OPTIMIZING BODY MASS FOR ENERGY EFFICIENCY:

A Study of Metabolic Rate and Kleiber's Law

TABLE OF CONTENTS

1. INTRODUCTION

Metabolism is a fundamental process in living organisms, governing the conversion of food into energy that fuels bodily functions (Hirsch, 2019). The basal metabolic rate (BMR) is the amount of energy expended by an organism at rest, and it plays a crucial role in understanding how different species manage energy. BMR is calculated through a calorimeter that involves analyzing the amount of oxygen and carbon dioxide a species consumes or expels . The BMR used in this exploration will be those that have already been calculated through external studies, the BMR is not a calculation that can be done through the means of common technology alone. The basal metabolic rate is measured in kilocalories/ per day or known more commonly as kcal/day (Davidson, 2022). This analysis will not take into consideration the differences between calories per species, but will rather consider all calories to be of equal significance to the correlation being tested at hand, and with energy efficiency that will be calculated nearing the end. Max Kleiber offers insight into the relationship between an organism's body mass and its metabolic rate (BMR). This Kleiber's Law suggests that the BMR of an endothermic (warm-blooded) animal scales with body mass to the power of approximately 3/4.

$$R = a \cdot M^{3/4}$$

Where 'R' is the Basal metabolic Rate, 'M' is the body mass and 'a' is a constant that is the scaling coefficient. The scaling coefficient is species specific as it accounts for a species physiological and environmental conditions. This formula suggests that as an organism's body mass increases, its metabolic rate increases. **Scaling** in biology refers to how various biological properties, such as metabolic rate, change in proportion to body size. Kleiber's Law suggests that metabolic rate does not scale linearly with mass but rather follows a power function, with the scaling exponent typically close to 3/4. This means that larger animals, though they consume more energy overall, use energy more efficiently per unit of body mass than smaller animals (Niklas et al, 2015).

This research focuses on understanding how well Kleiber's Law explains the relationship between body mass and BMR in three different organisms: humans, a small mammal (mouse), and a large mammal (elephant). By investigating this relationship, I aim to explore how accurate Kleiber's Law is and how metabolic scaling can be used to optimize energy efficiency across a range of body sizes. This includes analyzing the optimal weight for energy efficiency for a 50-year-old male, using data on BMR and body mass, to test this relationship in relation to my father.

On a personal level, this topic has intrigued me because of its potential to inform real-world applications such as health optimization and energy efficiency in various fields, including biotechnology and biomedical engineering, which are both fields of research that I wish to pursue. In addition, I am curious about how Kleiber's Law might apply to my own family, specifically my father. Over the past 2 years, I have observed how his weight fluctuates constantly and how that has impacted his energy levels, health, and well-being. Being a diabetes patient my father has suffered from unstable glucose levels that thereby affected his metabolic rate and thus energy levels. Thus, I will explore how my father's historical weight data aligns with Kleiber's Law to understand his optimal weight for energy efficiency, using BMR data of an average 50-year old man.

By investigating this question, I hope to better understand the importance of Kleiber's Law, especially in how we can apply this knowledge to optimize health and energy use in humans. Ultimately, this exploration will shed light on how metabolic scaling works across different species and body sizes, providing valuable insights into energy efficiency at both the individual and species levels.

2. ANALYSIS OF KLEIBER'S LAW IN HUMANS

To test Kleiber's law exponent coefficient 0.75, I will be using the variable 'b' as the exponent. Thereby making the following equation:

$$R = a \cdot M^{\frac{3}{4}} \quad \rightarrow \quad R = a \cdot M^{b}$$

2.1 Basal Metabolic Rate Calculations

The first species being assessed is the human species, specifically human males ages 18-29.9 years. The human male basal metabolic rate, and masses dataset is as follows:

Age (yrs)					Body mass (kg)					
	45	50	55	60	65	70	75	80	85	90
Men										
18 – 29.9		28.9	27.6	26.6	25.7	24.9	24.3	23.7	23.2	22.7

Figure 1: Partial Data of Men's Mass-Specific BMR at ages 18-29.9 taken from *Appendix A*

In order to calculate an organism's energy expenditure or metabolism total BMR must be considered without the influence of mass.

Hence, the formula for total BMR is as follows:

$$Mass - Specific\ basal\ metabolism\ rate\ = \frac{Total\ BMR}{Body\ Mass}$$

Kleiber's Law looks at how total BMR scales with body mass. However, mass-specific BMR only tells us how much energy is used per unit of body mass, not the total energy an organism needs. Therefore, the formula above must be rearranged to isolate for total BMR instead of mass-specific BMR. This allows us to properly test whether total metabolic rate will follow the predicted $M^{3/4}$ scaling pattern as suggested by Kleiber's Law (Thommen et al, 2019).

Rearranged Formula:

$$Total\ BMR\ =\ Mass - Specific\ basal\ metabolism\ rate\ \times\ Body\ Mass$$

Using the rearranged formula calculate **Total BMR** for age group **18- 29.9 years using data from *Appendix A*.**

Calculations were done in the following manner:

$$Total\ BMR\ =\ Mass - Specific\ basal\ metabolism\ rate\ \times\ Body\ Mass$$

$$Total\,BMR\ = 28.9\ \times\ 50$$

$$Total\,BMR\ = 1445\ kcal/day$$

ALL VALUES ARE ROUNDED TO THE **NEAREST WHOLE NUMBER** IN THE CHART

Mass (kg) → Variable M	Mass-Specific BMR (kcal/kg/day)	Total BMR (kcal/day) → Variable R
50	28.9	1445
55	27.6	1518
60	26.6	1596
65	25.7	1671
70	24.9	1743
75	24.3	1823
80	23.7	1896
85	23.2	1972
90	22.7	2043

Unfortunately, the study that the dataset was acquired from did not contain an estimate for the 'a' value. This value is based upon multiple environmental, and physiological processes. Factors such as age, gender, temperature, climate, activity level, organ function, etc. all play a role in the estimate of 'a'. Hence, there is no universal 'a' for all individuals or species, but 'a' value is rather based on the sample of participants used. While specific studies estimate this 'a' value, these values are not available for all scenarios such as in the study that the dataset originates. To overcome this obstacle, and to avoid the need to estimate the value of 'a', which cannot be done through the basis of this essay, we can simplify this exponential relationship into a linear relationship using Log Transformations (Zaoli et al, 2019). Then use linear regression to not only solve for ‘b’ but also indirectly solve for ‘a’.

2.2 Log Transformations

1. Take the natural log of both sides of the equation:

$$R = a \cdot M^{b} \rightarrow \qquad ln(R) = ln(a \cdot M^{b})$$

2. Use Logarithmic Product Rule → $ln(xy) = ln(x) + ln(y)$

Note: whenever ln is used the base of e is implied.

$$ln(R) = ln(a \cdot M^{b})$$

$$ln(R) = ln(a) + ln(M^{b})$$

3. Use Logarithmic Power Rule → $ln(x^{p}) = p\, ln(x)$

$$ln(R) = ln(a) + ln(M^{b})$$

$$ln(R) = ln(a) + bln(M)$$

After all the logarithmic Transformations, the exponential equation has been linearized.

2.3 Linear Regression

Linear function:

$$y = mx + b$$

A few characteristics of the equation of a linear function is that;

- ‘m’ = slope
- ‘b’ = y-intercept

With that being said, these same characteristics can now be seen within the newly transformed function of the previous exponential function;

New Transformed Function:

$$ln(R) = ln(a) + bln(M)$$

In this function however, the ‘m’ or slope is actually the variable ‘b’ which is the value we aim to estimate to determine the accuracy of Kleiber’s law. In addition, the ‘a’ value which was the species specific constant is now the y-intercept of this linear function. So, without needing to estimate the value of ‘a’ we can find the slope of the function or the value of ‘b’ using linear regression.

So far, the values obtained are as given (raw data obtained from *Appendix A*):

M (kg)	ln(M)	R(kcal/day)	ln(R)
50	$ln(50) = 3.912$	1445	$ln(1445) = 7.276$
55	$ln(55) = 4.007$	1518	$ln(1518) = 7.325$
60	$ln(60) = 4.094$	1596	$ln(1596) = 7.375$
65	$ln(65) = 4.174$	1671	$ln(1671) = 7.421$
70	$ln(70) = 4.248$	1743	$ln(1743) = 7.463$
75	$ln(75) = 4.317$	1823	$ln(1823) = 7.508$
80	$ln(80) = 4.382$	1896	$ln(1896) = 7.548$
85	$ln(85) = 4.443$	1972	$ln(1972) = 7.587$
90	$ln(90) = 4.500$	2043	$ln(2043) = 7.622$

To clarify the linear relationship being tested is

$$ln(R) = bln(M) + ln(a)$$

Where:

- $ln(R)$: Dependent variable (y)
- $ln(M)$: Independent variable (x)
- b: Slope of the line (m)
- $ln(a)$: y-intercept

Hence, now we can calculate the slope (b) and y-intercept (ln(a)) by performing linear regression on the ln(M) and ln(R) values calculated. [1]

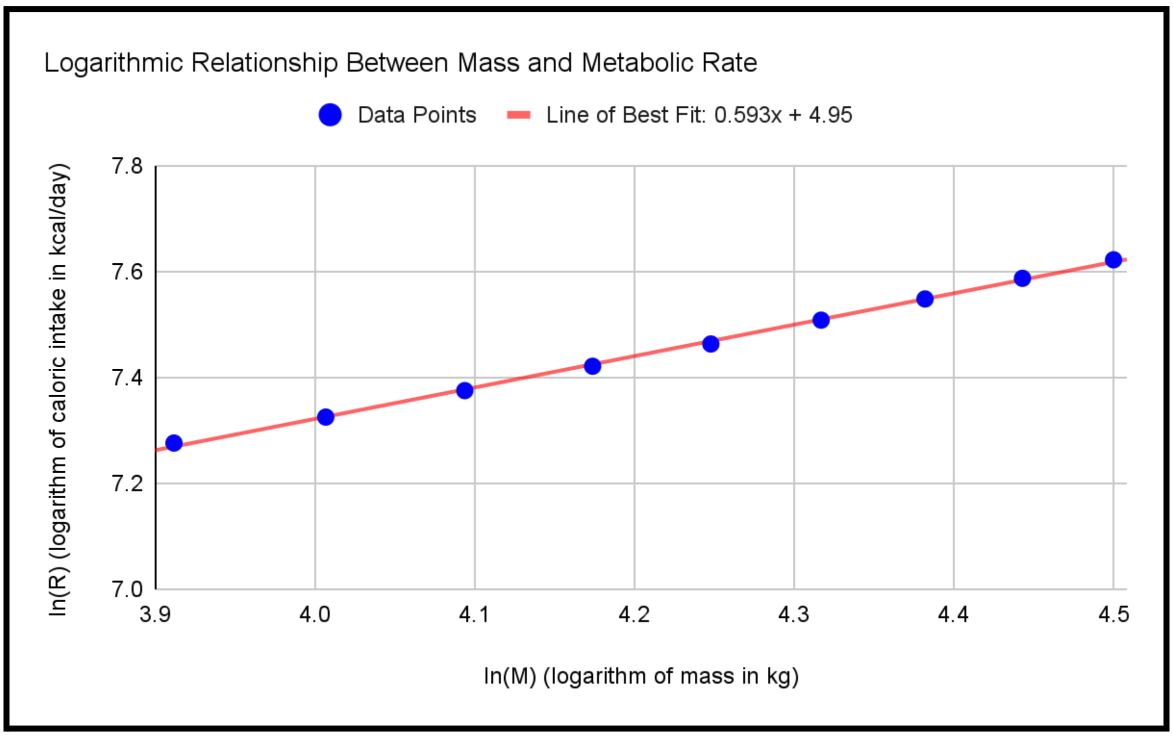

Figure 2: Graph of relationship between ln(R) and ln(M) created using googlesheets with data chart shown in *Appendix B*

This scatter plot demonstrates a clear linear relationship in the logarithmic space.

Regression line equation: $y = 0.593x + 4.95$

[1] Created on Google Spreadsheet

The regression analysis yielded a scaling exponent of b = 0.593, which deviates from the theoretical value suggested by Kleiber law of b = 0.75. The resulting scaling exponent represents the slope of the regression line and suggests that metabolic rate increases at a slower rate compared to body mass.

The BMR and body mass equation suggests a directly proportional relationship in the sense that as body mass increases so does the BMR. However since the scaling exponent of Body mass is less than 1 at 0.593, it indicates that the increase in metabolic rate is slower than the increase in body mass, meaning that metabolic rate does not increase at the same rate as body mass. Additionally, since 0.593 is <0.75, this suggests that the sample from the data set exhibits an even slower rate of increase in metabolic rate in relation to body mass than the prediction of Kleiber's law. The lower exponent value of 0.593 implies that the larger individuals are more energy efficient than the theoretical model would predict, requiring less energy per unit of body mass as their mass increases. In a broader biological context, the observed scaling exponent of 0.593, suggests that the relationship suggested by Kleiber's law may not hold true universally across all human populations under certain conditions.

2.4 Determining Strength of Relationship: Coefficient of Determination (R^2)

Finding R^2 for this dataset is crucial because it quantifies how well the linear regression model in *Figure 2* fits the data. In this study, R^2 will indicate how well the relationship between body mass and metabolic rate align with the predicted pattern suggested by Kleiber's law. Although the scaling coefficient b is not equal to Kleiber's predicted value, already suggesting the deviation from the Model, R^2 provides a crucial measure of how well the data fits the model by validating whether the linear relationship between ln(M) and ln(R) is strong enough to support the conclusions that there is still a significant linear relationship between body mass and metabolic rate. If R^2 is high, it means that the observed scaling exponent of 0.593 is still backed by Kleiber's model even though it deviated from the theoretical 0.75.

Coefficient of Determination Formula:

$$R^2 = 1 - \frac{SSE}{SST}$$

SSE = Sum of Squared Errors → Measures the variance that is not explained by the model

$$SSE = \Sigma(y_i - \hat{y}_i)^2$$

Where:

- y_i is the **actual** ln(R) value for each data point
- $\hat{y}_i$ is the **predicted** ln(R) value from the regression line

SST = Total Sum of Squares → Measures total variance in the observed values of ln(R)

$$SST = \Sigma(y_i - \bar{y})^2$$

Where:

- y_i is the **actual** ln(R) value for each data point
- $\bar{y}$ is the **mean** ln(R) of all the ln(R) values

Calculate R^2 with dataset values in *Appendix C* (Google Sheets was used to calculate)

SSE	SST
0.000123092079	0.113232

Now calculate using the formula:

$$R^2 = 1 - \frac{SSE}{SST}$$

$$R^2 = 1 - \frac{0.000123092079}{0.113232}$$

$$R^2 = 1 - 0.00108707855$$

$$R^2 = 0.99891292145$$

Therefore, an R2 value of 0.999 (rounded to 3 decimales) indicates that 99.9% of variance in ln(R) is explained by ln(M), this shows a very strong linear correlation between body mass and metabolic rate in the data set.

2.5 Evaluation

Hence, despite the deviation in the scaling exponent, there is still a strong relationship between body mass and metabolic rate in the dataset. The high R^2 value also indicates that body mass is a highly reliable predictor of metabolic rate, supporting the broader idea of metabolic scaling. This suggests that the general trend predicted by Kleiber's law is still largely applicable, even if the specific scaling exponent does not align. There are many factors that could have affected the results to produce such deviations. For instance, not all humans have the same physiological conditions, and factors such as genetics, fitness levels, and metabolic health can significantly influence energy usage. For example, someone with a higher level of physical fitness or more muscle mass might have higher metabolic efficiency, even at a higher weight. Hence, to address this further details about participant activity levels, body composition and health status would better account for the variability in metabolic rates among individuals. Additionally, the scaling constant 'a' is not explicitly provided in the dataset and it was indirectly estimated from the y-intercept of the log-log regression plot. The exponent 'b' can still provide a general understanding of the relationship between body mass and metabolic rate, but the precise value of 'a' is necessary to fully understand the scaling across different individuals especially within this dataset. The absence of this constant means that the relationship between metabolic rate and body mass might be incomplete or imprecise, especially when accounting for physiological differences mentioned previously. Furthermore, the sample used from the data set was solely focused on the younger individuals ages 18-29.9, and only males which could have further limited the relationship between metabolic rate and mass, as factors such as hormones, sleep, diet, stress, and many external factors could have also played a part in suggesting that metabolic rates were lower than expected by Kleiber's law. In addition to the human species in this study, it is also important to consider the metabolic scaling in other species to further explore how body mass affects metabolic rate across different organisms. For example, elephants and mice have stark differences in mass that can be analyzed to find similarities in their metabolic scaling.

3. ACCURACY OF KLEIBER'S LAW: Intraspecies vs. Interspecies Scaling

In addition to studying the human male BMR correlation with mass and how accurate Kleiber's law was with that intraspecies dataset, the question of how accurate the law is between interspecies was brought to my attention. During the last part of the study, the deviation in the scaling exponent raised questions about whether Kleiber's law is universally accurate across all endothermic species or if variations exist due to mass differences (Niklas et al, 2015). To explore this further, the study will extend its focus to interspecies comparisons by analyzing metabolic data from a small mammal such as a mouse, and a large mammal such as an elephant. All mammals used are endothermic species which is very significant to the purpose of this study. Endothermic species regulate their body temperature internally through metabolic processes, primarily driven by glucose metabolism. This ensures a consistent basis for comparison, as endotherms have predictable energy demand linking metabolic rate to body mass while accounting for heat production and dissipation (Seebacher, 2009). Hence this comparison will help determine whether the observed deviation in the human dataset is unique to humans or if similar variations occur across species, contributing to a deeper understanding of the limits and accuracy of Kleiber's law. Due to the lack of data on specific mass of mice and elephants along with BMR, only the average BMR as well as the average mass of these animals will be used for this section of the study.

3.1 Average Human Male Dataset

First, find the average total BMR, and average mass from the male human dataset used in the previous section found in *Appendix A*.

The following formula can be used;

$$Average\ Total\ BMR = \frac{\Sigma\, total\ BMR}{Number\ of\ Masses}$$

$$Average\ Total\ BMR\ = \frac{1445+1518+1596+1671+1743+1823+1896+1972+2043}{9}$$

$$Average\ \ Total\ \ BMR\ =\frac{15707}{9}$$

$$Average\ \ Total\ \ BMR\ \ =\ \ 1745.2222222222...$$

$$Average\ \ Total\ \ BMR\ \ = 1745\ kcal/day$$

(rounded to nearest whole number)

The average Mass can also be calculated for in a similar manner:

$$Average\ Body\ Mass\ \ =\ \ \frac{\Sigma\ total\ Body\ Mass}{Number\ of\ Masses}$$

$$Average\ Body\ Mass\ \ =\ \ \frac{50+55+60+65+70+75+80+85+90}{9}$$

$$Average\ Body\ Mass\ \ =\ \ \frac{630}{9}$$

$$Average\ Body\ Mass\ \ =\ \ 70\ kg$$

Hence, the average total BMR and mass that will be used for the human male datapoint is 1745 kcal/day at 70kg.

<u>3.2 Male Mice Dataset</u>

A journal published called “Factors Predicting Individual Variability in Diet-Induced Weight Loss in MF1 Mice” released data on the mice used for their study. I will be focusing on the baseline mice data from Data Chart in *Appendix D* with regards to only the mean mass and mean BMR recorded. (Vaanholt et al, 2012)

Mean Mass:

49.4 g → needs to be converted to Kg

$$\frac{49.4g}{1000} =\ 0.0494$$

= 0.049 kg

(rounded to 3 decimal places)

Mean Resting Metabolic Rate: 31.1 Kj/day

Resting metabolic rate and Basal metabolic rate are very similar in value. The only difference is that Basal metabolic rate is recorded after a 24h fast, while Resting Metabolic Rate is not. The BMR is usually around 10% lower than the RMR, with that being said I will be deducting 10% from the RMR given from the database (Frothingham, 2023).

31.1 Kj/Day → needs to be converted to kcal/day:

1 Kj = 0.239006 kcal

31.1 Kj = (0.239006 x 31.1) kcal

31.1 Kj = 7.433 kcal

$therefore,\ average\ RMR\ is\ 7.433\ kcal/day$

Next convert to BMR by reducing by 10%

$$BMR\ =\ 7.433\ kcal/day\ -\ (7.433\ \times\ 0.1)$$

$$BMR\ =\ 6.6897\ kcal/day$$

BMR = 6.690 kcal/day

(rounded to 3 decimal places)

Hence, the average mass and BMR for male mice is 6.690 kcal/day at 0.049 kg

3.3 Male Elephant Dataset

Finally, the Average mass and Average BMR for a male elephant is as follows:

24541.9 kcal/day average BMR at 4750 kg average mass (Lechowim, 2015)

3.4 Relationship Between All 3 Species

All data required taken from section 3.3:

Species (all male)	Average Mass (Kg)	ln (M)	Average BMR (kcal/day)	ln (R)
Mouse	0.049	$ln(0.049)\ =$ -3.016	6.690	$ln(6.690)\ =$ 1.901

Human	70	$ln(70) = 4.248$	1745	$ln(1745) = 7.465$
Elephant	4750	$ln(4750) = 8.466$	24541.9	$ln(24541.9) = 10.108$

Now, once again to find the scaling exponent ‘b,’ similar to part 1 of this exploration, I will be using the equation $R = a \cdot M^b$ and linearizing it by using logarithmic transformations to produce the equation $ln(R) = bln(M) + ln(a)$ as shown before. The values for the logarithmic function transformations are within the chart above. [2]

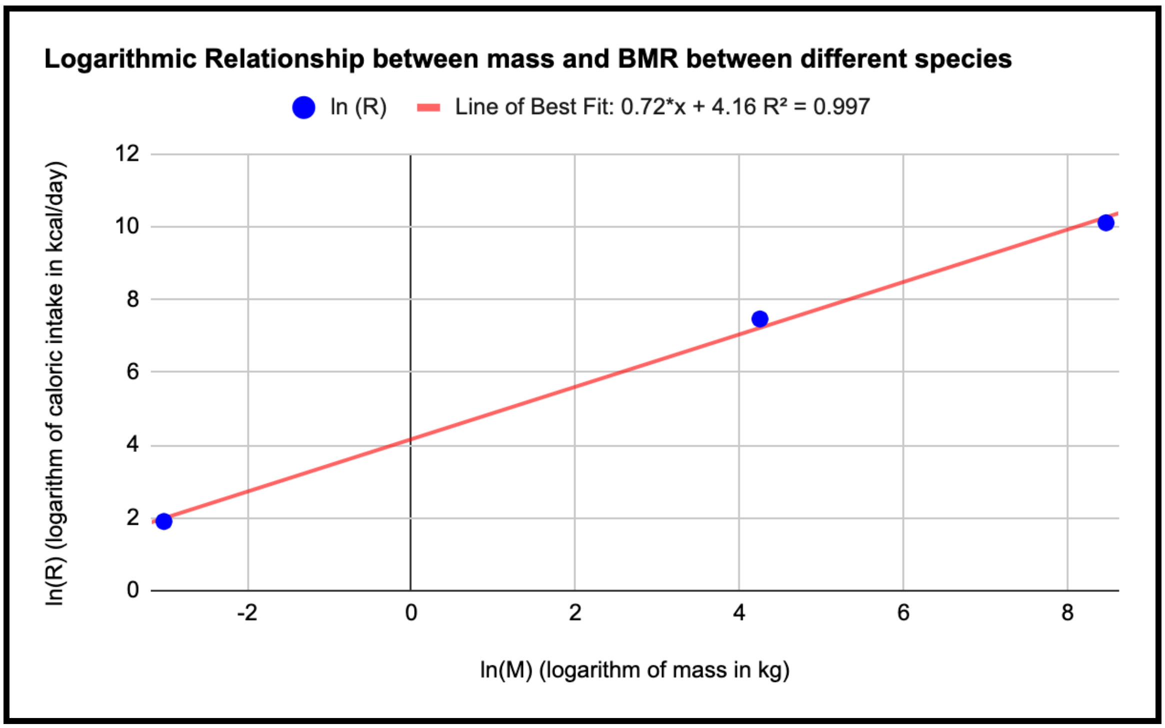

Figure 2: Graph of Interspecies Comparison of ln(R) against ln(M) created on google sheets.

The equation of the line of best fit is y = 0.72x + 4.16, with the value of ‘b’ being 0.72. After adding the new data points of mouse and elephant with the human data point averages, the slope of the regression line became closer to 0.75 which was suggested by Kleiber’s law. This suggests that the scaling law is essentially more accurate when considering a broader range of body masses or different species.

[2] Created on Google Spreadsheet

3.5 Evaluation

Hence, the analysis of metabolic data across different species has provided valuable insights into the applicability of Kleiber's law in interspecies scaling. The scaling exponent of 0.72, while slightly deviating from the expected 0.75, suggests that Kleiber's law holds reasonably well when comparing species of vastly different sizes from mice to elephants, including humans. This signfies that although the Kleiber Law was to work for all mammals, when considering intraspecies with a limited sample size, participant variability in regards to physiology and their environment plays a huge role in determining the scaling exponent. This helps explain why the previous exponent of 0.52 was much lower, as it was derived from a small range of human masses with considerable physiology variations. In contrast, the interspecies scaling exponent of 0.72 implies that having a wider range of masses minimizes the influence of individual variability on the analysis, resulting in a scaling exponent that aligns more closely with Kleiber's Law. Hence, we can conclude that Kleiber's law is more accurate when applied to interspecies comparisons, highlighting the important of considering the scope of data and variability when assessing biological scaling laws, offering a more comprehensive understanding of metabolic processes across species in realtion to body mass.

4. OPTIMIZING ENERGY EFFICIENCY

In this investigation, the concept of Kleiber's law is valuable in understanding the relationship between body mass and metabolic rate as it tells us that there is a correlation between the two. However, using the law's exact formula may not be the most accurate approach for the specific context of this section due to the results shown in previous sections that outlined this law's accuracy in only interspecies comparisons rather than within a restricted range of body masses that will be used in this section of the study.

As mentioned, the aim of this exploration was not only to analyze the accuracy of Kleiber's law, but also to essentially find the most optimal weight for maximum energy efficiency for my 50 year old father. Over the past few years I have witnessed my dad at a low weight of 85 kg, and as well as a high weight of 120kg. Seeing these drastic changes in his weight, I wondered how his energy levels or metabolic rate may have been affected, and what exactly would be the most optimal weight for a man his age, for his body's maximum energy efficiency metabolic rate.

The dataset provided previously in *Appendix A* will be used once again for this exploration, however this time the values will be of the 30-59.9 years old age group. Unfortunately, the data within the dataset provided only extends to 90 kg in mass. Thus, I will be finding the rate at which the mass-specific BMR changes within the mass range given in the dataset, and then applying this rate to find the mass-specific BMR at higher masses as well to align with my father's higher mass.

4.1 Linear Regression of Mass and Mass-specific BMR of 30-59.9 year old Males

x	Mass (kg)	50	55	60	65	70	75	80	85	90
y	Mass-specific BMR (kcal/kg/day)	28.9	27.3	26.0	24.9	23.9	23.1	22.4	21.7	21.2

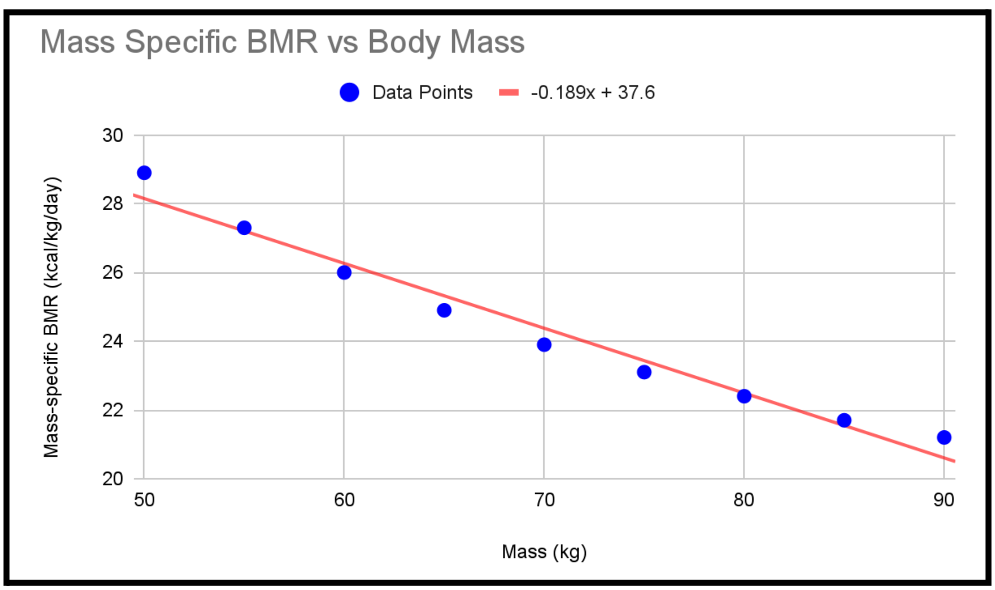

Figure 3: Graph of Mass plotted against Mass-specific BMR as taken from the [3]chart above, which contains data from *Appendix A*.

Hence the line of regression is:

$$y = -0.189x + 37.6$$

[3] Created on Google Spreadsheet

4.2 Pearson Correlation Coefficient

To determine the strength and direction of the linear relationship between body mass and mass-specific BMR in males aged 30-59.9 years, the Pearson correlation coefficient is being conducted. Since Kleiber's law already established that there is a relation between body mass and BMR, this test will ensure that there is a significant correlation between the two variables so that the masses being investigated can be expanded using the trend of relation seen in the graph above.

Pearson Correlation Coefficient Formula:

$$r = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2 \Sigma(y_i - \bar{y})^2}}$$

Where:

- r = pearson correlation coefficient
- x_i = Value of x (Mass in Kg)
- y_i = Value of y (Mass-specific BMR in kcal/kg/day)
- $\bar{x}$ = Mean of x values
- $\bar{y}$ = Mean of y values

Using the Calculations shown in *Appendix E*

$$r = \frac{-283}{\sqrt{1500 \times 54.93555556}}$$

$$r = \frac{-283}{\sqrt{1500 \times 54.93555556}}$$

$$r = \frac{-283}{287.059807949}$$

$$r = -0.9858572749$$

Since the resulting value r = -0.9859 (rounded) this indicates a strong negative correlation between body mass and mass-specific BMR. A correlation close to -1 suggests that as body mass increases, mass-specific BMR consistently decreases, and this aligns with the expectations based on Kleiber's law, which predicts that with decreasing mass-specific metabolic rate there is increasing body size. The high magnitude of r confirms that this trend is highly linear and thereby making it reliable basis for estimating the mass-specific BMR at higher masses beyond the dataset's original range to incorporate my fathers fluctuating weights at the highest.

4.3 Rate of Change

The rate of change of the Mass Specific BMR can be found using this equation of the regression line by finding the derivative of Mass specific BMR with respect to Mass.

B(M) → Mass specific BMR

$$B(M) =- 0.189M + 37.6$$

$$B'(M) = \frac{dB}{dM}$$

To find the derivative of this equation, we can use differentiation laws:

Power Rule: $\frac{d}{dx}(x^{n}) = nx^{n-1}$

Constant Rule: $\frac{d}{dx}(c) = 0$

$$\frac{dB}{dM} = (1) - 0.189M^{1-1} + 0$$

$$\frac{dB}{dM} =- 0.189$$

Hence, the rate of change and the derivative of B(M) is -0.189 kcal/kg meaning that for every 1 kg increase in body mass, the mass-specific BMR decreases by 0.189 kcal/kg. This constant rate is representative of the linear relationship shown through the graph above as well.

Total BMR → T(M)

This total BMR can be calculated using the equation T(M) = M x B(M)

Subbing in the B(M) from the previous step which was

$B(M) =- 0.189M + 37.6$

We can get; T(M) = $- 0.189M^2 + 37.6$M

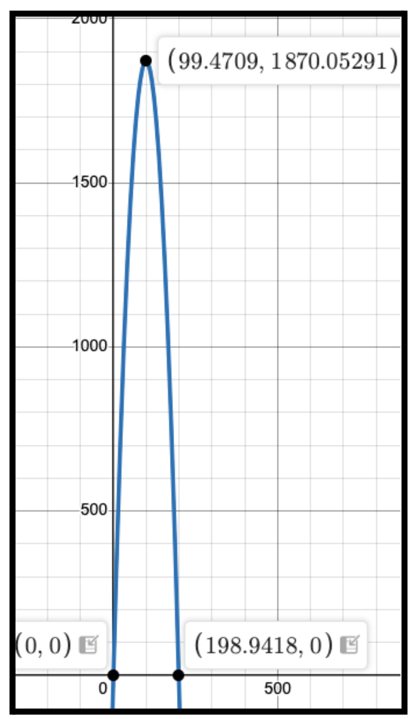

Figure 4: Graphical representation of the T(M) equation above. [4]

[4] Graphed on Desmos

Rate of Change of Total BMR

To find this rate of change, just like the previous step we can differentiate T(M) with respect to M using the same differentiation rules (power rule)

T'(M) = $-\ 0.378M\ +\ 37.6$

Hence, we can use this equation to find the rate of change at specific masses that surpass the range of masses provided in the dataset, by substituting in higher masses for *M*. Calculations shown in *Appendix F*

To represent this rate of change, we can visually graph the linear function:

Figure 5: Rate of Change [5]of BMR at higher weights graphed by differentiating T(M)

<u>4.4 Finding the Optimal Weight</u>

To find the mass where the total BMR is maximized, the slope of the tangent or derivative of T(M) has to be = 0, thereby we can use the T'(M) equation from before and equate it to 0 to isolate for *M*.

$$T'(M) = -\ 0.378M\ +\ 37.6$$

$$0\ =\ \ -\ 0.378M\ +\ 37.6$$

$$\frac{37.6}{-0.378} = M$$

$$99.5\ kg\ =\ M$$

[5] Graphed on Desmos

Therefore, the most optimal weight for maximum energy efficiency is approximately **99.5 kg** for my father. As for when mass increases above 99.5 kg, the total BMR starts to decrease as suggested by the negative kcal/day values shown in *Appendix 4*.

5. CONCLUSION

This exploration has provided valuable insights into the relationship between body mass and metabolic rate, helping me better understand how my father's weight fluctuations may have affected his energy levels if above or below the optimal weight of 99.5 kg. Although this may seem quite heavy, in the larger scheme of his weight fluctuations this is considered to be the optimal weight for him as a tall and muscular man. Beyond its personal relevance, this investigation has also deepend our understanding of Kleiber's Law. While the law effectively described interspecies scaling, its accuracy is limited when applied to a narrower range of masses within a single species. The findings suggest that individual physiological and environmental factors play a significant role, making the direct application of Kleiber's equation less precise for intraspecies comparisons.

Despite these limitations, the principles behind Kleiber's Law remain widely applicable. From optimizing nutrition and fitness plans to remain at the optimal weight for an individual to improving medical treatments. Understanding how metabolic rate scales with body mass has important implications in various fields, such as biotechnology and biomedical engineering, both of which I wish to venture into during post secondary education. This study highlights the importance of adapting general biological principles to specific contexts and how math plays a crucial role in the application of these principles, ensuring accurate and meaningful applications in personal and scientific settings.

References

Albert Thommen, Steffen Werner, Olga Frank, Jenny Philipp, Oskar Knittelfelder, Yihui Quek, Karim Fahmy, Andrej Shevchenko, Benjamin M Friedrich, Frank Jülicher, Jochen C Rink (2019) Body size-dependent energy storage causes Kleiber's law scaling of the metabolic rate in planarians eLife 8:e38187https://doi.org/

Bowes, H. M., Burdon, C. A., & Taylor, N. A. S. (2021). The scaling of human basal and resting metabolic rates. *European journal of applied physiology*, *121*(1), 193–208. https://doi.org/10.1007/s00421-020-04515-1

Davidson, K. (2022). Basal Metabolic Rate Calculator. Retrieved from https://www.healthline.com/health/how-to-calculate-your-basal-metabolic-rate#bottom-line

Frothingham, S. (2023). What Is Basal Metabolic Rate? Retrieved from https://www.healthline.com/health/what-is-basal-metabolic-rate#bmr-vs-rmr

Glazier D. S. (2008). Effects of metabolic level on the body size scaling of metabolic rate in birds and mammals. *Proceedings. Biological sciences*, *275*(1641), 1405–1410. https://doi.org/10.1098/rspb.2008.0118

Hirsch L. (2019) Metabolism (for Teens) | Nemours KidsHealth. Retrieved from https://kidshealth.org/en/teens/metabolism.html

Lechowim. (2015). Energy expenditure. Retrieved from https://asianelephantnutrition.wordpress.com/2015/02/25/big-body-lots-of-energy-maintenance/

Niklas, K. J., & Kutschera, U. (2015). Kleiber's Law: How the Fire of Life ignited debate, fueled theory, and neglected plants as model organisms. *Plant signaling & behavior*, *10*(7), e1036216. https://doi.org/10.1080/15592324.2015.1036216

S. Zaoli, A. Giometto, E. Marañón, S. Escrig, A. Meibom, A. Ahluwalia, R. Stocker, A. Maritan, & A. Rinaldo, Generalized size scaling of metabolic rates based on single-cell measurements with freshwater phytoplankton, Proc. Natl. Acad. Sci. U.S.A. 116 (35) 17323-17329, https://doi.org/10.1073/pnas.1906762116 (2019).

Seebacher F. (2009). Responses to temperature variation: integration of thermoregulation and metabolism in vertebrates. *The Journal of experimental biology*, *212*(18), 2885–2891. https://doi.org/10.1242/jeb.024430

Speakman J. R. (2005). Body size, energy metabolism and lifespan. *The Journal of experimental biology*, *208*(Pt 9), 1717–1730. https://doi.org/10.1242/jeb.01556

Vaanholt, L.M., Magee, V. and Speakman, J.R. (2012), Factors Predicting Individual Variability in Diet-Induced Weight Loss in MF1 Mice. Obesity, 20: 285-294. https://doi.org/10.1038/oby.2011.279

Wang, Z., Bosy-Westphal, A., Schautz, B., & Müller, M. (2011). Mechanistic model of mass-specific basal metabolic rate: evaluation in healthy young adults. *International journal of body composition research*, *9*(4), 147.

Yegian, A. K., Heymsfield, S. B., Castillo, E. R., Müller, M. J., Redman, L. M., & Lieberman, D. E. (2024). Metabolic scaling, energy allocation tradeoffs, and the evolution of humans' unique metabolism. *Proceedings of the National Academy of Sciences of the United States of America*, *121*(48), e2409674121. https://doi.org/10.1073/pnas.2409674121

APPENDICES

APPENDIX A: RAW DATA OF MASS-SPECIFIC BASAL METABOLIC RATE OF MEN AND WOMEN

Mass-specific basal metabolic rate (in kcal/kg per day) for men and women with diverse age and body mass.

Age (yrs)	**Body mass (kg)**									
	45	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**
Men										
18 – 29.9		28.9	27.6	26.6	25.7	24.9	24.3	23.7	23.2	22.7
30 – 59.9		28.9	27.3	26.0	24.9	23.9	23.1	22.4	21.7	21.2
≥60		23.5	22.4	21.5	20.8	20.1	19.5	19.1	18.6	18.2
Women										
18 – 29.9	25.6	24.6	23.7	22.9	22.3	21.8	21.3	20.9	20.5	
30 – 59.9	26.9	25.0	23.5	22.2	21.1	20.2	19.4	18.7	18.1	
≥60	23.7	22.3	21.1	20.1	19.2	18.5	17.9	17.3	16.8	

<u>APPENDIX B:</u> TABLE IN GOOGLE SPREADSHEETS OF LN(R) AND LN(M)

	A	B
1	ln(M)	ln(R)
2	3.912	7.276
3	4.007	7.325
4	4.094	7.375
5	4.174	7.421
6	4.248	7.463
7	4.317	7.508
8	4.382	7.548
9	4.443	7.587
10	4.5	7.622

<u>APPENDIX C:</u> SCREENSHOT OF TABLE IN GOOGLE SPREADSHEETS OF CALCULATIONS FOR R^2

	A	B	C
1	ln(M)	ln(R)	Predicted Values of ln(R)
2	3.912	7.276	7.269816
3	4.007	7.325	7.326151
4	4.094	7.375	7.377742
5	4.174	7.421	7.425182
6	4.248	7.463	7.469064
7	4.317	7.508	7.509981
8	4.382	7.548	7.548526
9	4.443	7.587	7.584699
10	4.5	7.622	7.6185
11		Mean ln(R) =	
12		7.458333333	

D	E
SSE [Squared Differences Between ln(R) and predicted ln(R)]	SST [Squared Differences Between ln(R) and Mean of ln(R)]
0.000038241856	0.03324544444
0.000001324801	0.01777777778
0.000007518564	0.006944444444
0.000017489124	0.001393777778
0.000036772096	0.00002177777778
0.000003924361	0.002466777778
0.000000276676	0.008040111111
0.000005294601	0.01655511111
0.00001225	0.02678677778
SUM OF ALL SSE	SUM OF ALL SST
0.000123092079	0.113232

APPENDIX D: SCREENSHOT OF TABLE FOR MICE DATA

Table 1. Descriptive statistics for male and female mice

	Sex	*n*	Mean	s.d.	Min	Max	Change
Baseline							
Food intake (g/d)	F	65	3.7	0.4	3.0	4.6	
	M	62	4.1	0.5	3.0	5.0	
Body mass (g)	F	65	42.1	4.4	33.1	53.7	
	M	62	49.4	4.2	38.2	60.3	
Fat free mass (g)	F	65	29.7	2.4	22.8	35.1	
	M	62	34.5	3.0	26.3	40.2	
FM (g)	F	65	12.9	3.6	6.1	22.3	
	M	62	15.1	3.4	8.4	26.2	
Resting metabolic rate (kJ/d)	F	65	29.4	5.3	21.0	48.9	
	M	62	31.1	4.5	22.7	41.5	
Activity ($\times 10^3$ counts/d)	F	63	11.8	4.9	5.5	34.8	
	M	60	9.4	2.5	5.2	17.8	
FAA (% of 24 h activity)	F	63	10.1	2.4	4.6	17.6	
	M	60	11.1	4.0	4.1	20.0	
Mean body temperature (°C)	F	63	36.9	0.3	36.2	37.5	
	M	60	36.6	0.3	35.0	37.8	
Min. body temperature (°C)	F	63	35.6	0.3	34.6	36.4	
	M	60	35.1	0.4	33.3	36.3	
Caloric restriction							
Food intake (g/d)	F	65	2.6*	0.3	2.1	3.2	–30
	M	62	2.8*	0.3	2.1	3.5	–30
Body mass (g)	F	65	35.6*	5.4	23.0	51.2	–16
	M	62	41.6*	6.0	27.6	54.1	–16
Fat free mass (g)	F	65	27.3*	2.8	20.6	35.9	–8
	M	62	31.4*	2.9	23.8	36.2	–9
Fat Mass (g)	F	65	8.6*	3.7	3.3	21.3	–34
	M	62	10.0*	4.0	2.9	18.3	–33
Resting metabolic rate (kJ/d)	F	65	21.8*	4.3	14.1	31.5	–26

APPENDIX E:

SCREENSHOT OF TABLE FROM GOOGLE SPREADSHEET CONTAINING r VALUE CALCULATIONS

x: Mass (kg)	y: Mass-Specific BMR (kcal/kg/day)	Deviation (Mass- Mean)	Deviation (BMR- mean)
50	28.9	-20	4.522222222
55	27.3	-15	2.922222222
60	26	-10	1.622222222
65	24.9	-5	0.5222222222
70	23.9	0	-0.4777777778
75	23.1	5	-1.277777778
80	22.4	10	-1.977777778
85	21.7	15	-2.677777778
90	21.2	20	-3.177777778
Mean Mass:	Mean (BMR):		
70	24.37777778		

Product of Deviations	Squared Deviation (Mass)	Squared Deviation (BMR)
-90.44444444	400	20.45049383
-43.83333333	225	8.539382716
-16.22222222	100	2.631604938
-2.611111111	25	0.2727160494
0	0	0.2282716049
-6.388888889	25	1.632716049
-19.77777778	100	3.911604938
-40.16666667	225	7.170493827
-63.55555556	400	10.0982716
Sum of Product of Deviations:	Sum of Squared Mass Deviations:	Sum of Squared BMR Deviations
-283	1500	54.93555556

APPENDIX F:

TABLE OF MASSES OUT OF THE ORIGINAL DATASET FROM APPENDIX A WITH CALCULATED BMR CHANGE FROM GOOGLE SPREADSHEET CONTAINING

Mass	BMR Rate of Change
85	5.47
90	3.58
95	1.69
100	-0.2
105	-2.09
110	-3.98
115	-5.87
120	-7.76