Fundamentals of Hypothesis Testing: One-Sample Tests

In the field of business, decision makers are continually attempting to find answers to questions such as the following:

- What **container shape** is most economical and reliable for shipping a product?
- Which **management approach best motivates employees** in the retail industry?
- How can the company’s retirement investment financial portfolio be diversified for **optimum performance**?
- What is the best way to **link client databases** for fast retrieval of useful information?
- What is the most **effective means of advertising** in a business-to-business setting?

In searching **for answers** to questions and in attempting to find explanations for business phenomena, **business researchers** often develop “**hypotheses**” that can be studied and explored. **Hypotheses are *<u>tentative explanations of a principle operating in nature</u>***.

Types of Hypotheses

1. *Research* hypotheses

2. *Statistical* hypotheses

3. *Substantive* hypotheses

Research Hypotheses

Research hypotheses are most nearly like hypotheses defined earlier.

A **research hypothesis** is *a statement of what the* ***researcher*** ***<u>believes</u>*** ***will be the*** ***<u>outcome</u>*** ***of an*** ***<u>experiment</u>*** ***or a study.***

Before studies are undertaken, business researchers often have some idea or theory based on **experience or previous work as** to how the study will turn out.

Some examples of **research hypotheses** in business might include:

■ Older **workers are more loyal to a company**.

■ Companies with more than $1 billion in **assets** spend a higher percentage of their annual **budget** on **advertising** than do companies with less than $1 billion in assets.

■ The implementation of a **Six Sigma quality** approach in manufacturing will result in **greater productivity**.

2

Statistical Hypotheses

In order to **scientifically test research hypotheses**, a more **formal** hypothesis **structure** needs to be set up using **statistical hypotheses**.

set of questions

5 no old

new

Suppose business researchers want to "prove" the research hypothesis that "older workers are more loyal to a company". A "loyalty" survey **instrument** is either developed or obtained.

Mean

If this instrument is administered to both older and younger workers, how much higher do older workers have to **score on** the "loyalty" instrument (assuming higher scores indicate more loyal) than younger workers to prove the research hypothesis?

What is the "proof threshold"? Instead of attempting to prove or disprove research hypotheses directly in this manner, business researchers convert their **research hypotheses to statistical hypotheses** and then **test the statistical hypotheses** using **standard procedures**.

All **statistical hypotheses** consist of two parts, a **null hypothesis and an alternative hypothesis**.

Null hypothesis *states that the "null"* ***condition exists; that is, there is nothing new happening, the old theory is still true, the old standard is correct, and the system is in control.***

The **alternative hypothesis**, on the other hand, *states* ***that the new theory is true, there are new standards, the system is out of control, and/or something is happening***.

As an example, suppose flour packaged by a manufacturer is sold by weight; and a particular size of package is supposed to **average 40 ounces**. Suppose the manufacturer wants to test to determine whether their packaging **process is out of control** as determined by the weight of the flour packages.

The null hypothesis for this experiment is that the average weight of the flour **packages is 40 ounces** (no problem). $H_0 = 40$

The alternative hypothesis is that the average is **not 40 ounces (process is out of control).**

$H_a \neq 40$

2)

As another example, suppose a company has held an 18% share of the market. However, because of an increased marketing effort, company officials believe the company's market share is now greater than 18%, and the officials would like to prove it. The null hypothesis is that the market share is still 18% or perhaps it has even dropped below 18%. Converting the 18% to a proportion and using p to represent the population proportion, results in the following null hypothesis: ~~H0~~ <= 0.18

1) Null hypo. →

2) alternati hypo

$H_0 = 0.18$ × $H_a \neq 0.18$

The alternative hypothesis is that the population proportion is now greater than .18: H_a: $p > 0.18$

H_0 → null

H_a → alternati

accept or Reject

Note that the "new idea" or "new theory" that company officials want to "prove" is stated in the alternative hypothesis

Because many business researchers only undertake an experiment to determine whether their new hypothesis is correct, they are hoping that the alternative hypothesis will be "proven" true.

However, if a manufacturer is testing to determine whether his process is out of control as shown in the flour-packaging example, then he is most likely hoping that the alternative hypothesis is not "proven" true thereby demonstrating that the process is still in control.

A **substantive result** is *when the outcome of a statistical study produces results that are important to the decision maker.*

However, an examination of the data revealed that on a five-point scale, their satisfaction ratings had gone up from 3.61 to only 3.63. Is going from a 3.61 rating to a 3.63 rating in one year really a substantive increase?

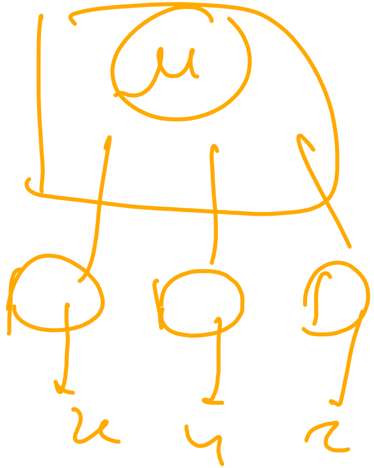
On the other hand, increasing the average purchase at a large, high-volume store from \$55.45 to \$55.50 might be substantive as well as significant if volume is large enough to drive profits higher.

0.05

Both business researchers and decision makers should be **aware that statistically significant results are not always substantive results.**

What is a Hypothesis?

- A hypothesis is a claim (assertion) about a population parameter:
  - population mean
  - population proportion



Example: The mean monthly cell phone bill in this city is μ = Rs 42

Example: The proportion of adults in this city with cell phones is π = 0.68

mean, proportions;
~~sta~~
variance

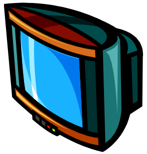
The Null Hypothesis, H_0

- States the claim or assertion to be tested

Example: The average number of TV sets in Indian Homes is equal to three ($H_0 : \mu = 3$)

Population mean

- Is always about a population parameter, not about a sample statistic



$$H_0 : \mu = 3$$

$$\cancel{H_0 : \bar{X} = 3}$$

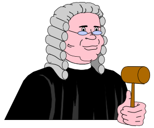
$H_0\ \mu = 3$

$H_a\ \mu \neq 3$

The Null Hypothesis, H_0

(continued)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty



- **Refers to the status quo or historical value**
- Always contains "=" , "≤" or "≥" sign
- May or may not be rejected

$=$

$\leq$

$\geq$

2) The Alternative Hypothesis, H_1

= $\neq$ > <

- Is the opposite of the null hypothesis

- e.g., The average number of TV sets in Indian homes is not equal to 3($H_1: \mu \neq 3$)

- **Challenges the status quo**

$H_0 \leftarrow$ Test $\rightarrow$ H_0 accept; $\rightarrow$ H_0 reject

- May or may not be proven

- **Is generally the hypothesis that the <u>researcher is trying to prove</u>**

Ex: H0: μ= 100 (the null hypothesis is that the population mean is 100)

H1: $\mu \neq 100$

H1: $\mu > 100$

H1: $\mu < 100$

$H_0: \mu = 100$
$H_1: \mu \neq 100$
$H_1: \mu > 100$
$H_2: \mu < 100$

The Hypothesis Testing Process

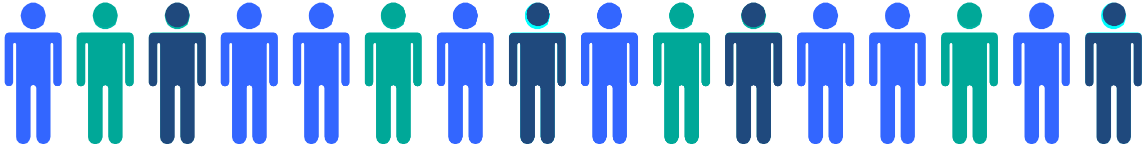
- Claim: The population mean age is 50.
  - H_0: $\mu = 50$, H_1: $\mu \neq 50$
- Sample the population and find sample mean.

$\mu = 50$

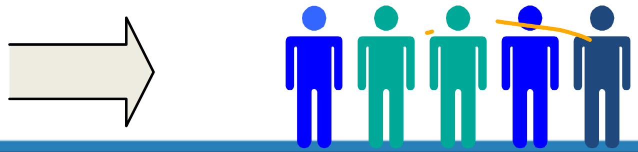
$\mu \neq 50$

mean

Population



Sample



avg = 20

The Hypothesis Testing Process

(continued)

- Suppose the sample mean age was $\overline{X} = 20$.

$\mu = 50$ $H_0 \rightarrow$ Reject

$\mu \neq 50$

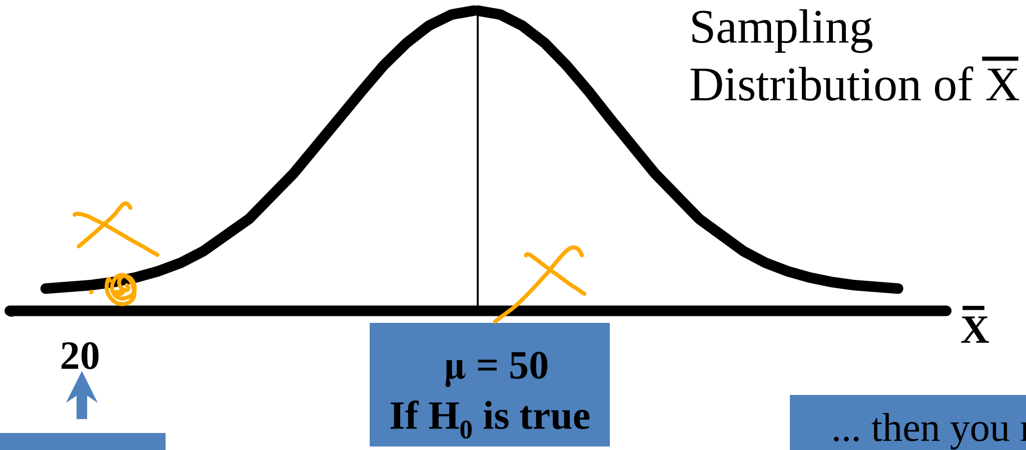
- This is significantly lower than the claimed mean population age of 50.

- If the null hypothesis were true, the probability of getting such a different sample mean would be very small, so you reject the null hypothesis .

- In other words, getting a sample mean of 20 is so unlikely if the population mean was 50, you conclude that the population mean must not be 50.

The Hypothesis Testing Process

(continued)



If it is unlikely that you would get a sample mean of 20.

When in fact 50 was the population mean…

... then you reject the null hypothesis that μ = 50.