Vortex Spin Down Using PySPH Solver

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This project involves simulation of a vortex spin down

1. INTRODUCTION

SPH

Smoothed Particle Hydrodynamics **(SPH)** is a mesh-free Lagrangian method (where the coordinates move with the fluid) used for simulating the dynamics of continuum media like fluid flows.

This method works by dividing the fluid into a set of discrete elements, referred to as particles. These particles have a spatial distance (known as the "smoothing length- ***h***" over which their properties are "smoothed" by a kernel function. So the physical quantity of any particle can be obtained by summing the relevant properties of all the particles which lie within the range of the kernel.

WCSPH

Recently SPH has been extended and used to simulate the motion of incompressible fluids. Incompressibility is approximated by assuming a compressible fluid with a large sound speed—typically a Mach number of $M \approx 0.1$ is used. This approach will be termed here "weakly compressible" SPH or "**WCSPH**" and results obtained using this approach have been acceptable for free surface and some low Reynolds number flows. Due to its simplicity and lower computational cost this method is widely used for various kinds of problems.

TVF

Standard SPH method suffers from tensile instability and it will occur where the pressure is negative and result in particle clumping. So Transport Velocity Formulation Method was introduced, characterized by the different utilization of a background pressure on computing the particle momentum velocity and the particle advection velocity. As a consequence of the new advection velocity, an additional term appears in the momentum equation that accounts for the difference between the motion of a particle and its averaged momentum.

2. PROBLEM DESCRIPTION

A standard problem of vortex spin down method is set up and solved in a Python module-PySPH using two different schemes – WCSPH and TVF.

Problem Statement:

Fluid: unit square with left bottom corner at origin, surrounded by walls on all four sides. Use 50 x 50 particle arrangement.

Re = 420, rho = 1.0, c (speed of sound) = 1.25, 2.5, 5

U = 0.25(y-0.5)

V = 0.25(0.5-x)

Simulation done for t=8,10 s

3. GOVERNING EQUATIONS

In SPH, particles are used to interpolate a continuous function A(x) at a position x. The contributing particles, relevant for a position, are determined by a kernel function W of finite support associated with each particle. For a fluid, the interpolation

$$\frac{d\rho_a}{dt} = \sum_b m_b \mathbf{v}_{ab} \nabla_a W_{ab}$$

is given as

$$A(x) = \sum_j mj\ Aj\ \rho j\ W(x - xj\ , h)$$

The value *mj* denotes the mass of a particle; *ρj* denotes the

$$W(\mathbf{r},h) = \frac{10}{7\pi h^2} \begin{cases} 1 - \frac{3}{2}q^2 + \frac{3}{4}q^3 & 0 \le q \le 1 \\ \frac{1}{4}(2-q)^3 & 1 \le q \le 2 \\ 0 & q \ge 2 \end{cases}$$

density of particle and *h* the particle radius. *W* is a interpolation kernel, here we use cubic spline kernel.

Following is normalized cubic spline for 2 D

$$\rho_a = \sum_b m_b W_{ab}$$

Continuity equation

The original SPH approach uses a summation formula to calculate the density:

Since the mass is carried by the particles, this form conserves mass exactly

The density is updated by the following relation

Momentum equation: In SPH form, the momentum equation is written as

$$\frac{d\mathbf{v}_a}{dt} = -\sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} \right) \nabla_a W_{ab} + \mathbf{g}$$

Equation of state: The following Tait's equation implements low density variation and it is efficient to compute

$$P = B\left(\left(\frac{\rho}{\rho_0} \right)^{\gamma} - 1 \right)$$

To determine values for B, we consider that compressibility effects scale with O(M^2) with M denoting the Mach number of the flow. This results in the following relation:

$$\frac{|\Delta\rho|}{\rho_0} \sim \frac{|\mathbf{v}_f|^2}{c_s^2}$$

Typically $\dfrac{\Delta\rho}{\rho_0}$ is set to 1% or 0.01 so M=0.1

So B is enforced as:

$$B = \frac{\rho_0 c_s^2}{\gamma}$$

Where c is speed of sound and ϒ=7 and so this give equation of state for WCSPH

For TVF, ϒ=1 which gives equation of state of TVF

$$p = c^2(\rho - \rho_0) = p_0\left(\frac{\rho}{\rho_0} - 1 \right)$$

Artificial viscosity:

In order to improve the numerical stability and to allow for shock phenomena, artificial viscosity is employed

$$\frac{d\mathbf{v}_a}{dt} = \begin{cases} -\sum_b m_b \Pi_{ab} \nabla_a W_{ab} & \mathbf{v}_{ab}^T \mathbf{x}_{ab} < 0 \\ 0 & \mathbf{v}_{ab}^T \mathbf{x}_{ab} \geq 0 \end{cases}$$

where $$\Pi_{ab} = -\nu\left(\frac{\mathbf{v}_{ab}^T\mathbf{x}_{ab}}{|\mathbf{x}_{ab}|^2+\varepsilon h^2}\right)$$

Energy equation: In SPH form, energy equation is written as follows:

Particle Advection velocity

$$\frac{De_i}{Dt} = \frac{1}{2}\sum_{j=1}^{N} m_j\left(\frac{p_i}{\rho_i^2}+\frac{p_j}{\rho_j^2}\right)v_{ij}^{\beta}\frac{\partial W_{ij}}{\partial x_i^{\beta}}.$$

The key element in TVF is the modification of the particle advection velocity. Different from moving the particles with the momentum velocity, a transport or advection velocity $\tilde{v}$ that is used to evolve the position of particles from one time step to the next by

$$\frac{d\mathbf{r}_i}{dt} = \tilde{\mathbf{v}}_i.$$

Using a constant background pressure field p_b, the particle motion takes advantage of the regularization and anti-clumping effect when advecting with the transport velocity $\tilde{v}$

$$\tilde{\mathbf{v}}_i(t+\delta t) = \mathbf{v}_i(t) + \delta t\left(\frac{\tilde{d}\mathbf{v}_i}{dt} - \frac{p_b}{m_i}\sum_j\left(V_i^2+V_j^2\right)\frac{\partial W}{\partial r_{ij}}\mathbf{e}_{ij}\right).$$

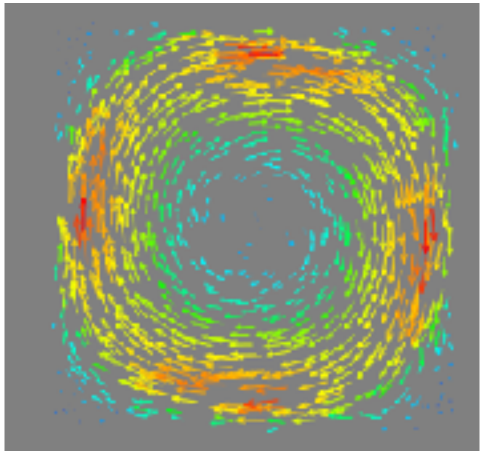
4. Time-integration scheme

To integrate equation of motion and momentum equation, EPEC model is used. In the Evaluate-Predict-Evaluate-Correct (EPEC) mode, the system is advanced using:

$$\begin{aligned}
F(y^n) &-- > Evaluate\\
y^{n+\frac{1}{2}} = y^n + F(y^n) &-- > Predict\\
F(y^{n+\frac{1}{2}}) &-- > Evaluate\\
y^{n+1} = y^n + \Delta t F(y^{n+\frac{1}{2}}) &-- > Correct
\end{aligned}$$

5. Comparison :

a) WCSPH



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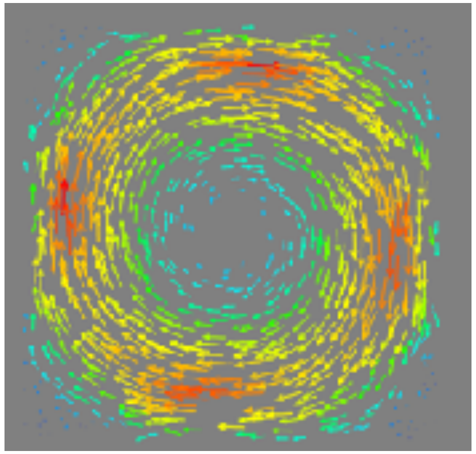

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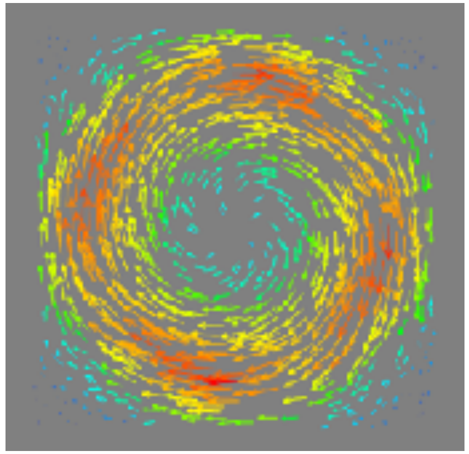

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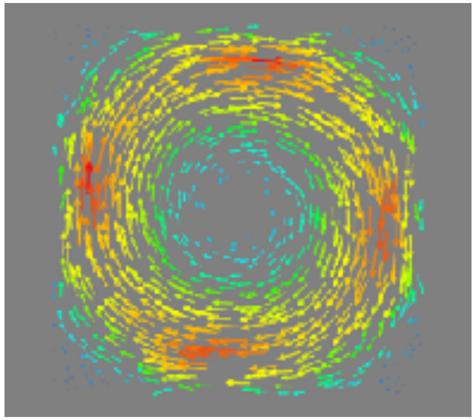

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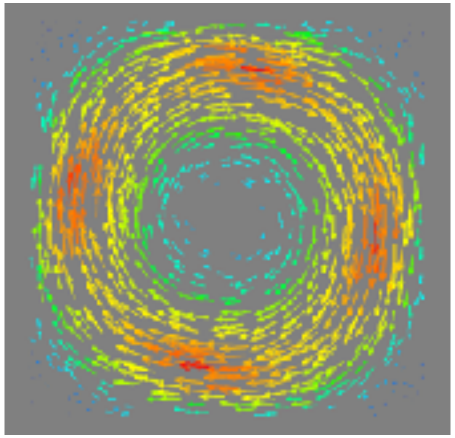

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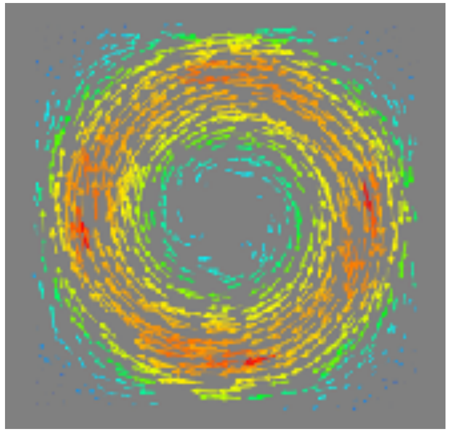

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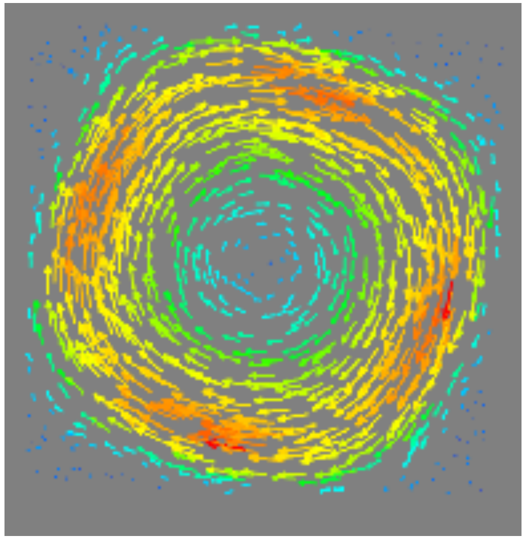

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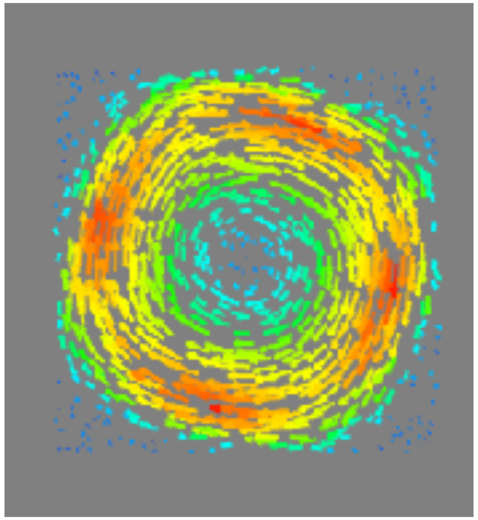

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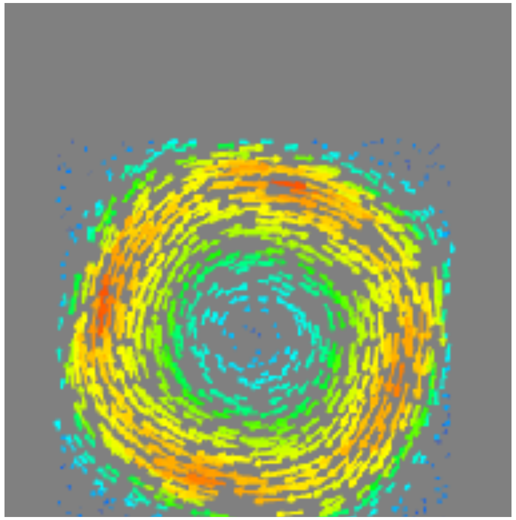

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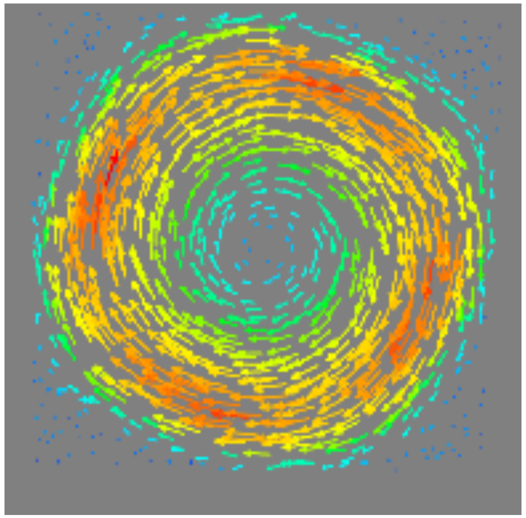

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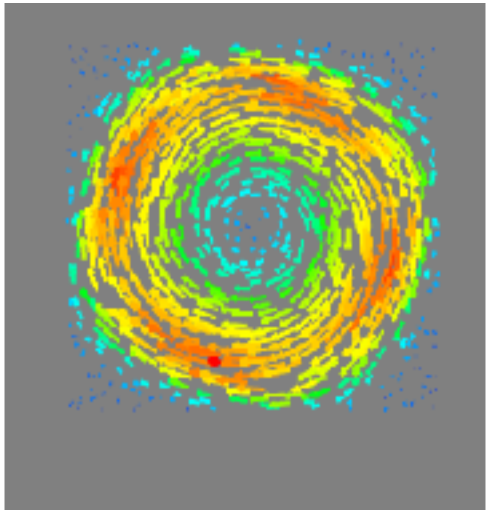

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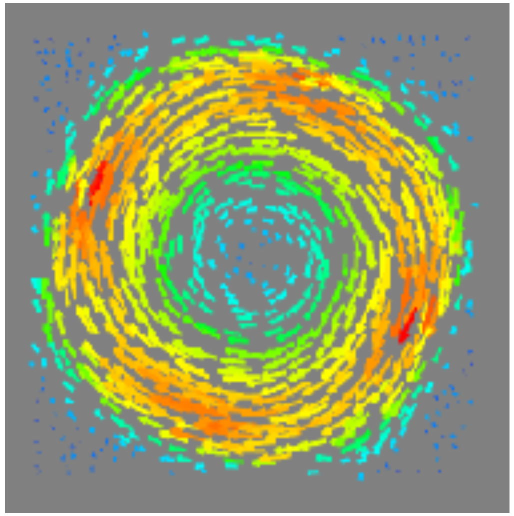

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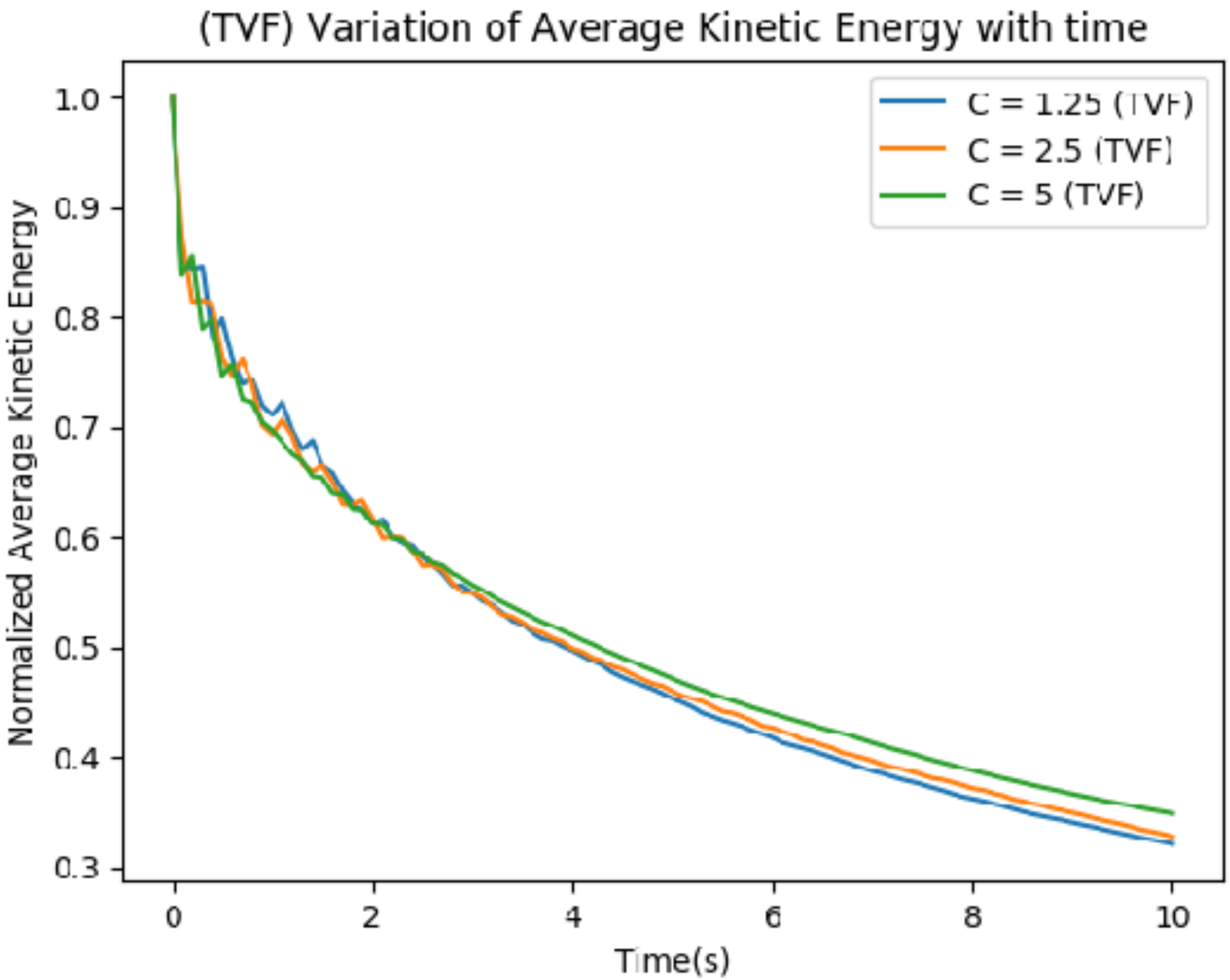
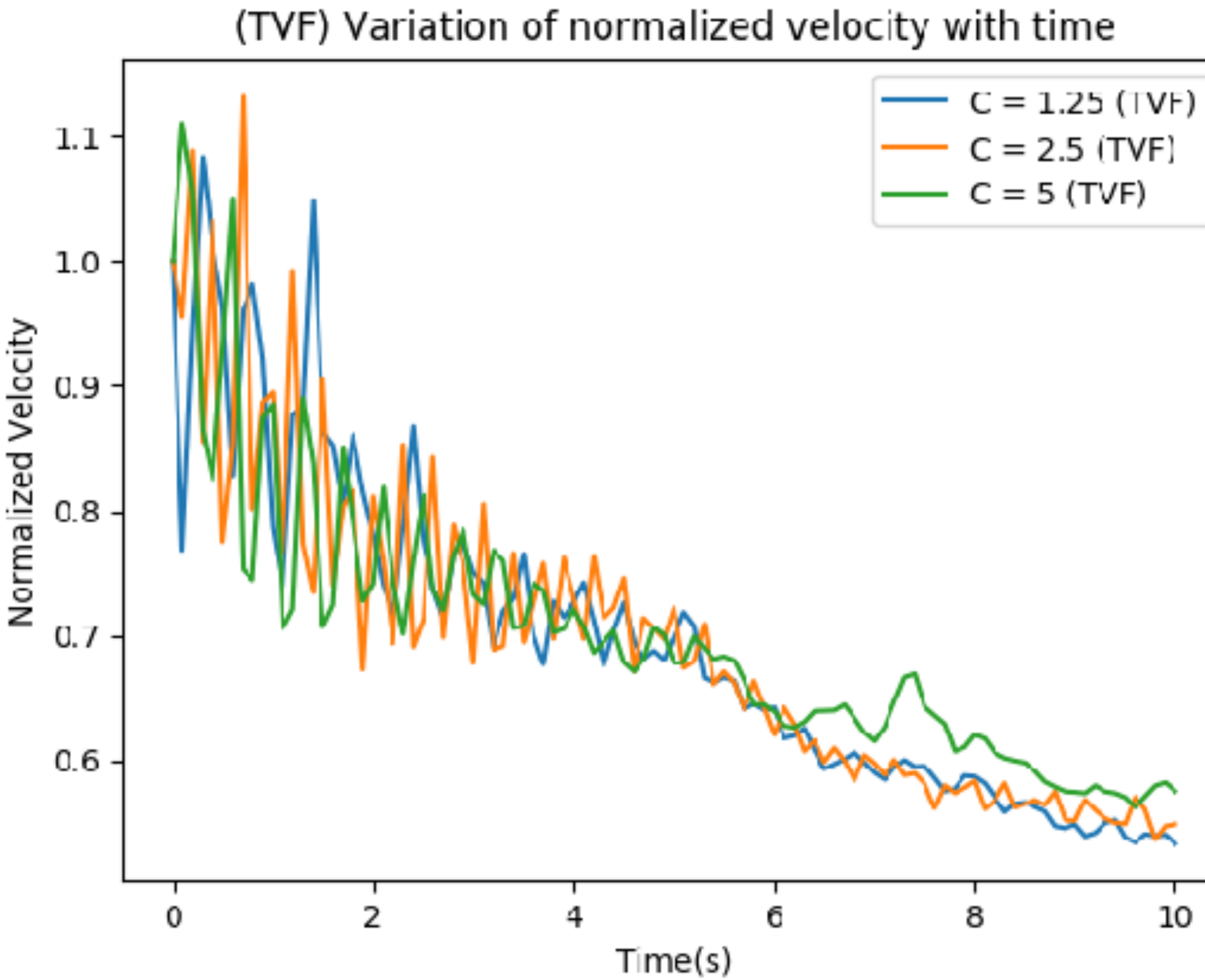
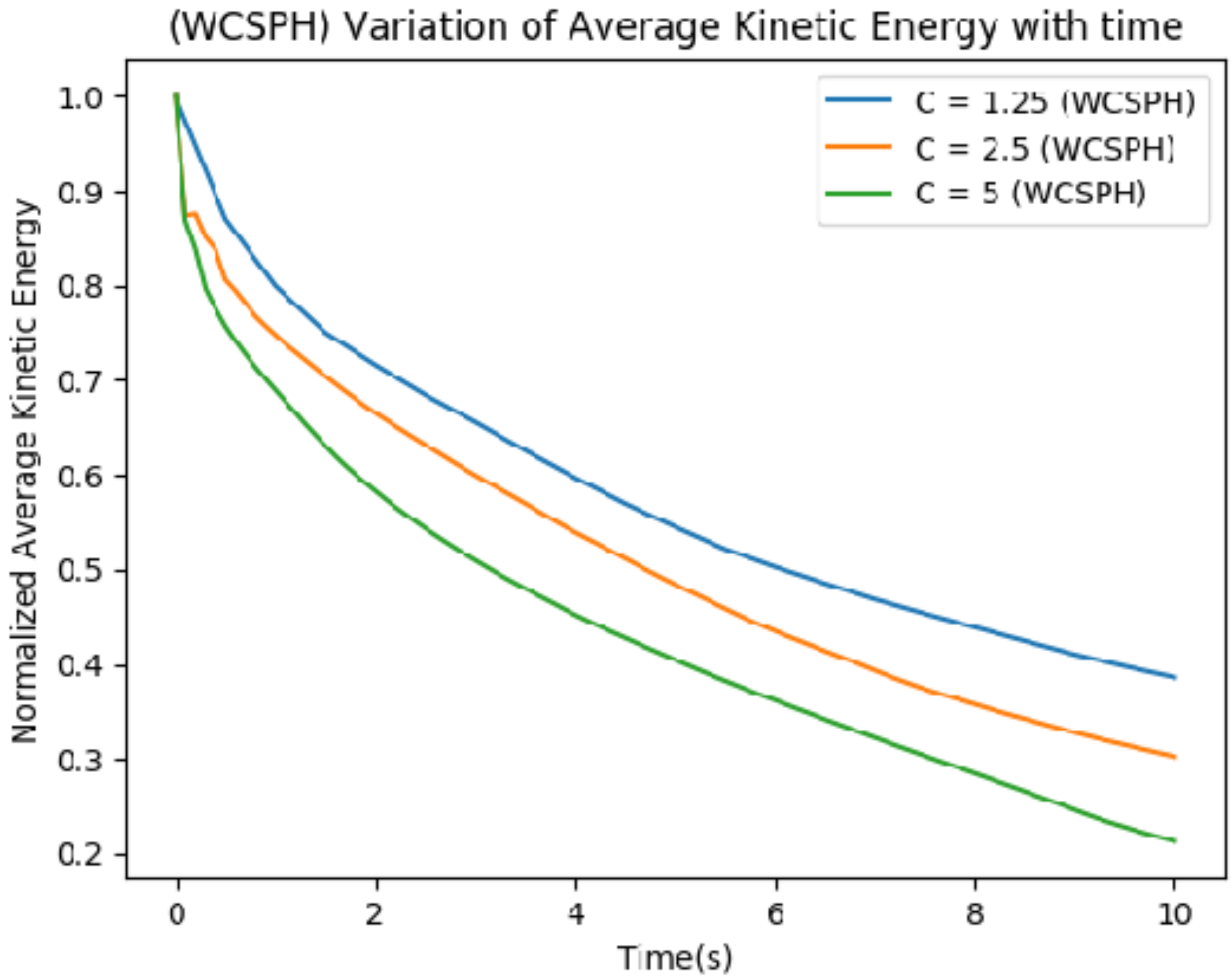
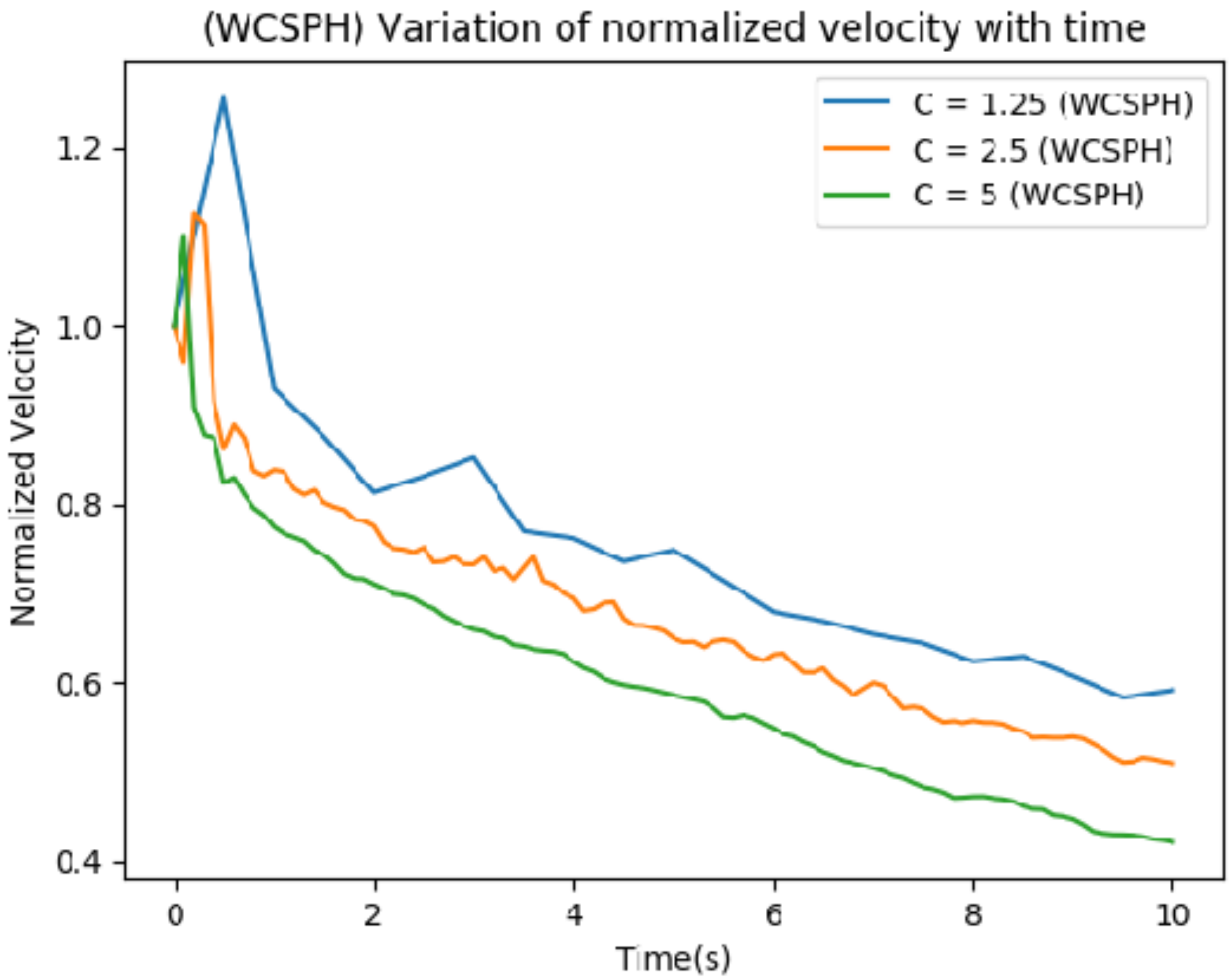
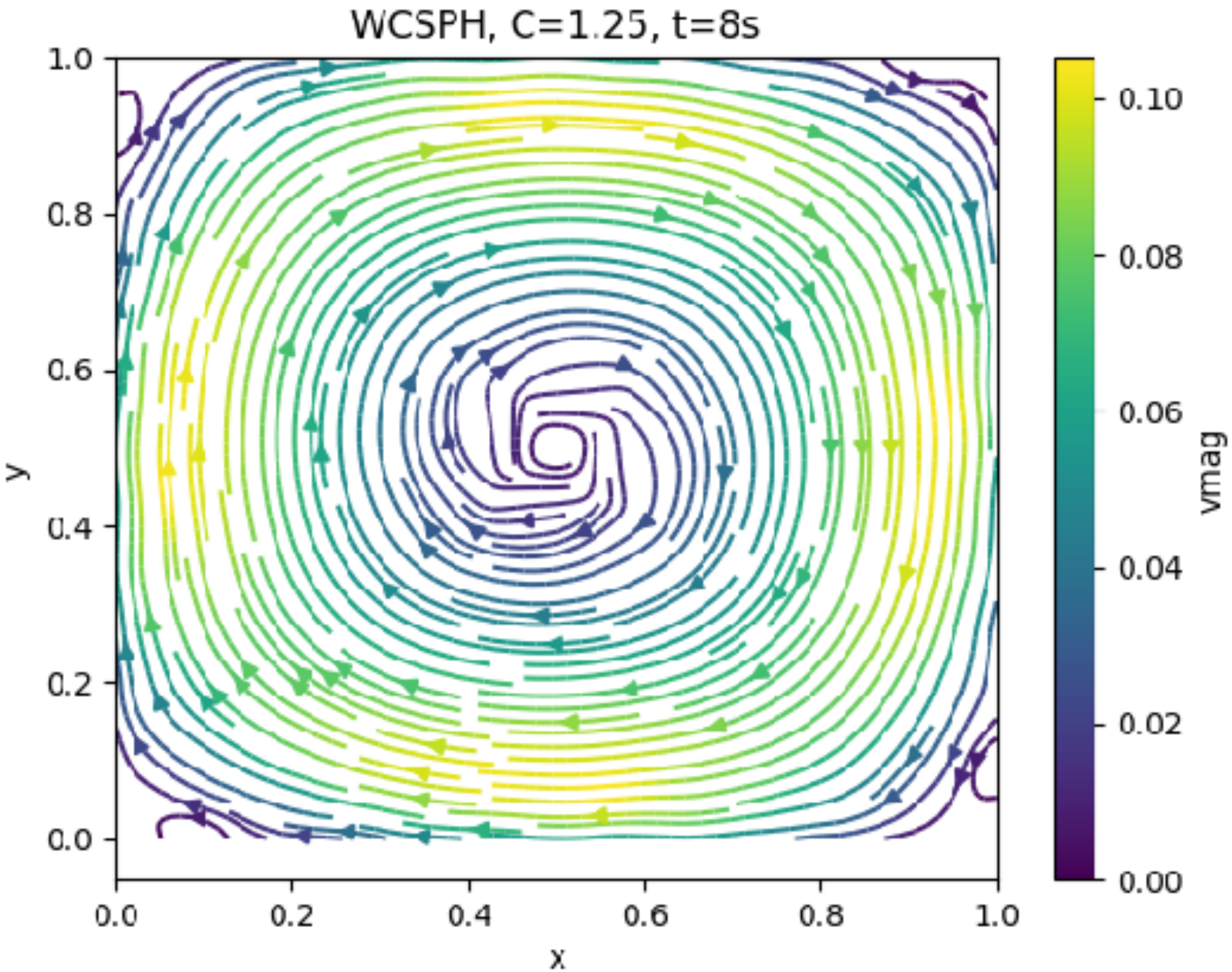

(TVF) Variation of Average Kinetic Energy with time
C = 1.25 (TVF)
C = 2.5 (TVF)
C = 5 (TVF)
Normalized Average Kinetic Energy
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
2
4
6
8
10
Time(s)


(TVF) Variation of normalized velocity with time
C = 1.25 (TVF)
C = 2.5 (TVF)
C = 5 (TVF)
Normalized Velocity
Time(s)
1.1
1.0
0.9
0.8
0.7
0.6
0
2
4
6
8
10

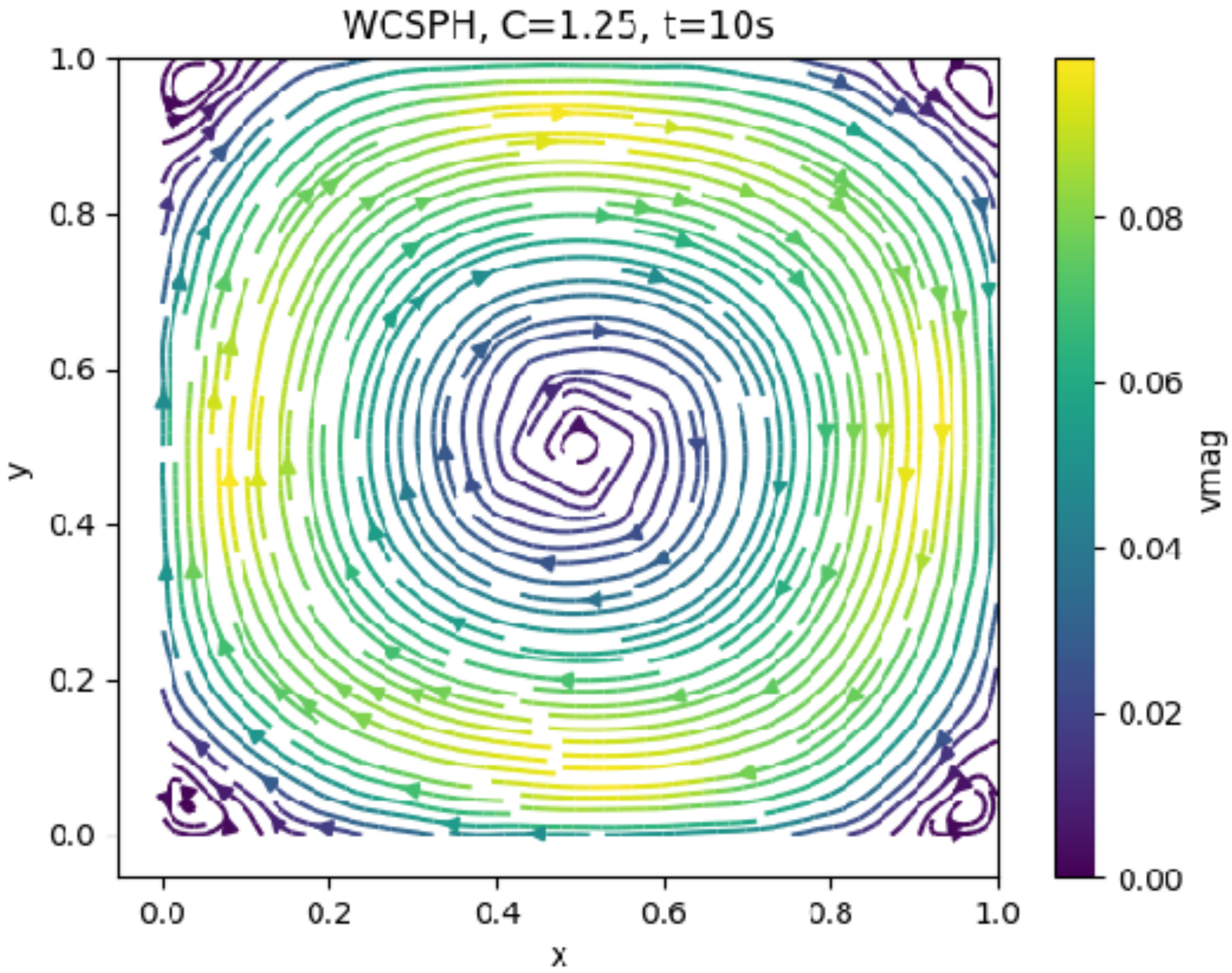

(WCSPH) Variation of Average Kinetic Energy with time
C = 1.25 (WCSPH)
C = 2.5 (WCSPH)
C = 5 (WCSPH)
Normalized Average Kinetic Energy
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
2
4
6
8
10
Time(s)

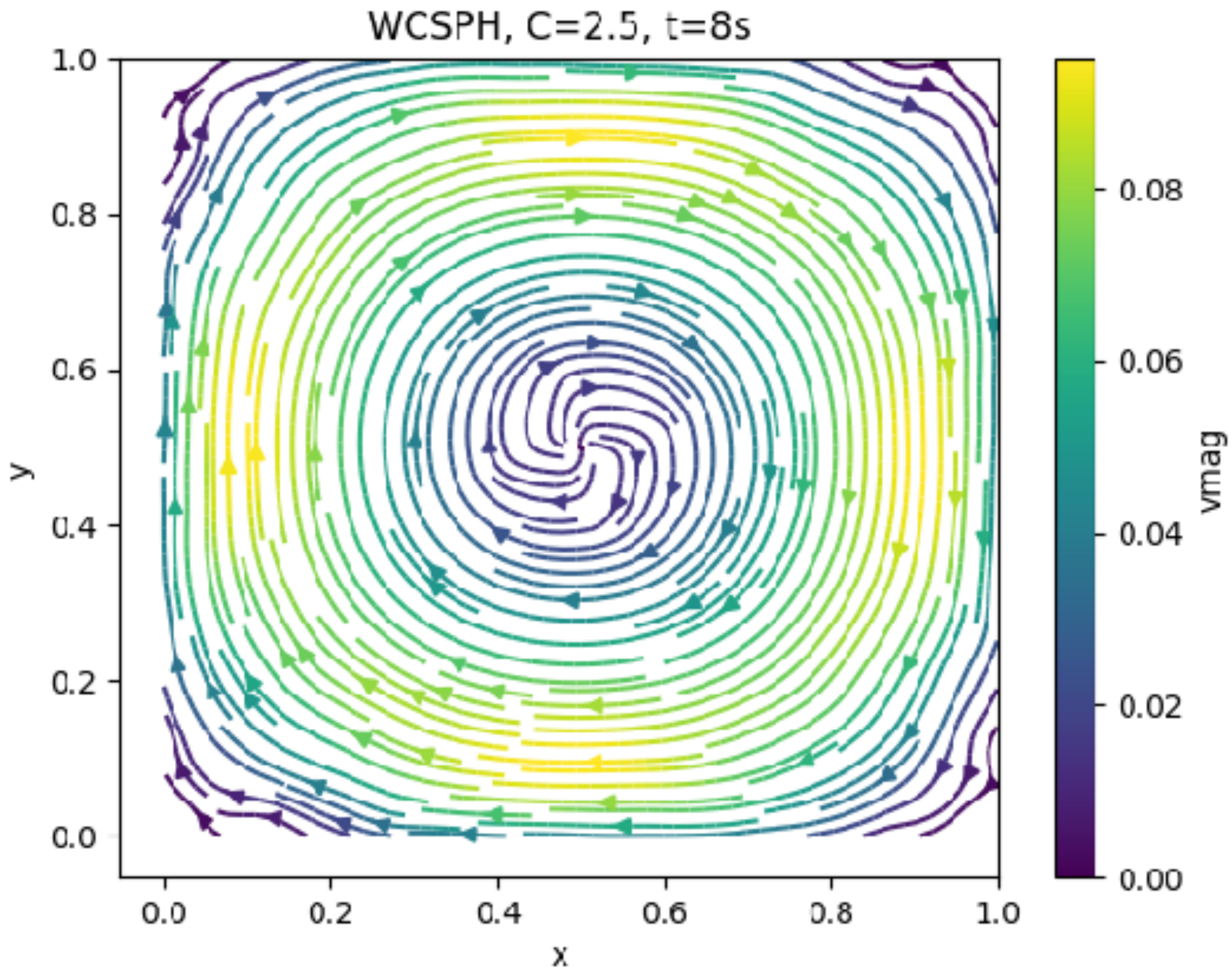


(WCSPH) Variation of normalized velocity with time
C = 1.25 (WCSPH)
C = 2.5 (WCSPH)
C = 5 (WCSPH)
Normalized Velocity
1.2
1.0
0.8
0.6
0.4
0
2
4
6
8
10
Time(s)



WCSPH, C=1.25, t=8s
y
x
vmag
0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.02
0.04
0.06
0.08
0.10


WCSPH, C=1.25, t=10s
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x
vmag
0.00
0.02
0.04
0.06
0.08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x
vmag
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0.2
0.4
0.6
0.8
1.0
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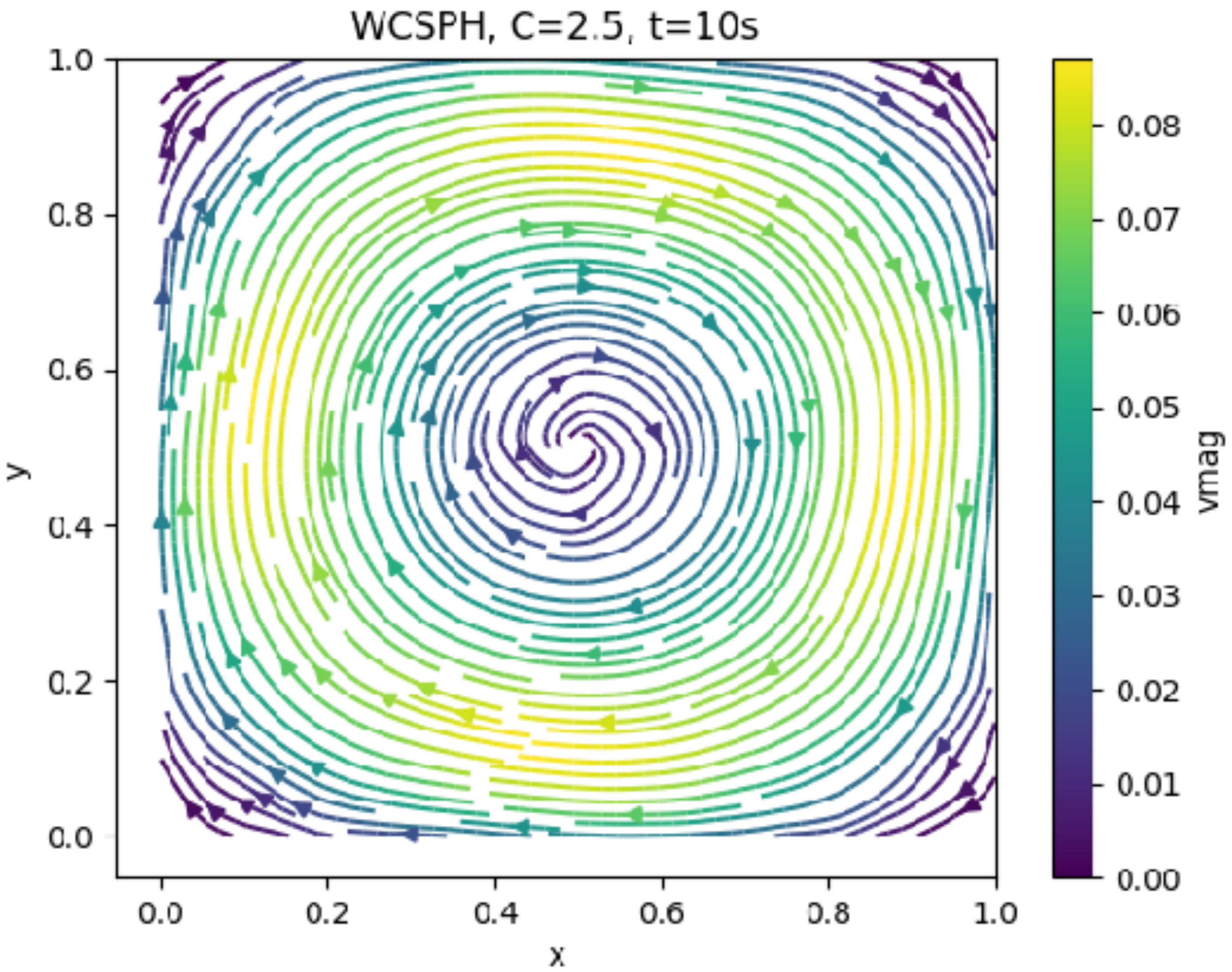
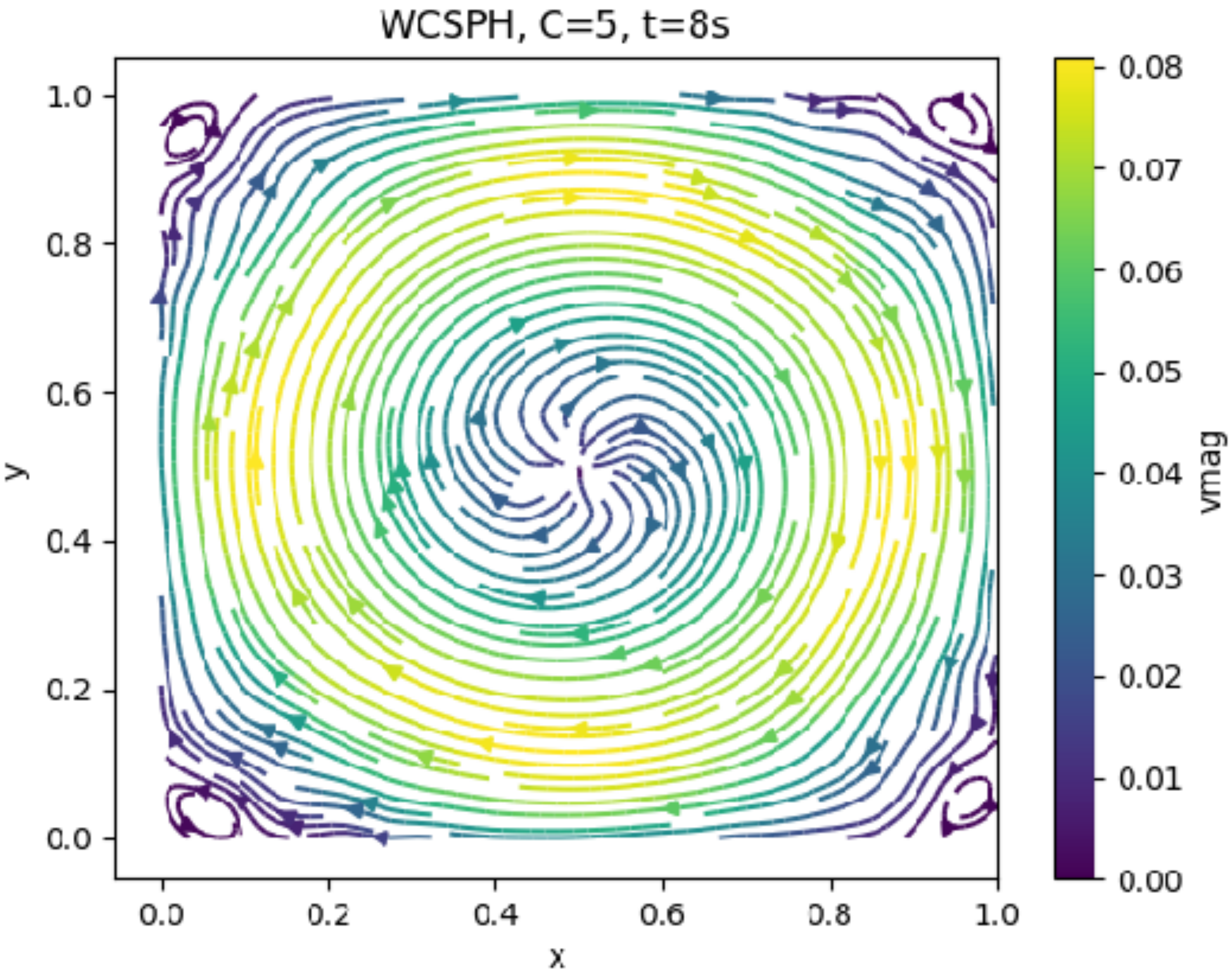
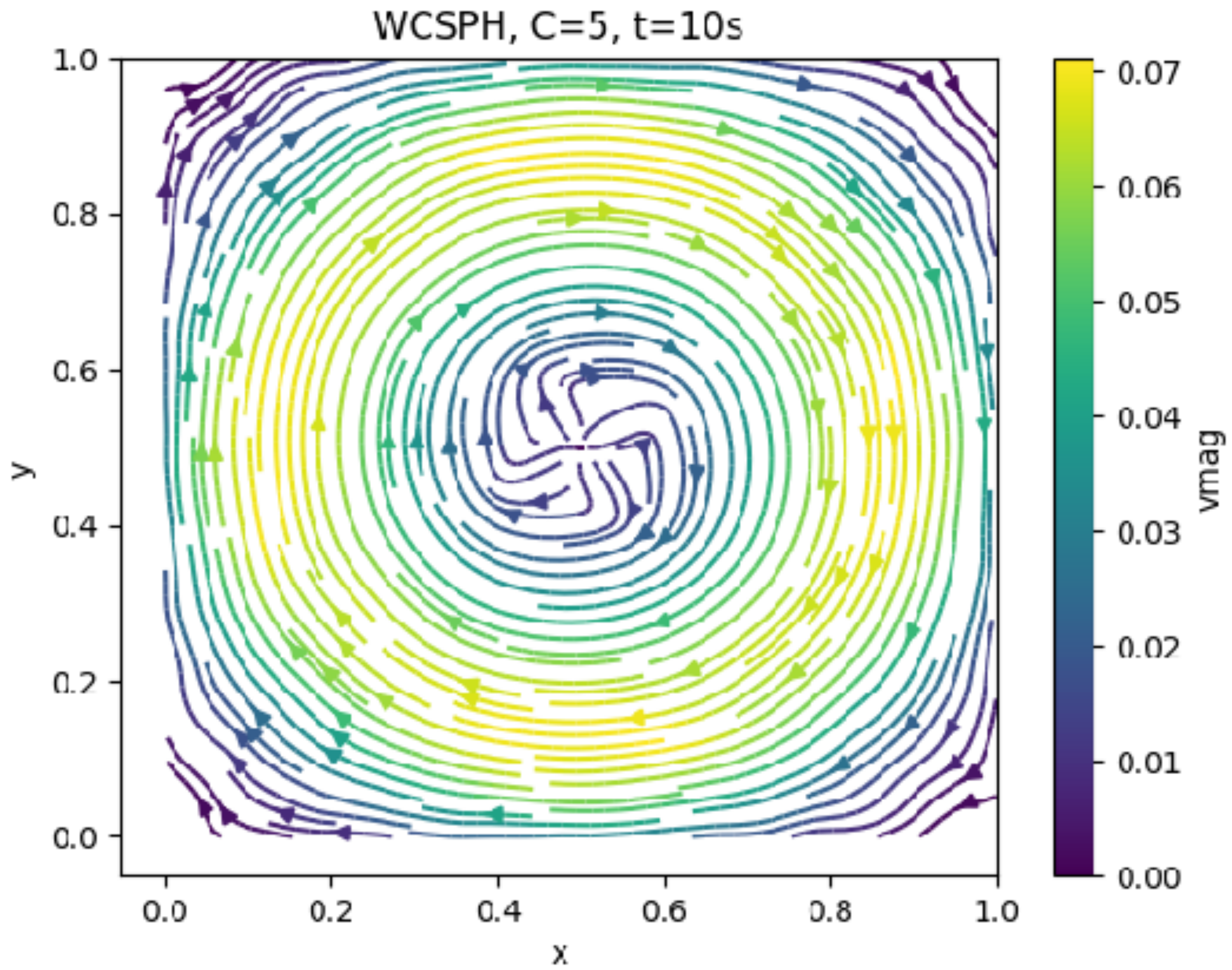
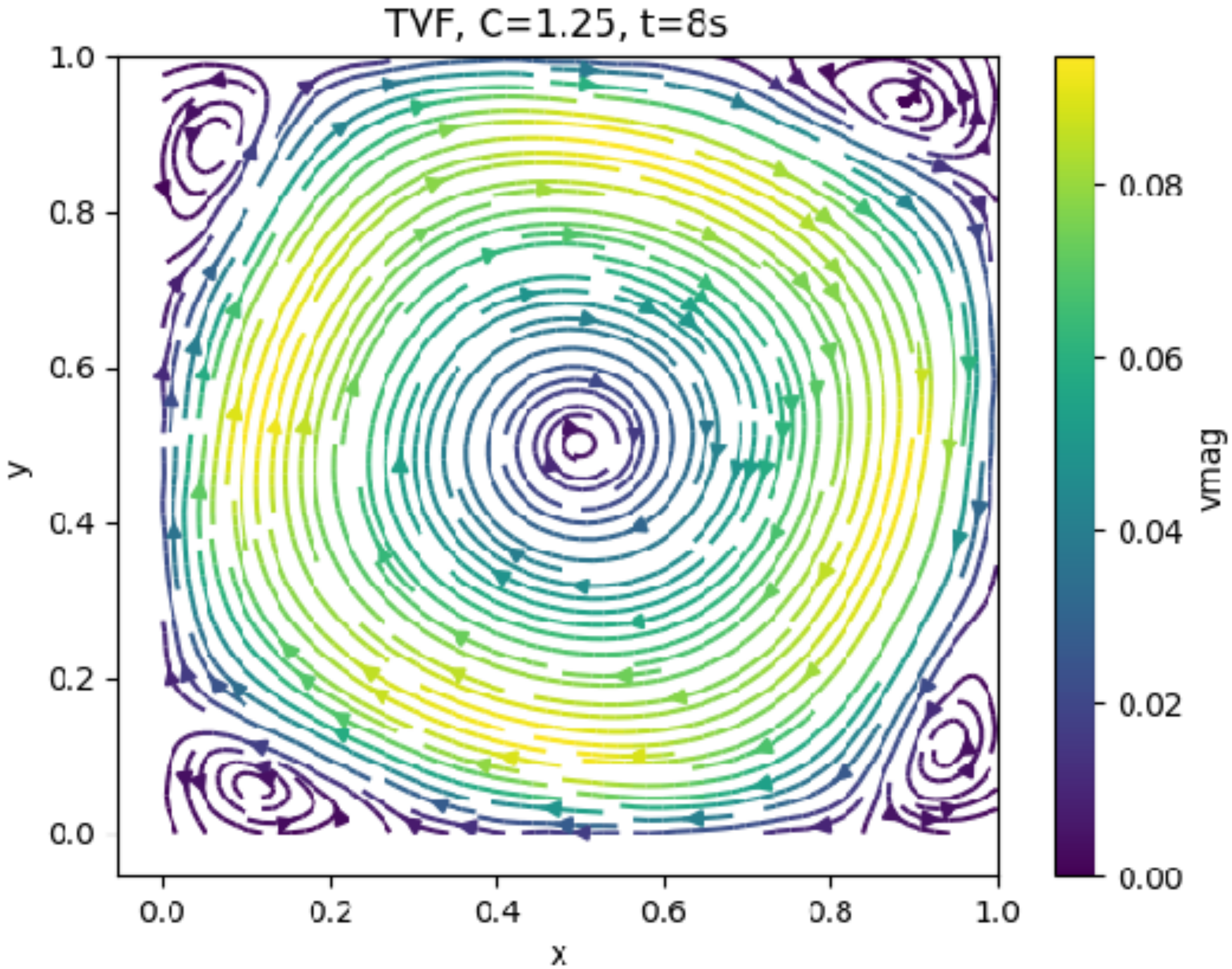
0.02
0.04
0.06
0.08


WCSPH, C=2.5, t=10s
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
vmag

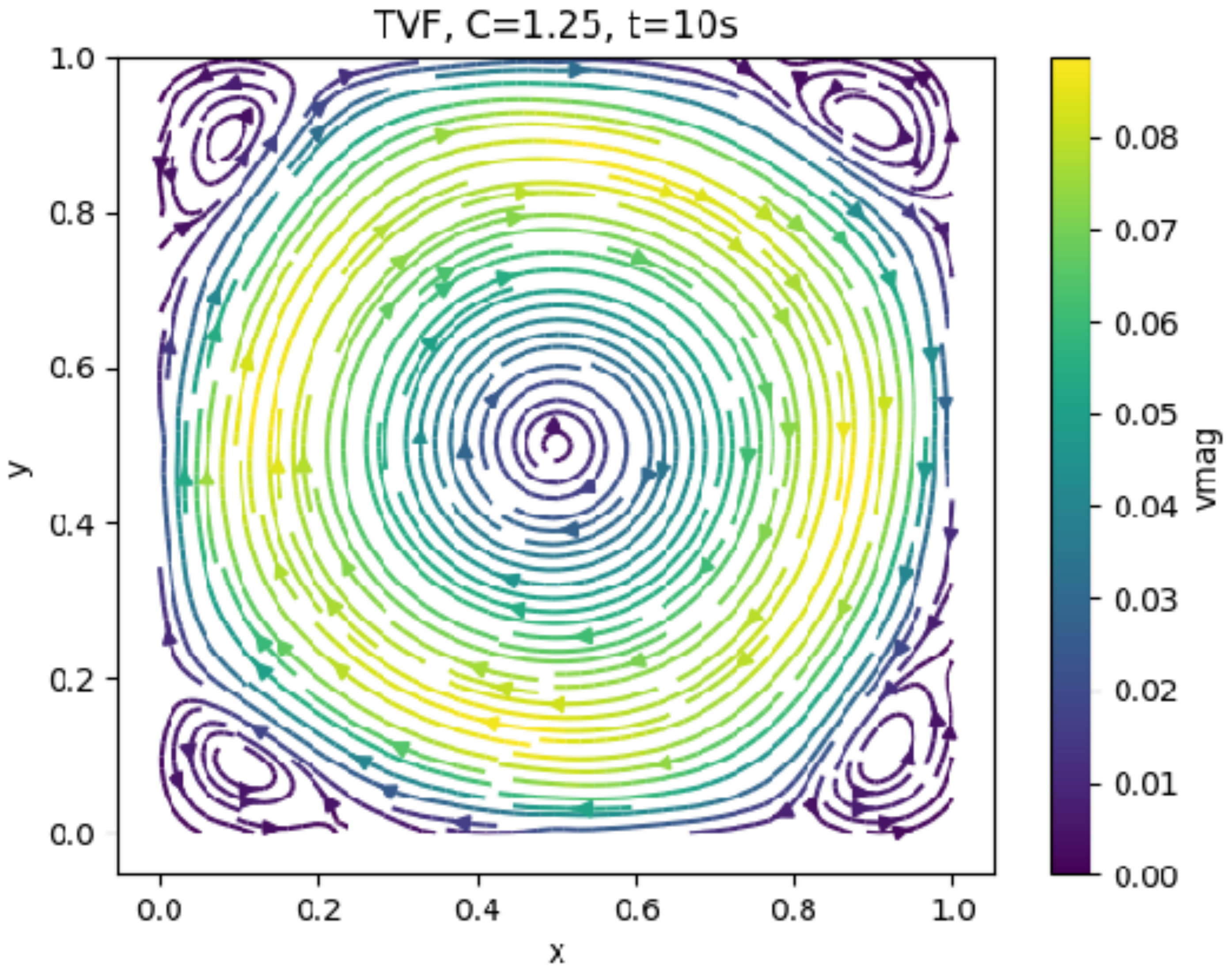


WCSPH, C=5, t=8s
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
vmag


WCSPH, C=5, t=10s
y
x
vmag
0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07



TVF, C=1.25, t=8s
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x
0.08
0.06
0.04
0.02
0.00
vmag


TVF, C=1.25, t=10s
y
x
vmag
0.0
0.2
0.4
0.6
0.8
1.0
0.00
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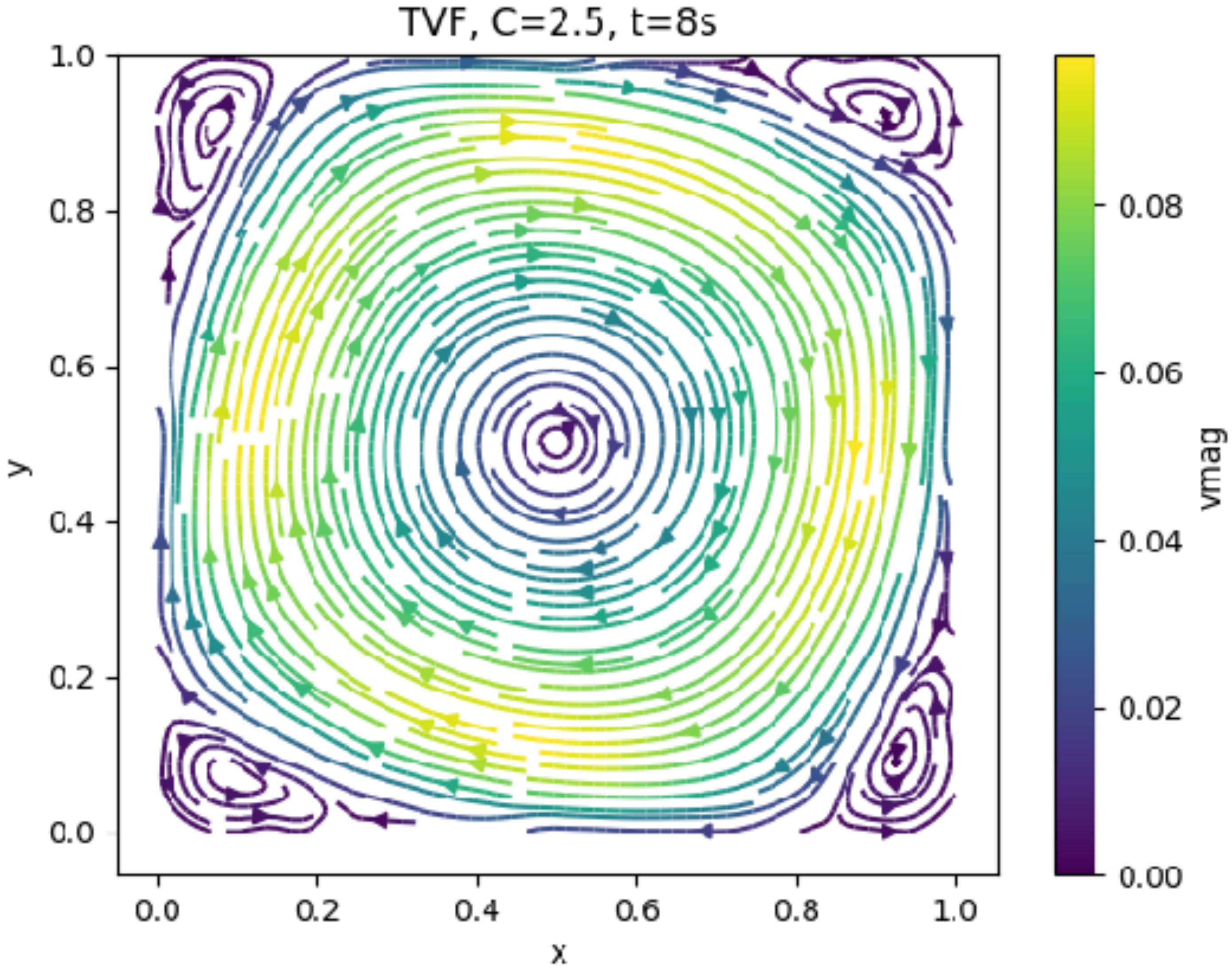
0.06
0.07
0.08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0.6
0.4
0.2
0.0
y
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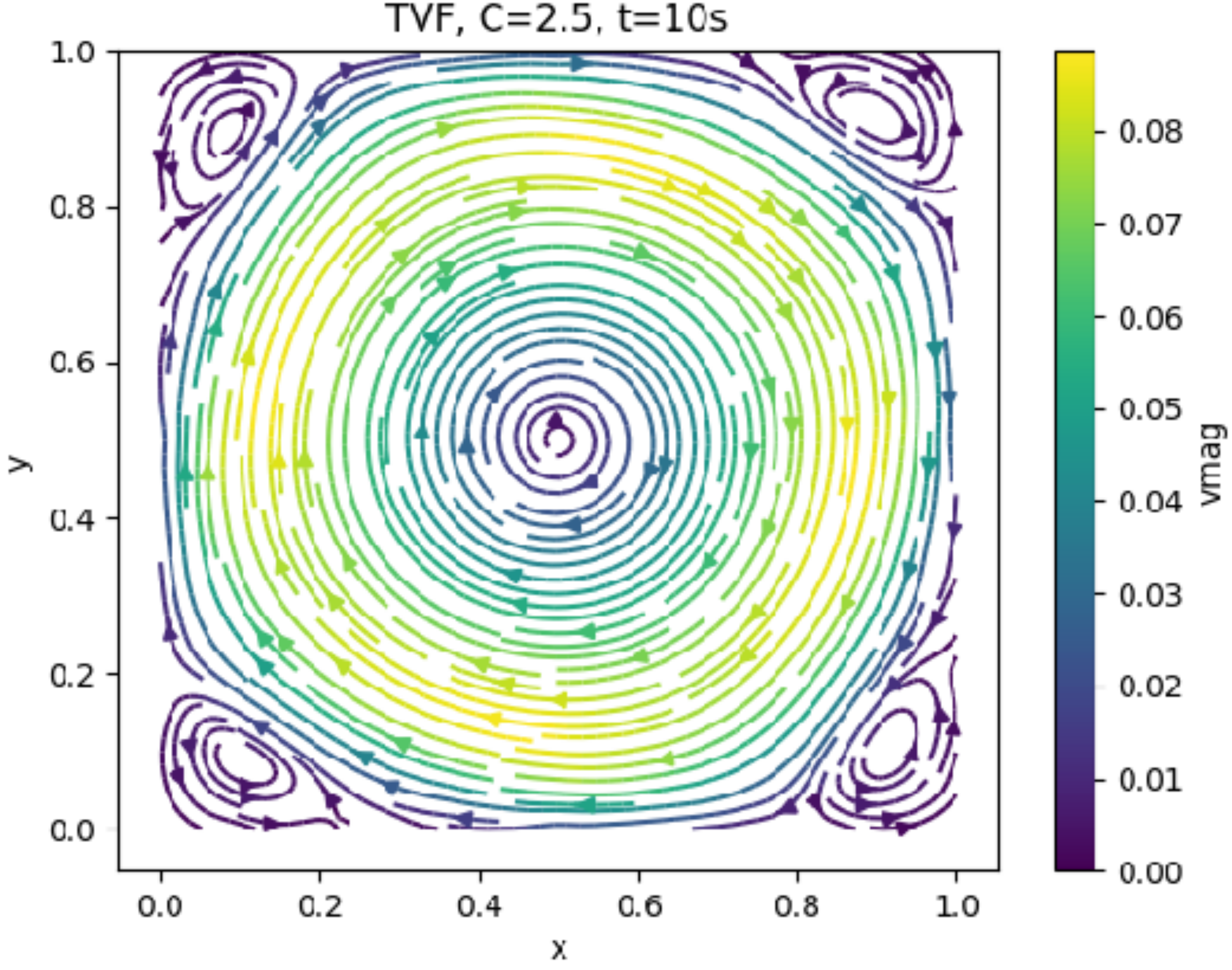
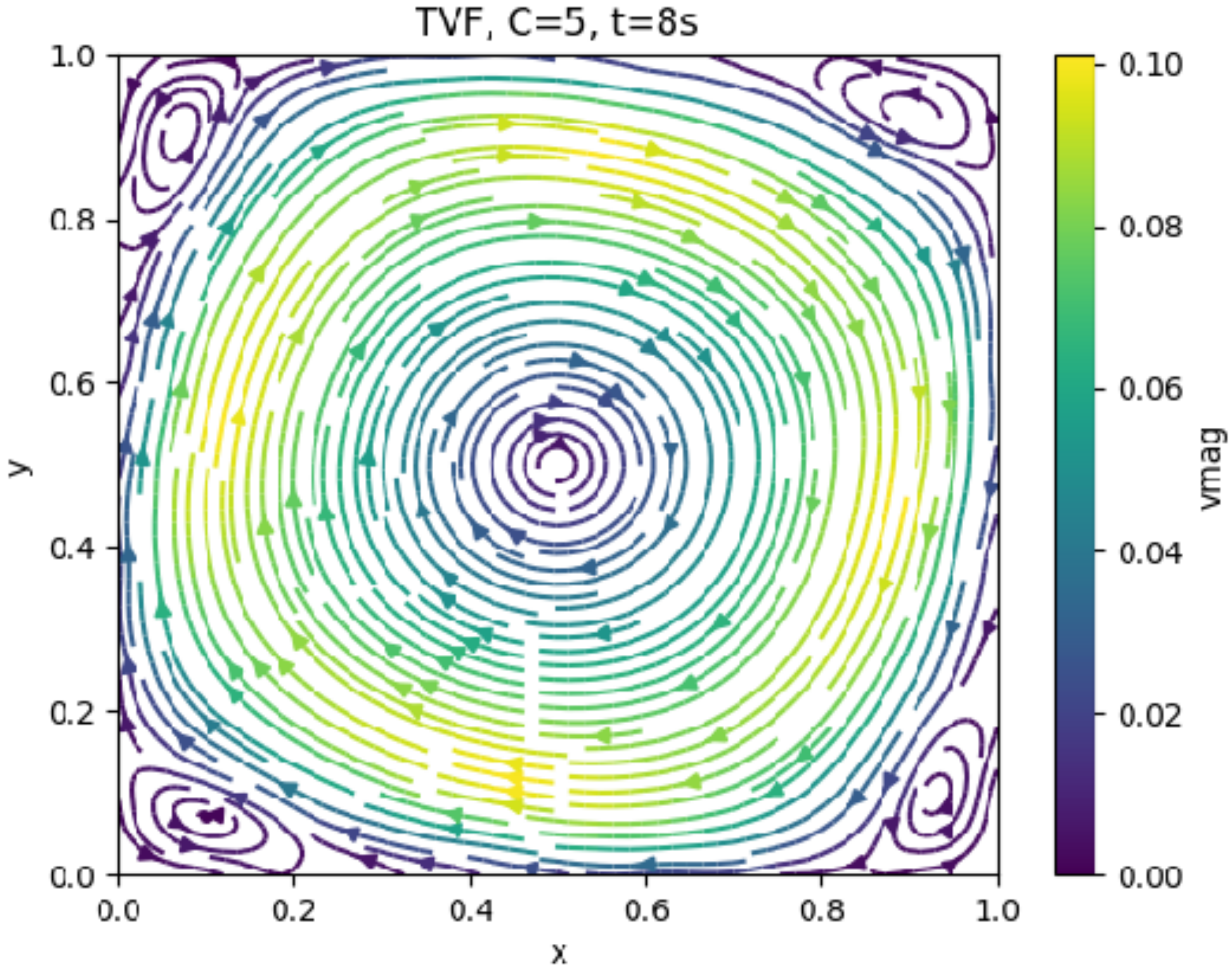
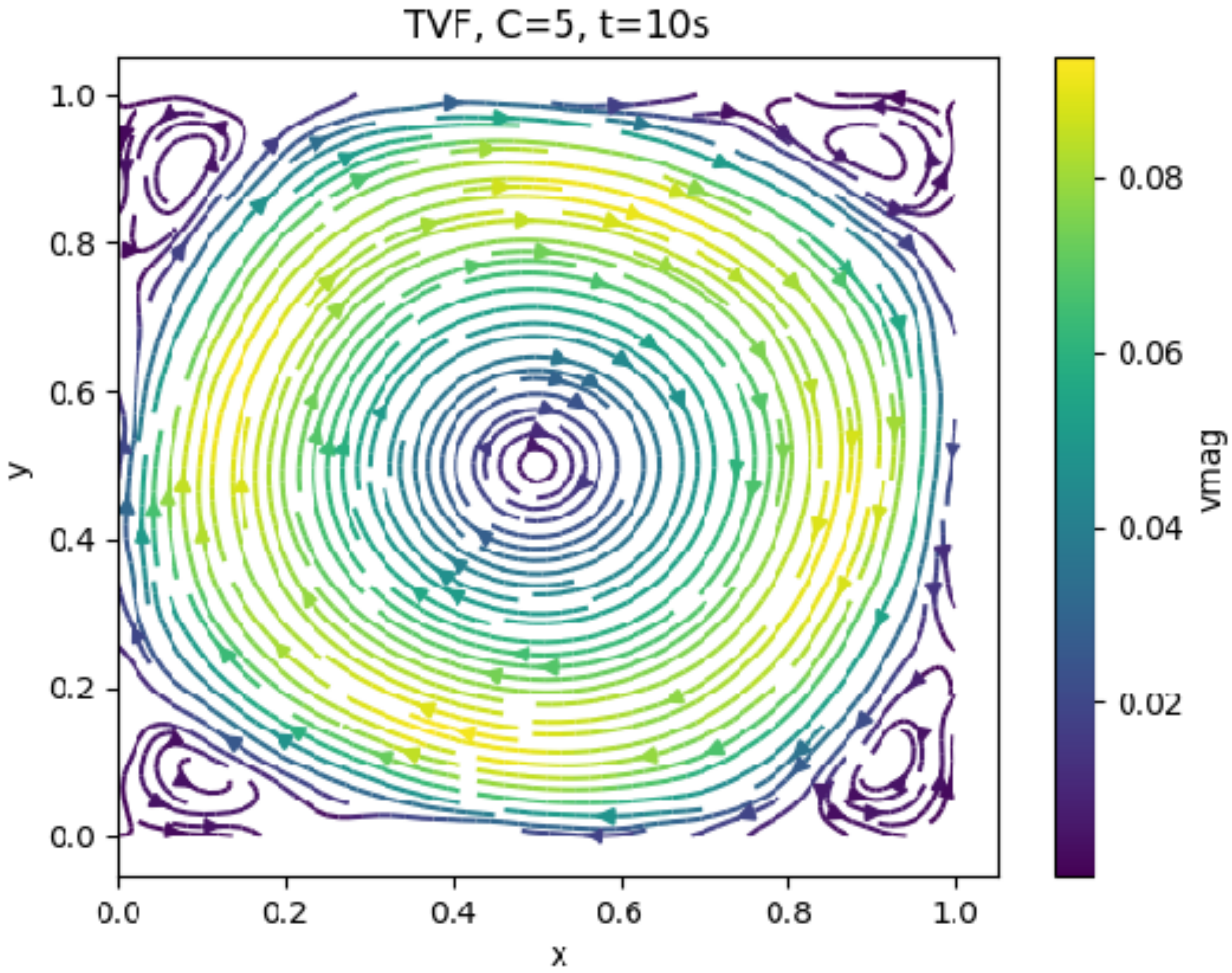
0.4
0.6
0.8
1.0
x
0.08
0.06
0.04
0.02
0.00
vmag



TVF, C=2.5, t=10s
y
x
0.0
0.2
0.4
0.6
0.8
1.0
vmag
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08

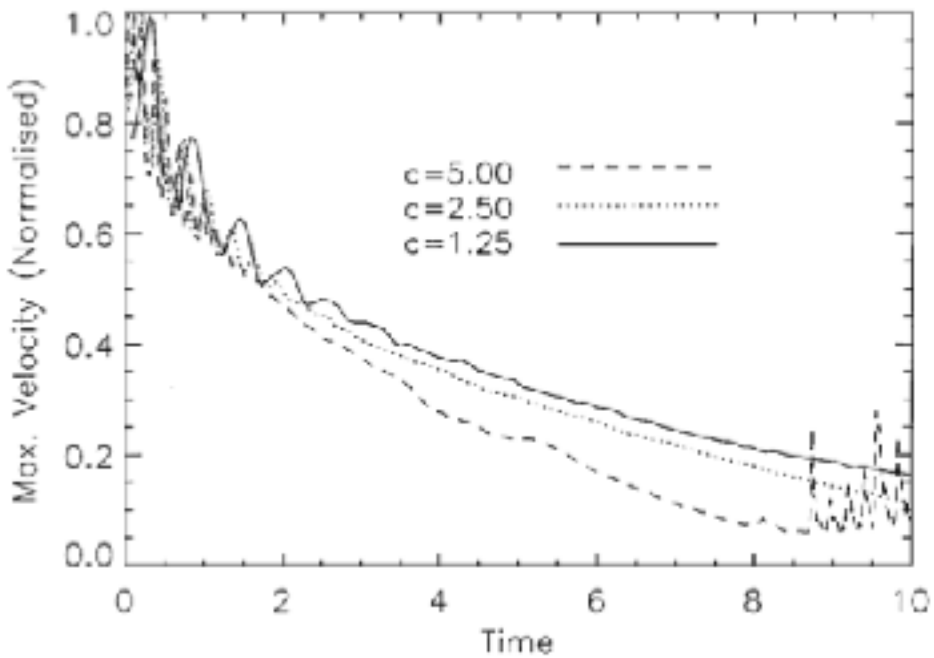

TVF, C=5, t=8s
y
x
vmag
0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.02
0.04
0.06
0.08
0.10



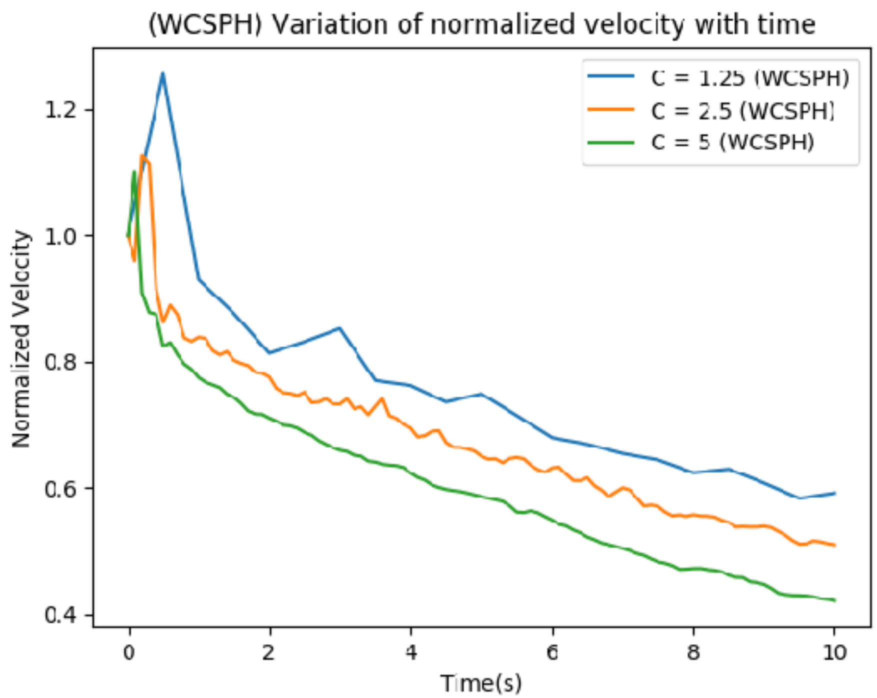
TVF, C=5, t=10s
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x
0.08
0.06
0.04
0.02
vmag

COMPARISONS

WCSPH RESULTS WITH LENNARD JONES BOUNDARY FORCE



WITHOUT INCORPORATION OF BOUNDARY FORCE



Conclusions

From the given graphs it can be concluded that the WCSPH scheme gives results which are more consistent with the ones presented in the research paper. Also, a general trend of dissipation of kinetic energy is observed which can be seen from the average kinetic energy and maximum velocity plots. Most importantly, we can conclude that SPH can handle such kind of problems fairly well and that too in an extremely simple and convenient manner, the code for WCSPH is very trivial and computed for time steps not that small, and yet the results are impressive.

References :

1) An SPH Projection Method - Sharen J. Cummins and Murray Rudman

2) Pysph docs

Pulkit - 150010059

Deep - 150010001

Shaurya - 150010060