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Friday  
JANUARY

006-359 Wk-01

JAN '23

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1	2	3	4	5	6	7	8	9	10	11	12	13	14
15	16	17	18	19	20	21	22	23	24	25	26	27	28
29	30	31											

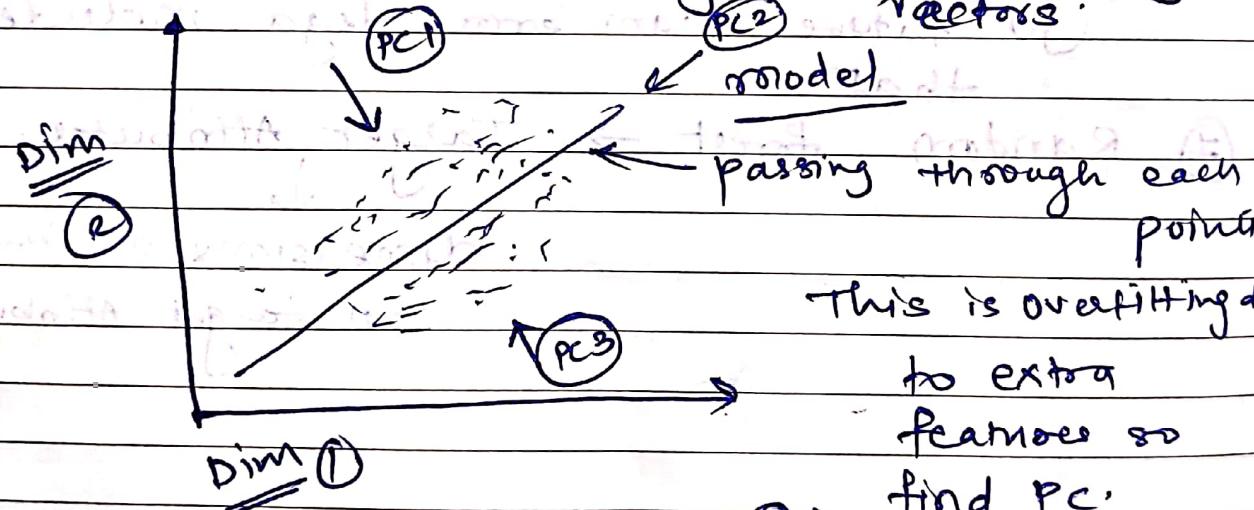
## Principal component Analysis

2023

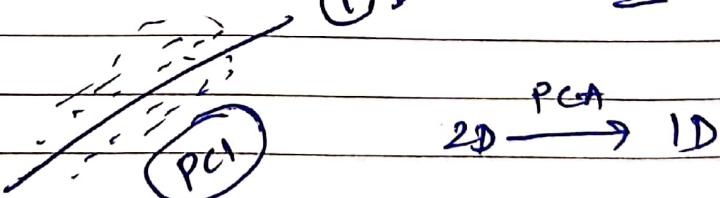
- It is unsupervised learning algo
- Used for dimensionality reduction in ML
- It is a statistical process that converts the observations of correlated features into a set of linear uncorrelated features with the help of orthogonal transformation
- It is used in
  - ① Image processing
  - ② Movie recommendation system
  - ③ Optimizing power allocation in various comm' channels.

It is based on ① Variance & covariance

② Eigen values & eigen vectors



This is overfitting due to extra features so find PC.



2023 Reduce dataset from 1D to 2D.

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	F	$E x_1$	$E x_2$	$E x_3$	$E x_4$
8	$x_1$	4	8	13	7
9	$x_2$	11	4	5	14

Step ① find mean

$$\bar{x}_1 = \frac{1}{4} (4 + 8 + 13 + 7) = 8$$

$$\bar{x}_2 = \frac{1}{4} (11 + 4 + 5 + 14) = 8.5$$

Step ② :- Covariance matrix

$$S = \begin{bmatrix} \text{cov}(x_1, x_1) & \text{cov}(x_1, x_2) \\ \text{cov}(x_2, x_1) & \text{cov}(x_2, x_2) \end{bmatrix}$$

$$\text{cov}(x_1, x_1) = \frac{1}{N-1} \sum_{k=1}^N (x_{1k} - \bar{x}_1)(x_{1k} - \bar{x}_1)$$

$$= \frac{1}{4-1} ((4-8)^2 + (8-8)^2 + (13-8)^2 + (7-8)^2) \\ = 14$$

$$\text{cov}(x_1, x_2) = \frac{1}{N-1} \sum_{k=1}^N (x_{1k} - \bar{x}_1)(x_{2k} - \bar{x}_2)$$

$$= \frac{1}{3} [(4-8)(11-8.5) + (8-8)(\frac{4-8.5}{Sunday 08}) + (13-8)(5-8.5) + (7-8)(14-8.5)]$$

$$= -11$$

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Monday  
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M	T	W	T	F	S	S	M	T	W	T	F	S
					1	2	3	4	5	6	7	8
JAN'23	9	10	11	12	13	14	15	16	17	18	19	20
	23	24	25	26	27	28	29	30	31		21	22

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$$8 \text{ Cov}(x_2, x_1) = \frac{1}{N-1} \sum_{k=1}^N (x_{2k} - \bar{x}_2)(x_{1k} - \bar{x}_1)$$

9

$$10 = \frac{1}{N-1} \sum_{k=1}^4 ( ) ( ) +$$

$$11 = 8 = ( ) ( ) + (-) (+) +$$

12

$$( ) ( )$$

1

$$= -11$$

$$2 \text{ Cov}(x_2, x_2) = \frac{1}{N-1} \sum_{k=1}^N (x_{2k} - \bar{x}_2)(x_{2k} - \bar{x}_2)$$

$$3 = \frac{1}{3} ((11-8.5)^2 + (4-8.5)^2 +$$

$$4 = ((5-8.5)^2 + (14-8.5)^2)$$

5

$$= 23$$

$$6 = \begin{bmatrix} 14 & -11 \\ -11 & 23 \end{bmatrix}$$

$$= (14 \cdot 23 - (-11) \cdot (-11)) / 23$$

$$= 11$$



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Step 3 :- Calculate eigen value of covariance matrix

The characteristic eqn of the covariance matrix is

$$10 \quad 0 = \det(S - \lambda I)$$

$$11 \quad 0 = \begin{vmatrix} 14 - \lambda & -11 \\ -11 & 23 - \lambda \end{vmatrix}$$

Note  $\Rightarrow I$  for  $2 \times 2$  matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix}$$

$$12 \quad 0 = (14 - \lambda)(23 - \lambda) - (-11) \times (-11) \quad S - \lambda I = \begin{bmatrix} 14 - \lambda & -11 \\ -11 & 23 - \lambda \end{bmatrix}$$

$$= \lambda^2 - 37\lambda + 20 \leftarrow \text{solve in calc}$$

$$13 \quad \lambda = \frac{1}{2} (37 \pm \sqrt{565})$$

$$14 \quad = 30.3849, 6.6151$$

Respectively,  $\lambda_1 = 30.3849$ ,  $\lambda_2 = 6.6151$

Step 4 :- Eigen vector calculation

$$15 \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$16 \quad \therefore \begin{bmatrix} 0 \\ 0 \end{bmatrix} = (S - \lambda I) v$$

$$17 \quad = \begin{bmatrix} 14 - \lambda & -11 \\ -11 & 23 - \lambda \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$18 \quad = \begin{bmatrix} (14 - \lambda)v_1 & -11v_2 \\ -11v_1 & (23 - \lambda)v_2 \end{bmatrix}$$

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JAN 23	1	2	3	4	5	6	7	8	9	10	11	12	13
	14	15	16	17	18	19	20	21	22	23	24	25	26
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8 eq<sup>n</sup> 1  $\rightarrow (14 - \lambda) u_1 - 11 u_2 = 0$

9 eq<sup>n</sup> 2  $\rightarrow -11 u_1 + (28 - \lambda) u_2 = 0$

consider eq<sup>n</sup> ①

10  $(14 - \lambda) u_1 - 11 u_2 = 0$

11  $\frac{u_1}{14 - \lambda} = \frac{u_2}{11} = t \dots \text{(say)}$

12  $14 - \lambda = 11t \quad 11 - u_1 = 11t \quad 11t = 11 \quad t = 1$

$14 - \lambda = 11 \quad \lambda = 14 - 11 = 3$

13 put.  $t = 1 \quad 14 - \lambda = 11$

14  $\therefore U = \begin{bmatrix} 11 \\ 14 - \lambda \end{bmatrix} = \begin{bmatrix} 11 \\ 11 \end{bmatrix} = R$

15 Note:- ① For PCA always consider largest value out of two eigen values

16  $\therefore \lambda_1 = 80.3849 \quad \lambda_2 = 6.6151$

17 ② If one PCA is asked consider largest value

18 ③ If multiple PC is asked consider 2nd eigen value

$$\begin{bmatrix} 11 & 14 - \lambda_1 & 11 \\ 11 & 14 - \lambda_2 & 11 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 11 \\ 11 & 11 & 11 \end{bmatrix}$$



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1	2	3	4	5	6	7	8	9	10	11	12		
13	14	15	16	17	18	19	20	21	22	23	24	25	26
27	28												

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$$\therefore \mathbf{v}_1 = \begin{bmatrix} 11 \\ 14 - \lambda_1 \end{bmatrix}$$

$$\begin{bmatrix} 11 \\ 14 - 30 \end{bmatrix} = \begin{bmatrix} 11 \\ -16 \end{bmatrix}$$

To find unit eigen vector, compute the length of  $\mathbf{v}_1$ .

$$\therefore \|\mathbf{v}_1\| = \sqrt{11^2 + (14 - \lambda)^2}$$

$$\begin{aligned} &= \sqrt{(11)^2 + (14 - 30 \cdot 3849)^2} \\ &= 19.73 \end{aligned}$$

$$\begin{aligned} \mathbf{e}_1 &= \begin{bmatrix} 11 / \|\mathbf{v}_1\| \\ (14 - \lambda_1) / \|\mathbf{v}_1\| \end{bmatrix} \\ &\text{1st eigen vector} \end{aligned}$$

$$= \begin{bmatrix} \frac{11}{19.73} \\ \frac{(14 - 30 \cdot 3849)}{19.73} \end{bmatrix} = \begin{bmatrix} 0.5574 \\ -0.8303 \end{bmatrix}$$

One more eigen value

$$\mathbf{v}_2 = \begin{bmatrix} 14 \\ 14 - \lambda_2 \end{bmatrix}$$

$$\begin{aligned} \therefore \|\mathbf{v}_2\| &= \sqrt{(11)^2 - (14 - 6.6151)^2} \\ &= \sqrt{\dots} = \dots \end{aligned}$$

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$$e_2 =$$

$$\begin{bmatrix} 11 \\ 11U_211 \end{bmatrix}$$

$$14 - \lambda_1$$

$$e_2 =$$

$$\begin{bmatrix} 11 \\ (x - \lambda_1) + U_211 \end{bmatrix}$$

$$0.8303$$

$$0.5574$$

SAP

1

$$\begin{bmatrix} 11 \\ 11U_211 \end{bmatrix} = 10$$

2

$$\begin{bmatrix} 11 \\ 11U_211 \end{bmatrix} = 10$$

3

$$\begin{bmatrix} 11 \\ 11U_211 \end{bmatrix} = 10$$

4

$$\begin{bmatrix} 11 \\ 11U_211 \end{bmatrix} = 10$$

5

$$\begin{bmatrix} 11 \\ 11U_211 \end{bmatrix} = 10$$

6

$$\begin{bmatrix} 11 \\ 11U_211 \end{bmatrix} = 10$$

SAP

$$\begin{bmatrix} 11 \\ 11U_211 \end{bmatrix} = 10$$

$$\begin{bmatrix} 11 \\ 11U_211 \end{bmatrix} = 10$$

$$\begin{bmatrix} 11 \\ 11U_211 \end{bmatrix} = 10$$



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S S  
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21 22

M	T	W	T	F	S	S	M	T	W	T	F	S	S
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13	14	15	16	17	18	19	20	21	22	23	24	25	26
27	28												

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Step 5 : Computation of first PC

$$\begin{aligned}
 e_1^T \begin{bmatrix} x_{1k} - \bar{x}_1 \\ x_{2k} - \bar{x}_2 \end{bmatrix} &= [0.5574 - 0.8303] \begin{bmatrix} x_{11} - \bar{x}_1 \\ x_{21} - \bar{x}_2 \end{bmatrix} \\
 &= 0.5574 (x_{11} - \bar{x}_1) - 0.8303 (x_{21} - \bar{x}_2) \\
 &= 0.5574 (4 - 8) - 0.8303 (11 - 8.5) \\
 &= -4.30535
 \end{aligned}$$

Computation of 2nd PC

$$\begin{aligned}
 e_2^T \begin{bmatrix} x_{1k} - \bar{x}_1 \\ x_{2k} - \bar{x}_2 \end{bmatrix} &= [0.5574 - 0.8303] \begin{bmatrix} x_{12} - \bar{x}_1 \\ x_{22} - \bar{x}_2 \end{bmatrix} \\
 &= 0.5574 - 0.8303 (x_{12} - \bar{x}_1) - \\
 &\quad 0.8303 (x_{22} - \bar{x}_2) \\
 &= 0.55(8 - 8) - 0.8303 (4 - 8.5)
 \end{aligned}$$

Computation for 3rd component

$$e_3^T \begin{bmatrix} x_{1k} - \bar{x}_1 \\ x_{2k} - \bar{x}_2 \end{bmatrix} = [0.5574 - 0.8303] \begin{bmatrix} x_{13} - \bar{x}_1 \\ x_{23} - \bar{x}_2 \end{bmatrix}$$

Sunday 15



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	1	2	3	4	5	6	7	8	9	10	11	12	13

14 15 16 17 18 19 20 21 22  
23 24 25 26 27 28 29 30 31

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Computation for 4th PC with original : 3905

8

$$e_1^T \begin{bmatrix} x_{1k} - \bar{x}_1 \\ x_{2k} - \bar{x}_2 \end{bmatrix} = [0.5574 \quad -0.8303] \begin{bmatrix} \bar{x} - 11.7 \\ \bar{x} - 4.08 \end{bmatrix} = x_{14} - \bar{x}_1$$

10

$$(2.8 - 11) \cdot 0.5574 - (8 - 4.08) \cdot -0.8303 =$$

11

$$(2.8 - 11) \cdot 0.5574 - (8 - 4.08) \cdot -0.8303 =$$

12

$$-8.2 \cdot 0.5574 - =$$

1

Feature	Ex 1.09	Ex 2	Ex 3	Ex 4
$(\bar{x} - x_1)^2$	4	8	$\bar{x} - 13.6$	$\bar{x} - 14.2$
$(\bar{x} - x_2)^2$	$(8 - 4)^2 = 16$	$(8 - 4)^2 = 16$	$(8 - 5)^2 = 9$	$(8 - 14)^2 = 36$
$x_2$	11	4	5	14
$(\bar{x} - x_1)^2$	$(8 - 2)^2 = 36$	$(8 - 2)^2 = 36$	$(8 - 5)^2 = 9$	$(8 - 14)^2 = 36$
Frost	-24.2052	3.7361	5.6928	-5.1238
Principle Components	$8.081 \cdot 0.5574 - (2 - 11) \cdot -0.8303$			

6

Inception for self training

$$\begin{bmatrix} \bar{x} - x_1 \\ \bar{x} - x_2 \end{bmatrix} = [8.081 \cdot 0.5574 - (2 - 11) \cdot -0.8303] \begin{bmatrix} \bar{x} - 11.7 \\ \bar{x} - 4.08 \end{bmatrix}$$



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