Bayesian

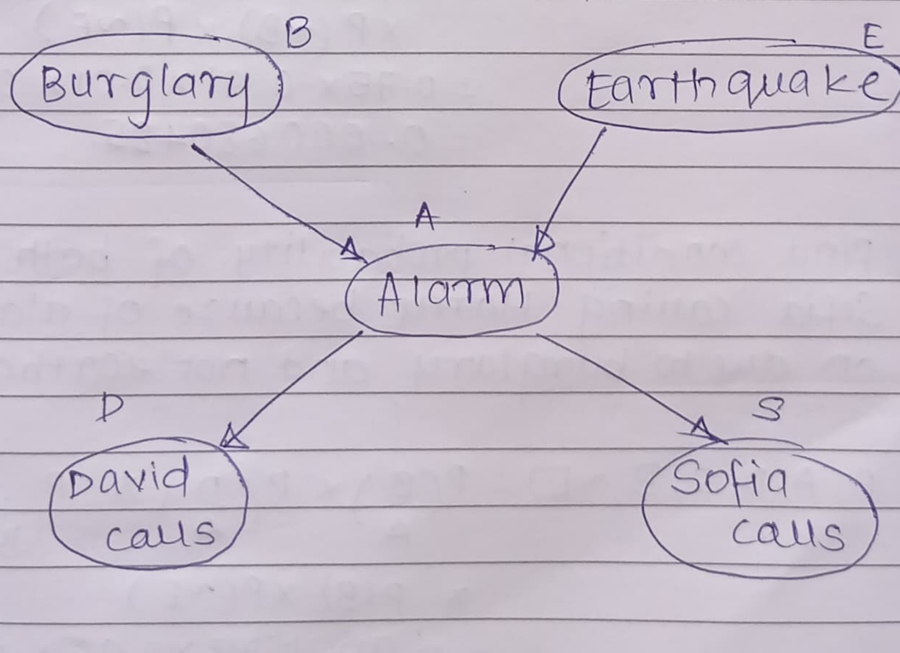
Q. Harry recently installed a new burglar alarm at his home to detect burglaries. The alarm is reliable at detecting burglaries but can also be triggered by minor earthquakes. Harry has two neighbours - David and sofia, who have aggreed to inform Harry when at work, whenever they hear the alarm. David tends to call Harry whenever he hears the alarm, but occasionly he calls by mistake when phone rings for other reasons. Sofia enjoys listening to loud music and occasionally misses to hear the alarm all together. Calculate the ~~bu~~ probability of burglary alarm.

Calculate the prob. that alarm has sounded but there is neither a burglary nor an earthquake, but David and Sofia both called Harry.

A(T) B(F) E(F)

D(T), S(T)

→



B

T	0.002
F	0.998

E

T	0.001
F	0.999

A

B	E	A(T)	A(F)
T	T	0.94	0.06
T	F	0.95	0.05
F	T	0.31	0.69
F	F	0.001	0.999

D

A	D(T)	D(F)
T	0.91	0.09
F	0.05	0.95

S

A	S(T)	S(F)
T	0.75	0.25
F	0.02	0.98

$$P(A, \sim B, \sim E, D, S) = P\left(\frac{B}{A}\right) \times P\left(\frac{D}{A}\right) \times P\left(\frac{A}{\sim B \wedge \sim E}\right) \times P(\sim B) \times P(\sim E)$$

$$= 0.75 \times 0.91 \times 0.01 \times 0.998 \times 0.999$$

$$= 0.000680435.$$

Q. Find conditional probability of both David and Sofia calling Harry because of alarm going on due to burglary and not earthquake.

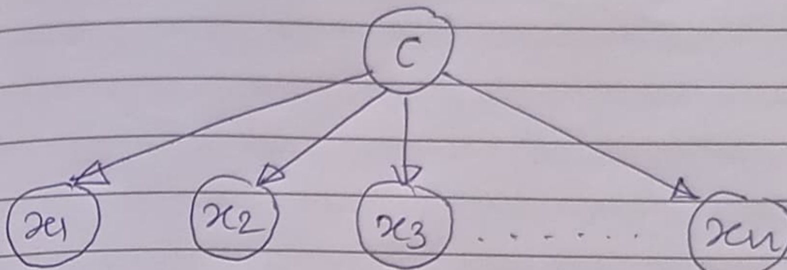
→

$$P(A, D, S, B, \sim E) = P\left(\frac{B}{A}\right) \times P\left(\frac{D}{A}\right) \times P\left(\frac{A}{B \wedge \sim E}\right) \times P(B) \times P(\sim E)$$

$$= 0.91 \times 0.75 \times 0.95 \times 0.02 \times 0.999$$

Naive Bayes Model
(conditional probability)

Q suppose we have N random variables, all of which are independent given another random variable C.

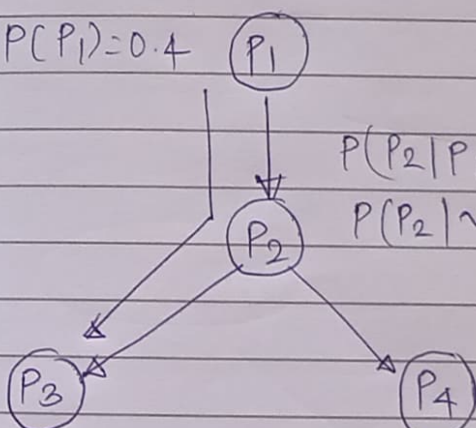


$$P(C, x_1, x_2, x_3, \ldots, x_n) = P(C) \cdot P\left(\frac{x_1}{C}\right) \cdot P\left(\frac{x_2}{C}\right) \cdots P\left(\frac{x_n}{C}\right)$$

$$= P(C) \cdot \prod_{i=1}^{n} P\left(\frac{x_i}{C}\right)$$

Q

$P(P_1) = 0.4$



$P(P_2 | P_1) = 0.8$

$P(P_2 | \sim P_1) = 0.5$

$P(P_4 | P_2) = 0.8$

$P(P_4 | \sim P_2) = 0.5$

$P(P_3 | P_2) = 0.2$

$P(P_3 | \sim P_2) = 0.3$

$$\begin{aligned} P(P_1, P_2, \sim P_3) &= P(P_1) . P(P_2 / P_1) . P(P_3 / P_2) + \\ &\quad P(P_1) . P(P_2 / P_1) . P(P_3 / \sim P_2) + \\ &\quad P(P_1) . P(P_2 / \sim P_1) . P(P_3 | P_2) + \\ &\quad P(P_1) . P(P_2 / \sim P_1) . P(P_3 / \sim P_2) \\ &= 0.26 \\ &= 1 - 0.26 \\ &= 0.75 . \end{aligned}$$

$$P(P_1, P_2 \mid \sim P_3) = P(P_1) * P(P_2 \mid P_1) \times P(P_3 \mid \sim P_2) + P(P_1) \times P(P_2 \mid \sim P_1) \times P(P_3 \mid \sim P_2)$$