(1)

Chap-7:-

(2) $f(x) = \begin{cases} \binom{3}{x}\left(\frac{2}{5}\right)^{x}\left(\frac{3}{5}\right)^{3-x} & x = 0,1,2,3 \\ 0 & \text{elsewhere} \end{cases}$

$Y = X^2$.

$y = x^2$

Since $x = 0,1,2,3$, $\Rightarrow y = 0,1,4,9$

$\boxed{x = \sqrt{y}}$

$g(y) = f(\sqrt{y}) = \begin{cases} \binom{3}{\sqrt{y}}\left(\frac{2}{5}\right)^{\sqrt{y}}\left(\frac{3}{5}\right)^{3-\sqrt{y}} & y = 0,1,4,9 \\ 0 & \text{elsewhere} \end{cases}$

(19) $p(x;\mu) = \frac{e^{-\mu}\mu^{x}}{x!}$ $x = 0,1,2,\ldots$

$M_X(t) = E[e^{xt}] = \sum_{x=0}^{\infty} \frac{e^{xt}e^{-\mu}\mu^{x}}{x!} = e^{-\mu}\cdot\sum_{x=0}^{\infty}\frac{(e^{t}\mu)^{x}}{x!}$

$= e^{-\mu}\cdot e^{\mu e^{t}} = e^{\mu(e^{t}-1)}$

(20) $M_X(t) = e^{4(e^{t}-1)}$

$p(x;\mu) = \frac{e^{-4}\mu^{x}}{x!} = \frac{e^{-4}4^{x}}{x!}$

$\mu = 4,$ $\sigma^2 = 4 \Rightarrow \sigma = 2$

$P(\mu - 2\sigma < X < \mu + 2\sigma) = P(0 < X < 8)$

$\mu - 2\sigma = 4 - 2\times 2 = 0$

$\mu + 2\sigma = 4 + 2\times 2 = 8$

$= \sum_{x=1}^{7} \frac{e^{-4}4^{x}}{x!}$

$= \sum_{x=1}^{7} p(x;4)$

$= \sum_{x=0}^{7} p(x;4) - p(0,4)$

$= 0.9489 - \frac{e^{-4}4^{0}}{0!}$

$= 0.9489 - 0.0183$

$= 0.9306$

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17 $f(x;k) = \begin{cases} \frac{1}{k} & x = 1, 2, \ldots, k \\ 0 \end{cases}$

$M_x(t) = E[e^{xt}] = \sum_{x=1}^{k} e^{xt} \cdot \frac{1}{k}$

$= \frac{1}{k} \sum_{x=1}^{k} e^{xt} = \frac{1}{k} \sum_{x=1}^{k} (e^t)^x$

$= \cancel{\frac{1}{k}\left(\frac{1-(e^t)^k}{1-e^t}\right) \cdot e^t}$

$1 + r + r^2 + \cdots + r^{n-1}$

$= \frac{1-r^n}{1-r}$

$= \frac{1}{k}\left((e^t)^1 + (e^t)^2 + \cdots + (e^t)^k\right)$

$= \frac{e^t}{k}\left(1 + (e^t)^1 + \cdots + (e^t)^{k-1}\right)$

$= \frac{e^t}{k}\left(\frac{1-(e^t)^k}{1-e^t}\right) = \frac{e^t(1-e^{kt})}{k(1-e^t)}$

Q18 (a) $f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$

$\frac{dx}{dy} = \frac{1}{2\sqrt{y}}$ $y = x^2$

$x = \sqrt{y}$ $0 < x < 1 \Rightarrow 0 < \sqrt{y} < 1 \Rightarrow 0 < y < 1$

(a) $g(y) = \begin{cases} 2(1-\sqrt{y}) \times \frac{1}{2\sqrt{y}} = (y^{-1/2} - 1) & 0 < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$

(b) $P(Y < \frac{500}{5000}) = P(Y < 0.1) = \int_{0}^{0.1} (y^{-1/2} - 1)\, dy$

do yourself

$e^{\log_e x} = x$

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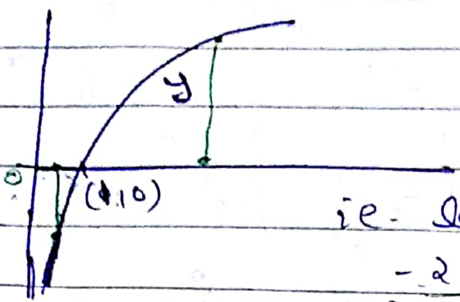
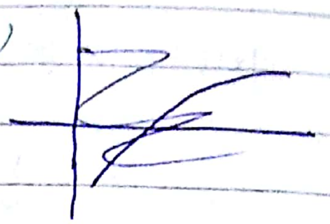
(5) $f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$

since $y = -2\ln x = f(x)$





i.e. $\log x < 0$ for $0<x<1$

$-2\log x > 0$ for $0<x<1$

So $y = -2\log x > 0$ for $0<x<1$

$\log x = -y/2 \Rightarrow x = e^{-y/2} = \omega(y)$

$\boxed{\omega(y) = e^{-y/2}}$

$\omega'(y) = e^{-y/2}\left(-\frac{1}{2}\right)$

$|\omega'(y)| = \frac{1}{2}e^{-y/2}$

$X \to f(X)$

$Y = X^2$

$f(x) \to \omega(y)$

$y = x^2$

$\boxed{x = \pm\sqrt{y}}$

So $g(y) = f(\omega(y))\,|\omega'(y)|$

$= 1 \cdot \frac{1}{2}e^{-\frac{y}{2}}$

$Y = f(x)$

$\boxed{x = \omega(y)}$

So $g(y) = \begin{cases} \frac{1}{2}e^{-y/2} & y \geq 0 \\ 0 & \text{elsewhere.} \end{cases}$

Since P.d.f. for chi-square distribution with n degree of freedom.

$f(x;n) = \begin{cases} \dfrac{1}{2^{\frac{n}{2}}\Gamma\frac{n}{2}}\, x^{\left(\frac{n}{2}-1\right)} e^{-\frac{x}{2}} & x > 0 \\ 0 & \text{elsewhere.} \end{cases}$

take particularly $n=2$

$f(x,2) = \begin{cases} \dfrac{1}{2^{\frac{2}{2}}\Gamma\frac{2}{2}}\, x^{\left(\frac{2}{2}-1\right)} e^{-\frac{x}{2}} = \frac{1}{2}e^{-\frac{x}{2}} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$

2×1

$= \sqrt{\pi}$

$\Gamma(n) \Rightarrow \text{int}$

$\Gamma n = (n-1)!$

So Y follows chi-square distribution with degree of freedom.