DIP Quiz-1

Aug 28th, Wednesday

1. **Question 1**: A filter f is defined as a linear filter if it satisfies the following two conditions:

$$f(A+B) = f(A) + f(B)$$
$$f(\alpha A) = \alpha f(A)$$

An averaging filter of size k X k is given as:

$$f(I(u,v)) = \frac{\sum_{i=-k/2}^{k/2} \sum_{j=-k/2}^{k/2} I(u+i, v+j)}{k^2}$$

Additivity:

$$f((I_1+I_2)(u,v)) = \frac{\sum_{i=-k/2}^{k/2} \sum_{j=-k/2}^{k/2} (I_1+I_2)(u+i, v+j)}{k^2}$$
$$= \frac{\sum_{i=-k/2}^{k/2} \sum_{j=-k/2}^{k/2} I_1(u+i, v+j) + \sum_{i=-k/2}^{k/2} \sum_{j=-k/2}^{k/2} I_2(u+i, v+j)}{k^2}$$
$$= f(I_1) + f(I_2)$$

Homogeneity:

$$f(\alpha I(u,v)) = \frac{\sum_{i=-k/2}^{k/2} \sum_{j=-k/2}^{k/2} \alpha I(u+i, v+j)}{k^2}$$
$$= \alpha \frac{\sum_{i=-k/2}^{k/2} \sum_{j=-k/2}^{k/2} I(u+i, v+j)}{k^2}$$
$$f(\alpha I(u,v)) = \alpha f(I(u,v))$$

2. **Question 2:**

METHOD 1:
Let $f(t)$ be a real and even function. Fourier Transform of $f(t)$ is defined as

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-2j\pi st} dt$$

$$= \int_{-\infty}^{\infty} f(t)[\cos(2\pi st) - j\sin(2\pi st)] dt$$

The product of an even and an odd function is odd. Also, for an odd function $f_o(x)$,

$$\int_{-a}^{a} f_o(x)dx = 0$$

$$\implies \int_{-\infty}^{\infty} jf(t)\sin{(2\pi st)}dt = 0$$

$$F(s) = \int_{-\infty}^{\infty} f(t)\cos{(2\pi st)}dt$$

Which is real and even.

METHOD 2:

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-2j\pi st}dt$$

$$F(-s) = \int_{-\infty}^{\infty} f(t)e^{2j\pi st}dt$$

Substitute $t = -x$

$$F(-s) = \int_{\infty}^{-\infty} f(-x)e^{-2j\pi sx}(-dx)$$

$$= \int_{-\infty}^{\infty} f(x)e^{-2j\pi sx}dx$$

$$F(-s) = F(s)$$

$F(s)$ is even

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-2j\pi st}dt$$

Take conjugate,

$$F^*(s) = \int_{-\infty}^{\infty} f^*(t)(e^{-2j\pi st})^*dt$$

$$= \int_{-\infty}^{\infty} f(t)e^{2j\pi st}dt = F(-s) = F(s)$$

$\implies$ $F(s)$ is real and even

3. **Question 3:**
   original range : $[a_{low}, a_{high}]$
   new range : $[a_{min}, a_{max}]$

$$f(a) = a_{min} + (a - a_{low}).\frac{(a_{max} - a_{min})}{(a_{high} - a_{low})}$$

$$f(a) = (a-1).\frac{63}{253-1}$$

$$f(a) = \frac{(a-1)}{4}$$

   rounded off to the nearest integer

120	188	40	112
180	55	253	190
7	102	100	1
6	11	150	77

30	47	10	28
45	14	63	47
2	25	25	0
1	3	37	19

4. **Question 4:**

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2} & 0 \leqslant r \leqslant L-1 \\ 0 & \text{otherwise} \end{cases}$$

$$S = T(r) = (L-1)\int p_r(r)dr$$

$$= \frac{r^2}{(L-1)}$$

$$T(3) = \frac{9}{(10-1)} = 1$$

5. **Question 5:**

$$f(t) = \cos{(2\pi\mu_0 t)} - j\sin{(2\pi\mu_0 t)}$$

$$f(t) = e^{-j2\pi\mu_0 t}$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

$$= \int_{-\infty}^{\infty} e^{-j2\pi\mu_0 t} e^{-j2\pi\mu t} dt$$

Symmetry Property of FT

$$f(t) <-> F(\mu)$$

$$F(t) <-> f(-\mu)$$

$$= \delta(\mu + \mu_0)$$