooms used by God's soln.

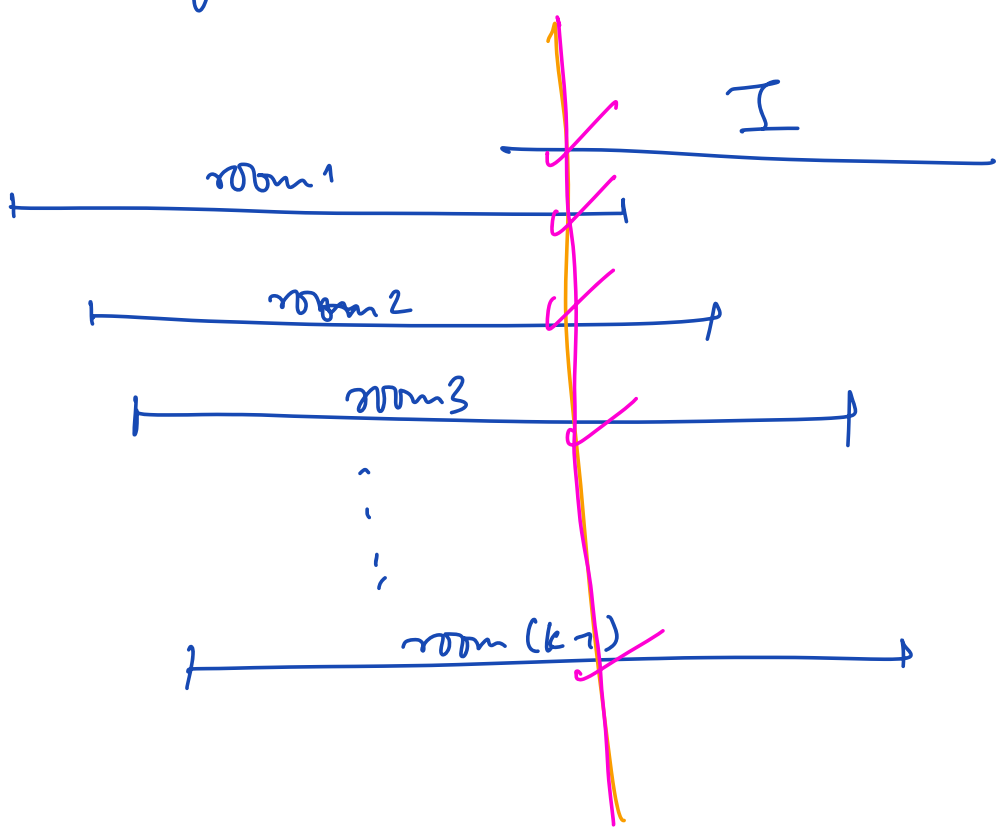
$\geq$ Size of the largest clique or max # mutually overlapping intervals.

Lemma: Our soln $\leq$ size of the largest clique.

Let k be the # rooms that our soln opens. In other words, let k be the # colors used to color the intervals s.t.

no two overlapping intervals are colored the same.

Let I be the first interval that is assigned to room k.



At the start time of I, these

are $k-1$ other intervals that are mutually overlapping thus resulting in a clique of size $\geq k$.

Thus God's opt $\geq k$.

Ex: Does the following alg. work?

- Choose the interval with min # overlapping intervals.
- Remove its conflicts.

Graph Algorithms
- BFS, DFS
- Appl. of BFS
- Appl of DFS

- Top 50m
- SCC.