Chap-5!

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(9) Let x denote no. of trucks which blowouts.

$P(S) = \frac{25}{100} = 0.25$, $n = 15$.

(a) $P(3 \le X \le 6) = \sum_{x=3}^{6} b(x, 15, 0.25) = \sum_{x=0}^{6} b(x,15,0.25) - \sum_{x=0}^{2} b(x,15,0.25)$

$= 0.9434 - 0.2361 = 0.7073$

(b) $P(X < 4) = \sum_{x=0}^{3} b(x, 15, 0.25) = \cancel{.3867} \cancel{.6418}\ .4613$

(c) $P(X > 5) = 1 - P(X \le 5) = 1 - \sum_{x=0}^{5} b(x, 15, 0.25) =$

$= 1 - 0.8516 = 0.1484$

(11) Given $n = 7$, $p = 0.9$

Let X be no. of patient whose operation survive.

$P(X = 5) = b(5, 7, 0.9) = \sum_{x=0}^{5} b(x, 7, 0.9) - \sum_{x=0}^{4} b(x, 7, 0.9)$

$= .1497 - 0.0257$

$= 0.1240$

(15) Given $n = 5$, $\cancel{p = 60\% = .6}$

Let X be no. of mich which contract the disease.

So $p = 1 - 0.6 = 0.4$ $\qquad q = 1 - p = 0.6$

(a) $P(X = 0) = b(0; 5, .4) = {}^{n}C_0\, p^0\, q^{n-0} = q^5 = (0.6)^5$

$= 0.07776$

$= 0.0778$

(b) $P(X < 2) = \sum_{x=0}^{1} b(x, 5, .4) = 0.3370$

(c) $P(X > 3) = \cancel{\Sigma}\ 1 - P(X \le 3) = 1 - \sum_{x=0}^{3} b(x, 5, .4)$

$= \cancel{1 - 0.6826}\ 1 - .9130 = 0.0870$

5.2

16 for M.1 Let X denote no. of engine run
$P = 0.6$

case (a) for 4-engine plane. no. of trial = 4

$P[\text{safe flight}] = P[X=2] + P[X=3] + P[X=4]$

(2 run engine run) i.e. prob of 3 engine run, prob 4 engine run

$= P\, b(2;4;0.6) + b(3,4,0.6) + b(4,4,0.6)$

$= \sum_{x=2}^{4} b(x;4,0.6) = \sum_{x=0}^{4} b(x;4,0.6) - \sum_{x=0}^{1} b(x;4,0.6)$

$= 1 - 0.1792 = 0.8208$

case (b) for 2-engine plane, no. of trial = 2

$P[\text{safe flight}] = P[X=1] + P[X=2]$ $\boxed{P = 0.6}$

prob of 1 engine run, prob of 2 engine run.

$= \sum_{x=1}^{2} b(x;2,0.6) = \sum_{x=0}^{2} b(x;2,0.6) - \sum_{x=0}^{0} b(x;2,0.6)$

$= 1 - .16 = 0.84$

Since prob of safe flight for 2 engine plane is greater than the 4-engine plane.
So 2-engine plane has the higher prob. for a successful flight.

M.2. Let X denote no. of engine fail. So $P = 0.4$

case (a): for 4-engine plane. no. of trial = 4

$P[\text{safe flight}] = P[X=0] + P[X=1] + P[X=2]$

i.e. prob of 0 engine fail. i.e. prob of 1 engine fail. prob of 2 engine fail
i.e. prob of 4 engine run. i.e. prob of 3 engine run ~~i.e. prob of 2 engine run.~~

$= \sum b(0;4,0.4) + b(1,4,0.4) + b(2;4,0.4)$

$= \sum_{x=0}^{2} b(x;4,0.4) = 0.8208$

Case (a): for 2-engine plane

$P[\text{safe flight}] = P[X=0] + P[X=1] = b(0;2,0.4) + b(1;2,0.4)$

i.e. prob of 0 engine fail i.e. prob of 1 engine fail.
i.e. prob of 2 engine run. i.e. prob. of 1 engine run

$= \sum_{x=0}^{1} b(x;2,0.4)$

$= 0.84$

||'ly here 2-engine plane has the higher prob. for a success flight.

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5-3

19. Let E_1 denotes events that a students encounters green light

E_2 " " " " " Yellow "

E_3 " " " " " Red "

So $P(E_1) = \frac{35}{35+5+60} = \frac{35}{100} = .35$

$P(E_2) = \frac{5}{100} = 0.05$

$P(E_3) = \frac{60}{100} = .6$

Let X_1: no of times E_1 occurs or (no of times he encounters green light)

X_2: " " " E_2 " or (" " " " " Yellow ")

X_2: " " " E_3 " or (" " " " " Red ")

Let n = no. of times he encounters to the traffic signal.

& i.e. we have here n trials.

So using multinomial Distribution.

$$f(x_1, x_2, x_3) = \binom{n}{x_1, x_2, x_3} P(E_1)^{x_1} P(E_2)^{x_2} P(E_3)^{x_3}$$

$$= \binom{n}{x_1, x_2, x_3} (0.35)^{x_1} (0.05)^{x_2} (0.6)^{x_3}$$

here $x_1 + x_2 + x_3 = n$

One sample point will form like.

$\underbrace{E_1 E_2 \cdot E_1}_{E_1 \text{ repeats } x_1 \text{ times}} \underbrace{\cdots}_{E_2 \text{ repeat } x_2 \text{ times}} \underbrace{E_3 E_2}_{E_3 \text{ repeat } x_3 \text{ times}}$

22. Red : black : White = 8 : 4 : 4 = $\frac{8}{16} : \frac{4}{16} : \frac{4}{16}$

Where $16 = 8 + 4 + 4$

$P(R) = \frac{1}{2}$, $P(B) = \frac{1}{4}$, $P(G) = \frac{1}{4}$

So prob of 5 red & 2 Black & 1 White ~~is~~

$$= \frac{8!}{5!\,2!\,1!} (P(R))^5 \, P(B)^2 \, (P(G))^1$$

$$= \frac{8!}{5!\,2!\,1!} \left(\frac{1}{2}\right)^5 \left(\frac{1}{4}\right)^2 \left(\frac{1}{4}\right)^1$$

5-4

31) $\boxed{4D + 2N}$ $N = 6$

D | N → 3

Let X denote the no. of doctors of the committee.

$X = 1, 2, 3$

$P(X=1) = \frac{\binom{4}{1}\binom{2}{2}}{\binom{6}{3}}$

prob of selecting 1 doctor

(If we take $x=0$ mean we are not selecting doctor. So we will have to select 3 Nurse. but we have only 2 Nurse

i.e. $n-(N-K)$
$= 3-(6-4)$
$= 3-2 = 1$

$\max\{1, 0\} = 1$

$P(X=2) = \frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}}$

$P(X=3) = \frac{\binom{4}{3}\binom{2}{0}}{\binom{6}{3}}$

$\boxed{\text{i.e. } P(X=x) = \frac{\binom{4}{x}\binom{2}{3-x}}{\binom{6}{3}} \text{ for } x = 1, 2, 3}$

$P(2 \le X \le 3) = h(2, 6, 3, 4) + h(3, 6, 3, 4)$

32) a) X: no. of non-defective missile.

$P(X=4)$ = prob of all 4 fire $= \frac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}} = \frac{7 \times 6 \times 5 \times 4}{10 \times 9 \times 8 \times 7} = \frac{1}{6}$

$h(4; 10, 4, 7)$

D	N
7	3
4	0

$N = 10$

$n = 4$

b) Let X: no. of defective missile

Prob of atmost 2 will no fire.

D	N
3	7
x	$4-x$

$N = 10$

$n = 4$

$= P[X=0] + P[X=1] + P[X=2]$

$= h(0; 10, 4, 3) + h(1; 10, 4, 3) + h(2; 10, 4, 3)$

$= \sum_{x=0}^{2} h(x; 10, 4, 3)$

$= \frac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}} + \frac{\binom{3}{1}\binom{7}{3}}{\binom{10}{4}} + \frac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}}$

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5.5

43. $2C + 3J + \boxed{5I} + 2G$ $N = 12$

$\downarrow$

C) J) I) G 4

(a) prob. of all nationalities $= \dfrac{\binom{2}{1}\binom{3}{1}\binom{5}{1}\binom{2}{1}}{\binom{12}{4}}$ (Here we will take each a student from each country)

(b) prob. of all nationalities except Italian = prob of selecting 2C & 1J & 1G + prob of selecting 1C & 2J + 1G + prob of selecting 1C & 1J + 2G

$= \dfrac{\binom{2}{2}\binom{3}{1}\binom{2}{1}}{\binom{12}{4}} + \dfrac{\binom{2}{1}\binom{3}{2}\binom{2}{1}}{\binom{12}{4}} + \dfrac{\binom{2}{1}\binom{3}{1}\binom{2}{2}}{\binom{12}{4}}$

44 $3G + 2B + 4R$ $N = 9$

$\downarrow$

G) B) R $n = 5$

G	B	R
~~2~~	2	1
3	1	1
2	1	2
1	2	2
2	2	1
1	2	2
0	2	3

So prob. 2 Blue balls & atleast 1 red ball

= prob of selecting 2 Blue ball & 1 Red Ball & 2 Green ball

+ prob of selecting 2 Blue ball & 2 Red Ball & 1 Green ball

+ prob of selecting 2 Blue ball & 3 Red ball & 0 Green ball.

$= \dfrac{\binom{2}{2}\binom{4}{1}\binom{3}{2}}{\binom{9}{5}} + \dfrac{\binom{2}{2}\binom{4}{2}\binom{3}{1}}{\binom{9}{5}} + \dfrac{\binom{2}{2}\binom{4}{3}\binom{3}{0}}{\binom{9}{5}}$

5.6

(49) Let x denote no. of person randomly interviewed in the city is k^{th} one to own a dog.

So here we will use negative Binomial distribution.

here $x = 10$, $k = 5$, $p = 0.3$

$b^*(x; k, p) = {}^{x-1}C_{k-1} \, p^k q^{x-k}$

$b^*(10; 5, 0.3) = {}^{9}C_{4} (0.3)^5 (0.7)^5$

i.e. $\boxed{b^*(10; 5, 0.3) = {}^{9}C_{4} (0.3)^5 (0.7)^5}$

(50) Let X denote no. of flip to get k th head.

$P(H) = P(T) = 0.5$

(a) third head on the seventh flip.

So $x = 7$, $k = 3$, $p = 0.5$

So using negative Binomial dis.

$b^*(7; 3, 0.5) = {}^{6}C_{2} (0.5)^3 (0.5)^4$

$= {}^{6}C_{2} (0.5)^7$

(b) first head on the fourth flip.

here $k = 1$, $x = 4$

(we can say here using N.B.D. or Geometric distribution)

$b^*(4, 1, 0.5) = {}^{3}C_{0} (0.5)^1 (0.5)^3$

$= 1 \cdot (0.5)(0.5)^3$

$= (0.5)^4$

(51) 1 2 3

HHH
TTT
} → here they will toss again.

HHT (in this 3rd person pay for coffee)
HTH
THH
HTT
THT (in this case 2nd person pay for coffee.
TTH

Experiments stop

$P(S) = P(HHT, HTH, THH, TTH, THT, HTT)$

$= P(HHT) + P(HTH) + P(THH) + P(TTH) + P(THT) + P(HTT)$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{6}{8}$

$= \frac{3}{4}$

So $P(F) = 1 - \frac{3}{4} = \frac{1}{4}$

(52) F F F F . . F S

here we will get sample pt. of type HHH or TTT

Ily here

here we'll get sample pt of type HHT, HTH, THH, HTT, THT, TTH.

~~X: denotes no toss to get first success.~~

So prob. that fewer than 4 tosses

$= P(X=1) + P(X=2) + P(X=3)$

$= g(1, p) + g(2, p) + g(3, p)$

$= pq^0 + pq^1 + pq^2$

$= p(1 + q + q^2)$

$(\because g(x, p) = pq^{x-1}$, $x = 1, 2, \ldots)$

$= \frac{3}{4}\left(1 + \frac{1}{4} + \frac{1}{16}\right) = \frac{3}{4}\left(\frac{16+4+1}{16}\right)$

$= \frac{3}{4} \times \frac{21}{16} = \frac{63}{64}$

(69) $\mu = np$

$= 4000 \times 0.001$

$= 4$

70

5.7

60

$\lambda = 12$

(a) $t = 1$

$P(x, \lambda t) = P(x, 12) = \frac{e^{-12}(12)^x}{x!}$

$P(X < 7) = \sum P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= \sum_{x=0}^{6} P(X=x) = \sum_{x=0}^{6} P(x, \lambda t) = \sum_{x=0}^{6} P(x, 12\times 1)$

$= \sum_{x=0}^{6} P(x, 12) = 0.0458$ (from the table)

(b) $n = 3, \quad x = 2$

$p = 0.0458$

prob. of on 2 of the next 3 acres inspected.

$= b(2; 3, p) = b(2; 3, 0.0458)$

$b(x; n, p) = {}^{n}C_{x} p^{x} q^{n-x}$

$= {}^{3}C_{2} (0.0458)^2 (0.9542)^{3-2}$

$= \frac{3\times \cancel{2}}{1\times\cancel{2}} (0.0458)^2 (0.9542)$

$= 0.0060$

5.8

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(70). Let X denoted no. of defective units.
So ~~to~~ $n = 100$

(a). $\mu = np$
~~Since~~ since given $p = 0.01$
$\mu = 100 \times 0.01 = 1$

(b) $\sigma^2 = npq$
Given $n = 100$, $p = 0.01$, $q = .99$
$\sigma^2 = 100 \times 0.01 \times .99$
$\boxed{\sigma^2 = .99}$

(47) Let X : no. of violation firm in the selected firm.

Vio	Non Vio
3	17

$N = 20$

0	5

$n = 5$

(a) prob. of inspection of 5 firms will find no violations

$$= \frac{\binom{3}{0}\binom{17}{5}}{\binom{20}{5}}$$

(b) prob that 2 violates out of 5

Vio	Non Vio
3	17

$N = 20$

2	3

$n = 5$

$$= \frac{\binom{3}{2}\binom{17}{3}}{\binom{20}{5}}$$