Indian Institute of Science Education and Research Kolkata

Department of Physical Sciences

PH1101 :: Mechanics I

End-semester Examination (Autumn 2024)

Marks: 50, Duration: 2 hours 30 minutes

1. Compute the following limits: [2+2]

$$(i) \quad \lim_{y \to 1} \frac{y^3 - 1}{y - 1} \ , \quad (ii) \quad \lim_{x \to 0+} x \log x \ .$$

2. Find the locations and the corresponding values of the finite maxima and minima of the potential function $U(x) = 2x^3 - 6x$. Sketch the graph of the potential and mark these extrema in the graph. [5+2]

3. Are these two vectors $u = ax$ and $v = ax + b$ linearly independent? Answer with proper justifications, if a and b are arbitrary constants. [3]

4. The orbital eccentricity of Phoebe, an irregular moon of the planet Saturn, is 0.16. In its elliptical orbit, Phoebe reaches as close as 10.9×10^6 km to Saturn's center. Find the semi-major and semi-minor axes of the orbit. What is maximum possible distance of Phoebe from the center of Saturn? [4+2]

5. Show that the total linear momentum of an isolated system of N interacting particles is a conserved quantity. [6]

6. Consider a system of two particles having mass $3m$ and m. Find the position of the center of mass if positions of the particles are given by $\vec{r}_1 = \cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}$ and $\vec{r}_2 = \cos(\omega t)\hat{i} - \sin(\omega t)\hat{j}$ respectively. [4]

7. Find the general solution for the position $x(t)$ of a particle whose motion is described by the equation $\ddot{x}(t) = 2\dot{x}(t) - 17x(t)$. If the particle satisfies the initial conditions $x(0) = 0$ and $\dot{x}(0) = 2$ then find the position of the particle at a time t and also sketch the position $x(t)$. [5+3+2]

8. Show that the orbit of planet Mercury of mass M moving around the Sun having mass $M_\odot$ is an ellipse. In polar coordinates, the acceleration vector is given to be $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\ \hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\ \hat{\theta}$. [10]