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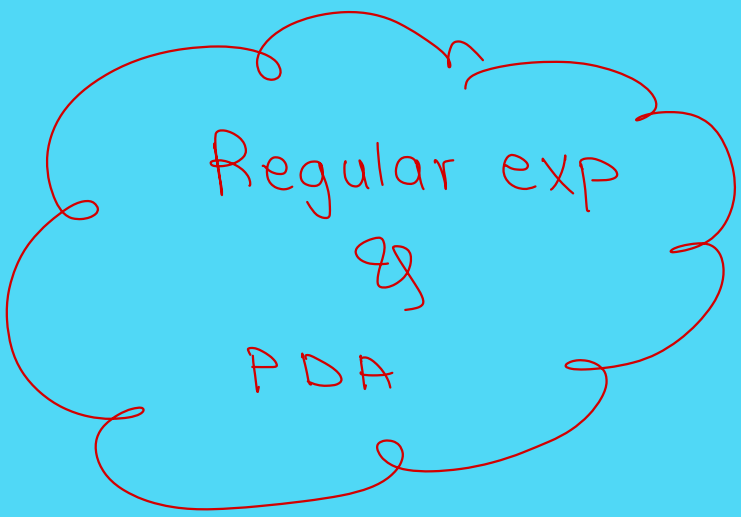
Class :- CMPN A

TOC



Practice

★ Regular expression ★

Generate the RE for given string that

Q.1 start with 1 and end with 0

Ans. $1\ (0+1)^*\ 0$

Q.2 Starts and ends with a and having any combination of b's in between.

Ans. $a\ b^*\ a$

Q.3 Starting with a but not having consucetive b's.

Ans. $(a + ab)^*$

Q.4 Contains any no. of a's, any no. of b's, any no. of c's.

Ans. $a^*\ b^*\ c^*$

Q.5 Contains even length of strings $(\Sigma = \{0\})$

Ans $(00)^*$

Q.6 Contains atleast one 0 and atleast one 1

Ans.

$|w_0| \geq 0$ $(0+1)^* 0 (0+1)^*$

$|w_1| \geq 1$ $(0+1)^* 1 (0+1)^*$

$$(0+1)^* 0 (0+1)^* 1 (0+1)^*$$
$$(0+1)^* 1 (0+1)^* 0 (0+1)^*$$
$$\Rightarrow (0+1)^* [\ 0 (0+1)^* 1 \ + \ 1 (0+1)^* 0\ ] . (0+1)^*$$

Q.7 Donot contain "01"

Ans. $L = \{ \epsilon, 0, 1, 10, 1010, 00, 11 \}$

$\Rightarrow 1^*\ 0^*$

Q.8 Every 0 is immidiately followed by 11

Ans $(011 + 1)^*$

✱ Regular expression (Practice)

Q.1 even no. of 1's

Ans. $0^*(10^*10^*)^*$

Q.2 odd no. of 1's

Ans $0^*(10^*\ 10^*)^*10^*$

Q.3 RE for even length string.

Ans $L=\{\epsilon, 00, 11, \ldots\}$

$\Rightarrow (00+10+01+11)^*$

Q.4 RE for odd length string

Ans. $L=\{0,1,000,111,\ldots\}$

$\Rightarrow (00+01+10+11)^*(0+1)$

Q.5 RE for odd no.s of 0's or odd no.s of 1's

Ans.

$$[1^*(01^*01^*)^*01^*]+[0^*(10^*10^*)^*10^*]$$

Q.6 RE $|w_a|$ = even

Ans.

$L=\{\epsilon, aa, aaaa\}$

$\Rightarrow (aa+b)^*$

Q.7 RE that does not contain substring "110".

Ans.

$$(0+10)^*1^*$$

Q.8 RE exactly contain 3 1's.

Ans.

$$0^*1\,0^*1\,0^*1\,0^*$$

Q.9 RE with atleast one a's

Ans $|w_a| \geq 1$

$$(a+b)^*\,a\,(a+b)^*$$

Q.10 RE with atleast two a's

Ans. $|w_a| \geq 2$

$$(a+b)^*\,a\,(a+b)^*a(a+b)^*$$

Q.11 RE with exact two b's.

Ans. $|w_b| = 2$

$$a^*\ b\ a^*\ b\ a^*$$

Q.12 RE with $|w_a| \geq 1$ & $|w_b| \geq 1$

Ans. $(a+b)^*(ab+ba)(a+b)^*$

Q.13 RE $\forall$ string divisible by 4.

Ans $((0+1)^4)^*$ //

expand this. //

if "n" then $(r0+1)^n)^*$

Q.14 RE must have 010 or 101

Ans. $(0+1)^*(010+101)(0+1)^*$

Q.15 RE containing even no. of 0's or even no. of 1's

Ans. (i) $1^*(01^*01^*)^*$

(ii) $0^*(10^*10^*)^*$

$\Rightarrow 1^*(01^*01^*)^* + 0^*(10^*10^*)^*$

Q.16 RE $\Rightarrow$ even length but starts with a

Ans.

$(ab+aa)(aa+ab+ba+bb)^*$

Q.17 RE $\Rightarrow$ even length but starts with "a" and ends with "b"

Ans.

$a(aa+ab+ba+bb)^*b$

Q.18 RE $\Rightarrow$ even length but ends with "aa"

Ans.

$(aa+ab+ba+bb)^*aa$

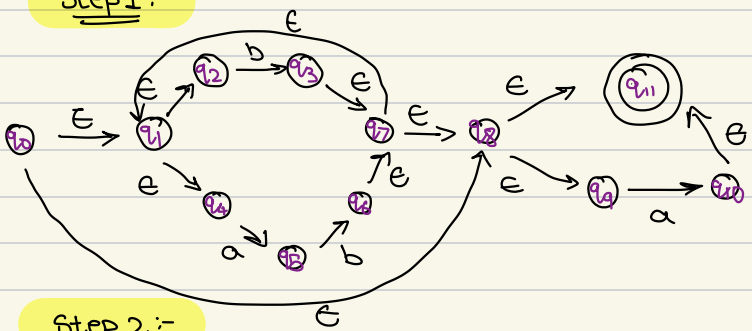
Q.19 RE of second last symbol is "1".

Ans. $(0+1)^*1(0+1)$ //

★ Convert Regular Expression to FA ★

Q.1 RE ⇒ $(b + ab)^* (\epsilon + a)$

Step 1 :-



Step 2 :-

DFA states	Set of NFA (X)	Move(X, a)	Move(X, b)
A	$\{q_0 : q_1, q_2, q_4, q_8, q_9, q_{11}\}$	$\{q_5, q_{10}\}$	$\{q_3\}$
B	$\{q_5, q_{10} : q_{11}\}$	ϕ	$\{q_6\}$
C	$\{q_3 : q_1, q_2, q_7, q_8, q_9, q_{11}, q_4\}$	$\{q_5, q_{10}\}$	$\{q_3\}$
D	$\{q_6 : q_1, q_2, q_4, q_7, q_8, q_9, q_{11}\}$	$\{q_5, q_{10}\}$	$\{q_3\}$
E	ϕ	ϕ	ϕ

Step 3 :-

$Q = (A, B, C, D, E)$

$F = (A, B, C, D)$

$\delta = Q \times \Sigma \longrightarrow Q$

$\Sigma = \{a, b\}$

$q_0 = \{A\}$

(I)

	a	b
A	B	C
B	E	D
C	B	C
D	B	C
E	E	E

convert "D" into "C" (Merge)

(II)

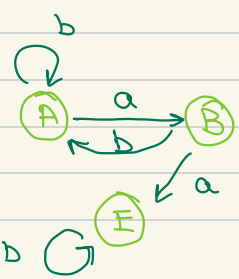
	a	b
A	B	C
B	E	C
C	B	C
E	E	E

convert "C" into "A" (Merge)

(III)

	a	b
A	B	A
B	E	A
E	E	E

Minimized DFA ⇒



Grammer

left linear → terminal elements on left side

Right linear → terminal elements on Right side

$G = \{V, T, P, S\}$

where,

V ⇒ finite set of variables

T ⇒ finite set of Terminals

P ⇒ finite set of production

S ⇒ start symbols.

RE ⇒ $00(0+1)^*00$

Q.1 →

* Note ⇒ if $S \to 00A00$, $A \to 0A \mid A1 \mid 00$ Here, Given language is non Regular grammer because there is no left and Right linear terminal.

(I) left linear grammer

$S \to 00A$

$A \to 0A \mid 1A \mid 00$

(II) Right linear grammer

$S \to A00$

$A \to A0 \mid A1 \mid 00$

Q.2 RE ⇒ $(\epsilon+1).(01)^*(\epsilon+0)$

(I) left linear G

$S \to A \mid 1A$

$A \to A01 \mid \epsilon \mid 0$

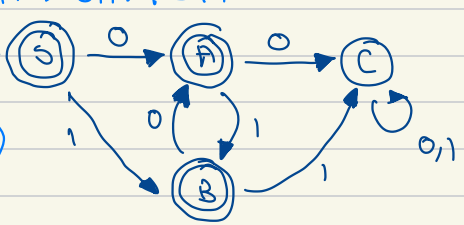
(II) Right linear G

$S \to A \mid A0$

$A \to 01A \mid \epsilon \mid 1$

Q.3 DFA to grammer →

DFA ⇒



$G: (\{S, A, B, C\}, \{0, 1\}, P, S)$

(I) Left linear Grammer

$S \to 0A \mid 1B \mid \epsilon$

$A \to 0C \mid 1B \mid \epsilon$

$B \to 1C \mid 0A \mid \epsilon$

$C \to 0C \mid 1C$

(II) Right linear Grammer

$S \to A0 \mid B1 \mid \epsilon$

$A \to C0 \mid B1 \mid \epsilon$

$B \to C1 \mid A0 \mid \epsilon$

$C \to C0 \mid C1$

change to State → I/P (Next state) Variable

Content-free Grammer

Q.1 Give CGF for given CFL

(i) $G = \{ 0^n 1^n , n \geq 0 \}$

→ $S \rightarrow 0S1 \mid \epsilon$

(ii) $G = \{ 0^n 1^{2m} 2^{3n} , n, m \geq 0 \}$

→ $S \rightarrow 0S222 \mid A$

$A \rightarrow 1A \mid \epsilon$

(iii) $G = \{ 0^n 1^{n-1} , n \geq 0 \}$

→ $S \rightarrow 0S1 \mid 1$

(iv) $G = \{ a^n b^{2n+1} c^m d^{3m-1} , n, m \geq 1 \}$

→ $ab^3 c\, d^2$

$S \rightarrow XY$

$X \rightarrow aXbb \mid abbb$

$Y \rightarrow cYddd \mid cdd$

(v) $G = \{ 0^{2n} 1^{3n} , n \geq 0 \}$

→ $S \rightarrow 00S111 \mid \epsilon$

(vi) $G = \{ a^i b^{i+j} c^j , i, j > 0 \}$

→ $S \rightarrow aSb \mid aAb$

$A \rightarrow bAc \mid bc$

(vii) $G = \{ a^i b^j c^{i+j} , i, j > 0 \}$

→

PDA (Pushdown Automata)

★ CFG (Context free Grammer)
↓ Generates
CFL (context free language)
↓ accepted by
PDA (Pushdown Automata)

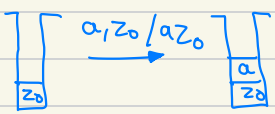
★ Defination =>

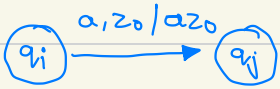
$M(Q, q_0, \delta, F, \Sigma, Z_0, \varepsilon)$

- Σ : Stack alphabet
- Z_0 : bottom / initial Stack Symbol

★ Operations performed on PDA

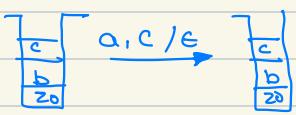
Push

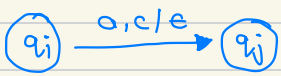




$\delta(q_i, a, z_0) = \delta(q_j, az_0)$

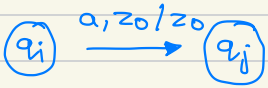
Pop





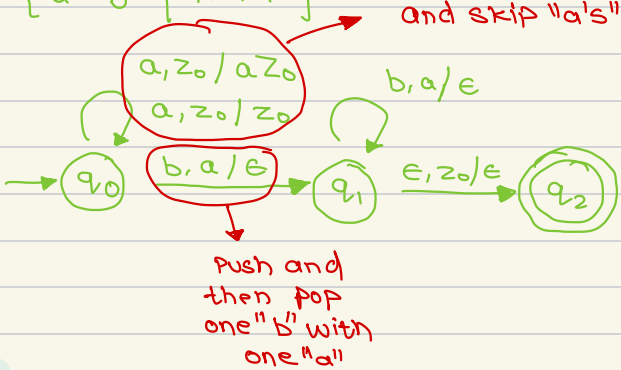
$\delta(q_i, a, c) = \delta(q_j, \varepsilon)$

Skip



Q.1 $L = \{ a^n b^n \mid n \geq 1 \}$

Ans.

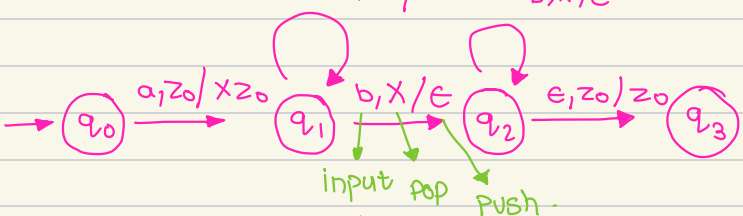


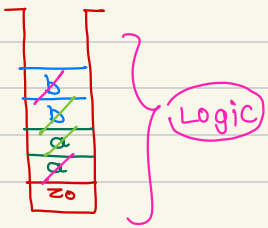
Q.2 $L = \{ a^n b^n \mid n > 0 \}$

Ans.

input — top — after pushing into stack.

$a, X / XX$ $\quad b, X / \epsilon$





Q.3 $L = \{ c^{2n} d^{3n+1} \mid n \geq 0 \}$

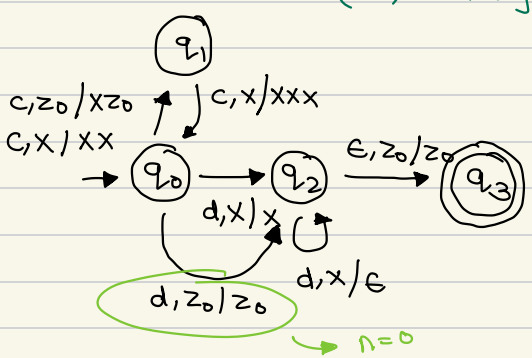
Ans

Logic → (i) Push X for C

(ii) Push XX for C

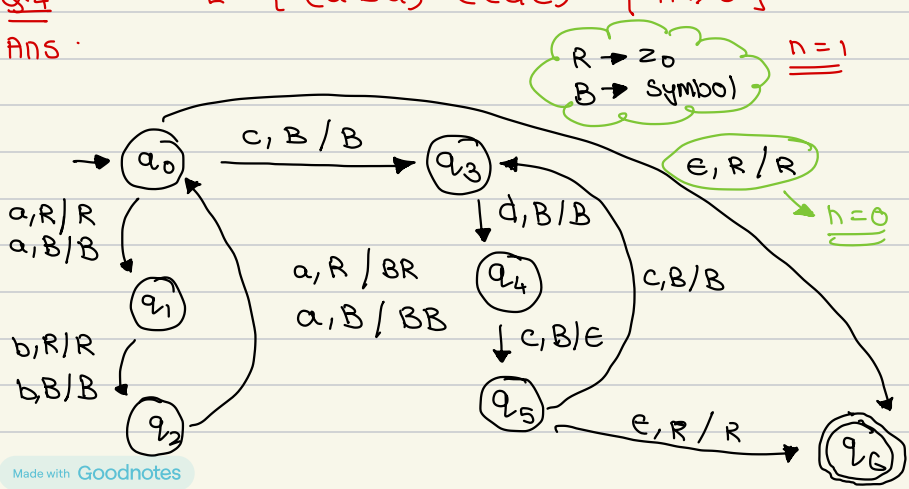
(iii) Pop X for d

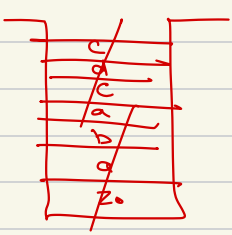
(iv) Lastly one "d" remains, do nothing.



Q.4 $L = \{ (aba)^n (cdc)^n \mid n \geq 0 \}$

Ans.

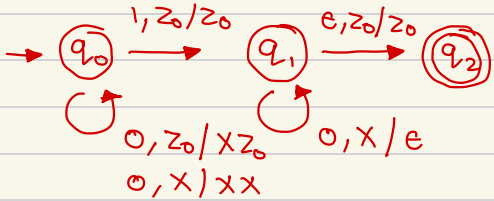


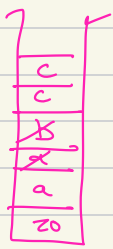


Q.5 $L = \{ 0^n \mid 0^n \mid n \geq 0 \}$

Ans

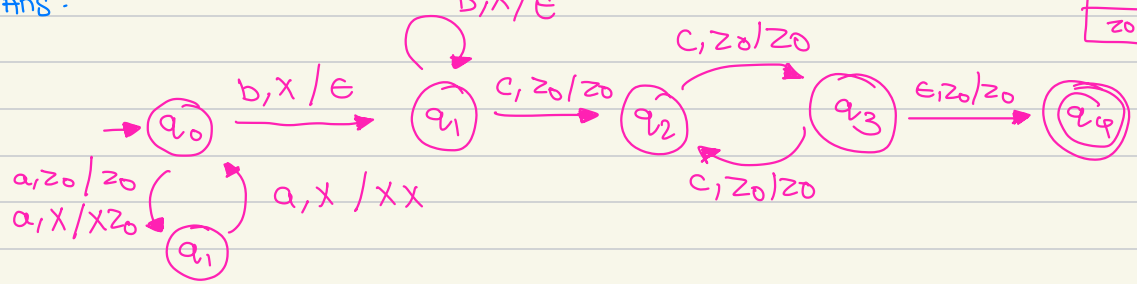
logic ⇒ (i) Push "0"
(ii) Skip "1"
(iii) pop "0" for "0"





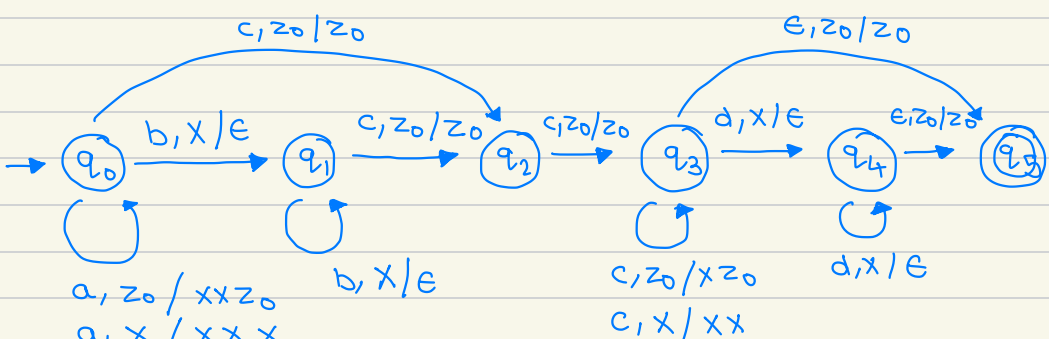
Q.6 $L = \{ a^{2n} b^n c^{2m} \mid n, m > 0 \}$

Ans.



Q.7 $L = \{ a^i b^{2i} c^{j+2} d^j \mid i, j \geq 0 \}$ for $i, j = 1$

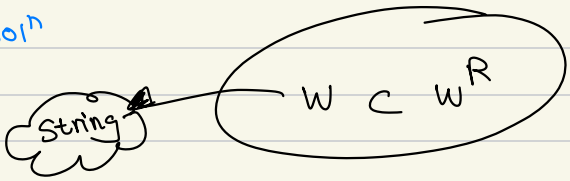
Ans.



logic →
(i) Push two "X" for one "a"
(ii) pop one "X" for one "b"
(iii) Skip two c's & push "c" as "Y"
(iv) pop one "X" with one "d".

Q. odd palindrome.

Soln

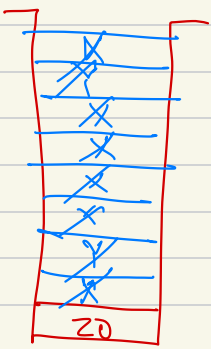


$w \Rightarrow (a+b)^*$

a b a a c a a b a.

$w \Rightarrow abaa$

$w^R = aaba.$



odd palindrome //

a → X

b → Y

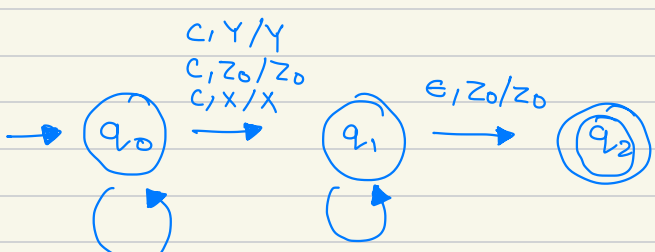
Logic → (i) a → x

b → Y

(ii) c → skip.

(iii) a → x pop.

b → Y pop //



Q. Even pallindrome

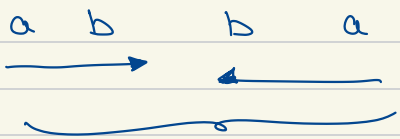
NPDA

Soln

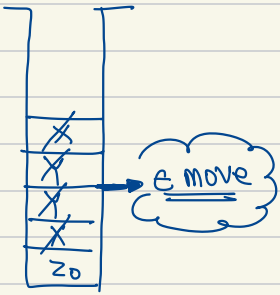
$L = \{ w w^R \mid w \in \{a, b\}^* \}$

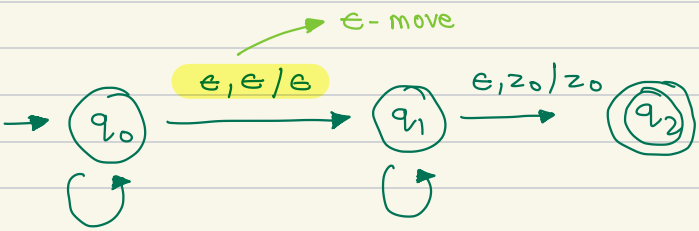
$w \Rightarrow ab$

$w^R \Rightarrow ba$



even pallindrome





$a, Z_0 / X Z_0$; $b, Z_0 / Y Z_0$

$a, X / XX$; $b, Y / YY$

$a, Y / XY$; $b, X / YX$

$a, X / \epsilon$

$b, Y / \epsilon$

Note

→ $L = \{ a^n b^n c^n \mid n \geq 0 \}$.

↳ Not CFL// as we can't relate with 3 relation.

→ $L = \{ a^i b^j c^i d^j \mid i, j \geq 0 \}$

↳ Not CFL.//

→ $L = \{ a^n b^n c^m \mid n, m \geq 0 \}$

↳ CFL//