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Concept of large sample confidence interval

If σ^2 is unknown and $n > 30$

$100(1-\alpha)\%$ C.I. for μ can be given by

$\left\{\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}},\ \bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}}\right\}$

(1)

(2)

$\sigma = 40$

$n = 30,\ \bar{x} = 780h.$

$100(1-\alpha)\% = 96\%$

$1-\alpha = .96$

$\boxed{\alpha = 0.04}$

$z_{\alpha/2} = z_{0.02}$

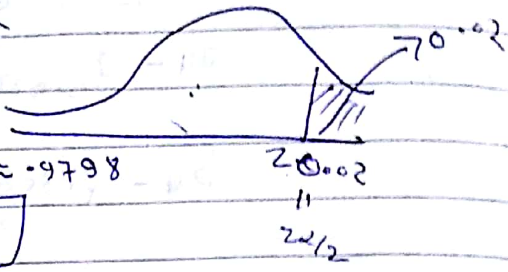
Since $P(z > z_{0.02}) = 0.02$

$1 - P(z > z_{0.02}) = 1 - 0.02$

$P(z < z_{0.02}) = 1 - 0.02$

$P(z < z_{0.02}) = .98 \approx .9798$

$\boxed{z_{0.02} = 2.05}$



confidence interval for μ

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$780 - (2.05)\frac{40}{\sqrt{30}} < \mu < 780 + (2.05)\frac{40}{\sqrt{30}}$$

$$\boxed{765.02 < \mu < 794.97}$$

(4)

$n = 50,\quad \bar{x} = 174.5,\quad s = 6.9$

(a) $100(1-\alpha)\% = 98\%$

$1-\alpha = .98$

$\boxed{\alpha = 0.02}$ $\boxed{\alpha/2 = 0.01}$

So $z_{\alpha/2} = z_{0.01}$

Since ~~$P(z > z_{\alpha/2})$~~

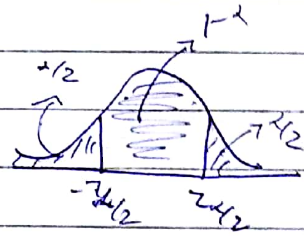
$P(z > z_{\alpha/2}) = \alpha/2$

$P(z > z_{0.01}) = 0.01$

So $P(z < z_{0.01}) = 1 - 0.01 = .99$

So $P(z < z_{0.01}) = .99 \approx .9901$

So $\boxed{z_{0.01} = 2.33}$



Since confidence interval for μ

$$\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$174.5 - \frac{2.33 \times 6.9}{\sqrt{50}} < \mu < 174.5 + \frac{2.33 \times 6.9}{\sqrt{50}}$$

$$173.226 < \mu < 176.77$$

$$\Rightarrow \boxed{173.23 < \mu < 176.77}$$

(2)

(b) (Th: If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that error will not exceed $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.)

i.e. $e < z_{\alpha/2}\frac{\cancel{s}}{\sqrt{n}}$ $\qquad z_{\alpha/2} = 2.33, \cancel{s} = 6.9, n = 50$

$e < 2.33 \cdot \frac{6.9}{\sqrt{50}}$

$e < 2.273$

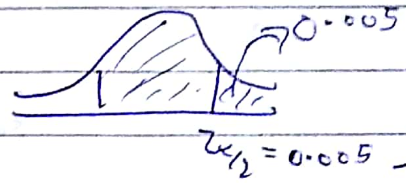
$\Rightarrow \boxed{e < 2.27}$

(5) $n = 100, \quad \bar{x} = 23{,}500, \quad \cancel{s} = 3900$

(a) $100(1-\alpha)\% = 99\%$

$1-\alpha = 0.99$

$\alpha = 0.01 \Rightarrow \alpha/2 = 0.005$



$P(Z > Z_{\alpha/2}) = 0.005$

$1 - P(Z \cancel{>} Z_{\alpha/2}) = 1 - 0.005$

$\boxed{P(Z < Z_{0.005}) = 0.995} \Rightarrow Z_{0.005} = 2.57$

$\bar{x} - \frac{\cancel{s}}{\sqrt{n}} Z_{\alpha/2} < \mu < \bar{x} + \frac{\cancel{s}}{\sqrt{n}} Z_{\alpha/2}$

$23500 - \frac{3900}{\sqrt{100}} \times 2.57 < \mu < 23500 + \frac{3900}{\sqrt{100}} \times 2.57$

$22{,}497.7 < \mu < 24{,}502.3$

(b) $e < Z_{\alpha/2}\frac{\cancel{s}}{\sqrt{n}}$

$\boxed{e < 2.57 \times \frac{3900}{\sqrt{100}} = 1002.3}$

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6. Given Thm: If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that error will not exceed a specified amount e when sample size is $n = \left(\frac{z_{\alpha/2}\,\sigma}{e}\right)^2$

$\sigma = 40$, $z_{\alpha/2} = z_{0.02} = 2.05$

$e = 10$

So $n = \left(\frac{z_{\alpha/2}\,\sigma}{e}\right)^2 = \left(\frac{2.05 \times 4\cancel{0}}{1\cancel{0}}\right)^2$

$n = 67.24$

$\boxed{n = 67}$

7. 95% confident. $\sigma = 0.0015$, $e = 0.0005$

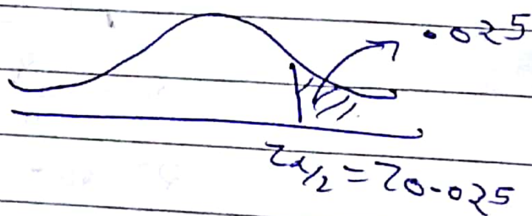
~~$z_{\alpha/2}$~~ $= 100(1-\alpha)\% = 95\%$

$\alpha = 0.05 \Rightarrow \boxed{\alpha/2 = 0.025}$

$P(Z > z_{\alpha/2}) = \alpha/2$

$P(Z > z_{0.025}) = 0.0.25$

$P(Z < z_{0.025}) = .975$



So $\boxed{z_{0.025} = 1.96}$

$n = \left[\frac{z_{\alpha/2}\,\sigma}{e}\right]^2 = \left(\frac{1.96 \times .0015}{.0005}\right)^2 = ~~34.57~~\ 34.57$

$\boxed{n = 34.57}$

$\boxed{n = 35}$

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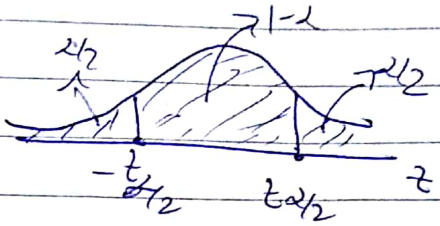
(12) $n = 10 \; (<30)$, $\bar{x} = 230$, $s = 15$. deg of freedom $= n-1 = 10-1 = 9$

$100(1-\alpha)\% = 99\%$

$\boxed{\alpha = 0.01}$ $\boxed{\alpha/2 = 0.005}$



$P(T > t_{\alpha/2}) = \alpha$

$t_{\alpha/2, n-1} = t_{0.005, 9} = 3.25$

confidence interval for μ

$$\bar{x} - t_{\alpha/2, n-1}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2, n-1}\frac{s}{\sqrt{n}}$$

$$230 - 3.25 \times \frac{15}{\sqrt{10}} \le \mu \le 230 + 3.25 \times \frac{15}{\sqrt{10}}$$

$$\boxed{214.58 \le \mu \le 245.41}$$

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Q2) Defination of Likelihood func:-

$$L(x_1, x_2, \ldots, x_n, \theta) = f(x_1, \theta) \cdots\cdots f(x_n, \theta)$$

$$\boxed{L(x_1, \ldots, x_n, \theta) = \prod_{i=1}^{n} f(x_i, \theta)}$$

So Likelihood func:- $\theta = (\mu, \sigma^2)$

$$L(x_1, \ldots, x_n; \mu, \sigma^2) = \prod_{i=1}^{n} f(x_i, \mu, \sigma)$$

$$\text{where } f(x, \mu, -) = \begin{cases} \frac{1}{\sqrt{2\pi}\,\sigma x} e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$L(x_1, \ldots, x_n, \mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma x_i} e^{-\frac{1}{2}\left(\frac{\ln x_i - \mu}{\sigma}\right)^2}$$

$$= \left(\frac{1}{\sqrt{2\pi}\,\sigma x_1} e^{-\frac{1}{2}\left(\frac{\ln x_1 - \mu}{\sigma}\right)^2}\right) \cdots \left(\frac{1}{\sqrt{2\pi}\,\sigma x_n} e^{-\frac{1}{2}\left(\frac{\ln x_n - \mu}{\sigma}\right)^2}\right)$$

$$= \frac{1}{(\sqrt{2\pi})^n} \frac{1}{\sigma^n \prod_{i=1}^{n} x_i} e^{-\frac{1}{2}\left[\frac{(\ln x_i - \mu)^2 + \cdots (\ln(x_n) - \mu)^2}{\sigma}\right]}$$

$\ln\left(\frac{m}{n}\right) = \ln m - \ln n$

$$= \frac{1}{(\sqrt{2\pi})^n} \frac{1}{\sigma^n \prod_{i=1}^{n} x_i} e^{-\frac{1}{2\sigma^2}\left[\sum_{i=1}^{n} (\ln x_i - \mu)^2\right]}$$

$$\ln L = \frac{-1}{2\sigma^2}\left(\sum_{i=1}^{n} (\ln x_i - \mu)^2\right) - n \ln(\sqrt{2\pi}) - \frac{n}{2} \ln \sigma^2 - \ln \prod_{i=1}^{n} x_i$$

$$\frac{\partial \ln L}{\partial \mu} = \frac{+1}{\cancel{2}\sigma^2} \cdot \cancel{2} \sum_{i=1}^{n} (\ln x_i - \mu)\,(+1) = 0$$

$$\sum_{i=1}^{n} (\ln x_i - \mu) = 0$$

$$(\ln x_1 - \mu) + \cdots + (\ln x_n - \mu) = 0$$

$$\sum_{i=1}^{n} \ln x_i - n\mu = 0$$

$$\mu = \frac{\sum_{i=1}^{n} \ln x_i}{n}$$

$$\text{So } \boxed{\hat{\mu} = \frac{\sum_{i=1}^{n} \ln x_i}{n}}$$

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$$\frac{\partial \ln L}{\partial \sigma^2} = \frac{-1}{2}\frac{(-1)}{(\sigma^2)^2}\left(\sum_{i=1}^{n}(\ln x_i-\mu)^2\right) - \frac{n}{2}\frac{1}{\sigma^2} = 0$$

$$-\frac{n}{2}\frac{1}{\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(\ln x_i-\mu)^2 = 0$$

$$\frac{1}{\cancel{2}\sigma^2}\sum_{i=1}^{n}(\ln x_i-\mu)^2 = \frac{n}{\cancel{2}}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n}(\ln x_i-\mu)^2}{n}$$

So $$\boxed{\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}(\ln x_i-\mu)^2}{n}}$$

85. $L(x_1,\dots,x_n,\theta) = f(x_1,\theta)\cdot \dots \cdot f(x_n,\theta)$

$$\boxed{U(0,\theta) = f(x_a,\theta) = \begin{cases}\frac{1}{\theta} & 0<x<\theta \\ 0 & \text{elsewhere}\end{cases}}$$

So $L(x_1,\dots,x_n,\theta) = \underbrace{\frac{1}{\theta}\cdots\frac{1}{\theta}}_{n\text{ times}} = \frac{1}{\theta^n}$

here
$\begin{cases} 0<x_1<\theta \\ 0<x_2<\theta \\ \vdots \\ 0<x_n<\theta \end{cases}$

i.e. $x_1<\theta$
$x_2<\theta$
$\vdots$
$x_n<\theta$
$\max\{x_1,\dots,x_n\}<\theta$

So Likelihood function of θ

$L(x_1,\dots,x_n,\theta) = \frac{1}{\theta^n}$, for $\max\{x_1,\dots,x_n\}<\theta<\infty$

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