22070108

MATHS ASSIGNMENT - IV

1. Given a stationary random process $X(t) = 10\cos(wt + \theta)$ where $\theta \in (-\pi, \pi)$ followed uniform distributed. Find the auto correlation of the process

To prove autocorrelation $X(t)$ is a wss we have to prove that

i) $E\{X(t)\} =$ a constant

ii) $R_{XX}(t, t+\tau) = E\{X(t)\, X(t+\tau)\} =$ a function of τ

i) $E\{X(t)\} = E\{10\cos(wt + \theta)\}$

Given $\theta \in (-\pi, \pi)$

$$f(\theta) = \frac{1}{\pi + \pi} \quad \text{in } (-\pi, \pi)$$

$$= \frac{1}{2\pi}$$

$$E\{X(t)\} = \int_{R\theta} X(t) f(\theta)\, d\theta$$

$$= \int_{-\pi}^{\pi} 10\cos(wt + \theta) \frac{1}{2\pi}\, d\theta$$

$$= \frac{10 \times 1}{2\pi} \left[\frac{\sin(wt + \theta)}{1}\right]_{-\pi}^{\pi}$$

$$= \frac{5}{\pi}\left[\sin(wt + \pi) - \sin(wt - \pi)\right]$$

$$= \frac{5}{\pi}\left[-\sin wt + \sin wt\right]$$

$= 0$ a constant

11) $R_{xx}(t, t+\tau) = E\{x(t)\,x(t+\tau)\}$ = a fn of τ

$R_{xx}(t, t+\tau)$

$E\{x(t)\,x(t+\tau)\}$

$x(t) = 10\cos(\omega t + \theta)$

$x(t+\tau) = 10\cos(\omega(t+\tau)+\theta)$

$x(t)\,x(t+\tau) = 10\cos(\omega t+\theta)\,10\cos(\omega t+\omega\tau+\theta)$

$E\{x(t)x(t+\tau)\} = E\{10\cos(\omega t+\theta\;10\cos(\omega t+\omega\tau+\theta)\}$

$= 100E\{\cos(\omega t+\theta)\cos(\omega t+\omega\tau+\theta)\}$

$= 100\int\cos(\omega t+\theta)\cos(\omega t+\omega\tau+\theta)\frac{1}{2\pi}d\theta$

$\cos(A+B)+\cos(A-B) = \cos A\cos B - \sin A\sin B + \cos A\cos B + \sin A\sin B$

$= 2\cos A\cos B$

$\cos A\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$

$A = \omega t+\theta \qquad B = \omega t+\omega\tau+\theta$

$\cos(\omega t+\theta)\cos(\omega t+\omega\tau+\theta) = \frac{1}{2}[\cos(\omega t+\theta+\omega t+\omega\tau+\theta)$

$+\cos(\omega t+\theta-\omega t-\omega\tau-\theta)]$

$= \frac{1}{2}[\cos(\omega t+\theta+\omega t+\omega\tau+\theta)+$
$\cos(-\omega\tau)]$

$E\{x(t)x(t+\tau)\} = 100\times\frac{1}{2\pi}+\frac{1}{2}\int_{-\pi}^{\pi}\cos(2\omega t+\omega\tau+2\theta)+$
$\cos(-\omega\tau)\,d\theta$

$= \frac{25}{\pi}\left[\int_{-\pi}^{\pi}\cos(2\omega t+2\theta+\omega\tau)\,d\theta + \int_{-\pi}^{\pi}\cos(-\omega\tau)\,d\theta\right]$

$= \frac{25}{\pi}\left[\left(\frac{\sin(2\omega t+2\theta+2\tau)}{2}\right)_{-\pi}^{\pi} + \cos\omega\tau\,[\theta]_{-\pi}^{\pi}\right]$

$$= \frac{25}{\pi}\left[\frac{(\sin(2wt+2\pi+wt)}{2} - \frac{\sin(2wt+2(-\pi)+w\tau}{2}\right]$$
$$+ \cos w\tau(\pi+\pi)$$

$$= \frac{25}{\pi}\left[\frac{\sin(2wt+w\tau)}{2} - \frac{\sin(2wt+w\tau)}{2} + 2\pi\cos w\tau\right]$$

$$= \frac{25}{\pi}(2\pi\cos w\tau) = 50\cos w\tau$$ a function of τ

$$R_{xx}(t, t+\tau) = 50\cos w\tau$$

2. Suppose that the customers arrive at a bank according to a poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 min

i) Exactly 4 customers arrive
ii) Less than 4 customers arrive
iii) more than 4 customers arrive

$\lambda = 3$

For a poisson process

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, 2, \ldots$$

$$\lambda t = (3)(2) = 6$$

$$P\{X(t)=n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \quad n = 0, 1, 2, \ldots$$

i) $$P\{X(2)=4\} = \frac{e^{-3(2)}(6)^4}{4!} = \frac{0.0025 \times 1296}{4!}$$

$$= 0.1350$$

ii) $P\{X(2) > 4\} = 1 - P\{X(2) \leq 4\}$

$= 1 - P\{X(2) = 0, 1, 2, 3, 4\}$

$= 1 - [P\{X(2) = 0 + P(X(2)) = 1) + P(X(2) = 3) + P(X(2) = 4)]$

$= 1 - \left[\frac{e^{-6}(6)^0}{0!} + \frac{e^{-6}(6)^1}{1!} + \frac{e^{-6}(6)^2}{2!} + \frac{e^{-6}(6)^3}{3!} + \frac{e^{-6}(6)^4}{4!}\right]$

$= 1 - \left[e^{-6}\left(1 + 6 + \frac{6^2}{2!} + \frac{6^3}{3!} + \frac{6^4}{4!}\right)\right]$

$= 1 - [0.0025(115)]$

$= 1 - 0.2875 = 0.7125$

iii) $P\{X(2) < 4\} = P\{X(2) = 0, 1, 2, 3 \ldots\}$

$= P[X(2) = 0 + P(X(2) = 1) + P(X(2) = 2) + P(X(3) = 3)]$

$= \frac{e^{-6}(6)^0}{0!} + \frac{e^{-6}(6)^1}{1!} + \frac{e^{-6}(6)^2}{2!} + \frac{e^{-6}(6)^3}{3!}$

$= e^{-6}\left[1 + 6 + \frac{6^2}{2!} + \frac{6^3}{3!}\right]$

$= 0.0025(61)$

$= 0.1525$