Homework 2 CPE 462 Deep Shah 2/12/25

2.1 (1A) Prove the multiplication property of DTFT

$$x[n]y[n] \longleftrightarrow \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$$

$$Z[n] = x[n]y[n] \longleftrightarrow Z(e^{j\omega})$$

<u>Note:</u> $\sum_{n=-\infty}^{\infty} y[n]e^{-j(\omega-\theta)n} d\theta = Y(e^{j(\omega-\theta)})$

$$Z(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]y[n]e^{-j\omega n}$$

$$Z(e^{j\omega}) = \sum_{n=-\infty}^{\infty}\left(\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})e^{j\theta n}d\theta\right)y[n]e^{-j\omega n}$$

$$Z(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi}\left(X(e^{j\theta})\sum_{n=-\infty}^{\infty} y[n]e^{-j(\omega-\theta)n}\right)d\theta$$

$$Z(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} X[e^{j\theta}]Y(e^{j(\omega-\theta)})d\theta$$

2.2) (1C) Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-2] + \delta[n-3]$, ~~$h[n] = \delta[$~~ $h[n] = \delta[n] + \delta[n-1]$. If $y[n] = x[n] * h[n]$ calculate the DTFT of $y[n]$. Note: $x[n] * h[n] \xleftrightarrow{DTFT} X(e^{j\omega})Y(e^{j\omega})$

Forward Transform: $X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty}(\delta[n] + 2\delta[n-1] - \delta[n-2] + \delta[n-3])e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty}(e^{-j\omega(0)} + 2e^{-j\omega(1)} - e^{-2j\omega} + e^{-3j\omega})$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty}(1 + 2e^{-j\omega} - e^{-2j\omega} + e^{-3j\omega})$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{+\infty}(\delta[n] + \delta[n-1]) = \sum_{n=-\infty}^{+\infty}(e^{-j\omega(0)} + e^{-j\omega(1)})$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{+\infty}(1 + e^{-j\omega})$$

Based on convolution property of DTFT, as $x[n] * h[n] \xleftrightarrow{DTFT} X(e^{j\omega})Y(e^{j\omega})$, and since $y[n] = x[n] * h[n]$, then $y[n]$ would also equal $X(e^{j\omega})Y(e^{j\omega})$.

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2.2 (continued) $y[n] = X(e^{jw})\,y(e^{jw})$ $\quad x(e^{jw}) = 1 + 2e^{-jw} - e^{-2jw} + e^{-3jw}$

$y[n] = (1 + 2e^{-jw} - e^{-2jw} + e^{-3jw})(1 + e^{-jw})$ $\quad y(e^{jw}) = 1 + e^{-jw}$

$y[n] = 1 + 2e^{-jw} - e^{-2jw} + e^{-3jw} + e^{-jw} + 2e^{-2jw} - e^{-3jw} + e^{-4jw}$

$\boxed{y[n] = 1 + 3e^{-jw} + e^{-2jw} + e^{-4jw}}$

I pledge my honor that I have abided by the Stevens Honor System. Deep A. Shah