

Radiometry

CMPSCI 670: Computer Vision

Grant Van Horn

February 6, 2024

College of
INFORMATION AND
COMPUTER SCIENCES



UMASS
AMHERST

Administrivia

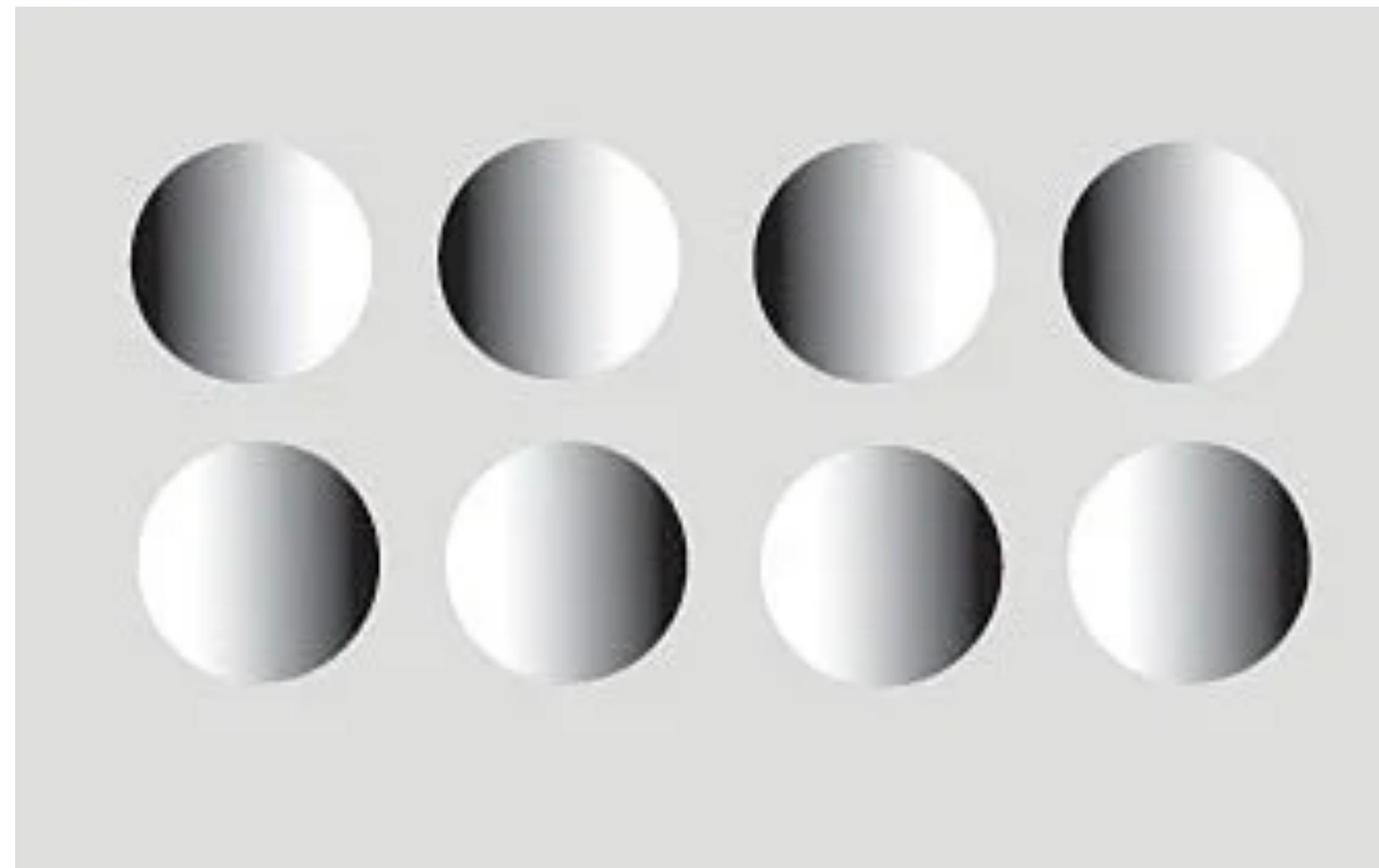
Homework 1 will be released today

Python tutorial – Friday 2/9, 2 - 3 pm, CS 150/151

TAs will cover python basics (e.g. numpy, matplotlib, scipy, skimage)

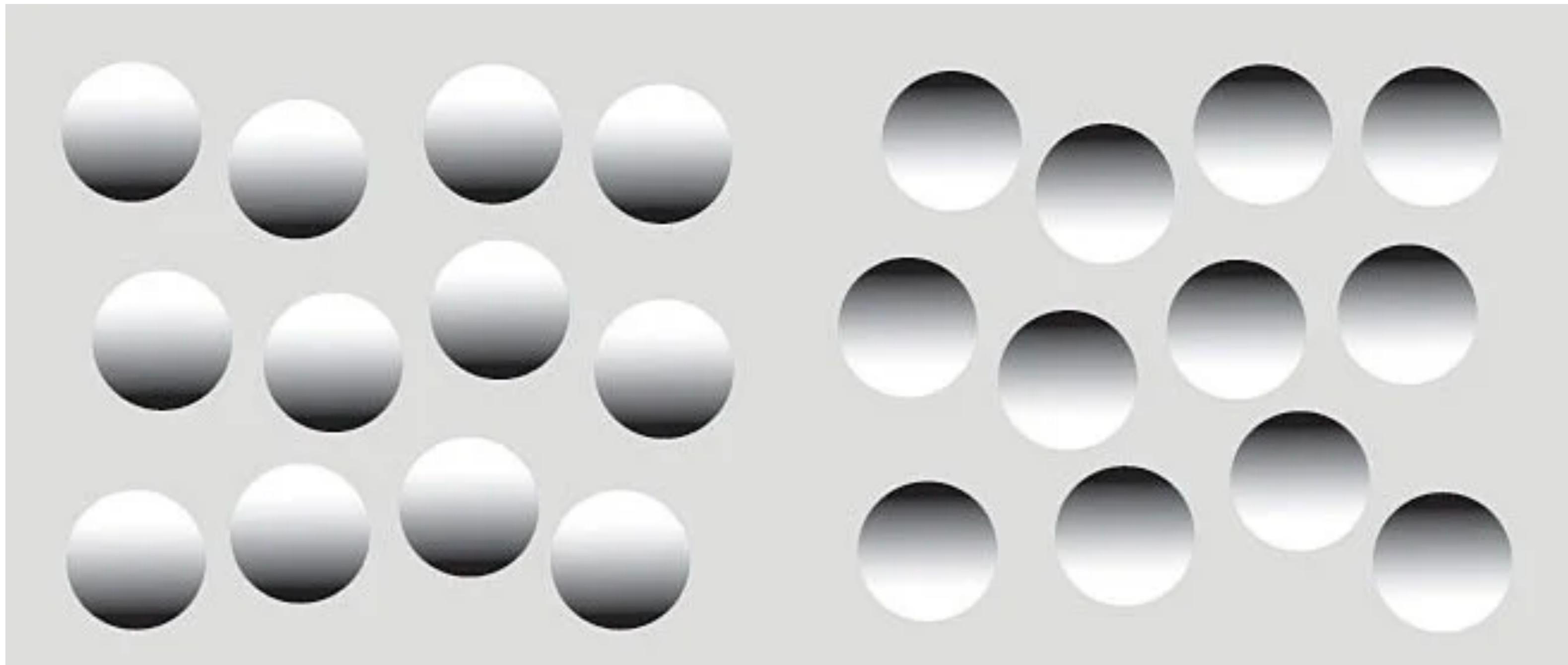
Project Poster Presentation May 10th, 2-4pm, LGRC A112.

Your Perception



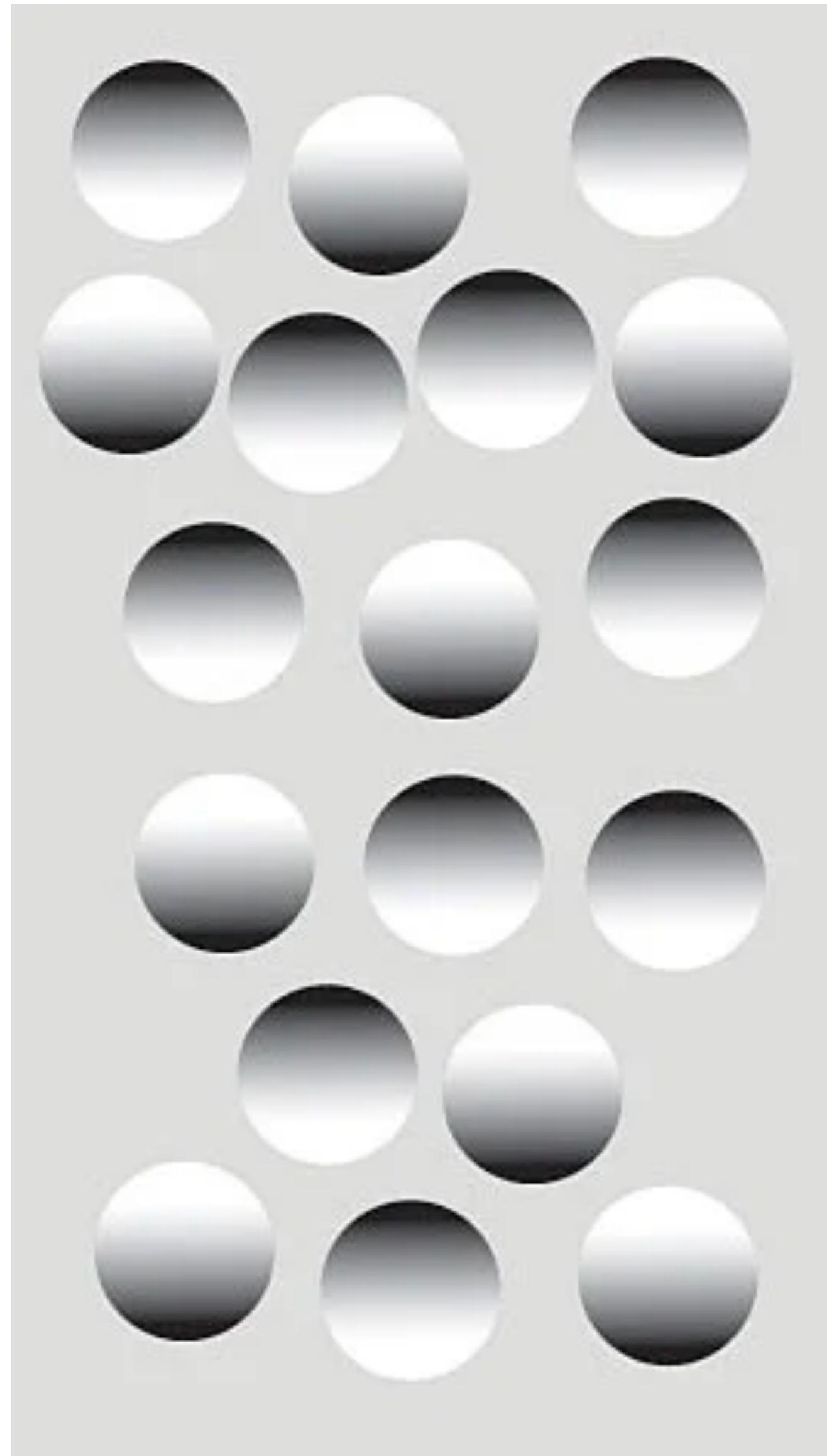
Figures from:
<https://www.scientificamerican.com/article/seeing-is-believing-aug-08/>

Your Perception



Figures from:
<https://www.scientificamerican.com/article/seeing-is-believing-aug-08/>

Your Perception



Figures from:
<https://www.scientificamerican.com/article/seeing-is-believing-aug-08/>

Your Perception



Figures from:
<https://www.scientificamerican.com/article/seeing-is-believing-aug-08/>

Your Perception



<https://macaulaylibrary.org/asset/557939521>

Your Perception



Radiometry

Questions:

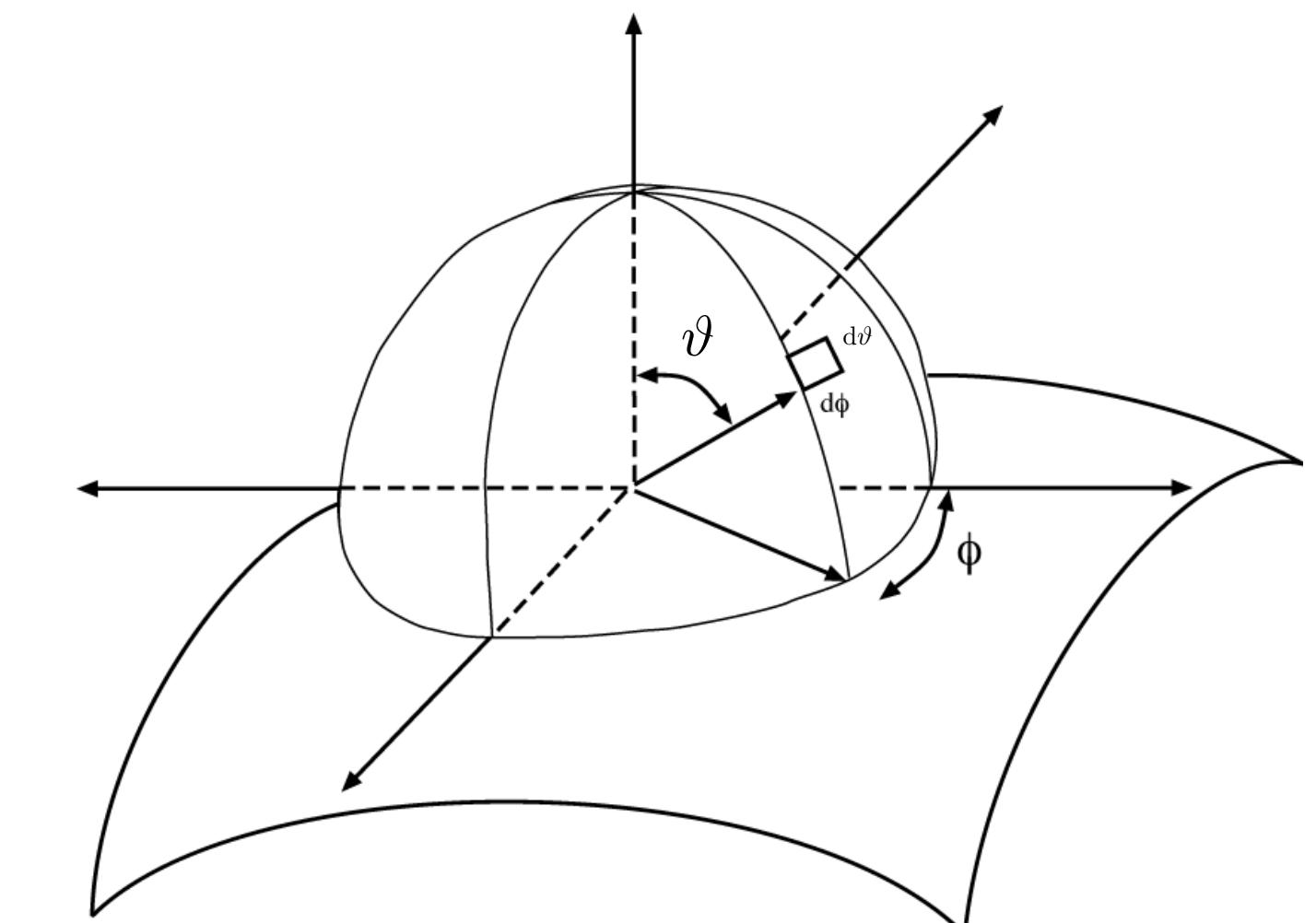
- How “bright” will surfaces be?
- What is “brightness”?
 - ▶ measuring light
 - ▶ interactions between light and surfaces

Core idea – think about light arriving at a surface around any point is a hemisphere of directions

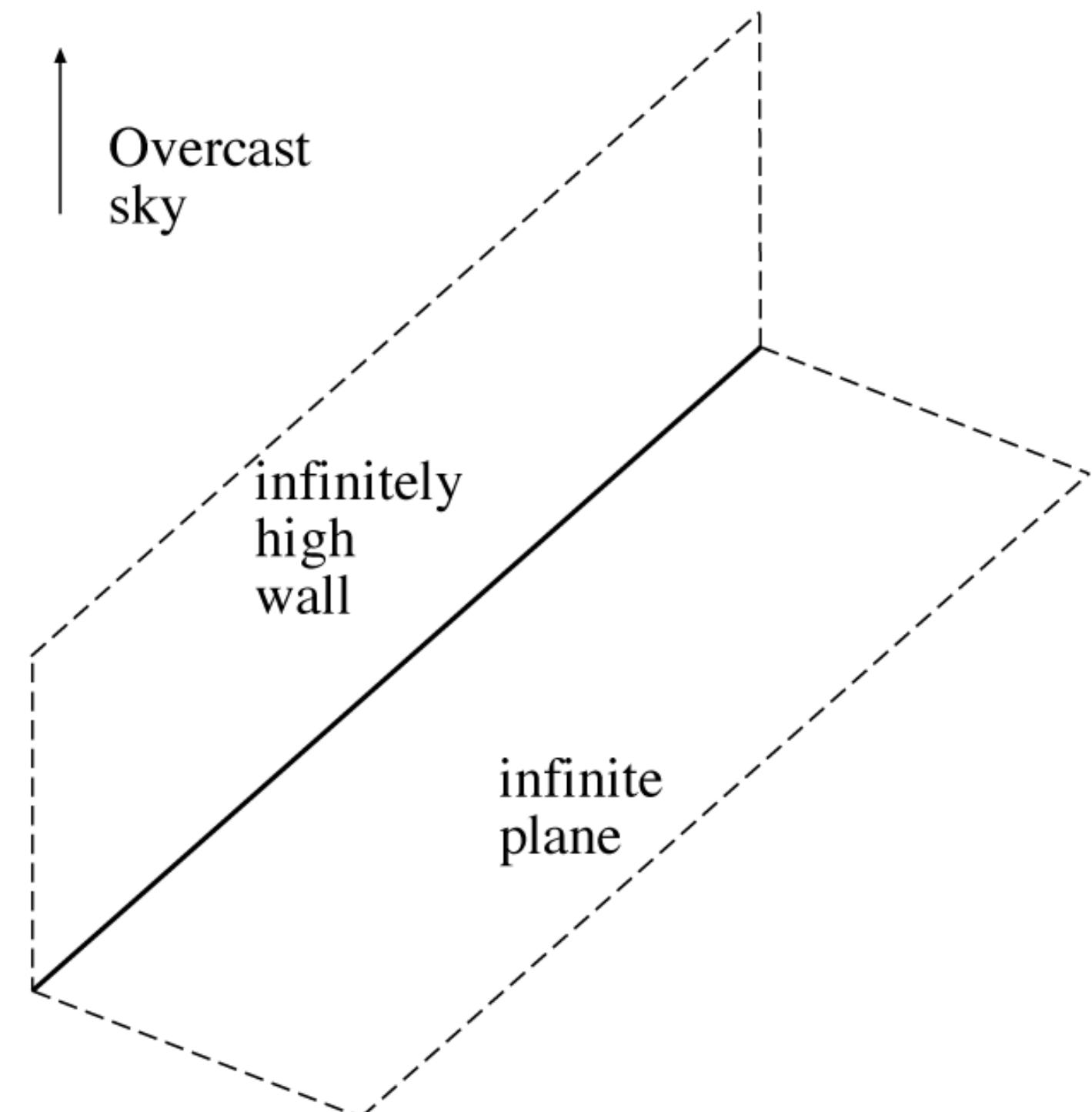
Simplest problems can be dealt with by reasoning about this hemisphere



source: <https://www.umass.edu/>

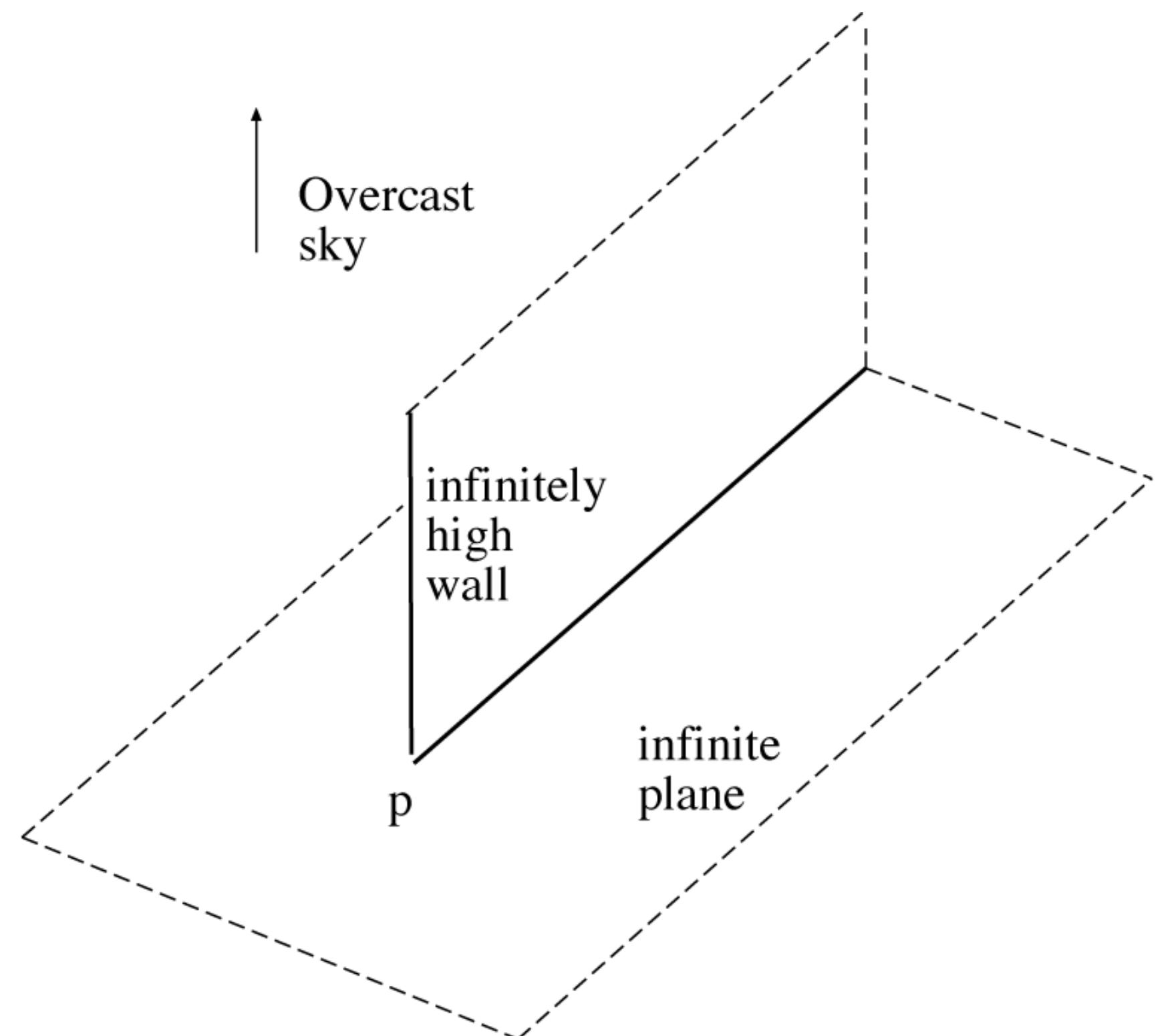


Lambert's wall



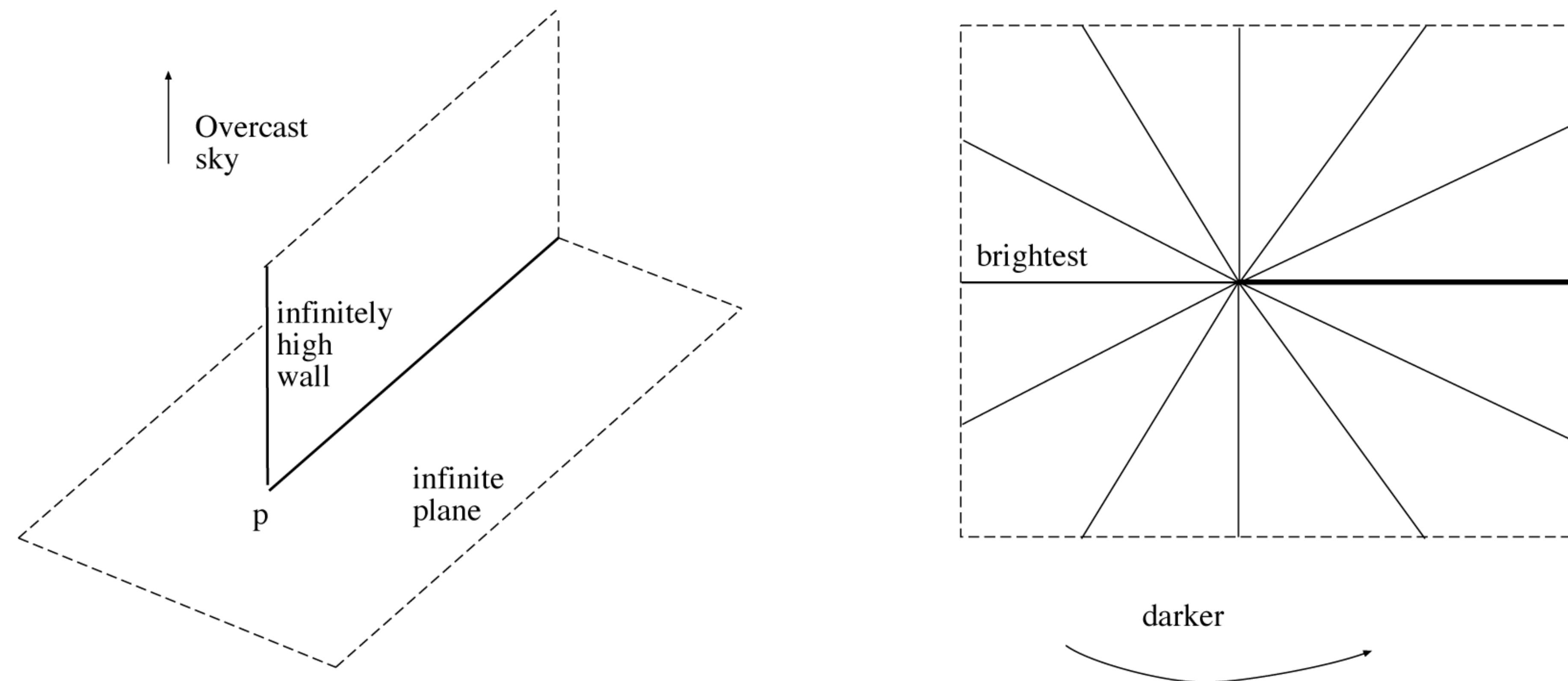
What is the distribution
of brightness on the ground?

More complex wall



Computer Vision - A Modern Approach
Set: Radiometry
Slides by D.A. Forsyth

More complex wall



Computer Vision - A Modern Approach
Set: Radiometry
Slides by D.A. Forsyth

Light at surfaces

What happens when a light ray hits a point on an object?

Some of the light gets **absorbed**

- converted to other forms of energy (e.g., heat)

Some gets **transmitted** through the object

- possibly bent, through refraction
- or scattered inside the object (subsurface scattering)

Some gets **reflected**

- possibly in multiple directions at once

Really complicated things can happen

- fluorescence

Fluorescence

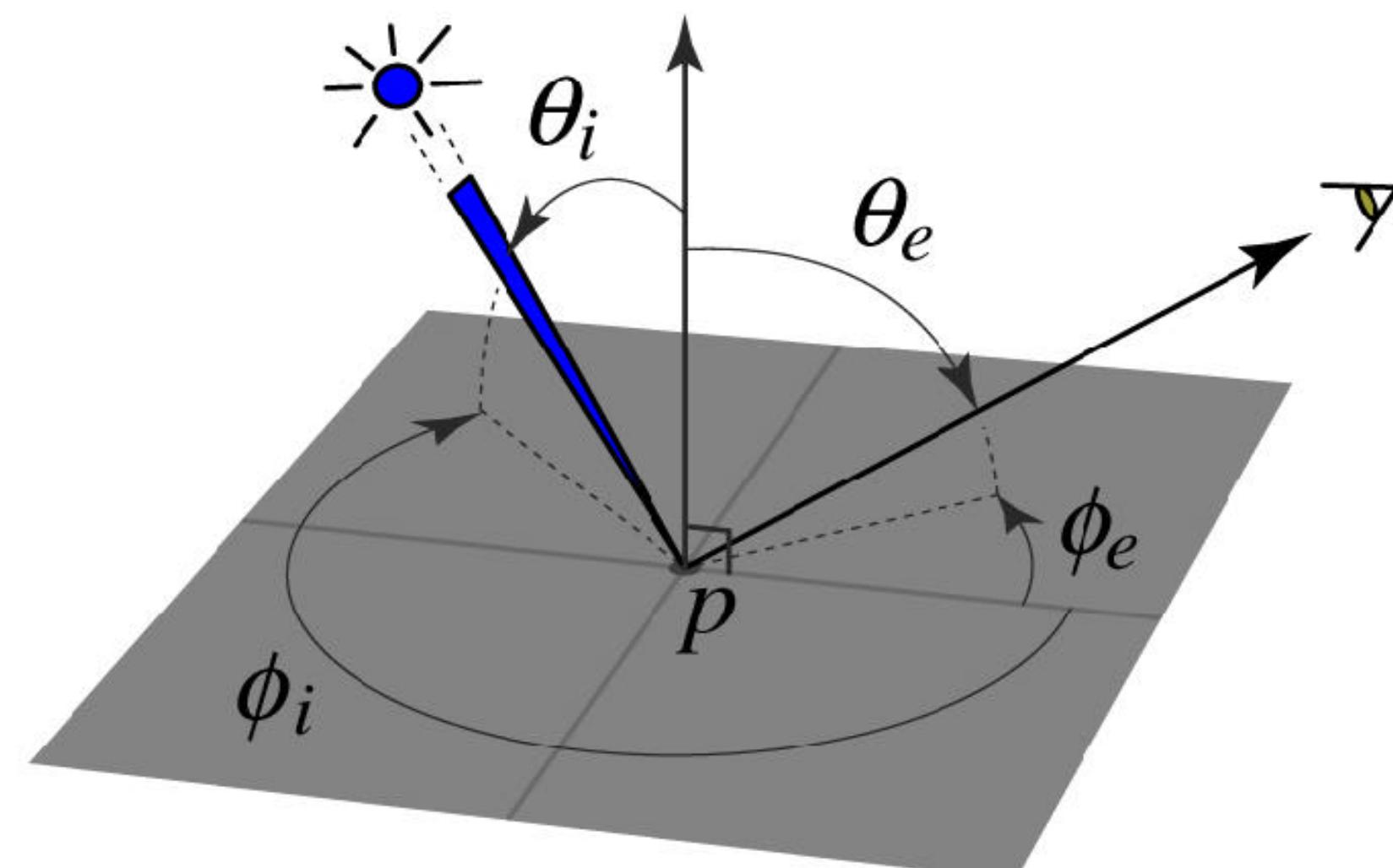


Modeling surface reflectance

Bidirectional reflectance distribution function (BRDF)

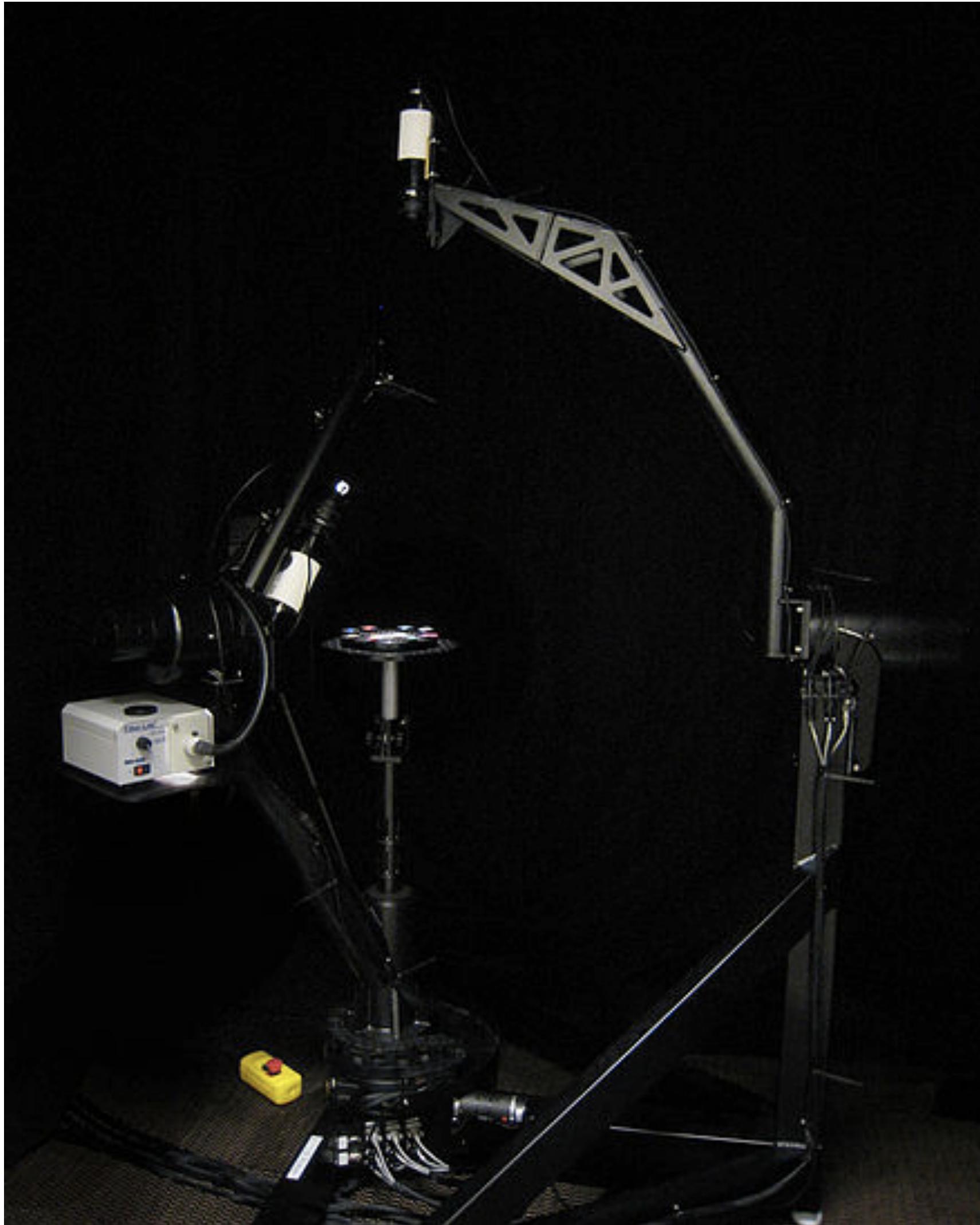
- How bright a surface appears when viewed from one direction when light falls on it from another
- **Definition:** ratio of the radiance in the emitted direction to irradiance in the incident direction

$$\rho(p, \theta_i, \phi_i, \theta_e, \phi_e) = \frac{L(p, \theta_e, \phi_e)}{L(p, \theta_i, \phi_i) \cos \theta_i d\omega}$$



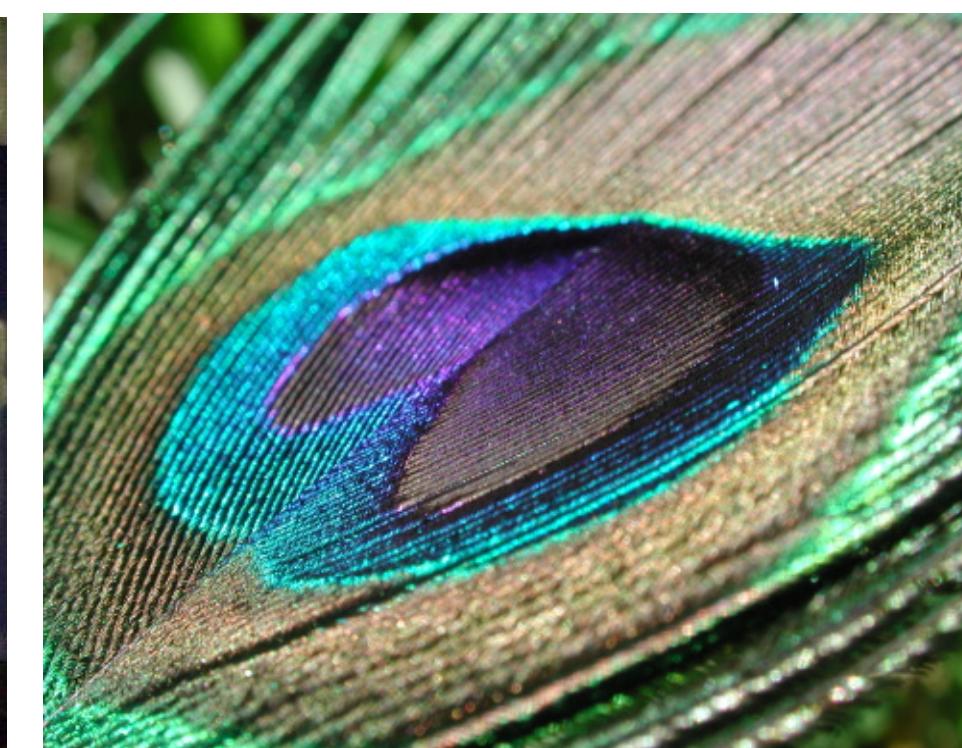
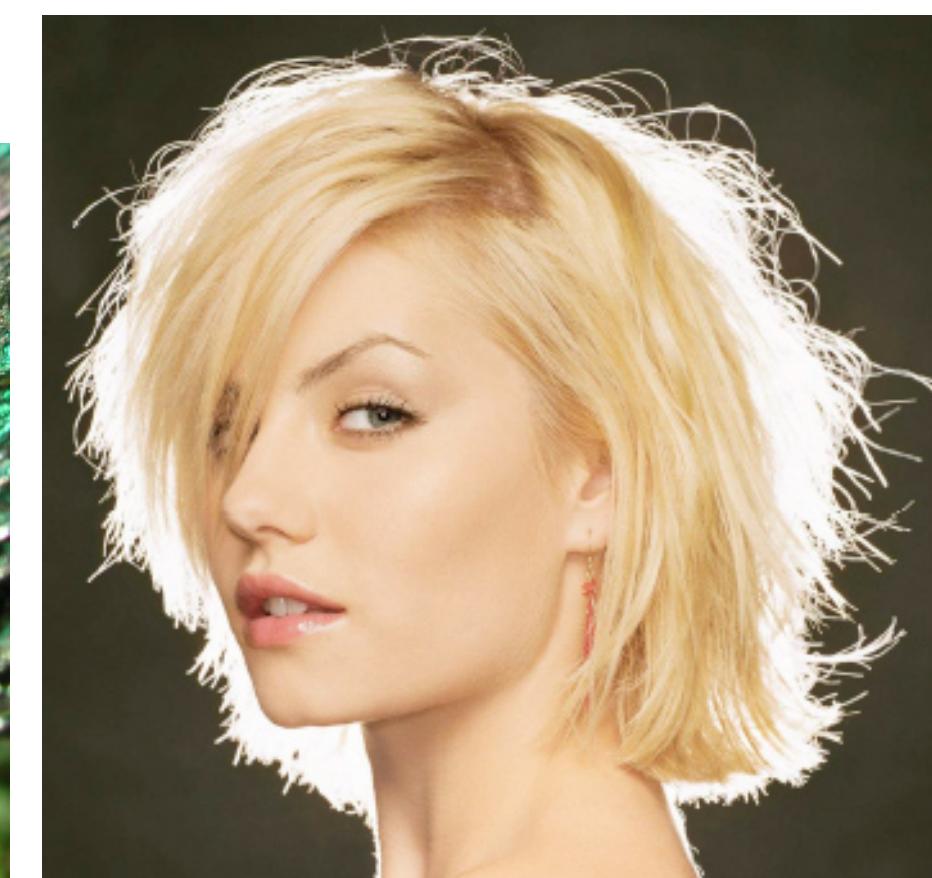
Simplifying assumptions
locality, no fluorescence,
does not generate light

Gonioreflectometer



The University of Virginia spherical gantry, an example of a modern image-based gonioreflectometer

BRDFs can be incredibly complicated...



Suppressing the angles in the BRDF

BRDF is a very general notion

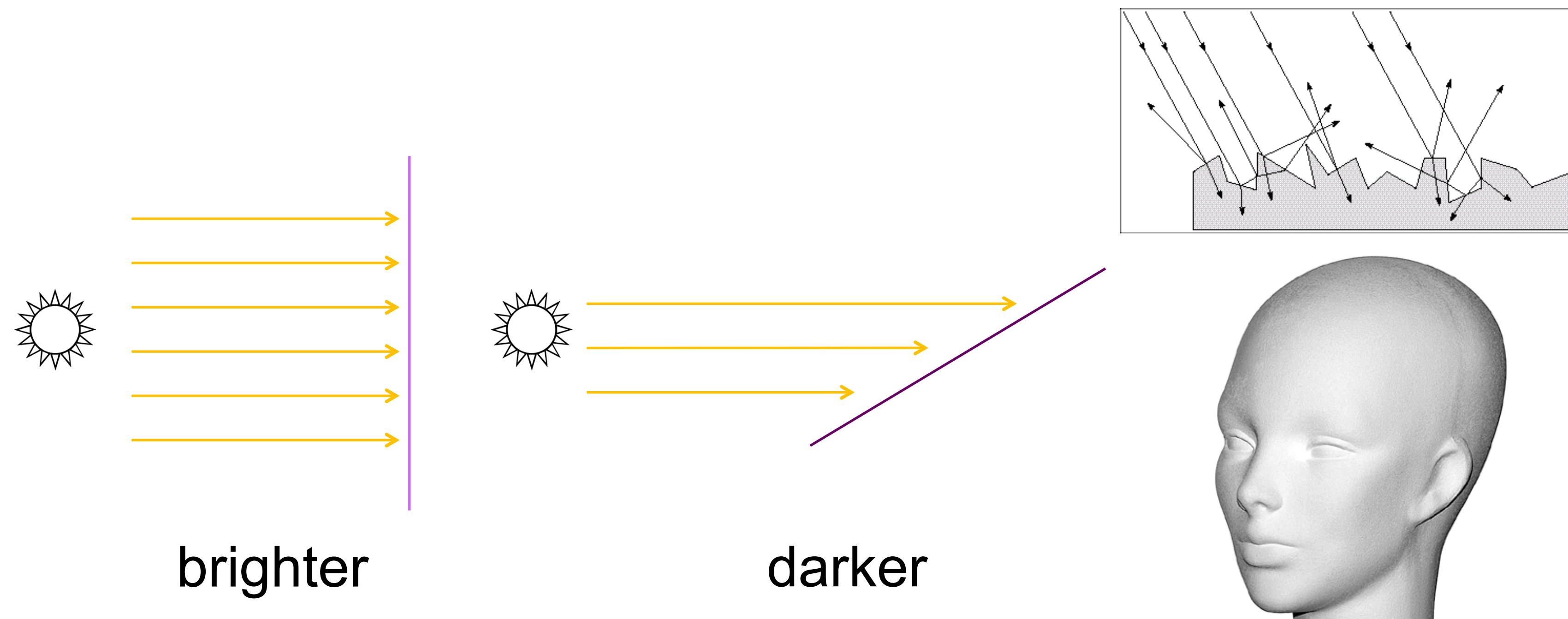
- some surfaces need it (underside of a CD; tiger eye; etc)
- very hard to measure — illuminate from one direction, view from another, repeat
- very unstable — minor surface damage can change the BRDF
 - e.g. ridges of oil left by contact with the skin can act as lenses
- However, for many surfaces, light leaving the surface is largely independent of exit angle
 - surface roughness is one source of this property

Special cases: Diffuse reflection

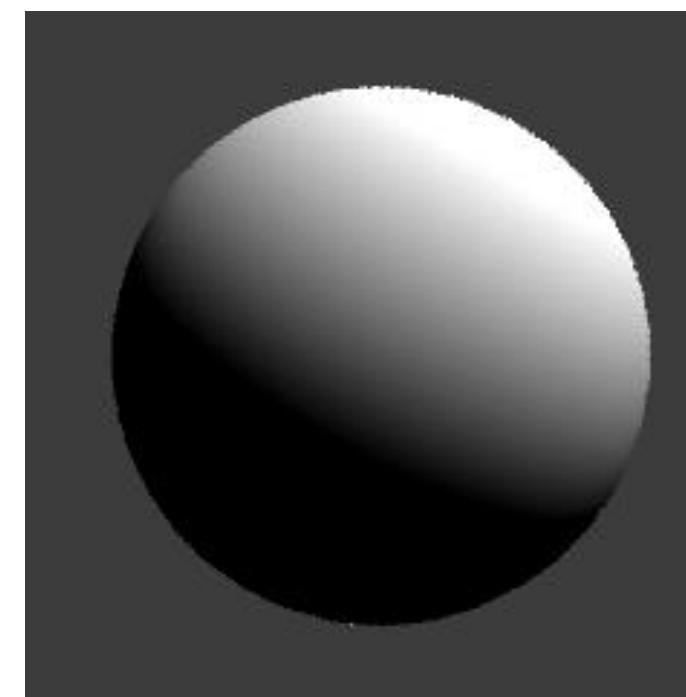
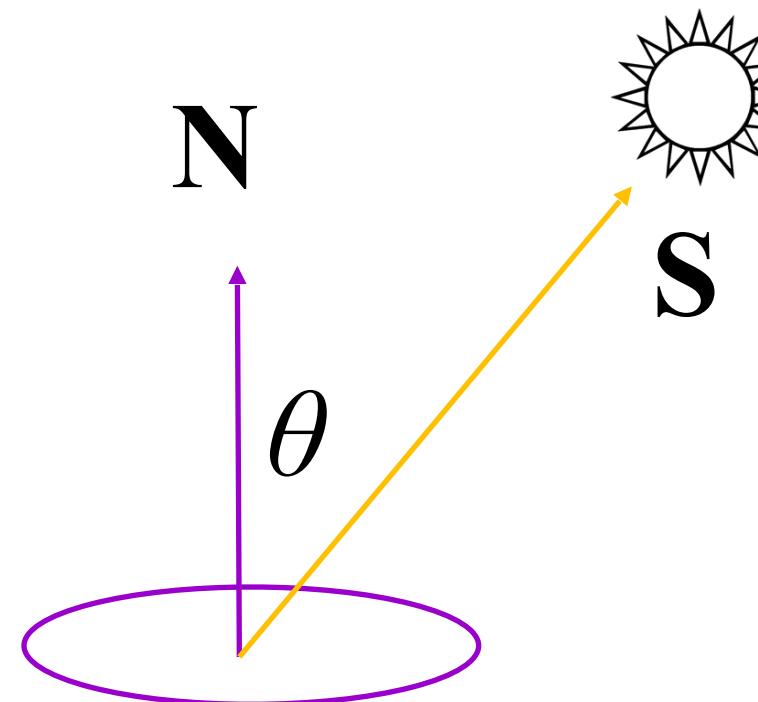
Light is reflected equally in all directions

- Dull, matte surfaces like chalk or cotton cloth
- Microfacets scatter incoming light randomly
- Effect is that light is reflected (approximately) equally in all directions

Brightness of the surface depends on the incidence of illumination



Diffuse reflection: Lambert's law



$$\begin{aligned}B &= \rho (\mathbf{N} \cdot \mathbf{S}) \\&= \rho \|\mathbf{S}\| \cos \theta\end{aligned}$$

B : radiosity (total power leaving the surface per unit area)

ρ : albedo (fraction of incident irradiance reflected by the surface)

N : unit normal

S : source vector (magnitude proportional to intensity of the source)

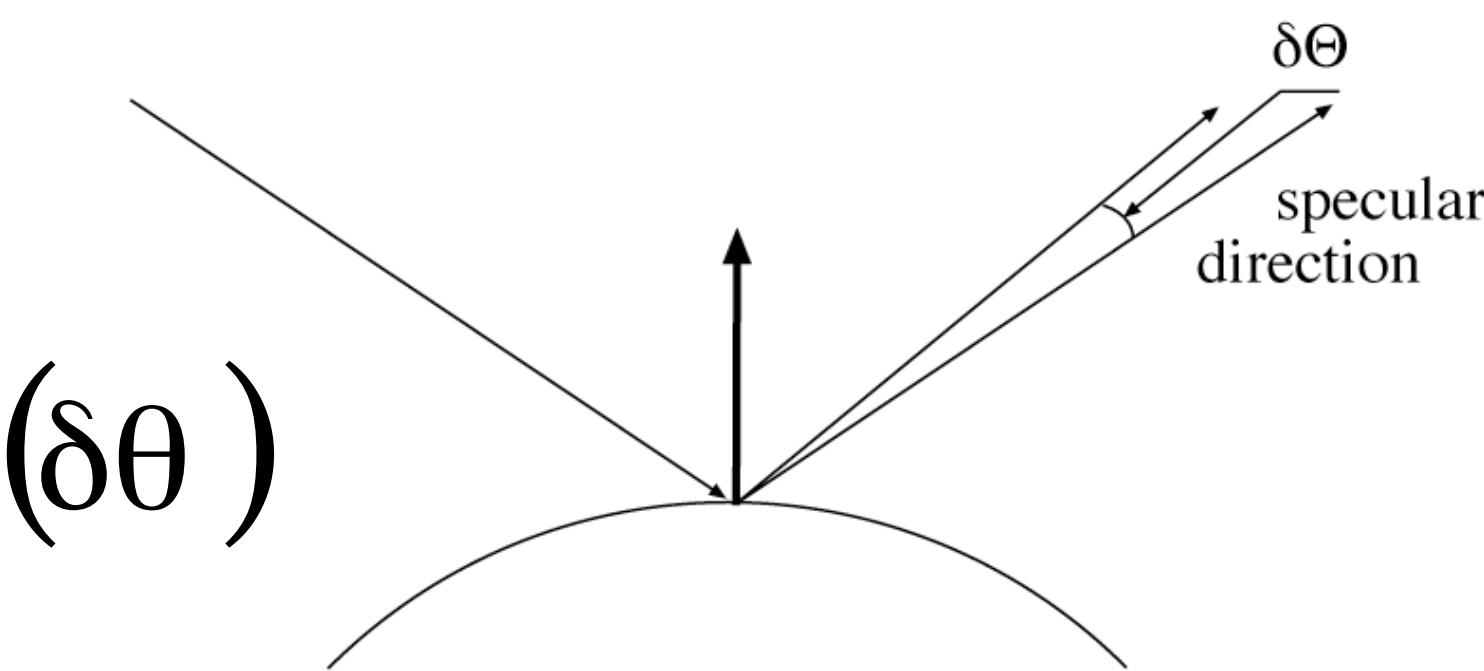
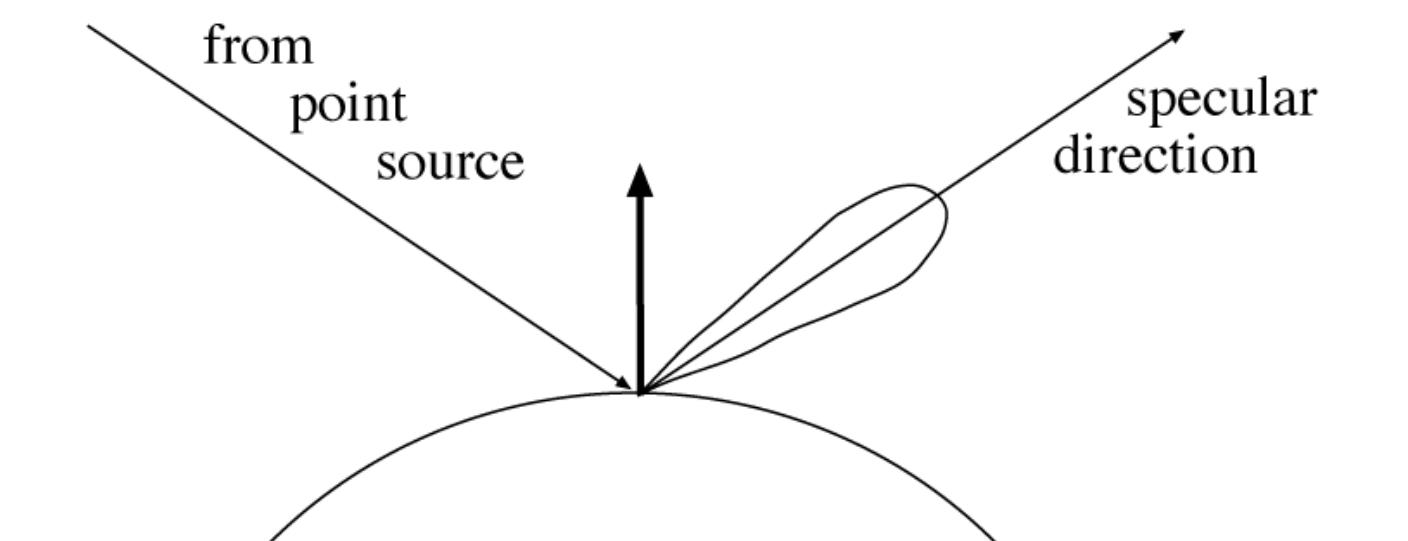
Specular reflection

Radiation arriving along a source direction leaves along the specular direction (source direction reflected about normal)

Some fraction is absorbed, some reflected

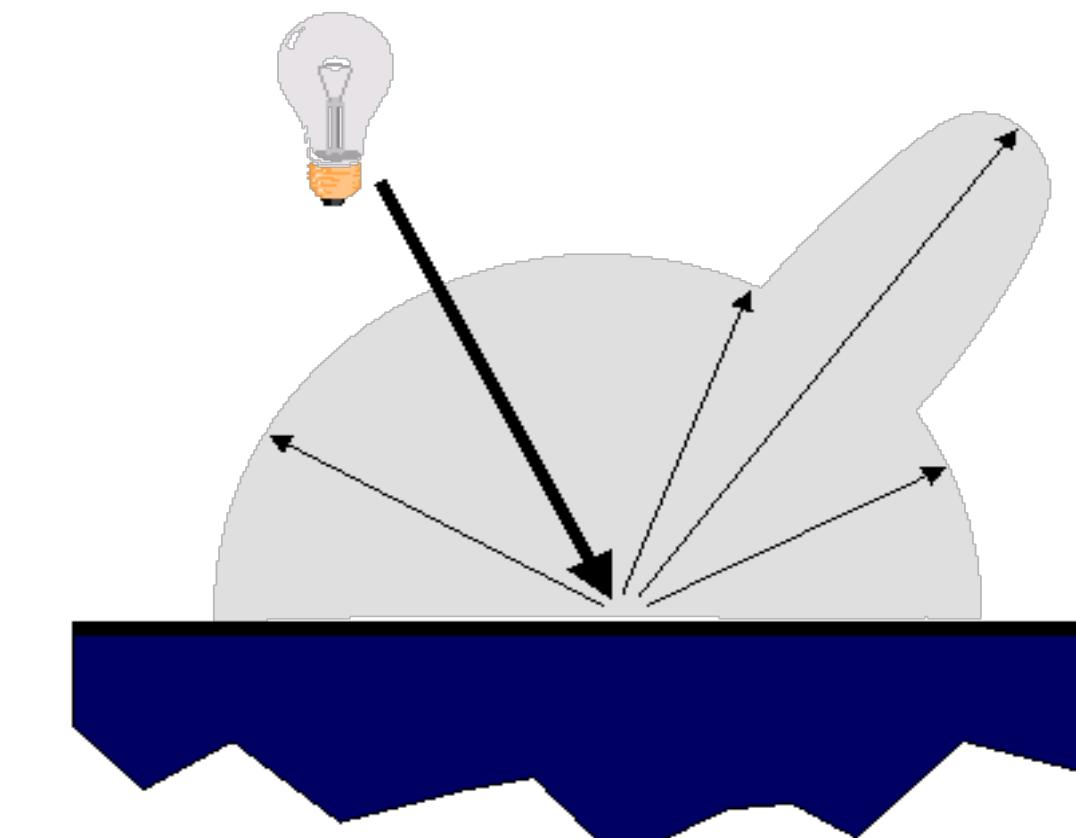
On real surfaces, energy usually goes into a lobe of directions

Phong model: reflected energy falls off with $\cos^n(\delta\theta)$

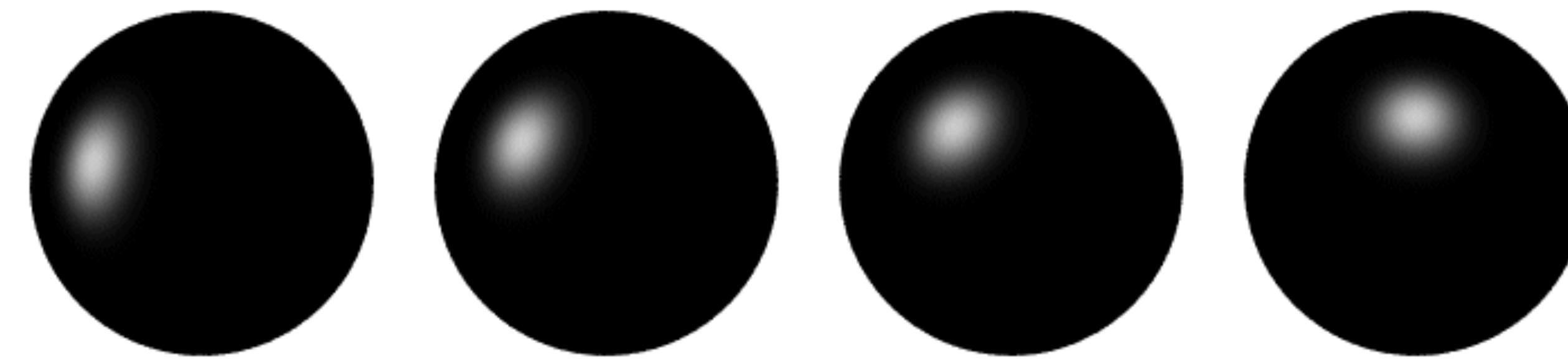


Lambertian + specular model: sum of diffuse and specular term

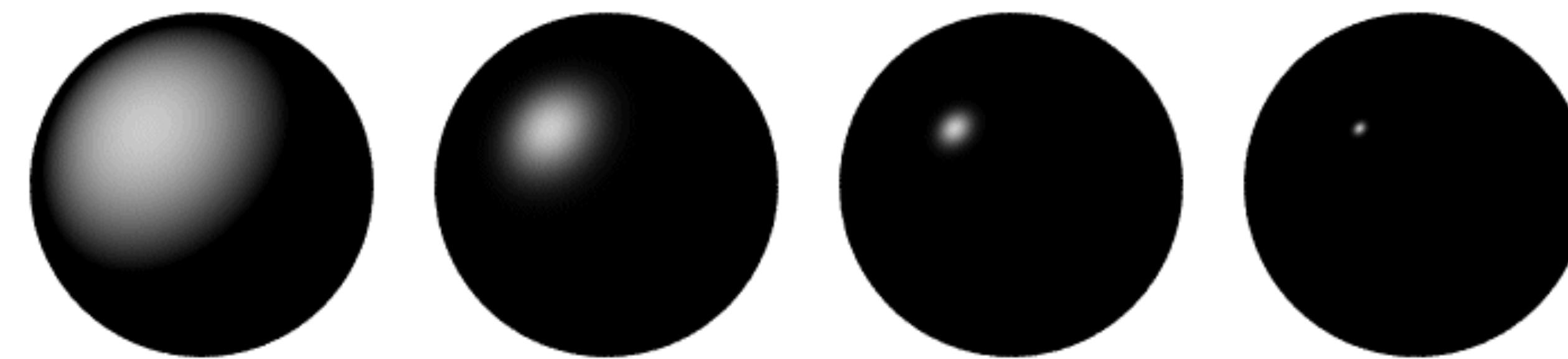
- a reasonable approximation to lot of surfaces we see



Specular reflection



Moving the light source



$$\cos^n(\delta\theta)$$

Changing the exponent

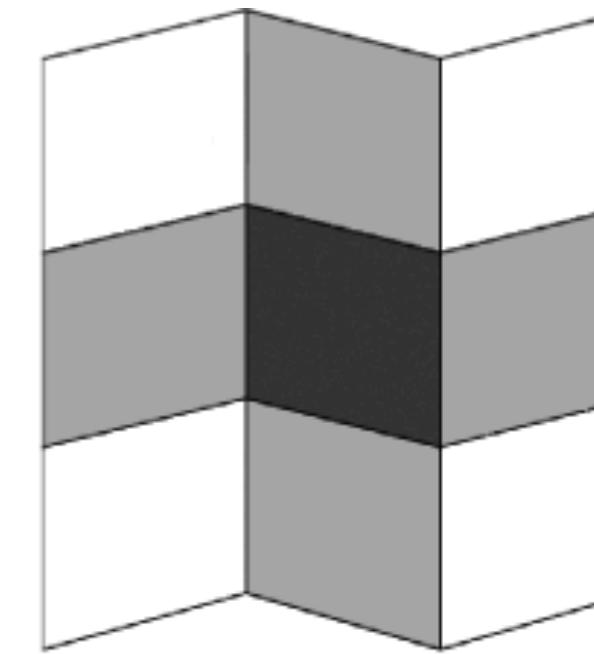
Photometric stereo

Can we reconstruct the shape of an object based on shading cues?

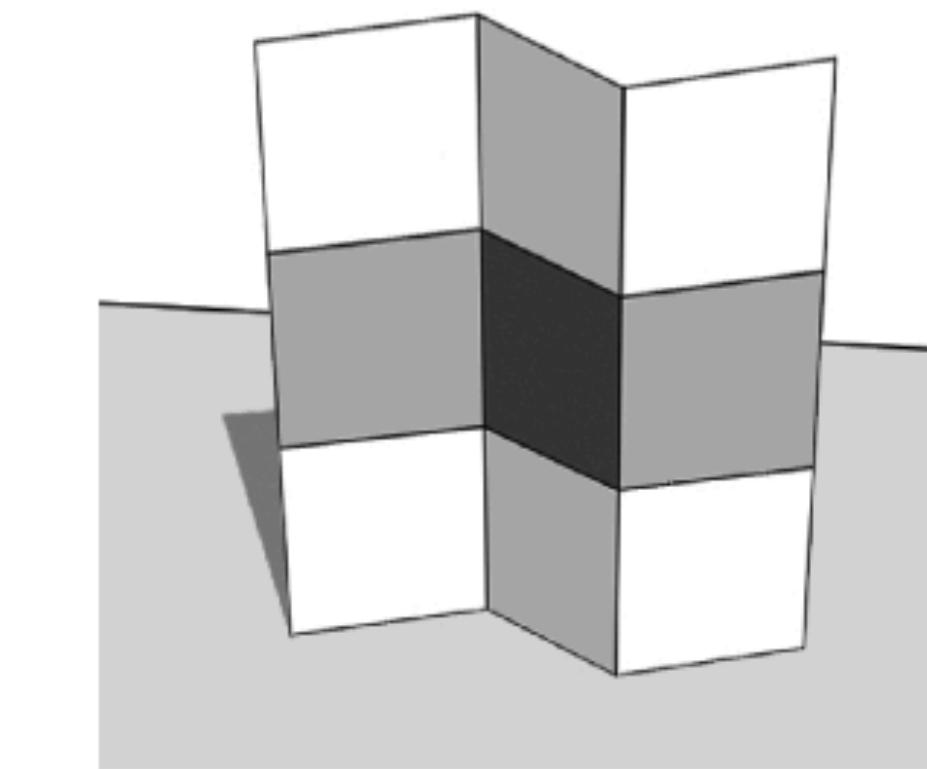


Luca della Robbia,
Cantoria, 1438

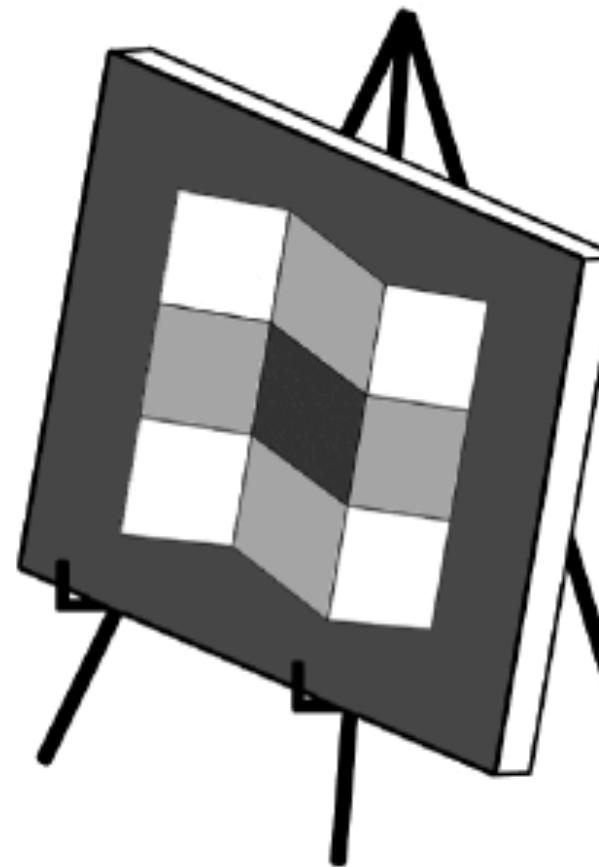
Ambiguities in shape and shading



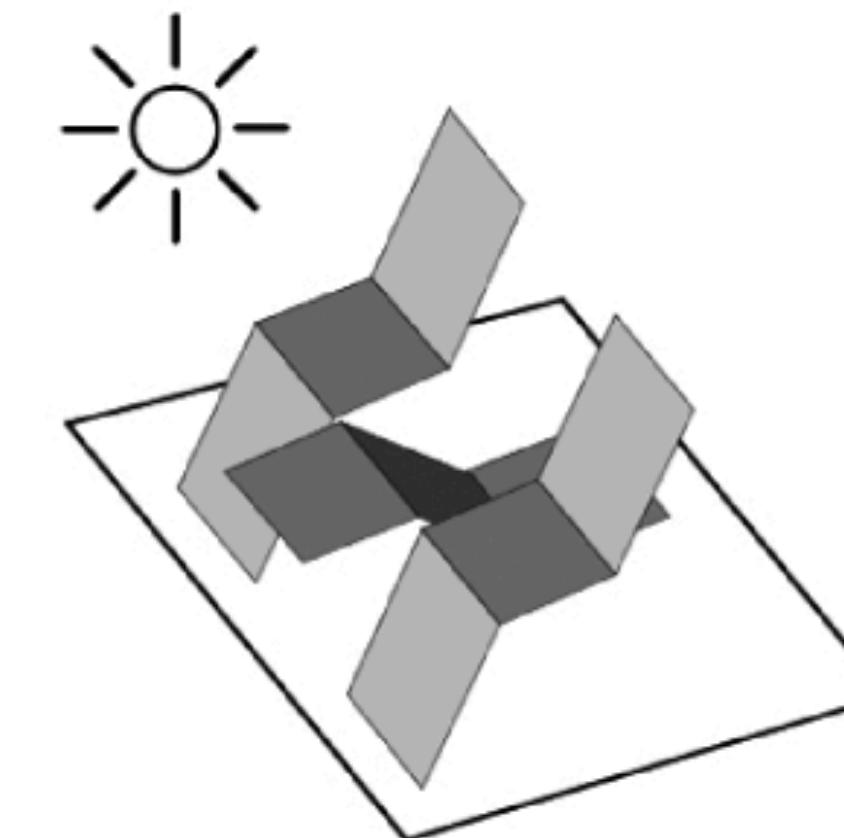
(a) an image



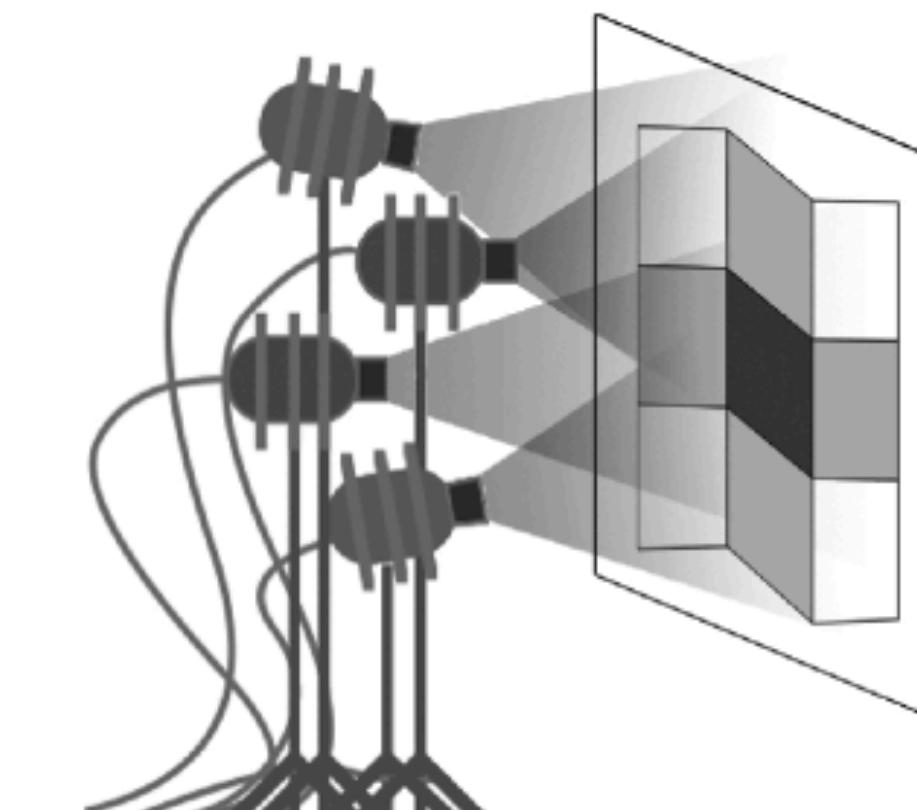
(b) a likely explanation



(c) painter's explanation



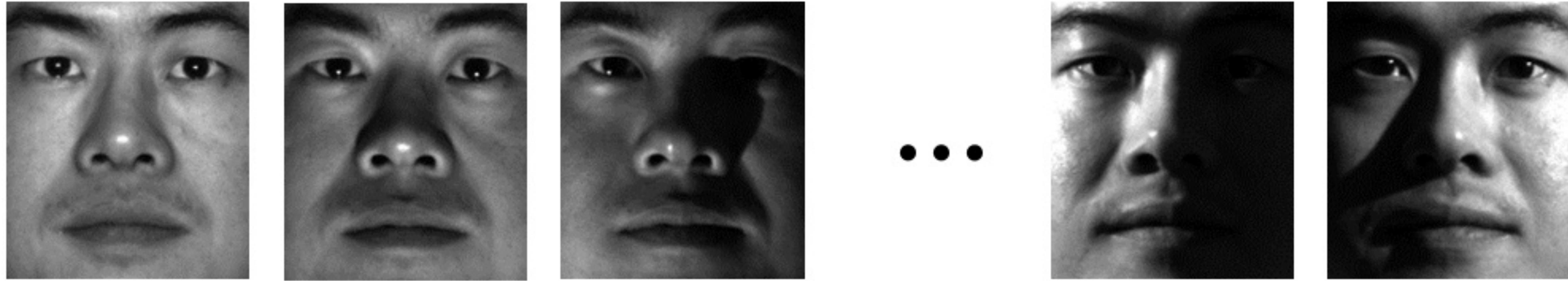
(d) sculptor's explanation



(e) gaffer's explanation

The workshop metaphor from Adelson and Pentland, 1996

Photometric stereo

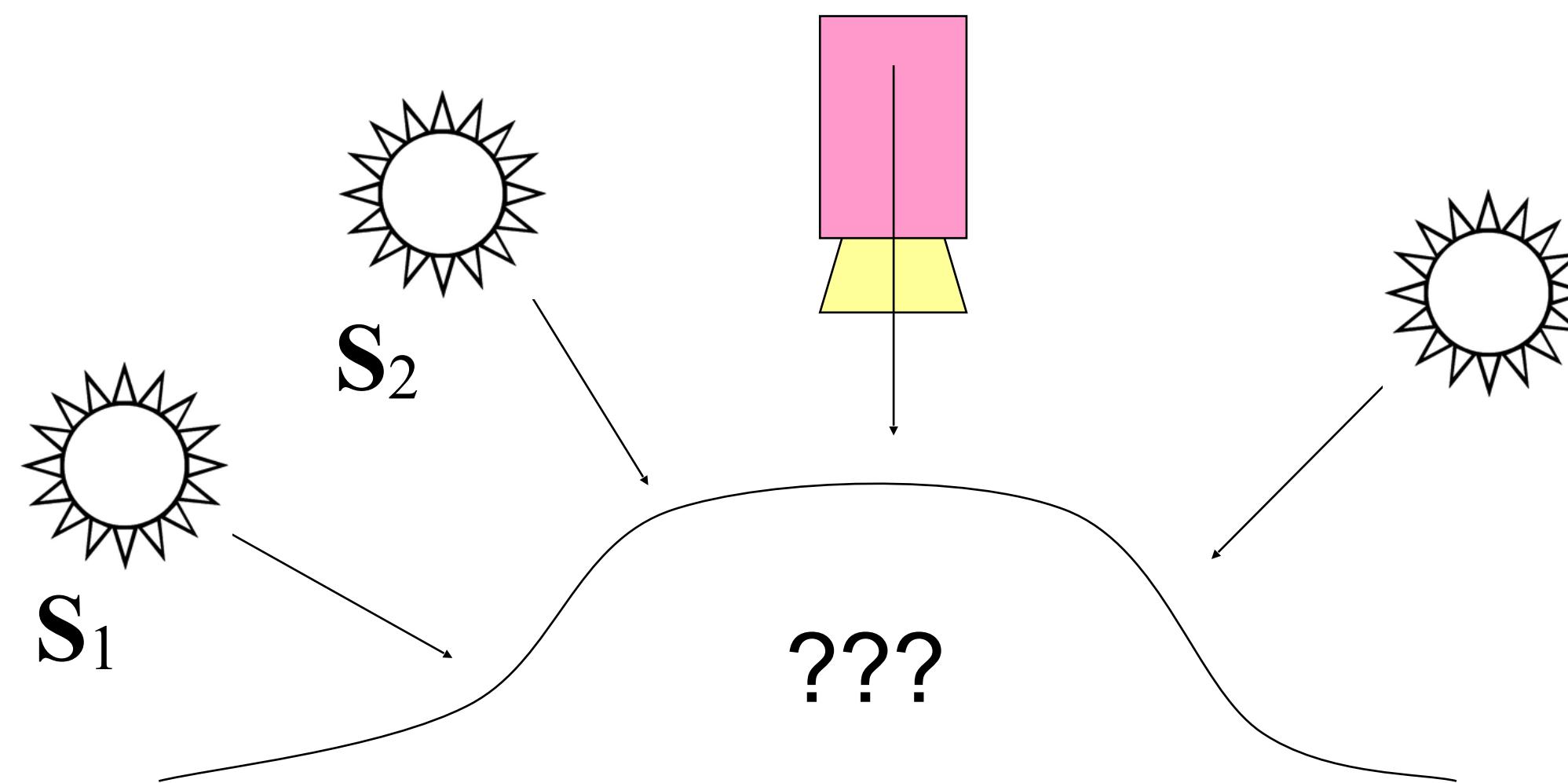


S_1

S_2

S_3

S_n

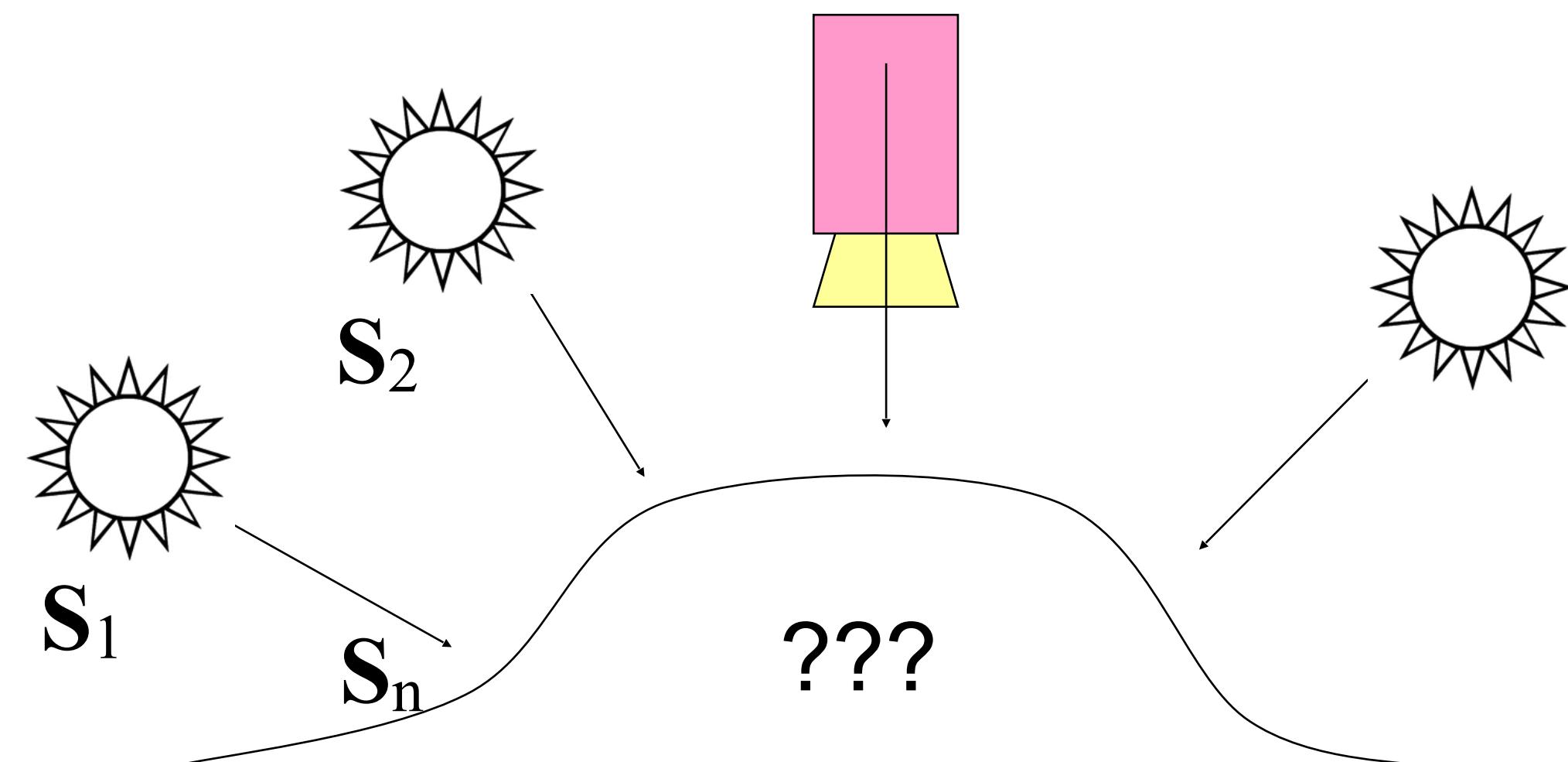


Photometric stereo

Assume:

- A Lambertian object
- A local shading model, i.e., each point on a surface receives light only from sources visible at that point
- A set of known light source directions
- A set of pictures of an object, obtained in exactly the same camera and object configuration but using different sources
- Orthographic projection

Goal: reconstruct object shape and albedo



Surface model: Monge patch

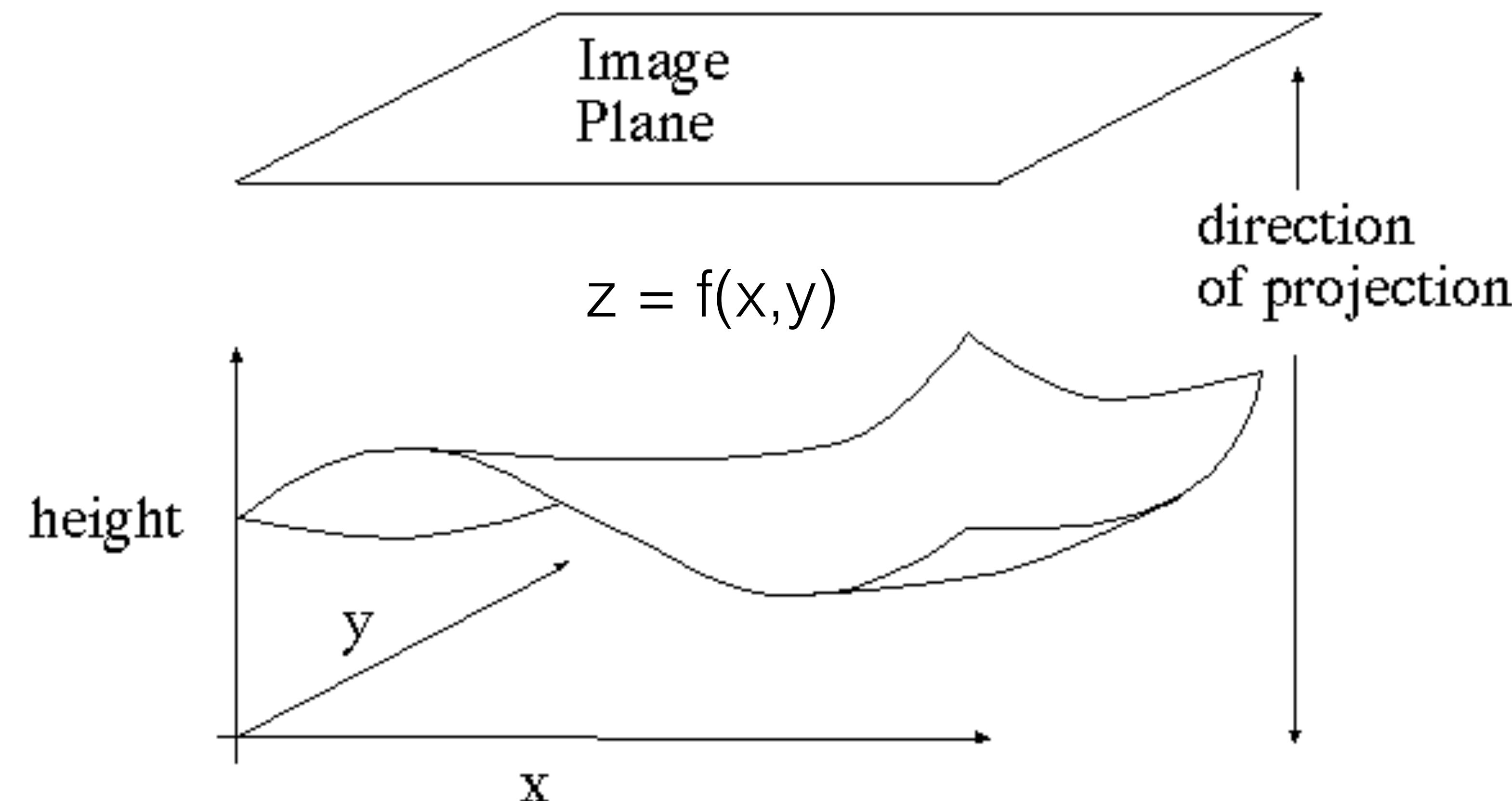


Image model

Known: source vectors \mathbf{S}_j and pixel values $I_j(x,y)$

Unknown: surface normal $\mathbf{N}(x,y)$ and albedo $\rho(x,y)$

Assume that the response function of the camera is a linear scaling by a factor of k

Lambert's law:

$$\begin{aligned} I_j(x,y) &= k \rho(x,y) (\mathbf{N}(x,y) \cdot \mathbf{S}_j) \\ &= (\rho(x,y) \mathbf{N}(x,y)) \cdot (k \mathbf{S}_j) \\ &= \mathbf{g}(x,y) \cdot \mathbf{V}_j \end{aligned}$$

Least squares problem

- ◆ For each pixel, set up a linear system:

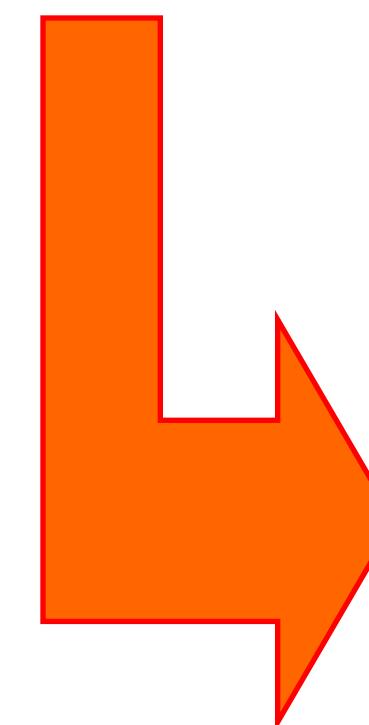
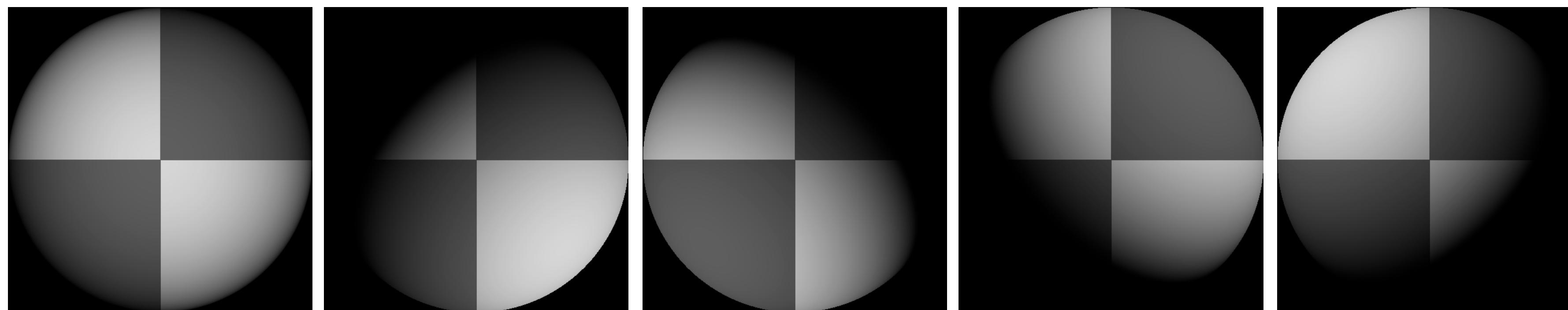
$$\begin{bmatrix} I_1(x, y) \\ I_2(x, y) \\ \vdots \\ I_n(x, y) \end{bmatrix}_{\substack{(n \times 1) \\ \text{known}}} = \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \\ \vdots \\ \mathbf{V}_n^T \end{bmatrix}_{\substack{(n \times 3) \\ \text{known}}} \mathbf{g}(x, y)_{\substack{(3 \times 1) \\ \text{unknown}}}$$

Obtain least-squares solution for $\mathbf{g}(x, y)$ defined as $\mathbf{N}(x, y) \rho(x, y)$

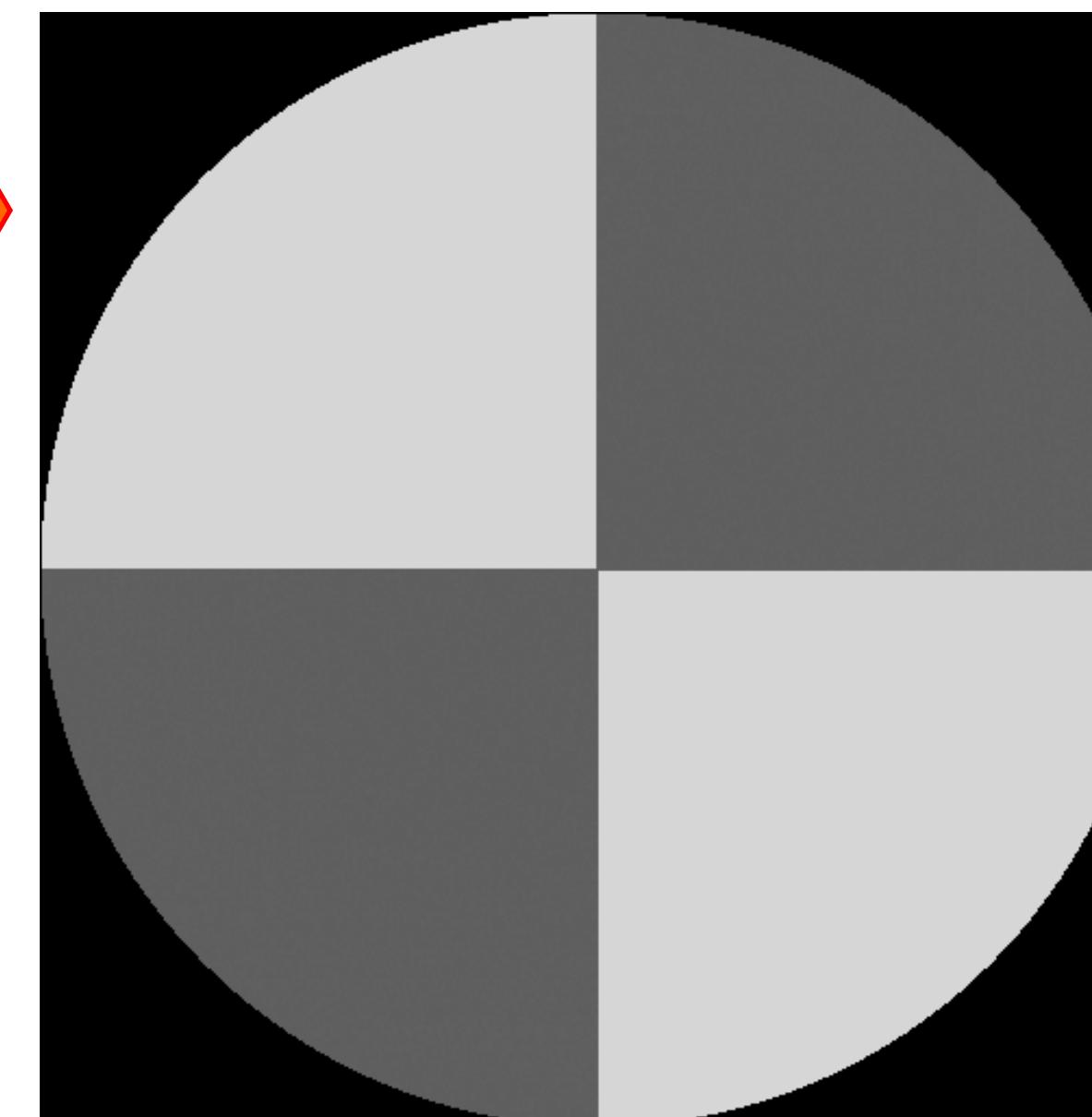
Since $\mathbf{N}(x, y)$ is the unit normal, $\rho(x, y)$ is given by the magnitude of $\mathbf{g}(x, y)$

Finally, $\mathbf{N}(x, y) = \mathbf{g}(x, y) / \rho(x, y)$

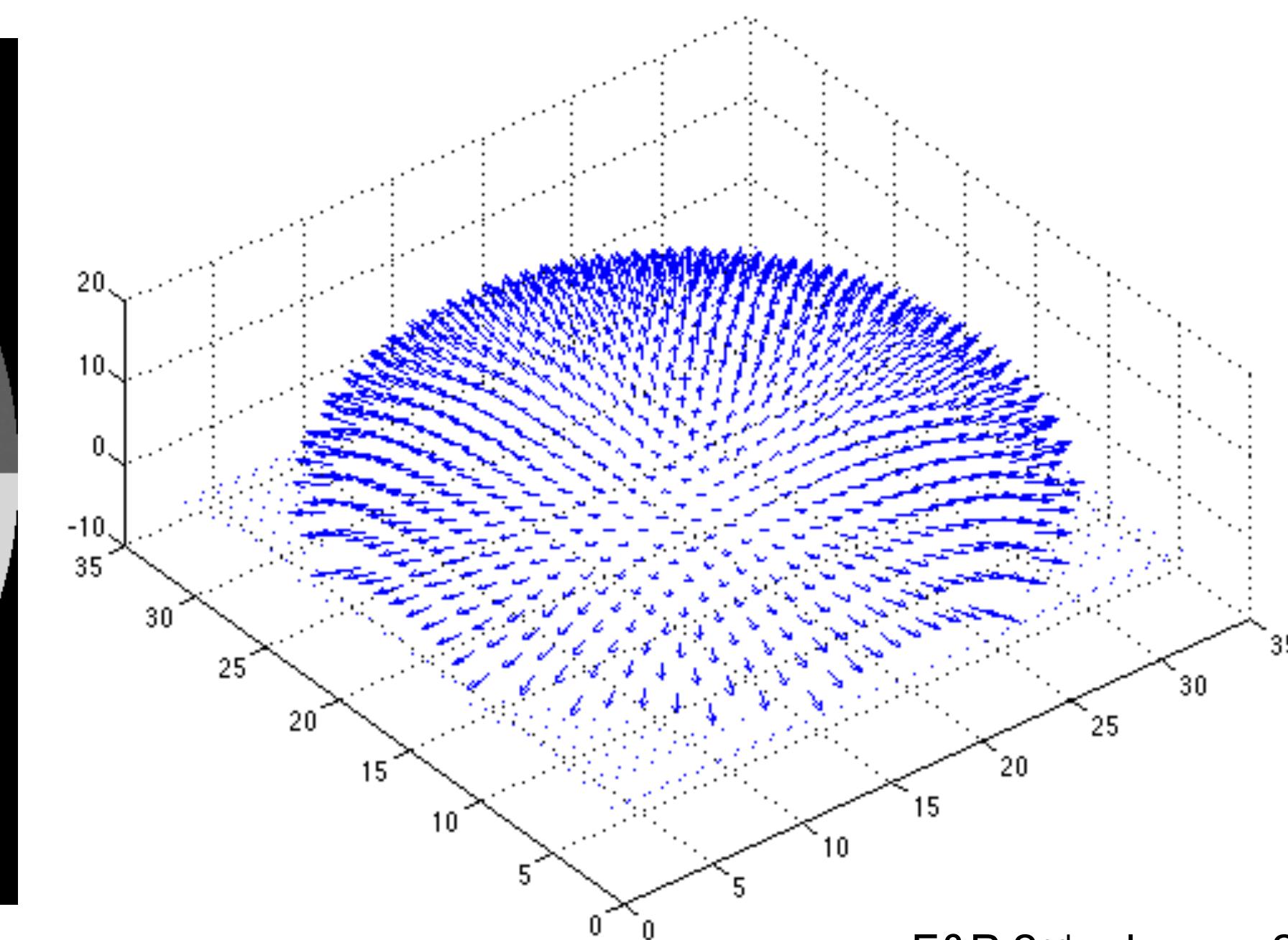
Example



Recovered albedo



Recovered normal field



F&P 2nd ed., sec. 2.2.4

Recovering a surface from normals

Recall the surface is written as

$$(x, y, f(x, y))$$

This means the normal has the form:

$$\mathbf{N}(x, y) = \frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \begin{pmatrix} f_x \\ f_y \\ 1 \end{pmatrix}$$

If we write the estimated vector g as

$$\mathbf{g}(x, y) = \begin{pmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{pmatrix}$$

Then we obtain values for the partial derivatives of the surface:

$$f_x(x, y) = g_1(x, y) / g_3(x, y)$$

$$f_y(x, y) = g_2(x, y) / g_3(x, y)$$

Recovering a surface from normals

Integrability: for the surface f to exist, the mixed second partial derivatives must be equal:

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \quad \text{i.e.,}$$

$$\frac{\partial}{\partial y} \left(g_1(x, y) / g_3(x, y) \right) =$$

$$\frac{\partial}{\partial x} \left(g_2(x, y) / g_3(x, y) \right)$$

In practice, they should be similar

We can now recover the surface height at any point by integration along some path

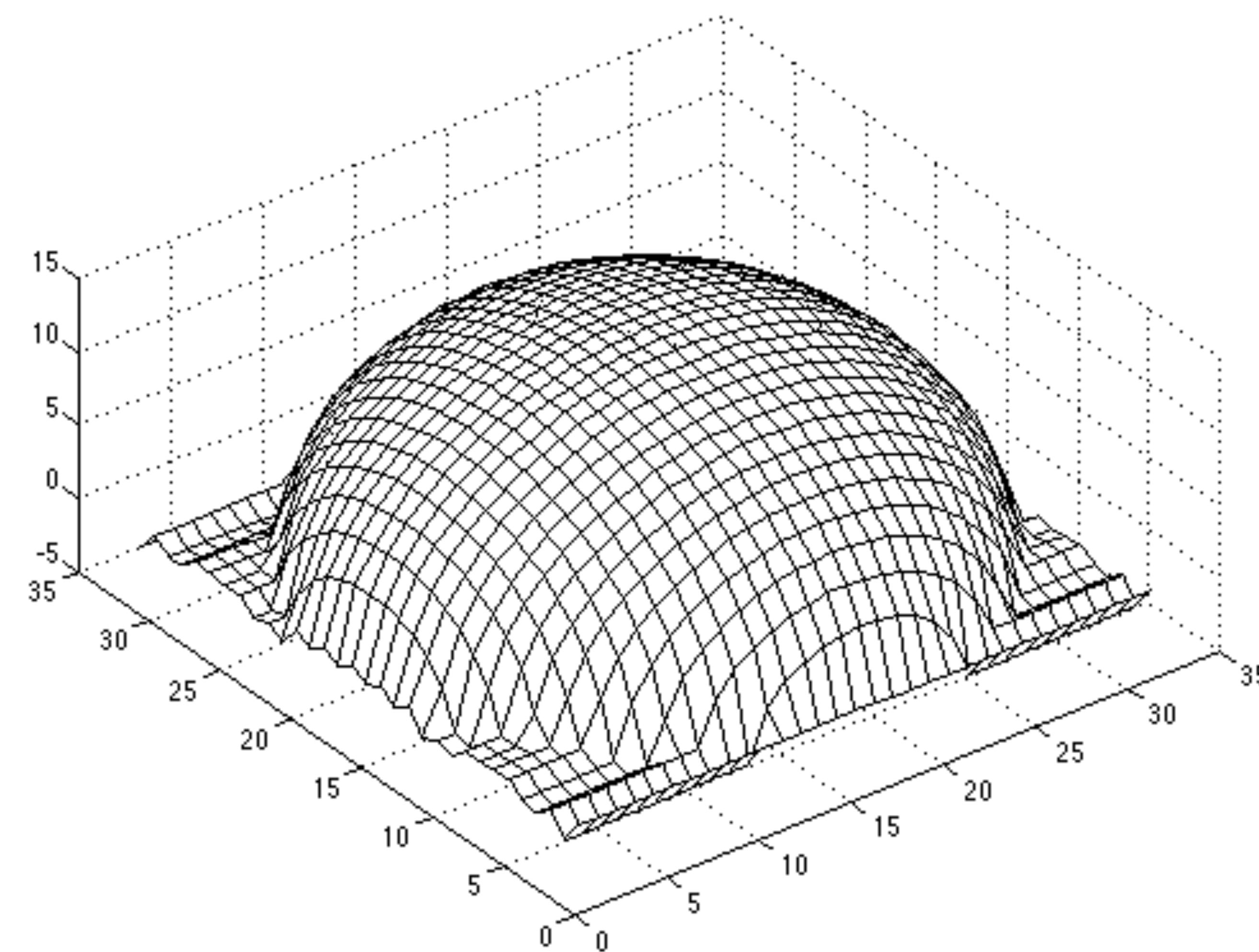
$$f(x, y) = \oint_C \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) \cdot d\mathbf{l} + c$$

For example:

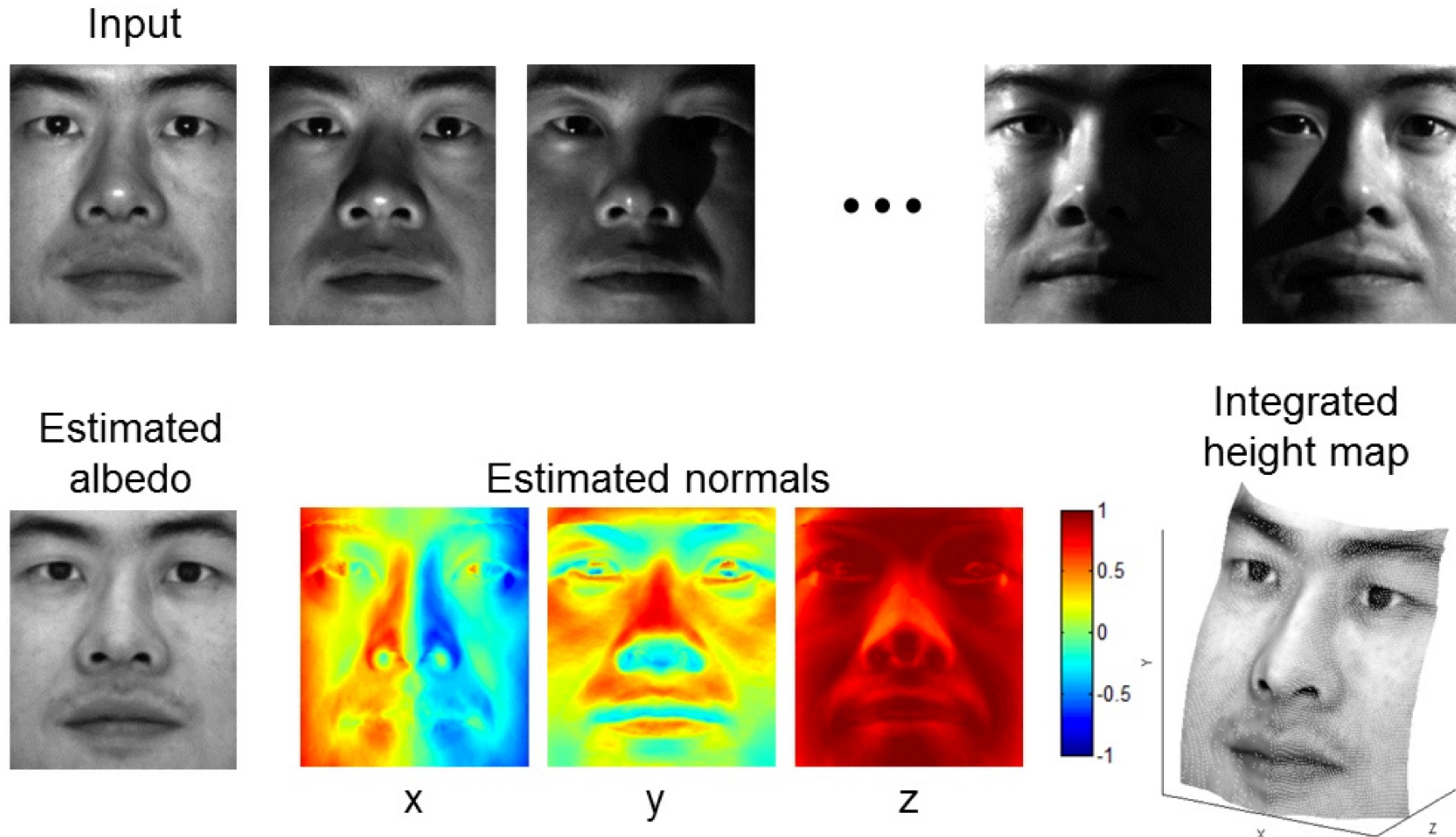
$$f(u, v) = \int_0^v \frac{\partial f}{\partial y}(0, y) dy + \int_0^u \frac{\partial f}{\partial x}(x, v) dx + c$$

For robustness, take integrals over many different paths and average the results

Surface recovered by integration



Works for more complicated surfaces



Limitations

Orthographic camera model

Simplistic reflectance and lighting model

No shadows

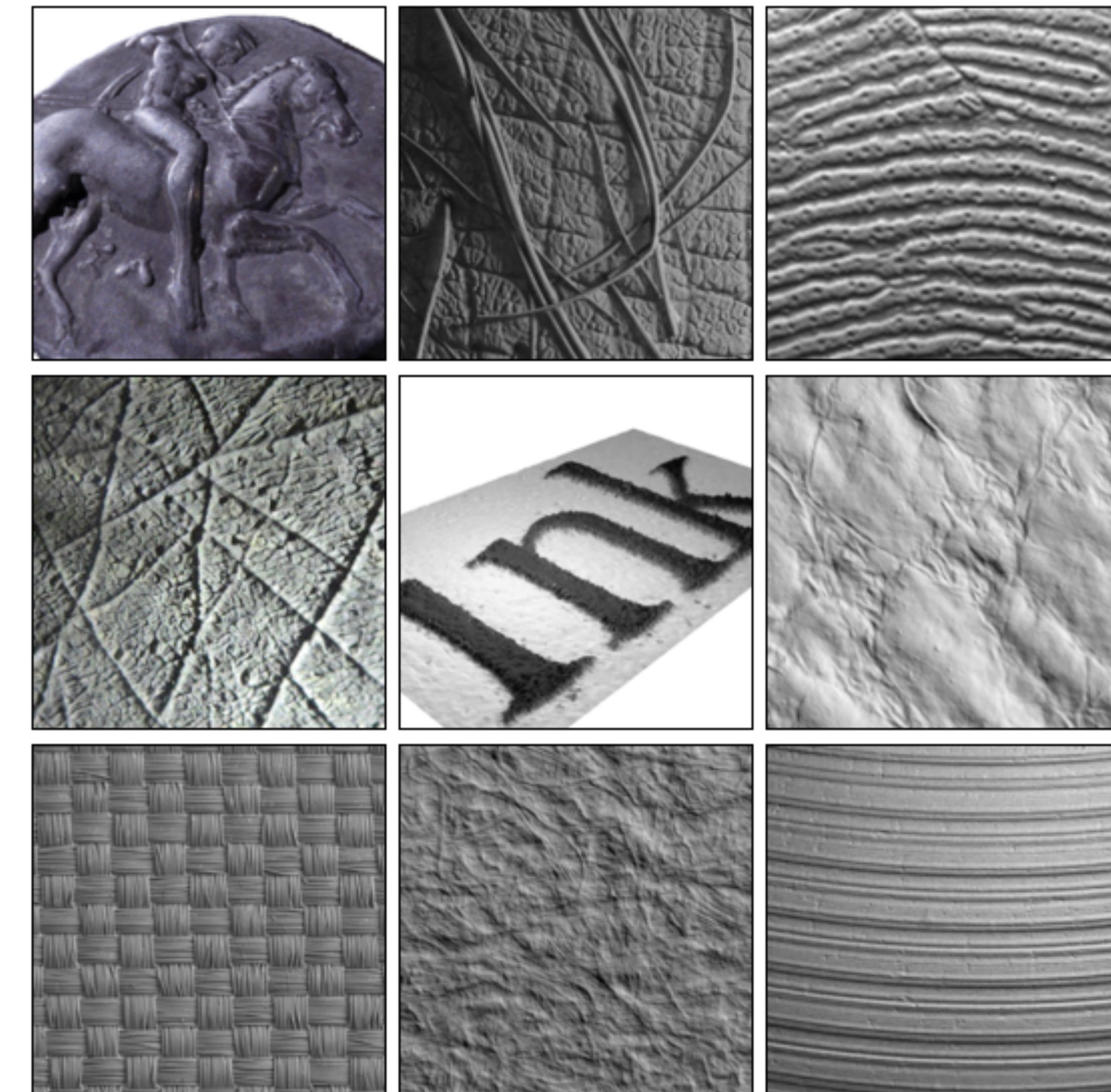
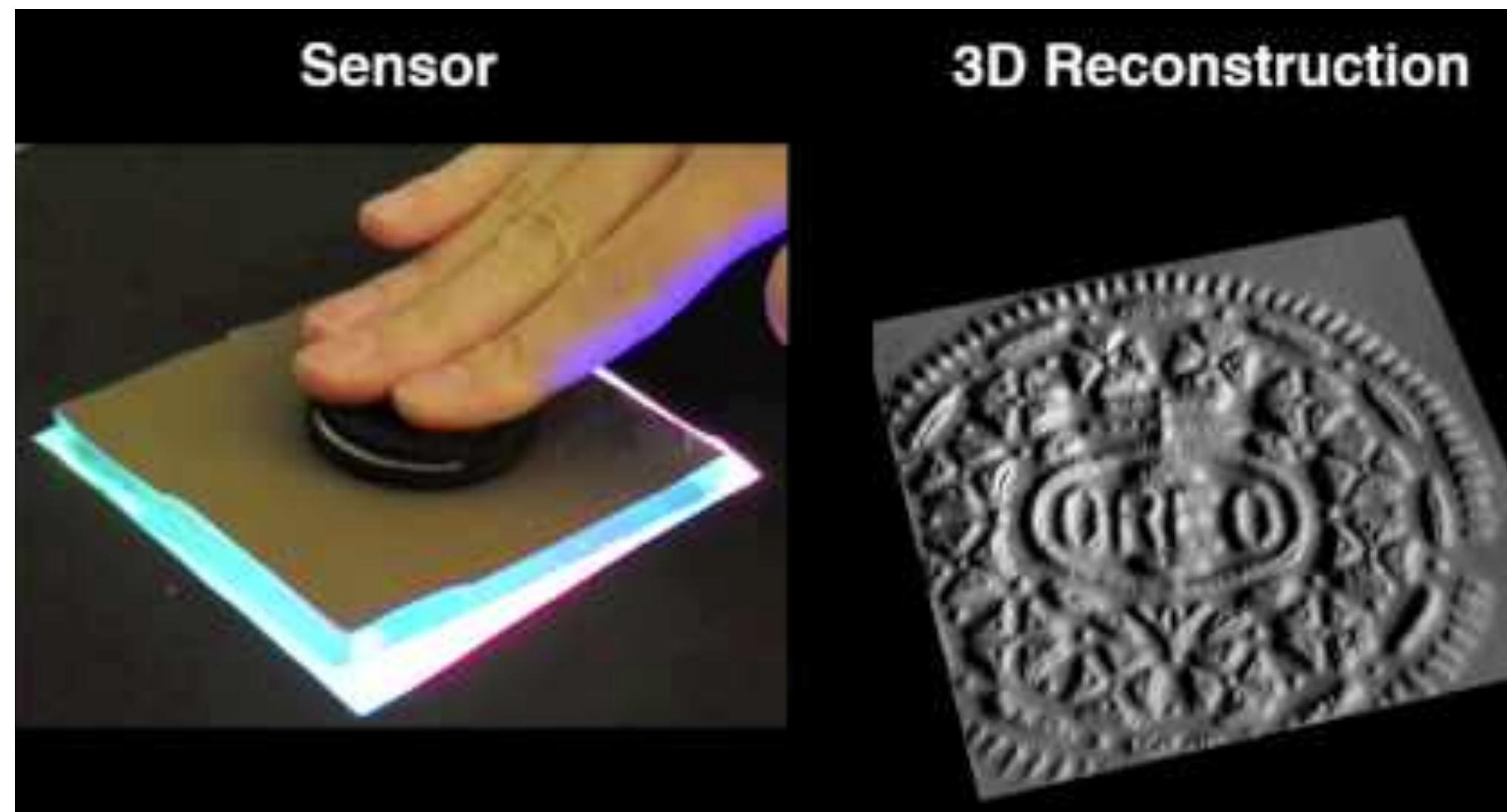
No inter-reflections

No missing data

Integration is tricky

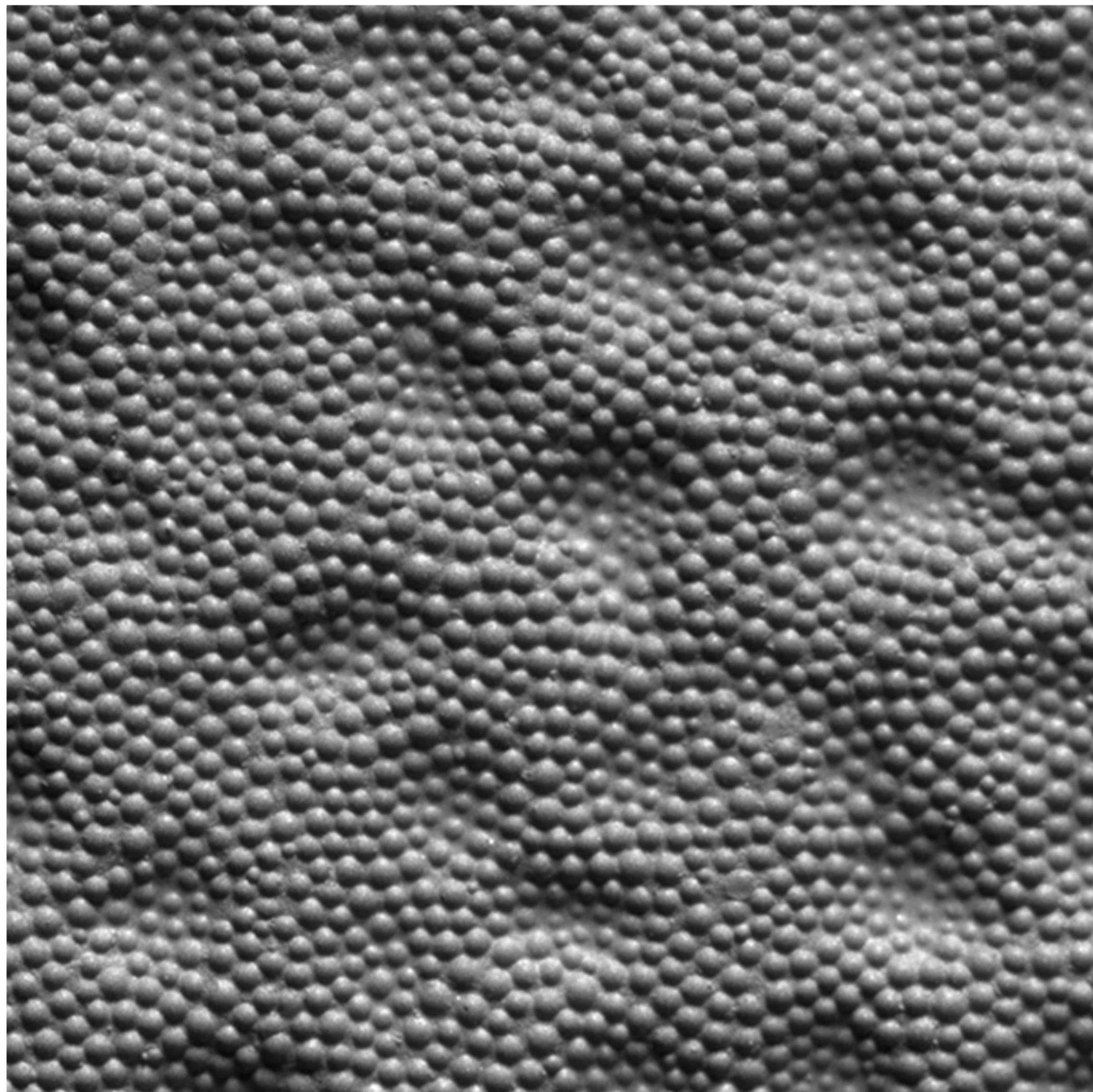
GELSIGHT

GELSIGHT



<https://www.youtube.com/watch?v=S7gXih4XS7A>

GELSIGHT



Retro-reflective fabric. Raw image using GelSight.



A fingerprint with visible pores. Raw image using GelSight.

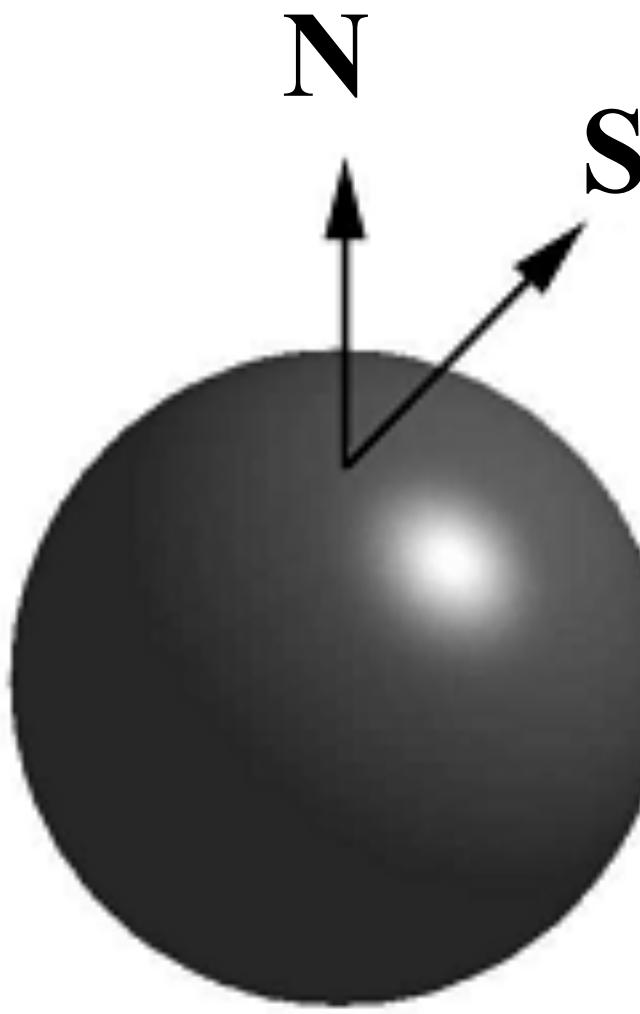
Finding the direction of the light source



P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Finding the direction of the light source

$$I(x,y) = \mathbf{N}(x,y) \cdot \mathbf{S}(x,y) + A$$



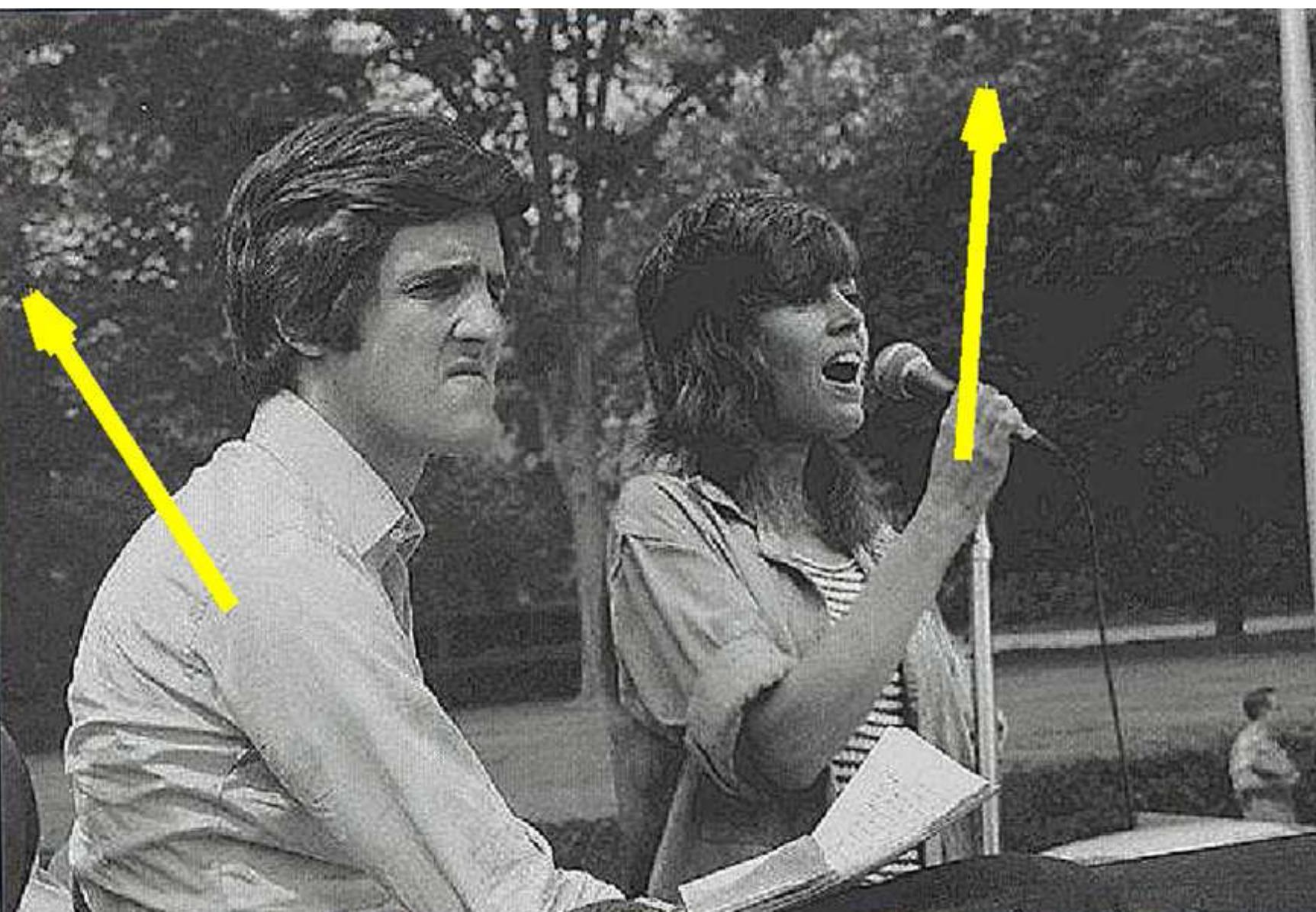
For points on the *occluding contour*:

$$\begin{pmatrix} N_x(x_1, y_1) & N_y(x_1, y_1) & 1 \\ N_x(x_2, y_2) & N_y(x_2, y_2) & 1 \\ \vdots & \vdots & \vdots \\ N_x(x_n, y_n) & N_y(x_n, y_n) & 1 \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ A \end{pmatrix} = \begin{pmatrix} I(x_1, y_1) \\ I(x_2, y_2) \\ \vdots \\ I(x_n, y_n) \end{pmatrix}$$

P. Nillius and J.-O. Eklundh, "Automatic estimation of the projected light source direction," CVPR 2001

Detecting composite photos

Fake photo



Real photo



M. K. Johnson and H. Farid, [Exposing Digital Forgeries by Detecting Inconsistencies in Lighting](#), ACM Multimedia and Security Workshop, 2005.

Neural radiance fields (NeRF)

Representing a scene as a continuous 5D function

$$(x, y, z, \theta, \phi) \rightarrow F_{\Omega} \rightarrow (r, g, b, \sigma)$$

Spatial
location

Viewing
direction

F_{Ω}

Output
color

Output
density

Fully-connected
neural network
9 layers,
256 channels

Neural radiance fields (NeRF)



<https://www.matthewtancik.com/nerf>