

Indian Institute of Science Education & Research Kolkata

MA1101: Mathematics I

First Internal Examination

INSTRUCTIONS

This is a **closed-book** exam.

17th September 2024

You have 90 minutes.

- The examination is scored out of 15 points.
- Answers without justification will receive NO credit.
- Even if you score more than 15 points, your score will be 15 points.
- If you are using a result/theorem proved in class, state it clearly before using it.

Good luck!

Problem 1 [2+4 points] Prove that the following sets are equal:

- (i) $S := \{x \in \mathbb{R} \mid x^2 + 1 = 0\}$ versus \emptyset .
- (ii) $A := \{n \in \mathbb{N} \mid n^2 \text{ is even}\}$ versus $B := \{n \in \mathbb{N} \mid n \text{ is even}\}$.

Problem 2 [4+2 points] Let R be the relation on the set of integers \mathbb{Z} defined as mRn if and only if $m + n + 1$ is odd.

- (i) Show that R is an equivalence relation.
- (ii) Find all the equivalence classes.

Problem 3 [4 points] Let A, B and C be sets. If $A \times B = A \times C$ and $A \neq \emptyset$ then prove that $B = C$.

Indian Institute of Science Education & Research Kolkata

MA1101: Mathematics I
Second Internal Examination

INSTRUCTIONS

This is a **closed-book** exam.

29th October 2024

You have **60 minutes**.

- The examination is scored out of **15 points**.
- Answers without justification will receive **NO credit**.
- If you are using a result/theorem proved in class, state it clearly before using it.

Good luck!

Problem 1 [2 + 3 points] (i) Let k be an integer. Show that if 2 divides k^3 , then 2 divides k .
(ii) Let $\sqrt[3]{2}$ be the cube root of 2. Show that it is not rational (via proof by contradiction). [Hint: *You may find (i) useful.*]

Problem 2 [6 points] Consider the function $f : \mathbb{N} \rightarrow \mathbb{Z}$ defined as follows

$$f(n) = \begin{cases} k & \text{if } n = 2k + 1 \text{ is odd} \\ -l & \text{if } n = 2l \text{ is even} \end{cases}$$

Show that f is a bijection.

Hint: *It may help to write out explicitly the first few values of f , in case the symbolic definition is confusing.*

Problem 3 [5 points] In a group of 2024 people, show that there are at least two people who have the same number of friends.

Note: *It is understood that being friends is a symmetric relation, i.e., if X is a friend of Y , then Y is also a friend of X .*

Indian Institute of Science Education & Research Kolkata

MA1101: Mathematics I
Third Internal Examination

INSTRUCTIONS

This is a **closed-book** exam.

19th November 2024

You have **60 minutes**.

- The examination is scored out of **15 points**.
- Answers without justification will receive **NO** credit.
- If you are using a result/theorem proved in class, state it clearly before using it.

Good luck!

Problem 1 [2+3 points] (i) Let $f : (a, b) \rightarrow \mathbb{R}$ be a function. Give the definition of “ f has a limit at $x_0 \in (a, b)$ ”.

(ii) Show that $f : (-1, 1) \rightarrow \mathbb{R}$, $f(x) = x^{\frac{1}{3}}$ has a limit at $x_0 = 0$.

Problem 2 [2+3 points] (i) Let $f : (a, b) \rightarrow \mathbb{R}$ be a function. Give the definition of “ f is continuous at $x_0 \in (a, b)$ ”.

(ii) Show that $f : (-1, 1) \rightarrow \mathbb{R}$, $f(x) = x^3$ is continuous at $x_0 = 0$.

Note: You may not use the fact that polynomials are continuous without proving it.

Problem 3 [2+4 points] (i) Let $f : (a, b) \rightarrow \mathbb{R}$ be a function. Give the definition of “ f is differentiable at $x_0 \in (a, b)$ ”.

(ii) Prove or disprove: The function $f : (-1, 1) \rightarrow \mathbb{R}$, $f(x) = x^{\frac{1}{3}}$ is differentiable at $x_0 = 0$.

Note: Using symbols of the form $\frac{d}{dx}x^{\frac{1}{3}} = \frac{1}{3}x^{-\frac{2}{3}}$ without proof is not acceptable.