# Lecture 04: Markov Chain

Part 3.

eq. $P(X_3 = 2, X_1 = 3, X_0 = 2)$
- start from state 2 (at time period 0)
- At 1 time period, move to state 3
- After 2 time period, move to state 2 again.

eg1. lect. 2 eg 2

(ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

$$\dashrightarrow 2 \longrightarrow 3 \longrightarrow 3 \longrightarrow 2$$

$= q_0(2) \times P_{23} \times P_{33} \times P_{32}$

$= 0.2 \times 0.2 \times 0.3 \times 0.4$

$= 0.0048.$

$\therefore \boxed{P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2) = 0.0048}$

eg2. lect 2 eg 3

(ii) $P(X_3 = B, X_2 = C, X_1 = B, X_0 = A)$

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

$q_0 = [0.3 \quad 0.4 \quad 0.3]$

$$\dashrightarrow A \longrightarrow B \longrightarrow C \longrightarrow B.$$

$= q_0(A) \times P_{AB} \times P_{BC} \times P_{CB}$

$= 0.3 \times 1 \times 1 \times \frac{1}{2}$

$= 0.15$

eg3. The TPM of Markove chain with three states 1, 2, 3 is.

$$P = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.2 & 0.3 & \boxed{\phantom{0}} \\ 2 & \boxed{\phantom{0}} & 0.6 & 0.3 \\ 3 & 0.4 & \boxed{\phantom{0}} & 0.3 \end{array}$$

And initial prob. is $(0.5, 0.3, 0.2)$. calculate

(i) $P(X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3)$

(ii) $P(X_3 = 3, X_1 = 1, X_0 = 3)$

(iii) $P(X_2 = 2)$ (iv) $P(X_3 = 2, X_1 = 0, X_0 = 2)$

→ sum of each row = 1

$$\therefore P = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.2 & 0.3 & 0.5 \\ 2 & 0.1 & 0.6 & 0.3 \\ 3 & 0.4 & 0.3 & 0.3 \end{array}$$

$$q_0 = \begin{bmatrix} 0.5 & 0.3 & 0.2 \end{bmatrix}$$

(i) $P(X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3)$

$$\dashrightarrow 3 \rightarrow 1 \rightarrow 2 \rightarrow 3$$

$$= q_0(3) \times P_{31} \rightarrow P_{12} \rightarrow P_{23}.$$

$$= 0.2 \times 0.4 \times 0.3 \times 0.3$$

$$= 0.0072$$

$$\therefore \boxed{P(X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3) = 0.0072}$$

(ii) $P(X_3 = 3, X_1 = 1, X_0 = 3)$

$$--\rightarrow 3 \rightarrow 1 \rightarrow 3$$

$$= q_0(3) \times P_{31} \times P_{13}$$

$$= 0.2 \times 0.4 \times 0.5$$

$$= 0.04$$

(iii) $P(X_2 = 2) = q_2(2) \qquad q_2 = q_0 P^2$

$$P^2 = \begin{bmatrix} 0.27 & 0.39 & 0.34 \\ 0.20 & 0.48 & 0.32 \\ 0.23 & 0.39 & 0.38 \end{bmatrix}$$

$$q_0 P^2 = [0.241 \quad 0.417 \quad 0.342]$$

$$q_2(2) = 0.417$$

$$\therefore \boxed{P(X_2 = 2) = 0.417}$$

(iv) $P(X_3 = 3, X_1 = 1, X_0 = 3)$

$$--\rightarrow 3 \rightarrow 1 \rightarrow 3$$

$$= q_0(3) \times P_{31} \times P^2_{13}$$

$$= 0.2 \times 0.4 \times 0.34$$

$$= 0.0272$$

$$\boxed{P(X_3 = 3, X_1 = 1, X_0 = 3) = 0.0272}$$

(v) Calculate $P(X_3 = 3, X_1 = 1)$

$\rightarrow$

$- - \rightarrow 1 \rightarrow 3$ * here initial state is 1 $\therefore q_0 = q_1$.

$= q_0(1) \times P_{13}^{(2)}$

$= 0.5 \times 0.34$

$= 0.17$