Homework 4 CPE 462: Introduction to Image Processing

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4.1 2-D DFT and DCT are seperable, and can be implemented through 1-D DFT or DCT along horizontal and vertical directional seperately. For the following 2x2 image block

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

4.1.1 Calculate its 2x2 DFT

Given $[A] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $N=2, N=2$

NxN 2-D DFT Formula: $Y[k_1,k_2] = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} x[n_1,n_2] e^{-j\frac{2\pi}{N}n_1k_1} e^{-j\frac{2\pi}{N}n_2k_2}$

$$Y[k_1,k_2] = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} x[n_1,n_2] e^{-j2\pi\left(\frac{n_1k_1}{N} + \frac{n_2k_2}{N}\right)}$$

Note: $Y[k_1,k_2] = \begin{bmatrix} (0,0) & (0,1) \\ (1,0) & (1,1) \end{bmatrix}$

$Y[0,0]: k_1=0, k_2=0, N=2$

$$Y[0,0] = \sum_{n_1=0}^{2-1} \sum_{n_2=0}^{2-1} x[n_1,n_2] e^{-j2\pi\left(\frac{0(0)}{2} + \frac{0(0)}{2}\right)} = \sum_{n_1=0}^{1} \sum_{n_2=0}^{1} x[n_1,n_2] e^{-j\pi(0)}$$

$$Y[0,0] = \sum_{n_1=0}^{1} [x[n_1,0] + x[n_1,1]]$$

$$Y[0,0] = [x[0,0] + x[0,1] + x[1,0] + x[1,1]]$$

$$Y[0,0] = [1+2+3+4]$$

$$\boxed{Y[0,0] = [10]}$$

$Y[0,1]: k_1=0, k_2=1, N=2$

$$Y[0,1] = \sum_{n_1=0}^{2-1} \sum_{n_2=0}^{2-1} x[n_1,n_2] e^{-j2\pi\left(\frac{n_1(0)}{2} + \frac{n_2(1)}{2}\right)} = \sum_{n_1=0}^{1} \sum_{n_2=0}^{1} x[n_1,n_2] e^{-j\cancel{2}\pi\left(\frac{n_2}{\cancel{2}}\right)}$$

$$Y[0,1] = \sum_{n_1=0}^{1} \sum_{n_2=0}^{1} x[n_1,n_2] e^{-j\pi n_2}$$

$$Y[0,1] = \sum_{n_1=0}^{1} x[n_1,0] e^{-j\pi(0)} + x[n_1,1] e^{-j\pi(1)}$$

$$Y[0,1] = \sum_{n_1=0}^{1} x[n_1,0] e^{0} + x[n_1,1] e^{-j\pi} \qquad e^0 = 1$$

$$Y[0,1] = \sum_{n_1=0}^{1} x[n_1,0] + x[n_1,1]e^{-j\pi}$$

$$Y[0,1] = x[0,0] + x[0,1]e^{-j\pi} + x[1,0] + x[1,1]e^{-j\pi}$$

$$Y[0,1] = 1 + 2e^{-j\pi} + 3 + 4e^{-j\pi}$$

$$Y[0,1] = 4 + 6e^{-j\pi}$$

$$Y[0,1] = 4 + 6(\cos\pi - j\sin\pi)$$

$$Y[0,1] = 4 + 6(-1-0)$$

$$Y[0,1] = 4 - 6$$

$$\boxed{Y[0,1] = -2}$$

Note: $e^{-j\theta} = \cos\theta - j\sin\theta$

$e^{-j\pi} = \cos\pi - j\sin\pi$

$\sin\pi = 0 \quad \cos(\pi) = -1$

$\sin 2\pi = 0$

$\sin 3\pi = 0$

<u>$Y[1,0]$: $K_1 = 1$, $K_2 = 0$, $N = 2$</u>

$$Y[1,0] = \sum_{n_1=0}^{2-1}\sum_{n_2=0}^{2-1} x[n_1,n_2]e^{-j2\pi\left(\frac{n_1(1)}{2} + \frac{n_2(0)}{2}\right)} = \sum_{n_1=0}^{1}\sum_{n_2=0}^{1} x[n_1,n_2]e^{-j\pi n_1}$$

$$Y[1,0] = \sum_{n_1=0}^{1} x[n_1,0]e^{-j\pi n_1} + x[n_1,1]e^{-j\pi n_1}$$

$$Y[1,0] = x[0,0]e^{-j\pi(0)} + x[0,1]e^{-j\pi(0)} + x[1,0]e^{-j\pi(1)} + x[1,1]e^{-j\pi(1)}$$

$$Y[1,0] = x[0,0]e^0 + x[0,1]e^0 + x[1,0]e^{-j\pi} + x[1,1]e^{-j\pi}$$

$$Y[1,0] = x[0,0] + x[0,1] + x[1,0](-1) + x[1,1](-1) \qquad e^{-j\pi} = -1$$

$$Y[1,0] = 1 + 2 + 3(-1) + 4(-1)$$

$$Y[1,0] = 3 + (-3) + (-4)$$

$$\boxed{Y[1,0] = -4}$$

<u>$Y[1,1]$: $K_1 = 1$, $K_2 = 1$, $N = 2$</u>

$$Y[1,1] = \sum_{n_1=0}^{2-1}\sum_{n_2=0}^{2-1} x[n_1,n_2]e^{-j2\pi\left(\frac{n_1(1)}{2} + \frac{n_2(1)}{2}\right)}$$

$$Y[1,1] = \sum_{n_1=0}^{1}\sum_{n_2=0}^{1} x[n_1,n_2]\, e^{-j\pi(n_1+n_2)}$$

Note: $e^{-j\pi} = -1$

$e^{-2j\pi} = \cos(2\pi) - j\sin(2\pi)$

$(1) - (0) = 1$

$$Y[1,1] = \sum_{n_1=0}^{1} x[n_1,0]\, e^{-j\pi(n_1+0)} + x[n_1,0]\, e^{-j\pi(n_1+1)}$$

$$Y[1,1] = x[0,0]e^{-j\pi(0+0)} + x[0,1]e^{-j\pi(0+1)} + x[1,0]e^{-j\pi(0+1)} + x[1,1]e^{-j\pi(1+1)}$$

$$Y[1,1] = x[0,0]\,e^{0} + x[0,1]e^{-j\pi} + x[1,0]e^{-j\pi} + x[1,1]e^{-2j\pi}$$

$$Y[1,1] = x[0,0] + x[0,1](-1) + x[1,0](-1) + x[1,1][\cos(2\pi) - j\sin(2\pi)]$$

$$Y[1,1] = 1 + 2(-1) + 3(-1) + 4(1-0)$$

$$Y[1,1] = 1-2-3+4$$

$$\boxed{Y[1,1] = 0}$$

Note: $e^{-jw} = -1$

$e^{-2jw} = 1$

$e^{-3jw} = -1$

$e^{-4jw} = 1$

$$\boxed{Y[k_1,k_2] = \begin{bmatrix} 10 & -2 \\ -4 & 0 \end{bmatrix}}$$

Using Kernel Matrix:

$$[K] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$[K]^T \rightarrow$ Transpose matrix

↳ A flipped version of the original matrix, created by switching the rows and columns

$$[K]^T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$[Y] = [K][A][K]^T$$

$$[Y] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 1(1)+1(3) & 1(2)+1(4) \\ 1(1)+(-1)(3) & 1(2)+(-1)(4) \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 4 & 6 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 4(1)+(6)(1) & 4(1)+(6)(-1) \\ -2(1)+(-2)(1) & -2(1)+(-2)(-1) \end{bmatrix} = \begin{bmatrix} 10 & -2 \\ -4 & 0 \end{bmatrix}$$

For the following 2x2 image block $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

4.1.2 Calculate its 2x2 DCT

$N=2,\ N=2$

NxN 2-D DCT is defined as

$$Y[K_1,K_2] = \sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1} c_1(K_1)\, c_2(K_2)\, x[n_1,n_2] \cos\left[\frac{(2n_1+1)K_1\pi}{2N}\right] \cos\left[\frac{(2n_2+1)K_2\pi}{2N}\right]$$

where $c_1(K_1) = \begin{cases} \sqrt{\frac{1}{N}} & \text{for } K_1=0 \\ \sqrt{\frac{2}{N}} & \text{otherwise} \end{cases}$ and $c_2(K_2) = \begin{cases} \frac{1}{\sqrt{N}} & \text{for } K_2=0 \\ \sqrt{\frac{2}{N}} & \text{otherwise} \end{cases}$

$$Y[K_1,K_2] = c_1(K_1)\, c_2(K_2) \sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1} \left\{ x[n_1,n_2] \cos\left[\frac{(2n_1+1)K_1\pi}{2N}\right] \cos\left[\frac{(2n_2+1)K_2\pi}{2N}\right] \right\}$$

Given: $N=2$

$$x[n_1,n_2] = \begin{bmatrix} x[0,0] & x[0,1] \\ x[1,0] & x[1,1] \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$Y[K_1,K_2] = \begin{bmatrix} Y(0,0) & Y(0,1) \\ Y(1,0) & Y(1,1) \end{bmatrix}$$

$Y[0,0]$: $K_1=0,\ K_2=0,\ N=2$ $\qquad \cos(0)=1$

$$Y[0,0] = c_1(0)\, c_2(0) \sum_{n_1=0}^{2-1}\sum_{n_2=0}^{2-1} \left\{ x[n_1,n_2] \cos\left[\frac{(2n_1+1)(0)\pi}{2(2)}\right] \cos\left[\frac{(2n_2+1)(0)\pi}{2(2)}\right] \right\}$$

$$Y[0,0] = c_1(0)\, c_2(0) \sum_{n_1=0}^{1}\sum_{n_2=0}^{1} \left\{ x[n_1,n_2] \cos(0) \cos(0) \right\}$$

$$Y[0,0] = \left(\sqrt{\tfrac{1}{2}}\right)\left(\sqrt{\tfrac{1}{2}}\right) \sum_{n_1=0}^{1} x[n_1,0] + x[n_1,1]$$

$$Y[0,0] = \tfrac{1}{2}\left[x[0,0] + x[0,1] + x[1,0] + x[1,1] \right]$$

$$Y[0,0] = \tfrac{1}{2}[1+2+3+4]$$

$$Y[0,0] = \tfrac{1}{2}[10]$$

$$\boxed{Y[0,0] = 5}$$

$\sqrt{\frac{2}{2}} = 1 \quad \cos\left[\frac{\pi}{4}\right] = \frac{\sqrt{2}}{2} \qquad \cos\left[\frac{3\pi}{4}\right] = -\frac{\sqrt{2}}{2}$

$\underline{Y[0,1] : K_1 = 0,\ K_2 = 1,\ N = 2}$

$$Y[0,1] = C_1(0)\,C_2(1) \sum_{n_1=0}^{2-1}\sum_{n_2=0}^{2-1} \left\{ x[n_1,n_2] \cos\left[\frac{(2n_1+1)(0)\pi}{2(2)}\right] \cos\left[\frac{(2n_2+1)(1)\pi}{2(2)}\right] \right.$$

$$Y[0,1] = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{2}{2}}\right) \sum_{n_1=0}^{1}\sum_{n_2=0}^{1} \left\{ x[n_1,n_2] \cos(0) \cos\left[\frac{(2n_2+1)\pi}{4}\right] \right.$$

$$Y[0,1] = \frac{1}{\sqrt{2}} \sum_{n_1=0}^{1}\sum_{n_2=0}^{1} x[n_1,n_2] \cos\left[\frac{(2n_2+1)\pi}{4}\right]$$

$$Y[0,1] = \frac{1}{\sqrt{2}} \sum_{n_1=0}^{1} x[n_1,0] \cos\left[\frac{(2(0)+1)\pi}{4}\right] + x[n_1,1] \cos\left[\frac{(2(1)+1)\pi}{4}\right]$$

$$Y[0,1] = \frac{1}{\sqrt{2}} \left[x[0,0]\cos\left[\frac{\pi}{4}\right] + x[0,1]\cos\left[\frac{3\pi}{4}\right] + x[1,0]\cos\left[\frac{\pi}{4}\right] + x[1,1]\cos\left[\frac{3\pi}{4}\right] \right]$$

$$Y[0,1] = \frac{1}{\sqrt{2}} \left[1\left(\frac{\sqrt{2}}{2}\right) + 2\left(-\frac{\sqrt{2}}{2}\right) + 3\left(\frac{\sqrt{2}}{2}\right) + 4\left(-\frac{\sqrt{2}}{2}\right) \right]$$

$$Y[0,1] = \frac{1}{\sqrt{2}} \left[\frac{\sqrt{2}}{2} - \sqrt{2} + \frac{3\sqrt{2}}{2} - 2\sqrt{2} \right]$$

$$Y[0,1] = \frac{1}{\sqrt{2}} \left[\frac{4\sqrt{2}}{2} - 3\sqrt{2} \right]$$

$$Y[0,1] = \frac{1}{\sqrt{2}} \left[2\sqrt{2} - 3\sqrt{2} \right]$$

$$Y[0,1] = \frac{1}{\cancel{\sqrt{2}}} \left[-\cancel{\sqrt{2}} \right]$$

$$\boxed{Y[0,1] = -1}$$

$\underline{Y[1,0] : K_1 = 1,\ K_2 = 0,\ N = 2} \qquad \sqrt{\frac{2}{2}} = 1$

$$Y[1,0] = C_1(1)\,C_2(0) \sum_{n_1=0}^{2-1}\sum_{n_2=0}^{2-1} \left\{ x[n_1,n_2] \cos\left[\frac{(2n_1+1)(1)\pi}{2(2)}\right] \cos\left[\frac{(2n_2+1)(0)\pi}{2(2)}\right] \right\}$$

$$Y[1,0] = \left(\sqrt{\frac{2}{2}}\right)\left(\sqrt{\frac{1}{2}}\right) \sum_{n_1=0}^{1}\sum_{n_2=0}^{1} \left\{ x[n_1,n_2] \cos\left(\frac{(2n_1+1)\pi}{4}\right) \cos(0) \right\}$$

$$Y[1,0] = \sqrt{\frac{1}{2}} \sum_{n_1=0}^{1}\sum_{n_2=0}^{1} \left\{ x[n_1,n_2] \cos\left(\frac{(2n_1+1)\pi}{4}\right) \right\}$$

4.12 (continued) $Y[1,0]: k_1=1,\ k_2=0$

$$Y[1,0]=\sqrt{\tfrac{1}{2}}\sum_{n_1=0}^{1}\left(x[n_1,0]\cos\left(\frac{(2n_1+1)\pi}{4}\right)+x[n_1,1]\cos\left(\frac{(2n_1+1)\pi}{4}\right)\right)$$

$$Y[1,0]=\sqrt{\tfrac{1}{2}}\left[x[0,0]\cos\left(\frac{(2(0)+1)\pi}{4}\right)+x[0,1]\cos\left(\frac{(2(1)+1)\pi}{4}\right)+x[1,0]\cos\left(\frac{(2(1)+1)\pi}{4}\right)+x[1,1]\cos\left(\frac{(2(0)+1)\pi}{4}\right)\right]$$

$$Y[1,0]=\sqrt{\tfrac{1}{2}}\left[1\left(\cos\left(\tfrac{\pi}{4}\right)\right)+2\left(\cos\left(\tfrac{\pi}{4}\right)\right)+3\cos\left(\tfrac{3\pi}{4}\right)+4\cos\left(\tfrac{3\pi}{4}\right)\right)$$

$$Y[1,0]=\sqrt{\tfrac{1}{2}}\left[\cos\left(\tfrac{\pi}{4}\right)+2\cos\left(\tfrac{\pi}{4}\right)+7\cos\left(\tfrac{3\pi}{4}\right)\right]$$

$$Y[1,0]=\frac{1}{\sqrt{2}}\left[\frac{\sqrt{2}}{2}+2\left(\frac{\sqrt{2}}{2}\right)+7\left(-\frac{\sqrt{2}}{2}\right)\right]$$

$$Y[1,0]=\frac{1}{\sqrt{2}}\left[3\left(\frac{\sqrt{2}}{2}\right)-7\left(\frac{\sqrt{2}}{2}\right)\right]$$

$$Y[1,0]=\frac{1}{\sqrt{2}}\left[-\cancel{4}^{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)\right]$$

$$Y[1,0]=\frac{1}{\cancel{\sqrt{2}}}\left[-2\cancel{\sqrt{2}}\right]$$

$$\boxed{Y[1,0]=-2}$$

$\underline{Y[1,1]: k_1=1,\ k_2=1,\ N=2}$

$$Y[1,1]=c_1(1)c_2(1)\sum_{n_1=0}^{2-1}\sum_{n_2=0}^{2-1}\left\{x[n_1,n_2]\cos\left[\frac{(2n_1+1)(1)\pi}{2(2)}\right]\cos\left[\frac{(2n_2+1)(1)\pi}{2(2)}\right]\right\}$$

$$Y[1,1]=\left(\sqrt{\tfrac{2}{2}}\right)\left(\sqrt{\tfrac{2}{2}}\right)\sum_{n_1=0}^{1}\sum_{n_2=0}^{1}\left\{x[n_1,n_2]\cos\left[\frac{(2n_1+1)\pi}{4}\right]\cos\left[\frac{(2n_2+1)\pi}{4}\right]\right]$$

$$Y[1,1]=(1)\sum_{n_1=0}^{1}\left\{x[n_1,0]\cos\left[\frac{(2n_1+1)\pi}{4}\right]\cos\left[\frac{(2(0)+1)\pi}{4}\right]+x[n_1,1]\cos\left[\frac{(2n_1+1)\pi}{4}\right]\cos\left[\frac{(2(1)+1)\pi}{4}\right]\right\}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \quad \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$Y[1,1] = x[0,0]\cos\left[\frac{(2(0)+1)\pi}{4}\right]\cos\left[\frac{(2(0)+1)\pi}{4}\right]$$
$$+ x[0,1]\cos\left[\frac{(2(0)+1)\pi}{4}\right]\cos\left[\frac{(2(1)+1)\pi}{4}\right]$$
$$+ x[1,0]\cos\left[\frac{(2(1)+1)\pi}{4}\right]\cos\left[\frac{(2(0)+1)\pi}{4}\right]$$
$$+ x[1,1]\cos\left[\frac{(2(1)+1)\pi}{4}\right]\cos\left[\frac{(2(1)+1)\pi}{4}\right]$$

$$Y[1,1] = 1\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) + 2\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{3\pi}{4}\right) + 3\cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$$
$$+ 4\cos\left[\frac{3\pi}{4}\right]\cos\left[\frac{3\pi}{4}\right]$$

$$Y[1,1] = \left(\frac{\sqrt{2}}{2}\right)\left(+\frac{\sqrt{2}}{2}\right) + 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{-\sqrt{2}}{2}\right) + 3\left(\frac{\sqrt{2}}{2}\right)\left(\frac{-\sqrt{2}}{2}\right) + 4\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$$

$$Y[1,1] = +\frac{2}{4} - 2\left(\frac{2}{4}\right) - 3\left(\frac{2}{4}\right) + 4\left(\frac{2}{4}\right)$$

$$Y[1,1] = \frac{2}{4} - \frac{4}{4} - \frac{6}{4} + \frac{8}{4}$$

$$Y[1,1] = -\frac{2}{4} + \frac{2}{4}$$

$$\boxed{Y[1,1] = 0}$$

$$\boxed{Y[K_1,K_2] = \begin{bmatrix} 5 & -1 \\ -2 & 0 \end{bmatrix}}$$

For 2x2 image

<u>2-D Using Kernel Method:</u> $[A] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $[K] = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

$$[Y] = [K][A][K]^T$$

$$[K]^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{1}{\sqrt{2}}(1) + \frac{1}{\sqrt{2}}(3) & \frac{1}{\sqrt{2}}(2) + \frac{1}{\sqrt{2}}(4) \\ \frac{1}{\sqrt{2}}(1) - \frac{1}{\sqrt{2}}(3) & \frac{1}{\sqrt{2}}(2) - \frac{1}{\sqrt{2}}(4) \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{4}{\sqrt{2}} & \frac{6}{\sqrt{2}} \\ -\frac{2}{\sqrt{2}} & -\frac{2}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

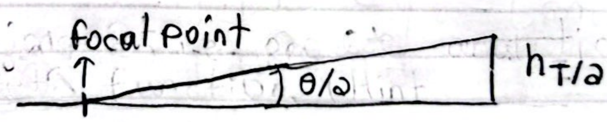
$$[Y] = \begin{bmatrix} \frac{4}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) + \frac{6}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) & \frac{4}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{6}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right)\right) \\ -\frac{2}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{2}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) & -\frac{2}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{2}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{4}{2} + \frac{6}{2} & \frac{4}{2} + \left(-\frac{6}{2}\right) \\ -\frac{2}{2} - \frac{2}{2} & \frac{-2}{2} + \left(\frac{2}{2}\right) \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 10 & -\frac{2}{2} \\ -\frac{4}{2} & 0 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 5 & -1 \\ -2 & 0 \end{bmatrix} \checkmark$$

4.2



(4.2.1) Waveform can be represented as
horizontal: $x[n_1] = 1$, vertical: $x[n_2] = \cos\left(\frac{2\pi}{64} n_2\right)$

Period: $T = \frac{512}{8} = 64$ pixels (8 cycles) $= \cos\left(\frac{\pi}{32} n_2\right)$

(4.1.2) Length of projection of 1 period on retina is:

$$h_T = \frac{d_f \times T}{d_o} = \frac{17 \times 2.5}{253} = 0.168\,mm$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{\frac{h_T}{2}}{r} = \frac{0.084}{10}$$

$\theta = 0.96^\circ \rightarrow$ 0.96 degrees per cycle

Frequency on retina: $f = \frac{1}{\theta} = \frac{1}{0.96} = 1.042$ cycles per degree on retina

$\boxed{f = 1.042 \text{ cycles per degree on retina}}$

I pledge my honor that I have abided by the Stevens Honor System. Deep A. Shah