

Algorithm Design

Homework Assignment 5

Given: March 10, 2023

Due: March 17, 2023

✓ 1. Give asymptotic upper and lower bounds for each of the following recurrences using the method of expansion (iteration). For all recurrences, assume that $T(n)$ is a constant for $n \leq 2$ and n is an exact power of 2.

✓ a. $T(n) = T(n/2) + n$

✓ b. $T(n) = 2T(n/2) + 2n$

✓ c. $T(n) = T(\sqrt{n}) + 1$

✓ d. $T(n) = 2T(n-1) + 1$

✓ 2. The input is a set S of n real numbers, and a real number z .

(a) Design $\theta(n \lg n)$ algorithm to determine whether or not there are two elements in S whose sum is exactly z .

(b) Suppose now that the set S is given in a sorted order. Design an algorithm to solve this problem in linear time.

✓ 3. Given a sorted array $A[1..n]$ of reals, give an efficient algorithm to determine whether A has a *majority* element or not. A majority element is an element that appears more than $n/2$ times.

✓ 4. Let $X[1..n]$ and $Y[1..n]$ be two arrays, each containing n numbers already in sorted order. Give an $O(\lg n)$ -time algorithm to find the median of all $2n$ elements in the array $X \cdot Y$, where $X \cdot Y$ is the concatenation of the arrays X and Y .