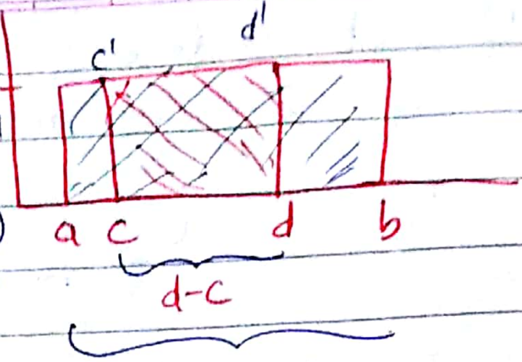
Chap-6:

(2)

(1)

So $P(c<x<d) =$ area of rectangle $cdc'd'$ $\quad f(x)=\frac{1}{b-a}$

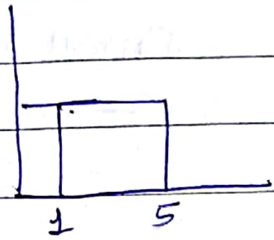


$= \text{base } (cd) \times \text{height} \left(\frac{1}{b-a}\right)$

$\boxed{P(c<x<d) = \frac{d-c}{b-a} = \cancel{(d-c)\, f(x)}}$

$= (d-c) f(x)$ $\boxed{\text{where } f(x) = \frac{1}{b-a}}$

So given



$P(X>2.5 \mid X\le 4)$ $\quad f(x) = \frac{1}{5-1} = \frac{1}{4}$

$= \frac{P(X>2.5 \cap X\le 4)}{P(X\le 4)}$

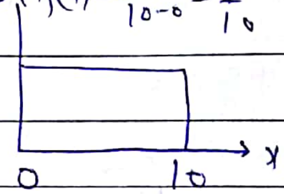
$= \frac{P(2.5<X\le 4)}{P(X\le 4)} = \frac{(4-2.5) f(x)}{P(1<X\le 4)} = \frac{1.5\, \cancel{f(x)}}{(4-1)\, \cancel{f(x)}} = \frac{1.5}{3}$

$= \frac{15}{30} = \frac{1}{2}$

(4) X : waiting time for a particular individual (with a cts uniform dis.)

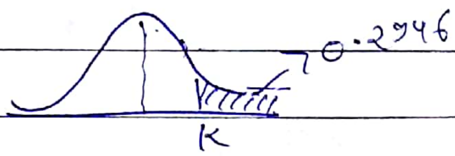
(a) $P(X>7) = P(7<X<10) = (10-7) f(x)$ $\quad f(x) = \frac{1}{10-0} = \frac{1}{10}$

$= 3 \times \frac{1}{10} = 0.3$



(b) $P(2<X<7) = (7-2) f(x) = 5 \times \frac{1}{10} = 0.5$

(7) (a) $P(Z>K) = 0.2946$



$1 - P(Z>K) = 1 - 0.2946$

$P(Z\le K) = 0.7054$

So $K = 0.54$ (from the table)

(b) $P(Z<K) = 0.0427$

So $K = -1.72$ from the table.

2

(c) $P(-0.93 < Z < K) = 0.7235$

$P(Z<K) - P(Z<-0.93) = 0.7235$

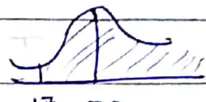
$P(Z<K) = P(Z<-0.93) + 0.7235$

$= .1762 + .7235 = 0.8997$

$\boxed{K = 1.28}$

8. $\mu = 30$, $\sigma = 6$

(a)



$P(X>17) = 1 - P(X<17)$

$= 1 - P(Z<-2.16)$

$= 1 - 0.0150$

$= 0.9850$

$X < 17$

$\cancel{Z = \frac{X-\mu}{}}$ $\frac{X-30}{6} < \frac{17-30}{6}$

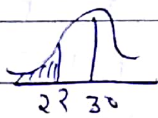
$Z = $

$Z < -2.16$

(b) $P(X<22) =$

$X < 22$

$Z = \frac{X-30}{6} < \frac{22-30}{6}$



$Z < -1.33$

$P(X<22) = P(Z<-1.33) = 0.0918$

(c) $P(32<X<41)$

$32 < X < 41$

$\frac{32-30}{6} < \frac{X-30}{6} < \frac{41-30}{6}$

$0.33 < Z < 1.83$

$P(32<X<41) = P(0.33<Z<1.83)$

$= P(Z<1.83) - P(X<0.33)$

$= 0.9664 - .6293 = 0.3371$

$\boxed{Z = \frac{x_2-\mu}{\sigma}} = K = Z$

(d) $P(Z<K) = 0.8$

(since $P(Z<0.84) = .7995 \approx 0.8$

So $\boxed{K = 0.84}$

$K = \frac{x-\mu}{\sigma}$

$x = K\sigma + \mu$

$x = (0.84)\times 6 + 30 = 35.04$

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$P(Z < Z_2)$
$P(Z < 1.14)$

(e) $P(Z_1 < Z < Z_2) = .75$

$P(Z < Z_2) - P(Z < Z_1) = .75$

since $0.8749 - .1251 = 0.7498 \approx .75$

(3)

So $P(Z < Z_2) - P(Z < Z_1) = 0.8749 - 0.7498$

i.e. $P(Z < Z_2) = 0.8749$, $P(Z < Z_1) = 0.7498$

So $Z_2 = 1.15$ $\qquad$ $Z_1 = -1.15$

$z_2 = \dfrac{x_2 - \mu}{\sigma}$ $\qquad$ $z_1 = \dfrac{x_1 - \mu}{\sigma}$

$x_2 = \sigma Z_2 + \mu$ $\qquad$ $x_1 = \sigma Z_1 + \mu$

$x_2 = 6 \times 1.15 + 30$ $\qquad$ $x_1 = 6 \times -1.15 + 30$

$= 36.9$ $\qquad$ $= -6.9 + 30$

$= 23.1$

10. So since we know from the chebychev's inequality

~~$P(X-3\mu < X$~~

$P(\mu - 3\sigma < X < \mu + 3\sigma) \geq \dfrac{8}{9} = 0.888$

$P(\mu - 3\sigma < X < \mu + 3\sigma) =$

Since $\mu - 3\sigma < X < \mu + 3\sigma$

$-3\sigma < X - \mu < 3\sigma$

$-3 < \dfrac{X-\mu}{\sigma} < 3 \Rightarrow -3 < Z < 3$

So $P(\mu - 3\sigma < X < \mu + 3\sigma) = P(-3 < Z < 3)$

$= P(Z < +3) - P(Z < -3)$

$= 0.9987 - 0.0013$

$= 0.9974$

15 (a) $\mu = 24, \ \sigma = 3.8$

(a) $P(X>30) = 1 - P(X \leq 30)$

$= 1 - P(Z < 1.58)$

$= 1 - .9429 = 0.0571$

$X \leq 30$

$X - 24 \leq 30 - 24 = 6$

$Z: \frac{X-24}{\sigma} \leq \frac{6}{3.8} = 1.5789$

$= 1.58$

X : ~~ano~~ amount of time to reach the office.

(b) $P(X>15) = 1 - P(X<15)$

$= 1 - P(Z<-2.37)$

$= 1 - 0.0089$

$= .9911$

i.e. lawyer will 99.11% late

$X \leq 15$

$\frac{X-24}{3.8} \leq \frac{15-24}{3.8}$

$Z \leq -2.368$

$\boxed{Z \leq -2.37}$

(c) 8:35 AM. $\longrightarrow$ 9:00 AM.

i.e. 25 min

$P(X>25) = 1 - P(X \leq 25)$

$= 1 - P(Z \leq .26)$

$= 1 - .6026$

$= 0.3974$

i.e. 39.74% he may miss the coffee.

$X \leq 25$

$\frac{X-24}{3.8} \leq \frac{25-24}{3.8}$

$Z \leq .2631$

$\Rightarrow \boxed{Z < .26}$

(d) $P(X>x) = 0.15$ find x ?

$1 - P(X>x) = 1 - 0.15$

$P(X<x) = .85$

i.e. $P(Z<z) = .85$ — (1)

relation b/w x & z

is $\boxed{z = \frac{x-\mu}{\sigma}}$ — *

Since $\boxed{P(Z<1.04) = .8508} \approx .85$ — (2)

comparing (1) & (2)

$z = 1.04$

put the value of z in (*)

$x = \sigma z + \mu$

$x = 3.8 \times 1.04 + 24$

$\boxed{x = 27.952}$

$z = 1.14$

$z = 1.15$

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15 (c) find prob. of 2 of the next 3 trips will take at least 1/2 hour

5

from the part (a) $p = 0.0571$

ways of choose 2 trips out of 3 trips $= \binom{3}{2}$

$n=3,\ x=2,\ p=0.0571$

using binomial distribution

$$b(x;n,p) = b(2;3,0.057) = \binom{3}{2}(0.057)^2(1-0.057)^{3-2}$$

= prob of 2 of the next 3 trips will take at least 1/2 hour.

24

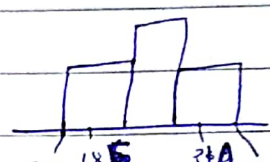
$n = 400$

x denote the no head

$p = P(H) = 1/2,\quad q = P(T) = 1/2$

$\mu = np = 400 \times \frac{1}{2} = 200$

$\sigma^2 = npq = 400 \times \frac{1}{2} \times \frac{1}{2} = 100 \Rightarrow \boxed{\sigma = 10}$



~~P(186 ≤ X ≤ 209)~~

(a) $P(185 \leq X \leq 210) \approx P(185-0.5 < X < 210+.5)$ for using normal ~~curve~~ approximation.

$$= P\left(\frac{(185-0.5)-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{(210+.5)-\mu}{\sigma}\right)$$

$$= P\left(\left(\frac{(185-0.5)-200}{10}\right) < Z < \left(\frac{(210.5)-200}{10}\right)\right)$$

$$= P(-1.55 < Z < 1.05)$$

= ~~.8531 − 0.0606 = .7925~~

$= .8531 - 0.0606 = .7925$

(b) $P(X = 205) \approx P(204.5 < X < 205.5)$ for using normal approximation

$$= P\left(\frac{204.5-200}{10} < \frac{X-200}{10} < \frac{205.5-200}{10}\right)$$

$$= P(.45 < Z < .55)$$

$$= P(Z < .55) - P(Z < .45)$$

$$= .7088 - .6736$$

$$= .0352$$

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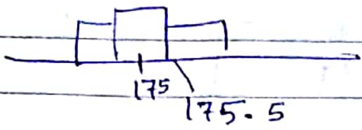
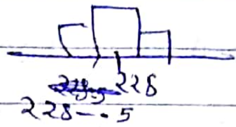
24 (c) $P(X<176 \text{ or } X>227)$

6

$= P(X<176) + P(X>227)$

$= P(X \le 175) + P(X \ge 228)$





$\approx P(X \le 175.5) + P(X \ge 228.5)$ for using normal approximation

$= P(X \le 175.5) + P(X \ge 227.5)$

$= P\left(Z \le \frac{175.5-200}{10}\right) + 1 - P(X \le 227.5)$

$= P(Z \le -2.45) + 1 - P(Z \le 2.75)$

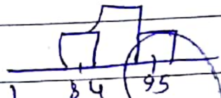
$= 0.0071 + 1 - 0.9970 = 0.0101$

26 Given $n = 100$, $p = \frac{10}{100} = \frac{1}{10}$, $q = \frac{9}{10}$

$\mu = np = 100 \times \frac{1}{10} = 10$

$\sigma^2 = npq = 100 \times \frac{1}{10} \times \frac{9}{10} = 9 \Rightarrow \boxed{\sigma = 3}$

X denote no. of defective.

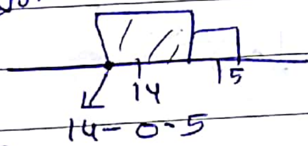


~~(a) $P(84 \le X \le 95) \approx P(84 - 0.5 \le X \le 95.5)$~~ ~~for normal~~ approximation

$=$

(a) $P(X>13) = P(X \ge 14)$ for normal approximation.

$\approx P(X \ge 14 - .5)$ for normal approximation



$= P(X \ge 13.5)$

$= P\left(\frac{X-10}{3} \ge \frac{13.5-10}{3}\right) = P\left(Z \ge \frac{3.5}{3}\right) = P(Z \ge 1.17)$

$= 1 - P(Z < 1.17)$

$= 1 - 0.8790 = 0.1210$

(b) $P(X<8) = P(X \le 7) \approx P(X \le 7.5)$ for normal approximation

$= P\left(\frac{X-10}{3} \le \frac{7.5-10}{3}\right)$

$= P(Z \le -0.83)$

$= 0.2033$

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7

(29) $n = 1000$, $p = \frac{20}{100} = 0.2$, $q = 0.8$

X denote no. of white telephone.

$\mu = np = 1000 \times \frac{20}{100} = 200$

$\sigma^2 = npq = 1000 \times \frac{20}{100} \times \frac{80}{100} = 160$

$\sigma = 12.6491106$

$\boxed{\sigma = 12.649}$

(a) $P(170 \le X \le 185) \simeq P(170 - .5 \le X \le 185 + 0.5)$

$= P(169.5 \le X \le 185.5)$ → for binomial normal approx.

$= P\left(\frac{169.5 - 200}{12.649} \le \frac{X - \mu}{\sigma} \le \frac{185.5 - 200}{12.649}\right)$

$= P(-2.41 \le Z \le -1.15)$

$= P(Z \le -1.15) - P(Z < -2.41)$

$= .1251 - 0.0080 = .1171$

(b) $P(210 \le X \le 225) \simeq P(210 - .5 \le X \le 225.5)$

$= P(209.5 \le X \le 225.5)$

$= P\left(\frac{209.5 - 200}{12.649} \le Z \le \frac{225.5 - 200}{12.649}\right)$

do yourself.

(8)

(34) $n = 180$, $p = \frac{6}{36} = \frac{1}{6}$ $q = \frac{5}{6}$

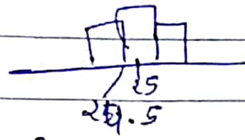
~~for total~~ $\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$ = possible sample point for getting total sum equal to 7.

So X denote ~~total~~ no. of times total sum occur 7.

$\mu = np = 180 \times \frac{1}{6} = 30$

$\sigma = \sqrt{npq} = \sqrt{180 \times \frac{1}{6} \times \frac{5}{6}}$

(a) $P(X \geq 25) \approx P(X \geq 25 - 0.5)$



$= P(X \geq 24.5)$ for normal approximation.

$= P\left(\frac{X-30}{\sigma} \geq \frac{24.5-30}{\sigma}\right)$

$= P\left(Z \geq \frac{24.5-30}{\sigma}\right)$ find yourself.

(b) $P(33 \leq X \leq 41) \approx P(33 - 0.5 \leq X \leq 41 + .5)$ for normal approximation

$= P(32.5 \leq X \leq 41.5)$

$= P\left(\frac{32.5-30}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{41.5-30}{\sigma}\right)$

$= P\left(\frac{32.5-30}{\sigma} \leq Z \leq \frac{41.5-30}{\sigma}\right)$

(find yourself.)

(c) $P(X = 30) \approx P(29.5 \leq X \leq 30.5)$ for normal approximation

$= P\left(\frac{29.5-30}{\sigma} \leq Z \leq \frac{30.5-30}{\sigma}\right)$

(find yourself)

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(41) Gamma distribution.

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\sqrt{\alpha}} x^{\alpha-1} e^{-x/\beta} & x>0 \\ 0 & \text{otherwise.} \end{cases}$$

$\alpha = 2,\ \beta = 1$

$\sqrt{2} = 1$

$$f(x) = f(x;1,2) = \frac{1}{1^2\sqrt{2}} x^1 e^{-\frac{x}{1}} = x e^{-x}$$

$$P[1.8 < X < 2.4] = \int_{x=1.8}^{2.4} f(x)\,dx = \int_{x=1.8}^{2.4} x e^{-x}\,dx.$$

(complete yourself)

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Chap-6

(54) Given mean $= \mu = 10$ | $\sigma = S.D. = \sqrt{50}$ | $\sigma^2 = \alpha\beta^2$

$\mu = \alpha\beta = 10$ | ~~$\alpha\beta^2 = \sqrt{50}$~~ | $\alpha\beta^2 = 50$

$$\frac{\alpha\beta^2}{\alpha\beta} = \frac{50}{10} \Rightarrow \boxed{\beta = 5}$$

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}} & x>0 \\ 0 & \text{elsewhere} \end{cases}$$

$$\alpha \cdot \frac{50}{10} = 10 \Rightarrow \boxed{\alpha = 2}$$

Let X denote the lifetime, in weeks of a transistor.

(a) $P(X \leq 50) = \int_{x=0}^{50} f(x)dx = \frac{1}{5^2\Gamma 2}\int_{x=0}^{50} x\, e^{-\frac{x}{5}}\,dx$

$(\Gamma 2 = 1)$ $= \frac{1}{25}\int_{x=0}^{50} x\, e^{-x/5}\,dx$

(do yourself)

(b) $P(X<10) = \frac{1}{5^2\Gamma 2}\int_{x=0}^{10} x\, e^{-\frac{x}{5}}\,dx$

$= \frac{1}{25}\int_{x=0}^{10} x\, e^{-\frac{x}{5}}\,dx$ (do yourself)