DSGT QB

Algebraic structure : (2, 7, 8, 12, 17, 18)

(1) Group : If $(A, *)$ is closure , associative, identity, inverse then it is called as group

Abelian group : If $(A, *)$ is closure, associative, identity, inverse and commutative then it is called as abelian group.

(2) $a * b = a + b - ab \quad a, b \in Q$

$$\cancel{a * e = a}$$
$$\cancel{a + e - ae = a}$$
$$\cancel{a - ae = a}$$
$$\cancel{a(1-e) = 0}$$
$$a \neq 0$$
$$\therefore \cancel{\boxed{1 = e}}$$

$$a * e = a$$
$$a + e - ae = a$$
$$e - ae = 0$$
$$e(1-a) = 0$$
$$e = 0 \qquad a \neq 1$$

Identity element $= 0$

(3) $G = \{1, -1, i, -i\}$

composition table :

X	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

(4) $G = \{1, \omega, \omega^2\}$

X	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

$w^1 = w * 1 = w$
$w^2 = w * w = w^2$
$w^3 = w^2 * w = w^3 = 1$

generator is w

(5) ~~(i) 01010 & 01010~~

(i) $x = 01010$
$y = 01010$

$x \oplus y = 00000$

$|x \oplus y| = 0$

(ii) $x = 10101$ & $y = 01110$

$x \oplus y = 11011$

$|x \oplus y| = 4$

(6) $B^2 \to B^6$
hamming distance $= \delta = 3$

$\therefore$ No. of errors detected $= \delta - 1 = 2$
$\therefore$ No. of errors corrected $= \frac{\delta - 1}{2} = \frac{3-1}{2} = \frac{2}{2} = 1$

(7)

(8)

(9) $G = \{1, -1, i, -i\}$

composition table

$\times$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	$+i$	$-i$	-1	1
$-i$	$-i$	i	1	-1

(i) $\forall\, a, b \in G \quad a * b \in G$

$\therefore$ It is closure

(ii) Multiplication of number is always associative

(iii) multiplication of first row and first column
= identity

$e = 1$

(iv) inverse

$1^{-1} = 1$

$-1^{-1} = -1$

$i^{-1} = -i$

$-i^{-1} = i$

$\therefore$ It is group

(10) $\begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix} \quad m \in Z$

(i) closure $\quad \forall\, a, b \in G \quad a * b \in G$

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix}$$

$a * b \in G$

2-Regular graph

(ii) Associative

matrix multiplication is always associative

(iii) Identity

Let $m = \begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix}$

$M * I = I * M = M$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is identity

(iv) Inverse

$A = \begin{bmatrix} 1 & M \\ 0 & 1 \end{bmatrix}$ $|A| = 1 \neq 0$

A matrix have its inverse

$A^{-1} = \begin{bmatrix} 1 & -M \\ 0 & 1 \end{bmatrix} \in G$

(v) commutative :

$a * b = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix}$

$b * a = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix}$

$\therefore$ It is abelian group

(11) $G = \{0, 1, 2, 3, 4, 5\}$ $\qquad a +_6 b = (a+b) \% 6$

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

(i) $\forall\ a, b \in G \quad a * b \in G$

$\therefore$ It is closure

(ii) addition modulo is associative

(iii) multiplication of first row and first column = identity

$e = 0$

(iv) Inverse

$0^{-1} = 0$

$1^{-1} = 5$

$2^{-1} = 4$

$3^{-1} = 3$

$4^{-1} = 2$

$5^{-1} = 1$

(v) It is commutative

(vi) $(1)^1 = 1$

$(1)^2 = 1 +_6 1 = 2$

$(1)^3 = 1^2 +_6 1 = = 2 +_6 1 = 3$

$(1)^4 = 4$

$(1)^5 = 5$

$(1)^6 = 0$

$1 \rightarrow$ generator

$\therefore$ It is cyclic group.

12. $G = \{1, 5, 7, 11, 13, 17\}$

(13) $e : B^2 \rightarrow B^5$

$e(00) = 00000$
$e(10) = 10101$
$e(01) = 01110$
$e(11) = 11011$

$N = \{e_0, e_1, e_2, e_3\}$ $(N, \oplus)$

$\oplus$	e_0	e_1	e_2	e_3
e_0	e_0	e_1	e_2	e_3
e_1	e_1	e_0	e_3	e_2
e_2	e_2	e_3	e_0	e_1
e_3	e_3	e_2	e_1	e_0

$\forall a, b \in G$ & $a * b \in G$
$\therefore$ It is closure

Multiplication of first raw and first column
γ = ~~idenity~~ identity $= e_0$

$\therefore$ It is subgroup

$\therefore$ It is group code

(14) $e : B^3 \rightarrow B^7$

$e(000) = 0000000$
$e(001) = 0010110$
$e(010) = 0101000$
$e(011) = 0111110$
$e(100) = 100010$
$e(101) = 1010011$
$e(110) = 1101101$
$e(111) = 1111011$

$N = \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$

$(N, \oplus)$

$\oplus$	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_0	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	e_1	e_0	e_3	e_2	e_5	e_4	e_7	e_6
e_2	e_2		e_0	e_1	e_6	e_7	e_4	e_5
e_3	e_3			~~e_0~~ e_0	e_7	e_6	e_5	e_4
e_4	e_4				e_0	e_1	e_2	e_3
e_5	e_5					e_0		
e_6	e_6						e_0	
e_7	e_7							e_0

$\oplus$	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_0	e_0	0_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	e_1	e_0	e_3	e_2	e_5	e_4	e_7	e_6
e_2	e_2	e_3	e_0	e_1	e_6	e_7	e_4	e_5
e_3	e_3	e_2	e_1	e_0	e_7	e_6	e_5	e_4
e_4	e_4	e_5	e_6	e_7	e_0	e_1	e_2	e_3
e_5	e_5	e_4	e_7	e_6	e_1	e_0	e_3	e_2
e_6	e_6	e_7	e_4	e_5	e_2	e_3	e_0	e_1
e_7	e_7	e_6	e_5	e_4	e_3	e_2	e_1	e_0

$\forall\ a, b \in N \qquad a * b \in N$

$\therefore$ It is closure

identity $= e_0$

$\therefore$ It is subgroup

$\therefore$ It is group code

14 let $M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$e_H : B^2 \to B^5$

$B^2 = \{00, 01, 10, 11\}$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$X = B \otimes H$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$e(00) = 00000$
$e(01) = 01011$
$e(10) = 10110$
$e(11) = 11101$

16. $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ $e_H : B^3 \to B^{\cancel{8}6}$

$B^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ $H^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$X = B \otimes H$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$e(000) = 000000$
$e(001) = 001111$
$e(\cancel{0}10) = 010011$
$e(011) = 011100$
$e(100) = 100100$
$e(011) = 011101$
$e(\cancel{1}10) = 110111$
$e(111) = 111000$

(17) $e_H : B^2 \to B^5$

$e(00) = 00000$ (i) 11110

$e(10) = 10101$ (ii) 10011

$e(01) = 01110$

$e(11) = 11011$

(i) Let $w = 11110$

$\delta(w, e_0) = 4$

$\delta(w, e_1) = 1$

$\delta(w, e_2) = 2$ $\delta(w, e_3) = 2$

$d(w) = e_0 = \cancel{01}\ 00$

(ii) Let $w = 10011$

$\delta(w, e_0) = 3$

$\delta(w, e_1) = 4$

$\delta(w, e_2) = 2$

$\delta(w, e_3) = 1$

$d(w) = e_3 = 01$

18 $e_H : B^3 \to B^5$

$e_0(000) = 00000$

$e_1(001) = 00110$

$e_2(010) = 01001$

$e_3(011) = 01111$

$e_4(100) = 10011$

$e_5(101) = 10101$

$e_6(110) = 11010$

$e_7(111) = 11100$

(i) 11001

let $w = 11001$

$\delta(w, e_0) = 3$
$\delta(w, e_1) = 5$
$\delta(w, e_2) = 1$
$\delta(w, e_3) = 3$
$\delta(w, e_4) = 2$
$\delta(w, e_5) = 2$
$\delta(w, e_6) = 2$
$\delta(w, e_7) = 2$

$d(w) = e_4 = 001$

(ii) 01010

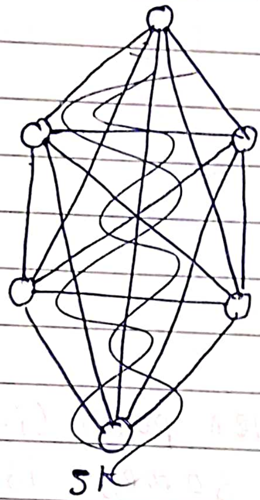
let $w = 01010$

$\delta(w, e_0) = 2$
$\delta(w, e_1) = 2$
$\delta(w, e_2) = 2$
$\delta(w, e_3) = 2$
$\delta(w, e_4) = 3$
$\delta(w, e_5) = 5$
$\delta(w, e_6) = 1$
$\delta(w, e_7) = 5$

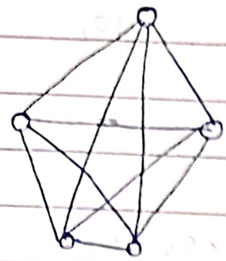
$d(w) = e_5 = 101$

Graph theory algebraic structure

(1)

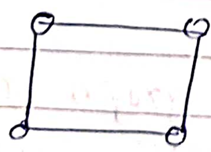


5K



5K

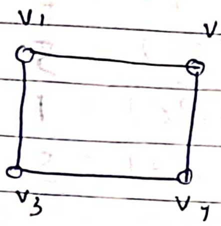
(2) (i) Regular Graph: A Graph G is said to be regular if every vertex has the same degree. If degree of each vertex of graph is k then it is called k-regular graph.



2-Regular graph

~~(ii) complete Bipartite Graph : A bipartite graph in which every vertices in one partition are connected to every other vertex in other partition~~

(ii) complete bipartite graph : A bipartite graph in which all vertices of one partition are connected to other partition.

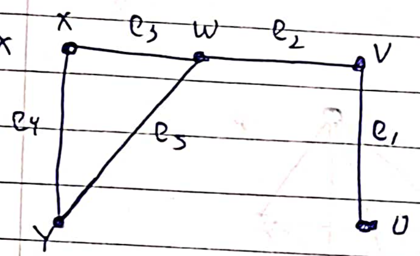


here $x = \{ v_1, v_4 \}$
$Y = \{ v_2, v_3 \}$

It is denoted by K_{22}

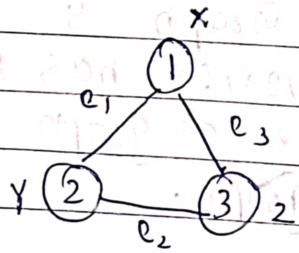
(3) Euler path : Traverse the graph in such a way that every edge is traverse exactly once is called Euler path.

EX



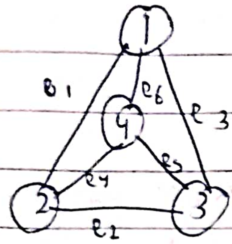
$u e_1 v e_2 w e_3 x e_4 Y e_5 w$

Euler circuit : The closed Euler path (i.e start and end vertices should be same) is called Euler circuit.



$x e_1 Y e_2 z e_3 x$

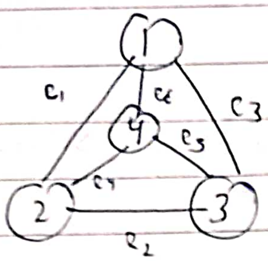
(4) Hamiltonian path : Traverse the graph in such a way that every vertex is traverse exactly one is called hamiltonian path.



$1 - e_1 - 2 - e_2 - 3 - e_5 - 4$

~~$1 - e_1 - 2 - e_2 - 3 - e_5 - 4 - e_6 - 1$~~

Hamiltonian circuit : The closed Hamiltonian path (i.e start and end vertices should be same) is called hamiltonian circuit



$1 - e_1 - 2 - e_2 - 3 - e_5 - 4 - e_6 - 1$

(5) Necessary condition for ~~Eule~~

(i) Euler graph : Degree of each vertex of graph G is even.

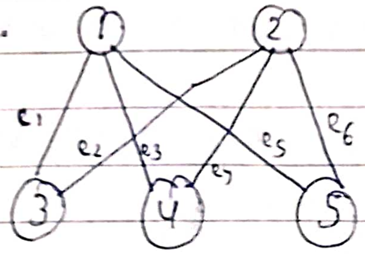
(ii) Euler path : Exactly two odd degree vertices

(6) Neccessary condition for

(i) Hamiltonian graph : Degree of each vertex is greater than equal to $n/2$.

(ii) Hamiltanian path : sum of the degrees of each pair of vertices is greater than equal to $(n-1)$.

(7) 1.

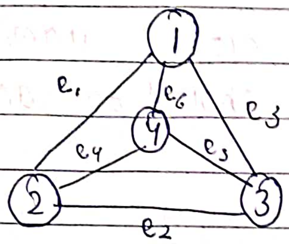


(i) Euler path : Exactly two vertices are odd degree
∴ path B 1 - e_1 - 3 - e_2 - 2 - e_4 - 4 - e_3 - 1 - e_5 - 5 - e_6 - 2

(ii) Hamiltonian path : Degree of each pair of vertices $\geq n-1 = 4$

path 3 - e_1 - 1 - e_5 - 5 - e_6 - 2 - e_4 - 4

2.

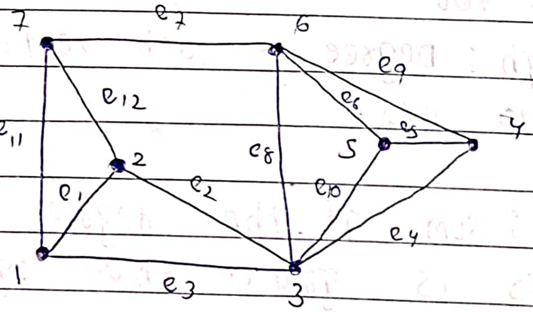


(i) Hamitonian path :
since degree of pair of vertex is $\geq n-1$
path : 1 - e_1 - 2 - e_2 - 3 - e_5 - 4

(ii) Hamitanian graph :
since degree of each vertex is (3) $\geq n/2 = 4/2 = 2$

circuit ~~1 - e_1 - 2 - e_2 - 3 - e_5 - e_6 - 1~~
1 - e_1 - 2 - e_2 - 3 - e_5 - 4 - e_6 - 1

2.



(i) Hamiltanian path
since degree of pair of vertix is $\geq n-1 \geq 6$

path : 1 - e_1 - 2 - e_{12} - 7 - B e_7 - 6 - e_9 - 4 - e_5 - 5 - e_{10} - 3

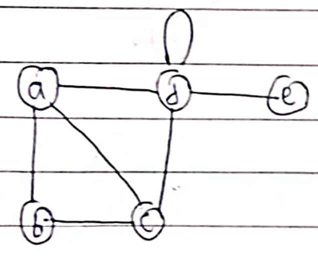
(8) Isomorphic graph :

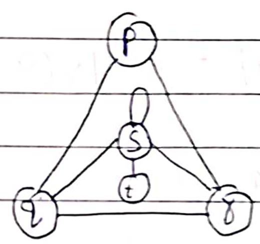
Two graphs G_1 and G_2 are said to be isomorphic if there is one to one correspondance between the edge set E_1 & E_2 in such a way that if e_1 is an ~~deg~~ edge with end vertices u_1 & v_1 in G_1, then the corresponding edge e_2 in G_2 has it's end point vertices u_2 & v_2 in G_2 which correspond to u_1 & v_1.

(9) ~~wat~~ 'Necessary condition for two graphs G_1 and G_2 is isomorphic

(i) The no. of vertices in G_1 and G_2 are same
(ii) The no. of edges in G_1 and G_2 are same
(iii) The degree sequence of G_1 and G_2 are same
(iv) vertex correspondance & edge correspondance valid

(10)





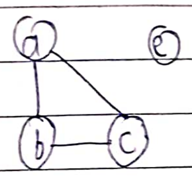
$|V(G_1)| = 5 = |V(G_2)|$

No. of edges in G_1 = ~~8~~ 7 = No. of edges in G_2

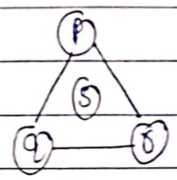
~~IV~~

In G_1 deg(d) = 5 and in G_2 deg(s) = 5
only vertices
$\therefore$ image(d) = s

$\therefore$ subgraph after removing vertices 'd' and 's' from G_1 and G_2



H_1



H_2

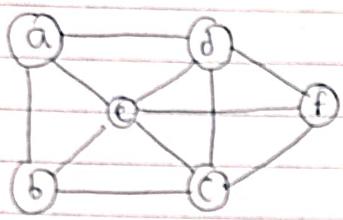
Hence $H_1 \cong H_2$

$\therefore G_1 \cong G_2$ and bijective

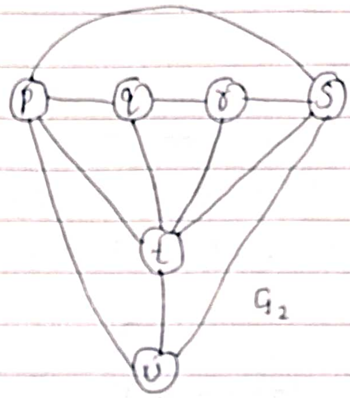
$f : V(G_1) \to V(G_2)$ as

Degree sequence	5	3	3	2	1
$V(G_1)$	d	a	c	b	e
$V(G_2)$	s	q	r	p	t

(ii)



G_1

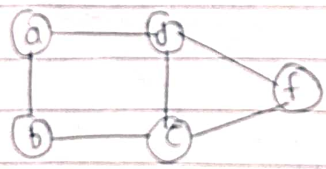


G_2

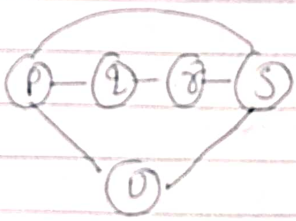
$|V(G_1)| = 6 = |V(G_2)|$

No. of edges in $G_1 = 11 =$ No. of edges in G_2

In G_1 ~~degree(e)~~ $\deg(e) = 5$ & in
G_2 $\deg(t) = 5$
$\therefore$ image$(e) = t$
$\therefore$ subgraph after removing vertices e and t from G_1 and G_2



H_1



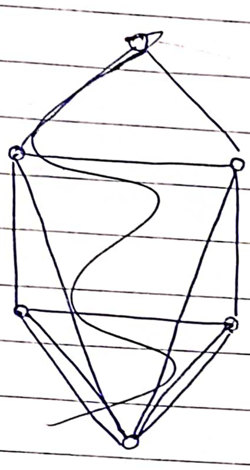
H_2

Hence $H_1 \cong H_2$
$G_1 \cong G_2$ and bijective
$f : V(G_1) \to V(G_2)$

Degree sequence	5	4	4	3	3	3
$V(G_1)$	e	c	d	u	a	b
$V(G_2)$	t	p	s	f	q	r

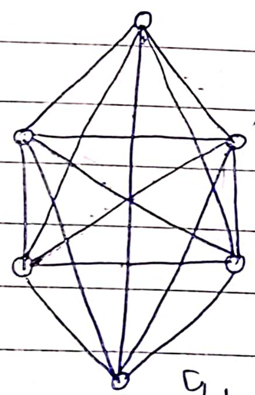
~~(12) same as 8 + E.~~

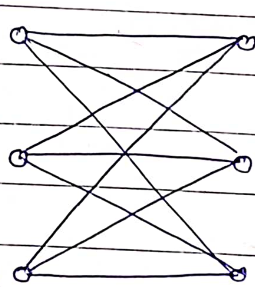
K_6



(12) same as 8 +

K_6 $K_{3,3}$





$|V(G_1)| = 6 \quad |V(G_2)| = 6$

No. of edges of G_1 and G_2 are not same

∴ They are not isomorphic