Indian Institute of Science Education & Research Kolkata

MA1101: Mathematics I

Mid-term Examination

INSTRUCTIONS

This is a **closed-book** exam. 30th September 2024

You have **1.5 hours.**

- The examination is scored out of **20 points.**
- Answers without justification will receive **NO** credit.
- Even if you score more than 20 points, your score will be 20 points.
- If you are using a result/theorem proved in class, state it clearly before using it.

Good luck!

"The answers are all out there,
we just need to ask the right questions."

Oscar Wilde

Problem 1 [2+3 points] Let A, B, C and D be sets.

(i) Show that $(A \setminus C) \times (B \setminus D) \subseteq (A \times B) \setminus (C \times D)$.

(ii) Prove or disprove: $(A \times B) \setminus (C \times D) \subseteq (A \setminus C) \times (B \setminus D)$.

(Note that "prove" means to provide a proof in the general case and not just a verification in an example. "Disprove" means to provide an explicit example where the claimed result does not hold.)

Problem 2 [1+3 points] This is a question on relations.

(i) Define the universal relation on a non-empty set X.

(ii) Let R be any equivalence relation on $X = \{a, b, c\}$. If $(a, b) \in R$ and $(a, c) \in R$, then show that R is the universal relation on X.

Problem 3 [2+2+3 points] Let $\mathbb{Q}$ and $\mathbb{R}$ denote the set of rational numbers and real numbers respectively.

(i) For $x, y \in \mathbb{R}$, define xRy if $x - y \in \mathbb{Q}$. Show that R is an equivalence relation on $\mathbb{R}$.

(i) Compute $[1]$, the equivalence class of 1.

(iii) For two integers $k < l$, show that the equivalences classes $[k\sqrt{2}]$ and $[l\sqrt{2}]$ are disjoint.

(You may use the fact that $\sqrt{2}$ is not in $\mathbb{Q}$.)

Problem 4 [3+2 points] Let $\mathbb{N}$ denote the set of natural numbers. Consider the function

$$f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}, \ f(m, n) := 2^m 3^n.$$

(i) Show that f is injective (or one-one).

(ii) Show that f is not surjective (or onto) by exhibiting three natural numbers not in the range.