Home work 4

Page No.
Date

Q1]

a] True

If we have a case in which m preffers w the most and w preffers the m most then they must be paired else the matching will not be stable, hence stable matching always contain some nice couple

b] True

If p(w,m) is not in s then it would not be a stable - matching

c] False

From the lemma discussed during the class (Multiple stable matching can exist)

d] False

If there is only one stable matching condition that mean if m_i has w_i as preference then w_i has M_i as first preference but set (w_i, m_i) is not disjoint.

Q2) let us consider 4 people P_1, P_2, P_3 and P_4

Preference list :

P_1 (P_2 , P_3 , P_4)
P_2 (P_3 , P_4 , P_1)
P_3 (P_4 , P_1 , P_2)
P_4 (P_1 , P_2 P_3)

If I assign P_1 and P_2 in one room and P_3 and P_4 in another then From the preference we can see the p (P_1, P_3) is more prefered than current pairing

Here the current is not a stable solution

Q3] → We have n ships and n-port.
→ This algorithm will run untill all the ship are ported
→ We will go to a port and check which is the last ship that arrives on the port. and that ship will remain there (for that month)
→ Now we will reduce the ship set and port set by 1 each removing the the ship that we ported in the previous step and the port on which it was ported
→ As the ship that we have selected in last step is ported it will never go to any other port.
→ Now we will repeat the previous step and chose a port and the last ship that arrive on that port from the (n-1) ships and (m-1) port.
→ At the end all the ships will be fixed on the port and our algorithm will stop

] Part 1 : Prove:

Suppose Z is not stable

Z contain ⇒ (m, w)
Unstability / blocking pair ⇒ (m, w')
i.e m preffer w' more than w
w' preffer m more than here current partner

without loss of generality, let X pair m with w' and Y pair m with w. Since X is a stable matching, w' prefers m to her partner in X, similary since Y is a stable matching, m prefers w to his partner in Y

Consider two cases

(1) w' prefer m to her partner in X, In this case (m, w') is a blocking pair in X, which is contradiction on stability of X

(2) w' prefers m to her partner in X. In this the man paired with w' is Z let him be m' must prefer w' to his partner in Y, since otherwise (m', w') would be a blocking pair in Y.
But then m' prefers w' to w, which means that w cannot be m's least preferred partner among w, which is not possible.

$\therefore$ Z is stable.

<u>Part 2</u> : <u>disprove</u>

<u>Counter example</u>

$m_1 : (w_1, w_2, w_3)$ $w_1 : (m_2, m_3, m_1)$
$m_2 : (w_2, w_1, w_3)$ $w_2 : (m_1, m_2, m_3)$
$m_3 : (w_2, w_1, w_3)$ $w_3 : (m_2, m_1, m_3)$

$X \Rightarrow (m, w_1)\ (m_2, w_2)\ (m_3 w_3)$
$Y \Rightarrow (m_1, w_2)\ (m_3, w_1)\ (m_2, w_3)$

$Z \Rightarrow (m_1, w_2)\ (m_2, w_3)\ (m_3, w_3)$

It is not even the possible as w_3 cannot be partner with 2 at same time

$\therefore$ Hence disproved

Q5]

a] $(3/2)^n \quad 3^{(n/2)} \quad 2^{(n/3)}$

$\Rightarrow \quad n \log(3/2) \quad \frac{n}{2} \log(3) \quad \frac{n}{3} \log 2$

$\therefore \quad (3/2)^n = (3^{n/2}) = 2^{(n/3)}$

b] $\log n = \ln(n) = \lg(n^2)$

c] $n^{\lg(4)} = 2^{\log n} = 2^{2\lg(n)}$

d] $\max(50n^2, n^3) = 50n^2 + n^3 \geq \min(50n^2, n^3)$

e] $\lceil n^2/20 \rceil = \lfloor n^2/20 \rfloor = n^2/20$

Page No.		
Date		

Q6]

a] → The outer most loop runs for n times
→ The second loop runs for j-i+1 time for every i^{th} iteration of the outer most loop
→ Then again we have to iterate through the array to add the values which would take $O(n)$
→ And storing will take constant time

$$\therefore \quad O(f(n)) = O(n \times n \times n)$$
$$= \underline{O(n^3)}$$

b] → As the first for loop will always run ~~from~~ n time
→ The second for loop wil run j-i+1 for every i^{th} iteration of first lop.
→ And the we have to add Ai to Aj for ever second loop

$$\therefore \quad = \sum_{i=1}^{n} (n-i) \sum_{j=}^{i} i$$

$$= \sum_{i=1}^{n} (n-1)\, \Omega(n)$$

$$= \Omega(n)\, \Omega((n)(n-1)/2)$$

$$= \Omega(n)\, \Omega(n^2)$$

$$= \Omega(n^3)$$

$$= \Omega(f(n))$$

c) $f(n) = n^3$

$$\lim_{n \to \infty} \frac{g(n)}{n^3} = 0$$

$\therefore$ The possible value of $(g(n)) = n^2$

```
For (i=1 ; i<=n ; i++)
    sym = 0 ;
    For ( j= i+1 ; j<n ; j++)
        sym = sym + Aj
        B[ij] = sym .
```

Time complexity $O(n^2)$