If $y[n] = x[n]$, then $h[n] * h_i[n] = \delta[n]$

Homework 1 CPE 462 Introduction to Image Processing
2/4/25 Professor Man 2/6/25

1.1 Determine if $y[n] = 3x[n] + 7$ is linear? time-invariant?

$y[n] = 3x[n] + 7$ Note: T transforms an input $x[n]$ into an output $y[n]$.

Additive

$x = (x_1[n] + x_2[n])$ and assume $y_1[n] = 3x_1[n] + 7$, $y_2[n] = 3x_2[n] + 7$

Then $y[n] = T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\}$

$T\{x_1[n] + x_2[n]\} = 3\{x_1[n] + x_2[n]\} + 7 = 3x_1[n] + 3x_2[n] + 7$

$T\{x_1[n]\} + T\{x_2[n]\} = 3\{x_1[n]\} + 7 + 3\{x_2[n]\} + 7$

$T\{x_1[n]\} + T\{x_2[n]\} = 3x_1[n] + 7 + 3x_2[n] + 7$

$T\{x_1[n]\} + T\{x_2[n]\} = 3x_1[n] + 3x_2[n] + 14$

$T\{x_1[n]\} + T\{x_2[n]\} \neq T\{x_1[n] + x_2[n]\}$

$3x_1[n] + 3x_2[n] + 14 \neq 3x_1[n] + 3x_2[n] + 7$

No, it is not additive.

$T\{x_1[n]\} = 3x_1[n] + 7$

Homogeneous: If $x = \alpha x_1[n]$, then $y[n] = T\{\alpha x_1[n]\} = \alpha T\{x_1[n]\}$

$T\{\alpha x_1[n]\} = (3\alpha x_1[n] + 7) = 3\alpha x_1[n] + 7$

$\alpha T\{x_1[n]\} = \alpha(3x_1[n] + 7) = 3\alpha x_1[n] + 7\alpha$

$T\{\alpha x_1[n]\} \neq \alpha T\{x_1[n]\}$

$3\alpha x_1[n] + 7 \neq 3\alpha x_1[n] + 7\alpha$

No, it is not homogeneous

~~Yes, it is homogeneous.~~

Since it is not additive and homogeneous, it is not linear.

Time-Invariant: If $x_1[n] = x[n-n_0]$, then $y_1[n] = T\{x[n-n_0]\}$
$= y[n-n_0]$

$y_1[n] = x_1[n](3) + 7 = x[n-n_0]3 + 7$

$y[n-n_0] = x[n-n_0]3 + 7$

$\boxed{y[n] = y[n-n_0]\text{, which implies that it is time invariant.}}$

1.2 Prove that convolution is commutative, i.e. $x[n] * h[n] = h[n] * x[n]$

$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = x[n] * h[n] = h[n] * x[n]$

Let $m = n-k$ When $k=\infty$, $m=-\infty$ Note: $\sum_{k=-\infty}^{+\infty} = \sum_{n-m=-\infty}^{+\infty} = \sum_{-m=-\infty}^{+\infty}$

$k = n-m$ $k=-\infty$ $m=\infty$ $\sum_{m=\infty}^{-\infty}$

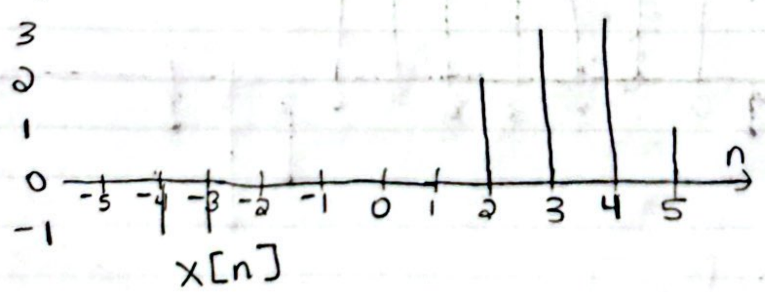
$x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \sum_{m=-\infty}^{+\infty} x[n-m]\,h[m]$

$h[n] * x[n] = \sum_{k=-\infty}^{+\infty} h[n-k]\,x[k] = \sum_{m=-\infty}^{+\infty} h[m]\,x[n-m]$

Note: $\sum_{m=-\infty}^{\infty} h[k]\cdot x[n-k]$ represents the convolution of two signals $h[n]$ and $x[n]$ $(h[n] * x[n])$

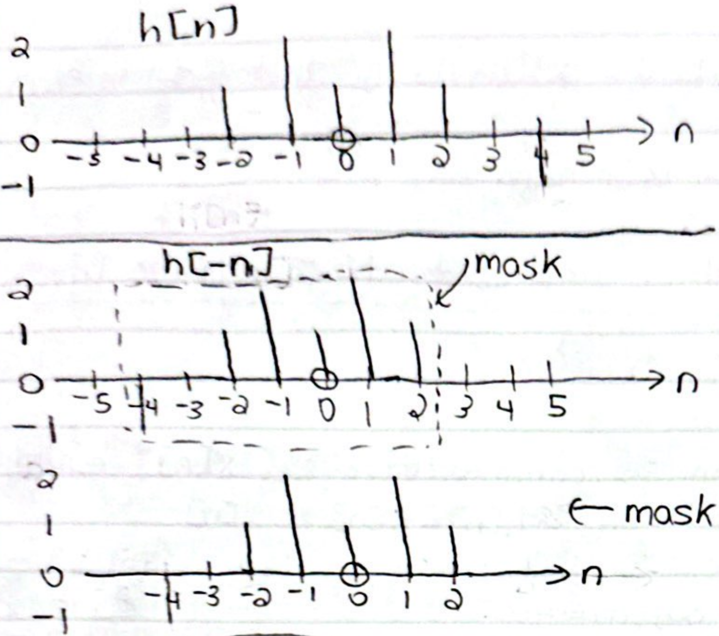
$\boxed{\text{Since } h[n] * x[n] = x[n] * h[n]\text{, this proves that convolution is commutative.}}$

1.3.1 Calculate the 1-D convolution $x[n] * h[n]$ using graphic approach, provide necessary intermediate steps.



Origin starts at -6 and goes to 9

I pledge my honor that I have abided by the Stevens Honor System. Deep A. Shah



n	h[n]	h[-n]	x[n]
-5	0	h[5] = 0	0
-4	0	h[4] = -1	-1
-3	0	h[3] = 0	-1
-2	1	h[2] = 1	0
-1	2	h[1] = 2	0
0	1	h[0] = 1	0
1	2	h[-1] = 2	0
2	1	h[-2] = 1	2
3	0	h[-3] = 0	3
4	-1	h[-4] = 0	3
5	0	h[-5] = 0	1