केन्द्रीय विद्यालय संगठन, राँची संभाग

KENDRIYA VIDYALAYA SANGATHAN, RANCHI REGION

अध्ययन सामग्री / STUDY MATERIAL

गणित (041) / MATHEMATICS (041)

कक्षा : 12 वीं / CLASS : XII

सत्र / SESSION : 2023-24

हमारे संरक्षक / Our Patron

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<u>उपायुक्त महोदय का संदेश</u>

केन्द्रीय विद्यालय संगठन, राँची संभाग के 12वीं कक्षा के विद्यार्थियों हेतु छात्र सहायता सामग्री प्रस्तुत करते हुए मुझे अपार हर्ष हो रहा है। बारहवीं कक्षा के छात्रों, यह सामग्री आपकी आवश्यकताओं को ध्यान में रखते हुए तैयार की गई है। जब आप परीक्षा की तैयारी के अंतिम चरण में होते हैं तब आप एक स्थान पर सभी संभावित प्रश्नों को देख अपने श्रम को एकाग्र कर पाते हैं। इस सहायता सामग्री से प्रश्नों को दोहराना और अभ्यास करना सहज होगा। जब आप आवंटित समय में प्रश्न पत्र पूरा करने की अपनी क्षमता का परीक्षण करना चाहते हों या जब आप अध्ययन करते समय कोई प्रश्न देखते हों तब यह सहायक सामग्री आपकी सहायता हेतु होगा। तैयारी करते समय कभी-कभी तत्काल उत्तर की आवश्यकता है, लेकिन पाठ्य-पुस्तक से खोजने और पढ़ने में समय लगेगा। वैसी स्थिति में जब आप कम समय में पूरी अवधारणा या विचार को समझना और जानना चाहते हैं तब यह पतली सी सामग्री आपको तुरंत परेशानी से बचा लेगी। यह पिछले सीबीएसई बोर्ड परीक्षा के पेपर और प्रतियोगी परीक्षा के किसी प्रश्न को जानने और समझने में मदद करेगा।

अपने विषयों में विशेषज्ञता रखने वाले समर्पित और अनुभवी शिक्षकों की एक टीम ने कड़ी मेहनत के बाद इस सामग्री को तैयार किया है। केवल उन्हीं वस्तुओं को शामिल करने का ध्यान रखा गया है जो प्रासंगिक हैं और पाठ्य-पुस्तक के अतिरिक्त हैं। इस सामग्री को एनसीईआरटी पाठ्य पुस्तक के विकल्प के रूप में नहीं लिया जाना चाहिए बल्कि इसे इसके पूरक के रूप में डिज़ाइन किया गया है। छात्रों की सहायता सामग्री में आपके लिए आवश्यक सभी महत्वपूर्ण पहलू हैं: प्रश्न पत्र का डिज़ाइन, पाठ्यक्रम, सभी इकाइयों/अध्यायों या बिंदुओं में अवधारणाएं, प्रत्येक अध्याय से नमूना परीक्षण आदि । मुझे यकीन है कि सहायक सामग्री का उपयोग छात्रों और शिक्षकों दोनों द्वारा किया जाएगा और मुझे विश्वास है कि यह सामग्री आपको अपनी परीक्षाओं में अच्छा प्रदर्शन करने में मदद करेगी । आनेवाली परीक्षा के लिए शुभकामनाओं के साथ आप यह अवश्य याद रखें मेहनत का कोई विकल्प नहीं है।

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सहायक आयुक्त का संदेश

Dear Students,

It's a matter of great pleasure that students of Ranchi region are being presented an exclusive as well as updated study material prepared by subject teachers of Ranchi region under the guidance and supervision of subject expert principals of Ranchi region.

The material has been designed keeping in mind latest curriculum and exam pattern of CBSE.A lot of effort has been undertaken to keep the study material precise, relevant and comprehensive so as to ensure conceptual clarity as well as minimum level of learning .

The content is crisp along with use of mind map and flow chart to enable quick and effective revision.
Hope this study material will help you prepare for the board exam with confidence .Work hard, manage your time prudently and be sincere in your efforts. ....Success is all yours .
Best of luck .

Sujata Mishra

Asst. Commissioner

KVS, RO, Ranchi

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SYLLABUS

CLASS-XII
(2023-24)

One Paper **Max Marks: 80**

No.	Units	No. of Periods	Marks
I.	Relations and Functions	30	08
II.	Algebra	50	10
III.	Calculus	80	35
IV.	Vectors and Three - Dimensional Geometry	30	14
V.	Linear Programming	20	05
VI.	Probability	30	08
	Total	240	80
	Internal Assessment		20

Unit-I: Relations and Functions

1. Relations and Functions **15 Periods**

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions **15 Periods**

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices **25 Periods**

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants **25 Periods**

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability **20 Periods**

Continuity and differentiability, chain rule, derivative of inverse trigonometric functions, *like* $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives **10 Periods**

Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals **20 Periods**

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px + q}{ax^2 + bx + c}dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}}dx, \int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2 - a^2}\, dx$$

$$\int \sqrt{ax^2 + bx + c}\, dx,$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals **15 Periods**

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

5. Differential Equations **15 Periods**

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + py = q, \text{ where p and q are functions of x or constants.}$$

$$\frac{dx}{dy} + px = q, \text{ where p and q are functions of y or constants.}$$

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors **15 Periods**

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry **15 Periods**

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming

1. Linear Programming **20 Periods**

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability **30 Periods**

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.

MATHEMATICS (Code No. - 041)

QUESTION PAPER DESIGN CLASS - XII

(2023-24)

Time: 3 hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
	Total	80	100

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper.

However, 33% internal choices will be given in all the sections

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

Note: For activities NCERT Lab Manual may be referred.

List of Basic Formulae

1. *A relation R in a set A is called*
   (i) ***reflexive*** *if* $(a,a) \in R$ *for every* $a \in A$.
   (ii) ***symmetric*** *if* $(a,b) \in R \Rightarrow (b,a) \in R$ *for every* $a, b \in A$.
   (iii) ***transitive*** *if* $(a,b) \in R$ *and* $(b,c) \in R \Rightarrow (a,c) \in R$ *for every* $a, b, c \in A$.
2. *A relation is* ***an equivalence relation*** *if it is Reflexive, Symmetric and Transitive.*
3. *A function* $f : X \subset R \to Y \subset R$ *is defined to be* ***one-one (or injective)*** *if the image of distinct elements of X under f are distinct. i. e. for every* $x_1, x_2 \in R$, $f(x_1) = f(x_2)$ *implies* $x_1 = x_2$. *Otherwise, f is called* ***many-one***.
4. *One may* ***disprove one-oneness*** *by using contrapositive statement of* $x_1, x_2 \in R$, $f(x_1) = f(x_2)$ *implies* $x_1 = x_2$, *which is* $x_1 \neq x_2$ *implies* $f(x_1) \neq f(x_2)$ *for some* $x_1, x_2 \in R$,
5. *The following table gives the inverse trigonometric function (principal value branches) along with their* ***domains*** *and* ***ranges:***

Functions	***Domain***	***Range (Principal Value Branches)***
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	R	$(0, \pi)$
$y = \operatorname{cosec}^{-1} x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$

6.

$\sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1} x,\ x \in R - (-1, 1)$	$\sin^{-1}(-x) = -\sin^{-1} x,\ x \in [-1, 1]$
$\cos^{-1}\frac{1}{x} = \sec^{-1} x,\ x \in R - (-1, 1)$	$\tan^{-1}(-x) = -\tan^{-1} x,\ x \in R$
$\tan^{-1}\frac{1}{x} = \cot^{-1} x,\ x > 0$	$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x,\ \lvert x \rvert \geq 1$
$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},\ x \in [-1, 1]$	$\cos^{-1}(-x) = \pi - \cos^{-1} x,\ x \in [-1, 1]$
$\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2},\ \lvert x \rvert \geq 1$	$\sec^{-1}(-x) = \pi - \sec^{-1} x,\ \lvert x \rvert \geq 1$

$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2},\ x\in R$	$\cot^{-1}(-x)=\pi-\cot^{-1}x,\ x\in R$

7. *If A be any given square matrix of order n, then* $A(adjA)=(adjA)A=\|A\|I$ *where I is the identity matrix of order n.*
8. *For matrices A & B, (i)* $(AB)^{-1}=B^{-1}A^{-1}$ *(ii)* $(AB)^T=B^TA^T$ *where* A^T *represents transpose of A.*
9. *A square matrix A is said to be singular if* $|A|=0$ *and vice versa. Otherwise it is non-singular.*
10. $|adjA|=|A|^{n-1}$ *where n is the order of a matrix A.*
11. *If* $|A|\neq 0$, *then we can find* A^{-1} *exists and* $A^{-1}=\frac{1}{|A|}adjA$.
12. *Any system of linear equation can be written as* $AX=D\Rightarrow X=A^{-1}D$ *provided A is a non-singular matrix.*
13. *Suppose* $f:A\subset R\to B\subset R$ *is a real valued function &* $a\in A$. *Then* f *is continuous at* a *iff* $\lim_{x\to a}f(x)=f(a)$

    which means $\underset{at\ x=a}{LHL}=\underset{at\ x=a}{RHL}=f(a)$

    where $\underset{at\ x=a}{LHL}=\lim_{x\to a^-}f(x)=\lim_{h\to 0}f(a-h)$

    and $\underset{at\ x=a}{RHL}=\lim_{x\to a^+}f(x)=\lim_{h\to 0}f(a+h)$
14. *Few most widely used trigonometric results:* $1+\cos 2x=2\cos^2 x$, $1-\cos 2x=2\sin^2 x$
15. *Suppose* f *is a real valued function and* a *is appoint in its domain. Then derivative of* f *at* $x=a$ *is defined as*

    $f'(a^-)=f'(a^+)$ *where* $f'(a^-)=\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$ *and* $f'(a^+)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

    $f'(a^-)$ *means LHD at* $x=a$ *and* $f'(a^+)$ *means RHD at* $x=a$
16. *Algebra of derivatives:*

$(u\pm v)'=u'\pm v'$	$(uv)'=u'v+uv'$	$\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$

17. *Few most widely used differentiation/integration formulaes:*

$\frac{d}{dx}(C)=0$ *Wh. C is a*	$\frac{d}{dx}x^{n+1}=(n+1)x^n$	$\frac{d}{dx}\left(\frac{1}{x}\right)=-\frac{1}{x^2}$	$\frac{d}{dx}(\sqrt{x})=\frac{1}{2\sqrt{x}}$
$\int(0)dx=C$	$\int x^n dx=\frac{x^{n+1}}{n+1}+C$	$\int\frac{1}{x^2}dx=-\frac{1}{x}+C$	$\int\frac{1}{\sqrt{x}}dx=2\sqrt{x}+C$
$\frac{d}{dx}(\log x)=\frac{1}{x}$	$\frac{d}{dx}(\sin x)=\cos x$	$\frac{d}{dx}(\cos x)=-\sin x$	$\frac{d}{dx}(\tan x)=\sec^2 x$
$\int\frac{1}{x}dx=\log x+C$	$\int\cos x dx=\sin x+C$	$\int\sin x dx=-\cos x+C$	$\int\sec^2 x dx=\tan x+C$

$\frac{d}{dx}e^x = e^x$	$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$	$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
$\int e^x dx = e^x + C$	$\int \sec x \tan x dx = \sec x + C$	$\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$	$\int \operatorname{cosec}^2 x dx = \cot x + C$

18. Few exponential/logarithmic formulaes:

$\log mn = \log m + \log n$	$\log(m/n) = \log m - \log n$	$A^B = e^{B \log A}$

19. Rate of change of Surface Area (S) is denoted $\frac{dS}{dt}$ *, Rate of change of Volume (S) is denoted* $\frac{dV}{dt}$ *.*

20. Let a function f be continuous on $[a,b]$ and differentiable on the open interval on (a,b). Then

(a) f is strictly increasing in (a,b) if $f'(x) > 0$ for each $x \in (a,b)$.

(b) f is strictly decreasing in (a,b) if $f'(x) < 0$ for each $x \in (a,b)$.

*(c) A function f will be increasing (decreasing) in **R** if it is so in every interval of **R***

21. Special integrals:

$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left\lvert \frac{x-a}{x+a} \right\rvert + C$	$\int \frac{dx}{a^2 - x^2} = -\frac{1}{2a} \log\left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$
$\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left\lvert x + \sqrt{x^2 + a^2} \right\rvert + C$	$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left\lvert x + \sqrt{x^2 - a^2} \right\rvert + C$

22. Most widely used integrals/forms:

$\int \tan x dx = \log\lvert \sec x \rvert + C$	$\int \sec x dx = \log\lvert \sec x + \tan x \rvert + C$
$\int \cot x dx = \log\lvert \sin x \rvert + C$	$\int \operatorname{cosec} x dx = \log\lvert \operatorname{cosec} x - \cot x \rvert + C$
$\int uv dx = u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx$	$\int e^x (f(x) + f'(x)) dx = e^x f(x) + C$

23. Forms of Partial Fraction:

S. No.	*Forms of Rational Functions*	*Forms of Partial Fraction*
1.	$\frac{px+q}{(x-a)(x-b)}, \quad a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ *where* x^2+bx+c can't be factorized further	$\frac{A}{x-a}+\frac{Bx+D}{x^2+bx+c}$

24. *Some definite integral properties:*

$\int_a^b f(x)dx=\int_a^b f(t)dt$	$\int_a^b f(x)dx=-\int_b^a f(x)dx$, *In particular* $\int_a^a f(x)dx=0$
$\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$	$\int_a^b f(x)dx=\int_b^a f(a+b-x)dx$
$\int_0^{2a} f(x)dx=\int_0^a f(x)dx+\int_0^a f(2a-x)dx$	$\int_0^a f(x)dx=\int_0^a f(a-x)dx$
$\int_0^{2a} f(x)dx=\begin{cases} 2\int_0^a f(x)dx & if \quad f(2a-x)=f(x) \\ 0 & if \quad f(2a-x)=-f(x)\end{cases}$	$\int_{-a}^{a} f(x)dx=\begin{cases} 2\int_0^a f(x)dx & if \quad f(-x)=f(x) \\ 0 & if \quad f(-x)=-f(x)\end{cases}$

25. ***Definition of Differential equations:***
An equation involving derivative (derivatives) of the dependent variable with respect to independent variable (variables) is called a differential equation.

26. ***Definition of Ordinary Differential equations:***
A differential equation involving ***derivatives of the dependent variable*** *with respect to* ***only one independent variable*** *is called an ordinary differential equation. e. g.*
$2\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ *is an ordinary differential equation.*

27. ***Order of a differential equation:***
Order of differential equation is defined as the ***order of the highest order derivative*** *involved in the given ODE.*

28. ***Degree of a differential equation:***
Degree of differential equation is defined as the ***degree of the highest order derivative*** *involved in the given ODE* ***when it is a polynomial in derivatives***.

29. *Method of solving Differential equation:*

S. No.	*Types/Forms*	*Methods*
1.	*Variable – Separable/*	$\int f(x)dx=\int g(y)dy$

2.	Homogeneous Differential Equation/	Use either $y = vx$ or $x = vy$, differentiate & substitute.
3.	Linear Differential equation/ (i) $\frac{dy}{dx} + P(x)dx = Q(x)$ (ii) $\frac{dx}{dy} + P(y)dy = Q(y)$	Solution is given by $y(IF) = \int (Q(x) \times IF)\, dx + C$ where $IF = e^{\int P(x)dx}$ $x(IF) = \int (Q(y) \times IF)\, dy + C$ where $IF = e^{\int P(y)dy}$

30. *If α, β and γ are the angles which a line makes with X-axis, Y-axis & Z-axis respectively. Then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the direction cosines and denoted by l, m and n respectively. i. e. $l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$. Also $l^2 + m^2 + n^2 = 1$.*

31. *Comparatives of Scalar (or dot) and Cross (or vector) product of two vectors:*

S.N.	Scalar (or dot) product	Cross (or vector) product
1.	The scalar (or dot) product of two non-zero vectors $\vec{a}$ and $\vec{b}$, denoted by $\vec{a}.\vec{b}$ is defined as $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$.	The vector (or cross) product of two non-zero vectors $\vec{a}$ and $\vec{b}$, denoted by $\vec{a}\times\vec{b}$ is defined as $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin\theta\, \hat{n}$ where θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.
2.	We define $\vec{a}.\vec{b} = 0$ if either $\vec{a}$ or $\vec{b} = \vec{0}$	We define $\vec{a}\times\vec{b} = \vec{0}$ if either $\vec{a}$ or $\vec{b} = \vec{0}$
3.	$\vec{a}.\vec{b}$ is real number.	$\vec{a}\times\vec{b}$ is a vector.
4.	For any two non-zero vectors, $\vec{a}.\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$	For any two non-zero vectors, $\vec{a}\times\vec{b} = 0 \Leftrightarrow \vec{a} // \vec{b}$
5.	If $\theta = 0$, then $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert$. In particular, $\vec{a}.\vec{a} = \lvert\vec{a}\rvert^2$ & $\vec{a}.(-\vec{a}) = -\lvert\vec{a}\rvert^2$, as θ in this case is 0 and π respectively.	In particular, $\vec{a}\times\vec{a} = \vec{0}$ and $\vec{a}\times(-\vec{a}) = \vec{0}$, as θ in this case is 0 and π respectively.
6.	Since $\hat{i}$, $\hat{j}$ and $\hat{k}$ are mutually unit perpendiculars. Then, $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$ & $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$	Since $\hat{i}$, $\hat{j}$ and $\hat{k}$ are mutually unit perpendiculars. Then, $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$ & $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$
7.	The angle between two no-zero vectors $\vec{a}$ and $\vec{b}$ Is given by $\theta = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}\right)$	In terms of vector product, The angle between two no-zero vectors $\vec{a}\times\vec{b}$ is given by $\theta = \sin^{-1}\left(\frac{\lvert\vec{a}\times\vec{b}\rvert}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}\right)$
8.	$\vec{a}.\vec{b} = \vec{b}.\vec{a}$ (follows commutativity)	$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$ (doesn't follow commutativity)

9.	*Projection of a vector **a** on other vector **b** is given by* $\vec{a}.\hat{b}$ *or* $\vec{a}.\frac{\vec{b}}{\lvert\vec{b}\rvert}$ *or* $\frac{1}{\lvert\vec{b}\rvert}\left(\vec{a}.\vec{b}\right)$	*For any two vectors **a** and **b**,* $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$

32. *If* $\vec{a}$ *and* $\vec{b}$ *represent two the two adjacent sides of a triangle. Then its area is given as* $\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$.

33. *If* $\vec{a}$ *and* $\vec{b}$ *represent two the two adjacent sides of a parallelogram. Then its area is given as* $\left|\vec{a}\times\vec{b}\right|$.

34. *Position vector of a point P(x, y, z) is given as a vector* $\boldsymbol{OP}\left(=\vec{r}\right)=x\hat{i}+y\hat{j}+z\hat{k}$ *and its magnitude is* $\left|\vec{r}\right|=\sqrt{x^2+y^2+z^2}$.

35. *The Vector Equation of a line through a given point* $P\left(\vec{a}\right)$ *and parallel to a given vector* $\vec{b}$ is given by $\vec{r}=\vec{a}+\lambda\vec{b}$.

36. *The Cartesian Equation of a line through a given point* $P\left(x_1, y_{1,} z_1\right)$*and parallel to a given vector having <a, b, c> as its direction ratio is given by*
$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$, *In particular, if l, m, n are direction cosines. Then the equation is* $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$.

37. *The Vector Equation of a line through two given points* $P\left(\vec{a}\right)$ *and* $Q\left(\vec{b}\right)$ is given by $\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right)$ *where* $\lambda\in R$.

38. *The Cartesian Equation of a line through given points* $P\left(x_1,\ y_1,\ z_1\right)$ *and* $Q\left(x_2,\ y_2,\ z_2\right)$*is given by*
$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

39. *The angle between two lines* $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ *and* $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$ *is given by* $\theta=\cos^{-1}\left(\frac{\vec{b}_1.\vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}\right)$

OR $\theta=\cos^{-1}\left(\frac{a_1.a_2+b_1b_2+c_1c_2}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}\right)$

40. *The above lines are perpendicular if* $a_1.a_2+b_1b_2+c_1c_2=0$ and *parallel if* $\frac{a_1}{.a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

41. *Skew lines: The two lines which are neither intersecting nor parallel are called skew lines. The shortest distance between them is given by*
$SD=\left|\frac{\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$ *In case of two parallel lines* $\vec{r}=\vec{a}_1+\lambda\vec{b}$ *and* $\vec{r}=\vec{a}_2+\lambda\vec{b}$
$SD=\left|\frac{\left(\vec{a}_2-\vec{a}_1\right)\times\vec{b}}{\left|\vec{b}\right|}\right|$.

42. *The line are intersecting if and only if* $\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)=0$.

43. *The probability of an event* A *given* B *is defined as*
$P\left(A/B\right)=\frac{P\left(A\cap B\right)}{P\left(B\right)}$ *where* $P\left(B\right)\neq 0$

44. *Multiplication Theorem: For any two events A and B*

$P(A \cap B) = P(A)P(B/A)$ provided $P(A) \neq 0$ and $P(A \cap B) = P(B)P(A/B)$ provided $P(B) \neq 0$.

For any three events A, B and C, $P(A \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)$

45. *Total Probability theorem: If* E_1, E_2 *and* E_3 *are mutually exclusive and exhaustive events and one event A is associated with all these three events. Then*

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$$

46. *Bayes' Theorem: For any three events,*

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} \quad .$$

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

Relation

A Relation R from a non-empty set A to a non-empty set B is a subset of the Cartesian product set A × B. i.e. R $\subset$ A × B. A relation in a set A is the subset of Cartesian product AXA. If a and b are elements of set A and a is related to b then we write as (a,b) ∈ R or aRb. The set of all first elements in the ordered pair (a,b) in a relation R, is called the domain of the relation R and the set of all second elements called images, is called the range of R.

Types of relations

Empty relation: A relation R in a set A is called *empty relation*, if no element of A is related to any element of A. i.e., $R = \phi \subset A \times A$.

Universal relation: A relation R in a set A is called *universal relation*, if each element of A is related to every element of A, i.e. R = A × A.

Equivalence relation. A relation R in a set A is said to be an *equivalence relation* if R is reflexive, symmetric and transitive

A relation R in a set A is called

(i) ***Reflexive***, if *(a, a)* ∈ R, for every *a* ∈ A,

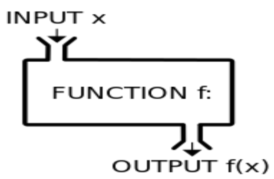
(ii) ***Symmetric,*** if *(a, b)* ∈ R implies that *(a, b)* ∈ R, for all *a*, *b* ∈ A.

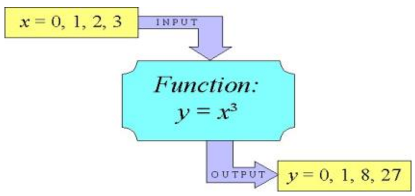
(iii) ***Transitive,*** if *(a, b)*∈ R and *(b, c)* ∈ R implies that *(a, c)* ∈ R *a, b, c* ∈ A.

Function

A relation f from a set A to a set B is said to be function if every element of set A has one and only one image in set B. The notation f : X →Y means that f is a function from X to Y. X is called the domain of f and Y is called the co-domain of f.

Given an element x ∈ X, there is a unique element y in Y that is related to x. The unique element y to which f relates x is denoted by f (x) and is called f of x, or the value of f at x, or the image of x under f. The set of all values of f(x) taken together is called the range of f or image of X under f.
Symbolically, Range of f = {y ∈ Y | y = f (x), for some x in X}.



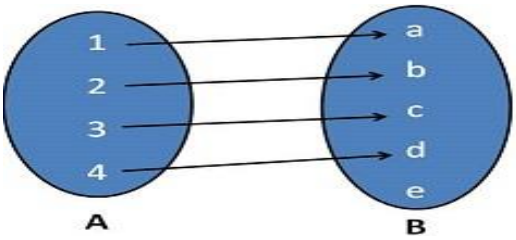


Types of functions:

One-one (or *injective*):

A function *f*: X → Y is defined to be *one-one* (or *injective*), if the images of distinct elements of X under *f* are distinct,
i.e., for every *a*, *b in* X, *f* (*a*) = *f* (b) implies *a* = *b*. Otherwise, *f* is called *many one*.

To disprove the result, we need to use its contrapositive statement:$a \neq b \Longrightarrow f(a) \neq f(b)$.



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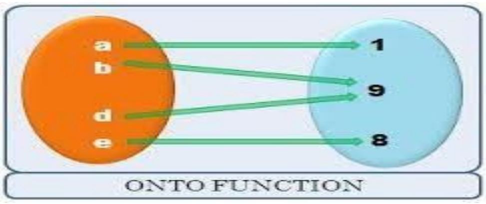
Onto (or surjective):

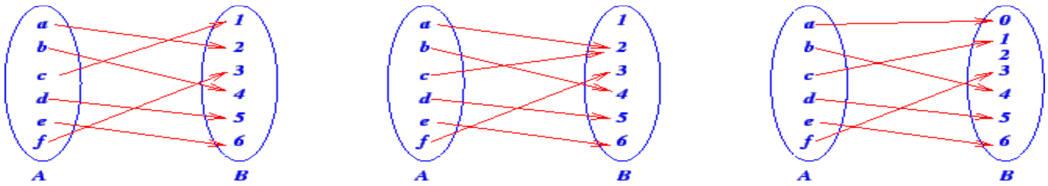
A function f : X →Y is said to be *onto* (or *surjective*), if every element of Y is the image of some element of X under *f*, i.e., for every *y in* Y, there exists an element *x* in X such that *f* (*x*) = *y*.

OR $Co - domf = Rangef$



Bijective function:

A function *f* : X→ Y is said to be *bijective* if *f* is both one-one and onto.



Important points on Relation & Functions:

1. *The total number of relations from a set A having 'm' elements to a set B having 'n' elements is* 2^{mn}.
2. *The total number of relations in a set A having 'n' elements is* 2^{n^2}.
3. *The number of reflexive relations in a set having n elements is* 2^{n^2-n}.
4. *The number of symmetric relations in a set having n elements is* $2^{\frac{n^2+n}{2}}$.
5. *The total number of equivalence relations in a set A having 'n' elements:*

   By Bell numbers

n	***1***			
1	*1*	***2***		
2	*2*	*3*	***5***	
3	*5*	*7*	*10*	***15***

4	*15 20 27 37* ***52***
	and so on

6. *The number of injective/one to one function(s) from a set A having 'm' to another set B having 'n' elements with* $n \geq m$ *is* $\frac{m!}{(m-n)!}$.
7. *The number of injective/one to one function(s) from a set A having 'm' to another set B having 'n' elements is* n^m.
8. *The number of onto functions from a set A having 'm' elements to a set B having 'n' elements* $m \geq n$ *is* $2^m - 2$
9. *The number of bijective functions in a set A having 'n' elements is n!*

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. If $A = \{5,6,7\}$ and let R = {(5,5), (6,6)), (7,7), (5,6), (6,5), (6,7), (7,6)}. Then R is
   A) Reflexive, symmetric but not Transitive
   B) Symmetric, transitive but not reflexive
   C) Reflexive, Transitive but not symmetric
   D) an equivalence relation
2. Let R be a relation defined on Z as follows:
   $(a, b) \in R$ If $a^2 + b^2 = 25$. $Then\ Domain\ of\ R$ is
   A) {3,4,5}
   B) {0,3,4,5}
   C) {0, ±3, ±4, ±5}
   D) None of these
3. The maximum number of equivalence relations on the set A= {1, 2, 3} is
   A) 1
   B) 2
   C) 3
   D)5
4. The number of elements in set A is 3. The number of possible relations that can be defined on A is
   A)8
   B)4
   C)64
   D)512
5. The number of elements in Set A is 3. The number of possible reflexive relations that can be defined in A is
   A) 64
   B) 8
   C)512
   D) 4

6. The number of elements in set P is 4. The number of possible symmetric relations that can be defined on P is

A) 16 B) 32

C)512 D)1024

7. Let P = {a, b, c}. Then the number of Equivalence relations containing (a, b) is

A) 1 B) 2

C) 3 D) 4

8. If f(x) = ax + b, where a and b are integers, f (–1) = – 5 and f (3) = 3, then a and b are equal to

A) a = – 3, b = –1 B) a = 2, b = – 3

C) a = 0, b = 2 D) a = 2, b = 3

9. If $f(x) = x^2 + 2$, $x \in R$, then the range of f(x) is

A) $[2, \infty)$ B) $(-\infty, 2]$

C) $(2, \infty)$ D) $(-\infty, 2) \cup (2, \infty)$

10. The domain of the function $f(x) = x/ (x^2 + 3x + 2)$ is

A) [-2, -1] B) R – {1, 2}

C) R – {-1, -2} D) R – {2}

11. The range of $f(x) = \sqrt{(25 - x^2)}$ is

A) (0, 5) B) [0, 5]

C) (-5, 5) D) [1, 5]

12. Let n (A) = m, and n (B) = n. Then the total number of non-empty relations that can be defined from A to B is

A) m^n B) $n^m - 1$

C) mn – 1 D) $2^{mn} - 1$

<u>ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)</u>

A Both A and R are true and R is the correct explanation of A
B Both A and R are true but R is NOT the correct explanation of A.
C A is true but R is false
D A is false but R is true

1. Assertion (A) *If n (A) =p and n (B) = q then the number of relations from Ato Bis 2^{pq}.*

Reason(R) *A relation from A to B is a subset of A X B.*

2. **Assertion (A)** *Domain and Range of a relation* $R = \{(x, y): x - 2y = 0\}$ *defined on the set* $A = \{1, 2, 3, 4\}$ *are respectively* $\{1, 2, 3, 4\}$ *and* $\{2, 4, 6, 8\}$

Reason(R) *Domain and Range of a relation R are respectively the sets* $\{a: a \in A \text{ and } (a, b) \in R.\}$ *and* $\{b: b \in A \text{ and } (a, b) \in R\}$

3. **Assertion (A)** The function f : {1,2,3,4} →{x,y,z,p } defined by f = {(1,x), (2,y) , (3,z)} is a bijective function.

Reason(R) The function f : {1,2,3,4} →{x,y,z,p } defined by f = {(1,x),(2,y) , (3,z)} is one one.

4. **Assertion (A)** A function f: A→ B, cannot be an onto function if n (A) < n (B).

Reason(R) A function f is onto if every element of co-domain has at leastone pre-image in the domain

5. **Assertion (A)** A, B are two sets such that n(A)=m and n(B)=n. The number of one-one functions from A onto B is nP_m if n ≥ m

Reason(R) A function f is one –one if distinct elements of A have distinct images in B

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. If the relation R be defined on the set N is given by a R b if 2a+3b=30, then find R.
2. Show that the relation R in the set {1,2,3} given by R = {(1,1),(2,2),(3,3),(1,2),(2,3)} is reflexive but neither symmetric nor transitive.
3. Let L be the set of all lines in the xy plane defined as R= $\{(L_1, L_2): L_1 \| L2\}$. Show that R is an equivalence relation.
4. Let T be the set of all triangles in a plane with R a relation in the set T given by R=$\{(T_1, T_2) : T_1 \cong T_2\}$. Show that R is an equivalence relation.
5. Let set A represents the set of all the girls of a particular class. Relation R on A is defined as R={(a,b) : difference between weights of a and b is less than 30 kg}. Show that relation R is a universal relation.
6. Show that the function f: R → R, defined as $f(x) = x^2$, is neither one-one nor onto.
7. Draw the Graphs of Signum function, Modulus function and Greatest integer function.
8. Show that the Signum function f : R→R, given by f(x) =$\begin{cases} 1, if\ x > 0 \\ 0, if\ x = 0 \\ -1, if\ x < 0 \end{cases}$ is neither one-one nor onto.
9. Show that the modulus function f: R→R, given by f(x) = $|x|$ is neither one-one nor onto.
10. Show that the greatest integer function f : R→R, given by f(x) = [x] is neither one-one nor onto.

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1. Show that the relation R in the set of real numbers, defined as R=$\{(a,b):a \le b^2\}$ is neither reflexive nor symmetric nor transitive.
2. Show that the relation S in the set R of real numbers, defined as S = $\{(a,b) : a,b \in R \text{ and } a \le b^3\}$ is neither reflexive, nor symmetric, nor transitive.
3. Let f : X→ Y be a function. Define a relation R in X given by R = {(a, b): f(a) = f(b)}. Examine if R is an equivalence relation.
4. Let Z be the set of all integers and R be the relation on Z defined as R={(a,b):a,b ϵ Z, and (a-b) is divisible by 5}. Prove that R is an equivalence relation.
5. Let A=R-{3} and B=R-{1}. Consider the function *f*: A→B defined by *f*(x) =$\frac{x-2}{x-3}$. Show that *f* is one-one and onto.

6. State whether $f: N \to N$ where N is set of natural numbers defined by

$$f(n) = \begin{cases} \dfrac{n+1}{2} & if \;\; n \; is \; odd \\ \dfrac{n}{2} & if \;\; n \; is \; even \end{cases} \qquad \text{for all } n \in N \quad \text{is bijective. Justify.}$$

7. Check whether a function f from the set of natural no. to set of integers defined by

$$f(n) = \begin{cases} \dfrac{n-1}{2} & if \;\; n \; is \; odd \\ \dfrac{-n}{2} & if \;\; n \; is \; even \end{cases} \qquad \text{is bijective or not? Justify.}$$

8. Let A and B be two sets. Show that $f: AXB \to BXA$ such that *f(a, b)=(b, a)* is bijective function.
9. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.
. (i) f: **R** →**R** defined by f (x) = 3 – 4x (ii) f : **R** →**R** defined by f (x) = 1 + x^2.

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

1. If R_1 and R_2 are two equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.
2. Show that the relation R in the set A of points in a plane given by
   R = {(P, Q): distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation.
   Further, show that the set of all points related to a point P ≠ (0,0) is the circle passing through P with origin as Centre.
3. Show that the relation R on the set A = {x ϵ Z : 0 ≤ x ≤ 12}, given by

   R = {(a,b): |a-b| is a multiple of 4} is an equivalence relation.

   Find the set of all elements related to 1 i. e. Find equivalence class [1].

4. Let R be the relation in $N \times N$ *defined by* $(a, b)\ R(c, d)$. If *a+d=b+c for (a,b), (c,d) in* $N \times N$. Prove that R is an equivalence relation.
5. Let N be the set of all natural number and let R be a relation on N×N defined by *(a,b)R(c,d)* ⇔*ad=bc for all (a,b),(c,d) ϵ N×N.* Show that R is an equivalence relation on N×N.
6. Let N denote the set of all natural numbers and R be the relations on N×N defined *by (a,b)R(c,d)* ⇔*ad(b+c)= bc(a+d).* Check whether R is an equivalence relation on N×N.
7. Show that $f: N \to N$ is given by $f(x) = \begin{cases} x+1, & if \;\; x \; is \; odd \\ x-1, & if \;\; x \; is \; even \end{cases}$ is both one-one and onto.

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

CASE STUDY 1

In two different societies, there are some school going students – including girls as well as boys. Satish forms two sets with these students, as his college project.

Society I   Society II

Let A = {a_1, a_2, a_3, a_4, a_5} and B = {b_1, b_2, b_3, b_4} where a_i's, b_i's are the school going students of first and second society respectively. Using the information given above, answer the following questions:

(i) Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible?

(ii) Let $R: A \rightarrow A$, $R = \{(x, y): x \text{ and } y$ are $\text{students of same sex}\}$. Then, Check if R is symmetric.

(iii) Satish and his friend Rajat are interested to know the number of symmetric relations. defined on both the sets A and B separately. Satish decides to find the symmetric relation on set A, while Rajat decides to find symmetric relation on set B. What is difference between their results?

OR

To help Satish in his project, Rajat decides to form bijective functions from set A to itself. How many such functions are possible?

CASE STUDY 2

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever. Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows: R = $\{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019$\}$

(i) Two neighbors X and Y$\in$ I. X exercised his voting right while Y did not cast her vote in general election – 2019. Is XRY?

(ii) Mr.'X' and his wife 'W'both exercised their voting right in general election -2019, Is it true that XRW and WRX?

(iii)Three friends F_1, F_2 and F_3 exercised their voting right in general election-2019, then Is it true that F_1RF_2 and $F_2R F_3$ implies F_1RF_3? Give reason.

OR

Mr. Shyam exercised his voting right in general election 2019, then find the equivalence class of Mr Shyam.

CASE STUDY 3

Priya(P) and Surya(S) are playing monopoly in their house during COVID. While rolling the dice their mother Chandrika noted the possible outcomes of the throw every time belongs to the set {1, 2, 3, 4, 5, 6}. Let A denote the set of players and B be the set of all possible outcomes. Then $A = \{P, S\}$ $\quad B = \{1, 2, 3, 4, 5, 6\}$.

Then answer the below questions based on the given information:

(i) Let $R: B \rightarrow B$ be defined by $R = \{(a, b)\ both\ a\ and\ b\ are\ either\ odd\ or\ even\}$. Is R an equivalence relation? Justify.

(ii) Chandrika wants to know the number of **functions** for A to B. How many numbers of **functions** are possible?

(iii) Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. State if R is reflexive symmetric or transitive?

OR

Chandrika wants to know the number of **relations** for A to B. How many numbers of **relations** are possible?

Answer key/Hints

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1	A	**2**	C	**3**	D	**4**	D	**5**	A
6	D	**7**	B	**8**	B	**9**	A	**10**	C
11	B	**12**	D						

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

1	A	**2**	D	**3**	D	**4**	A	**5**	A

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. R = {(3,8),(6,6),(9,4),(12,2)}
2. Given R = {(1,1),(2,2),(3,3),(1,2),(2,3)} defined on R : {1,2,3}→{1,2,3}
   For reflexive: As (1,1), (2,2), (3,3) ϵ R. Hence, reflexive
   For symmetric: (1,2) ϵ R but (2,1) ∉R. Hence, not symmetric.
   For transitive: (1,2) ϵ R and (2,3) ϵ R but (1,3) ∉ R. Hence, not transitive.
3. Easy to prove. Do yourself.
4. Try yourself
5. Let a,b ϵA then a-b < 30 kg always true for students of a particular class, i.e. aRb, for all a,b ϵ A. Hence, universal reaction.
6. Do yourself.
7. See NCERT.
8. Given function is $f(x) = \begin{cases} 1, if\ x > 0 \\ 0, if\ x = 0 \\ -1, if\ x < 0 \end{cases}$

let $x_1 = 2$ and $x_2 = 3$ then $f(x_1) = 1$ and $f(x_2) = 1$

i.e., $x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$.

So, function is not one-one.

Also, Range of function is {-1,0,1} and co-domain is set of real numbers R.
=> Range ⊆ co-domain.
Hence, function is not onto.

9. Given f : R→R, given by f(x) = │x│ Take x= 2 and -2
then f(2)=2 and f(-2)=2 that is image of two distinct elements is same. therefore R is not 1-1.
Also –ve elements in codomain have no pre image in the domain. Therefore R is not onto.
10. Given f : R→R, given by f(x) = [x] Take x= 1 and 1.5
then f(1)=1 and f(1.5)=1 that is image of two distinct elements of the domain is same. therefore R is not one-one.
Also, Range of function is set of integers and co-domain is set of real numbers R.
=> Range ⊆ co-domain. Therefore R is not onto.

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1. **For reflexive**: Let a=$\frac{1}{2}$,
(a,a) ϵ R =>½ ≤$(\frac{1}{2})^2$ =>$\frac{1}{2} \le \frac{1}{4}$, false, Hence, not reflexive.
For symmetric: Let (-1,2) ϵ R as -1 ≤ $(2)^2$,true.
Now (2,-1) ϵ R => 2 ≤ $(-1)^2$=> 2≤1, false,
As (-1,2) ϵ R ≠> (2,-1) ϵ R, Hence, not symmetric.
For transitive: Let (6,3),(3,2) ϵ R
(6,3) ϵ R => 6 ≤ $(3)^2$=> 6≤ 9, true
(3,2) ϵ R => 3≤$(2)^2$ => 3≤ 4, true
We have to show, (6,2) ϵ R
=>6 ≤ $(2)^2$ => 6 ≤ 4, false. So, not transitive.
2. *Given S = {(a,b) ϵ R │ a ≤ b^3}*
We can consider counter example.
For reflexive: Let (-2.2) ϵ S => -2≤ $(-2)^3$ => -2 ≤ -8, false, Hence, not reflexive.
For symmetric: Let (-1,2) ϵ S => -1 ≤ $(2)^3$ => -1 ≤ 8, true,
If symmetric then (2,-1) ϵ S

=>2 ≤ $(-1)^3$ => 2 ≤ -1, false, Hence, not symmetric.

For transitive: Let (25,3) ϵ S and (3,2) ϵ S
=> 25 ≤ $(3)^3$ and 3 ≤ $(2)^3$ => 25 ≤ 27 and 3 ≤ 8, true in both cases.
If transitive then (25,2) ϵ S => 25 ≤ $(2)^3$ => 25 ≤ 8, false,
Hence, not transitive.
3. Check the relation is reflexive, symmetric and transitive therefore it is an equivalence relation.
4. ***Try reflexivity & symmetricity***
For transitive*: Let (a,b) ϵ R and (b,c) ϵ R, for a,b,c ϵ Z*
=> (a-b) is divisible by 5 and (b-c) is divisible by 5
=> (a-b)+(b-c)=a-c is divisible by 5
=> (a,c) ϵ R
As (a,b) ϵ R and (b,c) ϵ R => (a,c) ϵ R
Hence, R is transitive.
Since R is reflexive, symmetric , transitive
=>R is equivalence relation.

5. *Given, A=R-{3},B=R-{1} and f(x)=* $\frac{x-2}{x-3}$.
For one-one: *Let for* $x_1, x_2 \in A$,
$f(x_1)=f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$
$\Rightarrow x_1x_2-2x_2-3x_1+6 = x_1x_2-3x_2-2x_1+6$
$\Rightarrow x_2 = x_1$
As $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Hence, function is one-one.*
For onto: Let for y ϵ B, there exists x ϵ A such that y = *f*(x) => y = $\frac{x-2}{x-3}$

$\Rightarrow xy-3y = x-2 \quad \Rightarrow xy-x = 3y-2$

$\Rightarrow x(y-1) = 3y-2 \Rightarrow x= \frac{3y-2}{y-1} \in A.$ Hence, onto.

6. *f:N→N is defined as for all* $n \in N$

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

It can be observed that:

$f(1)=\frac{1+1}{2} =1$ *and* $f(2)=2/2=1$

$\therefore f(1)=f(2)$

$\therefore$ *f is not one-one.*

Consider a natural number (n) in co-domain N

Case I: n is odd

$\therefore n=2r+1$ *for some* $r \in N$.

Then, there exists $4r+1 \in N$ *such that* $f(4r+1)= \frac{4r+1+1}{2} =2r+1$

Case II: n is even

$\therefore n=2r$ *for some* $r \in N$.

Then, there exists $4r \in N$ *such that* $f(4r)=4r/2/2=2r$

$\therefore$ *f is onto.*

Hence, f is not a bijective function

7. *Try Yourself.*

8. *Try yourself*

11. *(i) f: R→ R defined by f(x)=3−4x.*

For f(x)=f(y) , prove that x = y so as f is 1-1

For any real number y in R, there is $\frac{3-y}{4}$ in R such that

$f(\frac{3-y}{4})=3-4(\frac{3-y}{4})=y$ $\quad \therefore$ f is onto.

Hence, f is bijective.

(ii) Try yourself

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

1. *We have* $R_1 \cap R_2 = \{(a,b) \mid (a,b) \in R_1 \text{ and } (a,b) \in R_2\}$

For reflexive: *Let* $a \in A$, *then* $(a,a) \in R_1$ *and* $(a,a) \in R_2 \Rightarrow (a,a) \in R_1 \cap R_2$.

Hence, reflexive.

For symmetric: *Let* $(a,b) \in R_1 \cap R_2$

$\Rightarrow (a,b) \in R_1$ *and* $(a,b) \in R_2$

$\Rightarrow (b,a) \in R_1$ *and* $(b,a) \in R_2$

(as R_1 *and* R_2 *are equivalence relations)*

$\Rightarrow (b,a) \in R_1 \cap R_2$. *Hence, symmetric.*

For transitive: *Let* $(a,b),(b,c) \in R_1 \cap R_2$

$\Rightarrow (a,b),(b,c) \in R_1$ *and* $(a,b),(b,c) \in R_2$

$\Rightarrow (a,c) \in R_1$ *and* $(a,c) \in R_2$

(Because R_1 *and* R_2 *are equivalence relations)*

$\Rightarrow (a,c) \in R_1 \cap R_2$.

As (a,b), (b,c)ϵ $R_1 \cap R_2$ => (a,c)ϵ $R_1 \cap R_2$. So, transitive. Since relation $R_1 \cap R_2$ is reflexive, symmetric and transitive. Hence, $R_1 \cap R_2$ is an equivalence relation.

Consider set A as set of points in a plane, relation R = {(P,Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}

2. *Try reflexivity & transitivity*

For transitive: *Let(P,Q), (Q,T) ϵ R, for P,Q,T ϵ A*

=> The distance of the point P from the origin is same as distance of the point Q from the origin and also, the distance of the point Q and T from the origin is the same.

=>The distance of the point P from the origin is same as distance of the point T from the origin.

=> (P,T) ϵ R

As (P,Q) ϵ R, (Q,T) ϵ R => (P,T) ϵ R

Therefore, R is transitive.

Therefore, R is an equivalence relation.

Let, if O(0,0) is the origin and OP = r, then the set of all points related to P ≠ (0,0) are at distance 'r' f rom the origin. The set of all points related to P ≠ (0,0) will be those points whose distance from the origin is the same as the distance of point P from the origin.

Hence, these set of points from a circle with the centre as the origin and this circle passes through point P.

3. *We have, R= {(a,b) : $|a-b|$ is a multiple of 4}, Where a,b ϵ A = {x ϵ Z : $0 \le x \le 12$}*

We observe the following properties of relation R:

Reflexivity: *For any a ϵ A, we have $|a-a| = 0$, which is a multiple of 4.*

=> (a,a) ϵ R Thus, (a,a) ϵ R for all a ϵ A.

So, R is reflexive.

Symmetry: *Let (a,b) ϵ R. Then, (a,b)ϵ R*

=> $|a-b|$ is a multiple of 4

=> $|a-b| = 4\lambda$ for some $\lambda \in N$

=> $|b-a| = 4\lambda$ for some $\lambda \in N$

=> (b,a) ϵ R

So, R is symmetric.

Transitivity: *Let (a,b) ϵ R and (b,c) ϵ R. Then, (a,b) ϵ R and (b,c) ϵ R*

=> $|a-b|$ is a multiple of 4 and $|b-c|$ is a multiple of 4

=> $|a-b| = 4\lambda$ and $|b-c| = 4\mu$ for some $\lambda,\mu \in N$

=> $a-b = \pm 4\lambda$ and $b-c = \pm 4\mu$

=> $a-c = \pm 4\lambda \pm 4\mu$

=>a-c is a multiple of 4

=> $|a-c|$ is a multiple of 4

=> (a,c) ϵ R

Thus, (a,b) ϵ R and (b,c)ϵ R => (a,c)ϵ R

So, R is transitive.

Let x be an element of A such that (x,1) ϵ R. Then, $|x-1|$ is a multiple of 4

=> $|x-1| = 0,4,8,12$

=> x-1 = 0,4,8,12

=> x = 1,5,9 (Because 13 ∉A)

Hence, the set of all elements of A which are related to 1 is {1,5,9}.

4. **For Reflexive**

(a, b) R (a, b) ⇒ a + b = b + a which is true since addition is commutative on N.

⇒ R is reflexive.

For Symmetric

Let (a, b) R (c, d) ⇒ a + d = b + c ⇒ b + c = a + d ⇒ c + b = d + ⇒ (c, d) R (a, b)

⇒ R is symmetric.

For Transitive

for (a, b), (c, d),(e, f) in N × N

Let (a, b) R (c, d) and (c, d) R (e, f)
⇒ a + d = b + c and c + f = d + e
⇒ (a + d) – (d + e) = (b + c) – (c + f)
⇒ a - e = b - f ⇒ a + f = b + e
⇒ (a, b) R (e, f) ⇒ R is transitive.

Hence, R is an equivalence relation.

5. Do yourself
6. ***Try Reflexivity & Symmetricity***

Transitivity : *Let (a,b),(c,d),(e,f) ϵ N×N such that (a,b)R(c,d) and (c,d)R(e,f). then,*

(a,b)R(c,d) => ad(b+c) = bc(a+d) => $\frac{b+c}{bc} = \frac{a+d}{ad}$ => $\frac{1}{b}+\frac{1}{c} = \frac{1}{a}+\frac{1}{d}$ …(i)

And, *(c,d)R(e,f) => cf(d+e) = de(c+f) =>* $\frac{d+e}{de} = \frac{c+f}{cf}$ => $\frac{1}{d}+\frac{1}{e} = \frac{1}{c}+\frac{1}{f}$ …(ii)

Adding (i) and (ii), we get

$$\left(\frac{1}{b}+\frac{1}{c}\right)+\left(\frac{1}{d}+\frac{1}{e}\right) = \left(\frac{1}{a}+\frac{1}{d}\right)+\left(\frac{1}{c}+\frac{1}{f}\right)$$

$$\frac{1}{b}+\frac{1}{e} = \frac{1}{a}+\frac{1}{f} => \frac{b+e}{be} = \frac{a+f}{af}$$

af(b+e) = be(a+f) => (a,b)R(e,f)

Thus, (a,b)R(c,d) and (c,d)R(e,f) => (a,b)R(e,f) for all (a,b), (c,d), (e,f) ϵ N×N.

So, R is transitive on N×N.

Hence, R being reflexive, symmetric and transitive, is an equivalence relation on N×N.

7. *Given function f(x) =* $\begin{cases} x+1, if\ x\ is\ odd. \\ x-1, if\ x\ is\ even. \end{cases}$

Refer to Example 12 of Ch - 1

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

CS-1	i) 2^{20}	ii) Yes, Symmetric	iii) $2^{10}(31)$ OR 120
CS-2	i) $(X, Y) \notin R$	ii) Both are true.	iii) Yes (add reason)
CS-3	i) Yes (Justify)	ii) 36	iii) None OR 2^{12}

Ch – 2 Inverse Trigonometric Functions

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

*47. The following table gives the inverse trigonometric function (principal value branches) along with their **domains** and **ranges:***

Functions	***Domain***	***Range (Principal Value Branches)***
$y=\sin^{-1}x$	$[-1,\ 1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$y=\cos^{-1}x$	$[-1,\ 1]$	$[0,\ \pi]$
$y=\tan^{-1}x$	R	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
$y=\cot^{-1}x$	R	$(0,\ \pi)$
$y=\operatorname{cosec}^{-1}x$	$R-(-1,\ 1)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$
$y=\sec^{-1}x$	$R-(-1,\ 1)$	$[0,\ \pi]-\left\{\frac{\pi}{2}\right\}$

48.

$\sin^{-1}\frac{1}{x}=\operatorname{cosec}^{-1}x,\ x\in R-(-1,\ 1)$	$\sin^{-1}(-x)=-\sin^{-1}x,\ x\in[-1,\ 1]$
$\cos^{-1}\frac{1}{x}=\sec^{-1}x,\ x\in R-(-1,\ 1)$	$\tan^{-1}(-x)=-\tan^{-1}x,\ x\in R$
$\tan^{-1}\frac{1}{x}=\cot^{-1}x,\ x>0$	$\operatorname{cosec}^{-1}(-x)=-\operatorname{cosec}^{-1}x,\ \lvert x\rvert\geq 1$
$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2},\ x\in[-1,1]$	$\cos^{-1}(-x)=\pi-\cos^{-1}x,\ x\in[-1,\ 1]$
$\operatorname{cosec}^{-1}x+\sec^{-1}x=\frac{\pi}{2},\ \lvert x\rvert\geq 1$	$\sec^{-1}(-x)=\pi-\sec^{-1}x,\ \lvert x\rvert\geq 1$
$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2},\ x\in R$	$\cot^{-1}(-x)=\pi-\cot^{-1}x,\ x\in R$

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1.The Principal Value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

2. The Principal value of $\tan^{-1}(-1)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $\frac{-\pi}{4}$

3. If $\tan^{-1}\frac{3}{4}$ = x then Sin x is

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{3}{7}$

4. If $\sin^{-1}\frac{1}{2}$ = x then tan x is

(A) $\frac{1}{3}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{2}$ (D) $\frac{1}{5}$

5. If $\tan^{-1}x = \sin^{-1}\frac{1}{2}$ find the value of x is

(A) $\sqrt{3}$ (B) 1 (C) $\frac{1}{\sqrt{3}}$ (D) $n.d$

6. If $\cos^{-1}(\frac{1}{\sqrt{2}}) = \sin^{-1}x$ Find the value of x

(A) $\frac{1}{2}$ (B)1 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{2}}$

7. Evaluate $\sin[\frac{\pi}{6} - \sin^{-1}(\frac{-\sqrt{3}}{2})]$

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{-\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{2}$

8. Evaluate $\cos[\frac{\pi}{2} - \sin^{-1}(\frac{1}{2})]$

A) 1 (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) 0

9. The value of $\tan^{-1}1 + \cos^{-1}(\frac{-1}{2})$

(A) $\frac{11\pi}{12}$ (B) $\frac{\pi}{12}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{6}$

10. The value of $\mathrm{cosec}^{-1}(-1) + \cot^{-1}(\frac{-1}{\sqrt{3}})$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{5\pi}{12}$ (D) $\frac{\pi}{12}$

11. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to

(A) π (B) $\frac{-\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

12. The value of $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

(A) π (B) $\frac{-\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{3\pi}{4}$

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

Each of the following questions contains statement -1(Assertion) and statement - 2(Reason) and has following four choices (a), (b), (c), (d), only one of which is correct answer. Mark the correct choice

(a) Statement 1 is true; Statement 2 is true and 2 is correct explanation of 1

(b) Statement 1 is true; Statement 2 is true and 2 is not correct explanation of 1

(c) Statement 1 is true, Statement 2 is false

(d) Statement 1 is false, Statement 2 is true

1. **ASSERTION:** $\tan^{-1}x < \cot^{-1}x$ for $x<1$

   REASON: $\tan^{-1}x$ is an increasing function on R

2. **ASSERTION**: $\cos^{-1}x \geq \sin^{-1}x$ for all $x \in [-1,1]$

   REASON: $\cos^{-1}x$ is decreasing function on $[-1,1]$

3. **ASSERTION**: $\sin^{-1}x$ is an increasing function on $[-1,1]$ with least value $\frac{-\pi}{2}$ and greatest value $\frac{\pi}{2}$

   REASON: $\cos^{-1}x$ is decreasing function on $[-1,1]$ with the greatest value π and least value 0 respectively.

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. Draw the graph of the principal value branch of the function $f(x)= \cos^{-1}x$

2. Draw the graph of $f(x) = \sin^{-1}x$, $x \in [-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$ also write range of f(x)

3. Find the value of $\sin^{-1}(\sin\frac{3\pi}{4}) + \cos^{-1}(\cos\frac{3\pi}{4}) + \tan^{-1}1$

4. Evaluate: $\sin^{-1}(\sin\frac{3\pi}{4}) + \cos^{-1}(\cos\pi) + \tan^{-1}\frac{1}{\sqrt{3}}$

5. Find the value of $\sin^{-1}(\cos\frac{33\pi}{5})$

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1. Find the domain of $\cos^{-1}(2x-1)$

2. Compute the domain of $\sin^{-1}(x^2-4)$

3. Prove that $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right) = \frac{\pi}{4} - \frac{x}{2}$

   OR $\tan^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{x}{2}\cos^{-1}x$, $-\frac{1}{\sqrt{2}} \leq x \leq 1$

4. Write the following functions in the simplest form: $\tan^{-1}\left(\frac{x}{\sqrt{(a^2-x^2)}}\right)$

OR $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$

5.Write the following functions in simplest form $\tan^{-1}\left(\frac{\sqrt{a-x}}{\sqrt{a+x}}\right)$

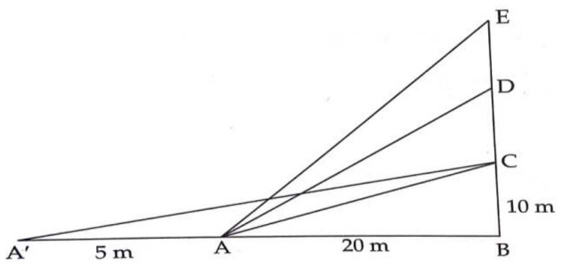
6.Prove that $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

7.Find the value of $\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}+\cos\left(\frac{1-y^2}{1+y^2}\right)\right)\right], |x|<1, y>0 \text{ and } xy<1$ $\left[Ans:\frac{x+y}{1-xy}\right]$

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

CASE STUDY 1

1. The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project.`A ` is considered to be a person viewing the hoarding board 20m away from building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at the three different locations namely C, D and E.`C` is at the height of 10m from ground level. For the viewer A, the angle of elevation of D is the double the angle of elevation C. The angle of elevation of E is triple the angle of elevation of C for the same viewer.



Look at the figure given above and based on the above information,answer the following questions:

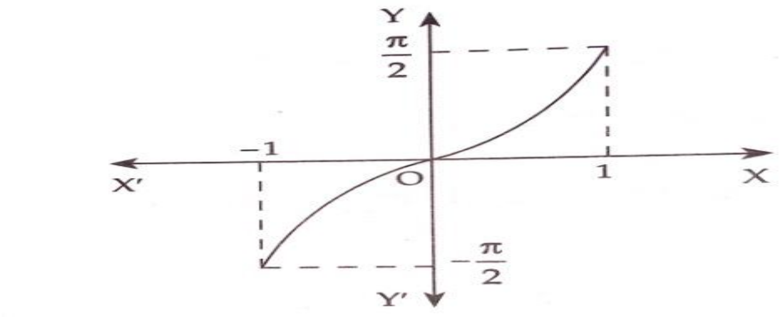
(i) Find the measure of angle CAB

(ii) Write the measure of angle DAB

(iii) What is the value of angle EAB OR

A` is another viewer standing on the same line of observation across the road. If the width of the road is 5 metres, then find the difference between angle CAB and CA`B`

2. Ram and Mohan are students of Class XII. One day, their mathematics teacher told them about the inverse trigonometric functions. Teacher sketched the graph of the function y= $\sin^{-1}x$ on the board as follows:



Based on the above information, answer the following questions:

(i) Write domain of the following function $\text{Sin}^{-1}x$

(ii) Write the range of the principal value branch of the function $\text{Sin}^{-1}x$ OR

(ii) write the range of one branch of the function $\text{Sin}^{-1}x$,other than principal value branch.

(iii) find the domain $\text{Sin}^{-1}(1-x)$.

Answer key/Hints

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1.B 2.D 3.B 4.B 5.C 6.D 7.A 8.C 9.A 10.A 11. B 12. D

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

1.D 2.A 3.D

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

3. $\frac{5\pi}{4}$ 4. $\frac{17\pi}{12}$ 5. $\frac{-\pi}{10}$

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1. [0,1] 2. X ε [√3,√5] 3. Put $\sqrt{1+Cosx}$=√{2(Cos^2x)} and $(1-Cosx)$=2 Sin^2x

4. Put x= a SinΘ 5. X= aCos2Θ

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

1.(i) $\text{Tan}^{-1}(\frac{1}{2})$ (ii) 2 $\text{Tan}^{-1}(\frac{1}{2})$ (iii) 3 $\text{Tan}^{-1}(\frac{1}{2})$ or CA`B= $\text{Tan}^{-1}(\frac{2}{5})$

1. Difference angle (CAB- CA`B) = $\text{Tan}^{-1}(\frac{1}{2})$- $\text{Tan}^{-1}(\frac{2}{5})$

(i) [-1,1] (ii) $[\frac{-\pi}{2},\frac{\pi}{2}]$ or $[\frac{\pi}{2},\frac{3\pi}{2}]$ (iii) X ε [0,2]

Ch – 3 Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

Matrix

A set of m×n numbers (weather real or complex) or functions arranged in the form of a rectangular array of m horizontal lines (called rows) and n vertical lines (called columns) is called a m × n matrix. A matrix is generally denoted by capital letter of English alphabet and an element of a matrix is denoted by a small letter. A matrix with m rows and n columns is called a matrix of order m by n.

Remark : (i) A matrix is an arrangement of numbers or functions and not a value.

(ii) Order of a matrix is written as m × n (read as m by n).

(ii) A general matrix of order m by n is written as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

i = 1, 2, 3, ……….., m and j = 1, 2, 3, ……….., n

a_{ij} is called the (i, j)th element of the matrix A.

Types of matrices:

1. Row matrix: A matrix having only one row is called a row matrix.

e.g. A = [1 3 5] is a row matrix of order 1× 3.

2. Column matrix: A matrix having only one column is called a column matrix. e.g. A = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a column matrix of order 3 × 1.

3. Zero or Null matrix : A matrix is said to be a zero matrix if all its elements are zero.

i.e. A = [a_{ij}] is null matrix if a_{ij} = 0 for all i,j.

Example: $O = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

4. Square matrix: A matrix $A = [a_{ij}]_{mxn}$ is said to be a square matrix if the no. of rows and columns in the matrix are same.

Example: $A = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$, $A = \begin{bmatrix} 2 & 0 & 5 \\ 0 & 5 & 6 \\ 4 & 0 & 8 \end{bmatrix}$

5. Diagonal elements of a square matrix : The elements a_{ij} are called diagonal elements of a square matrix $A = [a_{ij}]_{mxn}$ if $i = j$. i.e. $a_{11}, a_{22}, a_{33}, \ldots\ldots\ldots, a_{nn}$ are called the diagonal elements of a square matrix. The line containing the elements $a_{11}, a_{22}, a_{33}, \ldots\ldots\ldots, a_{nn}$ is called the principal diagonal.

6. Diagonal matrix: A square matrix is said to be a diagonal matrix if its non- diagonal elements are zeros.

e.g. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ is a diagonal matrix.

7. Scalar Matrix:

A square matrix is said to be a scalar matrix if its non- diagonal elements are zeros and all diagonal elements are equal.

As for example : $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

8.Unit matrix : A diagonal matrix is called a unit matrix if all the diagonal elements are unity.

As for example : $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

9.Equal Matrices: Two matrices $A = [a_{ij}]_{mx\,n}$ and $B = [b_{ij}]_{mx\,n}$ of the same order are equal if $a_{ij} = b_{ij}$ for all i, j

10. Upper Triangular matrix: A square matrix $A = [a_{ij}]_{nxn}$ is called an upper triangular matrix iff $a_{ij} = 0$ for all $i > j$ i.e. all elements below the principal diagonal are zero.

As for example : $A = \begin{bmatrix} 2 & 3 & 6 \\ 0 & 2 & 7 \\ 0 & 0 & 2 \end{bmatrix}$

11.Lower Triangular matrix: A square matrix $A = [a_{ij}]_{nxn}$ is called an lower triangular matrix iff $a_{ij} = 0$ for all $i < j$ i.e. all elements above the principal diagonal are zero.

As for example : $A = \begin{bmatrix} 2 & 0 & 0 \\ 4 & 2 & 0 \\ 1 & 3 & 2 \end{bmatrix}$

12.Triangular matrix: A square matrix $A = [a_{ij}]_{nxn}$ is called a triangular matrix if it is either upper triangular or lower triangular.

Addition of matrices

Let A and B be two matrices of the same order m $\times n$. Then their sum denoted by A + B is also a matrix of the same order m $\times n$ and is obtained by adding the corresponding elements of A and B.

If A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{m \times n}$ then A + B = $[a_{ij} + b_{ij}]_{m \times n}$

As for example :

If A =$\begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 5 & 1 \\ 9 & 0 \end{bmatrix}$, then A + B = $\begin{bmatrix} 13 & 8 \\ 15 & 2 \end{bmatrix}$,

Properties of matrix addition

If A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{m \times n}$ are of the same order then

(i) Matrix addition is commutative i.e. A + B = B + A
(ii) Matrix addition is associative i.e. (A + B) + C = A + (B + C)
(iii) Existence of additive Identity : If A is any matrix of order m $\times n$ then there exists a null matrix O of order m $\times n$ such that A + O = O + A = A
(iv) Existence of additive Inverse : A + (-A) = (-A) + A = O
(v) Cancellation Laws: A + B = A + C ⇒B =C and B + A = C + A ⇒ B =C

Properties of scalar multiplication

If A and B are two matrices of the same order and k, l are scalars then

(i) k(A + B) = kA + k B
(ii) (-k)A = -(k A) = k (-A)
(iii) I A = A
(iv) (-1) A = - A

Difference of two matrices

Let A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{m \times n}$ be two matrices of the same order m $\times n$. Then their difference denoted by A $-$ B is also a matrix of the same order m $\times n$ and is defined as

$$A - B = [a_{ij} - b_{ij}]_{m \times n}$$

Multiplication of Matrices

Two matrices A and B are said to be conformal for the product if the no. of columns in A is equal to the no. of rows in B.

Let A = $[a_{ij}]_{m \times n}$ and B = $[b_{jk}]_{m \times n}$ be two matrices then the product of A and B is a matrix C = $[c_{ik}]_{m \times p}$ where

$$c_{ik} = a_{i1}b_{1k} + a_{i2}b_{2k} + \ldots\ldots + a_{in}b_{nk} = \sum_{j=1}^{n} a_{ij}b_{jk}$$

Properties of matrix multiplication

(i) Matrix multiplication is not commutative i.e. AB ≠ BA
(ii) Matrix multiplication is associative i.e. (AB)C = A(BC)
(iii) Matrix multiplication is distributive over matrix addition i.e. A(B+C) = AB + AC

(iv) If A is an $m \times n$ matrix and I is the identity matrix of order $n \times n$ then I A = A = A I
(v) If A is an $m \times n$ matrix and O is the null matrix of order $n \times p$ then A O = O
(vi) In a matrix multiplication the product of two non-zero matrices may be a zero matrix.

Transpose of a matrix The transpose of a matrix A is denoted by A^T or A' which is obtained by interchanging its rows and columns.

Properties of Transpose of a Matrix

(i) $(A^T)^T = A$ (iii) $(A+B)^T = A^T + B^T$
(ii) $(KA)^T = K A^T$ (iv) $(AB)^T = B^T A^T$
(v) $(ABC)^T = C^T B^T A^T$

Symmetric matrix A square matrix A = $[a_{ij}]_{nxn}$ is said to be a symmetric matrix if transpose of A is equal to matrix A. i.e. if A^T = A.

For example: Let A= $\begin{bmatrix} 2 & 5 \\ 5 & 3 \end{bmatrix}$, then $\begin{bmatrix} 2 & 5 \\ 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 5 \\ 5 & 3 \end{bmatrix}$

∴ A is said is symmetric matrix.

Skew-symmetric matrix A square matrix A = $[a_{ij}]_{nxn}$ is said to be a skew symmetric matrix if $A^T = -$A.

For example:

Let A = $\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$, then $A^T = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$ = -A

∴ A is skew-symmetric matrix.

Note:

(1) All main diagonal elements of a skew symmetric matrix are zero.
(2) Every square matrix can be uniquely expressed as a sum of a symmetric and a skew-symmetric matrix.
(3) All positive integral powers of a symmetric matrix are symmetric.
(4) odd positive integral powers of a skew-symmetric matrix are skew-symmetric.

Invertible matrices

Let A be a square matrix of order n, If then there exist a square matrix B such that AB = I_n = BA, then B is called the inverse of A and is denoted by A^{-1}.

∴ $AA^{-1} = I = A^{-1}A$

If A and B are invertible square matrices of the same order then AB is also invertible and $(AB)^{-1} = B^{-1}A^{-1}$

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. A = $[a_{ij}]_{mxn}$ is a square matrix, if

a. m < n b. m > n c. m = n d. none of these

2. If A = $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and nϵN, then A^n is equal to

a. n A b. 2n A c. 2^{n-1} A d. 2^n A

3. The number of all possible matrices of order 3x3 with each entery 0 or 1 is:

a. 27 b. 18 c. 81 d. 512

4. If for a square matrix A , $A^2 - A + I = O$, then A^{-1} equals

a. A b. A + I c. I – A d. A – I

5. If A = $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, B = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and A = B^2 , then x equals

a. -1 b. 1 c. 2 d. -2

6. $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$ what is the value of x & y

a. 2, 9 b. -2, 9 c. 2, -9 d. -2, -9

7. If A is a square matrix and $A^2 = A$, then $(I + A)^2 - 3A$ is equal to:

a. I b. A c. 2A d. 3I

8. If A is a square matrix such that $A^2 = A$, then find $(2 + A)^3 - 19$ A.

a. 8I b. 2I c. I d. A

9. The numbers of all possible matrices of order 2x2 with each entry 1,2 or 3 is

a. 12 b. 64 c. 81 d. 7

10. The numbers of all possible matrices of order 2x3 with each enter 1 or 2 is

a. 12 b. 64 c. 36 d. 8

11. If A and B are square matrices of same order , then $AB' - BA'$ is a

a. skew – symmetric matrix b. symmetric matrix

c. null matrix d. unit matrix

12. If the matrix A is both symmetric and skew symmetric matrix, then

a. A is a diagonal matrix b. A is a zero matrix

c. A is a square matrix d. none of these

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). choose the correct answer out of the following choices.

a. Both A and R are true and R is the correct explanation of A.
b. Both A and R are true and R is not the correct explanation of A.
c. A is true but R is false.
d. A is false but R is true.

13. **ASSERTION**: $\begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{bmatrix}$ is a scalar matrix.

REASON: All the elements of the principal diagonal are equal, it is called a scalar matrix.

14. **ASSERTION**: $\begin{bmatrix} 0 & h & -g \\ -h & 0 & f \\ g & -f & 0 \end{bmatrix}$ is skew- symmetric matrix.

REASON: Every square matrix A can be expressed as sum of a symmetric and skew-symmetric matrix, $A = ½ (A + A^T) + ½(A – A^T)$.

15. **ASSERTION**: If A is a square matrix such that $A^2 = I$, then $(I + A)^2 – 3A = I$.

REASON: AI = IA = A , where I is the identity matrix

16. **ASSERTION**: If A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then A^n = $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$, for all n ϵ N.

REASON: If B = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then $(I + B)^n = I + n B$, for all n ϵ N.

17. **ASSERTION**: An identity matrix is a non- singular matrix.

REASON: Assertion is correct as for identity matrix I, modulus of I = 1 ≠ 0.

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

18. Find k if A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ satisfy the relation $A^2 = k A – 2I$.

19. If A = $\begin{bmatrix} sin\alpha & cos\alpha \\ -cos\alpha & sin\alpha \end{bmatrix}$, then verify that A'A = I.

20. If A' = $\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find (A + 2B)'.

21. If $\begin{bmatrix} 3x-2y & 5 \\ x & -2 \end{bmatrix}$ = $\begin{bmatrix} 3 & 5 \\ -3 & -2 \end{bmatrix}$, find the value of y .

22. If $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ = $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, find x,y,z.

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

23. Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

24. If $f(x) = \begin{bmatrix} cosx & -sinx & 0 \\ sinx & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that f(x) f(y) = f(x + y).

25. If $A^T = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, find $(A + 2B)^T$.

26. If $A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$, find A^4.

27. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, find 3A – C.

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

28. A trust fund has Rs. 30,000 that must be invested in two different types of bonds. the first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divides Rs. 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of.
    a. Rs. 1800 b. Rs. 2000

29. If $A = \begin{bmatrix} 0 & -tan\alpha/2 \\ tan\alpha/2 & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A) \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$$

30. Express the following matrix as the sum of a symmetric and a skew symmetric matrix

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

31. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k such that $A^2 = 8A + KI$

32. If $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, find f(A), where $f(x) = x^2 - 5x + 7$

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

33. Ram purchases 3 pens, 2 bags, and 1 instrument box and pays ₹ 41. From the same shop, Dheeraj purchases 2 pens, 1 bag, and 2 instrument boxes and pays ₹29, while Ankur purchases 2 pens, 2 bags, and 2 instrument boxes and pays ₹44.

Read the above information and answer the following questions:

(i) Find the cost of one pen. (1 mark)
(ii) What are the cost of one pen and one bag? (1 mark)
(iii) What is the cost of one pen & one instrument box? (2 marks)

34. Three friends Ravi, Raju and Rohit were buying and selling stationery items in a market. The price of per dozen of Pen, notebooks and toys are Rupees x, y and z respectively. Ravi purchases 4 dozen of notebooks and sells 2 dozen pens and 5 dozen toys. Raju purchases 2 dozen toys and sells 3 dozen pens and 1 dozen of notebooks. Rohit purchases one dozen of pens and sells 3 dozen notebooks and one dozen toys.
In the process, Ravi, Raju and Rohit earn ₹ 1500, ₹ 100 and ₹400 respectively.

(i) What is the price of one dozen pens? (1 mark)
(ii) What is the total price of one dozen pens and one dozen of notebooks? (1 mark)
(iii) What is the sale amount of Ravi? (2 marks)

Answer Key/Hints

<u>MULTIPLE COICE QUESTIONS (1 MARK EACH)</u>

Q.1 –C Q.2-D Q.3-D Q.4-C Q.5-B Q.6-A Q.7-C Q.8-A Q.9-C Q.10-C Q.11-A Q.12-B

<u>ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)</u>

Q.13- (a) Q.14- (b) Q.15-(a) Q.16-(a) Q.17- (b)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

Q.18- K = 1 **Q.20 -** $\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

Q.21 Since corresponding elements of equal matrices are equal.

$\therefore$ x= -3 and 3x - 2 y = 3 $\Rightarrow$ y = - 6

Q.22 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x+0+0 \\ 0-y+0 \\ 0+0+z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\Rightarrow$ x = 1, y = 0, z = 1.

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

Q.23 - $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

Q.25 A + 2B = $\begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

$\therefore$ $(A + 2B)^T = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$.

Q.26 $A^2 = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

$A^4 = A^2 . A^2 = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}$

Q.27 3A-C = $3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

Q.28 Type A – Rs. 15000, Rs. 15000

Type B – Rs. 5000, Rs. 25000

Q.31 $A^2 = A.A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$

$8\,A = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix}$

$K\,I = K\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$

Now $A^2 = 8A + KI \Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0+0 \\ -8+0 & 56+k \end{bmatrix}$

$\Rightarrow$ K = - 7

Q.32 $\because f(x) = x^2 - 5x + 7$

$\therefore f(A) = A^2 - 5A + 7I$

$A^2 = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 9-16 & 12-12 \\ -12+12 & -16+9 \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$

$5A = 5\begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}$

$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$$\therefore f(A) = A^2 - 5A + 7 = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & -20 \\ 20 & 15 \end{bmatrix}$$

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

Q.33 –

(i) ₹ 2 (ii) ₹17 (iii) ₹7

Q.34 -

(i) ₹ 100 (ii) ₹ 300 (iii)₹ 1200

Ch – 4 Determinants

Determinants of a square matrix (up to 3x3 matrices), minors, co-factors, and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency, and number of solutions of system of linear equations by example, solving system of linear equations in two or three variables (having unique solution) using inverse of matrix.

Determinants

Every square matrix can be associated to an expression or a number which is known as its determinant. If A is a square matrix then its determinant is donated by det A or |A|.

Note:

(i) Only square matrices have determinants. The determinant of non-square matrices is not defined.

(ii) A matrix is an arrangement of numbers and hence it has no fixed value while each determinant has a fixed value.

(iii) A determinant having n rows and having n columns is known as a determinant of order n.

Value of a determinant

Determinant of a matrix of order one

Let A = [a_{11}] be a square matrix of order 1 then |A| = $|a_{11}|$ and the value of determinant is the number itself. i.e. $|a_{11}| = a_{11}$

Remark: A determinant of order 1 should not be confused with the absolute value of the number a_{11}.

As for example: (i) If A= $[5]$, then $|5|$ = 5

(ii) If A= $[-2]$, then $|-2|$ = - 2

Determinant of a matrix of order 2

Let A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a square matrix of order 2,

then det A = $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ and its value is $a_{11}.a_{22} - a_{21}.a_{12}$

Note: The numbers $a_{11,}a_{12,}a_{21,}a_{22}$ are called the elements of the determinant.

As for example: (i) If A = $\begin{bmatrix} 3 & 2 \\ 4 & 6 \end{bmatrix}$, then |A| $= 3 \times 6 - 4 \times 2$ =10

(ii) $\begin{vmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{vmatrix}$ = $\cos\theta\,.\cos\theta - \sin\theta\,.(-\sin\theta)$ $= \cos^2\theta + \sin^2\theta$ = 1

Determinant of a matrix of order 3

Let A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a square matrix of order 3,

then det A = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

= $a_{11}(a_{22}.a_{33} - a_{23}.a_{32}) - a_{12}(a_{21}.a_{33} - a_{23}.a_{31}) + a_{13}(a_{21}.a_{32} - a_{22}.a_{31})$

As for example: (i) $\begin{vmatrix} 3 & -2 & 5 \\ 1 & 2 & -1 \\ 0 & 4 & 7 \end{vmatrix}$= $3\begin{vmatrix} 2 & -1 \\ 4 & 7 \end{vmatrix} - (-2)\begin{vmatrix} 1 & -1 \\ 0 & 7 \end{vmatrix} + 5\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix}$ = 3.18+27+5.4 = 88

(ii) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ = 3(0-5) –(- 1)(0+3) +(-2)(0-0) = - 15 + 3 +0 = - 12

IMPORTANT POINTS TO REMEMBER

1. Only square matrices have determinants.
2. In case of matrices, we take out any common factor from each element of matrix, while in the case of determinants we can take out common factor from any one row or any one column of the determinant.
3. If area is given, then we take both positive and negative values of the determinant for calculation.
4. If we want to prove that three points are collinear, we show that the area formed by these three points is equal to zero.
5. A square matrix of order n is invertible if it is non-singular.
6. If A is an invertible matrix of order n, then A^{-1}= $\frac{1}{|A|}$ adj A where $|A| \neq 0$
7. The inverse of an invertible symmetric matrix is symmetric.
8. If a system of equations has one or more solutions, then it is said to be a consistent system of equations otherwise it is in-consistent.
9. A system of linear equations may or may not be consistent.
10. A consistent system may or may not have a unique solution.
11. Every invertible matrix possesses a unique inverse.
12. If A' is transpose of a square matrix A then $|A'|$ = $|A|$

    As for example: If A = $\begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$, then $|A| = 2.4 - 5.3 = 8 - 15 = -7$

    Now A'= $\begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix}$, then $|A'| = 2.4 - 3.5 = 8 - 15 = -7$

13. If A =$[a_{ij}]_{n\times n}$ then | k A | = k^n $|A|$

    As for example:
    If A = $[a_{ij}]_{3\times 3}$, then $|kA|$ = k^3 $|A|$

14. If A is a non-singular matrix of order n, then $|adj\ A|$ = $|A|^{n-1}$
15. If A is a non-singular matrix of order n, then $|A^{-1}|$ = $\frac{1}{|A|}$
16. If A and B are non-singular matrices of same order, then$|AB|$=$|A||B|$ and hence $|A.adjA| = |A|^n$

Illustrations

1. If A is square matrix of order 3 with value of its determinant 4, then $|3A|$ = 3^3 $|A| = 27 \times 4 = 108$
2. If A is a square matrix order 3 such that $|A| = 4$, and $|KA|$= 500, find K.
   Solution: ∵ $|KA|$= 500
   ⇒ $k^3|A| = 500$
   ⇒ $k^3.4 = 500$
   ⇒ $k^3 = 500/4 = 125$
   ⇒ k = 5
3. Let A be a square matrix of order 3x 3, write the value of $|2A|$ where $|A|$ = 4

Solution: $\because |2A| = 2^3 . |A| = 8\times 4 = 32$

4. Given matrix A of order 3x3, find the value of k such that $|2A|$ =K$|A|$

Solution: $\because |2A| = 2^3 . |A| = K|A|$

$\Rightarrow$ K = 8

Area of a triangle

Area of a triangle with vertices (x_1, y_1) , (x_2,y_2) ,(x_3 ,y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Q. Find the value of K, if area of a triangle is 4 square unit when its vertices are (k,0) ,(4,0), (0,2).

(Ans: k=0 or 8)

Adjoint of a matrix

Let A = = $[a_{ij}]_{nxn}$ be a square matrix of order n and let A_{ij} denote the cofactor of a_{ij} in $|A|$. Then the adjoint of A denoted by adj A and is defined by adj A = $[A_{ji}]_{nxn}$. Thus, adj A is the transpose of the matrix of the corresponding cofactors of elements of $|A|$.

As for example: Let A =$\begin{bmatrix} 2 & -3 \\ 4 & 7 \end{bmatrix}$, then

$M_{11} = |7| = 7$; $M_{12} = |4| = 4$; $M_{21} = |-3| = -3$; $M_{22} = |2| = 2$

$A_{11} = (-1)^{1+1} M_{11} = |7| = 7$; $A_{12} = (-1)^{1+2} M_{12} = -|4| = -4$

$A_{21} = (-1)^{2+1} M_{21} = -|-3| = -(-3) = 3$ $A_{22} = (-1)^{2+2} M_{22} = |2| = 2$

adj A = transpose of $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ -4 & 2 \end{bmatrix}$

Q1. Find adjoint of a matrix $\begin{bmatrix} 3 & 1 \\ -5 & 4 \end{bmatrix}$

$A_{11} = (-1)^{1+1} M_{11} = |4| = 4$; $A_{12} = (-1)^{1+2} M_{12} = -|-5| = 5$

$A_{21} = (-1)^{2+1} M_{21} = -|1| = -1$ $A_{22} = (-1)^{2+2} M_{22} = |3| = 3$

$\therefore$ adj A = $\begin{bmatrix} 4 & -1 \\ 5 & 3 \end{bmatrix}$

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. If A is a Singular Matrix then A(adjA) is

(a) Scalar matrix (b) Null matrix (c) Identity matrix (d) None of these

2. The sum of the products of elements of any row with the co-factors of corresponding elements is equal

(a) Adjoint of the matrix (b) 0 (c)1 (d) Value of the determinant

3. If P is a square matrix of order 3,such that P(adjoint P) =10 I , then the determinant of adjoint P is equal to

(a) 0 (b) 1 (c) 10 (d) None of these

4. If A is an invertible square matrix, then

(a) (adjoint A)′ = (adjoint A′)   (b) adjoint A = O

(c) (adjoint A)′ = adjoint A   (d) None of these

5. The area of a triangle with vertices (-3,2), (5,4), (k,-6) is 42 sq units . What is the value of k?

(a) 6   (b) 5   (c) 7   (d) None of these

6. If A is a square matrix such that square of A =I then inverse of A is

(a) A   (b) 2A   (c) A/2   (d) None of these

7. A system of equations is said to be inconsistent if the solution

(a) exists   (b) is unique   (c) does not exist   (d) None of these

8. If A is a non- singular matrix of order 3 and determinant value of A is 3 then determinant value of (2A) is

(a) 24   (b) 12   (c) 40   (d) None of these

9. If A is a square matrix of order 3 and det A=7 what is the value of det (adjoint A) ?

(a) 39   (b) 49   (c) 30   (d) None of these

10. A square matrix is invertible if and only if

(a) A is not a non- singular matrix   (b) A is a singular matrix

(c) A is a non-singular matrix   (d) None of these

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(i) Both A and R are true and R is the correct explanation of A.
(ii) Both A and R are true and R is not the correct explanation of A.
(iii) A is true but R is false.
(iv) A is false but R is true.

11. **Assertion (A)** : For two matrices A and B of order 3,|A| =3,|B| = -4 then |2AB| = -96

Reason (R) : For a matrix A of order n and a scalar k det (k A) = k raised to the power n . (det A)

12. **Assertion (A)** : A inverse exists

Reason (R) : det A = 0

13. **Assertion (A)** : adj A is a non- singular matrix

Reason (R) : A is non – singular matrix

14. **Assertion (A)** : det Q = O

Reason (R) : Determinant of skew symmetric matrix is O

15. **Assertion (A)** :The value of k for which area of the triangle with vertices(1,1), (0,2), (k,0) is 3 square units

Reason (R) : We can use the determinant formula for finding area of triangle

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

16. What is the inverse of the matrix $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$?
17. If we find positive integral power of a symmetric matrix then we get which type of matrix- Symmetric or Skew symmetric?
18. Write the adjoint of matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$?
19. Find the corresponding value of determinant for given matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$
20. A is an invertible matrix of order 3 then the statement -The determinant of adjoint A is equal to square of the determinant of matrix A is True or False?

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

21. Find the value of y if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2y & 4 \\ 6 & y \end{vmatrix}$

22. A is an invertible matrix of order 3x3 and det A =7, then find det (inverse of A).

23. Solve for x: $\begin{vmatrix} x^2 & 0 & 3 \\ x & 1 & -4 \\ 1 & 2 & 0 \end{vmatrix} = 11$

24.Find the equation of the line joining (1,2) and (3,6) using determinants.

25. Verify A(adj. A) =(adj. A)A = (det A). I for A= $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

26. Solve using matrix method 2x-y = 1, 3x+2y=5

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

27. If A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$ then find inverse of A. Hence solve the system of equations

3x+4y+7z=14, 2x-y+3z=4, x+2y-3z=0

28. Solve system of equations

x-y+2z=7, 2x-y+3z =12, 3x+2y-z =5 using matrix method

29. Show that the points (a, 0) , (0, b) and (1,1) are collinear if a + b = ab

30. If A = $\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ show that A^2-12A+I = O

31.If A=$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Hence solve the given equations

$$2x - 3y + 5z = 11; \quad 3x + 2y - 4z = -5; \quad x + y - 2z = -3.$$

32. Given that A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB and use it to solve the system of equations: x – y + z = 4; x – 2y – 2z = 9; 2x + y +3z = 1

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

33. In the capital of a state an Educational Institute conducts classes for painting and dance and fees for two age- groups under -10 and above 10 years students. In painting there are 20 under 10 students and 5 students of the 10 years age group. The total monthly collection is Rs 900. I dance class there are 5 under 10 years students and 25 students of 10 years age group and the total monthly collection is Rs 2600. Now, based on the above information, answer the following questions.

(i) (a) If Rs x and Rs y be the fees for 10 years and under 10 age group children respectively (1 mark)

(b) What will be the matrix equation for the system of linear equations. (1 marks)

(ii) Calculate the fees for both age groups. (2 marks)

34. In a factory the social worker decided to distribute gifts to workers of a particular CABIN. If there were 8 workers less, everyone would have got Rs 10 more. Also, it was observed that if there were 16 students more, everyone would have got Rs 10 less.

(i) (a) If the number of workers in the CABIN be x and the social worker has decided to give Rs y to each worker, then find the system of linear equations for this situation. (1 mark)

(b) If AX=B, where A, X, B are matrices then what will be the value of X.

(ii) Find the number of workers and amount distributed by the social worker. (2 marks)

Answer Key/Hints

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. (b) 2. (d) 3. (c) 4. (a) 5. (c) 6. (a) 7. (c)

8.---(a) 9. (b) 10. (c)

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

11. (ii) 12. (iii) 13. (i) 14. (iii) 15. (iv)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

16.$\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ 17. symmetric 18. $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ 19. -7 20. True

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

21.+√3, -√3 22. 1/7

23. $\begin{vmatrix} x^2 & 0 & 3 \\ x & 1 & -4 \\ 1 & 2 & 0 \end{vmatrix} = 11 \Rightarrow x^2(0+8) - 0 + 3(2x-1) = 11$

$\Rightarrow 8x^2 + 6x - 3 = 11 \quad \Rightarrow 8x^2 + 6x - 14 = 0 \quad \Rightarrow \quad 4x^2 + 3x - 7 = 0 \Rightarrow \quad$ x= $-\frac{7}{4}$ or 1

24. 2x-y=0 25. Verification 26. x=1 y=1

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

27. x= -1 y = -1 z = -1 28.x=2, y=1, z=3 29. To use the area of triangle.

30 Take LHS and prove it.

31. $|A|$= 2(-4+4) - (-3)(-6+4)+5(3-2) = 2.0+3(-2)+5.1 = 0-6+5 = $-1 \neq 0$ $\therefore A^{-1}$exists.

Then, adj A = $\begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$$\therefore A^{-1} = \frac{\mathbf{1}}{|\boldsymbol{A}|}\ \textbf{adj A} = \frac{\mathbf{1}}{\mathbf{-1}}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations can be written as a single matrix equation

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

i.e. $AX = B$

$$\Rightarrow X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 0.11 + 1.(-5) + (-2).(-3) \\ -2.11 + 9.(-5) + (-23).(-3) \\ -1.11 + 5.(-5) + (-13).(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \therefore\ x = 1,\ y = 2,\ z = 3.$$

32.

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 8\ I_3$$

=> $\left(\frac{A}{8}\right)$ B = I => $B^{-1} = \frac{A}{8}$

=> $B^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

The given system of equations can be written as a single matrix equation

$\begin{bmatrix}1 & -1 & 1\\1 & -2 & -2\\2 & 1 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}4\\9\\1\end{bmatrix}$ i.e. B X = C => X = B^{-1} C = $\frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\-7 & 1 & 3\\5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\9\\1\end{bmatrix} = \frac{1}{8}\begin{bmatrix}24\\-16\\-8\end{bmatrix}$

=> $\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\-2\\-1\end{bmatrix}$ => x =2, y = -2, z= -1

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

33. (i) (a) 5x +20 y= 9000, and 25 x + 5y =26000

(b) $\begin{pmatrix}5x+20y\\25x+5y\end{pmatrix}=\begin{pmatrix}9000\\2600\end{pmatrix}$

(ii) Fees for under 10 and 10 year old students are respectively Rs 200 and Rs 1000.

Hint: A= $\begin{bmatrix}5 & 20\\25 & 5\end{bmatrix}$, Determinant A = 25-500 = -475

Inverse A =1/-475 $\begin{vmatrix}5 & 20\\-25 & 5\end{vmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix}$ = 1/-475 $\begin{pmatrix}45000-520000\\-225000+130000\end{pmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1000\\200\end{pmatrix}$

x= Rs 1000 and y= Rs 200

34. (i) (a) 5x-4y=40, 5x-8y+80 = 0

(b) X= (inverse of A)B

(ii) Rs. 32 and Rs 30.

Ch – 5 Continuity and Differentiability

Continuity and differentiability, chain rule, derivative of inverse trigonometric functions, like $sin^{-1}x$, $cos^{-1}x$, $tan^{-1}x$, derivative of implicit functions. Concept of exponential and logarithmic functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

Formulae for Limits

(a) $\lim_{x\to 0} cosx = 1$, (b) $\lim_{x\to 0}\frac{\sin x}{x} = 1$, (c) $\lim_{x\to 0}\frac{\tan x}{x} = 1$, (d) $\lim_{x\to 0}\frac{\sin^{-1}x}{x} = 1$

(e) $\lim_{x\to 0}\frac{\tan^{-1}x}{x} = 1$ (f) $\lim_{x\to 0}\frac{a^x-1}{x} = log_{e^a}, a > 0$ (g) $\lim_{x\to 0}\frac{e^x-1}{x} = 1$ (h) $\lim_{x\to 0}\frac{log_e(1+x)}{x} = 1$

(i)) $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$

Important terms and facts about Limits & Continuity of a function

(i) For a function $f(x)$, $\lim_{x\to m} f(x)$ exists if $\lim_{x\to m^-} f(x) = \lim_{x\to m^+} f(x)$

(ii) A function $f(x)$ is continuous at a point $x = m$ if,

$$\lim_{x\to m^-} f(x) = \lim_{x\to m^+} f(x) = f(m)$$

Where $\lim_{x\to m^-} f(x)$ *or* $\lim_{h\to 0} f(m-h)$ is **Left Hand Limit** of $f(x)$ at $x = m$ and

$\lim_{x\to m^+} f(x)$ *or* $\lim_{h\to 0} f(m+h)$ is **Right Hand Limit** of $f(x)$ at $x = m$ (0<h<<)

Also $f(m)$ is the value of the function $f(x)$ at $= m$.

Formulae for Derivatives

(a) $\frac{d}{dx}(x^n) = nx^{n-1}$ (b) $\frac{d}{dx}(k) = 0$, where k is any constant. (e) $\frac{d}{dx}(e^x) = e^x$

(d) $\frac{d}{dx}(log_e x) = \frac{1}{x}$ (e) $\frac{d}{dx}(\sin x) = \cos x$ (f) $\frac{d}{dx}(\cos x) = -\sin x$

(g) $\frac{d}{dx}(\tan x) = sec^2 x$ (h) $\frac{d}{dx}(\sec x) = \sec x \tan x$ (i) $\frac{d}{dx}(\cot x) = -cosec^2 x$

(j) $\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x$ (k) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$ (l) $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$

(m) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$ (n) $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$ (o) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

(p) $\frac{d}{dx}(\text{cosec}^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$ (q) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$ (r) $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

<u>MULTIPLE COICE QUESTIONS (1 MARK EACH)</u>

1. If function defined by $f(x) = \begin{cases} \frac{k\cos x}{\pi-2x}, & if\ \ x \neq \frac{\pi}{2} \\ 3 & , if\ \ x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of k is

(A) 2 (B) 3 (C) 6 (D) –6

2. The function $f(x) = [x]$, denotes the greatest integer function, is continuous at

(A) 4 (B) –2 (C) 1 (D) 1.5

3. The function $f(x) = \begin{cases} \frac{sinx}{x} + cosx, & if \ x \neq 0 \\ k & , if \ x = 0 \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of k is.

(A) 3 (B) 2 (C) 1 (D) 1.5

4. The function $f(x) = |x| + |x - 1|$ is

(A) Continuous at $x = 0$ as well as at $x = 1$

(B) Continuous at $x = 1$ but not at $x = 0$

(C) Discontinuous at $x = 0$ as well as at $x = 1$

(D) Continuous at $x = 0$ but not at $x = 1$

5. The function $f(x) = \begin{cases} \frac{e^{3x} - e^{-5x}}{x}, & if \ x \neq 0 \\ k & , if \ x = 0 \end{cases}$ is continuous at $x = 0$, then value of k is

(A) 3 (B) 5 (C) 6 (D) 8

6. The function $f(x) = tan\, x$ is discontinuous on the set

(A) $\{n\pi : n \epsilon Z\}$ (B) $\{2n\pi : n \epsilon Z\}$ (C) $\left\{(2n + 1)\frac{\pi}{2} : n \epsilon Z\right\}$ (D) $\left\{\frac{n\pi}{2} : n \epsilon Z\right\}$

7. The set of points where the function f given by $f(x) = |2x - 1|\, sin\, x$ is differentiable in

(A) R (B) $R - \{\frac{1}{2}\}$ (C) $(0, \infty)$ (D) none of these

8. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then $\frac{dy}{dx}$ is:

(A) $\frac{\cos\theta + \cos 2\theta}{\sin\theta - \sin 2\theta}$ (B) $\frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$ (C) $\frac{\cos\theta - \cos 2\theta}{\sin\theta - \sin 2\theta}$ (D) $\frac{\cos 2\theta - \cos\theta}{\sin 2\theta + \sin\theta}$

9. If $y = \log_e\left(\frac{x^2}{e^2}\right)$, then $\frac{d^2y}{dx^2}$ is equal to:

(A) $-\frac{1}{x}$ (B) $-\frac{1}{x^2}$ (C) $\frac{2}{x^2}$ (D) $-\frac{2}{x^2}$

10. If $\sin y = x\cos(a + y)$, then $\frac{dy}{dx}$ is:

(A) $\frac{\cos a}{cos^2(a+y)}$ (B) $\frac{-\cos a}{cos^2(a+y)}$ (C) $\frac{\cos a}{sin^2 y}$ (D) $\frac{-\cos a}{sin^2 y}$

11. If $(x^2 + y^2)^2 = xy$, then $\frac{dy}{dx}$ is:

(A) $\frac{y + 4x(x^2+y^2)}{4y(x^2+y^2) - x}$ (B) $\frac{y - 4x(x^2+y^2)}{x + 4(x^2+y^2)}$ (C) $\frac{y - 4x(x^2+y^2)}{4y(x^2+y^2) - x}$ (D) $\frac{4y(x^2+y^2) - x}{y - 4x(x^2+y^2)}$

12. If $sin^2 x + cos^2 y = 1$, then $\frac{dy}{dx}$ is :

(A) $\frac{sin^2 x}{cos^2 y}$ (B) $\frac{\sin 2x}{\cos 2y}$ (C) $\frac{\sin 2x}{\sin 2y}$ (D) $\frac{\cos 2x}{\cos 2y}$

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

(c) (A) is true, but (R) is false.
(d) (A) is false, but (R) is true.

13. **Assertion (A)**:$\frac{d\,(e^{\cos x})}{dx} = e^{\cos x}(-\sin x)$

Reason (R): $\frac{d\,(e^x)}{dx} = e^x$

14. **Assertion (A):** If $xy = e^{x-y}$, then $\frac{dy}{dx} = \frac{y(x-1)}{x(1+y)}$

Reason (R):$\frac{d}{dx}(uv) = u\frac{d}{dx}v + v\,\frac{d}{dx}$ u

15. **Assertion (A)**: The function $f(x) = \frac{|x|}{x}$ is continuous at $x = 0$.

Reason (R): The left- hand limit and right- hand limit of $f(x) = \frac{|x|}{x}$ are not equal at $x = 0$.

16. **Assertion (A):** $f(x) = \begin{cases} \frac{\sin 5x}{x}, & if\ \ x \neq 0 \\ \frac{k}{3} & ,if\ \ x = 0 \end{cases}$ is continuous at $x = 0\,, then\ k = 15$.

Reason (R): If $f(x)$ is continuous at a point $x = a$ in its domain, then $\lim\limits_{x \to a} f(x) = f(a)$.

17. **Assertion (A):** The function $f(x) = x|x|$ is everywhere differentiable.

Reason (R): The function $f(x) = x^2$ is everywhere differentiable.

<u>VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)</u>

18. Find the value of $k$for which $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & if\ \ x \neq 0 \\ k & ,if\ \ x = 0 \end{cases}$ is continuous at $x = 0$.
19. Find the value of k for which $f(x) = \begin{cases} \frac{x^2+3x-10}{x-2}, & if\ \ x \neq 2 \\ k & ,if\ \ x = 2 \end{cases}$ is continuous at $x = 2$.
20. Discuss the differentiability of $f(x) = x|x|$at $x = 0$.
21. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x .
22. If $x = a(2\theta - \sin 2\theta)$ and $= a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.

<u>SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)</u>

23. If the $f(x)$ given by $f(x) = \begin{cases} 3ax + b\,, & if\ x > 1 \\ 11 & ,\ if\ x = 1 \\ 5ax - 2b\,, & if\ x < 1 \end{cases}$ is continuous at $x = 1$,

find the values of a and b .

24. Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & ,\ if\ x < 0 \\ a & ,\ if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & ,\ if\ x > 0 \end{cases}$, Determine the value of a so that$f(x)$ is continuous at $x = 0$

25. If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1 - x^2)\frac{dy}{dx} + y = 0$.

26. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to x .

27. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+log x)^2}$

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

28. Let $f(x) = \begin{cases} \frac{1-sin^3 x}{3\,cos^2 x} & , \; if\ x < \frac{\pi}{2} \\ a & , \; if\ x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2}, & if\ x > \frac{\pi}{2} \end{cases}$ If $f(x)$ is continuous at $x = \frac{\pi}{2}$, find a and b.

29. Determine the k so that the given function is continuous.

$$f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & if -1 \le x < 0 \\ \frac{2x+1}{x-2} \;, & if\ \ 0 \le x \le 1 \end{cases}$$

30. For what value of k is the following function continuous at $= -\frac{\pi}{6}$?

$$f(x) = \begin{cases} \frac{\sqrt{3}\ \sin x + \cos x}{x\ +\ \frac{\pi}{6}}\ , & if\ \ x \ne -\frac{\pi}{6} \\ k \;, & if\ \ x = -\frac{\pi}{6} \end{cases}$$

31. Find $\frac{dy}{dx}$, if $y = e^{sin^2 x}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$.

32. Find $\frac{dy}{dx}$, if $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

33. Find $\frac{dy}{dx}$, if $y = (\tan x)^{cot\,x} + (\cot x)^{\tan x}$

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

34. A potter made a mud vessel, where the shape of the pot is based on $f(x) = |x-3| + |x-2|$, where $f(x)$ represents the height of the pot.

(i) When $x\ >\ 4$ What will be the height in terms of x ?

(ii) What is $\frac{dy}{dx}$at $x\ =\ 3$

(iii) If the potter is trying to make a pot using the function $f(x) = [x]$,will he get a pot or not? Why?

Or

When the x value lies between (2, 3) then the function is= ?

35. Reena started to read the notes on the topic 'Differentiability' which she has prepared in the class of Mathematics. She wanted to solve the questions based on this topic, which teacher gave as home work. She has written following matter in her notes:

Let $f(x)$ be a real valued function, then its Left Hand Derivative (LHD) is:

$$f'(a) = \lim_{h\to 0} \frac{f(a-h) - f(a)}{-h}$$

Right Hand Derivative (RHD) is: $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function $f(x)$ is said to be differentiable at $= a$, if its LHD and RHD at $x = a$ exist and one equal.

For the function $f(x) = \begin{cases} |x-3| , & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} , & x < 1 \end{cases}$

(i) Find the value of $f'(1)$.

(ii) Find the value of $f'(2)$.

(iii) Check the differentiability of the function at $x = 1$.

Or

Check the differentiability of the given function at $x = 1$.

$$f(x) = \begin{cases} x^3 - 1 , & 1 < x < \infty \\ x - 1 , & -\infty < x \leq 1 \end{cases}$$

Answer Key/Hints

<u>MULTIPLE COICE QUESTIONS (1 MARK EACH)</u>

1. (C) 6

$$k \lim_{x\to\frac{\pi}{2}} \frac{\sin(\frac{\pi}{2} - x)}{2(\frac{\pi}{2} - x)} = 3 \Rightarrow \frac{k}{2} \times 1 = 3 \Rightarrow k = 6$$

2. (D) 1.5 Greatest integer function is continuous except at integer points.
3. (B) 2

$$\lim_{x\to 0} \frac{\sin x}{x} + \lim_{x\to 0} cosx = k \Rightarrow 1 + 1 = k \Rightarrow k = 2$$

4. (A) Continuous at $x = 0$ as well as at $x = 1$
5. (D) 8

$$\lim_{x\to 0} \frac{e^{8x-5x} - e^{-5x}}{x} = k \Rightarrow 8 \times \lim_{x\to 0} e^{-5x} \times \lim_{x\to 0} \frac{e^{8x} - 1}{8x} = k \Rightarrow k = 8$$

6. (C) $\left\{(2n+1)\frac{\pi}{2} : n \epsilon Z\right\}$
7. (B) $R - \{\frac{1}{2}\}$
8. (B) $\frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$

$$\frac{dx}{d\theta} = -2sin\theta + 2sin2\theta \ and \ \frac{dy}{d\theta} = 2cos\theta - 2cos2\theta$$

9. (D)$-\frac{2}{x^2}$

$$y = 2\log_e x - \log_e e^2 \quad \Rightarrow \ y = 2\log_e x - 2$$

10. (A) $\frac{\cos a}{cos^2(a+y)}$

$$\frac{dx}{dy} = \frac{\cos(a+y)\cos y + siny\sin(a+y)}{cos^2(a+y)}$$

$$\frac{dx}{dy} = \frac{\cos[(a+y)-y])}{cos^2(a+y)} \Rightarrow \ \frac{dx}{dy} = \frac{\cos a}{cos^2(a+y)}$$

11. (C) $\frac{y - 4x(x^2+y^2)}{4y(x^2+y^2) - x}$

12. (C) $\frac{\sin 2x}{\sin 2y}$

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

13. (b)
14. (b)
15. (d)
16. (a)
17. (b)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

18. $\lim_{x\to 0}\frac{1-\cos 4x}{8x^2} = k \Rightarrow \lim_{x\to 0}\frac{2\,sin^2\,2x}{8x^2} = k \ \Rightarrow \lim_{x\to 0}(\frac{\sin 2x}{2x})^2 = k \ \ \Rightarrow k = 1$

19. $\lim_{x\to 2}\frac{(x-2)(x+5)}{x-2} = k \ \ \Rightarrow k = 7$

20. $f(x) = \begin{cases} x^2 \ , \ \ x \geq 0 \\ -x^2 \ , \ \ x < 0 \end{cases}$

LHD = $\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-}\frac{-x^2-0}{x-0} = \ 0$

RHD = $\lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+}\frac{x^2-0}{x-0} = \ 0$

21. f(x) $= \tan^{-1}\left(\frac{2\,cos^2\frac{x}{2}}{2\,sin\frac{x}{2}\cos\frac{x}{2}}\right) = \tan^{-1}\left(cot\frac{x}{2}\right) = \tan^{-1}\left[tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right] = \frac{\pi}{2}-\frac{x}{2}$

$$f'(x) = -\frac{1}{2}$$

22. $\frac{dx}{d\theta} = a(2 - 2cos\ 2\theta) = 4asin^2\theta \& \frac{dy}{d\theta} = 2asin\ 2\theta = 4a\ sin\theta\ cos\theta \ \ \Rightarrow \left.\frac{dy}{dx}\right]_{\frac{\pi}{3}} = \frac{1}{\sqrt{3}}$

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

23. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$$5a - 2b = 3a + b = 11 \ \ \Rightarrow \quad a = 3 \ and \ b = 2$$

24. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{1-\cos 4x}{x^2} = \lim_{x\to 0^-}\frac{2\,sin^2\,2x}{x^2} = 2\times 4\times \lim_{x\to 0^-}(\frac{\sin 2x}{2x})^2 = 8$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}\times\frac{\sqrt{16+\sqrt{x}}+4}{\sqrt{16+\sqrt{x}}+4} = \lim_{x\to 0^+}\sqrt{16 + \sqrt{x}} + 4 = 4+4 = 8$$

$\Rightarrow$ a = 8

25. $\frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{1+x}{1-x}} \times \frac{d}{dx}\left(\frac{1-x}{1+x}\right) \Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1+x}{1-x}} \times \frac{1}{(1+x)^2}$ [*Multiplying by* $(1-x^2)$]

26. Let $x = \tan\theta$ we get $y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{2sin^2\frac{\theta}{2}}{2sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$

$$\Rightarrow y = \frac{1}{2}\tan^{-1}x \Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{1+x^2}\right)$$

27. $e^{y\,logx} = e^{x-y} \Rightarrow y\log x = x - y \Rightarrow y = \frac{1}{1+logx} \Rightarrow \frac{dy}{dx} = \frac{\log x}{(1+logx)^2}$

<u>LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)</u>

28. $LHL = \lim_{x\to\frac{\pi}{2}^-} \frac{1 - sin^3 x}{3cos^2 x} = \lim_{x\to\frac{\pi}{2}^-} \frac{(1^3 - sin^3 x)}{3(1^2 - sin^2 x)} = \lim_{x\to\frac{\pi}{2}^-} \frac{(1-sinx)(1+sinx+sin^2 x)}{3(1-sinx)(1+sinx)} = \frac{1}{2}$

$$RHL = \lim_{x\to\frac{\pi}{2}^+} \frac{b(1-\sin x)}{(\pi - 2x)^2} = b\lim_{h\to 0} \frac{\left\{1 - \sin(\frac{\pi}{2}+h)\right\}}{\left\{\pi - 2\left(\frac{\pi}{2}+h\right)\right\}^2} = b\lim_{h\to 0}\frac{1-\cos h}{4h^2}$$

$$= \quad b\lim_{h\to 0}\frac{2sin^2\frac{h}{2}}{4h^2} = \frac{b}{8}$$

$$\therefore \frac{1}{2} = \frac{b}{8} = a \quad \Rightarrow a = \frac{1}{2} \quad and\ b = 4$$

29. $LHL = \lim_{x\to 0^-}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x} = \lim_{x\to 0^-}\frac{(\sqrt{1+kx}-\sqrt{1-kx})}{x} \times \frac{(\sqrt{1+kx}+\sqrt{1-kx})}{(\sqrt{1+kx}+\sqrt{1-kx})} =$

$\lim_{x\to 0^-}\frac{2kx}{x(\sqrt{1+kx}+\sqrt{1-kx})} = \frac{2k}{\sqrt{1}+\sqrt{1}} = k$

$$RHL = \lim_{x\to 0^+}\frac{2x+1}{x-2} = \frac{0+1}{0-2} = -\frac{1}{2} \therefore\ k = -\frac{1}{2}$$

30. $\lim_{x\to-\frac{\pi}{6}}\frac{2\left(\frac{\sqrt{3}}{2}\sin x + \cos x\,.\frac{1}{2}\right)}{\left(x+\frac{\pi}{6}\right)} = \lim_{x\to-\frac{\pi}{6}}\frac{2\left(\cos\frac{\pi}{6}\sin x + \cos x\ sin\frac{\pi}{6}\right)}{\left(x+\frac{\pi}{6}\right)} = \lim_{x\to-\frac{\pi}{6}}\frac{2sin\left(x+\frac{\pi}{6}\right)}{\left(x+\frac{\pi}{6}\right)} = 2 \quad \Rightarrow k = 2$

31. Let $x = \cos 2\theta$

We get $2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} = 2\tan^{-1}\sqrt{\frac{2sin^2\theta}{2cos^2\theta}} = 2\tan^{-1}(tan\theta) = 2\theta = \cos^{-1}x$

Hence $y = e^{sin^2 x}\cos^{-1}x \Rightarrow \log y = sin^2 x + \log(\cos^{-1}x)$

$$\Rightarrow \frac{1}{y}\times\frac{dy}{dx} = 2\sin x\cos x + \frac{1}{\cos^{-1}x}\times\frac{-1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = e^{sin^2 x}\cos^{-1}x\left[\sin 2x - \frac{1}{\cos^{-1}x\sqrt{1-x^2}}\right]$$

32. Let $u = (\sin x)^x \Rightarrow$ *Taking* log, *both sides* $\Rightarrow \log u = x\log\sin x$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x[x\cot x + \log\sin x]$$

$$and\ v = \sin^{-1}\sqrt{x} \quad \Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{x-x^2}}$$

$$\therefore \frac{dy}{dx} = (\sin x)^x[x\cot x + \log\sin x] + \frac{1}{2\sqrt{x - x^2}}$$

33. $\frac{dy}{dx} = (\tan x)^{cot\, x} cosec^2 x(1 - \log\tan x) + (\cot x)^{\tan x}\, sec^2 x(\log\cot x - 1)$

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

34. (i) $2x - 5$(ii) Function is not differentiable.
(iii) No, because it is not continuous. Or 1 .

35. (i) -1 (ii)-1
(iii) $f(x)$ is differentiable at $x = 1$. or $f(x)$ $is\ not$ differentiable at $x = 1$.

Ch – 6 Application of Derivatives

Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

RATE OF CHANGE OF QUANTITIES

$\frac{dy}{dx}$ represent the rate of change of y w.r.t. x.

Also $\left(\frac{dy}{dx}\right)_{x=x_0}$ represents the rate of change of y w.r.t. x. at $x = x_0$.

Marginal cost is the derivative of cost function i.e. MC = $\frac{d}{dx}\{C(x)\}$.

Marginal revenue is the derivative of revenue function i.e., MR = $\frac{d}{dx}\{R(x)\}$.

INCREASING AND DECREASING FUNCTIONS

DEFINITION

Let I be an interval contained in the domain of a real valued function f .Then f is said to be

(i) increasing on I if $x_1 < x_2$ in I => $f(x_1) \le f(x_2)$ for all $x_1, x_2 \in I$.
(ii) Strictly increasing on I if $x_1 < x_2$ in I => $f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$.
(iii) decreasing on I if $x_1 < x_2$ in I => $f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in I$.
(iv) Strictly decreasing on I if $x_1 < x_2$ in I => $f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$.

THEOREM: Let f be continuous on $[a, b]$ and differentiable on open interval (a, b) Then

(i) f is strictly increasing in $[a, b]$ if $f'(x) > 0$ for each $x \in$ (a, b)
(ii) f is strictly decreasing in $[a, b]$ if $f'(x) < 0$ for each $x \in$ (a, b)
(iii) f is constant function in $[a, b]$ if $f'(x) = 0$ for each $x \in$ (a, b)

MAXIMA AND MINIMA

Steps to solve a problem of maxima and minima

i) Formulate the given problem and find out $y = f(x)$.
ii) Find $\frac{dy}{dx}$.
iii) Equate $\frac{dy}{dx}$. to zero and find the solution of $\frac{dy}{dx} = 0$ .Let it is $c_1, c_2, c_3,$ ……..these points are called *CRITICAL POINTS.* Now we have to check which point will be point of maxima and which will be point of minima. For this purpose we can use First derivative test or Second derivative test

For first derivative test

Draw a number line and put critical points

(i) If $f'(x)$ changes sign from positive to negative as x increases through c, then c is a point of local maxima.

(ii) If $f'(x)$ changes sign from negative to positive as x increases through c, then c is a point of local minima.

(iii) If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima infact such a point is called point of inflection.

For second derivative test

find out second order derivative i.e $\frac{d^2y}{dx^2}$.

If $\frac{d^2y}{dx^2} < 0$ at a critical point say c_1 then c_1 will be point of maxima and the corresponding value will be maximum value.

If $\frac{d^2y}{dx^2} > 0$ at a critical point say c_2 then c_2 will be point of minima and the corresponding value will be minimum value.

<u>MULTIPLE COICE QUESTIONS (1 MARK EACH)</u>

1. The function$f(x) = x(x-3)^2$ decreases for the values of given by x

   (a) $1 < x < 3$ (b) $x < 0$ (c) $x > 0$ (d) $0 < x < \frac{3}{2}$

2. The function $f(x) = tanx - x$ is

   (a) Always increases (b) always decreases

   (c) never increases (d) sometimes increasing & sometimes decreasing

3. The function $f(x) = x^x$ decreases in the interval

   (a) $(0, e)$ (b) $(0, \frac{1}{e})$ (c) $(0,1)$ (d) $\left(\frac{1}{e}, e\right)$

4. The coordinates of the point on the ellipse $16x^2 + 9y^2 = 400$ where the ordinate decreases at the same rate at which the abscissa increases are

   (a) $(3, \frac{16}{3})$ (b) $(-3, \frac{16}{3})$ (c) $(3, -\frac{16}{3})$ (d) $(3, -3)$

5. If the rate of change of volume of a sphere is equal to the rate of change of radius, then its radius is equal to

   (a) 1 unit (b) $\sqrt{2\pi}$ units (c) $\frac{1}{\sqrt{2\pi}}$ units (d) $\frac{1}{2\sqrt{\pi}}$ units

6. A man of height 6 ft walks at a uniform speed of 9 ft/sec from a lamp post fixed at 15 ft height. The length of his shadow is increasing at a rate of-

   (a) 15 ft/sec (b) 9 ft/sec (c) 6 ft/sec (d) none of these

7. If the function $f(x) = x + cosx + b$ is strictly decreasing over R, Then

   (a) $b < 1$ (b) no value of b exists (c) $b \leq 1$ (d) $b \geq 1$

8. The interval in which $f(x) = x^3 + 6x^2 + 6$ is increasing, is

(a) $(-\infty, -4) \cup (0, \infty)$ (b) $(-\infty, -4)$ (c) $(-4, 0)$ (d) $(-\infty,\ 0) \cup (4,\ \infty)$

9. The function $f(x) = x - sinnx$ decreases for
   (a) All x (b) $x \leq \frac{\pi}{2}$ (c) $0 < x < \frac{\pi}{4}$ (d) no value of x

10. The real function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is
    (a) stricty increasing in $(-\infty,\ 2)$ and strictly decreasing in$(-2,\ \infty)$
    (b) strictly decreasing in$(-2,\ 3)$
    (c) stricty decreasing in $(-\infty,\ 3)$ and strictly increasing in$(3,\ \infty)$
    (d) stricty decreasing in $(-\infty, -2) \cup (3,\ \infty)$

11. The maximum value of $\left(\frac{1}{x}\right)^x$ is
    (a) e (b) e^e (c) $e^{\frac{1}{e}}$ (d) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

12. The least value of the function $f(x) = x^3 - 18x^2 + 96x$ in the interval [0, 9] is
    (a) 126 (b) 135 (c) 160 (d) 0

13. The function $f(x) = x^x$ has a stationary point at
    (a) $x = e$ (b) $x = \frac{1}{e}$ (c) $x = 1$ (d) $x = \sqrt{e}$

14. The minimum value of $x^2 + \frac{250}{x}$ is
    (a) 0 (b) 25 (c) 50 (d)75

15. The least value of function $f(x) = e^x + e^{-x}$ is
    (a) -2 (b) 0 (c) 2 (d) cannot be determined

16. If the function $f(x) = x^3 + ax^2 + bx + 1$ is maximum at x = 0 and minimum at x =1, then value of a and b is
    (a) $a = \frac{2}{3}, b = 0$ (b) $a = -\frac{3}{2}, b = 0$ (c) $a = 0, b = \frac{3}{2}$ (d) none of these

17. The point on the curve $y^2 = 4x$ which is nearest to the point (2, 1) is
    (a) $(1, 2\sqrt{2})$ (b) $(1, 2)$ (c) $(1, -2)$ (d) $(-2, 1)$

18. If the function $f(x) = x^2 - kx + 5$ is increasing on [2, 4], then k belongs to
    (a) $(2, \infty)$ (b) $(-\infty, 2)$ (c) $(4,\ \infty)$ (d) $(-\infty, 4)$

19. If $x + y = 8$ then the maximum value of xy is
    (a) 8 (b) 16 (c) 20 (d) 24

20. If x is real, the minimum value of $x^2 - 8x + 17$ is
    (a) -1 (b) 0 (c) 1 (d) 2

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

In the following question a statement of assertion (A) is followed by a statement of reason (R). Choose the correct option on the basis of A and R

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false.
(d) A is false but R is true.
(e) Both A and R are false.

1. Consider the function $f(x) = x^{\frac{1}{3}},\ x \in R$

   Assertion (A): $f(x)$ has a point of inflection at $x = 0$

   Reason(R): $\boldsymbol{f''}(0) = 0$

2. A function is defined as $f(x) = 4x + 3, x \in R$

   Assertion (A): $f(x)$ is decreasing function on R

   Reason(R): $\boldsymbol{f'}(x) = 4 > 0$

3. A function is defined as $f(x) = e^x$

   Assertion (A): $f(x)$ has no local maxima and no local minima

   Reason(R): The value of e is $2 < e < 3$

4. Function is defined as $f(x) = sinx$

   Assertion (A): $f(x)$ is strictly decreasing in $\left(\frac{\pi}{2},\ \pi\right)$

   Reason(R): $\boldsymbol{f'}(x) = cosx < 0$ in $\left(\frac{\pi}{2},\ \pi\right)$

5. A cylinder is inscribed in a sphere of radius R.

   Assertion (A): Height of cylinder of maximum volume is $\frac{2R}{\sqrt{3}}$ units.

   Reason(R): Maximum volume of the cylinder is $\frac{4\pi R^3}{\sqrt{3}}$.

6. Consider the function $f(x) = sin^4x + cos^4x$

   Assertion (A): $f(x)$is increasing function in$\left[0,\ \frac{\pi}{2}\right]$

   Reason(R): $f(x)$is decreasing function in$\left[\frac{\pi}{4},\ \frac{\pi}{2}\right]$

7. Function is $f(x) = [x(x-2)]^2$

   Assertion (A): $f(x)$ is increasing in (0, 1) U $(2, \infty)$

   Reason(R): $\boldsymbol{\frac{dy}{dx} = 0}$, when $x = 0, 1, 2$

8. The function $f(x)$ is defined as $f(x)= sinax + b$

   Assertion (A): $f(x)$ is decreasing on $x \in R$ for all values of a

   Reason(R): $f(x)$ is decreasing only if $a \in [1, \infty)$

9. Read the question.

Assertion (A): $f(x) = \sin(sinx)$ is defined for al real values of x.

Reason(R): Minimum and maximum values do not exist.

10. AB is the diameter of a circle and C is any point on the circle

Assertion (A): The area of triangle ABC is maximum when it is isosceles.

Reason(R): Triangle ABC is a right- angled triangle.

VERY SHORT/SHORT ANSWER TYPE QUESTIONS (2/3 MARKS EACH)

1. Find the interval in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is
   (a) strictly increasing (b) strictly decreasing.
2. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y coordinate is changing 8 times as fast as x coordinate.
3. The total cost C(x) in Rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$. Find the marginal cost when 17 units are produced.
4. Find the least value of 'a' such that the function f is given by $f(x) = x^2 + ax + 1$ is strictly increasing on (1, 2).
5. The total revenue received from the sale of x units of a product is given by $R(x) = 6x^2 + 13x + 10$. Find the marginal revenue when x = 10.
6. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.
7. A particle moves along the curve$y = \frac{4}{3}x^3 + 5$. Find the points on the curve at which Y-Coordinate changes as fast as x coordinate?
8. Find the interval in which the function $f(x) = 2x^3 - 3x$ may be strictly increasing.
9. Find absolute maxima and minima for the function $f(x) = x^3$ in the interval [-2, 2].
10. Find the maximum value, if any, without using derivatives of the following function
    (a) $f(x) = 5 + sin2x$ (b) $f(x) = -|x - 1| + 5$ for all $x \in$R
    (c) $f(x) = sin3x + 4$ for all $x \in (\frac{\pi}{2}, -\pi/2)$ (d) $f(x) = \sin(sinx)$ for all $x \in$R
    (e) $f(x) = -(x - 1)^2 + 10$ for all , $x \in$R
11. An edge of a variable cube is increasing at the rate of 10 cm/sec. How fast the volume of the cube is increasing when the edge is 5 cm long?
12. Show that the function f defined by $f(x) = (x - 1)e^x + 1$ is an increasing function for all x >0.
13. The volume of a cube is increasing at the rate of 8cm^3/sec. How fast is the surface area increasing when length of its edge is 12 cm?
14. Radius of the variable circle is changing at the rate of 5cm/sec. What is the radius of the circle at a time when its area is changing at the rate of 100 cm^2/sec?
15. A balloon which always remains spherical has a variable diameter $\frac{3}{2}(2x + 3)$.Find the rate of change of its volume w.r.t. x.
16. Find the intervals in which the function f given by $f(x) = sinx + cosx$, $0 \leq x \leq 2\pi$.is strictly increasing or strictly decreasing.

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

1. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a circle and the other into a square. What should be the lengths of the two pieces so that the combined area of the square and the circle is minimum.
2. Show that the surface area of a closed cuboid with a square base and given volume is maximum when it is a cube.
3. Show that the maximum volume of the cylinder which can be inscribed in a sphere of the radius $5\sqrt{3}$ cm is $500\pi\ cm^3$

4. Find the intervals in which following functions are strictly increasing or strictly decreasing

$$f(x) = (x+1)^3(x-3)^3$$

5. An open box with a square base in to be made out of a given quantity of sheet of area c^2 .Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$
6. The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}\ {cm^3}/{sec}$. Find the rate at which is side is increasing.
7. Sum of two numbers is 5. If the sum of the cubes of these numbers is least, find the sum of the squares of these numbers.
8. A window is in the form of a rectangle surmounted by a semi circle .If the total perimeter of the window is 30 m, find the dimensions of the window so that maximum light is admitted
9. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.
10. Find the intervals in which following functions are strictly increasing or strictly decreasing.

$$f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$$

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

1. An open tank with rectangular base and vertical sides is to be constructed so as to hold 8000 liters of water. The tank is 2m deep and material for bottom costs rupees 1800 per square meter and rupees 900 per square meter for vertical faces.

Now based on above information answer the following questions-

(i) If x and y represents the length and breadth of rectangular base, then find the total cost of construction of the base? (2M)

(ii) Find the least cost of construction of the tank? (2M)

2. A CEO in a company was hard working and always used new strategies for deployment of the employees of the company's betterment but so many times he was not able to produce a handsome profit for his company. He planned to change his strategies and formed a committee to of employees to work on it and finally his company produced profit and the total profit function is given by $f(x) = -5x^2 + 125x + 37500$

Where x is the production of company.

Now based on above information answer the following questions

(i) What will be the production when profit is maximum? (2M)

(ii) Find interval in which profit is strictly increasing? (2M)

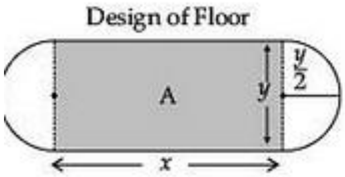
3. Suraj, s father wants to construct a rectangular garden using brick wall on one side (LENGTH) of the garden and wire fencing for the other three sides as shown in the figure. He has 200 meters of fencing wire

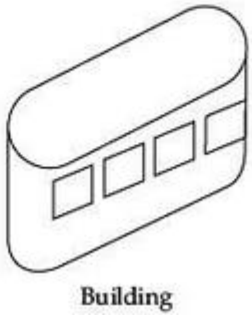
Based on the above information answer the following questions, (Board Exam-2023)

(i) Let x meters denote the length of the side of the garden perpendicular the brick wall and y meters denote the length of side parallel to the brick wall . Determine the relation representing the total length of fencing wire and also write A(x) area of the garden? (2M)

(ii) Determine maximum value of A(x)? (2M)

4. An architect designs a building for a multi- national company. The floor consists of a rectangular region with semicircular ends having a perimeter 200m.





Based on the above information answer the following questions,

(i) If x and y represents the length and breadth of the rectangular region, then find the relation between variable (2M)

(ii) Find The area of the rectangular region A expressed as a function of x (1M)

(iii) Find the Maximum value of area A (1M)

5. A rectangular sheet of metal when heated, its length decreases at the rate of 5 cm/sec and width increases at the rate of 4 cm/sec.

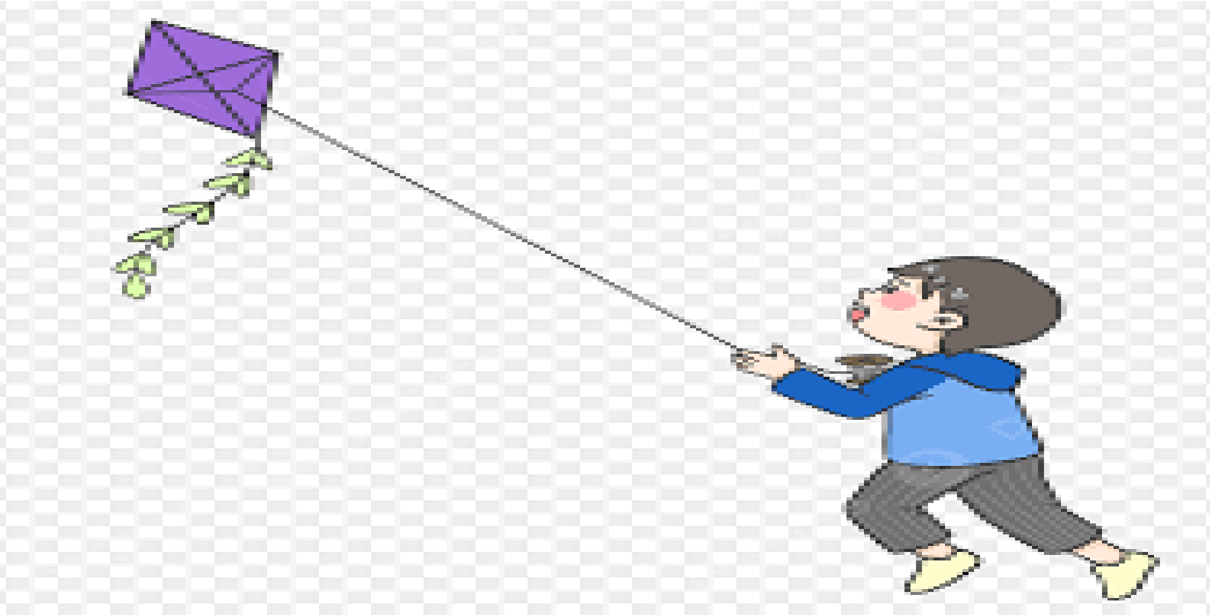
Based on the above information answer the following questions,

(i) When x=8 cm and y = 6 cm then find the rate of change of perimeter of the plate

(ii) When x=8 cm and y = 6 cm then find the rate of change of area of the plate

(iii) When x=8 cm and y = 6 cm then find the rate of change of diagonal of the plate

6. In a village a boy was fond of kite flying. In one morning he was flying kite which is moving horizontally at the height of 151.5 meters. The speed of kite is 10 m/sec and height of boy is 1.5 m,



Based on the above information answer the following questions,

(i) If at any time (when height is 151.5 m) length of string is y meters and horizontal distance of kite is x meters, then find rate of change of string in terms of x and y? (2M)

(ii) When length of string is 250m, then the rate of change of length of string? (2M)

7. The relation between the height of the plant (y in cm) with respect to exposure to sun light is governed by the following equation

$y = 4x - \frac{1}{2}x^2$ where x is number of days exposed to sun light

Based on the above information answer the following questions-

(i) What is the the rate of growth of the plant with respect to sun light? (1M)

(ii) What is the number of days it will take to complete for the plant to grow the maximum height? (2M)

(iii) What is the maximum height of the plant? (1M)

8. A cycle racer in the morning is cycling in a free cycling track and sometimes moving zig-zag. It is found that the path traced by the cyclist is given by the curve $f(x) = (x-1)(x-2)^2$.
. If at any part of time cyclist is at point (x, y).

Based on the above information answer the following questions-

(i) Find value of x for a stationary point? (2M)

(ii) Find the interval in which f(x) is increasing? (2M)

9. The shape of a toy is given as $f(x) = 6(8x^2 - x^2)$. To make the toy beautiful 2 sticks which are perpendicular to each other were placed at a point (2,3), above the toy.

(i) Find the abscissa of critical point?

(ii) Find the second order derivative of the function at $= 5$.

10. $p(x) = -5x^2 + 125x + 37500$ is the total profit function of a company, where x is the production of the company.

(i) What will be the production when the profit is maximum?

(ii) What will be the maximum profit?

Answer key/Hints

<u>MULTIPLE COICE QUESTIONS (1 MARK EACH)</u>

1.a, 2.a, 3.b, 4.a, 5.d, 6.c, 7.b, 8.a, 9.d, 10.b, 11.c, 12.d, 13.b, 14.d, 15.c, 16.b, 17.b, 18.d, 19.b, 20.c

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

1.c, 2.d, 3.b, 4.a, 5.c, 6.b, 7.b, 8.d, 9.c, 10.a

VERY SHORT/SHORT ANSWER TYPE QUESTIONS (2/3 MARKS EACH)

1. $f'(x)=0$ gives x=-2,3 check sign of $f'(x)$ for intervals $(-\infty,-2)$, $(-2,3)$, $(3, \infty)$
2. Given dy/dt = 8 dx/dt use 6dy/dt = $3x^2$ dx/dt get x= +4 and -4 and point
3. use $MC = \frac{d}{dx}\{c(x)\}$ and put x=17 MC=20.967
4. For strictly increasing -$f'(x)>0$ gives 2x+a>0 and get value of a at (1,2)
5. Use $MR = \frac{d}{dx}\{R(x)\}$ and put **x=10**
6. Given dy/dt = dx/dt use 2ydy/dt = 8 dx/dt get x and point
7. Given dy/dt = dx/dt use dy/dt =$4x^2$ dx/dt get x and point
8. $f'(x)=0$ gives x= $-1/\sqrt{2}$ and $1/\sqrt{2}$ check sign of $f'(x)$ for intervals $(-\infty,-1/\sqrt{2})$,$(-1/\sqrt{2},1/\sqrt{2})$, $(1/\sqrt{2},\infty)$
9. $f'(x)=0$ gives x=-0 get value of f(x) at x=0,-2,2 fix maxima and minima
10. (a) $4\le 5+\sin 2x\le 6$ (b) max=5 (c) $3\le \sin 3x+4\le 5$ (d) $-\sin 1\le f(x)\le \sin 1$ (e) max=10
11. use $v=x^3$ and dv/dt = $3x^2$dx/dt get ans=750cm^3/sec
12. get $f'(x)$ and show that $f'(x)>0$ for all x>0
13. $V=x^3$ dx/dt = $8/3x^2$ dS/dt = 32/x=8/3 cm^2/sec
14. Area $A=\pi r^2$ use dA/dt and dr/dt
15. Get radius r=3/4 (2x+3), volume V=4/3 $\pi[3/4\ (2x+3)]^3$ Now diff. w.r. to x and get rate
16. $f'(x)=0$ gives $x=\pi/4$ and $5\pi/4$ check in intervals $(0,\pi/4)$, $(\pi/4\ ,\ 5\pi/4)$, $(5\pi/4,\ 2\pi)$

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

$1 - Two\ pieces\ one\ x\ m\ anf\ other\ (28-x)m, x = 2\pi r, Total\ area$
$$= \frac{x^2}{4\pi} + \frac{1}{16}(28-x)^2, now\ put\ \frac{dA}{dx}$$
$$= 0\ get\ x\ and\ solve\ for\ maxima\quad Pieces\ will\ be\ \frac{28\pi}{4+\pi}, \frac{112}{4+\pi}$$

$2 - Use\ surface\ area\ s = 2(lb + bh + hl)\ and\ volume\ V = lbh\ and\ solve$

$3 - Use\ volume\ V = \pi r^2 h\ and\ radius\ of\ sphere\ R$

$4 - Stictly\ incraesing\ in (1,3) \cup (3,\infty) and\ strictly\ decreasing\ in\ (-\infty,-1) \cup (-1,1)$

$5 - Given\ x^2 + 4xh = c^2\ volume\ is\ given\ V = x^2 h\ now\ solve\ for\ maxima$

$6 - \frac{dm}{dt} = 2\sqrt{3}\ {}^{cm}/_{sec}\quad Also\ m^2 + \left(\frac{x}{2}\right)^2 = x^2\ now\ use\ these\ relation\ to\ get\ \frac{dx}{dt}$

$7 - use\ x + y = 5\ and\ S = x^3 + y^3\ to\ solve$

$8 - 2x + 2r + \pi r = 10, Area\ is\ given\ by\ A = 2xr + \frac{1}{2}\pi r^{2,}\ \frac{dA}{dx} = 0 gives\ r$
$= \frac{30}{\pi + x}\ \ now\ solve$

$10 - Use\ f'(x) > 0\ and\ f'(x) < 0\ and$

$get\ strictly\ increasing\ for(-3, 2) \cup (4, \infty) and\ strictly\ decreasing\ for\ (-\infty, -3) \cup (2, 4)$

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

$1 - (i) 1800xy - 3600(x + y) \qquad (ii) 21600$

$2 - (i) 12.5 \qquad (ii)(0,12.5)$

$3 - 2x + y = 200, \qquad A(x) = 2x(100 - x)$

$4 - Perimeter\ is\ given\ by\ p = 2x + 2(half\ of\ the\ circumference) = 2x + \pi y$
$= 200\ and\ area\ A = xy\ This\ will\ give\ (i) 2x\ + \pi y$
$= 200 \qquad (ii) \frac{2}{\pi}\ (100x - x^2\ ) \quad (iii)\ \frac{5000}{\pi}\ \ m^2$

$5 -\ (i) - 2$ cm/sec $\qquad (ii) 2$ cm2/sec $\qquad (iii) - 1.6$ cm/sec

$6 - By\ Pythagorus\ Theorem\ x^2 + y^2\ = 150^2\ Now\ diff. (i) \frac{dy}{dx} = 10 \frac{x}{y} \qquad (ii) At\ y$
$= 250, x = 200\ then\ \frac{dy}{dx} = 8\ {}^{cm}/_{sec}$

$7 - (i) first\ get \frac{dy}{dx} = 4 - x, Then(ii) \frac{dy}{dx} = 0\ will\ give 4 days \quad (iii) 8cm$

$8 - Get\ value\ of\ x\ by\ f'(x) = 0\ \ then\ get\ y,$

$Now\ apply\ the\ concept\ of\ inceasing\ and\ deacreasing\ function\ get\ intervals$

$(i) \left(2, \frac{4}{3}\ \right) \quad (ii) \left(-\infty, \frac{4}{3}\right) \cup (2, \infty)$

9.(i) $\pm \frac{1}{2}$ (ii) 3588

10.(i) 12.5 (ii) 38,281.25

Ch – 7 Integration

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2 - a^2}\, dx$$

$$\int \sqrt{ax^2 + bx + c}\, dx,$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

INTEGRATION AS AN INVERSE PROCESS OF DIFFERENTIATION

1. If f(x) is derivative of function g(x), then g(x) is known as antiderivative or integral of f(x).

   i.e., $\frac{d}{dx}\{g(x)\} = f(x) \Leftrightarrow \int f(x)dx = g(x) + c$

2. Derivative of a function is unique but a function can have infinite antiderivatives or integrals.

3. $\int f(x)dx = g(x) + c$ is known as indefinite integral, where C is constant of integration.

4. $\int dx = x + c.$

5. $\int c.f(x)dx = c\int f(x)dx.$

6. $\int\{f(x) \pm g(x)\}dx = \int f(x)dx \pm \int g(x)dx.$

7. $\int x^n\, dx = \frac{x^{n+1}}{n+1}, n \neq -1, n\ is\ a\ rational\ number. If\ n = 1, then \int \frac{1}{x}dx = \log|x| + c.$

8. $\int (ax + b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$, $n \neq -1$, is a rational number.

   If n=-1, then $\int \frac{1}{ax+b} dx = \frac{1}{a} log|ax + b| + c.$

9. $\int \sin ax\, dx = \frac{-\cos ax}{a} + c. If\ a = 1, \int \sin x\, dx = -\cos x + c.$

10. $\int \cos ax\, dx = \frac{\sin ax}{a} + c. If\ a = 1, \int \cos x\, dx = \sin x + c.$

11. $\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax + c. If\ a = 1 . then\ \int \sec x \tan x\, dx = \sec x + c.$

12. $\int \operatorname{cosec} ax \cot ax\, dx = \frac{1}{a}\operatorname{cosec} ax + c. If\ a = 1 . then\ \int \operatorname{cosec} x \cot x\, dx = -cosec\ x + c.$

13. $\int sec^2 ax\, dx = \frac{1}{a}\tan ax + c. If\ a = 1, then \int sec^2 x\, dx$= tan x+c.

14.$\int cosec^2 ax\, dx = \frac{1}{a}\cot ax + c. If\ a = 1, then \int cosec^2 x\, dx$= -cot x+c.

15.$\int e^{ax} dx = \frac{e^{ax}}{a}$ +c. If a=1 then $\int e^x dx = e^x$+c.

16.$\int a^{mx}\, dx = \frac{a^{mx}}{mlog_{e}a}$+c. If m=1 then $\int a^x\, dx = \frac{a^x}{log_{e}a}$+c.

INTEGRATION BY SUBSTITUTION

1.$\int f(x)dx \Leftrightarrow \int f\{g(x)\}g^{'}(x)dt$,if we substitute x =g(t) such that dx=g'(t)dt.

2.$\int \tan a\, xdx = -\frac{1}{a} log|\cos ax| + c\ or\ \frac{1}{a} log|\sec ax|$+c.

3.$\int \cot x\, dx$ =log$|\sin x|$+c.

4.$\int \sec xdx = log|\sec x + \tan x|$+c.

5.$\int cosec\ x\ dx = log|cosec\ x - \cot x|$+c.

INTEGRATION BY PARTIAL FRACTIONS

1. We must check that we are dealing with polynomials and degree of numerator is less than the degree of denominator and proceed for partial fraction. If not, divide numerator by denominator and write

$\frac{\text{Numerator}}{Denominator}$= quotient+$\frac{Remainder}{Denomiator}$ and proceed for partial fraction of $\frac{Remainder}{Denomiator}$.

i) when factor in denominator is linear and non-repeated.

$\frac{p(x)}{(x+a)(x+b)(x+c)}$=$\frac{A}{x+a} + \frac{B}{x+b} + \frac{C}{x+c.}$.

ii) When factor in denominator linear and repeated.

$\frac{p(x)}{(x-a)^2(x+b)}$=$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x+b}$.

iii) When factor in denominator is quadratic and non-repeated.

$$\frac{p(x)}{(x+a)(x^2+b)} = \frac{A}{x+a} + \frac{Bx+C}{x^2+b}.$$

2.To evaluate$\int \frac{ax+b}{px^2+qx+r} dx$. We write ax+b=$\lambda$(p$x^2$+qx+r) +$\mu$.

3. To evaluate $\int \frac{x^2}{(x^2+a^2)(x^2+b^2)}$dx.

Let x^2=y and proceed for partial fraction of $\frac{y}{(y+a^2)(y+b^2)}$ and after getting the partial fraction replace value of y as x^2 and then integrate.

INTEGRATION BY PARTS

Sometimes we get product of the functions which we cannot simplify in such cases we apply integration by parts. First check functions are in proper forms otherwise first reduce in proper form using substitution.

1. $\int\{f(x).g(x)\}dx = f(x).\int g(x)dx - \int\{\frac{d}{dx}f(x).\int g(x)dx\}dx.$

We can choose first and second functions according to ILATE, where I→Inverse trigonometrical function, L→Logarithmic function, A→Algebraic function, T→Trigonometric function and E→Exponential function.

DEFINITE INTEGRALS

$$\frac{d}{dx}\{g(x)\} = f(x) \Rightarrow \int_a^b f(x)dx = g(b) - g(a).$$

SOME PROPERTIES OF DEFINITE INTEGRALS

1. $\int_a^a f(x)dx = 0.$

2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, a < c < b.$

4. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx.$

5. $\int_0^a f(x)dx = \int_0^a f(a-x)dx.$

6. $\int_0^{2a} f(x)\,dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

7. $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, if\ f(2a-x) = f(x) and\ \int_0^{2a} f(x)dx = 0, if\ f(2a-x) = -f(x).$

8. $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx, if\ f(x) is\ an\ even\ function, i.e., f(-x) = f(x)$

$$\int_{-a}^a f(x)dx = 0. if\ f(x) is\ odd\ function\ i.e., f(-x) = -f(x).$$

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. $\int x^2e^{x^3}dx$ equals

(a) $\frac{1}{3}e^{x^3} + C$ (b) $\frac{1}{3}e^{x^2} + C$ (c) $\frac{1}{2}e^{x^3} + C$ (d) $\frac{1}{2}e^{x^2} + C$

2. $\int\frac{1}{sin^2xcos^2x}$ dx is equal to

(a) $sin^2x - cos^2x + c$ (b) -1 (c) tan x + cot x + c (d) tan x − cot x +c.

3. $\int\frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta}$ dx is equal to

(a) 2(sin x + x cosθ) + c (b) 2(sin x - x cosθ) + c (c) 2(sin x + 2x cosθ) + c (d) 2(sin x -2x cosθ) + c

4. $\int cot^2$x dx equals to

(a) cot x−x +c (b) cot x + x +c (c) −cot x + x +c (d) −cot x −x +c.

5. $\int \frac{\sin x+\cos x}{\sqrt{1+sin2x}}$dx, $\frac{3\pi}{4} < x < \frac{7\pi}{4}$ is equal to

(a) $\log|\sin x + \cos x|$ (b) $\log|x|$ (c) x (d) −x.

6. $\int \frac{3x^2+3^x \log 3}{3^x+x^3}$dx is equals to

(a) $3^x + x^3 + c$ (b) $\log|3^x + x^3|$ + c (c) $3x^2 + 3^x \log 3 + c$ (d) $\log|3x^2 + 3^x \log 3|$ + c.

7. If $\int sec^2(7-4x)dx = \text{a} \tan(7-4x) + c$, then value of a is

(a) 7 (b) −4 (c) 3 (d) $\frac{-1}{4}$

8. The value of K, for which $\int \frac{4x^3+K4^x}{4^x+x^4}$dx=$\log|4^x + x^4|$ + c is

(a) 1 (b) $\log_e 4$ (c) $\log_4 e$ (d) 4.

9. If $\int \frac{1}{\sqrt{4-9x^2}}$ dx = $\frac{1}{3}\sin^{-1}(ax)$+c, then value of a is

(a) 2 (b) 4 (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

10. If x = $\int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2}$ =ay , then value of a is equal to

(a) 3 (b) 6 (c) 9 (d) 1

11. $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$ equals to

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{2}$

12. $\int \frac{sin^6x}{cos^8x}\ dx$ is equal to

(a) $\frac{1}{6}\ tan^6x + C$ (b) $\frac{1}{7}\ tan^7x + C$ (c) $\frac{1}{8}\ tan^8x + C$ (d) $\frac{1}{4}\ tan^4x + C$

13. $\int \frac{1}{X^2(X^4+1)^{\frac{3}{4}}}$ dx is equal to

(a) $-(1+\frac{1}{x^4})^{\frac{1}{4}} + c$ (b) $(x^4+1)^{\frac{1}{4}} + c$ (c) $(1-\frac{1}{x^4})^{\frac{1}{4}} + c$ (d) $-(1+\frac{1}{x^4})^{\frac{3}{4}} + c$

14. $\int \frac{xe^x}{(1+x)^2}$ dx is equal to

(a) $\frac{e^x}{x+1} + c$ (b) $e^x(x+1) + c$ (c) $-\frac{e^x}{x+1} + c$ (d) $\frac{e^x}{x^2+1} + c$

15. If $\int \frac{2^x}{\sqrt{1-4^x}}$ dx = p.$\sin^{-1}(2^x) + c$, then p is equal to

(a) $\log_e 2$ (b) $\frac{1}{2}\log_e 2$ (c) $\frac{1}{2}$ (d) $\frac{1}{\log_e 2}$

16.The value of integral $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x} dx\ is$

(a) log 2 (b)$\frac{1}{20}\log 2$ $(c)\,\frac{1}{20}\log 3$ (d) log 5

17.The value of integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x.\log\left(\frac{1+x}{1-x}\right) dx$ is

(a) 0 (b) $\frac{1}{2}$ (c)$\frac{3}{2}$ (d) none of these.

18.The value of $\int_0^{\frac{\pi}{2}} \frac{1}{1+tan^3 x} dx$ is

(a) 0 (b)1 (c) $\frac{\pi}{4}$ $(d)\,\frac{\pi}{2}$

19. The value of $\int_8^{13} \frac{\sqrt{21-x}}{\sqrt{x}+\sqrt{21-x}}$dx is

(a) $\frac{21}{2}$ $(b)0$ $(c)\,\frac{5}{2}$ (d) none of these.

20.$\int \frac{\sqrt{\tan x}}{\sin x.\cos x}$dx is equal to

(a) $2\sqrt{\cot x}$ +c (b) $\frac{\sqrt{\tan x}}{2}$+c (c) $2\sqrt{\tan x}$ +c (d) none of these.

ASSERTION-REASON BASED QUESTIONS (1 MARK)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

21. **Assertion (A)**:$\int_0^{10} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}} dx = 10.$

Reason (R): $\int_0^a f(x)dx = \int_0^a f(a-x)dx.$

22. **Assertion (A)**:$\int \frac{e^{\tan^{-1}x}}{1+x^2}$dx =.$e^{\tan^{-1}x} + c$

Reason (R): $\frac{d}{dx} e^{\tan^{-1}x} = \frac{e^{\tan^{-1}x}}{1+x^2}$

23. **Assertion (A):**$\int_0^{2\pi} cos^5 x dx = 0.$

Reason (R): $\int_0^{2a} f(x)dx = 0. if\ f(2a-x) = -f(x).$

24. **Assertion (A):**$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx.$

Reason (R): $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = \frac{b-a}{2}$.

25. **Assertion (A):** $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}$dx= 2π

Reason (R): $\int_0^{2a} f(x)dx = \int_0^a f(x) + f(2a-x)dx$.

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

26. Evaluate: $\int_0^{\frac{\pi}{2}} e^x(\sin x - \cos x)dx$

27. Evaluate: $\int \frac{1}{\sqrt{9x-4x^2}}$ dx

28. Evaluate: $\int \frac{x+sinx}{1+cosx}$ dx.

29. Evaluate $\int \tan^{-1}\sqrt{x}\, dx$

30. If $\int \left(\frac{x-1}{x^2}\right) e^x dx = e^x f(x) + c, then\ write\ the\ value\ of\ f(x)$.

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

31. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{cosx}{(1+\sin x)\ (2+\sin x)}dx$

32. Evaluate: $\int \frac{\sin x+\cos x}{9+16\sin 2x}dx$.

33. Evaluate: $\int_{-1}^{2} |x^3 - x|dx$.

34. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$.

35. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. and hence prove that $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x}dx = \frac{\pi}{4}$.

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

36. Evaluate: $\int_0^{\pi} \frac{x}{a^2cos^2x+b^2sin^2x}dx$.

37. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x\sin x cos\, x}{sin^4x + cos^4\, x}dx$.

38. Evaluate: $\int \frac{x^2}{(x^2+1)(x^2+4)}\ dx$

39. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{cos^2x}{cos^2x+4sin^2x}dx$.

40. Prove that: $\int_0^{\frac{\pi}{2}} \log\sin x\ dx = \int_0^{\frac{\pi}{2}} \log\cos x\ dx = -\frac{\pi}{2}\log 2$.

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

41. The definite integrals are used to calculate the area bounded by the simple curve of the given function within the given limit. The definite integral is given by

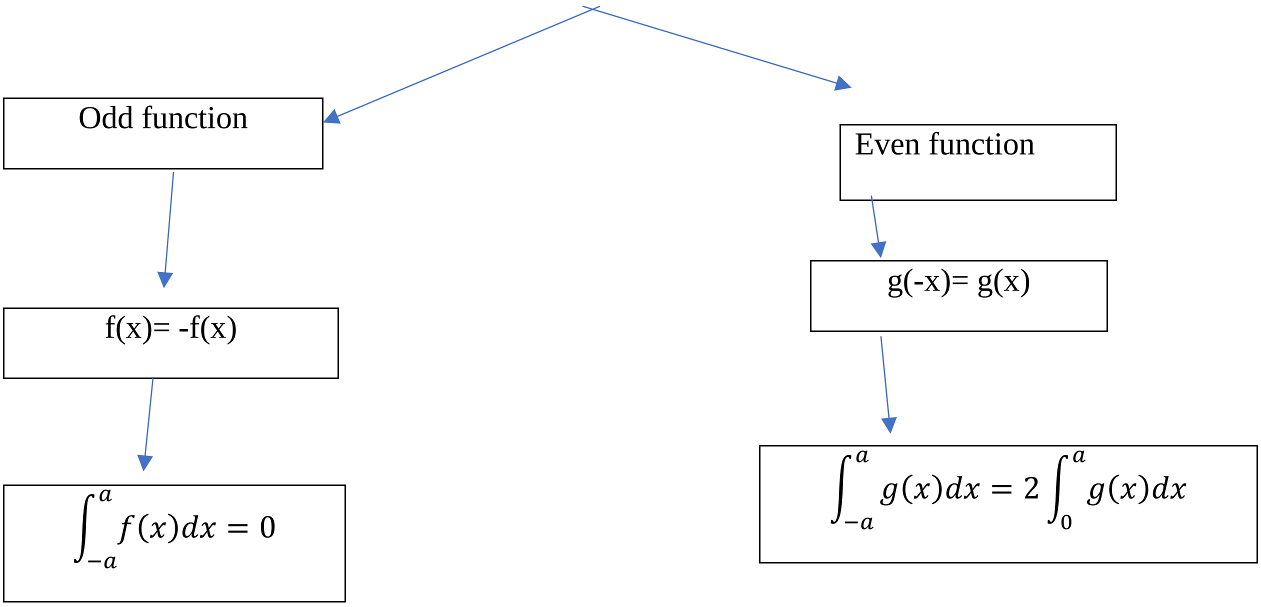
$\int_a^b f(x)dx$ =g(b)−g(a).

Based on the above information, answer the following.

(i) If a = b, then find the value of the definite integral.
(ii) If the function f(x) is an odd function and a = −b, then find the value of definite integral.
(iii)If the function f(x) is an even function and a = −b, then find the value of definite integral.

OR If a=0, then find the definite integral.

42. Read the following text and answer the following questions on the basis of the same:



(i) *Evaluate*: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^3x\ dx, using\ the\ above\ property.$

(ii) Evaluate: $\int_{-\pi}^{\pi} xsin\ xdx, using\ the\ above\ property.$

(iii) Is $\int_{-\pi}^{\pi} tanxsec^2x\ dx$ odd or even function, then what could be its value?

OR

Evaluate: $\int_{-\pi}^{\pi} xsin\ xdx, using\ the\ above\ property.$

43. Read the following text and answer the following questions on the basis of the same:

$$\int e^x[f(x) + f'(x)]dx = \int e^x f(x)dx + \int e^x f'(x)dx$$

$$= f(x)e^x - \int e^x f'(x)dx + \int e^x f'(x)dx$$

$$= e^x f(x) + c$$

(i). Evaluate $\int e^x(\sin x + \cos x)dx$

(ii) Evaluate: $\int_0^{\pi} e^x(\tan x + sec^2 x)dx$

(iii) Evaluate: $\int \frac{xe^x}{(1+x)^2}dx$

OR

Evaluate: $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

ANSWERS AND HINTS

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. (a)	2. (d)	3. (a)	4. (d)	5. (d)
6. (b)	7. (d)	8. (b)	9. (c)	10. (c)
11. (c)	12. (b)	13. (a)	14. (a)	15. (d)
16. (c)	17. (a)	18. (c)	19. (c)	20. (c)

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

21. (d)	22. (a)	23. (a)	24. (d)	25. (a)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

26. 1

27. $\frac{1}{2}\sin^{-1}(\frac{8x-9}{9}) + c$

28. $x\tan\frac{x}{2}$ + c

29. $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x}$ + c

30. $\frac{1}{x}$ + c

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

31. $\log\frac{4}{3}$

32. $-40\log\left[\frac{5+4(\cos x-\sin x)}{5-4(\cos x-\sin x)}\right] + c$

33. $\frac{11}{4}$

34. $\frac{\pi}{12}$

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

36. $\frac{\pi^2}{2ab}$

37. $\frac{\pi^2}{16}$

38. $-\frac{1}{3}\tan^{-1}x+\frac{2}{3}\tan^{-1}\frac{x}{2}+c$

39. $\frac{2}{3}\tan^{-1}2-\frac{\pi}{12}$

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

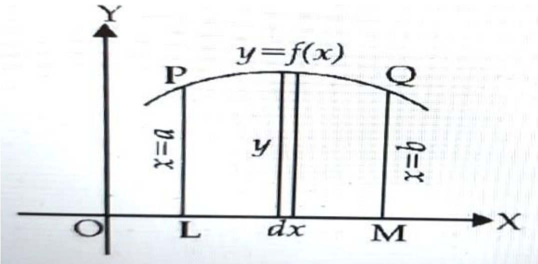
41. (i) 0	(ii)0	(iii)$2\int_0^b f(x)dx$ OR $\int_0^b f(b)dx$
42. (i) 0	(ii) 2π	(iii) 0 OR 0
43. (i) e^xsin x+c	(ii) 0	(iii)$\frac{e^x}{x+1}+c$ OR $e^x\tan\frac{x}{2}+c$

Ch – 8 Application of Integrals

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

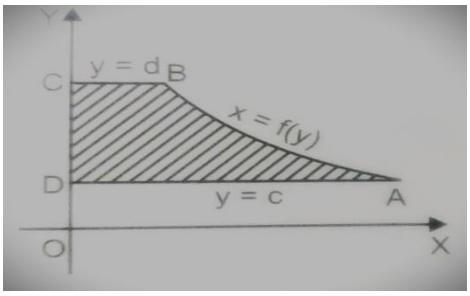
- One of the major applications of integrals is in determining the area under the curves.

1. Consider a function y = f(x), above the x-axis, between the ordinates x = a and x = b then the area is given as

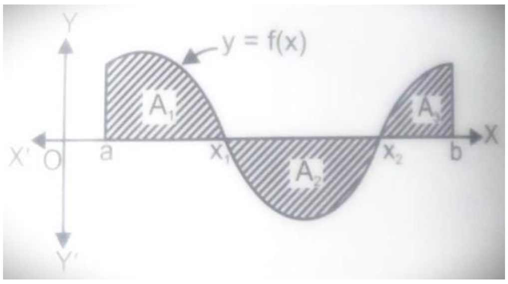


The area of the region $PQML = \int_a^b y dx = \int_a^b f(x) dx$

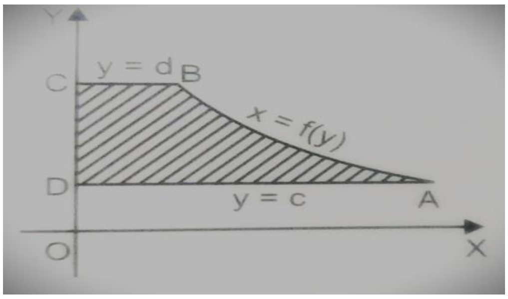
2. If the area is below the x-axis, then $A = \int_a^b f(x) dx$ is negative. Then the area is its magnitude. $\therefore A = \int_a^b -f(x) dx$.



3. If the curve $y = f(x)$ crosses x-axis to a number of times, then the area between the curve $y = f(x)$, and the ordinates x = a and x = b then the area is given as$A = A_1 + A_2 + A_3 + \cdots$



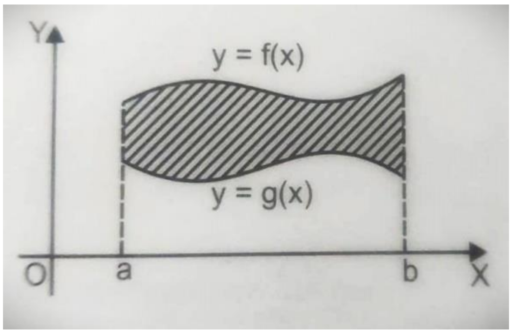
4. The area bounded by the curve x = f(y) and the abscissa y = c and y = d is to the right of y-axis is $A = \int_c^d x dy$.



5. The area bounded by the curve x = f(y) and the abscissa y = c and y = d is to the left of y-axis is $A = \int_c^d -x dy.$

6. Consider the two curves having equation of f(x) and g(x), the area between the region a, b of the two curves is given as

   dA = [f(x) – g(x)] dx, and the total area A can be taken as $Area = \int_a^b [f(x) - g(x)]dx$



MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. The area of the region bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$, and $x = 1$ is
   (a) -9 (b) $\frac{15}{4}$ (c) $-\frac{15}{4}$ (d) $\frac{17}{4}$
2. The area enclosed between the curve $y^2 = x$ and $y = |x|$, is
   (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1
3. The area of the region bounded by the curve $y = x|x|$, the x-axis and the ordinates $x = -1$, and $x = 1$ is
   (b) 0 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$
4. The area bounded by y-axis, $y = \cos x$ and $y = \sin x$ where $0 \le x \le \frac{\pi}{2}$ is
   (a) $2(\sqrt{2} - 1)$ (b) $\sqrt{2} - 1$ (c) $\sqrt{2} + 1$ (d) $\sqrt{2}$

5. The area (in sq. units) enclosed by the circle $x^2 + y^2 = 2$ is equal to
   (a) 4π (b) $2\sqrt{2}\,\pi$ (c) $4\pi^2$ (d) 2π
6. The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to
   (a) $\pi^2 ab$ (b) πab (c) $\pi a^2 b$ (d) $\pi\, ab^2$
7. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is
   (a) $\frac{32}{3}$ (b) $\frac{256}{3}$ (c) $\frac{64}{3}$ (d) $\frac{128}{3}$

8. The area of the region in the first quadrant enclosed by the x- axis, the line $y = x$ and the circle $x^2 + y^2 = 32$ is

   (a) $16\,\pi$ sq. unit (b) $4\,\pi$ sq. unit (c) $32\,\pi$ sq. unit (d) $24\,\pi$ sq. unit

9. The area of the region bounded by the curve $x = 2y + 3$, y-axis and the line y = 1 and y = -1 is

   (a) 4 sq. units (b) $\frac{3}{2}$ sq. units (c) 6 sq. units (d) 8 sq. units

10. The area of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

    (a) $\frac{3}{8}$ sq. units (b) 20π sq. units (c) $\frac{7}{8}$ sq. units (d) $\frac{9}{8}$ sq. units

ASSERTION – REASON QUESTIONS (1 MARK EACH)

Directions: Each of the following questions contains an assertion (A) followed by a reason (R). Read them carefully and answer the questions on the basis of the following options.

(a) Both A and R are true and the R is the correct explanation of A.
(b) Both A and R are true but the R is not the correct explanation of the A.
(c) A is true and R is false.
(d) A is false and R is true.

1. **Assertion (A)**: The area of the region bounded by the curve $y = x + 5$ and the lines $x = 1$ and $x = 4$ is

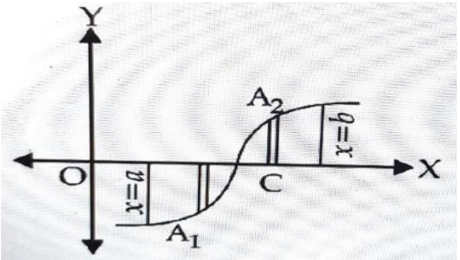
   $\frac{45}{2}$ square units.

   Reason (R): Required area is given by $A = \int_1^5 y\,dy$

2. **Assertion (A):** The region bounded by the curve $y^2 = 16x$, y- axis and the line $y = 2$ is $\frac{8}{3}$ square units.

   Reason (R): Required area $= \int_0^2 x\,dy$

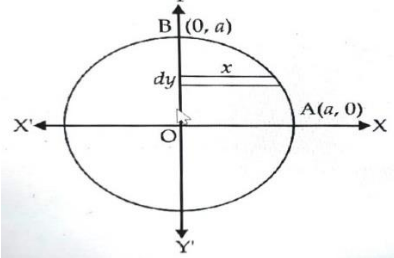
3. **Assertion (A):**



$$Area = |A_1| + |A_2|$$

Reason (R): It may happen that some portion of the curve is above x-axis, and some portion is below x-axis as shown in the figure. Let A_1 be the area below x-axis and A_2 be the area above the x-axis. Therefore, area bounded by the curve y = f(x), x-axis and the ordinates x = a and x = b is given by $Area = |A_1| + |A_2|$

4. **Assertion (A)**: The area enclosed by the circle $x^2 + y^2 = a^2$ is πa^2.



Reason (R): The area enclosed by the circle.

$$= 4\int_0^a x dy$$

$$= 4\int_0^a \sqrt{a^2 - y^2} dy$$

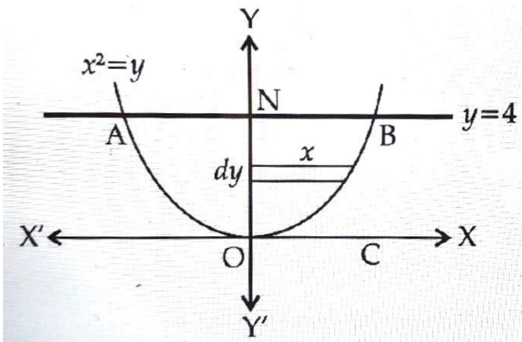
$$= 4\left[\frac{y}{2}\sqrt{a^2 - y^2} + \frac{a^2}{2}\sin^{-1}\frac{y}{a}\right]_0^a$$

$$= 4\left[\left(\frac{a}{2} \times 0 + \frac{a^2}{2}\sin^{-1} 1\right) - 0\right]$$

$$= 4\frac{a^2}{2}\frac{\pi}{2} = \pi a^2.$$

5. **Assertion (A):** The area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is $\frac{3}{32}$.

Reason (R):



Since the given curve represented by the equation $y = x^2$ is a parabola symmetrical about y-axis only, therefore from the figure, the required area of the region AOBA is given by

$$A = 2\int_0^4 x dy = 2\int_0^4 \sqrt{y} dy = 2 \times \frac{2}{3}\left[y^{\frac{3}{2}}\right]_0^4 = \frac{4}{3} \times 8 = \frac{32}{3}$$

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. Find the area bounded by the curve y = 3x, x-axis and between the ordinates x = 1 and x = 3.
2. Find the area bounded by the curve y = x, x-axis and the ordinates x = −1 and x = 2.
3. Find the area bounded by the curve $xy = 4$, x-axis and the line x = 1 and x = 4.
4. Find the area bounded by the curve $y = \cos x$ between 0 and 2π .
5. Find the area of the parabola bounded by $x^2 = 4y, y = 2, y = 4$ and $y - axis$ in the first quadrant.

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
2. Find the area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$.
3. Find the area of the region enclosed by the parabola $y^2 = 4ax$ and the line $y = mx$.

4. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

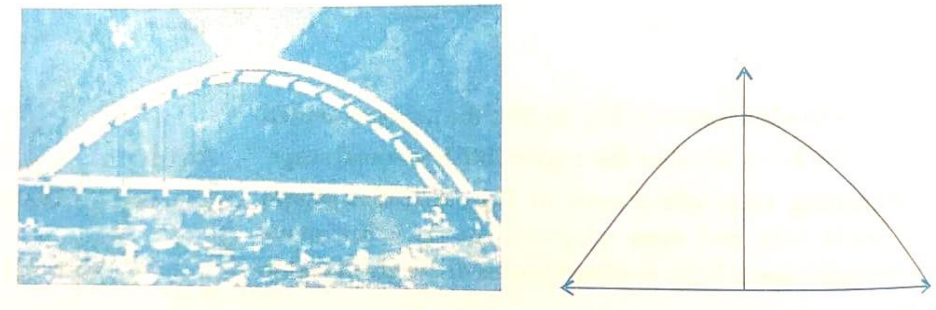
5. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

1. Make a rough sketch of the region $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$ and find the area of the region, using the method of integration.
2. Sketch the graph of $y = |x + 3|$ and evaluate $\int_{-6}^{0} |x + 3|\, dx$.
3. Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$.
4. Using integration, find the area of region bounded by the triangle whose vertices are (- 2, 1), (0, 4) and (2, 3).
5. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration.

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

1. The bridge connects two hills 100 feet apart. The arch of the bridge is 10 feet above the road at the middle of the bridge as seen in fig. Based on the above information answer the following question:

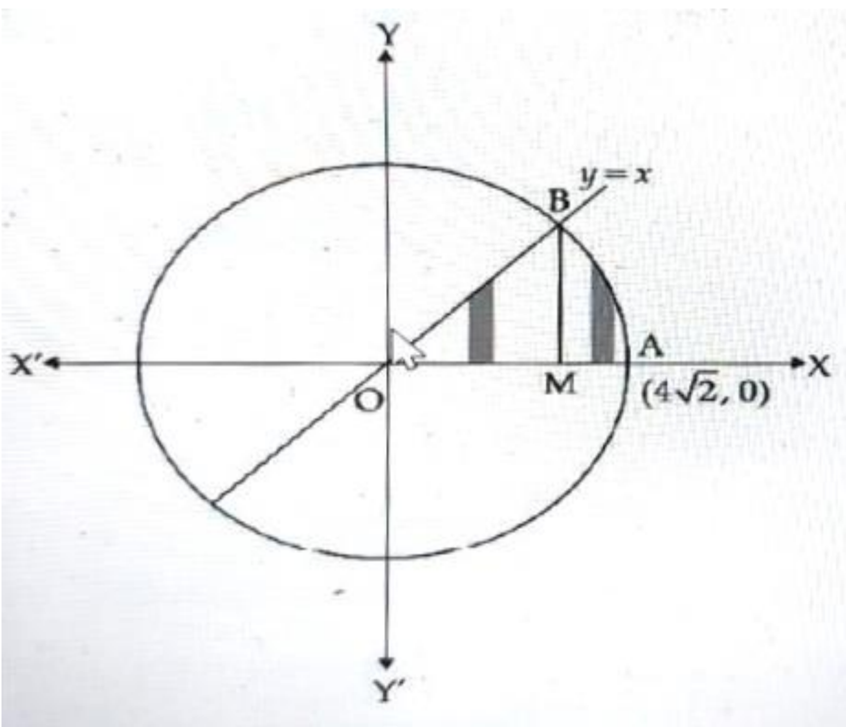


(i) Find the equation of the parabola designed on the bridge.

(ii) What is the value of the integral $\int_{-50}^{50} \frac{x^2}{250} dx$?

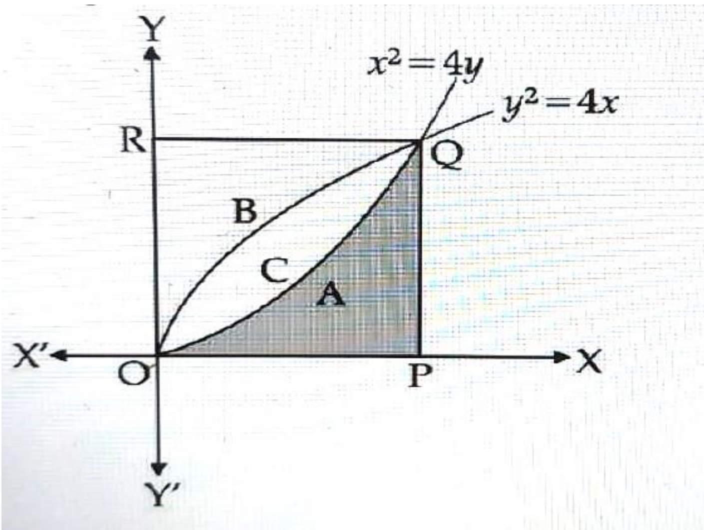
(iii) Find the area formed by the curve $x^2 = 250y, y\ axis, y = 2\ and\ y = 10$.

2. Read the following text and answer the following questions on the basis of the same: In the given figure O (0.0) is the centre of the circle. The line y = x meets the circle in the first quadrant at the point B.



(i) What is the equation of the circle?
(ii) What is the coordinates of the point B.
(iii) Find the area of the portion BAMB.
(iv) Find the area of the shaded region.

3. Read the following text and answer the following questions on the basis of the same:
A farmer has a square plot of land. Three of its boundaries are x = 0, y = 0 and y = 4. He wants to divide this land among his three sons A, B and C as shown in the figure.



(i) Find the coordinates of Q
(ii) Find the area received by son B.
(iii) Find the area received by son A.
(iv) What is the total area of the square field?

Answer key/Hints

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. (d) $\frac{17}{4}$ 2. (b) $\frac{1}{3}$ 3. (c) $\frac{2}{3}$ 4. (b) $\sqrt{2}-1$ 5. (d) 2π

6. (b) πab 7. (b) $\frac{256}{3}$ 8. (b) 4 π sq. unit 9. (c) 6 sq. Units 10. (d) $\frac{9}{8}$ sq. units

ASSERTION AND REASON BASED QUESTIONS (1 MARK EACH)

1. Option (c)
2. Option (d)
3. Option (a)
4. Option (a)
5. Option (d)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. 12 sq. units
2. $\frac{5}{2}$ sq. units

3. 4 log 4 sq. units
4. 4 sq. units
5. $\frac{8}{3}\left(4-\sqrt{2}\right)\ sq.units$

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1. πab square units.
2. π square units.
3. $\frac{8a^2}{3m^3}$ square units.
4. $\frac{1}{3}$ square units.
5. $\frac{a^2}{2}(\frac{\pi}{2}-1)$ square units.

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

1. $\frac{23}{6}$ square units.
2. 9 square units.
3. $\frac{13}{3}$ square units
4. 4 sq units
5. $\frac{4\pi}{3}$sq. units.

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

1.
 i. $x^2 = -250y$
 ii. $\frac{1000}{3}$
 iii. $\frac{1000}{3}$ sq. units

2.
 i. $x^2 + y^2 = 32$
 ii. (4, 4)
 iii. $4\pi - 8$
 iv. 4π

3.
 i. (4, 4)
 ii. $\frac{16}{3}$
 iii. $\frac{16}{3}$
 iv. 16

Ch – 9 Differential Equations

Definition, order, and degree, general and solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + py = q, \text{where p and q are functions of x or constants.}$$

$$\frac{dx}{dy} + px = q, \text{where p and q are functions of y or constants.}$$

Definitions:

An equation involving independent variable (variables), dependent variable and derivative or derivativesof dependent variable with respect to independent variable (variables) **is called a differential equation.**

Eg. $\frac{dy}{dx} - \sin x = 0$ and $\frac{d^3y}{dx^3} + x^2\left(\frac{d^2y}{dx^2}\right) = 0$ are differential equations, but 2x-3y=0 is not a differential equation as derivative or dependent variable(y) with respect to independent variable(x) is not present.

Order of a Differential equation

The order of the highest order derivative of dependent rankle with respect to independent variable involved in a differential equation, is called order of differential equation.

e. g. (i) $\frac{dy}{dx} - \sin x = 0$ (ii) $\frac{d^3y}{dx^3} + x^2\left(\frac{d^2y}{dx^2}\right) = 0$

Here, in e.g (i), equation has the highest derivative of first order and in e.g. (ii), equation has the highest derivative of third order. So, orders of the differential equations in e.g (i) and (ii) are I and 3, respectively.

Degree of a DifferentialEquation

The highest power (positive integral index) of the highest order derivative involved in a differentialequation, when it is written as a polynomial in derivatives, is called the degree of a differential equation.

e. g. $\left(\frac{d^{3y}}{dx^3}\right)^2 + x\left(\frac{d^3y}{dx^3}\right) + 3y\left(\frac{dy}{dx}\right) = 0$

In this, the highest order derivative is $\frac{d^3y}{dx^3}$ whose highest power is 2. So, degree of differential equation is 2.

Solution of a Differential Equation

Suppose a differential equation is given to us, in which y is dependent variable and x is independent
variable. Then the function □ □x□ will be its solution. If it satisfies the given differential equation, i.e.

when the function □ is substituted for the unknown y (dependent variable) in the given differential equation. LHS becomes equal to RHS. The solution of a differential equation is of two types, which are given below.

General Solution of a Differential Equation

If the solution of the differential equation of order n contains n arbitrary constants, then it is called a general solution. eg. The general solution of $\frac{d^2y}{dx^2}+y=0$ is $A\cos x+BSinx$. But $y=A\cos x+\sin x$ and $y=\cos x+B\sin x$ is not the solution of given differential equation, as it contains only one arbitrary constant.

Particular Solution of a Differential Equation

The solution of a differential equation obtained by giving Particular values to the arbitrary constants in thegeneral solution, is called the particular solution. Other words, the solution free from arbitrary constant is called particular solution.

The general solution of $\frac{d^2y}{dx^2}+y=0$ is $y=A\cos x+\sin x$. If A=B=1, then $y=\cos x+\sin x$ is a particular solution of the given differential equation.

Differential Equation with Variables Separable

A order and first degree differential equation $\frac{dy}{dx}=F(x,y)$ is in the form of variable separable, if the function F can be expressed as the product of the functions of x and the functions of y Suppose a first order and first degree differential equation is given to us, i.e. $\frac{dy}{dx}=F(x,y)$.

Now , expressed it as $\frac{dy}{dx}=h(y),g(x),if h(y)\neq 0$ it can be written as $\frac{1}{h(y)}dy=g(x)dx$.

Integrate both sides

$$\int \frac{1}{h(x)}dy=\int g(x)dx$$

It is the required solution of given differential equation.

Homogeneous Differential Equation

A function F(x,y) is said to be homogeneous of degree n, if $F(x,y)=x^n g\left(\frac{y}{x}\right) or\ \ y^n h\left(\frac{x}{y}\right)$

A differential equation of the form $\frac{dy}{dx}=F(x,y)$ is called a homogeneous differential equation, if F(x,y) is a homogeneous function of degree zero.

Linear Differential Equation

A differential equation of the form $A_0\frac{d^ny}{dx^n}+A_1\frac{d^{n-1}}{dx^{n-1}}+A_2\frac{d^{n-2}y}{dx^{n-2}}+.......+A_{n-1}\frac{dy}{dx}+A_n y=0$ where $A_0,A_1,A_2....A_{n-1},A_n$ are either constants or functions of independent variable x, is called a linear differential equation.

Linear Differential Equation of First Order

A first order differential equation in which the degree of dependent variable and its derivative is one and they do not get multiplied together, is called a linear differential equation of first order.

A differential equation of the form $\frac{dy}{dx}+Py=Q$, Where P and Q are constants of x only. e.g $\frac{dy}{dx}+2y=\sin x$, P = 2 and $Q=\sin x$

Integrating Factor (IF):

Linear differential equations are solved when they are multiplied by a factor, which is called the integrating factor, because by multiplying such factor the left hand side of the differential equationbecome exact differential of same function.

For differential equation,

$\frac{dy}{dx}+Py=Q$

$IF={}_{e}\int pdx$

Now, use the formula, $y-IF=\int(Q.F)dx+C$.

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. The order and the degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^2=4\frac{d^3y}{dx^3}$ respectively are:

   (a) $1,\frac{2}{3}$ (b) 3,1 (c) 3,1 (d) 1,2

2. Write the order and the degree of the differential equation $\left(\frac{d^4y}{dx^4}\right)^2=\left[x+\left(\frac{dy}{dx}\right)^2\right]^3$

   (a) 4,2 (b) 3,1 (c)3,3 (d)1,2

3. The degree of the differential equation

$$\left(\frac{d^{2y}}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2=x\sin\left(\frac{dy}{dx}\right) \text{ is } 0$$

   (a) 1 (b) 2 (c) 3 (d) not defined

4. The order of differential equation $\frac{d^4y}{dx^4}+\sin\left(\frac{d^2y}{dx^2}\right)=0is$

   (a) 2 (b) 4 (c) 1 (d) None of these

5. Integrating factor of the differential equation $(x+y)\frac{dy}{dx}=1$ is

   (a) $\cos x$ (b) $\tan x$ (c) $\sec x$ (d) $\sin x$

6. Which of the following differential equation satisfied by $y=e^{mx}$

(a) $\frac{dy}{dx}+my=0$ (b) $\frac{dy}{dx}-my=0$ (c) $\frac{d^2y}{dx^2}+m^2y=0$ (d) $\frac{d^2y}{dx^2}-m^2y=0$

7. The General solution of differential equation $(x+y)\frac{dy}{dx}=1$ is

(a) $2x+y=ce^{-y}$ (b) $x+y=ce^{-y}$ (c) $(x+y+1)=ce^{y}$ (d) $x-y=ce^{-y}$

8. The degree of $2x^2\frac{d^2y}{dx^2}=3\frac{dy}{dx}+y=0$ is

(a) 2 (b) 1 (c)) (d) not defined

9. General solution of the differential equation are $\frac{ydx-xdy}{y}=0$ is

(a) $xy=c$ (b) $x=cy^2$ (c) $y=cx$ (d) $y=cx^2$

10. If $x\frac{dy}{dx}=y(\log y-\log x+1)$ then the solution is

(a) $\log\left(\frac{x}{y}\right)=cy$ (b) $\log\left(\frac{y}{x}\right)=cx$ (c) $x\log\left(\frac{y}{x}\right)=cy$ (d) $y\log\left(\frac{x}{y}\right)=cx$

11. The sum of order and degree of differential equation $\left(\frac{d^2y}{dx}\right)^2+x^2\left(\frac{dy}{dx}\right)^3=0$ is

(a) 3 (b) 4 (c) 5 (d) 7

12. The number of solution of $\left(\frac{dy}{dx}\right)=\frac{y+1}{x-1}$ *when* y(1) =2 is

(a) 1 (b) 2 (c) infinite (d) none

13. The order of the differential equation $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ is

(a) 2 (b) 1 (c) 0 (d) not defined

14. The general solution of $\frac{dy}{dx}\neq y\tan x=\sec x$ is

(a) $y\sec x=\tan x+c$ (b) $y\tan x=\sec x+c$

(c) $\tan x=y\tan y+c$ (d) $x\sec x=\tan y+c$

15. The general solution of the differential equation $e^x dy+\left(ye^x+2x\right)dx=0 is$

(a) $xe^y+x^2=c$ (b) $xe^y+y^2=c$ (c) $ye^x+x^2=c$ (d) $ye^y+x^2=c$

16. A homogenous differential equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution

(a) $y = vx$ (b) $v = yx$ (c) $x = xy$ (d) $x = v$

17. The general solution of the differential equation 18. Which of the following differential equations has $y = c_1e^x + c_2e^{-x}$ has the general solution

(a) $\frac{d^2}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$ (c) $\frac{d^2y}{dx^2} + 1 = 0$ (d) $\frac{d^2y}{dx^2} - 1 = 0$

18. The number of arbitrary constants in the general solution of a differential equation of fourth order are

(a) 0 (b) 2 (c) 3 (d) 4

19. The number of arbitrary constants in the particular solution of a differential equation of third order are

(a) 3 (b) 2 (c) 1 (d) 0

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

Direction: Each of these questions contains two statement, Assertion and Reason. Each of these questions also has fur alternative choices have to select one of the codes (a), (b), (c) and (d) given below-

(a) Assertion is correct, Reason is correct Reason is a correct explanation for Assertion.
(b) Assertion is correct, Reason is correct Reason is a correct. Reason is not correct explanation for Assertion
(c) Assertion is correct, Reason is wrong
(d) Assertion is incorrect, Reason is correct.

20. **Assertion(A)** : Order of the differential equation whose solution is

$$y = C_1e^{x+c_2} + C_3e^{x+c4}$$ is 4

Reason(R): Order of the differential equation is equal to the number of independent arbitrary constant mentioned in the solution of differential equation.

21. **Assertion(A)**: The Degree of the differential equation $\frac{d^2y}{dx} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2x}{dx^2}\right)$ is not defined.

Reason(R): If the differential equation is a polynomial in terms of its derivatives then its degree is defined.

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. Write the sum of the order and degree of the differential equation.

$$1+\left(\frac{d^2y}{dx^2}\right)^5=7\left(\frac{d^3y}{dx^3}\right)^4$$

2. The integrating factor of the differential equation $x\frac{dy}{dx}-y=\log x$ is ?
3. The solution of the differential equation $x\frac{dy}{dx}+y=e^x$ is ?
4. The difference of degree and order of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}=\frac{d^2y}{dx^2}$.
5. Show that the differential equation given by $x^2\frac{dy}{dx}=x^2-2y^2+xy$ is Homogeneous.
6. Write the degree and the order of the differential equation $y^m+y^2+e^y=0$
7. The integrating factor of $\sin x\frac{dy}{dx}+(2\cos x)y=\sin x.\cos x$ is ?
8. The General solution of $\frac{dy}{dx}=\sqrt{4-y^2}$ where -2<y<2
9. Form the differential equation of the family of the curves $y=a\sin(x+b)$ where a.b are arbitrary constants.
10. Find the differential equation of a curve passing through the point (0,-2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x - coordinate of the point.

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1. Find a particular solution of the differential equation ; given y=0 when x=1
   -
2. Solve the differential equation $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$. subject to the initial condition y(0)=0
3. Find the solution of the differential equation $\log\left(\frac{dy}{dx}\right)=ax+by$
4. Find the general solution of the differential equation $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x}$
5. Solve the differential equation $(e^x+1)ydy=e^x(y+1)dx$
6. Find the particular solution of the differential equation $\frac{dy}{ds}=y\tan x$ when y(0)=1
7. Solve the differential equation $x(x^2-1)\frac{dy}{dx}=1$, y=0 when x=2
8. Find the particular solution of the differential equation $xdx-ye^y\sqrt{1+x^2}dy=0$, given that y=1 when x=0
9. Solve the differential equation $\frac{dy}{dx}+2xy=y$

10. Solve the differential equation $x cox\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

11. Find the particular solution of the differential equation $x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$

12. Find the general solution of the differential equation $x\left(y^3 + x^3\right)dy = \left(2y^4 + 5x^3y\right)dx$

13. Find the general particular solution of the differential equation $\frac{dy}{dx} + y\sec x = \tan x$ where $x \in \left[0, \frac{\pi}{2}\right]$ given that $y = 1$ when $x = \frac{\pi}{4}$

14. Solve the differential equation $\frac{dy}{dx} = \frac{x+y}{x-y}$

15. Solve the differential equation $\left(1+x^2\right)dy + 2xydx = \cot xdx$

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

1. Find the particular solution of the differential equation $(x+y)\frac{dy}{dx} = (x+2y)$
2. Find the particular solution of the differential equation $x\frac{dy}{dx} + y - x + xy\cot x = 0$
3. Solve the differential equation $\left(\tan^{-1}y - x\right)dy = \left(1+y^2\right)dx$ given that $x = 1$ when $y = 0$
4. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ given that $y = 1$ when $x = 0$
5. Find the particular solution of the differential equation $\left(3xy + y^2\right)dx + \left(x^2 + xy\right)dy = 0$ for $x = 1$, $y = 1$.

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

CASE STUDY -1

1. An equation involving derivatives of the dependent variable with respect to the equation. A differential equation of the form $\frac{dy}{dx} = F(x,y)$ is said to be homogeneous if F(x,y) is a homogeneous function of degree zero whereas a function F(x,y) is a homogeneous function of degree n.

   If $F\left(\lambda x, xy) = \lambda^n F(x,y)\right)$ To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x,y) = g\left(\frac{y}{x}\right)$.

   We make the substitution y=vx and the separate the variables. Based on the above, answer the following questions:

(i) Show that $\left(x^2-y^2\right)dx+2xy=0$ is differential equation of the type $\frac{dy}{dx}=g\left(\frac{y}{x}\right)$

(ii) Solve the above equation to find its general solution.

CASE STUDY -2

2.

A first order – first degree differential equation is of the form $\frac{dy}{dx}=F(x,y)$

If F(x,y) can be expressed as product of g(x).h(y) where g(x) is a function of x and h(y) is a function

of y then

$$\frac{dy}{dx}=g(x).h(x) \Rightarrow \int \frac{1}{h(x)}dy=\int g(x).dx$$

The solution of differential equation by this method is called variable separable.

Based on the above information answer the following questions.

(i) Find the general solution of differential equation : $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

(ii) What is solution of differential equation

$$\frac{dy}{dx}=-4xy^2$$

Answer key / Hints

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1-(b)	2-(a)	3-(d)	4-(b)	5-(c)	6-(d)	7- (c)	8-(a)	9-(c)	10-(b)
11-(b)	12-(1)	13-(a)	14-(a)	15-(c)	16-(c)	17-(a)	18-(b)	19-(d)	

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

20-(d) 21-(a)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. Order = 3, degree = 4, $\therefore$ Sum = 7

2. $IF = e^{-\int \frac{1}{x} dx} = e^{\log \frac{1}{x}} \therefore IF = \frac{1}{x}$

3. $IF = x$, $\therefore$ $yx = \int \frac{e^x x}{x} dx$

$y = \frac{e^x}{x} + \frac{C}{x}$

4. Squaring both the sides $\left(1+y^2{}_1\right)^3 = \left(y_2\right)^2$ order = 2, degree = 2

$\therefore$ difference = 0.

5. In each term sum of powers of variable (x, y) is same equal to 2.

6. Order = 3, degree not defined

7. $IF = e^{\int P dx}$ $\therefore$ $IF = e^{2\int \cot x dx}$

$= e^{\log(\sin x)^2}$

$= (\sin x)^2$

8. $\int \frac{dy}{\sqrt{4-y^2}} = \int dx \therefore \sin^{-1}\left(\frac{y}{2}\right) = x + C \therefore y = 2\sin(x+C)$

9. $y_1 = a\cos(x+b)$ Differentiate with respect to x again $y_2 = -a\sin(x+b)$

$\therefore y_2 = -y \therefore \frac{d^2y}{dx^2} + y = 0$.

10. $y\frac{dy}{dx} = x$

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1.

$$\frac{dy}{dx}=1+x+y+xy$$

$$\frac{dy}{dx}=(1+x)(1+y)$$

$$\int\frac{dy}{1+y}=(1+x)\,dx$$

$$\log\left|(1+y)\right|=x+\frac{x^2}{2}+C$$

Put $y=0$ and $x=1 \therefore C=\frac{3}{2}$

$$\therefore 2\log\left|(1+y)\right|=2x+x^2-3$$

2. $\left(1+x^2\right)\frac{dy}{dx}+2xy-4x^2=0$ is given differential equation.

$$\therefore \frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{4x^2}{1+x^2}$$

$$IF=e^{\int\frac{2x}{1+x^2}dx}$$

$$\therefore \quad =e^{\log\left(1+x^2\right)}$$

$$=1+x^2$$

$$\left(1+x^2\right)y=\int 4x^2dx$$

$$\therefore$$

$$\left(1+x^2\right)y=\frac{4}{3}x^3+C$$

Put $x=0$ and $y=0$

$\therefore C=0$

Solution $y=\dfrac{4x^3}{3\left(1+x^2\right)}$

3. $\log\left(\frac{dy}{dx}\right)=ax+by$ given

$$\therefore \frac{dy}{dx}=e^{ax+by}$$

$$\therefore e^{-by}dy=e^{ax}dx$$

$\therefore \dfrac{e^{ax}}{a}+\dfrac{e^{-by}}{b}+C=0$ is the required solution.

4. We have $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x} \Rightarrow \therefore \frac{dy}{dx}=\frac{e^x-1}{x}$

$\therefore \int \frac{dy}{e^y-1}=\int \frac{dx}{x}$

$\therefore \log\left(1-e^{-y}\right)=\log C$

$\therefore e^{-y}=1-Cx$

Or $-y=\log(1-Cx)$

$\therefore y+\log(1-Cx)=0$

5. Given DE is $\left(e^x+1\right)dy=e^x\left(y+1\right)dx$

So, on separating variables and integrating both sides

$\int \frac{y}{y+1}dy=\int \frac{e^x}{e^x+1}dx$

Or $\int\left[1-\frac{1}{y+1}\right]dy=\int \frac{e^x}{e^x+1}dx$

$\therefore y-\log\left(y+1\right)=\log\left(e^x+1\right)+C$

6. We have $\frac{dy}{dx}=y\tan x$

On separating variable and integrating both sides

$\therefore \int \frac{dy}{y}=\int \tan x dx$

$\therefore \log y=\log(\sec x)+\log C$

$\therefore y=C\sec x$

Put $x=0$ and $y=1 \therefore C=1$

$\therefore$ Required Solution will be $y=\sec x$.

7. So given differential equation is $x\left(x^2-1\right)\frac{dy}{dx}=1$, $y=0$ and $x=2$.

$\therefore \frac{dy}{dx}=\frac{1}{x\left(x^2-1\right)}=\frac{1}{x(x+1)(x-1)}$

$\therefore \frac{1}{x(x+1)(x-1)}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$ Hint $x=0$, $x=-1$, $x=1$

Putting $x=0$, $x=-1$ and $x=1$ in above equation.

In Alternate sequence we get $A=-1, B=\frac{1}{2}$ and $C=\frac{1}{2}$.

$$\therefore dy = \int\left(-\frac{1}{x}\right)dx + \frac{1}{2}\int\frac{dx}{x+1} + \frac{1}{2}\int\frac{dx}{x-1}$$

$\therefore y = -\log|x| + \frac{1}{2}\log|x+1| + \frac{1}{2}\log|x-1| + C$, Putting $x=2$ and $y=0$

$$\therefore C = \log\frac{2}{\sqrt{3}}$$

$$\therefore y = \log\left(\frac{2\sqrt{3}-\sqrt{x^2-1}}{3x}\right)$$

8. Given DE is $xdx - ye^{y}\sqrt{1+x^2}\,dy = 0$ given that $y=1,\ x=0$

$$\therefore \frac{1}{2}\int\frac{2x}{\sqrt{1+x^2}}dx = \int \underset{I}{y}\,\underset{II}{e^{y}}\,dy = \left[y\int e^{y}dy - \int\left(\frac{d}{dy}(y)\cdot\int e^{y}dy\right)dy\right]$$

$$\therefore \left(1+x^2\right)^{1/2} = e^{y}\left(y-1\right)+C$$

Put $y=1$ and $x=0$

$$\therefore C=1$$

$\therefore$ Required solution will be $\left(1+x^2\right)^{1/2} = e^{y}\left(y-1\right)+1$.

9. Given DE is $\frac{dy}{dx} + 2xy = y$ .... (1)

Comparing $\frac{dy}{dx} + Py = Q$ there is no result because Q is not function of x.

$\therefore$ From equation (1) $\frac{dy}{dx} + (2x-1)y = 0$

Now comparing above with $\frac{dy}{dx} + Py = Q$,

$P = 2x-1$ and $Q = 0$

$$\therefore IF = e^{\int Pdx}$$

$$= e^{x^2-x}$$

$$\therefore y\cdot IF = \int Q\cdot IF dx + C$$

$$\therefore ye^{x^2-x} = C$$

$$\therefore y = Ce^{x-x^2}$$

10. Given DE is $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

$\therefore \frac{dy}{dx} = \dfrac{y\cos\left(\frac{y}{x}\right)+x}{x\cos\left(\frac{y}{x}\right)}$ is Homogeneous

$\therefore$ Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \int\cos v\,dv = \int\frac{dx}{x}$$

$\sin v = \log|x| + \log|C|$

$\therefore \sin\left(\frac{y}{x}\right) = \log|Cx|$

Which is required solution.

11	$\frac{dy}{dx} + \frac{1}{x}y = -\frac{1}{x(1+x^{2)}}$ I.F = $\int_e \frac{1}{x}dx = e^{\log x} = x$ Solution is y.x = $\int -\frac{1}{1+x^2}dx + c$ xy= $-\tan^{-1}x + c$ y(1)=0 $c = \frac{\pi}{4}$ Particular solution is $ny = \frac{\pi}{4} - \tan^{-1}x$
12	Given D.E can be written as $\frac{dy}{dx} = \frac{2y^4 + 5x^3y}{xy^3 + x^4}$ ---------------------------(i) let y=vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ Eq (i) becomes $v + x\frac{dv}{dx} = \frac{2v^4 + 5v}{dx} = \frac{2v^4 + 5v}{v^3 + 1}$ $\Rightarrow x\frac{dv}{dx} = \frac{v^4 + 4v}{v^3 + 1}$ $\Rightarrow \int \frac{4v^3 + 4}{v^4 + 4v}dv = 4\int \frac{dx}{x}$ $\Rightarrow \log\left|v^3 + 4v\right| = \log(x)^4 + \log c$ $\Rightarrow \log\left|\frac{y^4 + 4yx^3}{x4}\right| = \log Cx^4$

	$\therefore\ y^4+4yx^3=cx^8$
13	Given D.E is $\frac{dy}{dx}+y\sec x=\tan x$ If =e $\int_e pdx=\int_e \sec xdx=e^{\log(\sec x+\tan x)}=\sec x+\tan x$ Required solution, $y(\sec x+\tan x)=\int \tan x(\sec x+\tan x)dx+c$ = $\int \tan x.\sec xdx+\int \tan^2 xdx+c$ = $\sec x+\int(\sec^2 x-1)dx+c$ $y(\sec x+\tan x)=\sec x+\tan x-x+c$ y= 1 when $x=\frac{\pi}{4}$ $\Rightarrow 1\left(\frac{1}{\sqrt{2}}+1\right)=\frac{1}{\sqrt{2}}+1-\frac{\pi}{4}+c \qquad \Rightarrow \frac{1}{\sqrt{2}}+1=\frac{1}{\sqrt{2}}+1-\frac{\pi}{4}+c$ i. e. $c=\frac{\pi}{4}$ Required solution = $y(\sec x+\tan x)=\sec x+\tan-x+\frac{\pi}{4}$
14	$\frac{dy}{dx}=\frac{x+y}{x-y}=\frac{1+\frac{y}{x}}{1-\frac{y}{x}}$ Say $y=vx \ \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ $\therefore v+x\frac{dv}{dx}=\frac{1+v}{1-v}$ $\Rightarrow x\frac{dv}{dx}=\frac{1+v}{1-v}-v=\frac{1+v-v+v^2}{1-v}$ $\Rightarrow \int\frac{1-v}{1+v^2}dv=\int\frac{dx}{x}$ $\Rightarrow \int\frac{1}{1+v^2}dv-\frac{1}{2}\int\frac{2v}{1+v^2}dv=\int\frac{dx}{x}$ $\Rightarrow \tan^{-1}v=\frac{1}{2}\log\|1-v^2\|+\log\|x\|+C$

	$\Rightarrow \tan^{-1}\dfrac{y}{x}=\dfrac{1}{2}\log\left\|\dfrac{x^2+y^2}{x^2}\right\|+\log\|x\|+C$ $\therefore \tan^{-1}\dfrac{y}{x}=\dfrac{1}{2}\log\left\|x^2+y^2\right\|+C$
15	$(1+x^2)dy+2xydx=\cot x dx$ $\Rightarrow \dfrac{dy}{dx}+\dfrac{2x}{1+x^2}y=\dfrac{\cot x}{1+x^2}$ I.F = $\int_E \dfrac{2x}{1+x^2}dx = {}_e\log\left\|1+x^2\right\|=1+x^2$ Sol. is $y.(1+x^2)=\int\cot x dx=\log\|\sin x\|+C$ or, $y=\dfrac{1}{1+x^2}\log\|\sin x\|+\dfrac{c}{1+x^2}$

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

1	$\dfrac{dy}{dx}=\dfrac{x+2y}{x-y}=\dfrac{1+\frac{2y}{x}}{1-\frac{y}{x}}$ $y=xV$ $\Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$ $\Rightarrow v+x\dfrac{dv}{dx}=\dfrac{1+2v}{1-v}$ $\Rightarrow x\dfrac{dv}{dx}=-\dfrac{1+2v-v-v^2}{v-1}$ $\Rightarrow \int\dfrac{2v+1}{v^2+v+1}dv$ $\Rightarrow \dfrac{v-1}{v^2+v+1}dv=-\dfrac{dx}{x}$

	$\Rightarrow \int \frac{2v+1}{v^2+v+1}dv = \int -\frac{2}{x}dx$ $\Rightarrow \frac{2v+1}{v^2+v+1}dv - 3\int \frac{1}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} - dv = -\int \frac{2dx}{x}$ $\Rightarrow \log\left\|v^2+v+1\right\| - 3\frac{2}{\sqrt{3}}\tan^{=1}\left(\frac{2v+1}{\sqrt{3}}\right) = -\log\|x\|^2 + C$ $\Rightarrow \log\left\|y^2+xy+x^2\right\| - 2\sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3x}}\right) = C$ $\Rightarrow$ x=1 , y=0 $C = -2\sqrt{3}\frac{\pi}{6} = -\frac{\sqrt{3}}{3}\pi$ $\log\left\|y^2xy+x^2\right\| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3x}}\right) + \frac{\sqrt{3}}{3}\pi = 0$
2	$x\frac{dy}{dx} + y - x + xy\cot x = 0 \qquad x \neq 0$ $\Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$ I.F = $\underset{e}{\int} pdx = \underset{e}{\int}\left(\frac{1}{x}+\cot x\right)dx {=}_e \log\|x\| + \log\|\sin x\|$ = $x\sin x$ Sol of D.E. $y.x\sin x = \int x\sin xdx$ $xy.\sin x = -x\cos x + \sin x + C$ When $x = \frac{\pi}{2}, y = 0, C = -1$ Required Sol. $y. = \frac{1}{x} - \cot x - \frac{1}{x\sin x}$
3.	$\frac{dx}{dy} = \frac{\tan^{-1}}{1+y^2} - \frac{x}{1+y^2}$

	$\Rightarrow \frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$ Proceed to obtained $x.e^{\tan^{-1}}y=e^t(t-1)+C$ $x=\tan^{-1}y-1+\frac{c}{\tan^{-1}y}$ If x=1 y=0 gives C=2 Sol. $x=(\tan^{-1}y-1)+2/e^{\tan-1y}$
4	$\frac{dy}{dx}=\frac{xy}{x^2+y^2}=\frac{\frac{y}{x}}{1+\left(\frac{y}{x}\right)^2}$ Let y =Vx $\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ $\Rightarrow v+x\frac{dv}{dx}=\frac{v}{1+v^2}$ $\Rightarrow \int\left(\frac{1}{v^3}+\frac{v^2}{v^3}\right)dv=-\int\frac{dx}{x}$ $\Rightarrow \frac{-x^2}{2y^2}+\log\lvert v\rvert=-\log\lvert x\rvert+C$ $\Rightarrow \frac{-x^2}{2y^2}+\log\lvert y\rvert=C$ When x=0 y=1 C=0 So, $\frac{-x^2+2y^2\log\lvert y\rvert}{2y^2}=0$ $\Rightarrow 2y^2\log\lvert y\rvert-x^2=0$
5	Do yourself When x=1, y=1 Sol. $x^2(y^2+2x)=3$

<u>CASE STUDY BASED QUESTIONS (4 – MARKS EACH)</u>

1	(i) $\left(x^2 - y^2\right)dx + 2xydy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} = \frac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)} = g\left(\frac{y}{x}\right)$
	(ii) $y = vx$ $\Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$ $\Rightarrow v + x\frac{dy}{dx} = \frac{v^2 - 1}{2v} - v = \frac{-1 - v^2}{2v}$ $\Rightarrow \int \frac{2v}{1+v^2}dv = -\int \frac{dx}{x}$ $\Rightarrow \log\left\|1+v^2\right\| + \log\|x\| = \log C$ $\Rightarrow x\left(1 + \frac{y^2}{x^2}\right) = C$ $\therefore x^2 + y^2 = Cx$
2.	(i) $\tan^{-1} y = \tan^{-1} x + C$ (ii) $y = \frac{1}{2x^2 - c}$

Vectors and scalars, magnitude, and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in given ratio. Definition, Geometrical Interpretation, properties, and application of scalar (dot) product of vectors, vector (cross) product of vectors.

Important Points

Vector: A directed line segment is called a Vector. It has a magnitude and direction. Any number is called a scalar.

If $\vec{a} =$ a $\hat{\imath}$ + b $\hat{\jmath}$ + c $\hat{k}$, then $|a| = \sqrt{a^2 + b^2 + c^2}$

Unit Vector*:* A vector with unit length along any vector $\vec{a}$ is called a unit vector in the direction of a and it is denoted by $\hat{a}$

Thus $\hat{a} = \frac{\vec{a}}{|a|}$

Collinear Vector*:* Two or more non-zero vectors are said to be collinear if they are parallel to the same line. Two vectors $\vec{a}$ and $\vec{b}$ are collinear if $\vec{a} = \gamma\, \vec{b}$ for some scalar γ.

Position Vector of a Point: Let O be the origin and OA= $\vec{a}$ we say that the position vector of A is $\vec{a}$

$\overrightarrow{AB}$=(Position vector of B) – (Position vector of A)

Section Formula: Let A and B be two points with position vector $\vec{a}$ and $\vec{b}$ respectively and let P be a point which divides AB in the ratio m: n. Then position vector of

P(r) = $\frac{m\vec{b}+n\vec{a}}{m+n}$

Component of a Vector: If $\vec{A}$= a $\hat{\imath}$ + b $\hat{\jmath}$ + c $\hat{k}$ we say that the component of $\vec{A}$ along X-axis, Y- axis and Z-axis are a,b,c respectively.

Dot or Scalar Product*:*

If θ is the angle between $\vec{a}$ and $\vec{b}$ then $\vec{a}.\vec{b} = |a|\ |b|\cos\theta$

Vector Projection of $\vec{a}$ on $\vec{b} = (\frac{\vec{a}.\vec{b}}{|b|^2})\,\vec{b}$

Cross or Vector Product:

If θ is the angle between $\vec{a}$ and $\vec{b}$ then $\vec{a}$x$\vec{b} = |a|\ |b|\sin\theta\ \hat{n}$ where $\hat{n}$ is a unit vector perpendicular to $\vec{a}$ and $\vec{b}$

Area of a parallelogram with sides $\vec{a}$ and $\vec{b}$ = $|\vec{a}x\vec{b}|$

Area of a parallelogram with diagonals $\vec{a}$ and $\vec{b}$ $= \frac{1}{2}\left|\vec{a} x \vec{b}\right|$

Area of a Δ ABC = $\frac{1}{2}\left|\overrightarrow{AB}\ X\ \overrightarrow{AC}\ \right|$

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. Find a unit vector in the direction of vector a$\vec{a}$ = 6$\hat{\imath}$ +2$\hat{\jmath}$ +3$\hat{k}$

(A) $\frac{6\hat{\imath}+2\hat{\jmath}+3\hat{k}}{|7|}$ (B) $\frac{6\hat{\imath}+2\hat{\jmath}+3\hat{k}}{|6|}$ (C) $\frac{6\hat{\imath}+2\hat{\jmath}+3\hat{k}}{|5|}$ (D) $\frac{6\hat{\imath}+2\hat{\jmath}+3\hat{k}}{|1|}$

2. Write direction ratio of the vector $\vec{a}$ = $\hat{\imath}$ +$\hat{\jmath}$ -2$\hat{k}$

(A) $(1,1,-2)$ (B) $(1,1,2)$ (C) $(1,-1,2)$ (D) $(-1,1,2)$

3. Write the value of($\hat{\imath}$ x $\hat{\jmath}$) .$\hat{k}$ + ($\hat{\jmath}$ x $\hat{k}$). $\hat{\imath}$ + ($\hat{k}$ x $\hat{\imath}$). $\hat{\jmath}$

(A) 1 (B) 3 (C) 2 (D) 0

4. Find $|\vec{x}|$ if for a unit vector $\vec{a}$, $(\vec{x} - a^{\rightarrow}).(\vec{x} + a^{\rightarrow})$ =15

(A) ± 1 (B) ± 4 (C) ± 2 (D) ± 3

5. Find the angle between $\vec{a}$ &$\vec{b}$ if $\left|\vec{a}.\vec{b}\right|=\left|\vec{a}x\ \vec{b}\right|$

(A) 0^0 (B) 30^0 (C) 60^0 (D) 90^0

6. Find Projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}.\vec{b}$ =8, $\vec{b}$ =6$\hat{\imath}$ +2$\hat{\jmath}$ +3$\hat{k}$

(A) $\frac{8}{3}$ (B) $\frac{8}{5}$ (C) $\frac{8}{7}$ (D) $\frac{8}{9}$

7. Find the angle between two vectors vector$\vec{a}$ on the vector$\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}.\vec{b}. = \sqrt{6}$

(A) $\frac{1}{5}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

8. Let the vector $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|$ =3 and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$ and the angle between $\vec{a}$ and$\vec{b}$. So that $\vec{a}\ x\vec{b}$ is a unit vector.

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{5}$ (D) $\frac{\pi}{4}$

9. Find $|\vec{x}|$ if for a unit vector $\vec{a}$, $(\vec{x} - a^{\rightarrow}),(\vec{x} + a^{\rightarrow})$ =15

(A) ± 1 (B) ± 4 (C) ± 2 (D) ± 3

10. What are the direction cosines of line which makes equal angles with the coordinate axis.

(A) $l = m = n = \pm\frac{1}{\sqrt{3}}$ (B) $l = m = n = \pm\frac{1}{\sqrt{7}}$ (C) $l = m = n = \pm\frac{1}{5}$

(D) $l = m = n = \pm\frac{1}{\sqrt{2}}$

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

Each of the following questions contains statement -1(Assertion) and statement-2(Reason) and has following four choices (a),(b),(c),(d), only one of which is correct answer.Mark the correct choice

(a) Statement 1 is true,Statement 2 is true and 2 is correct explanation of 1

(b) Statement 1 is true,Statement 2 is true and 2 is not correct explanation of 1

(c) Statement 1 is true,Statement 2 is false

(d) Statement 1 is false,Statement 2 is true

1. **ASSERTION**: In triangle ABC, $\overrightarrow{AB} + \overrightarrow{AB} + \overrightarrow{AB} = 0$

   REASON: If $\overrightarrow{OA} = \vec{a}$ $\overrightarrow{OB} = \vec{b}$ then $\overrightarrow{AB} = \vec{a} + \vec{b}$

2. **ASSERTION**:

$\vec{a} = \hat{\imath} + p\,\hat{\jmath} + 2\hat{k}$, $\vec{b} = 2\,\hat{\imath} + 3\hat{\jmath} + q\hat{k}$ are parallel vectors if p=3/2, q=4

   REASON:

If $\vec{a} = a\,\hat{\imath} + b\,\hat{\jmath} + c\hat{k}$, $\vec{b} = d\,\hat{\imath} + e\hat{\jmath} + f\,\hat{k}$ are parallel if a/d = b/e = c/f

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. Write the position vector of the mid-point of the vector joining the points P(2 ,3 ,4) and Q(4 ,1 ,-2)
2. If $\hat{\imath} + \hat{\jmath} + \hat{k}$, $2\hat{\imath} + 5\hat{\jmath}$, $3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\hat{\imath} - 6\hat{\jmath} - \hat{k}$ are the position vectors of the points A ,B ,C and D , Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.
3. Find the position vectors of the point R which divides the line joining two points P and Q whose position vectors are $(2\overrightarrow{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively in the ratio 1:2 .

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1. What is the angle between vectors $\vec{a}$ & $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively? Given $\vec{a} \cdot \vec{b} = 3$
2. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ & $\vec{b}$ externally in the ratio 1:4.
3. If $\vec{a} = \hat{\imath} + \hat{\jmath}$, $\vec{b} = \hat{\jmath} + \hat{k}$, $\vec{\imath} = \hat{k} + \hat{\imath}$, find a unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$.
4. Let $\vec{a}$ & $\vec{b}$ be two vectors such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector. Then what is the angle between $\vec{a}$ & $\vec{b}$?
5. Write the value of p for which $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + p\hat{\jmath} + 3\hat{k}$ are parallel vectors and perpendicular vectors.

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

1. Geetika house is situated at Kanke at point o, for going to Alok's house she first travels 8 km by bus in the East. Here at point A a hospital is situated, from Hospital, Geetika take an auto and goes

6 km in the North, here at point B school is situated. From school she travels by bus to reach Alok's house which is 30^0 East,6km from B

Based on the above information, answer the following questions

(i) What is the vector distance between Geetika's house and school?

(ii) What is the Vector distance Geetika's house and school? (ii)How much Geetika travels to reach school? Ans: 14km

(iii) What is the vector distance from school to Alok's house? Ans: $3\sqrt{3}\hat{\imath}$ $+3\hat{\jmath}$,

OR

What is the total distance travelled by Geetika for her house to Alok's house? Ans: 20km

2. A plane started from airport at o with a velocity of 120km/s towards East. Air is blowing at a velocity of 50km/s towards the North. The plane traveled 1 hr in OP direction with the resultant velocity, from P To R the plane traveled 1hr keeping velocity of 120m/s and finally landed at R.

Based on the above information, answer the following questions:

(i) What is the resultant velocity from O to P?

(ii) What is the direction of travel from O to P ?

(iii) What is the Displacement from O to P?

Answer key/Hints

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. a 2.a 3.b 4.b 5.c 6. c 7.d 8.d 9.b 10.a

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

1. a 2. a

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. Let $\vec{a}$ and $\vec{b}$ be the position vector of points P(2 ,3 ,4) and Q(4 ,1 ,-2) respectively .

$\vec{a}$ $=2\hat{\imath}$ $+3\hat{\jmath}$ $+4\hat{k}$,$\vec{b}$ $=4\hat{\imath}$ $+\hat{\jmath}$ $-2\hat{k}$

Position Vector of the mid-point of P and Q $=\frac{(\vec{a}+|\vec{b})}{2}$ $=(6\hat{\imath}$ $+4\hat{\jmath}$ $+2\hat{k})/2$ $=3\hat{\imath}$ $+2\hat{\jmath}$ $+\hat{k}$

2. $\overrightarrow{AB}$ = $\hat{\imath}$ $+4\hat{\jmath}$ $-\hat{k}$ $\overrightarrow{CD}$ $=-2\hat{\imath}$ $-8\hat{\jmath}$ $+2\hat{k}$ = $-2(\hat{\imath}$ $+4\hat{\jmath}$ $+\hat{k})$ = -2 $\overrightarrow{AB}$

$\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel vectors.

So, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear and angle between them is zero .

3.P.V of R i.e $\overrightarrow{OR}$ $=\frac{(\vec{a}-3\vec{b})-2(2\vec{a}+\vec{b})}{1-2}=\frac{\vec{a}-3\vec{b}-4\vec{a}-2\vec{b}}{-1}$ $=3\vec{a}+5\vec{b}$

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1. Given $\vec{a} \cdot \vec{b} = 3$

- ⇨ $|\vec{a}|.\left|\vec{b}\right| \cos\theta = 3$
- ⇨ $\sqrt{3} \, . \cos\theta = 3$
- ⇨ $\cos\theta = \frac{\sqrt{3}}{2}$
- ⇨ $\theta = \frac{\pi}{6}$ (Ans)

2. $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath} + \hat{k}$

Position vector, $\vec{r} = \dfrac{1x(-\hat{\imath} + \hat{\jmath} + \hat{k}) - 4(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})}{1-4}$

$= \dfrac{(-9\hat{\imath} - 11\hat{\jmath} - 15\hat{k})}{-3}$

$= 3\hat{\imath} + \frac{11}{3}\hat{\jmath} + 5\hat{k}$ (Ans)

3. $\vec{a} = \hat{\imath} + \hat{\jmath}$, $\vec{b} = \hat{\jmath} + \hat{k}$, $\vec{c} = \hat{k} + \hat{\imath}$

Let $\vec{x} = \vec{a} + \vec{b} + \vec{c} = 2(\hat{\imath} + \hat{\jmath} + \hat{k})$

Therefore, unit vector along $\vec{x}$

$\frac{\vec{x}}{|\vec{x}|} = \frac{2(\hat{\imath} + \hat{\jmath} + \hat{k})}{2\sqrt{1+1+1}} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

4. Given $\vec{a}$ x $\vec{b}$ is a unit vector $\left|a^{\rightarrow} \, x \, b^{\rightarrow}\right| = 1$

$=> \left||a^{\rightarrow}|.|b^{\rightarrow}| \, sin\,\theta\right| = 1$

$=> 3 - \frac{\sqrt{3}}{2} \, sin\,\theta = 1$

$=> sin\,\theta = \frac{1}{\sqrt{2}}$

$=> \theta = \frac{\pi}{4}$

5. Since vectors are parallel

$\frac{3}{1} = \frac{2}{p} = \frac{9}{3}$ $=> 3 = \frac{2}{p}$ $=> p = \frac{2}{3}$

Since vectors are perpendicular

3+2p+27=0

=>2p=-30 =>p=-15

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

1. (i) Ans : $8\hat{\imath} + 6\hat{\jmath}$, (ii) 14km (iii) $3\sqrt{3}\hat{\imath} + 3\hat{\jmath}$, OR (iii) 20km

2. (i) Ans: 130m/s (ii) Ans : $\tan^{-1}(\frac{5}{12})$ (iii) 468km

Ch – 11 Three-Dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Key Points

DIRECTION COSINES **-** The direction cosines of a line are defined as the direction cosines of any vector whose support is the given line.

A(x_1,x_2,x_3) B(y_1,y_2,y_3)

If A(x_1,x_2,x_3) and B(y_1,y_2,y_3) are two point on a line L, then its direction cosines are < $\frac{y_1-x_1}{AB}, \frac{y_2-x_2}{AB}, \frac{y_3-x_3}{AB}$ > where AB= distance between points A and B = | $\overrightarrow{AB}$|

In vector if $\overrightarrow{AB} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is position vector. The angle α, β *and* γ made by the vector $\vec{r}$=($\overrightarrow{AB}$) with the positive directions of x,y and z axes respectively are called its ***direction angles*** the cosine values of these angles $\cos\alpha$, $\cos\beta$ *and* $\cos\gamma$ are called direction cosines of $\overrightarrow{AB}$ usually denoted by l, m and n.

If $\overrightarrow{AB} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then $|\overrightarrow{AB}| = \sqrt{x^2 + y^2 + z^2}$

Direction cosines are given as follows:

$$l = \cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}}, \quad m = \cos\beta = \frac{y}{\sqrt{x^2+y^2+z^2}}, \quad n = \cos\gamma = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

Relation between l, m & n is : $l^2 + m^2 + n^2$=1

DIRECTION RATIOS: **-** The direction ratios of a line are proportional to the direction ratios of any vector whose support is the given line.

If A (x_1,y_1,z_1) and B (x_2,y_2,z_2) are two points on a line, then its direction ratios are proportional to x_2-x_1, y_2-y_1, z_2-z_1.

A (x_1,y_1,z_1) B (x_2,y_2,z_2)

If $\overrightarrow{AB} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then d.r are x, y and z.

NOTE-Direction ratios (d.r) and direction cosines (d.c) of a line passing through two points A (x_1,y_1,z_1) and B (x_2,y_2,z_2) .

A (x_1,y_1,z_1) B (x_2,y_2,z_2)

d. r's are < x_2-x_1, y_2-y_1, z_2-z_1. >

D.C's are < $\frac{x_2-x_1}{AB}, \frac{y_2-y_1}{AB}, \frac{z_2-z_1}{AB}$ > Where AB=$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

EQUATION OF LINE IN DIFFERENT WAYS

1. Equation of a `line passing through (x_1,y_1,z_1) and direction ratios < a,b,c>.

   is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

   A (x_1,y_1,z_1) d.r's < a,b,c>

2. Equation of a line passing through two given points A (x_1,y_1,z_1) and B (x_2,y_2,z_2)

   is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

   A (x_1,y_1,z_1) B (x_2,y_2,z_2)

3. Equation of a line through a given point (x_1,y_1,z_1) and parallel to a given vector $\overrightarrow{B} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

` A(x_1,x_2,x_3) D.r. a,b,c $\overrightarrow{B} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

4. If l,m,n are d.c. and a given point (x_1,y_1,z_1) of the line then the equation of the line is $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$.

SKEW LINES: Two straight line in space which are neither parallel nor intersecting are called skew lines.

SHORTEST DISTANCE: The shortest distance between two lines l_1 and l_2 is the distance PQ between the points P and Q where the lines of shortest distance intersects the two given lines.

LINE OF SHORTEST DISTANCE: If l_1 and l_2 are two skew –lines ,then there is one and only one line perpendicular to each of lines l_1 and l_2 which is known as the line of shortest distance. (1) If lines intersect then shortest distance between them is zero. (2) If lines are parallel then the shortest distance between them is the distance between the two lines.

SHORTEST DISTANCE BETWEEN TWO SKEW LINES (Vector Form):-The shortest (S.D.) between two non-parallel lines $\overrightarrow{r} = \overrightarrow{a_1} + \alpha\, \overrightarrow{b_1}$ and $\overrightarrow{r} = \overrightarrow{a_2} + \alpha\, \overrightarrow{b_2}$ is given by

S.D. = $\left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

SHORTEST DISTANCE BETWEEN TWO SKEW LINES (Cartesian form)

If given equations in Cartesian form as below

$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_1}{l_2} = \frac{y-y_1}{m_2} = \frac{z-z_1}{n_2}$ then for shortest distance

First we change this equation in vector form like as

$\overrightarrow{r} = \overrightarrow{a_1} + \alpha\, \overrightarrow{b_1}$ and $\overrightarrow{r} = \overrightarrow{a_2} + \beta\, \overrightarrow{b_2}$

Where $\overrightarrow{a_1} = x_1\,\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$, $\overrightarrow{b_1} = l_1\,\hat{\imath} + m_1\hat{\jmath} + n_1\hat{k}$,

$\overrightarrow{a_2} = x_2\,\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$, $\overrightarrow{b_2} = l_2\,\hat{\imath} + m_2\hat{\jmath} + n_2\hat{k}$

And use formula S.D. in vector form

S.D. = $\left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

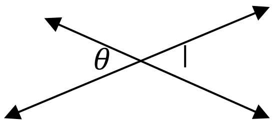
Shortest distance between the parallel lines $\overrightarrow{r} = \overrightarrow{a_1} + \alpha\, \overrightarrow{b_1}$ and $\overrightarrow{r} = \overrightarrow{a_2} + \beta\, \overrightarrow{b_1}$

S.D. = $\left|\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}_1}{|\vec{b}_1|}\right|$

ANGLE BETWEEN TWO LIES (VECTOR FORM)

$\overrightarrow{r} = \overrightarrow{a_1} + \alpha\, \overrightarrow{b_1}$ and $\overrightarrow{r} = \overrightarrow{a_2} + \beta\, \overrightarrow{b_2}$

COSθ = $\left|\frac{(\vec{b}_1.\vec{b}_2)}{|\vec{b}_1||\vec{b}_1|}\right|$



ANGLE BETWEEN TWO LIES *(Cartesian form)*

$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_1}{l_2} = \frac{y-y_1}{m_2} = \frac{z-z_1}{n_2}$

COSθ = $\left|\frac{l_1l_2+m_1m_2+n_1n_2}{\sqrt{l_1^2+m_1^2+n_1^2}.\sqrt{l_2^2+m_2^2+n_2^2}}\right|$

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1. Direction ratio of line joining (2,3,4) and (-1,-2,1) are

a) (-3,-5,-3) b) (-3,-5,-3) c)(-1,-5,-3) d) -3,3,2)

2. The direction cosines of the y-axis are

a) (-1,0,0) b) (1,0,0) c) (0,1,0) d) (0,0,1)

3. The angle between the lines 2x=3y= -z and 6x = -y = -4z

a)90^o b) 45^o c) 60^o d) 0^o

4. Angle between the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z-2}{1}$ and $\frac{x-1}{1}=\frac{y}{2}=\frac{z-1}{3}$ is

a)90^o b) 45^o c) 60^o d) 0^o

5. Direction ratios of the line $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z-2}{1}$ is

a)-5,2,-2 b) 7,-5,1 c) 5,-2,2 d)-7,5,-1

6. The line of shortest distance between two skew-lines is ...... to both the line

a) Perpendicular b) parallel c) interest d) none of these.

7. If a line makes angles 90^o, 60^o and θ with x, y and z axes respectively, where θ is acute then the value of θ is

a)30^o b) 60^o c) 90^o d) 45^o

8. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect at a point, then the value of k is

a) 9/2 b) 2/9 c) 2 d) 3/2

9. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-2}{k}=\frac{z}{1}$ are Perpendicular at a point, then the value of k is

a) 2 b) - 3 c) 4 d) -2

10. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{8}=\frac{y-2}{k}=\frac{z}{16}$ are parallel then the value of k is

a) 8 b) 16 c) 12 d) 24

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

1. **Assertion (A)**: The points A(2,9,12), B(1,8,8), C(2,11,8) and D(1,12,12) are the vertices of a rhombus.

Reason(R): AB=BC=CD=DA and AC=BD

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).
(c) (A) is true (R) is false.
(d) Both (A) and (R) false

2. **Assertion (A):** Lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-2}{k}=\frac{z}{1}$ are perpendicular at a point.

Reason(R): 2x1+3xk+4x1=0 or if k=-2

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true and (R) is not the correct explanation of (A).
(c) (A) is true (R) is false.
(d) Both (A) and (R) false.

SECTION-B

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

Q1 Find direction ratios equation of line $\frac{x-3}{5}=\frac{y-2}{12}=\frac{2z-1}{16}$.

Q2 Find the value of K if lines $\frac{x-5}{7}=\frac{y+2}{K}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are perpendicular.
Q3 Find the intercepts cut off by the plane 2x+y-z=5.
Q4 Write equation of line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ in vector form.
Q5 Write direction cosines of line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$.

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

Q1 Find the equation of the line passing through the point (-1,2,1) and parallel to the line $\frac{2x-1}{4}=\frac{3y+5}{2}=\frac{2-z}{3}$.

Q2 Find the equation of the line passing through the point (-1,2,1) and perpendicular to the line $\frac{2x-1}{4}=\frac{3y+5}{2}=\frac{2-z}{3}$

Q3 Find the equation of `line passing through (1,2,3) and direction ratios <4,5,6>.

Q4 Find the equation of line through a given point (1, 2, 3) and parallel to a given vector.

$\overrightarrow{B}=4\hat{\imath}+5\hat{\jmath}+6\hat{k}$.

Q5 Find the image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

Q6 Find the coordinates of foot of the perpendicular drawn from the point (1, 2, 3) to the line

$\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$ also find the length of perpendicular from the given point to the given line.

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

Q1 A line passes through (2, -1, 3) and is perpendicular to the line.
$\overrightarrow{r}=\hat{\imath}+\hat{\jmath}-\hat{k}+\alpha(2\hat{\imath}-2\hat{\jmath}+\widehat{k)}$ and $\overrightarrow{r}=\widehat{2\imath}-\hat{\jmath}-3\hat{k}+\beta(\hat{\imath}+2\hat{\jmath}+2\widehat{k)}$. Obtain its equation.

Q2 Q1 Find the distance between the lines L1 and L2 given by:
$\overrightarrow{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\alpha(2\hat{\imath}+3\hat{\jmath}+6\widehat{k)}$
$\overrightarrow{r}=\widehat{3\imath}+3\hat{\jmath}-5\hat{k}+\beta(2\hat{\imath}+3\hat{\jmath}+6\widehat{k)}$

Q3 Find the vector equation of a line passing through a point with position vector. $2\hat{\imath}-\hat{\jmath}+\hat{k}$ and parallel to the line joining the points $-\hat{\imath}+4\hat{\jmath}+\hat{k}$ *and* $\hat{\imath}+2\hat{\jmath}+2\hat{k}$. Also, find the cartesian equivalent of this equation.

Q4 Find the value k so that the lines l_1:$\frac{1-x}{3}=\frac{7x-14}{2k}=\frac{z-3}{2}$ and l_2 : $\frac{7-7x}{3k}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angle. Also,
find the equations of a line passing through the point (3, 2, -4) and parallel to line l_1.

Q5 Find the shortest distance between the lines: $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ *and* $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$.

Q6 An insect is crawling along the line $\bar{r}=6\imath+2\jmath+2\hat{k}+(\imath-2\jmath+2\hat{k})$ and another insect is crawling along the line $\bar{r}=-4\imath-\hat{k}+(3\imath-2\jmath-2\hat{k})$. At what points on the lines should they reach so that the distance between them is the shortest. Find the shortestpossible distance between them.

Q7. The equations of motion of a rocket are: $x=2t, y=-4t, z=4t$, where the time t is given in seconds, and the coordinates of a moving point in km. What is the path of the rocket? At what distances will the rocket befrom the starting point O (0, 0, 0) and from the following line in 10 seconds?

$\vec{r}=20\imath-10\jmath+40\hat{k}+(10\imath-20\jmath+10\hat{k})$

ANSWER KEY

MULTIPLE COICE QUESTIONS (1 MARK EACH)

Q NO.1	a
Q NO 2	c
Q NO.3	a
Q NO.4	a
Q NO.5	b
QNO.6	a
Q NO.7	a
Q NO.8	a
Q NO.9	d
Q NO.10	c

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

Q NO.1	d
Q NO 2	a

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

Q NO.1	Ans- 5,12,8
Q NO 2	Ans K= -5
Q NO.3	Ans 5/2,5,-5
Q NO.4	Ans- $\overrightarrow{r} = \hat{\imath} - \hat{\jmath} + \hat{k} + \alpha(2\hat{\imath} + 3\hat{\jmath} + 4\widehat{k)}$
Q NO.5	Ans- $\frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}$

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

Q NO.1	Ans-$\frac{x+1}{2} = \frac{y-2}{\frac{2}{3}} = \frac{z-1}{-3}$
Q NO 2	Ans-$\frac{x+1}{\frac{1}{2}} = \frac{y-2}{\frac{3}{2}} = \frac{z-1}{\frac{-1}{3}}$
Q NO.3	Ans- $\frac{x-1}{4} = \frac{y-2}{5} = \frac{z-3}{6}$
Q NO.4	Ans-$\frac{x-1}{4} = \frac{y-2}{5} = \frac{z-3}{6}$
Q NO.5	Ans (1,0,7)

Q NO.6	Foot of perpendicular (3,5,7) length=7 units

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

Q NO.1	$\overrightarrow{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \alpha(-6\hat{i} - 3\hat{j} + 6\widehat{k})$. (Hints. the required line is Perpendicular to given lines. $\overrightarrow{r} = \hat{i} + \hat{j} - \hat{k} + \alpha(2\hat{i} - 2\hat{j} + \widehat{k})$ and $\overrightarrow{r} = \widehat{2\imath} - \hat{j} - 3\hat{k} + \beta(\hat{i} + 2\hat{j} + 2\widehat{k})$ So, it is parallel to vector $(2\hat{i} - 2\hat{j} + \widehat{k}) \times (\hat{i} + 2\hat{j} + 2\widehat{k}) = - 6\hat{i} - 3\hat{j} + 6\hat{k}$ Therefore, equation of line passes through the point (2, -1, 3) and parallel to vector- $6\hat{i} - 3\hat{j} + 6\hat{k}$ is $\overrightarrow{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \alpha(-6\hat{i} - 3\hat{j} + 6\widehat{k})$.)
Q NO 2	Ans: √293/7 (Hints since given lines are parallel therefore shortest distance between two parallel line S.D. = $\left\lvert\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}_1}{\lvert\vec{b}_1\rvert}\right\rvert$ Where $\vec{a}_2 - \vec{a}_1 = (\widehat{3\imath} + 3\hat{j} - 5\hat{k})$-$(\hat{i} + 2\hat{j} - 4\hat{k}) = 2\hat{i} + \hat{j} - \hat{k}$ $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$ $\therefore \quad \lvert (\vec{a}_2 - \vec{a}_1)\times \vec{b}_1\rvert = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = \lvert 9\hat{i}-14\hat{j}+4\hat{k}\rvert = \sqrt{293}$ and $\lvert\vec{b}_1\rvert = 7$
Q NO.3	$\overrightarrow{r} = 2\hat{i} - \hat{j} + \hat{k} + \alpha(2\hat{i} - 2\hat{j} + \widehat{k})$. In Cartesian form $\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}$
Q NO.4	Ans. k=70/11 (Required equation of line $\frac{x-3}{-3} = \frac{y-2}{20/11} = \frac{z+4}{2}$)
Q NO.5	Ans s. d.= $\frac{1}{\sqrt{6}}$ (Hints 1st change equation in vector form $\overrightarrow{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \alpha(2\hat{i} + 3\hat{j} + 4\widehat{k})$ $\overrightarrow{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + \beta(3\hat{i} + 4\hat{j} + 5\widehat{k})$ and use S.D. = $\left\lvert\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{\lvert\vec{b}_1\times\vec{b}_2\rvert}\right\rvert$
Q NO.6	9 units
Q NO.7	Path of the rocket is line $\frac{x}{2} = \frac{y}{-4} = \frac{z}{4}$, Distance= 10 √3 km.

Ch – 12 Linear Programming Problems

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

TYPES OF QUESTIONS THAT MAY BE ASKED FROM THE UNIT

1. Questions related to the feasible region. (1m/2m)
2. Questions related to the feasible solutions. (1m/2m)
3. Question related to the optimal solution or optimal feasible solution. (1m/2m)
4. Question related to finding the optimal value. (1m/2m)
5. Questions related to bounded and unbounded region. (1m/2m)
6. Questions related to the formation of the constraints and objective function (2m/3m)
7. Questions related to solving the linear inequalities (constraints) graphically (3m/5m)
8. Questions related to solving LPP. (3m/5m)

PRE-REQUISITE KNOWLEDGE THAT WILL HELP IN BETTER REVISION OF THE UNIT

OBJECTIVE FUNCTION: The linear function that has to be optimized i. e. either to be maximized or minimized is called an objective function.

LINEAR CONSTRAINTS: The linear inequalities that represent the restrictions that are mentioned in the LPP are called the linear constraints.

NON-NEGATIVE RESTRICTIONS: $x \geq 0, y \geq 0$ are called the non negative constraints as the decision variables must be always greater than or equal to zero.

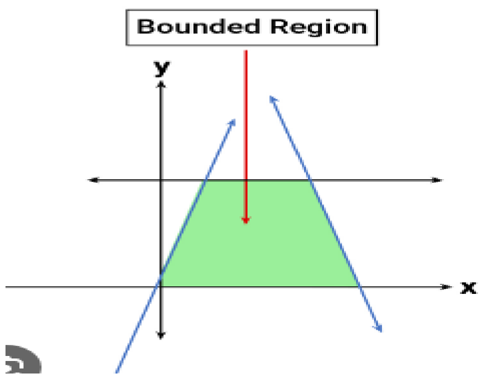
OPTIMAL SOLUTION: The point in the feasible region which optimizes the objective function is called optimal solution.

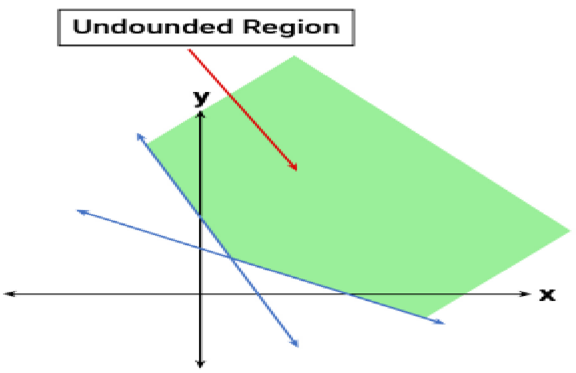
OPTIMAL VALUE: The maximum or the minimum value of the objective function which is obtained by putting the optimal solution in the objective function is called optimal value.

FEASIBLE REGION:

Bounded: A closed region which consists of all the possible solutions of the given system of constraint is called a feasible bounded region.

Unbounded: an open region which consists of all the possible solutions of the given system of constraints is called a feasible unbounded region.



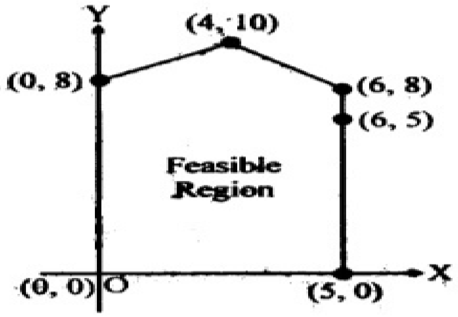


FEASIBLE SOLUTIONS: A solution of a LPP which satisfy the non negativity restrictions of a problem is called its feasible solutions.

MULTIPLE COICE QUESTIONS (1 MARK EACH)

Q1.

The feasible region for a LPP is shown shaded in the figure. Let $Z = 3x - 4y$ be the objective function. Minimum of Z occurs at



a) (6, 10) b) (0, 0) c) (5, 0) d) (0, 8)

Q2. Corner points of the bounded feasible region for an LP problem are A (0,5); B (0,3); C(1,0); D(6,0).

Let $z=-50x + 20y$ be the objective function. Minimum value of z occurs at ______ center point.

a) (0, 5) b) (0, 3) c) (1, 0) d) (6, 0)

Q3. Maximize $Z = 3x + 5y$, subject to $x + 4y \leq 24$, $3x + y \leq 21$, $x + y \leq 9$, $x \geq 0$, $y \geq 0$

a) 20 at (1, 0) b) 37 at (4, 5) c) 33 at (6, 3) d) 30 at (0, 6)

Q4. A feasible solution to an LP problem,

a) Must optimize the value of the objective function.
b) Must satisfy all of the problem's constraints simultaneously.
c) Must be a corner point of the feasible region.
d) Need not satisfy all of the constraints, only some of them.

Q5. In solving the LPP: "minimize $f = 6x + 10y$ subject to constraints $x \geq 6$, $y \geq 2$, $2x + y \geq 10$, $x \geq 0$, $y \geq 0$" redundant constraints are

a) $x \geq 6$, $y \geq 2$ b) None of these
c) $x \geq 6$ d) $2x + y \geq 10$, $x \geq 0$, $y \geq 0$

Q6. The optimal value of the objective function is attained at the points:

a) Corner points of the feasible region b) On X-axis
c) On Y-axis d) None of these

Q7. Feasible region in the set of points which satisfy.

a) All of the given constraints b) None of these
c) Some the given constraints d) The objective functions

Q8. The objective function of a linear programming problem is:

a) None of these b) Function to be optimized
c) A relation between the variables d) A constraint

Q9. The maximum value of $Z = 4x + 2y$ subject to the constraints $2x + 3y \leq 18$, $x + y \geq 10$, $x, y \leq 0$ is

a) 40 b) None of these c) 30 d) 36

Q10. Z = 8x + 10y, subject to 2x + y ≥ 1, 2x + 3y ≥ 15, y ≥ 2, x ≥ 0, y ≥ 0. The minimum value of Z occurs at

a) (1.5, 4) b) (0, 7) c) (4.5, 2) d) (7, 0)

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

Q1) **Assertion(A):** Feasible region is the set of points which satisfies all of the constraints.

Reason (R): The optimal value of the objective function is attained at the points on X axis only.

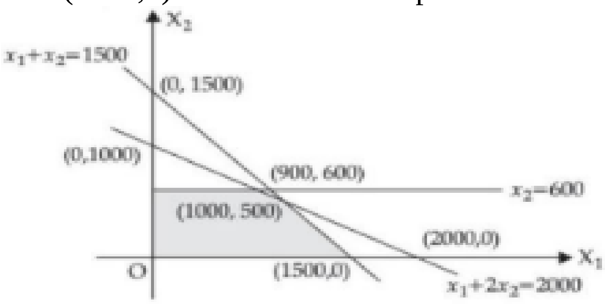
a) Both A and R are true and R is the correct explanation of A
b) Both A and R are True but R is Not the correct explanation of A
c) A is true but R is false.
d) A is false But R is true.

Q2) **Assertion(A):** It is necessary to find objective function value at every point in the feasible region to find optimum value of the objective function.

Reason (R): For the constraints $2x + 3y \leq 6, 5x + 3y \leq 15, x \geq 0$ and $y \geq 0$ corner points of the feasible region are (0,2), (0,0) and (3,0).

a) Both A and R are true and R is the correct explanation of A
b) Both A and R are True but R is Not the correct explanation of A
c) A is true but R is false.
d) A is false But R is true.

Q3) Assertion(A): For the constraints of a LPP problem given by $x_1 + 2x_2 \leq 2000$ $x_1 + x_2 \leq 1500, x_2 \leq 600$ and $x_1, x_2 \geq 0$ the points (1000,0), (0,500) (2,0) lie in the positive bounded region, but points (2000,0) does not lie in the positive bounded region.



Reason (R):

a) Both A and R are true and R is the correct explanation of A
b) Both A and R are True but R is Not the correct explanation of A
c) A is true but R is false.
d) A is false But R is true.

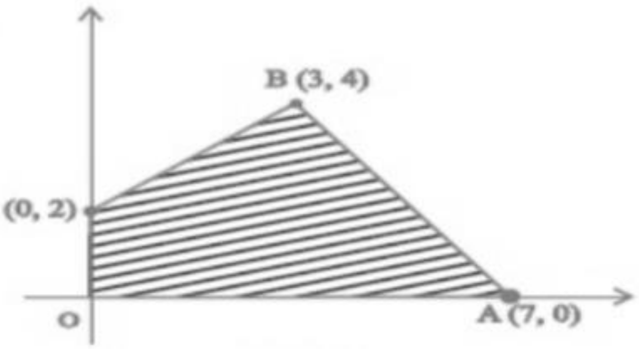
VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

Q1. Draw the graph of the following LPP: $5x + 2y \leq 10, x \geq 0, y \geq 0$

Q2. Maximize $Z = 3x + 4y$ subjected to the constraints:
$x + y \leq 4, x \geq 0, y \geq 0.$

Q3. Determine the maximum value of $Z = 11x + 7y$ subjected to the constraints:
$2x + y \leq 6, x \leq 2, x \geq 0, y \geq 0.$

Q4. Feasible region for the LPP is shown Maximize $Z = 5x + 7y.$



Q5. The feasible region for the LPP is shown below. Find the minimum value of $Z = 11x + 7y$

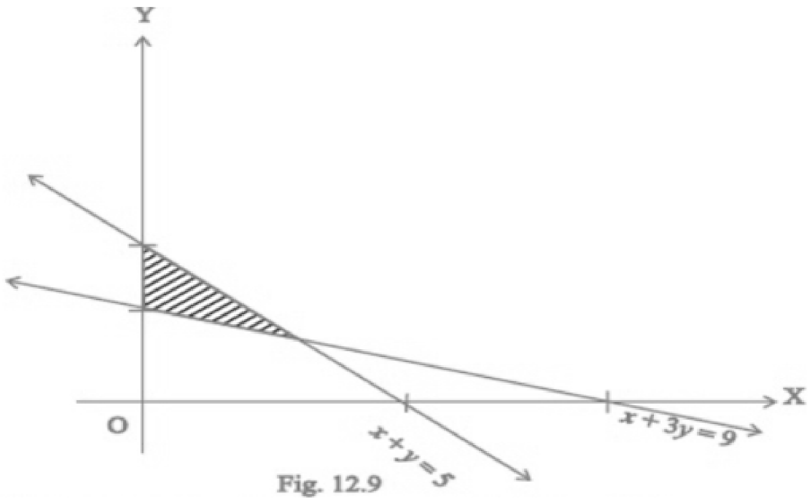
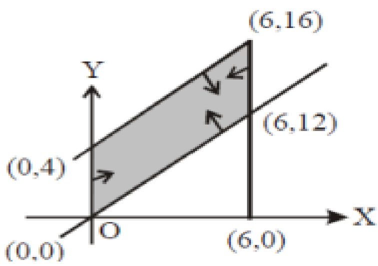


Fig. 12.9

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

Q1. The feasible region for LPP is shown shaded in the figure. Let Z = 3 x – 4 y be the objective function, then write the maximum value of Z



Q2. Solve the following LPP graphically:

Minimize Z = 5x + 10y subject to the constraints
$x + 2y \leq 120$
$x + y \geq 60$,
$x - 2y > 0$ and $x, y \geq 0$

Q3. Maximize and minimize Z = x + 2y subject to the constraints
$x + 2y \geq 100$
$2x - y \leq 0$
$2x + y \leq 200$
$x, y \geq 0$
Solve the above LPP graphically.

Q4. Minimize $Z = 13x - 15y$, subjected to the constraints:
$x + y \leq 7, 2x - 3y + 6 \geq 0, x, y \geq 0$

Q5. Determine the maximum value of $Z = 3x + 4y$ *if the feasible* region for the LPP is shown below.

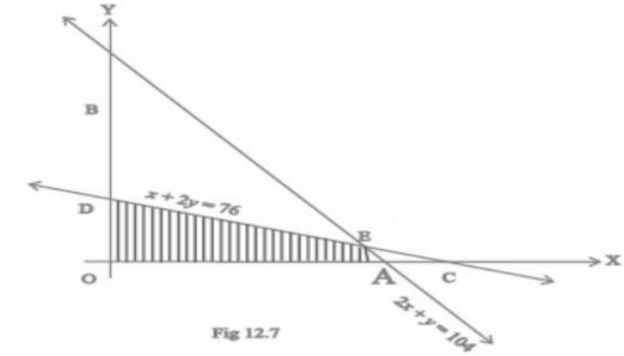
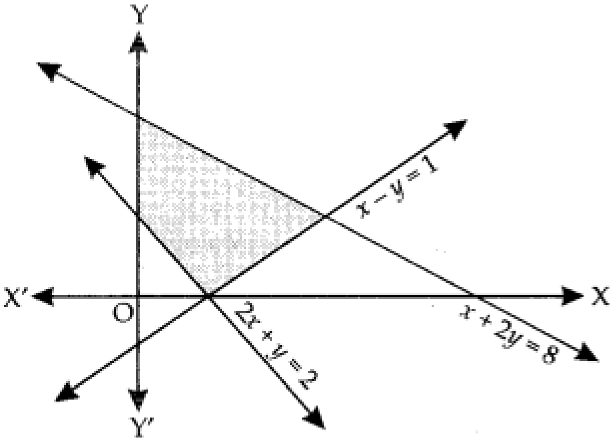


Fig 12.7

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

Q1. Find the maximum value of Z = 3x + 4y subjected to constraints $x + y \leq 40$, $x + 2y \leq 60$, $x \geq 0$ and $y \geq 0$.

Q2. Find the linear constraints for which the shaded area in the figure below is the solution set:



Q3. Maximize $Z = 5x + 3y$

Subjected to the constraints: $3x + 5y \leq 15, 5x + 2y \leq 10\ , x \geq 0\ , y \geq 0$

Q4. Minimize and maximize $Z = 5x + 2y$ subjected to the following constraints:

$x - 2y \leq 2, 3x + 2y \leq 12, -3x + 2y \leq 3, x, y \geq 0$

Q5. Maximize $P = \frac{8x}{100} + \frac{10y}{100}$ subjected to the constraints.

$x + y \leq 12000, x \geq 2000, y \geq 4000\ , x, y \geq 0$

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

Q1. An aeroplane can carry a maximum of200 passengers. A profit of rupees 1000 is made on each executive class ticket and a profit of rupees 600 is made on each economy class ticket. The airlines reserve at least 20 seats for the executive class. However, at least 4 times as many passengers prefer to travel by economy class, than by executive class. It is given the number of executive class tickets is x and that of economy ticket is y.

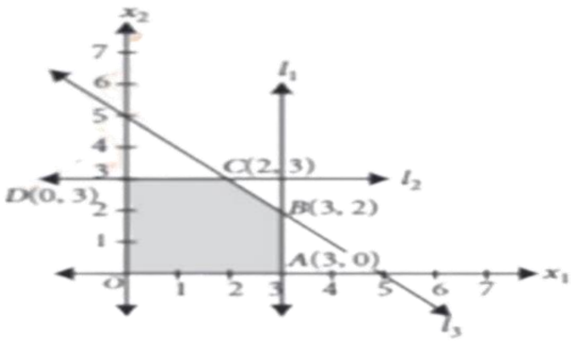
For the above the LPP is to

Maximize $Z = 1000x + 600y$

Subjected to $x + y \leq 200, y \geq 4x, x \geq 20, y \geq 0$
On the basis of the above information answer the following questions

(i) What is corner points of the feasible bounded region.
(ii) What will be the maximum value of Z
(iii) According to the optimal solution what will be the value of $x + y$
OR (iii) The profit when x=20 and y=80 is

Q2) Corner points of the feasible region for the LPP are (0,3) , (5,0) ,(6,8) , (0,8). Let $Z = 4x - 6y$ be the objective function.



Based on the above information answer the following Questions.
(i) The minimum value of Z occurs at.
(ii) The maximum value of Z occurs at
(iii) Maximum of Z- Minimum of Z=
OR (iii) The corner points of feasible regions determined by the system of linear inequalities are.

ANSWER KEY

MULTIPLE COICE QUESTIONS (1 MARK EACH)

Q NO.1	d
Q NO 2	d
Q NO.3	b
Q NO.4	b
Q NO.5	d
QNO.6	a
Q NO.7	a
Q NO.8	b
Q NO.9	b
Q NO.10	a

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

Q NO.1	C
Q NO 2	D
Q NO.3	A

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

Q NO.1	Drawing correct graph
Q NO 2	Z=16 at (0,4)
Q NO.3	Maximum value is 42 at (0,6)
Q NO.4	Maximum value is 43 at (3,4)
Q NO.5	Minimum value is 21 at (0,3)

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

Q NO.1	-16
Q NO 2	minimum at A (60,0)and minimum value is 300
Q NO.3	maximum at (0,200)and maximum value is 400.
Q NO.4	minimum at 90,2) and the minimum value is − 30.
Q NO.5	maximum is 196.

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

Q NO.1	140 at (20,20)
Q NO 2	$x - y \leq 1, 2x + y \geq 2, 2x + 2y \leq 8, x \geq 0$ and $y \geq 0$
Q NO.3	maximum at $\left(\frac{20}{19}, \frac{45}{19}\right)$ and the value is $\frac{20}{19}$
Q NO.4	maximum at $\left(\frac{7}{2}, \frac{3}{4}\right)$ and the value 19 $minimum\ at\ (0,0) and\ the\ minimum\ value\ is\ 0$
Q NO.5	maximum value is 1160 at (2000,10000)

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

Q NO.1	I) (20,180), 940,160), (20,80) (ii) 136000 at (40,160) (iii) 200 **OR** 68000
Q NO.2	(i) -48 aat (0,8) (ii) -20 at (5,0) (iii) 68 **OR** (3,0), (3,2) ,(2,3), (0,3)

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable, and its probability distribution, mean of random variable.

- ***Conditional Probability***

If E and F are two events associated with the same sample space of a random experiment, then the conditional probability of the event E under the condition that the event F has occurred, written as P (E | F), is given by:

$$P(E \mid F) = \frac{P(E\cap F)}{P(F)}, P(F) \neq 0$$

- ***Properties of Conditional Probability***

Let E and F be events associated with the sample space S of an experiment. Then:

(i) P (S | F) = P (F | F) = 1

(ii) P [(A ∪ B) | F] = P (A | F) + P (B | F)-P [(A∩B | F)], where A and B are any two events associated with S.

(iii) P (E' | F) = 1-P (E | F)

- ***Multiplication Theorem on Probability***

Let E and F be two events associated with a sample space of an experiment. Then

$$P(E \cap F) = \begin{cases} P(E)\,P(F \mid E), & P(E) \neq 0 \\ P(F)P(E \mid F), & P(F) \neq 0 \end{cases}$$

If E, F and G are three events associated with a sample space, then.

P (E ∩ F ∩ G) = P(E) P (F | E) P (G | E ∩ F)

- ***Independent Events***

Let E and F be two events associated with a sample space S. If the probability of occurrence of one of them is not affected by the occurrence of the other, then we say that the two events are independent. Thus, two events E and F will be independent, if

P(F | E) = P(F), provided P(E) ≠ 0 and P(E | F) = P(E), provided P(F) ≠ 0

Using the multiplication theorem on probability, we have P (E ∩ F) = P(E) P(F)

Three events A, B and C are said to be mutually independent if all the following conditions hold.

P (A ∩ B) = P(A) P(B)

P (A ∩ C) = P(A) P(C); P (B ∩ C) = P(B) P(C) and P (A ∩ B ∩ C) = P(A) P(B) P(C)

- ***Partition of a Sample Space***

A set of events $E_1, E_2, \dots \dots E_n$ is said to represent a partition of a sample space S if

$(a)\ E_i \cap E_j = \phi, i \neq j; i, j = 1,2,3 \ldots, n$

(b) $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and

(c) Each $E_i \neq \phi, i.e., P(E_i) > 0 i = 1,2,3 \ldots$, n

- ***Theorem of Total Probability***

Let $\{E_1 E_2, \ldots, E_n\}$be a partition of the sample space S. Let A be any event associated with S, then

P(A) = $\sum_{j=1}^{n} P(E_j)P(A|E_j)$

- ***Bayes' Theorem***

If $E_1, E_2, \ldots \ldots E_n$ are mutually exclusive and exhaustive events associated with a sample space, and A is any event of non-zero probability, then. $P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{i=1}^{n} P(E_i)P(A|E_i)}$

- ***Random Variable and its Probability Distribution***

A random variable is a real valued function whose domain is the sample space of a random experiment. The probability distribution of a random variable X is the system of numbers.

X:	x_1	x_2	....	x_n
P(X):	p_1	p_2	....	p_n

where, p_i > 0; i = 1, 2, ..., n, $\sum_{i=1}^{n} P_i = 1$.

MULTIPLE COICE QUESTIONS (1 MARK EACH)

Q.1) If A and B are two events such that, P (A) =0.2, P (B) =0.4 and P (AUB) = 0.6 then, P (A/B) is equal to:

a. 0 b. 0.5 c. 0.3 d. 0.8

Q.2) If P(A∩B) = ½ and P (A'∩ B') = 1/3, P(A) = P, P(B)=2P(A) then the value of P is

a.1/9 b. 1/3 c. 4/9 d. 7/18

Q.3) If A and B are two independent events such that P (A U B') = 0.8 and P (A)=0.3 then P(B) is:

a. 2/3 b. 2/7 c. 1/8 d. 3/8

Q.4) 5 Boys and 5 Girls are sitting in a row randomly. The probability that boys and girls sit alternately:

a. 5/126 b. 4/126 c. 3/126 d. 1/126

Q.5) Three persons A, B and C fire a target in turn. Their Probabilities of hitting the target one 0.4; 0.3 and 0.2 respectively. The probability of two hits is:

a. 0.024 b. 0.188 c. 0.336 d. 0.452

Q6.) A and B toss 3 coins Probability that they both obtain the same number of heads is:

a. 1/9 b. 3/16 c. 5/16 d.3/8

Q7.) For the random variable X, E(X)=3 and E (X^2) =11 then the variance of X is equal to:

a.8 b. 5 c. 2 d. 1

Q8.) X speaks truth in in 60% and Y in 50% of the cases. The probability contradicts each other while narrating the same fact in

X	30	10	-10
P(X)	1/5	3/10	1/2

a. 1/4 b. 1/3 c. 1/2 d. 2/3

Q9.) The probability distribution of a discrete random variable X is given below. Then E(X) is equal to

a.6 b.4 c.3 d. -5

Q10) A bag contains 5 brown and 4 black socks. A man pulls out two socks. The probability that these one of the same colors is:

a. 5/108 b.18/108 c. 30/108 d. 48/108

Q11.) If A and B are two events such that P (A) $\neq$ 0 and P (B/A) = 1 then:

a. ACB b. BCA c. B= ϕ d. A= ϕ

Q12.) If P (A/B) > P(A) then which of the following is correct

a. P(B/A) < P(B) b. P(A∩B) < P(A). P(B) c. P(B/A) > P(B) d. P(B/A) = P(B)

Q13.) If A and B are any two events such that P (A) + P(B) – P(A∩B) = P(A) then:

a. P(B/A) =1 b. P(A/B) =1 c. P(B/A) =0 d. P(A/B) =0

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

Q14) If P(A) =0.4, P(B)=0.8 and P(B/A) = 0.6 then find P(A ∪ B).

Q15) Mother, father and son line up at a random for a family photo then find the probability of father should be in the middle when his son on one end.

Q16) Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively if both are trying to solve the problem independently then find the probability of that

(i) The problem is solved (ii) Exactly one of them solves the problem

Q17) The random variable X has a probability distribution $p(X)$ of the following form, where $'k'$is same number

$$p(X = x) = \begin{cases} k, & if\ x = 0 \\ 2k, & if\ x = 1 \\ 3k, & if\ x = 2 \\ 0 & otherwise \end{cases}$$

$then\ determine\ the\ value\ of\ k.$

Q18) The probability that it will rain on any particular day is 50%. Find the probability that it rains on the first 4 days of the week.

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

Q19) A refrigerator box contains 2 milk chocolates, and 4dark chocolates 2 chocolates are drawn at random
one by one without replacing. Find the probability distribution of the number of milk chocolates. What is the most likely outcome?

Hint: let x denotes the no. of milk chocolates drawn then $X = 0,1,2$ $p(X = 0) = \frac{12}{30}$, $p(X = 1) = \frac{16}{30}$, $p(X = 2) = \frac{2}{30}$ most likely chocolate of each kind.

Q20) Given that E and F are events such that P(E)=0.8 P(F)=0.7, P(E∩F) =0.6, then find $P(\bar{E}/\bar{F})$ =?

HINT: $P(\bar{E}/\bar{F}) = \frac{P(\bar{E}\cap\bar{F})}{P(\bar{F})} = \frac{P(\overline{E\cup F})}{P(\bar{F})} = \frac{1-P(E\cup F)}{1-P(F)}$

Q21) A die marked 1,2,3 in red and 4,5,6 in green is tossed. Let A be the event "numbers even" And B be the event" numbers are marked red". Find whether the event A and B are independent or not.

Q22) Suppose that 5 men out of 100 and 25 women out of 1000 are good orators assuming that there are equal nos. of men and women, find the probability of choosing a good orator.

Q23) The probability that a husband and wife will be alive 20 years from now are 0.7 and 0.8 find that 20-years's husband will be widower.

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

Q24) Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed, A reports the head appears. What is the probability that actually it was head?

Q25) An insurance company insured 2000 scooters drivers ,4000 cars drivers and 6000 truck drivers. The probabilities of accidents are 0.01,0.03,0.15 respectively one of the insured people meets an accident.
What is the probability that he is a scooter driver?

Q26) The probability distribution of a discrete variable X is given under:

X	1	2	4	$2p$	$3p$	$5p$
$p(X)$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Calculate:(i) the value of p if $E(X) = 2.94$ (ii) Variance of X

Q27) The probability function of a random variable x is given by

$$p(X) = \begin{cases} 2p, & if\ x = 1 \\ p, & if\ x = 2 \\ 4p, & if\ x = 3 \\ 0 & otherwise \end{cases}$$

Where p is constant find the value of

(i) P (ii) $p(0 \le x < 3)$ (iii) $p(x > 1)$

Q28) Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the mean and the variance of the number of red cards drawn.

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

In the following questions a statement of assertion (A) is followed by a statement of reason (R) choose the correct explanations of answer out of the following choices

(e) Both (A) and (R) are true and (R) is the correct explanation of (A).
(f) Both (A) and (R) are true and (R) is not the correct explanation of (A).
(g) (A) is true (R) is false.
(h) (A) is false (R) is true.

Q29) **ASSERTION(A):** Two coins are tossed simultaneously the probability of getting two heads if it is known that at least one head comes up is $\frac{1}{3}$.

REASON(R): Let E and F be two events with a random experiment, then $P\left(\frac{E}{F}\right) = \frac{P(E\cap F)}{P(F)}$

Q30) **ASSERTION (A):** A family has two children's the probability that both children are boys. If it is known that the elder child is boy is $\frac{1}{3}$.

REASON(R): Let E and F be two events with a random experiment then.

$$P(E\cup F) = P(E) + P(F) - P(E \cap F).$$

Q31) **ASSERTION (A):** Mother father and son line up in a random for a family picture. If E: son on one end and F: father in middle, then $P\left(\frac{E}{F}\right) = \frac{1}{2}$

REASON(R): Let E and F be two events with a random experiment then $P\left(\frac{E}{F}\right) = \frac{P(E\cap F)}{P(F)}$.

Q32) **ASSERTION (A):** If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$ then

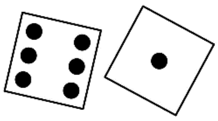
$$P(A' \cap B') = \frac{2}{9}$$

REASON(R): Let A and B be two events with a random experiment and A and B are independent. then P(A∩B) = P(A) . P(B)

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

Q33) A biased die is tossed and respective probabilities for various faces turn up are the following:

Face	1	2	3	4	5	6
Probability	0.1	0.24	0.19	0.18	0.15	k



Based on the information, answer the following:

(i) Then find the value of k
(ii) If face is showing an even number has turned up then, the probability it is the face with two or four

Q34) At the start of a cricket match coin is tossed and the team winning the toss has the opportunity to choose ball or bat. Such is coin in unbiased with equal probabilities of getting head and tails.:

Based on the information, answer the following:

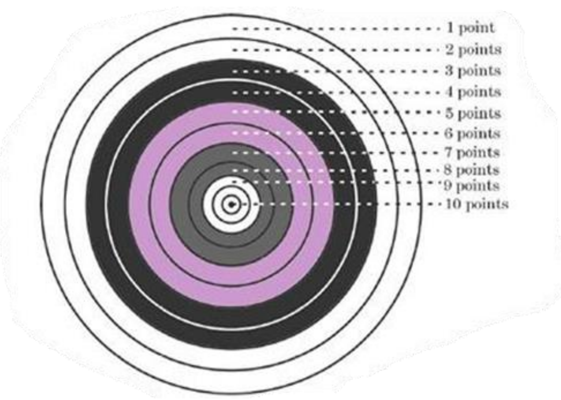
(i) If such a coin is tossed 2 times, then find the probability distribution of numbers of tails.
(ii) Find the probability of getting at least one head in three tosses of such coin.

Q35) A shopkeepers sells three types of flower seeds A_1 , A_2 & A_3. They are sold in the form of a mixture where the properties of these seeds are 4:4:2 respectively. The germination rates of the three types of seeds are 45%,60%and 35% respectively.

Based on the information, calculate the probability that

(i) A random seed chosen with germinate.

Q36) In a game of Archery, each ring of the Archery target is valued. The centermost ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards. Archer A is likely to earn 10 points with a probability of 0·8 and Archer B is likely the earn 10 points with a probability of 0·9. Based on the above information, answer the following questions:



If both of them hit the Archery target, then find the probability that

(i) exactly one of them earns 10 points.
(ii) both of them earn 10 points

ANSWERS

SECTION A

1. (a)
2. (d)
3. (b)
4. (d)
5. (b)
6. (c)
7. (c)
8. (c)
9. (b)
10. (d)
11. (a)
12. (c)
13. (b)

SECTION B

14.(0.96)

15. 1

16. (i) 2/3 (ii) ½

17. k=1/6

18.1/128

SECTION -C

19. hint given

20.1/3

21. not independent

22.($\frac{2}{3}$)

23. 0.14

ASSERTION-REASON

29. (a)

30 (d)

31.(d)

32.(a)

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

24. 4/5

25.1/52

26.(i)3 (ii) 10.41

27.(i) 1/7 (ii)3/7 (iii) 5/7

28. Mean = 3/2 Variance= 3/4

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

33.(i)0.14 (ii) ¾

34. (i)

X	0	1	2
P(X)	1/4	1/2	1/4

(ii) 7/8

35. (i) 49/100 (ii)24/49

Answer Key/ Hints

MULTIPLE CHOICE QUESTIONS (1 MARK EACH)

Q1) $P(A \cap B) = P + P(B) - P(A \cup B)$

$P(A \cap B) = 0 \qquad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = 0$

$Option\ (a)$

Q2) $(A' \cap B'^{)} \ = \ 1 - P(A \cup B)\ , \frac{1}{3} = 1 - [P(A) + P(B) - P(A \cap B)]$

$$\frac{1}{3} = 1 - \left[P + 2P - \frac{1}{2}\right]$$

$$\frac{1}{3} - 1 = 3P + \frac{1}{2} \quad => \ -\frac{2}{3} - \frac{1}{2} = \ -3P$$

$$P = \frac{7}{18}$$

Option (d)

Q3) $P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$$0.8 = 0.3 + P(B') - P(A).P(B')$$

$$0.3 = P(B')[1 - 0.3\} \quad => P(B') = \frac{5}{7} \therefore P(B) = \frac{2}{7}$$

Option (b)

Q4) 5 boys and 5girls

Total no. of ways of sitting in a row total ways $= 10!$

No. of ways to sitting alternatievly $= 5! \text{ x } 5! \text{ x } 2$

Probablity $= \frac{5!\text{x}5!\text{x}2}{10!} = \frac{2}{126}$

Option (d)

Q5. P(A) P(B) P(C) + P(A) P(B) P(C) + P(A) P(B)P(C)

= (0.4) (0.3) (0-8) + (0-4) (0.7) (0.2) + (0.6) (0.3) (0.2)

= 0.096 + 0.056 + 0.036 = 0.188

Option (b)

Q6) A got 1 head and B got 1 head or A got 2 head and B got 2 heads or A got 3 heads and B got 3 heads or A got 0 heads and B got 0 heads

$$\frac{3}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{3}{8} + \frac{1}{8} \times \frac{1}{8} + \frac{1}{8} \times \frac{1}{8} = \frac{9+9+1+1}{64} = \frac{20}{64} = \frac{5}{16}$$

Option (c)

Q7) Variance $= E(X^2) - E(X)^2$

$$= 11 - 9 = 2$$

Option (c)

Q8) $P(\bar{X})$ P(Y)+ P(X) P($\bar{Y}$) = $\frac{40}{100} \times \frac{50}{100} + \frac{60}{100} \times \frac{50}{100} = \frac{50}{100} = \frac{1}{2}$

Option (c)

Q9) $E(X) = 6 + 3 + (-5) = 4$

Option (b)

Q10) $\frac{5C_2 + 4C_2}{9C_2} = \frac{\frac{5}{2!\,3!} + \frac{4!}{2!\,2!}}{\frac{9}{2!\,7!}} = \frac{5\times2+2\times3}{9\times4} = \frac{\cancel{16}}{9\times4} = \frac{4}{3} \therefore \frac{48}{108}$

Option (d)

Q11) $\frac{\boldsymbol{P(B\cap A)}}{\boldsymbol{P(A)}}$=1 ∴ P (A∩B = P (A) ACB

Option (a)

Q12.) $\frac{P(A\cap B)}{P(B)}$ > P(A) P(B/A) = P$\frac{(A\cap B)}{PCA}$ $\frac{P(A\cap B)}{P(A)}$ > P(B) (P(B/A) > P (B)

(option – c)

Q13.) B (A∩B) = P (B) = $\frac{P(A\cap B)}{P(B)}$ = 1 = P(A/B) = 1

(option – b)

Q14.) P(B/A) = $\frac{P(A\cap B)}{P(A)}$ 0.6 * 0.4 = P (A ∩ B) ∴ P(A U B) = P(A) + P(B) – P (A∩B)
= 0.4+0.8-0.24

= 1.2 – 0.24

= 0.96

Q15.) n(s)= {(MFS), (MSF) (SM), (SFM), (SMF), (FMS)}

n(F)= {(SMA, (FMS)}
n(E)= {(SMF), (FMS)}

∴ P(E/F) = $\frac{P(E\cap F)}{P(F)}$ = $\frac{2}{2}$ = 1

Q16.) (i) P $(\overline{\mathbf{A}})$ P(B) + P(A) P $(\overline{\mathbf{B}})$ + P(A). P(B)

$\frac{1}{2}$ x $\frac{1}{3}$ + $\frac{1}{2}$ x $\frac{2}{3}$ + $\frac{1}{2}$ x $\frac{1}{3}$ = $\frac{2}{3}$

(ii) P $(\overline{\mathbf{A}})$ P(B) + P(A). P$(\overline{\mathbf{B}})$

$\frac{1}{2}$ x $\frac{1}{3}$ + $\frac{1}{2}$ x $\frac{2}{3}$ = $\frac{1}{2}$

Q17) $k + 2k + 3k + 0 = 1$ So $k = \frac{1}{-}$

Q18) $P(A) = (\frac{1}{2})^4 \times (\frac{1}{2})^3$

$= \frac{1}{128}$

HINT: 1ST rain,2nd rain,3rd rain, 4th rain,5th no rain, 6th no rain,7th no rain

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

Q19) and Q20 hint already given in question.

Q21) $A = \{2,4,6\}$ $B = \{1,2,3\}$

$P(A) = \frac{3}{6} = \frac{1}{2}$ $P(B) = \frac{1}{2}$ $P(A) \times P(B) = \frac{1}{4}$

$n(A \cap B) = \{2\}$ $P(A \cap B) = \frac{1}{6}$ $\therefore P(A \cap B) \neq P(A) \times P(B)$

$\therefore not\ independent$

Q22) A = good orator

E_1 = no. of men good orator

E_2 = no. of women good orator

$\therefore P(E_1) = P(E_2) = \frac{1}{2}$, $\quad P\left(\frac{A}{E_1}\right) = \frac{5}{100}$, $\quad P\left(\frac{A}{E_2}\right) = \frac{25}{1000}$

$$\therefore P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E1}\right)}{P(E_1) \times P\left(\frac{A}{E1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)} = \frac{2}{3}$$

Put the values and get Ans : 2/3.

Q23) P (E) $= P(H \cap \overline{W})$

$= \text{P(H). P}(\overline{W}) = P(H).[1 - P(W)]$

$= 0.7 \times 0.2$

$= 0.14$

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

Q24.) E_1:A speaks truth

E_2:A speaks false

Let X be the event that a head appears.

P(E_1)= $\frac{4}{5}$

P(E_2:)=1−P(E_1)= $1 - \frac{4}{5} = \frac{1}{5}$

If a coin is tossed, then it may result in either head (H) or tail (T).

The probability of getting a head is $\frac{1}{2}$ whether A speaks truth or not.

P(X/E_1)=P(X/E_2)=$\frac{1}{2}$

The probability that there is actually a head is given by P(E1|X).

$$P(E_1|X) = \frac{P(E_1).P(X|E_1)}{P(E_1).P(X|E_1) + P(E_2).P(X|E_2)}$$

$$= \frac{\frac{4}{5}\cdot\frac{1}{2}}{\frac{4}{5}\cdot\frac{1}{2} + \frac{1}{5}\cdot\frac{1}{2}}$$

$$= \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{10}} = \frac{\frac{2}{5}}{\frac{5}{10}} = \frac{4}{5}$$

$$\Rightarrow P(E_1|X) = \frac{4}{5}$$

Q25) Let E_1, E_2 and E_3 be the events of a driver being a scooter driver, car driver and truck driver. respectively. Let A be the event that the person meets with an accident.

There are 2000 insured scooter drivers, 4000 insured car drivers and 6000 insured truck drivers.

Total number of insured vehicle drivers = 2000 + 4000 + 6000 = 12000

$$\therefore P(E_1) = \frac{2000}{12000} = \frac{1}{6}, P(E_2) = \frac{4000}{12000} = \frac{1}{3}, P(E_3) = \frac{6000}{12000} = \frac{1}{2}$$

Also, we have:

$P(A|E_1) = 0.01 = \frac{1}{100}$, $P(A|E_2) = 0.03 = \frac{3}{100}$, $P(A|E_3) = 0.15 = \frac{15}{100}$

Now, the probability that the insured person who meets with an accident is a scooter driver is $P(E_1|A)$.

Using Bayes' theorem, we obtain:

$$P(E_1 | A) = \frac{P(E_1)\times P(A|E_1)}{P(E_1)\times P(A|E_1) + P(E_2)\times P(A|E_2) + P(E_3)\times P(A|E_2)}$$

$$= \frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100} + \frac{1}{3}\times\frac{3}{100} + \frac{1}{2}\times\frac{15}{100}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + 1 + \frac{15}{2}}$$

$$= \frac{1}{6}\times\frac{6}{52}$$

$$= \frac{1}{52}$$

Q26) (i) $E(X) = 1\times\frac{1}{2} + 2\times\frac{1}{5} + 4\times\frac{3}{25} + 2P\times\frac{1}{10} + 3P\times\frac{1}{25} + 5P\times\frac{1}{25}$

$2.94 = \frac{69}{50} + \frac{26P}{50}$

$2.94 = 1.38 + 0.52P$

$1.56 = 0.52P$

$P = 3$

(ii) Variance $= 1^2\times\frac{1}{2} + 2^2\times\frac{1}{5} + 4^2\times\frac{3}{25} + 6^2\times\frac{1}{10} + 9^2\times\frac{1}{25} + 15^2\times\frac{1}{25} - (2.94)^2$

$= 19.06 - 8.6436$

$= 10.4164$

≈ 10.41

Q27) $2P + P + 4P + 0 = 1$

(i) $P = \frac{1}{7}$

$(ii) P(0 < x < 3) = 2P + P$

$= 3P = \frac{3}{7}$

$$(iii)\ P(X > 1) = P + 4P + 0$$
$$= 5P = \frac{5}{7}$$

Q28.)

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

Probability of red card= 26/52 = 1/2

Mean = 0x1/8 + 1x3/8 + 2x3/8 + 3x1/8= 1.5

Variance = $0^2 * \frac{1}{8} + 1^2 * \frac{3}{8} + 2^2 * \frac{3}{8} + 3^2 * \frac{1}{8} - (1.5)^2$ = 3-2.25 = 3/4

Q29.) The reason is a general rule for conditional probability and is correct. However, it is not directly related to the assertion.

To find the probability of getting two heads when at least one head comes up, we can use the formula for conditional probability:

P(A/B) = P(A and B) / P(B)

Here, let A be the event of getting two heads and B be the event of getting at least one head. Then, we have: P (A and B) = P(A) = 1/4, since there is only one way to get two heads out of four possible outcomes when two coins are tossed.

P(B) = 3/4, since there are three possible outcomes out of four when at least one head comes up (HH, HT, TH).

Therefore, the conditional probability of getting two heads given that at least one head comes up is: P(A/B) = P(A and B) / P(B) = (1/4) / (3/4) = 1/3

So, the assertion is true and the reason is also true, but the reason does not explain or justify the assertion. Hence, the correct option is (a).

Q30) $A: elder\ boy\ child: \{BB, BG\}$

$$B: both\ are\ boys: \{BB\}$$

$$A \cap B: \{BB\}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P\ (A)} = \frac{1}{2}$$

Option (d) – (A) is false (R) is true.

The total number of ways in which Mother(M), Father(F) and Son(S) can be lined up at random in one of the following ways:

{MFS, MSF, FMS,FSM,SFM,SMF}

A={SMF, SFM,MFS,FMS} and B={MFS,SFM}

Therefore, A∩B= {MFS, SFM}; n(A∩B) =2 and n(B)=2

Therefore,

Required probability = P(A/B) =n(A∩B) / n(B) = $\frac{2}{2} = 1$

Is false (R) is true

Option – (d)

31.)

Q32.) $P(A'\cap B')=1-P(A\cup B)=1-[P(A)+P(B)-P(A\cap B)]=1-[\frac{3}{5}+\frac{4}{9}-\frac{3}{5}\times\frac{4}{9}]$ $\quad [P(A\cap B)=P(A)\cdot P(B)]$

$=1-[\frac{27+20-12}{45}]=1-\frac{35}{45} = \frac{10}{45}=\frac{2}{9}$

Option (a)

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

Q34) (i) Table is already given.

(ii) $P(E) = 1 - P(no\ head)$

$= 1 - P(TTT)$

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$

Q35.) We have, A1: A2: A4=4:4:2

P(A1) $=\frac{4}{10}$, P(A2) $=\frac{4}{10}$ and P(A3) $=\frac{2}{10}$

where A1, A2 and A3 denote the three types of flower seed.

Let E be the event that a seed germinates and $\bar{E}$ be the event that a seed does not germinate.

∴P(E/A1)$=\frac{45}{100}$,P(E/A2)$=\frac{60}{100}$,P(E/A3)$=\frac{35}{100}$

And ∴P($\bar{E}$/A1) $=\frac{55}{100}$, P($\bar{E}$/A2)$=\frac{40}{100}$,P($\bar{E}$ /A3)$=\frac{65}{100}$

(i) ∴P(E)=P(A1)·P(E/A1)+P(A2)·P(E/A2)+P(A3)·P(E/A3)

$=\frac{4}{10}.\frac{45}{100}.+\frac{4}{10}.\frac{60}{100}+\frac{2}{10}.\frac{35}{100}$

$=\frac{180}{1000}+\frac{240}{1000}+\frac{70}{1000}=\frac{490}{100}=0.49$

Q36) $P(A) = 0.8$

$P(B) = 0.9$

$(i) P(B) \times P(\bar{B}) + P(\bar{A}) \times P(B) = 0.8 \times 0.1 + 0.2 \times 0.9$

$= 0.26$

$(ii) P(A) \times P(B) = 0.8 \times 0.9$

$= 0.72$

SAMPLE QUESTION PAPER

Class:-XII

Session 2023-24

Mathematics (Code-041)

Time: 3 hours **Maximum marks: 80**

General Instructions:

1. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** of 4 marks each with sub-parts.

Section –A
(Multiple Choice Questions)

Each question carries 1 mark

Q1. If $A=\left[a_{ij}\right]$ is a square matrix of order 2 such that $a_{ij}=\begin{cases}\mathbf{1}, \text{ when } i\neq j\\ \mathbf{0}, \text{ when } i=j\end{cases}$, then A^2 is

(a) $\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}_{2\times2}$ (b) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}_{2\times2}$ (c) $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}_{2\times2}$ (d) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}_{2\times2}$

Q2. If A and B are invertible square matrices of the same order, then which of the following is not correct?

(a) $\left|AB^{-1}\right|=\dfrac{|A|}{|B|}$ (b) $\left|(AB)^{-1}\right|=\dfrac{1}{|A|\ |B|}$

(c) $(AB)^{-1}=B^{-1}A^{-1}$ (d) $(A+B)^{-1}=B^{-1}+A^{-1}$

Q3. If the area of the triangle with vertices $(-3,0)$, $(3,0)$ and $(0,k)$ is **9 sq units,** then the value/s of k will be

(a) 9 (b) ± 3 (c) -9 (d) 6

Q4. If $f(x)=\begin{cases}\dfrac{kx}{|x|}, & \text{if } x<0\\ 3, & \text{if } x\geq 0\end{cases}$ is continuous at $x=0$, then the value of k is

(a) -3 (b) 0 (c) 3 (d) any real number

Q5. The lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(6\hat{i} + 9\hat{j} - 18\hat{k}\right)$; (where λ & μ are scalars) are

(a) coincident (b) skew (c) intersecting (d) parallel

Q6. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$ *is*

(a) 4 (b) $\frac{3}{2}$ (c) 2 (d) Not defined

Q7. The corner points of the bounded feasible region determined by a system of linear constraints are $(0,3),(1,1)$ and $(3,0)$. Let $Z = px + qy$, where $p, q > 0$. The condition on p and q so that the minimum of Z occurs at $(3,0)$ and $(1,1)$ is

(a) $p = 2q$ (b) $p = \frac{q}{2}$ (c) $p = 3q$ (d) $p = q$

Q8. $ABCD$ is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$ equals to

(a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BD}$ (d) $2\overrightarrow{AD}$

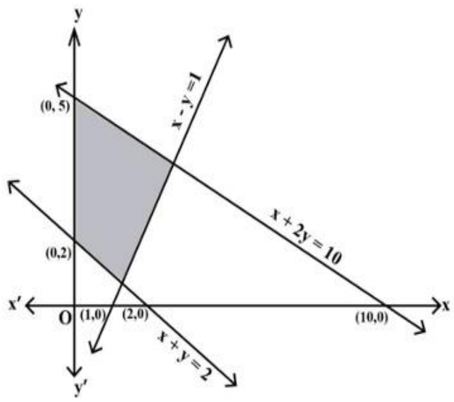
Q9. For any integer n, the value of $\int_{-\pi}^{\pi} e^{\cos^2 x} \sin^3(2n+1)x \, dx$ is

(a) -1 (b) 0 (c) 1 (d) 2

Q10. The value of $|A|$, if $A = \begin{bmatrix} 0 & 2x-1 & \sqrt{x} \\ 1-2x & 0 & 2\sqrt{x} \\ -\sqrt{x} & -2\sqrt{x} & 0 \end{bmatrix}$, where $x \in \mathbb{R}^+$, is

(a) $(2x+1)^2$ (b) 0 (c) $(2x+1)^3$ (d) $(2x-1)^2$

Q11. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.



Which of the following is not a constraint to the given Linear Programming Problem?

(a) $x + y \geq 2$ (b) $x + 2y \leq 10$ (c) $x - y \geq 1$ (d) $x - y \leq 1$

Q12. If $\vec{a} = 4\hat{i} + 6\hat{j}$ and $\vec{b} = 3\hat{j} + 4\hat{k}$, then the vector form of the component of $\vec{a}$ along $\vec{b}$ is

(a) $\frac{18}{5}\left(3\hat{i} + 4\hat{k}\right)$ (b) $\frac{18}{25}\left(3\hat{j} + 4\hat{k}\right)$ (c) $\frac{18}{5}\left(3\hat{i} + 4\hat{k}\right)$ (d) $\frac{18}{25}\left(4\hat{i} + 6\hat{j}\right)$

Q13. Given that A is a square matrix of order 3 and $|A| = -2$, then $|adj(2A)|$ is equal to

(a) -2^6 (b) $+4$ (c) -2^8 (d) 2^8

Q14. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the events of their solving the problem are independent then the probability that the problem will be solved, is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Q15. The general solution of the differential equation $ydx - xdy = 0$; (Given $x, y > 0$), is of the form

(a) $xy = c$ (b) $x = cy^2$ (c) $y = cx$ (d) $y = cx^2$;

(Where 'c' is an arbitrary positive constant of integration)

Q16. The value of λ for which two vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + \lambda\hat{j} + \hat{k}$ are perpendicular is

(a) 2 (b) 4 (c) 6 (d) 8

Q17. The set of all points where the function $f(x) = x + |x|$ is differentiable, is

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, \infty)$

Q18. If the direction cosines of a line are $< \frac{1}{c}, \frac{1}{c}, \frac{1}{c} >$, then

(a) $0 < c < 1$ (b) $c > 2$ (c) $c = \pm\sqrt{2}$ (d) $c = \pm\sqrt{3}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

Q19. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x)) = (x-1)^3(x-3)^2$, then

ASSERTION (A): $f(x)$ has a minimum at $x = 1$.

REASON (R): When $\frac{d}{dx}(f(x)) < 0, \forall x \in (a-h, a)$ and $\frac{d}{dx}(f(x)) > 0, \forall x \in (a, a+h)$; where '$h$' is an infinitesimally small positive quantity, then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.

Q20. **ASSERTION (A)**: The relation $f:\{1,2,3,4\}\to\{x,y,z,p\}$ defined by $f=\{(1,x),(2,y),(3,z)\}$ is a bijective function.

REASON (R): The function $f:\{1,2,3\}\to\{x,y,z,p\}$ such that $f=\{(1,x),(2,y),(3,z)\}$ is one-one.

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

Q21. Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$.

OR

Find the domain of $\sin^{-1}\left(x^2-4\right)$.

Q22. Find the interval/s in which the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=xe^x$, is increasing.

Q23. If $f(x)=\frac{1}{4x^2+2x+1}$; $x\in\mathbb{R}$, then find the maximum value of $f(x)$.

OR

Find the maximum profit that a company can make, if the profit function is given by

$P(x)=72+42x-x^2$, where x is the number of units and P is the profit in rupees.

Q24. Evaluate : $\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx$.

Q25. Check whether the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x^3+x$, has any critical point/s or not ? If yes, then find the point/s.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

Q26. Find : $\int\frac{2x^2+3}{x^2\left(x^2+9\right)}dx$; $x\neq 0$.

Q27. The random variable X has a probability distribution $P(X)$ of the following form, where 'k' is some real number:

$$P(X)=\begin{cases} k, & \text{if } x=0 \\ 2k, & \text{if } x=1 \\ 3k, & \text{if } x=2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the value of k.

(ii) Find $P(X<2)$.

(iii) Find $P(X > 2)$.

Q28. Find : $\int \sqrt{\frac{x}{1-x^3}}\,dx;\quad x \in (0,1)$.

OR

Evaluate: $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx$.

Q29. Solve the differential equation: $ye^{\frac{x}{y}}\,dx = \left(xe^{\frac{x}{y}} + y^2\right)dy,\ (y \neq 0)$.

OR

Solve the differential equation: $(\cos^2 x)\frac{dy}{dx} + y = \tan x;\quad \left(0 \leq x < \frac{\pi}{2}\right)$.

Q30. Solve the following Linear Programming Problem graphically:

Minimize: $z = x + 2y$,

subject to the constraints: $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0$.

OR

Solve the following Linear Programming Problem graphically:

Maximize: $z = -x + 2y$,

subject to the constraints: $x \geq 3, x + y \geq 5,\ x + 2y \geq 6, y \geq 0$.

Q31. If $(a+bx)e^{\frac{y}{x}} = x$ then prove that $x\frac{d^2y}{dx^2} = \left(\frac{a}{a+bx}\right)^2$.

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

Q32. Make a rough sketch of the region $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$ and find the area of the region, using the method of integration.

Q33. Let $\mathbb{N}$ be the set of all natural numbers and R be a relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a,b)\,R\,(c,d) \Leftrightarrow ad = bc$ for all $(a,b),(c,d) \in \mathbb{N} \times \mathbb{N}$. Show that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$. Also, find the equivalence class of $(2,6)$, i.e., $[(2,6)]$.

OR

Show that the function $f : \mathbb{R} \to \{x \in \mathbb{R} : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in \mathbb{R}$ is one-one and onto function.

Q34. Using the matrix method, solve the following system of linear equations :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2.$$

Q35. Find the coordinates of the image of the point $(1, 6, 3)$ with respect to the line $\vec{r}=\left(\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$; where '$\lambda$' is a scalar. Also, find the distance of the image from the $y-$ axis.

OR

An aeroplane is flying along the line $\vec{r}=\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$; where '$\lambda$' is a scalar and another aeroplane is flying along the line $\vec{r}=\hat{i}-\hat{j}+\mu\left(-2\hat{j}+\hat{k}\right)$; where '$\mu$' is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them.

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

Q36. Read the following passage and answer the questions given below:

In an Office three employees Jayant, Sonia and Oliver process incoming copies of a certain form. Jayant processes **50%** of the forms, Sonia processes **20%** and Oliver the remaining **30%** of the forms. Jayant has an error rate of **0.06** , Sonia has an error rate of **0.04** and Oliver has an error rate of **0.03** .

Based on the above information, answer the following questions.

(i) Find the probability that Sonia processed the form and committed an error.

(ii) Find the total probability of committing an error in processing the form.

(iii) The manager of the Company wants to do a quality check. During inspection, he selects a form at random from the days output of processed form. If the form selected at random has an error, find the probability that the form is **not** processed by Jayant.

OR

(iii) Let E be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Jayant, Sonia and Oliver processed the form. Find the value of $\sum_{i=1}^{3} P(E_i|E)$.

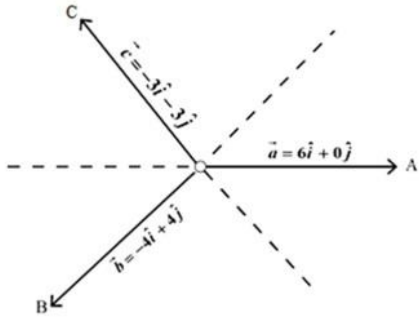
Q37. Read the following passage and answer the questions given below:

Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area.

Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}\ kN$,

Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}\ kN$,

Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}\ kN$,



(i) What is the magnitude of the force of Team A?

(ii) Which team will win the game?

(iii) Find the magnitude of the resultant force exerted by the teams.

OR

(iii) In what direction is the ring getting pulled?

Q38. Read the following passage and answer the questions given below:

The relation between the height of the plant ('y' in cm) with respect to its exposure to the sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where 'x' is the number of days exposed to the sunlight, for $x \le 3$.

(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

(ii) Does the rate of growth of the plant increase or decrease in the first three days? What will be the height of the plant after 2 days?

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