By: Glen Schmidt

The Newsvendor Model

When I lived in the Washington DC area, I would pass a newsvendor selling the Washington Post on the way to my office. Let's assume the newsvendor sells the papers for $1.00, after picking up the papers that morning for 70 cents for each. Further assume that if he has any leftover papers, he sells them at a discount, at 20 cents each. There is uncertainty in how many people will buy a newspaper – He estimates he will definitely sell at least 35 papers, but no more than 40. More specifically, he estimates he will sell 35 papers with probability 0.10, 36 papers with probability 0.15, 37 papers with probability 0.25, 38 papers with probability 0.25, 39 papers with probability 0.15, and 40 papers with probability 0.10.

Let "Q" denote the number of papers the newsvendor picks up every morning for subsequent resale. How should he solve for Q?

Application

While you may not aspire to selling newspapers on the street-corner upon graduation, you may face many other possible problems where an equivalent framework applies. For example, we might rephrase the problem as follows. Assume you are responsible for ordering jackets for the South Face retail store in Chicago. The South Face orders a fall jacket from overseas, with a lead time of 16 weeks. Because of this long lead-time, it has only one chance of ordering for the fall season. It buys jackets for $70 each, and sells jackets for $100. Units remaining at the end of the season must be heavily discounted, selling for only $20. The demand distribution is the same as given previously for the newspapers. How many jackets should the South Face order?

Given this setup, South Face's problem is essentially equivalent to the newsvendor's problem, with the only exception being that the monetary amounts are in dollars rather than in cents. To generalize, we can say that we have a newsvendor type problem when the framework is as follows: You have one chance to make a decision regarding the amount of "product" to purchase, and the demand for that product is uncertain. There is a known marginal profit to be gained for each additional unit sold, and there is a known marginal loss experienced for every unit you buy but are unable to sell (at normal price).

Marginal Analysis

To solve the newsvendor's problem, we apply the following logic. Let Q denote the number of units we stock. Referring back to our initial example, we will buy the Q^{th} newspaper if we expect to realize a profit by doing so. If we buy the Q^{th} newspaper, there is the possibility we will be able to sell it, and receive a marginal profit (*MP*) of 30 cents (the $1.00 sales price minus the 70 cent cost). There is also the possibility we will *not* be able to sell it, in which case we experience a marginal loss (*ML*) of 50 cents (our cost of 70 cents minus the salvage value of 20 cents). We should buy the Q^{th} newspaper only if the *expected* marginal profit exceeds the *expected* marginal loss.

Let P denote the probability that the Q^{th} newspaper is *not* sold (so that $1 - P$ is that probability that it *is* sold). Expected marginal profit is equal to $(1 - P)$, the probability that the Q^{th} newspaper is sold, multiplied by MP, the marginal profit from that sale. Expected marginal loss is equal to P, the probability that the Q^{th} newspaper is *not* sold, multiplied by ML, the marginal loss. Thus, we buy the Q^{th} newspaper only if:

$$(1 - P)(MP) \geq P(ML)$$

Rearranging terms, and solving for P, we find that our expected profit from the Q^{th} unit is positive only if the probability P of not selling the Q^{th} unit is less than $(MP) / (MP + ML)$. That is:

$$P < (MP) / (MP + ML)$$

We define the ratio $(MP) / (MP + ML)$ to be the "critical ratio," denoted as P_c:

Equation 1. $P_c = (MP) / (MP + ML)$

In other words, we buy the Q^{th} newspaper only if $P < P_c$. Applying Equation 1 to the newsvendor's problem at hand, we can calculate the critical ratio:

$$P_c = (MP) / (MP + ML) = (\$1.00 - \$0.70) / [(\$1.00 - \$0.70) + (\$0.70 - \$0.20)] = 0.375.$$

To determine if the probability P of *not* selling the Q^{th} unit is les than the critical ratio $\boldsymbol{P_c}$, we construct Table 1.

Table 1. Probabilities of selling and not selling each unit.

Q = # of units	Probability that demand = Q	$1 - P$ = Probability of selling the Q^{th} unit	P = Probability of NOT selling the Q^{th} unit	$(1 - P)$ ($0.30) Expected profit from selling the Q^{th} unit	(P) 0.50 = Expected loss of NOT selling the Q^{th} unit	Expected net profit from Q^{th} unit
< 35	0.00					
35	0.10	1.00	0.00	0.30	0.00	0.30
36	0.15	0.90	0.10	0.27	0.05	0.22
37	0.25	0.75	0.25	0.225	0.125	0.10
38	0.25	0.50	0.50	0.15	0.25	– 0.10
39	0.15	0.25	0.75	0.075	0.375	– 0.30
40	0.10	0.10	0.90	0.03	0.45	– 0.42
> 40	0.00	0.00	1.00	0.00	0.50	– 0.50
	1.00					

From Table 1, we see that the requirement $P < \boldsymbol{P_c} = 0.375$ is met for all units up to and including the 37^{th} unit. Thus, it is optimal to buy 37 newspapers.

In fact, Table 1 takes the analysis a step further, completing the calculation of expected net profit for each incremental unit. This is done to confirm that expected net profit is positive for units 37 and below, and negative for units 38 and above.

Cost of Underage of Cost of Overage

Before proceeding, an extension to the previous analysis will be introduced, resulting in a change in notation. In some problems, the impact of not carrying enough product may be more than suggested thus far by the *MP* term. For example, if we fail to have enough newspapers today, a customer who expected to buy one today, but couldn't because we ran out, may decide to take a different route to work tomorrow, to insure that she is able to buy the paper she wants tomorrow. Thus, we have lost not only today's sale, but have also lost some potential future profit or "customer goodwill." In other words, when we under-buy on the number of newspapers, we may lose more than the immediate marginal profit. To allow for this possibility, we will introduce the term "cost of underage," to include not only the immediate lost marginal profit, but to also include any lost customer goodwill, and other costs associated with holding too few units in inventory.

In other words, we will use the cost of underage (denoted by c_u) as a more inclusive term than *MP*, where c_u includes all potential "costs" associated with not buying enough product to satisfy eventual customer demand. Similarly, the cost of overage (denoted by c_o) will replace the term *ML* and will include all potential "costs" associated with over-buying, that is, all costs associated with buying an amount that exceeds eventual customer demand. Making these substitutions, Equation 1 becomes:

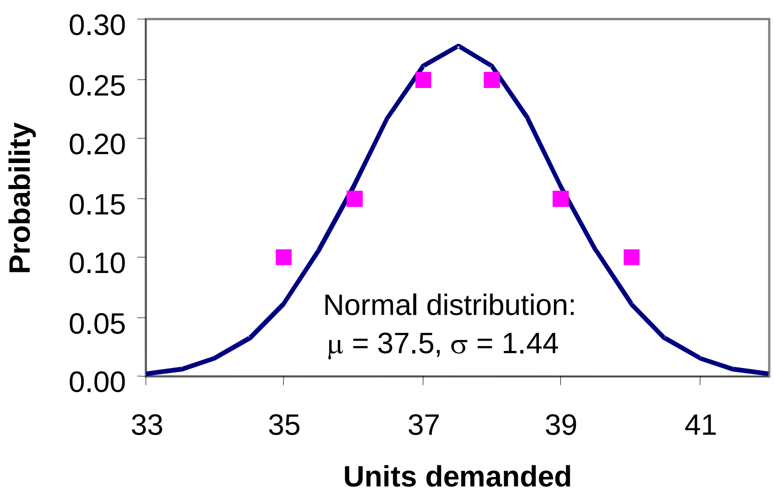
Equation 2. $P_c = (c_u) / (c_u + c_o)$

In the newsvendor example considered thus far, we have assumed there is no lost customer goodwill. So for the example considered thus far, $c_u = MP$, and $c_o = ML$.

Continuous Probability Distribution

The above example used a discrete probability distribution to characterize uncertain customer demand (the probabilities were given at discrete levels of 35, 36, 37, 38, 39, and 40 units). However, it is often more convenient to express our anticipated demand in terms of a continuous probability distribution, particularly when the possible number of units demanded might be large. For example, rather than assigning probabilites to discrete numbers of units from 35 to 40, the newsvendor might believe his demand is normally distributed with a mean of 37.5 newspapers, and a standard deviation of 1.44 newspapers. This continuous demand distribution (along with the previously described discrete distribution) are illustrated in Figure 1. Again, the newsvendor's problem is to determine how many newspapers to order for the upcoming day.

Figure 1. Continuous and discrete probability distributions for newspaper demand.

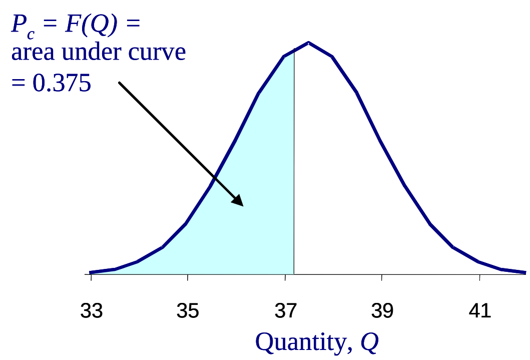


As Figure 1 might suggest, it turns out this alternate problem, based on the continuous distribution, is essentially equivalent to the original problem based on the discrete distribution. To show this, we proceed similarly to the procedure just outlined. Namely, as before, we calculate the critical ratio:

$$\boldsymbol{P_c = (c_u) / (c_u + c_o) = (\$1.00 - \$0.70) / [(\$1.00 - \$0.70) + (\$0.70 - \$0.20)] = 0.375.}$$

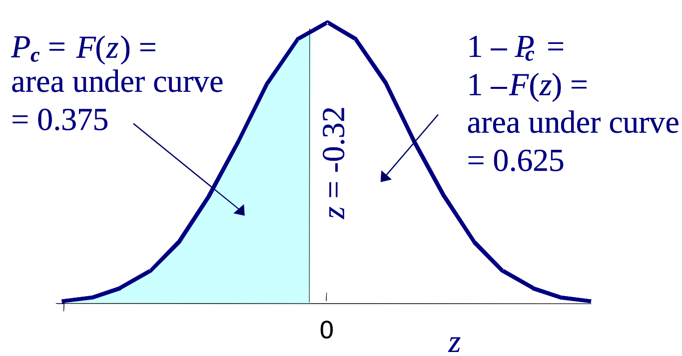
Next, as before, we set out to determine those values of Q (where Q represents the number of newspapers) for which the probability of selling the Q^{th} unit is less than this critical ratio, P_c. Note that we will sell the Q^{th} unit if and only if the realized demand is equal to or more than Q. In other words, the probability P of *not* selling the Q^{th} unit is simply the probability that the realized demand is less than Q. Recall that for a continuous probability distribution $f(Q)$, the probability that we will realize a value less than Q is equal to the area under the probability distribution curve to the left of Q, also known as the left-hand tail area, or cumulative distribution function $F(Q)$. Thus, for the problem at hand, our task is to find the value Q for which the left-hand tail area is equal to the critical value of 0.375 (refer to Figure 2).

Figure 2. The probability of not selling the Q^{th} unit is the area under the curve to the left of Q.



Since we are dealing with a normal distribution (while the general newsvendor framework can be applied given any distribution, we will always assume normality), a standard normal "z" table can be used to find the "z" value associated with the appropriate area (the left-tail area for this problem is 0.375). Any standard normal "z" table can be used, however, a "z" table giving left-hand tail areas is most convenient. Such a table is attached at the end of this note. By interpolating within this table, we find that a "z" value of –0.32 is associated with a left tail area of 0.375 (see Figure 3).

Figure 3. Using the normal curve and "z" table .



Recall that the "z" value indicates the number of standard deviations that Q lies away from the mean (a negative z value indicates Q lies below the mean, and a positive z value indicates Q lies above the mean), and recall that z is calculated as $z = (Q - \mu) / \sigma$, where μ and σ are the mean and standard deviation, respectively. Simply re-writing this relationship in terms of Q, we find:

Equation 3. $$\boldsymbol{Q = z\,\sigma + \mu}$$

For the problem at hand, we find: $Q = z\,\sigma + \mu = -\,0.32\ (1.44) + 37.5 = 37.04$. Since we must buy an integer number of newspapers (we can't buy a fraction of a paper), we buy 37 newspapers.[1]

Performance Measures

If the newsboy stocks 37 newspapers, his in-stock probability is the probability he *doesn't* run out:

In-stock probability = $\boldsymbol{P_c}$ = 37.5%.

The probability that he *does* run out is his stockout probability:

Stockout probability = 1 – $\boldsymbol{P_c}$ = 62.5%.

Next he wants to know how many sales he expects to lose by not stocking more papers. To figure this out, he runs a 100-day test during which time he always stocks a very large number of papers, such that he never runs out. He finds that on 38 of the 100 days he sold 37 or less (recall that there was a 37.5% chance of selling not running out if he stocked 37), but he ignores these 38 days in his calculation because he would have lost no sales on these 38 days. Looking at the other 62 days on which he sold 38 or more, he finds that on a number of these days he sold exactly 38 papers (one more than 37), on a few days he sold 39 (two more than 37), and so forth. For each of these 62 days he calculates the number sold over 37, and adding these all up he finds that in total he sold 83 more papers than if he had stocked only 37. In other words, over the 100 days, on average he sold 0.83 more papers per day than if he had stocked only 37. Said another way, each day's loss associated with a stocking level of 37 would have been 0.83 papers, which is the equivalent of 0.58 standard deviations (recall that 1 standard deviation was 1.44, or 0.83 / 1.44 = 0.58).

Had the newsvendor known how to use something called a "loss function table" (or had he known how to calculate the loss function in Excel), he would not have had to run the 100-day test. Since we found that a stocking level of 37 is associated with a z value of –0.32, we can go to the loss function table (attached at the end of this note) and find the loss function, denoted by $L(z)$, is 0.579. This means that the expected loss is 0.579 standard deviations, essentially equivalent to what the newsvendor's 100-day test showed. That is:

Expected lost sales = $\boldsymbol{\sigma L(z)}$ = 1.44 (0.579) = 0.83.

The newsvendor is an inquisitive fellow! He also wants to know how many papers he expects to sell each day if he stocks 37. The demand expected each day is 37.5, but from this he has to subtract the 0.83 units in sales he expects to lose each day. Thus his expected sales are:

Expected sales = $\boldsymbol{\mu}$ – expected lost sales = 37.5 – 0.83 = 36.67.

Further, he wants to know how many papers he expects to have left over each day. Since he stocked 37 and expects to sell 36.67 units, his expected leftover inventory is:

Expected leftover inventory = $\boldsymbol{Q}$ – expected sales = 37 – 36.67 = 0.33.

[1] Technically, there is a "rounding rule" associated with the Newsvendor problem but we will simply round to the nearest whole number (the uncertainty inherent in describing the demand distribution and in measuring the costs of underage and overage probably overshadow this rounding subtlety).

Additionally, he wants to know what fraction of demand he expects to fill – this is called his *fill rate*. (Can you describe how this differs from the stockout probability?)

Expected fill rate = (expected sales) / (expected demand) = 36.67 / 37.5 = 97.8%.

Finally, he wants to know what profit to expect each day:

Expected profit=(price–cost) (expected sales) – (cost–salvage value) (expected leftover inventory)
= ($1.00 – $0.70) (36.67) – ($0.70 – $0.20) (0.33) = $10.84.

Zenith TV Capacity

Here is another example. Let's say you feel as though this OM class is completely irrelevant, since you are a marketing major. Assume that upon graduation, you take a position as a marketing manager for a Fortune 500 company. You have identified a customer need for a new product, and have convinced your company to ramp up production for the coming year. As marketing manager, you are asked by the factory manager to give her an estimate of what the customer demand for the product will be, so she can set the factory capacity level accordingly.

Just such a real-life problem was actually faced by Zenith TV, as described in Harvard Business School case 9-674-026. In 1966, color televisions were replacing black and white sets in American households (sales had doubled each of the previous 3 years). Zenith Radio Corp. was expanding, and needed to decide how big its factory should be. A plant with a capacity of 1,000 sets per day required an investment of $3 million, with interest and amortization running at 15% annually. Color TVs sold for $370, with an after-tax profit of 7%. Industry demand was estimated at 8.5 million sets per year. Zenith estimated it could achieve 20% market share, but because of considerable uncertainty surrounding these estimates, Zenith put the standard deviation associated with its estimated demand at 0.5 million sets per year. Zenith's question was, How big of a factory should we build (what should its capacity be)?

It may not be immediately obvious that the Zenith problem is a newsvendor type problem. The key to seeing this is to observe that the "product" being purchased by Zenith is not the color TVs themselves, but rather, the product Zenith is buying is the *plant capacity* to *build* color TVs.

Zenith's cost of underage (its cost of building a plant that has too little capacity) is equal to the lost marginal profit of 7% of the sales price of $370, such that c_u = $25.90 per set. To calculate the cost of overage (the cost of building a plant that has "excess" capacity), begin by noting that it costs $3 million for a plant with a capacity of 1,000 sets per day. If we assume 250 production days per year, then 1,000 sets per day equates to 250,000 sets per year. This suggests it costs $12 for each incremental unit of production capacity ($3 million / 250,000). But we don't need to recoup this full $12 in one year, since we may be able to use this capacity in future years even if we don't use it this year. We only need to recoup the interest and amortization expense of 15% of $12, such that c_o = $1.80 per set.

Thus, we calculate the critical ratio as follows: $P_c = c_u / (c_u + c_o)$ = $25.90 / ($25.90 + $1.80) = 0.935. Consulting a "*z* table" suggests the *z* value associated with this critical ratio is $z = 1.51$. Calculating the *Q* value, we find $Q = z\,\sigma + \mu$ = 1.51 (0.5 million sets/yr) + 0.20 (8.5 million sets/yr) = 2.46 million sets per year.

Note that we find it optimal to build a plant with a capacity of 2.46 million sets per year, whereas our "best" estimate of the demand is only 1.7 million sets per year (20% of 8.5 million sets per year).

Insights

Even if you never do another newsvendor problem in your future career, there are some insights that you can take away from this analysis. In fact, this may be the most important reason for doing mathematical problems such as these. The end goal is not necessarily the answer itself, but to understand WHY you get the answer you get.

For example, we saw in the first example involving the newsvendor that it was optimal to order slightly *less* than the mean demand (the mean demand was 37.5 papers, and the optimal number to order was 37). On the other hand, for Zenith, the optimal capacity was significantly *more* than the mean demand: 2.46 million sets per year, versus a mean demand of only 1.7 million sets per year.

The intuition behind these results is as follows. If there is more “upside potential,” then we will want to have on hand *more* than the mean demand. In other words, if we “lose” a lot of potential profit by falling short of product (i.e., the cost of underage is greater than the cost of overage), then we don’t want to run out of product.

Conversely, if there is more “downside potential,” then we stock *less* than the mean demand. That is, if the cost of ending up with leftover product overshadows the cost of falling short (i.e., the cost of overage is greater than the cost of underage), then we want to avoid holding too much product, so we stock less than the mean demand.

The model also offers some insight as to why some Marketing Managers might be notorious for being overly optimistic in their sales projections. In fact, here we find some evidence to suggest that such “excessive” optimism may, in some situations, be quite rational.

For example, suppose Zenith’s management had been a zealous believer in Einstein’s adage that states you should “make everything as simple as possible . . .” In following with this adage to keep things simple, suppose Zenith asked its Marketing Manager to simply provide the company with her best estimate of the mean demand for TVs, indicating that Zenith would then set its TV production capacity level precisely at the Marketing Manager’s estimate.

A “smart” Marketing Manager might inflate the true mean estimate of 1.7 million sets per year to the higher figure of 2.46 million sets per year calculated herein, knowing that this higher number is in fact the optimal capacity level for Zenith. She might further (diplomatically) inform management that they should acknowledge the more complete text of Einstein’s quote, which states you should “make everything as simple as possible, but not simpler.” In other words, a demand estimate that gives only the mean demand level is *too* simple – a good demand estimate also provides an indication of the dispersion around that mean.

Newsvendor Summary

Does it apply? The newsvendor model applies when you have one chance at making a stocking decision (if you "know what you are doing," you may be able to judiciously apply the model to a given specific situation that may not precisely meet this stipulation).

Cost of underage c_u: the profit you could have made had you not you run out of inventory, including possible goodwill loss, for example (typically, c_u = goodwill loss + full sales price – cost).

Cost of overage c_o: the loss you take on any item left over that didn't sell at "full price" and had to "salvage" (typically, c_o = cost – salvage value).

Critical ratio P_c: $P_c = (c_u) / (c_u + c_o)$.

In-stock probability: the probability that demand is less than the optimal order quantity Q, i.e., the probability that you *don't* stock out of items if you order the optimal quantity Q. The in-stock probability is simply equal to P_c.

Stockout probability: the probability that demand is equal to or greater than the optimal order quantity Q, i.e., the probability that you *do* stock out of items if you order the optimal quantity Q. The stockout probability is simply equal to $1 - P_c$.

***z* value:** the number of standard deviations that the optimal order quantity Q is away from the mean, where a negative z means Q is z standard deviations less than the mean and a positive z means Q is z standard deviations greater than the mean. You find z by using a z-table that gives left-hand tail areas; to use the z-table you look up the z associated with a left-hand tail area of P_c.

Order quantity Q: $Q = z\,\sigma + \mu$
where μ is the mean demand and σ is the standard deviation of demand.

Expected lost sales: the number of units that you expect to be short (this number can be a fraction).
Expected lost sales = $\sigma L(z)$,
where σ is the standard deviation of demand and $L(z)$ is the loss function.
You find $L(z)$ in a loss function table by looking up the $L(z)$ associated with z. Or, use the Excel formula "=NORMDIST(z,0,1,0) – z [1-NORMSDIST(z)]".

Expected sales: the number of units you expect to sell (it can be a fraction).
Expected sales = μ – expected lost sales; where μ is the mean demand.

Expected leftover inventory: the number of units you expect to have left over (it can be a fraction).
Expected leftover inventory = Q – expected sales.

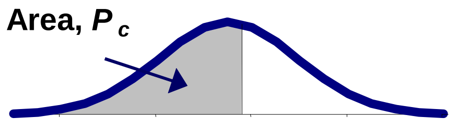
Expected fill rate: the fraction of demand that you expect to satisfy (i.e., supply or "fill").
Expected fill rate = (expected sales) / (expected demand).

Expected profit: (price–cost) (expected sales) – (cost–salvage value) (expected leftover inventory)

Steps in Calculating the Newsvendor Quantity Q

1) Calculate c_u and c_o.
2) Calculate $P_c = (c_u) / (c_u + c_o)$.
3) Find z using a z-table or using the Excel function formula "=NORMSINV(P_c)".
4) Calculate $Q = z\,\sigma + \mu$.

z values for left-tail areas
z = (*Q* - mean) / (std. deviation)
z = NORMSINV(area)



P_c	z	P_c	z	P_c	z	P_c	z	P_c	z	P_c	z	P_c	z
0.001	-3.090	0.145	-1.058	0.289	-0.556	0.433	-0.169	0.577	0.194	0.721	0.586	0.865	1.103
0.004	-2.652	0.148	-1.045	0.292	-0.548	0.436	-0.161	0.580	0.202	0.724	0.595	0.868	1.117
0.007	-2.457	0.151	-1.032	0.295	-0.539	0.439	-0.154	0.583	0.210	0.727	0.604	0.871	1.131
0.010	-2.326	0.154	-1.019	0.298	-0.530	0.442	-0.146	0.586	0.217	0.730	0.613	0.874	1.146
0.013	-2.226	0.157	-1.007	0.301	-0.522	0.445	-0.138	0.589	0.225	0.733	0.622	0.877	1.160
0.016	-2.144	0.160	-0.994	0.304	-0.513	0.448	-0.131	0.592	0.233	0.736	0.631	0.880	1.175
0.019	-2.075	0.163	-0.982	0.307	-0.504	0.451	-0.123	0.595	0.240	0.739	0.640	0.883	1.190
0.022	-2.014	0.166	-0.970	0.310	-0.496	0.454	-0.116	0.598	0.248	0.742	0.650	0.886	1.206
0.025	-1.960	0.169	-0.958	0.313	-0.487	0.457	-0.108	0.601	0.256	0.745	0.659	0.889	1.221
0.028	-1.911	0.172	-0.946	0.316	-0.479	0.460	-0.100	0.604	0.264	0.748	0.668	0.892	1.237
0.031	-1.866	0.175	-0.935	0.319	-0.470	0.463	-0.093	0.607	0.272	0.751	0.678	0.895	1.254
0.034	-1.825	0.178	-0.923	0.322	-0.462	0.466	-0.085	0.610	0.279	0.754	0.687	0.898	1.270
0.037	-1.787	0.181	-0.912	0.325	-0.454	0.469	-0.078	0.613	0.287	0.757	0.697	0.901	1.287
0.040	-1.751	0.184	-0.900	0.328	-0.445	0.472	-0.070	0.616	0.295	0.760	0.706	0.904	1.305
0.043	-1.717	0.187	-0.889	0.331	-0.437	0.475	-0.063	0.619	0.303	0.763	0.716	0.907	1.323
0.046	-1.685	0.190	-0.878	0.334	-0.429	0.478	-0.055	0.622	0.311	0.766	0.726	0.910	1.341
0.049	-1.655	0.193	-0.867	0.337	-0.421	0.481	-0.048	0.625	0.319	0.769	0.736	0.913	1.359
0.052	-1.626	0.196	-0.856	0.340	-0.412	0.484	-0.040	0.628	0.327	0.772	0.745	0.916	1.379
0.055	-1.598	0.199	-0.845	0.343	-0.404	0.487	-0.033	0.631	0.335	0.775	0.755	0.919	1.398
0.058	-1.572	0.202	-0.834	0.346	-0.396	0.490	-0.025	0.634	0.342	0.778	0.765	0.922	1.419
0.061	-1.546	0.205	-0.824	0.349	-0.388	0.493	-0.018	0.637	0.350	0.781	0.776	0.925	1.440
0.064	-1.522	0.208	-0.813	0.352	-0.380	0.496	-0.010	0.640	0.358	0.784	0.786	0.928	1.461
0.067	-1.499	0.211	-0.803	0.355	-0.372	0.499	-0.003	0.643	0.366	0.787	0.796	0.931	1.483
0.070	-1.476	0.214	-0.793	0.358	-0.364	0.502	0.005	0.646	0.375	0.790	0.806	0.934	1.506
0.073	-1.454	0.217	-0.782	0.361	-0.356	0.505	0.013	0.649	0.383	0.793	0.817	0.937	1.530
0.076	-1.433	0.220	-0.772	0.364	-0.348	0.508	0.020	0.652	0.391	0.796	0.827	0.940	1.555
0.079	-1.412	0.223	-0.762	0.367	-0.340	0.511	0.028	0.655	0.399	0.799	0.838	0.943	1.580
0.082	-1.392	0.226	-0.752	0.370	-0.332	0.514	0.035	0.658	0.407	0.802	0.849	0.946	1.607
0.085	-1.372	0.229	-0.742	0.373	-0.324	0.517	0.043	0.661	0.415	0.805	0.860	0.949	1.635
0.088	-1.353	0.232	-0.732	0.376	-0.316	0.520	0.050	0.664	0.423	0.808	0.871	0.952	1.665
0.091	-1.335	0.235	-0.722	0.379	-0.308	0.523	0.058	0.667	0.432	0.811	0.882	0.955	1.695
0.094	-1.317	0.238	-0.713	0.382	-0.300	0.526	0.065	0.670	0.440	0.814	0.893	0.958	1.728
0.097	-1.299	0.241	-0.703	0.385	-0.292	0.529	0.073	0.673	0.448	0.817	0.904	0.961	1.762
0.100	-1.282	0.244	-0.693	0.388	-0.285	0.532	0.080	0.676	0.457	0.820	0.915	0.964	1.799
0.103	-1.265	0.247	-0.684	0.391	-0.277	0.535	0.088	0.679	0.465	0.823	0.927	0.967	1.838
0.106	-1.248	0.250	-0.674	0.394	-0.269	0.538	0.095	0.682	0.473	0.826	0.938	0.970	1.881
0.109	-1.232	0.253	-0.665	0.397	-0.261	0.541	0.103	0.685	0.482	0.829	0.950	0.973	1.927
0.112	-1.216	0.256	-0.656	0.400	-0.253	0.544	0.111	0.688	0.490	0.832	0.962	0.976	1.977
0.115	-1.200	0.259	-0.646	0.403	-0.246	0.547	0.118	0.691	0.499	0.835	0.974	0.979	2.034
0.118	-1.185	0.262	-0.637	0.406	-0.238	0.550	0.126	0.694	0.507	0.838	0.986	0.982	2.097
0.121	-1.170	0.265	-0.628	0.409	-0.230	0.553	0.133	0.697	0.516	0.841	0.999	0.985	2.170
0.124	-1.155	0.268	-0.619	0.412	-0.222	0.556	0.141	0.700	0.524	0.844	1.011	0.988	2.257
0.127	-1.141	0.271	-0.610	0.415	-0.215	0.559	0.148	0.703	0.533	0.847	1.024	0.991	2.366
0.130	-1.126	0.274	-0.601	0.418	-0.207	0.562	0.156	0.706	0.542	0.850	1.036	0.994	2.512
0.133	-1.112	0.277	-0.592	0.421	-0.199	0.565	0.164	0.709	0.550	0.853	1.049	0.997	2.748
0.136	-1.098	0.280	-0.583	0.424	-0.192	0.568	0.171	0.712	0.559	0.856	1.063	0.999	3.090
0.139	-1.085	0.283	-0.574	0.427	-0.184	0.571	0.179	0.715	0.568	0.859	1.076		
0.142	-1.071	0.286	-0.565	0.430	-0.176	0.574	0.187	0.718	0.577	0.862	1.089		

Loss function for *z* values

$L(z)$ = normdist(z,0,1,0) – z [1-normsdist(z)]

z	*L(z)*	*z*	*L(z)*	*z*	*L(z)*	*z*	*L(z)*	*z*	*L(z)*	*z*	*L(z)*	*z*	*L(z)*	*z*	*L(z)*
-2.00	2.008	-1.50	1.529	-1.00	1.083	-0.50	0.698	0.00	0.399	0.50	0.198	1.00	0.083	1.50	0.029
-1.99	1.999	-1.49	1.520	-0.99	1.075	-0.49	0.691	0.01	0.394	0.51	0.195	1.01	0.082	1.51	0.029
-1.98	1.989	-1.48	1.511	-0.98	1.067	-0.48	0.684	0.02	0.389	0.52	0.192	1.02	0.080	1.52	0.028
-1.97	1.979	-1.47	1.501	-0.97	1.058	-0.47	0.677	0.03	0.384	0.53	0.189	1.03	0.079	1.53	0.027
-1.96	1.969	-1.46	1.492	-0.96	1.050	-0.46	0.670	0.04	0.379	0.54	0.186	1.04	0.077	1.54	0.027
-1.95	1.960	-1.45	1.483	-0.95	1.042	-0.45	0.664	0.05	0.374	0.55	0.183	1.05	0.076	1.55	0.026
-1.94	1.950	-1.44	1.474	-0.94	1.033	-0.44	0.657	0.06	0.370	0.56	0.180	1.06	0.074	1.56	0.026
-1.93	1.940	-1.43	1.464	-0.93	1.025	-0.43	0.650	0.07	0.365	0.57	0.177	1.07	0.073	1.57	0.025
-1.92	1.930	-1.42	1.455	-0.92	1.017	-0.42	0.644	0.08	0.360	0.58	0.174	1.08	0.071	1.58	0.024
-1.91	1.921	-1.41	1.446	-0.91	1.009	-0.41	0.637	0.09	0.356	0.59	0.171	1.09	0.070	1.59	0.024
-1.90	1.911	-1.40	1.437	-0.90	1.000	-0.40	0.630	0.10	0.351	0.60	0.169	1.10	0.069	1.60	0.023
-1.89	1.901	-1.39	1.427	-0.89	0.992	-0.39	0.624	0.11	0.346	0.61	0.166	1.11	0.067	1.61	0.023
-1.88	1.892	-1.38	1.418	-0.88	0.984	-0.38	0.617	0.12	0.342	0.62	0.163	1.12	0.066	1.62	0.022
-1.87	1.882	-1.37	1.409	-0.87	0.976	-0.37	0.611	0.13	0.337	0.63	0.161	1.13	0.065	1.63	0.022
-1.86	1.872	-1.36	1.400	-0.86	0.968	-0.36	0.605	0.14	0.333	0.64	0.158	1.14	0.063	1.64	0.021
-1.85	1.863	-1.35	1.391	-0.85	0.960	-0.35	0.598	0.15	0.328	0.65	0.155	1.15	0.062	1.65	0.021
-1.84	1.853	-1.34	1.382	-0.84	0.952	-0.34	0.592	0.16	0.324	0.66	0.153	1.16	0.061	1.66	0.020
-1.83	1.843	-1.33	1.373	-0.83	0.944	-0.33	0.585	0.17	0.320	0.67	0.150	1.17	0.060	1.67	0.020
-1.82	1.834	-1.32	1.364	-0.82	0.936	-0.32	0.579	0.18	0.315	0.68	0.148	1.18	0.058	1.68	0.019
-1.81	1.824	-1.31	1.355	-0.81	0.928	-0.31	0.573	0.19	0.311	0.69	0.145	1.19	0.057	1.69	0.019
-1.80	1.814	-1.30	1.346	-0.80	0.920	-0.30	0.567	0.20	0.307	0.70	0.143	1.20	0.056	1.70	0.018
-1.79	1.805	-1.29	1.337	-0.79	0.912	-0.29	0.561	0.21	0.303	0.71	0.140	1.21	0.055	1.71	0.018
-1.78	1.795	-1.28	1.327	-0.78	0.905	-0.28	0.554	0.22	0.299	0.72	0.138	1.22	0.054	1.72	0.017
-1.77	1.785	-1.27	1.319	-0.77	0.897	-0.27	0.548	0.23	0.294	0.73	0.136	1.23	0.053	1.73	0.017
-1.76	1.776	-1.26	1.310	-0.76	0.889	-0.26	0.542	0.24	0.290	0.74	0.133	1.24	0.052	1.74	0.017
-1.75	1.766	-1.25	1.301	-0.75	0.881	-0.25	0.536	0.25	0.286	0.75	0.131	1.25	0.051	1.75	0.016
-1.74	1.757	-1.24	1.292	-0.74	0.873	-0.24	0.530	0.26	0.282	0.76	0.129	1.26	0.050	1.76	0.016
-1.73	1.747	-1.23	1.283	-0.73	0.866	-0.23	0.524	0.27	0.278	0.77	0.127	1.27	0.049	1.77	0.015
-1.72	1.737	-1.22	1.274	-0.72	0.858	-0.22	0.519	0.28	0.274	0.78	0.125	1.28	0.047	1.78	0.015
-1.71	1.728	-1.21	1.265	-0.71	0.850	-0.21	0.513	0.29	0.271	0.79	0.122	1.29	0.047	1.79	0.015
-1.70	1.718	-1.20	1.256	-0.70	0.843	-0.20	0.507	0.30	0.267	0.80	0.120	1.30	0.046	1.80	0.014
-1.69	1.709	-1.19	1.247	-0.69	0.835	-0.19	0.501	0.31	0.263	0.81	0.118	1.31	0.045	1.81	0.014
-1.68	1.699	-1.18	1.238	-0.68	0.828	-0.18	0.495	0.32	0.259	0.82	0.116	1.32	0.044	1.82	0.014
-1.67	1.690	-1.17	1.230	-0.67	0.820	-0.17	0.490	0.33	0.255	0.83	0.114	1.33	0.043	1.83	0.013
-1.66	1.680	-1.16	1.221	-0.66	0.813	-0.16	0.484	0.34	0.252	0.84	0.112	1.34	0.042	1.84	0.013
-1.65	1.671	-1.15	1.212	-0.65	0.805	-0.15	0.478	0.35	0.248	0.85	0.110	1.35	0.041	1.85	0.013
-1.64	1.661	-1.14	1.203	-0.64	0.798	-0.14	0.473	0.36	0.245	0.86	0.108	1.36	0.040	1.86	0.012
-1.63	1.652	-1.13	1.195	-0.63	0.791	-0.13	0.467	0.37	0.241	0.87	0.106	1.37	0.039	1.87	0.012
-1.62	1.642	-1.12	1.186	-0.62	0.783	-0.12	0.462	0.38	0.237	0.88	0.104	1.38	0.038	1.88	0.012
-1.61	1.633	-1.11	1.177	-0.61	0.776	-0.11	0.456	0.39	0.234	0.89	0.102	1.39	0.037	1.89	0.011
-1.60	1.623	-1.10	1.169	-0.60	0.769	-0.10	0.451	0.40	0.230	0.90	0.100	1.40	0.037	1.90	0.011
-1.59	1.614	-1.09	1.160	-0.59	0.761	-0.09	0.446	0.41	0.227	0.91	0.099	1.41	0.036	1.91	0.011
-1.58	1.604	-1.08	1.151	-0.58	0.754	-0.08	0.440	0.42	0.224	0.92	0.097	1.42	0.035	1.92	0.010
-1.57	1.595	-1.07	1.143	-0.57	0.747	-0.07	0.435	0.43	0.220	0.93	0.095	1.43	0.034	1.93	0.010
-1.56	1.586	-1.06	1.134	-0.56	0.740	-0.06	0.430	0.44	0.217	0.94	0.093	1.44	0.034	1.94	0.010
-1.55	1.576	-1.05	1.126	-0.55	0.733	-0.05	0.424	0.45	0.214	0.95	0.092	1.45	0.033	1.95	0.010
-1.54	1.567	-1.04	1.117	-0.54	0.726	-0.04	0.419	0.46	0.210	0.96	0.090	1.46	0.032	1.96	0.009
-1.53	1.557	-1.03	1.109	-0.53	0.719	-0.03	0.414	0.47	0.207	0.97	0.088	1.47	0.031	1.97	0.009
-1.52	1.548	-1.02	1.100	-0.52	0.712	-0.02	0.409	0.48	0.204	0.98	0.087	1.48	0.031	1.98	0.009
-1.51	1.539	-1.01	1.092	-0.51	0.705	-0.01	0.404	0.49	0.201	0.99	0.085	1.49	0.030	1.99	0.009