Homework 3 Deep Shah CPE 462 2/27/25

3.1 (1A) Given: $x_1[n] = \delta[n] - 2\delta[n-1]$, $x_2 = 2\delta[n] + \delta[n-1] - \delta[n-2]$

$x_1[n] = \delta[n] - 2\delta[n-1]$

$\delta[n]$ is unit impluse function that is 1 at $n=0$

$-2\delta[n-1]$ is a unit impulse function that is -2 at $n=1$.

$x_1[n] = \{1, -2\}$ $n=2$, length of first signal

$x_2 = 2\delta[n] + \delta[n-1] - \delta[n-2]$

$2\delta[n]$ is a unit impulse function that is 2 at $n=0$.

$\delta[n-1]$ is a unit impulse function that is 1 at $n=1$.

$-\delta[n-2]$ is a unit impulse function that is -1 at $n=2$.

$x_2 = \{2, 1, -1\}$ $m=3$, length of second signal

Length of resulting convolved signal: $L = n+m-1$

$2+3-1 = 4$

$$
\begin{array}{lrrrr}
x_2(n) \rightarrow & & 2 & 1 & -1 \\
* \; x_1(n) \rightarrow & & & 1 & -2 \\
\hline
 & & -4 & -2 & +2 \\
+ & 2 & 1 & -1 & 0 \\
\hline
 & 2 & -3 & -3 & 2
\end{array}
$$

(3.1.1) Compute the linear convolution

$y[n] = x_1[n] * x_2[n]$

$\boxed{y[n] = \{2, -3, -3, 2\}}$

(3.1.2) Compute 4-point DFT: $X_1[k] = DFT\{x_1[n]\}$ and $X_2[k] = DFT\{x_2[n]\}$

Linear Convolution using DFT Steps

1) Calculate value of L using $L = n+m-1$
2) Make length of $x(n)$ and $h(n)$ equal to L
3) Calculate DFT of $x_1(n)$ that means $X_1(k)$
4) Calculate DFT of $x_2(n)$ that means $X_2(k)$

5) Multiply $X_1(K)$ and $X_2(K)$ to get $Y(K)$

$$Y(K) = (X_1(K))(X_2(K))$$

6) Obtain IDFT of $Y(K)$ that means $y(n)$

$$X_1(K) = \begin{bmatrix} -1 \\ 1+2j \\ 3 \\ 1-2j \end{bmatrix} \quad X_2(K) = \begin{bmatrix} 2 \\ 3-j \\ 0 \\ 3+j \end{bmatrix}$$

$L = n + m - 1 = 2 + 3 - 1 = 4 \quad L = 4$

$x_1[n] = \{1, -2, 0, 0\}$

$x_2[n] = \{2, 1, -1, 0\}$

$$X_1(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-2+0+0 \\ 1+2j+0+0 \\ 1+2+0+0 \\ 1-2j+0+0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1+2j \\ 3 \\ 1-2j \end{bmatrix}$$

$$X_2(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2+1-1+0 \\ 2-j+1+0 \\ 2-1-1+0 \\ 2+j+1+0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3-j \\ 0 \\ 3+j \end{bmatrix}$$

$Y(K) = (X_1(K))(X_2(K)) \qquad j^2 = -1$

$Y(0) = (X_1(0))(X_2(0)) = -1(2) = -2$

$Y(1) = (X_1(1))(X_2(1)) = (1+2j)(3-j) = 3+6j-j-2j^2 = 3+5j-2(-1)$

$Y(1) = 3+5j+2 = 5+5j$

$Y(2) = (X_1(2))(X_2(2)) = (3)(0) = 0$

$Y(3) = (X_1(3))(X_2(3)) = (1-2j)(3+j) = 3-6j+j-2j^2 = 3-5j-2(-1)$

$Y(3) = 3-5j+2 = 5-5j$

$Y(K) = \{-2, 5+5j, 0, 5-5j\}$

$$y(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & 1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} -2 \\ 5+5j \\ 0 \\ 5-5j \end{bmatrix} = \begin{bmatrix} -2(1)+(5+5j)(1)+(0)(1)+(5-5j)(1) \\ -2(1)+(5+5j)(j)+(0)(-1)+(5-5j)(-j) \\ -2(1)+(5+5j)(-1)+(0)(1)+(5-5j)(-j) \\ -2(1)+(5+5j)(-j)+(0)(-1)+(5-5j)(j) \end{bmatrix}$$

$j^2 = -1$

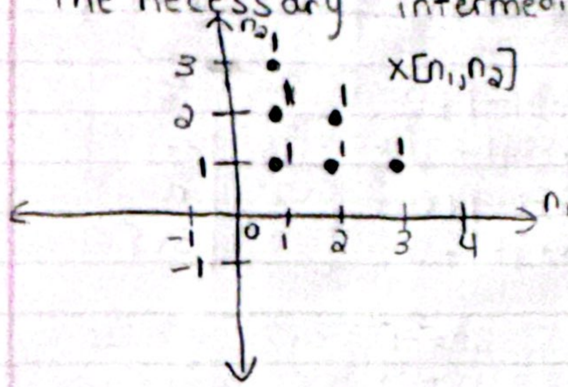
$$y(n) = \frac{1}{4}\begin{bmatrix} -2+5\cancel{+5j}+0+5\cancel{-5j} \\ -2\cancel{+5j}+5j^2+0\cancel{-5j}+5j^2 \\ -2-5\cancel{-5j}+0\cancel{+5j}+5j^2 \\ -2\cancel{-5j}-5j^2+0\cancel{+5j}-5j^2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ -2+10(-1) \\ -2-5+5(-1) \\ -2-5(-1)-5(-1) \end{bmatrix}$$

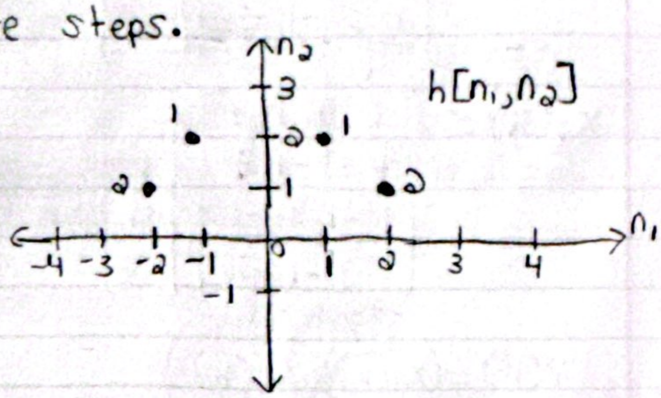
$$y(n) = \frac{1}{4}\begin{bmatrix} 8 \\ -12 \\ -12 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -3 \\ 2 \end{bmatrix}$$

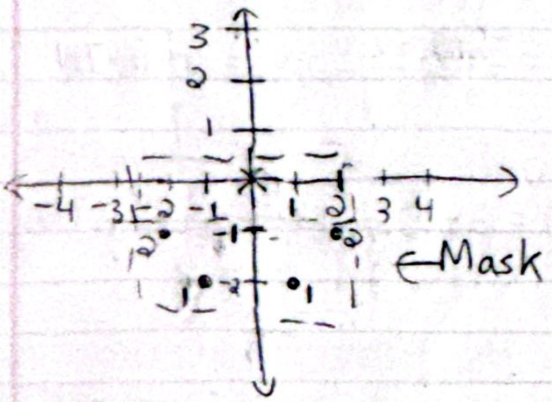
$$\boxed{y(n) = \{2, -3, -3, 2\}}$$

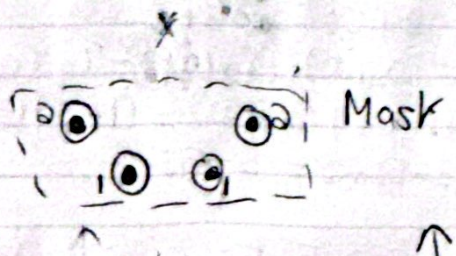
$y[n,n] = x[n,n] ** h[n,n]$

3.a(ia) Calculate the 2-D convolutions $x[n,n] ** h[n,n]$, show all the necessary intermediate steps.



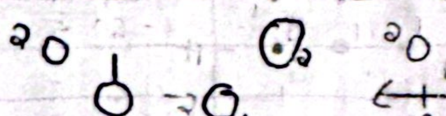


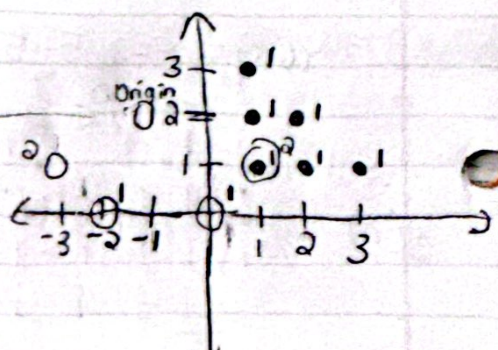




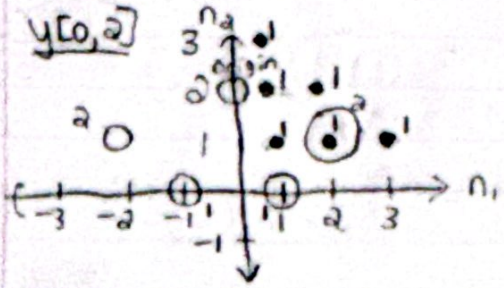
$y[-1,2]$

$2(1) = \boxed{2}$



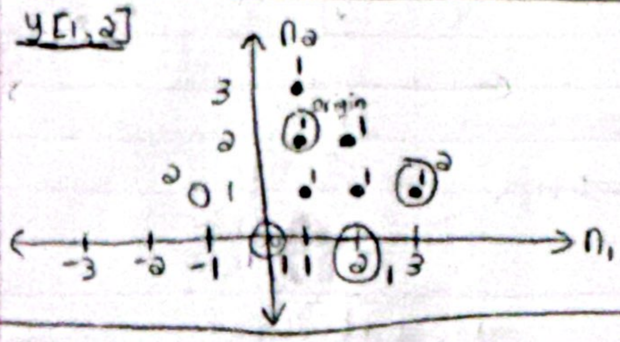


y[0,2]



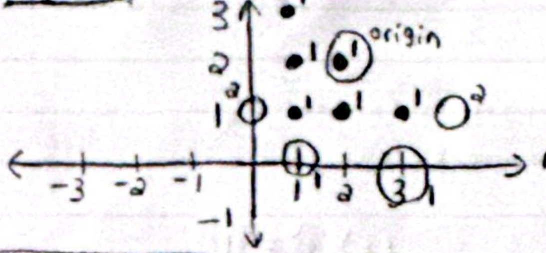
$2(1) = \boxed{2}$

y[1,2]



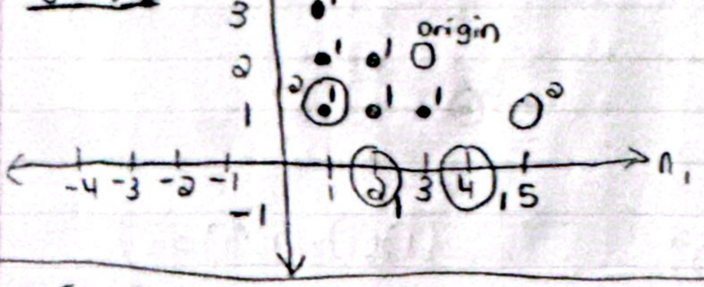
$2(1) = \boxed{2}$

y[2,2]



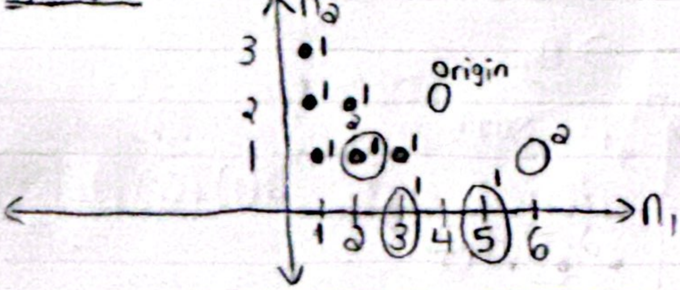
$\boxed{0}$

y[3,2]



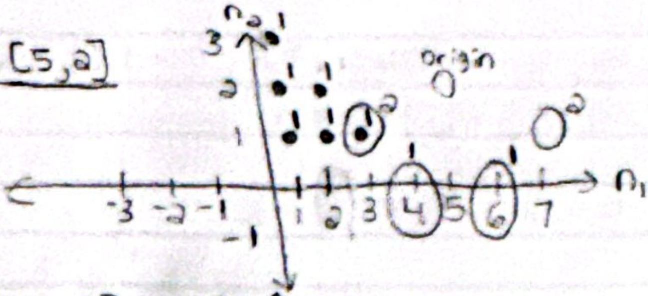
$2(1) = \boxed{2}$

y[4,2]



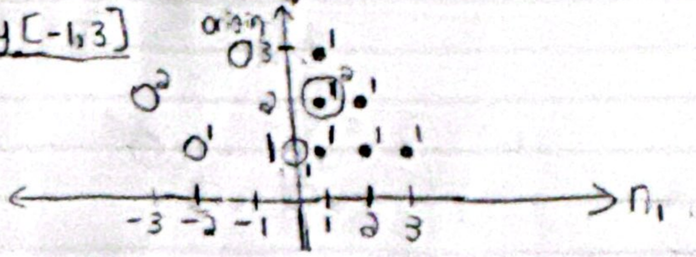
$2(1) = \boxed{2}$

y[5,2]



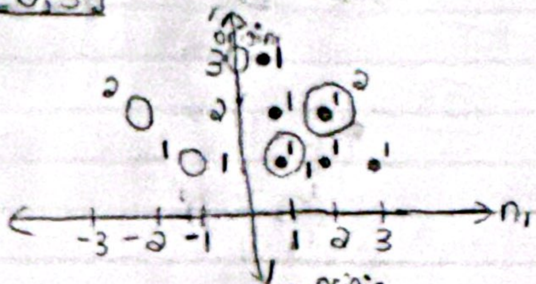
$2(1)=\boxed{2}$

y[-1,3]



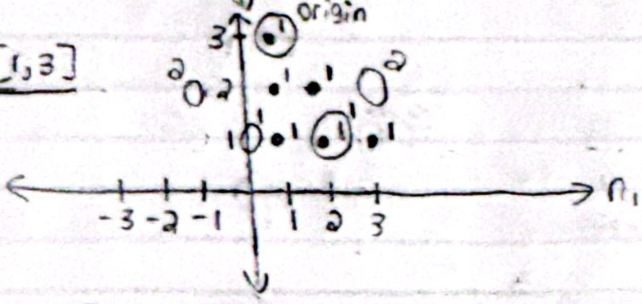
$2(1)=\boxed{2}$

y[0,3]



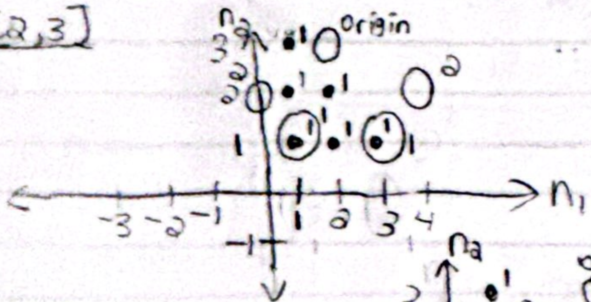
$2(1)+1(1)=\boxed{3}$

y[1,3]



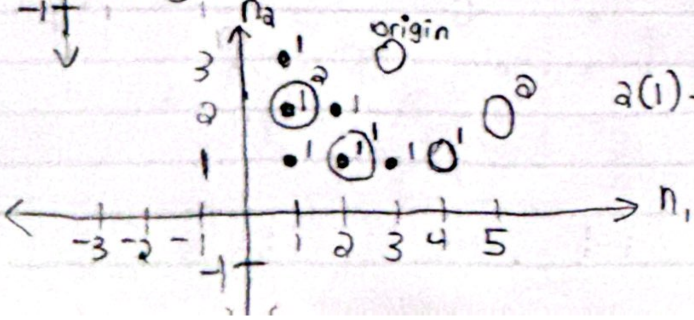
$(1)(1)=\boxed{1}$

y[2,3]



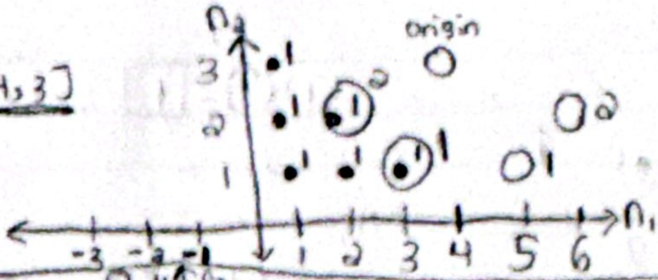
$(1)(1)+(1)(1)=\boxed{2}$

y[3,3]



$2(1)+(1)(1)=\boxed{3}$

y[4,3]

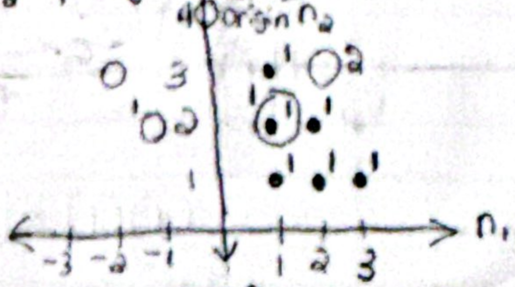


$2(1)+1(1)=\boxed{3}$

y[-1,4]

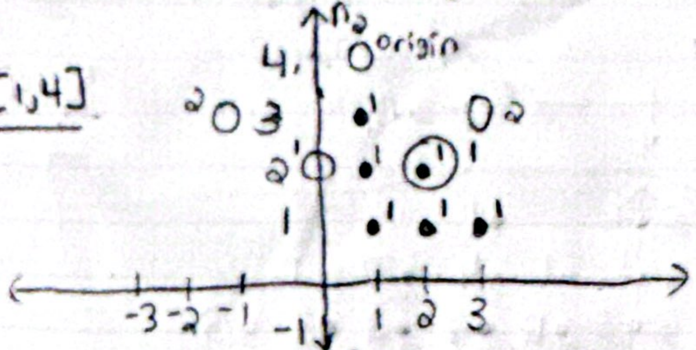
$2(1)=\boxed{2}$

y[0,4]



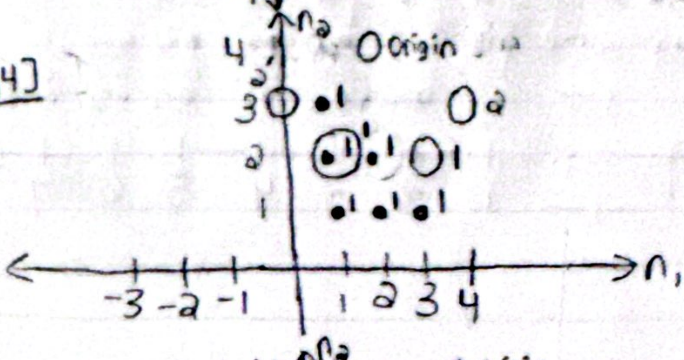
$(1)(1)=\boxed{1}$

y[1,4]



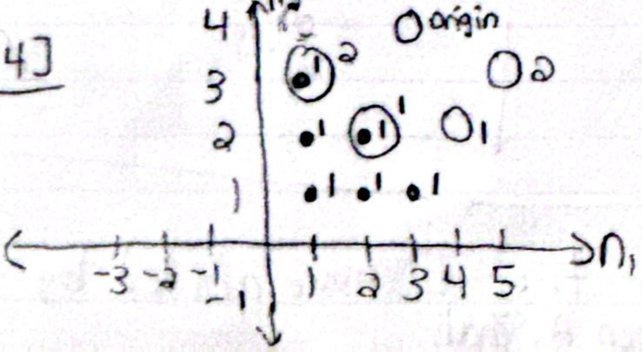
$(1)(1)=\boxed{1}$

y[2,4]



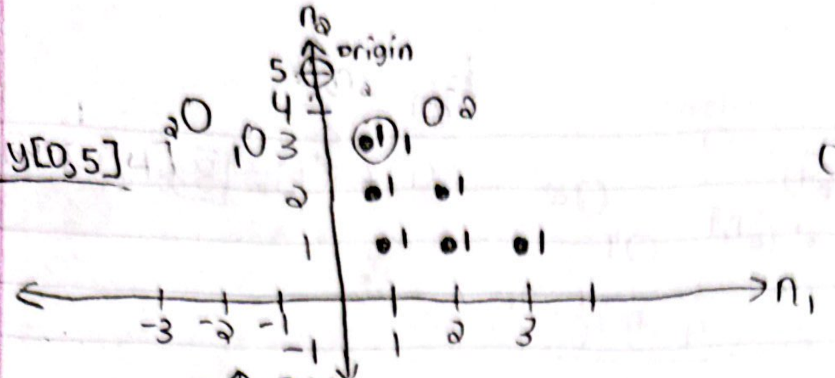
$(1)(1)=\boxed{1}$

y[3,4]



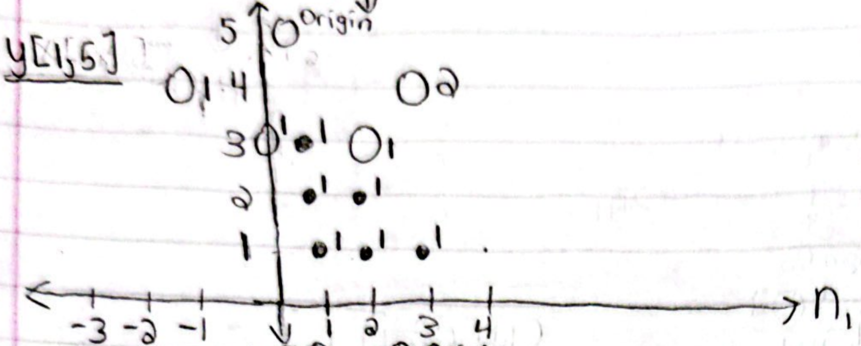
$(2)(1)+(1)(1)=\boxed{3}$

$y[0,5]$



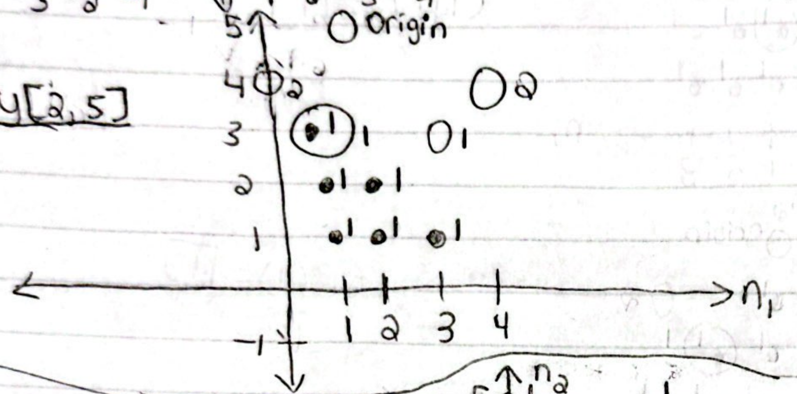
$(1)(1)=\boxed{1}$

$y[1,5]$



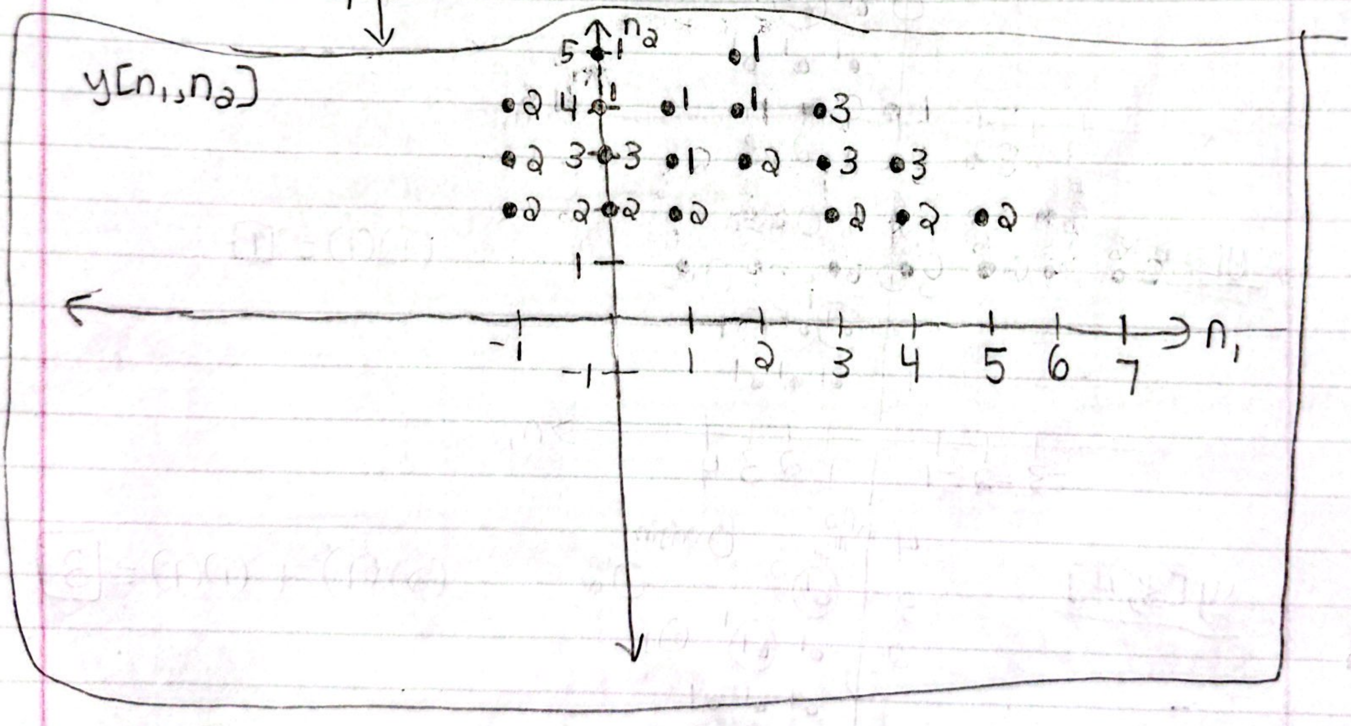
$\boxed{0}$

$y[2,5]$



$(1)(1)=\boxed{1}$

$y[n_1,n_2]$



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